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A Mechanical Modeling Approach

Tacx, Paul; Teurlings, Matthijs; Habraken, Roel; Witvoet, Gert; Heertjes, Marcel; Oomen, Tom

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Spatio-Temporal Analysis of Overactuated Motion Systems: A Mechanical Modeling Approach ⋆

Paul Tacx * **Matthijs Teurlings** * **Roel Habraken** *,**** **Gert Witvoet** *,**** **Marcel Heertjes** *,*** **Tom Oomen** *,**

* *Department of Mechanical Engineering, Eindhoven University of Technology, Eindhoven, the Netherlands.*
** *Faculty of Mechanical, Maritime, and Materials Engineering, Delft University of Technology, Delft, the Netherlands.*
*** *ASML, Mechatronic Systems Development, Veldhoven, the Netherlands*
**** *TNO, Optomechatronics Department, Delft, the Netherlands*

Abstract: Flexible dynamics in motion systems lead to inherent spatio-temporal system behavior. The aim of this paper is to develop an unified approach for the identification of modal models of spatio-temporal overactuated systems. The approach exploits the modal modeling framework and the overactuated setting to enhance the estimation of the spatial system behavior. The proposed approach is applied in an experimental case study. The case study considers an experimental overactuated stage and illustrates the effectiveness of the proposed approach.

Keywords: Mechatronic systems, Motion Control Systems, Identification for Control, Identification and control methods, Modeling.

1. INTRODUCTION

Stringent demands regarding performance in mechatronic systems require the flexible dynamic behavior to be addressed explicitly in the control design (Oomen, 2018). A crucial application includes adaptive optics in satellite communication and astronomy (Kuiper et al., 2018; Beckers, 1993; Booth, 2007; Roddier, 1999). Adaptive optics is used to compensate for atmospheric distortions in the incoming wavefront (Yu and Verhaegen, 2017). A key component in adaptive optics is the deformable mirror, see Figure 1. The deformable mirror contains a large number of spatially distributed actuators that enable the deformation of the mirror face sheet. Traditionally, the deformable mirror can be accurately described by an influence matrix, i.e. static gains, in the frequency range that is relevant for control (Yu and Verhaegen, 2017). When this static assumption is valid, static decentralized controllers can be used. However, the stringent performance requirements and the increasing size of the deformable mirrors results in the flexible dynamics being present within the control bandwidth. As a consequence, the static modeling assumption is no longer valid and the flexible dynamics need to be addressed explicitly in the control design (Balas and Doyle, 1994).

The flexible dynamics in next-generation motion systems lead to spatio-temporal system behavior that needs to be controlled with a large number of spatially distributed actuators. At the same time, the measured positions do not necessarily coincide with the positions of interest. Furthermore, the number of sensors that measure the deformation of the deformable mirror is limited with respect to the number of actuators. An approach that deals with such spatial-temporal control problems is inferential control (Oomen et al., 2014a). This control approach necessitates accurate models that incorporate the spatio-temporal system behavior. Inevitably, inferential control relies on accurate modeling techniques that capture the spatio-temporal nature of the flexible dynamic behavior (Voorhoeve et al., 2020; Tacx and Oomen, 2022; Voorhoeve et al., 2016).

A key challenge for next-generation motion systems is the modeling of the spatio-temporal flexible dynamics (da Silva et al., 2008). Traditional parametric and non-parametric identification approaches aim to identify the temporal behavior of the flexible dynamics (Van de Wal et al., 2002). Consequently, the response of the system is modeled at the limited number of sensor locations. As a result, the flexible dynamic behavior is estimated at a limited spatial grid which limits the understanding of the position-dependency of the flexible dynamic behavior.

Spatio-temporal systems are modeled with LPV techniques in Van Wingerden and Verhaegen (2009); Bamieh and Giarre (2002). Alternatively, in Voorhoeve et al. (2020), an approach is proposed to identify spatio-temporal mechanical models. However, a key drawback of these LPV techniques is that the spatial component is solely based on the limited amount of sensor locations.

