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A novel framework for probabilistic model updating using load test data and Copula-Based Bayesian Networks

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Abstract. Accurate model updating is essential for reliable structural assessment and performance prediction of civil infrastructure. Classical Bayesian model updating approaches, while powerful, are often limited by their reliance on predefined parametric distributions, and by the computational burden of sampling-based inference, which becomes prohibitive as the number of updating parameters increases. These limitations hinder their applicability to complex structures where high-dimensional parameter spaces and uncertain parameter-output relations are prevalent. The restriction on the number of updating parameters often leads to a mismatch between the global-level formulation of the updating problem and the local structural behavior or failure mechanisms of interest. To address these limitations, this paper proposes a novel model updating framework that integrates load test data, Finite Element Modelling (FEM), and Copula-Based Bayesian Networks (CBBNs). The approach uses load test measurements to calibrate FEM parameters, and employs CBBNs to efficiently capture complex dependencies among model variables. This paper presents the framework's formulation and implementation, along with key verification steps. The flexibility and computational tractability of the CBBN-based framework make it particularly suited for structural engineering applications, contributing to more reliable data-driven asset management.

Keywords: Structural Identification, Bayesian Model Updating, Bridge Load Testing, Bayesian Networks

Introduction

Aging bridge infrastructure motivates the deployment of data-driven assessment solutions to obtain direct insight into actual structural performance. System identification (SI), commonly referred to as structural identification in a civil engineering context, is the process of using measured system responses to update a numerical model through estimation of structural properties, local weaknesses, and ultimately residual service life. Models for reassessment are typically constructed from design documentation; SI connects these models to observed behavior to produce updated, evidence-based representations of the structure.

Accurate model updating is essential for reliable assessment and performance prediction of civil infrastructure. Classical Bayesian model updating approaches [1, 2] provide a principled framework but are often limited by restrictive parametric assumptions and by computational costs that grows rapidly with the number of updating parameters. These limitations reduce their effectiveness on complex structures characterized by high-dimensional parameter spaces and localized failure mechanisms, producing a mismatch between global model updates and the local behavior of interest [3, 4, 5]. Consequently, there is a need for scalable, computationally efficient updating methods that can represent localized phenomena and accommodate large or nonparametric parameter spaces.

Besides, using measurement data from load tests for statistically sound model updating is challenging because measured responses reflect the interaction of loads and system parameters (load + system parameters = system



response) and these dependency structures change with load scenario. The dependency structure between uncertain parameters and outputs therefore varies with scenario. This scenario-dependent variability complicates straightforward interpretation of measurements: it requires statistical models that can separate marginal behavior from joint dependencies and that can integrate information across multiple parameters and multiple load scenarios.

Copula-Based Bayesian Networks (CBBN) [6] are proposed as the modelling framework for this purpose: by representing loads, system parameters and monitored responses as nodes in a Bayesian Network (BN), and by modelling their dependencies with copulas (in particular a Gaussian copula formulation as used in this work), the network provides a structured probabilistic basis to update parameter estimates when new measurement data become available. In this view, the system parameters (i.e. the quantities that characterise the structural model) act as the updating (unknown) variables in the BN, while measured structural responses from load testing supply the observational data that inform posterior inference of those parameters; dependencies that are most informative for identification can be retained (and less informative links sparsified) to focus the updating on useful relations and scenarios — selecting the relations and load scenarios that are most informative for estimating particular parameters and integrating them within a unified probabilistic model.

1. Solution framework

Bayesian Networks are identified as a suitable family of models to perform this task. In practice, however, model updating must contend with limited informative relations between specific parameters and outputs under particular scenarios; the outlined approach therefore advocates optimizing the network to include only those relations and load scenarios that are helpful for identifying particular parameters, and—where appropriate—sparsifying the BN by retaining only the strongest dependencies per scenario so as to reduce complexity while focusing inference on the most informative links.

