WTHD 41

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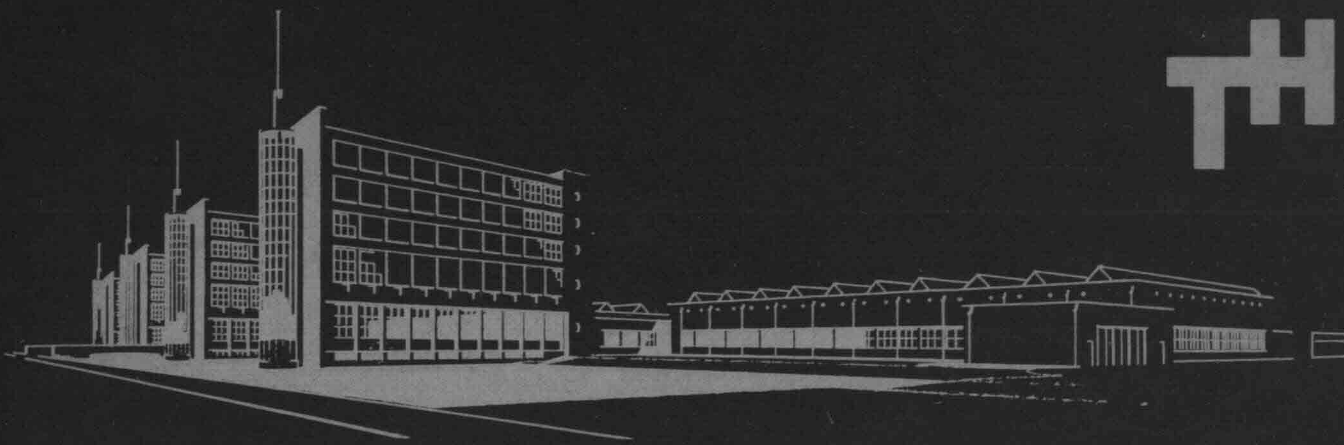


AFDELING DER WERKTUIGBOUWKUNDE

MEMORY EFFECT IN A TURBULENT BOUNDARY-LAYER FLOW DUE TO A RELATIVELY STRONG AXIAL VARIATION OF THE MEAN-VELOCITY GRADIENT

BY

J. O. HINZE, R. E. SONNENBERG

AND

P. J. H. BUILTJES

September
1972

LABORATORIUM VOOR
AERO- EN HYDRODYNAMICA
DELFT

41

Hinze, Builtjes (Vrijdag 2 ~~maart~~ april 1976) — KIVI lezing

Gehengeneffecten in turb.

Hinze: → Deeltje in turb. past zich niet direct volledig aan omgeving aan : → voorgeschiedenis telt nog.

correlaties $f(t)\,f(t-\tau)$

beperking hier tot impuls als overdraagbare eigensch.

Boussinesq : gradient van loc. gemid. ~~vel~~ snelh. → Reyn. schuifsp.

↑ Locaal = toch wel groot gebied

Gehengenafst = Lagr. integr. tijdschaal { × gem. snel → in hoofdstr. richting
{ × turb. " → ⊥ "=wervel"schaal. dus veel groter

te bepalen in 1ste orde ben. uit Lagr. snelh. correlatie

Soms geeft 1ste orde benad → neg diff. coef ⊥ hoofdr.
neg turbulentie productie
fysisch ~~on~~ niet toelaatbaar.

Deisler: korte grenslaag : sterk versneld of vertraagd : schuifsp. blijf zelfde langs stroomlijn → gehengen hele grenslaag door.

2de orde naast bouss. $-\overline{v_2 v_1} = \varepsilon_m \frac{\partial U_1}{\partial x_2} - \frac{1}{2}\varepsilon_m^{0} \, L_{\phi} \frac{\partial^2 U_1}{\partial x_1 \partial x_2}$ blv.

⇑ door reeksontw.

evt ook gehengenfunctie (algemeen) ervoor in te voeren.

2de term dan $\int_0^{t_1} \ldots G(t_1)\,dt_1$

$-\overline{v_2 v_1} = -\left(\overline{v_2 v_1}\right)_{\text{omg}} - \left(\overline{v_2 v_1}\right)_{\text{geh}}$ ← door gehengen eff.

↳ aangepast aan omg

exponentiele vorm.

$G(x_1 - x_1') = \frac{1}{\Lambda_1} \exp - \frac{(x_1 - x_1')}{\Lambda_1}$

G is opl. van relaxatieverg. — gehengenlengte

NB. ~~...~~ $\Lambda = \Lambda(x_1)$

Gehengeneffecten b.v. te verwaarl. als schuifsp langs hoofstr. richting weinig verandert in afst Λ_1 (?)
Bepaling Λ_1 moeilijk

↳ gehengen afstand: hang af van lagr. int. tijdschaal.

⇑ slecht theor. te gebruiken.

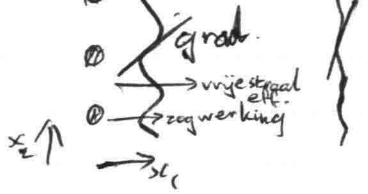
⇑ wel een uitdrukking voor (exp) naar Eulersedingen.

In ongestoorde grensl. blijken de gehengeneff. verwaarloosbaar. Storing door cilinders etc. geeft wel soms belangrijke gehengeneff. N.B. Ook op te lossen door niet const. ε_m.
↑ boussinesq

NB. In gelijkv. stroming kan je geen gehengen eff. meten, zit verstopt in ε_m. Overal hobbelt turb. evenveel ter de werkelijke stroming aan.
Eig. bij volledig ontw. zogstroming: gehengeneffect b.v. 20% van ε_m.

NB. Gehengeneff. ook toepasbaar (analoog) op overdr. van warmte en stof etc.

<u>Builtjes</u>: toegespitst op roosterturb.
Ver achter rooster wel schuifsp geen snelh.grad
⟹ typisch sprake van gehengeneff.



rooster erg nauwkeurig gemaakt v goed homogene turb.

Ook ver achter het rooster u_1' is ±25% groter dan u_2' → niet isotroop.
↑ hangt van o.a. aard van rooster af.

$-\overline{u_1'u_2'}$ = schuifsp. valt al gauw tot nul terug.

~~Op plaats waar~~
als schuifsp.tensor
$$\begin{pmatrix} \overline{u_1'^2} & \overline{u_1u_2} & 0 \\ \overline{u_1'u_2'} & \overline{u_2'^2} & 0 \\ 0 & 0 & \overline{u_3'^2} \end{pmatrix} = \begin{pmatrix} \overline{u_1'^2} & 0 & 0 \\ 0 & \overline{u_2'^2} & 0 \\ 0 & 0 & \overline{u_3'^2} \end{pmatrix}$$
↑ draaien assen.