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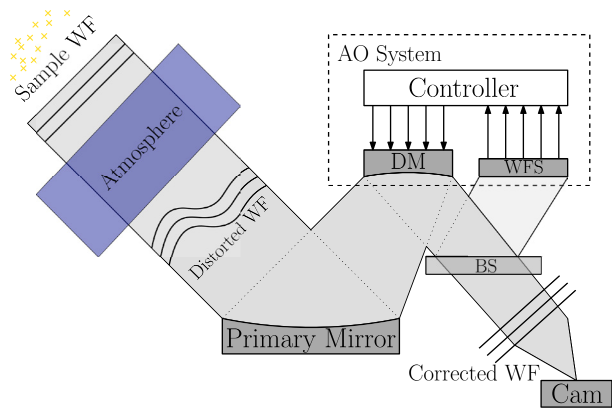

Fig. 1. Working principle of adaptive optics (AO) in a ground-based telescope. The incoming sample wavefront is distorted due to atmospheric turbulence. These atmospheric distortions are corrected by an adaptive optics system consisting of a deformable mirror (DM) and a wavefront sensor (WFS) before the imaging takes place.

This limitation in existing linear parameter varying modeling techniques for motion systems underlines the importance of exploiting prior knowledge in the identification of the spatio-temporal behavior of next-generation motion systems.

Although identifying the spatio-temporal behavior of motion systems is essential for the control of next-generation motion systems, and several results are present to identify such models, at present, a method that allows for accurate and practical identification of the spatio-temporal behavior with a limited number of sensors is not available. The aim of this paper is to introduce a unified approach for the identification of the spatio-temporal system behavior for the control of next-generation overactuated motion systems with a limited amount of sensors. This is achieved by exploiting prior mechanical system knowledge, i.e. exploiting the overactuated setting of next-generation motion systems. Related work includes the field of experimental modal analysis and the Maxwell–Betti reciprocal work theorem, see e.g. (Maxwell, 1864; Betti, 1872; Gawronski, 2004; Ghali and Neville, 1972). In contrast to the field of experimental modal analysis, this paper aims to identify and reconstruct the spatio-temporal behavior for spatio-temporal control of overactuated systems with a limited number of sensors.

The main contribution of this paper is the development of a unified approach for the identification and reconstruction of spatio-temporal system dynamics in next-generation motion systems with a limited number of sensors. In addition, an experimental case-study with an experimental overactuated beam setup which is representative of next-generation overactuated motion systems confirms the effectiveness of the proposed approach.

The paper is organized as follows. Section 2 describes the industrial application motivation and the problem formulation. In Section 3, the proposed modeling framework is introduced. An experimental case study is presented in Section 4 to illustrate the proposed approach. The conclusions are provided in Section 5.

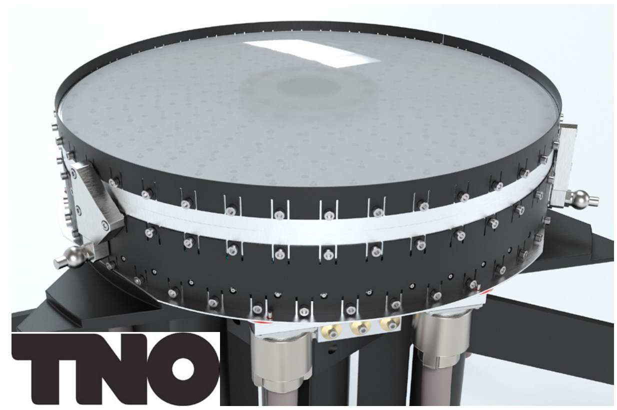

Fig. 2. Computer render of a deformable mirror designed by TNO. The deformable mirror is designed for the University of Hawai'i 88-inch telescope.

