

A comparison of explicit continuous and discontinuous Galerkin methods and finite differences for wave propagation in 3D heterogeneous media

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ABSTRACT

In seismic imaging, the need of efficient numerical techniques for solving the wave equation is of vital importance when using techniques such as Reverse Time Migration or Full Waveform Inversion. Finite-difference methods (FDMs) are widely used because they are relatively easy to implement and computationally efficient. Moreover, the use of explicit time stepping is natural. However, they rely on Cartesian structured grids and that makes them less suitable for complex heterogeneous geological settings. When abrupt changes in the properties of the material arise or in the presence of complex topography, the scheme loses its accuracy. This can be amended by using very fine grids at the expense of computational time. Finite-element methods (FEMs) have some advantages over FDMs because they rely on unstructured meshes and can more easily handle geometric or structural discontinuities. The use of the finite-element Galerkin method for the semi-discretization of the second-order wave equation leads to a system of second-order ordinary differential equations that needs to be solved with a suitable time-discretization scheme. In particular, when an explicit time stepping is used, the mass matrix arising from the spatial discretization has to be inverted at each time step, which has dramatic impact on the efficiency of the scheme. In this work we consider two different finite-element formulations that can be used to overcome the problem. The first consists in using continuous finite elements with mass lumping. In this case, the mass matrix is replaced by a diagonal one that is trivial to invert. This technique is straightforward for piecewise linear elements but requires particular quadrature rules and additional discretization nodes for higher-order schemes in order to preserve the accuracy [2]. As a second approach, we consider the Symmetric Internal Penalty Discontinuous Galerkin (SIPDG) method (see [3] for details), where each element of the mesh is decoupled from its neighbours, leading to a block-diagonal mass matrix that is easily inverted. In both cases, we use a second-order time-integration scheme. This choice leads to an energy conservative numerical scheme for the discontinuous formulation [1]. The stability limit values are computed on the reference tetrahedron and they are comparable for the two methods except for linear polynomials where SIPDG is much more restrictive. We analyze the accuracy and efficiency of the two methods and compare their performances in terms of CPU time. To this end, we solve an initial value problem (no source term) with known exact solution on a regular 3-D cubic domain with Neumann boundary conditions. The results in Fig. 1 show that both techniques have

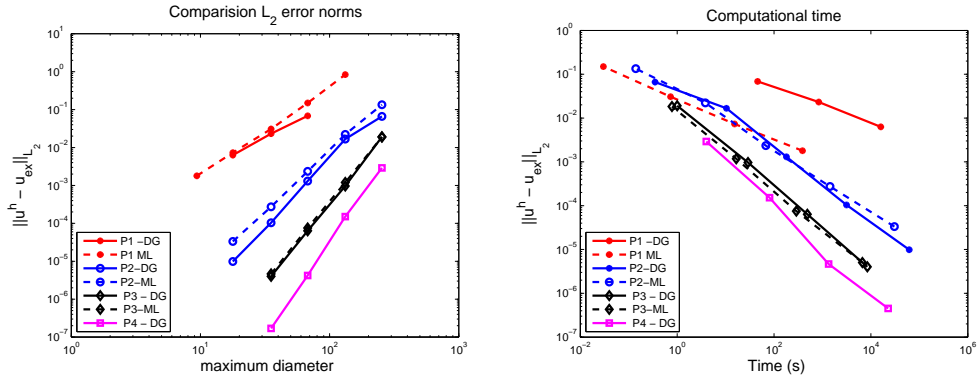


Figure 1: L_2 error norms, accuracy and computational times for the continuous and discontinuous FEMs.

a similar performance in terms of accuracy and computational time. In fact, the continuous methods requires additional discretization points in the interior of the faces and the elements, increasing the computational cost. The DG formulation uses the classical set of nodes but requires the computation of additional fluxes on the faces of each element in order to preserve the continuity of the solution.

We finally address a more realistic 3 dimensional setting as shown in Fig. 2. In this example, there is a velocity inversion at depth containing a salt inclusion with high reflectivity. We solve the wave field on a set of meshes with decreasing maximum diameter by means of the two FEMs as well as a FDM of 8th order in space. We estimate the relative errors as a measure of the accuracy and we compare the seismograms of the solutions. The cubic finite elements start to outperform the finite-difference method when considering a relative error smaller than 0.01. From the analysis of the seismograms, we observe how the finite elements avoid the so-called “stair-case effect” typical for finite-difference grids, as well as the effect of the air layer that needs to be incorporated into the finite-difference method but is embedded in the boundary conditions of the FEM.

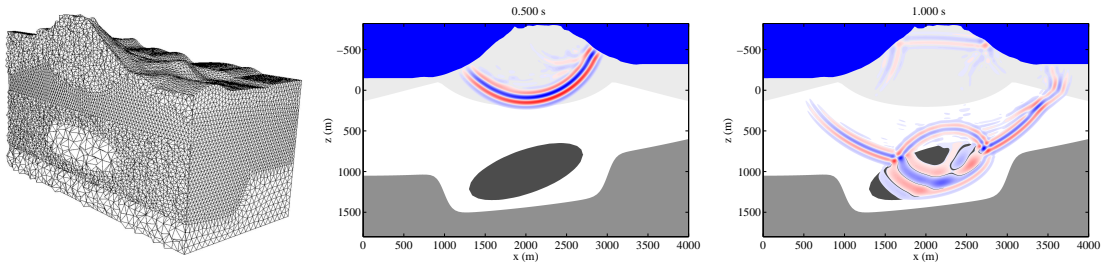


Figure 2: Example of wave propagation in a heterogeneous medium.

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