

Modelling and Control of a Dynamical Labour Market System

An Economic-Engineering Approach

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An Economic-Engineering Approach

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Abstract

Labour market economics has been gaining more and more attention over the years, due to increasing job insecurity. There is a growing demand for the improvement of labour market models. Labour market models are representations of the labour market system used to provide insight to the effects of government policy. Current labour market models have been criticised due to their complexity. Moreover, the models do not provide adequate insight on the relation between the model's assumptions and their outcome.

This thesis applies Economic Engineering to the labour market system. The system is modelled using Economic-Engineering analogs and the government is modelled as a controller. The Economic-Engineering methodology adds understanding to the labour market system itself and the performance of the government's policy.

This thesis presents both a micro- and macro-scale interpretation of the labour market. It is shown how system characteristics determine the system's response to market shocks. Frequency domain analysis is used to connect the dynamical behaviour on both scales to the system's response. This gives increased insight in the labour market's system dynamics.

The government is treated as a controller. A PID- and a robust \mathcal{H}_∞ -controller are designed to provide policy advice. The control objective is to minimise the effects of international disturbances on Dutch unemployment rates. The control input of both controllers is in compliance with current economic theory. It is shown in what sense the PID controller differs from the \mathcal{H}_∞ -controller and under which conditions which controller is advisable. This depends on knowledge on the type of uncertainty and disturbances acting on the system.

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Chapter 1

Introduction

1-1 Labour Market Modelling in the Netherlands

Labour market economics has been one of the key topics in economic debate for decades [2, 3]. Especially in recent years, interest in labour market phenomena has increased. The digital revolution has led people to worry about their job security [4]. At the same time, decreasing job security is a main cause for mental health damage to working people [5]. As a consequence, political parties seek ways to decrease unemployment rates [6] and by doing so create more job security. This political pressure leads to a constant demand for better performing labour market models [7].

Current labour market research in the Netherlands is mainly done by the Dutch Bureau for Economic Policy Analysis (from now on denoted by the Dutch abbreviation CPB). The CPB's macroeconomic models form the basis of most of the Dutch political debate and decision-making. However, critics claim the current models are overly complex and too influential [8, 9]. Especially the complexity of the models is a problem the CPB acknowledges [10]. The CPB is not always aware of which parameter or equation in the model has which effects on the outcome of the model. This lack of insight in the models is considered to be problematic by both critics and the CPB itself.

1-2 Economic Engineering

This thesis presents an Economic-Engineering solution to the problem of this lack of insight. This is done by quantifying economic behaviour using engineering concepts.

The Economic-Engineering approach of this thesis is the product of a new research group at the Delft Center of System and Control. The group seeks to apply engineering methods to economic systems [11]. One of the benefits of using engineering methods is the understanding it gives in complicated economic dynamics.

The analysis of dynamical systems is a common task in the field of Systems and Control Engineering [12]. Many tools have been developed to model and control a variety of mechanical and electrical systems. The Economic-Engineering group adds economic systems to the range of dynamical systems which can be analysed using Systems and Control methods.

This thesis consists of two parts: a modelling part and a control part. The modelling part will focus on the labour market system itself. The modelling is discussed in Chapter 3 and Chapter 4. Chapter 5 will focus on the role of the government as a controller and the implications of policy decisions.

1-3 Modelling

The modelling part consists of two parts on itself: a micro-scale model in Chapter 3 and a macro-scale model in Chapter 4.

Micro-Scale

In Chapter 3, the micro-scale Economic-Engineering labour market model is presented and analysed. This micro-scale model consists of a representative household.

Analogies between physics and labour market economics are given and used to formulate the model in Section 3-2. The results of the model are discussed in Section 3-3.

To get from a micro-scale to a macro-scale model, the aggregation problem needs to be tackled. A theoretic basis to deal with the aggregation problem using techniques that are commonly applied in physics, is given in Section 3-4.

Macro-Scale

In Chapter 4, the macro-scale labour market system is discussed. The macro-scale labour market system is an aggregated version of the micro-scale system. The dynamics are comparable but the system now deals with macroeconomic quantities.

This thesis compares the dynamic behaviour of the macro-scale Economic-Engineering labour market system with the behaviour of the CPB's most important macroeconomic model: SAFFIER II. The CPB uses SAFFIER II to predict labour market behaviour. Market shocks are also used by the CPB to check whether their system is behaving as expected. The Economic-Engineering model's response to market shocks is compared with the response of SAFFIER II.

In the documentation the CPB has published on SAFFIER II, an interesting problem is stated when market shocks are discussed [1]:

“The time it takes the economy in SAFFIER II to reach a new equilibrium, 15-20 years, appears to be too long. This is a point of further research.”

This thesis jumps into this research gap and investigates whether Economic-Engineering tools are a suitable way to deal with these large settling times.

1-4 The Controller

In chapter Chapter 5, the government is viewed as a controller. Policy goals are formulated as control objectives and the control output is interpreted as a policy advise. The control part of this thesis too consists of two parts: a PID-control design and a robust control design procedure.

PID-Control

Section 5-3 presents a PID-controller that minimises international effects on the Dutch unemployment rates. The control output is chosen to be the money spent on allowances. The control output is investigated to check whether the PID-controller's policy advise follows basic labour market laws.

Robust Control

In Section 5-4, uncertainty is introduced. Uncertainty is a common phenomenon in macroeconomic modelling, because of the probabilistic nature of society [13]. Robust control is a control engineer's way of dealing with uncertainty. While robustness is hard to quantify in economics, control engineering offers a clear definition. It is checked whether the PID-controller is already robust. Moreover, a \mathcal{H}_∞ -controller is designed to deal with society's uncertainty. The controllers are compared from the perspective of performance and robustness. The implications of robust policy advise to the performance are discussed.

Chapter 2

Background

2-1 Introduction

This chapter provides the background material that is used in this thesis.

Section 2-2 is a description of the Economic Engineering tradition of which this thesis is a product. It shows which engineering techniques are relevant to the labour market system and points out why these techniques are useful. Section 2-3 gives a description of the economic literature that has been essential in defining the problem statement. It states the knowledge gap and shows how this gap can be filled using Economic Engineering.

2-2 Economic-Engineering Background

Economic Engineering is a relatively new branch of Control Engineering in the Delft Center of Systems and Control (DCSC). Economic Engineering seeks to apply engineering methods to dynamical economic systems. This idea is not new to economics, but has never been done in the way that is proposed by Mendel [11]. Already back in the 70's and 80's, engineering techniques were used to define and analyse economic systems. One of the pioneers in the field was John W. Brewer. However, Brewer chose different analogs than the Economic Engineering group at DCSC. In his 1982 paper [14], Brewer stated that in his model, force is viewed as analogous to price per commodity and flow is defined as the time rate of a flow of orders. Despite the fact that this idea led to mathematical and conceptual difficulties, most of the researchers that use engineering equivalents for economic systems use Brewer's analogs.

Mendel defined flow in a similar manner. He however did not view the price per commodity as a force, but as a momentum. This had implications for the interpretation of the entire economic engineering framework, and is already researched by Mendel and the other MSc-candidates in the Economic Engineering group. This thesis assesses the application of economic engineering in labour market modelling. Therefore in this section an elaborate description on the theories and analogs from economic engineering that are relevant to labour market economics will be given.

2-2-1 Analogs

The Economic Engineering group uses analogs between economics and engineering fields such as mechanics and electronics. These analogs are based on the similarities between physical laws and economic laws. The similarities between different engineering domains are already well-known [15]. In mechatronic systems design for example, the equivalents between mechanical and electrical systems are widely used. The aim of the Economic Engineering group is to add economic analogs to the table of engineering analogs. Two equivalent second-order systems are depicted in Figure 2-1.

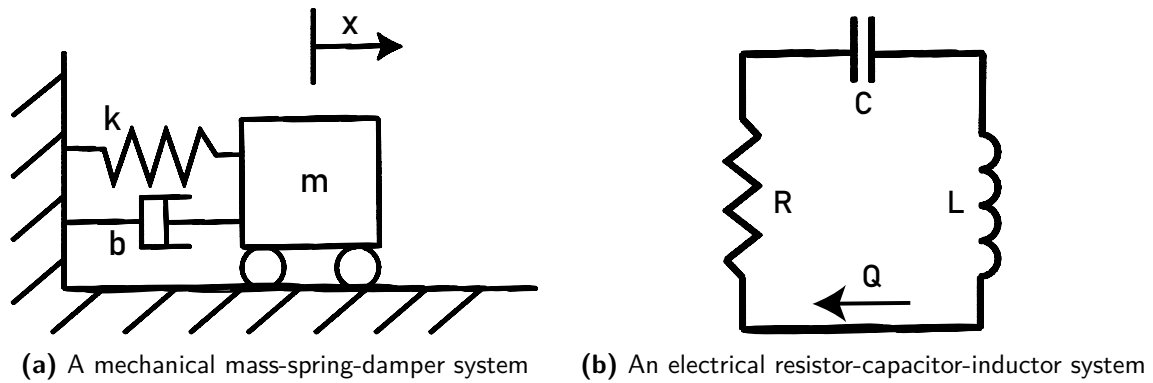


Figure 2-1: Schematic figure of equivalent mechanical and electrical systems

The power and energy variables (in a general sense: effort, flow, momentum, displacement, power and energy) of both systems in Figure 2-1 will behave exactly the same when an effort or flow is applied to the system. The similarity in behaviour is the reason both systems are said to be equivalent. In the Economic-Engineering group it is assumed that economic variables act similar to the power and energy variables of Figure 2-1's systems. In Table 2-1, the power and energy variables in the domains of mechanics and electronics [15] are listed.

Table 2-1: Analogs in different energy domains

General	Mechanics		Electronics	
	Variable	Unit	Variable	Unit
Effort (e)	Force (F)	N	Voltage (U)	V
Flow (f)	Velocity (v)	$\frac{m}{s}$	Current (I)	I
Momentum (p)	Momentum (p)	Ns	Flux linkage (λ)	Vs
Displacement (q)	Displacement (x)	m	Charge (Q)	As
Power (P)	$F(t)v(t)$	$\frac{Nm}{s}$	$U(t)I(t)$	$VA = \frac{Nm}{s}$
Energy (E)	$\int^x F dx, \int^p V dp$	Nm	$\int^Q e dQ, \int^\lambda I d\lambda$	$VAs = Nm$

In his reader [11], Mendel has defined the variables in Table 2-1 from an economic engineering point of view. The analogs are listed in Table 2-2. As can be seen, not all variables are relevant in labour market economics and essential labour market variables are missing. Because of this, this thesis seeks to add labour market related phenomena to the economic engineering framework in order to build a dynamical labour market system based on existing economic

engineering theory. In Table 2-2 the unit # denotes the number of assets. This can be any asset. Apples, pears, houses, oil, etc. It can be a combination of multiple types of assets too.

Table 2-2: Economic Engineering analogs

General	Economics	
	Variable	Unit
Effort (e)	Costs (\dot{p})	$\frac{\$}{\#yr}$
Flow (f)	Flow (\dot{q})	$\frac{\#}{yr}$
Momentum (p)	Price (p)	$\frac{\$}{\#}$
Displacement (q)	Assets (q)	#
Power (P)	Growth (G)	$\frac{\$}{yr^2}$
Energy (E)	Earnings (E)	$\frac{\$}{yr}$

In Mendel's reader, flow is mostly considered to be a flow of assets.

2-2-2 Bond Graphs

A way of modelling energy-domain neutral systems is by using bond graph theory. The book on Systems Dynamics by Karnopp, Margolis and Rosenberg [15] was a useful source on bond graph theory. A bond graph is a graphical representation of a dynamical system, which can be described by, among others, a state space representation, a transfer function, etc. Brewer too used bond graph theory to build his system. The reason bond graph theory is so useful in economic engineering is because of its domain-neutrality. This means that both systems in Figure 2-1 are represented by the same bond graph. This bond graph is shown in Figure 2-2. The foundation of economic engineering is the analogs, so it makes sense to look for domain-neutral system design techniques. Bond graph theory is such a technique.

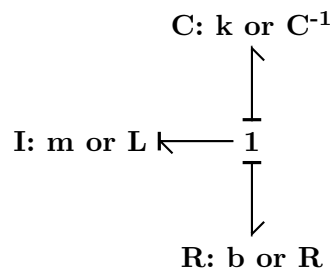


Figure 2-2: The bond graph that represents both systems in Figure 2-1

In bond graph theory, bonds are elements representing two signals at once (as opposed to a line in a block diagram): a flow and an effort (see Table 2-1). These bonds are connected through different elements, which form the configuration of the system as a whole. Three types of elements exist: 1-port elements, 2-port elements and 3-port elements.

A bond is represented as follows: $\text{┌───} \nearrow$. The direction of the half-arrow represents the direction of the power, while the stroke at one side represents the direction of the effort (the causality). The bonds connect the elements.

The first kind of element is the 1-port element. One type of 1-port elements is the source element. Examples are voltage sources and current sources in electronics, or external forces or velocities in mechanics. They feed the system with either a flow or an effort. The other types of 1-port elements have already been depicted in Figure 2-2. In bond graph theory, they are called C-, I- and R-elements and all three have different effects on the system. The relation between the variables and the C-, I- and R-elements is represented in the tetrahedron of state: see Figure 2-3.

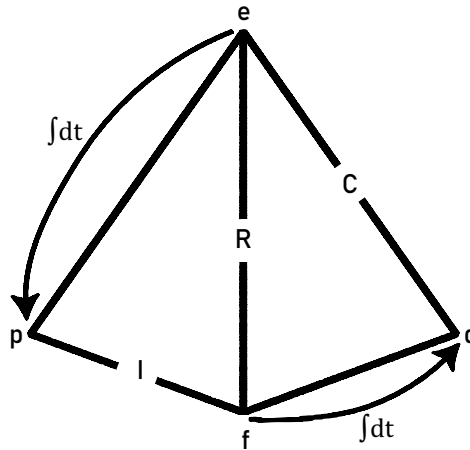


Figure 2-3: The tetrahedron of state

Mathematically, this leads to the relations in Equation (2-1).

$$\begin{aligned}
 C : q &= C \cdot \dot{p} \\
 R : \dot{p} &= R \cdot \dot{q} \\
 I : p &= I \cdot \dot{q}
 \end{aligned}
 \tag{2-1}$$

The meaning of all 1-port elements in mechanics and electronics is given in Table 2-3.

Table 2-3: 1-port elements in mechanics and electronics

	Mechanics	Electronics
C (Capacitor)	Spring (k)	Capacitor (C)
I (Inductor)	Mass (m)	Inductor (L)
R (Dissipation)	Damper (b)	Resistor (R)
S _e (Effort source)	External force (F)	Voltage source (U)
S _f (Flow source)	External displacement (\dot{x})	Current source (I)

The second type of element is the 2-port element. 2-port elements change specific attributes of the effort and flow, as long as power is conserved. They can change the energy domain of subsystems, they can change direction, they can shift from large effort to large flow and vice versa, etc. In Table 2-4 some examples of 2-port elements are given.

Table 2-4: 2-port elements in mechanics and electronics

Elements	Examples
Transformer	Levers, gear pairs, electrical transformers, etc.
Gyrator	Mechanical/electrical gyrators, voice coil transducers

The mathematical relation of a transformer is given in Equation (2-2).

$$\begin{aligned}\dot{p}_1 &= m \cdot \dot{p}_2 \\ \dot{q}_2 &= m \cdot \dot{q}_1\end{aligned}\tag{2-2}$$

Where m is the transformer modulus, which gives the relationship between the signals. The mathematical description of a gyrator is given in Equation (2-3).

$$\begin{aligned}\dot{p}_1 &= r \cdot \dot{q}_2 \\ r \cdot \dot{q}_1 &= \dot{p}_2\end{aligned}\tag{2-3}$$

Where r is the gyrator modulus.

The economic interpretations of the 1- and 2-port elements differ for each type of economic system. In this thesis, the interpretation of both types of elements in the domain of labour market economics is given in Chapter 3 and Chapter 4.

