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Calvert, Simeon

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Stochastic Macroscopic Analysis and Modelling for Traffic Management

Simeon Craig Calvert

Delft University of Technology

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voorzitter van het College voor Promoties,

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Simeon Craig CALVERT

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Dit proefschrift is goedgekeurd door de promotor:
Prof. dr. ir. S.P. Hoogendoorn

Copromotors:
Dr. ir. H. Taale
Dr. M. Snelder

Samenstelling van de promotiecommissie:

Rector Magnificus	Voorzitter
Prof. dr. ir. S.P. Hoogendoorn	Technische Universiteit Delft, promotor
Dr. ir. H. Taale	Technische Universiteit Delft, copromotor
Dr. M. Snelder	Technische Universiteit Delft, copromotor

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Preface

The opportunity to perform this research wouldn’t have been possible without the support of both sponsors: TNO and TrafficQuest. As a TNO employee I have had the pleasure of undertaking projects with both a strong research content as well as being very practical. The chance to undertake a more theoretical research project in the form of a PhD is extremely appreciated. Also TrafficQuest, a joint TU Delft, Rijkswaterstaat and TNO collaboration, are due many thanks as joint-sponsors of the research. TrafficQuest not only supported the work financially, but also substantially through various encouraging conversations with its members.

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Outside of those who directly influenced my work, were many who supported me in many other ways, not least my room-mates. Bernat, Xavi and Yaqing: you guys travelled (sometimes literally) with me in this journey and we have had a ball in the process. I think we can all agree that sanity is most definitely nowhere to be found in our room, and you would probably say especially behind my desk. Especially Bernat, you have been a friend beyond

what I can give you credit for here, thank you! The Transport & Planning department must be unique for so many reasons, but maybe one reason would be the crazy table tennis battles. There are too many to thank from the department, so let this be a thank you to you all! And finally a special word for Hans van Lint. Hans, you weren't directly involved in my PhD, but you have been a great encouragement throughout many years, even before I graduated for my Masters under your supervision. Thank you for your guidance and support!

Being a part-time PhD student also means that I have colleagues at TNO. Many of you have been a great deal of help with data, thinking about modelling solutions and with other aspects of the research. All have been an encouragement throughout! Special thanks goes to Michiel Minderhoud and Taoufik Bakri for your technical assistance along the way.

And of course I would not have ever started out on this work if I had not have had the support of my darling wife: Els! Your hard work at home has given me the space and allowed me to pursue this research work at the expense of some of your own aspirations. Maybe you understand more than I what it really means to be a team. Ian, Lisa and Abby: Daddy has finally finished playing with the cars in his computer! As I often ask you to, it is now time for me to tidy them up.

It would not do Him justice for me to just thank God, for He is more than a support or encouragement. Life is so much more than all of us see and can comprehend and the cogs that make it work and give birth to us all are outside what we can comprehend. Father, yours is the glory above all and so much more than I can write in words. Completion of a PhD is a big step in the career of an academic, but in the wake of things it found its source in the curiosity and drive You gave me. From an eternal perspective it doesn't mean anything, other than to be part of the path You have set out for me and I have chosen to accept. This path goes beyond this research, my work, my studies and my own effort and intelligence, but all these things are woven into the journey that I am grateful to be on. For those who ask how I know you exist, I can only say that I just know from having You in my heart and knowing Your Spirit within me. This is not something that can be proven by clever arguments or mathematical equations. But for now I can only offer a deep heartfelt thank you for these past five years and all that has happened both in this research and outside of it.

I have learnt many things since starting out with this research, so here is a piece of wisdom I have picked up along the way: Life is not what we make it, it is the way we choose to face the things that come our way. Our achievements should be enjoyed, but never rested upon as a guarantee to future glories. Achievements are by definition completed and in the past, and future achievements are tomorrows past. Real triumphs are only found in grace that will last for eternity.

Simeon Calvert, May 2016

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Chapter 1

Introduction

When congestion becomes a problem on a road or road network, there are generally three main solution areas available to tackle it: construction, pricing or traffic management. For a long time road authorities could reasonably keep up with increasing traffic demand through expansion of the road network. However, this is a finite solution as space and resources are limited. While pricing can often be politically difficult, traffic management became an increasingly preferred option towards the end of the twentieth century as an alternative to construction in many cases. Traffic management proves a more efficient alternative and focusses on influencing traffic flows such that the existing road and network capacity is more effectively utilised resulting in a reduction in congestion.

The effectiveness of traffic management is dependent on the ability to influence traffic flow. As the term suggests, traffic can be considered as a flow, but unlike the flow of fluids, traffic consists of larger individual particles, namely the vehicles, which can be influenced. The particles portray a relatively large amount of stochastic behaviour, which is connected in part to human driving behaviour. The fluctuations that occur in traffic flow due to this stochastic behaviour have a large effect on the effectiveness of traffic management. The investigation of these fluctuations and their relevance to traffic management is the main subject of this thesis.

In this chapter a basis is laid for the thesis, containing an introduction to the research topic, the objectives of the research and the relevance of the research. In section 1.1, the context is given as a backdrop for the research. In section 1.2, the research objectives and questions are stated as well as the scope of the research. The main scientific and practical societal contributions are given in section 1.3, followed by the research approach and an outline of the thesis in sections 1.4 and 1.5 respectively.

1.1 Research context

It has long been known that the average distance travelled by individuals is linked to prosperity. As personal travel budgets increase, the willingness to travel longer distances for work and other motives increases, leading to an increase in travelled distance (Zahavi et al., 1981). At the same time the worldwide population continues to grow, which in turn leads to an increase in potential travellers. The combined effect is a total net increase in the travel demand. As the growth of public transport and other non-car related travel remains relatively low compared to road travel, this means that the majority of the growth in travel is undertaken on roads. The growth in car and truck travel in the United States compared to the Gross Domestic Product (GDP) is shown in Figure 1.1 and demonstrates this principle. Also for the Netherlands such graphs can be constructed, as is shown for recent decades in Figure 1.2.

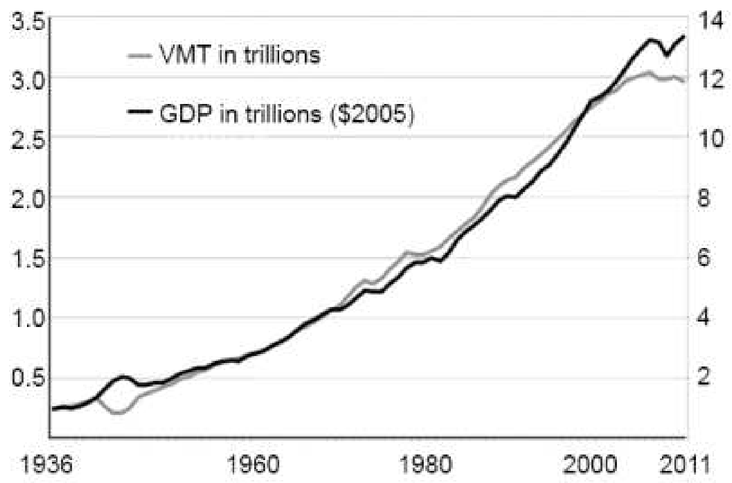


Figure 1.1: Total car and truck Vehicle Miles Travelled (VMT) and Gross Domestic Product (GDP in billions) for 1936-2011 in the US (Ecola and Wachs, 2012)

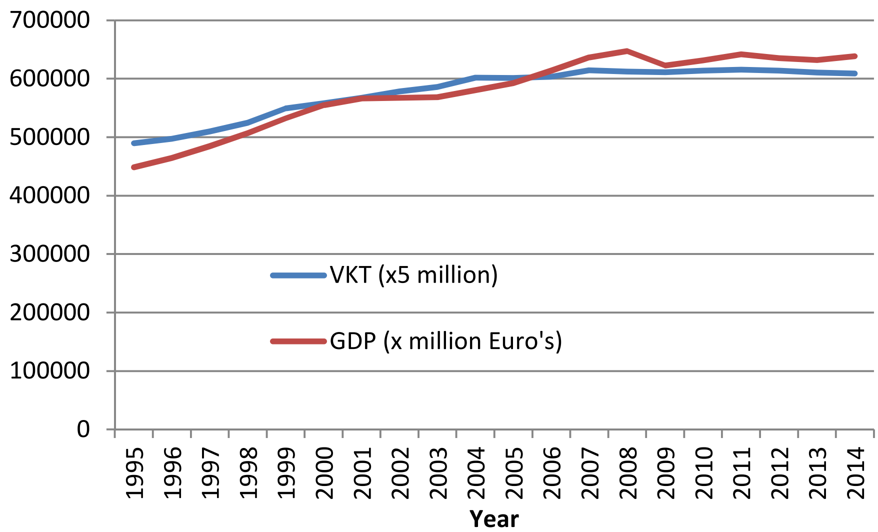


Figure 1.2: Total Vehicle Kilometres Travelled (VKT) by motorized vehicles in the Netherlands between 1995-2014 (data derived from CBS, Statistics Netherlands)

However the growth of road travel has also led to an increase in congestion as the expansion of road networks continuously lag behind the increase in travel demand. A further difficulty in network and road expansion lies in scarcity, especially spatially and financially, but also increasingly due to environmental restrictions. For these reasons the application of traffic management has steadily increased in past decades. Traffic management involves the utilisation of existing road capacity through influencing traffic flows to improve overall network performance. Often network performance will be measured in the extent of delay in a network. Influencing traffic flow can be performed in many ways, but to be effective it must consider the inherent characteristics of traffic.

Traffic flow comprises of the aggregated interaction of all vehicles on a specific section of road. General traffic flow theory has been derived that explains the macroscopic flow of traffic under varying traffic states, from free flow into congested traffic flow. As traffic flow is influenced by individual driver behaviour, this behaviour is also of importance. Human behaviour is typified by stochastic fluctuations on all sorts of levels. This behaviour also enters a drivers' driving behaviour and influences traffic flow. Differences between drivers also introduce further stochastic variations into traffic flow. Therefore traffic exists of different behaviour from different drivers and varied behaviour in time from all drivers, not to mention differences in vehicle capabilities. It has previously been shown that stochastic heterogeneous traffic has the potential to lead to congestion at lower flow rates than the maximum flow and therefore increase total delays (Brilon and Zurlinden, 2003, Elefteriadou et al., 1995). This heterogeneity in traffic therefore also influences the effectiveness of traffic management, as traffic management explicitly aims to influence the flow of traffic. Often the effect of a traffic management measure will only be visible once a measure has been taken, by which time costs have been made and a decision has already been made how a measure is applied. Therefore, it is important that the effect of traffic management can be predicted in advance. Traffic models are often used to a-priori determine the effects of traffic management measures. However, herein lies a problem: most traffic models cannot or do not consider stochastics in traffic flow and its influence on traffic management measures and are therefore not capable to determine the real effects.

In many traffic models, stochastic variations are ignored or assumed to be of limited importance to the outcome of simulations. In many cases reducing the input of a traffic model to average or representative values, rather than considering stochastic variations, can have detrimental effect on the simulation results. It may even lead to biased outcomes in relation to what may be found from empirical data (Calvert et al., 2012, Mahmassani et al., 2012, van Lint et al., 2012). An increase in this realisation has occurred in the past decades and has led to some pioneering research in this area (Brilon and Zurlinden, 2003, Elefteriadou and Lertworawanich, 2003). Traffic models that consider certain stochastic elements of traffic flow have also been developed. It is argued that the stochasticity in traffic cannot be reduced to a single representative value prior to traffic flow simulation. Results of simulations also cannot be expected to give the same outcome with stochastic input compared to a reduced representation of the input as a representative value. Instability in traffic, including network effects in congestion, lead to a non-linear propagation of stochastic variation, especially for the more extreme cases. In turn greater traffic flows and congestion will lead to higher values for travel times and delays than can be derived from averaged or representative input values (Calvert et al., 2012). It is therefore imperative to explicitly consider stochastic variation in traffic flow modelling, when this variation is present in the considered scenarios and networks. Some traffic models have been developed in recent years to address this problem. However, most models have limitations when considering stochastic behaviour for traffic management applications. This is discussed later in this thesis in Chapter 2.

To address the issue of the influence of stochastic variation in traffic flow for traffic management applications, a greater understanding is required on the influence that traffic flow stochastics have on the effectiveness of traffic management. Application of these insights in various modelling applications, allowing the effects of traffic measures to be a-priori determined, is also required to allow measures to be effectively evaluated and designed for optimal application on road networks.

1.2 Research objectives and scope

1.2.1 Research objectives

The main objective of the research presented in this thesis is to give insight into the stochastic fluctuations and uncertainty in traffic flow for the application of traffic management measures and to propose tools that allow these effects to be analysed and subsequently modelled. Stochastic processes are considered as *uncertainty*, which describes day-to-day uncertainties *between* traffic flows, and *fluctuations*, which describe microscopic variability *in* the traffic flow. This main objective is broken down into three sub-objectives, which focus on:

a) The analysis of uncertainty and fluctuations in traffic

b) Modelling uncertainty and traffic fluctuations

c) The development of visual aids for effective communication of uncertainty in traffic

The research questions addressed in this thesis are derived from the sub-objectives. The first questions refer to the analysis of uncertainty and are formulated as:

1. *Which variables have a substantial influence on stochasticity in traffic flow?*

2. *How can the distributions of the stochastic variables in traffic flow be quantified?*

There are a large number of variables that influence variability in traffic flow on various levels. On the highest level these influences can be considered on the level of influence on traffic demand and capacity. At the lowest level the influence is on individual driving behaviour. The first two research questions aim to summarise which variables have the greatest influence on traffic flow and the stochasticity thereof. Methodologies are sought that allow the uncertainty of variables to be quantified in the form of probability distributions. Key variables should be demonstrated in these methodologies for which distributions are derived as generic distributions that may be applied in a scenario based macroscopic modelling approach.

The following questions consider modelling of uncertainty and of stochastic fluctuations in traffic and are formulated as:

3. *What are currently the main issues for modelling stochastics in traffic flow?*
4. *How can uncertainty scenarios in traffic be modelled effectively?*
5. *How can stochastic fluctuations in traffic flow be modelled macroscopically?*

Stochastics in traffic flow are separated into uncertainty of traffic conditions, such as on a day-to-day scenario level, described here as macroscopic stochastics, and into stochastic fluctuations in traffic flow, such as between vehicles, and is described here as microscopic stochastics. Research questions 3 and 4 consider the main issues that exist when modelling uncertainty in traffic and consider possibilities to improve scenario-based uncertainty modelling in macroscopic models. Research question 5 addresses the question of modelling microscopic stochastic fluctuations in macroscopic traffic flow, in which vehicle interactions are present. Analysis and modelling of stochastics in traffic is however useless if one is not able to adequately visualise and communicate the outcomes. Therefore the final research question is formulated as:

6. *What are effective options to visualise and communicate uncertainty from probabilistic traffic models?*

This question addresses the main difficulties in visualising and communicating uncertainty results from stochastic traffic models. This considers options for effective visualisation and the cognitive processing of different visual cues and a person's ability to process these cues to effectively make use of probabilistic model results.

1.2.2 Research scope

This thesis presents research on the stochastic effects on traffic flow for the analysis and modelling of traffic flow for traffic management applications. Although many of the analyses, modelling techniques and considerations presented in the thesis may be more widely applicable, the main considerations remain for their application in traffic management. A main difference with non-traffic management applications lies in the way that the application of traffic management affects traffic flow. Often traffic management is applied to influence traffic flow under extreme conditions and may often specifically target the extent of homogeneity in traffic flow. Both extreme conditions and homogeneity are directly predisposed to the effects of uncertainty and stochastic fluctuations in traffic flow. Therefore, most cases and examples used in the thesis also relate to the application of traffic management.

While microscopic models are often applied to analyse local effects of traffic management, macroscopic models are far more effective in considering the network effects. However, much less is known about the modelling capabilities and possibilities of macroscopic modelling for traffic management applications on a wider network scale. In this research, the focus is exclusively on macroscopic modelling with an exclusive consideration of unidirectional uninterrupted flow for motorway traffic. Although traffic management may be applicable for urban networks, its application is often different to that of motorway network applications. Furthermore, only single class traffic flows are considered. This is by choice to limit the span of the research and retain focus, while it is recognised that multi-class consideration of traffic flow is relevant for the considered subject and application.

The term stochastic is extensively used in traffic modelling to describe different aspects of models. However, in many cases it does not refer explicitly to traffic flow itself, but rather to certain aspects that affect traffic flow, such as route choice method, equilibrium conditions or choice of scenario. In this research, stochastic refers directly to traffic flow influencing random factors. Two different levels of stochastic influence are considered: macroscopic and microscopic stochastics. Macroscopic stochastics are defined as uncertainties in a traffic system and can be viewed as day-to-day or time-to-time scenarios. An example of this is the uncertainty in traffic demand on a network for a specific day. Microscopic stochastics are defined as stochastic fluctuations in time dependant traffic flow, often due to instantaneous behaviour. The fluctuations in time-headway between two vehicles are an example of microscopic stochastics in traffic flow.

Application for traffic management purposes obviously implies a practical implementation of measures and therefore the use of analysis techniques and models is considered likewise. The presented and developed models are explicitly considered with practical applications in mind. While many approaches and models exist that may be more elaborate and may produce better results, many of these are constructed purely theoretically and have drawbacks when it comes to application in practice. Therefore, only approaches that can easily be applied by practitioners are considered and demonstrated. A number of the presented and developed models make use of parameters for calibration and fine-tuning. It is not in the scope of this thesis to give refined parameters settings for each application of these methods. The models are demonstrated using applicable parameter settings for the considered cases, without detailed analysis of the considered parameter settings. The objective of the research is focussed rather on methodological approaches for practice in relation to traffic management.

1.3 Thesis contributions

1.3.1 Scientific contributions

Science is broadly defined as: “The observation, identification, description, experimental investigation, and theoretical explanation of phenomena” (American Heritage, 2011). The main scientific contribution of this thesis is the advancement in the understanding of the role of uncertainty and stochastic fluctuations in traffic flow, especially in relation to traffic management. For each contribution, the corresponding research question is given in brackets. The contributions are further summarised as follows:

- Demonstration and argumentation of the necessity to consider traffic flow stochastically for evaluation of traffic management. *(Question 3, 4 & 5)*

- Insight into the main traffic flow influencing stochastic variables and a quantification of these variables. *(Question 1 & 2)*

- Identification of relevant modelling issues for modelling uncertainty in stochastic traffic flow in practice. *(Question 3)*

- Advancing uncertainty modelling in traffic models. Performed through the demonstration of the advantages of Advanced Monte Carlo sampling in uncertainty modelling in traffic models, and the development of the scenario-based Core Probability Framework (CPF). *(Question 4)*

- Development of a methodology to consider microscopic stochastic fluctuations in traffic flow in a first-order macroscopic model environment. The methodology considers individual vehicles characteristics in macroscopic flow modelled in a Lagrangian system also capturing the capacity drop and other traffic phenomena. *(Question 5)*

- New methodology to evaluate the resilience level of road sections. The methodology is based in part on traffic heterogeneity as an important variable for traffic breakdown. *(Question 4 & 5)*

- Development and proof of visualisation possibilities for communicating uncertainty from probabilistic traffic models. *(Question 6)*

1.3.2 Practical contributions

The practical societal contribution of this thesis is to aid the reduction of congestion and improve traffic throughput and reliability on crowded motorway networks. This is achieved through a number of different contributions given in this thesis, which are summarised as:

- Development of a practical framework for demand and capacity estimation and generic base values for probabilistic capacity under different conditions. *(Question 1 & 2)*

- Highlighting the relevance of traffic management and widened scope of its application. It is demonstrated that traffic management can have a greater positive effect on traffic flow than previously realised. *(Question 3, 4 & 5)*

- Development and demonstration of models to a-priori evaluate the effects of traffic management under different conditions. These models increase the accuracy and reliability of forecasts for the application of traffic management *(Question 4 & 5)*

- Development of the Link Performance Indicator for Resilience (LPIR) to be applied as a quick-scan approach for network evaluation. *(Question 4 & 5)*

- Presentation of visualisations that allow the results from probabilistic traffic models to be easily communicated in practice. *(Question 6)*

1.4 Research approach

The approach followed in this research follows the same line as the presented objectives and focusses on answering the earlier presented research questions. The general flow diagram of the various parts of each of these areas presented in this research is shown in Figure 1.3.

a) The analysis of uncertainty and fluctuations in traffic

To derive patterns and distributions of traffic flow influencing variables and their stochastic nature, data processing methodologies must first be reviewed, refined and applied. These methodologies should explicitly consider and display the stochastic uncertainty in traffic. The methodologies allow input for stochastic models to be constructed along with a set of feasible traffic management measures, but also for independent data analysis.

b) Modelling uncertainty and traffic fluctuations

Identification of road sections requiring attention can be performed in different ways. Analysis on ways to quantify the vulnerability of road sections is performed with a focus on improving resilience and developing a methodology to indicate locations requiring attention based on heterogeneity of traffic flow. Modelling day-to-day uncertainty in traffic requires a different modelling approach to modelling fluctuations in traffic flow. Therefore, different approaches are sought to effectively model uncertainty on one hand, and the microscopic fluctuations between vehicles on the other hand. In both cases the ability for practical application must be considered.

c) The development of visual aids for effective communication of uncertainty in traffic

It is necessary that the results from the uncertainty models can be conversed to strategic and operational road managers, policy makers, and others requiring insight into the options of applying traffic management. Therefore, a cognitive visual analysis is performed to evaluate effective methods and visualisation options to aid this process.

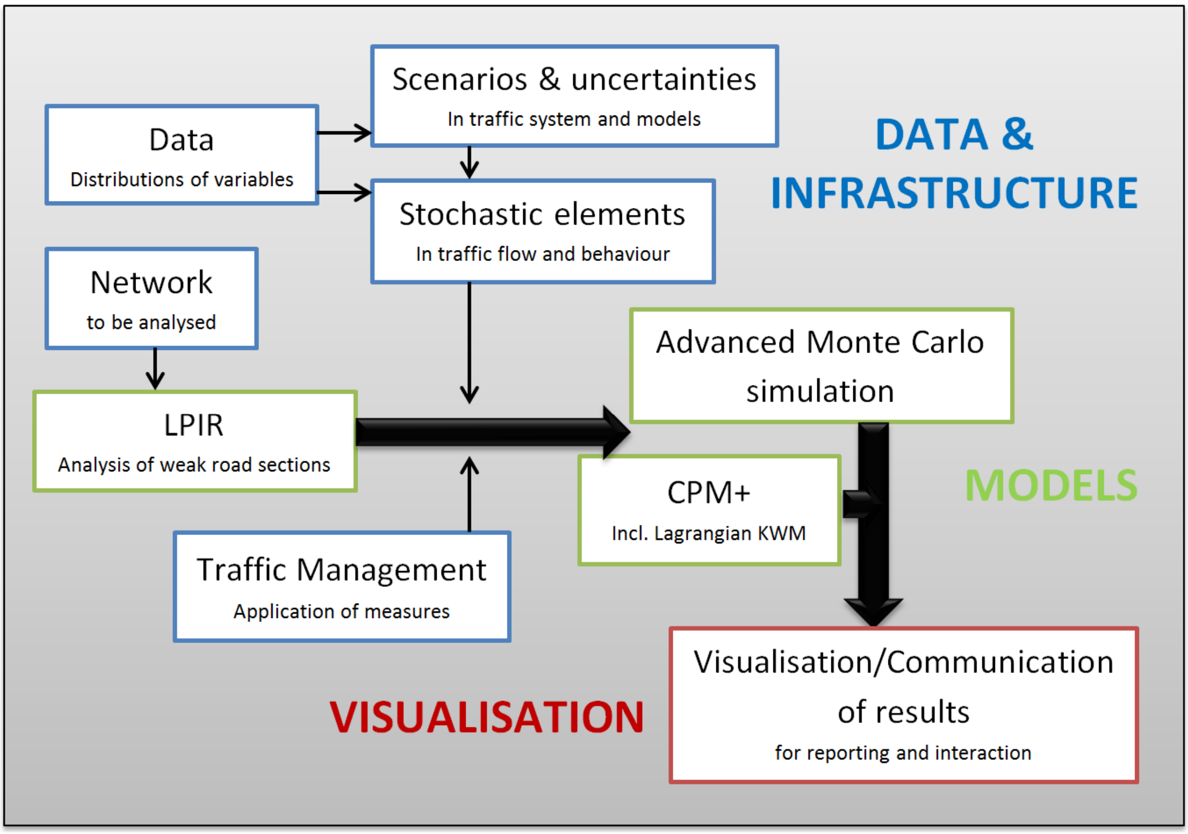


Figure 1.3: Research approach

1.5 Thesis outline

The structure of this thesis comprises of six parts as shown in Figure 1.4. Part one sets out the current practice and necessity for considering stochastics in traffic flow modelling. The main content reflects the three objectives and their research questions stated in section 1.2.1, namely the data part, model part and visualisation part, and further include a section on the practical application in a comprehensive case study. Research questions 1 and 2 are answered in Chapter 3. In the model part, research question 3 is dealt with in Chapter 2, research question 4 is answered in Chapters 4 and 5, and research question 6 is answered in Chapter 6. The application part gives a demonstration of the methodologies in Chapters 7 and 8. Chapter 9 answers research question 6. Each chapter is introduced separately on its own title page and, where applicable, the source publication(s) are given that make up the chapter. In the final part, the conclusions and recommendations flowing from this thesis are given.

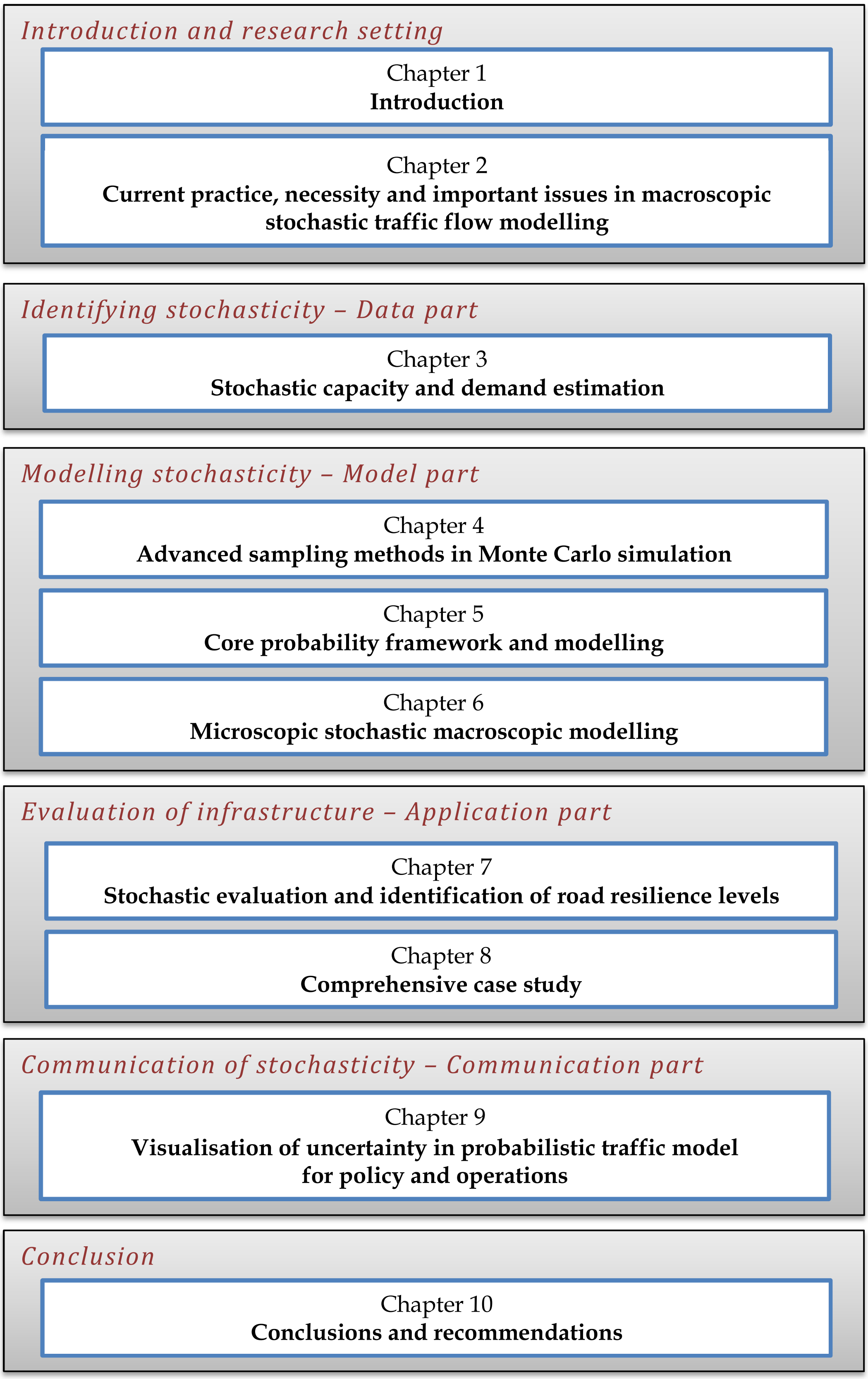


Figure 1.4: Thesis outline

Chapter 2

Current practice, necessity and important issues in macroscopic stochastic traffic flow modelling

Since traffic modelling became a mainstream area of scientific research halfway through the last century, continuous developments have taken place in order to improve performance and eradicate shortcomings of models. Since the turn of the century an increase in research regarding stochasticity and probability in traffic modelling has occurred. The realisation that simple presumptions and basic stochastic elements are insufficient to give accurate modelling results has grown.

The purpose of this chapter is to give a demonstration of the necessity to consider stochastics in traffic models and to highlight a number of issues that require further development. Firstly, a concise overview of the current state of the art in the area of macroscopic and stochastic modelling is given, as well as some of the shortcomings of these models (sections 2.1 and 2.2). The case for the necessity of stochastic modelling is then argued, and demonstrations are given and discussed, for two cases in which deterministic approaches are shown to be inferior compared to a stochastic approach (section 2.3) Challenges for further development of stochastic models are given in sections 2.4 and 2.5.

This chapter is an edited version of the articles:
Calvert, S. C., Taale, H., Snelder, M., & Hoogendoorn, S. P. (2012). Probability in traffic: a challenge for modelling. In *DTA2012: 4th International Symposium on Dynamic Traffic Assignment, Martha's Vineyard, USA, 4-6 June 2012.*

Calvert, S. C., Taale, H., & Hoogendoorn, S. P. (2014). Introducing the Core Probability Framework and Discrete-Element Core Probability Model for efficient stochastic macroscopic modelling. In *DTA 2014: 5th International Symposium on Dynamic Traffic Assignment, Salerno, Italy, 17-19 June 2014*.

2.1 Macroscopic traffic modelling in general

This thesis focusses on stochastic variation in macroscopic models. This does not mean that stochastic behaviour is not and should not be present in microscopic models. However, this is much easier to achieve and is already mature. In macroscopic models, this is currently not the case and therefore the focus of the research is on macroscopic models. Before focusing the necessity of stochastic macroscopic models, it is necessary to first understand what macroscopic are and their current level of development. This is performed in this section.

2.1.1 State-of-the-art macroscopic traffic flow models

Various types of traffic models exist, each with their specific purposes and applications. A well accepted distinction is based on the level of detail and differentiates between macroscopic, mesoscopic and microscopic models (Hoogendoorn and Bovy, 2001). Another categorisation focuses on the deterministic level of the model. This indicates the extent to which a model incorporates variation in its calculations and distinguishes between deterministic and stochastic models (Hoogendoorn and Bovy, 2001). Within these categories further differentiation can be made, also between the categories further differentiation is possible.

Macroscopic traffic models do not consider individual vehicles, but rather describe the flow from the collective behaviour of vehicles and are therefore more readily applied to larger networks. In essence the vast majority of macroscopic traffic models are deterministic. Deterministic traffic models presume that no stochastic variability is present in traffic, while stochastic traffic models do presume certain levels of variations. A distinction in macroscopic models is generally made between first order models and higher order models. Lighthill and Whitham (1955) were among the first to propose a first order approach based on fluid dynamics from the field of continuum mechanics. This group of models, known as LWR models, makes use of the law of conservation, combined with a fundamental relation between the main traffic quantities, density, volume and speed, and makes use of the numerical Godunov scheme to solve the model equations (Godunov, 1959, Lebacque, 1996b). This creates a nonlinear discrete time dynamical system which solves the partial differential equations from the LWR. Later Daganzo proposed an extension to the LWR-model in the form of the Cell Transmission Model (CTM) (Daganzo, 1994, Daganzo, 1995a). In this work, shockwaves are automatically incorporated in the applicable equations, which avoids the necessity of considering shockwaves as an external case.

Higher order traffic models make use of multiple differential equations to describe traffic flow. One of the first higher order models to be proposed was by Payne (1971) in which the LWR-model was extended with a dynamic speed equation. This addition solved a number of difficulties with the original first order models, which occurred at the boundaries of traffic states. Such a difficulty is the inability to create start-stop waves, as a first order model presumes instantaneous speed correction from vehicles. Despite the improvements, higher order models initially received a fair amount of criticism, partly due to the explicit level of complexity in solving them. And while methods have been developed to perform the task of solving the equations (Papageorgiou, 1998), the greater level of complexity makes completely understanding the mathematical properties of these models a rigorous task (Hoogendoorn and Bovy, 1998), which can lead to instability in their implementation (Daganzo, 1995a). However, further developments by Aw and Rascle (2000b) and Zhang (2002) eradicated many deficiencies, such as the violation of the anisotropic character of traffic (Lebacque et al., 2007b), and opened the door for further developments. Aw and Rascle (2000a) proposed adjustments to the original definition by replacing the space derivative with a convective derivative. Zhang (2002) described this similarly and explicitly state that traffic flow moves with the velocity along the trajectory and is therefore described as a Lagrangian quantity.

Lebacque et al. (2007b) applied the same rationale to generalise the ARZ models (Aw and Rascle, 2000a, Zhang, 2002). The ARZ models apply an invariant term to represent the relative speed of vehicles which is connected to these vehicles. Lebacque et al. (2007b) define this term as a general invariant that can also be related to global flow properties and therefore represent other characteristics of microscopic flow. The model is described as a generic second order model (GSOM) after the flexibility one has to define an invariant that can take on many different purposes. This approach has been applied in a number of consequential publications (Costeseque and Lebacque, 2014, Costeseque and Lebacque, 2015, Lebacque and Khoshyaran, 2013). One such application describes the invariant term as a stochastic driver attribute describing the random driver interactions of a driver with other drivers (Lebacque and Khoshyaran, 2013). Their Stochastic GSOM describes the stochastic behaviour as a Brownian process and white noise process and if further defined in Lagrangian coordinates. While the GSOM also allows a first order description to be formulated (Lebacque et al., 2007b, Lebacque and Khoshyaran, 2013), applications of the GSOM are generally not found in first order formulations.

The majority of applied macroscopic traffic models make use of first order theory, or an adaptation thereof, especially for models applied in practice. Second order models are gaining in popularity and possibilities, but remain less established than first order models, mainly due to difficulties in practical application and complexity. Our main focus will lie with the first order approaches, but the work described in this chapter may in many cases also apply to second order models.

Both macroscopic and microscopic models can be stochastic. Application of stochasticity in traffic models entails the inclusion of variability in the manner in which traffic flow is modelled. Contrary to deterministic models, in which one set situation is modelled, variables in stochastic models may vary due to stochastic effects. Although this adds complexity, it represents the real world to a better extent. Including stochasticity in macroscopic models, in which a wide range of variables are varied, is generally performed in two ways: by means of repetitive simulations, and secondly by including variation in the model core. Both of these methods are described and discussed in the following sections. The focus here is on stochastic traffic flow modelling, therefore stochastics in route and other type of choices are not discussed in this chapter.

2.2 Stochastic macroscopic traffic modelling

In this section a concise overview of stochastic traffic flow models is presented that can be found in literature. A discussion is also given on the main application areas for these models and acts as a step-up to a demonstration of the necessity of stochastic models, which is described in section 2.3.

2.2.1 State-of-the-art stochastic traffic flow models

Since the 1990's there has been a gradual increase in effort towards improving traffic flow modelling through the explicit inclusion of stochastic variation. Initially, focus was on Monte Carlo simulation and later the focus shifted more towards internalised stochastics. In Monte Carlo simulation various input values for the traffic variables are sampled and applied in simulation for a *N* number of simulations to approach a distribution of possible outcomes. Although Monte Carlo simulation has been widely applied, mainly due to its relative simplicity and effectiveness, the method has its drawbacks. Main concerns in traffic modelling in the past have been the computational load of the method (Chang et al., 1994, Chen et al., 2002, Sumalee et al., 2011) and the presence of correlation between input variables. The incorporation of variance reduction methods, such as Importance sampling or Latin Hypercube sampling, have helped to reduce the computational effort of such models as well as the use of more powerful computers (Calvert et al., 2014c, Hess et al., 2006, Jonnalagadda et al., 2001, van Lint et al., 2012). Furthermore, recent developments in marginal simulation approaches offer an alternative solution to a heavy computational load in Monte Carlo approaches (Corthout et al., 2011). In marginal simulation a significant overlap between traffic flow in successive simulation iterations is presumed. By only simulating the marginal difference in traffic flow, repetitive network loading with a full dynamic macroscopic model is not required. The marginal simulation method only requires a single full initial model simulation and thereafter simulates the marginal differences with a first-order based kinematic model, leading to a gain in computational efficiency. Correlation between input variables may be considered prior to simulation at the sampling stage (Chen et al., 2002). Variables with dependencies may also have probabilities which rely on the values sampled from other variables. In this way, correlation between two or more variables is included and allows for a realistic simulation. However, calculating non-bias outcomes in situations in which correlations are more complex and, furthermore, have dependencies on variables in the model, becomes much more difficult (Chang et al., 1994). In many approaches the extent of bias is presumed to be limited and therefore little attention is spent on this difficulty.

An analytical approach to probability in the model core, or simply one shot, stochastic traffic modelling approach has proven an extremely difficult undertaking. Clark and Watling (2005) proposed a method for travel time reliability based on day-to-day variations in the travel demand matrix. Their framework computes a total travel time distribution based on the multivariate moments of a link flow vector. This was successfully demonstrated, however the method only considered a single random variable, namely the traffic demand, and therefore has limited difficulties with correlation. Others propose a more numerical approach to analytically incorporating stochasticity in the model core. Recent developments include Sumalee et al. (2011), who proposed a stochastic cell transmission model (S-CTM) which makes use of fives operational modes depending on the states of traffic flow. Each mode incorporates a set of stochastic conditions to describe probability in each mode. Others who proposed using multiple functions as dictated by the traffic state, include Muñoz et al. (2003) and Sun et al. (2003). A main reason for considering multiple traffic states is the avoidance of nonlinearity in the fundamental relation, which is difficult to quantify otherwise. More recently Jabari and Liu (2012) argued that presuming non-linearity, while being mathematically beneficial, may lead to inconsistency with the original deterministic dynamics. Therefore Jabari and Liu (2012) proposed to include stochasticity as a function of the uncertainty in the driver gap choice, represented by the random vehicle headway. In doing so, they argue that non-linearity is avoided in continuous time as all traffic dynamics may be derived from the longitudinal car following behaviour. Boel and Mihaylova (2006) similarly proposed an extension to the CTM with stochastic elements. Rather than reconstructing the CTM as piece-wise structure based on traffic states, they defined the sending and receiving functions from the CTM as random variables in which the dynamics of the average speed in each cell is stochastically varied. The purpose was to incorporate stochasticity in the heart of the model at link level, which may propagate through an entire network through cell interaction. However, as their approach only considers a single stochastic scenario at a time, repetitive simulations are required to compose a probability distribution of the outcomes.

Stochasticity can also be included in (macroscopic) traffic models by means of a stochastic fundamental diagram. Li et al. (2009) make a strong argument that a simple, but effective manner of stochastic modelling is to make use of a stochastic fundamental diagram. Such a diagram is constructed through a flux function obtained from random elements observed from speed-density data. Kim and Zhang (2008) also previously described stochasticity in the fundamental diagram by defining the growth and delay of perturbations from random fluctuations in both the gap time and transitions between traffic states. In their work they closely examined fluctuations in car following to derive their defined gap time.

Advances in approaches bringing probability to the core of a model have generally been performed as extensions of existing methods. This has the obvious advantage that sound theory may be further elaborated on. The extension of the cell transmission model (CTM) is therefore a logical one. While disadvantages of applying such non-linear approaches are brought forward (Jabari and Liu, 2012), the question remains to which extent this has a detrimental effect on the outcomes. Jabari and Liu (2012) argue that most models are nonlinear and therefore handle traffic propagation inconsistently, and that stochastic variables are often applied as mere white noise. Application of stochastic variables as a representation of an underlying function rather than white noise therefore should lead to a reduction in error, substantiating Jabari and Liu’s claims. While possibly guaranteeing consistency when avoiding nonlinearity, there may be an issue in relation to accuracy as nonlinear models have a greater ability to generalize and freedom to fit the complex dynamics of traffic flow (Vlahogianni et al., 2005). The case for linearity against nonlinearity is therefore a complex one in which nonlinear solutions continue to gain in strength, even if complexity issues increase. Well posed approaches have been proposed, but still do not claim to satisfy all properties of traffic dynamics (Aw and Rascle, 2000b).

Table 2.1 gives a concise overview of the main types of models, some example references and a summary of their strengths and weaknesses. The majority of the presented methods, while applying stochasticity, do this based on presumptions of random variables. In many cases, random distributions may be acceptable, however a number of random variables which presume a nominal distribution will show a persistent error when empirically challenged. These errors transpire from the difference between the nominal distributions and the underlying distributions, which can be obtained empirically (Knospe et al., 2004, Lin, 2001). To this extent the random variables are not pure stochastic, in the sense that they represent real empirical fluctuations, as the random variables do not always accurately correspond to empirically derived probabilities. A further major difficulty that is only partially addressed is that of dependence between random variables (Chen et al., 2002, Sumalee et al., 2011, van Lint et al., 2012). These correlations are often presumed non-existent for the ease of modelling (Sumalee et al., 2011), or are simplified by means of presumptions or transformations (Clark and Watling, 2005, Jabari and Liu, 2012). While some research does consider correlations between random variables, these alternative models are often restricted to less elaborate modelling approaches.

Table 2.1: Classification of stochastic traffic flow models

Class		Reference example	Characteristics
Monte Carlo	*With & without advanced sampling*	(Calvert et al., 2014b)	+ Simple + Accurate and effective - Time & computationally heavy
	Marginal	(Corthout et al., 2014)	+ Limited additional computational load - Dependent on a base run - Poorly effective for large changes - Interactions at network borders difficult to estimate
First over model generalisations	*Stochastic element*	(Boel and Mihaylova, 2006, Sumalee et al., 2011)	+ Based on sound and proven theory + Easy application in practice - Loss of accuracy due to simplification of probability & correlations
	Fundamental diagram	(Kim and Zhang, 2008, Li et al., 2009, Muñoz et al., 2003)	+ Based on sound and proven theory + Simple & effective approach - Loss of accuracy due to simplification of probability & correlations
Second order model		(Khoshyaran and Lebacque, 2009)	+ Mathematically sound & correct + High potential level of accuracy - Difficult to implement in practice - Calibration & validation of variables cumbersome
Other analytical		(Clark and Watling, 2005)	+ Mathematically sound & correct - Difficult to implement in practice - Computationally heavy (in some cases)

Some advances have been recently made in stochastic core modelling, as shown. The majority of these models are developed for very specific purposes with possibilities for larger scale implementation. However the, sometimes complex, formulations may provide difficulty for implementation of methods in a complete macroscopic or mesoscopic framework. To the knowledge of the authors, no model has yet been developed that is capable of matching the accuracies of the computationally heavy repetitive simulation through a one-shot approach on a comprehensive network, and is able to be extensively applied in practice. In section 2.4, the main issues for stochastic macroscopic flow modelling are considered.

2.2.2 Application range

While the need for a greater element of probability and consideration of variability has been shown, this does not apply for all applications in which traffic models are required. In many cases, deterministic models will work just as well. It is therefore necessary to evaluate under which conditions variability should be considered, while considering potential drawbacks of including variation in traffic.

In general, the main advantages of using deterministic models are the relatively short calculation time and the limited amount of input data required. The advantages of using stochastic models are an increased accuracy with consideration of numerous situations, as demonstrated in the experimental cases in the next section, and the possibility of giving results with a reliability score or sensitivity. It is easy to see that a stochastic model will always be preferred if it can be just as easily applied as a deterministic model, however in reality this is not the case. It is therefore necessary to review the goals and requirements of a model analysis before performing calculations. This is a step that is too often omitted in practice, mainly due to practical issues or understandable unawareness from the viewpoint of the user. Considering the aforementioned advantages of the models, a concise overview of conditions under which both model types should be used, is given in Table 2.2.

Table 2.2: Application range for stochastic models versus deterministic models

Stochastic modelling	Deterministic modelling
Applicable for…	***Applicable for…***
Variation in input variables	Negligible variation in input variables
Distribution of input variable is reliable and can be easily determined	Distribution is unreliable and cannot be easily determined
Variation in input variables has an amplified effect on model outcome	Variation in input variables has a limited or linear effect on model outcome
Congested network with high congestion volatility	Uncongested network or congested with low congestion volatility
Comprehensive overview of network performance	General indication of network performance

Variations in the input variables lead to a primary source of variation in the model results. When these variations are negligible or non-existent, there is no need to apply a stochastic approach and a deterministic approach suffices. However, for limited variations, the results from model runs may show greater discrepancy than for large input variations between the stochastic and deterministic cases. This is shown in the next section. This is due to sensitivities in the vicinity of critical road sections in a network, which show congestion and therefore delay for a limited part of the possibility set, but are not sufficiently captured by the deterministic case. In turn, a greater error is found for the deterministic case. However when variations are large, extreme congestion or quiet traffic will often also be captured by the 'deterministic' case to a large extent, which leads to a smaller error between the stochastic and deterministic cases. When the variation in input variables has a direct linear correlation with the outcome of a model, the model results will show a similar level of variation, as in both cases a similar 'representative' situation will result from both a distributed input and a mean or median input. In an uncongested network this will often be the case, as traffic can propagate at (near) desired speeds without too much disruption, resulting in a stable model output. In these scenarios, stochastic modelling may not show much difference to the deterministic case. Furthermore, it goes almost without saying that when probability distributions or functions cannot be accurately constructed, one should apply a known variable in deterministic model rather than applying inaccurate presumptions of a distribution function. Finally the main application of stochastic modelling should be to give an accurate and comprehensive overview of traffic on network under a wide variety of conditions. If one is merely interested in a general indication of network performance then a deterministic model again suffices.

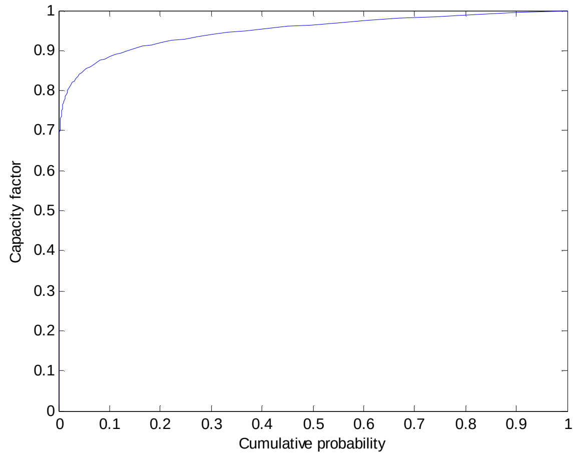
2.3 Need for stochastic models

Often there is a specific and sometimes urgent need to use stochastic models. This is argued in many of the papers presented here thus far. The application of simple stochastic or deterministic models in some cases may be unintentionally deceiving policy-makers with biased results. However, it is not always apparent when stochastic models should be applied and what the extent is of errors made by applying non-stochastic models. Tampère and Viti (2010) remarked on this and included questions relating to the reliability of dynamic modelling and the lack of most current models to properly consider stochastic elements. van Lint et al. (2012) experimentally demonstrated the importance of not ignoring variations in traffic by showing biases in results that occurred by not considering stochastic variations. Many recent contributions propose elaborate analytical solutions for the application of probability in modelling. However, most remain incomplete from the point of view of practical widespread implementation. Tampère and Viti (2010) and Jabari and Liu (2012) also argue that randomness is often applied in an imperfect and incomplete fashion. This may be through merely adding stochastic 'noise' or presuming an inaccurate distribution, for example. For many of the proposed models this remains the case, which can lead to the misrepresentation of reality and wrong conclusions based on the outcome of such models.

In the following paragraphs, two experimental cases are given to demonstrate area's in which deterministic modelling has shortcomings and a stochastic approach is required. This gives a demonstration of the necessity to consider the stochastic character of traffic flow when modelling.

2.3.1 Experimental demonstrations

To demonstrate potential situations in which modelling, without consideration of variation in traffic quantities, can lead to biased results, two small scale experimental cases are considered. These cases each have a focus on a specific contribution of stochastic modelling. The goal of the experiments is to show that considering variations as probabilities gives substantially different results than by considering a single deterministic run. In each case, the capacity of the road sections is varied according to an arbitrary, but probable distribution. Variations in capacity are applied to all road sections as a blanket factor, which may represent the reduction in operational capacity from i.e. weather conditions, luminance conditions, etc. The applied distributions are logarithmic functions and are shown in Figure 2.1. These distributions resemble the distribution of empirical observations. Different distributions are used to demonstrate the influence of the input distribution in the cases. To avoid the necessity to derive correlation between capacity and demand variation, only the capacity is varied, which is more than sufficient to give an indication of the effects of modelling traffic variability. In each case, use is made of the dynamic macroscopic traffic assignment model INDY, which is based on the link transmission model, as developed by Yperman (2007). Route choice in the applied version of INDY is path based, with path fractions derived iteratively until equilibrium over all OD-routes is achieved. For each simulation iteration, this equilibrium is recalculated to correspond to the new traffic conditions. This presumes that during the peak period drivers have a good knowledge of traffic conditions and are aware of irregularities in this daily pattern and can anticipate this (Peeta and Yu, 2005). This presumption is an imperfect simplification of reality. The other extreme would suggest that drivers have no prior knowledge of the network and changes in daily traffic flows, which is not realistic either, as drivers making work related trips should be presumed to have a greater knowledge of the traffic system. The true equilibrium state will most probably lie in between these two events, and has been discussed in a number of contributions in recent years (Gao et al., 2011, Guo et al., 2010, Ng and Waller, 2012). With a lack of certainty on the equilibrium state, the choice is made to presume a new equilibrium for each simulation based on driver knowledge of the traffic system. The model is applied to a section of the Amsterdam network, as shown in Figure 2.2.



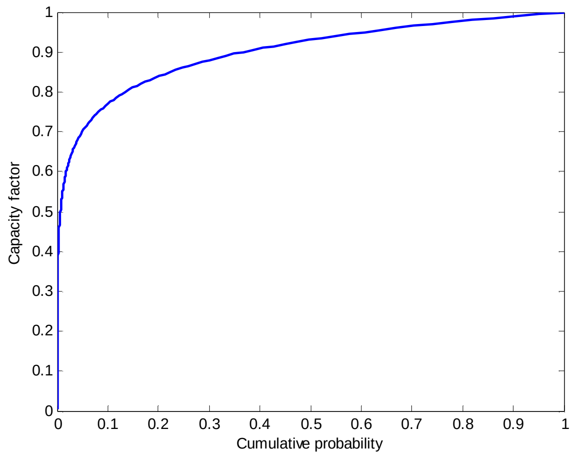


Figure 2.1: Capacity factor functions for model input: case 1 (left) and case 2 (right)

Figure 2.2: Network used for the experimental cases in INDY, representing the south ring of Amsterdam

The outcomes of the experiments are analysed using the *total experienced delay* as performance indictor on the entire network compared to free-flow conditions, and are expressed in total vehicle hours. In the case studies, the *averaged travel time* over route AB (see Figure 2.2) is also analysed. Other result indicators may also be used, such as the travel time over other specified trajectories or the average network speed, among others. For the demonstration here, it is not of great importance which indicators are chosen, merely that the network can be evaluated. The mean average and the median of the distributed results are compared with that of a single model run for the median situation, which represents a deterministic model run.

In general, the *total experienced delay* T_{delay} is defined as:

$$T_{delay} = \sum_{veh=1}^{\infty} (tt_{scen.veh} - tt_{ff.veh}) \tag{2.1}$$

where	veh	= vehicles
	$tt_{scen.veh}$	= travel time in the scenario
	$tt_{ff.veh}$	= travel time in free flow

In the macroscopic model, where vehicles are not modelled individually, the *total experienced delay* T_{delay} is calculated by:

$$T_{delay} = \sum_{t=1}^{\infty} \sum_{link=1}^{\infty} \left(q_{link.t} \cdot \left(\frac{l_{link}}{v_{link.t}} - \frac{l_{link}}{v_{\mathrm{ff}.link}} \right) \right) \tag{2.2}$$

where	t	= time
	$q_{link.t}$	= traffic flow on link at time t
	l_{link}	= length of link

$v_{link.t}$	= mean cell speed on link at time interval t:t-1
$v_{ff.link}$	= cell speed on link in free-flow

The averaged travel time over route AB is the average of all travel times during the simulation on the route, and is defined as:

$$TT_{AB} = \sum_{t=1}^{n} \frac{\sum_{link=1}^{\infty}\left(\frac{l_{linkAB}}{v_{linkAB.t}}\right)}{n} \tag{2.3}$$

where		
	TT_{AB}	= travel time between origin A and destination B
	l_{linkAB}	= length of a link, between origin A and destination B
	$v_{linkAB.t}$	= cell speed on link at time t
	n	= number of time steps

2.3.2 Case setup

In the first experimental case a near-critical level of traffic flow is present on the network. This could represent a situation in a busy peak hour period on a well-designed network, which nicely meets the extreme level of demand. In the reference scenario, the capacities are set to the median value of all possible capacity values corresponding to the capacity distribution; this is the 'representative' situation. The stochastic scenario takes a sample from the capacity distribution (Figure 2.3a) and applies these values to all links in the network. This is iterated for 40 simulations and is performed for Latin hypercube and systematic sampling, to verify that the sampling method does not bias the tendency of the result. Both systematic and Latin hypercube sampling are both advanced sampling methods that systematically sample from ordered sub-selections. For more information on these methods see (Iman and Conover, 1980, McKay et al., 1979). These methods are chosen, as they represent the input distributions in the outcomes much better than for simple random sampling for a low number of samples (Black, 2009, Iman and Conover, 1980, McKay et al., 1979).

The second experimental case considers the event that the variability of the capacity is extensive. This may be the case in a period in which extreme weather is present in varying severity over an extended period of time. The capacity distribution as in Figure 2.3b is applied, which shows a greater variation in capacity value compared to case 1. Again for the reference scenario, the capacities are set to the median value of the all possible capacity values corresponding to the capacity distribution. The stochastic scenario takes a sample from the capacity distribution and applies it to the network, which is repeated for 40 iterations. This is also performed for Latin hypercube and systematic sampling.

2.3.3 Results of experimental cases

The results from the experimental cases are shown in the form of histograms, as well as the numerical values for each sampling method. The outcome of the median input value, which is used to represent the deterministic case, is also given.

Case 1

The results of case 1 show that, depending on the sampled capacity value, a skewed distribution is produced with an average network delay of a little under 18000 vehicle hours in the network (Figure 2.3 and Table 2.3). This is considerably higher than the deterministic situation, modelled with the input median, which produced a total network delay little over 9000 vehicle hours. In the stochastic case, the near-critical level of traffic flow on capacity will be breached in many cases in which the capacity is marginally below the critical level of traffic flow. And while this may not happen in the majority of cases, when it does, widespread congestion can occur in the network and a greater total network delay for vehicles is registered. The average capacity remains above that of the critical traffic demand. Because the 'representative' situation, as modelled in a deterministic approach, does not trigger widespread congestion, the total network delay is significantly lower which gives a misleading outcome. When considering the travel times over route AB, a similar outcome is obtained (Figure 2.4 and Table 2.4). The deterministic value (18.2 minutes) lies very close to the left side of the distribution, while travel times well above these 18 minutes are recorded in many cases.

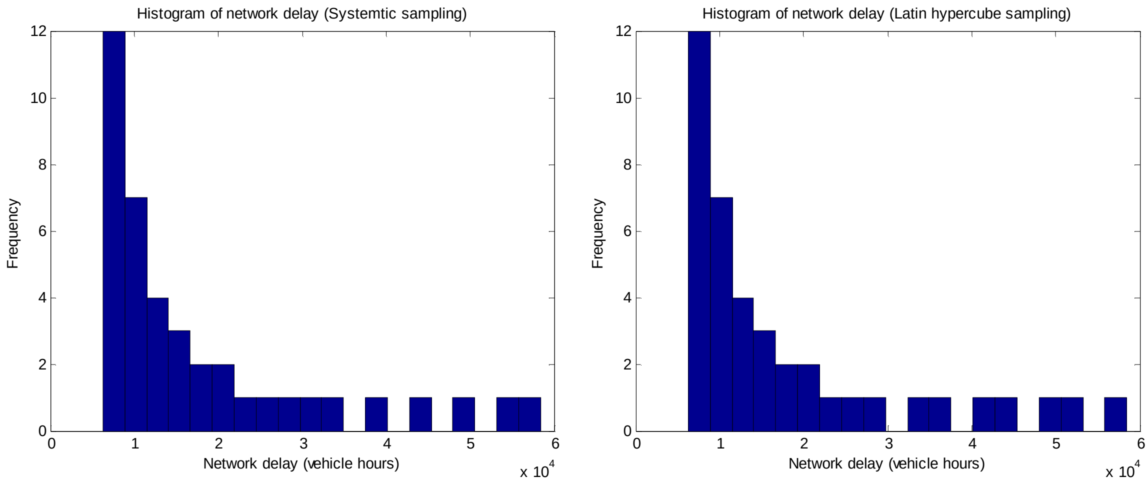


Figure 2.3: Network delay for case 1. Sampled as systematic (a-left) and as Latin hypercube (b-right) sampling

Table 2.3: Network delay of case 1 in vehicle hours

Sampling method	Median Network delay (vehicle hours)	Average Network delay (vehicle hours)
Latin Hypercube	12164	17990
Systematic	12166	17986
Median input	9113	9113

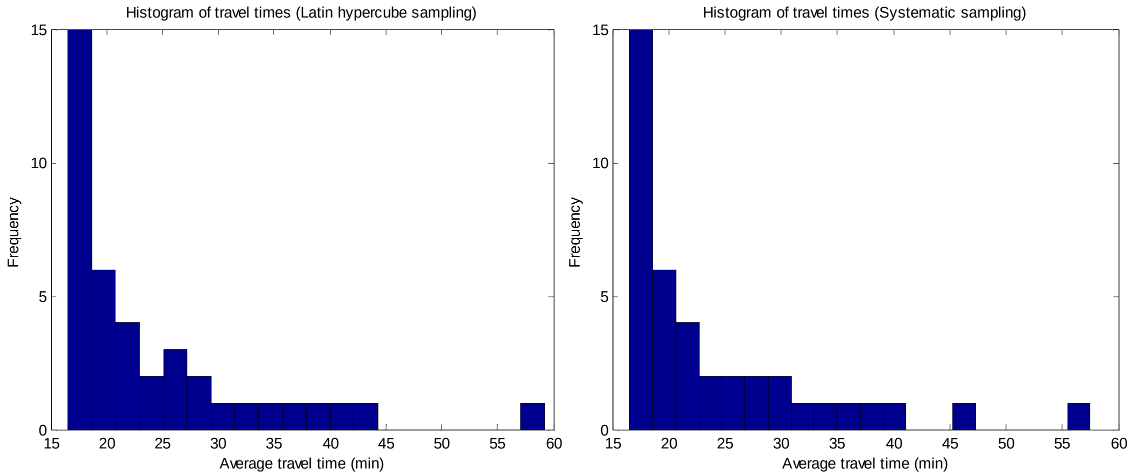


Figure 2.4: Averaged travel times on route AB (see Figure 2.2) for case 1. Sampled as systematic (left) and as Latin hypercube (right) sampling

Table 2.4: Averages travel times for case 1 on route AB (see fig. 2)

Sampling method	Median Travel times (minutes)	Average Travel times (minutes)
Latin Hypercube	20.23	23.98
Systematic	20.23	23.95
Median input	18.16	18.16

Case 2

The results of case 2 show similar distributions to that of the first case. The average total network delay of the repetitive simulations is around 18,000 vehicle hours, while the deterministic run produces just over 12,000 vehicles hours (Figure 2.5 and Table 2.5). For this experiment a larger variation is applied to the input capacity variable. This results in a larger spread of values for the total network delay for the stochastic approach. The deterministic approach also shows a greater number total network delay due to a greater average capacity drop in the input, which allows the traffic demand to exceed capacity to a greater extent. As capacity in this case is low enough to become critical, the difference between the stochastic and deterministic outcomes is smaller, while the average stochastic outcomes are only slightly higher for case 2 than in case 1. This further shows the sensitivity of the deterministic approach to small changes in the input, while these are easily captured by the stochastic approach.

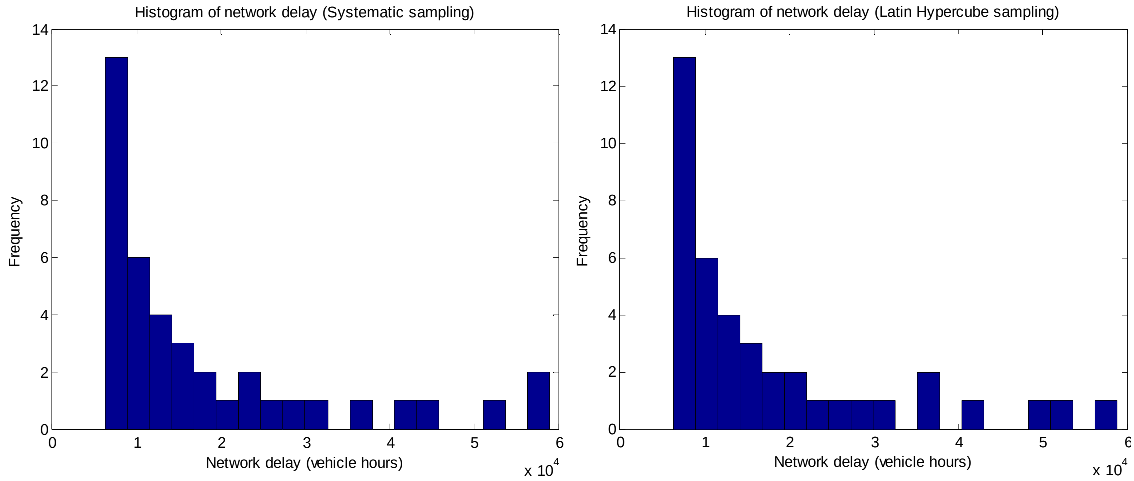


Figure 2.5: Network delay of case 2. Sampled as systematic (left) and as Latin hypercube (right) sampling

Table 2.5: Network delay of case 2 in vehicle hours

Sampling method	Median Network delay (vehicle hours)	Average Network delay (vehicle hours)
Latin Hypercube	12136	17845
Systematic	12164	18481
Median input	12359	12359

By considering a complete distribution of probable input values, a complete distribution of outcomes can be considered for the stochastic approach. In the model, a small deterioration in road capacity has an amplified effect on the experienced traffic delay, a characteristic that is not picked up by the deterministic approach. We have therefore demonstrated a major deficiency of deterministic and simple stochastic models that do not consider variable traffic flow. The inability to consider anything other than an average situation and the sensitively to variations in 'real' input variables, by presuming single values rather than distributions, leads to a considerable chance of model results giving unreliable and biased outcomes.

2.4 Challenges for further development of stochastic models

Although research in stochastic traffic flow modelling is gaining momentum, a number of significant challenges remain for the further development of stochastic macroscopic modelling. And while many of these challenges have been addressed individually or in part in research, a further challenge remains in bringing each part together to form a complete and operational stochastic model. In the previous chapter, the state of the art in relation to stochastic modelling of traffic variations is given. From the reviewed literature, a number of issues are described that have still to be satisfactorily solved, such that they can be included in a fully operational traffic model for practical application. These issues are summarised and described in this chapter. The main challenges discussed are:

1. Computational efficiency
2. Correlations and spatiotemporal dependency
3. Data gathering and processing
4. Stochastic propagation of probability
5. Generality of stochastic variation
6. Driving behaviour in macroscopic traffic

An additional challenge may be mentioned in the form of the implementation, however this affects each challenge individually, and does not explicitly affect the core workings of the model. The described issues give a basis for development of solutions in the rest of the thesis to address the main issues. Although six issues are mentioned, it remains too extensive to address each one completely within this thesis. However, when tackling a number of the challenges from the issues, attention is given to the complete set of issues to avoid a solution for one leading to the aggravation of another. In the rest of this chapter, each issue is described and is concluded through the formulation of a challenge for each issue.

2.4.1 Computational efficiency

Consideration of computational efficiency applies to the computational load of a model on the applied hardware, but also the speed at which calculations can be made as a consequence of the applied calculations. Macroscopic models in their application are almost always applied to larger networks and therefore demand computational power, which is severely compromised by including probability due to consideration of a larger number of scenarios or uncertainty. The computational load of models in general has been seen as a problem in the past (Chang et al., 1994, Chen et al., 2002, Sumalee et al., 2011). However, nowadays this problem is diminishing with the increase in computational power of hardware (Chang et al., 1994). Nevertheless the possibilities of increased computational power always seem to be tested to the limit as advancement of modelling techniques continually demand greater computational power (Bliemer and Taale, 2006). For both macroscopic stochastic methods mentioned in this thesis: repetitive simulations and one-shot analytical solutions, there are difficulties relating to scientific advancement in terms of the computational efficiency.

As described in section 2.2, repetitive, or rather Monte Carlo, simulation techniques have increasingly applied greater computing power to tackle the lack of applied variables and the complexity of the variables functions (Chang et al., 1994). Greater numbers of random variables are considered in the input, and model, in an attempt to describe the traffic system to a more realistic extent. This however means that correlation between considered variables becomes of greater importance. This is because the effect of correlation becomes greater as one considers larger numbers of dependent events. Determining correlation functions is already difficult, however calculating them also leads to a greater demand of hardware resources. The other mentioned innovation in repetitive simulation, marginal simulation, offers an alternative that reduces the computational load, while still allowing a complete course of Monte Carlo simulations (Corthout et al., 2011). By only simulating the marginal traffic flows, the spatiotemporal range that is simulated can be limited, such that a complete network and all traffic flow do not require resimulation, but only those differing from a previous simulation. Limitations of marginal simulation lie mainly in the approximation that is made regarding the affected network and traffic flows. Assumptions are made to the extent that the initial base simulation is affected and which parts of the network are activated for marginal simulation. The approach has proven to be efficient in various cases, however developments continue and generic proof of efficiency without significant loss of accuracy is still in the process of being unveiled. Nevertheless, this category of models has much potential and has already proven to have its uses (Corthout et al., 2009, Frederix et al., 2011).

The development of one shot models, which largely does away with the necessity for repetitive simulations has a great potential to allow for stochastic simulation at a lesser computational cost. Such models as the S-CTM (Sumalee et al., 2011) and that of Jabari and Liu (2012) are at the forefront of these developments. A danger however is that a simplification of the stochastic input or propagation may be required to allow one shot models to be effective. In this, a simple rule that the more elaborate the solution, the greater the computational load, is evident.

Recent developments should be applauded, but come in many cases with large drawbacks. The challenge for researchers in this field is therefore: not only to develop elegant solutions for stochastic modelling, but to do this in a manner that allows easy and efficient application in computational terms. Furthermore, with a greater efficiency, comes a larger network that can be calculated, shorter calculation times, and a greater robustness of the model.

2.4.2 Correlations and spatiotemporal dependency

When applying stochastic modelling, it is necessary to consider multiple random variables as both input and in the model itself, depending on the applied approach. In the simplest terms, one has at least the traffic demand and supply as input variables, however these may consist of many other variables, such as weather effects, general randomness in demand, and others. These all have some level of dependence which cannot be ignored (Chang et al., 1994). In deterministic modelling, one has only to consider single values, which relate directly to one another. Within random variables, not every permutation will be possible in conjunction with another from a separate random variable. A simple example of this is a high speed of 100 km/h which will never occur simultaneously with a high traffic density of say 40 veh/hr/lane, while both may be present as part of the probability of their random variables. A limited number of solutions have been proposed to deal with correlations in (Berdica, 2002, Chang et al., 1994), however these and similar approaches are complex or may only deal with specific dependent relations. While offering some sort of solution, a difficulty remains and is connected to the challenges from the previous paragraph, in that the applicability of the methods in an operational model may be cumbersome due to their complexity. To this extent, there remains a challenge to develop a global approach to consider correlation between random variables in a manner that can be easily implemented and that does not substantially detract from the efficiency of the model.

Incorporation of spatial and temporal dependent fluctuations from different sources brings a further issue of correlation on a number of levels. On a temporal plane, it is clear that a stochastic element will affect traffic during a certain time frame, possibly with differing severity. A basic example is that of an accident that reduces road capacity. At the time an accident occurs, the capacity is affected differently than during the aftermath and the clean-up, but nevertheless the capacity reduction is correlated in time, as a natural consequence of a chain of events. In the same way, there is also a spatial correlation. The capacity reduction affects the location of the accident, but due to congestion propagation, also affects upstream traffic flow. A further complexity in dependence comes from not only considering a single stochastic influence variable, such as the capacity, but also the traffic demand. In the case of an accident, drivers may reroute, shift departure time, etc. This does not only affect traffic flow in time, but also in space. Furthermore, correlation effects also exist between the traffic demand and road capacity in some instances. When considering a greater number of variables, the dependency relations explode.

In many cases, some of these dependencies are presumed non-existent for ease of modelling (Clark and Watling, 2005, Sumalee et al., 2011). Especially for the interdependent correlations between variables this is readily the case, while spatiotemporal dependencies must be considered on some level to avoid disutility of a model. Even then, these correlations may be simplified by means of presumptions or transformations (Clark and Watling, 2005, Jabari and Liu, 2012). It should not immediately be presumed that a less than full consideration of dependency will have large detrimental effects on model outcomes, as there are cases in which this is clearly the case (Calvert et al., 2012), however the possibility thereof should always be considered. The challenge resulting from this issue is therefore, to develop models that sufficiently consider the main effects of correlations between variables, while allowing a model to not become overly loaded with too complex levels of internal dependency between variables.