2. SYSTEM DESCRIPTION

In Figure 2, a next-generation deformable mirror is depicted. Compared to an earlier version of the deformable mirror see e.g. Hamelinck et al. (2008), the deformable mirror depicted in Figure 2 has an increased diameter which leads to flexible dynamics being present within the control bandwidth. Also, the number of actuators increased to 207. Furthermore, for a future observatory engineers are planning to build a deformable mirror with approximately 2000 actuators.

The increasing complexity necessitates the need for modeling the flexible dynamic behavior explicitly. If the deformable mirror is integrated into a telescope, the large grid of the wavefront sensor can be used for system identification. However, to focus specifically on the flexible dynamic behavior and to validate the deformable mirror design, the deformable mirror is identified before integration in the telescope. As a consequence, a limited amount of position sensors can be located in front of the mirror face sheet during experimentation. In addition, global control performance is required on the mirror surface. As a consequence, the performance variables and the measured variables do not necessarily coincide. As a consequence, a spatio-temporal control problem is encountered. This motivates the identification of a spatio-temporal model with a limited amount of sensors.

The aim of this paper is to identify the spatio-temporal behavior of overactuated motion systems by exploiting prior mechanical system knowledge. A frequency-domain-based approach is pursued, and the structure of mechanical systems is exploited to reconstruct the temporal behavior at an increased set of spatial locations.

3. APPROACH

In this section, the method for identifying the spatio-temporal behavior in next-generation motion systems is introduced. First, the modeling of flexible structures is discussed. Second, the modal form of mechanical systems is introduced. Lastly, the approach for identifying the spatio-temporal behavior of next-generation motion systems is introduced, which constitutes contribution C.1.

3.1 Modeling Flexible Structures

The aim of this paper is to address high-precision motion systems, which are typically designed for linear system dynamics, e.g. low damping, lack of backlash, and lack of friction. For this reason, the flexible dynamics are dominating the overall system dynamics which allow these systems to be modeled as a flexible body.

The key variable is the out-of-plane deformation $u(\rho, t) \in \mathbb{R}$ which is defined by partial differential equations Gawronski (2004). The spatial domain $\mathcal{S} \in \mathbb{R}^3$ of the variable ρ is the three-dimensional point in the geometry of the flexible body. The spatio-temporal behavior is typically described by space-time-separated basis functions

$$u(\rho, t) = \sum_{k=1}^{n_m} w_k(\rho) q_k(t). \tag{1}$$

The temporal contribution is determined by the generalized coordinates $q_k(t)$ and the spatial contribution is determined by $w_k(\rho)$. The solution in Eq. (1) converges for $n_m \to \infty$ for appropriate basis functions. Analytical solutions are not available in general and only exist for specific cases. For this reason, the solution often is limited to finite element method-based models that use a finite set of points in space. Given the separation of space and time in Eq. (1), the dynamics can be formulated as a coupled set of second-order ordinary differential equations

$$M\ddot{q} + D\dot{q} + Kq = f(t) \tag{2}$$

where the mass matrix $M \in \mathbb{R}^{n_m \times n_m}$ is positive definite, $D \in \mathbb{R}^{n_m \times n_m}$ denotes the damping matrix, $K \in \mathbb{R}^{n_m \times n_m}$ the stiffness matrix, and $f(t) \in \mathbb{R}^{n_\mathrm{a} \times 1}$ the input function at actuation locations $\bar{\rho}_\mathrm{a} = \{\rho_{\mathrm{a},1}, ..., \rho_{\mathrm{a},n_\mathrm{a}}\} \subset \mathcal{S}$.