Our solution framework comprises the following elements:

1. Evaluation of the structural behaviour of interest
2. Identification of the key uncertainties in structural behaviour and the most influential uncertain structural parameters
3. Development of a load test setup to measure the structural response
4. Physical modelling of the structure and generation of synthetic data
5. Evaluation of the dependency structure
6. Bayesian network development
7. Bayesian network validation
8. Bayesian network inference based on load test data

The proposed approach, integrating structural modelling, load testing and a BN statistical model, is schematically presented in Figure 1. The 3 levels in the BN structure are, from top to bottom, related to the structural (system) parameters, the calculated response variables, and the measured response variables, respectively. The latter two levels are separated to allow for measurement errors or noise.

The proposed sparsified Copula-Based Bayesian Network provides a structured, probabilistic basis for updating structural parameters from monitoring data while controlling model complexity and facilitating validation.

The present conference paper scope is limited to the key elements 5 to 7, being evaluation and modelling of dependency, and Bayesian network development and validation.

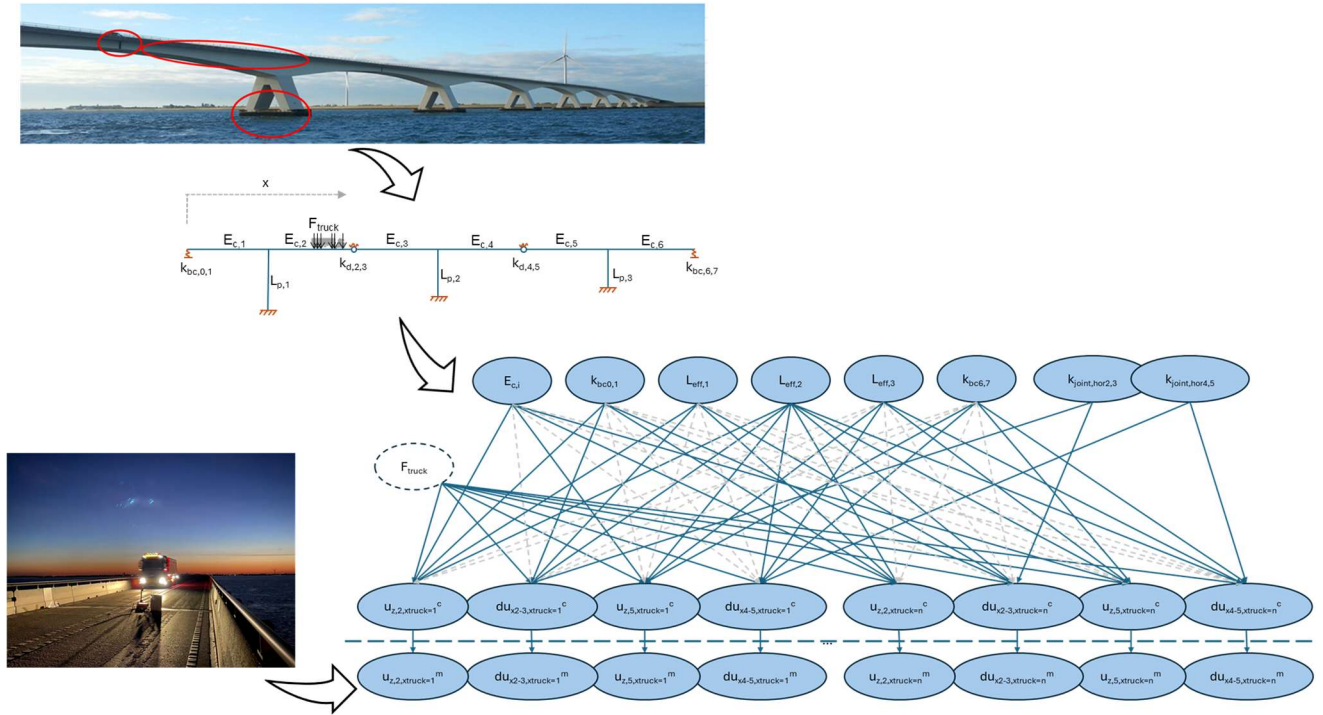


Fig. 1. Solution framework schematically presented for the Zeeland Bridge, where $E_{c,i}$ refers to the concrete Young's modulus of halfspan i , $k_{joint,i,j}$ refers to the stiffness of the joint between halfspans i and j , $L_{p,i}$ refers to the freestanding pile length of pile I , and $k_{bc,i,j}$ refers to the model boundary interaction stiffness representing interaction with adjacent spans.