2.4.3 Data gathering and processing

Probabilistic or stochastic models, by definition, work with a wide variety of possible values for the considered random variables. The outcomes of these models will often be given as a distribution, and the input will often encompass an even greater spread of data points. In some cases input for stochastic models will be explicitly applied from empirically collected data, and other cases will be applied from an empirically derived or presumed analytical function. In either case, there is a need for large amounts of data to form a generically valid distribution or to validate the presumed function. The specific type of data depends heavily on the manner in which an approach is applied. However, for approaches which try to include multiple variations of traffic influencing variables, such as weather conditions, gathering and processing the required data is not a trivial task. If we consider weather and even the effects of snow, it must be pointed out that a great number of permutations are possible. One can distinguish between snowfall and lying snow on the road surface, between the first snowfall of the year and snow two weeks later when drivers have already become accustomed to the conditions. Also various combinations of weather conditions can be considered, such as strong winds, poor luminance, and low sunshine, all in combination with snow. Each situation needs consideration to be able to determine specific causation of events and correlations between the events. This requires years of data, and even then this may be insufficient. This challenge obviously applies for many other variables, besides the weather. And once sufficient data has been gathered, it still needs to be processed. The principal difficulty of this is processing the data in such a way that dependencies between variables are correctly reflected in the random variables, or as a correlation function.

To address these issues, the application or development of concise methodologies is required, which will allow for an efficient and comprehensive data processing and result in accurate distributions. This therefore is also the challenge that is presented with this issue: to development and apply techniques that produce representative distributions based on empirical findings for application in models, without the need to rely heavily on heuristics and arbitrary assumptions.

2.4.4 Stochastic propagation of probability

In traffic flow models, it is commonplace for traffic to propagate through a link and network. However, upon including stochastic probability in traffic flow modelling, the probabilities of traffic values also propagate in time and space with traffic (Hoogendoorn et al., 2008, Lebacque et al., 2007a). For Monte Carlo simulation, this is not an issue, as each simulation is a single probability value and therefore no probability value is required to be considered. For one shot models there is a challenge to propagate probability information without compromising model accuracy or one of the other important issues, such as computational efficiency.

In models, which apply stochastic effects through the fundamental diagram, traffic flow is presumed to propagate in an identical fashion to that of a regular flow model. In a stochastic fundamental diagram, probabilities are stochastically applied in the shape of the diagram. In the S-CTM, for example, median and standard deviations of traffic variables are propagated through time and space, dependent on the relevant traffic state. It is not uncommon to only consider a median and standard deviation, as this requires the least computational effort and still gives a good estimation of variational spread. However, more in-depth analysis is harder as the underlying distribution is not preserved. Furthermore, such an approach often presumes probability distributions to be symmetrical according to a presumed shape, which is not always the case. In such a case, biases are allowed, which may not accurately represent the underlying distribution. It should however be noted that these biases may be small compared to the overall error level.

This issue is not one that will hinder the working of a model, but can have a substantial effect on the results produced by a model and therefore requires attention. The defined challenge for this issue is to construct a methodology that propagates the probabilities of variables with traffic flow, such that these probabilities or variations are maintained without losing their inherent descriptive power over the variables.

2.4.5 Generality of stochastic variation

Generality of stochastic variation refers to the applicability of parametric distributions to represent the underlying empirical distributions. Inclusion of stochastic variation does not only demand solid and accurate modelling, but also realistic and correct model input. The level of stochastic input depends on which variables are considered stochastic. These may be the time headway (or gap time) between vehicles, capacity values, traffic demand values, or even 'lower level' variables, such as vehicle population or probability of accidents. Depending on how a model processes the stochastic variables, these may be offered to the model as a complete distribution, either of a specific form or empirical, or as a description of variations, such as median, standard deviation and possibly a shape parameter. The difficulty with this issue is that of generality. A set parametric shape of probable values for a set variable may not be valid for every location on a network or under certain other conditions. Furthermore, such variables may not pertain to a set distribution type. Often presumptions are made to how general distributions or variations are. In many instances, white noise may be applied to known representative values to imitate variation (Helbing et al., 2001, Jabari and Liu, 2012). The validity of such approaches is not often considered and is taken as a model assumption. However, there is also room for improvement, when applying stochastic variation to traffic flow models. In the case of stochastic fundamental diagrams, the difficulty of generality may also arise. In some cases allowing specific local data to influence the extent of stochastic variation can help solve this.

It may be acceptable to presume parametric distributions in many cases, however in many more this may be an unwanted source of error. The challenge described in relation to this issue is therefore, to develop a method or framework to test the correctness of presumed distributions, but more so, to allow non-parametric distributions to be applied in a stochastic propagation model without undue side effects when this is required.

2.4.6 Driving behaviour in macroscopic traffic

Driving behaviour is at the heart of traffic flow and traffic flow theory. In many cases traffic flow is presumed deterministic, however driving behaviour is far from this and contains a great deal of stochastic fluctuations. These fluctuations are present for a single vehicle, in their desired speed, ability to maintain that speed, lateral positioning, acceleration capabilities and behaviour, for example. Fluctuations also exist between vehicles, such as car-following behaviour, lateral interactions, such as lane changes, etc. Each of these elements affects traffic flow. In a microscopic model, it is easy to include these aspects, when a suitable algorithm is present. In macroscopic modelling, these aspects are not explicitly considered as they are hidden in the macroscopic aggregation of traffic flow. However, it is known that these stochastic fluctuations do affect traffic flow and therefore also macroscopic traffic flow and should be considered in some way (Helbing et al., 2001).

Traffic characteristics and states of traffic in a homogeneous traffic flow will often differ from largely heterogeneous traffic flows, such that capacity values and other road variables will differ for identical demands. Such stochastic fluctuations from driving behaviour are currently not included in macroscopic modelling and here lies the challenge: to include microscopic driving behaviour stochastics in macroscopic modelling such that the effect on traffic flow is well represented.

2.5 Summary of issues

It is of course the case that each issue influences the others in some way. This is a main reason why individual solutions for each issue do not necessarily yield an overall solution for all the issues. Figure 2.6 gives a high-level description of the dependencies between the issues discussed in this chapter. We derive that especially the manner of stochastic propagation of probability in traffic is a key issue. There is a strong influence from this issue to both the manner in which the spatiotemporal dependency is influenced and the extent to which stochastic variables can be dealt with generically. It may be that certain presumptions for dealing with uncertainty propagation may limit how stochastic variables are defined. Furthermore, each issue affects the computation time of a model and in most cases contributes to a lower computational efficiency. There are situations possible that may lead to shorter computational times, such when a process inherently or even implicitly allows for parallelisation. When setting out on tackling one of the issues, the effect on the others should not be ignored, moreover the effect should explicitly be considered for model usefulness.

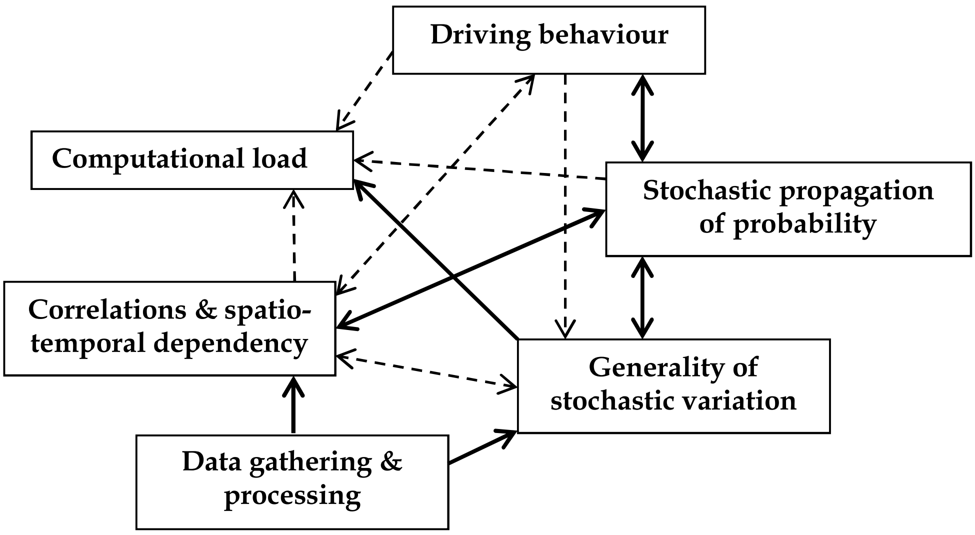


Figure 2.6: Interrelations between the main modelling issues (continuous and dashed lines indicates strong and weak relationships respectively)

2.6 Conclusions

In this chapter, the case for considering stochastic variation in macroscopic traffic modelling was argued. This begins with a description of current practices in traffic flow modelling, and more importantly, in stochastic traffic flow modelling. It is shown that currently two main avenues of models are utilised: repetitive Monte Carlo simulation, and the analytical consideration of probability in the core of a model. Current and recent research developments on both of these approaches are discussed. While classically, the Monte Carlo approach has been applied, the advancement of various analytical approaches has increased, with a number extensions of deterministic models being proposed.

Too often stochastic variation in models is not considered in practice, either for application or the necessity for development. Focussing on deterministic or simple stochastic models has the danger of closing ones eyes to inaccuracies caused by an incorrect choice of modelling approach. To demonstrate this, two experimental cases are given in which the application of a deterministic approach is shown to yield substantially biased results in comparison to a stochastic approach. While stochastic models can be seen as more 'complete' than deterministic models, their application is not recommended in every situation. A short investigation is therefore performed on the application range of stochastic models.

There is a necessity, but also many challenges for the scientific and consultancy worlds to further the development and application of stochastic modelling in traffic analysis. A realisation must arise of the detrimental effects of blindly applying non-stochastic models where probability is rife. It is the joint responsibility of both worlds to address this and make further developments in this area of research possible.

While the case for macroscopic traffic flow modelling is strong in theory, the application of such modelling approaches is only possible with sufficiently developed models. However, there are still certain challenges to be addressed in probabilistic and stochastic modelling before a widespread implementation is likely. These have been discussed in this chapter and are considered in the rest of thesis. It goes beyond the scope of this thesis to extensively and explicitly address all of the issues, however when addressing the main challenges and objective in this thesis, the described issues must be kept in mind.

Chapter 3

Stochastic capacity and demand estimation

In this chapter a methodological framework with a conceptual model for practical stochastic capacity estimation is presented and a quantification of motorway capacity variation is given. Furthermore, a methodology for stochastic demand estimation combined with stochastic capacity is also given. A quantification of the capacity is given in the form of a Weibull capacity estimation fit for each type-of-day scenario. Further consideration of the implications and applications of the framework are also given.

One of the most influential external and commonly occurring influences on traffic flow is the weather. Weather conditions affect both traffic demand as well as road capacity. The capacity estimation framework is applied on weather as part of a holistic approach for simultaneous influence on both the demand and supply. Furthermore, a case is made to quantify such outcomes stochastically.

The chapter starts by considering the three main variables considered in the chapter. Section 3.2 gives a review of capacity definitions followed by a conceptual model for capacity variation in section 3.3. In sections 3.4 and 3.5 the methodologies for stochastic capacity estimation, and for a combined stochastic capacity-demand estimation are given. These methodologies are demonstrated in case studies in sections 3.6 and 3.7. Section 3.8 finally gives a discussion of the results, followed by the conclusions in section 3.9.

This chapter is an edited version of the articles:
Calvert, S. C., Taale, H., and Hoogendoorn, S. P. (2015) Quantification of motorway capacity variation: influence of day type specific variation and capacity drop. *Journal of Advanced Transportation, doi: 10.1002/atr.1361.*

Calvert, S. C., & Snelder, M. (2016). Influence of Weather on Traffic Flow: an Extensive Stochastic Multi-effect Capacity and Demand Analysis. *European Transport, 60(4), 2016*

3.1 Introduction

3.1.1 Capacity

In traffic flow theory and modelling there are few variables that are as fundamental as road capacity. Road capacity is applied in modelling for the likes of infrastructure planning and the evaluation of traffic measures. The capacity of a road has a direct influence on the traffic state in reality as well as in models. It is therefore important that correct capacity values are applied when modelling traffic. There are however a number of challenges for the estimation of reliable capacity values. These are related to aspects such as *capacity definitions*, the *stochastic nature of capacity*, and *traffic instability* in the critical traffic states.

The first aspect is the capacity definition. The various different definitions for road capacity all have a specific purpose, while each estimation method makes use of a different approach to detect and calculate capacity. Common capacity definitions relate to the traffic state, such as the undersaturated or breakdown capacity, discharge capacity, and nominal capacity. In section 3.2, a more detailed description is given of the capacity definitions and how their values can be calculated. It is obvious that applying the correct definition is paramount as well as consistently applying the same definition for comparison.

Secondly, traffic flow and road capacity are strongly dependent on driver behaviour. Human behaviour is well known for pertaining a great deal of (unexplained) *stochasticity*, which understandably extends to traffic flow. It is therefore not reasonable to state that there is one definitive capacity for a specific section of road. Furthermore, road capacity is increasingly seen as stochastic with a probable value and a standard deviation around that value (Brilon et al., 2005, Calvert et al., 2012, Lorenz and Elefteriadou, 2000). This however is problematic in traffic models that make use of deterministic and fixed capacity values. Although these values will often give a good representation of the most probable capacity value, they do not consider the spread in capacity values and the resulting influence that this fluctuation in capacity values has on traffic flow. Taking these variations into account leads to results that are not linearly correlated to the outcomes of a deterministic traffic model (Calvert et al., 2012, Mahmassani et al., 2012, van Lint et al., 2012).

The third challenge of reliable capacity estimation is the uncertainty of traffic performance in unstable traffic near critical flow levels (Chen et al., 2013). As traffic is a stochastic system, the traffic states near to the traffic breakdown threshold cannot be easily defined (Weng and Yan, 2015). This makes it difficult to accurately determine current traffic states and can lead to unclear capacity estimations, as it becomes difficult to distinguish between pre-breakdown capacity and discharge capacity. (Bigazzi and Figliozzi, 2011, Tu et al., 2007).

Uncertainty of road capacity in traffic flow has led to a growth in stochastic modelling. These models take (a part) of this uncertainty into account to improve accuracy and reliability of traffic simulations (Ryu et al., 2015). Many of these models make use of arbitrary stochastic variations in either or both the capacity and traffic demand (Tampère and Viti, 2010). In some cases a distribution of capacity is considered, however in such a way that it does not always accurately resemble capacity variations in reality, and therefore may introduce additional variational errors. The availability of such distributions for specific circumstances is limited. Furthermore, a quantitative relationship between capacity variation and various contributing factors is yet unsubstantiated.

3.1.2 Demand

Traffic demand is arguably much more stochastic than road capacity. It is easily understood that the number of vehicles requiring use of infrastructure is subject to fluctuations, but all the more when the daily and inter-daily trends in demand are considered. Estimation of traffic demand is a vast area of research for which each subdomain has a specific purpose. For economical purposes, demand is often linked to elasticities and, given monetary value, compared to a wide range of variables (Graham and Glaister, 2004). A far more relevant area of research for this contribution is that of origin-destination (OD) estimation. Here, the goal is to link demand to an origin to give insight into local traffic demand. This is primarily performed in three ways: through large scale population surveys, through empirical observations of traffic flow, or through a combination of both (Bera and Rao, 2011). In this research, we are interested in local demand variations and less so in explicit OD-relations. Furthermore, the goal is to determine these local variations in demand patterns using vast amounts of traffic flow data, rather than population data. Therefore, methods that explicitly look at deriving demand from traffic flows are most suited. Within this category a distinction may be made between methods that consider the effects of congestion on demand estimations and those that do not consider congestion effects. In Bera and Rao (2011), among others, a detailed review of various OD-estimation methods is given.

The discussion between congested and uncongested estimation is an important one. When deriving demand, one may expect that traffic flow resembles demand where congestion is not present, as traffic has the ability to reach a road section more or less unhindered. When congestion is present a few effects occur that introduce a bias to this reasoning. Firstly, traffic is delayed and is therefore dispersed over time so that traffic with identical demand in time arrive at a location at different times. A second effect is that traffic may reroute to avoid congestion leading to different travel times and also passing of other locations than expected without congestion. A third effect is that of departure time shifts. If some traffic is not bound to a set departure time, shifts in the departure time may occur as drivers attempt to reduce their travel times by avoiding congestion. So although demand estimation for one specific road section may seem trivial, there are external effects, such as congestion, that should not be ignored. These effects are taken into consideration in the developed method for the demand estimation in section 3.5 to reduce a possible bias.

In previous research, it has been argued that the influence of relevant variables should be considered as stochastic (Lorenz and Elefteriadou, 2001, Van Stralen et al., 2015). In this chapter, a demonstration is given with an investigation using weather as the considered influence. It is apparent from literature that each weather type is viewed for its influence on either capacity or on demand and rarely on the combination of both. Also, many studies show single values, rather than stochastically as a distribution of values. This is a gap in literature that is not unimportant, as it is not just the demand or supply that influences traffic flow dynamics, but rather the combination thereof. Also, the stochastic character of both weather and traffic should not be presumed to be captured by single valued observations, but rather by the underlying distributions. This was considered previously in Van Stralen et al. (2015), however they estimated demand from a stated preference experiment and did not observe it, whereas here we introduce a methodology which extracts the influence of demand from data.

3.1.3 Effect of weather on traffic

In this chapter, a specific focus will be placed on the influence of the weather as an important variable. It is well known that weather influences many dynamic processes in traffic flow on multiple levels (Agarwal et al., 2005, Böcker et al., 2013). In operational and tactical analysis, as well as in the planning thereof, there may often be requirements to consider the influence that weather conditions have on traffic flow. Fluctuations in traffic flow on both an operational hour-to-hour as well as on a tactical day-to-day level need to be accurately considered. It has been shown that weather has an influence on both traffic demand and capacity and is therefore a key variable and one that should be closely considered. It is therefore important that strong methodologies exist that allow fluctuations in various weather effects to be determined for an entire traffic system and, furthermore, that a base quantification exists of the possible influences. In past decades, research has been performed on a number of separate weather conditions for their effects on both capacity as well as traffic demand, such as rainfall, snowfall, wind, temperature and mist (Böcker et al., 2013, Snelder and Calvert, 2016). Here, we will focus on the first four weather conditions.

Precipitation, both in the form of rain and snow, has probably been most extensively researched out of all weather conditions. Research on the effects of rain on capacity is generally performed for large rain intensity intervals and is compared to dry weather conditions. Agarwal et al. (2005), Calvert and Snelder (2013a), Cools et al. (2010), Hranac et al. (2006), Smith et al. (2004) and van Stralen et al. (2014) are just some who have estimated capacity reduction due to rain and have found varying values in different regions varying in general from 4-30% capacity reduction depending on rain intensity. Dutch guidelines estimate the reduction to be 5% for moderate and 10% for heavy rainfall (Rijkswaterstaat, 2015). Changes in traffic demand due to rain have also been found, generally indicating a reduction in traffic demand in the region of 0-5% also depending on rain intensity in most cases (Chung et al., 2005, Hogema, 1996, Keay and Simmonds, 2005, Vukovic et al., 2013).

The effect of snowfall on capacity reduction has been found to be in between 3-30% capacity reduction depending on the intensity (Agarwal et al., 2005, Hranac et al., 2006). The effects on traffic demand of snowfall are somewhat more pronounced than for rain and have been found by a number of researchers to be anywhere up to 50% (Al Hassan and Barker, 1999, Hanbali and Kuemmel, 1993).

Previous research into the effects of wind has widely remained inconclusive. In Kwon et al. (2013) no significant effects of wind were found on the capacity. Agarwal et al. (2005) also found limited effects of 2% at most for above 32 kph. Other research has also shown the effects to be limited. No conclusive research was found on the demand effects of high winds. However, it should also be noted that local wind conditions can lead to substantial decreases in capacity, such as on bridges or along a coastline.

The effects of cold temperatures were found to predominantly be present for the more extreme temperatures and only really for freezing temperatures. In Agarwal et al. (2005) values of 2% capacity were found for temperatures down to -20 degrees Celsius and up to 10% for more extreme cold. Other research has confirmed the reduction for non-extreme temperatures to be limited or non-existent (Kwon et al., 2013).

3.1.4 Focus and objectives

In this chapter, the main contribution is the construction of a methodological framework with a conceptual model for practical stochastic capacity estimation and consideration of stochastic demand with its combined influence on traffic flow. In addition, insight is given into the extent of day-type specific variation in capacity values and a quantification of the combined stochastic effects of certain weather conditions on traffic flow. The application of stochastic approaches is not common in practice, while the necessity is greater than is realised. Therefore, the described methodology also gives practitioners tools to aid the application in practice. The relationship between the variations in the capacity distribution for a number of scenarios is investigated and this is quantified where a relationship is present. This is performed for the scenarios: workdays, weekend days, and holidays, of which capacity values have been previously proven to significantly differ (Cools et al., 2007, Hilbers et al., 2004, Thomas et al., 2008, Yeon et al., 2009). Also, the corresponding capacity drop is analysed. On top of this, a stochastic quantification of various weather effects on traffic is performed, considering the effects of both stochastic capacity and demand.

This section is followed by a discussion on capacity definitions and the conceptual model for capacity variation (sections 3.2 and 3.3). The methodology for stochastic capacity is given in section 3.4, followed by the methodology for stochastic demand in section 3.5. The experimental cases and results for stochastic capacities (section 3.6) and for stochastic demand and capacities for weather conditions (section 3.7) are then given. Thereafter, the discussion and conclusions section conclude the chapter.

3.2 Capacity Definitions

Various definitions exist for the capacity of a road. Some of these are conflicting, while most refer to a specific traffic state (e.g. free flow or congestion) and are therefore complementary. It is important when applying capacity that the correct definition is chosen for the relevant purpose. Failure to do so may lead to incorrect capacity values and other undesired effects in data-analysis and especially in modelling. In general, capacity definitions can be arranged into two groups: *design (or nominal) capacity,* and *operational capacity* (Minderhoud et al., 1997). The design capacity is the foreseen capacity that is considered for planning and road design purposes. The well-known definition of capacity from the Highway Capacity Manual

(2010) is considered as a design capacity. It defines the capacity of a freeway as the maximum flow rate that can reasonably be expected to traverse a uniform segment of road under prevailing roadway, traffic and control conditions. However, this is still a rather generic definition of capacity with a number of aspects that are open for interpretation. For traffic operations the operational capacity, defined by Minderhoud et al. (1997) as "the actual flow values at which traffic breakdown occurs on a specific road under certain conditions", is far more relevant. *Operational capacity* values are generally based on direct-empirical capacity methods with for dynamic traffic (Minderhoud et al., 1997, van Arem and van der Vlist, 1992). In this chapter, only operational capacities are considered. In the rest of this section, the main definitions relating to operational capacity will be explained. A simple taxonomy of these definitions is given in Figure 3.1.

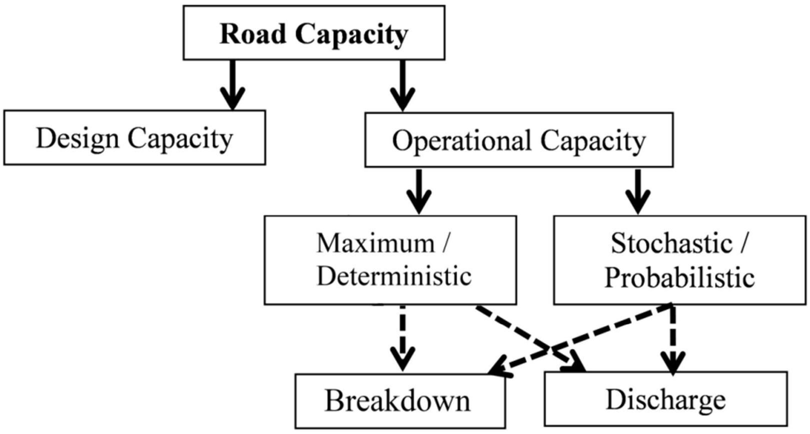


Figure 3.1: Classification of capacity definitions

3.2.1 Maximum versus stochastic capacity

Traditionally capacity, referred to here as the *maximum capacity*, is defined as:

> *"the maximum traffic flow on a section of road under fluent traffic conditions".*

This view of capacity considers capacity as a deterministic entity that has a single value for any given time. However, it has been argued in recent decades that a single value for the capacity does not exist and therefore a reference to multiple values for the capacity should be considered (Brilon et al., 2005, Lorenz and Elefteriadou, 2000, Lorenz and Elefteriadou, 2001). As traffic is an extensively stochastic system, capacity as a result is also stochastic and has multiple values. However, certain capacity values will occur more frequently than others and therefore describing capacity as a probability function becomes an obvious choice. A definition of the operational capacity as a *stochastic capacity* is given by Lorenz and Elefteriadou (2000) as:

> *"the rate of flow along a uniform freeway segment corresponding to the expected probability of breakdown deemed acceptable under prevailing traffic and roadway conditions in a specific direction".*

Here, the reference to capacity as a dynamic entity with certain probable values is evident, as traffic breakdown can occur at different flow values under similar conditions.

3.2.2 Breakdown versus discharge capacity

Distinguishing between capacity as stochastic or maximum offers different approaches to describe capacity. However, it has been demonstrated that there are actually two capacity regimes that should be considered, and can be described in such a fashion, namely the breakdown capacity and the discharge capacity (Banks, 1991, Hall and Agyemang-Duah, 1991). The breakdown capacity follows the traditional definition which states that the capacity of a freeway is:

> *"the maximum flow rate that can reasonably be expected to traverse a uniform segment of road under prevailing roadway, traffic and control conditions"*.

The term 'reasonably' here indicates that this will not always be the highest observed flow, even though that will often be presumed to be the case. Furthermore, the breakdown capacity will most often be observed on the transition from an undersaturated traffic state to an oversaturated state, or rather an uncongested to congested state. It should be noted that although there are various levels of saturation, there is a definitive oversaturated point in traffic flow, which can be observed when traffic flow no longer can increase and starts to decrease with additional demand. The *discharge capacity*, on the other hand, follows from the realisation that oversaturated traffic flow yields a reduced flow compared to undersaturated traffic flow. Therefore, the *discharge capacity* is defined as the:

> *"maximum flow rate that can reasonably be expected on a uniform segment of road in an oversaturated traffic state under prevailing roadway, traffic and control conditions"*.

A graphical demonstration of the capacity definitions is given in Figure 3.2 in a fundamental diagram. The fundamental diagram shows the relation between traffic flow and density in the traffic domain. Both the breakdown and the discharge capacity can be described as either stochastic or maximum depending on the method used to calculate and describe them.

The difference between the capacity before breakdown and the discharge capacity on a road section is known as the *capacity drop*, referring to the fall in capacity frequently observed after traffic breakdown between observations in a critical undersaturated traffic state and an oversaturated traffic state. In many cases, the capacity before breakdown is taken as the maximum observed capacity. However, the breakdown capacity is also used as the maximum capacity and is dependent on an incidental observation that may increase for a longer observation time. The capacity drop is a phenomenon that can arise once congestion occurs on a road (Banks, 1991, Daganzo et al., 1999, Hall and Agyemang-Duah, 1991, Kerner and Rehborn, 1997, Kim and Coifman, 2013). The occurrence of the capacity drop is due to the so called hysteresis effect that is not explained here, but can be found in the suggested literature.

In summary, in this chapter a distinction is made in capacity definitions between:

- *Maximum Breakdown Capacity*
- *Maximum Discharge Capacity*
- *Stochastic Breakdown Capacity*
- *Stochastic Discharge Capacity*

The main focus in this chapter is on stochastic capacity estimations for both breakdown and discharge capacity types. We will refer to the *stochastic breakdown capacity* when the capacity is derived using a stochastic or probabilistic capacity method and derived from stochastic data. We furthermore refer to the *maximum breakdown capacity* for a pre-breakdown capacity indicating the traditional definition: the maximum flow which can reasonably be expected. Furthermore, the capacity drop will be referred to as the *stochastic capacity* drop when one determines it using the *stochastic breakdown capacity* as the initial capacity value.

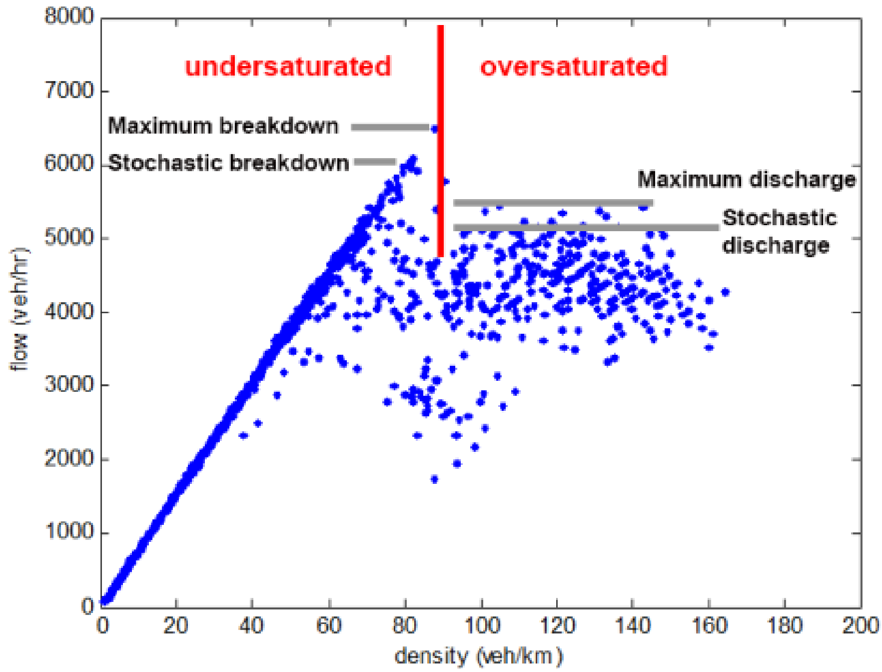


Figure 3.2: Graphical overview of capacity definitions

3.3 Conceptual Model of Capacity Variation

Variations in capacity stem directly from stochastic driver behaviour, not only from individual drivers, but also between drivers. Furthermore, a drivers' behaviour can also vary in time and space. The mathematical description of capacity is directly linked to that of the traffic flow and is inversely proportionate to the average time headway of traffic. The capacity of a road is then the traffic flow for the smallest mean time headway before traffic flow breakdown, for the breakdown capacity, or after breakdown in case of the discharge capacity. From the relationship between the time headway and flow, it is evident that there is a direct relationship between driver behaviour and capacity. As the actions of a driver are variable, the ability of a driver to traverse a road at a certain time headway to their predecessor is also variable. Moreover, this ability is also subject to the prevailing conditions of both the driver and the driving conditions. Therefore, one can clearly derive that the capacity of a road is also subject to an accumulation of these conditions.

There are a number of known factors that directly or indirectly influence road capacity. For some of these factors, (exploratory) quantitative research has been performed, for others the quantitative relationship is less well researched. In Figure 3.3, an overview of the main known variables is given, including an indication of which variables influence each other.

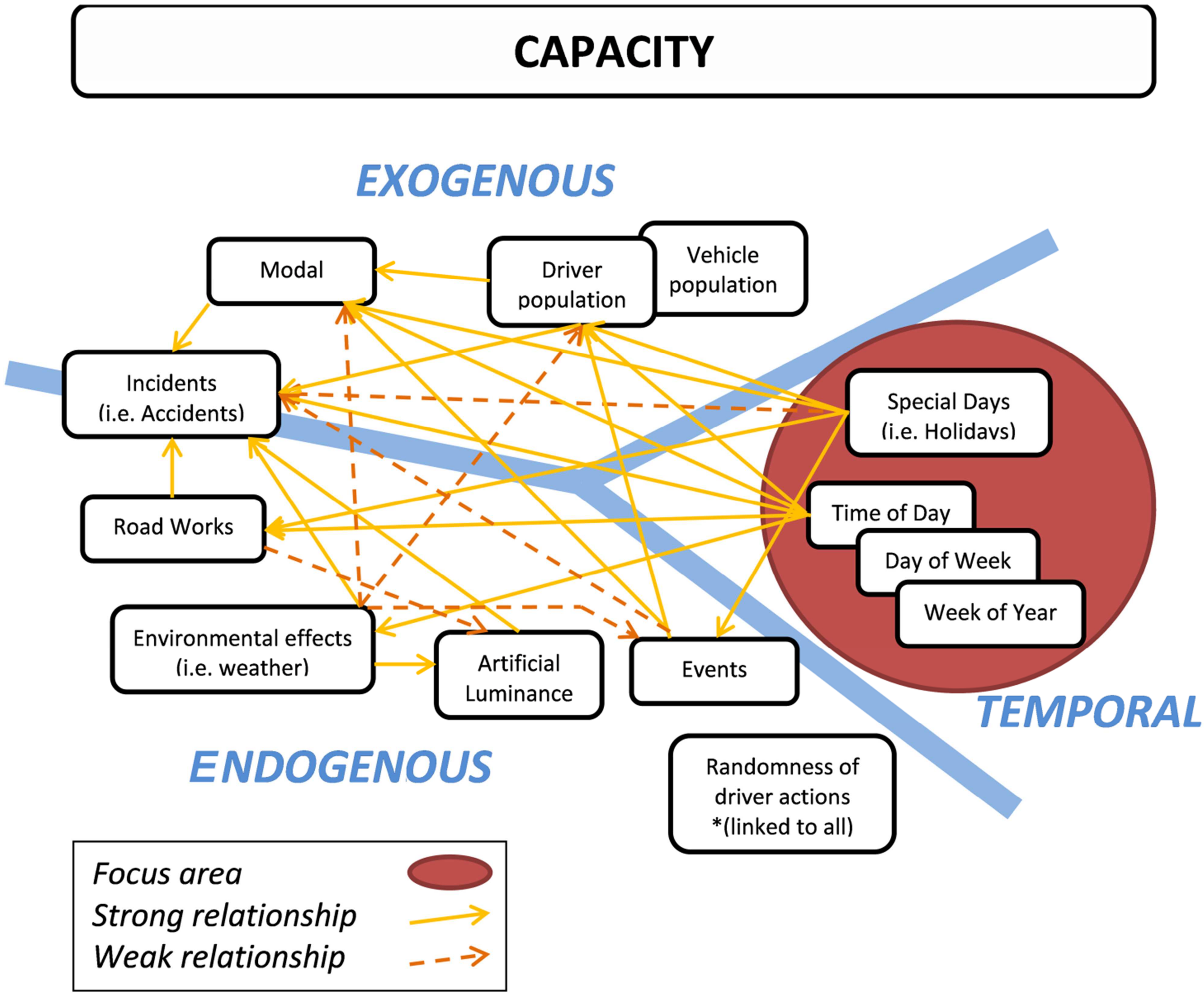


Figure 3.3: Overview of important capacity influencing variables and their relations

Besides the variables shown in Figure 3.3, there are many more unknown variables that may have a (small) effect on capacity. Due to their limited influence, most variables can be ignored or summarised in a general stochastic variable. For most variables mentioned in the figure, literature exists that indicates the qualitative influence on road capacity and in some cases also small scale explorative quantification. Weather effects, road works and incidents in particular have been well studied for their capacity reduction influence, while others have to a lesser extent. In this chapter, the focus lies on capacities on holiday days and particular days of the week and on the effects of weather. Apart from incidents and road works, which are filtered out in this research, these specific days are largely affected by the modal split, the driver and vehicle population, and, to a lesser extent, the presence of major events and certain weather conditions.

Previous research has shown that capacity values for the days of the week can differ significantly (Cools et al., 2007, Yeon et al., 2009). Yeon et al. (2009) even demonstrated the possibility that capacity values may be different during a day and intra-day. However, their research also showed that the trends can be extremely location dependent, and that opposite trends can be found if one considers only a subset of the considered locations. Nevertheless, the research demonstrated the importance of considering different capacity values at different moments during a week. The research described in this chapter expands this analysis to include weekend days and designated holiday days.

A further distinction in location can be made using bottleneck types. A large number of papers have investigated capacity values for specific bottleneck types (Boyles et al., 2011, Calvert and Minderhoud, 2012, Elefteriadou and Heaslip, 2008, Laval, 2006, Roess and Ulerio, 2009, Yeon et al., 2009). The bottleneck types are indicated in this research, but are not explicitly considered for their effect on capacity values due to the limited sample size per segment type.

3.4 Methodology for stochastic capacity estimation

Construction of probability distributions of the day-specific capacity is performed on the basis of extensive empirical analysis with data. The general methodology of data processing and capacity estimation follow the steps given in below. The methodology exists of two main processes: data processing and capacity estimation which are explained in more detail in section 3.4.1 and 3.4.2. These processes are repeated for each of the considered variables, such as demand and capacity at the highest level, to give an outcome for each variable in the form of two probability distributions, for which a distribution fit is made. The steps are as follows:

Step 1. Bottleneck selection
Known freeway bottlenecks

Step 2. Traffic state detection
Flowing, Breakdown and Congestion

Step 3. Data filtering
Scenario based

Step 4. Capacity estimation
Stochastic breakdown and discharge capacity

Step 5. Distribution fitting
Distribution parameters

3.4.1 Data processing

Data from 23 known bottleneck locations on the Dutch motorway network is gathered for a three year period. Flow and speed data, as well as lane availability data, is collected per minute at and near each location from induction loops. The data is filtered and checked for missing data and validity. The locations are selected from 11 different motorways throughout the Dutch motorway network and such that the locations are separated by major interchanges to avoid substantial interchange effects.

The traffic states upstream and downstream of each bottleneck location are recorded at a location as close to the bottleneck as possible, generally at a distance of 100-300 meters. An aggregation level of 5-minute intervals is chosen to reliably capture traffic states without the period becoming too large. Three traffic states are defined in the labelling process: *free flow traffic* (F), *breakdown conditions* (B), and *congested traffic* (C). These are defined as (Brilon et al., 2005):

> *Free flow traffic* (F): Traffic is in a free flow traffic state for speeds above 60 km/hr in the considered time interval t and remains in an uncongested traffic state in the following time interval t +1.
>
> *Breakdown conditions* (B): Traffic is in an uncongested traffic state in the considered time interval t, however it is in a congested state in the following time interval t +1. A congested state is assumed when traffic speed drops below 60 km/hr for the entire interval t.
>
> *Congested traffic* (C): Traffic is in a congested traffic state upstream of the active bottleneck in the considered time interval t, and remains in congested state in the following time interval t+1. Traffic flow downstream of the bottleneck is uncongested.

A threshold of 60 km/hr is applied as traffic breakdown on motorways generally results in a prompt decrease in traffic speed from 70 km/hr to 50 km/hr from the undersaturated to oversaturated traffic state, therefore the chance of erroneous labelling is kept low by using the 60 km/hr threshold. Furthermore, dynamic speed limits are applied in congestion of 50 km/h through Variable Message Signs (VMS). However, compliance is low and these speed restrictions are generally reactive to traffic conditions.

For each location at every time interval, a large number of characteristics are recorded regarding the weather conditions (i.e. rain, snow, temperature, (natural) luminance), and the type of day (i.e. day-of-week, holiday, season, peak periods). Data is filtered corresponding to the scenarios using the labelling, which filters the relevant traffic data during the considered period. Details on the filtering and data labelling can be found in Calvert and Snelder (2013b). This results in a data set based on a collective variable for that specific scenario.

3.4.2 Capacity estimation

A number of capacity estimation methods exist, which make use of different assumptions and capture capacity values in different ways. For an overview of many of these methods, see

Minderhoud et al. (1997), and more recently on stochastic methods: Geistefeldt and Brilon (2009).

In this research, a distribution of the capacity for each scenario and each bottleneck location is derived from the filtered data, for both stochastic breakdown and discharge capacity. This is performed for the stochastic breakdown capacity through application of the Product Limit Method (PLM) as described by Brilon et al. (2005) and recommended in the mentioned capacity estimation reviews. Traffic flow observations in free flow traffic (F) and of breakdown traffic (B) observations are used. Using data of non-breakdown events (F), as *censored data*, that are nevertheless greater than traffic flows that have led to a breakdown improves one's ability to accurately determine a capacity distribution. The method makes use of a probability function which is used to estimate the probability of traffic breakdown, with the median being the presumed capacity with an uncertainty margin given by the shape of the distribution. Function *F(q)* is defined as the probability that a detected traffic flow value reaches a state of congestion. The method is described by two main equations:

$$F(q) = 1 - Prob(q_c \leq q) \tag{3.1}$$

$$F(q) = 1 - \prod_{q_i} \frac{K_{q_i} - 1}{K_{q_i}} \quad with \;\; q_i \in \{B\} \tag{3.2}$$

Where K_{qi} = total number of observations with intensity q_i larger than the congestion threshold intensity q_c

B = set of breakdown observations

The distribution of the discharge capacity is determined from the discharge flow of traffic through the bottleneck during a congested traffic state (C) following a traffic breakdown. The discharge capacity is much simpler to calculate as this can be continuously observed in the bottleneck or at the outflow of the bottleneck location (Minderhoud et al., 1997):

$$G(q) = q \quad with \;\; q \in \{C\} \tag{3.3}$$

Where q = traffic intensity

C = set of congested observations

The entire set of discharge flow observations are used to construct the probability distribution of the discharge capacity for that specific location. For the stochastic breakdown capacity, all uncongested data is also considered, but only uncongested data that exceeds the lowest breakdown observation is applied in the PLM. In previous research (Brilon et al., 2005, Brilon and Zurlinden, 2003), it was shown that the Weibull distribution gives a good fit to probabilistic capacity distributions on freeways. The Weibull distribution is similar to a Gaussian distribution in shape, but has a greater flexibility towards the tails of the distribution. This allows for a greater power to fit empirical data. Weibull distributions make use of a scale and a shape parameters, and are defined as:

$$F(x) = 1 - e^{-\left(\frac{x}{\beta}\right)^{\alpha}} \text{ for } x \geq 0 \tag{3.4}$$

Where α = shape parameter

β = scale parameter

To test the presumption that the Weibull distribution is suitable to fit the considered datasets, a goodness-of-fit test is carried out. There are different statistical tests available for the goodness-of-fit to a distribution. Jia et al. (2010) correctly argue that the Kolmogorov-Smirnov (KS) test is best suited to test capacity distributions in such a way, as it quantifies a distance between the empirical distribution of the sample and the cumulative distribution of a reference distribution. More importantly, the KS test is distribution free and therefore makes no assumption with respect to the underlying distribution (Chakravarti and Laha, 1967, Jia et al., 2010). The KS test is also an exact test while some other commonly applied tests, such as the chi-squared-test, depend on an adequate sample size to validate approximations (Jia et al., 2010, Ross, 2009). For details on the workings of the KS-test, the reader is referred to one of the many statistical textbooks on the subject, such as: Ross (2009).

The KS-test is carried out on four distributions types: *Normal*, *Weibull*, *Gamma* and *Lognormal* distributions. These distributions types have been previously found to have the potential to show a good resemblance to fit empirical capacity data on various motorways (Brilon et al., 2005, Chow et al., 2009, Elefteriadou and Heaslip, 2008, Jia et al., 2010, Kondyli et al., 2013, Minderhoud et al., 1997). The capacity data from each location is fitted with each distribution type to produce a distribution, which is compared with the empirical distribution from the data. The resulting KS-test values are shown for each location for both the stochastic breakdown capacity and the discharge capacity in Table 3.1a & 3.1b.

Table 3.1a-b: Kolmogorov-Smirnov goodness of fit tests for empirical capacity data

K-S scores: Stochastic Breakdown Capacity				
	Distribution			
Location	Normal	Weibull	Gamma	Lognormal
1	0.127	0.079	0.133	0.139
2	0.097	0.071	0.102	0.107
3	0.264	0.262	0.269	0.271
4	0.123	0.078	0.129	0.131
5	0.152	0.165	0.148	0.144
6	0.135	0.155	0.138	0.144
7	0.080	0.027	0.094	0.100
8	0.063	0.028	0.069	0.074
9	0.166	0.154	0.172	0.178
10	0.081	0.046	0.092	0.097
11	0.098	0.099	0.106	0.112
12	0.192	0.177	0.197	0.201
13	0.101	0.091	0.107	0.112

K-S scores: Discharge Capacity				
	Distribution			
Location	Normal	Weibull	Gamma	Lognormal
1	0.077	0.136	0.074	0.076
2	0.044	0.039	0.057	0.064
3	0.086	0.046	0.098	0.106
4	0.068	0.024	0.082	0.088
5	0.131	0.103	0.156	0.168
6	0.057	0.034	0.070	0.076
7	0.030	0.064	0.023	0.029
8	0.126	0.075	0.150	0.162
9	0.095	0.045	0.108	0.115
10	0.067	0.039	0.083	0.092
11	0.062	0.037	0.076	0.084
12	0.259	0.284	0.214	0.192
13	0.059	0.041	0.072	0.078

14	0.111	0.082	0.116	0.121
15	0.214	0.261	0.211	0.216
16	0.107	0.068	0.115	0.119
17	0.173	0.138	0.180	0.186
18	0.206	0.207	0.207	0.206
19	0.139	0.115	0.145	0.150
20	0.187	0.140	0.193	0.195
21	0.080	0.070	0.088	0.093
22	0.137	0.114	0.142	0.146
23	0.117	0.110	0.124	0.129

14	0.292	0.314	0.248	0.226
15	0.048	0.033	0.060	0.067
16	0.049	0.055	0.060	0.066
17	0.097	0.047	0.110	0.117
18	0.034	0.043	0.049	0.056
19	0.041	0.032	0.053	0.058
20	0.023	0.051	0.036	0.043
21	0.087	0.036	0.098	0.103
22	0.050	0.047	0.063	0.070
23	0.097	0.055	0.118	0.130

From the table, it is shown that the Weibull distribution was confirmed to fit well for most surveyed locations. While each road location is unique and has its own characteristics, the goodness-of-fit test shows that for the considered locations, the Weibull distribution is a valid distribution. An optimization of the Weibull parameters is performed using the root mean squared error (RMSE) as performance indicator for fitting, denoted by:

$$RMSE = \sqrt{\frac{1}{n}\sum_{j=1}^{n}(y_j - \hat{y}_j)^2} \tag{3.5}$$

Where n = number of estimated observations

y_j = observed capacity estimation at probability interval j

$\hat{y}_j$ = Weibull fitted capacity estimation at probability interval j

The RMSE is applied here as a simple and effective test of the consistency between the median values from the Weibull distribution, rather than for the goodness-of-fit test which has already been performed using the Kolmogorov-Smirnov test. For such an optimization of the parameters, such a test more than suffices.

The entire procedure produces an empirical distribution and Weibull parameters which best fit the empirical distribution. This is performed for each scenario (see section 3.4.3) at each bottleneck location, and for both the stochastic breakdown capacity and discharge capacity. An explicit example of the PLM methodology can be found in (Brilon et al., 2005) and is therefore not given here.

3.4.3 Day-specific capacity case scenarios

The scenarios considered in this research are *workdays*, *weekend days*, and *national holidays.* The *stochastic capacity drop* in each scenario is also included in the analysis, for which the median (the 50th percentile) from the capacity distribution is applied. Workdays are defined as week days from Monday through Friday that do not fall in holiday weeks or on national holidays. Both Saturdays and Sundays are considered for weekend days. Holiday days are defined as official national school holidays, which normally also correspond to a significant reduction in work traffic. Data is gathered and used for the years 2007-2009, due to availability and quality of the data from those years.

3.5 Methodology for combined stochastic capacity-demand estimation

3.5.1 Framework

The applied methodology makes use of a combined approach using some existing methodological elements from both traffic theory and data analysis, while introducing some new methods. Figure 3.4 gives a complete overview of the main parts of the methodological framework. On one side, a comprehensive capacity estimation is performed using the adapted Product Limit Method in which 25 bottleneck locations are considered during a three year period and from which capacity estimations are made in the test case. The capacity estimations also include a stochastic estimation of the probability of various capacity values. On the other side, an estimation is made of traffic demand following a traffic cordon inflow approach. This approach records the inflow of all traffic into a specific network area during a set time period and derives the traffic demand therefrom. In this research, a 142 location cordon is applied and considered during a 4 year period. Both the capacity and demand parts are fed with detailed traffic data with minute-to-minute accuracy for both the traffic flow and speed. Furthermore, detailed hourly weather data is acquired for all periods indicating a wide range of weather conditions and their corresponding data. For capacity estimations, a further source of data is available in the form of minute-to-minute radar data with an accuracy of approximately 1 kilometre, allowing specific capacity estimation, as capacity is moment-in-time observation. Finally, both the capacity and demand estimations are combined to give an estimation of the effect on traffic fluency. This is performed through a simple division of the change in capacity by the change in traffic demand, which means that if both variables change at the same rate, that the effect on traffic fluency will remain identical. The traffic fluency is given by:

$$qf = \frac{q_{cap}/q_{cap.ref}}{D/D_{ref}} \tag{3.6}$$

Here, qf is the traffic fluency, $q_{cap.ref}$ is the reference capacity under average conditions, while q_{cap} is the capacity under the specified conditions. Similarly, D_{ref}, is the reference traffic demand, while D is the scenario demand. While a lower capacity value may reduce traffic flow, a reduction in demand can counteract the ability of traffic to flow fluently. Therefore, only a combination of both gives an accurate estimation on the actual effect on traffic fluency. Note that traffic fluency here denotes the level-of-service or ability of traffic to flow rather than a quantitative value of flow. In the following subsections, a more detailed description is given of the various parts of the methodology.

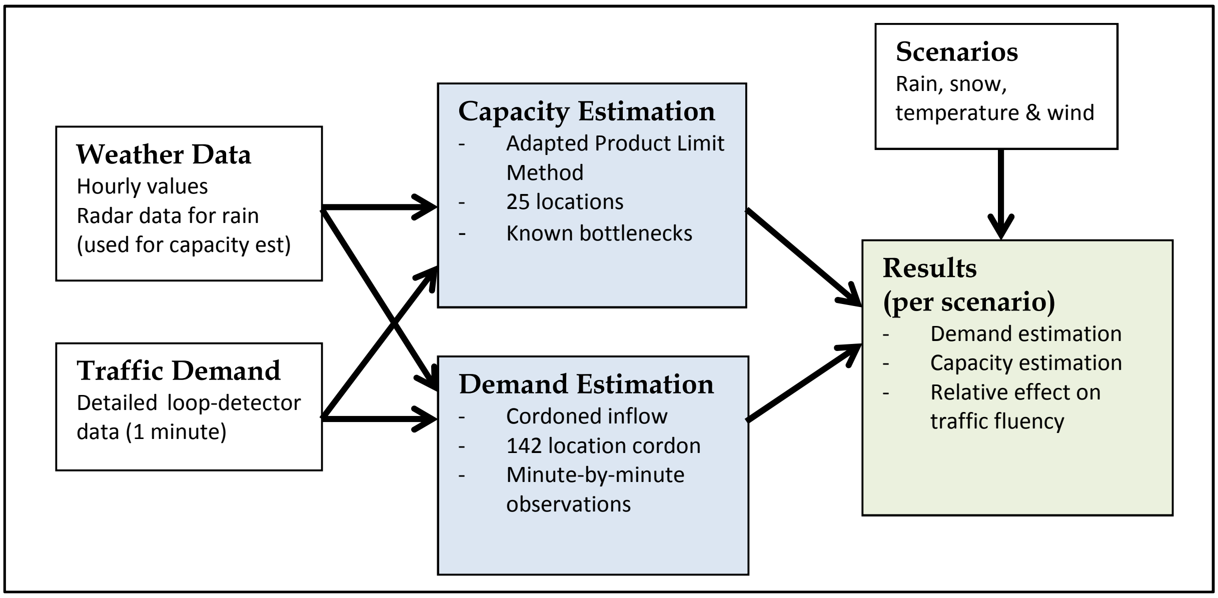


Figure 3.4: Applied framework for stochastic demand-capacity estimation

3.5.2 Capacity analysis

In this research, a stochastic capacity estimation method is applied that allows probability distributions of capacity to be constructed which envelop the full range of possible capacity values. The applied method is based on the Product Limit Method (PLM) as described by Brilon et al. (2005) and adapted from Kaplan and Meier (1958). The method was already explained in section 3.4.

3.5.3 Demand analysis

Calculation of changes in the traffic demand is performed through empirical data analysis of a cordoned area of a motorway network in a region. Maintaining a cordon around the entire network reduces external issues that may bias the demand results as previously described. Such biases may occur from rerouting to other parts of the same network or from certain areas of the network reacting differently to other areas. Although the approach cannot entirely rule out small disturbances, the approach substantially decreases the chances thereof. An example of the cordon used in this chapter is given in Figure 3.5.

The total daily demand is calculated for a desired time period: starting at time t_s and ending at time t_e. It may be relevant for example for research to just collect demand during the morning peak period, as this gives a good indication of the total demand on that day. It should be noted that the traffic demand is not identical to the observed flow, as traffic may be delayed either in the considered network or in the approach to the network. To reduce this effect so that the actual demand resembles the measured flow, the times t_s and t_e should be chosen such that no or very limited congestion exists on the network and especially on the cut-off points of the network. For no or limited congestion, it should be expected that most if not all traffic that demanded, has had the opportunity to enter the network.

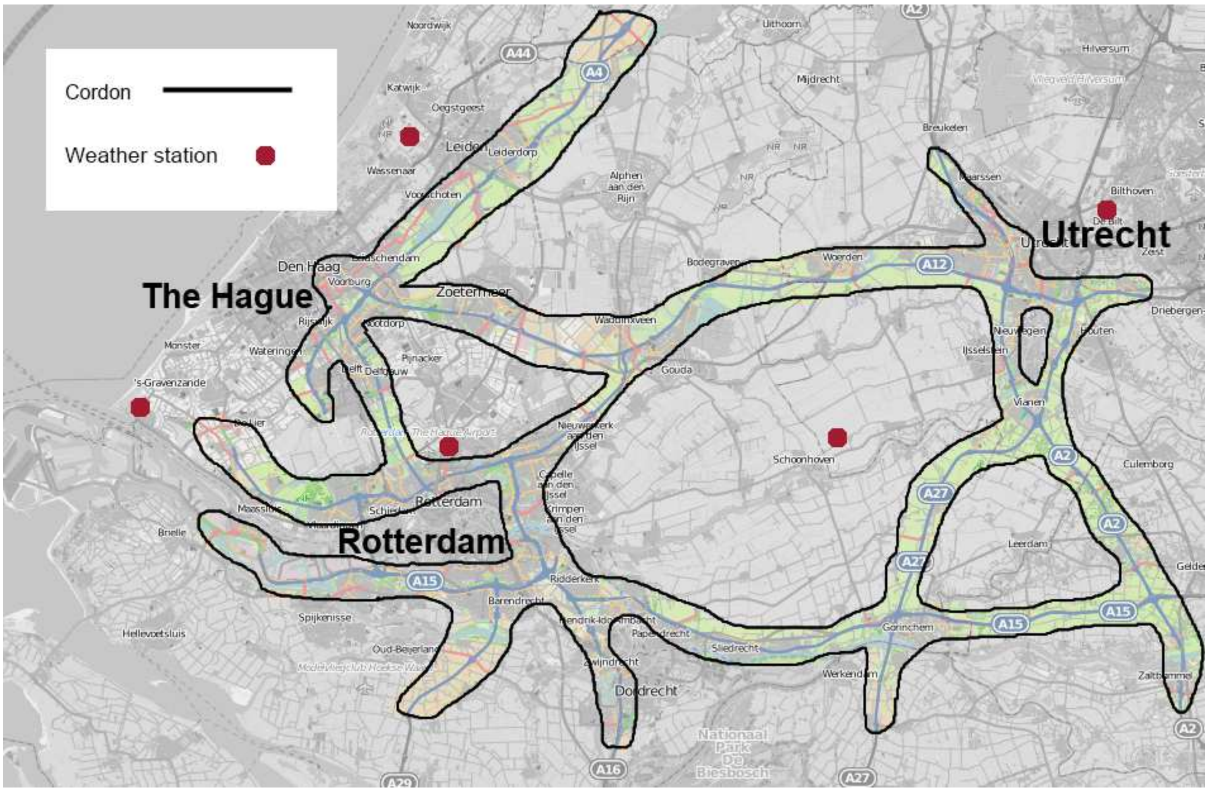


Figure 3.5: Considered network for the data analysis

The inflow into the network q_i, is collected at each inflow location i, and time moment t, into the network at both cut-off points on motorways as well as motorway junctions (if one is considering only a motorway network). The sum of all locations at a single time step t, gives the total inflow into the network, however this is not yet the demand as congestion delays the arrival of traffic in time. However, summation over time, for which no congestion is longer present and in which delayed vehicles have the chance to pass the detectors, allows a reliable estimation to be made of the demand. For a scenario k, on an arbitrary day d, the total demand $D_{k,d}$ in a time period, $[t_s , t_e]$, is given by:

$$D_{k,d} = \sum_{t=t_s}^{t_e} \sum_{i} q_{i,t} \tag{3.7}$$

On its own, the value of $D_{k,d}$ does not have any significant meaning, as a network or scenario can be arbitrarily chosen. Therefore, a demand $D_{k,d}$ is considered as part of a coherent scenario k for which the main scenario characteristics are kept identical. Careful selection of the scenario characteristics is important to be able to make a fair comparison between various days of $D_{k,d}$ within k. A careful consideration of the main variables, such as the type of day (week, weekend, holiday, etc.) or of other important characteristics should be made. Availability of all selected detection locations should also be consistent for all days that are to be compared to avoid inconsistent measurements in the collected demand. Once multiple days of a single scenario have been gathered, one may construct an empirical distribution of the observations of that scenario:

$$D_k = \{D_{k,1}, D_{k,2}, \dots, D_{k,n}\} \tag{3.8}$$

Here, n denotes the number of observation days for the considered scenario. In most cases, it will be desired to compare scenarios to gain insight into the effects of certain characteristics on the traffic demand. Therefore, a reference scenario should be defined that is considered as a 'neutral' scenario. For example, in the analysis in the following sections, dry weather conditions are considered as a base scenario which against other, sometimes overlapping, scenarios are compared. Comparison between the considered scenario, D_k, and the reference scenario, D_{kr}, is performed such that a ratio, r, between the scenarios is derived:

$$r = \frac{median(D_k)}{median(D_{kr})} \tag{3.9}$$

The calculated ratios are then applied for comparison between scenarios and as a strong indication of the effects that a scenario has compared to the reference scenario.

3.5.4 Stochastic capacity and demand for weather

In the previous sub-sections, the methodology for determination of the capacity and demand were presented. Here, the applied characteristics of the methodology in this research are given. This starts with the locations and data sources and is followed by the considered weather scenarios.

Demand

Quantification of traffic demand is performed for an enclosed area of the motorway network in the west and central areas of The Netherlands, which includes the cities of The Hague, Rotterdam and Utrecht (see Figure 3.5). The total area is approximately 1200 square kilometres in size. At the 'cut-off' points on the motorways and on motorway entrances[1] and junctions along each motorway data is collected of the total inflow with a minute accuracy from loop-detectors. The vast majority of all junctions were able to be analysed and totalled 142 locations in all. The data used for the demand calculations is taken from the years 2009-2013 (for 2013 only until June). The demand values are collected for two different periods. The first considers the demand throughout the whole day between 5 AM and 10 PM. The second only considers the demand during the morning peak period between 6 AM and 10 AM. A distinction is also made between the time of the day for which the weather classification is performed: either for just the morning or the entire day. This results in four demands per scenario: Day weather with morning or day demand, and morning weather with morning or day demand. A further filtering is applied to the collected data. A minimum of 20

[1] Often the flow at entrances will need to be measured indirectly by subtracting the downstream flow from the upstream flow before the entrance location.

observations (days) are taken per scenario to sufficiently make an accurate estimate of the demand profile for that scenario.

Capacity

The capacity analysis is carried out in the same region of the Netherlands at known and proven bottleneck locations. In total, 30 bottleneck locations were initially selected from which capacity data could be accurately collected through the use of double loop-detectors according to the previously described methods. Of these 30 locations, a further five locations were later rejected as the data was not consistently able to produce a sufficient number of reliable capacity estimations, leaving 25 locations that produced reliable and accurate capacity estimations. These included 20 2-lane motorway sections and five 3-lane sections. The data used for the capacity estimations is taken from the years 2007-2009. It was not easily possible to extend to later years as the data collection algorithm had changed for the later years, which may give undesired discrepancies in the data between the years.

Weather scenarios

In this research, four main types of weather conditions are considered, namely *rain*, *snow*, *temperature* and *wind*. The focus is on each individual weather type separately, rather than a combination of various types in one scenario. This means that correlation is not explicitly considered between the results. An example is snowfall that is recorded for temperatures below 2 degrees Celsius. While the scenario snow will overlap with low temperatures, the opposite will not necessarily be the case. Rather than search for causality, these are accepted in this research. It is a subject of later research to look closer at the specific correlations.

The previously described methods for capacity and demand estimation obviously must make use of data on weather and climatological conditions. For this, use is made of stationary weather stations administered by the Royal Netherlands Meteorological Institute (KNMI). The KNMI makes use of more than 30 high quality meteorological stations throughout The Netherlands which relay accurate and extensive hourly and daily data on wind, temperature, sunshine, radiation, precipitation, air pressure, visibility, humidity, and other categorical weather observations. The five weather stations in the considered area are shown in Figure 3.5. For each category, maximum and average hourly and daily values are collected as well as descriptive information relating to these values. Detailed information on the exact measurement apparatus and techniques can be found in KNMI (2014).

From the four weather conditions, ten scenarios are defined. In each scenario, the weather conditions are considered for the hours between 5 AM and 10 PM. This is also the period for which quantities are observed. For a day to be considered for a weather condition, the average value of that weather condition must be present for at least 3 hours during that day at, at least, three of the five weather stations. This last condition is almost always met due to the close proximity of the weather stations, and conditions are nearly identical on a day-to-day basis. As an example, if rain category 1.4-1.9 mm is considered, then this intensity must be found for at least 3 hours during the day. For the demand data, only the weather during the morning peak, the hours between 6 AM and 10 AM, are considered. For the morning demand, only a single hour average needs to fit the relevant weather condition category. Furthermore, only data is considered for weekdays and for non-holiday days to avoid pollution of the data with possible trends from these day types. Seasonal trends are implicitly allowed, also due to the fact that the scenarios are explicitly correlated to certain seasons.

The scenarios are defined as:

1. Dry (Reference scenario)
2. Rain: 0-0.1 mm
3. Rain: 0.2-1.3 mm
4. Rain: 1.4-1.9 mm
5. Rain: >=2.0 mm
6. Snow: >0 mm
7. Temperature: <2C
8. Temperature: >=2C
9. Wind: <40 kph (<6 knots)
10. Wind: >=40 kph (>=6 knots)

For clarity, the data processing steps for the weather conditions are reiterated: The prevailing weather conditions for each day are labelled against that day according to the above scenario's and the described prerequisites. This is performed for both time windows: 6-10 AM and 5AM-10PM. The demand analysis and capacity analysis are performed for each day, as described in the previous sections per day. For each scenario, the days that contain the relevant weather condition are viewed. The median of corresponding demand and capacity values over these days is taken and presented in section 3.6.

It is reiterated that the category values are hourly totals for rain and snow, and hourly averages for temperature and wind. In relation to precipitation, this means that it is highly probable that higher values were found during that hour, but were averaged out. Therefore, we cannot speak of precipitation intensities, but rather of the quantity of precipitation. A conversion table is given in section 3.7.2 to allow global comparison of the results with other literature. It is presumed that travellers will perceive a day (or part of a day) to be of a certain weather category, rather than focus on a specific precipitation intensity at one single moment. This does not apply to the capacity estimates, as they are coupled to radar data that gives minute-to-minute and kilometre precise rain observations. This is necessary as the influence of precipitation cannot be averaged over an hour for capacity, as capacity observation is a 'moment-in-time' observation.

3.5.5 Capacity hypotheses

In this chapter, the focus lies on the methodology for the quantification of the probability distribution of capacity values in operational day-specific traffic. Elefteriadou and Lertworawanich (2003) recommended that capacity distributions should be constructed to indicate the extent of variations in capacity values. They also recognised that these capacity distributions are subject to change under various conditions. Research has resulted in various capacity distributions for a number of variables. However, a generic value for day specific variation, based on a sufficiently large set of data, does not exist. Here, the aim is to also give a quantification of day-specific capacity variation for workdays, weekend days and holiday days and also to give distributions of the capacity drop for the situation on the Dutch motorway network. Four hypotheses are constructed and tested in relation to the capacity shifts between the considered scenarios. These hypotheses are:

1. Mean capacity values on weekend days and holidays are lower than for workdays.
2. On weekend days there is a lower mean capacity than on holiday days.
3. The variation of the capacity, measured in standard deviation, is greater on weekend days and holidays than for workdays.
4. There is no significant change in the capacity drop for weekend days and holiday days in relative terms in comparison to workdays.

3.6 Results: Day-specific capacity variation

The results of the data analysis for day-specific capacity variation are given by capacity type (stochastic breakdown or discharge) and are compared for capacity shifts between scenarios. A quantification of the capacity and the variations is given. The results are shown in three parts:

1. Comparison of weekend and holiday capacity estimations versus workday capacity and standard deviations. Hypothesis 1, 2 and 3 are tested.
2. Estimation of the variation in the stochastic capacity drop between the three scenarios. Hypothesis 4 is tested.
3. Mean stochastic capacity and stochastic capacity drop values and the estimated Weibull distribution fit for the each scenario.

The average values of the median capacity of each motorway location and standard deviation thereof, as well as both Weibull parameters and the stochastic capacity drop, are given in Appendix 3.A. This is performed for both the stochastic breakdown and the discharge capacity estimations for all three scenarios. The individual results are also given in Appendix 3.A for the workday reference scenario.

3.6.1 Day-type specific capacity estimation

In this section, hypotheses 1 through 3 are tested and mean values are given for the estimated capacity over all considered locations. A comparison is also made between the considered scenarios: *Workday traffic*, *Weekend traffic*, and *Holiday traffic*. These are statistically tested for significance using Levene's test for equality and the t-test for equality of means with a 95% confidence level. The hypothesis is tested for significant differences in the means of the workday, weekend day and holiday capacity values. The test is performed for each of the capacity types for the mean as well as the standard deviation of the results. The t-test presumes normality, which is also presumed here between locations. As the samples are median values from different locations, the collective set of all samples should verge towards a normal distribution according to the central limit theorem and therefore confirm normality. The analysis of the capacity results are summarised for the stochastic breakdown and discharge capacity over all the observed locations and are given as follows:

1. Distributions of *workday* traffic show a median capacity value for the stochastic breakdown and discharge capacity of 2260 and 1793 veh/hr/lane (-26%).

2. The *weekend* traffic returns average capacity values of 2131 and 1622 veh/hr/lane (-31%).

3. *Holiday* traffic results in average capacity values of 2265 and 1719 veh/hr/lane (-32%).

All these values are reasonable values for Dutch motorways. For the sake of comparison, workday traffic is taken as a base against which the weekend and holiday traffic capacity values are compared. The distribution of the median stochastic breakdown capacity values from the PLM-analysis for the scenarios is shown in the boxplot in Figure 3.6a as well as for the standard deviation in Figure 3.6b. From Figure 3.6 hypothesis 1-3 can be answered:

1. The mean stochastic breakdown capacity on weekend days is 4% lower compared to workdays, however the capacity on holidays is not significantly lower (with 95% confidence) compared to workdays.

2. The mean stochastic breakdown capacity is found to be significantly lower on weekend days in comparison to holiday days. The difference is again in the range of 4%.

3. The variation of the capacity, measured in standard deviation, is found to be significantly lower for both weekend traffic and holiday traffic compared to workday traffic. The difference in the median standard deviation is circa 15% and 3.5% respectively.

The results of the average median discharge capacity values and the average standard deviations are given in Figure 3.6c-d. The discharge capacity for weekend days is again lower than that of workdays. The reduction in the discharge capacity is approximately 8% compared to the discharge capacity during workdays. The discharge capacity for holidays is also lower in comparison to workdays by approximately 3-4%. The reduction in the standard deviation is less extreme for the discharge capacity on weekend days, while the standard deviation on holidays does not significantly change over all locations. These results are further discussed in the section 3.8.

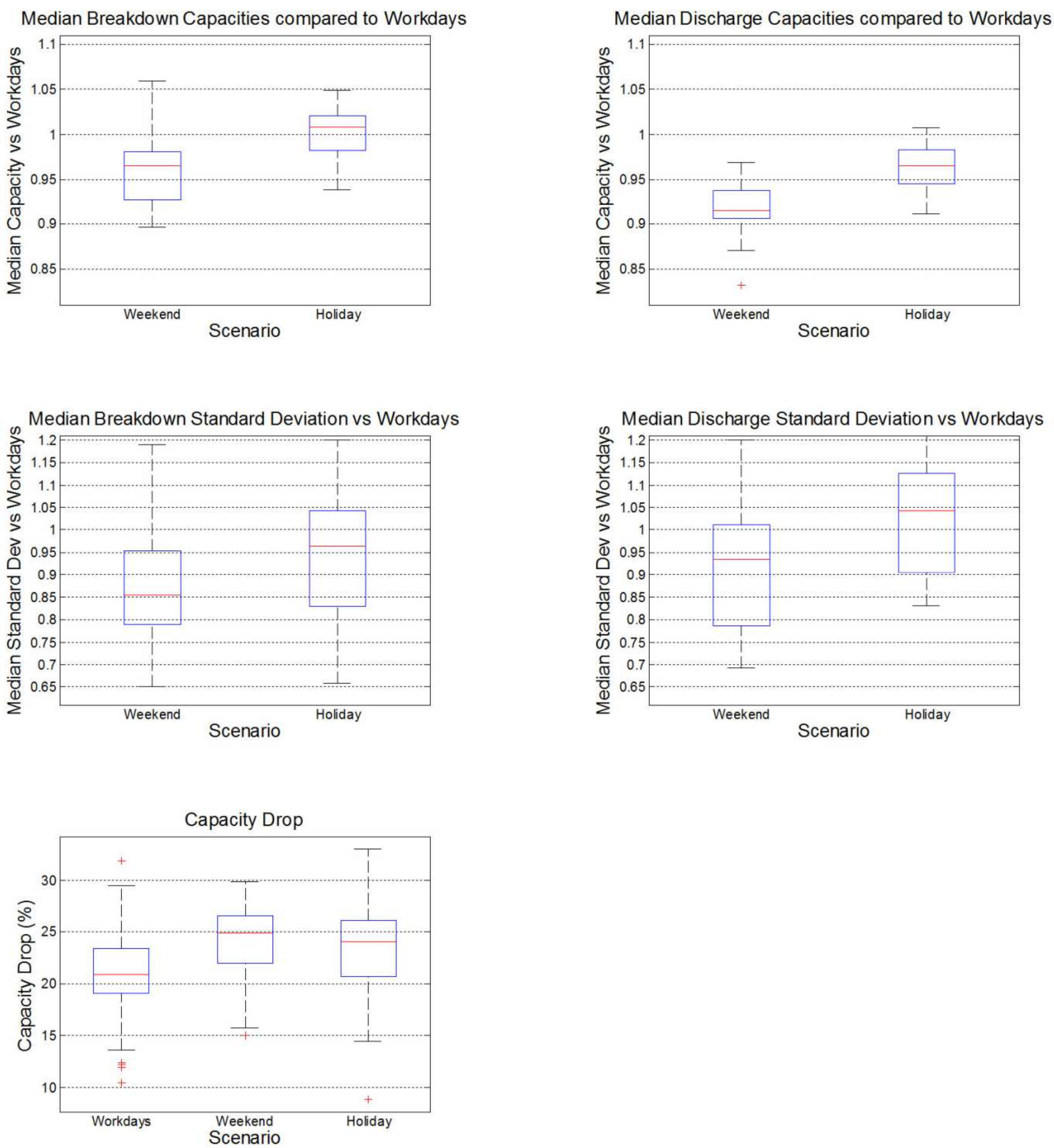


Figure 3.6a-e: Capacity estimations in comparison to the workday scenario

3.6.2 Capacity drop estimation

Hypothesis 4 states that there will be no significant difference in the stochastic capacity drop between workday traffic and weekend and holiday traffic. The stochastic capacity drop over all 23 locations for the three scenarios is shown in the boxplot in Figure 3.6e. The median value for the stochastic capacity drop over each scenario is estimated at 20.5%, 24.9% and 23.6% for workdays, weekend days, and holidays respectively. The difference between workdays and weekend days is shown to be a significant difference. As the discharge capacity was shown to decrease by a greater amount than the breakdown capacity, this also indicated a widening between the capacity types and an increase in capacity drop. The spread of the observed values in each scenario is relatively similar in each scenario. It should be noted that correlation is present in the results. For example, a location with a lower capacity drop in one scenario may be expected to also have a relatively low value in another scenario. This has to do with the intrinsic characteristics of bottleneck locations.