∠α gedraaid.
$$\alpha = \frac{\overline{u_1'u_2}}{\overline{u_1'^2} - \overline{u_2'^2}}$$

45° t.o.v. hoofdassen: max. schuifsp.

eindje achter rooster

Als $\overline{u_1'u_2'} = 0$ $\alpha = 0$ toch niet u_1' u_2' ontkoppeld n.l. nog wel max schuifsp.

wel schuifsp. doch in richting 45° op hoofdassen. terwijl in geen richting snelh. grad.

NB. 2 vgl'en op te stellen 1 relaxatie vgl. voor verschil in normaalsp. $\overline{u_1'^2} - \overline{u_2'^2}$

1 rel " " schuifsp.

⇒ hieruit af te leiden: er moeten geheugen effecten zijn

(uit verband van) deze vgl'en.

als geen geheugen effect $\alpha \neq \alpha(x_1)$ doch experimenteel wel $\alpha = \alpha(x_1)$

Echter hier ~~lengte~~ tijd schaal ~~no~~ voor geheugeneffect in schuifsp. moet hier uit schuifsp. komen.

Lagrange tijdschaal ~~uit~~ van schuif.

(Uit ruimte-tijd corr. van $\overline{u_1'u_2'}$)

$\overline{u_1'u_2'(x_A, 0, 0, t)\, u_1'u_2'(x_B, 0, 0, t+\tau)}$

Gehengeneffect van schuifsp. klein hier

Maar toch ~~α verandert~~ schuifsp. verderop, komt dus uit geheugeneffect $\overline{u_1'^2} - \overline{u_2'^2}$

DEPARTMENT OF MECHANICAL ENGINEERING

Delft University of Technology

the Netherlands

Memory effect in a turbulent boundary-layer flow due to a relatively strong axial variation of the mean-velocity gradient.

by

J.O. Hinze

R.E. Sonnenberg

P.J.H. Builtjes

WTHD No.41

September 1972

Laboratory for Aero- and Hydrodynamics,

Rotterdamseweg 145 - Delft.

Abstract:

Measurements have been made of the distributions of mean velocity, turbulence intensities and turbulence shear-stress in a turbulent boundary-layer downstream of a hemi-spherical cap attached onto the plane rigid wall. The eddy-viscosity, when computed in the classical way according to Boussinesq's concept from the lateral gradient of the mean velocity and the turbulence shear-stress, showed a very strong non-uniform lateral distribution, also across the outer region of the boundary-layer. Moreover, the non-dimensional values of the eddy viscosity, using the wall-friction velocity and the boundary-layer thickness as the velocity scale and length scale respectively, were higher than those for the boundary-layer when not disturbed by the wake of the spherical cap.

However, when account is taken of an axial memory effect of the streamwise variation of the lateral gradient of the mean-velocity, the values of the non-dimensional eddy viscosity are close to those for the undisturbed boundary-layer.

1. Introduction. In predominantly one-dimensional turbulent flows like free-turbulent shear-flows and boundary-layer flows, with the gradient of a mean property mainly in the transverse direction, the concept of a gradient-type of turbulence diffusion appears to be a reasonable assumption for describing the distribution of this mean property. An eddy-diffusion coefficient is introduced, which may be obtained from a Lagrangian description of the transport of a transferable property in this transverse direction. Memory effects then are restricted to the relative short distances, over which the gradient of the mean property is practically constant. In free-turbulent flows and in the outer region of the boundary-layer flows the eddy diffusion coefficient turns out to be almost constant, when at the same time the effect of intermittency of the turbulence is taken into account. When the eddy-diffusion coefficient is rendered dimensionless with a suitably chosen velocity scale and length scale, the non-dimensional value is the same throughout the whole self-preserving part of these flows.

Less satisfactory results may be obtained if the concept of a gradient-type of diffusion is applied to cases where the transverse gradient of the mean property may not be considered constant over the effective diffusion distances, as determined by the Lagrangian integral time-scale and a transport velocity. For the transport in transverse direction usually the intensity of the turbulence velocity component in this direction is taken. Such effects as transport up the gradient instead of down the gradient may then be obtained, which in most cases can not be accepted on physical grounds. Well known in the case of momentum transport is the situation where the mean-velocity gradient and the shear-stress have an opposite sign, as observed by a number of investigators in small regions of turbulent flows with a non-symmetric distribution of the mean-velocity, e.g. wall jets. In such a situation the concept of a gradient-type of diffusion would lead to a negative eddy-viscosity. [1,2,3].

Also, less satisfactory results may be obtained if the transverse gradient of the mean property does not change slowly enough in streamwise direction with respect to the Lagrangian integral time scale.
This may be expected to occur e.g. in the region downstream of a point of reattachement of a turbulent boundary-layer, or just downstream of a sudden change in wall roughness. In general, in flow regions where self-preservation has not yet been attained.
In the self-preserving part of the flow, because of the self-similarity of the flow pattern in down-stream sections, the axial memory-effect only influences the value of the non-dimensional eddy-diffusion coefficient. For instance, if the non-dimensional eddy-diffusion coefficient is constant in a cross section, it has the same constant value in any section of the whole self-preserving part of the flow.

In order to include axial memory effects in the calculation of e.g. developing boundary layers, present activities are focussed on methods where the Reynolds' equations are complemented by additional transport equations for the Reynolds' stresses. Which, however, require assumptions regarding the unavoidable closure problem [4,5,6,7,8].

It may be expected that a number of cases exists where, on the one hand the local eddy-viscosity concept can no longer be applied with success, on the other hand that it may not be necessary to consider the above additional transport equations. Provided that the eddy-viscosity is modified with a suitably built-in memory effect.

The authors believe that such a case is presented by the flow considered here, with a relatively fast changing mean-velocity profile in streamwise direction. It has been observed to occur in the wake flow of a hemi-spherical cap attached onto a rigid plane wall, placed in a flow with uniform free-stream velocity. The turbulent boundary-layer at the station of the hemi-spherical cap was relatively thick with respect to the height of the cap. Downstream of the cap the distortion of the mean-velocity profile compared with that of the undisturbed boundary-layer, and due to the wake of the cap, reduces at a much faster rate than the changes of the undisturbed mean-velocity profile across the same axial distance. When according to the Boussinesq's concept of an eddy viscosity, the shear-stress was expressed in terms of the local gradient of the distorted mean-velocity profile, a strongly non-uniform distribution of the eddy-viscosity across the boundary-layer was obtained. This might be partly due to the direct wake-action, producing a higher degree of turbulence than in the undisturbed boundary-layer, but also partly due to some axial memory-effect. It has been assumed that the shear-stress at a certain station should not only be related to the local mean-velocity gradient, but also to the upstream mean-velocity gradients, due to the relatively rapid stream-wise changes of this gradient.