The third group is the 3-port elements. These elements are called the 0- and 1-junctions. These junctions represent the two ways the other elements can be connected to each other. They follow basic physical laws in all energy domains. The relationship between the signals in a 0-junction is given in Equation (2-4).

$$\begin{aligned}\dot{p}_1 &= \dot{p}_2 = \dot{p}_3 \\ \sum \dot{q}_{in} &= \sum \dot{q}_{out}\end{aligned}\tag{2-4}$$

And the relationship between the signals in a 1-junction is given in Equation (2-5).

$$\begin{aligned}\dot{q}_1 &= \dot{q}_2 = \dot{q}_3 \\ \sum \dot{p}_{in} &= \sum \dot{p}_{out}\end{aligned}\tag{2-5}$$

Mendel has given an economic interpretation to the 0- and 1-junctions that is valid in every economic domain [11]. A 0-junction in an economic sense (given the variables in Table 2-2) is a junction where the efforts (costs or benefits) are equal and the flows of goods add up to zero. According to Mendel, this represents a market clearing: the amount of bought goods is equal to the amount of sold goods. The goods are being traded for a certain market price

Table 2-5: 3-port elements in mechanics, electronics and economics

	Mechanics	Electronics	Economics
0-junction	$F_1 = F_2 = F_3, \sum v = 0$	Kirchhoff's 1 st law	Market clearing
1-junction	$v_1 = v_2 = v_3, \sum F = 0$	Kirchhoff's 2 nd law	Budget constraint

(the costs). The 1-junction is an element in which the flows of goods are equal and the efforts (costs and benefits) add up to zero. This is interpreted as a budget constraint: the money coming in is spent (going out) on a certain flow of goods. The two junctions and their meaning in the mechanical, electronic and economic domain are shown in Table 2-5.

Using bond graphs ultimately leads to an economic system representation which can be analysed using a variety of engineering tools. Engineers are able to do time and frequency analysis, use control methods and simulate the entire system in a quick and computationally not so demanding way.

2-2-3 Lagrangian Mechanics

In Section 3-4, the Lagrangian mechanics of the labour market system are researched. Most background on Lagrangian mechanics was derived from Landau [16]. Also, the MSc-thesis of C. Hutters was useful for the economic interpretation of Lagrangian mechanics [17].

In Lagrangian mechanics, the Lagrangian function contains all dynamical information of the system. It is defined by the the kinetic energy minus the potential energy:

$$L(\dot{q}, q) = T(\dot{q}) - V(q) \quad (2-6)$$

Where q is the displacement, \dot{q} is the flow and L is the Lagrangian. Lagrangian mechanics offers a broader formulation of dynamical behaviour than Newtonian mechanics. Newtonian mechanics can be derived from Lagrangian mechanics by introducing the concept of action, which is defined as follows:

$$S = \int_a^b L(\dot{q}, q) dt \quad (2-7)$$

The principle of least action [16] says that any moving body always takes the path that takes the least action, which means the following:

$$\delta S = 0 \quad (2-8)$$

Solving this optimisation problem leads to the famous Euler-Lagrange equation:

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}} = \frac{\partial L}{\partial q} \quad (2-9)$$

Solving this differential equation leads to the equations of motion of the body. In classical mechanics this leads to the Newtonian laws of motion, and as Hutters [17] has shown, it

applies to microeconomic systems as well. In a microeconomic environment, the Lagrangian is analogous to the utility function. In economics this principle is called the principle of maximum utility (in a mathematical sense, maximisation and minimisation are interchangeable by adding a minus sign) and it is one of the most fundamental microeconomic laws [18].

In Section 3-4 the analogy between the relationship between micro- and macroeconomics and quantum and classical mechanics is researched. In his revolutionary work from 1965, Richard Feynman used Lagrangian mechanics to formulate a quantum interpretation of the principle of least action [19]. His solution was the introduction of the path integral. The usage of the path integral in economic engineering systems is handled in Section 3-4. This path integral is Feynman's way of using Lagrangian mechanics to connect quantum and classical mechanics. This thesis will examine whether the path integral can be used to connect micro- and macroeconomics too.

2-2-4 Control

In Chapter 5, two types of controllers are presented. The first one is PID-control. PID-control is one of the most commonly used control methods. A 1989 Japanese study showed that 90% of the controllers used in the process industry was a PID-controller [20]. PID has proven itself to function in a broad range of control problems. PID-control is based on a feedback loop which uses the error between an output and a desired reference signal. This error is multiplied with a proportional gain, an integral gain and a derivative gain and the sum of these signals form the control output. The standard PID-configuration is given in Figure 2-4.

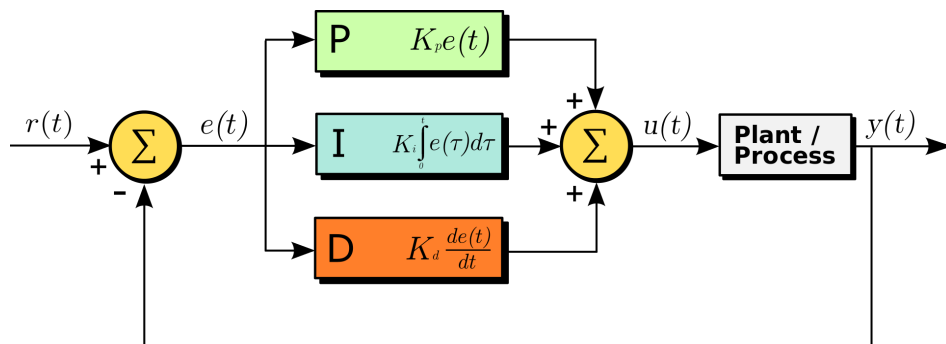


Figure 2-4: A standard PID-configuration.

Source: en.wikipedia.org/wiki/PID_controller

However, when the plant is uncertain, PID-control is not always the best option [21]. Uncertainty is a common phenomenon in the field of economics [13]. In control engineering, the field of robust control is concerned with uncertain behaviour. In this thesis, a robust \mathcal{H}_∞ -controller is designed to deal with uncertainty. From the 1980's on, \mathcal{H}_∞ -control has gained popularity in the field of robust control. \mathcal{H}_∞ -control uses a combination of closed-loop transfer functions. Weight functions need to be selected to get to the desired robust control output. The \mathcal{H}_∞ -controller is based on the concept of minimising the \mathcal{H}_∞ -norm of the combination of these closed-loop transfer functions. The result of this optimisation problem is a robust controller.

2-3 Labour Economic Background

In the Netherlands, the most influential labour market models are developed by the CPB. The CPB's online documentation has been an important source for this thesis and the researchers that built their main labour market model (SAFFIER II) were receptive to answering my questions and pointing out the problems they are facing. Many types of labour market models are used by different institutions. Each of them has upsides and downsides. This thesis compares the macro-scale Economic-Engineering labour market model with the currently used SAFFIER II-model, especially when it comes to SAFFIER II's shortcomings.

2-3-1 SAFFIER II

The CPB calls the SAFFIER II-model their “working horse for making short-term projections, medium-term scenarios and analyses of coalition agreements and policy options” [1]. It is a comprehensive model that includes every macroeconomic variable and coefficient the CPB deems important, in two qualities (a quarterly and yearly version) and three applications (short, medium and long term). A schematic view of the size of SAFFIER II is depicted in Figure 2-5.

The core of the model exists of 25 behavioural equations. These equations are empirically determined production functions, consumption functions and other functions that describe the main economic relations between agents in the model. Apart from that, there are 270 ‘rules of thumb’. These are mostly institutionally determined relationships such as taxes, insurances, allowances and so on. Those equations do not need to be estimated because they are a premise to the system. However, the number of rules of thumb already further complicates the system. The third layer of relationships consist of 1455 so-called ‘identities’. These are definitions and technical relationships which are assumed to be always true (like the labour force = the number of unemployed + the number of employed).

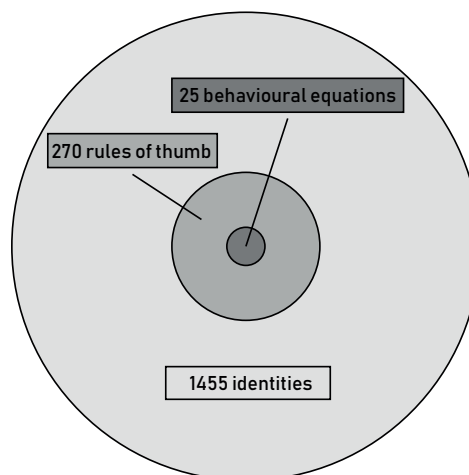


Figure 2-5: SAFFIER II's size [1]

The only equations that require estimation are the 25 behavioural equations. This does not mean the model is simple. In total the model consist of 1750 equations, all of which are

influencing each other. Apart from that, most of these equations are non-linear relationships between many coefficients and variables. The most important relationships are given in Figure 2-6.

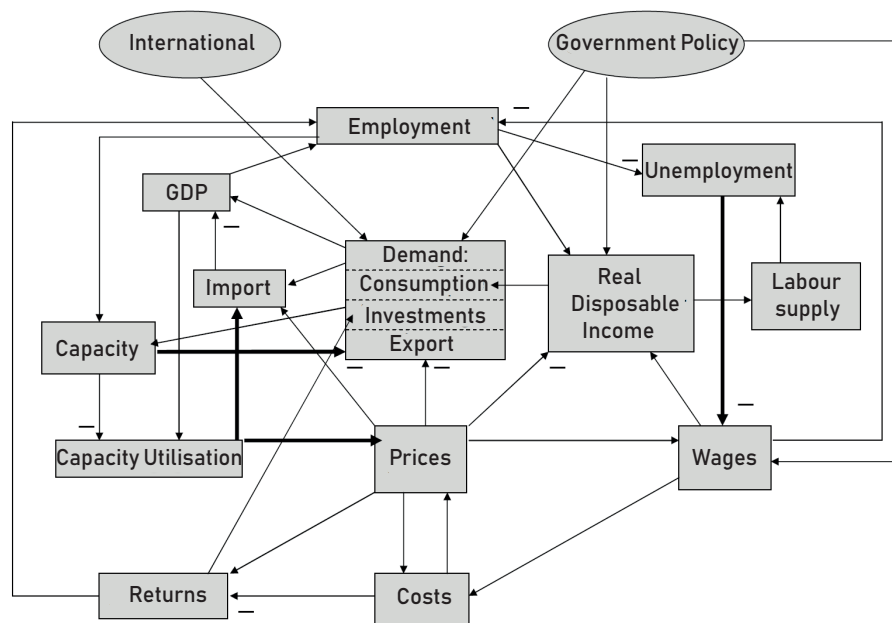


Figure 2-6: SAFFIER II's most important relations [1]

The relations in Figure 2-6 are depicted in a simplified manner in Figure 2-7. International effects are exogenous.

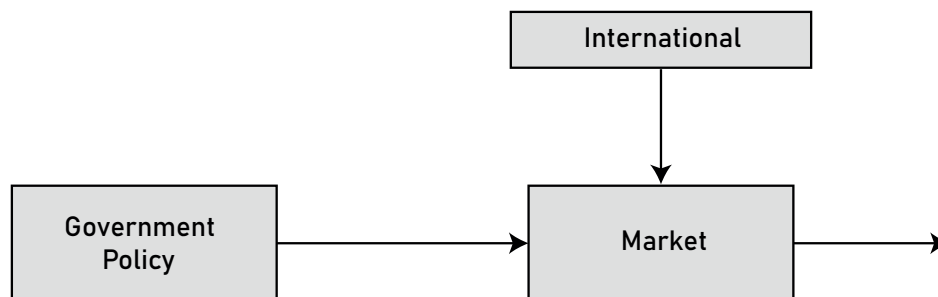


Figure 2-7: SAFFIER II as an open-loop control system

The output of the system is not defined. This depends on the aims of the government policy at stake.

The CPB runs their discrete model for each time step separately to gain knowledge on the behaviour in the time domain. Doing this for one time step is already computationally intensive, let alone over a longer period. This is one of the biggest challenges the CPB is facing. Looking at the response of a simple market shock takes a long time and a lot of computation power.

Another problem is the fact that the response of the system does not easily follow from the system dynamics. When the model shows unexpected behaviour after applying a shock to

the system, it is not clear where it comes from and there is a risk of overfitting [10]. In other words, a simpler system has multiple benefits over the current model.

2-4 Conclusions

This chapter provided the background material that is used throughout this thesis.

Section 2-2 contains the Economic-Engineering background that has been developed in Mendel's group at DCSC. Labour market phenomena are not yet fully included, but the existing Economic-Engineering framework consists of tools that are able to solve some of the problems in current labour market models. Bond graph theory is used to model the labour market system from a micro- and macro-scale perspective. PID- and \mathcal{H}_∞ -control methods are used to model the government as a controller.

The macro-scale labour market system is compared with the CPB's SAFFIER II model. In Section 2-3, an overview of SAFFIER II is given. SAFFIER II is an extensive model that takes many macroeconomic phenomena into account. One of the CPB's challenges is to gain knowledge on the connection between the model's assumptions and its behaviour. This connection has become more difficult to quantify due to the complicated nature of the model. Economic-Engineering tools have proven themselves to be able to gain more understanding to this type of connection.

A Micro-Scale Economic-Engineering Labour Market Model

3-1 Introduction

In this chapter, the micro-scale Economic-Engineering labour market model is presented and analysed. The micro-scale model functions as a building block for Chapter 4's macro-scale Economic-Engineering model, which is compared with the CPB's SAFFIER II. The micro-scale model is analysed using control engineering methods. Frequency analysis and impulse responses increase insight in the micro-scale system.

Section 3-2 explains the elements of which the model consists by pointing out the analogs between physics and microeconomic labour market phenomena. The choice of the analogous elements and variables follows from laws from both physics and labour market economics.

In Section 3-3, the system's outcome is analysed using a technology shock. Economic shocks are a commonly used way to verify economic behaviour [22]. The shock is modelled as an impulse response. In addition, the poles and zeros are analysed. It is shown that the household system is stable by design due to the conservation of energy. An economic interpretation is given for the natural frequency of the micro-scale labour market system.

In Section 3-4, the aggregation problem is discussed. It is shown how the aggregation problem is inevitable when using the micro-scale labour market system as a building block for the macro-scale labour market system. It is shown how quantum mechanics helps to understand the relation between the micro-scale model and the macro-scale one.

3-2 The Model

This section presents the micro-scale Economic-Engineering system model: the representative household.

3-2-1 Microeconomic Analogs

The most fundamental variables in bond graph theory are the power and energy variables [15]. First, these need to be interpreted from the perspective of a representative household.

- **Displacement:** q

The “displacement” represents the most significant aspect of the dynamical system: what is actually moving? In mechanics this is a location and in electronics it is a charge. The question is: what is moving in a household? The answer to this is twofold. Obviously the consumption of the household is an important aspect of the household’s economic activity. On the other labour needs to be addressed. Households are only able to buy stuff if they have performed labour and received wage in return (for now the concept of allowances is neglected). This interaction between consumption and labour is a fundamental assumption in this model. The displacement variable has two distinctive meanings: the amount of goods that are being bought by the household and the **amount of labour** (in men). From a microeconomic perspective this may look trivial but still it makes sense to look at it this way. A labourer has 1 of itself to offer in return for a wage. Later, it is shown how to switch from labour to goods and vice versa in one microeconomic system. For the other variables, the displacement is interpreted as the amount of labour, because the meaning of the variables when displacement represents the amount of goods is already discussed in Section 2-2.

- **Momentum:** p

The units of momentum in Mendel’s economic engineering framework are $\frac{\$}{\#}$, where the dollar is the currency and the # is the amount of goods. For the labour market model euros are chosen as the unit of money to compare results more easily to the SAFFIER II model, which is also measured in euros. The # now represents the number of men. Adam Smith, one of the founding fathers of modern economics, stated in his ‘*Wealth of Nations*’ [23] that the original value of goods came from the value of the labour that was needed to produce the goods. In Marxist theory, moreover, the value of labour is the *only* thing that should create value of goods. Every deviation in the price of a good from the value of the labour needed to produce that good was considered by Marx as the exploitation of the working class [24, part III]. While there is discussion on the moral judgement of whether a deviation in labour value and price is exploitation or not, most economists acknowledge the importance of the value of labour. Momentum in a labour market sense will therefore be chosen to be the **value of labour** ($\frac{\text{€}}{\text{man}}$).