2. Evaluation and modelling of dependency in the context of BN based SI

2.1 Quantifying dependency and copulas

Dependencies between system parameters and structural response variables under load testing can be exploited to infer system parameters from measured responses. These dependencies generally vary with the applied load scenario. Therefore, investigating dependency structures through physics-based simulations is a key step in developing and optimizing the statistical models used for parameter inference.

An example for the Zeeland Bridge is shown in Figure 2. The horizontal axis represents the load position of a test vehicle, while the vertical axis shows the correlation coefficient between selected structural parameters and the vertical deformation response at a midspan joint. Each curve corresponds to a different structural parameter. Parameters $E_{c,i}$ denote the concrete elastic modulus, $L_{eff,i}$ the effective pile length supporting the bridge piers, and $k_{bc,i}$ the boundary stiffness representing interaction between adjacent spans. The figure illustrates that certain load positions are more informative than others, and that this varies across parameters. In the proposed approach, this information is used to reduce the number of arcs in the Bayesian Network, retaining only statistically significant dependencies.

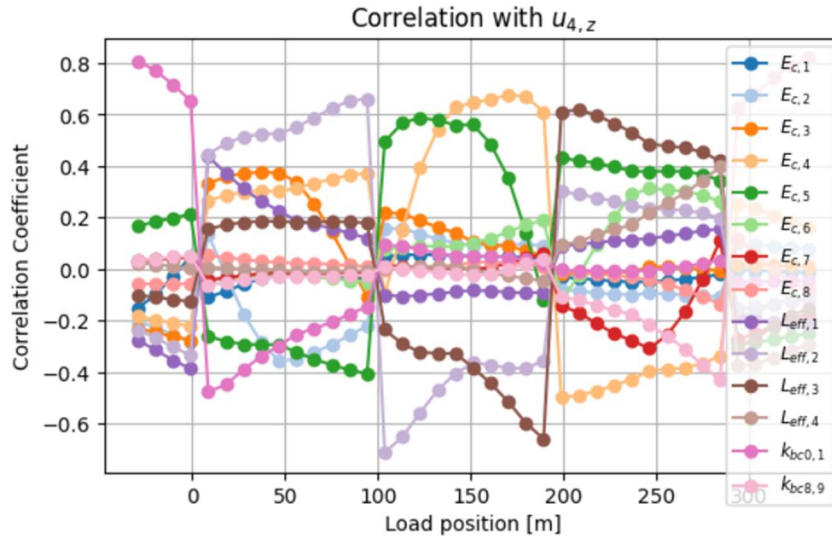


Fig. 2. Dependency of vertical deformation response to the bridge modelled stiffness parameters Zeeland Bridge

The Pearson correlation coefficient (Pearson's r or ρ) is commonly used to quantify linear dependence between variables. However, it is limited to linear relationships and may fail to capture nonlinear dependencies. Rank-based measures, such as Spearman's ρ and Kendall's τ , instead quantify monotonic relationships and are more robust to nonlinear behavior and outliers.

Copulas provide a convenient framework for analyzing such dependencies independently of marginal distributions [7]. A copula is a joint distribution defined on the unit square with uniform marginals. For two random variables X and Y , with cumulative distribution functions F_X and F_Y , the joint distribution can be expressed as:

$$F_{X,Y}(x,y) = C_\theta(F_X(x), F_Y(y))$$

where C_θ denotes the copula function parameterized by θ .