2. Theory. Assume in the plane, predominantly one-dimensional, flow the x_1-coordinate in the main-flow direction, and the x_2-coordinate in the transverse direction. Let $\mathcal{P}$ be a transferable property. Then the turbulence transport of this property through a streamwise control plane and in the x_2-direction is given by the expression:

$$\overline{u_2 p} = \frac{1}{T}\int_0^T dt_o \; u_2(t_o)\mathcal{P}(t_o) \tag{1}$$

where u_2 is the turbulence velocity in the x_2-direction and p the turbulent fluctuation of $\mathcal{P}$. Let $y_1(t_o,t)$ and $y_2(t_o,t)$ be the distances travelled during a time t in the x_1-direction and x_2-direction respectively, by a fluid particle that crosses the control plane at time t_o.
A series expansion of $\mathcal{P}(t_o)$ yields

$$\mathcal{P}(t_o) = \bar{\mathcal{P}} - y_1(t_o,t)\frac{\partial\bar{\mathcal{P}}}{\partial x_1} - y_2(t_o,t)\frac{\partial\bar{\mathcal{P}}}{\partial x_2} +$$

$$\frac{1}{2}y_1^2(t_o,t)\frac{\partial^2\bar{\mathcal{P}}}{\partial x_1^2} + \frac{1}{2}y_2^2(t_o,t)\frac{\partial^2\bar{\mathcal{P}}}{\partial x_2^2} + y_1y_2(t_o,t)\frac{\partial^2\bar{\mathcal{P}}}{\partial x_1\partial x_2} + \ldots \tag{2}$$

Assume the terms with $\partial\bar{\mathcal{P}}/\partial x_2$ and $\partial^2\bar{\mathcal{P}}/\partial x_1\partial x_2$ to be dominant.

With

$$y_2(t_o,t) = \int_0^t dt' \; u_2(t_o-t'),$$

Eq.(1) becomes

$$- \overline{u_2 \rho} = \int_0^t dt' \, \overline{u_2(t_o) \, u_2(t_o - t')[\frac{\partial \bar{\rho}}{\partial x_2} - y_1(t_o, t) \frac{\partial^2 \bar{\rho}}{\partial x_1 \partial x_2}]}$$

Now put

$$\frac{\partial \bar{\rho}}{\partial x_2} - y_1(t_o, t) \frac{\partial^2 \bar{\rho}}{\partial x_1 \partial x_2} = \int_0^t dt'' \frac{\partial \bar{\rho}}{\partial x_2} (t_o - t'').M(t''). \tag{3}$$

where M(t") is a memory function satisfying the condition

$$\int_0^\infty dt'' \, M(t'') = 1 \tag{4}$$

When t is sufficiently large we may introduce an effective eddy-diffusion coefficient, defined by

$$- \overline{u_2 \rho} = \epsilon_\rho^* \int_0^\infty dt' \frac{\partial \bar{\rho}}{\partial x_2} (-t') \, M(t') \tag{5}$$

[When ρ is the turbulence momentum component u_1, Eq.(5) is similar to that for the shear-stress of a visco-elastic fluid [9]].

Further assume for the memory function a simple, exponential behaviour

$$M(t) = \frac{1}{T_m} \exp.[-t/T_m] \tag{6}$$

where T_m is a relaxation time.

With

$$\frac{\partial \bar{\rho}}{\partial x_2} (-t') = \frac{\partial \bar{\rho}}{\partial x_2} - t' \frac{\partial^2 \bar{\rho}}{\partial t' \partial x_2} , \tag{7}$$

Eq.(5) yields

$$- \overline{u_2 \rho} = \epsilon_\rho^* [\frac{\partial}{\partial x_2} - T_m \frac{\partial^2 \bar{\rho}}{\partial t' \partial x_2}] \tag{8}$$

When the relative turbulence intensity is sufficiently small to justify the application of Taylor's hypothesis of a "frozen" turbulence, Eq.(8) may be written

$$- \overline{u_2 \rho} = \epsilon_\rho^* [\frac{\partial \bar{\rho}}{\partial x_2} - \bar{U}_1 T_m \frac{\partial^2 \bar{\rho}}{\partial x_1 \partial x_2}], \qquad (9)$$

since $t' < 0$, and $\partial/\partial t' = \bar{U}_1 \partial/\partial x_1$ for a fluid particle.
In the case considered we deal with the turbulence shear-stress, for which the corresponding expression reads

$$- \overline{u_2 u_1} = \epsilon_m^* [\frac{\partial \bar{U}_1}{\partial x_2} - \bar{U}_1 T_m \frac{\partial^2 \bar{U}_1}{\partial x_1 \partial x_2}] \qquad (10)$$

When applying this expression to the mean-velocity profile of the boundary-layer, distorted by the wake flow of the hemi-spherical cap, two aspects have still to be considered. Namely, first the intermittent nature of the turbulence in the outer part of the boundary-layer, and second, the evaluation of the relaxation time.

For the intermittent region Eq.(10) should apply to the periods that the flow is turbulent. If we assume the non-turbulent regions to be irrotational, then under the condition valid for the boundary-layer flow the shear-stress in the irrotational part of the flow is zero, and the left-hand-side of Eq.(10) should read $(- \overline{u_2 u_1})/\Omega$, where Ω is the intermittency factor. Also we have to consider $(\partial \bar{U}_1/\partial x_2)_{rot}$.
Except near the crests of the turbulent-nonturbulent interface, according to experimental evidence the value of this velocity gradient is close to that of $\partial \bar{U}_1/\partial x_2$ [see e.g. ref. 10]. Consequently the right-hand-side of Eq.(10) needs not to be corrected for the intermittency.