- **Flow:** \dot{q}

The flow is defined as the time derivative of the displacement. In this case, this will be a flow of labour. In a microeconomic sense this is not a straightforward quantity. A way of looking at this is by thinking of an unemployed person. An unemployed person might have the potential to generate value, but this potential is not transferred in actual work. It is comparable with a mass that is not moving. A mass has the potential to contain a lot of kinetic energy, but if there is no momentum to get it moving, it will just stand still. The same goes for the unemployed person. If there are no prices to create an incentive to start working, people will not work and the labour flow will be zero. Another way of looking at it is through the employer. If a company has work that

needs to be done, it might be advisable to hire an employee and get the labour flow starting. This is why the flow of labour is chosen to be the **employment** ($\frac{\text{men}}{\text{wk}}$). The time unit is chosen to be weeks.

- **Effort:** \dot{p}

According to Mendel, the effort has units of $\frac{\$}{\# \text{wk}}$. Combining this with the previous definitions, this means in a labour market sense the units are $\frac{\text{€}}{\text{man} \cdot \text{wk}}$. For a labour market economist, this is a common unit, representing a weekly wage. The labour market analog of effort is **wage** ($\frac{\text{€}}{\text{man} \cdot \text{wk}}$). This wage is the theoretical wage. The actual wage is not easy to model, because it depends on when contracts are being signed, inflation, and more.

- **Energy:** E

The unit of economic energy is $\frac{\$}{\text{wk}}$. It represents the money in circulation. For a household, this can be interpreted as the income. Normally, you can only spend a euro you received. It is also possible to view it from the other way around: economic energy as negative income, so as the expenses. The income can be spent on consumption, taxes, savings, etc. In this thesis, economic energy in a microeconomic sense will be the **income** ($\frac{\text{€}}{\text{wk}}$) of a household.

- **Power:** P

The economic power is the derivative of the economic energy. This is positive if the income of a household increases and negative if it decreases. Most household would want their economic power to be positive. Although unusual, I call this the household's **income growth** ($\frac{\text{€}}{\text{wk}^2}$).

Summarising, the microeconomic analogs are given in Table 3-1. The symbols of the variables are adjusted to economic consensus.

Table 3-1: Analogous variables in the labour domain

General	Representative household	
	Variable	Unit
Displacement	Labour (L)	men
Momentum	Labour value ($\int^t w dt$)	$\frac{\text{€}}{\text{men}}$
Flow	Employment (\dot{L})	$\frac{\text{men}}{\text{wk}}$
Effort	Wage (w)	$\frac{\text{€}}{\text{man} \cdot \text{wk}}$
Energy	Income (I)	$\frac{\text{€}}{\text{wk}}$
Power	Growth (G)	$\frac{\text{€}}{\text{wk}^2}$

In order to build a bond graph representation of a household, the bond graph elements need to be interpreted. Section 2-2 gave the mechanical and electronic meaning of all bond graph elements. The economic interpretations were purposely left out, because they depend on the application. To interpret the meaning of the elements, their definitions [15] are investigated and they are linked accordingly to the economic variables as listed in Table 3-1.

- C (*Capacitor*)

The C-element is determined by

$$L = C \cdot w \quad (3-1)$$

So the C-element determines the relation between the labour and the wage. It makes sense to think of this ratio as the **labour supply**. The labour supply is normally defined as the work (in time units) someone is willing to do for a certain wage. In this case the labour is formulated as the number of men, but the principle is the same. The C-element determines how much a labourer gets in exchange for his or her labour.

- I (*Inertia*)

The I-element is determined by

$$\int^t w dt = I \cdot \dot{L} \quad (3-2)$$

So the I-element determines the relation between the value of labour and the employment. This ratio is related to the market. The more value your labour has, the higher the stakes are you are employed. It is a measure of the demand of your specific brand of labour, which is why the element represent the **labour demand**.

- R (*Dissipation*)

The R-element is determined by

$$w = R \cdot \dot{L} \quad (3-3)$$

The effect of the R-element is the effect of dissipation. A resistor in electrical systems subtracts electrical energy from the system and a damper subtracts mechanical energy from a system. This means the economic R-element should subtract economic energy from the system. In a household, lots of causes exist for a loss of income (economic energy). Examples are taxes, transaction costs, financial gifts, etc. But from a microeconomic perspective, consumption too can be viewed as friction, because consumption is not necessarily used to increase labour or in any other way influence the economic activity of the system other than just spendings. In accordance to the mechanical analog, I will call all these effects **friction**.

- TF (*Transformer*)

One of the roles a transformer can have in a multi-energy domain system is to switch between energy domains. These transformers are called transducers. A labourer's production is measured in the goods domain, instead of the labour domain. This means, a switch in units from labour to goods is necessary. A transformer transducer is able to do so. In this case, the transformer transducer represents the productivity, because productivity gives the ratio between labour and goods [25, p. 503]. It is important to notice that on a microeconomic level, this is a simplification. The assumption is that the goods a household consumes are the same as the goods the labourers in that household produce. Of course this is not the case. But for a representative household this is a safe assumption. When aggregated, the households are connected to all goods and labour markets. I will use the q as the economic output, in this case measured in goods. Then the **productivity** (P) as a transformer is given by:

$$L \cdot P = q \quad (3-4)$$

- 0- and 1-junctions
See Section 2-2.

This leads to the following bond graph elements:

Table 3-2: Analogous elements in the labour domain

General	Representative household
C (Capacitor)	Labour supply (S_L)
I (Inductor)	Labour demand (D_L)
R (Dissipation)	Friction (R)
S_e (Effort source)	Allowances (Al)
TF (Transformer)	Productivity (P)
0-junction	Market clearing
1-junction	Budget constraint

The only thing left is the elements in the goods domain instead of the labour domain. The interpretation of the variables and elements in the goods domain is given in Table 3-3.

Table 3-3: Variables and elements in the goods domain

General	Goods producer			
	Variable	Unit	Elements	Goods producer
Displacement	Goods (q)	#	C (Capacitor)	Stock of goods (S_g)
Momentum	Price (p)	$\frac{\text{€}}{\text{#}}$	I (Inductor)	Goods demand (D_g)
Flow	Goods flow (\dot{q})	$\frac{\text{#}}{\text{week}}$	R (Dissipation)	Friction (R)
Effort	Rent (\dot{p})	$\frac{\text{€}}{\text{#week}}$	0-junction	Market clearing
Energy	Cash flow (E)	$\frac{\text{€}}{\text{week}}$	1-junction	Budget constraint
Power	Growth (G)	$\frac{\text{€}}{\text{week}^2}$		

In Table 3-3, the variables are no longer viewed from the perspective of the household, but from the perspective of the firms that hire labourers to produce goods. Most of this is derived from Mendel's reader [11]. Now the defining 'displacement' variable is the goods q , which are being traded for a certain price p . The effort is no longer a wage, but a rent. The rent is the money a firm pays for keeping its goods in stock. It is, like wage, a periodic payment. All other elements and variables follow from these definitions. The C-element changed because it is no longer a variable that is related to the stock of labour (the labour supply) but the stock of goods. All other elements have similar interpretations as in Table 3-2.

3-2-2 The Bond Graph

The bond graph representing the microeconomic labour market system is given in Figure 3-1. All the elements present in the bond graph are listed in Table 3-4. These elements correspond to the elements in Table 3-2 and Table 3-3. The household consists of two labourers.

Table 3-4: Microeconomic bond graph elements

Element	Meaning
D_{l1}	Labour Demand for labourer 1
S_{l1}	Labour Supply of labourer 1
R_1	Friction of labourer 1
P_1	Productivity of labourer 1
D_{g1}	Goods Demand for goods produced by labourer 1
S_{g1}	Goods Supply of goods produced by labourer 1
R_2	Friction of labourer 1's firm
D_{l2}	Labour Demand for labourer 2
S_{l2}	Labour Supply of labourer 2
R_3	Friction of labourer 2
P_2	Productivity of labourer 2
D_{g2}	Goods Demand for goods produced by labourer 2
S_{g2}	Goods Supply of goods produced by labourer 2
R_4	Friction of labourer 2's firm
R_{extra}	Friction for the entire household

In Figure 3-1, the blue box represents the first labourer in the household with his own labour value, labour demand, friction and productivity. The red box contains his employer. This box represents the goods market that comes with the goods that are produced by employer 1. This goods market consists of the goods in stock, the demand of goods and also friction. This friction might consist of bureaucracy, wasted hours, or other ways in which the cash flow of a company is spent on processes that do not contribute to the production of goods. The labour demand in the blue box (D_{l1}) is actually more of an attribute that belongs to the goods producer and the goods demand (D_{g1}) more to the labourer but for the sake of the clarity the bond graph is visualised like this.

The content of the green box is identical to the blue and red box combined, but represents all aspects (in the labour and goods domain) relevant to the second labourer in this household. The R_{extra} element represents friction that applies to both labourers, like shared consumption and taxes. The location of the R-elements has no impact on the response because the effect is always the same (extracting energy). 5 friction elements are present anyway for conceptual completeness. In state space form, the states are $\mathbf{x} = [L_1 \ q_1 \ L_2 \ q_2 \ \int^t w_1 dt \ p_1 \ \int^t w_2 dt \ p_2]^T$. The system then looks as follows, where R_{extra} is abbreviated to R_x :

$$\dot{\mathbf{x}} = A\mathbf{x} \quad (3-5)$$

With

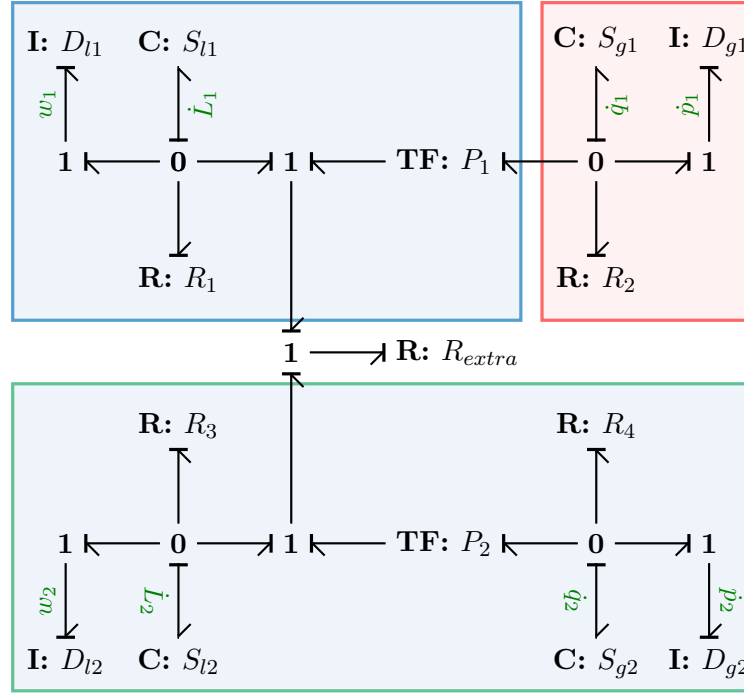


Figure 3-1: Household bond graph with two people working

$$A = \begin{bmatrix} -\frac{1}{R_1 S_{l1}} - \frac{1}{R_x S_{l1}} & -\frac{1}{P_1 S_{g1} R_x} & -\frac{1}{S_{l2} R_x} & -\frac{1}{P_2 S_{g2} R_x} & -\frac{1}{D_{l1}} & 0 & 0 & 0 \\ -\frac{1}{P_1 S_{l1} R_x} & -\frac{1}{R_2 S_{g1}} - \frac{P_1^2 S_{g1} R_x}{1} & -\frac{1}{P_1 S_{l2} R_x} & -\frac{1}{P_1 P_2 S_{g2} R_x} & 0 & -\frac{1}{D_{g1}} & 0 & 0 \\ -\frac{1}{S_{l1} R_x} & -\frac{1}{P_1 S_{g1} R_x} & -\frac{1}{S_{l2} R_3} - \frac{1}{S_{l2} R_x} & -\frac{1}{P_2 S_{g2} R_x} & 0 & 0 & -\frac{1}{D_{l2}} & 0 \\ -\frac{1}{S_{l1} R_x P_2} & -\frac{1}{P_1 P_2 S_{g1} R_x} & -\frac{1}{P_2 S_{l2} R_x} & -\frac{1}{S_{g2} R_4} - \frac{P_2^2 S_{g2} R_x}{1} & 0 & 0 & 0 & -\frac{1}{D_{g2}} \\ \frac{1}{S_{l1}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{S_{g1}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{S_{l2}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{S_{g2}} & 0 & 0 & 0 & 0 \end{bmatrix} \quad (3-6)$$

An example of the derivation of the A-matrix from the bond graph is given in Appendix A. The state space representation of the micro-system is more complex due to 4 extra states when compared with the macro-system of the example in Appendix A, but the rules are the same.

No output is given yet. When analysing the system, possible outputs are further examined.

3-3 Analysis

Market shocks are a common tool for investigating an macro-scale economic model [1, 22]. This tool will also be used in this section to review the micro-scale model. The market shock is modeled using the impulse response of a state-space representation of the micro-scale labour market model.

3-3-1 Economic Shocks as Impulse Responses

An economic shock is defined as an event that occurs outside of the system, and has a certain effect on the variables in the system. In vector autoregression models in economics (VAR-models), economic shocks are modelled by an impulse response [22]. VAR-models have a similar form as state space representations in Systems and Control. This is convenient because it is possible to make assumptions about the model that is discussed in this thesis based on assumptions in VAR-model analysis: in this case the assumption that states it is valid to model economic shocks using impulse responses.

There are multiple economic shocks that might be of interest to a household. In this thesis, technology shocks will be examined because these are common and extensively researched. Moreover, the effects of a technology shock are intuitive. The outputs are chosen to be the wage of one of the labourers and the household's consumption.

The system is modelled as a state-space representation that follows from the bond graph. The state-space representation has the form given in Equation (3-7), where the A-matrix and the state-vector \mathbf{x} are given in Section 3-2.

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t) \\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t) + \mathbf{D}u(t)\end{aligned}\tag{3-7}$$

From now on, I will leave out the (t) -arguments for the sake of brevity. All states, inputs and outputs are time-dependent functions.

The B-matrix is determined by the choice of the input channel. The immediate effect of a technology shock is an increase in the supply of goods, because the productivity of the firm increases. The shock acts on \dot{q}_1 . This means that

$$\mathbf{B} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T\tag{3-8}$$

The outputs are chosen to be the wage of one of the labourers and the household's consumption. The wage of labourer 1 is w_1 and the consumption is the flow that flows through R_x . R_x is a dissipative element that acts in the labour domain. In a strict sense this flow has to be scaled down by a certain ratio to change from the labour domain to the goods domain (consumption is in the goods domain), but for the form of the response this does not matter. Moreover, the value of R is not determined yet, and will depend on this ratio. From the bond graph, it follows that the flow through R_x is:

$$\dot{q}_x = \begin{bmatrix} \frac{1}{S_{l1}R_x} & \frac{1}{P_1S_{g1}R_x} & \frac{1}{S_{l1}R_x} & \frac{1}{P_2S_{g2}R_x} & 0 & 0 & 0 & 0 \end{bmatrix} \mathbf{x}\tag{3-9}$$

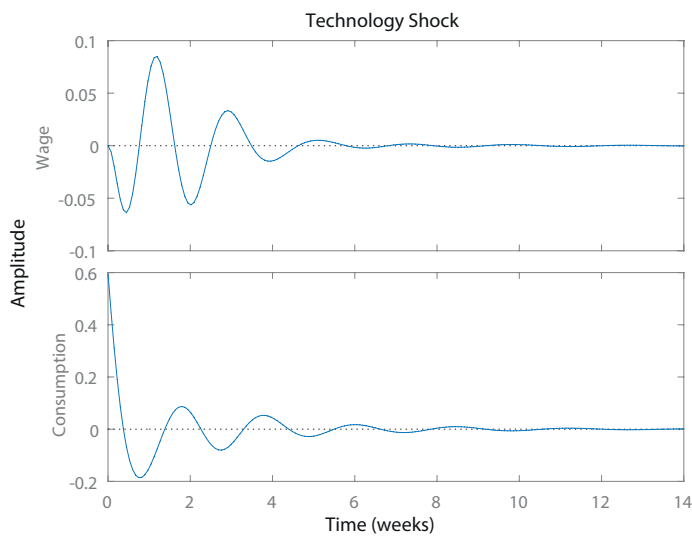
This means that for $\mathbf{y} = \begin{bmatrix} w_1 & \dot{q}_x \end{bmatrix}^T$ (wage and consumption), the C-matrix then is given by:

$$\mathbf{C} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \frac{1}{S_{l1}R_x} & \frac{1}{P_1S_{g1}R_x} & \frac{1}{S_{l1}R_x} & \frac{1}{P_2S_{g2}R_x} & 0 & 0 & 0 & 0 \end{bmatrix}\tag{3-10}$$

Table 3-5: Representative values for the micro-scale elements

Element	Value	
	Configuration 1	Configuration 2
D_{l1}	0.4	0.1
S_{l1}	0.25	0.1
R_1	2	2
P_1	0.8	0.8
D_{g1}	0.3	0.2
S_{g1}	0.35	0.1
R_2	3	3
D_{l2}	0.4	0.4
S_{l2}	0.45	0.45
R_3	4	4
P_2	0.9	0.9
D_{g2}	0.5	0.5
S_{g2}	0.55	0.55
R_4	5	5
R_{extra}	6	6

For a representative set of values for the elements given in Table 3-5 using configuration 1, this state space representation leads to the impulse response in Figure 3-2.