3.6.3 Generic capacity distribution estimation

A goal of this chapter is to derive reliable distributions for capacity variations dependent on day-specific scenarios for both the stochastic breakdown and discharge capacities. The observed empirical capacity results are fitted using a Weibull distribution for the 20 two-lane motorway locations and are aggregated per scenario in Figure 3.7. Specific differences per location can be found in Appendix 3.A. From the figure, a number of conclusions can be drawn. Firstly, the stochastic capacity drop for each scenario can be clearly seen. This is the difference between the blue lines to the right and the red lines to left for each scenario. The stochastic breakdown capacity distributions, shown in blue to the right, show that the distribution for workday traffic and holiday traffic hardly deviate from one another, while the capacity distribution for weekend traffic is substantially lower. In a similar way, the differences in discharge capacity are obvious from the red distribution lines to the left. The distributions for the scenarios do show a good estimation of capacity variation for Dutch motorways and may later be proven by analysis with an ever larger dataset to be pretty generic. The corresponding parameter values for the Weibull distributions and median capacity values are given for completeness in Table 3.2.

Table 3.2: Estimated capacity distribution Weibull parameters for two-lane motorways

	Weibull parameter:	α	β	Median Capacity
Workdays	**Stochastic breakdown capacity**	14,9	4647	4520
	Discharge capacity	12,7	3699	3584
Weekend days	**Stochastic breakdown capacity**	16,4	4396	4263
	Discharge capacity	12,4	3334	3244
Holiday days	**Stochastic breakdown capacity**	16,0	4652	4530
	Discharge capacity	11,3	3564	3441

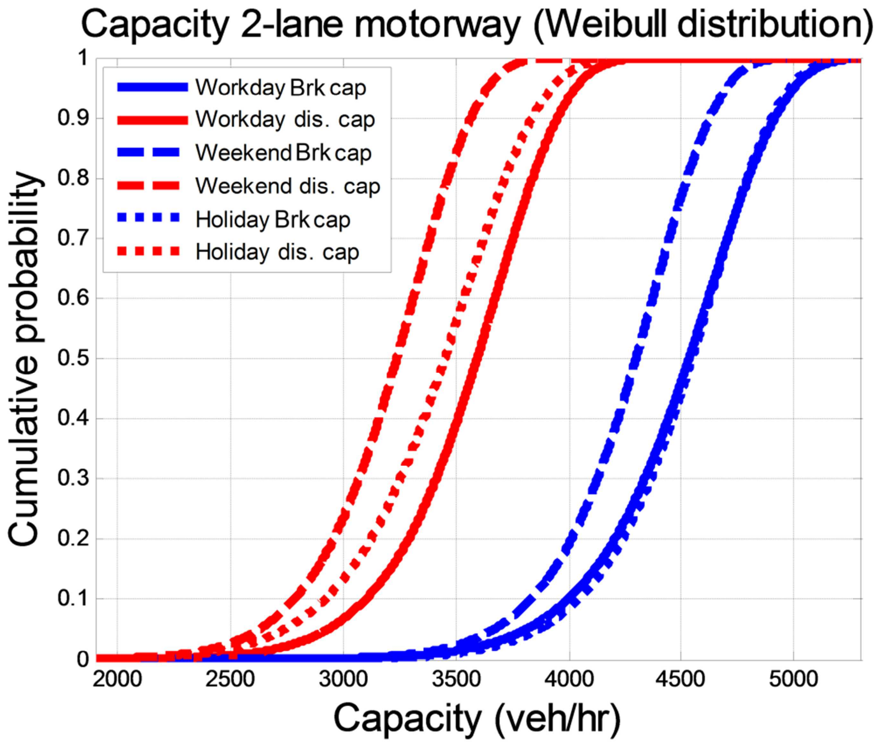


Figure 3.7: Overall capacity results as Weibull distributions

3.7 Results: Stochastic capacity and demand for weather

The results of the combined stochastic capacity and demand analysis for weather conditions are given in this section. First, the main results of the capacity and demand analysis are presented in section 3.7.1. A transformation of the results to intensity for precipitation is given in section 3.7.2 to allow comparison with literature. The final combined stochastic results of the analysis are given in section 3.7.3.

3.7.1 Main results

The results of the entire analysis are shown in Table 3.3 for both the capacity effects and all demand calculations. The capacity results are shown per lane and considered for 2-lane and 3-lane motorways respectively, including the ratio compared to the reference scenario. The demand results are shown as a ratio compared to the reference scenario for each of the considered time window combinations for the demand.

The results for the capacity show that for an increasing quantity of rainfall the capacity of both 2- and 3-lane motorways fall with increasing rates. For a limited rainfall of under 1.4 mm in an hour, the drop in capacity is limited to less than 2%. However, for the two greater categories, the drop in capacity is greater at 4-6% and 7% respectively. At the same time, an increase in rainfall has an overall negative effect on the traffic demand. The effect for a wet day with less rainfall is nearly non-existent, while for higher rain quantities the drop in demand is around 4.5%. Interestingly, considering rainfall only during the morning peak period shows a greater drop in demand: approximately 1% and 4% for the lower rain categories, while up to 9% reduction is found for the 1.4-1.9 mm category. For the largest rain category, insufficient data was available for the morning peak on the considered days to make an accurate prediction. It proved difficult to accurately determine the effect of snowfall using the available data with the described methodology. Locally, values could be derived, but a total trend from the dataset according to data-driven approach proved impossible due to a lack of snow observations. It was possible to derive an estimation of the effect on demand for days classed as ‘snowy’ in which a reduction in demand was found of 15-17%. For temperatures above 2 degrees Celsius, no real difference is found in capacity, as may be expected, however a slightly lower demand is found albeit only 1%. The demand for temperature below 2 degrees does not drop, while the capacity is found to be nearly 7% lower for cold conditions. Although there is a small overlap with snow conditions, the vast majority of ‘cold’ observations are made under dry but cold weather conditions. The effect of windy weather on capacity is shown to be present but limited to 3-4%, while the demand on windy days is not found to substantially change.

Table 3.3: Capacity and demand influence of weather conditions

Scenario	**Capacity results**				**Demand results**			
	2-lane cap	***2-lane ratio***	***3-lane cap***	***3-lane ratio***	***Day weather AM demand ratio***	***Day weather Day demand ratio***	***AM weather AM demand ratio***	***AM weather Day demand ratio***
Reference (Dry)	2291	1.000	2243	1.000	1.000	1.000	1.000	1.000
Rain 0-0.1mm*	2287	0.998	2243	1.000	0.995	0.997	0.992	0.994
Rain 0.2-1.3mm*	2258	0.986	2233	0.996	0.994	0.998	0.988	0.993
Rain 1.4-1.9mm*	2153	0.940	2152	0.959	0.944	0.956	0.911	0.959
Rain >=2.0mm*	2132	0.931	2079	0.927	0.941	0.956	-	-
Snow >0mm	-	-	-	-	0.838	0.852	-	-
Temp <2C	2139	0.934	2091	0.932	1.000	1.000	1.000	1.000
Temp >=2C	2282	0.996	2237	0.997	0.989	0.987	0.989	0.987
Wind <40kph	2282	0.996	2253	1.004	0.999	1.000	0.996	0.999
Wind >40kph	2229	0.973	2153	0.960	0.999	1.000	0.999	1.000

** The method collects the rainfall rather than rain intensity. A conversion can be made for comparison to other data (see section 3.7.2)*

3.7.2 Rainfall-intensity transformation

To allow a comparison with other literature and data, a transformation can be made of the corresponding rainfall into the probable rain intensity which would be found in the same period. Note that this is rough transformation, but gives a general basis for order of magnitude comparisons. The corresponding values for rainfall versus rain intensity are found in Table 3.4.

Table 3.4: Conversion table for rainfall versus rain intensity

Rainfall (mm in an hour)	Intensity (mm/h)
Rain 0-0.1mm	0-0.5 mm/h
Rain 0.2-1.3mm	0.5-5 mm/h
Rain 1.4-1.9mm	5-7 mm/h
Rain >=2.0mm	>7 mm/h

These are derived through consideration of two characteristics found in the data. These are the duration of rainfall in an hour, and the volatility of the rainfall (i.e. difference between the highest and lowest intensities). It was found that the volatility equates to a peak intensity in a range of 2-3 times the average rainfall when precipitation is actually falling. This hardly differs as a function of the total rainfall in an hour. Furthermore, a comparison was made between the duration of rainfall in an hour and the total rainfall in that hour. A duration correction factor is derived which indicates this relationship. For hours in which it rains for half the time, a factor of 2 is given, for an hour in which it only rains for a third of the time the correction factor is 3, etcetera. Figure 3.8 shows the relationship found from the rain data and the derived equation. Application of both a volatility factor of approximately 2.5 and a duration correction factor according to Figure 3.8, gives the estimated values for the corresponding rain intensities in Table 3.4.

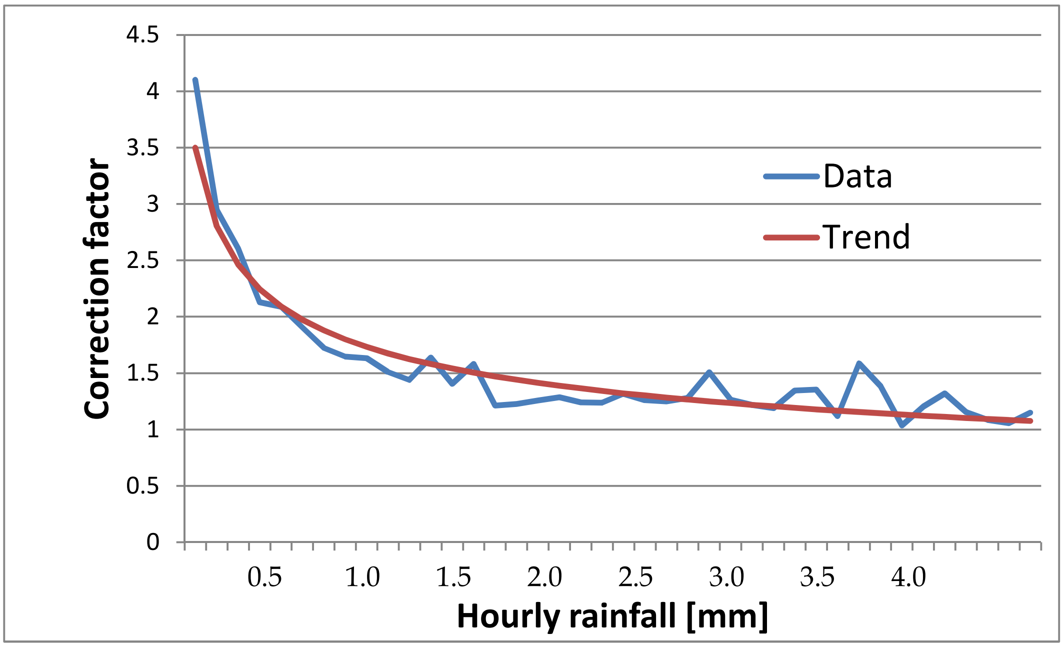


Figure 3.8: Rainfall duration correction factor

3.7.3 Combined stochastic demand-capacity results

For each of the scenarios, distributions of the results are constructed. The reference scenario is shown in Figure 3.9 as an example of the distributions. From the distributions, it becomes apparent that the spread in the capacity distributions for each scenario do not show any substantial differences between scenarios. This can be easily derived from the shape parameter values (b-value) of the capacity Weibull distribution, which all in the range between 15-17. The distributions for the demand (on the left) are shown for the first considered demand window (Day weather with morning peak demand). The demand distributions are not significantly normally distributed. Maximum Likelihood analysis showed that the distributions best fitted a t-location scale distribution or a logistic distribution. The corresponding parameter values for the demand distributions are given in Appendix 3.B. While each scenario has a different median value, again the general shape of each distribution is within a similar range, which indicates that stochasticity of demand, regardless of the scenario, exists within a certain range.

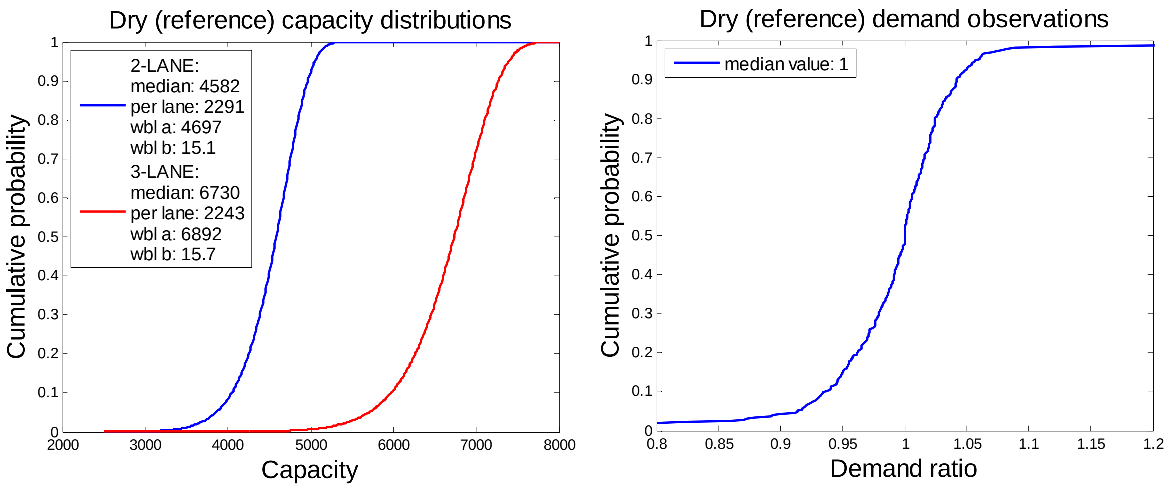


Figure 3.9: Capacity distributions (left) and demand distribution (right) for the reference scenario (dry)

Although the effects of capacity changes and demand changes individually give an impression of the effect on traffic fluency, it is only when both are combined one can gain a true picture of the effect of a weather condition on traffic fluency and an indication of the level of service. If, for example, a scenario causes a reduction in capacity, but causes an even greater reduction to demand, traffic flow as a whole may benefit from this, while only considering capacity changes would suggest otherwise. Therefore, the combined capacity-demand results are shown in Table 3.5. This is done for the 2-lane motorway capacity estimations (although the difference between two or three lanes was near negligible). The 'day weather condition' and 'peak hour weather conditions with peak morning demand' are taken as representative reference demand estimation periods.

Despite what the individual capacity and demand results say, the combined effect on traffic shows a different trend. The effect of rainfall has a limited negative effect on traffic fluency which remains for all rain categories below 2%. The effect of cold temperatures on traffic fluency is indicated to be one of the more important factors. Behind this lingers a maintained traffic demand, while the capacity is estimated to be lower resulting in a negative effect on traffic fluency as whole. This may also explain to a large extend some of the seasonal effects that are often observed during the winter months. Furthermore, the effect of high winds also shows an increased negative effect on traffic fluency, and that more so than rainfall, reaching a fall of 2.6%.

Table 3.5: Combined effect of weather on traffic fluency

Scenario	Effect on traffic fluency (Capacity/Demand)	
	2-lane capacity with Day weather & AM demand	*2-lane capacity with AM weather & AM demand*
Reference (Dry)	1.000	1.000
Rain 0-0.1mm*	1.003	1.006
Rain 0.2-1.3mm*	0.992	0.998
Rain 1.4-1.9mm*	0.996	1.032
Rain >=2.0mm*	0.989	-
Snow >0	-	-
Temp <2C	0.934	0.934
Temp >=2C	1.007	1.007
Wind <40kph	0.997	1.000
Wind >40kph	0.974	0.974

** The method collects the rainfall rather than rain intensity. A conversion can be made for comparison to other data (see section 3.8.2)*

3.8 Discussion on stochastic capacity

3.8.1 Capacity & Capacity drop values

An analysis has been given showing the extent of day-specific stochastic capacity variation for workdays, weekend days and holiday days. The results shown confirm the hypotheses that there is a significant difference in capacity between these scenarios. Fair comparison of stochastic capacity against results in literature is difficult, especially for Dutch motorways, as this is one of the first publications to investigate them stochastically in The Netherlands. Comparison with maximum capacities is not a fair comparison as the capacities are derived in different ways; however they still give an indication of the order of magnitude and spread of capacity values. In previous research for the Dutch capacity handbook, CIA (Capaciteit-Infrastructuur-Autosnelwegen), maximum breakdown capacity values were recommended of 4300 veh/hr for a two-lane motorway irrespective of the type of day (Rijkswaterstaat, 2015). This recommendation was also made using capacity estimations. A wide range of capacity drop values were also found, ranging from 0-30%, with most being between 10-15% and an average value of approximately 15%. The lower value capacity drop derived from the CIA may be a consequence of the generic properties that the CIA handbook must consider and therefore may intentionally be a conservative estimate. Other research comparing capacity estimations found in the CIA also suggests this (Tu et al., 2010). The capacity drop values found are in a similar range to this research, however the gravity point is lower. From Tu et al. (2010) an even greater spread of capacity drop was found for The Netherlands, between 4-

55%, with a mean value of 19%. Between locations substantial differences can be found in capacity values, likely caused by the infrastructural characteristics, such as geometry, type of asphalt, interweaving traffic flows, etc. This has also been found by the research mentioned here. However, a definitive trend is present, captured by the distribution of the capacity values from the different locations. For application in other regions or countries, there may be differences, related to driving behaviour and physical geographical differences. In any case, each location wherever it is, does have location specific characteristics. It should not be expected that mountainous roads or roads with unlimited speed limits will have identical (stochastic) capacity values for example. When traffic or infrastructure characteristics deviate, such as in these examples, the results in this chapter should not be too heavily relied on. However, as a general guide or starting-point for similar conditions, the estimates given here are expected to give a good indication prior to the estimation of local values.

3.8.2 Distribution fit

In Brilon et al. (2005), an attempt was made to estimate Weibull parameters to observations for three-lane freeways in Germany. The typical range found for the shape-parameter was between 9-15 with an average of 13. This is lower than the range found for workdays of 11-17 and an average of 15 here. For these values, an indication is given of the spread of the stochastic capacity estimations, from the shape parameter. The differences between the results may be caused by the difference in traffic stability between three and two lane motorways, and may also be caused by the different traffic characteristics between The Netherlands and Germany. Further research is needed to give clarity on this issue. The authors are aware of other possible distributions, such as investigated in Jia et al. (2010), however the Weibull fit is deemed best suited to the Dutch traffic situation, which has a closer resemblance to German traffic as originally discussed by Brilon et al. (2005). The goodness-of-fit test performed, with the results shown in Table 3.1, backed up the assumption that the Weibull distribution is suitable for use in this analysis.

3.8.3 Brief qualitative discussion

A brief reflection is given on the results in relation to some of the causes behind the results. The goal of this research is not qualitative; therefore the discussion is speculative and based on the findings.

- On weekend days, traffic is shown to have a higher probability of breakdown at lower flow values compared to workdays, but not necessarily with a greater probability spread.
- Drivers on workdays, during peak periods when congestion usually occurs, are generally more experienced drivers, especially on the considered routes. This greater familiarity leads in theory to a greater efficiency in traffic flow and stability (Tu et al., 2007).
- It is presumed that there is no substantial difference between workday capacities, as the driver population is in general identical on most days.

- On weekend days a greater number of irregular trips are made, and also by drivers that may on average be less experienced. Without going too deep into the explanatory details behind this, this leads in theory to longer headways and a slightly less stable and efficient traffic flow. This is backed up by the results in this chapter.

- Holiday traffic is harder to intuitively predict for stability, as the considered days are constructed out of a wide range of holidays on which different types of traffic may be present on the road. For example, a holiday day in the summer may have many drivers heading to resort locations, while Christmas day traffic on the other hand will have a completely different composition. This is maybe one of the reasons that the estimation of the breakdown capacity for these days does not show such a large difference to workdays. Nevertheless, the results do show some differences, especially for the discharge capacity. In congestion, the capacity of the road is significantly lower than for workdays. Characteristics such as the vehicle population may be possible causes. Vehicles performing holiday trips, may also be heavier loaded or even pulling a trailer or caravan. Furthermore, there may be less of a delay penalty for holiday traffic under congested conditions compared to workday traffic.

While these thoughts give some possible explanations behind the results, they are not conclusive, and are not meant to be. Further qualitative research following on from these quantitative results may be a good continuation for this research area.

3.8.4 Relevance and application of stochastic capacities

The relevance and applicability of the results in this chapter is of interest especially in two main areas, namely for traffic flow modelling, and for capacity analysis and highway planning. The necessity to consider uncertainties in modelling was highlighted in the introduction. True stochastic traffic flow models are not common in practice, however uncertainty is often still applied through scenario modelling (multiple model runs with set values, i.e. a median and the standard deviation values) or Monte Carlo simulation. Results, in which stochastic capacity values are used, can be applied to give indications of the performance of the traffic system with various probabilities. Especially in the case of a stochastic model or Monte Carlo simulation, it is possible to construct a whole distribution for the model results for a network. These results then give an overview of the probability of certain values for the capacity uncertainty. It is of course also possible and recommended to include the traffic demand uncertainty in the same analysis.

The use of stochastic capacities is also relevant for planning purposes. This can be viewed on two levels. Firstly, when planning new infrastructure, the desired level of service is directly connected to the capacity of a road. When considering a road with a large capacity distribution, a larger proportion of probability of congestion is present for traffic demand below the median capacity. A road authority can take this into consideration and may decide to plan for more capacity. The decisive conditions may vary for such a decision; therefore the use of different day-types becomes relevant. Secondly, changes to existing roads can be considered to solve flow problems by observing the distribution of the capacity. For example, the application of various traffic management measures is often applied at bottleneck locations to increase homogeneous flow and thus reducing the probability that congestion occurs due to an instantaneous exceedance of capacity, such as on weaving sections. While stochastic capacities are not explanative for the cause of heterogeneity (large distribution spread), they can show the extent of heterogeneity and can be used to evaluate measures that are targeted at such bottlenecks.

These application areas give an impression of how stochastic capacities may be applied. For each location it is recommended to determine the local capacity distribution, as these can vary per location. This can be performed with the presented framework in this chapter. However, if this is not possible, i.e. due to a lack of data or financial constraints, a similar location may be analysed or generic values from this research may be selected that represent the considered situation. The use of a generic value will not always be 100% accurate for every location, but will allow stochastic effects of capacities to be considered, which in most cases will assist in the accuracy of traffic flow analysis or road planning. In previous research, a discussion was also given of limitation for the application of stochastic capacities and demands, which is also valid for the methodology given in this contribution (Calvert et al., 2012). On networks with large variations in traffic demand and driver behaviour, the application of stochastic modelling is preferred. This is also the case for more integrate networks in which secondary congestion effects are more likely as well as for high levels of congestion. Consideration of stochastic variations in modelling also has limitations. On simpler networks, networks with little congestion or on which the extent of variations in road capacity and traffic demand cannot be easily determined, the use of a stochastic approach is not as necessary. As a stochastic approach requires more effort and is more time consuming it would be more desirable to perform deterministic modelling in these cases.

3.9 Conclusions

In this chapter, a methodological framework with a conceptual model for practical stochastic capacity estimation is presented. Furthermore, a methodology is presented that considers the combined effects of stochastic demand and capacity, expressed through the influence of weather on traffic fluency. The first methodology, on stochastic capacities, makes use of a number of different analysis tools and is designed to give practitioners and researchers a concise and easy to follow approach for stochastic capacity estimation. In addition, insight is given into the extent of day-type specific variation in capacity values. The analysis is performed for three different scenarios: for workday, weekend, and for holiday traffic. A stochastic estimation of road capacity for these scenarios for both the stochastic breakdown capacity and discharge capacity are made. These are produced using a Weibull distribution, which was shown to resemble the empirical data, based on a goodness-of-fit test and has previously been shown to yield good capacity probability fits. The analysis is performed using induction loop data from 23 locations on the Dutch motorway network. Extensive filtering for day specific characteristics and capacity estimation using the Product Limit Method was applied to reach empirical estimation results for the median and variation in the capacity for the considered scenarios.

The results indicate that there is a reduction in stochastic breakdown motorway road capacity on weekend days of 4% in comparison to workdays. Furthermore, a decrease of 8% is found for the discharge capacity in comparison to workdays. The analysis showed that the stochastic breakdown capacity on holidays is not significantly lower than on workdays, while the discharge capacity does drop 3% to 4%. An estimation is also made of the spread in stochastic capacity drop. This shows that the capacity drop grows, on average, on both weekend days and holiday days in comparison to workdays. For workdays an average stochastic capacity drop is found of 21% with a spread between 13% and 29%. For weekend days and holiday days this is 25% and 24%, with a similar spread around the average values. The results are in line with comparable research.

The second methodology is applied to give quantitative insight into the combined stochastic effects of demand and capacity, and is applied for weather on traffic and to furthermore highlight the necessity of considering the effects simultaneously on both traffic supply and demand. The methodology allows both the capacity and demand to be calculated and combined to give an indication of the effects of weather on traffic. An extensive data-driven analysis is performed applying the described method in which the effects of rain, snow, temperature and wind are analysed for their influence on traffic. The analysis was performed for motorways in a large 1200 kilometre square area in the urbanised west of The Netherlands. The results show that increasing reductions of both capacity and demand are found for precipitation in the form of rainfall. Despite the reduction, the overall influence of rain on traffics ability to flow fluently is not substantially reduced. Insufficient data for the described approach meant that capacity estimation could not be made for snowfall, while a reduction in demand for snow was found of more than 15%. The influence of cold temperatures proved to be substantial on traffic fluency. Demand was found not to vary significantly, while capacity is reduced leading to a greater chance of a reduction in level-of-service of roads. Similarly, high winds were found to also reduce the quality of traffic fluency, although at a lower level of approximately 2-3%.

A further quantification of the stochastic distributions of the results is derived for each weather scenario. This showed that the distribution shape of each weather type does not significantly differ and was found to yield similar shape-parameters when fitted for a Weibull distribution. The shape of the demand distributions also showed a close resemblance and was found to adhere to a t-location-scale and logistic distributions. The resulting distributions may be used for a number of future purposes, such as application of uncertainty and sensitivity analysis both in data-analysis and modelling of traffic effects during weather to name two.

It is concluded that the difference in types of day has a significant effect on road capacity and that this variation in capacity can quantitatively be derived, as demonstrated in this chapter. The derived distributions for the specific day types give both a quantification of the mean and the spread of the relevant capacities and are therefore applicable for use as input in stochastic traffic models. Applications for motorways or freeways in other countries and under other conditions may differ from the results found here. However, the same methodology as applied in this chapter can be easily applied to these other situations and locations to give local capacity estimations. Further research is recommended to gain a greater qualitative explanation of how these differences in capacity occur, rather than just the quantification as shown here. In such a way, a greater causality may be given to certain variations found from the results. Further research following the quantification of weather effects lies primarily in quantification of correlated weather effects on traffic flow, such as a combination of rain and high winds for example. Further research also lies in quantification of other weather effects as well as the development of a refined methodology for widespread data analysis of the effects of snow of traffic flow for limited observations.

Appendix 3.A: Specific capacities per location

Mean values

		Free-flow Capacity				Discharge Capacity			
		Median	Stnd Dev	Weib a	Weib b	Median	Stnd Dev	Weib a	Weib b
Work Days	2 lanes:	2260	202	14.9	4647	1792	190	12.7	3699
	3 lanes:			15.5	6796			11.6	5478
Weekend Days	2 lanes:	2131	174	16.4	4396	1622	168	12.4	3334
	3 lanes:			18.4	6301			11.6	5082
Holidays	2 lanes:	2265	191	16.0	4652	1720	268	11.3	3564
	3 lanes:			16.7	6819			10.8	5251

All values

Work Days

Road	Lanes	Bottleneck Type	Free-flow Capacity					Discharge Capacity					Cap Drop
			Median	per lane	Stnd Dev	Weib a	Weib b	Median	per lane	Stnd Dev	Weib a	Weib b	
4-1	2	junction	4500	2250	400	14.9	4616	3550	1775	360	12.8	3657	21.1%
4-2	2	junction	4340	2170	330	17.3	4438	3820	1910	400	12.7	3930	12.0%
9-1	2	lane drop	4840	2420	360	18	4935	3900	1950	320	16.2	3994	19.4%
9-2	2	junction	4920	2460	440	14.8	5041	3940	1970	390	13.1	4056	19.9%
12-1	3	junction	7120	2373	600	15.7	7289	5540	1847	610	11.8	5719	22.2%
15-1	2	junction	4220	2110	510	10.8	4365	3700	1850	370	13.2	3803	12.3%
20-1	2	lane drop	4060	2030	470	11.3	4196	3210	1605	460	8.9	3343	20.9%
20-2	3	speed reduction / bend in road	6090	2030	490	16.5	6224	5260	1753	540	13	5407	13.6%
27-1	2	junction	4000	2000	380	13.8	4109	3580	1790	340	14.1	3672	10.5%
50-1	2	weaving section	4230	2115	360	15.6	4333	3410	1705	400	11.1	3528	19.4%
50-2	2	bridge	4170	2085	400	13.7	4280	3660	1830	370	13.1	3763	12.2%
2-1	2	junction	4470	2235	440	13.5	4591	3520	1760	390	12	3628	21.3%
2-2	2	bridge	4470	2235	370	15.8	4577	3380	1690	350	12.6	3480	24.4%
27-1	2	weaving section	5010	2505	360	18.2	5116	4040	2020	330	16.1	4133	19.4%
27-2	2	bridge	5080	2540	520	12.8	5225	3460	1730	380	11.8	3571	31.9%
27-3	2	weaving section	4270	2135	300	19.2	4348	3160	1580	470	8.7	3292	26.0%
27-4	2	bridge	4420	2210	370	16	4519	3580	1790	340	14	3671	19.0%
27-5	2	junction	4370	2185	390	14.8	4476	3210	1605	390	10.8	3319	26.5%
27-6	2	weaving section	5030	2515	480	13.8	5136	3920	1960	410	12.4	4041	22.1%
1-1	2	junction	4950	2475	510	12.7	5093	3490	1745	390	11.7	3602	29.5%
1-2	2	junction	4580	2290	400	15.5	4685	3650	1825	310	15.7	3732	20.3%
1-3	2	junction	4750	2375	390	16.3	4857	3660	1830	350	13.5	3764	22.9%
16-1	3	junction	6700	2233	610	14.4	6874	5120	1707	670	9.9	5308	23.6%

Appendix 3.B: Demand distribution parameters

Scenario	Demand distribution fit: parameter values	
	t-location-scale [mu-sigma-nu]	***Logistic [mu-sigma]***
Reference (Dry)	[1.000 0.028 1.886]	[0.996 0.030]
Rain 0-1mm*	[0.997 0.034 2.115]	[0.996 0.034]
Rain 2-13mm*	[0.997 0.032 1.746]	[0.996 0.037]
Rain 14-19mm*	[0.954 0.046 2.236]	[0.948 0.043]
Rain >=20mm*	[0.957 0.035 1.815]	[0.951 0.038]
Snow >0	[0.849 0.051 2.663]	[0.841 0.043]
Temp <2C	[1.000 0.031 2.249]	[1.000 0.030]
Temp >=2C	[0.987 0.036 2.678]	[0.982 0.030]
Wind <40kph	[0.999 0.0312 2.111]	[0.998 0.031]
Wind >40kph	No best fit	[0.993 0.041]

Chapter 4

Advanced sampling methods in Monte Carlo simulation

In an effort to improve performance and speed up stochastic calculation, advanced sampling techniques have been developed in the past century. These techniques are investigated in this chapter for their ability to reduce the computational load in traffic modelling with variable input values. A comparison is made between these techniques and that of Crude Monte Carlo simulation. The objective of the chapter is to demonstrate the efficiency of the methods for use in traffic modelling. This has not previously been demonstrated for traffic modelling, and their application is shown in several experimental cases.

The applied techniques and applied methodology are discussed and explained in section 4.2. In section 4.3 case studies are presented, which are used to demonstrate the effectiveness of the advanced sampling techniques and for which the results are given in section 4.4. Section 4.5 provides the conclusions of this chapter.

This chapter is an edited version of the article:
Calvert, S. C., Taale, H., Snelder, M., & Hoogendoorn, S. P. (2014). Application of advanced sampling for efficient probabilistic traffic modelling. *Transportation Research Part C: Emerging Technologies, 49, pp. 87-102.*

4.1 Introduction

In traffic models, assumptions are made to simplify the complex decision processes and interactions that rule the dynamics of traffic and transport system. This is necessary as factors that influence traffic flow are extensive, and not every variable can be considered. It is commonplace in traffic modelling that equilibrium states are sought that give a good average representation of the dynamics of traffic. Traffic modelled as a deterministic system, meaning that there is no randomness involved in the development of future states, implies that assumptions are made to describe the randomness that exists. Examples of these assumptions are related to the demand and supply, the behaviour of drivers and the characteristics of vehicles. Each of these variables is a stochastic system which is often reduced to an average value in a deterministic representation (e.g. average demand, average supply, average desired speed, average maximum acceleration, etc.). However, it must be realized that traffic in reality is hardly ever ‘average’ (Ernst et al., 2012). Therefore, modelling traffic as if it were always in a deterministic state is not realistic and will lead, in many cases, to biased outcomes (Calvert et al., 2012, van Lint et al., 2012). Consideration of stochastic dynamics in traffic modelling allows for a much more realistic representation of the traffic system (Clark and Watling, 2005, van Lint et al., 2012).

Modelling traffic flow with stochastic input is often performed in traffic models through one of two main methods: analytically or by replicative simulations through Monte Carlo simulation or a derivative thereof. Analytical approaches have seen an increased development in recent years, but remain largely complicated models, which are not readily applied in practice by practitioners unless incorporated in a simulation package (Davidson, 2011). Therefore, the approach using Monte Carlo simulation remains an attractive option, despite the requirement of a relatively high computational effort (Chang et al., 1994).

Monte Carlo simulation has been widely applied in various sciences to help describe stochastic systems, as well as in the traffic domain in many applications, such as the construction of probabilistic models. The method was first conceived in the 1940’s (Metropolis and Ulam, 1949) and has since grown in popularity. For multivariate problems, the technique makes use of predefined probabilities for each of the input variables, indicating the probability of occurrence and the corresponding value (Ang and Tang, 2007).

In traffic modelling, the Monte Carlo method has been applied in many ways. Its extent is such that a complete overview of all literature is not given here, but merely an indication of some recent publications. The method has been applied in traffic modelling in traffic assignment & route choice (Zhang et al., 2008), mode choice (Jonnalagadda et al., 2001), traffic propagation (Chang et al., 1994, Szeto et al., 2011), strategic scenario assessment (Salling and Leleur, 2011), sensitivity analyses, and reliability studies (Murray-Tuite and Mahmassani, 2004, Tampere et al., 2007b, van Lint et al., 2012).

The application of Monte Carlo simulation in this contribution relates to sensitivity analysis, and, to a certain extent, reliability of traffic flow and networks. The Monte Carlo routine is often performed prior to traffic assignment and on input variables such as network capacities and the traffic demand. Application of the routine for sensitivity analysis in such a manner is not uncommon. Examples of such applications are Chen et al. (2002) and Tampere et al. (2007b), who investigated reliability. In these studies link capacity is varied using Monte Carlo simulation. Szeto et al. (2011), Zhang et al. (2008) and van Lint et al. (2012) applied Monte Carlo simulation to vary multiple input variables. Taking van Lint et al. (2012) as an example, the values for network capacity and demand used in traffic simulation are constructed considering multiple influencing factors. These factors, such as weather effects, random traffic demand and incidents are applied as an adjustment factor over the base capacities and traffic demand. In a similar way, this is also applied in this chapter and is explained later on.

4.2 Methodology for advanced sampling

Inclusion of stochasticity in traffic modelling using stochastic input through Monte Carlo simulation can be performed according to the approach shown in Figure 4.1. This is also the approach applied in this study. The basis for the approach takes the base capacity values, or deterministic capacity values, from the defined network capacities and applies capacity reduction factors leading to the cumulative probability density function (CDF) of the capacity distributions. In a similar fashion, individual values from a CDF for the traffic demand are applied to a base set of traffic demands in the traffic network. From the resulting capacity and traffic demand CDF's, a random sample is taken, one for both the capacity values and one for the traffic demand values. Depending on the applied Monte Carlo technique, these are either dependently or independently sampled. A simulation run is performed with the traffic model, resulting in a single output result. The process of sampling and modelling is repeated until a complete distribution of results is constructed. This approach neglects correlations which can exist between capacity and demand variations, but rather focusses on the performance of the sampling techniques in the traffic models.

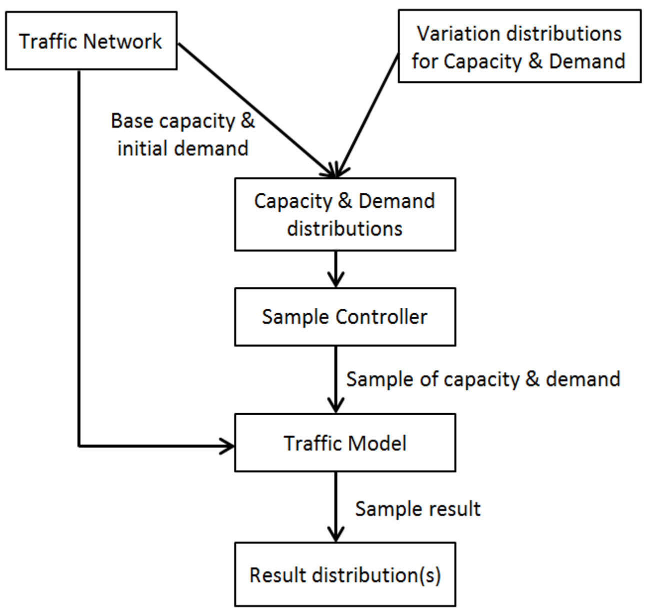


Figure 4.3: Approach to Monte Carlo simulation for stochastic input in traffic modelling

The various parts of this process are described in greater detail in the remainder of this section. The networks used in the test cases are described in the following section with the description of the case study setup.

4.2.1 Traffic modelling

The applied traffic model in this study is the dynamic macroscopic traffic model INDY (Bliemer et al., 2004). This model is used as it is strategic model and dynamic in time, and is a good representation of the type of model that may be used in practice by practitioners. INDY makes use of the Link Transmission Model (LTM) for dynamic traffic propagation, as described by (Yperman, 2007). The LTM is a macroscopic Dynamic Network Loading model which applies first order kinematic wave theory as described by Newell (1993). For further details of the LTM see (Tampère et al., 2011, Yperman, 2007). Route assignment in INDY is performed through a dynamic path based approach. Distribution of path flows is based on travel costs, predominantly determined by link travel times. An equilibrium state in route choice is solved in INDY using a simple iterative process, applying the method of successive averages. The routes are generated from a route set generation model described by Bliemer and Taale (2006).

4.2.2 Application of stochastic variation

Stochastic variation is applied using a probability distribution for the capacity and traffic demand respectively. These distributions give the probability of certain influence factors on the capacity and traffic demand. Multiplication of the sampled factor values from the distributions with the base values, either or both the capacity and traffic demand, results in the corresponding values used in each individual simulation. The sampled factor values are applied identically to all road sections and all zones respectively for a single iteration. The applied distributions in two of the three test cases in this study are derived from real traffic data from the A4 and A12 motorways in The Netherlands. At various locations, along the 2 to 4 lane motorways, the relative variation in traffic flow for 28 Tuesdays and Thursdays in 2008 are fused through weighted averaging to represent the local demand. The resulting distributions are shown in Figures 4.2a-b. Both distributions closely resemble distributions for most motorway locations and can be generically applied as the distributions are constructed as a relative factor rather than absolute capacity values. In the other test case, use is made of artificially constructed distributions, resembling real distributions in more extreme cases (i.e. with greater variations and less favourable values). The distributions are deliberately very different to those shown in Figure 4.2 to demonstrate the differences in outcome. These distributions are shown in Figure 4.3a-b.

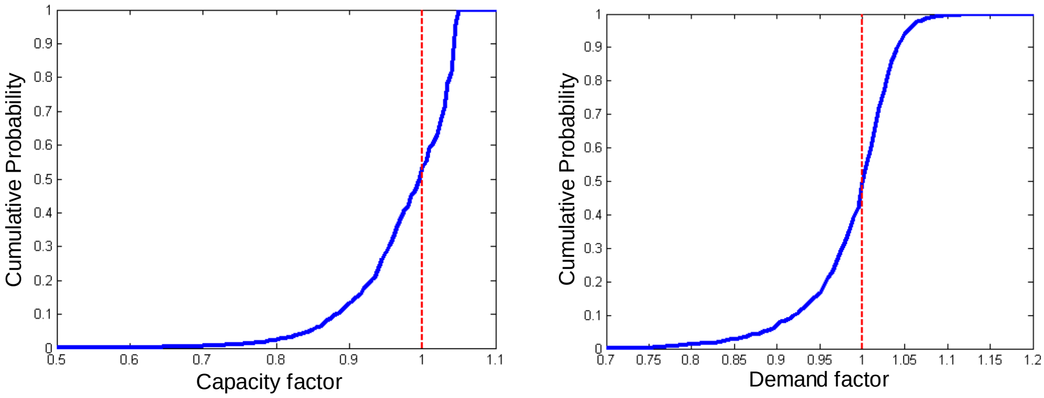


Figure 4.2a-b: Cumulative Density Function derived from the A4 and A12 motorways and applied in test case 1 and 2 for a) the capacity and b) the traffic demand.

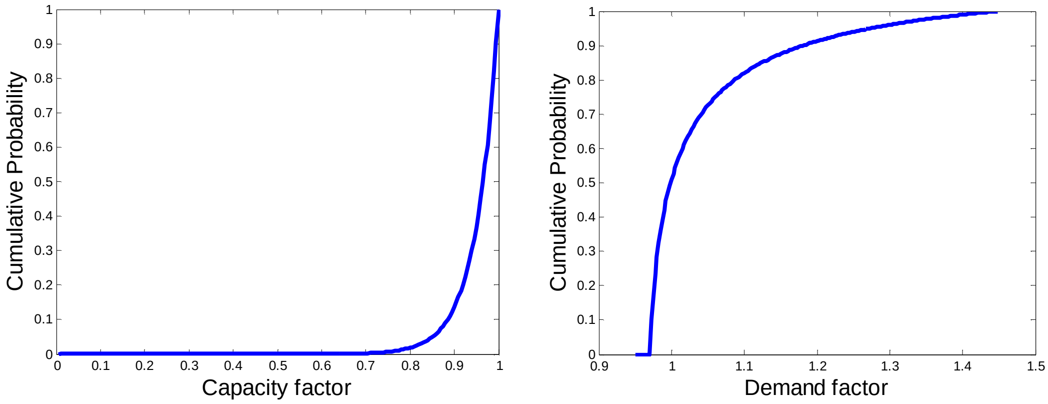


Figure 4.3a-b: Cumulative Density Function factor (left) as applied in test case 3 for a) the capacity and b) the traffic demand.

4.2.3 Sampling Methods

To reduce the number of required sample iterations when performing Monte Carlo simulations, techniques are applied which reduce the estimation error in a sampling process. While reducing errors, these techniques must also maintain the shape of the original output distribution. In practice the estimation of the original distribution will always deviate from the true solution due to statistical fluctuations (Bekaert et al., 2000). The goal of variance reduction techniques is therefore not to improve the quality of the estimation, but rather to reach the same level of performance while requiring fewer samples.

An increasing number of variance reduction techniques have been developed, each with a different approach, but also a certain degree of complexity for implementation and effectiveness. Previously, a number of these techniques have been applied in traffic modelling for various goals, as explained in the previous section. In Kroese et al. (2011), a qualitative analysis is given of the complexity and of the potential effectiveness over various variance reduction techniques. An overview of this is shown in Figure 4.4. There it can be seen that the range of effectiveness of different techniques can have a large variation and that for larger improvements the level of complexity to implement the technique will most likely increase.

The extent of the effectiveness depends heavily on the type of applied distributions, the number of considered variables, but also the set-up of the chosen technique.

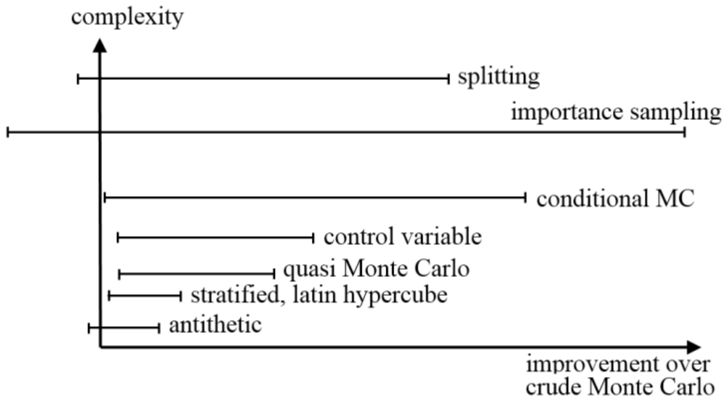


Figure 4.4: Complexity versus effectiveness of variance reduction techniques (from Kroese et al. (2011))

Two of the considered sampling techniques applied in this contribution are Importance sampling and Latin Hypercube sampling. To a certain extent, this choice is somewhat arbitrary, but is made from the viewpoint of potential effectiveness against complexity, and is further backed up by previous research (van Lint et al., 2012) in which these techniques are recommended for further investigation. In theory, Latin Hypercube Sampling is less complex, but more robust, while Importance Sampling is more complex and is potentially more powerful, but may also lead to a reduction in convergence, hence it is less robust. It is beyond the scope of this research to evaluate every technique, therefore the set will be limited to these two techniques.

A third technique considered comes from the area of quasi-random Monte Carlo methods: Sobol quasi-random sequence. Where the previous methods are truly random, that is sampling without explicit selection, quasi-random sequences are constructed from well distributed selection by sequential functions. This should therefore allow for a much faster convergence to the true underlying distribution compared to the crude Monte Carlo method (Chi et al., 2005). The comparison of random variance reduction methods with quasi-random sequences in practise is difficult because of convergence issues and does not definitively show one to consistently outperform the other (Berman, 1997, Dror et al., 2002). Besides the mentioned sampling techniques, simulations will also be applied for Crude Monte Carlo sampling as a reference for the two variance reduction techniques and the quasi-random MC method. Further details on sampling theory can be found in one of the many pieces of literature on the subject (Govindarajulu, 1999, Knottnerus, 2003, Kroese et al., 2011).

Crude Monte Carlo Sampling

In Monte Carlo procedures, it will often be the traditional Crude Monte Carlo (CMC) sampling that is applied. The technique is described as simple, because the randomly generated samples are directly taken without further processing from the considered distribution. The technique of CMC sampling is straight forward and produces an estimation based on the generation of *N* samples *H* from a predefined distribution *h*:

$$\boldsymbol{H}_1, \dots, \boldsymbol{H}_N \in h \tag{4.1}$$

These are consequently applied in the CMC technique to gain an estimation $\widehat{M}$ of the original distribution h as follows:

$$\widehat{M} = \frac{1}{N}\sum_{k=1}^{N} h(\boldsymbol{H}_k) \tag{4.2}$$

Importance Sampling

Importance sampling (IS) is a technique used in Monte Carlo simulation that gives extra consideration to the outlying sections of a distribution which have a lower probability of being sampled, but have a relatively large influence on the output variable (Kroese et al., 2011). By assigning the extremities of a distribution a greater probability than they originally have, creates a higher chance of extreme values being sampled and therefore the speed at which the output distribution is 'complete' is greater. There are multiple variations of IS, such as *Minimum-Variance Density, Variance Minimization Method, Cross-Entropy Method and Sequential Importance Sampling* (Cappé et al., 2007, Kroese et al., 2011, Smith et al., 1997). The technique of IS applied here is that of the Weighted Importance Sampling method (Bekaert et al., 2000). Weighted IS predicts the extent that the distribution of a set of samples deviates from the original source distribution by assigning weights to the samples corresponding to their probability and deviation from the source distribution. The sample weights denote the ratio between the source distribution and the estimator distribution. The estimator distribution is chosen, such that it increases the probability of extreme input values being selected, which have an amplified effect on the final model output. The technique is mathematically described as follows:

Consider a random variable *M* for some real function with values *H* and probability density function *f*:

$$M = E_f H(\boldsymbol{x}) = \int H(\boldsymbol{x}) f(\boldsymbol{x}) d\boldsymbol{x} \tag{4.3}$$

A transformation of *M(x)* is applied, by introducing an estimator distribution *g(x)* to get:

$$M = \int H(\boldsymbol{x}) \frac{f(\boldsymbol{x})}{g(\boldsymbol{x})} g(\boldsymbol{x}) d\boldsymbol{x} = E_g H(\boldsymbol{x}) \frac{f(\boldsymbol{x})}{g(\boldsymbol{x})} \tag{4.4}$$

The estimation of *M* after sampling is therefore:

$$\widehat{M} = \frac{1}{N}\sum_{k=1}^{N} H(\boldsymbol{x})w(\boldsymbol{x}) \tag{4.5}$$

Where:

$$w(\boldsymbol{x}) = \frac{f(\boldsymbol{x})}{g(\boldsymbol{x})} \tag{4.6}$$

in which *w(x)* is the weight known as the likelihood ratio estimator. Furthermore the expectation of the sample *M*, should reflect the distribution *M;* $E(\widehat{M}) = M$. See Rubinstein (1981) for proof of this.

Latin Hypercube Sampling

Latin Hypercube sampling (LHS) is a stratified sampling technique that, other than general stratified sampling, ensures that the entire sample space for multiple input variables is sufficiently covered (Iman and Conover, 1980, McKay et al., 1979). The technique is an extension of quota sampling. The basic technique sees variables evenly sampled from the sample spaces, also known as a d-dimensional hypercube, in which each random variable is attributed a dimension. Combinations of the samples are randomly generated, such that a good spread of samples is achieved to form a single target function. This can be applied on any number of dimensions of variables, but is applied in this research in two dimensions.

The definition of LHS is as follows:

The input variables X_k are divided into *N* strata of equal probability

$$P(\boldsymbol{x_k}) = \frac{1}{N} \tag{4.7}$$

A single sample $X_{k.j}$ is taken from each stratum *j*. These form the component Xk:

$$X_k = X_{k.j} \qquad \text{with k =1..K, j =1..N} \tag{4.8}$$

For each input variable X_k, the components are matched at random to form a *K*-dimensional sample cube. Graphically, the technique for a two-dimensional cube for *N* = 200 samples, with *K* = 10 strata per dimension is shown in Figure 4.5.

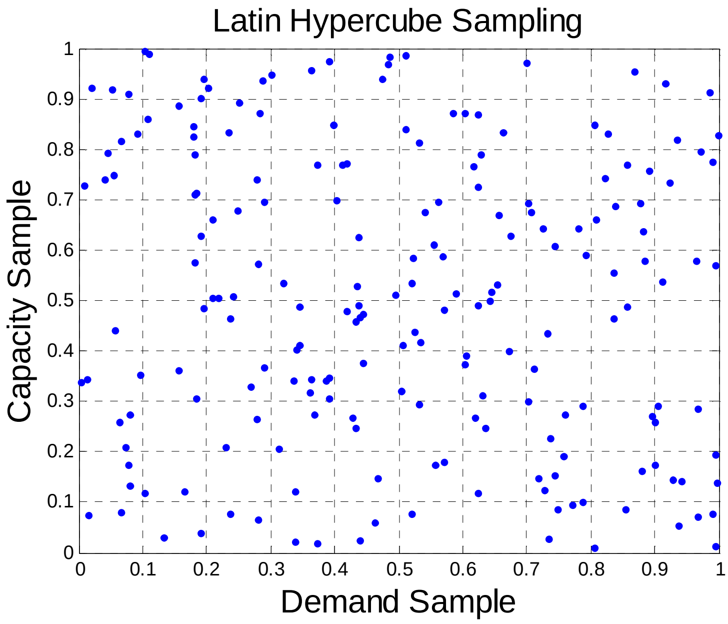


Figure 4.5: Latin Hypercube sampling of a two-dimensional space for Capacity and traffic demand samples. *N* = 200 samples, with *K* = 10 strata per dimension.

Figure 4.5 demonstrates the samples taken in one of the case scenarios described later in the chapter for the capacity and demand factors. Although not every joint stratum is evenly covered, the coverage over each stratum is well distributed, including the random spread within each stratum. The distribution over each individual marginal stratum is evenly distributed though. This results in a superior spread of samples in most spaces, especially for those with a limited number of samples (Iman and Conover, 1980, Larson et al., 2005, McKay et al., 1979, Minasny and McBratney, 2006). It further results in the capture of sampled input values that would otherwise have been insufficiently utilized, especially where their probability is relatively low.

Sobol Quasi-random Sequence

The Sobol Quasi-random Sequence (SQS) is a type of quasi-random number sequence (QRNS) (Sobol, 1967). QRNS are deemed to improve on random sampling by explicitly "sampling" the probability space more uniformly (Bratley and Fox, 1988, Joe and Kuo, 2003, Joe and Kuo, 2008a). Unlike true sampling methods, sequential numbers are predefined numbers that are not explicitly random, but due to their well distributed coverage in sequence of the considered set, can be considered as near-perfectly distributed random numbers and therefore a good converge is gained much faster than with crude Monte Carlo (Chi et al., 2005). The choice to apply SQS here, rather than another QRNS is due to the combined advantage of simplicity and efficiency of the SQS (Chi et al., 2005).

SQS are constructed from a primitive polynomial using direction numbers. A primitive polynomial of some degree sj is chosen:

$$x^j + a_{1,j}x^{j-1} + a_{2,j}x^{j-2} + \cdots + a_{s,j}x + 1 \tag{4.9}$$

where the coefficients $a_{1,j}$, $a_{2,j}$, . . . , $a_{sj-1,j}$ are either 0 or 1. We define a sequence of positive integers $\{m_{1,j}$, $m_{2,j}$, . . .$\}$ by the recurrence relation:

$$m_{k,j} = 2a_{1,j}m_{k-1,j} \oplus 2^2a_{1,j}m_{k-1,j} \oplus \dots \oplus 2^{sj-1}a_{sj-1,j}m_{k-sj+1,j} \oplus 2^{sj}m_{k-sj,j} \oplus m_{k-sj,j} \tag{4.10}$$

Here $\oplus$ denotes the bitwise operator. The initial *values* $m_{1,j}$, $m_{2,j}$, . . . , $m_{sj,j}$ are restricted such that each $m_{k,j}$, $1 \leq k \leq s_j$, must be odd and less than 2^k. The direction numbers $\{v_{1,j}$, $v_{2,j}$, . . .$\}$ are defined as:

$$v_{k,j} = \frac{m_{k,j}}{2^k} \tag{4.11}$$

Then $x_{i,j}$, the *j*-th component of the *i*-th point in a Sobol sequence, is given by

$$x_{i,j} = i_1v_{1,j} \oplus i_2v_{2,j} \oplus \dots \tag{4.12}$$

where i_k is the *k*-th digit from the right when *i* is written in binary $i = (\ldots i_3i_2i_1)_2$. For more details on the method, see (Joe and Kuo, 2003, Joe and Kuo, 2008b, Joe and Kuo, 2008a).

4.3 Case Study for advanced sampling

To demonstrate the effectiveness of the considered sampling techniques, three case studies are considered. Each case study considers a different network, and therefore different capacities and demands, and two different stochastic distributions are applied over the three cases. For each network, representative routes along which the travel-time variations are calculated are selected such that good route coverage is achieved and the main corridors are considered. The applied indicators are given in section 4.3.2 and the results of the cases are presented and discussed in section 4.4.

4.3.1 Test networks

The case studies consider convergence of the travel time over multiple routes and the overall delay times of three different motorway networks located in the Netherlands. For each of these variables, travel times and network delays, the rate of convergence is compared for the application of the two variance reduction techniques and the Sobol Quasi-random Sequence technique (SQS), and is further compared against the Crude Monte Carlo technique as a reference. For each case, 200 simulations are carried out with varying sample values from which the rate of convergence becomes evident. Each applied network has a different structure, traffic volumes, degree of congestion and routing options. Furthermore, bottlenecks on each network exist in different forms such that each network represents a uniquely different scenario. The characteristics of each network, including the considered routes for travel time, are described in the following sub-sections.

Rotterdam Ring network

The motorway ring network around Rotterdam is shown in Figure 4.6. Inner city Rotterdam is situated to the centre of the ring, with suburbs to the outside of the ring. The network has been reduced to the main motorway stretches and their connections to the city. This allows for a purely motorway analysis of the effects of variations without inner-city route choice changes. The network coverage takes in an area of approximately 30 km by 20 km. The network exists of 37 zones, situated at the peripherals and motorway junctions, and 610 network links. The network and traffic demand are calibrated for the morning peak period from 6-10 AM. In this period congestion occurs at numerous points on the ring under normal circumstances.

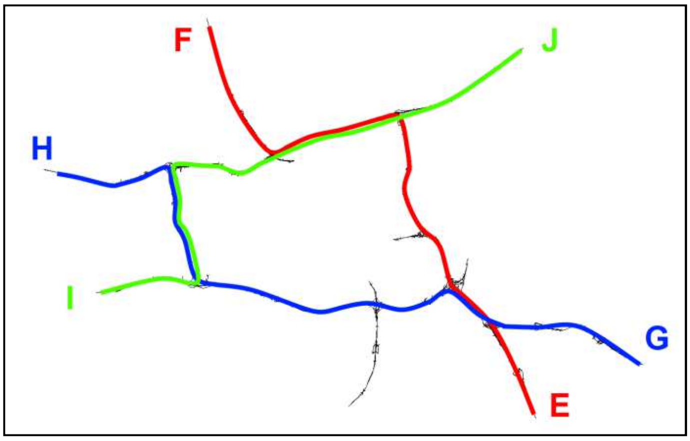


Figure 4.6: Rotterdam Ring Network for traffic simulations, showing routes E-F, G-H and I-J for travel time observations

The Hague – Gouda network

The Hague – Gouda network (Figure 4.7) is, similarly to that of the Rotterdam Ring, reduced to the main motorway and provincial roads. The main route from Gouda to The Hague is the route shown by K-L. There are alternative routes, but all at a greater travel time cost under free flow conditions. The network covers an area of approximately 30 km by 30 km and exists of 43 zones and 639 network links. The network and demand are calibrated for the morning peak period from 6-10 AM. In this period, congestion is often present on the route leading into The Hague towards location L. Along the other routes, M-N and O-K congestion is also present at certain locations during a normal morning peak period.

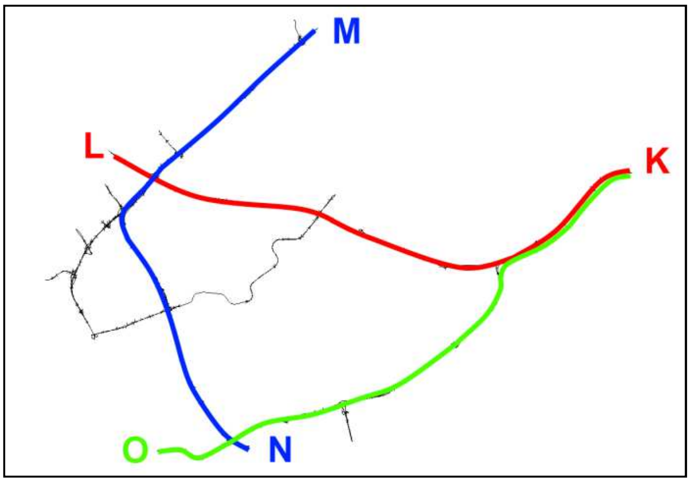


Figure 4.7: The Hague – Gouda Network for traffic simulations, showing routes K-L, M-N, O-K and K-O for travel time observations

Amsterdam South-East network

The Amsterdam South-East network comprises of four main motorway stretches and the surrounding local roads to the south-eastern side of the city (Figure 4.8). The network covers an area of approximately 20 km by 15 km and consists of 72 zones and 624 links. The network and traffic demand are calibrated for the evening peak period between 4-8 PM in which the main traffic volume is situated on the ring (along point D) and out of the city towards the south and east.

In this network, there are a number of route choices available for motorway traffic. Furthermore local roads are included in the network to give additional routing options.

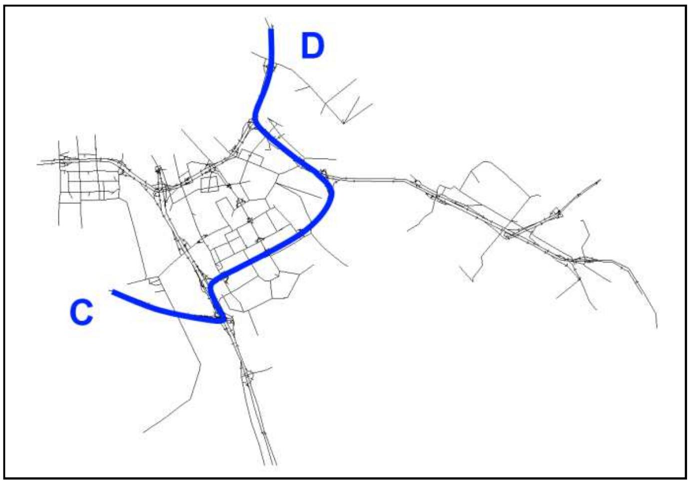


Figure 4.8: Amsterdam South-East Network for traffic simulations, showing route C-D for travel time observations

4.3.2 Performance Indicators

Network Performance Indicators

The output of the dynamic macroscopic traffic model is presented as the average travel time over the various routes and the total network delay per network, on which the capacity and the traffic demand are applied as variable input in the model. The average travel time is defined as the unweighted average of all realized travel times during the simulation on the route, and is defined as:

$$TT_{AB} = \sum_{t=1}^{n} \frac{\sum_{link=1}^{m} (\frac{l_{link.AB}}{v_{link.AB.t}})}{n} \quad (4.13)$$

Where	TT_{AB}	= travel time between origin *A* and destination *B*
	l_{linkAB}	= length of a link, situated between origin *A* & destination *B*
	$v_{linkAB.t}$	= cell speed on link at time *t*
	n	= number of time steps
	m	= number of links

In general, the total network delay Tlost is defined as:

$$T_{lost} = \sum_{veh>0} (tt_{scen.veh} - tt_{ff.veh}) \quad (4.14)$$

Where	*veh*	= vehicles
	$tt_{scen.veh}$	= travel time in the scenario
	$tt_{ff.veh}$	= travel time in free flow

In a macroscopic model, where vehicles are not modelled individually, the *total experienced delay* T_{lost} is calculated by:

$$T_{lost} = \sum_{t=1}^{n} \sum_{link=1}^{m} \left(q_{link.t} \left(\frac{l_{link}}{v_{link.t}} - \frac{l_{link}}{v_{ff.link}} \right) \right) \quad (4.15)$$

Where	t	= time
	$q_{link.t}$	= traffic flow on link at time *t*
	l_{link}	= length of link
	$v_{link.t}$	= cell speed on link at time *t*
	$v_{ff.link}$	= cell speed on link in free-flow
	n	= number of time steps
	m	= number of links

Convergence Estimator

Convergence of the network performance indicators for each Monte Carlo technique is performed using the *estimated relative error* (ER-Error). The ER-error is a method that is closely related to that of the root mean squared error (RMSE) and is often applied to determine the rate of convergence in simulations studies (Kroese et al., 2011). In many simulation studies, the true relative error cannot be determined, because the true state can either not be determined or does not exist. The ER-Error is therefore an estimator of the *true relative error* in such cases. It is also a method that is well suited for application in convergence testing (Anton, 2010). The ER-Error is defined as:

$$ERE(\bar{Y}) = \frac{\bar{\sigma}}{\bar{Y}\sqrt{N}} \tag{4.16}$$

Where $\bar{\sigma}$ = estimator of the standard deviation
$\bar{Y}$ = estimator of the unknown distribution X
N = number of samples

From this the similarity with the RMSE can be seen, which is defined as:

$$\mathrm{RMSE} = \frac{\sigma}{\sqrt{\mathrm{N}}} \tag{4.17}$$

4.4 Results

4.4.1 Convergence of input samples

Prior to the simulations with the traffic model, an analysis is performed on the samples taken as input for the model. This gives an initial indication of the convergence before the effects of the traffic network are included. In the traffic model, both the factors for the capacity values and the traffic demand are applied, such that performance indicators can be extracted from the model runs. Therefore, it is also necessary to combine the input factors for both variables when analysing the convergence of the input values. The capacity and traffic demand have an inversely proportional effect to each other, therefore the convergence of the input values is tested using the value of the capacity divided by the demand for each sample. In Figure 4.9a-b the combined division of the capacity factors by the demand factors is shown for each of the sampling techniques. The complete unsampled distribution is also plotted to show the deviation in the sampling process per technique. Note furthermore that the result of this division gives an indication of the effect on traffic demand and therefore possible congestion. For example, higher demand and lower capacity will obviously lead to congestion more rapidly, than the opposite case. This is captured in the applied measure. The applied transformation for the IS in case 1 and 2 is quadratic for the demand ($g(\boldsymbol{x}) = \frac{f(x)^2+1.89}{2}$) and a square root for the capacity ($g(\boldsymbol{x}) = \frac{f(x)^{0.5}+1}{2}$). For case 3, both the demand and capacity transformation is linear. The cumulative forms are visible in Figure 4.9a-b. From the figures it is evident that the joint samples of the LHS and SQS techniques represent the true joint distribution to a better extent than CMC. Quantitatively the mean squared error of the CMC distribution is 2.1 times higher than that of LHS, and of a similar magnitude for SQS. Especially for values further from 1.0 the error in sampling is higher (1.0 indicates no change to the capacity-demand ratio). For the IS technique, it is difficult to state how the performance is prior to modelling, as IS makes use of a 'dummy' distribution or estimator distribution for sampling which is later corrected using the corresponding weights. For this reason, it cannot be compared to the other distribution in Figure 4.9a-b. It is nevertheless included to demonstrate the difference in approach and what this means for the applied distributions. The shape of the estimator distribution is also visible from the sample distribution of IS in Figure 4.9a-b.

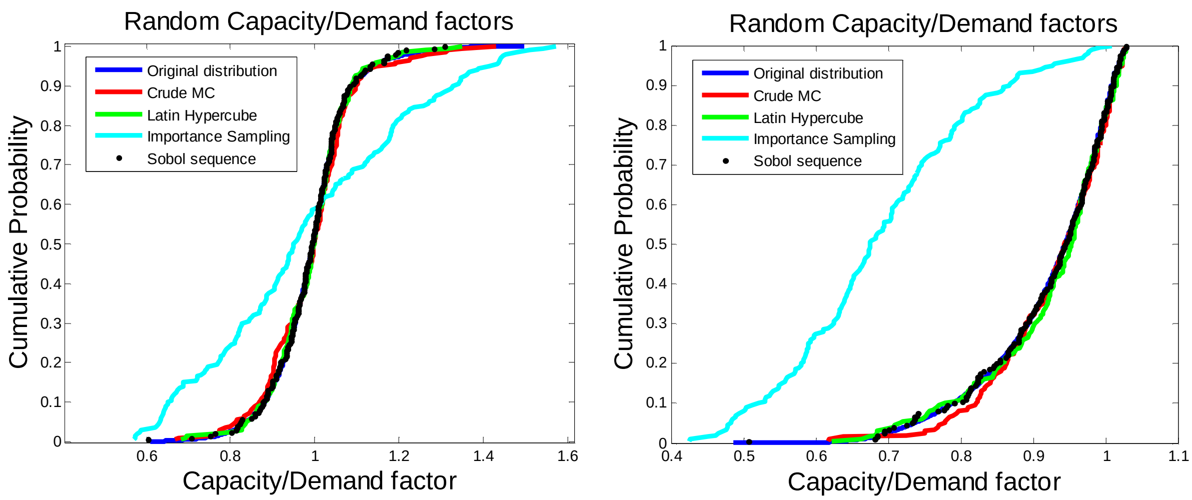


Figure 4.9 a-b: Cumulative distribution function of the joint capacity factor/traffic demand factor, including the sample distributions, as applied in a) case 1 & 2: The Hague-Gouda and Rotterdam Ring and b) case 3: Amsterdam South-East

The ER-error is calculated for the joint input samples, and is shown as a function of the sample size for convergence towards the original joint distribution. These results are shown in Tables 4.1 and 4.2. For the samples from case 1 and case 2, the rate of convergence of the combined input values is greater for both LHS and SQS than for the CMC. The results are shown for up to 100 iterations, as between 100-200 iterations further convergence was discovered to be marginal. The initial convergence of the IS technique is similar to that of CMC. For the combined input samples for case 3, the greater rate of convergence for both the LHS and SQS techniques in comparison to that of CMC is evident. The convergence of IS however is poor in comparison. For both sets of samples, the LHS and SQS techniques show a greater distribution of samples, which in turn leads to a greater rate of convergence. The difference in performance of the IS technique between the cases is most probably due to the application of the initial estimator distribution which is identical for both samples sets, while the initial distributions for the capacity and demand are different in both sets. To demonstrate the importance of a correct estimator distribution, the applied distribution is not further improved for the distributions used for the IS technique in case 3. This means that we expect IS to work well in case 1 and 2, but poorer in case 3.

Table 4.1: Estimated Relative error of the joint input samples, applied to case 1 and 2

Iterations	CMC	LHS	SQS	IS
10	0.0284	0.0095	0.0108	0.0244
20	0.0146	0.0057	0.0056	0.0172
50	0.0048	0.0037	0.0022	0.0072
100	0.0030	0.0015	0.0006	0.0046

Table 4.2: Estimated Relative error of the joint input samples, applied to case 3

Iterations	CMC	LHS	SQS	IS
10	0.0246	0.0082	0.0168	0.0756
20	0.0122	0.0048	0.0044	0.0534
50	0.0062	0.0022	0.0018	0.0286
100	0.0056	0.0012	0.0004	0.0198

4.4.2 Convergence of model results

The sampled input values for the capacity and traffic demand are applied to the dynamic macroscopic traffic model, from which the travel time along the defined routes and the total network delay are captured for each of the three cases. Both indicators have an (unknown) real distribution to which the Monte Carlo results should converge with increasing iterations, N. The rate of convergence shown as the ER-error for the travel time and the total network delay are shown for each case. It should further be noted that the raw random samples applied on the distributions for the sampling techniques are identical for all three sampling techniques per case. Therefore, a fair comparison can be made without needing to consider random differences between techniques. The Sobol sequence samples are obviously not identical to the other techniques, but are applied identically over each case. The main difference between the techniques lies in the way the four techniques apply the samples to form input values for the model from the capacity and demand distributions.