The evaluation of the relaxation time T_m presents more difficulties. It may be a good guess to take for it the Lagrangian integral time-scale $\mathfrak{J}_L$. Since the Lagrangian auto-correlation has not been determined, $\mathfrak{J}_L$ has been estimated from the Eulerian longitudinal integral length scale Λ_f, by making use of the approximate relation [11,12].

$$\mathfrak{J}_L \simeq 0.4 \Lambda_f / u_1' \qquad (11)$$

In this expression the intensity u_1' has to be corrected for the intermittency.
Finally, Λ_f may be assumed to be roughly constant in the outer region of the boundary-layer, and equal to $\sim 0.4\ \delta_{99}$ for the case of a smooth wall[13], and decreasing linearly with the distance to the wall for the inner region. The expression used for the evaluation of ϵ_m^* thus reads

$$\epsilon_m^* = \frac{-\overline{u_2 u_1}/\Omega}{\frac{\partial \bar{U}_1}{\partial x_2} - \bar{U}_1 \mathfrak{I}_L \frac{\partial^2 \bar{U}_1}{\partial x_1 \partial x_2}} \qquad (12)$$

3. Description of the experiments. These have been carried out in a low-turbulence windtunnel of the closed circuit type. The working section is 4.5m long, and has a cross section of 0.8 × 0.7 m^2. The boundary-layer studied was on one side of a glass plate, put vertically in streamwise direction in a plane of symmetry of the working section. Transition from laminar to turbulent flow of the boundary-layer was fixed at 0.6 m from the leading edge by means of a trip-wire placed spanwise at a short distance from the wall. A hemi-sphere of 40 mm diameter was attached onto the glass plate in a centre position at 3.65 m from the leading edge. The free-stream velocity during most of the experiments was 10.5 m/s. The thickness of the turbulent boundary-layer at the location of the hemi-sphere was about 50 mm.

Mean-velocity and turbulence measurements have been carried out with a constant-temperature hotwire anemometer. A platinum-coated tungsten-wire of 5 μm diameter has been used. The length of the sensitive part was 2 mm. The distance between the prongs 10 mm.

The measurements included: the three components of the mean-velocity (the mean-flow in the wake of the cap is three-dimensional), the three components of the turbulence intensity, the turbulence shear-stresses, spatial velocity correlations and one-dimensional energy spectra of the axial turbulence velocity. Wall shear-stresses have been measured with a Preston tube. Measurements with this tube in the undisturbed boundary-layer have been compared with the wall shear-stresses obtained from the change in integral momentum, and from the mean-velocity gradient at the wall. The agreement was satisfactory. It was concluded that the measurements with the Preston tube in the wake-flow region of the hemi-spherical cap would yield acceptable values of the wall shear-stress, at least for downstream distances well beyond the point of reattachement of the flow behind the cap.

The coordinate system was taken with the origin in the centre of the base of the cap, with the x_1-coordinate in streamwise direction, the x_2-coordinate perpendicular to the plate and the x_3-coordinate in spanwise direction.

The above measurements have been made at a number of x_1-stations and in planes with different values of x_3, including such a large value of x_3 that the undisturbed boundary-layer at a given x_1-distance could be investigated.

4. Experimental results. From the measurements of the mean velocities the picture of the mean-flow pattern as shown in Fig. 1 has been obtained. The point S on the wall marks the region of reattachment. The two streamwise trailing vortices originate from the corner eddies present in the corner formed by the cap and the wall, and which extend from the upstream stagnation point to the points of separation of the flow along the cap. Though the velocities induced by these trailing vortices are rather weak, the maximum values being 1 to 2 percent of the free-stream velocity, they have a noticeable effect in e.g. the flow in the plane of symmetry, $x_3 = 0$. The centres of the trailing vortices move slightly outward in both spanwise and transverse directions. In the region between $x_1 = 0.25$ m and 0.50 m the x_3-coordinate of these centres varies roughly from $\pm$ 23 mm to $\pm$ 27 mm, the x_2-coordinate varies from 10 mm to 20 mm. In order to minimize the effect of these trailing vortices the considerations presented in the following are restricted to the measurements of the $\bar{U}_1$-velocity component, the turbulence intensities u_1' and the turbulence shear-stress $(-\overline{u_2u_1})$ in the cross-sections $x_1 = 0.25$ m, $x_3 = 20$ mm and $x_1 = 0.50$m, $x_3 = 20$ mm. For the evaluation of $\partial^2\bar{U}_1/\partial x_1\partial x_2$, also the $\bar{U}_1$-distributions in the plane $x_3 = 20$ mm at $x_1 = 0.15$ m and 0.35 m have been used. For the corresponding measurements in the undisturbed boundary-layer, those made in the plane $x_3 = -0.15$ m have been taken.

Since no intermittency measurements have been made, the distribution of the intermittency factor Ω across the boundary layer as measured by Klebanoff [14] has been used in our calculations.

Figures 2 and 3 show the distributions of $\bar{U}_1$, Ω(from Klebanoff), $u_1'/\Omega u^*$, $-\overline{u_2u_1}/\Omega u^{*2}$ and $\epsilon_m/\Omega u^*\delta$ for the undisturbed boundary-layer, measured in plane $x_3 = -0.15$ m, and for $x_1 = 0.25$ m and 0.50 m respectively.

Figures 4 and 5 show the distributions of $\bar{U}_1$, u_1' and $-\overline{u_2u_1}$ for $x_1 = 0.25$ m and 0.50 m respectively. It has been assumed that the same distribution of the intermittency factor as for an undisturbed boundary-layer of the same thickness could be used. Note that the boundary-layer thickness at the same x_1-station is slightly greater for the disturbed than for the undisturbed case. The data points shown are those determined from the smoothed, directly measured distributions of $\bar{U}_1$, u_1' and $-\overline{u_2u_1}$.

Note that the wall shear-stresses as measured with the Preston tube are higher than the values of the turbulence shear-stresses measured at the points closest to the wall. As may be concluded from the mean-velocity distributions, the flow close to the wall accelerates in downstream direction, which corresponds with a negative lateral gradient of the shear stress at the wall.

5. Eddy-viscosity calculations. From the smoothed $\bar{U}_1$-distributions and $(-\overline{u_2u_1})$ distributions first the eddy-viscosity ϵ_m has been computed according to the classical way, namely from $(-\overline{u_2u_1})/(\partial\bar{U}_1/\partial x_2)$. As Figs. 6 and 7 show the ϵ_m-distributions vary strongly across the boundary-layer, even when corrected for the intermittency. This in contrast with the distribution in the undisturbed boundary-layer. The distributions have a pronounced maximum at $x_2/\delta \simeq 0.3$, roughly in the same region where the turbulence shear-stress has its maximum.
However, when the eddy-viscosity ϵ_m^* is calculated according to Eq.(12), its distribution in the outer region becomes much more uniform, and resembles more that for the undisturbed boundary-layer. The agreement with the undisturbed boundary-layer also holds true quantitatively, when the eddy-viscosities ϵ_m/Ω and ϵ_m^* for the undisturbed and the disturbed boundary-layer respectively, are rendered dimensionless with the wall-friction velocity u^* and the boundary-layer thickness δ. This is shown in Fig. 8. For comparison the distribution of $\epsilon_m/\Omega u^*\delta$ as computed from Klebanoff's [14] shear-stress measurements is included in this Fig. 8.