**Figure 3-2:** Impulse response of a technology shock

First, consumption is considered. The technology shock was an impulse on the supply of goods. This immediately results in a peak in the consumption. This is due to the market clearing principle. Goods that are supplied are consumed as well. This is clearly visible in Figure 3-2. The supply of the goods reacts to the change in demand and vice versa, which

eventually leads to the equilibrium. This behaviour makes sense because a market equilibrium is assumed.

The other output is the wage. First the wage of labourer 1 decreases. This can be explained by the fact that his labour has lost a little of its value because of the increase in technology. However, after a short time the wage starts increasing again. The reason for this is the increased goods demand. The economy reacts to the shock and the wage does so too. In the long run, the wage stabilises again.

So the general form of the impulse response can be explained with simple economic phenomena. This means the model is able to deal with economic shocks in an intuitive manner. The exact values however are unknown. It is not known how much the wage and consumption deviate from the steady state, or what the settling time is. This will change from household to household. The important notion is that basic microeconomic principles have been met.

Still it is interesting to evaluate the dynamical effects of the different coefficients from the bond graph in Figure 3-1. The effects are investigated through a pole-zero analysis.

3-3-2 Pole-Zero Analysis

The transfer function is derived from the state-space matrices in Equation (3-7) using Equation (3-11).

$$H(s) = C(sI - A)^{-1}B + D \quad (3-11)$$

Using the state space matrices from the previous section, this results in:

$$\begin{aligned} H_{wage}(s) &= \frac{a_0s^5 + a_1s^4 + a_2s^3 + a_3s^2 + a_4s + a_5}{s^8 + b_0s^7 + b_1s^6 + b_2s^5 + b_3s^4 + b_4s^3 + b_5s^2 + b_6s^2 + b_7s + b_8} \\ H_{consumption}(s) &= \frac{c_0s^7 + c_1s^6 + c_2s^5 + c_3s^4 + c_4s^3 + c_5s^2 + c_6s + c_7}{s^8 + b_0s^7 + b_1s^6 + b_2s^5 + b_3s^4 + b_4s^3 + b_5s^2 + b_6s^2 + b_7s + b_8} \end{aligned} \quad (3-12)$$

Finding the analytical expression of a_i , b_i and c_i in terms of the elements in Figure 3-1 is computationally challenging. All the coefficients in Equation (3-12) will be dependent on the variables in the state space matrices. However, there is no need to do so. The transfer function shows an 8th-order system, which makes sense from the definition of the model in Figure 3-1.

The system is stable for all positive real values of the elements. This follows from the physical laws on which the bond graph is built. The law of conservation of energy tells us that as long as there are damping elements present and there is no external energy source, the system is stable.

In Figure 3-3, the poles and zeros of a set of possible system configurations with positive values are plotted. The fact that all of the poles and zeros are in the left half-plane corresponds with the idea that the system is stable for all values for the elements.

When relating the elements in the bond graph back to their mechanical equivalent, the effects of changing the value of each element becomes clear. In mechanics the oscillatory effects of

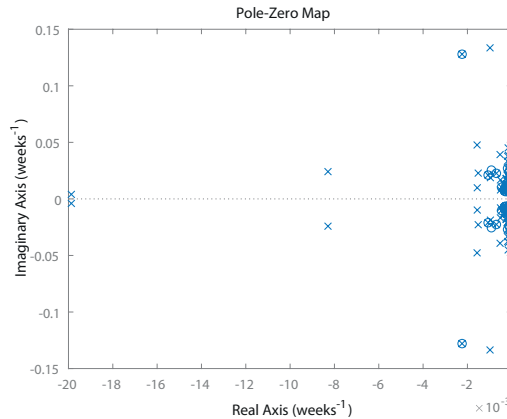


Figure 3-3: Pole-Zero Map of a large set of possible micro-scale system configurations. Poles are x's and zeros o's.

the masses and springs depend on each other. The same is true for an economic system. In mechanical systems this relation between mass and spring is defined by the natural frequency Equation (3-13)

$$\omega_n = \sqrt{\frac{k}{m}} \Leftrightarrow \sqrt{\frac{1}{SD}} \quad (3-13)$$

Where k and m are the spring constant and mass, and S and D are the supply and demand elements. Note that supply is analogous to the inverse of the spring constant. This becomes clear when comparing Equation (3-1) to Hooke's law:

$$F = k \cdot x \quad (3-14)$$

This is not a problem: capacitance is also analogous to the inverse of the spring constant.

The natural frequency is the frequency at which a system oscillates if there is no friction [16]. In the case of the household, this value determines how responsive the household is to changes in the market. This too will change from household to household, depending on the system parameters. Examples of two configurations and their poles and zeros are given in Figure 3-4. Figure 3-4a has element values of configuration 1 in Table 3-5. Figure 3-4b shows the response with the values of configuration 2. Configuration 2 has lower values for the S - and D -elements of the first labourer. This results in a higher 'natural frequency' or responsiveness.

Figure 3-4 shows that the household with the second configuration has faster poles (with a real part at -3), which corresponds with the faster response.

Changing the values of the elements affect the response of the system. However, the system cannot be tuned because no data exists on the microeconomic behaviour of the representative household. Still, the findings are an important basis for the following chapters of this thesis.

The relation between the poles and zeros to the response is a contribution to existing economic literature. Existing models have problems distinguishing which element has which effect and

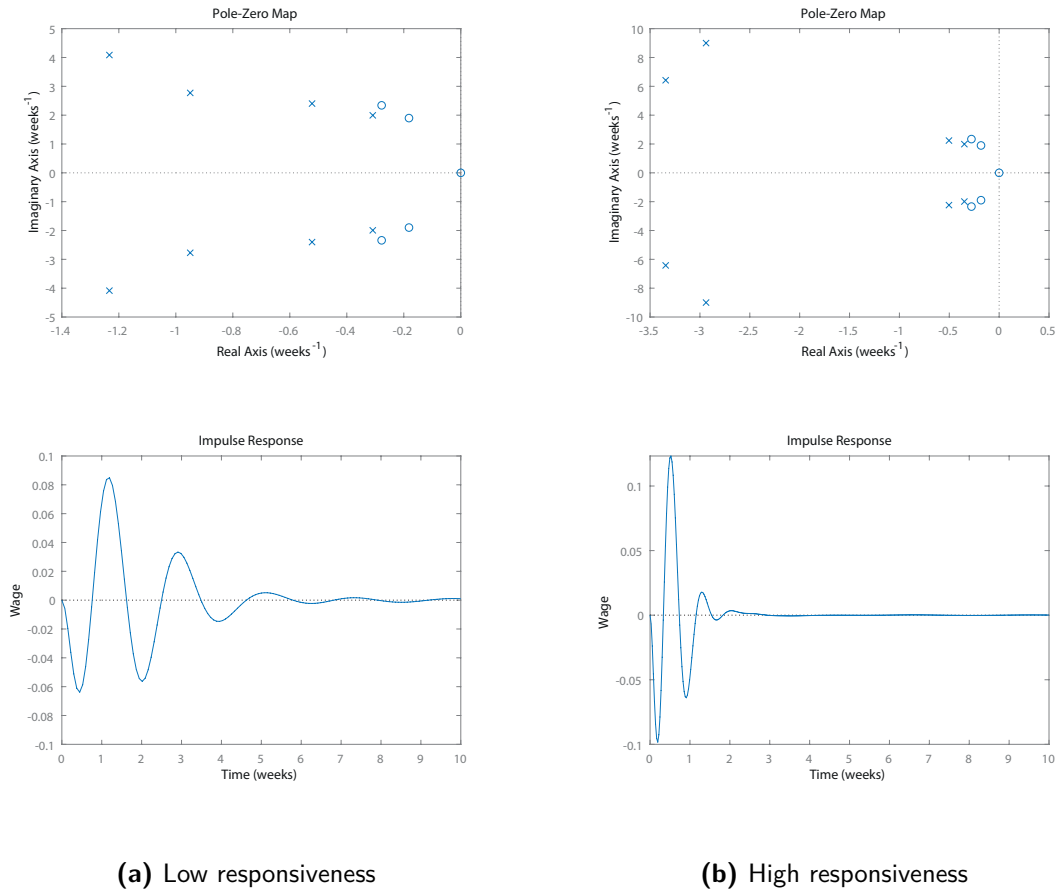


Figure 3-4: Comparing the response to a market shock of two different household configurations. Poles are plotted as x's and zeros as o's.

from knowledge on engineering concepts like masses, dampers and springs it is known how the bond graph elements will effect the poles and zero's and therefore the system response.

3-4 The Aggregation Problem

In this chapter, the micro-scale Economic-Engineering labour market sytem is modelled. The question now rises how this model can serve as a building block to model a macro-scale labour market system. Doing so is not straightforward, due to the aggregation problem. This problem is stated as follows [26].

Aggregation Problem

How does macroeconomic behaviour follow from microeconomic preferences?

This problem is extensively researched in economic literature [26, 13, 27, 25] and relevant to this thesis, because a microeconomic representative household is used to model the labour

market system. In this thesis, a model based on bond graph theory and therefore on physical laws is proposed. As a consequence, physical aggregation techniques can be used as well. In this section, a theoretical quantum approach is presented to deal with the aggregation problem.

3-4-1 The Aggregation Problem in Economics and Physics

One way of solving the aggregation problem is by using the representative household. Multiplying all variables by the number of households in society should lead to a macroeconomic description of the economy. An important argument against this use of the representative household is the fact that societies are not homogeneous [26]. Societies consist of many different types of households. Households with different compositions, backgrounds, job types, consumption behaviour, etc. Using a representative household for modelling the labour market implies that households that differ a lot from the representative household might not be taken into account by the model. If the government would use such a model, it is inevitable some types of households will not be taken into consideration.

The aggregation problem is most fundamentally a simplification problem. It is evident that it is impossible to model all possible households that exist in society, so simplification is inevitable.

Not only is this a challenge in economics. In physics too, simplifications are widely used. Scientists are able to make statements on physical laws on an aggregate level, without having to model the behaviour of all individual particles. This is why it might be interesting to view the relationship between particles and large bodies as analogous to the relationship between micro- and macroeconomics.

One of the fields where this relationship becomes apparent, is in quantum mechanics. Quantum mechanics is based on “ideas of motion that are fundamentally different from those of classical mechanics” [28].

The model that was proposed in Chapter 3 is based on bond graph rules. This means physical laws are obeyed and the labour market system can be viewed as if it were a physical system. Because of this, it is possible to use quantum mechanics to solve the aggregation problem.

3-4-2 Quantum Mechanics vs. Microeconomics

In the beginning of the twentieth century experimental physics was confronted with many examples of the inadequacy of classical physics in explaining certain (quantum) phenomena. The solution to this problem was found in quantum mechanics [19]. The theory of quantum mechanics states that on a quantum scale, the outcome of experiments is fundamentally unpredictable. An example used by physicist Richard Feynman [29] is a glass window that passes through 96% of the light. If you send one photon to the window, will the photon be transmitted or reflected? This is a matter of odds (the odds are 4% that the photon will be reflected).

The analog with microeconomics is that microeconomic behaviour too is a matter of odds. Will a certain household indeed buy a new car if their wage increases with 1%? There is no way to tell, but there is a certain probability that they will.

Feynman used the path integral to describe quantum mechanics. The path integral is based on the principle of least action, as explained in Section 2-2. This principle states that nature always moves objects using the least possible action, where action is defined as the integral of the Lagrangian over time. Dynamical equations can be derived from the principle of least action. Feynman states that for individual particles, the principle of least action is not necessarily true. A particle can take every path to another position, even move to the edge of the universe and come back. However, the odds are small that a particle will do this. In Feynman's theory, the path integral is the integral over all the possible paths a particle can take to move from one place to another. In practice, the contribution of the paths of little action to the odds is considerably higher than the contribution of paths that use a lot of action. This results in the observation that beams of light and bodies of mass (aggregated quantities) follow the principle of least action.

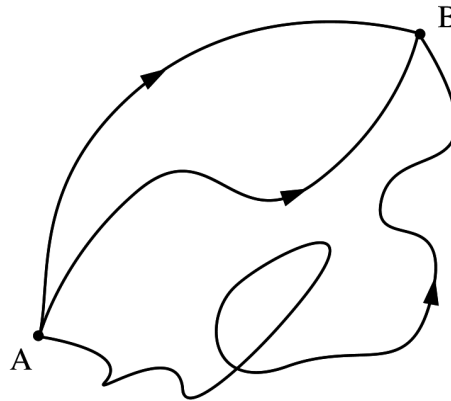


Figure 3-5: Three paths from A to B. The shortest path (the path of least action) is the path that is observed on an aggregate level.

Source: en.wikipedia.org/wiki/Path_integral_formulation

The path integral is a mathematical tool to analytically determine quantum states. The probability amplitude to travel from one state to another is specified by the propagator, which is calculated using the path integral. The formulation of the propagator (K) is given in Equation (3-15).

$$K(b, a) = \int_a^b e^{\frac{i}{\hbar} S[b, a]} \mathcal{D}x(t) \quad (3-15)$$

In which the particle travels from a to b. Here, \hbar denotes Planck's constant, S is the classical action and $\mathcal{D}x(t)$ is the notation of the integral over all the paths.

The economic analog of the propagator is an important notion when solving the aggregation problem using the path integral. The economic analogs of the phenomena in Equation (3-15) are given in Table 3-6.

Table 3-6: Economic-Engineering Analogs in the definition of the propagator $K(b,a)$

Physical meaning (symbol)	Economic analog
Propagator (K)	Function of the odds on how much the utility in a household will change after a certain event
Start- and endpoint path (a,b)	Present and future states
Action (\mathcal{S})	Minus the household's utility integrated over time
Planck's constant (\hbar)	Cent
Path integral ($\mathcal{D}x(t)$)	The integral over all possible changes in utility

K is the functional that specifies the probability amplitude [19]. The probability amplitude represents the odds a household's utility will change with a certain amount. a and b are the points in time in which this will take place. The economic interpretation of classical action is minus the integrated utility, as is shown in R. Smit's MSc thesis from DCSC's Economic-Engineering group [30]. The minus is present because utility in microeconomics is maximised, while action in physics is minimised. Planck's constant represents the smallest quantum of action [31]. The economic analog of action, which is utility, is measured in €'s, as follows from [30]. The smallest portion of money possible is 1 cent. This means the economic analog of Planck's constant is a cent. The path integral then is the integral over all possible changes in utility. This means the propagator K consists of all possible utility changes the household may have. The household may make the craziest of decisions, although unlikely. That path will not contribute as much to the total probability function as the more likely decisions (the ones maximising utility).

All of this means that when more and more households are investigated, the aggregate will behave more and more as expected. This idea corresponds to the idea that when more and more quanta are investigated, the mechanical system starts to behave more and more according to classical mechanics. The path integral is the bridge between the micro-scale labour market system and the macro-scale labour market system.

The only thing that is needed to use the path integral for connecting the representative household to the macro-scale labour market system is the definition of the action, which is defined by the integral of the utility Lagrangian. The utility Lagrangian of an Economic-Engineering system is not easy to determine though, due to the dissipation elements (R). Lagrangian mechanics is not commonly used for dissipative systems. In the Economic-Engineering group, the utility Lagrangian is found by solving the condition in Equation (3-16) [17].

$$-\frac{d}{dt} \frac{\partial L}{\partial \dot{q}} + {}_a\mathcal{D}_t^{\frac{1}{2}} \frac{\partial L}{\partial \check{q}} + \frac{\partial L}{\partial q} = 0 \quad (3-16)$$

Where ${}_a\mathcal{D}_t^{\frac{1}{2}}$ denotes the half time derivative and \check{q} denotes the half time derivative of q . Solving this equation for the 8th-order micro-scale labour market system is mathematically difficult. Applying this Lagrangian to the path integral is even harder. This section gave a theoretical foundation for using the representative household for the macro-scale labour market model. However, this relationship is not used in the final macro-scale model.