Case 1: The Hague – Gouda

The rate of convergence of the ER-error of the total network delay for case 1 (The Hague – Gouda) is shown in Figure 4.10 and Table 4.3. The results of the delay values given by the LHS technique show a better convergence, which is a factor 2 better after 50 iterations and a factor 3 after 100 iterations. The SQS achieves similar results after 100 iterations, but appears to reach this level at a slower rate. The convergence of IS does not improve on that of CMC for the delay in case 1.

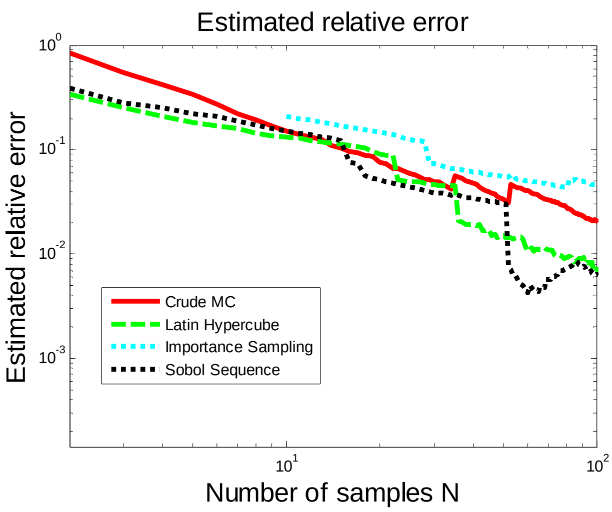


Figure 4.10: Convergence of the total network delay indicator case 1: The Hague – Gouda network

Table 4.3: Estimated relative error of the total network delay case 1: The Hague - Gouda

Iterations	CMC	LHS	SQS	IS
10	0.1518	0.1312	0.1516	0.2225
20	0.0746	0.0917	0.0512	0.1573
50	0.0336	0.0152	0.0306	0.0525
100	0.0206	0.0072	0.0064	0.0430

Convergence of the ER-errors for the travel times on the defined routes in The Hague – Gouda network is shown in Table 4.4. On each of the routes the LHS technique shows a strong rate of convergence with error values varying per route, but generally in the order of 2 times lower than CMC. Interestingly SQS performs substantially better on all the routes in comparison to LHS, except route O-K, despite the convergence in delay being similar between the two. In this case IS also performs poorer than CMC for the route travel time convergence.

Table 4.4: Estimated relative error of the route travel times case 1: The Hague - Gouda

Route	Iterations	CMC	LHS	SQS	IS
K-L	20	0.1030	0.0608	0.0348	0.1973
	50	0.0196	0.0385	0.0084	0.1190
	100	0.0322	0.0217	0.0038	0.0410
M-N	20	0.0460	0.0222	0.0132	0.0748
	50	0.0114	0.0105	0.0050	0.0493
	100	0.0112	0.0057	0.0022	0.0183
O-K	20	0.0342	0.0625	0.0306	0.1188
	50	0.0218	0.0030	0.0128	0.0675

	100	0.0164	0.0045	0.0020	0.0393
K-O	20	0.0600	0.0415	0.0286	0.1888
	50	0.0200	0.0242	0.0094	0.0915
	100	0.0114	0.0138	0.0050	0.0515

Case 2: Rotterdam Ring

In case 2 on the Rotterdam Ring network, the same input samples are applied as for case 1. However, as both networks are different, accompany different traffic dynamics and have different bottleneck locations and severity, the outcome of the delay from the network can also differ. The convergence of the ER-error of the delay values is shown in Figure 4.11 and Table 4.5. Both the LHS and SQS techniques show a swift convergence, in which SQS convergences at an exceptionally fast rate of ca. 3-4 times faster than CMC. The convergence of LHS is less impressive than SQS, but remains good compared to CMC. The IS technique convergences at a similar rate to CMC. Peaks in the graph for low sample numbers are a consequence of the cumulative character of the ER-error, which is more susceptible to extreme values for the lower sample iterations.

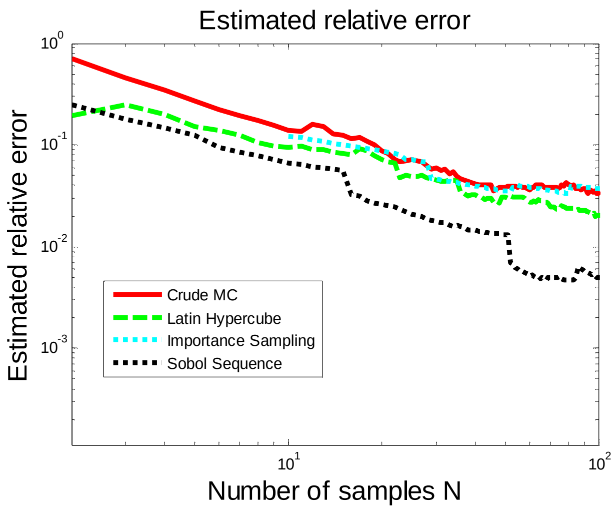


Figure 4.11: Convergence of the total network delay indicator for case 2: Rotterdam

Table 4.5: Estimated Relative error of the total network delay for case 2: Rotterdam

Iterations	CMC	LHS	SQS	IS
10	0.1414	0.0957	0.0670	0.1403
20	0.0872	0.0713	0.0260	0.0993
50	0.0378	0.0305	0.0132	0.0403
100	0.0332	0.0200	0.0050	0.0303

The convergence of the ER-errors for each of the defined routes in case 2 are given in Table 4.6. The convergence of the travel times resembles that of the delay for LHS and SQS, in which both converge well and, especially the SQS, well outperforms CMC. Interestingly, IS also shows an improvement in convergence for all routes compared with CMC. This would suggest that there is a large spread in the travel times, which IS, as well as the other techniques, are able to represent much better than CMC.

Table 4.6: Estimated relative error of the route travel times for case 2: Rotterdam ring

Route	Iterations	CMC	LHS	SQS	IS
E-F	20	0.0734	0.0573	0.0182	0.0695
	50	0.0340	0.0285	0.0088	0.0323
	100	0.0320	0.0195	0.0044	0.0285
G-H	20	0.1898	0.1333	0.0290	0.1773
	50	0.0820	0.0803	0.0106	0.1320
	100	0.0790	0.0498	0.0040	0.0645
I-J	20	0.1670	0.1097	0.0226	0.2090
	50	0.0836	0.0743	0.0084	0.1235
	100	0.0748	0.0477	0.0036	0.0323

Case 3: Amsterdam South-East

In case 3 for the Amsterdam South-East network, a different capacity and demand distribution is applied as to case 1 and case 2. From the convergence of the input value errors (Figure 4.12), it may be expected that the performance should be similar to that of case 1 and 2. However, different network characteristics can lead to different outcomes. Especially the IS technique may perform worse than in case 1 and 2, as indicated from the input value errors. The results of the convergence in network delay are shown in Table 4.7. The convergence of the travel times on route C-D is given in Table 4.8. The LHS shows for the delay an initial convergence greater than CMC, while SQS easily outperforms the reference CMC. As expected, the IS technique performs very poorly and lacks well behind CMC. A similar result is found for the convergence of the travel times on route C-D, in which SQS shows excellent convergence, while LHS also performs well.

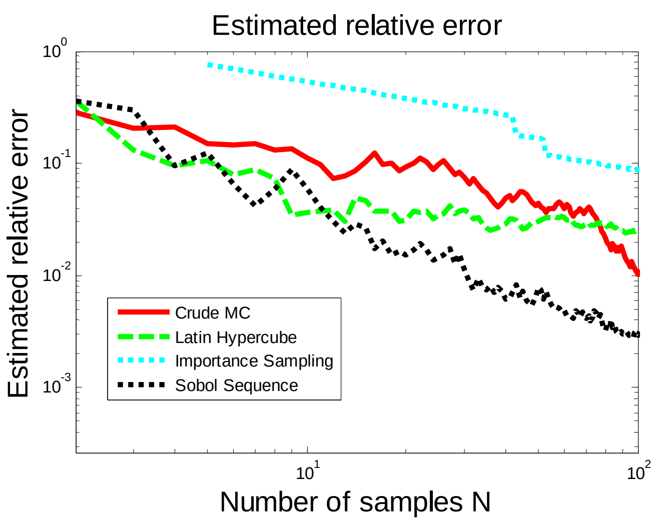


Figure 4.12: Convergence of the total network delay indicator for case 3: Amsterdam

Table 4.7: Estimated relative error of the route travel times for case 3: Amsterdam South-East

Iterations	CMC	LHS	SQS	IS
10	0.1114	0.0370	0.0586	0.4868
20	0.0916	0.0310	0.0154	0.3442
50	0.0444	0.0302	0.0078	0.1489
100	0.0102	0.0243	0.0030	0.0837

Table 4.8: Estimated relative error of the route travel times for case 3: Amsterdam South-East

Route	Iterations	CMC	LHS	SQS	IS
C-D	20	0.0908	0.0432	0.0108	0.2875
	50	0.0576	0.0202	0.0056	0.1413
	100	0.0474	0.0122	0.0028	0.0815

4.4.3 Discussion of results

The results demonstrate that variance reduction techniques can substantially improve convergence in stochastic and reliability modelling using Monte Carlo simulation for traffic modelling. This is evident in all three cases, in which either one or more of the techniques improved on the reference technique of Crude Monte Carlo (CMC), despite the differences between the three networks and the defined routes.

Latin Hypercube Sampling (LHS) shows a good ability to improve convergence of the performance indicators from the traffic model for all considered scenarios. LHS is seen as a stable and reliable technique that is especially powerful for multiple variables, but here even with two variables (capacity and demand) showed its power. In each of the scenarios the improvements shown in the ER-error values are significant in comparison to CMC, and in many cases even with half the error or more for the same number of iterations.

The Sobol Quasi-random Sequence (SQS) method is especially designed to sample such that a comprehensive coverage of values is achieved even from a relatively small number of iterations. This ability was clearly shown in each of the cases in which the method demonstrated error values multiple times lower than CMC, and often also compared to LHS, for the convergence of the network delay and travel time distributions. Of all the considered methods here, the SQS clearly performed the best over all cases.

Importance Sampling (IS) is a technique that is especially applicable when stochastic distributions show a large degree of variation (Kroese et al., 2011, van Lint et al., 2012). The technique is however dependent on the applied estimator or ‘dummy’ distribution. This dependence is clearly seen in this research between the two differently applied sets of distributions with the same initial estimator distribution for IS. In case 1 and 2, the rate of convergence was similar to that of Crude Monte Carlo. For case 3, using a less distributed and more symmetrical distribution, the IS technique performed worse in comparison to CMC. The overall performance of IS was poor, but in this we have also demonstrated the importance of applying an optimized estimator distribution. Various techniques exist to assist the choice and estimation of the estimator distribution, which are not dealt with here, but still require the necessary expertise and effort to apply correctly. Herein, the sensitivity of the technique also becomes evident.

4.5 Conclusion

Advances in Monte Carlo simulation techniques in past decades have led to a substantial potential for increased sampling efficiency. However, the development of variance reduction techniques has struggled to find its way into stochastic traffic modelling. In this chapter, it has been demonstrated that the incorporation of variation in traffic modelling through advanced variance reduction techniques in Monte Carlo simulation has the ability to substantially reduce computational load by improving convergence to a representative state. The ability to increase the rate of convergence using Latin Hypercube Sampling showed a decrease in the number of simulations required to achieve comparable error levels from that traditional Crude Monte Carlo simulation. Latin Hypercube Sampling is most effective for multiple input variables. In the considered cases, there were two stochastic variables which proved to be sufficient for this stratified technique to substantially improve convergence. Sobol Quasi-random Sequences, just like Latin Hypercube Sampling, sample with an explicit spread from a set, however they also explicitly consider the consequential construction of the samples using an analytical sequence. This was clearly shown to be the most effective technique in the presented cases. For most indicators, the error level was a multifold smaller compared to Crude Monte Carlo and, in most cases, also compared to Latin Hypercube.

Importance Sampling has a great potential to decrease computational load through capturing the extremities of a distribution, especially when the traffic system has an amplified effect on the outcome, as is often the case in congestion. The technique however is dependent on the applied estimator distribution. Application of an estimator method to optimize the estimator distribution is therefore essential. In this contribution, the importance of a reliable estimator function is shown from the difference between the cases.

In this chapter, variance reduction techniques have clearly shown to be stable and consistently able to improve convergence of samples to a true distribution allowing for a reduction in computational load and to make stochastic and reliability analyses with Monte Carlo simulation in traffic modelling more applicable and efficient. Especially the application of a sequential technique, such as Sobol Quasi-random sequencing, has significant potential to allow faster Monte Carlo simulation in traffic modelling. Also other variance reduction techniques also yield good results, such as Latin Hypercube sampling, and likely others not explicitly considered in this contribution.

Chapter 5

Core probability framework and modelling

From previous chapters, it was made clear that it is imperative to explicitly consider stochastic variation in traffic flow modelling, when this variation is present in the considered scenarios and networks. In this chapter, a new stochastic macroscopic framework is introduced which, combined with the relevant dynamic network loading (DNL) models, tackles many challenges in macroscopic modelling and is developed with a view for easy and efficient application in practice.

The Core Probability Framework (CPF) is a probabilistic framework for modelling multi-dimensional variations in capacity and traffic demand in dynamic macroscopic traffic flow. The CPF extends a base model, such as the Cell Transmission Model (CTM), by considering each traffic variable as a stochastic variable denoted as a probability distribution of the chance of values for each traffic variable. The CPF is accompanied by the Discrete-Element Core Probability Model (DE-CPM) as an example of a possible DNL model. The DE-CPM is introduced as an internalisation of the Monte Carlo routine in the core of the traffic model.

A description of the conceptual framework of the CPF and the application of the DE-CPM DNL model is given in sections 5.1-5.5. In sections 5.6, an explanation is given how the model addresses some of the issues mentioned in Chapter 2. Section 5.7 shows a demonstration case of the model in practice and the potential calculation time gains for two networks.

This chapter is an edited version of the article:
Calvert, S. C., Taale, H., & Hoogendoorn, S. P. (2014). Introducing the Core Probability Framework and Discrete-Element Core Probability Model for efficient stochastic macroscopic modelling. In *DTA 2014: 5th International Symposium on Dynamic Traffic Assignment, Salerno, Italy, 17-19 June 2014*.

5.1 Core Probability Modelling

In this section, the framework for the Core Probability modelling and the underlying assumptions are explained. The Core Probability Framework (CPF) extends an existing macroscopic traffic flow model to allow uncertainty scenarios in traffic to be *internalised* in the traffic flow model which it extends. Internalisation here refers to a single model execution in which uncertainty is considered, without multiple simulations. Monte Carlo simulation is a clear example of external stochastic influence to this extent. Initial application of the CPF makes use of the Cell Transmission Model (CTM) as base model. The basic premise entails replacing single traffic variables in time and space, such as the density, in a model with a distribution of that same traffic variable, also in space and time. The distribution, denoted as a vector, consists of predefined probabilities of various possible values of the considered traffic variable at a certain time and location, therefore transforming the traffic variables into stochastic variables. The general dynamics of the base model are kept the same as the deterministic version of the base model. In such a way, traffic is propagated through a link (or network) considering possible valid values of each traffic variable with a set probability, using already validated traffic flow dynamics from the base model. The input distributions are empirically determined for specific locations and/or scenarios or from generic empirical analysis (Calvert et al., 2014a, van Stralen et al., 2014).

The framework allows different probabilistic models for propagation of the stochastic traffic flows to be developed and applied. In this chapter, we further present the Discrete-Element Core Probability Model as one option for use of the framework. A more detailed description of the framework and this model are given in the subsequent subsections. This begins with a short explanation of the applied base model (5.2). The concept of the CPF is given in 5.3 and is followed in section 5.4 by the description of the manner in which probability is included in the DE-CPM, how it is propagated, and how congestion and traffic states are dealt with. A simple numerical example is shown to conclude the section (5.5).

5.2 Base model

The Core Probability Framework makes use of a base model, which describes the manner in which traffic flow propagates, and considers stochastic probabilities in the core of a macroscopic traffic model. The base model applied here is the first order Cell Transmission Model (CTM) (Daganzo, 1994, Daganzo, 1995a). The CTM describes traffic using a discretised form of the Lighthill-Whitham-Richards (LWR) model (Lighthill and Whitham, 1955). The LWR model is governed by the law of conservation of vehicles equation (5.1), and the fundamental relation equation (5.2):

$$\frac{\partial k(x,t)}{\partial t} + \frac{\partial q(x,t)}{\partial x} = 0 \tag{5.1}$$

$$q(x,t) = Q_E(k(x,t)) \tag{5.2}$$

Here, $\partial k(x,t)/\partial t$ denotes the change in density in time t; $\partial q(x,t))/\partial x$ denotes the change of the same for the flow rate over space x, while Q_E is the fundamental relation between the density and flow, which is explained in more detail later on.

In the CTM, the continuous model is used as a basis for the description of the flow propagation in discretized time and space. With respect to the spatial discretization, cells m are considered. In the CTM, the traffic flow at the interfaces between two cells, q, is determined by a sending and receiving function, denoted here as the demand, D, and supply, S, which closely represent the available capacity in a cell and the desired traffic flow into a cell:

$$q^{x_m \to x_{m+1}}(k(x,t)) = \min(D_m(k(x,t)), S_{m+1}(k(x,t))) \tag{5.3}$$

The demand function D is calculated by the largest flow or capacity of cell m in relation to equation (5.2), and the supply function S by the desired outflow from the previous cell according to the fundamental traffic characteristics of the preceding cell. The base model is applied in its discrete form for use in the Core Probability Framework and governs the main dynamics of traffic flow.

5.3 Core Probability Framework

The main premise of the Core Probability Framework (CPF) is the incorporation of uncertainty in the core of the model as probability distributions. While regular Monte Carlo simulation applies uncertainty through multiple simulation iterations, for the CPF these are internalised. This approach allows for a *one shot* simulation run and an increased efficiency in simulation. The uncertainty is applied in the form of (discrete) empirical probability distributions, which describe the variations in traffic variables in the model and are primarily applied as cumulative probability functions of the traffic demand at the origins and the capacity of each cell. A graphical description of the Core Probability Framework is shown in Figure 5.1b, alongside the general framework of a Monte Carlo routine as a comparison over a similar macroscopic traffic model for a simple three cell road stretch (Figure 5.1a).

The CPF is in its self not a DNL, but rather the framework which states that distributions are explicitly propagated through time and space in combination with the dynamics of the base model. The Discrete-Element Core Probability Model (DE-CPM), in combination with the CTM, is given in this contribution as a possible DNL model that may be applied in the framework, which describes how the distributions of the stochastic traffic variables are propagated through the network.

Figure 5.1 clearly shows the evasion of multiple simulations in the case of the CPF in comparison to a Monte Carlo routine over the same base model. In the rest of this section, the CPF is explained for the application of the DE-CPM. Other core probability models may also be applied to the CPF, but are not discussed in this chapter.

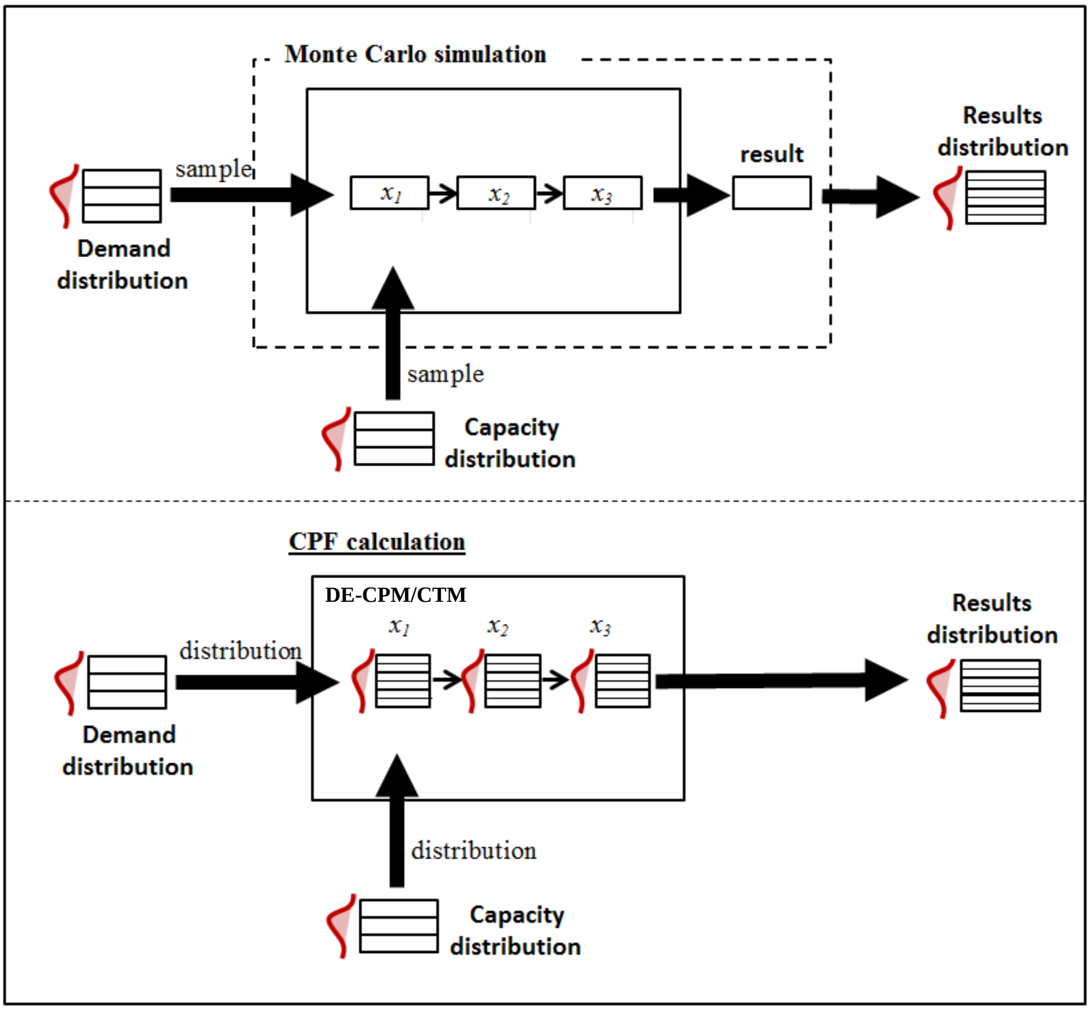


Figure 5.1a-b: Conceptual overview of the (a - above) Monte Carlo traffic simulation framework and (b - below) the Core Probability Framework

5.4 Discrete-Element Core Probability Model

5.4.1 Concept

The Discrete-Element Core Probability Model (DE-CPM) is a DNL model that makes use of the Core Probability Framework to propagate traffic through a link and network. The DE-CPM describes the traffic variables as a distribution, denoted as a vector, which consists of static probabilities of various possible values of the considered traffic variable at a certain time and location. For each variable at each time step, identical static probability elements are used in the distribution. Each discrete element in the distribution is explicitly kept from interaction with other elements as the flow distributions are propagated through the network. This approach basically creates an internalisation of the Monte Carlo routine, in which each discrete element or ‘scenario’ is kept separate. In such a way, traffic is propagated through a link (or network) considering possible valid values of each traffic variable with a set probability, using already validated traffic flow dynamics from the base model.

In the following paragraphs the Core Probability Framework is defined for application of the Discrete-Element CPM as network loading model.

5.4.2 Inclusion of probability

In classical first order models, each variable is represented by a single value for each point in time, t, and space, x. In the core-probability approach, a further variable is added, which represents the probability of the density occurring, and sequentially the traffic flow, $\boldsymbol{q}$, and the speed, $\boldsymbol{v}$. This further transforms the variables from a single value in time and space into a probability distribution in the same time and space, represented by their corresponding vector.

Presuming static values for the probability elements avoids the necessity to explicitly define the probabilities of the values corresponding to the probabilities for each cell (*m*) in each time step (*n*).

Initially in the continuation of the description, for the reason of clarity, a further presumption is made that each value in the probability vector has identical probability. This assumption also entails that the discrete probability values for each probability element are set for the entire simulation for all time steps (*n*), cells (*m*), and for each variable (*k*, *q*, *v*):

$$\boldsymbol{p}\text{:}\ p_1 = p_2 = \ldots = \ p_i \quad \forall(k, q, v) \tag{5.4}$$

Now let the random variable $K(x,t)$ denote the density on a cell [x, x+dx] and at time t. Let $p_i(x,t)$ denote the accompanying probabilities. Such a relation is given as:

$$P(K(x,t) = k) = p_i(x,t) \tag{5.5}$$

Note that the values of k are discrete and hence a discrete probability function can be used. However, such a notation indicates a variable probability as a function of given densities. The CPM presumes set probability elements, and therefore the random density variable $K(x,t)$ is defined as a function of set probabilities instead.

So for example, $K(x,t)$, now written as vector $\boldsymbol{k}(x,t\,;\boldsymbol{p})$, denotes all possible values of the density for a moment in time and a location, given the probabilities of these densities. The density vector can also be written as:

$$\boldsymbol{k}(x,t\,;\boldsymbol{p}) = \begin{Bmatrix} k_1(x,t)\ with\ probability\ p_{d.1} \\ k_2(x,t)\ with\ probability\ p_{d.2} \\ \ldots \\ k_i(x,t)\ with\ probability\ p_{d.i} \end{Bmatrix} \tag{5.6}$$

This notation is similar to the one applied in Fuzzy Logic, in which a crisp number is denoted as having multiple possible values, each with their own probability (Buckley, 2005). Here, the notation is borrowed from Fuzzy Logic Theory, while applying General Probability Theory, which states in this case that k is a stochastic variable, which has various values with predefined probabilities.

From now on, we will only use the short form for the density vector, rather than the description on the right hand side of equation (5.6). The addition of the vector $\boldsymbol{p}$ includes all possible values of the appropriate variable with identical probabilities of each value in time and space, so that:

$$\boldsymbol{p} = p_1 + p_2 + \dots + p_i = 1 \tag{5.7}$$

Here, i is further limited to a finite value, which is applied as an input parameter of the model. The equations for the conservation of vehicles equation (5.1) and the fundamental relation equation (5.2) now incorporate a further dimension for the probability in time and space, and become dependent on the probability of their value:

$$\frac{\partial \boldsymbol{k}(x,t\,;\boldsymbol{p})}{\partial t} + \frac{\partial \boldsymbol{q}(x,t\,;\boldsymbol{p})}{\partial x} = 0 \tag{5.8}$$

$$\boldsymbol{q}(x,t\,;\boldsymbol{p}) = Q_E(\boldsymbol{k}(x,t\,;\boldsymbol{p})) \tag{5.9}$$

The conservation of vehicles therefore remains intact by definition, as each considered element in the probability distribution vector acts as an individual case of the CTM for which conservation has been proven (Daganzo, 1994).

6.4.3 Application of stochastic demand, capacity and traffic propagation

Stochastic traffic demand is applied in the model at the peripherals of a network on the inflowing cells. From there on, traffic may propagate applying equation (6.8) and equation (6.9) according to the dynamics of the base model. The initial traffic demand contains j_d times j_c number of elements in the probability vector $\boldsymbol{p}$, where j_d is the number of probability elements in the vector for the demand and j_c is the number of probability elements for the capacity, such that each probability vector $\boldsymbol{p}$ is constructed of all possible combinations of p_{jd} and p_{jcd}. The initial flow at the network origins is therefore:

$$\boldsymbol{q}(x_0,t_1\,;\boldsymbol{p}) = \{q_{p1}, q_{p2}, \dots q_{p(jd\cdot jc)}\} \tag{6.10}$$

where the probability vector $\boldsymbol{p}$ exists of j_d times j_c elements. This multiplication is performed to accommodate sufficient elements in the discrete probability distribution for the outcomes of each combination of traffic demand and capacity.

The variation in the capacity of the network is applied for each cell corresponding to the probability of the capacity of that cell in a similar way to the traffic flow $\boldsymbol{q}$. In a simplified case only bottleneck cells will have varied capacity values, with the other cells yielding identical capacity values for each element in $\boldsymbol{p}$. The capacity contains j_c probability elements for the capacity in both time and space, although for most cells, variation in the capacity has little to no influence where flow is sub-critical:

$$\boldsymbol{q}_{cap}(x_m, t_n\,; \boldsymbol{p}) = \begin{Bmatrix} q_{cap.1}(x,t)\ with\ probability\ p_{c.1} \\ q_{cap.2}(x,t)\ with\ probability\ p_{c.2} \\ \dots \\ q_{cap.i}(x,t)\ with\ probability\ p_{c.i} \end{Bmatrix} \tag{5.11}$$

Once the traffic from the stochastic scenarios is on the network, the traffic propagates through the network dependent on the corresponding demand and following the dynamics as previously shown in equation (5.8) and equation (5.9).

Spatiotemporal dependence is applied as a conditional probability at the entrance of a network, between the initial demand (applied to connector links to get initial densities) and capacity variables. Propagation of this dependence entails that each element in the probability vector of the density corresponds to the same place in the probability vector of the density of the following time step. This is described as the *chain-rule*, as graphically shown in Figures 5.2 & 5.3, and is further described later in this paragraph and is given in equation (5.15). The *chain-rule* ensures an identical number of elements in the resulting probability vector for propagation through the network, and therefore avoids an explosion of marginal probability elements. Basically, this creates a set of values which can be seen as scenarios of unique traffic demand and capacity combinations.

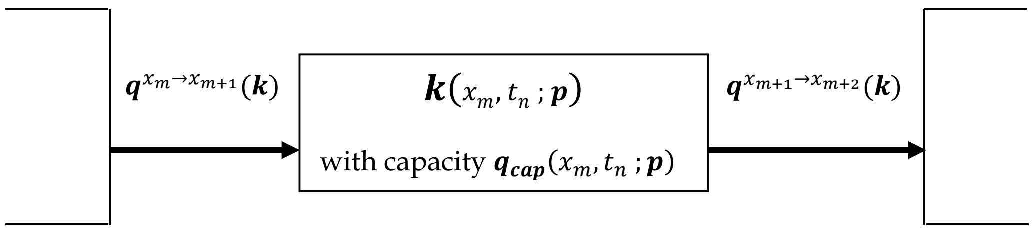


Figure 5.2: Traffic propagation in the DE-CPM

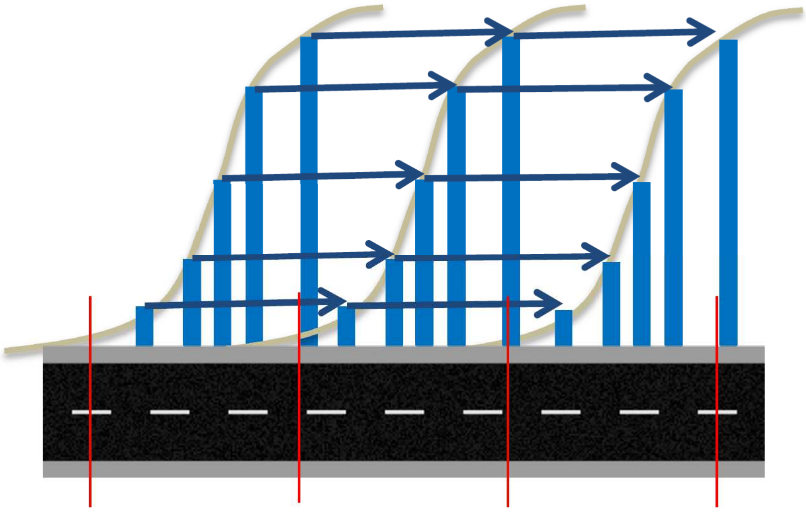


Figure 5.3: Chain-rule for propagation of traffic variables as discrete elements of a distribution in the DE-CPM

The process is explained as such: there is a traffic demand $q(x_1, t_1)$ with a set of possible values, q_p, corresponding to certain probabilities:

$$q(x_1, t_1\,; p) = \{q_{p1}, q_{p2}, \dots q_{pi}\} \tag{5.12}$$

Calculations in the model are performed using the density, therefore q is transformed using equation (5.2) to:

$$k(x_1, t_1\,; p) = \{k_{p1}, k_{p2}, \dots k_{pi}\} \tag{5.13}$$

In the following time step, there is a new q and k at location x_1, in line with traffic flow in and out of the cell and in keeping with the conservation of vehicles equation (5.1) :

$$k(x_1, t_2\,; p) = \{k_{p1}, k_{p2}, \dots k_{pi}\} \tag{5.14}$$

However, the position of each element in the $(x_1, t_2\,; p)$ corresponds only to that of the element in the same position in the following time step in $\boldsymbol{k}(x_1, t_2\,; \boldsymbol{p})$, so that for each element, i, applies:

$$q(x_1, t_2\,; p_i) \;\rightarrow\; q(x_1, t_1\,; p_i) \tag{5.15}$$

This strict *'chain-rule'*, that demands that for each location in consecutive time steps the same probability must apply, protects the validity of the initial conditional dependence between the capacity and traffic demand in both time and space.

Although the CTM base model, and therefore also CPF / DE-CPM, calculates traffic using the density, it is often required to translate this to the traffic flow $\boldsymbol{q}(x,t\,;\boldsymbol{p})$, for determination of the flux for example. This is performed using the fundamental relation shown in equation (5.2), in which each value of q is transformed using a deterministic fundamental diagram. The resulting values of $\boldsymbol{q}(x,t\,;\boldsymbol{p})$ from $\boldsymbol{k}(x,t\,;\boldsymbol{p})$ maintain the same probabilities for each time step and cell in space.

In the same way, the traffic flow on the subsequent cells is also calculated. The only difference is that the supply and demand refer to those of the following cells, x_j. In such a way, one can speak of multiple scenarios in a single procedure, as each element of the marginal probabilities are considered individually for a single variable.

5.4.4 Determination of Congestion

The sending and receiving functions, or rather demand and supply, $\boldsymbol{d}$ and $\boldsymbol{s}$, are in part determined by the traffic state. Traffic states are in turn determined by the density of traffic in a cell at a specific time. Under congestion, the demand function is equal to the capacity, and the supply function of the outgoing traffic flow:

$$\boldsymbol{d}(x_m,t_n\,;\boldsymbol{p}) = \boldsymbol{q_{cap}}(x_{m-1},t_n\,;\boldsymbol{p}) \tag{5.16}$$
$$\boldsymbol{s}(x_m,t_n\,;\boldsymbol{p}) = \boldsymbol{q}(x_m,t_n\,;\boldsymbol{p}) \tag{5.17}$$

For uncongested states, the demand function is the incoming traffic flow, and the supply function is the available capacity:

$$\boldsymbol{d}(x_m,t_n\,;\boldsymbol{p}) = \boldsymbol{q}(x_{m-1},t_n\,;\boldsymbol{p}) \tag{5.18}$$
$$\boldsymbol{s}(x_m,t_n\,;\boldsymbol{p}) = \boldsymbol{q_{cap}}(x,t\,;\boldsymbol{p}) \tag{5.19}$$

For the Core Probability Framework without capacity variation, congestion is determined by comparison between the probable density and the critical density of a cell:

$$\boldsymbol{Cong}(x,t\,;\boldsymbol{p}) = \boldsymbol{k}(x,t\,;\boldsymbol{p}) \geq k_{crit}(x,t) \tag{5.20}$$

However, when capacity is also varied, the congestion equation states a distribution vector on either side of the operator:

$$\boldsymbol{Cong}(x,t\,;\boldsymbol{p}) = \boldsymbol{k}(x,t\,;\boldsymbol{p}) \geq \boldsymbol{k_{crit}}(x,t\,;\boldsymbol{p}) \tag{5.21}$$

5.4.5 Network flow over nodes

For modelling traffic in networks, it is imperative to consider traffic flow over the nodes. This is usually performed using a node model which deals with the manner in which traffic propagates at convergence and divergence points in a network, but also how other traffic waves, such as congestion may propagate in an upstream direction. This contribution does not aim at developing a stochastic node model, and therefore they will not be reviewed here. For an overview of the state-of-art of node models we refer to (Tampère et al., 2011). The inherent characteristics of the chain-rule, as used in the DE-CPM for the propagation of distributions as an internalisation on the Monte Carlo routine, determine that just about any arbitrary node model that is applicable for the base model may be applied in the CPF.

This is demonstrated for the merge model as described by Daganzo (Daganzo, 1995a) for an uncongested flow. The merge model describes the maximised flow, Q, from two incoming links, $i = 1,2$, into a single outgoing link 3. As seen already from the CTM, sending flows perpetuate from the upstream links (see equation (5.3)). These flows are constrained by the maximum flows that may leave each link: S_1, S_2. Likewise, the receiving downstream link also has a maximum flow that it is capable of receiving: R_3. Therefore, we can easily see that traffic flow is constrained by either the traffic demand from the inflowing links or the supply of capacity from the receiving link according to:

$$q_i \leq S_i \qquad \forall i \in \{1\,,2\} \tag{5.22}$$

$$\sum_{i=1,2} q_i \leq R_3 \tag{5.23}$$

Considering the constraints and convergence of the flow from equations (5.22) and (5.23), it becomes apparent that the flow into the receiving downstream link for uncongested circumstances is:

$$\mathrm{Q} = \min\{S_1 + S_2\,; R_3\} \tag{5.24}$$

Extension of the node model for use in the DE-CPM extends equation (5.22) and equation (5.23) by considering each variable as a discrete stochastic variable in which the chain-rule is valid between the corresponding elements of the variables. Hence, equations (5.22-5.24) become:

$$\boldsymbol{q}_i(t_1\,;\boldsymbol{p}) \leq \boldsymbol{S}_i(t_1\,;\boldsymbol{p}) \tag{5.25}$$

$$\sum_{i=1,2} \boldsymbol{q}_i(t_1\,;\boldsymbol{p}) \leq \boldsymbol{R}(t_1\,;\boldsymbol{p}) \tag{5.26}$$

$$\mathbf{Q}(t_1\,;\boldsymbol{p}) = \min\{\boldsymbol{S}_1(t_1\,;\boldsymbol{p}) + \boldsymbol{S}_2\,(t_1\,;\boldsymbol{p}); \boldsymbol{R}(t_1\,;\boldsymbol{p})\} \tag{5.27}$$

In equations (5.25-5.27), $\boldsymbol{p}$ indicates the entire distribution vector for which is valid p *for* $\forall\, p \in \boldsymbol{p}$ according to the previously defined chain-rule for an arbitrary variable, *f*: $f(p_i) \rightarrow$

$f(p_i)$. Graphically, it is very easy to observe how the propagation of traffic in the DE-CPM does not require special attention for nodes beyond the introduced theory that is also applicable for stretches. Convergence and divergence of traffic flow at a node are again dealt with according to the dynamics of the base node modal, where each element from the stochastic variables is processed independently.

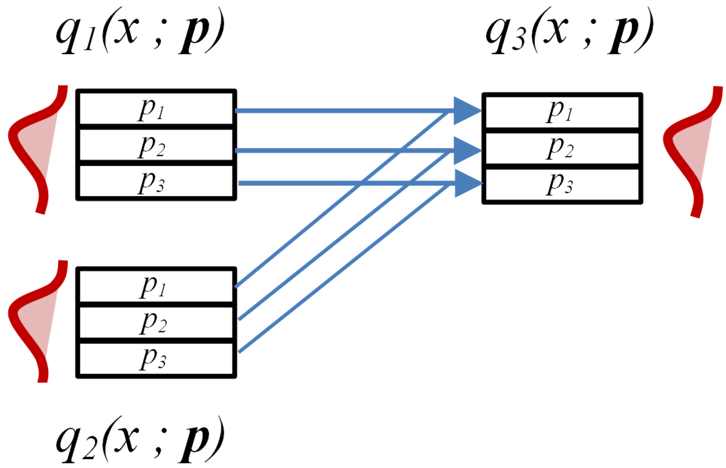


Figure 5.4: Graphical representation of the DE-CPM for a node merge

The same simple extension applies to other node models and the additional equations that describe the congested states in the node models for application in the DE-CPM. As the chain-rule explicitly keeps the individual elements of the discrete distribution separated for calculation, these act in the same fashion as the deterministic case for which the models are already developed.

5.5 Simple numerical example (both capacity and demand varied)

To demonstrate the manner in which the DE-CPM works, a simple numerical example is given as demonstration. A more elaborate demonstration is given in section 5.5. The traffic demand at the network peripherals is given as a flow with a set probability. In this example there is a 50% chance of two different inflow values, and there is 50% of two different capacity values. Therefore, there are 4 elements in the demand vector, because the size of $\boldsymbol{q}(x, t\,; \boldsymbol{p})$ is equal to j_d times j_c (see equation (5.10)):

$$\boldsymbol{q}\left(x_1, t_1, \boldsymbol{p} = \begin{Bmatrix} 0.25 \\ 0.25 \\ 0.25 \\ 0.25 \end{Bmatrix}\right) = \begin{Bmatrix} 1900 \\ 1900 \\ 2200 \\ 2200 \end{Bmatrix} \tag{5.28}$$

The capacity values of the cell are also given in the, j_d times j_c number of elements, capacity flow vector:

$$\boldsymbol{q_{cap}}\left(x_1, t_1, \boldsymbol{p} = \begin{Bmatrix} 0.25 \\ 0.25 \\ 0.25 \\ 0.25 \end{Bmatrix}\right) = \begin{Bmatrix} 2100 \\ 2300 \\ 2100 \\ 2300 \end{Bmatrix} \tag{5.29}$$

Note that the sequences for the values of the flow in the demand vector equation (5.28) are differently arranged over the j_d times j_c elements in comparison to the capacity flow vector equation (5.29).

This flow vector, $\boldsymbol{q}(x,t\,;\boldsymbol{p})$, in equation (5.28) is transformed to a density vector, $\boldsymbol{k}(x,t\,;\boldsymbol{p})$, using the fundamental relation $q = Q_E(k)$ in which the critical density is $k_{crit} = 25$. This gives:

$$\boldsymbol{k}\left(x_1,t_1,\boldsymbol{p}=\begin{Bmatrix}0.25\\0.25\\0.25\\0.25\end{Bmatrix}\right)=\begin{Bmatrix}22\\20\\26\\24\end{Bmatrix} \tag{5.30}$$

The probability of congestion is calculated using equation (5.20):

$$\boldsymbol{Cong}\left(x_1,t_1,\boldsymbol{p}=\begin{Bmatrix}0.25\\0.25\\0.25\\0.25\end{Bmatrix}\right)=\boldsymbol{k}(x,t\,;\boldsymbol{p})\geq k_{crit}(x,t)=\left[\begin{Bmatrix}22\\20\\26\\24\end{Bmatrix}\geq 25\right]=\begin{Bmatrix}0\\0\\1\\0\end{Bmatrix} \tag{5.31}$$

Therefore, based on equation (5.16) through equation (5.19), the demand *D* and supply *S*, can be calculated as:

$$\boldsymbol{D}\left(x_1,t_1,\boldsymbol{p}=\begin{Bmatrix}0.25\\0.25\\0.25\\0.25\end{Bmatrix}\right)=\begin{Bmatrix}1900\\1900\\2200\\2200\end{Bmatrix}\quad\text{and}\quad\boldsymbol{S}\left(x_1,t_1,\boldsymbol{p}=\begin{Bmatrix}0.25\\0.25\\0.25\\0.25\end{Bmatrix}\right)=\begin{Bmatrix}2100\\2300\\2100\\2300\end{Bmatrix} \tag{5.32}$$

The flux between two cells is defined and given as:

$$\boldsymbol{q}^{x_i\to x_{i+1}}\left(x_1,t_1,\boldsymbol{p}=\begin{Bmatrix}0.25\\0.25\\0.25\\0.25\end{Bmatrix}\right)=\min\big(\boldsymbol{D}(\boldsymbol{k}),\boldsymbol{S}_{i+1}(\boldsymbol{k})\big)=\begin{Bmatrix}1900\\1900\\2100\\2200\end{Bmatrix} \tag{5.33}$$

The density therefore in the current and following cells in the following time step, t_2, is given by the previous density adjusted by the flux into and out of that cell, during the size of the time step, h. Here we presume an identical inflow into cell x_1 for t_2 as in t_1:

$$\boldsymbol{k}\left(x_1,t_2,\boldsymbol{p}=\begin{Bmatrix}0.25\\0.25\\0.25\\0.25\end{Bmatrix}\right)=\boldsymbol{k}(x_1,t_1\,;\boldsymbol{p})+(\boldsymbol{q}^{x_0\to x_1}-\boldsymbol{q}^{x_1\to x_2})\cdot h=\begin{Bmatrix}22\\20\\26\\24\end{Bmatrix}+\left(\begin{Bmatrix}1900\\1900\\2200\\2200\end{Bmatrix}-\begin{Bmatrix}1900\\1900\\2100\\2200\end{Bmatrix}\right)\cdot h \tag{5.34}$$

Similarly, the flow into the yet unoccupied cell x_1 is calculated:

$$\boldsymbol{k}\left(x_1, t_2, \boldsymbol{p} = \begin{Bmatrix} 0.25 \\ 0.25 \\ 0.25 \\ 0.25 \end{Bmatrix}\right) = \boldsymbol{k}(x_1, t_1\, ; \boldsymbol{p}) + (\boldsymbol{q}^{x_0 \to x_1} - \boldsymbol{q}^{x_1 \to x_2}) \cdot h = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} + \left(\begin{Bmatrix} 1900 \\ 1900 \\ 2100 \\ 2200 \end{Bmatrix} - \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}\right) \cdot h \tag{5.35}$$

This same process repeats itself for each cell in each time step and so on.

5.6 Addressing the main issues

In Chapter 2, six important issues relating to stochastic traffic flow modelling were presented and described. Four of these issues are addressed in the development of the CPF and are explained here.

For **computational efficiency**, the main challenge is to reduce computational load and in doing so, do it in a way that the model is not reduced in stochastic and modelling accuracy. Compared to a Monte Carlo simulation, the CPF does not require multiple repetitive simulations before arriving at a distribution, as the distribution of the traffic variables is explicit to the methodology. Therefore, the computational load will be lighter if a single (DE-) CPM simulation run is quicker than the sum of the required number of Monte Carlo simulations on the same base model. It is hypothesised that this is the case, as the DE-CPM has a single computational overhead for the entire distributions, while a Monte Carlo simulation has a computational overhead for each simulation iteration. Furthermore, a lower detail of discretisation is hypothesised to be required for the DE-CPM as the model calculates using distributions throughout. Simplification and reduction of distributions would lead to higher errors and therefore require more samples to attain the same level of accuracy. In section 5.7.2 a demonstration is given of the potential computational gains. Monte Carlo simulation makes use of less efficient random process of sampling, which reduces the completeness of a distribution and therefore requires a greater number of simulations to reach the same level of accuracy, therefore increasing the computational load. On a simple network or a corridor, the efficiency effect will be limited, however for larger networks and for a greater spread of variation the gains should be greater. It should be noted that Monte Carlo simulation allows for parallelisation, which can significantly improve computation time.

Spatiotemporal dependency is catered for in the DE-CPM through the explicit consideration of correlations at the peripheral of the model and maintenance thereof in propagation through the chain-rule. For other DNL models in the CPF, the manner in which the dependency is dealt with may vary. Reduced to two dependant variables, the traffic demand and road capacity, correlations between possible values of both are explicitly considered in the distributions entering a network at the peripherals. Values in the initial distribution vector of the traffic demand entering the network correspond on an element-to-element bases to that of values of the capacity distribution vector at the same element location. By explicitly maintaining this chain-rule throughout the traffic propagation, independency between traffic demand and capacity is maintained. Dependency in time for both the demand and capacity is also explicitly dealt with outside the model. Input values for certain elements in the distribution vectors follow those of the preceding time step and therefore already consider a logical and dependant propagation from the input vectors in time. Spatial dependency is dealt with in the same way as in the base model and therefore requires no further attention. Simplified, each element in a distribution vector may be seen as a single input value for a single Monte Carlo simulation, therefore it may also be considered as independent from other elements just as a single Monte Carlo iteration is from another Monte Carlo iteration.

Stochastic propagation of probability in traffic flow is performed as described in section 5.4 for the DE-CPM and is also touched upon in the previous issue on spatiotemporal dependency. Dealing of this issue is also DNL model dependent and not generic for the CPF. A complete distribution of possible values per traffic variable is present as a distribution in the form of a vector. This vector exists of more elements than is necessary, so to allow each possible value of that vector to correspond to the elements of other vectors and therefore to avoid correlation difficulties. As these distribution vectors are propagated in space and time, there is no need to reduce variables to a representation of the distribution using a set distribution type, median, standard deviation, shape parameter or such like. Although this may lead to a higher computational effort, it maintains a guaranteed accuracy of the propagation of the traffic variables and their probabilities, as the distributions remain intact in the process of propagation. Therefore, a greater accuracy can be achieved in comparison to methods that do transform distributions to characteristics of the distribution, mostly to some parametric form.

For the CPF, the question of **generality** is one that is less relevant to the model itself, but rather to the quality of the data and distributions that it is fed with. As the CPF performs calculations using discrete distributions, a reduction of the input data may only happen in the case of rediscretisation for the sake of computational efficiency. Therefore, the necessity to apply accurate input distributions for the traffic demand and road capacity is applicable for the local circumstances or from a general distribution if the local situation is not known. Construction of generic input distributions for this purpose, taken from wide spread empirical analysis, makes it easy to apply the CPF without requiring extensive data analysis for each application of the model (Calvert et al., 2014a, van Stralen et al., 2014). Nevertheless, this issue is one that is less explicit to the model, as the quality of input data is relevant and independent to all models. However, the manner in which a model deals with accurate input is important. The CPF does not simplify input by moulding it to a parametric function, therefore maintaining high level of accuracy and avoiding additional unnecessary biases, contrary to many other models. The CPF makes use of empirical distributions which maintain the characteristics of each distribution as it propagates through a network.

5.7 Test cases DE-CPM

In this section, a demonstration of the application and validity of the Discrete-Element Core Probability Model (DE-CPM) is performed in a number of test cases. The first test case aims to show that traffic propagation along a road section in the DE-CPM can accurately resemble traffic flow found from empirical observations. As the case is carried out on a single stretch, there is not much that can be said about the computational efficiency. This is considered in the following sub-section. A second test case is performed on two small test networks to demonstrate the application in networks and to further demonstrate potential computational gains of the framework and model.

5.7.1 Traffic propagation on a single road section

The test case is carried out for the A12 motorway in The Netherlands between Utrecht and The Hague (see Figure 5.5). On this motorway in 2009[2], a lane drop was present from four to three lanes, which acted as a structural bottleneck at location A. Daily congestion starting at this location near the town of Woerden would be present, especially during the evening peak period. A section of 11 kilometres is considered, of which 10 km upstream and 1 km downstream of the bottleneck. The DE-CPM is fed with data from 63 afternoon peak period observations of the traffic flow between 2 PM and 9 PM from 2009 as a representation of the probability of certain traffic flows appearing. The input for the model is taken exclusively from the most upstream location. Therefore, the validation is that of the stochastic traffic propagation. Each observation is considered as an equal probability of a real traffic demand for this location and is therefore given a 100/63 = 1.6% probability for the input at the inflow of the corridor. These traffic flows are fed into the network at the most upstream location. The traffic demand derived from data that is fed into the model is given in Figure 5.6.

Figure 5.5: Bottleneck location near Woerden at the considered road section on the A12 used in the case study

A comparison is made based on the ability of the model to accurately predict the propagation of the probabilities of traffic flow and corresponding traffic states between the outcome of the DE-CPM simulation and the empirical data. For this, the unfiltered traffic states in time and space are gathered on the entire corridor. The comparison focusses on the time of traffic breakdown, congestion duration, spill-back distance, and the specific speed values in time and space. This is shown for the median probability (most likely traffic situation) and a further

[2] Since 2009, this location has been upgraded to four lanes along the entire stretch to eradicate the bottleneck.

demonstration of the results is given in the form of a 3D congestion probability plot. The results of the median probability are shown in the time-space Figure 5.7.

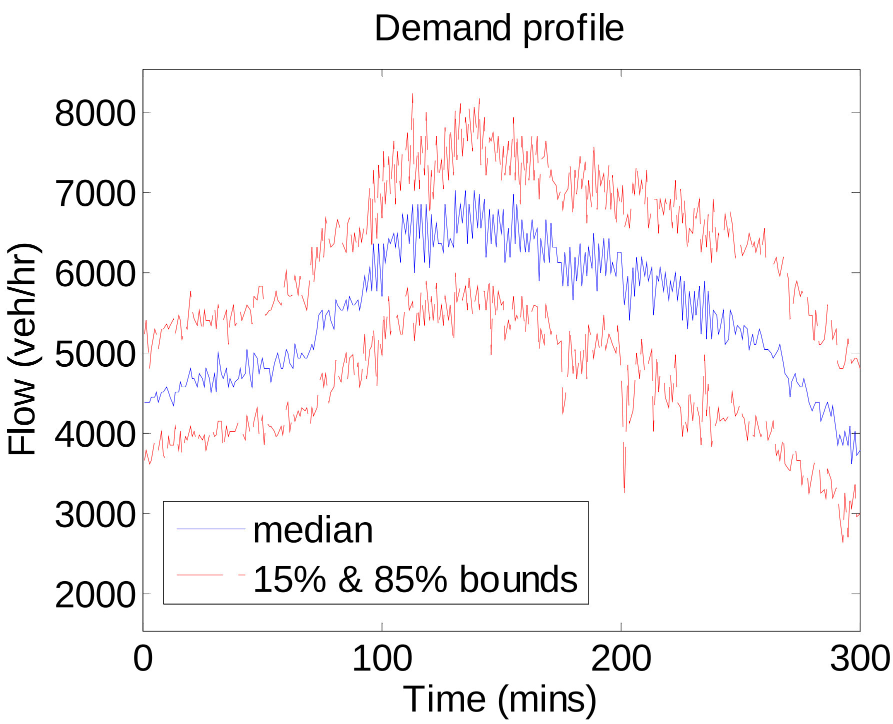


Figure 5.6: Demand profile for the A12 with confidence bandwidths

The initial results, shown in Figure 5.7, show the simulated median (50%) results from the model, with the median from the empirical data shown in Figure 5.8. The speed values are shown as these give a good indication of where congestion is present, how extreme congestion is and how traffic flow changes the time. Initially, the extent of congestion appears to be relatively well modelled. Nevertheless, there are certain deviations in comparison to the empirical data. The onset of congestion occurs approximately 10 minutes earlier in the simulation, while congestion lasts for 158 minutes compared to 190 minutes in the data. However, the spillback of congestion in both is of a similar magnitude and deviates no more than 200 meters over a distance of some 9 kilometres. The speed in the heavily congested area of traffic is lower in the empirical data compared to the model (ca. 30 kph versus 40 kph). This may also be a main reason why the duration of congestion differs, as traffic in the simulation may proceed at a slightly higher speed and therefore let congestion disperse earlier. Despite these minor deviations, this initial test case gives cause for optimism. A further fine-tuning of the model parameters when applied in practice may easily compensate for the observed differences.

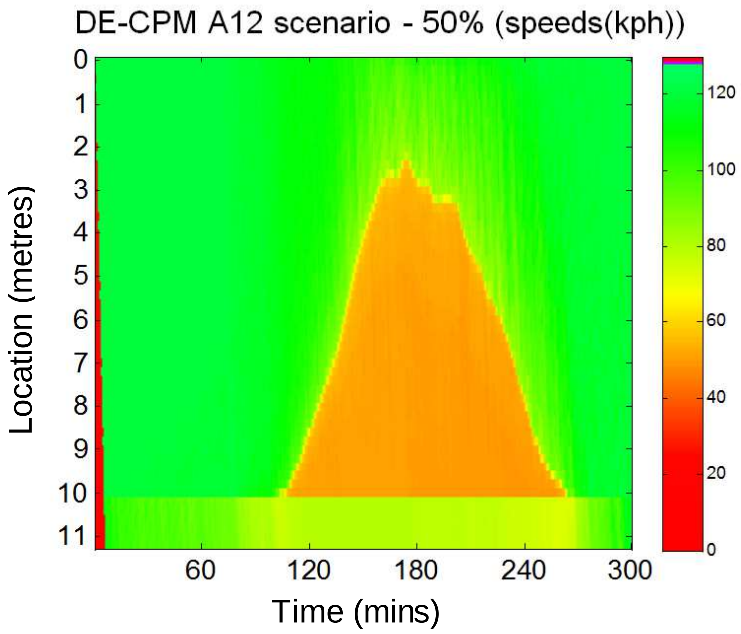


Figure 5.7: Modelled speed diagram for the median probability in the A12 test case

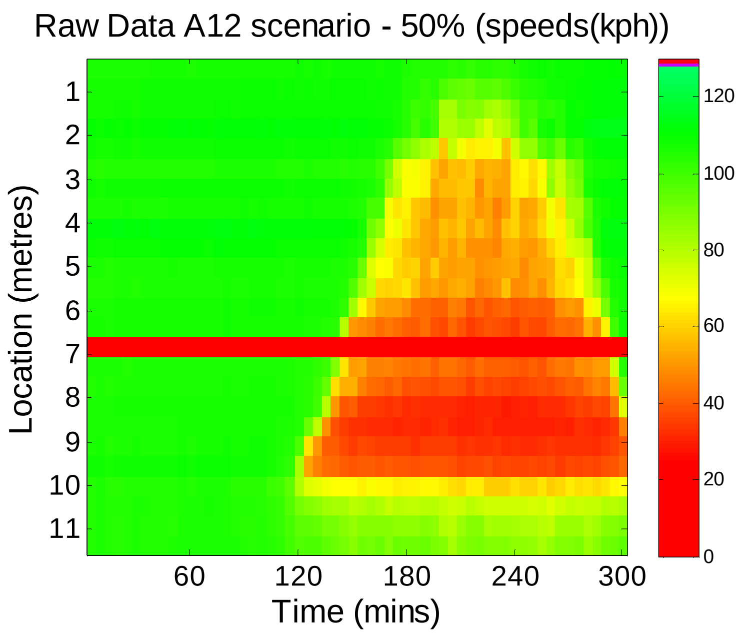


Figure 5.8: Empirical speed data for the median observation in the A12 test case [3]

The CPF allows a vast amount of data to be produced and presented as a probability distribution or in another forms as a direct consequence of the way the CPF works. As each traffic variable is considered as a distribution of possible values, each can therefore be calculated or shown as such at each time step and location. This is demonstrated in Figure 5.9 in which the congestion probability at each location and for every time step is given. Congestion is defined as such when the critical density is exceeded, while the probability thereof indicates the frequency that congestion is expected to occur for an arbitrary location

[3] The red horizontal line indicates a location at which a faulty detector is present. The speed at this location is returned as null.

and time along the corridor. It is possible to show more complex results in a greater number of dimensions, i.e. including the probability as a variable in a diagram, however this leads to difficulties in the interpretation of diagrams. Nevertheless, broad analyses are made much easier and more extensive with the results from the CPF. Significant computational gains are not found on a single corridor, but rather are expected for networks and for greater variations in stochastic variables. This is further looked at in the following sub-section.

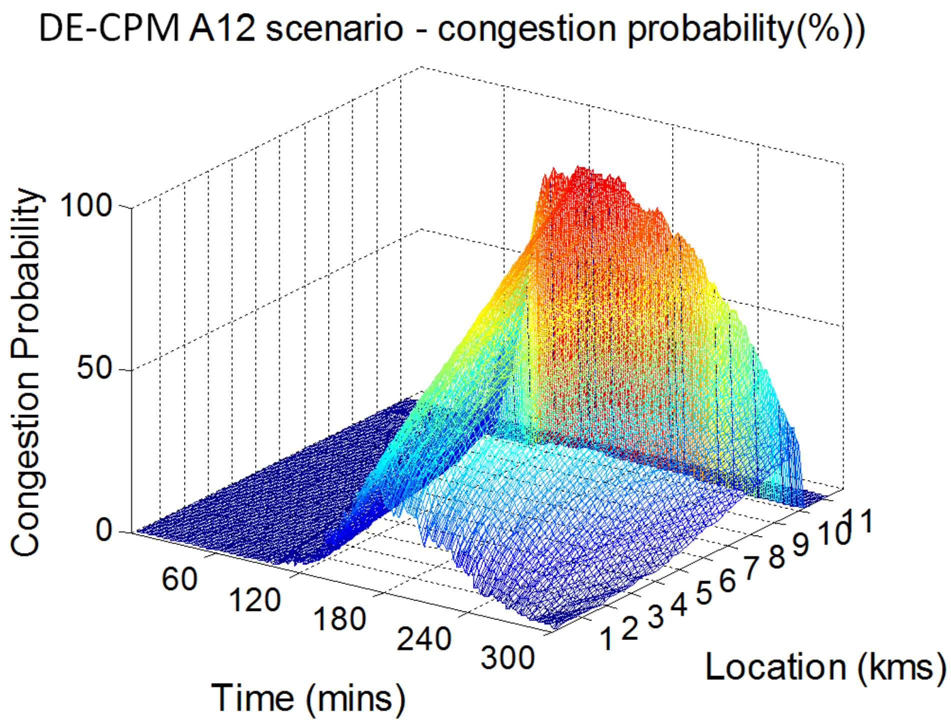


Figure 5.9: Modelled congestion probability in time and space for the A12 test case

5.7.2 Network computational performance

Performance of the DE-CPM for computational efficiency is tested on two simple networks. In comparison to the previously considered road stretch, variation in traffic flow can interact much more as it propagates through a network and will also include network effects. The considered networks are shown in Figure 5.10 and 5.11. Network 1 is a 5 link network with two origins and one destination, while network 2 is constructed from 7 bi-directional links with four origins and destinations. A comparison is made between the application of identical input distributions and capacity distributions in the DE-CPM against a CTM Monte Carlo simulation on the same networks in a MATLAB implementation. In both models the main CTM code is identical, naturally with the addition of the core probability components for the DE-CPM model. Furthermore, both models make use of exactly the same route model, which presumes static turning fractions and all other variables and parameters are kept identical in both cases.

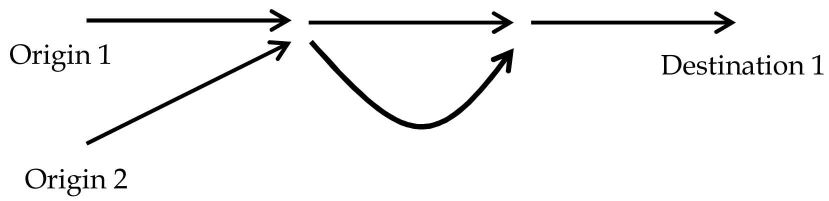


Figure 5.10: Test network 1

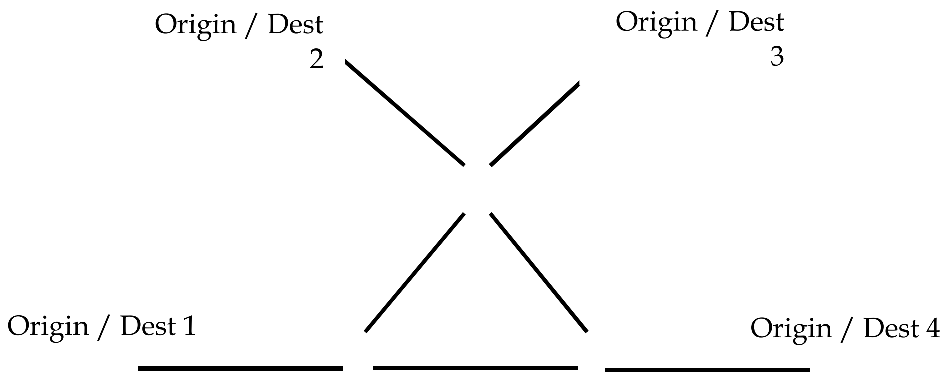


Figure 5.11: Test network 2

Input for the models is the *network definition*, which includes network characteristics and geometry, *stochastic dynamic demand matrices*, and *stochastic capacity values*. The demand and capacity distributions are kept to a limited number of discrete elements, which also act as the input for the DE-CPM and as each combination for the Monte Carlo routine. The input distributions therefore do not require further discretisation. Besides tests on two different networks, various 'total number of time steps' and various 'number of discrete elements in the input distributions' are applied, as shown in Table 5.1. For each scenario, at least five simulations are performed of which the average computation times are given in the last two columns of Table 5.1. The reason for multiple simulations is to be sure that there are no or limited variations caused by the computer. Although five simulations are performed for each scenario and model, the differences in calculation time for the five simulations in all cases on the same machine consequentially varied minimally, generally below 2%.

Table 5.1: Computational speed tests for the DE-CPM

Network	Links (incl inflow)	Origin	Destination	Nodes (excl. connectors)	Time step size (sec)	Time steps	Cap stoch elements	Demand stoch elements	Unique combin-ation of elements	MC run time (sec)	DE-CPM run time (sec)
Netw 1	7	2	1	2	6	200	10	4	40	25	1.1
Netw 1	7	2	1	2	6	400	10	4	40	51	2.3
Netw 1	7	2	1	2	6	200	5	4	20	13	1.0
Netw 1	7	2	1	2	6	800	10	4	40	101	4.3
Netw 1	7	2	1	2	6	800	5	4	20	51	3.9
Netw 1	7	2	1	2	6	200	5	2	10	7.0	1.0
Netw 1	7	2	1	2	6	400	5	2	10	13	2.0
Netw 1	7	2	1	2	6	400	5	4	20	25	2.1
Netw 1	7	2	1	2	6	800	5	2	10	26	3.8
Netw 2	17	4	4	3	6	200	10	4	40	66	2.7
Netw 2	17	4	4	3	6	400	10	4	40	131	5.5
Netw 2	17	4	4	3	6	800	10	4	40	261	11
Netw 2	17	4	4	3	6	200	5	4	20	33	2.6
Netw 2	17	4	4	3	6	200	5	2	10	16	2.5

The results of the computation time tests, as a function of the number of time steps and discrete elements from the distributions, show some interesting trends. A graphical representation of the results is shown in Figures 5.12 and 5.13 for the CTM Monte Carlo and DE-CPM respectively for network 1. The relationship between the number of time steps and the calculation time is approximately linear for both models and has its origin near to a time of zero. The relationship between the number of discrete elements and the calculation time is also approximately linear in both cases. However, there is a significant difference between the CTM Monte Carlo and DE-CPM for the incremental increase in relation to the number of discrete elements from the distributions. As may be expected, the CTM Monte Carlo model increases linearly with an origin near to time zero. This is expected as each Monte Carlo simulation makes exactly the same calculations for each combination of inputs, with each calculation taking approximately the same amount of time. The DE-CPM, however, requires a relatively shorter additional time to calculate additional number of discrete elements from the input (and here in the propagation). This is found for both networks and can be clearly observed in Figures 5.12 and 5.13. Also, it appears that the linear increase with the number of elements does not originate at zero seconds, which indicates some sort of small start-up time. Comparison between the two networks would indicate that the start-up time is dependent on the size of the network.

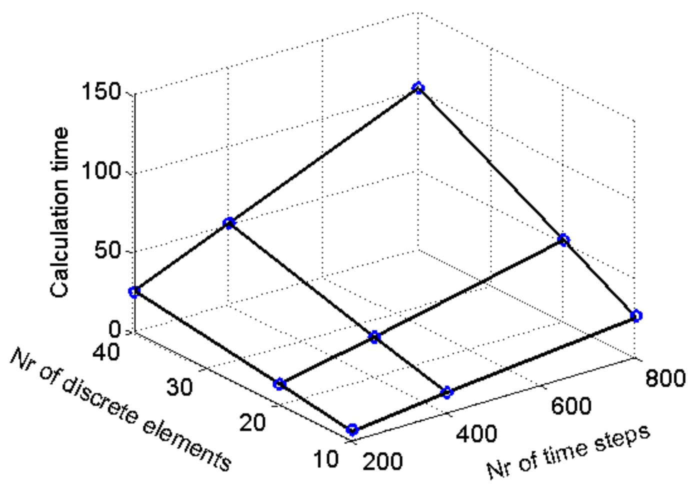


Figure 5.12: Calculation time CTM Monte Carlo for network 1

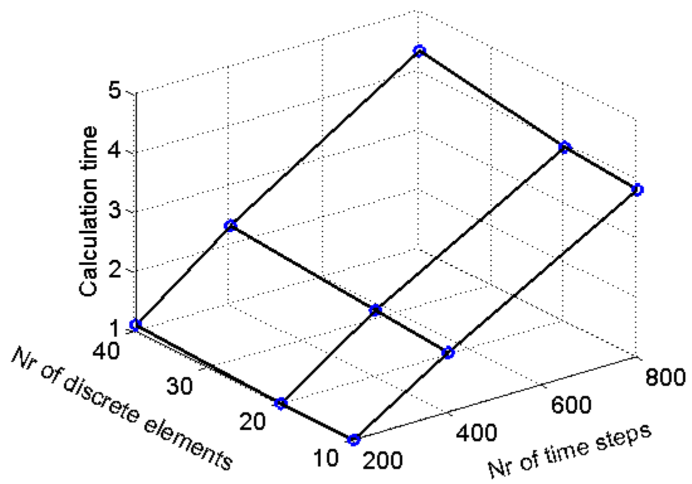


Figure 5.13: Calculation time DE-CPM for network 1

The consequence of this low coefficient for increasing number of discrete elements is that the DE-CPM is far more capable of efficiently dealing with traffic flows with large amounts of stochasticity in comparison to the compared Monte Carlo routine. This allows simulations to be carried out in which a greater detail of uncertainty may be incorporated at a marginal cost to the computational time.

The used samples for the Monte Carlo simulation are identical to the input percentiles used in the DE-CPM, to ensure identical outcomes. Therefore, the results of the DE-CPM simulations, in terms of flow and density values in time and space, are identical to that of the Monte Carlo simulations. This is by definition, following the earlier described chain-rule that maintains each internalised scenario, as if it were a Monte Carlo simulation. Even when the Monte Carlo samples would be completely random, the only difference in results would be the result of different samples, rather than an inherent deviation in calculation method.

The computation times in absolute terms for the DE-CPM outperform those for the CTM Monte Carlo by a factor 5-20 depending on the size of the network and number of stochastic elements. The results from Table 5.1 show that for larger networks and for a greater number of discrete elements that the DE-CPM outperforms the Monte Carlo routine to a greater extent. This is in line with the expectations that this model shows its effectiveness best for larger networks and under greater levels of uncertainty. The possibility of parallelisation for Monte Carlo routines can reduce the computation time, however even compared to parallelisation such gains of 20 times or more for larger networks with the DE-CPM may even be competitive in comparison.