In order to address the global system dynamics explicitly, the modeling of mechanical systems should focus specifically on global structure variables, i.e. the modal parameters. The modal parameters include the eigenfrequencies ω_k and modeshapes $\phi_k(\rho) : \mathcal{S} \mapsto \mathbb{R}$. These modal parameters are obtained by solving the undamped generalized eigenvalue problem

$$\left[K - \omega_k^2 M\right] \bar{\phi}_{\mathrm{a},k} = 0. \tag{3}$$

The eigenvalues, ω_k^2, are the squared undamped eigenfrequencies and the eigenvectors $\bar{\phi}_{\mathrm{a},k}$ denote the modeshape sampled at the actuator locations, i.e. $\bar{\phi}_{\mathrm{a},k} = [\phi_k(\rho_{\mathrm{a},1}), ..., \phi_k(\rho_{\mathrm{a},n_\mathrm{a}})]$. Throughout this paper, mass-normalized generalized eigenvectors are considered.

The coupled set of differential equations in Eq. (2) can be decoupled by introducing the coordinate transformation to modal coordinates, i.e. $q = \Phi\eta$, where $\Phi_\mathrm{a} = \left[\bar{\phi}_{\mathrm{a},1}, ..., \bar{\phi}_{\mathrm{a},n_\mathrm{a}}\right]$. Substituting the coordinate transformation and multiplying Eq. (2) with $\Phi_\mathrm{a}^\top$ leads to

$$I\ddot{\eta} + D_m\dot{\eta} + K_m\eta = \Phi_\mathrm{a}^\top f(t), \tag{4}$$

$$u(\rho, t) = \sum_{k=1}^{n_m} \phi_k(\rho)\eta_k(t) \tag{5}$$

where $D_m = \Phi_\mathrm{a}^\top D \Phi_\mathrm{a} = \mathrm{diag}\left(d_{m,1}, ..., d_{m,n_\mathrm{a}}\right)$, $K_m = \Phi_\mathrm{a}^\top K \Phi_\mathrm{a} = \mathrm{diag}\left(\omega_1^2, ..., \omega_{n_\mathrm{a})^2}\right)$. In this paper, modal damping is considered which leads to the decoupled set of differential equations in Eq. (4) and which is known to be representative of many lightly-damped systems in practice.

Motion systems including deformable mirrors measure the spatio-temporal system behavior at a limited amount of sensor locations. Consider a motion system $G(s) \in \mathbb{R}^{n_\mathrm{s} \times n_\mathrm{a}}$ with n_a actuators at the actuator locations $\bar{\rho}_\mathrm{a}$ and n_s sensors located at $\bar{\rho}_\mathrm{s} = \{\rho_{\mathrm{s},1}, ..., \rho_{\mathrm{s},n_\mathrm{s}}\} \subset \mathcal{S}$. The finite number of sensor locations and the application of the modal expansion theorem allows the reformulation of the system of equation in Eq. (4) to a summation of spatio-temporal contributions

$$G(s) = \sum_{k=1}^{n_m} \frac{R_k}{s^2 + d_{m,k}s + w_k^2} \tag{6}$$

where R_k denotes the modal participation matrix that is based on the sampled modeshape vectors

$$R_k = \bar{\phi}_{\mathrm{s},k}\bar{\phi}_{\mathrm{a},k}^\top. \tag{7}$$

Here, $\bar{\phi}_{\mathrm{a},k}$ denotes the modeshape sampled at the actuator locations, and $\bar{\phi}_{\mathrm{s},k} = [\phi_k(\rho_{\mathrm{s},1}), ..., \phi_k(\rho_{\mathrm{s},n_\mathrm{s}})]$ denotes the modeshape sampled at the sensors.

Summarizing, the modal system description of mechanical systems provides a significant amount of insight into the flexible dynamics since the resulting set of decoupled equations is directly related to the underlying flexible dynamic behavior. Specifically, the modeshapes express the manner in which the flexible dynamic behavior is position dependent.