For simplification reasons, society is assumed to be homogeneous. This means the representative household is aggregated as if all households were the same. In Chapter 5, uncertainty

is included to compensate for problems concerning the assumption of a homogeneous society.

3-5 Conclusions

In this chapter, the micro-scale labour market model was presented and analysed.

It was found that the model responds to a technology shock in a reasonable manner. It was shown how the poles and zeros of the system affect the form of the response. This gives a direct connection between the structure of the system, the choice of its parameters and its output. This knowledge is a contribution to existing labour market literature.

Furthermore, a theoretical basis was given to connect the micro-scale labour market model to the macro-scale one. Due to the complexity of the mathematics of the model, in practice this relation was not modelled.

The micro-scale labour market model is assumed to act as a representative household in a homogeneous society.

Analysis of the micro-scale system in the time and frequency domain has showed that the system acts reasonable and is a good candidate as a building block for the macro-scale system.

A Macro-Scale Economic-Engineering Labour Market Model

4-1 Introduction

In this chapter, the macro-scale Economic-Engineering labour market system is presented, analysed and compared with the labour market model currently used by the government: SAFFIER II.

Section 4-2 presents the macroeconomic analogs. The macro-scale labour market system is modelled as an aggregate version of the micro-scale system. It is shown how the aggregate system is built up using the same elements as in Chapter 3. The macro-scale model is simpler than the micro-scale version due to the fact that there are no longer multiple agents present in the same system (the system represents all agents in society).

In Section 4-3 the stability is proven and the poles and zeros are analysed. A market shock is applied to the system and the result is compared with the result of a market shock in the CPB's SAFFIER II model. It is shown how sticky wages are viewed in the Economic-Engineering system as inertia. Quantification of stickiness has been a matter of debate in economic models [32] and in this chapter, a quantification approach is proposed using the analog between stickiness and inertia.

In Section 4-4, uncertainty is added to the system's parameters and it is shown how this affects the system behaviour.

4-2 The Model

In this chapter, a macroeconomic model is designed that is able to model certain phenomena the CPB is currently modelling the SAFFIER II model with. These phenomena are mentioned in Section 2-3.

4-2-1 Macroeconomic Analogs

The macroeconomic analogs differ slightly from the microeconomic ones:

- **Displacement:** q

In the microeconomic model, displacement represents labour, measured in the amount of men. As a macroeconomic variable, this variable makes a lot more sense. The amount of working men is a relevant quantity in macroeconomics. The labour force in society consists of two parts: the employed and the unemployed. The displacement variable represents **the amount of employed people** in society (men).

- **Momentum:** p

The microeconomic analog of momentum is the value of labour. The value of labour is a quantity that applies to a certain process. How much money is an employee worth to a firm for a certain process? On an aggregate level, the value of labour for all processes are included. I will interpret it as an average of all processes. This quantity is not measurable because there is no such thing as an average process. However, not all variables need to be measurable. The same goes for momentum. One can do a variety of calculations on a mechanical system without having to measure the momentum (measuring the energy, velocity, force, etc. is enough to accurately model the system). This is also the case for the macroeconomic equivalent. So the momentum will be the **average value of labour** ($\frac{\text{€}}{\text{man}}$).

- **Flow:** \dot{q}

The microeconomic analog of flow is the employment. In the micro-scale model this variable was hard to interpret. The macroeconomic analog is more intuitive, although the unit of men per year is unusual in current macroeconomic literature. The flow is a measure of how much of the defining quantity (in this case labour) is moving around. On a macroeconomic scale this means that the flow will increase if there is more economic activity. The intuition that the flow is directly related to the employment still holds. The employment rate - the measure for employment on a macroeconomic scale - is normally measured as a percentage of the total labour force. In this thesis it is assumed that the labour force is constant. This means that the flow is interpreted as a fraction of a maximum flow. In physics too there is a maximum flow, which is the speed of light. However, normally velocity is not measured as a percentage of the speed of light. The same goes for the employment in this thesis. A maximum employment is assumed and the flow of labour is interpreted as a direct measure for the **employment rate** ($\frac{\text{men}}{\text{yr}}$). On a macroeconomic scale, time is normally measured in quarters or years instead of hours, because most measurements are in terms of quarters or years. In this thesis, the macroeconomic unit of time is years. Years are chosen because the CPB's responses are also measured in years.

- **Effort:** \dot{p}

Microeconomic effort represents wage. The CPB uses wage too as a macroeconomic measure. This wage is interpreted as an average wage, sometimes for different professions, sometimes for society as a whole. In this thesis, macroeconomic wage is interpreted as **the average wage** ($\frac{\text{€}}{\text{man}\cdot\text{yr}}$) in society.

- **Energy: E**

The microeconomic equivalent of energy is the income. The macroeconomic equivalent is well known: the GDP. The gross domestic product is a measure for the money circulating in a society. This not only consists of wages, but also allowances, dividend, gifts, or any other way someone might earn money. Summarising: the macroeconomic energy is **GDP** ($\frac{\text{€}}{\text{yr}}$).

- **Power: P**

The derivative of the energy, the power, is a relevant quantity for policy makers. The change in GDP is normally referred to as the **economic growth**. The growth is important to politicians, because critics tend to judge (at least part of) the policy of a certain administration by its economic growth.

The macroeconomic analogs and their symbols are listed in Table 4-1.

Table 4-1: Macroeconomic labour market analogs

General	Representative household	
	Variable	Unit
Displacement	Labour (L)	men
Momentum	Avg. labour value ($\int^t w dt$)	$\frac{\text{€}}{\text{men}}$
Flow	Employment rate (\dot{L})	$\frac{\text{men}}{\text{yr}}$
Effort	Avg. wage (w)	$\frac{\text{€}}{\text{manyr}}$
Energy	GDP (GDP)	$\frac{\text{€}}{\text{yr}}$
Power	Economic Growth (G)	$\frac{\text{€}}{\text{yr}^2}$

Not a lot changes to the interpretation of the bond graph elements. The important difference is that now, the effects of the elements work on a macro-scale. Still, all the elements and their meaning will shortly be mentioned.

- **C (*Capacitor*)**

In the microeconomic model, the capacitor denotes the **labour supply**. On a macro scale, this relates to a common concept: more unemployed people lead to lower wages and vice versa. This phenomenon is one of the most fundamental ideas of demand and supply in a labour market sense [25, p. 253].

- **I (*Inertia*)**

Something similar is true for inertia. In a microeconomic sense, the inertia element is the demand of your type of labour. In a macroeconomic sense it is the demand of labour in general. During economic recession, people are not buying a lot of stuff, which reflects on the value a labourer has to a firm. This will harm the average wage in society. This effect is caused by the market. So still, the inertia element represents the **labour demand**.

- **R (*Dissipation*)**

The dissipation element has a slightly different interpretation compared to the microeconomic model. Before, all expenses that are flowing out of the household are modelled

as dissipation. The difference in the macro-scale case is that some of the consumption is modelled in the goods demand element. Still, some of the consumption is represented by the R-element. The flow of q in the goods demand element is the part that stays within the economic system, while the consumption of goods that are outside the economic system (for example expenses on foreign goods) is modelled as dissipation. So the dissipation elements denote all the money that is flowing to processes that are not contributing to economic activity in that society. Transaction costs, savings, taxes and investments abroad are such processes. I will still call these elements **friction**. The effects of friction are sometimes called leakage in economics [25, p. 332 – 333]. Leakage can be caused by all types of frictional effects, and is an intuitive term because it denotes the flow of money that leaks out of the system.

- TF (*Transformer*)
The transformer has the same interpretation on a macroeconomic scale and on a microeconomic scale: **labour productivity** [25, p. 503]. Labour productivity relates labour to goods and vice versa. Labour productivity is a common macroeconomic concept.
- 0- and 1-junctions
The junctions are the key link between the microeconomic interpretation and the macroeconomic interpretation of the entire model. The junctions are the result of microeconomic preferences and are assumed to still hold on an aggregate level.
- Se (*Effort source*)
No flow source is used. However, on a macroeconomic scale the government is introduced. In the aggregated system, the government is modelled as a provider of allowances, which are like a ‘free’ wage. An effort source has exactly that effect. It is an external force to the economy that effects the economic activity.

4-2-2 The Bond Graph

In the household that was examined in Chapter 3, two labourers were present. Now the entire labour force is modelled in an aggregated manner. This means that half of the bond graph is redundant and the labour-elements are assumed to be an aggregated quantity that applies to all labourers combined. The new bond graph is depicted in Figure 4-1. The elements are listed in Table 4-2.

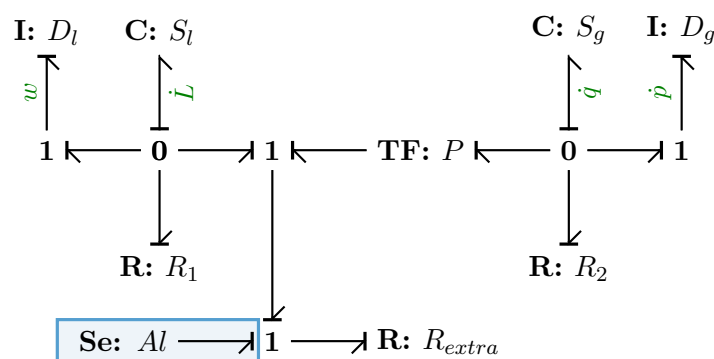


Figure 4-1: Aggregated household with allowances as an effort source element.

Table 4-2: Macroeconomic bond graph elements

Element	Meaning
D_l	Labour Demand
S_l	Labour Supply
R_1	Labour Friction
P	Average Productivity
D_g	Goods Demand
S_g	Goods Supply
R_2	Industry Friction
R_{extra}	Consumption/Taxes
Al	Allowance spendings

Allowances are added as a government policy tool. Allowances are a common tool to deal with unemployment [33] so it is a good candidate for a control input. All the benefits the government gives to households are modelled by the ‘Allowance’ block. The allowances act as an input to the economy, similar to the way the CPB views it: see Section 2-3.

The A-matrix of the system looks as follows:

$$A = \begin{bmatrix} -\frac{1}{R_1 S_l} - \frac{1}{R_x S_l} & -\frac{1}{R_x S_g P} & -\frac{1}{D_l} & 0 \\ -\frac{1}{R_x S_l P} & -\frac{1}{R_x S_g P^2} - \frac{1}{R_2 S_g} & 0 & -\frac{1}{D_g} \\ \frac{1}{S_l} & 0 & 0 & 0 \\ 0 & \frac{1}{S_g} & 0 & 0 \end{bmatrix} \quad (4-1)$$

With the state vector

$$\mathbf{x} = \begin{bmatrix} L & q & \int^t w dt & p \end{bmatrix}^T \quad (4-2)$$

4-3 Analysis

In this section, the macro-scale model is analysed. It is compared with the CPB’s modelled to check whether the responses are comparable. A state-space representation is given using input-output configurations that are also considered in the CPB’s documentation on SAFFIER II.

4-3-1 Stability

The structure of the macro-scale Economic-Engineering labour market model is similar to the micro-scale version. This means that again, due to the damping elements, the system will be stable for all possible positive parameters values. This is shown in Figure 4-2.

In Figure 4-2, poles and zeros of the uncertain macro-scale Economic-Engineering labour market model is given. The values of the elements vary from 0 to 200. No matter what values

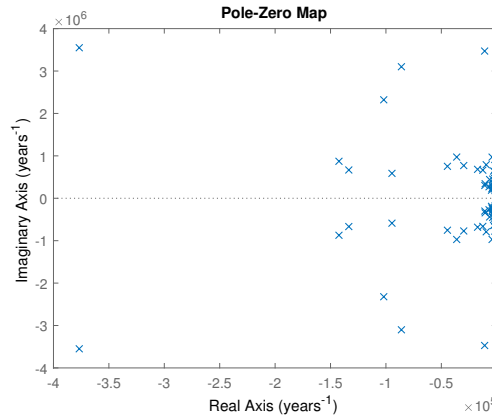


Figure 4-2: The poles and zeros of a large set of systems with elements varying from 0-200.

are chosen, the poles and zeros always lie in the LHP. This indicates that the open-loop system is stable for all element values. This behaviour is equivalent to the behaviour of the damped harmonic oscillator, which is also stable for all spring-, damper- and mass-values.

4-3-2 Wage Shock

In economics it is not possible to apply a shock to the economy and see what happens. If that would be possible, it would be easy to check whether the model is predicting reality realistically. So the CPB is forced to just check whether their response to the market shock makes sense, not so differently from the analysis in Chapter 3. One important shock in particular is addressed. This is the effect of a wage shock to the GDP and the prices. The wage shock is modelled as a step input, because it is permanent. Wage shocks in reality can be caused by increased labour market flexibility or a change in the law on dismissal. In Figure 4-3, the response of the SAFFIER II model is compared with the response of the Macro-Scale Economic Engineering labour market system, using the parameters in Table 4-3 and the state space representation of Equation (4-3). In the state-space representation given in Equation (4-3), the outputs are price and flow of goods. For the GDP response, those two are multiplied with each other.

In Equation (4-3), a step on the wage is the input. This means the wage is chosen to be the input. Allowances are ignored in when looking at this input/output pairing because they do not influence the form of the response of the step on the wages.

$$\begin{aligned}
\dot{\mathbf{x}} &= \underbrace{\begin{bmatrix} -\frac{1}{R_1 S_l} - \frac{1}{R_x S_l} & -\frac{1}{R_x S_g P} & -\frac{1}{D_l} & 0 \\ -\frac{1}{R_x S_l P} & -\frac{1}{R_x S_g P^2} - \frac{1}{R_2 S_g} & 0 & -\frac{1}{D_g} \\ \frac{1}{S_l} & 0 & 0 & 0 \\ 0 & \frac{1}{S_g} & 0 & 0 \end{bmatrix}}_A \mathbf{x} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}}_B u \\
\mathbf{y} &= \underbrace{\begin{bmatrix} -\frac{1}{R_x S_l P} & -\frac{1}{R_x P^2 S_g} - \frac{1}{R_2 S_g} & 0 & -\frac{1}{D_g} \\ 0 & 0 & 0 & 1 \end{bmatrix}}_C \mathbf{x} + \underbrace{\begin{bmatrix} 0 \end{bmatrix}}_D u
\end{aligned} \tag{4-3}$$

Table 4-3: Configuration of the Macro-Scale Economic Engineering labour market system used for comparison with the CPB's SAFFIER II.

Element	Meaning
D_l	1.5
S_l	0.3
R_1	0.8
P	0.8
D_g	0.12
S_g	45
R_2	0.06
R_{extra}	40

The prices are given by p . The response is given in Figure 4-3d. The GDP is the economic equivalent of the energy in the system on the goods part. This means the energy is determined by the product of the flow of goods (\dot{q}) and the prices (p). The response to the wage shock is depicted in Figure 4-3b.

The units of GDP and price by the CPB the percentage deviation from the current position. The amplitude in the Economic-Engineering model can be factorised by any scalar to get to the same result. However, the form of the response is what is to be compared, so the units and the magnitude on the y-axes are left out.

First, the price response of SAFFIER II is compared with the Economic-Engineering price response. According to the CPB's documentation [1, p. 88-90], prices increase due to the increase in wages. In return, the increase in price leads to an even bigger increase in wages. This phenomenon is called the wage-price spiral [34]. However, in Figure 4-3c, the prices are already 0.3% higher than normal at the start of the simulation. This means the CPB's model is not including wage stickiness. In a real economy, prices need time to adjust to the increase of wages and vice versa. This phenomenon is referred to as stickiness or rigidity and introduced by the economist John Maynard Keynes [35]. SAFFIER II is not including this sticky behaviour, but the Economic-Engineering model is. In Section 4-3-3 the Economic-Engineering interpretation of sticky wages will further be discussed.