5.8 Conclusions

In this chapter, the Core Probability Framework (CPF) has been introduced with the application of the Discrete-Element Core Probability Model (DE-CPM) as a new DNL for dynamic macroscopic modelling of stochastic traffic flow. An initial validation case has been also been shown as well as an indication of the computational performance on networks. The CPF extends current deterministic traffic flow models by redefining traffic variables in the core of the model as distribution vectors of probable values for each traffic variable. In such a way stochastic variation in traffic is internalised in the model and does away with the necessity of repetitive Monte Carlo simulation. Furthermore, a greater degree of flexibility in analysis is obtained, as each individual traffic variable in time and space may be given as a function of their probability. Moreover, the underlying distribution of each traffic variable in space and time is preserved such that the introduction of distribution fitting errors is limited to a minimum. Important issues facing stochastic traffic flow modelling are given, and are identified as *computational efficiency*, *spatiotemporal dependency*, *stochastic propagation of probability*, and *stochastic generality*. The DE-CPM addresses each of these issues through element based calculation using the chain-rule and in doing so demonstrates the ability to advance developments in the area of stochastic traffic modelling. In particular, the CPF aims to further the possibilities for reliable, accurate, efficient, and most of all, practically applicable stochastic macroscopic traffic flow modelling. The outcome of the calculation time tests on simple networks compared to a CTM Monte Carlo model showed that the DE-CPM has great significant potential to reduce computation times, especially for larger networks and for greater stochasticity. This is mainly due to the small marginal computational costs incurred when increasing the level of uncertainty in the discrete model. With the DE-CPM DNL model, a first step within the framework is taken. Further expansions in the form of more advanced model developments within the framework are recommended for future work and focus on the propagation of the stochastic variables as distributions without the application of the chain-rule. These developments have the potential to deal with stochastics to a more efficient extent.

Chapter 6

Micro-stochastic macroscopic modelling

In this chapter, a new model to include stochastic vehicle specific behaviour and interaction, described as microscopic stochasticity, in traffic flow modelling is presented. The First Order Model with Stochastic Advection (FOMSA) is a first order macroscopic kinematic wave model in a platoon-based Lagrangian coordinate system. The use of Lagrangian coordinates allows characteristics of specific vehicles or vehicle-groups to propagate along with the traffic flow using a vehicle (group) specific invariant. The invariant reflects how vehicle specific characteristics propagate with the vehicles and influence the local behaviour on a macroscopic level and in interaction with other surrounding vehicles. The application of bounded acceleration and vehicle reaction time to improve accuracy and assist the generation of the capacity drop is also demonstrated in two cases.

An introduction to the topic is given in section 6.1. The modelling principles applied in the approach are first explained in section 6.2. The developed approach is then described in section 6.3, including the assumptions made and the limitations. In section 6.4, a first experimental case is given to demonstrate the approach without explicit capacity drop, in which a further comparison is made with a non-stochastic reference case to demonstrate the necessity of considering stochastic driving behaviour in macroscopic modelling. Further experimental cases are given in section 6.5 for the incorporation of bounded acceleration and reaction to induce the capacity drop. Finally the conclusions are given in section 6.6.

This chapter is an edited version of the articles:
Calvert, S., Taale, H., Snelder, M., & Hoogendoorn, S. (2015). Vehicle Specific Behaviour in Macroscopic Traffic Modelling through Stochastic Advection Invariant. *Transportation Research Procedia*, *10*, 71-81.

Calvert, S. C., Snelder, M., Taale, H., Wageningen-Kessels, V., & Hoogendoorn, S. P. (2015). Bounded acceleration capacity drop in a Lagrangian formulation of the kinematic wave model with vehicle characteristics and unconstrained overtaking. In *IEEE 18th International Conference on Intelligent Transportation Systems, Santa Catalina, Gran Canaria, 15-18 September 2015;* IEEE.

6.1 Introduction

Traffic is a highly dynamic and complex system, which encompasses human behaviour through the act of driving. Human driving behaviour is complex in itself and exists of a general behavioural aspects related to the general rules of driving, i.e. traversing a lane in a certain direction at a certain speed without collision, and of intrinsic behavioural aspects that can be driver specific (Fuller, 2005, Toledo, 2007). A general aggregation of this behaviour is seen as something that can be understood, observed and reproduced in macroscopic models. However, individual driver behaviour is somewhat harder to capture and reproduce. Efforts to capture and understand stochastic driver behaviour have been successful and have described many aspects of driving behaviour. In this chapter, the focus is on vehicle movement, but which is of course influenced by driver behaviour. In this respect, we will refer to vehicle-driver-unit behaviour as driver-vehicle behaviour in which the effect of different vehicle capabilities is also considered. With an increase in microscopic modelling, and especially agent-based models, much stochastic behaviour of individual vehicles and interaction between vehicles has been included in modelling. This allows stochastic behaviour in longitudinal and lateral movements to be included by simply adding terms describing this to a vehicles behavioural algorithm (Arasan and Koshy, 2005, Mallikarjuna and Rao, 2009, Treiber et al., 2006). However, in macroscopic traffic modelling each individual vehicle is generally considered to adhere to identical or similar behaviour. This is especially the case in deterministic modelling. Although this has a number of advantages and often seems to produce acceptable results for most purposes, interaction between vehicles is generally ignored. However, observations of traffic flows show that considering differences between vehicles and their stochastic behaviour is relevant and necessary, especially for constrained or critical traffic states (Kerner, 2013, Persaud et al., 1998, Polus and Pollatschek, 2002). This is also demonstrated later in section 6.4.

Capturing such fluctuations in behaviour between vehicles in macroscopic traffic flow however, demands certain levels of disaggregation of the macroscopic flow, which is not traditionally inherent to such models. In this chapter, we aim to overcome this difficulty to allow stochastic behaviour from vehicles and between vehicles to be modelled in a first order macroscopic setting. This is achieved through the use of a Kinematic Wave Model, which considers the movement of vehicles according to first order traffic theory in a platoon-based Lagrangian coordinate system (Leclercq et al., 2007). Consideration of the stochastic behaviour of vehicles is included through the application of a vehicle specific invariant term that describes local stochastic characteristics of vehicles and drivers within and between individual vehicles or platoons. These characteristics implicitly describe aspects of driver behaviour such as desired time headway. The use of Lagrangian coordinates allows the vehicle specific invariant term to propagate along with the vehicles for which it is valid and thus avoids numerical diffusion of driver behaviour variables (Leclercq et al., 2007, van Wageningen-Kessels et al., 2009). This approach is unique to first order macroscopic models, and is generally found in the more elaborate second order models and is explained in the chapter.

The simplicity of first order models is a major advantage over second order models and therefore many extensions have been proposed to help capture more traffic dynamics while retaining much of the simplicity (Leclercq, 2007b). One such advancement is the introduction of techniques to include the capacity drop, such as bounded acceleration. Bounded acceleration was previously introduced by Lebacque (2003) and has been applied and further developed by various researchers (Lebacque, 2005, Leclercq, 2007a, Leclercq et al., 2011, van Lint et al., 2008). The approach is relatively simple, but effective, and involves bounding the accelerative ability of vehicles by preventing speeds that exceed a pre-set acceleration value as vehicles propagate. In the basic kinematic wave models, vehicles may accelerate at an unrealistic speed. By bounding this acceleration, a more realistic description of real traffic flow is given adhering to the physical capabilities of vehicles. The effect is especially visible for acceleration of vehicles from low speeds such as out of congestion. The use in Lagrangian coordinates is especially advantageous for use with bounded acceleration as the speed of traffic is the resultant from the fundamental equation used in the Lagrangian system, rather than traffic flow. The use of Lagrangian coordinates also allows a vehicle specific invariant term to propagate along with the vehicles for which it is valid and thus avoids numerical diffusion of driver behaviour variables (Leclercq et al., 2007, van Wageningen-Kessels et al., 2009).

This chapter offers a unique approach based on proven theories to include vehicle specific behaviour in first order macroscopic modelling, filling a void that has been previously solved for microscopic models, but that is still lacking in macroscopic models. A demonstration of inclusion of the capacity drop through bounded acceleration and driver reaction times in a Lagrangian formulation of the KWM with vehicle characteristics through an advection invariant is given. This explores the capability to reproduce the capacity drop in traffic with unconstrained overtaking.

6.2 Modelling principles

6.2.1 Kinematic Wave Model

The kinematic wave model (KWM) captures the aggregated propagation of traffic flow described as the propagation of traffic waves and the adhering traffic characteristics. The concept of modelling kinematic waves of traffic was first introduced by Lighthill and Whitham (1955) and by Richards (1956) and is therefore often referred to as the LWR model. Since the introduction of the KWM various extensions have been proposed, however the underlying theory as originally described remains intact. Construction of the kinematic waves is achieved through use of the fundamental relationship of traffic flow which is generally described by the relationship between the density ρ and the flow q of traffic. The model further relies on the conservation equation and initial boundary conditions. The conservation equation and the fundamental relation are denoted by:

$$\partial_t \rho + \partial_x q = 0 \qquad (6.1)$$

$$q = Q(\rho) \tag{6.2}$$

in which ρ is the traffic density in time t and q is the flow in space x. $Q(\rho)$ denotes the form of the fundamental relation. As the KWM is a macroscopic model, it makes use of aggregation of individual vehicles and describes an aggregated flow. van Wageningen-Kessels et al. (2014) point out that empirical density–flow plots usually show wide scatter, which is not captured by an aggregated flow. Macroscopic models presume some sort of equilibrium, which results in crisp steady state conditions in flow regimes. However, van Wageningen-Kessels et al. (2014) go on to point out that the scatter is a consequence of not all data representing such a steady state condition.

6.2.2 Lagrangian Coordinates

In traditional macroscopic modelling, Eulerian coordinates are usually applied which state that for a specific time and location, a flow, such as traffic, will pass with certain characteristics (Helbing and Treiber, 1999, van Wageningen-Kessels et al., 2009). In this case, it is the flow which moves in relation to the coordinate system. Lagrangian coordinates in contrast are not fixed in space, but are given the freedom to transform with the resulting flow. This can be described such that particles in the flow are explicitly considered in individual consecutive cells. Therefore, the coordinates follow the flow rather than the flow following the coordinates. A graphical demonstration is shown in Figure 6.1. The Eulerian formulation of the KWM was given in equations (6.3)-(6.4). When describing the KWM in Lagrangian coordinates the same equations are formulated slightly differently. The conservation equation is given as:

$$\partial_t s + \partial_n v = 0 \tag{6.3}$$

Here, s denotes the mean space headway of vehicles in a single cell. v denotes the mean speed of vehicles, while n is the vehicles number, which decreases in the driving direction. The fundamental relation in Lagrangian coordinates makes use of the speed v in relation to the density ρ, which is derived from the mean headway spacing s: $s = 1/\rho$. The fundamental relation is denoted as:

$$v = V(s) \tag{6.4}$$

The use of Lagrangian coordinates has been proven to lead to more accurate results as a result of a reduction in numerical diffusion that occurs in the transfer of flows between cells in Eulerian coordinates, but is almost non-existent in Lagrangian coordinates (Leclercq et al., 2007, van Wageningen-Kessels et al., 2010, van Wageningen-Kessels et al., 2009).

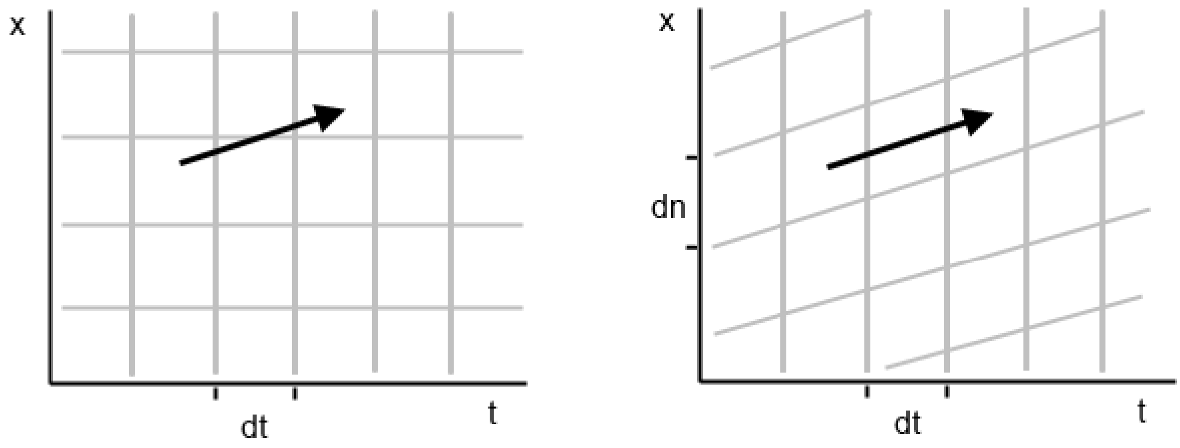


Figure 6.1: Comparison of Eulerian (left) and Lagrangian (right) coordinates. The arrow represents the direction of traffic flow.

Lagrangian coordinates were introduced to traffic flow from the domain of hydrodynamics (Makigami et al., 1971, Moskowitz, 1965). Jin et al. (2014) state that Lagrangian coordinates can be incorporated into continuum traffic flow modelling by either establishing moving boundary conditions for Euler formulations (Claudel and Bayen, 2010, Herrera and Bayen, 2010) or by application of hydrodynamic flow (Leclercq et al., 2007). The latter has been the more forthcoming in relation to advancement and application and was shown by Leclercq et al. (2007) to be able to be derived by using a space function based on variational theory (Daganzo, 2005). More recently, Laval and Leclercq (2013) further applied the theory of Hamilton-Jacobi to KWM in which the theory is applied to three two-dimensional coordinate systems, which included Eulerian and Lagrangian systems and is continuing to be extended by a number of researchers (Jin et al., 2014). This however goes beyond the scope of this thesis in which the Lagrangian description as given in equations (6.3)-(6.4) is applied and which was also previously described (van Wageningen-Kessels et al., 2010, van Wageningen-Kessels et al., 2009).

6.2.3 Advection

The difficulty following from problems in the aggregated representation of flows in macroscopic models has previously been widely acknowledged. These difficulties have not only been identified in the lack of stochastic behaviour and its consequence, but even more in the detailed introduction of vehicle attributes and driver behaviour, such as anticipation of drivers or the consideration of vehicle diversity. The main effort to describe such driver behaviour in a macroscopic model has been performed in second order macroscopic models. Application of between-vehicle stochastics has generally not been applied in first order macroscopic models. While first order models describe the conservation of vehicles according to equation (6.1), second order models also consist of a second differential equation that describes the velocity dynamics. There are different formulations present, Aw and Rascle (2000a) formulate it as:

$$\partial_t(v + p(\rho)) + v\partial_x(v + p(\rho)) = 0 \tag{6.5}$$

in which $p(\rho)$ is a pressure term. Originally, second order models were criticised for resulting in some inacceptable behaviour, such as vehicles being able to move backwards (Daganzo, 1995b). However, further developments resolved this issue. Aw and Rascle (2000a) proposed adjustments to the original definition by replacing the space derivative with a convective derivative. Zhang (2002) described this similarly and explicitly state that traffic flow moves with the velocity along the trajectory and therefore it becomes a Lagrangian quantity.

Lebacque et al. (2007b) applied the same rationale to generalise the ARZ models (Aw and Rascle, 2000a, Zhang, 2002). The ARZ models apply an invariant term to represent the relative speed of vehicles which is connected to these vehicles. Lebacque et al. (2007b) define this term as a general invariant that can also be related to global flow properties and therefore represent other characteristics of microscopic flow. The model is described as a generic second order model (GSOM) after the flexibility one has to define an invariant that can take on many different purposes. The conservation of vehicles is as equation (6.5), while the conservation of the invariant term and description of the fundamental relation with invariant are given by:

$$\partial_t(\rho I) + \partial_x(\rho v I) = \rho\varphi(I) \qquad \text{Dynamics of a driver attribute} \tag{6.6}$$

$$v = V(\rho, I) \qquad \text{Fundamental relation} \tag{6.7}$$

in which x is the position, I is the invariant term, and the V is the fundamental relation.

This approach has been applied in a number of successive publications (Costeseque and Lebacque, 2014, Costeseque and Lebacque, 2015, Lebacque and Khoshyaran, 2013). One such application describes the invariant term as a stochastic driver attribute describing the random driver interactions of a driver with other drivers (Lebacque and Khoshyaran, 2013). Their Stochastic Generic 2nd Order Model describes the stochastic behaviour as a Brownian process and white noise process and if further defined in Lagrangian coordinates. While the GSOM also allows a first order description to be formulated (Lebacque et al., 2007b, Lebacque and Khoshyaran, 2013), applications of the GSOM are generally not found in first order formulations. First order models on the other hand have advantages due to their relative computational efficiency in practice.

6.2.4 Bounded Acceleration

It has been claimed that application of bounded acceleration can make it possible to model the capacity drop in first order models (Laval, 2004, Srivastava and Geroliminis, 2013). The capacity drop is defined as the difference between the breakdown capacity and the discharge capacity on a section of road and can frequently be observed after traffic breakdown between observations in a critical undersaturated traffic state and an oversaturated traffic state. The occurrence of the capacity drop is generally attributed to the so called hysteresis effect

(Banks, 1991, Daganzo et al., 1999, Hall and Agyemang-Duah, 1991). The hysteresis effect occurs in part due to differing driving behaviour as vehicles enter and exit congested traffic states (Farrell, 1999) and is most commonly captured in macroscopic models in second order formulations. In these models, an additional equation is given that describes the dynamics of vehicle flow. There have also been attempts to include the capacity drop in first order models (Laval, 2004).

Commonly, the capacity drop is included in first order models through an explicit reduction of the constrained flow around a bottleneck location. This however focusses on the effect rather than the cause. This furthermore leads to discrepancies in modelling boundaries due to assumptions that are made to allow flows to be constrained. Here, we further analyse the application of the capacity drop through a bounded acceleration of vehicles and through the introduction of driver reaction times.

6.3 First order model with stochastic advection

The formulation of the First Order Model with Stochastic Advection (FOMSA) is presented in this section. Firstly, the general formulation of FOMSA is presented along with the applied model discretisation. Section 6.3.3 describes the application of the vehicle specific invariant term in the model. Sections 6.3.4 and 6.3.4 give the description of the methods applied to capture the capacity drop: bounded acceleration, and the driver reaction time.

6.3.1 Model formulation

The first order model with stochastic advection (FOMSA) is a discrete first order macroscopic model based on the conservation of vehicles and adhering to the fundamental relation according to Lighthill and Whitham (1955) and Richards (1956) and given in equations (6.1)-(6.2). However contrary to the KWM, the FOMSA makes use of a different definition of the fundamental relation defining it in terms of traffic speed, which is more in line with equation (6.7). The main difference is the inclusion of an additional invariant term I, which describes the stochastic nature of traffic. This term is also conserved in space and time. The model is described by:

$$\partial_t \rho + \partial_x(\rho v) = 0 \qquad \text{Conservation of vehicles} \qquad (6.8)$$

$$\partial_t \rho I + \partial_x(\rho v I) = 0 \qquad \text{Conservation of invariant} \qquad (6.9)$$

$$v = V(\rho, I) \qquad \text{Fundamental relation} \qquad (6.10)$$

Here the invariant, I, is the vehicle specific invariant, a term that denotes a vehicle dependent adjustment factor that directly influences the density ρ for each vehicle or group of vehicles depending on the level of discretisation. The vehicle specific invariant acts as a descriptive term that describes driving-style in relation to other vehicles, in variables such as the time headway. This is explained in more detail in section 6.3.3.

6.3.2 Model discretisation

The Godunov scheme is a commonly applied approach for the discretisation of macroscopic models (Lebacque, 1996a). The FOMSA is defined in Lagrangian coordinates rather than the traditional Eulerian coordinates. van Wageningen-Kessels et al. (2009) previously described how the Godunov scheme in Lagrangian coordinates is reduced to an upwind scheme, independent of traffic state with conservation equation:

$$\partial_t s + \frac{v\left(s^j(t)\right) - v\left(s^{j-1}(t)\right)}{\Delta n} = 0 \tag{6.11}$$

where Δn is the vehicle group size and $s^j(t) = s(j\Delta n, t)$ is the space headway of the $j\Delta n$-th vehicle at time t. van Wageningen-Kessels et al. (2009) and van Wageningen-Kessels et al. (2013) then define the Lagrangian formulation in time as an explicit semi-discretised scheme. As explicit time stepping is used, the semi-discretised scheme from equation (6.11) is made explicit for application and is given by (van Wageningen-Kessels et al., 2009):

$$\frac{s^{j,k+1} - s^{j,k}}{\Delta t} + \frac{v\left(s^{j,k}\right) - v\left(s^{j-1,k}\right)}{\Delta n} = 0 \tag{6.12}$$

where Δt is the time step and $s^{j,k}$ is the space headway at the position of the $j\Delta n$-th vehicle group at time $t = k\Delta t$. (van Wageningen-Kessels et al., 2013) also describe the scheme implicitly, that has an added advantage that it is relatively easy to solve as it only relies on traffic states in one direction and does not need to consider the propagation of traffic state changes with the flow as these are implicitly considered with the movement of vehicles, which follow the traffic flow. It is however the explicit scheme that is applied in this chapter as this is consistent with the applied extension of invariant advection.

6.3.3 Vehicle specific invariant

A main contribution of the FOMSA is the inclusion of driver-vehicle behaviour in relation to inter-vehicular interaction and behaviour. This is achieved through the vehicle specific invariant term. This term is derived from previous work by Lebacque et al. (2007b), who introduced a generic invariant term which allows numerous descriptive variables to be propagated with traffic flow in a second order macroscopic model. In the FOMSA, an invariant term is introduced as a first order Lagrangian model, which retains the relatively simplicity of first order modelling approaches. The vehicle specific invariant is a term that influences the density of traffic and is vehicle (group) specific and is applied in the fundamental equation. In traffic, different drivers harbour different driving behaviour and levels of aggressiveness. This can often be described by the desired headways maintained, which is what the influence of the density directly describes, as:

$$s = \frac{1}{\rho} \tag{6.13}$$

Here, s is the space headway and ρ is the density of traffic. As adjustment of the density values would directly violate the law of traffic conservation, the invariant is applied to the deterministic critical density $\rho_{crit.0}$ and jam density $\rho_{max.0}$ in the fundamental relation $v = V(\rho, I)$:

$$\rho_{crit} = I\rho_{crit.0} \tag{6.14}$$

$$\rho_{max} = I\rho_{max.0} \tag{6.15}$$

Empirical analysis has shown that driver behaviour and therefore also vehicle-driver combination is also influenced by the traffic state, i.e. a driver may be less aggressive in congestion as this may have little advantage. This is accounted for by a traffic state term f applied to the equation (6.14)-(6.15) that is dependent on an adjusted ratio of the current density and the critical density, such that the formulation becomes:

$$\rho_{crit} = I^{f(\rho(t),\rho_{crit},\rho_{max})}\rho_{crit.0} \tag{6.16}$$

$$\rho_{max} = I^{f(\rho(t),\rho_{crit},\rho_{max})}\rho_{max.0} \tag{6.17}$$

Differences between behaviour of vehicles and vehicle-groups may be presumed to be randomly distributed in space. For example, it is not likely to have all aggressive drivers followed by all conservative drivers. However, we hypothesise that drivers can also influence other drivers in the direct vicinity and that some clustering may occur. In this case, it cannot be presumed that the distribution of driver types (indicated by their vehicle specific invariant value) is perfectly random. This is considered in the model through the addition of a transition term β, that describes how the vehicle specific invariant is distributed over vehicles or vehicle groups in space:

$$I(n) = I(n-1)\begin{cases} -\,min(I(n-1) - X, \beta) & for\ I(n-1) > X \\ +\,min(X - I(n-1), \beta) & for\ I(n-1) < X \end{cases} \tag{6.18}$$

$$X \sim U([1-\alpha, 1+\alpha]) \tag{6.19}$$

where $I(n)$ is the value of the vehicle specific invariant, which is dependent on the value $I(n-1)$ of the previous vehicle group n (note that a vehicle group may contain one single vehicle or multiple vehicles as a platoon). X is a random number between $[1-\alpha, 1+\alpha]$ in which α is the stochastic boundary parameter which indicates the maximum extent of the stochastic influence. Parameter β is the transition parameter that indicates the maximum change in I between consecutive vehicle groups. Parameter β is in itself also dependant on the size of vehicle group sizes, if a vehicle group n is not equal to a single vehicle. The vehicle specific invariant, I, is assigned to each vehicle or platoon at the entrance of a network according to equations (6.18)-(6.19). In this chapter, perfect values for α and β are not analysed. This is recommended for later research.

6.3.4 Bounded Acceleration

The first method applied to induce the capacity drop makes use of bounded acceleration. The concept of bounded acceleration involves a limitation of vehicle capabilities in a model, such that it resembles the capabilities of real traffic flow. The formulation applied here resembles that described in (Lebacque, 2003) with some adjustments. The KWM conservation equation given in equation (6.5) remains valid, while a limitation is given to the fundamental relation:

$$v = min(V(\rho, I), V_{BA}) \tag{6.20}$$

$$V_{BA} = v_{t-1} + a_{max}\Delta t \tag{6.21}$$

where V_{BA} is the bounded speed for a vehicle (group) at a certain time and space; v_{t-1} is the speed of the considered vehicle (group) in the previous time step; $a_{\max}$ is the maximum acceleration allowed, while Δt indicates the time step.

The speed that is then applied to calculate the new location of the vehicle (group) in the numerical scheme in the Lagrangian formulation may therefore be limited when a vehicle (group) has the possibility to accelerate faster than the maximum acceleration rate, a_{max}. This approach therefore does not make changes to the numerical scheme, but rather the input of the vehicles speed into the scheme.

6.3.5 Driver reaction time

The second method applied to induce the capacity drop considers the reaction time of drivers before commencing an acceleration action in reaction to a predecessor. The capacity drop is induced here through the application of the Reaction Time (T_r) *of drivers to downstream speed increases in combination with heterogeneous traffic. The reaction time of drivers in acceleration is a cause for the capacity drop* (Kesting and Treiber, 2008). In a Lagrangian KWM, speeds v, are updated at each discrete time step. However, reaction times are generally much shorter than a time step. To include the T_r within a time step Δt, and allow an update of the speed, v, and location x, an updated location of the following vehicle is calculated based on the location of a vehicle without reaction time, x^*. The principle is shown in Figure 6.2.

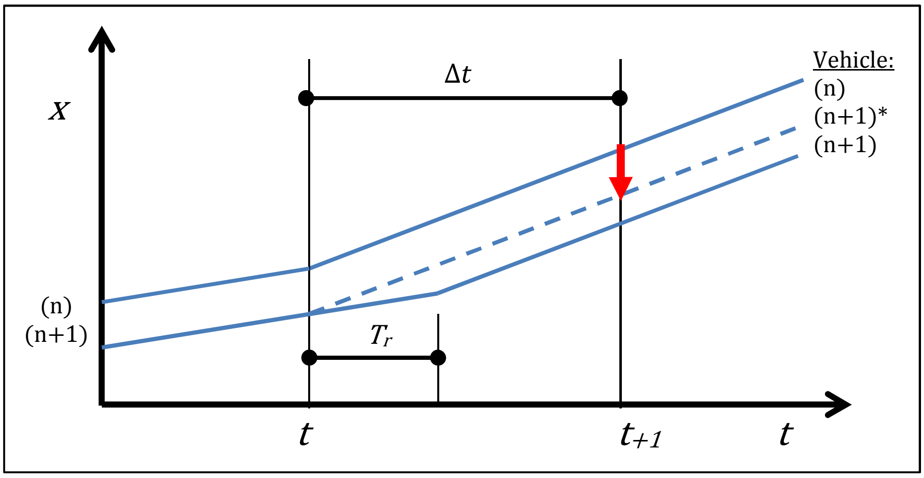


Figure 6.2: Method principle; modified vehicle location and spacing

The location of the following vehicle at time step $t+1$ is given by:

$$x_{t+1}^{(n+1)} = v_t^{(n+1)}.T_r + v_{t+1}^{(n+1)}.(\Delta t - T_r) \tag{6.22}$$

where $x_{t+1}^{(n+1)}$ is the location of vehicle (group) $(n+1)$ at $t+1$; $x_{t+1}^{(n+1)}$ is the speed of vehicle (group) $(n+1)$ at $t+1$; Δt is the time step and T_r is the reaction time

For use in the model, this should be represented as the space headway, s, which is given in the model by:

$$\mathrm{s}_t^{(n)} = x_t^{(n)} - x_t^{(n+1)} \tag{6.23}$$

Here, $\mathrm{s}_t^{(n)}$ is the space headway at time t for vehicle (n) and is updated with:

$$s_{t+1}^{(n+1)} = s_t^{(n+1)} - \frac{\Delta t}{\Delta n}\left(v_t^{(n+1)} - v_t^{(n)}\right) \tag{6.24}$$

where Δn is the vehicle group size.

As the non-reaction time location at $t+1$, $x_{t+1}^{*(n+1)}$, can easily be calculated, and the difference with updated location with reaction time, $x_{t+1}^{(n+1)}$, is the space headway $s_{t+1}^{(n+1)}$. The time-headway at $t+1$ with reaction time can then be easily described by:

$$s_{t+1}^{(n+1)} = s_t^{(n+1)} - \frac{\Delta t - T_r}{\Delta n}\left(v_t^{(n+1)} - v_t^{(n)}\right) \quad for \;\; v_t^{(n+1)} > v_t^{(n)} \tag{6.25}$$

6.4 FOMSA experimental case

6.4.1 Setup and results

The first order model with stochastic advection is demonstrated in an experimental case. In the first case, this is performed without explicit capacity drop. The experimental case is setup for a single highway corridor of 11 kilometres on which two bottleneck locations are present. The first bottleneck is less severe and has a reduced capacity of 8% compared to the rest of the corridor, while the second bottleneck further downstream has a capacity reduction of 15%. Traffic flow into the corridor is maintained at a constant flow of 2000 veh/hr, which is sufficient to lead to congestion in deterministic traffic flow equivalent to a bottleneck with an 11% capacity reduction. Therefore, in the deterministic case the first bottleneck will not be activated, while the second will always be activated. The applied time step Δt is 5 seconds, while the values for the stochastic boundary parameter α and the transition parameter β are randomly assigned to vehicles to show their effects, but remain static in time for a single vehicle. α is varied in the range [0.1, 0.4], leading to a I value of [0.95:1.05, 0.6:1.4], and β is varied in the range [0.1, 0.3]. The occurrence of a traffic state influence, f, is ignored in this experimental case and will be examined in later research.

Two simulation runs with different random values for the vehicle specific invariant are shown as an example in Figure 6.3. From this, it is clear that the effect of the behavioural term has a considerable influence on the occurrence of congestion, as in both cases the same traffic demand is applied, however the characteristics of each vehicle group is different. In Figure 6.3a, both bottlenecks are activated while the case in Figure 6.3b shows that only the second more severe bottleneck is activated, which would indicate that in the second case the vehicles have a higher invariant value and therefore vehicle at closer proximity to each other. Note that for a deterministic simulation run, only the second bottleneck would have been activated. This therefore demonstrates that consideration of stochastic variations in driver-vehicle behaviour between vehicles can have a detrimental effect of traffic flow, as is also the case in real life.

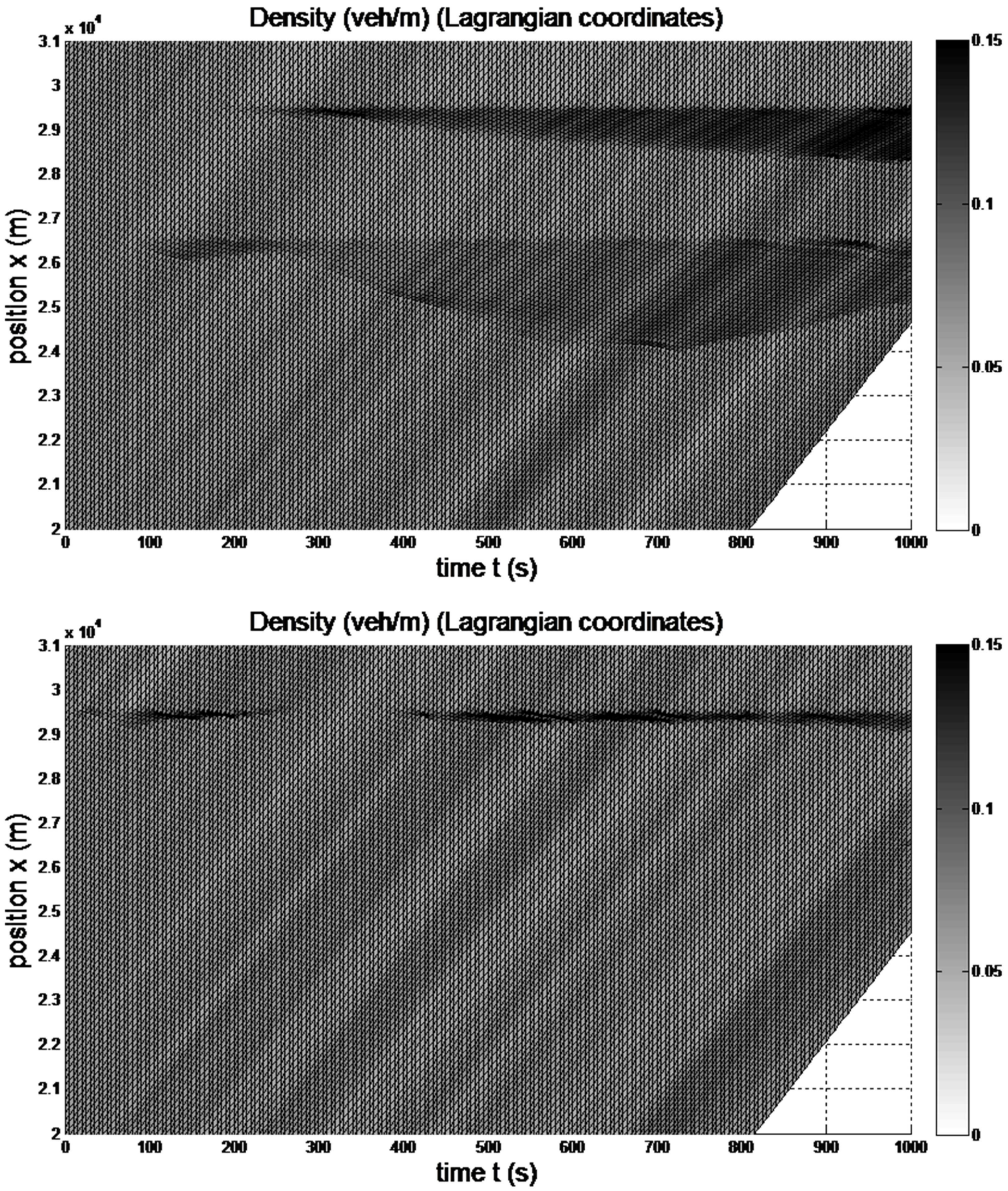


Figure 6.3a-b: Simulation results of the FOMSA model for a dual bottleneck case for two different random procedures with settings α=0.2 and β=0.1

It is not the goal of this chapter to fine-tune the applied parameters. However, further simulations are performed to demonstrate the effects of changes to parameter values. In Figure 6.4 the value of β is held at 0.1, while the boundary parameter α is given a value of 0.05 and 0.4 for Figure 6.4a and 6.4b respectively.

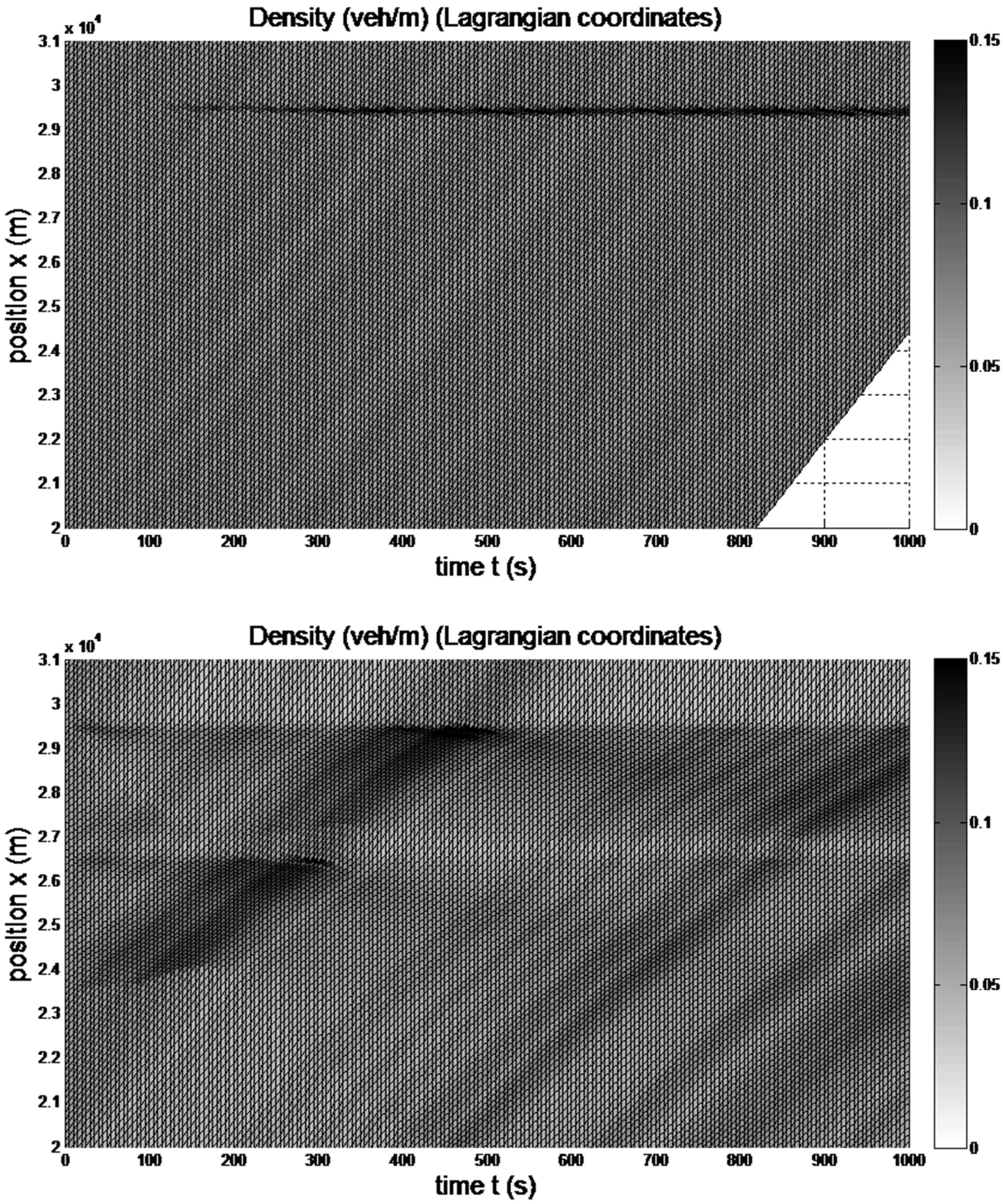


Figure 6.4a-b: Simulation results of the FOMSA for a dual bottleneck case for two different random procedures with settings α=0.05 (a) / 0.40 (b) and β=0.1

A low value of α means a low level of stochasticity in the assignment of vehicle specific invariant values and therefore means that the simulation result will be close to the deterministic case (Figure 6.3a). Performing multiple simulations with these values, showed very little difference between the outcomes. Each simulation also resulted in the second bottleneck being activated, while the first bottleneck was never activated. This is due to the boundary values for the vehicle specific invariant being out with of the required deviation to trigger the less severe bottleneck or to prevent the second bottleneck from being activated. The figure also shows little variation in the free flow densities. A high value for α on the other hand means a high level of stochasticity as seen in Figure 6.4b. As may be expected this leads to extensive congestion in comparison to a lower level of stochasticity. Both bottlenecks are activated and congestion is widespread and propagates quickly. This was found for the majority of the simulations, while a lower number of simulations for α=0.4 also resulted in both bottlenecks not being triggered, which is possible when the random values for the vehicle specific invariant are consistently low. However, as the probability of the value remaining low is small, the case in which congestion occurs is greater.

In Figure 6.5, a demonstration is given of the effect of changes to the transition parameter β. β is given a value of 0.3, while α retains the same value as in Figure 6.2 of 0.2. From Figure 6.5, it is clear that an increased boundary for the transition of the vehicle specific invariant value between vehicle groups leads to greater changes between consecutive vehicles. This increases the randomness of traffic flow and reduces homogeneity. However, as the effect of a high invariant value at a moment in time can immediately be counteracted by an equally strong low value from the following vehicle, when congestion occurs at the first bottleneck it is often of limited severity and does not last for a long time. Therefore, the effect seen from multiple simulations is that congestion occurs more readily compared with the deterministic case, however the severity is similar to other values of β.

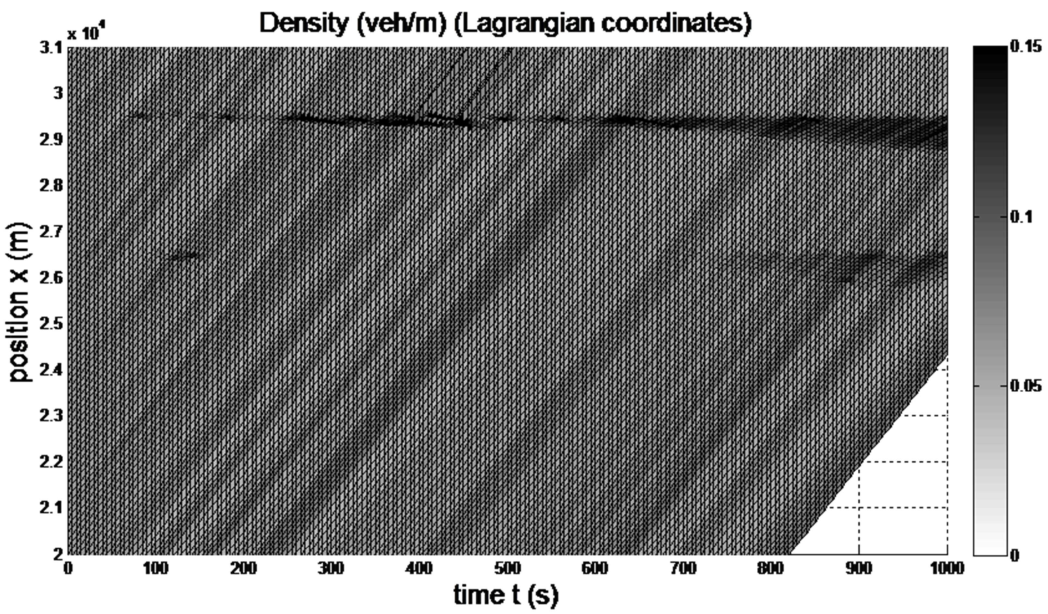


Figure 6.5: Simulation results of the FOMSA for a dual bottleneck case for two different random procedures with settings α=0.2 and β=0.3

6.4.2 Discussion

The values applied in these simulations for α and β are estimates of realistic values, however are not explicitly based on empirical observations. Further research is recommended to determine which values are most suited for these parameters and also to confirm the hypothesis that these parameters are of influence to traffic flow in way described. Values for α and β can be derived from empirical observations of vehicle interaction. The boundary parameter α can be observed from the distribution of vehicle headways in traffic, for which α is the relative deviation from the average observed headway. Transition parameter β can be derived from the change in headways of consecutive vehicles. Vehicles that are platooning are expected to yield similar values of α, therefore limiting the level of transaction β. Even in free flow without platooning, the hypothesis states that there may be a certain amount of correlation between following vehicles, due to space constraints and car-following behaviour.

The test case has demonstrated the face validity of the model and has further shown that vehicle specific behaviour can lead to situations in which a bottleneck sometimes will be activated and at other times will not be activated under identical traffic flow. The difference is in the characteristics of individual vehicles or platoons, which leads to local anomalies in traffic flow and a local reduction of the critical density, which increases the chance of traffic breakdown. These effects are also seen in real life on roads and confirm the face validity of the approach. The value for the boundary parameter, α, is found to be important for the probability of traffic breakdown and the level of congestion severity. This is not surprising as a large reduction in the critical density leads by definition to a higher probability of traffic breakdown. Even with the probability of higher density values, once congestion occurs, capacity is reduced through the capacity drop and therefore has a greater detrimental effect on traffic flow. The value for transition parameter, β, on the other hand indicates regimes in traffic flow from behaviour and gives a quantity for the interaction between vehicles. A higher value indicates independent driver-vehicle behaviour, while a low value increases the presumed interaction effects. It further shows that a better distribution of vehicle and driver types (aggressive and conservative drivers) can lead to a reduction in congestion severity. However, there is some uncertainty of the validity of such a parameter. The term is included as a hypothetical effect that can explain some characteristics of traffic flow, but has still to be validated against empirical data. This is also a recommended for later research.

6.5 Experimental case: capturing the capacity drop

Two different approaches are applied to explicitly introduce the capacity drop in FOMSA. The first makes use of bounded acceleration, while the second considers a drivers' reaction time when accelerating. Both approaches are considered in separate experimental cases, described in this section. Tests with the application of both approaches together did not show much difference to just applying a reaction time.

6.5.1 Setup for bounded acceleration case

The first experimental case is carried out for an 11 kilometre highway corridor with a single bottleneck location. The corridor is modelled in a first order kinematic wave model (KWM) in Lagrangian coordinates with three lanes, with the bottleneck set at a capacity reduction of 20% compared to the rest of the road. The KWM and numerical scheme allow for unconstrained overtaking of vehicles in case of a predecessor catching up with another vehicle group. Traffic flow into the corridor is initially set at an increasing rate from 1000 up to 3000 veh/hr/lane and retracts to 2000 veh/hr/lane at a set time. The driver specific characteristics of the traffic flow means that congestion may occur sooner for a vehicle specific invariant value lower than 1.0 and less readily for values above 1.0. The vehicle specific invariant is set rather liberally such that values between 0.8-1.2 are possible, with one scenario also allowing values between 0.6-1.4. Furthermore, the value of *I* remains identical for each vehicle (group) and does not change in time of with traffic state. This simplification does not affect the demonstration of the bounded acceleration. Assignment of invariant values is carried out randomly, using the same random seed for all scenarios. The time step applied in the simulations is 2.0 seconds, while the maximum speed limit is set at 100 kph, which meets the CFL condition and eliminates any numerical diffusion issues. Different scenarios are modelled in which two variables are varied, namely the acceleration bound [m/s^2] and the invariant value. The considered scenarios are given in Table 6.1.

Table 6.1: Scenario variable values

Scenario number	Acceleration bound [m/s^2]	Invariant bounds
0 (reference)	n/a	n/a (1.0)
1	2.0	n/a (1.0)
2	n/a	0.8-1.2
3	2.0	0.8-1.2
4	2.0	0.6-1.4
5	1.0	n/a (1.0)
6	1.0	0.8-1.2
7	0.75	0.8-1.2
8	0.5	0.8-1.2

The results of the scenarios are given in the form of flow-density fundamental diagrams, which allows insight into the spread of traffic values. Selected trajectory-space-time plots are given for relevant scenarios and a final comparison of the different levels of bounded acceleration are shown in a cumulative flow diagram.

6.5.2 Bounded Acceleration

The introduction of vehicle specific invariant values was previously introduced to increase realism in modelling, especially aiding stochastic breakdown. Here, bounded acceleration is added to the model. In Figure 6.6, the density/trajectory plots are given of selected scenarios in which the consequence of various values of the varied variables is shown. The selection is made taking into consideration that the outcome of scenarios 0, 1 and 5 were nearly identical, as were scenarios 2, 3 and 6, which were also similar.

The reference scenario 0 shows traffic flow increasing until at a certain point congestion is triggered. Each vehicle group homogenously transverses with the presiding flow and density values without fluctuations in flow. In scenario 2 and 4, the invariant is randomly applied (with an identical random seed for both for comparisons sake). The difference between vehicle groups is obvious and also makes is easy for one to see the trajectories of traffic, especially when a vehicle group reaches a congested road section. More importantly, higher bandwidths for the invariant show greater degrees of congestion. This can be seen between scenario 4, 2 and 0. In scenario 2 and 8, the same invariant values are applied, however with different bounded acceleration. In scenario 8, an extreme and unrealistic value of 0.5 m/s^2 is applied. This leads to a much denser congestion and longer congestion. However, when applying a value of 1.0 m/s^2 to the same case (not shown), little difference was found in the degree of congestion compared to a value of 2.0 m/s^2.

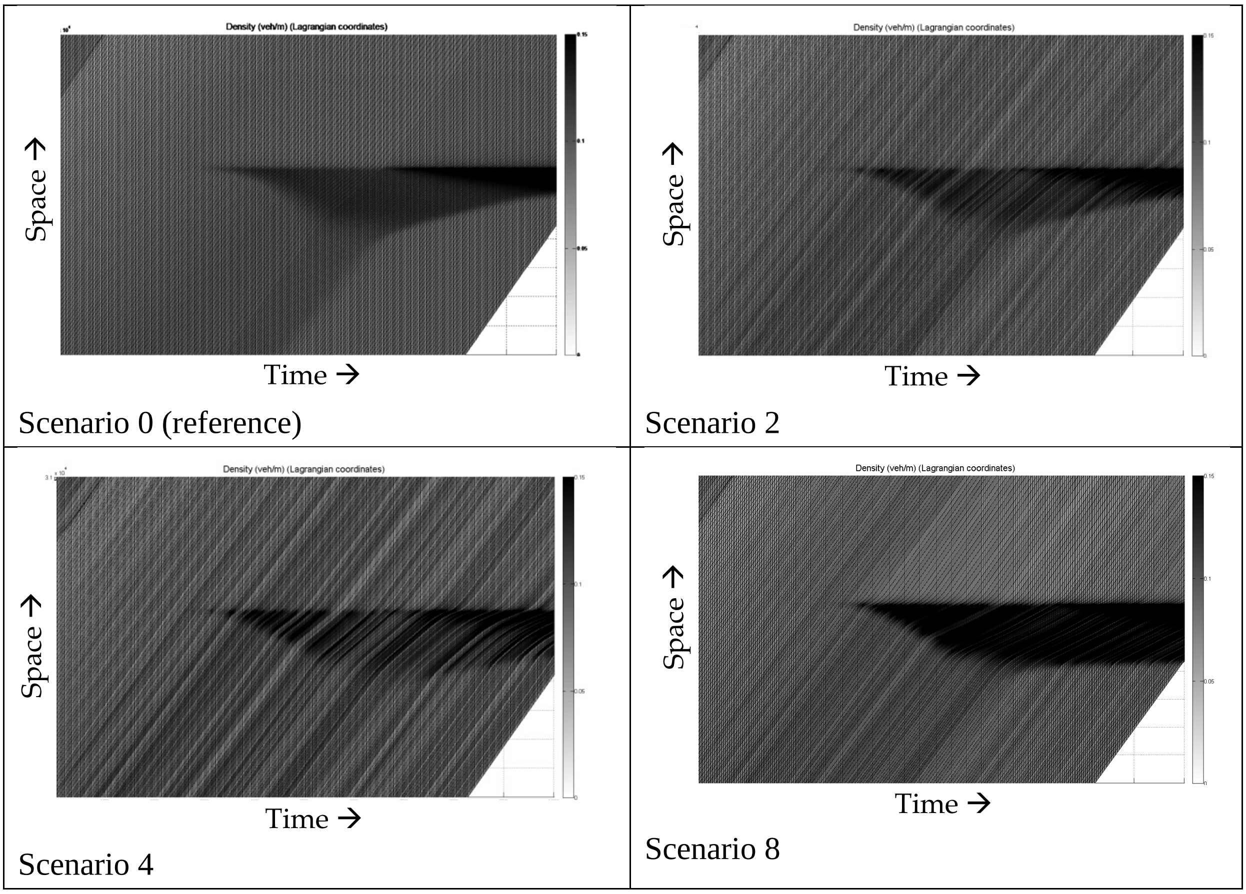


Figure 6.6: Trajectory-space-time diagrams for scenario 0, 2, 4 and 8

The fundamental diagrams for each scenario are shown in Figure 6.7. These are captured at a location directly downstream of the bottleneck. This gives further insight into the spread of traffic flow values and the extent of congestion. A first obvious observation is the spread in points on the congested arm for scenarios with invariant values (scenarios 2, 3, 4, 6, 7 and 8). As the invariant is setup to allow 'more aggressive' vehicle groups to drive at a smaller time headway (larger density), a higher flow rate can be achieved for identical speeds, while for 'less aggressive' vehicles the opposite is the case. This can be further noted from the greater spread for scenario 4 with the higher invariant bounds. The resulting fundamental diagrams represent the 'cloud' seen from empirical representations more realistically than the straight lines seen form scenarios 0, 1 and 5 without the invariant, resembling synchronized flow found by Kerner (2000). Another observation from Figure 6.7 is the severity of congestion, which is represented by the resulting densities. In the case of a broader invariant value, congestion is found to be slightly more severe, while for very low acceleration bounds an even higher density is found.

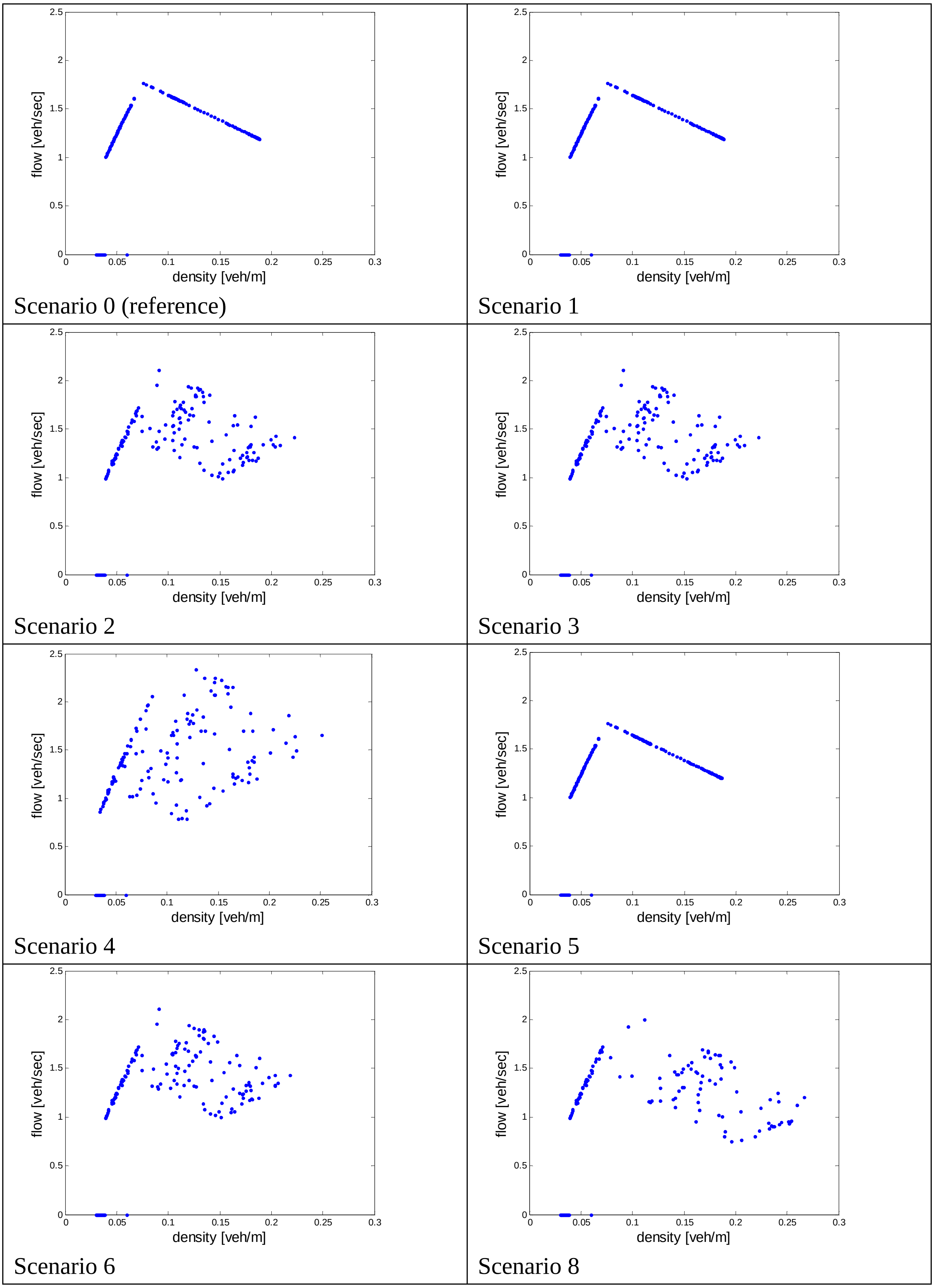


Figure 6.7: Fundamental diagram, from the bottleneck location

Bounded acceleration (BA) is tested in the first experimental case in four scenarios (3, 6, 7 and 8) for which the acceleration is bounded at 2.0, 1.0, 0.75, and 0.5 m/s^2. A realistic value for road vehicles lies between 1.0 and 2.0 m/s^2, and therefore the values below 1.0 are more demonstrative rather than realistic. From Figures 6.6 and 6.7, the effect of BA on the traffic throughput in congestion remained very limited for the two scenarios with 2.0 and 1.0. This is further demonstrated in Figure 6.8, in which the cumulative number of vehicle groups that pass the bottleneck location are shown in time. A lower number of vehicles indicates that capacity is relatively low and therefore a greater capacity drop is present. Only a very marginal difference is found between an acceleration bound of 2.0 and 1.0 m/s^2. In the hypothetical case that the BA is set to 0.75 m/s^2 and 0.5 m/s^2 an increasingly lower throughput is found in the flow.

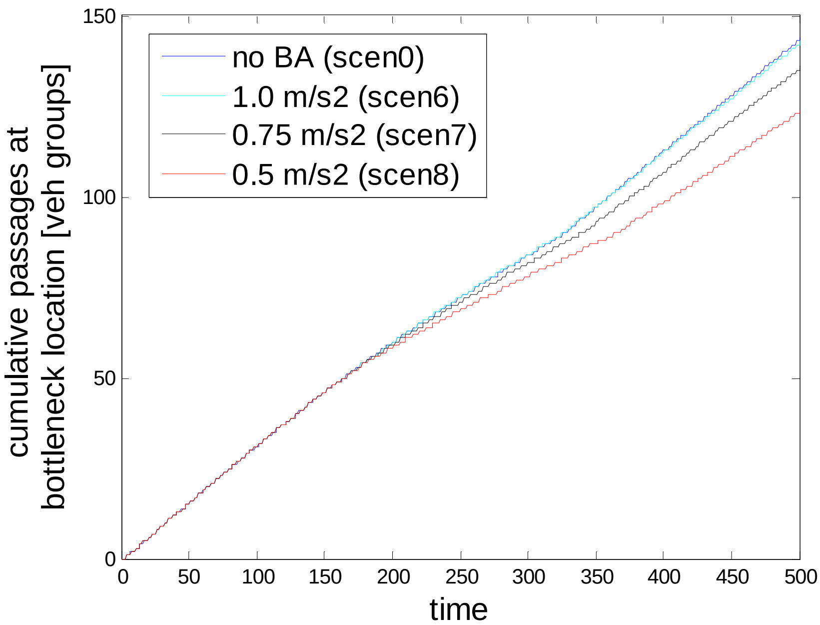


Figure 6.8: Cumulative throughput at the bottleneck location

From these results, it can be concluded that the application of bounded acceleration in a driver-specifically modelled flow with unconstrained overtaking does not lead to a substantial capacity drop for realistic values of BA. A drop in throughput is found for more extreme values, which does indicate that BA does directly contribute to some extent, however this is not large enough to be able to contribute the drop to BA of individual vehicle (groups). Therefore, a further hypothesis is constructed that the interaction of vehicle characteristics in a constrained manner is more important to reproducing the capacity drop from bounded acceleration. This is therefore also recommended as further research. A further improvement may be achieved by also including driver reaction times when exiting congestion. This is considered in the second experimental case in this section.

6.5.3 Reaction time

In the second experimental case, the effect of a drivers' reaction time in acceleration, T_r, is shown on the same 11 kilometre single bottleneck corridor. In this case, the traffic demand is increased to above the bottleneck capacity and later decreased to show the effect of the capacity drop. T_r values of 0, 0.5, 1.0 & 1.5 seconds are applied. Traffic heterogeneity is set at 0, 10% & 20% deviation for the vehicle specific invariant, *I*, for time headways. An identical capacity profile and demand profile is applied in each case. The applied time step is 3 seconds with a vehicle group size of $3^1/_3$ vehicles. The capacity drop is measured using flows downstream of the bottleneck with a 5 minute moving average aggregation. Aggregation is required for the heterogeneous cases. Figure 6.9 shows the trajectories of a reference case without T_r and in homogenous traffic (left), and on the right a case with T_r=1.0 and *I* =[0.8;1.2]. The capacity drop for the considered T_r and heterogeneity values are shown in Table 6.2. The capacity drop values are given against the highest flow pre-breakdown for each scenario, and in brackets compared to the reference case with no heterogeneity or reaction time, as in heterogeneous traffic flow, lower capacity values are found. Note that T_r=1.5 may not be a realistic value, however does give insight into the effectiveness of the method in extreme cases.

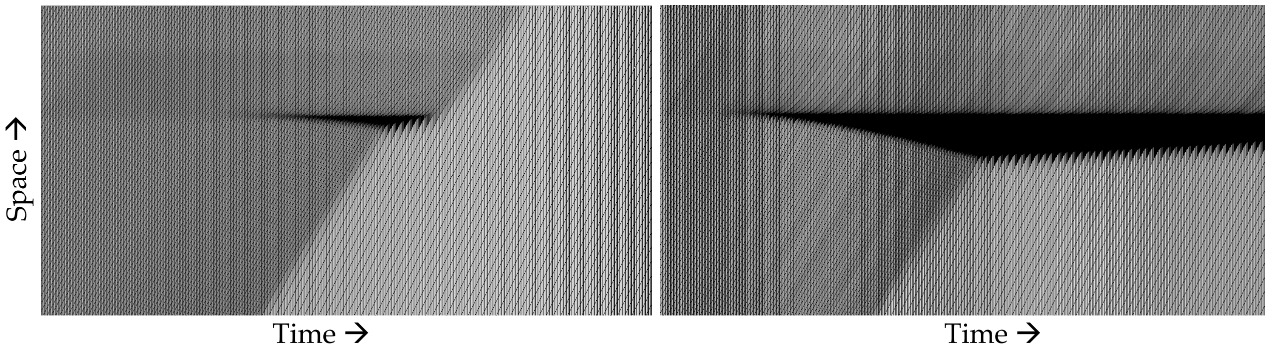


Figure 6.9: Trajectories and densities of the reference case (left) and case with I=10%, T_r=1.0s (right)

Table 6.2: Capacity drop results per case

Traffic heterogeneity I	*Reaction time* $\boldsymbol{T_r}$			
	0 sec	0.5 sec	1.0 sec	1.5 sec
0%	0%	9% (14%)	23% (27%)	40% (43%)
10%	0%	9% (14%)	24% (30%)	41% (46%)
20%	0%	6% (16%)	24% (32%)	41% (49%)

The results shown in Table 6.2 show that higher reaction times give increasingly higher capacity drop values, as is expected, and therefore the ability of the proposed method to capture the capacity drop is successfully demonstrated. Increased heterogeneity in traffic flow does not show an increase in capacity drop compared to the capacity of the same scenario. However, increased heterogeneity did result in lower capacities. If the discharge capacity is compared to the homogeneous case, then a higher capacity drop is found for higher rates of heterogeneity.

6.6 Conclusions

Capturing micro-stochastic driving behaviour in a macroscopic model is important to accurately describe traffic flow phenomena on a macroscopic level. A first order stochastic macroscopic model formulation is introduced in this chapter that makes use of first order traffic flow theory in conjunction with an additional invariant term, the vehicle specific invariant, that describes the heterogeneous effect of driver-vehicle behaviour and the level of aggressiveness of drivers and represents the vehicle specific change to a deterministic density value. This is performed in the Lagrangian coordinate system, which allows the invariant term to propagate along with the vehicles for which it is valid and thus avoids numerical diffusion of driver-vehicle behaviour variables. The use of Lagrangian coordinates have previously been shown to lead also to more accurate numerical results. The vehicle specific invariant is defined as an adaptation of a deterministic density as a function of two further parameters: α, which represent a stochastic boundary parameter that describes the limitations in variance between vehicles, and a transition parameter β that describes the interaction between driver-vehicle behaviour and gives a quantity of the change in the vehicle specific invariant in time. The described model offers the advantages of including driver-vehicle behaviour with an increased accuracy due to reduced diffusion effects, while doing this in a first order setting and therefore avoiding some of the complexity involved in second order model that are often applied to incorporate driver-vehicle behaviour in macroscopic modelling.

The model is demonstrated in an experimental case on a corridor with two bottlenecks present. The case demonstrates the face validity of the model and offers insight into the effects of different values for the model parameters. A further calibration of the model parameters based on empirical data is recommended as further research, as well as investigating the effects of other types of bottlenecks.

The capacity drop is applied in an extension of the model and is demonstrated. This is achieved through two different approaches: bounded acceleration and driver reaction times. The investigation of bounded acceleration found that the application in the model under constrained conditions has a limited contribution to a capacity drop. Only under low acceleration bounds was there a substantial capacity drop visible. This leads to the conclusion that the capacity drop is not merely a consequence of a restriction in the acceleration ability of vehicles on an individual basis, when vehicles are unconstrained by surrounding vehicles and are easily able to overtake one another. Additional approaches may include greater vehicle interaction and reaction times. In the second approach, the effect of reactions times is analysed. This approach successfully captured capacity drops for increasing reaction times. It also showed that the influence of heterogeneous traffic, through use of the invariant term, leads to lower capacities, while the capacity drop compared to a deterministic scenario is not increased. However, the capacity drop under heterogeneous traffic is greater as the base capacity is lower.

Chapter 7

Stochastic evaluation and identification of road resilience levels

Major and minor disturbances can have a considerable impact on the performance of road networks. In this respect, resilience is considered as the ability of a road section to resist and to recover from disturbances in traffic flow. In this chapter an indicator is presented, the Link Performance Index for Resilience (LPIR), which evaluates the resilience level of individual road sections in relation to a wider road network based on traffic flow stochastics. The indicator can be used to detect the least resilient road sections and to analyse which underlying road and traffic characteristics cause this non-resilience. The method adds to related concepts like robustness and vulnerability by also considering recovery from congestion events explicitly and by focussing on everyday operational traffic situations rather than just on disasters or major events.

This chapter starts with an introduction to the topic of resilience in traffic networks. In section 7.2, a detailed look is taken at performance concepts commonly used in traffic and related fields and considers their various definitions. This is followed in section 7.3 by an overview of commonly applied components and indicators corresponding to the described performance concepts. The proposed LPIR methodology is described in section 7.4, followed by a demonstration of the methodology in an experimental case in section 7.5. The chapter concludes with the overall conclusions and discussions in section 7.6.

This chapter is an edited version of the article:
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7.1 Introduction

While it is clear that major calamities and disasters can have a considerable effect on traffic and transport systems, there is awareness that more minor disturbances in traffic and transport systems can also play an important part in reducing the efficiency of such systems. A large number of effects have been proven to influence driving behaviour and with that the ability of traffic to maintain certain speeds, and also a certain serviceability, which in turn depletes traffic flow locally, but also on a network level. The effect of weather is probably one of the variables most commonly researched for its effect on road capacity and speed reduction (Calvert and Snelder, 2016, Hranac et al., 2006, Snelder and Calvert, 2016). Precipitation as rain as well as snow, wind, temperature and mist have all been considered (Agarwal et al., 2005, Calvert and Snelder, 2016, Cools et al., 2010, Maze et al., 2006). Also the influence of the local infrastructure can have an effect on traffic flow, where poor road surfaces, (incorrect) road geometry, different speed regimes, etcetera, can often lead to disturbances in traffic flow. Locations on a road network where interweaving traffic occurs are well known for their pertinent ability to disrupt smooth traffic flow and often with an unknown and erratic uncertainty of their time of occurrence (Calvert and Minderhoud, 2012, Sarvi, 2013, Shawky and Nakamura, 2007). Obviously stochastic driver behaviour, sometimes in combination with vehicle population, is often recognised for its stochastic characteristics and with that its disturbance of homogeneous traffic flow (Wagner, 2012, Wu, 2013). However, fluctuations between drivers and within one's own driving behaviour can be instable and difficult to quantify. Furthermore, the effects of driving behaviour are often combined and exacerbated together with other local disturbances. A number of other variables can also be identified.

Disturbances do not only affect local road sections, but by definition also (complete) networks. While local effects of disturbances are often considered, it is actually the network effects that are more profound and important to recognise as this is where the greatest delays occur. The two should not be considered entirely separately as local disturbances influence network flow and network flows in turn influence local conditions. However, the causes behind network disturbances are most often found in a local disturbance. While disturbances will often not be the core cause of congestion, they will often be a catalyst to hasten the onset of congestion. Network performance in relation to disturbances has been researched on a number of different levels. Reliability, robustness, vulnerability, accessibility and resilience are just some concepts that can be considered of a network. Especially reliability and vulnerability of networks has attracted much attention in recent decades, often in relation to travel-times and the ability to maintain a level-of-service. In the following sections, we will consider the differences and overlap between these concepts and give the applicable definitions. This is performed to clarify the distinction between the concepts. However, it is the concept of resilience with a close focus on traffic flow that is the main focus of this chapter. The reason for focussing on resilience is argued in section 7.2.4.

The focus on *resilience* is not commonly made in traffic flow analysis. In case of disturbances on roads, traffic flow will often be adversely affected, also commonly leading to congestion. Many measures of disturbances on the traffic consider either the probability of disturbances or the consequence of the disturbance, or both. However, in many cases small disturbances may not lead to congestion, while the balance between congestion and no congestion may be small. Furthermore, once congestion occurs traffic flow deteriorates, the duration before traffic returns to its original level-of-service is important to be able to quantify how widespread the adverse effect of the disturbance becomes. In both cases, road sections and networks recover from disturbances and have a direct relation to the overall performance of the network. The ability to recover from a disturbance is often referred to as resilience. Resilience research is not common within the traffic flow domain, and is found more readily in other transport domains, such as supply-chain management and logical operations (Chen and Miller-Hooks, 2012, Cox et al., 2011, Ishfaq, 2012).

In this chapter, a methodology is presented, the Link Performance Index for Resilience (LPIR), which evaluates the resilience level of individual road sections in relation to a wider road network. In such a way, the ability of a road section to deal with traffic disturbances can be quantified. The proposed methodology is constructed with an application to detect poorly resilient road sections. A main contribution of the methodology is the consideration of both resistance and recovery from a traffic heterogeneous point of view.

These road sections are considered for their ability to avoid traffic breakdown, however if congestion occurs also their ability to recover from a disturbance to normal operations. Limited ability to facilitate local disturbances and recover can lead to a greater traffic disruption more so than sections that do have the ability to more easily recover. Herein we aim to fill the gap in knowledge in relation to resilience for road traffic networks, and do this much more from a traffic flow perspective rather than a general network perspective.