3.2 Identifying Mechanical Systems

The modeling of the spatio-temporal system dynamics requires the identification of a parametric modal model $\hat{G}(\theta, s)$ which is defined by the modal form in Eq. (6). The parameterization is fully defined by the parameter vector

$$\theta = \mathrm{vec}\left\{\bar{d}_m, \bar{\omega}_m, \Phi_\mathrm{a}, \Phi_\mathrm{s}\right\} \tag{8}$$

Here, the eigenfrequencies $\bar{\omega}_m = [\omega_{m,1}, ..., \omega_{m,n_m}]$, the damping constants $\bar{d}_m = [d_{m,1}, ..., d_{m,n_m}]$, and the modeshape vectors from the actuators Φ_a and the sensors $\Phi_\mathrm{s} = \left[\bar{\phi}_{\mathrm{s},1}, ..., \bar{\phi}_{\mathrm{s},n_\mathrm{s}}\right]$.

The key identification aim is to find the parameter vector θ that minimizes the Frobenius norm-based cost function

$$\hat{\theta} = \arg\min_{\theta} \sum_{v=1}^{N} \left\| W(j\omega_v) \circ \left(\tilde{G}(j\omega_v) - \hat{G}(\theta, j\omega_v)\right)\right\|_F^2. \tag{9}$$

Here, $W(j\omega_v)$ denotes a frequency-dependent weighting matrix and $\circ$ denotes the Hadamard product. The modal form $\hat{G}(\theta, s)$ is a non-linear function of the parameter vector θ. For this reason, Eq. (9) is solved iteratively using Sanathanan-Koerner iterations and Gauss-Newton iterations, see e.g. Voorhoeve et al. (2020) for the details.

In view of identification for control, low-order models are desired that are sufficiently accurate in the frequency range of interest for control. This means that only a limited number of modal coordinates n_m are required. In sharp contrast, the modeling of the spatial component, i.e. modeshape, $\phi_k(\rho)$ in Eq. (7) typically leads to high model complexity. At the same time, the deformable mirror contains only a limited number of n_s sensor locations. Thus, a limited portion of the spatial domain $\mathcal{S}$ is covered which limits the spatial modeling of the flexible dynamics of the mechanical systems. In the next subsection, an approach is proposed that allows for the evaluation of the modeshape at an increased set of spatial locations by combining sensor and actuator data.

3.3 Extending Modal Models

The main objective of this section is to introduce a unified approach for the construction of spatio-temporal models of overactuated motion systems with a limited number of sensors. The key drawback of conventional spatio-temporal modeling techniques is that these techniques fully rely on the sensor data. The key idea in this paper is that in the modal description, e.g. Eq. (6), the modeshape is encountered twice. Specifically, the modeshape is sampled at the sensor and actuator locations. To enhance the spatial density of the estimated modeshape, the modeshapes sampled by the sensors and actuators are combined

$$\phi_{\text{ext},k} = \phi_k(\rho_{\text{ext}}) \tag{10}$$

where $\bar{\rho}_{\text{ext}} = \bar{\rho}_{\text{a}} \cup \bar{\rho}_{\text{s}}$ denotes the combined actuator and sensor location vector. The extended modeshape $\phi_{\text{ext},k}$ provides additional information about the spatial nature of the flexible dynamic behavior to the control and design engineers.

Moreover, the combined sampled modeshape allows the reconstruction of the spatio-temporal system behavior by enhancing the spatial density of the modal description, i.e. Eq. (6). This is achieved by the introduction of the extended plant

$$\hat{G}_{\text{ext}}(s) = \sum_{k=1}^{n_m} \frac{R_{\text{ext},k}}{s^2 + d_{m,k}s + w_k^2}, \tag{11}$$

$$R_{\text{ext},k} = \phi_{\text{ext},k}\phi_{\text{a},k}^{\top}. \tag{12}$$

The key step in the formulation of the extended plant is to uniquely identify $\phi_{\text{s},k}$ and $\phi_{\text{a},k}$ from the rank-one matrix R_k in Eq. (6). The key requirement is that at least one collocated sensor is required to uniquely identify the mass-normalized modeshapes. This means that $\bar{\rho}_{\text{a}} \cap \bar{\rho}_{\text{s}}$ should be a non-empty set. An additional requirement is that the collocated sensor should not be located at a node of any relevant modeshape.