Another difference between Figure 4-3c and Figure 4-3d, is SAFFIER II's prices dropping more heavily after the peak in 2015 than the Economic-Engineering prices. This is explained in the CPB's documentation by the weakened Dutch price competitiveness. Export will suffer

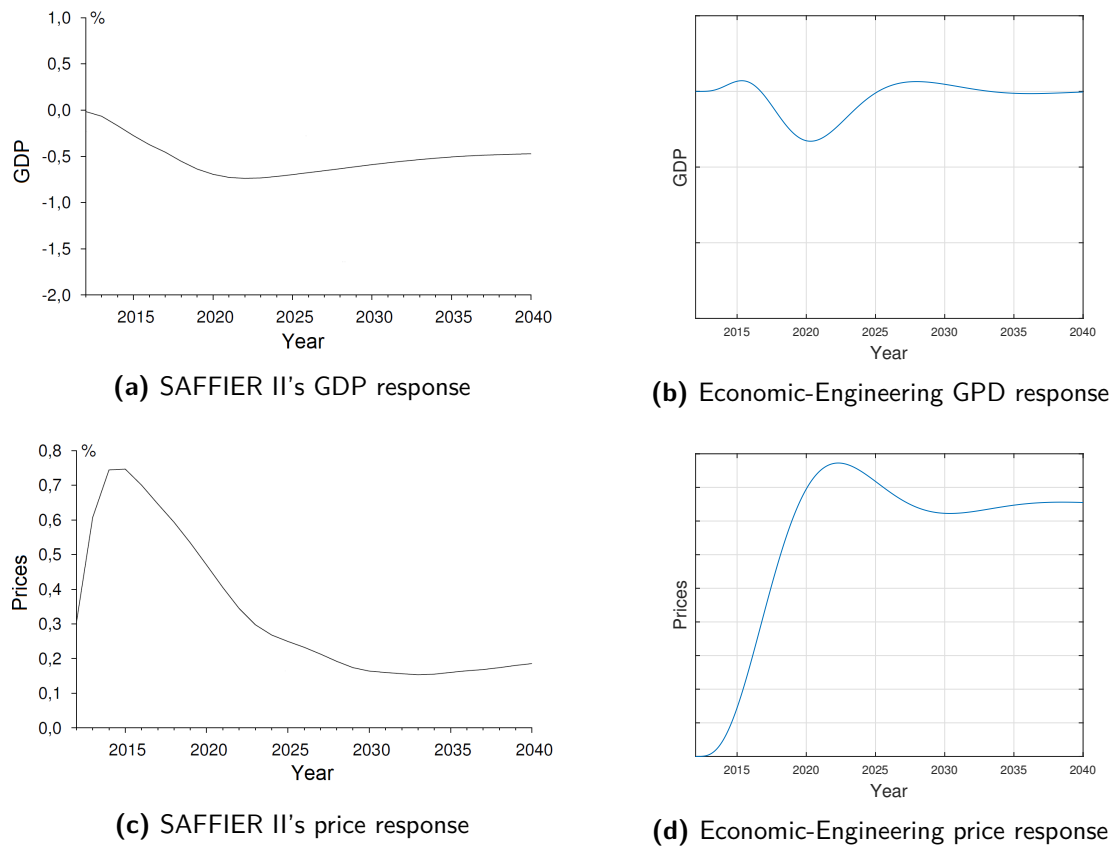


Figure 4-3: Comparison of SAFFIER II's response with the Macro-Scale Economic-Engineering model's response after a **wage shock**

from the high prices in the Dutch economy. Decreasing export on the long term results in more unemployment, which leads to lower prices. However, the macro-scale Economic-Engineering model based on the bond graph in Figure 4-1, is an isolated system. This means export is not included in the model, so the large fall in prices in SAFFIER II's response is absent in the Economic-Engineering's response.

The same behaviour is visible when looking at the GDP. In SAFFIER II's short term, the positive effects of the wage-price spiral are about as much as the negative effects due to the decreased export. On the long term however, the negative effects outweigh the positive ones, eventually leading to a structurally lower GDP. In the Economic-Engineering model there is undershoot caused by the sticky interaction between labour demand and goods demand (see Section 4-3-3). This causes a dip in 2020. On the long term however, the GDP remains the same as before the wage shock, due to the fact that the system is isolated and there is conservation of energy.

In the introduction, one of the problems the CPB mentions themselves was quoted [1]:

“The time it takes the economy in SAFFIER II to reach a new equilibrium, 15-20 years, appears to be too long. This is a point of further research.”

Figure 4-3 shows these settling times of 15-20 years. The CPB solves this problem by giving the following disclaimer:

“This means the effects as depicted in the figures in for example 2040 should not be linked to that year in a literal sense.”

However, in the Economic-Engineering model this problem is easily solvable. By scaling down the I- and C-elements with a factor of three (thereby increasing the system’s natural frequency), the responses in Figure 4-4 were found.

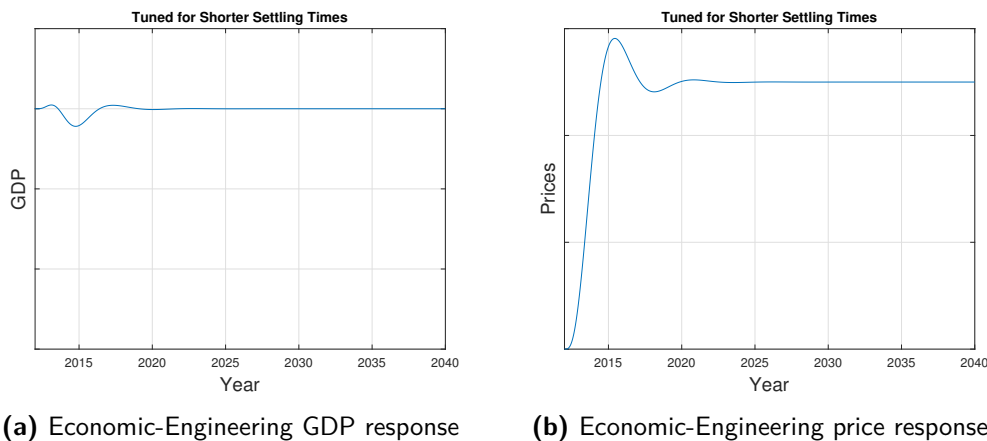


Figure 4-4: The Economic-Engineering responses to a wage shock when tuned for a shorter settling time

Figure 4-4 shows that by adjusting 4 values, the CPB’s problem of decreasing settling times is solved. This shows the relative ease at which the Economic-Engineering system can be tuned is an advantage.

4-3-3 Sticky Wages as Inertia

Sticky wages and prices are a concept introduced by the economist John Maynard Keynes [35]. The idea is that both wages and prices do not change immediately after a market shock. This idea is related to the mechanical concept of inertia. Inertia is the concept that says mechanical bodies are resistant to a change of state. Stickiness has the same meaning in labour market economics.

As is shown in Section 4-3-2, SAFFIER II has trouble modelling sticky wages. The SAFFIER II model is nonlinear and discrete: the states are calculated for every time-step separately. This makes it difficult to tune time-dependent behaviour like stickiness. For a control engineer however, tuning systems in order to change the rise time or settling time is relatively easy.

Using the inertia elements of the bond graph (the demand elements) already leads to common responses in economics. An example is given in Figure 4-3d.

Figure 4-3d shows a common response in control engineering. It is however a perfect example of the effect of sticky prices. A step input is applied to the wage, and on the long run this

leads to higher prices, but this effect is not taking place immediately. It takes time until the prices have a new equilibrium. It is easy to tune the speed at which a new equilibrium is reached. And so is it to determine what the new equilibrium will be.

4-4 Uncertainty

Because the heterogeneity of society was not included in the macroeconomic model, we are faced with a problem. Macroeconomic quantities like GDP and unemployment will not behave like the aggregate of an assumed representative household. So uncertainty needs to be included. The government's task is to function as a robust controller that is able to deal with this uncertainty. An uncertainty of 50% on every element is assumed in both positive and negative direction to show the effect the uncertainty has on the system. The magnitude of the uncertainty in the labour market system is unknown.

The response of a wage shock is investigated again, but now with the beforementioned uncertainty. The uncertain response is given in Figure 4-5.

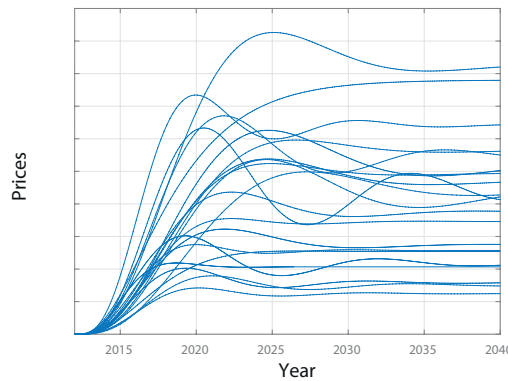


Figure 4-5: The prices after a step on the wage with uncertainty

This gives an idea of how well the economic future can be predicted. The problem is we do not know what will happen, but the government needs to be prepared anyway. This is why a robust controller is an accurate way of modelling government control.

4-5 Conclusions

In this chapter, the macro-scale Economic-Engineering labour market model was presented and analysed.

SAFFIER II's response to a wage shock differs from the Economic-Engineering response. However, the differences are perfectly understandable. SAFFIER II takes international competitiveness into account while the Economic-Engineering model does not. However, the Economic-Engineering model is capable of modelling sticky behaviour more accurately. Furthermore, the Economic-Engineering system's settling times can be tuned to the desired

outcome. For SAFFIER II, decreasing the settling time is currently not possible. In general, the Economic-Engineering model is easier to tune and has a more straightforward relation between assumptions and outcome.

Moreover, it was shown that uncertainty has a large effect on the system's response. In economics, almost all the parameters are uncertain, so the introduction of uncertainty is an important aspect of analysing the labour market system.

Government as a Controller

5-1 Introduction

In this chapter, the labour market system and its relation with the government is viewed as a control problem. The government is adjusting policy to data, which is similar to the role of a controller in a closed-loop configuration. There is increasing interest in applying this adaptivity, sometimes called evidence-based policy making, to unemployment and other labour market problems [36, 37, 38].

Furthermore, the response of the controller gives insight in the effects of policy decisions on the system's output.

Because of these two reasons, the government is modelled as a controller. The control action is interpreted as a policy advice.

In Section 5-2, the control objective is stated. The control objective is to minimise unemployment using government spendings on allowances as control output.

Section 5-3 presents a PID-controller. The PID-controller is used to model government policy behaviour. The control output is analysed and compared with common policy decisions concerning unemployment.

In Section 5-4, an \mathcal{H}_∞ -controller is introduced. \mathcal{H}_∞ -control is a robust control method. The reason this method is chosen is because of the presence of uncertainty in economic systems. It is shown to what extent uncertainty can be dealt with and how robustness and performance relate to each other. An economic interpretation of robustness and performance is given.

5-2 Control Objective

In this section, the control objective is stated. As was mentioned in Section 2-3, the government has many possible goals. Different political parties will have different priorities and different approaches to deal with unwanted market behaviour.

Most political parties fortunately have similar goals considering the labour market: to minimise the unemployment. Although choosing different means, the aim of policy makers is most of the time to decrease unemployment rates. This disagreement on the means of how to do so is exactly what the CPB is for in the Netherlands. Most of the Dutch political parties send their policy decisions to the CPB, after which the CPB publishes which policies are most effective. The party that achieves the lowest unemployment rates in their election promise, will (mostly during campaigns) be proud of the fact that they are the ‘jobs champion’ [6].

Which means a political party chooses to minimise unemployment is a matter of political preference. A party might choose, among others, a fiscal approach or a monetary approach [39].

The effect of increasing or decreasing allowances to unemployment rates is investigated. In the model, allowances are a broad range of government investments in households. They also include unemployment benefits. The effects of unemployment benefits on (the incentive to) work has been extensively researched [33, 37, 40]. This scientific interest indicates that there is political pressure to gain more knowledge on the effects of allowance and benefits on the unemployment. This is the reason allowance spendings are chosen as a control input.

The control objective is to decrease international disturbances on the employment rates. As was shown in Section 2-3, international effects act as a disturbance on the SAFFIER II model. The Economic-Engineering macro-scale labour market system views the international effects in such a way too. The allowances are used to decrease the effects of international disturbances on unemployment rates.

The system’s state space representation is given in Equation (5-1). The A-matrix is identical to the A-matrix in Chapter 4. The derivation of this specific state space representation is given in Appendix A.

$$\begin{aligned} \dot{\mathbf{x}} = & \underbrace{\begin{bmatrix} -\frac{1}{R_1 S_l} - \frac{1}{R_x S_l} & -\frac{1}{R_x S_g P} & -\frac{1}{Dl} & 0 \\ -\frac{1}{R_x S_l P} & -\frac{1}{R_x S_g P^2} - \frac{1}{R_2 S_g} & 0 & -\frac{1}{D_g} \\ \frac{1}{S_l} & 0 & 0 & 0 \\ 0 & \frac{1}{S_g} & 0 & 0 \end{bmatrix}}_A \mathbf{x} + \underbrace{\begin{bmatrix} \frac{1}{R_x} \\ \frac{1}{R_x} \\ 0 \\ 0 \end{bmatrix}}_B u \\ y = & \underbrace{\begin{bmatrix} \frac{1}{R_1 S_l} + \frac{1}{R_x S_l} & \frac{1}{R_x S_g P} & \frac{1}{Dl} & 0 \end{bmatrix}}_C \mathbf{x} + \underbrace{[0]}_D u \end{aligned} \quad (5-1)$$

With

$$\mathbf{x} = \begin{bmatrix} L & q & \int^t w dt & p \end{bmatrix}^T \quad (5-2)$$

It is important to notice that again the system is stable already for all positive values of the elements.

The control system as depicted in Figure 5-1 was used to test the control behaviour.

Figure 2-7 in Section 2-3, which depicted SAFFIER II schematically, had an almost identical form. The difference is that in Figure 5-1, the loop is closed. International effects are assumed

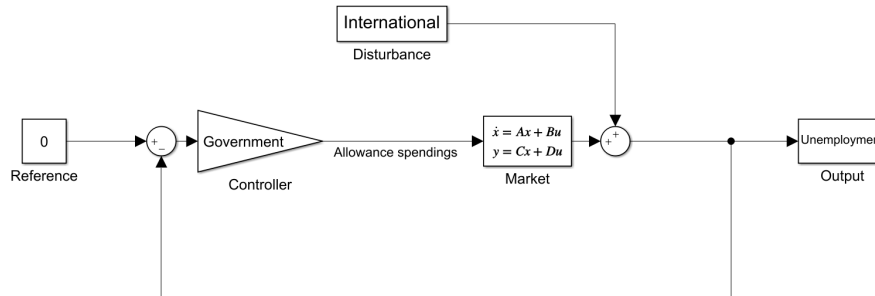


Figure 5-1: The feedback control-loop

to be a disturbance on the output, which in this case is the unemployment rate. The spendings on allowances are the government's tool to decrease the international effects on employment rates. The unemployment rate from now on is expressed as a percentage. We are allowed to do so by simply comparing it to a desired unemployment rate. This is done in economics as well. The unemployment rate is typically quantified as a percentage deviation from the desired situation where the entire labour force is employed.

5-3 Minimising Unemployment using PID-Control

Figure 5-1 depicts a disturbance rejection problem. The PID-controller is one of the most widely used control methods [20], and relatively easy to implement and interpret. The system is already stable which makes it easy to tune: the control is only needed to speed up the disturbance rejection. To interpret the PID-controller from an economic point of view, the units of the signals are considered. In Figure 2-4 in Section 2-2-4, the standard configuration of a PID-controller is given. In Figure 5-2, a detailed visualisation of the PID-controller is given.

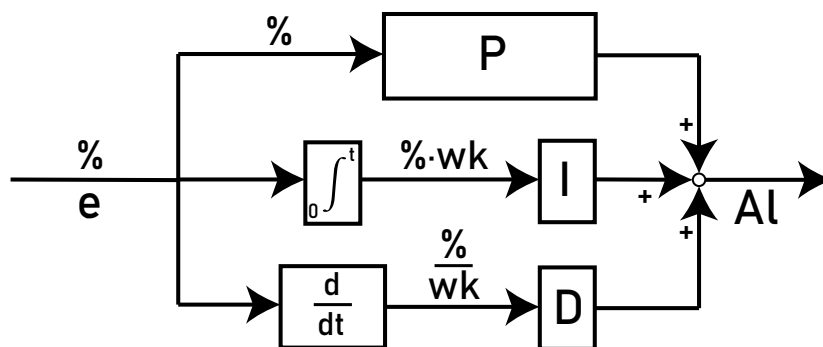


Figure 5-2: Detail of the PID-controller. The control input is the error between desired and actual unemployment and the control action is the allowance spendings

The PID-control consists of three signals that are summed to get to the advised allowance spendings:

- **P-action**

The signal that is multiplied with the proportional gain is the error itself. This means the government is looking at the current unemployment rate, and considers to adjust the allowance spendings according to this rate.