6. Discussion. The results presented in Fig. 8 seem to support the conclusion that for the case considered in the above, memory effects are mainly responsible for the non-uniform distribution in the outer region of the disturbed boundary-layer. Consequently, corrections for these effects are necessary. Yet it must be admitted that in the application of the theory to the experimental results there are a number of uncertain points. One of the major points is the estimation of the relaxation time T_m and the Lagrangian integral time scale $\mathfrak{J}_L$ as presented. Another difficulty of practical nature is the determination of $\partial^2\bar{U}_1/\partial x_1\partial x_2$ from the measured mean-velocity distributions, since it may be appreciated that carrying out twice a graphical differentiation is not an accurate procedure. Moreover it turned out that this second derivative was not constant along the distances compa-

rable with $\bar{U}_1 \mathfrak{J}_L$. In the outer region $\bar{U}_1 \mathfrak{J}_L$ had values from 0.2 to 0.3 m, i.e. from 4 to 5 times δ, which is of the magnetude of the large eddies. Some weighted average value of $\partial^2\bar{U}_1/\partial x_1 \partial x_2$ to be used in Eq.(12) had to be taken. Eq.(7) may be too approximate, and it might be that higher order terms in the series expansion in this equation are required. Also the admissibility of the neglect of a number of terms in the series expansion of Eq.(2) has to be looked at closer.

As mentioned earlier the flow in the plane x_3 = 20 mm has been chosen to minimize the effect of the secondary current caused by the trailing vortices. At this plane the $\bar{U}_2$-component was very small indeed. However, very close to the wall there was still a $\bar{U}_3$-component up to 2% of the free-stream velocity. At the x_2-distances considered the $\bar{U}_3$-component was again small, though. Similar calculations of ϵ_m^* have been carried out for the measurements made in the plane of symmetry $x_3 = 0$. The distribution of ϵ_m^* was still peaked in the region x_2/δ = 0.2 to 0.3, probably because in this region the direct local wake-effect was still not negligibly small. Though for higher values of x_2/δ the distribution was again more uniform than that of ϵ_m/Ω.

Acknowledgement. The authors want to express their thanks to Mr. H. Leijdens for his cooperation and advise during the experimental part of the investigations.

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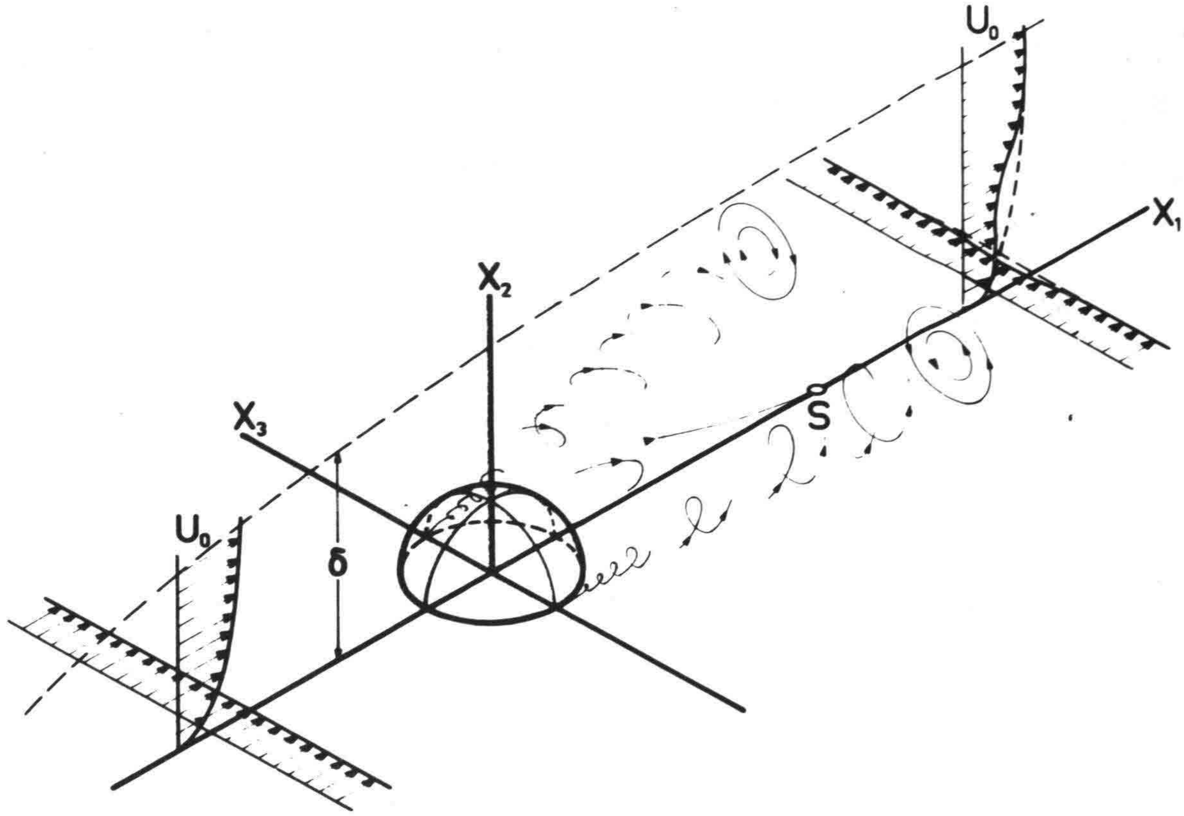

Fig. 1. Mean flow pattern downstream of the hemi-sperical cap.