This formulation separates marginal behavior from dependence structure by transforming the variables to the uniform scale via their cumulative distribution functions. Spearman's rank correlation can then be interpreted as the Pearson correlation coefficient applied to these transformed variables, i.e., to $F_X(X)$ and $F_Y(Y)$. In this sense, rank correlation measures linear dependence in the copula (uniform) space rather than in the original data space.

Figure 3 illustrates an example of the dependency between a boundary stiffness parameter (k_{bc}) and a stress response in an adjacent span, shown both in the original data space and in the copula-transformed space. While the relationship appears nonlinear in the original space, the copula representation isolates the dependence structure from marginal effects.

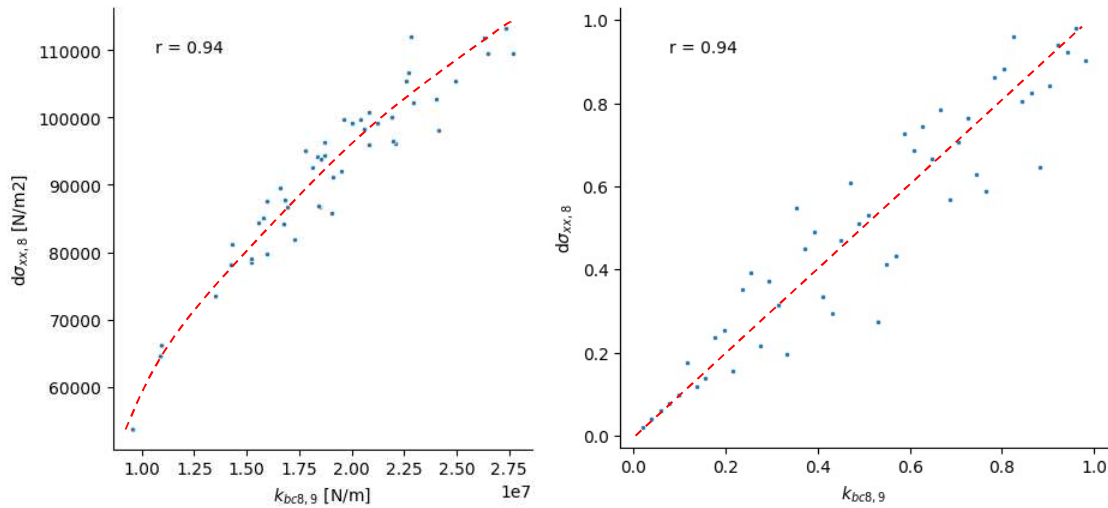


Fig. 3. Example of system parameter to response variable dependency presented on actual and copula scales

2.2 Symmetry and the normal copula assumption in dependency modelling

Copulas enable flexible modelling of dependence structures independently of marginal distributions. The Gaussian (normal) copula is often adopted due to its computational efficiency and compatibility with Bayesian Network formulations. However, its applicability depends on the nature of the underlying dependencies.

A key limitation of the Gaussian copula is its symmetry: it assumes that dependence is identical in all regions of the distribution, including both tails. In practice, dependencies may exhibit asymmetry, such as stronger association in extreme values (tail dependence) or direction-dependent behavior. In such cases, the Gaussian copula may provide an inadequate representation of the true dependence structure.

This issue is illustrated in Figure 3, where nonlinear dependence and indications of tail dependence are visible in the simulated data. Accurately capturing such features may require more flexible copula families.

The Cramér–von Mises (CvM) statistic provides a goodness-of-fit measure for evaluating copula models. It quantifies the discrepancy between the empirical copula and a fitted theoretical copula by integrating the squared difference over the unit hypercube. Assessing CvM statistics for key dependencies is therefore an important step in validating the suitability of a Gaussian copula within a Bayesian Network model for structural identification. Previous studies have shown that approximating dependencies using Gaussian copulas may be acceptable within certain limits [6, 8], but the impact on model accuracy should be carefully evaluated for each specific application.