The significance of this chapter is twofold: The method allows for identification of road sections which are susceptible to traffic breakdown. These locations therefore require more attention as also stochastic fluctuations can cause these locations to show weakness. Furthermore, the method allows for analysis of the consequences of network locations with volatile traffic flow. This involves characteristics of the road infrastructure, such as surface conditions or curvature, and vehicle characteristics, such as traffic composition. This can lead to a greater understanding of the variables that most affect resilience and possibly approaches that can lead to a limitation of stochasticity and improved resilience.

7.2 Performance concepts and definitions

When considering the performance of traffic flow on a road or in a network there are a number of performance concepts that need to be considered. It is important to be clear on the precise definition of each concept, as these vary slightly between scientific domains and even within domains. This concise overview of concepts is given to clarify the difference in definitions and describe why the focus in this chapter is on resilience. Here we will first consider the main concepts and highlight important and recent contributions. This is followed by the considered definitions in this chapter and the relationship between the concepts. The four concepts considered here are: *reliability*, *vulnerability*, *robustness* and *resilience.*

7.2.1 Reliability

The *Reliability* concept is well established in traffic and network analysis on a number of levels. In general, one of the most accepted definitions of reliability is given by Wakabayashi and Ilda (1992) as *“the probability that a system or a unit will perform its purpose adequately for the period of time intended under the operating conditions encountered.”* From this definition it is clear that reliability is concerned with the performance of a system, in our case a road or network, while it still satisfactorily functions. It is however important to note here that the study of reliability focusses on probability of this. Berdica (2002) even goes as far as to state that “reliability studies are generally concerned with probabilities only”. This gives a very definitive explanation of what reliability studies aims to achieve. However, it is argued that such a technical definition does not consider perception of users (Nicholson, 2007, Nicholson et al., 2003). It is important to identify expectations of users as they will only evaluate a system as reliable if their expectations are met (Nicholson et al., 2003). For this it is also important to realise that both the frequency and the consequence of a disturbance are relevant in an individual’s evaluation process. Jenelius et al. (2006) make a further distinction by stating that from an individual’s perspective a system can be seen as a binary decision: it is either reliable or not, while from an aggregate point of view some users will find a system reliable, while others will not. This also underlines a strong subjective aspect of reliability analysis. A wide range of reliability measures have been developed in the past decades. These differ on one hand for their application area and in their approach to reliability analysis and often consider slightly different definitions of reliability. One may consider capacity reliability (Chen et al., 1999, Chen et al., 2002, Church and Scaparra, 2007), connectivity or terminal reliability (Bell and Iida, 1997, Chen et al., 2007, Grubesic et al., 2007, O’Kelly and Kim, 2007, Wakabayashi and Ilda, 1992), and travel time or cost reliability (Bell and Schmöcker, 2002, Bell, 1999, Carrion and Levinson, 2012, Chen et al., 2003, Tu et al., 2012), most of which can be applied to either individual road sections or on network level. Other classes of reliability to be identified are also behavioural reliability (Clark and Watling, 2005, Lo and Tung, 2003, Mirchandani and Soroush, 1987, Yin and Ieda, 2001) and Potential reliability (Bell, 2000, Bell and Cassir, 2002, Berdica, 2002, Clark and Watling, 2005).

7.2.2 Vulnerability

When discussing reliability, one is considering the proper working of a system. Vulnerability on the other hand considers the improper working of a system. However, it may not entirely be seen as the opposite of reliability. To expand, a well-regarded definition of vulnerability in a road transportation system is that *“vulnerability is a susceptibility to incidents that can result in considerable reductions in road network serviceability”* (Berdica, 2002). Husdal (2004) goes on to state that serviceability then describes the possibility to use a system during a given period. Susceptibility in this definition on the other hand indicates a probability of an occurrence. Hence, vulnerability may be considered a two-component concept in which probability and consequence are the two main attributes, in short: probability of susceptibility, with a consequence for the serviceability. A similar view is also argued by Jenelius et al. (2006), in which some disadvantages of this approach, as also mentioned by Sarewitz et al. (2003), are mentioned. The main disadvantage being that estimation of probabilities of uncertain events is very difficult as some events are too rare to accurately derive from empirical data. However, when considering more regular disturbances in traffic flow, this difficulty dissipates somewhat. In another definition of vulnerability by Taylor and D'este (2003) only the consequence of an incident is considered, while the probability of a disturbance is ignored or presumed unquantifiable.

7.2.3 Robustness

Robustness is a concept that has more recently been developed for road traffic networks. A general definition of robustness is the *"the ability of a system to resist change without adapting its initial stable configuration"* (Wieland and Wallenburg, 2012). For road networks, a definition of robustness is given by Snelder et al. (2012) as *"the extent to which, under pre-specified circumstances, a network is able to maintain the function for which it was originally designed"*. Both Snelder et al. (2012) and Berdica (2002) state that robustness is an interchangeable opposite of vulnerability in relation to road networks. However, this is only true up to the point that vulnerability must place a greater emphasis on probability as it considers the occurrence of disturbances, while robustness considers the prevention of detrimental effects of disturbances. It is possible to only consider the effects of a disturbance, but more often than not one will also want to know its rate of recurrence. A robust network has the capability to compensate for disruptions on network links with relative ease and with only a small deterioration of performance Sullivan et al. (2010). Therefore, a major difference compared to reliability is that robustness considers how a network can maintain its function while suffering a disturbance and therefore focusses more on the effects of a disturbance, while reliability is more concerned with the probability of a disturbance. Following from the definition, a robust network can allow a decline in performance as long its function is maintained, and while probability is not the main focus, the term *'extent to which'* indicates a clear possibility to quantify robustness (Snelder et al., 2012).

7.2.4 Resilience

The final concept to be considered here for road and network performance is resilience. *Resilience* is a concept that has been recognised a number of times within the traffic domain to be of possible relevance without much research being performed (Berdica, 2002, Nicholson, 2007). In other transportation domains, resilience is more recognised, such as in the transport related areas of logistics and supply-chain management (Chen and Miller-Hooks, 2012, Cox et al., 2011, Ishfaq, 2012). Chen and Miller-Hooks (2012) define a resilient network as a network that is able to *recover from disruptions*. This ability depends on the network structure and activities that can be undertaken to preserve or restore service in the event of a disaster or other disruption. Goldberg (1975) states that two main attributes are relevant for resilience, namely the level of disturbance and the speed at which the system can recover from the disturbance. Berdica (2002) further states that resilience could be described as the capability of reaching a new state of equilibrium, however in the case of traffic flow, a new equilibrium state may resemble or equate to the original undisturbed state. Bankes (2010) states that it is tempting to define robustness and resilience synonymously. However, he goes on to say that robustness can be generally understood as the ability to withstand or survive external shocks; to be stable in spite of uncertainty. Resiliency involves the ability of a system to recover from disturbances. Recovery implies a failure of robustness on a shorter time scale than that at which the system is judged to be resilient. This means that a system may be deemed as not being robust, whilst it may be considered resilient.

In this research, the choice is made to focus on the concept of resilience. Much has previously been performed on robustness and reliability; however the concept of resilience, as defined here, also includes the recovery of the traffic system. In stochastic traffic flow, there is an increased probability of traffic flow breakdown and loss of function of a traffic system. It is also hypothesised that the recovery of traffic flow is hindered by heterogeneity in traffic. Therefore, the focus should lie on a concept that includes both. In this research, we therefore focus on and define resilience as *"the ability of a system to cope with disturbances and recover after a loss of function"*. Here, the term 'to cope with' indicates that to measure resilience does not require a state of 'functional failure' to be measured and can be evaluated when still properly functioning. 'Loss of function' refers to a reduction in performance of the system. In the case of traffic network, a commonly applied performance indictor is the 'level of service'. Similarly, the onset of congestion constitutes a main loss of function, with higher density and lower speed values indicating an increase in functional loss. A system that can easily cope with a disturbance may be deemed more resilient than a system that only just manages to cope, as a different more extreme disturbance may cause the latter to lose function in any case. However, when a system experiences functional loss, it may still be deemed resilient, albeit to a lesser extent, if it is able to promptly recover.

7.2.5 Overview

Form the various descriptions, it should be apparent that although there are varying definitions for the described concepts, there is a general level of consensus on their meaning. The main characteristics of the concepts are given in Table 7.1 to more easily distinguish between the application areas of the concepts. Figure 7.1, taken from (Wang et al., 2014), gives a good overview of the interdependent relations between the aforementioned concepts.

Table 7.1: Overview of performance concepts and their definitions

	Reliability	**Vulnerability**	**Robustness**	**Resilience**
Description	Probability of serviceability	Susceptibility of serviceability loss	Ability to maintain serviceability	Ability to maintain and recover serviceability
Disturbance relevance	Probability of occurrence of…	Not withstand the effects of…	Withstand the effects of…	Withstand and if necessary recover from…
Probability relevance	Main focus – Indicates proximity to perfect performance	Facilitating – Indicate chance of function loss	Facilitating – Indicate chance of function loss	Facilitating – indicate recovery ability
Effect relevance	N/A	Quantification of effects	Quantification of effects	Quantification of effects
General application	Both locally & on network	Mainly on network level, but also locally applicable	Mainly on network level, but also locally applicable	Mainly local, but also applicable on network level

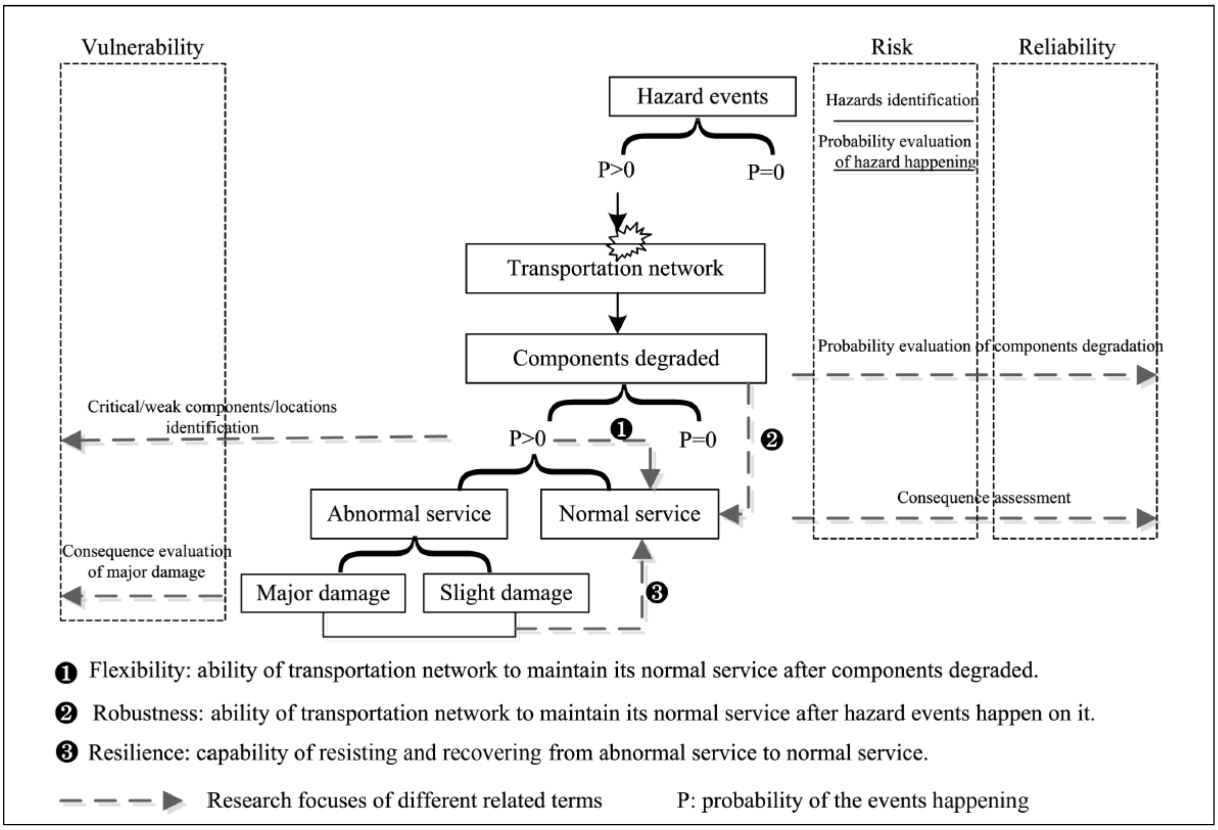


Figure 7.1: Relationships between main concepts (taken from Wang et al. (2014))

7.3 Performance components and indicators

In section 7.2, main concepts were described, which are related to resilience. While the focus is on resilience, there is much overlap between the concepts and therefore various components from the other closely related concepts can be valuable for resilience. In this section, components and indicators from robustness, vulnerability and other resilience approaches are reviewed.

7.3.1 Robustness and vulnerability

From the previous section, is should be obvious that resilience is much closer related to robustness and vulnerability than to reliability. There is a sufficient similarity for it to be useful to review components of both robustness and vulnerability before looking at the relevant components for resilience. Both robustness and vulnerability will be considered together as they are near enough each other opposites and therefore will generally make use of the same components and indicators.

When reviewing literature, it becomes quickly apparent that there are a wide range of definitions and descriptions for the same attributes. Here we will try to use the most generic terminology, but will often refer to authors' own definitions of components.

Different approaches are found to classify vulnerability and robustness. On the one hand, accessibility and network efficiency are applied as main indicators in which the network geometry is seen as a more important factor (Chen and Miller-Hooks, 2012, Jenelius et al., 2006, Taylor and D'Este, 2007). On the other hand, some apply an approach which considers the importance or criticality of links to be focal point (Scott et al., 2006). Jenelius et al. (2006) make a distinction between exposure and criticality on a network level. The exposure indicator covers the position of links and the connectivity of links on a network, while the criticality gives an indication of how important or critical a link is. Srinivasan (2002) states that there are four types of factors: *deterministic, quantitative time-varying, qualitative measures* and *random factors.* These factors describe various attributes that may be classified in four categories: *network characteristics, traffic flow, threats* and *neighbourhood attributes* (El-Rashidy and Grant-Muller, 2014, Srinivasan, 2002). Within these categories, a similar trend is found with different descriptions; Networks and infrastructure characteristics account for the supply characteristics of network links, traffic flow basically entails the demand on a network, while threats identifies weaknesses in a network and neighbourhood attributes the connectivity or accessibility of network links. Snelder et al. (2012) consider robustness more as an umbrella concept, which includes resilience among other parts. However, here we will refer to robustness as a single concept which overlaps, but does not enclose resilience.

Table 7.2: Components used in performance concepts

Traffic dynamics & demand	*Disturbances/Threats*	*Network Characteristics*
Travel time (ratio) [1,2] **- Speed of movement** [1] **(=traffic volume x average speed)** **Traffic speed (ratio)** [2] **Delay (ratio)** [1,2] **- Incl. relative delay rate** [1] **Distance covered** [2] **Traffic demand (ratio)** [2] **Volume capacity ratio** [2,3,4,5] **- Volume capacity ratio for low capacity links** [4] **- Increase in volume versus capacity (for low capacity links)** [4] **- Arrival rate at end of queue** [3,6] **Accessibility** [1,2] **- Arrival rate for a set period** [3,4]	**Effect of disturbance** [2,3,4] - Number of effected vehicles [2] **Spare capacity** [2,4] **Congested travel (density)** [1] - Number of vehicles in congestion times congestion length [1] **Duration of disturbance effect** [1] **Distance of disturbance effect** [1] **Probability and effect** [2,3,4] **(for low capacity links)** [4]	**Redundancy** [2] - Alternative routes [2,4,5] **Connectivity** [2] **Link capacity** **Node degree** [2] **Distance** [2] - distribution of distance **Node centrality** [2] **Node coreness** [2] **Compartmentalisation** [2,4,5] - i.e distance between ramps or junctions

[1](Berdica, 2002) [2](Snelder et al., 2012) [3](Knoop et al., 2012) [4](Tampere et al., 2007a) [5](Li, 2008) [6](Tamminga et al., 2005)

Much available literature primarily considers the components for an overall ‘vulnerability’ or ‘robustness’ indicator. These individual components are discussed making use of the categories given by (Srinivasan, 2002) in which network characteristics and neighbourhood attributes are considered as a single category. A complete overview of a number of found components is shown per category in Table 7.2 in which the relevant references are also given.

The *traffic dynamics category* includes a wide range of traffic related indicators. Comparable components are grouped for clarity, such as travel time and (average) speed, as these are convertible through distance. Many components are similar, but are defined such that it gives an additional quantification. For example, travel time is considered, but may also be adjusted to indicate the ‘speed of movement’, which is a multiplication of the average speed with the traffic volume. For most traffic dynamics components, both the component itself and the ratio of the component compared to a base or reference value are suggested. The advantage of using ratio’s is that it easily allows for normalisation, which can be a preferred approach as attributes are on a closed interval and can be more easily compared, either with or without weighting (El-Rashidy and Grant-Muller, 2014). The *disturbances category* includes suggestions for a quantification of the effect of disturbances on traffic flow, such as quantification of the number of affected vehicles, of congested travel, and of the distance or duration of a disturbance. The probability of a disturbance or threat is also an attribute to be considered. The *network characteristics category* mainly considers the attributes related to the physical infrastructure and the way the various network links are connected. When specifying specific characteristics for vulnerability and robustness, not a great deal of attributes were found in literature for the physical road characteristics. It maybe that these are not considered relevant or, probably more likely, that they are the underlying variables linked to other attributes. For example, a poor road surface may lead to lower speeds and therefore a higher travel time, both of which have already been considered.

There are a wide range of existing performance indicators for both robustness and vulnerability. To give an indication of some of these indicators, a short overview is given in Table 7.3 with a description of the indicator.

Table 7.3: Some recently applied performance indicators for robustness and vulnerability

Described	Indicator	Applied to…	Description
Snelder et al. (2012)	Delay	Robustness	Delay encountered under pre-defined disturbances (incidents). Delay on specific links, routes, or (sub)networks indicates the robustness level.
-	Volume to Capacity (V/C)	Robustness & Vulnerability	Ratio between the traffic volume and available capacity
-	Gamma index	Robustness & Vulnerability	Connectivity index indicating how well a network is connected. A value of 1 indicates a completely connected network.
Scott et al. (2006)	Network Robustness Index (NRI)	Robustness	Change in total travel time over a given interval resulting from the re-assignment of traffic in the system when a specific link is removed from the network. Measures how critical a given link is to the overall network.

Sullivan et al. (2010)	Network Trip Robustness (NTR)	Robustness	Applies an 'importance' factor to the NRI of routes based on the demand over specific links. NTR is calculated as the summation of NRI over all links and dividing that by the total trip demand in the network.
El-Rashidy and Grant-Muller (2014)	Link & Network Vulnerability Index (LVI & NVI)	Vulnerability	Makes use of weighted multiple attributes. The applied attributes are: 1. Link traffic flow in relation to link capacity 2. Impact of link flow compared to capacity 3. Inverse of time for congestion to reach upstream junction 4. Link capacity compared to maximum link capacity in the network 5. Link length 6. Importance of link: number of time that a link is on the shortest path between OD pairs
Jenelius et al. (2006)	Importance & Exposure	Vulnerability	Importance calculated as the consequence of a link closure for travel cost and for unsatisfied demand. Exposure is the accessibility dependant on the expected increase in travel cost. Both include arbitrary weighing of link reflecting a links position in a network.

7.3.2 Resilience

While resilience is sometimes mentioned in relation to traffic flow and networks, research into descriptive methods is limited. Some authors describe resilience from an organisational and economical perspective (Bruneau et al., 2003, Nicholson, 2007, Reggiani et al., 2002, Rose, 2009), while resilience is discussed more explicitly in other domains. Within road network research, there is also an area of research that involves resilience in case of disasters (Faturechi and Miller-Hooks, 2014). In these works, the focus tends to be more on decision frameworks, and therefore we will not focus on this area of research here. A few suggestions for more generic attributes in resilience are given here based on some of these other transport related domains. These are merely meant as an indication from other disciplines, rather than an exhausted review of resilience in the whole transportation domain.

In their review of transport security, Reggiani (2013) cite four dimensions for resilience: *robustness*, *redundancy*, *resourcefulness* and *rapidity* (Bruneau et al., 2003). Robustness demonstrates the need to consider the avoidance of serviceability for a disturbance as part of resilience as a whole, where redundancy of unused capacity may be addressed. However, when serviceability is affected, resourcefulness and rapidity become relevant. Resourcefulness relates to stabilising measures, either from within a system itself or externally applied (such as traffic management in traffic). Rapidity relates to the importance of a rapid return to an acceptable level of service. It is further stipulated that the main aspects to consider should aim *to reduce probability of failures*, *the consequences from failures*, and *the time to recovery*.

In intermodal freight transport, Chen and Miller-Hooks (2012) present a resilience indicator. The main premise applied considers the "the level of effort (cost, time, resources) required to return the network to normal functionality (or a fixed portion thereof)". Here the main focus is on the recovery process and the ability to achieve a return to required level of functionality or serviceability. From this, it is also clear that a complete return to the same level of serviceability is not required, but rather a pre-defined acceptable level of serviceability. The occurrence of (major) disturbances is considered as an unknown random effect that occurs, therefore less attention is spent on prevention of a disturbance leading to a loss in serviceability. Some variables applied are:

- *Recovery activities*
- *Change in capacity after implementation of recovery activities*
- *Travel time (incl. Maximum travel time)*
- *Time to implement recovery activities (incl. Maximum implementation time)*
- *Cost of recovery (incl. Maximum allowable cost)*
- *Network connectivity*

In other research on transportation network, (Murray-Tuite, 2006) describes a simulation approach for resilience in which a system optimum approach is compared to a user equilibrium approach. In her research, she identifies ten main dimensions to be considered for resilience:

- *Redundancy*
- *Diversity*
- *Efficiency*
- *Autonomous components*
- *Strength*
- *Collaboration*
- *Adaptability*
- *Mobility*
- *Safety*
- *Ability to recover quickly*

Some of these attributes are more relevant for transportation networks rather than traffic networks, such as collaboration or autonomous process components. However, other attributes and the general premise give a good insight into the type of attributes that should be considered.

7.4 Methodology for resilience analysis

Many of the previously described measures and components are keyed very much towards network performance even if many calculate local road section performance to obtain a network score. As defined in a previous section, a main application area here for resilience is very much on the performance of local road sections. This is due to the stochastic breakdown effects that materialise on a microscopic level locally rather than for a whole wider network. As traffic management is mainly applied to a local area, it is most relevant to focus on this local level for the level of traffic homogeneity. In this research, there is a greater emphasis on the determinants of certain attributes, rather than only on the resulting effects. A previous example of a poor road surface is an example of such a determinant, while a lower speed for that road section is the resulting effect. As we define resilience as *"the ability of a system to cope with disturbances and recover after a loss of function"*, it may be seen as an extension of robustness/vulnerability as it considers the ability of a system to cope with disturbances. It does however differ in the sense that it also considers the recovery process explicitly and as an important part of the concept. Moreover, the focus in this chapter is more on traffic flow rather than network infrastructure.

We start by stating therefore that resilience exists out of two main parts: *resistance* and *recovery*, as is found in the majority of the cited literature. The *resistance* part incorporates the extent to which a road section or network is robust and can resist functional loss under stress and is comparable to robustness. The *recovery* part of resilience is what sets the concept apart from robustness/vulnerability and describes the ability of a road section to return to an acceptable level-of-service.

7.4.1 Resistance

We define the ability of the traffic system to resist a disturbance (resistance) as *"the ability to avoid going into a state of congestion"*. To this extent, we quantify this as the ability of a road section to maintain a density lower than the critical density: $k < k_{crit}$. Writing this as an index which represents stability below a value of 1, gives:

$$index = \frac{k}{k_{crit}} \tag{7.1}$$

The density and the critical density can be derived from a number of other components. In traffic flow, in relation to the influence of disturbances, we have identified the following components for the density and the critical density in an uncongested flow, which are explained in the rest of this sub-section:

Table 7.4: LPIR Resistance components

Density	**k**	**Critical Density**	**kcrit**
q	flow	q_{cap}	road capacity
v	speed	g	road characteristics
ψ^q	volatility of flow	h	traffic characteristics
		ψ^{cap}	volatility of capacity
		f	temporal capacity reductions (i.e. incidents)

In traffic flow theory, the fundamental relation is given by: $k = {}^{q}/_{v}$. While the flow of traffic can be aggregated to set value, q, in reality traffic flow is stochastic with a certain bandwidth, $q \pm \psi^q$. Here ψ^q is the traffic volatility, defined by:

$$\psi^q = \frac{1}{2}(q_{max} - q_{min}) \tag{7.2}$$

For $q - \psi^q$, traffic will remain in an uncongested state if q is also uncongested, however for $q + \psi^q$, traffic may enter a congested state if the flow is near to capacity. This is therefore critical and therefore the density is reformulated using the fundamental relation and equation (7.2) to give:

$$k = \frac{q + \psi^q}{v} \tag{7.3}$$

In a similar way, the critical density can also be reformulated to indicate the critical state to resist congestion as:

$$k_{crit} = \frac{q_{cap}(g,h).f + \psi^{cap}}{v_{crit}} \tag{7.4}$$

Here, three notable differences can be seen compared to equation (7.2). The flow and speed variables are now the critical capacity values, rather than the current traffic state values. Furthermore, the capacity is also dependant on the variables (g, h), which indicate the influence of road and traffic characteristics. Thirdly, there may be temporal capacity reduction to be considered, which is indicated by f. Substitution of the components from equation (7.3) and equation (7.4) into equation (7.1) gives the *derived resistance equation*:

$$Resistance = \frac{\left[\frac{q + \psi^q}{v}\right]}{\left[\frac{q_{cap}(g,h).f + \psi^{cap}}{v_{crit}}\right]} \tag{7.5}$$

The equation is valid for a set time interval, *T*. The dependence on time is excluded from the equation for readability. Here we see in the numerator the density given by the 'volatile flow' divided by the speeds, which follows from the fundamental relation of traffic flow: $k = {}^{q}/_{v}$. The volatility of traffic flow describes the traffic flow increased with a measure of volatility, describing the stochastic behaviour of the flow in a predefined period, for the time interval, T, for example 15 minutes. The volatility is an indication of the bandwidth of traffic flow in this time window and is therefore defined by:

Note that $(q + \psi)$ does not need to correspond with a maximum value of q in the considered time period, as the gravity of the values may be skewed higher or lower. Fluctuations in the speed can also be included in this volatility factor, however are expected to follow the fluctuations in the flow and are therefore not required. In the denominator, the critical density is given, which also incorporates the fundamental relation. The speed is the critical speed by definition, and is dependent on the road and traffic characteristics. The critical flow is described as the capacity reduced by a temporal capacity reduction factor and also includes a volatility component. It is given as a function of the road and traffic characteristics. The volatility for the capacity is collected for an entire period and is not time dependant as the flow volatility. The volatility of the capacity is given here as:

$$\psi^{cap} = \frac{1}{2}\left(q_{cap.max} - q_{cap.min}\right) \tag{7.6}$$

The road characteristics component represents the influence of the infrastructure and depends on variables, such as the maximum speed limit, number of lanes, lane width, gradient, curvature, road surface, and etcetera. The traffic characteristics represent variables such as vehicle types and characteristics, vehicle dimensions, driver types, etcetera. A further quantification of these components is not given here, but is rather recommended for later research.

7.4.2 Recovery

Corresponding to the definition given of the resistance part, the recovery part is defined as "*the ability to come out of a state of congestion*". This is quantified as the ability of a road section to regain a density lower than the critical density from its current state: $k > k_{crit}$. This index allows use of the same equation (7.1). The main additional components identified as relevant for determination of the recovery are given as:

Table 7.5: LPIR recovery components

Recovery components			
Δq	flow change, $q_{in} - q_{out}$	q_{in}	inflow
q_{cd}	capacity drop (absolute)	q_{out}	outflow
$v_{eq}(q)$	speed, derived from fundamental diagram		

The recovery equation is derived in a similar fashion to the resistance equations, making use of the fundamental relation and a further expansion of the underlying variables, but in this case for a congested traffic state. The two main traffic variables that influence the recovery of a road section are found to be the resulting *capacity drop* in a section and the difference between the in- and outflow of traffic into a road section. From equation (7.5), it is clear that a higher capacity drop will reduce the ability to recover, as well as a higher inflow compared to the outflow. The density in congestion makes use of the congested speed, which is dependent on the traffic flow and can be calculated from the fundamental diagram to give $v_{eq}(q)$. The applied fundamental diagram is given in equation (7.10). To recover from congestion, traffic inflow must reduce if capacity does not increase. This is derived for a single road section through the change in flow: $q_{in} - q_{out}$. Combined these variables together with the fundamental relation give:

$$k = \frac{q + \Delta q}{v_{eq}(q)} \tag{7.7}$$

The critical density in congestion is similar to equation (7.4) from the pre-congested state. There is however one important difference, that when congestion occurs a capacity drop often also occurs. This is added to the equation, while the volatility of the capacity is less interesting as traffic breakdown has already occurred. Therefore, the equation for the critical density becomes:

$$k_{crit} = \frac{q_{cap}(g,h).f - q_{cd}}{v_{crit}} \tag{7.8}$$

Substitution of equation (7.7) and equation (7.8) in equation (7.1) results in the recovery equation is given by:

$$Recovery = \frac{\left[\frac{q + \Delta q}{v_{eq}(q)}\right]}{\left[\frac{q_{cap}(g,h).f - q_{cd}}{v_{crit}}\right]} \tag{7.9}$$

Again, the equation is valid for a set time interval for which the dependence on time is excluded from the equation for readability. Here, $v_{eq}(q)$, further represents the speed derived from the fundamental diagram with input: q. Written in full, this corresponds to:

$$v_{eq}(q), = \frac{q}{\left[k_{crit} + (1 - \frac{q}{q_{cap} - q_{cd}}).(k_{jam} - k_{crit})\right]} \tag{7.10}$$

where k_{jam} is the jam density and k_{crit} is the critical density.

7.4.3 General Link Performance Indicator for Resilience

As we define resilience in traffic flow as the combination of both resistance and recovery, the combination of the previously described equations results in the Link Performance Indicator for Resilience (LPIR) and is given by:

$$LPIR = \frac{1}{T}\sum_{t=0}^{T}\left(\frac{\left[\frac{q + \psi^{q}}{v}\right]}{\left[\frac{q_{cap}(g,h)f + \psi^{cap}}{v_{crit}}\right]} 1_{k \le k_{crit}} + \frac{\left[\frac{q + \Delta q}{v_{eq}(q)}\right]}{\left[\frac{q_{cap}(g,h)f - q_{cd}}{v_{crit}}\right]} 1_{k > k_{crit}}\right) \tag{7.11}$$

Combining both parts is allowed as it gives the entire range of possible density values, from uncongested to congested density values. Note that each variable is valid for a set time interval. For readability, the notation of the dependence on t has been omitted from the equation. The total LPIR score per road section is the average over all time intervals for the considered period.

The LPIR can be applied to any road section to give an indication of the relative resilience of that road section compared to other road sections. Focus on local homogeneity means that network connectivity is deliberately avoided to allow the index to describe the local vulnerability. Obviously local traffic conditions are influenced by network effects, however the index is not greatly sensitive to this as the evaluation of a road section considers local flow and local capacities. A value of LPIR =< 1 indicates that a road section is able to resist a significant drop in level-of-service and therefore remain uncongested and by definition must be considered resilient as well as robust. However, a road section that does suffer a drop in level-of-service, but can recover promptly should also be considered resilient as resilience considers the ability to recover from a disturbance or loss of service. However, in the latter case, the road section may not be considered robust, as a failure event occurred. One cannot state that a value above LPIR > 1 is always non-resilient. Normalisation of the LPIR may be applied, as this may make comparison between values from different road sections easier. However, this has the drawback that the quantitative interpretation of the index is lost and is not performed in the experimental case.

7.4.4 Stochastic Link Performance Indicator for Resilience

The presented description of the LPIR given in equation (7.11) is a deterministic score for resilience. However, increasingly the importance of explicitly considering stochastic fluctuations in traffic is being seen as relevant and often necessary. Therefore, a stochastic representation of the LPIR is also relevant. Incidentally, it is not that difficult to transform LPIR for a stochastic representation. The variables representing the flow from the original LPIR should be described as random variables rather than deterministic and must be further condensed, resulting in:

$$LPIR = \frac{1}{T}\sum_{t=0}^{T}\left(\frac{\left[\frac{q}{v}\right]}{\left[\frac{q_{cap}(g,h)f}{v_{crit}}\right]}1_{k\leq k_{crit}} + \frac{\left[\frac{q+\Delta q}{v_{eq}(q)}\right]}{\left[\frac{q_{cap}(g,h)f - q_{cd}}{v_{crit}}\right]}1_{k>k_{crit}}\right) \tag{7.12}$$

Note that the main changes relate to the representation of q, which is now the random variable q. Furthermore the volatility variables become obsolete in a stochastic version, as they were used as a measure of variability, which is now incorporated in the random variables of the flows and capacities. It is also possible to represent the incident reduction factor as a random variable, as well as the speed and critical speed. However, it is chosen not to do that here and consider stochasticity only from the flow and capacity variables.

7.4.5 Considerations and component sensitivity

The presented methodology differs in its approach to many other methods that have previously been presented for similar measures, mainly in the area of robustness. The first main difference is the focus on specific road sections, rather than on a network performance. The second one is the explicit consideration of traffic flow dynamics, where many other methods consider more static descriptive variables.

In relation to consideration of local road sections, an implicit consideration of the influence of other bottlenecks and connectivity to the rest of the network is present: downstream congestion that reaches an arbitrary road section will affect the LPIR score of that section in conjunction with the severity of the congestion. However, the opposite does not apply. That is the network effect of congestion caused by a considered road section on the rest of the network. This is a drawback when one wishes to expand the method to be used to calculate a network index. In relation to consideration of traffic flow dynamics, this method aims to seek out the core reasons behind resilience or the lack of, and offers the possibility to connect the resilience score to the causes. At the highest level, this is only calculated from traffic data, while further adding detail to the g and h terms, denoting road and traffic characteristics, allows explicit causality to be derived. This is not performed in this chapter though.

The variables applied in the method have been tested for their sensitivity, while a few other variables that were considered have been shown not to be of great relevance. The choice of the time interval, T, has been analysed for its effect on the results. The time interval is mainly relevant for the volatility variables (ψ), including the delta flow variable (Δq). The outcome of the analysis shows that the absolute value of LPIR does shift slightly, but in relative terms there is a limited effect. In any case, this is not sufficiently large enough to influence the analysis of the road sections. When delta flow is not included, the LPIR shows a higher sensitivity for higher T values (T=15), while for lower T-values (i.e. T=2), the exclusion of delta flow does not influence the LPIR score. As the influence of delta flow requires a higher T-value and the relative difference is not large between T-values, a value of 15 minutes is viewed as a suitable value, as this allows variation in flows to be considered in LPIR. The analysis of this variable is shown in Appendix A. Besides the Δq and T variables, a further volatility term for congested traffic was considered as well as a volatility value for the speed and critical speed. Congested traffic is more stable than uncongested traffic and the Δq term already includes the relevant variations in recovery, such that the inclusion of a volatility term for the recovery does not have a large effect. Including a volatility terms for the speed and critical speeds in the resistance equation also did not possibly influence the scores. The traffic speeds were found to include too much noise to be included as they made the results messy, while the flow volatility already captured many of the fluctuations, but in a more stable manner. The critical speed was found to be rather stable for most locations and between different breakdown events (consistently between 70-75 km/hr) and therefore added little to the overall method. Therefore these additional volatility variables were not applied.

7.5 Experimental study results

In this section, the LPIR is demonstrated in an experimental case study for a part of a heavily congested network north of the city of Rotterdam. Demonstration of the validity of the method is achieved through comparison with other indices related to robustness and recovery and with a comparison with the qualitative causes and effects of regular congestion on the considered network.

7.5.1 Setup and network

A demonstration of the Link Performance Indicator for Resilience (LPIR) is given making use of a real network. The purpose of the demonstration is to show the applicability of the method using existing and accessible traffic data. The demonstration also acts as an indicative validation of the methodology.

This is achieved by comparison with two simple measures for both robustness and resilience, namely the *time to recovery* and the *total delay time*. The time to recovery, *TR*, per road section is defined as:

$$TR = \frac{\sum_n T_n^{recover}}{N} for\ N \geq 10 \tag{7.13}$$

Here, $T_n^{recover}$ is the recovery time of a single congestion event, n, defined as the time from the start of congestion to the end of congestion. N is the number of congestion events per road section, while a minimum number of 10 congestion events is required to give an estimate.

The total delay time, TD, per road section, is defined as:

$$TD = \sum_{t=0}^{t=e} \frac{veh(t)}{v_{free} - v_{obs}(t)} \tag{7.14}$$

Here, $veh(t)$ is the number of vehicles on a road section in a time interval, v_{free} is the free-flow speed, which corresponds to the maximum speed limit, and v_{obs} is the observed average speed of all vehicles during the time period. In total there are a number of time periods.

Figure 7.2: Considered network of the A13 and A20 motorways

The considered network exists of two interconnecting motorway stretches to the north of the city of Rotterdam in The Netherlands (see Figure 7.2). The motorways are the A13 and the A20 motorways and vary in width between two to four lanes and include several junctions and interchanges. The network is regularly congested in the peak periods with known bottlenecks at multiple locations. The total distance of the roads is approximately 55 kilometres long.

The data used for the considered network is taken from an extensive collection of induction loops at a distance of approximately 300-500 metres. The induction loops relay one minute aggregated data on the traffic flow and the speed of traffic.

7.5.2 LPIR calculation

The Link Performance Indicator for Resilience (LPIR) is calculated for the network shown in Figure 7.2. This is performed using an aggregation time interval of 15 minutes, as argued in section 7.4.5. Data for the entire year of 2009 is used in the experiment. Road sections are defined as the section of road between two correct working loop detectors. In this test case the jam density of traffic is assumed as 130 vehicles per kilometre per lane. Incidents are not explicitly considered, meaning that the incident reduction term is unused and has a value of 1. Capacity values are pragmatically estimated from data by taking the 99.9th percentile value for each road section. At bottleneck locations, this will resemble the real capacity, while at non-bottleneck locations the value will be less important as traffic flow will either remain uncongested (captured by the traffic speed) or will be influenced by an external bottleneck with a lower capacity value.

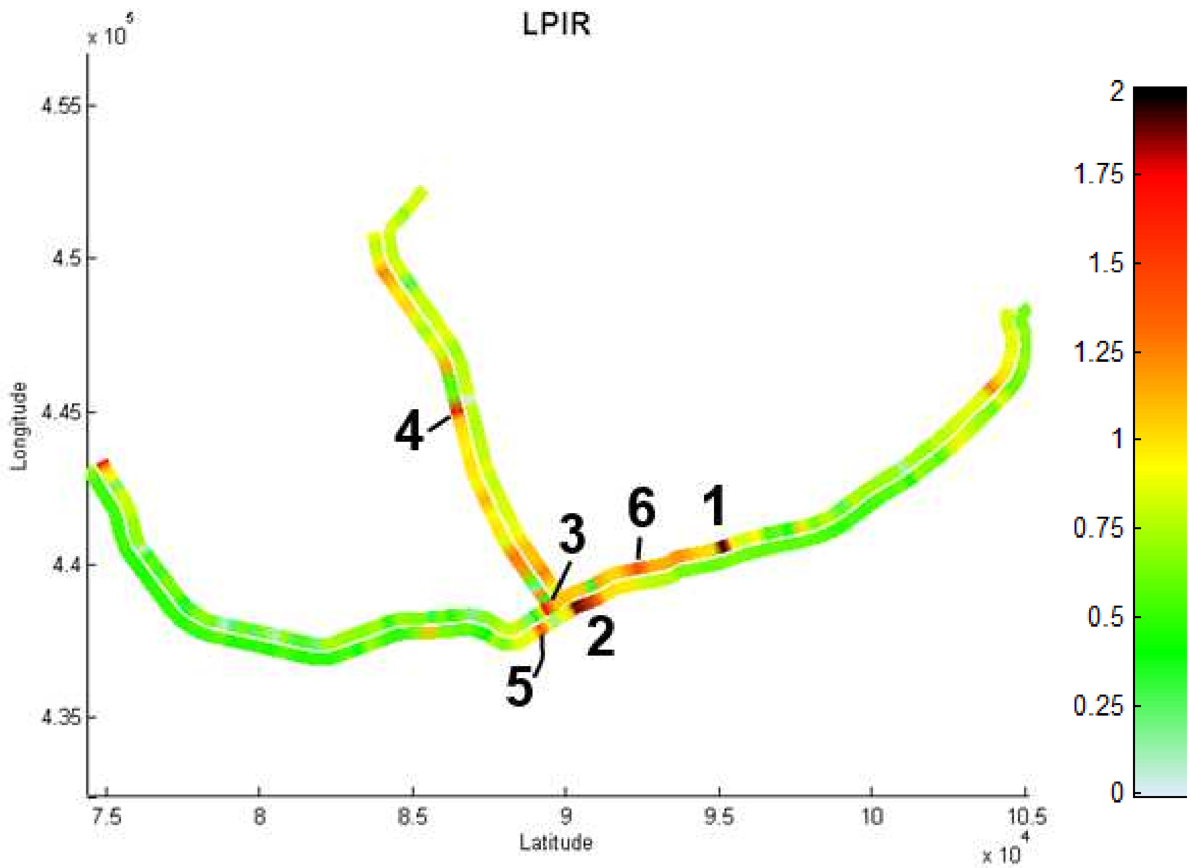


Figure 7.3: LPIR score per road section

The primary LPIR results of the experiment are shown in Figure 7.3 on the considered network. Values are shown to generally vary between 0-1.4, with one section in particular reaching a LPIR value of 2.0. Road sections with higher values are sections that should be viewed in more detail and are the sections that should be most readily considered for improvement to improve the traffic throughout and in turn the network performance, even if the network performance is not directly calculated. In Figure 7.3, road sections that appear with a red colour or darker are the least resilient. These are road sections that have a LPIR score equal to or above 1.2, with orange indicating values around 1.0, and yellow and green indicating values below 1.0, which are deemed to be road sections that have a lesser priority in comparison to the higher scoring road sections.

Using the results from the LPIR analysis, a priority list can be constructed which indicates which road sections should be addressed with which urgency by road authorities. This list is given in Table 7.6, with the numbered sections shown in Figure 7.3. A manual check based on expert judgement is performed to give an indication of the possible reasons of each section belonging to the list and the causality of the low resilience score. Causality can be added to the analysis by making use of the traffic characteristics and road characteristics terms from equation (7.11). This would exist of adding data from further relevant variables, such as data on the road surface, infrastructure geometry, traffic composition, and many more. This more detailed analysis is not performed here, therefore causality is left to expert judgement.

Table 7.6: Least resilient road sections from the A13-A20 analysis

Section nr (see Fig. 3)	LPIR value	Location description	Section type	Estimation of problem (expert judgement)
1	2.0	A20L Terbregseplein		Joining flows after interchange
2	1.9	A20R Centrum	Section with onramp	Narrow lanes, gradient and inflowing traffic on short onramp
3	1.7	A20L Kleinpolderpolein	Weaving section	Weaving section
4	1.6	A13R Delft-Zuid	Onramp	Joining flow with a bend in the road
5	1.4	A20R Kleinpolderplein	Weaving section	Weaving traffic at interchange split
6	1.4	A20L Centrum	Off-ramp	Short uphill off-ramp

A deeper analysis of the results is shown in Figure 7.4 for the A20 motorway in the westbound direction. The figure shows the traffic speeds during an arbitrary work week along with the LPIR scores.

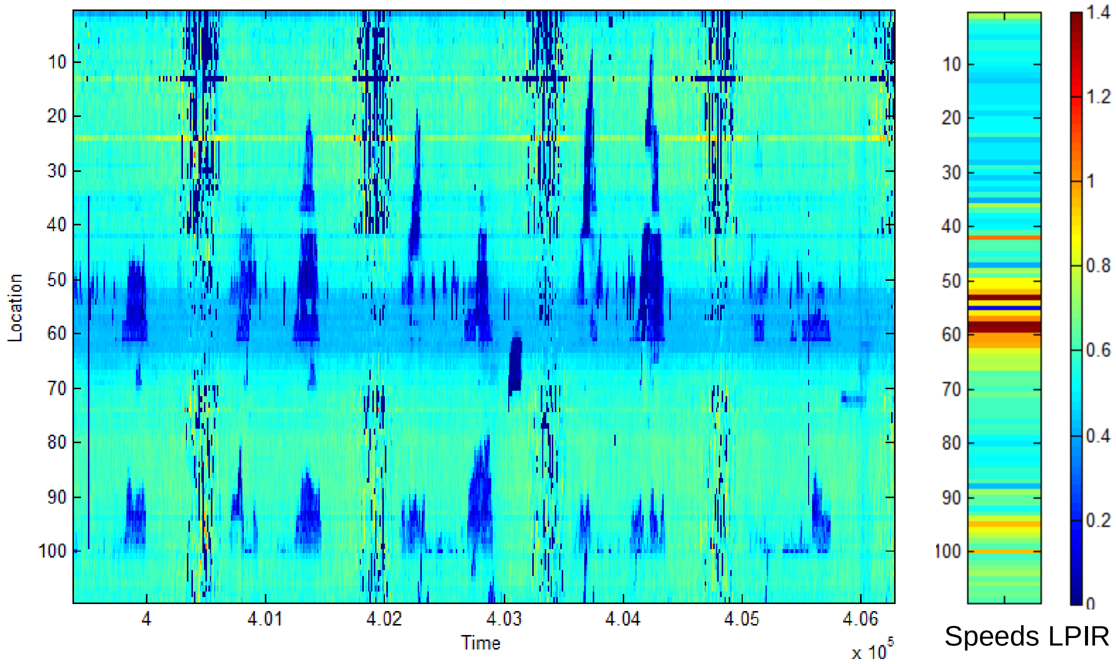


Figure 7.4: Comparison between speeds (left) and LPIR values (right) on the A20R

From Figure 7.4, it quickly becomes apparent that the LPIR score does not simply replicate traffic speeds, but rather focusses on the main areas in which congestion occurs. Moreover, the method also aims to gives an indication of the ability of a road section to recover from disturbances. Road sections which suffer congestion, and are especially the cause of congestion, and cannot readily recover receive higher index scores, representing this. This can be derived at a number of places from Figure 7.4. The congestion in the middle of the road (around section nr 60) is more severe and lasts longer and even leads to secondary congestion upstream. In comparison, the congestion observed at the bottom of the figure (near section 100) occurs regularly during a week, but is less severe and has a tendency in a number of cases to lead to limited spillback and to dissolve faster. This is represented in the LPIR score, which is close to 1.0, therefore indicating a road section that may need attention, but has a limited negative effect.

7.5.3 Comparison with other measures

In many disciplines, the resilience of a system is measured by the required recovery time. The recovery time is then a measure for the recovery. Although recovery is only seen as part of the resilience definition here, a comparison with the LPIR score can be insightful. In Figure 7.5, the average recovery times are shown for a road section to exit congestion. It is expected that a number of locations that have a long recovery time are part of the higher LPIR locations. However, there are also a few that do not score high on the LPIR. One such example is that at the coordinates [9.8; 4.42]. Alternatively, some locations with relatively low recovery times, are shown to have relatively high LPIR scores, even if they are not among the highest LPIR scores. These effects are down to the combined effect of both recovery and resistance in the LPIR. If one of these aspects is low, then the overall LPIR will also be relatively low. This shows that the LPIR is a typical impact index. Despite some difference, the majority of the least resilient road sections are also amongst the road sections with the highest recovery times. The ability of the index to react to both weakly resistant and road section with weak recovery is a particular strength of the index and an important contribution.

Another measure used to compare the LPIR results is the network delay, for which the results are shown in Figure 7.6 per kilometre distance. The total network delay is a measure that can be used to indicate robustness and therefore mainly reflects the resistance part of traffic flow. The total network delay includes a further element compared to the LPIR and the recovery time, which is the total flow. This acts as a sort of weight for negative effects of congestion and indicates also a combined effect of the number of vehicles affected and the length of a delay. However, the indicator focusses on the effect of traffic breakdown and not on the causality, which is a more important part of the LPIR. To that extent, the locations shown are slightly different to the LPIR. The presence of congestion in the LPIR does not necessarily lead to the highest LPIR score. And although the network delay does indicate where most delays are recorded, it fails to pinpoint the main weaknesses in the network.

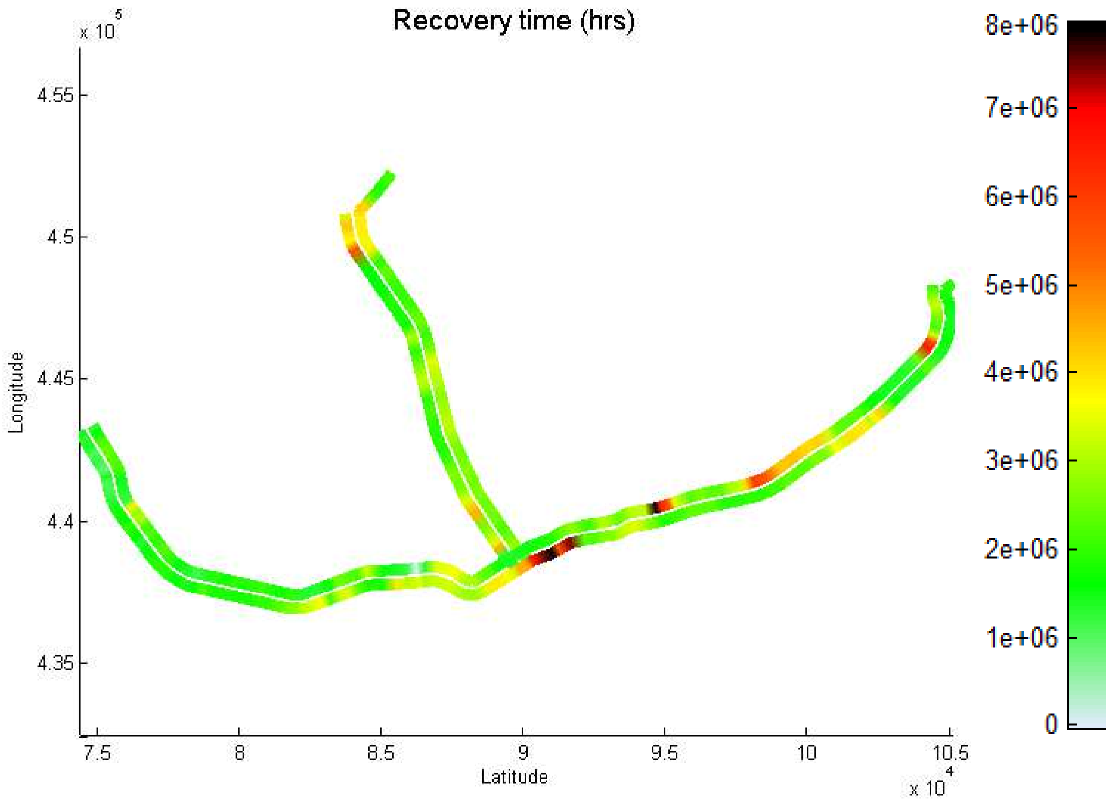


Figure 7.5: Average yearly recovery times per road section [hrs]

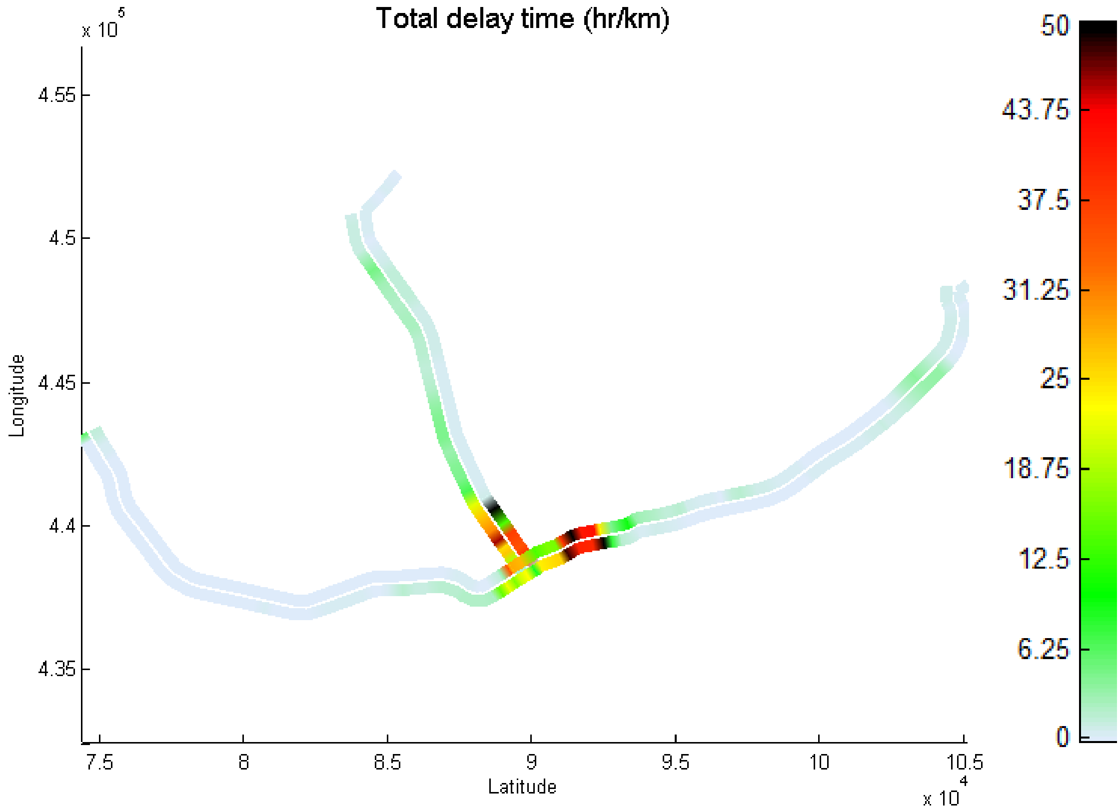


Figure 7.6: Total network delay in 2009 per road section [hours/km]

7.6 Conclusions and discussion

In this chapter, the Link Performance Index for Resilience (LPIR) is presented as a new methodology to evaluate the resilience level of road sections in relation to the surrounding network, where resilience was defined as *"the ability of a system to cope with disturbances and recover after a loss of function"*. The methodology offers a powerful tool that allows road authorities and alike to perform analyses of their road network and identify the weak links, which may demand the higher priority when considering investment. The focus of the methodology is on resilience and is therefore wider than robustness, as it also considers the ability of road sections to recover from disturbances as well as the classical robustness itself. To this extent, a distinction is made between a resistance part and a recovery part as part of the entire methodology. Contrary to many other works, the basis for the methodology does not focus on the network as a whole or as a generic description of the network and its parts against a certain measure. Rather, the resilience is calculated in relation to the traffic flow characteristics at a flow level and the ability of road sections to maintain their predefined purpose to serve vehicles without overly experiencing congestion. The focus on homogenous and volatile traffic flows also leads many of the considered components to relate closely to traffic flow characteristics.

Prior to the explanation of the method, an extensive literature review was performed to set the scene for the LPIR, but also to indicate where most efforts have been performed in the past. This showed that much has been done and is being done in reliability and vulnerability and increasingly in robustness analysis. Resilience is found in many transportation related disciplines, such as transport networks, freight movements and logistics, but it not explicitly commonplace in traffic flow analysis. This is where the niche and the main contribution of the LPIR method lies. The relevance of resilience analysis in traffic flow stems from the importance of road section not only to resist degradation of function, but to also recover promptly as a consequence of traffic flow stability.

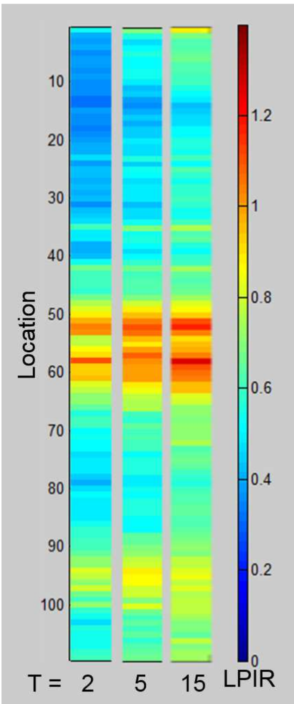
The effectiveness and validity of the methodology is demonstrated in an experimental case for a small network of two interconnecting motorways to the north of the city of Rotterdam in The Netherlands. This showed that the LPIR is able to detect weak and poorly resilient locations by calculating the relative resilient value of individual road sections. For the road sections with the highest LPIR value a manual causality is given as a further demonstration of how a road authority may be able to use the results to determine poorly resilient road sections. The calculated LPIR values are further compared with the results of two other measures for resilience and robustness, namely the ‘recovery time’ and ‘total delay’. Many locations that performed poorly in the LPIR were also highlighted in the other measures, however there were also important differences that further showed the strength of focussing on resilience. The recovery time merely shows locations that can quickly recover from a congestion event after a disturbance, while the occurrence rate is not considered and therefore says little about the overall impact during a longer period. On the other hand, the total delay experienced on a road section does give an overall indication of the negative effect of congestion on a road section. In comparison to the LPIR, this lacks as it does not sufficiently take into consideration where bottlenecks are present and therefore the road sections which are the cause of congestion. Congestion on a road section for a bottleneck further downstream is unfairly penalised due to the weakness of another road section. Although one may argue that this is also a part of resilience, it does not accurately contribute to the purpose of identification of the main problem areas for disturbed traffic flow and recovery from congestion and therefore a lack of resilience. We therefore argue that the analysis of the resilience offers a deeper insight into the way road sections are judged for weakness and that resilience analysis offers a complementary tool to robustness. This is especially the case when the analysis concentrates on the influence of disturbances on traffic flow at the level of traffic rather than at a higher abstraction level.

The LPIR methodology also allows for a deeper analysis of the casualty of a poorly resilient road section. This is performed through additional data analysis. This part of the LPIR was not further elaborated on in this chapter and was also not part of the experimental case. The consideration of incidents was also not part of the considered case. Both of these elements are given as recommendations for further research. Especially the analysis of resilience causality is an interesting area that can be a strong addition to the presented method, as it does not only return road sections that require attention, but also gives a strong indication of the reasons behind the lack of resilience allowing a road authority to act more precisely.

Appendix 7.A: Sensitivity of the time interval parameter *T*

The parameter for the time interval, *T*, is relevant for the considered period in which the volatility and extreme values of the flow are measured. There is however no required value, therefore an analysis of appropriate values is carried out to test the influence of different values. This is also combined with a test of the necessity of the delta flow variable, which indicates the difference between the incoming and outgoing flows on a congested road section.

An upper bound is set of 15 minutes for *T*, as a higher value would lead to a less representative observation of the traffic states. It might even be suggested that 15 minutes is already too high, however such a value is not an uncommon aggregation level in traffic flow theory and modelling. Figure 7.7 shows LPIR values for the A20R (westbound) for three *T*-values: 2, 5, and 15 minutes. From the figure, each result shows that the locations of higher values correspond between *T*-values, which is not surprising. Higher *T*-values show a higher LPIR score. This is also not surprising as longer time intervals allow a larger range of flows to be observed, which in turn will lead to a higher LPIR. Further analysis shows that the scores between the three tested values are relatively similar. Therefore, for relative comparison there is little difference. As the LPIR is applied as a relative index between road sections, we conclude that there is not a strong preference for the choice of *T* value based on its own sensitivity alone



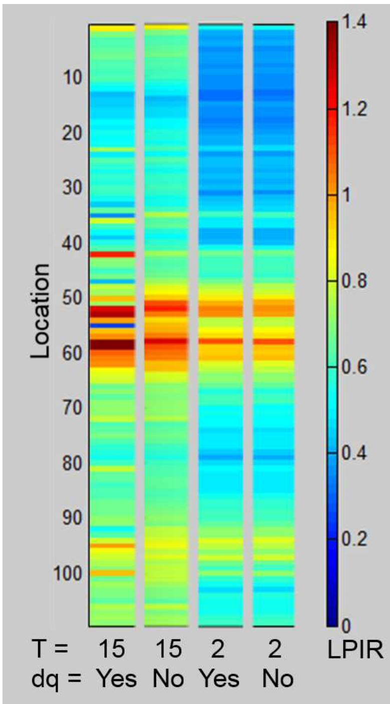


Figure 7.7: (left) Comparison between time interval values, from left to right: 5 mins, 15 mins, 2 mins

Figure 7.8: (right) Comparison between time interval values and the application of delta flow, from left to right: *T*=15 mins with Δq, 15 mins without Δq, 2 mins with Δq, and 2mins without Δq

In Figure 7.8, the effect of the delta flow variable is considered together with different *T*-values: 2 and 15 minutes. This comparison shows that the value of *T* does matter for the results of LPIR when delta flow is included. This can be seen in the difference between the first and second result, with or without the use of the delta flow term. However, when $T = 2$, there is no difference between the LPIR scores with or without delta flow, which can be seen from the third and fourth column in Figure 7.8. This makes sense as there are only two observations for $T = 2$, and therefore the maximum and minimum value will always be one of those values.

The results of this analysis show that the main differences are absolute shifts, rather than relative shifts in the scores. Nevertheless the use of T =15 while retaining the delta flow term gives more pronounced results, as the absolute values are higher. A more pronounced result makes it easier to distinguish between roads sections and therefore a preference is made to use a $T = 15$, with the delta flow term. This also gives more observations to make an estimate of the volatility, which is limited by a smaller *T*-value. While stating this, we recognise that the use of a lower *T*-value would not necessarily be an incorrect approach.

Chapter 8

Comprehensive case study

This chapter gives a demonstration of the applicability and usefulness of the developed methodologies in a comprehensive case study. In the case study, the application of traffic management to improve traffic flow is analysed. Section 8.1 describes the applied framework and its steps. The considered network is presented in section 8.2. Execution of the framework steps are performed from section 8.3, concluding with the assessment of the results of the case in sections 8.6 and 8.7.

This chapter is an edited version of the article:
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8.1 Framework

To integrally demonstrate the developed tools described in this thesis, a comprehensive case study is performed in which the complete chain of tools, to analyse and apply traffic management in an uncertain traffic system, is shown. Additionally, a second goal is defined to demonstrate the necessity of considering traffic as stochastic for traffic management. The case is carried out on a network with the aim to derive weakly resilient locations, offer traffic management solutions for these locations and predict the positive effect that the traffic management measures are expected to have. These three steps are described in greater detail in this section. In section 8.7, we will also demonstrate the necessity of considering uncertainties and traffic flow fluctuations when estimating the future effects of specific traffic management measures.

Step 1: Network resilience scan

The first step involves a resilience scan of the considered network using the Link Performance Indicator for Resilience (LPIR). The LPIR was previously described in Chapter 8, where more details are given on the indicator.

As resilience is defined in traffic flow as the combination of both resistance and recovery (see section 8.4), both elements are combined in the Link Performance Indicator for Resilience (LPIR), given by:

$$LPIR = \frac{1}{T}\sum_{t=0}^{T}\left(\frac{\left[\frac{q+\psi^{q}}{v}\right]}{\left[\frac{q_{cap}(g,h)f+\psi^{cap}}{v_{crit}}\right]}1_{k\le k_{crit}} + \frac{\left[\frac{q+\Delta q}{v_{eq}(q)}\right]}{\left[\frac{q_{cap}(g,h)f-q_{cd}}{v_{crit}}\right]}1_{k>k_{crit}}\right) \tag{8.1}$$

Here, for each time interval t, q is the traffic flow, v the traffic speed, v_{crit} the critical speed just before traffic breakdown, k the traffic density, q_{cap} the estimated capacity, f a temporal reduction factor for the capacity (i.e. due to incidents) and q_{cd} the estimated capacity drop. ψ^{q} and ψ^{cap} are volatility variables that give an indication of the extent of homogeneity for the traffic flow and capacity respectively. g and h represent the road and traffic characteristics and influence the capacity. In Chapter 7, a more extensive explanation of the build-up of the equation is given.

Recall that each variable is valid for a set time interval *[t, t+dt*). For readability, the notation of the dependence on *t* has been omitted from the equation. The total LPIR score per road section is the average over all time intervals for the considered period. In this case, the considered period is a complete year of data for the A20 and A13 motorways in the year 2009, due to availability. The data is taken from an extensive collection of induction loops at a distance of approximately 300-500 metres. The induction loops relay one minute aggregated data on the traffic flow and the speed of traffic.

The LPIR gives an indication of the relative resilience of that road section compared to other road sections. A value of $LPIR \leq 1$ indicates that a road section is able to resist a significant drop in level-of-service and therefore remain uncongested and by definition must be considered resilient as well as robust, as seen in Chapter 7. However, a road section that does suffer a drop in level-of-service, but can recover promptly should also be considered resilient as resilience considers the ability to recover from a disturbance or loss of service. However in the latter case, the road section may not be considered robust, as a failure event occurred.

Step 2: Design of set of Traffic management measures

In the first decade of the 21st century a coherent framework was developed in The Netherlands for the deployment and decision making processes surrounding traffic management (In Dutch: 'Gebiedsgericht benutten'). The framework is applied as the basis on which the majority of integral traffic management decisions are taken (Rijkswaterstaat, 2003). In the framework, depicted in Figure 8.1, a distinction is made between services and measures for the application of traffic management. Services relate to the network wide objective that is being sought through traffic management for an identified problem. A service is in general a description of actions intended to achieve the desired effect for certain traffic flows, locations, or roads (e.g. limit the flow of incoming traffic, increase the capacity at the bottleneck). On the other hand measures relate to the physical application of an action that directly influences the traffic system. In general, measures are derived from services, where the measures are the actions that achieve the objectives set out in the services. The services categories are defined as: influencing throughput, redistribution of traffic flow, influencing demand, influencing capacity, and general network-services. In the past decade, a number of additional services and measures may be added to the list, such as personalised in-car travel information and cooperative ITS.

Although the traffic management framework gives a good overall indication of the majority of possible measures, new options have been developed since its finalisation which is very relevant to the considered case. One aspect that is considered is that of network wide integrated traffic management. Although there are many ways such an approach can be defined and implemented, we will focus here on the definitions and approach as described in (Hoogendoorn et al., 2015, Hoogendoorn et al., 2014, Landman et al., 2010). In this approach four main principles are applied:

- Spare capacity in the network is optimally utilised given the prevailing traffic conditions
- Capacity drop is prevented for as long a time as possible
- Traffic flows in the network should not be unnecessarily hindered (secondary congestion)
- A bottleneck should be resolved at the level at which it manifests.

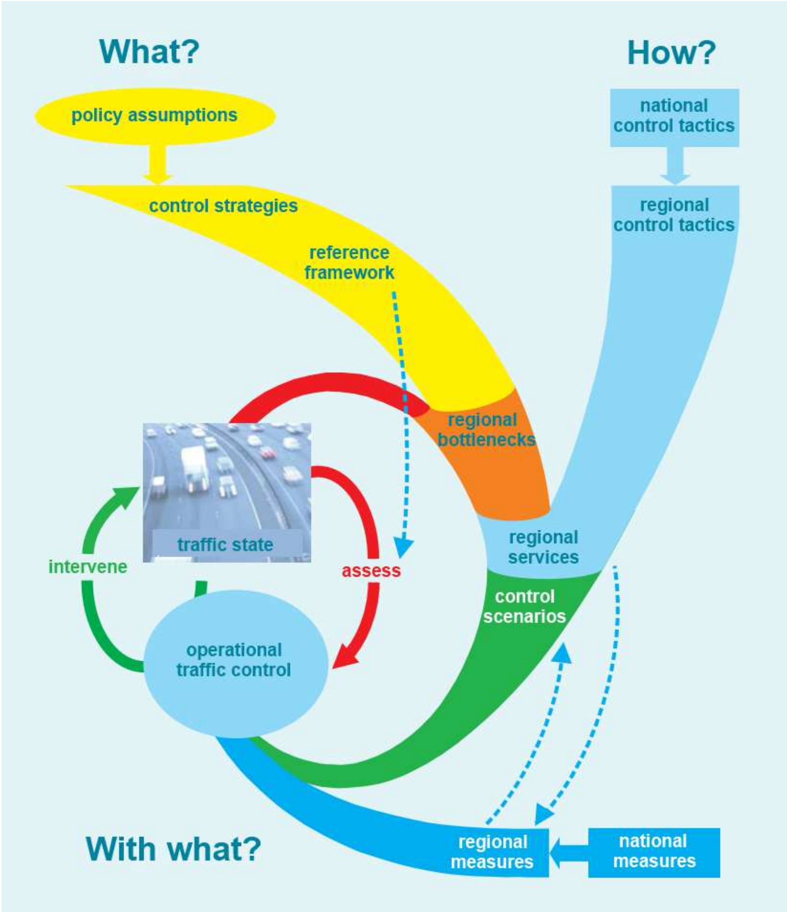


Figure 8.1: 'Gebiedsgericht benutten' (GGB) traffic management strategy framework (Rijkswaterstaat, 2003)

These principles are applied in practice on a network such that multiple bottlenecks can be tackled without a solution at one location leading to secondary problems at another and also allowing multiple correlated bottlenecks to be simultaneously addressed. In most instances, the measures that can be applied exist in the previously described framework; however the setup and application of the measures are controlled such that each one considers the setup of the other measures and such each measure does not work individually, but rather as part of an integrated system.

In this case, a service solution will be defined for each identified location from which one or more traffic management measures can be selected. The selected measures will then be analysed for their effectiveness as described in the following step.

Step 3: Evaluation of measures

Forecasting of the effect of traffic management measures for each carriageway is carried out using simulation models in which the stochastic character of traffic flow is considered on different levels. Two models are applied for this purpose: INDY-MonteCarlo and FOMSA. INDY-MonteCarlo is a dynamic macroscopic traffic model based on the LTM and enriched with advanced Monte Carlo sampling algorithms for uncertainty modelling, as described in

Chapter 4. INDY-MonteCarlo is suited for use with uncertainty analysis and network scenarios. The considered scenario and uncertainties are given in the following subsection.

The FOMSA model is a Lagrangian based dynamic semi-macroscopic model based on first order traffic flow theory with additional invariant terms to consider stochastic driver behaviour. The use of Lagrangian coordinates allows vehicles and vehicle-groups to be individually followed and be assigned specific characteristics. This model is described in Chapter 6 of this thesis. For specific locations and traffic management measures, a more detailed analysis of the traffic flow may be required. An analysis on the level of vehicles and vehicle interaction can give insight into the level of effectiveness of traffic management measures. This may be the case where there are multiple interacting traffic flows that cannot as easily be captured in a regular macroscopic model. In such a case, the FOMSA model is suited and can be applied using a single vehicle platoon (therefore microscopically) or on a platoon basis.

8.2 Case study network

The case study is performed for the A20 motorway, which forms the North Ring of Rotterdam motorway network. The network of greater Rotterdam is shown in Figure 8.2. The network covers the city of Rotterdam including surrounding cities and towns, such as Delft, Dordrecht and Zoetermeer and includes the major motorways and interconnecting roads down to a local level.