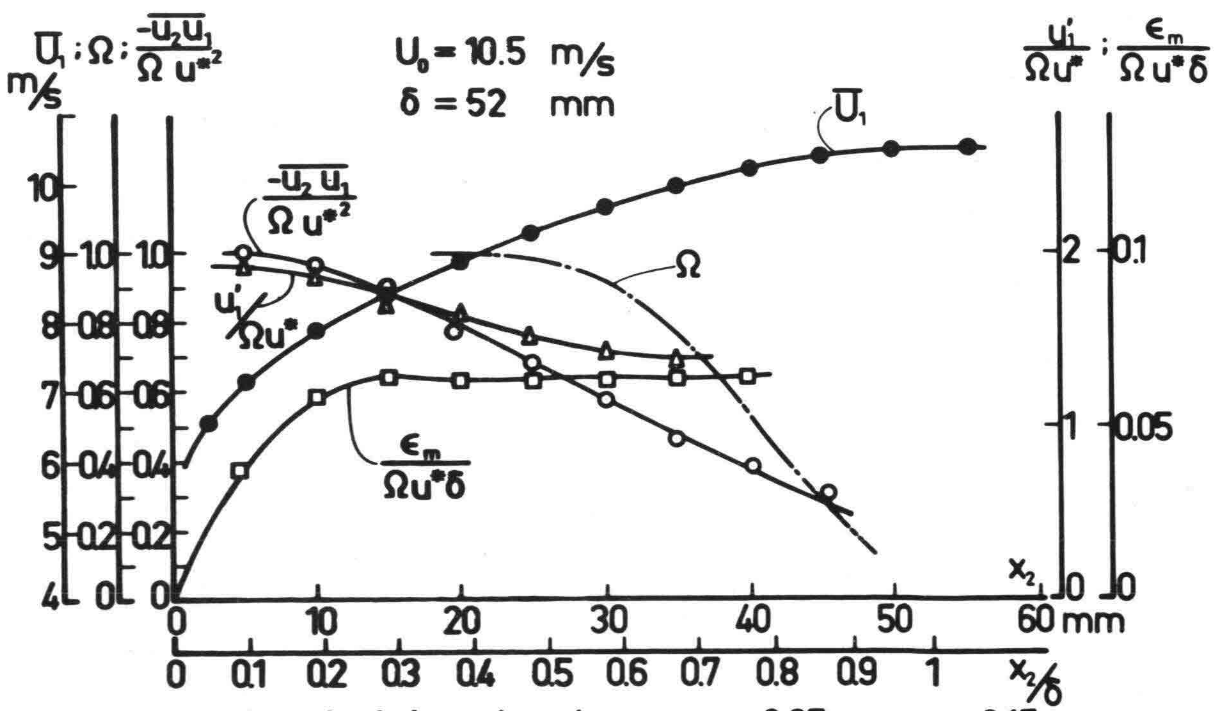

Fig. 2. Undisturbed boundary-layer. $x_1 = 0.25$m ; $x_3 = -0.15$m. Distributions of: $\overline{U}_1$, Ω (Klebanoff), $u_1'/\Omega u^*$, $-\overline{u_2 u_1}/\Omega u^{*2}$ and $\epsilon_m/\Omega u^* \delta$.

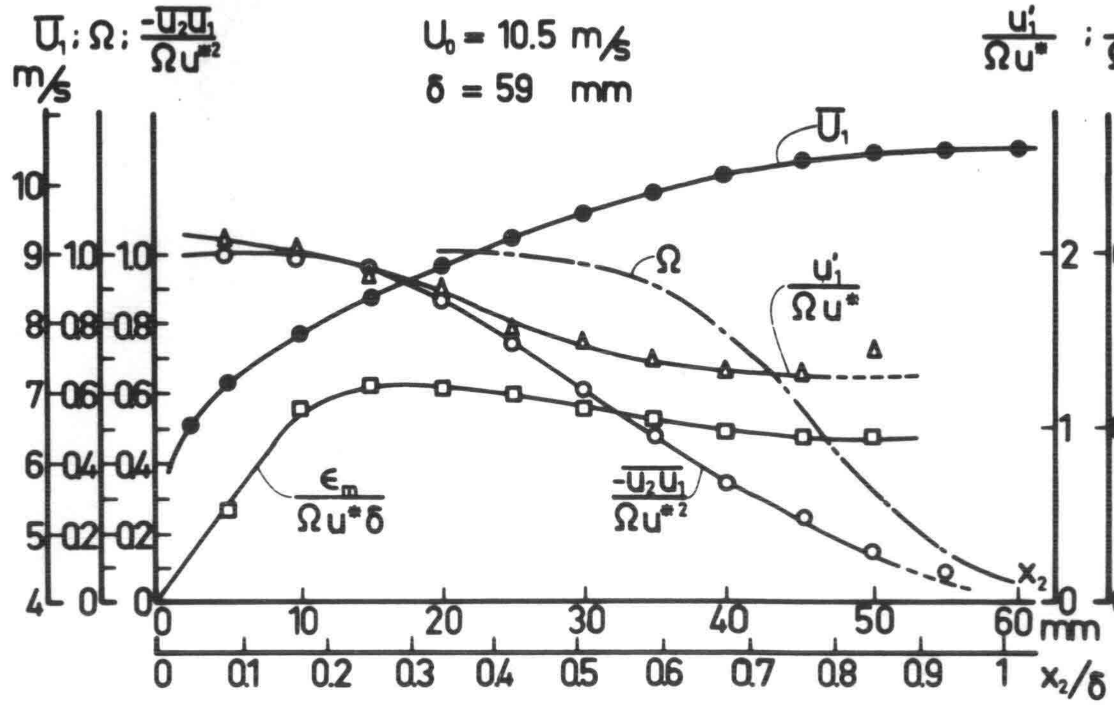

Fig.3. Undisturbed boundary-layer. x_1=0.50 m x_3= -0.15 m
Distributions of : $\overline{U}_1$, Ω (Klebanoff), $u_1'/\Omega u^*$,
$-\overline{u_2u_1}/\Omega u^{*2}$ and $\epsilon_m/\Omega u^*\delta$.