4. EXPERIMENTAL CASE STUDY

In this section, the method proposed in this paper is illustrated in an experimental case study. The case study includes an experimental overactuated beam setup, see Figure 3. The full-system behavior will be identified while only using a single sensor. The case study encompasses all steps from frequency response function identification to the formulation of the extended plant. The experimental setup is explained first. Second, the identification procedure is discussed. Lastly, the extended plant is estimated and the results are discussed. This section constitutes contribution C.2.

4.1 Setup

The experimental overactuated beam setup is depicted in Figure 3. The considered system is designed to exhibit out-of-plane flexible dynamic behavior. For this reason, this system is considered to be representative of next-generation motion systems, including future deformable mirrors. The system consists of a flexible beam with dimensions $2 \times 20 \times 500$ mm. The system operates in 2 degrees of freedom, one translation, and one rotation. Four degrees of freedom are constrained by wire flexures. Due to the limited out-of-plane stiffness of the beam, the system contains a significant amount of flexible dynamics. The system is operating with a sampling frequency of 4 kHz. The setup is equipped with three voice-coil actuators and five collocated fiber-optical sensors. To facilitate exposition, only the three collocated sensors are considered in the experimental case study.

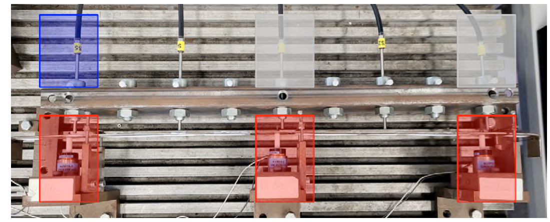

Fig. 3. Experimental overactuated beam setup. Three degrees of freedom are constrained by four vertical wire flexures, and one degree of freedom is constrained by a horizontal wire flexure on the left. The beam is actuated by three voice coil actuators (red), one position sensor (blue) is considered by the proposed method and 2 sensors (grey) are used for validation purposes.

To illustrate the effectiveness of the proposed approach, an overactuated setting with limited sensing capabilities is created. Specifically, only the first position sensor is used by the proposed method. It is emphasized that the remaining two sensors are only used for validation purposes.

4.2 Identification Procedure

The aim of this section is to identify the full system behavior

$$G_o = [u_1 \ u_2 \ u_3]^{\top} \mapsto [y_1 \ y_2 \ y_3]^{\top} \tag{13}$$

while only having access to the first sensor, i.e.

$$G_l = [u_1 \ u_2 \ u_3]^{\top} \mapsto [y_1]^{\top}. \tag{14}$$

This is achieved by applying the method proposed in Section 3.

Step 1: Non-parametric model A frequency domain-based procedure is pursued to identify a parametric model of G_l. To identify the model, first, a non-parametric model is estimated. Since the system is stable, open-loop experiments are performed. A full-MIMO random phase multisine with a flat amplitude spectrum is injected into all inputs up to a frequency of 1000 Hz. The identified element-wise Bode magnitude plot of the non-parametric model of G_l and the full system G_o is depicted in Figure 4.

Step 2: Modal model This subsection aims to identify a parametric modal model of G_l. Modal models are estimated of the form

$$\hat{G}_l = \sum_{k=1}^{3} \frac{\hat{R}_k}{s^2 + d_{m,k}s + \omega_k^2}. \tag{15}$$