- **I-action**

The second signal is the signal entering the integral (I-) gain. This signal is the integral over the error. This corresponds with the government looking at the past. If for example the unemployment rates are rapidly decreasing in the past weeks, one would expect that behaviour to continue and it might be advisable to keep the allowance spendings at the current level.

- **D-action**

The third signal is going into the derivative (D-) gain. This signal represents an estimation of future behaviour. By looking at the current rate of the error, it is possible to predict what the error will be one time-step in the future. This derivative signal is comparable with the government trying to account for predictions on future unemployment rates.

It follows from the interpretation of the three gains that the government is already basing their policy decisions on the same signals as the PID-controller does. The PID-control is an intuitive way of modelling the government. However, in reality, the government is a discrete controller with a large sampling time. Allowances are not adjusted on a weekly level. This is not a problem, because the controller in this thesis is used to model a policy advice, not actual policy.

Some representative parameters for the bond graph elements are given in Table 5-1.

Table 5-1: Representative values for the bond graph elements

Element	Value
R_1	0.7
R_2	0.7
R_x	10
S_l	0.8
S_g	0.8
D_l	0.8
D_g	0.8
P	0.8

Table 5-2: PID control gains

Gain	Value
K_P	25
K_I	6
K_d	0.5

Using MATLAB's PID-tuner the control gains from Table 5-2 were found. The PID-controller was tuned given that overshoot needs to be minimal and fast response is preferred.

5-3-1 Analysis and Interpretation

Now the response of the system is analysed. The disturbances are assumed to be exogenous, international cyclical effects. This is why they are modelled as a sinusoid. The disturbance

(sinusoidal unemployment), system response (the unemployment rate) and control input (allowance spendings) are shown in Figure 5-3.

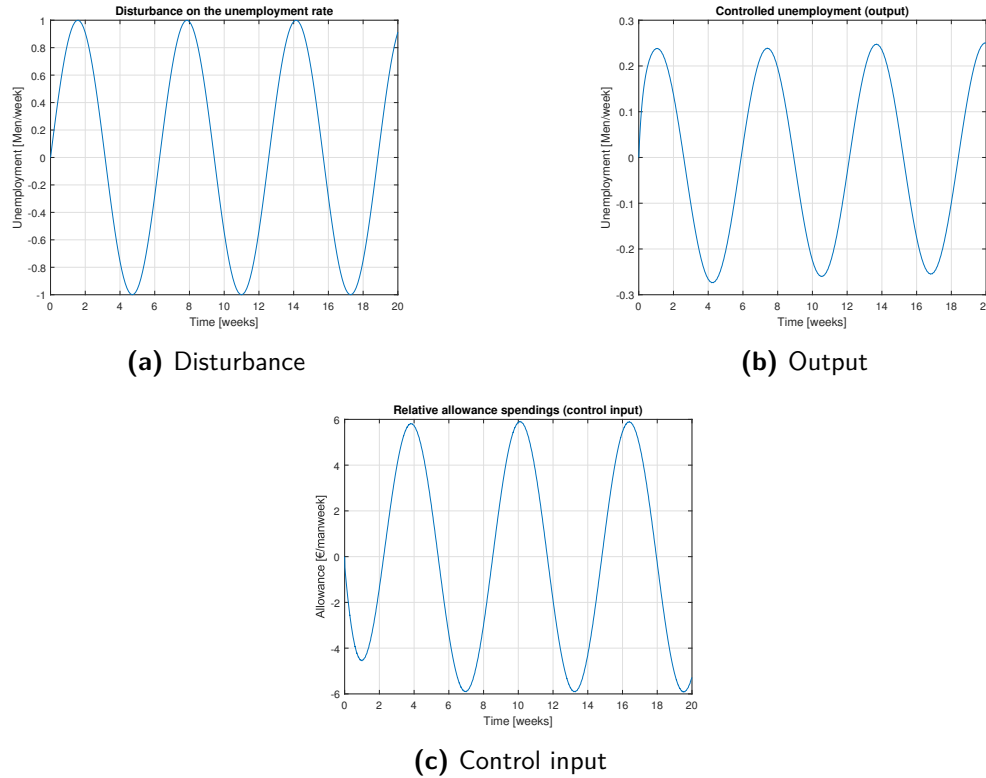


Figure 5-3: Plots of a cyclical disturbance, the output and control input. All values are relative to zero. The values on the y-axes are not related to real data.

In Figure 5-3a, the disturbance on the output is plotted. The disturbance is chosen to be a sinus with a frequency of 1 rad/week ($d = \sin(t)$). It is assumed that the world economy behaves oscillatory on the short term (weeks) and this has a sinusoidal effect on the Dutch unemployment rates.

The control problem is to minimise the external effects on the unemployment rates. In Figure 5-3b, it is shown that the effect of the disturbance has decreased from an amplitude of 1 to an amplitude of 0.25.

The control input is shown in Figure 5-3c. When international effects lead to an increased unemployment rate, it is advisable to decrease the allowances. Decreasing allowances is a stimulus to start working. The inverse is also true. When there is low unemployment, the economy is flourishing and more money can be spent on allowances. Until the point that people are not able to find work because the labour market is satisfied and there is plenty of money to spend on allowances. This cycle of decreasing and increasing allowances and unemployment repeats itself.

The negative relationship between spendings on unemployment benefits (higher allowance spending) and unemployment rates comply with economic research [40].

The government is not likely to decrease and increase their allowances every other week. Still, the response gives an interesting insight in the behaviour of the labour market. How much

the government spends on allowances is dependent on a lot of things that are not included in this model. Still, the direct effect of a change in allowances to the exogenous labour market disturbances is clear.

Including more external effects into the system will lead to more insight in the effects at stake. The aim of this control problem is not to solve unemployment in general, or even to model unemployment in an exact and quantifiable manner. The aim is to give insight in what effects government policy has on labour market phenomena. Those effects are clear from Figure 5-3.

5-4 Robust Control of an Uncertain Labour Market

The field of Robust Control is concerned with the control of uncertain systems. Macroeconomic predictions are to a large extent subject to uncertainty [13]. Many effects and types of behaviour are hard to model and it is important to beware of worst-case scenarios. Policies based on too optimistic predictions lead to unwanted results. This means robust policy design is desirable [41]. Robustness is a term that is already used in theory concerning the analysis of policy decisions. However, in social and economic sciences robustness is not a quantifiable concept. In control engineering there are ways to quantify robustness.

This section will discuss the uncertainty in the macro-scale Economic-Engineering labour market model and present an \mathcal{H}_∞ -controller that is able to give robust policy advice. The nominal values that are used to evaluate the robustness of the system are the same as in Table 5-1.

In Section 5-4-1 and Section 5-4-2, the concept of uncertainty in dynamical control systems is explained. In Section 5-4-3, the economic interpretation is given. Section 5-4-4 explains the procedure of desinging the \mathcal{H}_∞ -controller and the results are presented and analysed in Section 5-4-5.

5-4-1 Defining Uncertainty

Most types of uncertainty can be grouped into two main classes [21]:

1. **Parametric (real) uncertainty**

Here the structure of the model is known, but the parameters are uncertain.

2. **Dynamic (frequency-dependent) uncertainty**

Here the model is incorrect due to missing dynamics (mostly at high frequencies).

In this section, parametric uncertainty will be discussed. The structure that was discussed in Chapter 4 forms the foundation of the model. This structure is assumed to be true, so no dynamic uncertainty is included.

In macroeconomics it is impossible to be certain about model's parameters, as was mentioned in Chapter 4. The exact same model as in Chapter 4 was used, but now all parameters are modelled as uncertain parameters using the MATLAB-command `ureal`. The uncertainty for all parameters is assumed to be 70% in positive and negative direction.

5-4-2 Control Goals: Robustness and Performance

In the field of Robust Control, robustness and performance are quantified by means of four criteria. Those criteria are defined in “Multivariable Feedback Control” by S. Skogestad and I. Postletwaithe as follows [21]:

- **Nominal Stability (NS).** The system is stable with no model uncertainty.
- **Nominal Performance (NP).** The system satisfies the performance specifications with no model uncertainty.
- **Robust Stability (RS).** The system is stable for all perturbed [uncertain] plants about the nominal model up to the worst-case model uncertainty.
- **Robust Performance (RP).** The system satisfies the performance specifications for all perturbed plants about the nominal model up to the worst-case model uncertainty.

The nominal system is analysed in Chapter 4. It was concluded that for all values of the elements, the system is stable. This means there is Nominal and Robust Stability. To check for Nominal Performance, performance specifications are necessary.

The criterion for Nominal Performance as formulated by Skogestad and Postletwaithe [21] is as follows:

$$NP \Leftrightarrow |W_p S| < 1 \quad \forall \omega \quad (5-3)$$

Where the sensitivity function S is defined as:

$$S = (I + GK)^{-1} \quad (5-4)$$

With a plant G (the market) and a controller K (the government).

5-4-3 The Economic Interpretation of Performance

Stability is satisfied for every configuration of the system. This means stability analysis does not add any knowledge to the economic system. The interesting system property is its performance. In control engineering, the engineer formulates certain requirements for the output. These requirements are the foundation of the performance criterion. Examples of such requirements are decisions on the magnitudes of disturbances, a maximum magnitude of the control output or a maximum deviation from the desired system output [21]. The economic interpretation of such a set of requirements is a set of policy objectives. A certain administration might have different priorities than their predecessors. Those priorities will reflect on their policy objectives. From the control engineer’s perspective, this means the new administration has different weighting functions.

For now, nominal performance is assumed for the system with PID-control. This means the results from the previous section are considered to be satisfactory. Using the definition of

Equation (5-3), the weighting function W_p is derived. This function represents the performance requirement based on the performance of the PID-controller. A politician might argue that the disturbances should be rejected more aggressively than the PID-controller does. The weighting function should be shaped accordingly in that case.

W_p is shaped using the Bode Plot of S of the closed-loop system with PID-control. This plot is given in Figure 5-4.

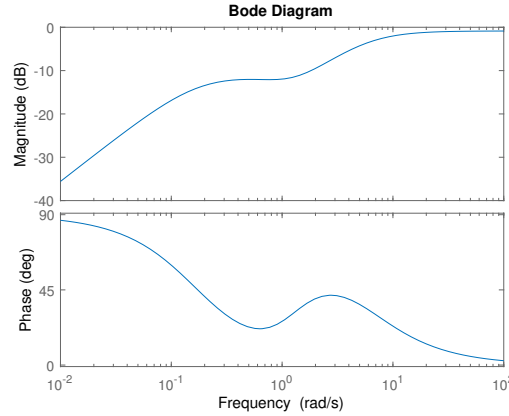


Figure 5-4: The Bode Plot of the sensitivity function of the nominal system with PID-control

The magnitude of the inverse weighting function W_p needs to be above the magnitude in Figure 5-4 for all ω to guarantee NP (Equation (5-3)). The weighting function typically has the following form:

$$W_p = \frac{s/M + \omega_B^*}{s + \omega_B^*A} \quad (5-5)$$

Where ω_B^* is the desired bandwidth, A is a number as close as possible to 0 and $1 \leq M \leq 2$. For a good performance, we want the bandwidth to be as small as possible. The chosen parameters are given in Table 5-3.

Parameter	Value
A	0.01
M	1.1
ω_B^*	0.001

Table 5-3: The parameters of weighting function W_p

Now the inverse of the weighting function is compared with the magnitude of S . The magnitude plots are depicted in Figure 5-5a.

In Figure 5-5a it is shown that the magnitude of the sensitivity function is underneath the magnitude of W_p^{-1} , which means there is Nominal Performance. In Figure 5-5b the same plots are depicted, but now with parametric uncertainty. For some systems, the performance requirement is not met. This means there is no Robust Performance. An \mathcal{H}_∞ -controller is proposed to achieve Robust Performance.

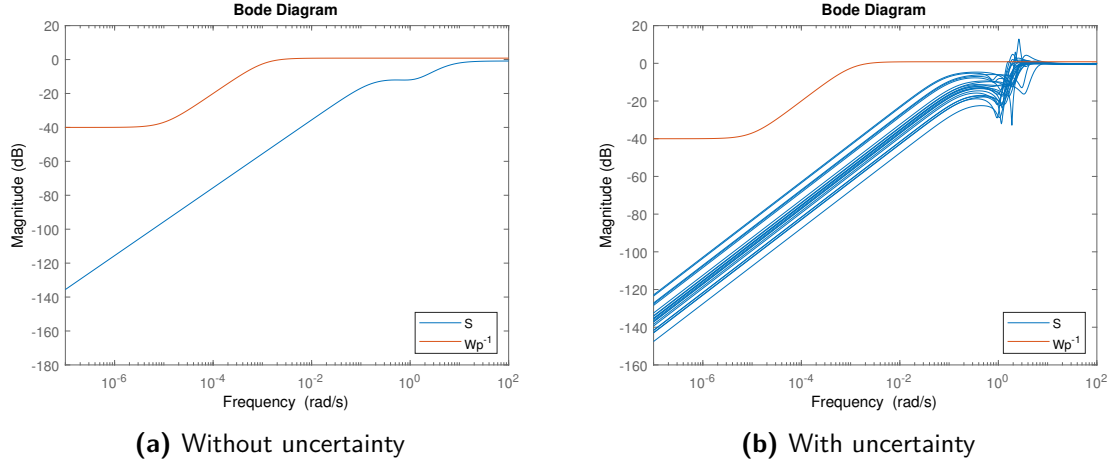


Figure 5-5: Magnitude plots of the sensitivity function and the inverse weighting function

5-4-4 \mathcal{H}_∞ -Control Design

The configuration of the system and controller is the same as in Figure 5-1. A disturbance acts on the uncertain system. This means the system can be written as

$$\begin{aligned} y &= \underbrace{(I + GK)^{-1}}_S d \\ u &= -KSd \end{aligned} \quad (5-6)$$

Where d the disturbance, y is the output and u the plant input. $G(s)$ and $K(s)$ are the market and the government respectively. The argument (s) is left out for the sake of readability.

The combined control objective is to reject the disturbance under a specified parametric uncertainty. These kinds of problems are typically formulated as mixed-sensitivity problems. This means a controller needs to be found that minimises [21]

$$\left\| \begin{bmatrix} W_p S \\ W_u K S \end{bmatrix} \right\|_\infty \quad (5-7)$$

Where S is given in Equation (5-6).

The bounds of the closed-loop transfer functions S and T are defined by the weights W_p and W_u . W_p is given in Equation (5-5) and Table 5-3. W_u is chosen to be equal to 0.2.

Shaping S and KS is needed to achieve both robustness and performance. Also, by shaping KS through W_u , a bound is given for the control output. This is a preliminary from the government's perspective. It is not possible to spend unlimited sums of money on allowances, so it makes sense to bound the control output.

The controller was determined using the MATLAB-command `mixsyn`, using the plant from Equation (5-1) and the weighting functions W_p and W_u that were given in this section.

5-4-5 Analysis and Interpretation

First the Bode Plot of the S of the \mathcal{H}_∞ -controller is investigated. The result is given in Figure 5-6.

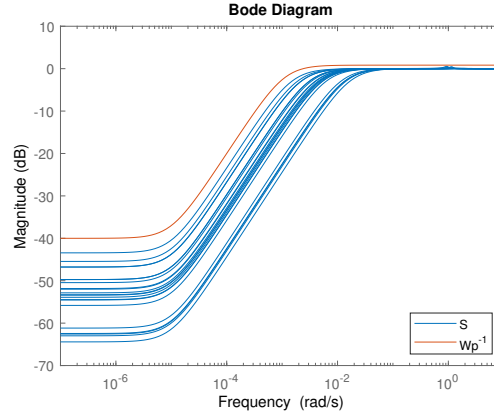


Figure 5-6: The magnitude plot of the sensitivity function of the uncertain system with \mathcal{H}_∞ -control

In Figure 5-6, it is shown that the RP requirement is met. To check performance in the time domain, the same disturbance is considered as in Section 5-3. The results of both PID- and \mathcal{H}_∞ -control are given in Figure 5-7.