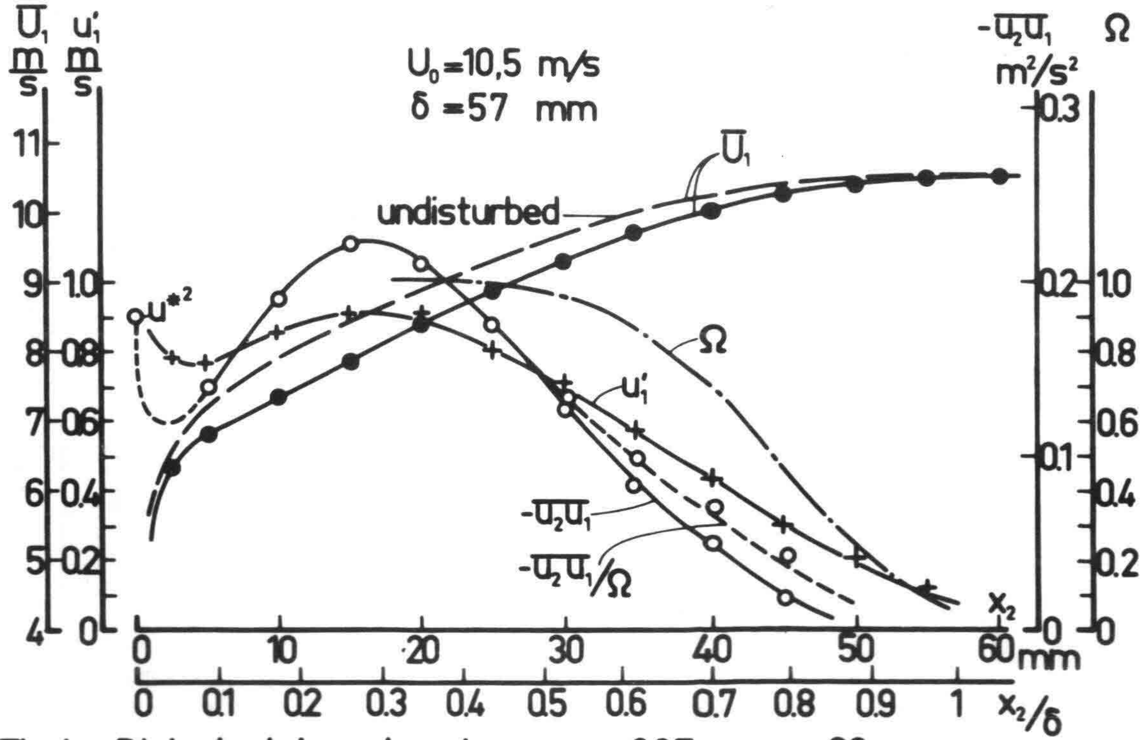

Fig.4. Disturbed boundary-layer. x_1= 0.25 m ; x_3=20mm.
Distributions of : $\overline{U}_1$, Ω (Klebanoff), u_1' and $-\overline{u_2u_1}$

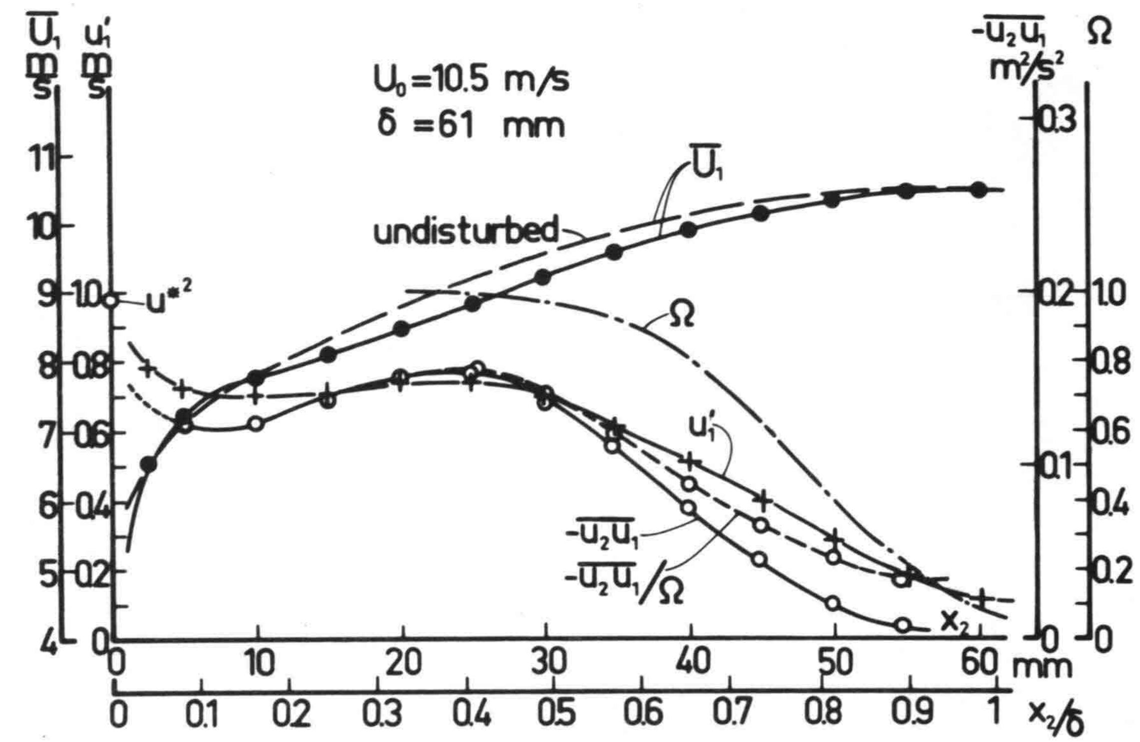

Fig. 5. Disturbed boundary-layer. x_1=0.50 m ; x_3=20 mm
Distributions of : $\overline{U}_1$, Ω (Klebanoff), u_1' and $-\overline{u_2u_1}$.