3. Bayesian network optimization and validation

3.1 Bayesian network basics

Bayesian networks (BNs), also called Bayesian nets, belief nets, or probabilistic graphical models, are statistical models represented by directed acyclic graphs (DAGs). Nodes correspond to random variables and directed arcs express direct dependency relations between variables. The power of Bayesian networks lies in their ability to encode conditional independence relations compactly, enabling an efficient representation of high-dimensional probability distributions. A BN over $\mathbf{X} = \{X_1, \dots, X_n\}$ consists of a DAG structure together with local conditional probability distributions; it encodes the joint distribution by the factorization:

$$P(\mathbf{X}) = \prod_{i=1}^n P(X_i | Pa(X_i))$$

where $Pa(X_i)$ denotes the parents of X_i in the DAG.

Definitions and fundamentals for Bayesian networks are given in Pearl [8]. Working with BNs as statistical models allows us to explicitly encode dependencies in our statistical model which are physically relevant based on simulated or measured data. Integrating expert judgement and data to set the dependencies overcomes the issues of limited training data, a typical constraint in structural identification for infrastructure assets, which may arise for purely data-driven techniques.

3.2 Bayesian network optimization

When constructing a Bayesian network from ordinal (i.e. categorical statistical ranked) multivariate data without a known DAG, one can directly use empirical one-dimensional marginals and assume a normal copula for the joint distribution [9]. Transforming each marginal to a standard normal yields a joint normal distribution preserving the original rank correlation structure. The goal of learning the BN typically would be to establish a joint normal copula well representing the multivariate data, but resulting in a sparsified model with dependencies which can be physically justified.

For load test based structural identification this implies that input-input dependencies between system parameters may be implemented based on literature or material test data or be chosen in accordance with expert opinions. Input-output relations would then be derived from load test data. Here the issue of convergence? in the BN, i.e., numerical problems caused by child nodes having many parent nodes, should be addressed. In order to overcome convergence problems we follow the concept presented in [10], by defining a sub-system as a basis for our structural identification problem. A sub-system consists of an isolated part of a structure including its local system parameters, and boundary conditions covering the interaction with adjacent part of the structure. Subsystems can partially overlap, depending on the structure and the extend of load-effects in a specific structure. This approach allows us to limit the number of parameters and especially parent nodes in an isolated identification problem.

3.3 Bayesian network validation

Aiming for a BN model which is valid, learning and validation of the model are closely connected. Validation of the model involves two key steps. Apart from verifying that the normal copula adequately represents the data's multivariate dependence, as discussed earlier in this paper, there is a need of checking that the network structure with its conditional independence relations is an adequate model of the saturated graph. In [6] the determinant of the correlation matrix is proposed as a measure of multivariate dependence. The arcs in a CBBN are described by conditional rank correlations, for which the corresponding partial correlations may be computed. From the partial correlations and the (conditional) independence statements of the DAG the correlation matrix and its determinant may be computed. Two determinants are compared when validating the normal copula assumption: DER, from empirical rank correlations, and DNR, from transformed normal rank correlations derived via Pearson's transformation linking rank and product-moment correlations. Statistical testing involves resampling DNR to create confidence bands and assessing whether DER lies within these bounds. Then, starting from a skeletal CBBN, by adding arcs to the DAG based on largest rank correlation between pairs of variables, the CBBN structure including the main dependencies can be obtained. Here the CBBN can be validated by comparing the determinant of the normal rank correlation matrix (DNR) to the determinant of the CBBN rank correlation matrix. The determinant factorizes over CBBN arcs, emphasizing strong conditional rank correlations and reflecting the conditional independences encoded in the network. It is important to note here that for small data sets, the normal copula assumption seldom rejects; for large data sets, the determinant test can be stringent. In the context of structural identification this favours the definition of sub-systems as a basis for BN-based inference.

4. Conclusions and outlook

This paper presents a framework for load test based structural identification using Bayesian Networks as statistical models to infer structural parameters based on the measured response of a structure. The key elements of the proposed approach are introduced and the concepts of dependencies, modelling dependencies through copula and integrating copula in Copula Based Bayesian Networks, are explained. Finally, some important validation steps for CBBNs are briefly discussed in relation to structural identification applications.

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