Figure 8.2: Road network for Greater Rotterdam with the considered A20 motorway highlighted

The objective of the study is to evaluate the traffic operations on and throughput of the A20 motorway on the north ring of Rotterdam (see Figure 8.2) and consider traffic management improvements to improve traffic flow conditions on that corridor and the surrounding network. The A20 on the North Ring of Rotterdam has a number of bottleneck locations with spillback often reaching other bottleneck locations. There are a lot of intertwining traffic flows, both local and national. The congestion problems on the road have been a major concern for a while and continue to form a challenge, especially as there is very little space to expand the infrastructure to increase capacity. Therefore traffic management potentially has an important role to play.

8.3 Network scan for weakness

The first step of the approach entails scanning the network for weak elements. To this end, the Link Performance Indicator for Resilience (LPIR) is calculated for the network shown in Figure 8.2. This is performed using an aggregation time interval of 15 minutes. Data for the entire year of 2009 is used in the experiment. Road sections are defined as the section of road between two correct working loop detectors. In this case the critical density of traffic is assumed as 25 vehicles per kilometre per lane. Incidents are not explicitly considered, meaning that the incident reduction term is unused and has a value of 1. Upper bounds for the traffic flow are pragmatically estimated from data by taking the 99.9th percentile value of the flows for each road section. At bottleneck locations this will resemble the real capacity minus outliers, while at non-bottleneck locations the value will be less important as traffic flow will either remain uncongested (captured by the traffic speed) or will be influenced by an external bottleneck with a lower capacity value.

The LPIR results of the experiment are shown in Figure 8.3 on the considered network. Values generally vary between 0.0-1.4, with one section in particular reaching a LPIR value of 2.0. Road sections with higher values are sections that should be viewed in more detail and are the sections that should be most readily considered for improvement with traffic management to improve the traffic throughout and in turn the network performance. In Figure 8.3, road sections that appear with a red colour or darker are the least resilient. These are road sections that have a LPIR score equal to or above 1.2, with orange indicating values around 1.0, and yellow and green indicating values below 1.0, which are deemed to be road sections that have a lesser priority in comparison to the higher scoring road sections.

Using the results from the LPIR analysis a priority list can be drawn up that indicates which road sections should be addressed with priority by road authorities. This list is given in Table 8.1, with the numbered sections shown in Figure 8.3. A plausibility check based on expert judgement is performed to give an indication of the possible reasons of each section belonging to the list and the causality of the low resilience score. Causality can be added to the analysis by making use of the traffic characteristics and road characteristics terms from equation (8.1). Data is added from other relevant variables, such as data on the road surface, infrastructure geometry, traffic composition, and many more. This more detailed analysis is not performed in this contribution, therefore causality is left to expert judgement.

Table 8.1: Locations with the highest LPIR values

Section nr (see Fig. 8.3)	LPIR value	Location description	Section type	Initial estimation of problem
1	2.0	A20L Terbregseplein		Joining flows after interchange and lane drop
2	1.9	A20R Centrum	Section with onramp	Narrow lanes, gradient and inflowing traffic on short onramp
3	1.7	A20L Kleinpolderplein	Weaving section	Weaving section
4	1.4	A20R Kleinpolderplein	Weaving section	Weaving traffic at interchange split
5	1.4	A20L Crooswijk	Weaving section	Weaving section

Figure 8.3: Network and results of the LPIR analysis

8.4 Design of traffic management solutions

In the second step of the proposed framework, traffic management solutions are constructed for selected locations. The quick-scan resilience analysis of the network returned a number of locations that are found to be the least resilient. These locations have been prioritised as shown in Table 8.1. An initial estimation of the reasons behind the lack of resilience is also given for each location. Using this analysis, a selection of feasible traffic management measures can be drawn up to tackle the problem locations.

Two sub-cases are considered to allow both modelling techniques to be demonstrated. On the westbound carriageway (A20L) the FOMSA model is applied, as this corridor shows multiple interacting bottlenecks, which can be suitably analysed by this model. On this stretch, it is ill advised to consider a single location as the occurrence of multiple bottlenecks do not stand alone, rather a coordinated traffic management approach is required. On the eastbound carriageway (A20R) advanced sampling Monte Carlo is applied using the INDY-MonteCarlo model. The second sub-case considers location 2 from Figure 8.3 at which will be referred to as location A20R31. The results from the LPIR analysis into the resilience of the motorway are considered to a focus on especially weak areas on the carriageways.

8.4.1 Sub-case 1: Westbound carriageway of the A20 Ring Rotterdam

The westbound carriageway of the A20 Ring Rotterdam has a long standing problem during peak periods due to multiple bottleneck locations. There is very limited space available for the realisation of extra capacity and many traffic management measures thus far have not eradicated the expansive congestion problems.

Location: A20L from Terbregseplein interchange to Kleinpolderplein interchange

Problem: Multiple bottlenecks in succession:

- Merging flows after an interchange (Terbregseplein interchange)
- Inflow of HGV onto main highway together with bend on road
- Busy on-ramp (Crooswijk)
- Two weaving sections in quick succession (Crooswijk-Rotterdam Centrum-A13).

Solution*: Coordinated traffic management with multiple solutions.

- Facilitate merging
- Maximize bottleneck capacity
- Limit traffic flow

Possible measures:

- (Dynamic) Lane drop prior to merge
- Lane drop on the outside lane instead of inside lane
- Prevent left lane changes in merge area
- Lane choice advice
- Ramp-metering

Solution approach scenario 1:

- Dynamic lane drop prior to merge location. (Terbregseplein interchange)

The first scenario considers the reduction of traffic flow into the motorway corridor. Due to congestion at the merging section at Terbregseplein, vehicle interaction can lead to congestion for both inflowing traffic flows (see Figure 8.4). This combined with the lane drop leads to an increased reduction of the local capacity. By moving the lane drop upstream before the merge, removes the necessity to merge over all lanes and means that any congestion resulting from the lane drop only affects one of the incoming flows. However, applying the lane drop upstream will lead to a reduced utilisation of capacity, as the flow with the lane drop can no longer make use of any spare capacity on the two lanes from the other inflowing carriageway. It is unclear to what extent this plays a major role. This measure is focussed on reducing the inflow of vehicles into the problem area and thus reducing the chance of secondary congestion on the Rotterdam Ring.

Figure 8.4: Terbregseplein motorway merge considered in scenario 1 of sub-case 1

Solution approach scenario2:

- Construction of the considered A16-A13 bypass extension

While creating extra capacity on the A20 is not possible, for a number of decades there have been plans to build a bypass extension to the A16 motorway, which would reduce the size of the traffic flow on the A20 (see Figure 8.5). This is not really a traffic management option, but is considered as it has long been seen as a viable and attractive option. However, it should be noted that it comes at a far greater monetary expense. The construction of the bypass diverts traffic from the A16 and A20 that have their destination north of Rotterdam away from the city ring road and therefore reduces the pressure on the North Ring (A20).

Figure 8.5: Planned A16/A13 bypass considered in scenario 2 of sub-case 1 (Rijkswaterstaat, 2015)

Solution approach scenario 3:

- Ramp-metering (Crooswijk on-ramp)

The third considered scenario involves focussing on the most significant downstream bottleneck location on the carriageway. As congestion moves in an upstream direction, the most downstream bottlenecks are of most significance as spillback will influence the greatest area. The on-ramp and weaving section at Crooswijk (location 5 in Figure 8.3) is one of the most downstream bottlenecks on the corridor. The inflow of traffic at this onramp is high during peak periods and has a disruptive effect on the main carriageway. Therefore ramp-metering is applied on the on-ramp to reduce the inflow and level of disruption on the main carriageway and therefore lead to a lower level of congestion and upstream spillback into other bottleneck locations.

This scenario also requires network traffic management for the secondary roads that connect to the on-ramps. This additional secondary buffering is required as there is limited space available on the on-ramp for buffering and additional spillback onto the urban roads is undesired. The secondary buffering limits traffic throughput to the onramp on urban roads and therefore prevents the onset of additional congestion at the start of the on-ramp. This methodology has been previously described by (Hoogendoorn et al., 2015, Hoogendoorn et al., 2014). However the secondary network traffic management is not modelled in the case and is presumed possible.

8.4.2 Sub-case 2: Eastbound carriageway of the A20 Ring Rotterdam

Similarly to the westbound carriageway, the eastbound carriageway also has extensive congestion problems with few options for capacity expansion. However, there is one clear bottleneck location at which the majority of congestion occurs. This allows a more focussed approach to the problem. Another difficulty that is not tackled here, but is of relevance, is regular spillback from connecting motorways. However, we will focus on congestion occurring from the A20 itself in this sub-case.

<u>**Location: A20R Centrum (A20R31)**</u>

Problem: Narrow lanes, gradient and inflowing traffic on short onramp

- Busy onramp with short merge distance onto a carriageway on a gradient with narrow lanes.

Solution*: Restrict flow / Buffer traffic

Possible measures:

- NB: Keep your lane is already in operation!
- Ramp-metering
- Traffic buffering at subsequent upstream intersection on secondary network

Solution approach:

- Ramp-metering (Redesigned with coordinated traffic controls for secondary roads)

A ramp-metering installation is already present at the onramp, but not in use, partially due to the spillback onto the secondary road network and partially due to the limited effectiveness. The proposed measure will make use of the ramp-metering installation with an increased buffer-area. As the buffer area will still be insufficient and it is infeasible to allow traffic to buffer on the upstream roundabout, coordinated traffic control is proposed from traffic onto the roundabout for the directions heading to the onramp (Hoogendoorn et al., 2015, Hoogendoorn et al., 2014). The exact control setup will not be considered in the case, however the effect on the roundabout will. The effect of ramp-metering should delay the on-set of congestion, which has a positive effect through a reduction of the duration of the capacity drop and the reduction in secondary effects from the spillback from congestion on the motorway network. The effect of a reduced capacity-drop duration is estimated at 2% during the entire peak period on the upstream bottleneck link (Zhang and Levinson, 2003).

Figure 8.6: Rotterdam Centrum onramp considered in sub-case 2

8.5 Model set-up and scenarios

In this step, we will discuss the (experimental) set-up of the two models that will be used for the respective subcases. Also, the applied distributions and scenario application is discussed in this section. The correct choice and set-up of appropriate modelling tools is essential for the correct assessment of the measures that have been put forward in section 8.4. For subcase 1, we have opted to use FOMSA to model the interaction between bottlenecks, as the model considers microscopic fluctuations in traffic and therefore allows interactions between bottlenecks to be visible as trajectories are followed. For subcase 2, we elect to make use of INDY-MonteCarlo because of its ability to consider scenario based uncertainties that are present on the A20R at the considered bottleneck location.

8.5.1 FOMSA model setup (sub-case 1)

The first sub-case considers the westbound A20L carriageway over a distance of 11.6 km from the onramp at Capelle to the Giessenbrug bridge. The Lagrangian model is setup with the correct number of lanes for each road section, including the presence of peak hour lanes. Onramps and off-ramps are not considered in the number of lanes, unless they are weaving sections, as vehicles will ‘appear’ or ‘disappear’ from the carriageway at these sections in the model. At on-ramp locations vehicles are forcibility added to the road and the surrounding vehicles on the carriageway have the opportunity to adjust their speed and headway to accommodate the new vehicle.

The basic setup of the traffic demand is derived from traffic data collected from the motorway at the relevant in- and outflow locations. During calibration of the model these values were adjusted in a conservative manner to create an accurate congestion pattern for the morning peak period, which is the dominant peak period. At locations on the carriageway where further capacity reductions are present, an additional capacity reduction is applied, which directly influences flow through the fundamental diagram. An example of this is at a location prior to the Crooswijk onramp where there is a sharp bend in the carriageway together with a gradient.

The applied fundamental diagram has a nominal jam density of 140 veh/km and a critical density of 25 veh/km. The maximum speed limit is 100 km/hr, the critical speed is set at 85 km/hr and the minimal spacing at standstill is 7.5 m. A time-step of 0.5s is applied to comply with the number of lanes and traffic density and the vehicle group size is 2 vehicles per group. A forced capacity drop value is applied of 10% for congestion, while the advection invariant is set at a value of 0.2 and the maximum acceleration bound is 1.0 m/s^2.

The simulation is carried out for a time period of 60 minutes, in which the traffic flow is gradually increased up to the desired level and maintained for 15 minutes after which it is reduced to a lower level to allow congestion to dissipate. This is sufficiently long to demonstrate the build-up and dissipation of congestion. A short increase and decrease is chosen as a controlled way to demonstrate the effects of congestion and the performance of the motorway stretch. Use of real demand profile data proved complicated and overly time-consuming for the sake of the required demonstration and was not chosen. As different random sampling of the vehicle characteristics can lead to different results, a single identical sample is taken which is applied identically to each scenario for the sake of comparison.

8.5.2 INDY-MonteCarlo model setup (sub-case 2)

The applied network for sub-case 2 is shown in Figure 8.2. The network consists of 8200 links and 285 zones and is calibrated for an afternoon peak period between 2-8 PM. The network is derived from the Dutch national model and therefore has accredited speed and capacity values. The assignment model is INDY, which is a dynamic macroscopic model, which makes use of the Link Transmission Model (Yperman, 2007). The network is calibrated for the evening peak period, which is the dominant period for the carriageway. The model is run with time steps of 10 seconds.

The applied Monte Carlo routine makes use of Sobol numbers on the two input variables: demand and capacity, to construct a well distributed set of samples. Sobol quasi-random numbers were previously shown to give a good distribution of samples and 20 samples proved also sufficient to obtain and good distribution. Identical sample values are applied for both the reference and traffic management scenario for sake of comparison. The sampled distributions are shown later in this sub-section.

8.5.3 Scenarios and boundary conditions

Scenarios and stochastic fluctuations in traffic flows are considered in the analysis. Scenarios are defined as uncertainties on a day-to-day level or even on a greater time horizon, such as over multiple years. Scenarios reflect the possibility of a set of conditions being present for a longer period of time during a day, such as weather conditions, the present of an incident or road works, the day of the week, the presence of a major event and so on. Fluctuations are defined as inherent stochastic changes dynamically during a relatively short time period. Such fluctuations are often difficult to exactly predict in advance and are often the consequence of local conditions combined with external influences from the current scenario or scenarios.

The main uncertainties can be reduced to variations of the traffic demand (on a day-to-day basis and for scenarios) and variations in capacities as demonstrated in Chapter 3 of this thesis. Figure 8.7 gives an overview of how capacity and demand variations are influenced by scenarios and fluctuations in the traffic system. When considering day-to-day uncertainties, external temporal conditions play an important role, such weather effects, day of the week, etc. For stochastic fluctuations between vehicles, behavioural aspects are far more important, such as time-headways and level of aggressiveness. Traffic demand and infrastructure characteristics have a substantial effect on both uncertainty and fluctuations.

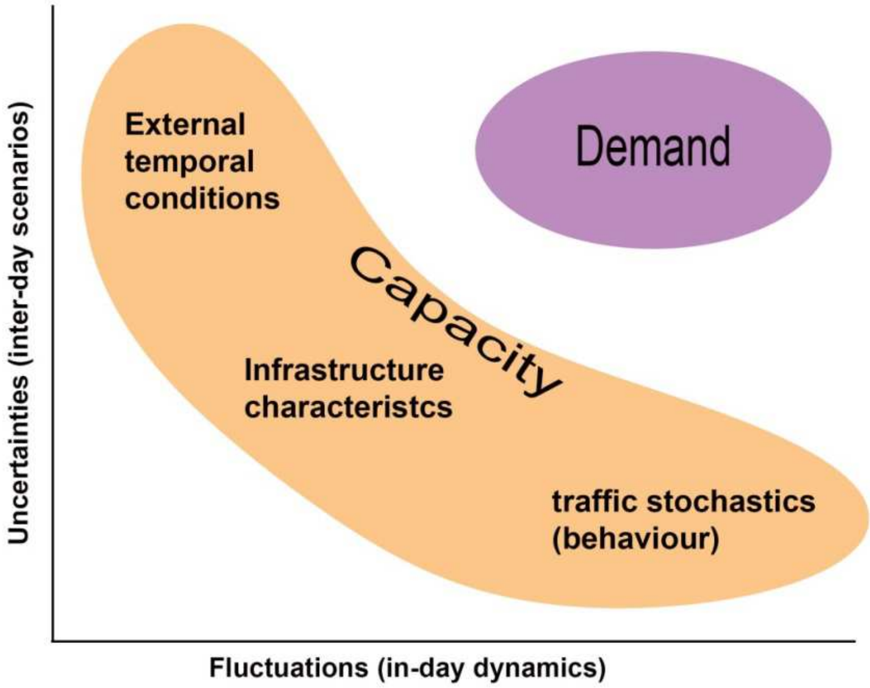


Figure 8.7: Relationship between uncertainty and fluctuations in traffic demand and capacity

In sub-case 2, a choice is made in relation to the scenarios to be considered for each location. The scenarios determine the demand profile for traffic and the base capacity levels for the network. For example, if a scenario is considered for a weekend day in wet weather, the traffic demand distribution will represent a set of feasible demand for a weekend day and the road capacity will represent a distribution of empirically obtained capacity values in wet weather. Dynamic in-day fluctuations of the traffic demand and actual capacity fluctuations are applied to the demand profiles and capacity values for sub-case 1. Doing this completes the distributions to be applied in the model analysis.

The goal of this case study is to evaluate the effect of traffic management on the A20, primarily during regular peak periods and demonstrate the applied models. For this reason, the scenarios and the applied distributions are taken from non-holiday days. The demand distributions give an indication of the level of demand and the spread of the demand. A relative demand distribution is derived for each day of the week separately. These are relative distributions as the model already harbours absolute values which have been pre-calibrated for the applied network. The level of demand is derived through the selection and analysis of a set of five locations spread out across the network at a major motorway on which no or little congestion is present. The presence of congestion prohibits an accurate demand estimation, as capacity is exceeded and therefore the measured levels do not resemble the true demand. The selected locations at which the demand is measured are given in Figure 8.8.The following assumptions are made for the data-processing to construct the distribution:

- Only week days are considered, outside of the holiday periods
- Capacity and demand data is captured for the month September through November 2014 as this is a coherent and continuous period with only a single holiday week.
- Both carriageways are analysed separately.
- Capacity variation is only applied locally to the considered bottleneck location which is being analysed.
- Global capacity variations are not applied.

Demand distributions are applied to all Origin-Destination pairings equally as a generic indication of business.

Figure 8.8: Locations used to determine the demand distribution

The distributions for the demand are shown in Figure 8.9 and have been smoothed to 15-minute intervals for effective use in the model analysis.

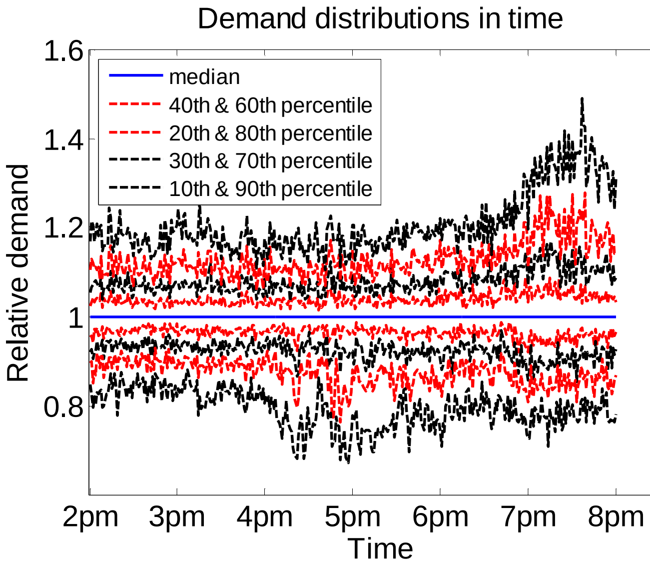


Figure 8.9 Traffic demand distributions

Capacity scenarios and distributions

Two types of capacity distributions can be applied: local capacity distributions that affect a single link, or global distributions that influence the entire network. Global capacity variation may be used for situations such as weather conditions that affect a whole (sub)network in a similar way. In this case, only local capacity variation is applied and in particular for the bottleneck locations that are specifically considered. The capacity distributions are derived using an adapted Product Limit Method, described in Calvert et al (2015). The method is not expanded on here, as it is already explained in Chapter 3. As driving behaviour is a major factor that influences capacity, a distinction is also made for the day of the week for the two bottleneck locations. The distributions for the local capacity variation are shown in Figure 8.10.

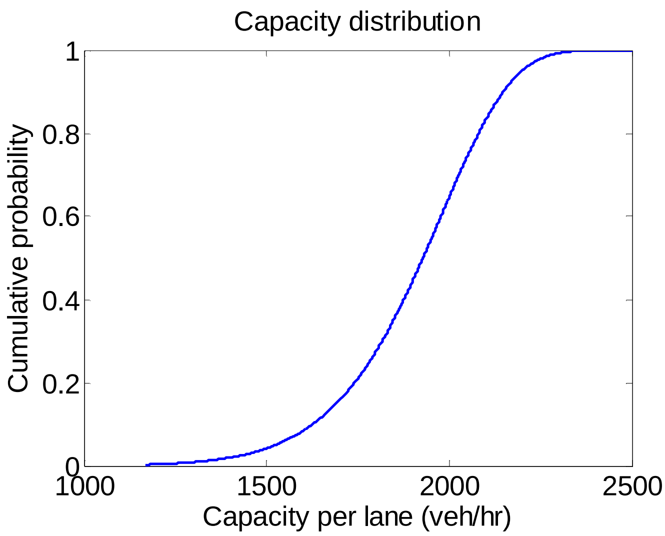


Figure 8.10 local capacity distribution

Traffic flow stochastics

While uncertainties relate to scenarios, within scenario traffic flow remains a stochastic process with fluctuations that are often caused by differences between drivers. These fluctuations can lead to premature congestion and therefore an incomplete utilisation of capacity. Stochastic fluctuations are analysed for the two locations and a demonstration is given of improvements in homogeneity of traffic through the application of the proposed traffic measures.

The initial parameter values are derived using the available data from the above distributions for the median day and calibration of the FOMSA model to represent the level of congestion. New parameter values are derived for the new situation with traffic management measure by sampling traffic stochasticity at a nearby reference location which has similar characteristics to the new situations. The parameter values are derived through comparison with the initial calibrated parameters prior to simulation.

Network adjustments

Implementation of the scenarios in the models requires adjustments of the network and to the traffic flows on the network. These adjustments are given in Table 8.2.

Table 8.2: Model adjustments per scenario

Scenario	***Network/Flow changes***
Sub-case 1: Westbound (FOMSA)	
- Scenario 1 (lane drop)	Inflow reduced from 2+2 lanes to 2+1 lanes. Inflow rate from 3100 -> 2200 veh/hr on reduced road.
- Scenario 2 (bypass)	Inflow from A16 to A20L reduced from 3100 - > 1000 veh/hr
- Scenario 3 (ramp-metering)	Outflow from A20L to A16/A13 increased from 4100 -> 5100 veh/hr Inflow on on-ramp decreased from 1000 -> 500 veh/hr
Sub-case 2: Eastbound (INDY-MonteCarlo)	Capacity on-ramp reduced from 2052 -> 900 veh/hr Capacity weaving section increased from 5888->6006 veh/hr (+2%)

The first sub-case contains three scenarios, which influence two different locations. Scenario 1 and 2 are applied to the Terbregseplein interchange (see Figure 8.3, location 1). Scenario 1 reduces the inflow onto the Rotterdam ring through a lane drop prior to the lane merge and therefore aims to reduce secondary spillback and reduce traffic volume on the ring at the cost of possibly more congestion entering the ring from the east. Scenario 2 applies a planned bypass of the entire A20L north ring. This results in a major reduction in the traffic volume on the A20L. Scenario 3 applies ramp-metering at the Crooswijk on-ramp (Figure 8.3, location 5) to specifically target an important bottleneck. Scenarios 1-3 are implemented in the FOMSA model. The scenario for sub-case 2 also targets an on-ramp in the A20R direction eastbound. Implemented in INDY-MonteCarlo, the adjustments for the scenario involve a reduction of the inflow onto the main carriageway from the on-ramp located at the Centrum junction (Figure 8.3, location 2). A capacity increase of 2% is presumed on the main carriageway due to a reduction in weaving movements (Zhang and Levinson, 2003).

8.6 Analysis and assessment of measures

The goal of the case is to demonstrate the effectiveness of the developed models in a real case for traffic analysis and effectiveness of traffic management. Additionally, a further demonstration of the necessity of considering stochasticity in traffic flow for these analyses is sought. The latter goal is demonstrated in section 8.7 and the prior is addressed in this section. The traffic management scenarios are aimed at reducing congestion on the A20 motorway and increasing throughput. With this in mind, the total delay and travel time, as well as congestion length are considered as three relevant performance indicators. As both models are setup for different types of analysis, the applied indicators differ and are applied as follows:

Sub-case 1: Congestion length and spillback:

$$L_{cong} = \max(L_{cong.end} - L_{cong.start}) \tag{8.2}$$

Travel time

$$TT = \frac{\sum T_{B,i,t} - T_{A,i,t}}{N_{veh,i,t}} \tag{8.3}$$

The congestion length, L, is the largest distance from the start of congestion to the end of congestion or at a specific time. The travel time, *TT*, considers the average actual travel time of all vehicles between a two locations, A and B.

The results of the second analysis allow for a more in-depth qualitative analysis. This is carried out for the effects of congestion spillback over the various bottleneck locations for the three scenarios in this sub-case.

Sub-case 2: Total (network) delay:

$$TD = \sum_{t=0}^{t=e} \frac{veh(t)}{v_{free} - v_{obs}(t)} \tag{8.4}$$

Average peak travel time:

$$\overline{TT_{AP}} = \frac{\sum \frac{\sum T_{B,i,t} - T_{A,i,t}}{N_{veh,i,t}}}{\max(t)} \; for \; t = 1..4 \tag{8.5}$$

The total delay, *TD*, indicates the delay experienced by all vehicles during a specified time period, $t=0..e$, in relation to free-flow traffic conditions denoted by the speed, v_{free}. The average peak travel time, $\overline{TT_{AP}}$, considers the actual travel time for all vehicles during the main nominal peak period, $t = 1..4$.

8.6.1 Sub-case 1 (FOMSA)

The first sub-case considers three different scenarios to improve traffic flow on the westbound carriageway (A20L). The resulting trajectory plot of the reference is given in Figure 8.11a. Additionally the traffic speed diagrams for the three scenarios are given in Figure 8.12a-d. Comparison of the levels of congestion is made in relation to the reference scenario, shown in Figure 8.12a, for which no additional traffic management measures are taken. The numbers shown in Figure 8.12a represent the two locations where the traffic measures are applied, while in the scenario figures in the locations are given with an arrow. In the reference scenario, congestion occurs relatively early at the Crooswijk onramp (location 1 in Figure 8.11) and propagates upstream. At Terbregseplein interchange (location 2) congestion also occurs and is later exacerbated by the spillback from Crooswijk.

In scenario 1 (Figure 8.12b), the lane drop at Terbregseplein is moved upstream to before the merge with inflowing traffic from the adjoining motorway (A16). This has three consequences for the congestion pattern. Firstly, the downstream activation of the Crooswijk bottleneck is avoided due to a reduction in the traffic flow that passes the merge point. The second consequence is that no congestion propagates upstream towards the A16 from the merge point, as congestion is triggered prior to the merge point. The third consequence is an increase in the severity of congestion on the upstream flow from the A20. However, Figure 8.12b shows that the congestion remains limited due to the available upstream capacity to temporarily buffer the traffic.

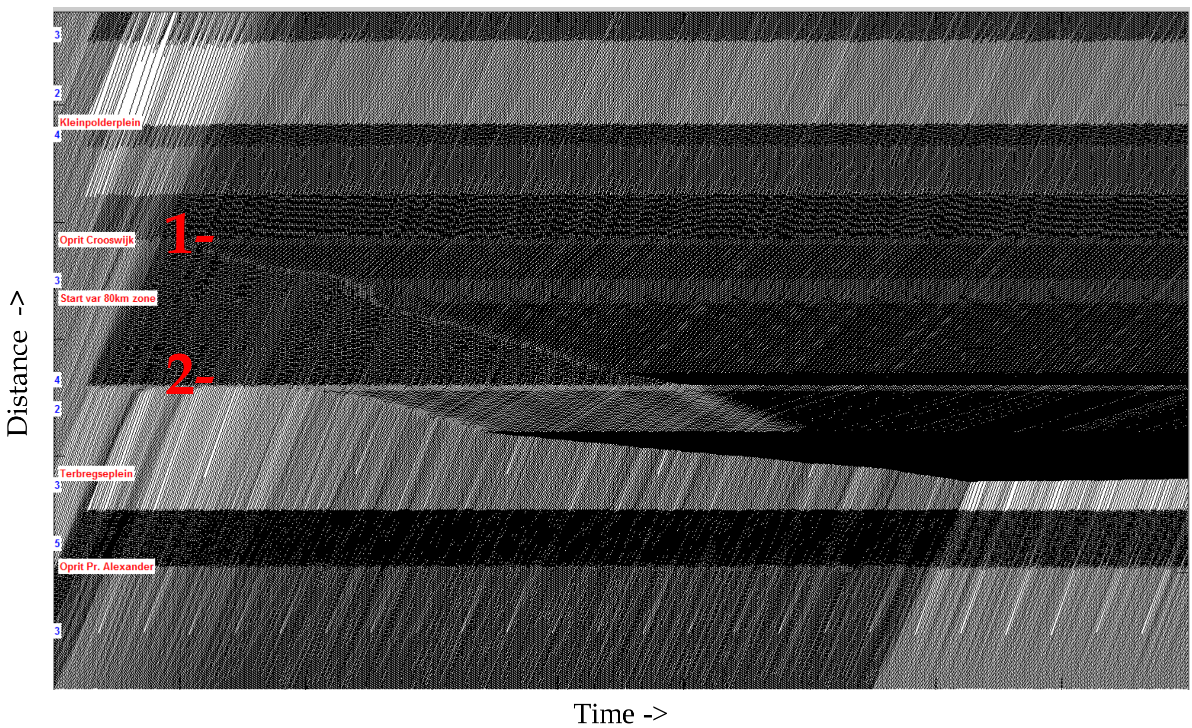


Figure 8.11: FOMSA model results given as trajectories the reference scenario

Scenario 2 (Figure 8.12c) considers the presence of the A16/A13-bypass, substantially reducing the traffic flow onto the A20L. From Figure 8.11c it is clear that this has a large effect on the occurrence of congestion on the road. At all potential bottleneck locations, traffic flow is sufficiently reduced to prevent congestion occurring.

Scenario 3 (Figure 8.12d) focusses on the reduction of congestion at Crooswijk through ramp-metering. Inflowing traffic is reduced to 500 vehicles per hour at the onramp. This leads to a delay in the onset of congestion at the onramp and also leads to less severe congestion and a slower spread of congestion to upstream bottleneck locations. This also has a positive effect on the congestion that occurs at Terbregseplein, as can been seen in comparison to the reference. Further analysis of the bottleneck at Crooswijk showed that a reduction to approximately 200 vehicles per hour would be required to prevent congestion occurring at the on-ramp.

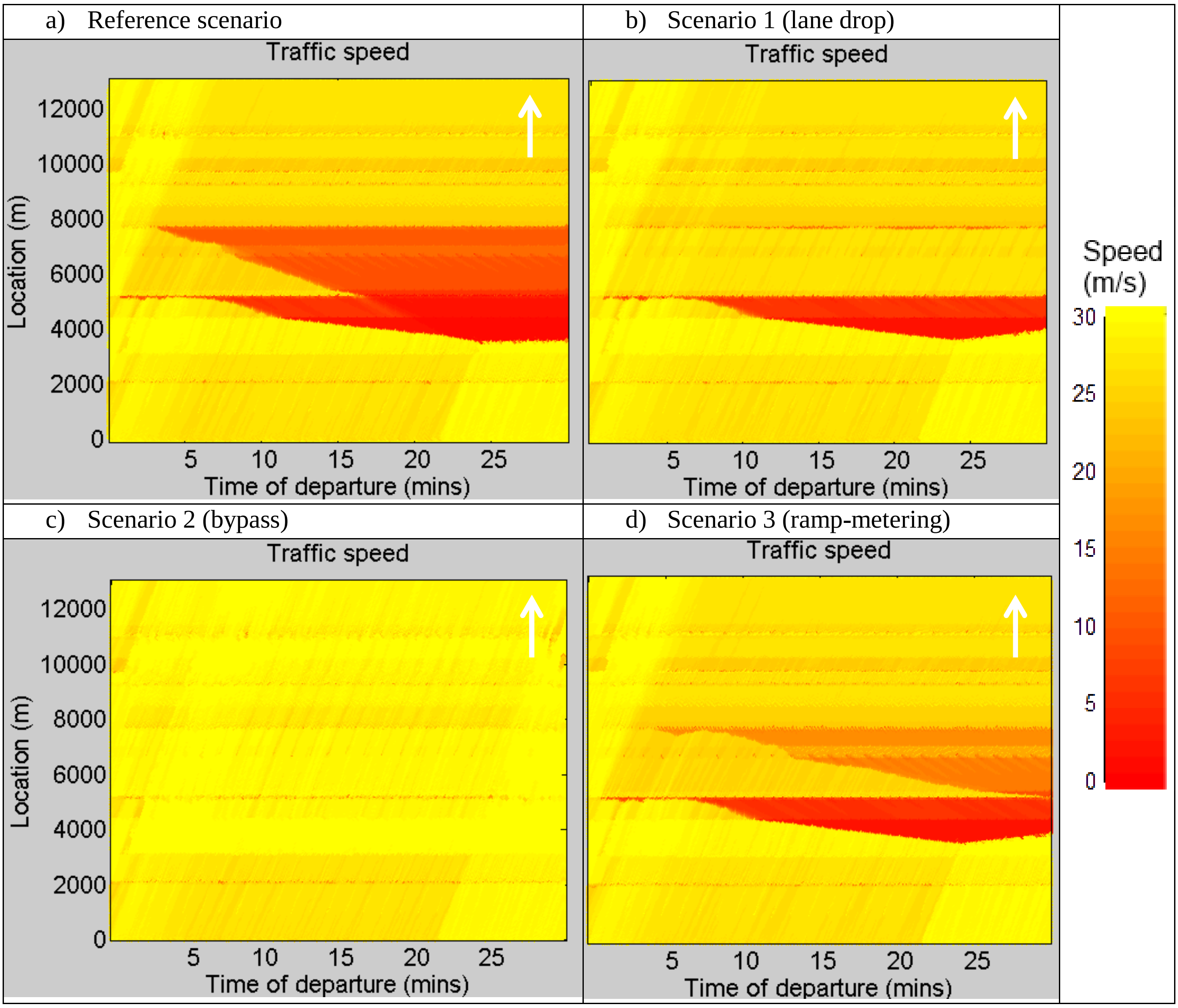


Figure 8.12: FOMSA model results given as the speeds for all scenarios of sub-case 1

A further analysis of the results of the scenarios is given by the developments of travel times and is shown in Figure 8.13. These are the actual travel times of vehicles that entered the motorway at the most upstream location and exited 11.5 km later at the most downstream location, which is not the case for all vehicles. The reference scenario shows an increasing travel time until the traffic demand is reduced and only a slight decrease in travel time once the inflow demand is reduced. This is due to the extensive congestion that occurs. The line for the reference scenario also finishes earlier as vehicles with later starting times spend too long in congestion to be able to exit the motorway stretch before the end of the 60 minute simulation. Scenario 1 (lane drop prior to merge) and Scenario 3 (ramp-metering) both show similar travel time patterns. For the higher inflow rate, the travel time gradually grows as congestion increases, however at a much lower rate than the reference. Once the traffic inflow is reduced, congestion starts to dissipate and travel times quickly drops towards the free-flow travel time, which is approximately 6.5 minutes. Scenario 2 (Bypass) shows a slight increase in the travel time to 7 minutes when traffic is heavier, however as no congestion occurs, the travel-time remains low throughout.

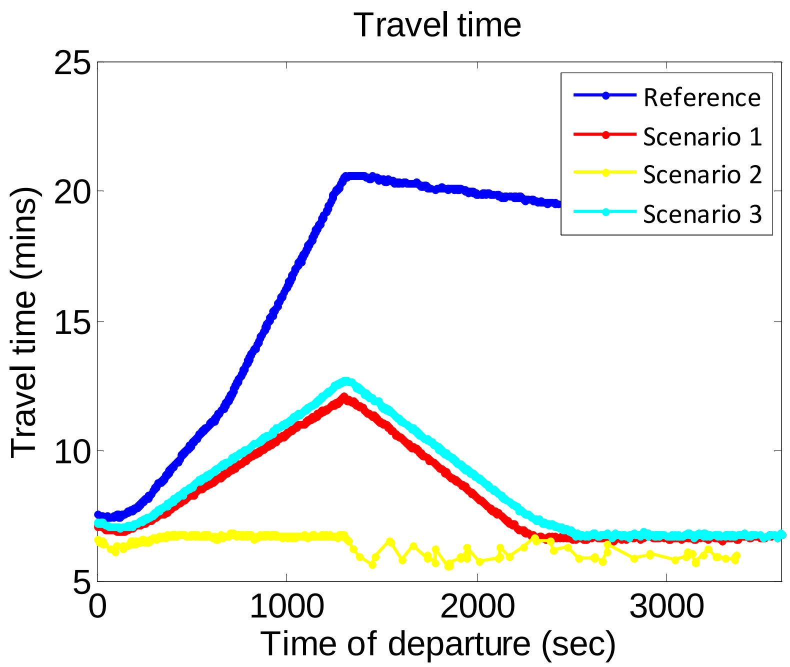


Figure 8.13: Travel-times in the sub-case 1 scenario's

8.6.2 Sub-case 2 (INDY-Monte Carlo)

The results of the total delay time of the 20 Monte Carlo simulations for the reference (blue) and scenario (red) are given in Figure 8.14. The results are in sorted ascendingly to give an indication of the distribution of the delay. The yellow bars show the percentage difference between the two. From this, it is clear that there is an exponential distribution of the delay probability for the network. This means that in some extreme cases very high delays are present for certain traffic conditions, while in most cases there is some sort of an average delay, which corresponds to the extent of the traffic conditions. In the Monte Carlo simulations, two variables are applied, namely the global demand and the local capacity. Figure 8.15a-b shows the sampled demand and capacity factors respectively in comparison to the total delay time. The figures show that the effect of the traffic demand is much greater on the total delay than the change in capacity value. There is a very definitive increase for the demand samples, while the capacity samples shows a wider distribution with a small tendency for a higher total delay for lower capacity values, as may be expected. An explanation for this can be found in part by the fact that the demand factor is applied globally to the entire network, while the capacity factor is only applied to the analysed bottleneck location.

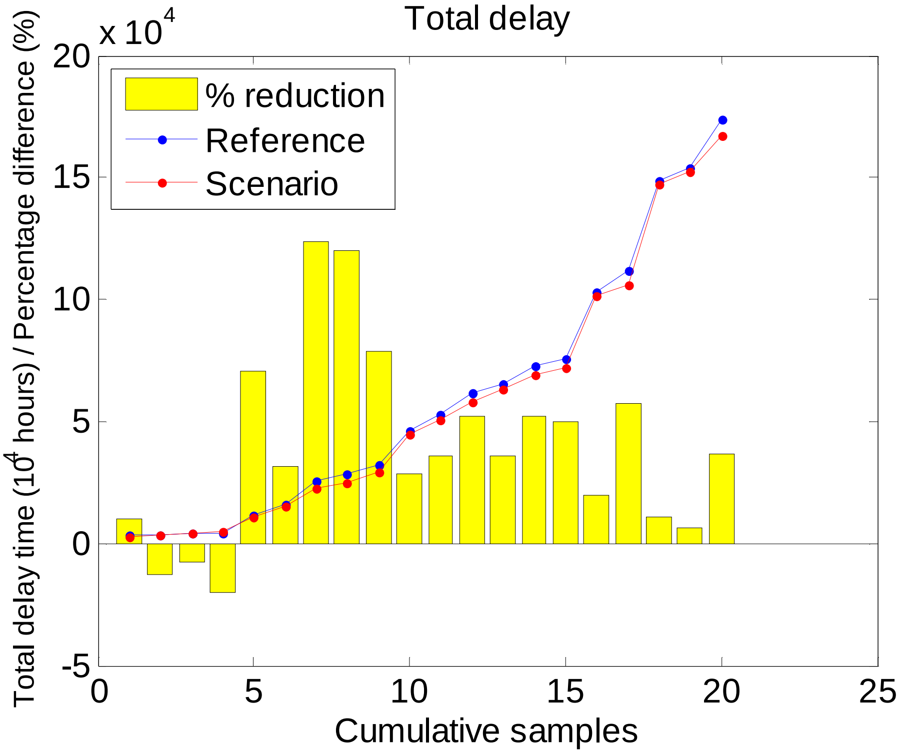


Figure 8.14. Total network delay for sub-case 2

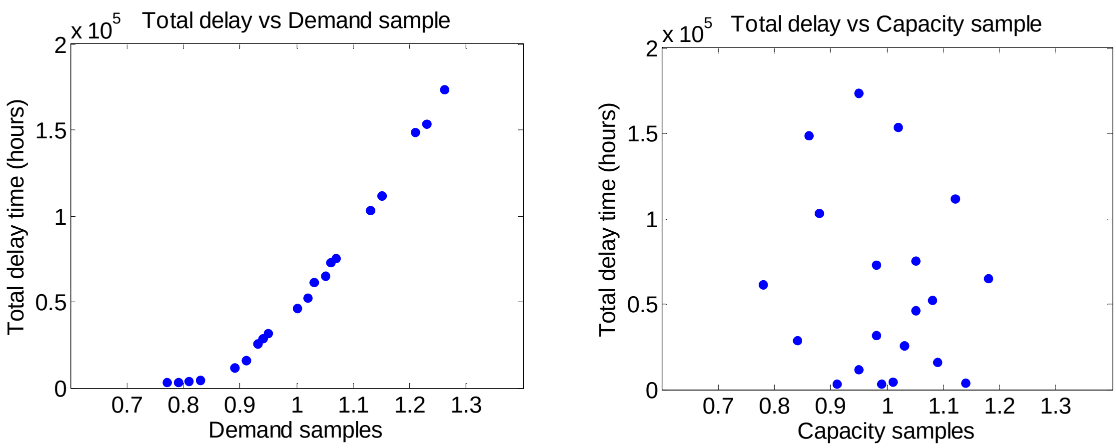


Figure 8.15 a-b. Total network delay versus demand sample for sub-case 2

The yellow bars in Figure 8.14 indicate the percentage difference between the reference scenario and considered scenario of the network delay. From this, the effectiveness of the traffic management measure is indicated. The results show that the traffic management measure is effective with an improvement in the total delay of 2-12% for the majority of the samples, with a median improvement of 3.7%. The trend of the reduction in absolute terms is uniform over all samples, which results in a declining relative improvement for higher total delays. This can be expected as ramp-metering has a set bandwidth in which it is effective. Once traffic flow exceeds the upper bounds, congestion will occur and the improvement on traffic flow reaches its optimum.

The distribution of the travel times along the A20R motorway is given in Figure 8.16 for the reference (scenario) and scenario (red). The distribution of the travel times shows a much greater linearity than the delay time. This is due to the measurement of the travel time on the A20R only, while the total delay is calculated over the entire network. Therefore, secondary delays, as a consequence of congestion on the A20R, are captured by the total delay time, which works exponentially for greater degrees of congestion at the considered bottleneck location.

The effect of the traffic management measure for the improvement in travel time is found to be in the range of 6-11% with a median value of 7.5%. The improvement in travel time along the considered road stretch is also more linear in relative terms, but does decrease slightly for the higher travel time samples.

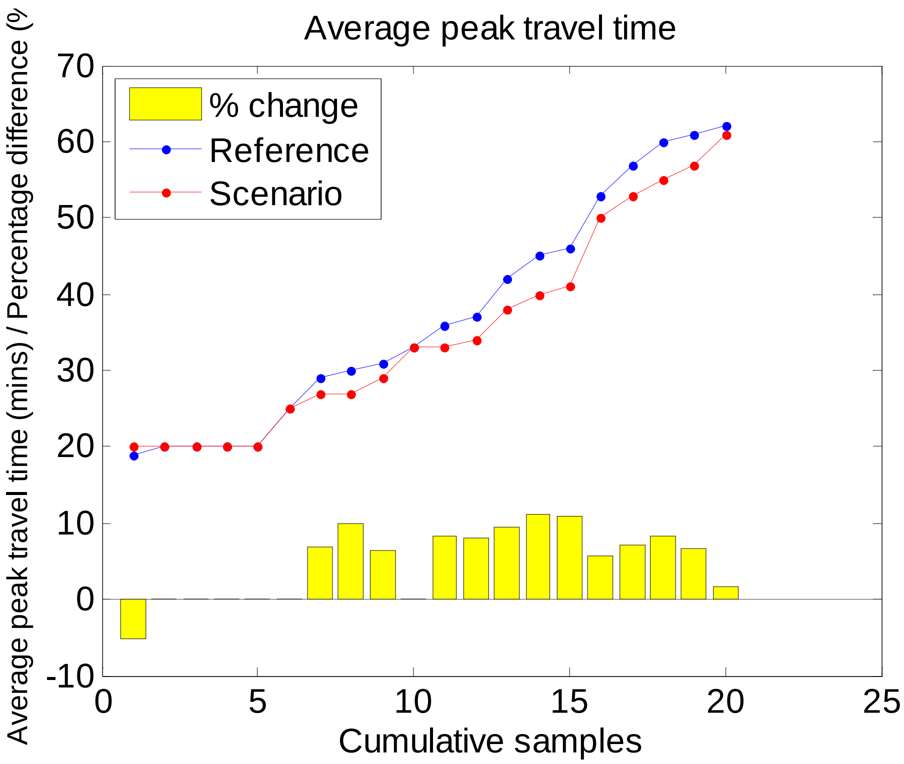


Figure 8.16. Average peak travel times for sub-case 2

In summary, the application of ramp-metering as a the traffic management measure for the Rotterdam-Centrum on-ramp is effective in reducing the network delay on the Rotterdam Ring (3.7%) and reducing the travel time on the A20R motorway (7.5%) and may be considered for implementation. The practical implementation of additional buffering and coordinated traffic signals on the connecting urban and provincial roads is not considered however. This should be reviewed before the ramp-metering can be applied to prevent secondary problems on the local road network.

8.7 Assessing the influence of stochastic characteristics

As part of step 3, consideration of the influence of variations in traffic flow is given to address the second goal of this case. The models in this case are designed and applied to consider uncertainty and stochastic variations in traffic flow on different scales. The INDY-MonteCarlo model considers uncertainty in traffic flow and capacity values on a day-to-day level in which each individual day a different pattern is visible. The FOMSA model focusses on inter-vehicle stochastics in which each vehicle or vehicle group shows different behaviour and therein influences traffic flow. In this section, the relevance of considering these stochastics is demonstrated by offering the alternative approach in which a deterministic approach is applied. When considering the real stochastic variations, one is considering the effects that are also present in reality on roads. Consideration of a non-existence average case as a deterministic calculation deviates from the real values which would be found in practice, which is shown in the next paragraphs.

8.7.1 Sub-case 1 (FOMSA)

The first sub-case, carried out with the FOMSA model, considers stochastic behaviour between vehicles, rather than their macroscopic day-to-day influence. Two parameters are adjusted to show their influence in the model, namely the advection invariant, which describes the following times, and the bounded acceleration rate. The case with no invariant value and no acceleration bound is shown Figure 8.17a. Figure 8.17b shows the same reference scenario with an invariant value of 0.4 and a bounded acceleration of 0.5 m/s^2. Interestingly, the ‘stochastic’ case yields less congestion than the ‘deterministic’ case. Analysis of the results shows that this is mainly due to the ability of merging traffic to accommodate inflowing vehicles better when natural gaps are present, such as in the stochastic case. When all vehicles drive with identical gaps, inflowing vehicles force additional gaps when merging, which lead to a reduction of capacity. In numbers, there is little difference between both cases on the upstream bottleneck, however on the downstream bottleneck congestion in the deterministic case takes 904 seconds to reach the second upstream bottleneck. In the stochastic, case this is 1143seconds, which is 26% longer. This causes the congestion spillback in the stochastic case to reach more than 200 metres further upstream than the deterministic case before stabilising and slowly dissipating. The effect on the travel time is found to be less than 1% during the congestion build-up. The influence of only considering the bounded acceleration is limited, as has been shown in previous research (Calvert et al., 2015). The individual characteristics of vehicles, when decelerating and accelerating, is not considered here and may change the outcome of the results as it can be hypothesised that it may lead to a quicker onset of congestion due to greater heterogeneity in the traffic flow at bottlenecks. Furthermore, there may also be additional capacity drops effects which are limited here.

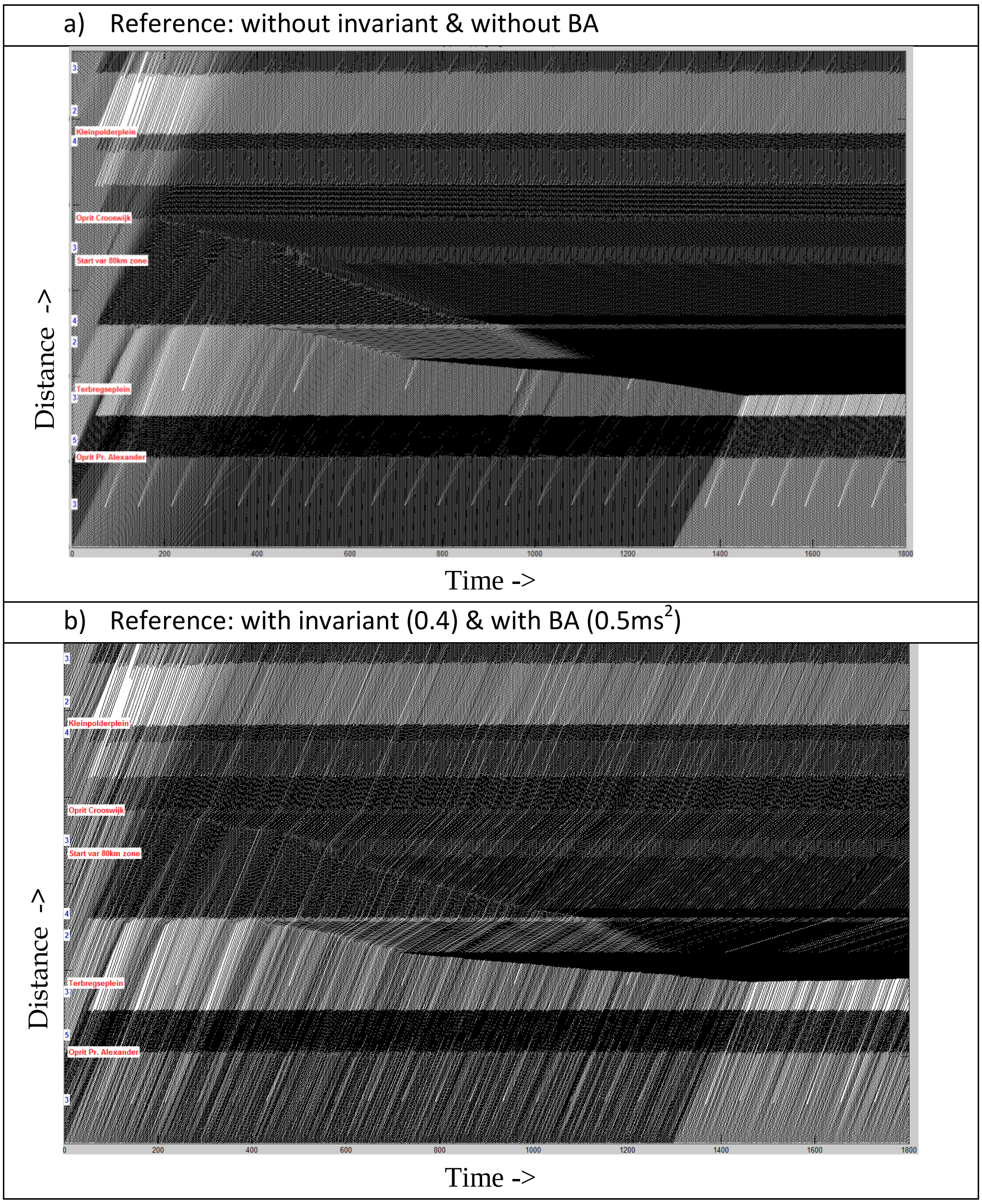


Figure 8.17a-b: Sub-case 1 comparison for modelling a) deterministically b) stochastically

8.7.2 Sub-case 2 (INDY-MonteCarlo)

A single INDY-MonteCarlo simulation is run for the median input values of the traffic demand and road capacity. This is the deterministic case and is performed for both the reference scenario and the traffic management scenario. The results of the deterministic runs are compared with the stochastic case in Figure 8.18. Here the deterministic results are given with dashed lines for both the reference and scenario for the total delay. The stochastic case here shows a reduction in total delay due to the traffic management measure to be 3.7%, while the reduction in the deterministic case is only 1.2%. From the results it is very clear that the deterministic case underestimates the improvements, which is due to the inability of an 'average' input value to consider the entire distribution of all possible values. This is especially the case for the more extreme ends of the distribution. This result is not a surprise and has been previously found in other examples (see Chapter 2 of this thesis). Nevertheless, it once again demonstrates the importance to consider stochastic elements of traffic flow, especially when applying measures that are aimed at addressing extreme delays in traffic.

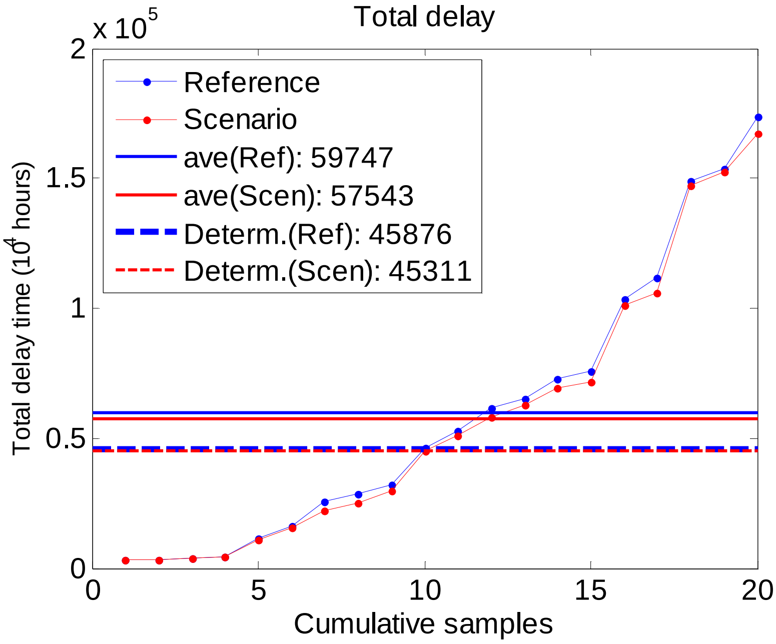


Figure 8.18: Comparison between stochastic and deterministic modelling for sub-case 2

8.8 Conclusions

In this case study, the entire chain for the application of stochastic effects in traffic modelling to aid the application of traffic management has been demonstrated. This was performed for the A20 motorway, the northern part of the Rotterdam Ring Road. The Link Performance Indicator for Resilience was first applied as a quick-scan method to indicate weak sections of a road network requiring attention. Weak sections on the network were identified and their sources were identified as possible locations to apply traffic flow improving traffic management measures. The application and selection of traffic management measures was applied in part using the 'Gebiedsgericht benutten' (GGB) methodology, developed by Rijkswaterstaat in The Netherlands for the application of traffic management. In two subcases a set of measures were selected. The first sub-case focussed on the eastbound A20L motorway using the FOMSA model to analyse the knock-on effects of individual vehicle behaviour on multiple interrelated bottleneck locations. The second sub-case focussed on the westbound A20R using the INDY-MonteCarlo model to consider day-today stochastic variations in the local capacity and global demand. The analysis in both cases is not related, but shows the application of the different models and why one model is more suited to one case, while another may be suited to another.

Ramp-metering at a critical location (Rotterdam Centrum onramp) on the westbound (A20R) carriageway was shown to be effective in reducing delays by 2-12% depending on the day. On the eastbound (A20L) carriageway, ramp-metering at the Crooswijk onramp was found to have a positive effect on the reduction and delay of congestion. The most effective measures on this carriageway were found to be related to the reduction in traffic flow onto the North Ring. A change to the configuration of Terbregseplein merge between A20 and A16 traffic flows, showed that congestion on the A20L can be nearly eliminated by moving the lane drop prior to the merge. The construction of the A13/A16 bypass of the A20 Ring North was also considered and showed that such a measure would eradicate congestion as it would divert a sufficiently high amount of traffic from the A20L. However, it comes at a much greater financial cost and it not strictly a traffic management measure.

In the application of the case, the models showed they are able to perform well and demonstrated their value for their specific purposes and their ability to a priori evaluate potential traffic management measures for sensitive road sections and carriageways. The importance of consideration of the stochastic influence of traffic is further demonstrated for both day-to-day variations as well as intraday and inter-vehicle stochastics for the outcome of studies. Failure to consider the stochastic effects would of have resulted in a bias of 26% for the speed of congestion spillback in sub-case 1 and of 200% for the delay in the second sub-case. A further recommendation is made in relation to the GGB methodology. The methodology remains extremely effective and relevant, however is in need of updating, especially in relation to the possibilities of floating devices and social media. The increase in possibilities for communication and traffic flow guidance has further developed in past decades and should be further included in a revised version of the GGB methodology.

Chapter 9

Visualisation of uncertainty in probabilistic traffic models for policy and operations

This chapter investigates different methods to visualise uncertainty in static graphical representations of probabilistic traffic model predictions on road networks. Throughout the chapter, probabilistic may be seen as synonymous with stochastic. Although various graphical cues may be used to represent uncertainty, it is not a-priori clear which of them are most suited for this purpose, since their legibility, intelligibility and the degree to which they interfere with other graphical elements in a representation differ widely. Several graphical uncertainty representations were therefore developed and analysed in expert sessions. A selection of the initial set of uncertainty visualisations has further been evaluated in a cognitive alternative task-switching experiment. This chapter also presents an overview of possible graphic uncertainty representations and the considerations involved when applying them to uncertainty in traffic model visualisations.

The first section gives a description of the main challenge which is tackled in this chapter. Section 9.2 looks at the main elements to be considered in visualisation of traffic flows in models. In section 9.3, a number of visualisations are presented, followed by the results of a cognitive experiment for a selection of the developed visualisations in section 9.4. The results are discussed in section 9.5 and conclusions are drawn in section 9.6.

This chapter is an edited version of the article:
Calvert, S. C., Rypkema, J., Holleman, B., Azulay, D., & de Jong, A. (2015). Visualisation of uncertainty in probabilistic traffic models for policy and operations. *Transportation*, 1-29. DOI: 10.1007/s11116-015-9673-3

9.1 Introduction

Developments in road traffic modelling have led to a greater consideration of various uncertainties, which are present in traffic systems. These models, referred to here as probabilistic traffic models, consider uncertainties and stochastic fluctuations in traffic flows, which are of importance to accurately represent traffic flow and determine correct performance indicator values (Calvert et al., 2012). While the focus has traditionally been on the development of probabilistic traffic models, the representation of their output and, particularly the uncertainty therein, has grown in importance. In contrast to interactive visualisations, that allow a wide range of interactions, the static representation of the output of probabilistic traffic models is much more challenging since this requires that all information is contained in a single figure. In this chapter, the focus is on feasible options to represent the results of probabilistic traffic models which contain uncertainty data in road networks. A selection of five of these visualisations is tested in an experiment. This considers visualisations for printed communication and includes a description of the needs and shortcomings of existing traffic model visualisations for this objective and.

9.1.1 Probabilistic traffic models

In traffic modelling, a number of distinctions can be made between models. A primary distinction is on the level of detail: microscopic models consider the movements of individual vehicles, while macroscopic models consider the aggregated movement of all vehicles on a specific road section. Some models are dynamic, which indicates that they make use of a time component, while static models are not time dependent. But it is primarily at the level of determinism where probabilistic traffic models are defined. Deterministic traffic models make use of uniformity in traffic behaviour and do not consider stochastic variations in the traffic flow. Stochastic traffic propagation models consider fluctuations in traffic flows, while probabilistic models explicitly consider a quantification of fluctuations in terms of uncertainty or probability. This is typically reflected in probabilities of accuracy or of calculated and displayed values. While there are many different possibilities for probabilistic modelling, a flow of traffic for a specific road section may for example include either a feasible probability distribution of the traffic flow or in a simplified approach show a confidence bandwidth of certain standard deviations or percentiles.

Traditionally probabilities of uncertainty are applied in modelling through the use of Monte Carlo simulation (Calvert et al., 2014b). Multiple simulations are performed with random input values taken from distributions of the underlying the considered variables, which also result in a distribution of results. However, such an approach is usually not applied unless there is a specific need to consider uncertainty, since the associated computation times are greater. Even as probabilistic models continue to develop and no longer have to (entirely) rely on Monte Carlo simulation, their application is still limited often due to monetary or time constraints (Binder and Heermann, 2010, Calvert et al., 2014b). Hence there is a need to develop new ways to visualise their results (Batterman et al., 2014).

9.1.2 Difficulties in probabilistic traffic model visualisation

Results from traffic models are typically reported by projecting the informative variables onto a road network. In general, this approach allows up to two variables to be represented. Normally this is performed for a single moment in time, as transforming a time dimension into a spatial dimension is not feasible for a static visual representation. The use of colours is most commonly applied to identify the value of different variables projected onto a network. These may indicate one of many different variables, from the speed of traffic, traffic flow, delays, but also other variables, such as environmental variables for example. When a second variable is applied on a single one-dimensional network link, this is often performed by adding a further spatial dimension to the one-dimensional network link. Network links are constructed using vectors to describe their geographical location. A network link vector may remain unchanged, while additional spatial dimensions orthogonally can vary in size representing different values for the considered variable.

In traffic modelling, it is uncommon for a third variable to be depicted in a single figure. However, when one considers the uncertainty of one of two variables, it may be desirable to include an additional visual cue to represent this variable. This however not only offers challenges in designing such a cue, but more so in designing one that is comprehensible. This means that the visualisation should be intuitive and should fit in the current methods of visualisation, such that a complete interpretation can be achieved without so called switching difficulties.

9.2 Visualisation considerations

9.2.1 Common visualisations

When considering visualisation of traffic models we will focus mainly on the macroscopic class of models, as these models are aggregate and are therefore more easily applicable for use with probabilistic visualisation. Even probabilistic results of microscopic models must be aggregated, as one cannot consider the distributions of all vehicles simultaneously, which may be performed in a similar visual surrounding to macroscopic models. As described in the introduction, macroscopic models will tend to make use of up to two dimensions for visualisation of results. It is common for these dimensions to depict the variables speed and traffic flow. In Figure 9.1 an example is given of the OmniTRANS model package (OmniTRANS, 2015), which depicts traffic flow as the thickness of a line and the speed as the colour of that line. In this chapter, this approach is taken as the basis for expansion into probabilistic visualisations, as it is the most commonly applied method.

Although it is not commonplace, it is not unheard of for more than two quantitative dimensions to be applied on a network. However, many cases in which more dimensions are added tend to be purpose-built for a specific type of variable and typically not for uncertainty. There is also a trend to externalise probability results outside of the network representation. This gives a great deal of flexibility as there is no need to maintain a close link to a network. However, this poses two difficulties; the first being that a large area is required to visualise the uncertainty of a whole network externally, unless a user is only interested in a specific location. The second difficulty is that the physical connection between the visualisation and network is lost. Furthermore, there may be issues involving interference between a larger number of visual cues in a visualisation. Therefore such approaches, although useful, are not considered as ideal for static reporting of probabilistic traffic model results. Later in section 9.5 it is shown that this approach can be useful for dynamic interaction.

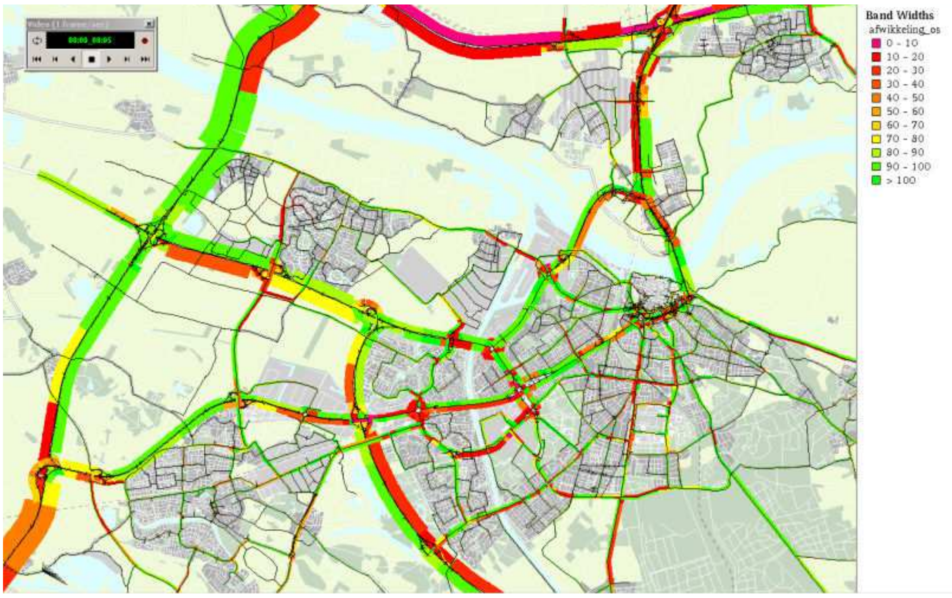


Figure 9.1: A regular macroscopic traffic model visualisation (OmniTRANS (2015))

When a network is considered, and the proximity of the graphical representation to the network is relaxed, additional visual cues have been known to be applied beyond the strict network locations. For example, characteristics of a network link may be shown at a (small) distance relative to the physical location of a network link. This is especially the case in environmental representations of additional variables for road networks. An example is given in Figure 9.2 for air pollution as a consequence of road traffic (Batterman et al., 2014). In Figure 9.2, three variables are applied by representing traffic flow by the line type, vehicle classification volumes by the size of a circle and the volume of the classification types by colour of the circle. This example demonstrates some difficulties of combing multiple variables. Overlapping visual cues make some information difficult or even impossible to interpret, and the limited interval scale also limits the precision.

Anwar et al. (2014) use luminance to highlight locations where road traffic incidents have occurred (see Figure 9.3). Often such visual cues are applied to attract visual attention of users. The colour of a circle indicates the type of traffic incident, while the network lines denote traffic states. In many cases, the visualisation of uncertainty in printed reporting is given as an additional variable. This is applied for each road section on a network. However, a continuous representation demands a different approach from an incidental one.

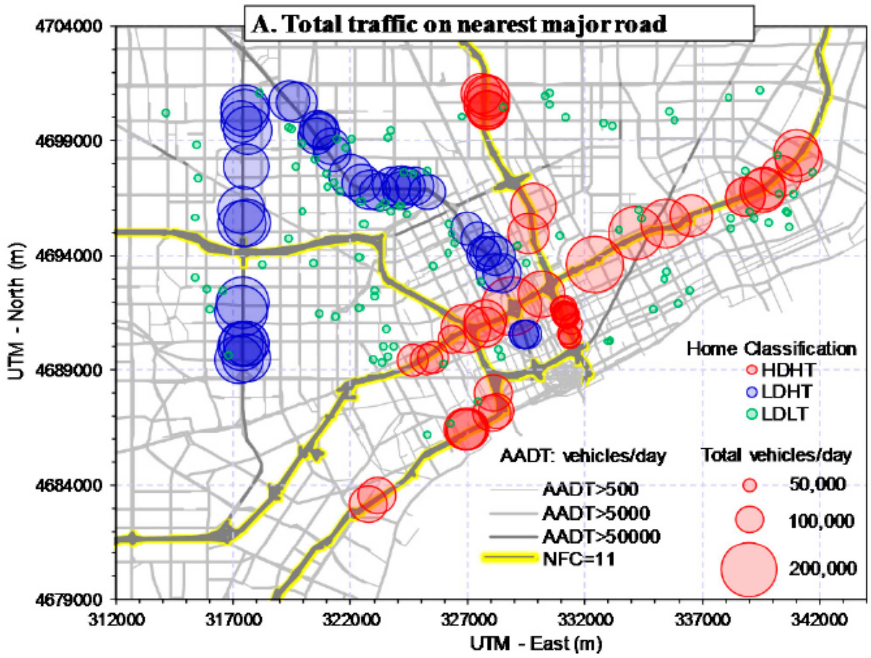
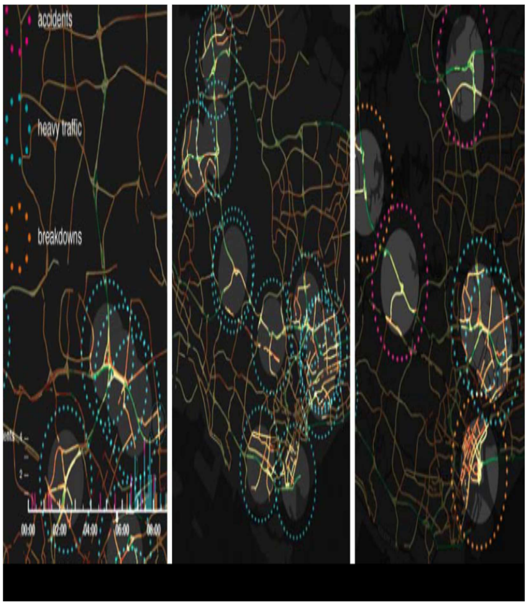


Figure 9.2: Exposure metrics for traffic-related air pollutants (adapted from Batterman et al. (2014))



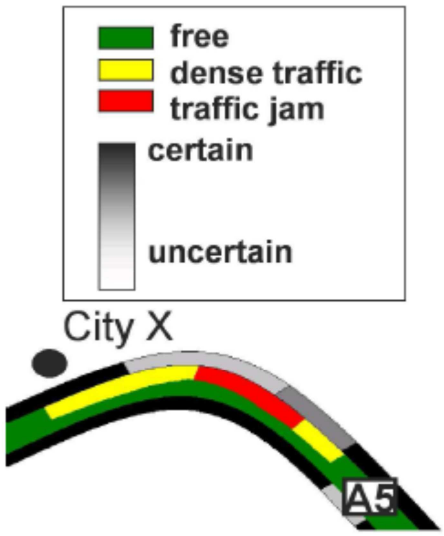


Figure 9.3: Incident frequency displayed in a luminance cue (Anwar et al., 2014)

Figure 9.4: Uncertainty in traffic visualisation by spatial colour extension (Bender et al., 2005)

Continuous representation of traffic state uncertainty on a network scale is demonstrated by Bender et al. (2005) (see Figure 9.4). This makes use of a double line for road sections, with the inner coloured line indicating the traffic state and the outer grey-scaled line indicating the certainty of the traffic state. While this solution can clearly be applied continuously over an entire network and will also work for a ratio-scale, it remains a two dimensional solution and therefore lacks flexibility to add additional variables. Most current traffic models consider speed and traffic flow through link colour and line width, while here the width of the line is more difficult to vary due to the presence of a double line. Nevertheless, this solution is one of few proposed and applied solutions that gets close to offering a solution to adding probability to the existing visualisation of traffic model results on a traffic network.