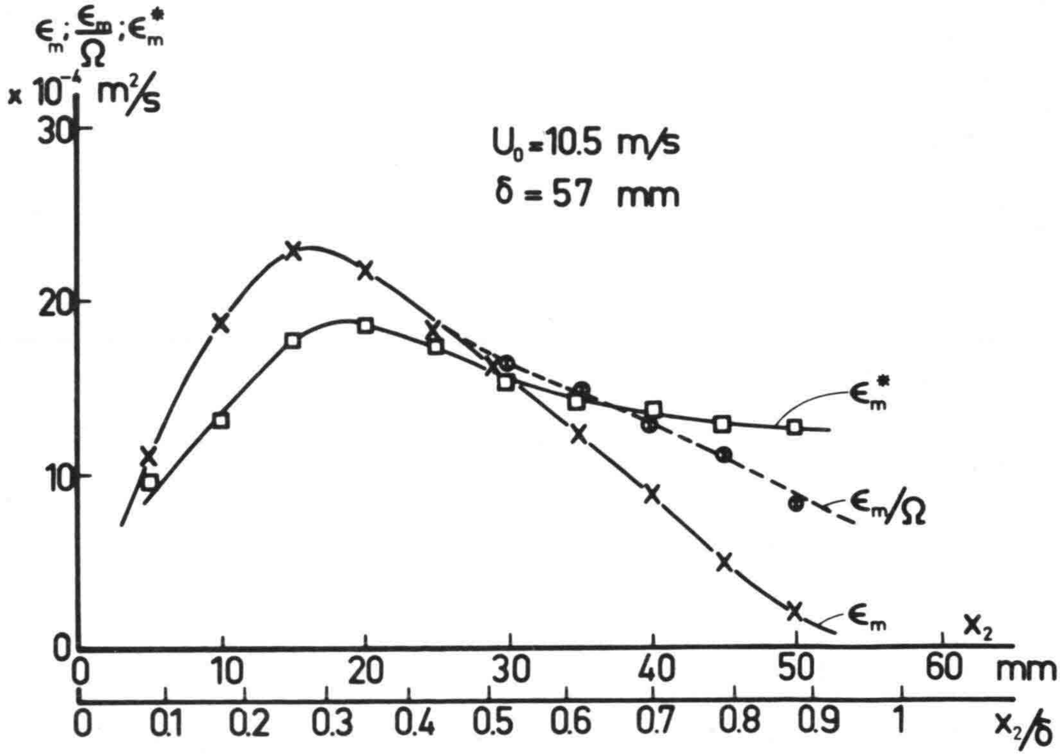

Fig. 6. Disturbed boundary-layer. x_1=0.25m ; x_3=20 mm .
Distributions of the eddy-viscosity, computed from the measured mean-velocity and shear-stress distributions.

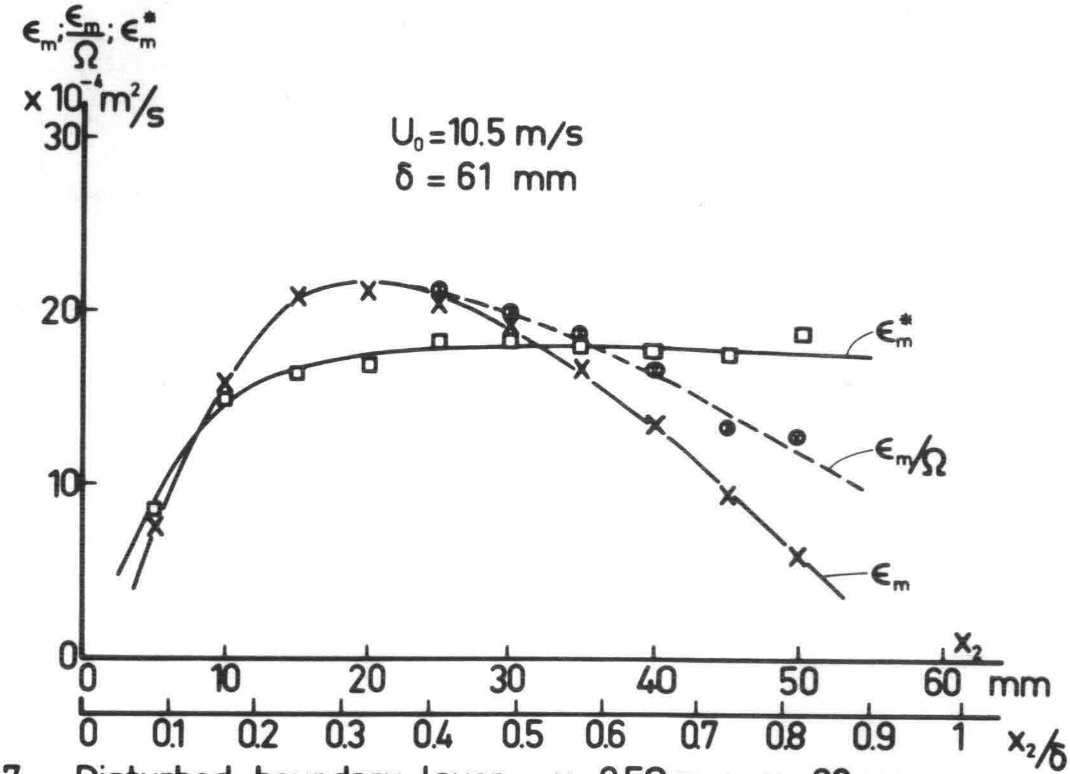

Fig. 7. Disturbed boundary-layer. $x_1 = 0.50$ m ; $x_3 = 20$ mm
Distributions of the eddy-viscosity, computed from the measured mean-velocity and shear-stress distributions.

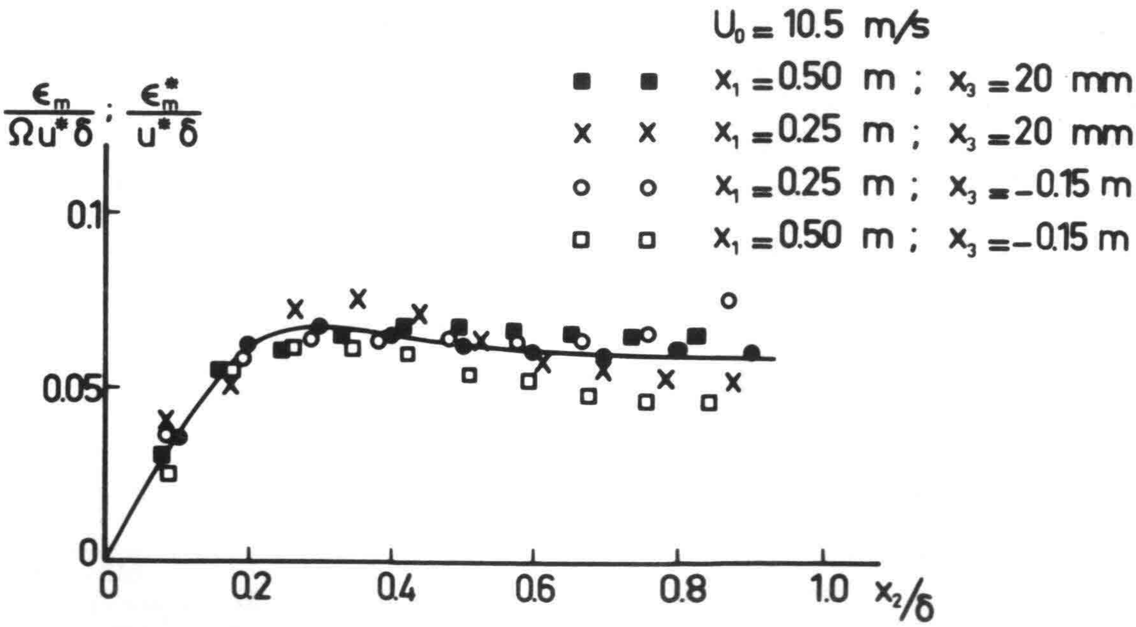

Fig. 8. Distributions of the non-dimensional eddy-viscosities.
●—● Calculated from shear-stress data measured by Klebanoff[9].