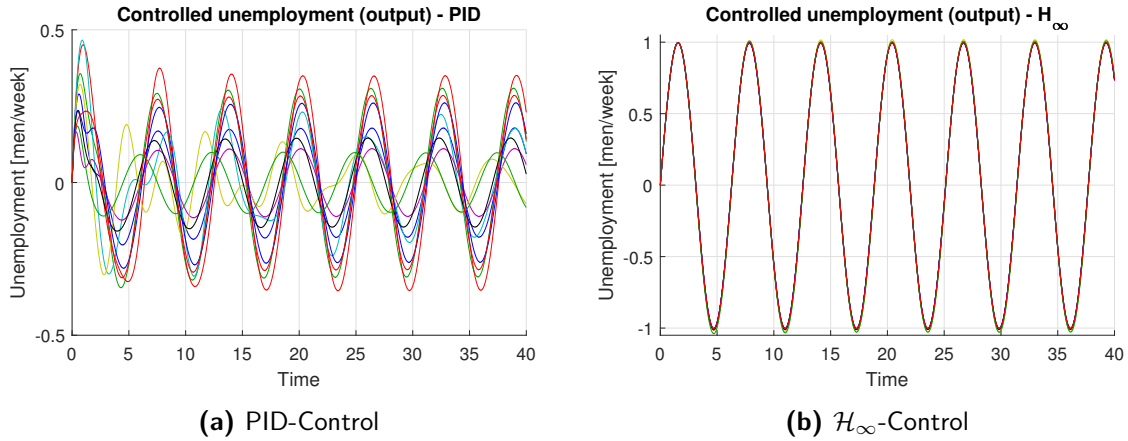


Figure 5-7: Output of the system with both controllers when a sinusoidal disturbance is applied.

The disturbance again has a frequency of 1 rad/week. It follows from Figure 5-4 and Figure 5-6, that at a frequency of 1 rad/week, the \mathcal{H}_∞ -controller behaves way more robust than the PID-controller. However, the gains at 1 rad/week are lower for the PID-controller, which means the PID-controller does a better job rejecting the disturbance. This is clearly visible in Figure 5-7. In this case choosing a \mathcal{H}_∞ - or PID-controller is a clear choice between performance (disturbance rejection) or robustness.

This is not the case for every type of disturbance though. In Figure 5-5b, there is a peak in

$|S|$ at approximately 3.5 rad/week for most uncertain systems. It is interesting to compare both controllers for a disturbance with a frequency of 3.5 rad/week. The result is depicted in Figure 5-8.

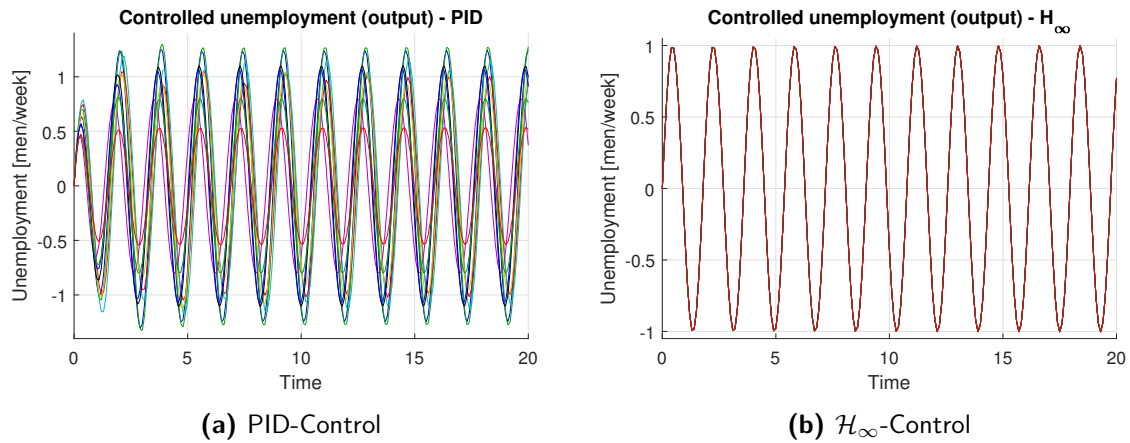


Figure 5-8: Output of the system with both controllers when a sinusoidal disturbance with a frequency of 3.5 rad/week is applied.

In this case, the PID-controller is not only not robust, the performance is also worse than the performance of the H_∞ -controller for some of the systems within the uncertainty set. Choosing between PID- and H_∞ -control depends on knowledge on both the system and the type of disturbance.

Analysis in the frequency domain has proven to be useful. It follows from the Bode Plots at which frequencies the PID-controller might underperform. If the government knows that disturbances at 3.5 rad/week are unlikely, choosing a PID-controller for policy advice is reasonable.

5-5 Conclusions

In this chapter, the government was modelled as a controller.

It was shown that PID-control gives an intuitive interpretation of government policy decision-making. The PID-controller's output provides reasonable policy advice. The PID-controller that is developed in this chapter is able to decrease exogenous effects on the unemployment rates with 75%.

When the system's parameters are uncertain, PID-control is not always performing satisfactory. H_∞ -control is developed to deal with the system's uncertainty. The H_∞ -controller can be used to take care of either robustness or performance or both, depending on the government's priorities.

Chapter 6

Discussion

In this chapter, the results are discussed and it is reviewed how the results should be interpreted.

The Micro-Scale Labour Market Model

The micro-scale labour market model gives a reasonable response to market shocks. The non-existence of the representative household however leads to problems [26]. Because no actual representative household exists, it is fundamentally impossible to ever verify if the model is ‘correct’. No data is available on representative households so the numerical values of the elements that are used are always a guess.

Still, the response to the market shocks showed that the micro-scale system is behaving reasonable. This supports the idea that the micro-system is useful as a building block for the macro-scale labour market system. Despite the fact that no data could be fitted and the micro-scale household has a fictitious nature, the model is still useful. In economic literature, this way of working is common [7, 26, 30, 40].

The Macro-Scale Labour Market Model

The macro-scale system too is not fitted. The CPB uses macroeconomic data to fit their models as well as they can. However, the output of the SAFFIER II model is not public. The CPB does not want to share their data because of risk that political parties might get their hand on it and tune their policy in such a way that their programs always get the best result.

However, verification of the SAFFIER II model is not always based on real data too. Market shocks for example represent a fictitious input to the system. This means it is possible to compare the macro-scale Economic-Engineering labour market model to SAFFIER II to some

extent. In their documentation, the CPB gives some plots and the Economic-Engineering result was quite close to the response of the SAFFIER II model. The differences are understood and explained in Chapter 4.

The macro-scale model too reacted to market shocks in a reasonable manner. Also, the problems the CPB is facing concerning too large settling times were solved using the Economic-Engineering approach.

Control

The role of the government's decisions on allowance spendings was investigated, when trying to minimise international disturbances on unemployment rates.

Two types of controllers were presented with different results. The \mathcal{H}_∞ -controller was more robust, but did not always do the best job considering performance. The PID-controller had difficulty controlling the system at all possible disturbance frequencies and for all (uncertain) configurations. Choosing between the two depends on the knowledge on the system and the disturbance acting on it.

Conclusions and Recommendations

This chapter presents the conclusions based on the findings of the previous chapters and offers recommendations for further research.

7-1 Conclusions

The aim of this thesis was to apply System and Control engineering tools to labour market modelling.

On a micro-scale, it is shown that the Economic-Engineering model is able to correctly model basic household phenomena like consumption and budget considerations. It is shown how micro-economic behaviour can be modelled using bond graph theory. Analogs between micro-economics and physics have been established as a foundation for the model. It is shown how poles and zeros affect the system and how one is able to tune the system by placing the poles differently. The natural frequency of the system is a measure on how responsive a household is to changes in the goods market. Also, it is shown how on a theoretical basis, the path integral can be used to aggregate a micro-scale Economic-Engineering system to a macro-scale.

On a macro-scale, it is shown that the Economic-Engineering system is able to generate similar responses as the CPB's currently used macroeconomic model SAFFIER II. One of the issues with SAFFIER II, too large settling times, was solved by the Economic-Engineering model. Furthermore, it was shown that sticky wages can be quantified by Economic-Engineering tools due to the fact that demand elements are assumed to be analogous to masses. Wage stickiness is interpreted as the interaction between the two I-elements that represent the goods and the labour demand. This follows from the concept of inertia. Inertia, similar to stickiness, is the resistance to a change of state. Quantifying stickiness using the analog with inertia is new in economic literature. Moreover, it is shown that adding uncertainty to the system influences the system's response to market shocks, although the system is stable by design.

(Robust) Performance requirements offer a useful way of evaluating a government's policy decisions. Two controllers with different control outputs were compared: a PID- and an \mathcal{H}_∞ -controller. Both can be used to provide policy advice.

It is shown that the policy advice generated by the PID-controller is reasonable when compared to current beliefs on the effect of the government's allowance spendings on unemployment rates. It is also shown that an \mathcal{H}_∞ -controller can be used to ensure both performance and robustness, whereas the PID-controller does not meet the Robust Performance-criterion. However, robustness and performance are a trade-off. It depends on the priority of the government whether a PID- or a \mathcal{H}_∞ -controller is preferable and how both controllers should be tuned. Quantifying economic robustness and performance by the control engineer's Robust Performance criterion is new in economic modelling.

7-2 Recommendations

This thesis presented a basic labour market model with the government as a controller. There are many ways in which the model or the controller can be improved. One way is by adding more elements to the bond graph. Bond graph theory is capable of linking different subsystems to each other [15]. The financial market is an example of a market that is not included in the system as proposed in this thesis. This makes it hard to get satisfactory results when reviewing for example the decision whether to save or spend money.

Linear models are highly unusual in labour market modelling. The system can be expanded by using non-linear bond graph elements instead of the linear C-, I- and R-elements that were considered in this thesis. More types of dynamical behaviour can be included. Modelling the government as a non-linear controller would be an interesting and more realistic representation of reality, although outside of the scope of this thesis.

In Section 3-4, the aggregation problem was mentioned. A theoretical basis was given to apply the path integral to the micro-scale Economic Engineering labour market model using the utility Lagrangian. It was outside the scope of this thesis to practically do so. In future research, it would be interesting to further examine the analogy of the relation between quantum and classical mechanics to micro- and macroeconomics.

Another addition to the model and its controller is by expanding it to a Multiple Input Multiple Output (MIMO-)system. The government has lots of possible policy goals. Other policy goals might be to maximise wages, or to maximise consumption in order to stimulate economic activity. Furthermore, the government has more ways of controlling the labour market apart from just the allowances. Taxes could be incorporated as a control output, or the height of the minimum wage. Combining all these policies leads to an extensive MIMO-system. Analysing this MIMO-system might show how certain policy goals will influence each other and give insight in the choices a government has when looking at conflicting goals.

Appendix A

Derivation State-Space Representation

In this appendix, the state-space representation is derived from the macro-scale system's bond graph.

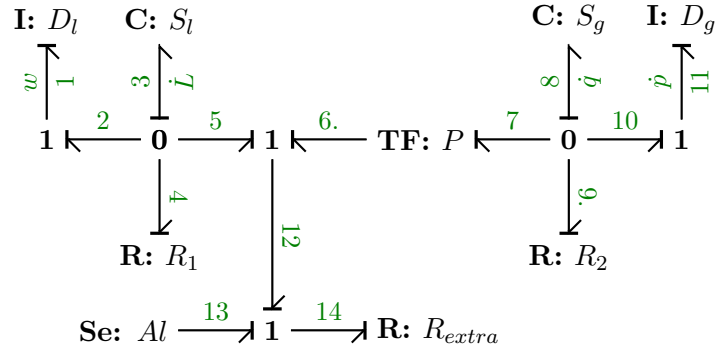


Figure A-1: The macro-scale model. All bonds are numbered.

R_{extra} is abbreviated to R_x . The following identities follow from the 0- and 1-junctions.

$$\begin{aligned}
 w_1 &= w_2 = w_3 = w_4 = w_5 \\
 w_{12} &= w_5 + w_6 \\
 w_{14} &= w_{13} + w_{12} \\
 \dot{L}_1 &= \dot{L}_2 = -\dot{L}_3 - \dot{L}_4 - \dot{L}_5 \\
 \dot{L}_5 &= \dot{L}_6 = \dot{L}_{12} = \dot{L}_{13} = \dot{L}_{14} \\
 \dot{p}_7 &= P \cdot w_6 \\
 P \cdot \dot{q}_7 &= \dot{L}_6 \\
 \dot{p}_{11} &= \dot{p}_{10} = \dot{p}_9 = \dot{p}_8 = \dot{p}_7 \\
 \dot{q}_{11} &= \dot{q}_{10} = -\dot{q}_8 - \dot{q}_9 - \dot{q}_7
 \end{aligned} \tag{A-1}$$

The following relations follow from the definitions of the 1-port elements:

$$\begin{aligned}
\int^t w dt &= D_l \cdot \dot{L}_1 \\
L_3 &= S_l \cdot w_3 \\
w_4 &= R_1 \cdot \dot{L}_4 \\
p_{11} &= D_g \cdot \dot{q}_{11} \\
L_8 &= S_g \cdot w_8 \\
w_9 &= R_2 \cdot \dot{L}_9 \\
w_{13} &= Al = u \\
w_{14} &= R_x \cdot \dot{L}_{14}
\end{aligned} \tag{A-2}$$

The state vector was chosen to be $\mathbf{x} = \begin{bmatrix} L_3 & q_8 & \int^t w_1 dt & p_{11} \end{bmatrix}^T$. To find the A-matrix, the expressions of $\dot{\mathbf{x}} = \begin{bmatrix} \dot{L}_3 & \dot{q}_8 & w_1 & \dot{p}_{11} \end{bmatrix}^T$ need to be found. These were found using the identities in Equation (A-1) and Equation (A-2). This led to the following economic equations of motion:

$$\begin{aligned}
\dot{L}_3 &= -\left(\frac{1}{R_1 S_l} + \frac{1}{R_x S_l}\right) \cdot L_3 - \frac{1}{R_x S_g P} \cdot q_8 - \frac{1}{D_l} \cdot \int^t w_1 dt + \frac{1}{R_x} \cdot Al \\
\dot{q}_8 &= -\frac{1}{R_x S_l P} \cdot L_3 - \left(\frac{1}{R_x S_g P^2} + \frac{1}{R_2 S_g}\right) \cdot q_8 - \frac{1}{D_g} \cdot p_{11} + \frac{1}{R_x} \cdot Al \\
w_1 &= \frac{1}{S_l} \cdot L_3 \\
\dot{p}_{11} &= \frac{1}{S_g} q_8
\end{aligned} \tag{A-3}$$

With the chosen input- and output-configuration of Chapter 5 (unemployment as output), this leads to the state space representation of Equation (A-4). The C-matrix follows from the employment (\dot{L}).

$$\begin{aligned}
\dot{\mathbf{x}} &= \underbrace{\begin{bmatrix} -\frac{1}{R_1 S_l} - \frac{1}{R_x S_l} & -\frac{1}{R_x S_g P} & -\frac{1}{D_l} & 0 \\ -\frac{1}{R_x S_l P} & -\frac{1}{R_x S_g P^2} - \frac{1}{R_2 S_g} & 0 & -\frac{1}{D_g} \\ \frac{1}{S_l} & 0 & 0 & 0 \\ 0 & \frac{1}{S_g} & 0 & 0 \end{bmatrix}}_A \mathbf{x} + \underbrace{\begin{bmatrix} \frac{1}{R_x} \\ \frac{1}{R_x} \\ 0 \\ 0 \end{bmatrix}}_B u \\
y &= \underbrace{\begin{bmatrix} \frac{1}{R_1 S_l} + \frac{1}{R_x S_l} & \frac{1}{R_x S_g P} & \frac{1}{D_l} & 0 \end{bmatrix}}_C \mathbf{x} + \underbrace{[0]}_D u
\end{aligned} \tag{A-4}$$

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Glossary

List of Acronyms

CPB	Centraal Planbureau (English: <i>Netherlands Bureau for Economic Policy Analysis</i>)
SAFFIER II	Update of SAFFIER I, based on the models SAFE and JADE
PID	Proportional-Integral-Derivative Control
DCSC	Delft System for Systems and Control
GDP	Gross Domestic Product
VAR	Vector Autoregression
NS	Nominal Stability
NP	Nominal Performance
RS	Robust Stability
RP	Robust Performance