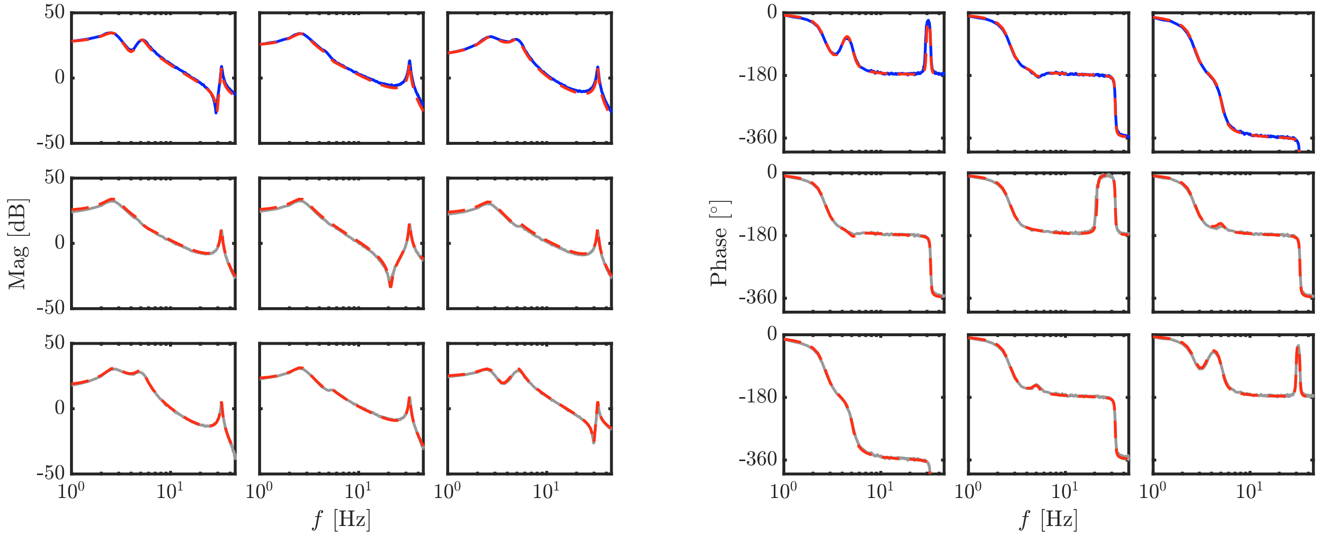

Fig. 4. Element-wise Bode magnitude (left) and phase (right) plot of the non-parametric estimate of the full system G_o and the subset G_l (grey), the parametric modal model of the subsystem $\hat{G}_l$ (blue), and extended plant $\hat{G}_{\text{ext}}$ (dashed red). It is emphasized that the extended plant is estimated using the non-parametric estimate of the subsystem G_l only and the full system G_o is only visualized for validation purposes.

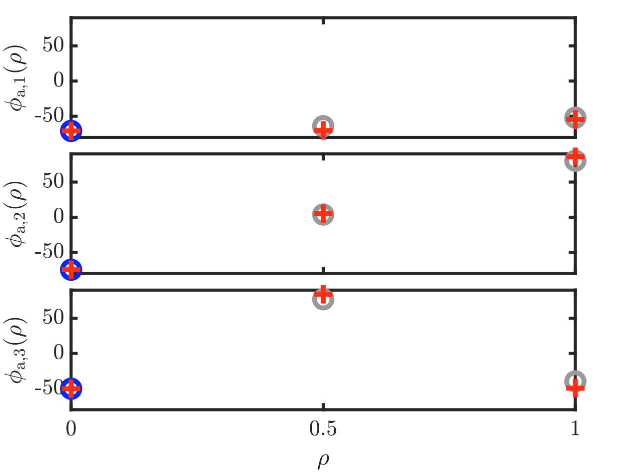

Fig. 5. The first (top), second (center), and third (bottom) modeshape of the flexible beam. Visualized are the modeshape based on the full system $\phi_{s,o,k}$ (grey), the modeshape based on G_l in view of the actuators $\phi_{a,l,k}$ (red) and sensors $\phi_{s,l,k}$ (blue). The position coordinate ρ is normalized with respect to the length of the beam.

Three modes are considered since the aim of this paper is to identify the spatio-temporal behavior for the control of next-generation motion systems. Specifically, the setup considered in the case study contains three actuators, hence, at most three modes can be explicitly controlled. In addition, for control, low-order models are desired that are sufficiently accurate in the control-relevant frequency range Oomen et al. (2014b). For these reasons, the first three modes are considered in this case study.

To identify the modal model in Eq. (15), the optimization algorithm based on a weighted Frobenius norm-based cost function in Eq. (9) is used. The weighting filter in Eq. (9) is selected to emphasize accurate identification of the flexible dynamic behavior. This is achieved by an inverse weighting filter, see e.g. Voorhoeve et al. (2020). Alternatively, control-relevant approaches may be considered to identify low-order models in view of the control goal, see e.g. Oomen et al. (2014b).

The resulting Bode plot of the modal model $\hat{G}_l$ is depicted in Figure 4. The Bode plot reveals that the modal model accurately fits the frequency response measurement. However, the analysis of the modal model $\hat{G}_l$ in the current form is limited to a temporal analysis at the first sensor. As a consequence, the spatial component of the flexible dynamics is unclear.

Step 3: Extended model The key step in the reconstruction of the full system G_o using the subsystem G_l is to exploit the modal modeling framework. Since the first sensor is collocated with the first actuator, the matrix $\hat{R}_k$ in Eq. (15) is uniquely decomposed into modeshape vectors

$$\hat{R}_k = \bar{\phi}_{s,k}\bar{\phi}_{a,k}^\top \tag{16}$$

where $\bar{\phi}_{s,k} \in \mathbb{R}$ and $\bar{\phi}_{a,k} \in \mathbb{R}^3$. These modeshape vectors are visualized in Figure 5. The extended plant is formulated by combining the modeshapes in Eq. (16)

$$\hat{G}_{\text{ext}} = \sum_{k=1}^{n_m} \bar{\phi}_{\text{ext},k} \frac{1}{s^2 + d_{m,k}s + w_k^2} \bar{\phi}_{a,k}^\top \tag{17}$$

where $\bar{\phi}_{\text{ext},k} = \bar{\phi}_{a,k}$. The resulting Bode plot is depicted in Figure 4.

Spatio-temporal analysis When analyzing the non-parametric estimate of G_l in Figure 4, the analysis of the flexible dynamic behavior is limited to a temporal analysis in view of the first sensor only. In sharp contrast, the approach proposed in this paper allows identifying the full response G_{ext} by exploiting the modal framework. In particular, the approach allows analyzing the spatio-temporal behavior with limited sensing capabilities with Figure 5 and 4.

Firstly, it is observed that the first mode occurs at 2.5 Hz in Figure 4. The first modeshape in Figure 5 indicates that the flexible beam is not deforming, i.e. a rigid-body motion. Specifically, the beam is mainly translating with a limited rotation which is an effect of the wire flexures connected to the left of the beam, see Figure 3. Secondly, the second mode occurs at 4 Hz which corresponds to the second rigid-body mode. Figure 5 confirms the rigid-body motion and indicates that the system is mainly rotating. Interestingly, the second mode is almost not observable in the second row and column of the Bode plot. This can be explained by the second actuator being located at a node of the second modeshape, see Figure 5. The third mode occurs at 50 Hz. This mode corresponds to the first bending mode of the flexible beam which is confirmed in Figure 5.

5. CONCLUSIONS

The identification of the spatio-temporal behavior is an essential step for the control of future motion systems, especially motion systems with a limited sensing capacity. This paper provides a unified approach for the identification the spatio-temporal system behavior of next-generation motion systems, including the identification of the flexible dynamic behavior of deformable mirrors, with a limited number of sensor locations. An essential step in the proposed approach is to exploit prior mechanical systems knowledge which is incorporated into the modal modeling framework. In particular, the overactuated setting is used to gather additional knowledge regarding the spatial system behavior. An experimental case study with an experimental overactuated motion system is used to illustrate the proposed approach.

The proposed modeling approach can be applied in various applications including, e.g., position-dependent modeling techniques, inferential control, and global spatio-temporal feedforward control. The main benefit of the proposed approach for these spatio-temporal modeling and control techniques is the enhanced estimation of the spatial component of the spatio-temporal system behavior, especially for systems with a limited number of sensors.

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