From these examples, it is apparent that there are challenges for the visualisation of uncertainty in traffic modelling. The main challenges relate to: *dimensionality and clarity, visual perception* and *cognitive processing*:

1. *Dimensionality and clarity* convey the ability to present the desired variables in a clear an intuitive manner. In this research that refers to the application in a traffic network. There are limitations to the application of certain types of visual cues and the number of dimensions. These limitations have been shown in part in the previous examples.

2. In relation to *visual perception*: A visualisation must be able to be easily interpreted. Overloading happens when too many visual cues are applied. Overloading, as seen in Figure 9.2, solves the first challenge, but makes certain variables difficult or nearly impossible to extract. This is especially the case when the visualisations of different variables need to be compared: such as the uncertainty of another variable.

3. *Cognitive processing* of probabilistic visualisations does not only relate to a correct perception of results, as in the second challenge, but also means that visualisations should be able to be processed in an efficient manner. This relates to the way results can be used by a user to draw conclusions and give an interpretation of the results with as little difficulty as possible.

The mentioned challenges are simultaneously addressed in the development of possible options for the visualisation of probabilistic traffic model results.

9.2.2 Classification of graphical variables for uncertainty

Information visualisation is defined as “the act or process of interpreting in visual terms or of putting information in visual form” (Theis, 2011) and involves *symbolization* and *comprehension.* It also offers tools to explore and analyse data sets and therefore facilitates the discovery and extraction of relevant information through graphical means. Furthermore, the application of uncertainty visualisation can help minimize the effects of uncertainty on analysis and decision making (Thomson et al., 2005), but is not straightforward. A visualisation is graphically perfect when it gives the viewer the greatest number of ideas in the shortest time (Tufte and Graves-Morris, 1983). However, different types of data require different representations of uncertainty and specific visualisation techniques are usually designed with a specific data-dimensionality in mind (Sanyal et al., 2009). As data dimensionality increases, the amount of visual cues available for displaying uncertainty becomes limited (Potter et al., 2012, Sanyal et al., 2009) which leads to an increased difficulty in quantifying, representing and understanding it (Thomson et al., 2005).

While cognitive theory contains a large body of work related to human visual attention, empirical research on the attentional aspects of uncertainty visualisation is limited (MacEachren et al., 2012). A main challenge is that interference may arise between different visual channels under certain circumstances when they are utilised in the same display (Acevedo and Laidlaw, 2006, Carswell and Wickens, 1990, MacEachren et al., 2012). Bertin (1974) compiled a classification system in which he assesses visual variables according to the characteristics of *selectiveness, associativeness*, *orderedness*, *quantitativeness* and *length*. *Selectiveness* indicates whether a variable facilitates immediate perceptual group perception. *Associativeness* indicates the way in which a variable is grouped perceptually. *Orderedness* indicates the natural perceptual ordering of the variable. *Quantitativeness* indicates whether a variable facilitates quantitative comparisons. And finally, *length* indicates the number of distinct value-levels a variable can assume (Zuk and Carpendale, 2006, Tufte and Graves-Morris, 1983).

In this research, the main focus is on the quantitativeness variable, since we are dealing with numeric values. According to Bertin, there are seven distinct classes of visual variables: *position, size, shape, value, colour, orientation,* and *texture*. Another applicable categorization of visual attributes is presented by Ware (2012), in which *colour, orientation, size, contrast* and *motion or blinking* are stronger cues, while effects such as line curvature are weaker. Wolfe and Horowitz (2004) consider ’undoubted attributes’ to be those features supported by a wide body of literature, such as colour, motion, orientation and size (including length and spatial frequency). *Luminance onset (flicker), luminance polarity, vernier offset, stereoscopic depth and tilt, pictorial depth cues, shape, line termination, closure, topological status* and *curvature* also belong to this category. A summary of the regarded cues is given in Table 9.1. There is a wide range of categorizations possible, while it is also important to recognise that while some visual cues stand out better than others, combinations must be considered for application to different variables in a single visualisation. The main feasible combinations from the cited literature are *space and colour, stereoscopic depth and colour, stereoscopic depth and motion, luminance polarity and shape, convexity/concavity* and *colour* (shape from shading), and finally *motion and shape* (Ware, 2012).

Table 9.1: Summary of applicable visual cues

Position	Orientation	Luminance polarity	Line termination
Size	Texture	Vernier offset	Topological status
Shape	Contrast	Stereoscopic depth	Curvature
Value	Motion	Stereoscopic tilt	
Colour	Blinking	Depth (pictorial)	

Since uncertainty estimations in traffic model output are inherently numeric, graphical cues should be able to convey absolute magnitudes and to enable relative comparisons with other uncertainty estimations. Furthermore, traffic model visualisations are generally displayed on a background of road-networks, presenting additional sources of potential visual interference.

9.2.3 Task-switching

Commonly, analysis of visual representations is performed through task-switching (Monsell, 2003), as is also the case in this research. Consideration of both visual attention and combinations of different visual features leads to analysis of a user's ability to actively switch between different visual cues. This is studied in the area of Task-switching (Monsell, 2003). When subjects switch between tasks that have different cognitive requirements, such as switching between naming a digit and then reporting if it is even or odd, they incur a *switching cost*. Subjects generally take longer and show higher error rates when responding on *switch-trials*, in which they switch from a different kind of task, than on *non-switch-trials*, where the current task is identical to the previous one (Monsell, 2003). The task-switching paradigm relies on the concept of the task-set: the configuration of cognitive resources in preparation for the execution of a specific task (Monsell, 2003). For each task, there is a preparation effect and a residual cost. The preparation effect is a reduction in effort of an upcoming switch task due to advance knowledge. Residual costs are the additional effort required at a task-switch, possibly quantified as a time-delay. Residual costs may indicate that some parts of the reconfiguration process cannot be completed until after a stimulus has been presented (Rogers and Monsell, 1995).

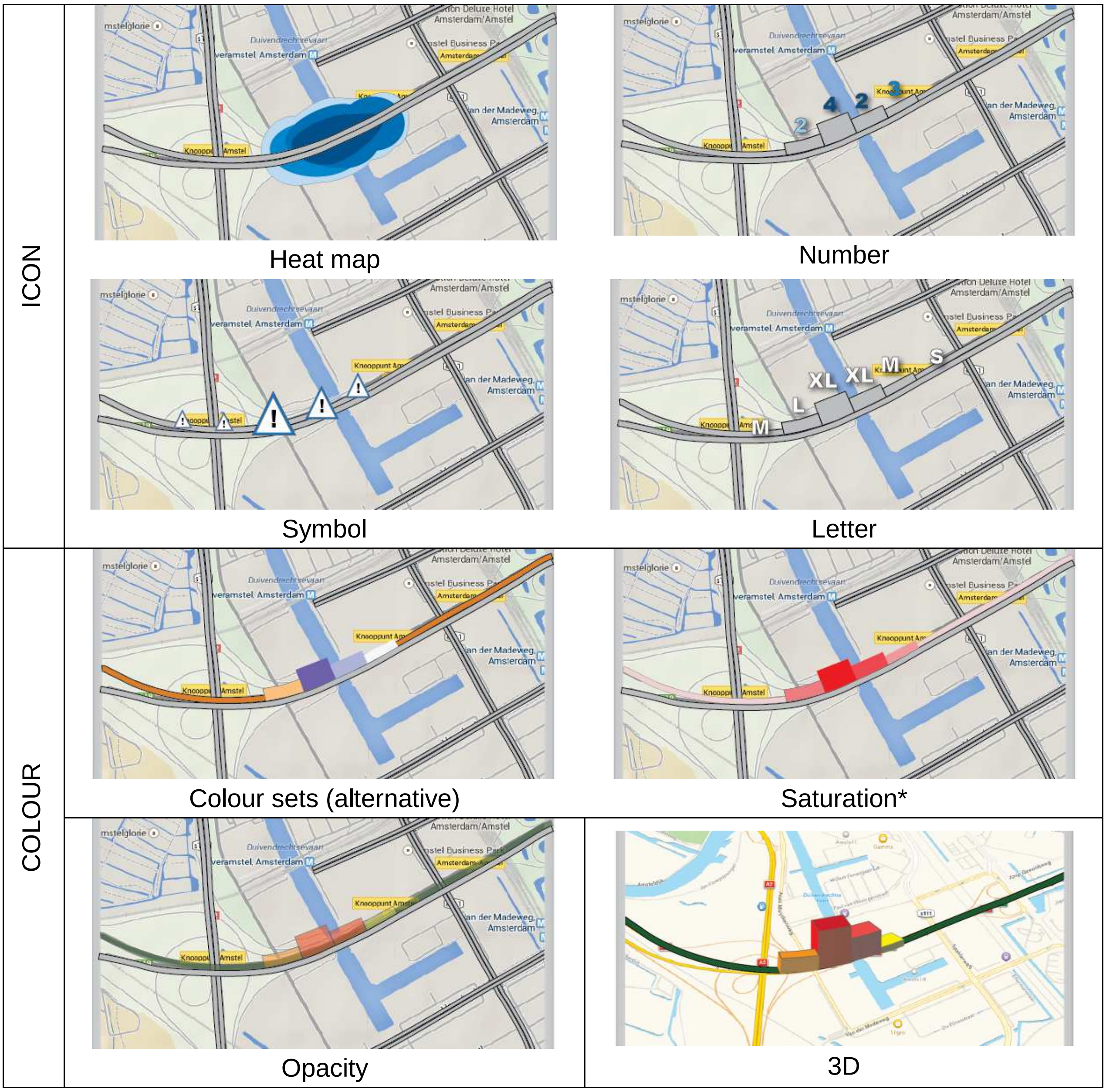
The task-switching paradigm provides a framework in which response times and accuracy can systematically be measured in users for visual cue dimensions. Higher switch costs will in this case be indicative of more interference between the uncertainty dimension and the other cue dimensions. This is an indication that the cue dimensions are not pre-attentively processed well together. This therefore allows different combinations of visual cues to be tested for pre-attentive processing, to test the ability of a user to efficiently interpret the displayed results.

9.3 Multi-dimensional probabilistic visualisation

The construction of a set of feasible visualisations is carried out through an approach that consists of multiple expert sessions and is followed by a visual experiment in which a selected number of graphical representations are preliminary tested as described in section 9.4. Based on literature of uncertainty visualisation and on brainstorm sessions with both cognitive and traffic experts, a collection of initial options for visualising uncertainty are constructed within the boundaries of a traffic network and as a continuation of existing traffic model visualisation. The goal of the expert sessions was to construct a range of visualisations that include representations of traffic speeds, traffic flow and uncertainty variables.

The traffic speeds and traffic flow correspond with the traditional traffic model results, while the uncertainty variable is a new visual cue that gives an indication of the uncertainty, or rather probability, of the value of one of the traffic variables for each road section. It is also possible to seek an additional uncertainty variable, which may be referred to as the impact variable. This variable is a more flexible variable that allows either an incidental indication or a continuous spatial indication of a certain quantity. This may be the probability of incidents at specific locations, a metric for evaluation traffic signals, or merely the uncertainty/probability estimation of the second traffic variable.

The initial set of graphical representations are separated into different categories and all make use of the existing basis of macroscopic models as a requisite as described earlier. The applied categories are *symbols*, *colour*, *texture* and *spatial dimension (3D)* and are shown in Figure 9.5. A qualitative comparison can be made using specific attributes. An example of suitable attributes may be taken from Ravden and Johnson (1989), who present a checklist of attributes for which visual cues should be evaluated. The considered attributes are: visual clarity, consistency, compatibility, informative feedback, explicitness, appropriate functionality, flexibility and control, error prevention and correction, user guidance and support. Partially based on these attributes, five graphical representations are chosen to be applied in the experiment: boxplot, line style, texture pattern, blur and saturation/opacity. Note that blur is not present in Figure 9.5, as it is not easily visible on the presented scale and can be seen in Figure 9.7.



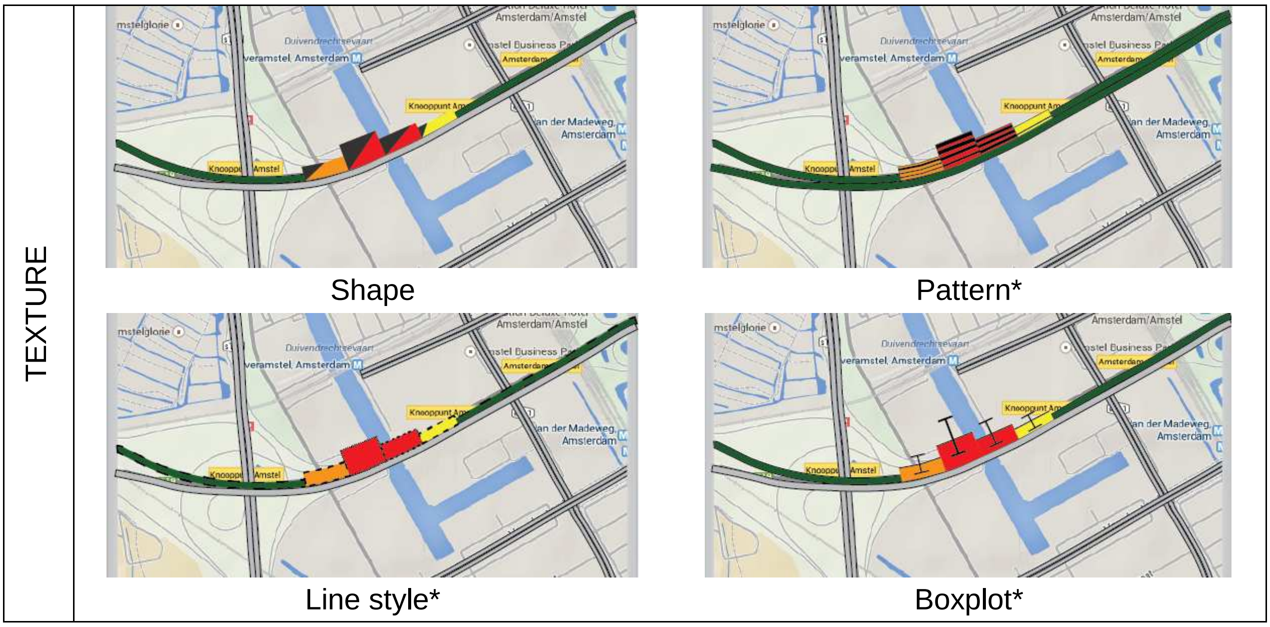


Figure 9.5: Graphical representations for uncertainty and impact

9.4 Task-switching experiment

9.4.1 Experimental design and set-up

As interpretation of visual cues is a task that involves subconscious processing of information to interpret data and make decisions, an experiment in which a select number of graphical representations from the original expert session is performed. The experiment is designed to test the *pre-attentive processing* of a visualisations presented together with other (possibly interfering) cues. This indicates how well a user can interpret the model results, and tests the ability of a user to perform task-switches. This further gives an indication of a user's ability to process different types of information in conjunction, i.e. consider the uncertainty of a traffic variable as well as its value.

The experiment is a forced choice bi-alternative task-switching experiment using multiple stimulus blocks. This entails that participants were shown two visualisations beside each other (see Figure 9.6). Each visualisation has three visual cues and contains information on traffic speed, traffic flow and an uncertainty value. In all visualisations, the traffic speed was represented by the colour and the traffic flow was represented by the height. The third visual cue changed from block to block and represented the level of uncertainty. A participant is asked to state, in as shortest time as possible, if the quantity of a certain queried variable is 'equal' or 'non-equal'. The quantity for which the participant must make an estimate for was given in text above the visualisations and is one of: *speed, flow or uncertainty*.

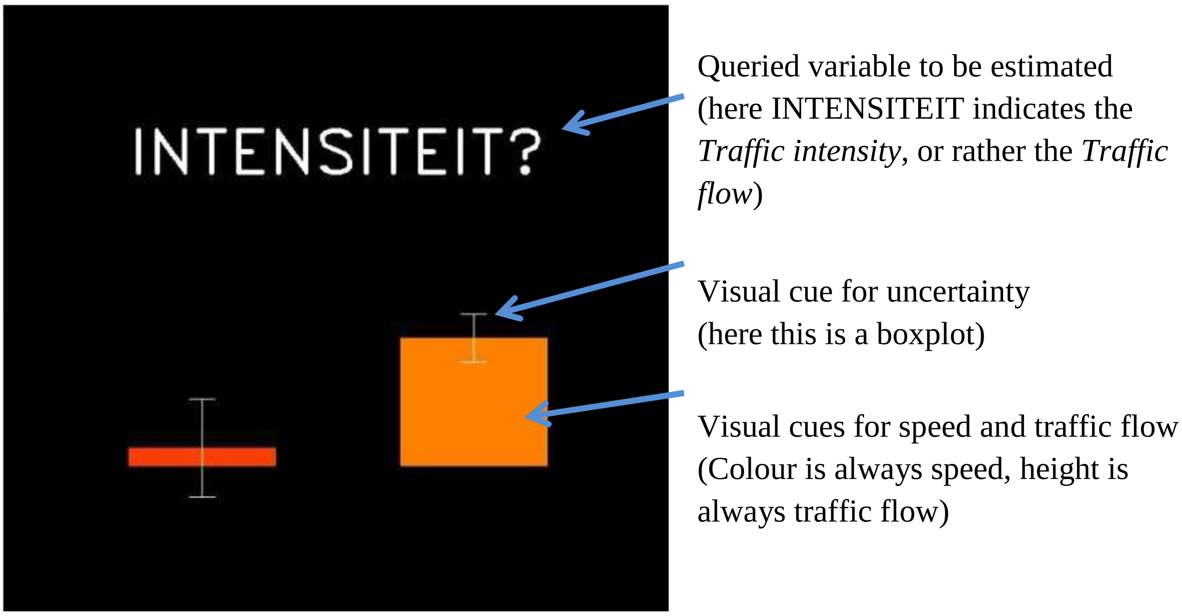


Figure 9.6: An example of a single trial in the visual experiment

Participants were requested to complete 10 consecutive blocks consisting of 16 trials. The entire experiment took the average participant approximately 10 minutes. In each trial, a participant is given a single choice between two different visualisations for which a choice has to be made. The participants made their choice by pressing one of two keyboard keys, with the keys representing ‘equal’ or ‘not equal’. Prior to the start of the experiment, examples were given of the uncertainty cues and a training block containing 16 trials was completed by the participant to ensure the participant understood the procedure. Each uncertainty cue was applied for two entire blocks of trials. The order of the 10 blocks in relation to the applied uncertainty cue was randomised for each participant in an attempt to minimize carry-over and learning effects. This means that the order of the uncertainty cue was different for each participant and that for a single participant each block contained an unknown and randomised uncertainty cue, with the restriction that an uncertainty cue was not applied in more than two blocks. Task-switching was randomly applied. Non task-switch blocks had identical queries for consecutive trials. The visualisation types for the uncertainty variable changed between trial blocks and was either *blur*, *line-style*, *saturation*, *boxplot or texture pattern* (see Figure 9.7). For each trial in each of the 10 blocks, the *time-to-answer* and the *correctness* of an answer was recorded by the computer.

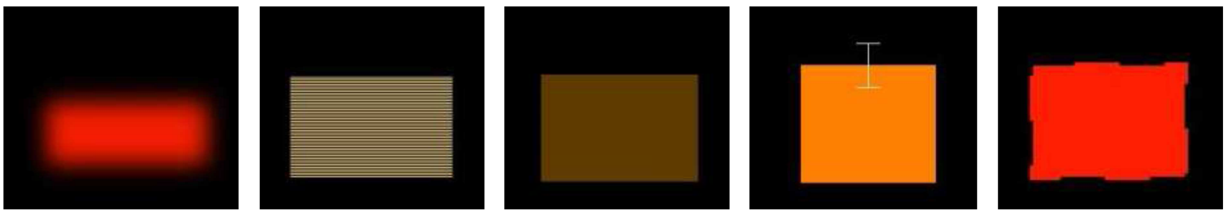


Figure 9.7: Graphical representations tested in the experiment: blur, texture pattern, saturation/opacity, boxplot, and line style

Participants

In total 48 participants took part and completed the trial. Invited participants for the experiment were a combination of traffic experts and non-traffic experts. Traffic related participants were specifically selected and approached by email to participate in the experiment. Other participants were attracted by a general advertisement, mainly made among students. On one hand, traffic experts were invited to judge the graphical representations as potential users in traffic models. On the other hand, the graphical representations should be generic enough that one's profession should not make a difference. Of all participants, 49% had a professional background in a traffic or transport related area, while the rest were not professionally involved in traffic related subjects. The ages of participants were: 34% 18-25 yo, 32% 26-31 yo, 12% 32-40 yo, 8% 41-50 yo, and 14% were 51+ years old. 63% had completed a university education, with 16% having completed a higher college education and 22% having high school as their highest completed form of education. Participants performed the experiment on computers using a web-based application that controls the experiment and collects the required information and results. Participants are first instructed in a few screens on what is expected of them and are asked to give feedback if they understand. Then they commence with the example trials before starting the experiment in earnest. A separate analysis of the results from both groups of participants (traffic and non-traffic related) did not show any significant or substantial differences, which allows both groups to be analysed together.

9.4.2 Measures

Pre-attentive processing is tested by making use of the time required for a user to react to a visual cue and correctly give feedback in relation to that cue. Task-switching is tested in a similar way, by recording the time required to react to a visual cue in a correct manner for the case that a visual cue is changed compared with the previous cue. The results of the experiment are analysed using the following indicators:

> *Effectiveness* is tested by means of the percentage of correctly (PC) answered trials. This gives an indication of effectiveness as the ability of a person to correctly distinguish between different scores of the same variable.
>
> *Efficiency* of a visual cue is tested using the response time (RT), which is the time required to give a correct answer. This gives an indication of efficiency as a user's ability to promptly analyse, interpret and react to the visual cue of a variable.
>
> *Inverse Efficiency Score (IES)* is a combined metric in order to simultaneously assess the two measures: effectiveness and efficiency. The International Standards Organisation (ISO) define efficiency as reflecting the amount of resources required, such as time, to complete a task, and effectiveness as reflecting the extent to which a process can be completed without error. Therefore, effectiveness is mirrored by accuracy, and that efficiency is mirrored by response time. The Inverse Efficiency Scores (Bruyer and Brysbaert, 2011) is defined as:

$$IES(n) = \frac{RT(n)}{PC(n)} \tag{9.1}$$

Here, *RT* is response time and *PC* is proportion of correct answers, both of a specific trial n. This means that better scores are given by lower IES outcomes. In addition to the experiment, a few questions in the survey query participants on the visual cues in which participants can also explicitly state their preference for a visual cue to represent the uncertainty variable. Participants are also asked to give a score for each graphical representation in a range of 1-5 for its ability to convey uncertainty information. In this survey, general information regarding a participant's education, age, experience and area of work was also asked.

9.5 Results

9.5.1 Effectiveness

The accuracy of the answers given in the task-switching experiment indicates how well visual cues can be analysed in a short time. It is necessary to note again that participants were requested to respond as quickly as possible, which may have reduced the accuracy levels. The results of the effectiveness are given in Table 9.2 and are separated into switch and non-switch trial accuracies.

Table 9.2: Effectiveness results of the visual experiment [percentage score]

	method	Mean	Std. Error
1. Non-switch trial	1. Blur	0.88	0.33
	2. Texture	0.76	0.43
	3. Saturation	0.81	0.39
	4. Boxplot	0.94	0.23
	5. Line style	0.88	0.32
2. Switch trial	1. Blur	0.84	0.37
	2. Texture	0.88	0.32
	3. Saturation	0.88	0.33
	4. Boxplot	0.93	0.26
	5. Line style	0.85	0.35

The effectiveness results show that the boxplot is most often correctly estimated for switch and non-switch trials with a combined accuracy of approximately 94%. Blur and line style score similarly for both accuracies. The saturation and texture were both found to yield relatively low accuracies. Their accuracies for the non-switch trials were significantly lower than for switch trials for both.

9.5.2 Efficiency

The efficiency scores, which are measured by the reaction time to correctly give an answer, are given in Table 9.3 with the values being the average values over all relevant trails in milliseconds. Outliers are removed for which the reaction time was lower than 300 ms or higher than 8000 ms. The lower boundary indicates the reactive ability of a participant that will never be below 300 ms. The higher bound is selected as a reasonable value in which most participants easily make a choice. There were 81 low-end outliers and 109 high-end outliers from the 7680 trials. The efficiency score reflects the ability of a participant to correctly interpret the shown graphical representations within a given time frame.

Table 9.3 Efficiency results of the visual experiment [reaction time in seconds]

Switch type	method	Mean	Std. Error	95% Confidence Interval	
				Lower Bound	Upper Bound
1. Non-switch trial	1. Blur	3447	1635	864	6006
	2. Texture	2644	1787	968	6145
	3. Saturation	3457	1730	870	6282
	4. Boxplot	1956	1082	889	4384
	5. Line style	2009	1118	925	4129
2. Switch trial (to uncertainty)	1. Blur	2927	1308	1425	5526
	2. Texture	2644	1318	1076	5674
	3. Saturation	2694	1486	1063	5965
	4. Boxplot	2683	1489	1168	5959
	5. Line style	2459	1146	1248	4731

The reaction times for switch trials are found to be similar for all graphical representations with all values found to be around 2500-2900 ms and not significantly different. The boxplot and line style were found to yield the lowest reaction times and therefore the highest efficiency, which are significantly better than the other graphical representations. The blur, texture and saturation scored worse when considering both the switch and non-switch trials.

9.5.3 Inverse Efficiency

While the efficiency of a graphical representation for a visual cue is measured by the reaction time, it is a requisite that the accuracy of a cue is maintained. If a participant reacts to a visual cue effectively, but does so with a high inaccuracy, then that graphical representation cannot be considered efficient. Therefore, the Inverse Efficiency Score (IES) is applied for which the results of all trials are given in Table 9.4.

Table 9.4: Inverse Efficiency (IES) score for the visual task-switching experiment

Switch type	method	Mean	Std. Error	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
1. Non-switch trial	1. Blur	3921	1720	983	5864
	2. Texture	3478	1934	1273	5983
	3. Saturation	4268	1846	1074	5943
	4. Boxplot	2072	1148	941	4335
	5. Line style	2275	1233	1048	4044
2. Switch trial (to uncertainty)	1. Blur	3480	1455	1695	4751
	2. Texture	3002	1421	1222	4837
	3. Saturation	3078	1589	1215	5547
	4. Boxplot	2891	1455	1258	5503
	5. Line style	2884	1581	1463	5500

For the switch trial, the IES is similar for all visualisations, with values roughly between 2900-3400. The boxplot is found to score the best when both accuracy and effectiveness are simultaneously considered in the IES, closely followed by the Line style. Blur also scores reasonably well, in part due to a high level of effectiveness. The ability of participants to interpret the saturation and texture cues appears to be relatively poor.

9.5.4 Task-switching

For the previous indicators, all task-switching combinations were combined as an overall indication of the effectiveness and efficiency of each graphical representation. Here we investigate the different task-switching combinations in more detail. A task switch means that different variables need to be assessed on two consecutive trials. For three variables, six different task-switches are possible. The scores for task-switches between each of three variables are shown in Figure 9.8. The variable to which was switched was found to be relevant, while the previous variable was less relevant, therefore we can suffice with showing the switches to the three variables.

The segregation of the results gives some further insights into the overall performance of the graphical representations. The accuracy for switches to both traffic flow and traffic speed was relatively high, at approximately 90-95% and 85-90% with no difference between switch or non-switch trials. Two interesting observations can be made for the boxplot and texture representations. The boxplot scores higher for non-switches than other for the speed cue, while slightly worse for the traffic flow. It may be that the colour cue, indicating speed, is not affected by the boxplot, while it does interfere with one’s estimate of the magnitude of the traffic flow cue. Texture scores poorly for non-switch trials for the traffic flow. It is not entirely clear why this is, but it could be caused by cognitive overload in combination with the perception of the magnitude of a cue.

Figure 9.8: Detailed task-switching results

A switch to the uncertainty cue shows a much greater disparity in scores. Both the texture and saturation accuracy scores are well below the scores for the other visual representations at just below or above 70% correct for non-switch trials. Blur scores especially poor for switch trial to the uncertainty cue, while better for non-switch trials, which indicates that this visualisation requires more effort from a user to adjust to than to the other representations. The efficiency score for texture, saturation and blur, also show that users need more time to interpret these graphical representations while achieving less accuracy, especially for the non-switch trials. This suggests a certain negative adaptation to these graphical representations.

9.5.5 Questionnaire

Participants were asked to rank each graphical representation from 1-5, with 5 being the best score, for its ability to convey uncertainty information. The results are shown in Figure 9.9.

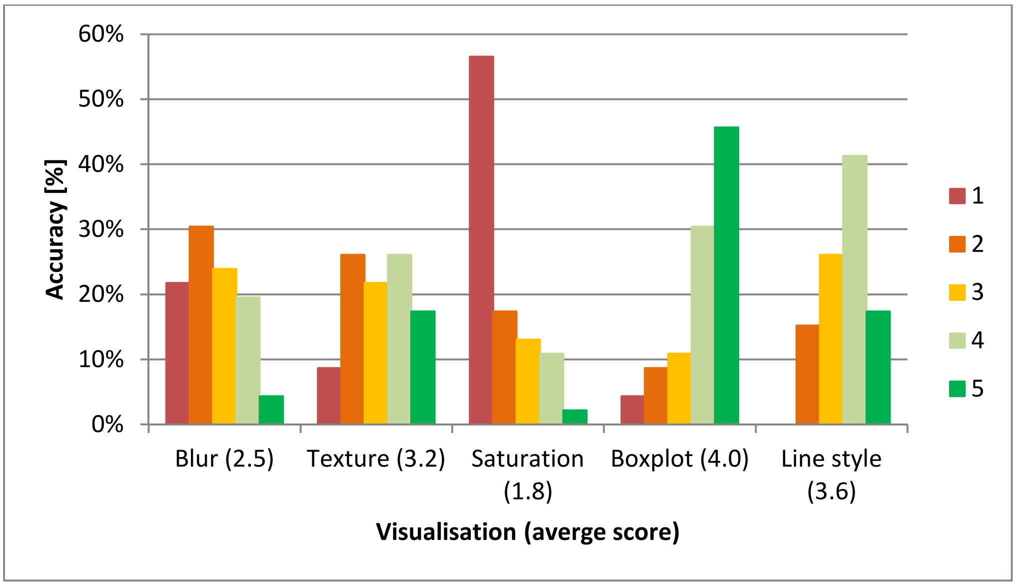


Figure 9.9: Assessment survey scores (5 is the best score)

The boxplot appeared to be easy to interpret with nearly half of the participants giving it the highest score and less than 15% giving it one of the two lowest scores. Line style also scored favourably with nearly 60% giving it one of the top two scores. Participants reacted varied to texture, with an average overall score of 3.2. Blur was given a generally poor assessment with over half of the participants giving it one of the lowest two scores. Finally, saturation received by far the lowest scores, with an average of 1.8 and some 57% giving it a score of 1.

Comparison of the results of the questionnaire with those from the experiment shows an incredible amount of similarity in the relative scores between the analysed graphical representations. From both, it is clear that the boxplot scored best, while line style also performs well. At the other end of the scale both saturation and texture both scored poorly in both the experiment as well as in the survey. In the following section, these results, as well as the outcome of the expert sessions, are discussed and clarified, where possible.

9.5 Discussion

Visualising additional dimensions for the uncertainty of the results of probabilistic traffic models is not straightforward and no one solution can be said to be best for all circumstances. The results of the experiment clearly showed that the boxplot performed best among the five tested graphical representations for the uncertainty cue, both objectively (in terms of processing efficiency and accuracy) and subjectively (through positive assessments). Line style also scored well, even though its performance was rated slightly lower than that of the boxplot. Both of these cues have the advantage that interaction with the cues of other displayed variables, the traffic flow (height) and speed (colour), is limited: neither cue distorts the colour of a network link. The effect on the traffic flow cue seemed also to be limited. Although the individual task-switching results may suggest that the boxplot shows a slight interference with the traffic flow, it was not enough to substantially reduce its score. Comments made by participants indicated that a conflict occurred for saturation and texture with the colour cue representing the speed. This was especially the case for saturation, which can also easily be seen in the low accuracy obtained for uncertainty estimations from this cue. But also for texture, participants struggled to accurately and promptly distinguish between different levels of texture intensity. Both of these graphical representations are therefore less useful in their current form, because of their limited ability to convey quantitative information and because they interact with the speed cue. Participants also found it difficult to distinguish different gradations of blur, however this representation did not suffer from interference with the other graphical representations to the same extent.

The results of this study clearly show that the appropriateness of a given uncertainty visualisation depends on the condition in which it is deployed. We identified three important issues that should be considered before selecting a graphical cue to represent uncertainty in traffic flow model outputs:

Avoid interaction between graphical dimensions. Both the results of the experiment and the outcome of accompanying questionnaire indicated that graphical cues are more difficult to process when they are used in conjunction (i.e. when they occupy the same physical space, e.g. a combination of saturation and colour).

Use visualisations that allow explicit comparison or quantification. The experts judged that certain graphical representations can be used to indicate a scenario, but are less suited for comparison of their underlying values. Blur is a good example of this. Participants had difficulties in judging the extent of the blur. On the other hand, the boxplot proved to be easier to interpret, possibly due to its crisp scale. There are certain categories of visual representations that are less suited for explicit comparison or quantification, and this should be considered if this is seen as a requirement for the visual cue, as is the case here for uncertainty.

Avoid visual clutter. In the introduction on model visualisations, this was already mentioned and is again confirmed in this study. Also when combining different visual cues in a single visualisation, one must be careful not to overload the visualisation and therein limit the clarity and ability of a user to promptly interpret a visualisation. It is suggested that certain combinations of visual cues and graphical representations do not mix well in this regard. This is especially the case when multiple cues attract the attention of a user and even prohibit the view of other graphical representations and the underlying network or background.

While these considerations are mainly relevant to static visualisations, they should not entirely be ignored for interactive dynamic visualisations. However, in an interactive visualisation there are more options available to select various cues and therefore to limit the number of cues shown at once. This does not only limit the information load, but also allows certain explicit and easily comprehendible graphical representations to be used alternatively.

While the proposed graphical representations in the previous sections are designed with static reporting in mind, there are also a number of additional possibilities when the considered visualisations are placed in an interactive framework. There are increasingly more interactive environments becoming available for hands-on interaction with model results and data. An example of a platform using some of the visualisations from this research is the CommonSense platform (TNO, 2014), shown in Figure 9.10. The three visualisation options shown in Figure 9.10 give some insight into the way static visual cues may be deployed in a dynamic interactive environment where the specific purpose of the overall visualisation dictates which type of visualisation is most suitable.

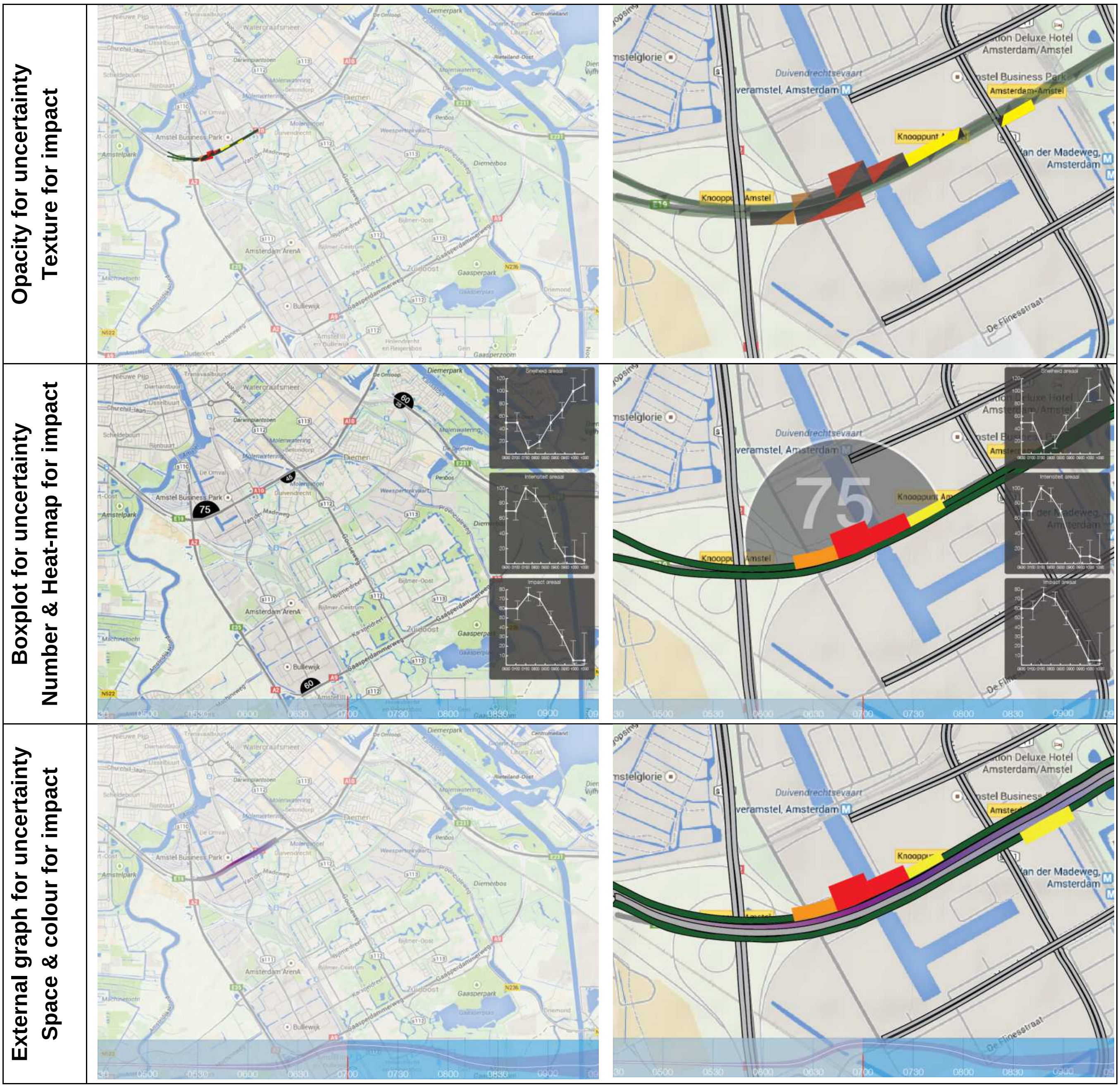


Figure 9.10: Graphical representation examples for dynamic interactive visualisations of probabilistic traffic flow

9.6 Conclusions

In conclusion, this chapter presents a number of candidate graphical representations that can be applied to represent uncertainty in traffic model visualisations. Although the focus was on uncertainty visualisations, the graphical representations may also be applied to represent other dimensions. The applicability and quality of these representations was initially assessed through a series of expert workshops. In a follow-up visual cognitive experiment, a selection of the graphical representations was further evaluated. It appeared that certain graphical representations performed better than others (a boxplot and a line style representation outperformed the other representations). An additional finding was that participants were able to assess graphical representations relatively accurately. This was found by comparing the (objective) results of the experiment with the (subjective) preferences reported in response to an accompanying questionnaire. It appears that the actual choice of a graphical uncertainty representation strongly depends on the desired type, speed and level of information disclosure and is therefore model and scenario dependant. Further analysis showed that three main considerations should be taken into account when designing model visualisations. It is important (1) to avoid interference between the visual cues on different dimensions, (2) to apply cues that allow explicit comparison or quantification where required, and (3) to be aware of the detrimental effects of clutter or overloading visualisations with too much information. It is evident that no single graphical representation will serve to represent uncertainty in all different traffic model visualisations. A set of feasible candidates are presented together with some essential issues that need to be considered when deploying uncertainty representations to traffic model visualisation.

Chapter 10

Conclusions and recommendations

The research presented in this thesis has been performed with the motivation to improve the understanding and analysis of the stochastic effects of traffic flow with respect to the application and impact of traffic management. Three main areas are highlighted and considered, namely the analysis of uncertainty and fluctuations in traffic, modelling of uncertainty and fluctuations to a-priori evaluate traffic management measures and the visualisation and communication of uncertainty in traffic. The main conclusions and recommendations of the presented research are given in this chapter, as well as some practical implications of the research results.

The chapter starts with the main findings and conclusions of the research in section 10.1 and is followed by the practical implications of the results for application in section 10.2. Section 10.3 offers some recommendations for further research following the developments made in the presented research in this thesis.

10.1 Main findings and conclusions

The main findings and conclusions of this thesis are presented in this section. This is performed using the same structure as applied to present the research objectives in the introduction chapter, which considers the topics of *analysis, modelling and visualisation*. The modelling section 10.1.2 is split into three parts, focussing on: *background, the models, and practice*. In each sub-section, the main relevance of the specific topic is firstly highlighted, and then the main findings are given followed by the conclusions found in this thesis.

10.1.1 The analysis of uncertainty and fluctuations in traffic

Analysis of stochastic characteristics of the variables that influence traffic flow is necessary in order to understand and model uncertainty and fluctuations. These variables lead to a greater understanding of the extent of the stochastics and can act as input for models. Both demand and capacity are directly affected by such variables. Demand fluctuations strongly depend on the local network and scenario. Capacity fluctuations do too; however they tend to be more generic and less easy to quantify in model calibration. Therefore, the main focus for analysis was on capacities.

In Chapter 3, two methodological frameworks were presented for stochastic analysis of traffic; one based on stochastic capacity and the other on combined stochastic demand and capacity. In Chapter 3, both research questions (questions 1 and 2) regarding the analysis of uncertainty are addressed. The first methodological framework is a conceptual model for practical stochastic capacity estimation that allows the stochastic nature of capacities to be captured and quantified. The methodology is designed to give practitioners and researchers a concise and easy to follow approach for stochastic capacity estimation. The stochastic capacity estimation part of the framework is based on the adapted Product Limit Method (PLM) by Brilon et al. (2005) to quantify capacity as a distribution. This approach was found to be robust, but also effective as it uses both pre-breakdown, as well as breakdown observations, of traffic to construct a probability distribution of capacity. Furthermore, the approach can easily be applied under different scenarios to construct capacity distributions for different conditions, such as for incidents, with different weather conditions, or for major events. A quantification of day-type specific variation in capacity values was given in the form of a Weibull capacity estimation fit for each type-of-day scenario. In the case, weekdays, weekend days and holiday days were considered and served as a demonstration of the applied methodology and gave a generic capacity distribution applicable for The Netherlands motorway network. In addition to the framework, an extensive overview of capacity influencing variables was described and presented in a diagram. This showed that variables can be categorised as exogenous, endogenous or temporal.

The second methodological framework, also presented in Chapter 3, considered the joint stochastic effect of demand and capacity on traffic flow. The methodology applied the same capacity approach as the first method and derived demand distributions through an empirical process of cordoned observations. This was performed for the influence of the endogenous variable environmental effects in the form of weather conditions, as one of the most influential external and commonly occurring variables on traffic flow. Weather conditions affect both traffic demand as well as road capacity. The capacity estimation framework was applied on weather as part of a holistic approach for simultaneous influence on both the demand and supply in an extensive data-driven analysis for the effects of rain, snow, temperature and wind for their influence on traffic. The results showed that for increasing precipitation in the form of rainfall, both the capacity and demand decreased. Despite the reduction, the overall influence of rain on traffics ability to flow fluently was not substantially reduced, due to the interaction between the demand and capacity. Insufficient data for the described approach meant that capacity estimation could not be made for snowfall, while a reduction in demand for snow was found of more than 15%. The influence of cold temperatures proved to be substantial on traffic fluency. Demand was found not to vary significantly, while capacity decreased leading to a greater chance of a reduction in level-of-service of roads. Similarly, high winds were found to also reduce the quality of traffic fluency, although at a lower level of approximately 2-3%. For each weather scenario, stochastic distributions were derived. These showed that the distribution shape of each weather type does not significantly differ and was found to yield similar shape-parameters when fitted for a Weibull distribution. The shape of the demand distributions also showed a close resemblance to each other and was found to adhere to a t-location-scale and logistic distributions. From the case study, we conclude that the methodology can be applied to give estimation of the effects of varied demand and capacity variables in a single analysis. The resulting distributions may be useful for a number of future purposes, such as application of uncertainty and sensitivity analysis both in data-analysis and modelling of traffic effects during weather to name just two.

10.1.2 Modelling uncertainty and traffic fluctuations: the background

Before being able to construct new approaches to answer the research questions, it was first necessary to understand what the current state of the art is in macroscopic and stochastic modelling. Furthermore, insight was also required on the necessity of considering traffic stochastically, which without there is no need to seek methods to analyse and model it. Both the necessity and the background of stochastic macroscopic modelling were presented in Chapter 2. It was found that two main avenues of models are utilised: *repetitive Monte Carlo simulation* and the *analytical* consideration of probability in the core of a model. While classically, the Monte Carlo approach has been applied, the advancement of various analytical approaches has increased, with a number of extensions of deterministic models being proposed. It was found that too often stochastic variation in models is insufficiently considered in practice, either for application or the necessity for development. Models that did include stochastic elements, did so with various successes, but often succumb to one of the two following shortcomings: too high complexity or computational effort for easy application in practice, or oversimplification, resulting in inaccuracies and a limited consideration of the real stochastic effects in traffic flow. To demonstrate the necessity of considering uncertainty in traffic modelling, two experimental cases were given in Chapter 2, in which the application of a deterministic approach was shown to yield substantially biased results in comparison to a stochastic approach. A further deliberation concluded that stochastic models can be seen as more accurate than deterministic models, as they represent reality better, however their application is not recommended where there is little congestion, little variability in variables or when a general indication of network performance is sought. In most other cases, stochastic variability in traffic should be considered. Is was argued that there is a clear necessity, but also many challenges for the scientific and consultancy communities to further develop and apply stochastic modelling in traffic analysis. It is the joint responsibility of both communities to address this and make further developments in this area of research possible and realise that blindly applying non-stochastic models where probability is rife can have detrimental effects.

Research question 3, the first research question in relation to modelling stochastics, addressed the main issues that still exist. In Chapter 2, the main issues to modelling stochastics, derived from the literature research, were presented as: *Computational efficiency, Correlations and spatiotemporal dependency, Data gathering and processing, Stochastic propagation of probability, Generality of stochastic variation,* and *Driving behaviour in macroscopic traffic*. For each issue, a challenge was formulated to tackle it. In Chapter 2, it was derived that especially the manner of stochastic propagation of probability in traffic is a key issue. There is a strong influence from this issue to both the manner in which the spatiotemporal dependency is influenced and the extent to which stochastic variables can be dealt with generically. It may be that certain presumptions for dealing with uncertainty propagation may limit how stochastic variables are defined. Furthermore, each issue was found to affect the computation time of a model and in most cases contributes to a lower computational efficiency. During the development of the modelling approaches in this thesis, the described issues were taken into account as much as possible, while it was beyond the scope of the thesis to extensively and explicitly address all of the issues individually.

10.1.3 Modelling uncertainty and traffic fluctuations: the models

Two levels of modelling stochastics are considered in this thesis: *uncertainty* is considered in macroscopic stochastics, which describes day-to-day uncertainties *between* traffic flows in scenarios, and *fluctuations* is considered in microscopic stochastics, which describes microscopic variability *in* the traffic flow between vehicles. This distinction is necessary as both have inherently different consequences and require different modelling approaches, even if the source of the stochasticity is the same.

Two types of uncertainty modelling were presented in this thesis and reflect the main modelling approaches found in Chapter 2: *Monte Carlo modelling* and *'one shot' analytical modelling*. These approaches give an answer to research question 4: *How can uncertainty scenarios in traffic be modelled effectively?* The first type considered the application of Advanced Monte Carlo simulation to include uncertainty (Chapter 4). These are Monte Carlo techniques that make use of algorithms to spread samples for simulation and therefore require fewer samples to give a representative distribution. These techniques were investigated for their ability to reduce the computational load. A comparison was made in several experimental cases between these techniques and that of regular Monte Carlo simulation. Three approaches were analysed: *Latin Hypercube Sampling*, *Importance Sampling*, and *Sobol Quasi-random Sequencing*. The techniques were clearly shown to be stable and consistently able to improve the convergence of samples to a true distribution allowing for a reduction in computational load and to make stochastic and reliability analyses with Monte Carlo simulation in traffic modelling more applicable and efficient. This had not previously been demonstrated for traffic modelling. Sobol Quasi-random Sequences was clearly shown to be the most effective technique in the presented cases. The technique samples with an explicit spread from a set, however it also explicitly considers the consequential construction of the samples using an analytical sequence. For most indicators, the error level was a multifold smaller compared to Crude Monte Carlo. Latin Hypercube Sampling was most effective for multiple input variables. In the considered cases, two stochastic variables were considered, which proved to be sufficient for this stratified technique to substantially improve convergence, however not as well as the Sobol technique. Importance Sampling has a great potential to decrease computational load through capturing the extremities of a distribution, especially when the traffic system has an amplified effect on the outcome, as is often the case in congestion. The technique however is dependent on the applied estimator distribution, and did not perform as well in the presented cases. Application of an estimator method to optimize the estimator distribution is therefore essential. In this thesis, the importance of a reliable estimator function was demonstrated for Importance Sampling. Quasi-random sequencing is concluded to be most effective in general, due to a good spread in samples and a robust and simple application. However, other techniques may be applied and may be more effective depending on the specific application and variables considered.

The second type of uncertainty modelling considered a new modelling framework: the Core Probability Framework (Chapter 5). The Core Probability Framework (CPF) is a probabilistic framework for modelling multi-dimensional variations in capacity and traffic demand in dynamic macroscopic traffic flow. The CPF makes use of a propagation model, the *Discrete-Element Core Probability Model (DE-CPM)* that extends a base model, such as the Cell Transmission Model (CTM), by considering each traffic variable as a stochastic variable denoted as a probability distribution of the chance of values for each traffic variable. The CPF and DE-CPM extend current deterministic traffic flow models by redefining traffic variables in the core of the model as discrete distribution vectors of probable values for each traffic variable. Each discrete element in the distribution represents a single plausible scenario. In such a way, stochastic variation in traffic is internalised in the model and does away with the necessity of repetitive Monte Carlo simulation. Furthermore, a greater degree of flexibility in analysis is obtained, as each individual traffic variable in time and space may be given as a function of their probability. Moreover, the underlying distribution of each traffic variable in space and time is preserved such that the introduction of distribution fitting errors is limited to a minimum. Important issues facing stochastic traffic flow modelling: *computational efficiency*, *spatiotemporal dependency*, *stochastic propagation of probability*, and *stochastic generality*, were shown to be tackled by the CPF. The outcome of the calculation time tests on simple networks, compared to a CTM Monte Carlo model, showed that the DE-CPM has great potential to reduce computation times, in most cases by a factor 5-20, especially for larger networks and for greater levels of stochasticity. This is mainly due to the small marginal computational costs incurred when increasing the level of uncertainty in the discrete model. The DE-CPM addresses the other mentioned issues through the element based calculation using the so-called chain-rule, which requires the dependencies between variables to be dealt with externally and then explicitly maintains distributions of scenarios in the propagation of traffic through a network. We therefore conclude that the CPF and DE-CPM offer an alternative approach that tackles many existing issues in modelling uncertainty in traffic.

Modelling of fluctuations in traffic flow is presented in Chapter 6. There, a microscopic stochastic method to include stochastic vehicle specific behaviour and interaction was presented, which addresses research question 5: *How can stochastic fluctuations in traffic flow be modelled macroscopically?* The First Order Model with Stochastic Advection (FOMSA) is presented as a first order macroscopic kinematic wave model in a platoon-based Lagrangian coordinate system. Capturing micro-stochastic driving behaviour in a macroscopic model is important to accurately describe traffic flow phenomena on a macroscopic level. The proposed model makes use of first order traffic flow theory in conjunction with an additional invariant term, the vehicle specific invariant, which describes the heterogeneous effect of vehicle behaviour and the level of aggressiveness of drivers and represents the vehicle specific change to a deterministic density value. The use of Lagrangian coordinates was shown to allow characteristics of specific vehicles or vehicle-groups to propagate along with the traffic flow using the vehicle specific invariant and had previously been shown to lead also to more accurate results. The described model offers the advantages of including vehicle behaviour with an increased accuracy due to reduced diffusion effects, while doing this in a first order setting and therefore avoiding some of the complexity involved in second order model that are often applied to incorporate vehicle behaviour in macroscopic modelling. The model was demonstrated in an experimental case on a corridor with two bottlenecks present. The case demonstrated the face validity of the model and offered insight into the effects of different values for the model parameters. To include the effects of the capacity drop, further analysis was performed through two different approaches: *bounded acceleration* and *driver reaction times.* The investigation of bounded acceleration found that the application in the model under constrained conditions has a limited contribution to a capacity drop. Only under low acceleration bounds was there a substantial capacity drop visible. This led to the conclusion that the capacity drop is not merely a consequence of a restriction in the acceleration ability of vehicles on an individual basis. In the second approach, the effect of reactions times for accelerating vehicles out of congestion was analysed and successfully captured capacity drops for increasing reaction times. It also showed that the influence of heterogeneous traffic, through use of the invariant term, leads to lower capacities, while the capacity drop compared to a deterministic scenario is not increased.

10.1.4 Modelling uncertainty and traffic fluctuations: in practice

Putting the developed models into practice for traffic management requires a mandate for the necessity of traffic management at a location or area. While the locations of some problems are obvious, others are less so, especially in complex networks with highly heterogeneous traffic flow. In Chapter 7, a methodology is presented that evaluates the resilience level of road sections based on traffic flow stochastics. The methodology applies the *Link*

Performance Index for Resilience (LPIR), which evaluates the resilience level of individual road sections in relation to the wider road network. The focus of the methodology is on resilience and is therefore wider than robustness, as it also considers the ability of road sections to recover from disturbances as well as the classical robustness itself. Resilience is found in many transportation related disciplines, such as transport networks, freight movements and logistics, but it not explicitly commonplace in traffic flow analysis, while reliability and vulnerability and increasingly in robustness analysis are. Also when considering the effect of stochastics in traffic flow for performance, resilience is considered most relevant. A distinction was made between a resistance part and a recovery part as part of the entire methodology with a focus on homogenous and volatile traffic, which plays an important role in resilience. The resilience was calculated in relation to the traffic flow characteristics at a flow level and the ability of road sections to maintain their predefined purpose to serve vehicles without overly experiencing congestion. The method explicitly aimed at capturing the level of traffic heterogeneity. The effectiveness and validity of the methodology was demonstrated in an experimental case for a small network of two interconnecting motorways. This demonstrated the LPIRs ability to detect weakly resilient locations by calculating the relative resilient value of individual road sections. The calculated LPIR values were compared with the results of two other measures for resilience and robustness, namely the 'recovery time' and 'total delay'. Many locations that performed poorly in the LPIR were also highlighted in the other measures, however there were also important differences that further showed the strength of focussing on resilience. It was therefore concluded that the analysis of the resilience offers a deeper insight into the way road sections are judged for weakness compared to current approaches and that resilience analysis offers a complementary tool to robustness. This is especially the case when the analysis concentrates on the influence of disturbances on traffic flow at the level of traffic rather than at a higher abstraction level. The LPIR methodology also allows for a deeper analysis of the casualty of a poorly resilient road section. This can be performed through additional data analysis.

In Chapter 8, a demonstration of the entire model suite is given in a comprehensive case study for a real network and problem case. The case study was performed for the application of stochastic effects in traffic modelling to aid the application of traffic management. In this, the LPIR, Advanced Monte Carlo simulation, and FOMSA models were all applied. The application and selection of traffic management measures was applied in part using the Dutch 'Gebiedsgericht benutten' (GGB) methodology for the application of traffic management. In the case, the models showed they are able to perform well and demonstrated their value for their specific purposes and their ability to a-priori evaluate potential traffic management measures for sensitive road sections and carriageways. The importance of consideration of the stochastic influence of traffic was further demonstrated for both day-to-day variations as well as intraday and inter-vehicle stochastics for the outcome of studies. Failure to consider the stochastic effects would of have resulted in a bias of 26% for the speed of congestion spillback in the first sub-case and of 200% for the delay in the second sub-case.

10.1.5 Visual aids for effective communication of uncertainty in traffic

The sixth and final research question referred to the visualisation of uncertainty in traffic and asks: *What are effective options to visualise and communicate uncertainty from probabilistic traffic models?* An answer to this question was given in Chapter 9 of this thesis. Here, the results are given of an investigation into different methods to visualise uncertainty in static representations of macroscopic stochastic traffic model predictions on road networks. Several graphical uncertainty representations were developed and analysed in expert sessions. Following this, a selection of the initial set of uncertainty visualisations was evaluated in a cognitive alternative task-switching experiment. A conclusion from this was that the actual choice of a graphical uncertainty representation strongly depends on the desired type, speed and level of information disclosure and is therefore model and scenario dependant. Nevertheless, it was possible to find appropriate representations and it was shown that boxplot and line-style representations of uncertainty were generally favourable additions to current macroscopic model visualisations. An additional finding from the experiment was that participants were able to assess graphical representations relatively accurately. Further analysis on designing model visualisations showed that three main considerations should be taken into account. Firstly, it is important to avoid interference between the visual cues on different dimensions, secondly, to apply cues that allow explicit comparison or quantification where required, and thirdly, to be aware of the detrimental effects of clutter or overloading visualisations with too much information. The chapter concluded with a presentation of a set of feasible candidate visualisation for uncertainty in traffic model visualisation.

10.2 Practical implications

The research presented here is performed to aid the analysis and implementation of traffic management in practice. To that extent, the results of the research have evidential practical implications. The main practical implications for practice are discussed in this section.

10.2.1 Data analysis implications

The presented data analysis frameworks allow stochastic capacity and demand estimations to be made. The practical implementations from these frameworks are described as:

- Application of the frameworks in practice allows capacities to be estimated with greater detail and above all stochastically. Increased accuracy of capacity estimation can aid model application as more accurate capacity values should lead to more accurate predictions and easier calibration of models.

- Analysis of traffic networks and systems can be performed while taking the joint effects of capacity *and* demand variation into account. This is important, as the joint effects will often lead to unexpected, but more realistic results, as was shown with the precipitation case in Chapter 3. Such approaches can be applied for network broad analyses of traffic flow, such as for the analysis of existing traffic management measures, expansion of a network, or merely for an accurate analysis of the performance of a network.

- The performed cases in this thesis also have practical relevance. The stochastic capacity distributions found for the capacity for different day-types, and for the different weather types can be applied straight into models or in analyses requiring this type of data.

10.2.2 Modelling implications

In this thesis, three different modelling approaches and a network evaluation approach were presented. These all have separate practical implications as well as a combined relevance:

- The combination of the resilience methodology and both types of modelling approaches, for uncertainty and fluctuations, gives a complete procedure for analysing and a-priori evaluating the application of traffic management on a road network (see Figure 10.1). Their combined application can be applied by road authorities to improve traffic flow and network performance through analysis of roads and the evaluation and application of traffic management measures.

- Prior to the application of traffic management, it is useful to first evaluate the possible gains. As variations in traffic play an important part in this effectiveness, the application of Advanced Monte-Carlo simulations or the Core Probability Framework to estimate the effectiveness allows road authorities to more accurately apply traffic management measures than using models that do not consider uncertainty. This greatly improves their ability to come to an appropriate decision for action.

- Consideration of uncertainty in traffic modelling is not only relevant for traffic management, but for all forms of traffic modelling that considers the performance of a traffic network and considers different scenarios. For example, Cost Benefit Analyses often requires analysis of networks for differing scenarios, in which the presence of uncertainty is rife and is therefore suitable to apply one of the developed approaches.

- Besides uncertainty, also fluctuations in traffic flow between vehicles plays a role. Capturing and modelling these fluctuations with FOMSA allows analysis on a microscopic stochastic level to be performed of traffic flow, however using a macroscopic model. This allows stochastic dynamics in traffic flow to be modelled without the necessity of multiple simulations with differ seeds and also in a much shorter computation time. The model especially has potential for practical applications for multiple interacting disturbances in traffic flow and for the analysis of traffic flow phenomenon on networks, such as congestion shockwaves.

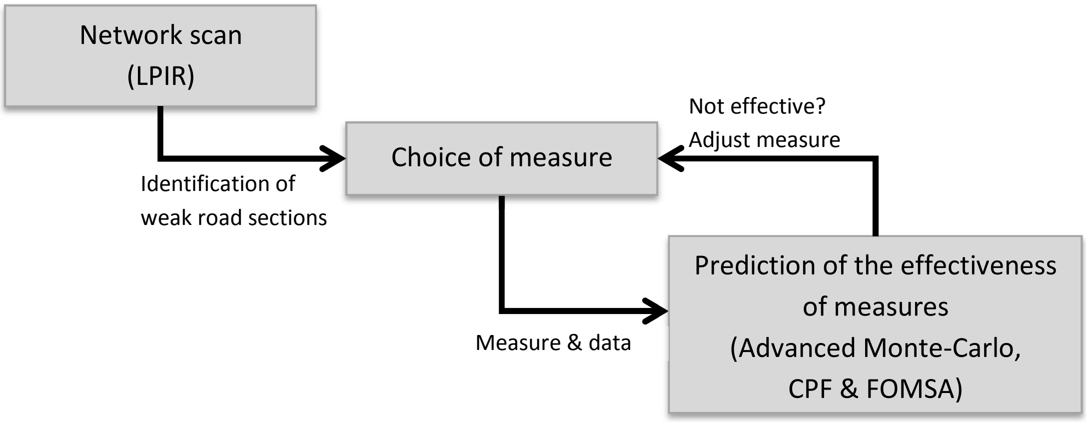


Figure 10.1: Analysis procedure for a-priori measure evaluation

10.2.3 Communication implications

Analysis and modelling with stochasticity is one thing, but being able to communicate the results of these processes is another, which is important if the approaches are to have practical value.

- The presented research on visualisation for uncertainty modelling of traffic gives tangible options to visualise the extent of uncertainty in static reports. This allows the results of uncertainty analysis to be presented to road authorities, managers and other decisions makers when making a case for specific measures.

- A number of considerations for communicating uncertainty are given in Chapter 9. These have practical implications for the consideration of communication techniques when attempting to communicate results of uncertain events or scenarios. Often the way results are presented plays just as large a role as the content of the results, therefore careful consideration of how they are presented can have a large influence on the outcome of a process.

10.3 Recommendations for future research

While certain questions have been answered and new insights gained on various aspects on the stochastic effects of traffic on traffic management, this research has also led to new questions and paths of thought that did not fit in the scope or time constraints of the research. These are presented in this section as recommendations for future research. The recommendations are again presented following the structure of the research objectives of this thesis.

10.3.1 The analysis of uncertainty and fluctuations in traffic

The main recommendations for analysis of uncertainty in the variables that influence traffic flow relate to the relationships between vehicles. While methodologies exist and analyses have been performed to give insights into single or a few variables, there is a need to consider the wider interdependencies of variables for their influence on traffic flow. This is also the case in reference to the uncertainties and fluctuations in these variables; how do these stochastic elements influence each other and finally the traffic system?

10.3.2 Modelling uncertainty and traffic fluctuations

In this thesis, a set of issues related to stochastic macroscopic traffic modelling were given. While solutions and improvements were given to tackle a number of them, further advances are still required and are recommended. Especially the issues: *correlations and spatiotemporal dependency, stochastic propagation of probability,* and *driving behaviour in macroscopic traffic,* still require further attention.

The possibilities of advanced Monte Carlo sampling were reviewed. However, there is still a need to further review which technique is most suited under which conditions. It may be that Quasi-random sequences are found to be most effective in all situations, however this has still to be investigated and further research on specific application of various techniques is therefore recommended.

The other uncertainty approach, the Core Probability Framework (CPF), introduced a novel approach to consider uncertainty in macroscopic models. The application of the Discrete-Element Core Probability Model (DE-CPM) showed promise, but is only one example of possible execution of the framework. Further research into extensions, additional models and more integrate approaches for the CPF is recommended with the hope that computation time may be further decreased and more complex modelling problems with more extensive consideration of uncertainty can be tackled.

The FOMSA model demonstrated a good ability to consider microscopic fluctuation in macroscopic traffic flow. Two expansions for future research are recommended. The first relates to a greater understanding and description of microscopic traffic dynamics in macroscopic modelling. Traffic flow dynamics can be complex, even on a microscopic scale. Scaling up to a macroscopic level can be even more challenging and will require further research to include more elements that are already known in microscopic modelling. Also, the changing dynamics of traffic flow with different vehicle populations and possibilities form a challenge, such as consideration of increased vehicle automation. The second challenge refers more directly to FOMSA. At the moment, the model is designed for motorway corridors. Expansion of the model to be efficiently applicable for (complex) networks and for urban networks is recommended. This is in theory very possible, but requires careful implementation and consideration of a number of other traffic aspects, such as interweaving traffic and intersections.

The Link Performance Indicator for Resilience (LPIR) was presented as an approach to evaluate resilience levels and identify hot spots for traffic management application. In the framework of the approach, a description was given of an additional aspect of the LPIR that allows causality of resilience weakness to be derived from data analysis. This was not further expanded on in this research and is left as a recommendation for further research. The analysis of resilience causality is an interesting area that can be a strong addition to the presented method, as it does not only return road sections that require attention, but also gives a strong indication of the reasons behind the lack of resilience allowing a road authority to act more precisely.

Finally, the comprehensive case study applied the models and used the GGB methodology for traffic management measure selection. The methodology was found to be extremely effective and relevant, however is in need of updating, especially in relation to the possibilities of floating devices, social media, and cooperative and automated driving. The increase in possibilities for communication and traffic flow guidance has further developed in past decades. Therefore, a recommendation is made to revise the current methodologies to include traffic fluctuations and the latest developments in traffic modelling.

10.3.3 Visual aids for effective communication of uncertainty in traffic

In this thesis, existing visualisation options were viewed for communication of uncertainty. It was beyond the scope of the thesis to design new visual cues for uncertainty representation for macroscopic models; however this is an interesting and maybe important area that should be considered. Making use of the main considerations for designs, it is recommended that new designs are made that comply with the findings presented in this research and can further aid the communication of uncertainty in traffic modelling. Furthermore, the focus in this research was on static reporting of model results. However, increasingly interactive platforms and methods of communication are advancing and it is therefore also recommended that a similar study is performed into the effectiveness of the visualisation of uncertainty for such platforms of communication.

Bibliography

Acevedo, D. & Laidlaw, D. H., 2006. Subjective quantification of perceptual interactions among some 2D scientific visualization methods. *Visualization and Computer Graphics, IEEE Transactions on,* 12**,** 1133-1140.

Agarwal, M., Maze, T. H. & Souleyrette, R., 2005. Impacts of weather on urban freeway traffic flow characteristics and facility capacity. Proceedings of the 2005 mid-continent transportation research symposium.

Al Hassan, Y. & Barker, D. J., 1999. The impact of unseasonable or extreme weather on traffic activity within Lothian region, Scotland. *Journal of Transport Geography,* 7**,** 209-213.

Ang, A. & Tang, W. 2007. *Probability Concepts in Engineering*, John Wiley & Sons, Inc.

Anton, H. 2010. *Elementary linear algebra*, Wiley.

Anwar, A., Nagel, T. & Ratti, C., 2014. Traffic Origins: A Simple Visualization Technique to Support Traffic Incident Analysis. Pacific Visualization Symposium (PacificVis), 2014 IEEE. IEEE, 316-319.

Arasan, V. T. & Koshy, R. Z., 2005. Methodology for modeling highly heterogeneous traffic flow. *Journal of Transportation Engineering,* 131**,** 544-551.

Aw, A. & Rascle, M., 2000a. Resurrection of "second order" models of traffic flow. *Siam Journal on Applied Mathematics,* 60**,** 916-938.

Aw, A. & Rascle, M., 2000b. Resurrection of" second order" models of traffic flow. *Siam Journal on Applied Mathematics***,** 916-938.

Bankes, S. C., 2010. Robustness, Adaptivity, and Resiliency Analysis. AAAI Fall Symposium: Complex Adaptive Systems.

Banks, J., 1991. The two-capacity phenomenon: some theoretical issues. *Transportation Research Record,* 1320**,** 83-90.

Batterman, S., Burke, J., Isakov, V., Lewis, T., Mukherjee, B. & Robins, T., 2014. A comparison of exposure metrics for traffic-related air pollutants: Application to epidemiology studies in Detroit, Michigan. *International journal of environmental research and public health,* 11**,** 9553-9577.

Bekaert, P., Sbert, M. & Willems, Y. D., 2000. Weighted importance sampling techniques for monte carlo radiosity. *Rendering Techniques***,** 35-46.

Bell, M., 2000. A game theory approach to measuring the performance reliability of transport networks. *Transportation Research Part B: Methodological,* 34**,** 533-545.

Bell, M. & Schmöcker, J., 2002. Network reliability: Topological effects and the importance of information. Actes de la conférence ICTTS2002, Guilin, China.

Bell, M. G., 1999. Measuring network reliability: a game theoretic approach. *Journal of Advanced Transportation,* 33**,** 135-146.

Bell, M. G. & Cassir, C., 2002. Risk-averse user equilibrium traffic assignment: an application of game theory. *Transportation Research Part B: Methodological,* 36**,** 671-681.

Bell, M. G. & Iida, Y. 1997. *Transportation network analysis,* Chichester, West Sussex, Wiley.

Bender, P., Mulle, J. A. & Schepperle, H., 2005. Enhancing user acceptance of traffic services uing imperfect information. *Proceedings of 16th European Regional Conference, Porto, Portugal***,** 4-6.

Bera, S. & Rao, K., 2011. Estimation of origin-destination matrix from traffic counts: the state of the art. *European Transport/Trasporti Europei* 49, 2-23.

Berdica, K., 2002. An introduction to road vulnerability: what has been done, is done and should be done. *Transport Policy,* 9**,** 117-127.

Berman, L., 1997. Accelerating Monte Carlo: quasirandom sequences and variance reduction. *Computational Finance,* 1 - 2.

Bertin, J. 1974. *Graphische Semiologie: Diagramme, Netze, Karten*, Walter de Gruyter.

Bigazzi, A. & Figliozzi, M., 2011. The impacts of stochastic capacity on freeway traffic flow benefits and costs: A model and a case study from Portland, Oregon. Integrated and Sustainable Transportation System (FISTS), 2011 IEEE Forum on. IEEE, 245-250.

Binder, K. & Heermann, D. 2010. *Monte Carlo simulation in statistical physics: an introduction*, Springer Science & Business Media.

Black, K. 2009. *Business statistics: Contemporary decision making,* Wiley.

Bliemer, M., Versteegt, H. & Castenmiller, R., 2004. INDY: a new analytical multiclass dynamic traffic assignment model. Proceedings of the TRISTAN V conference, Guadeloupe.

Bliemer, M. C. J. & Taale, H., 2006. Route generation and dynamic traffic assignment for large networks. Proceedings of the First International Symposium on Dynamic Traffic Assignment (DTA 2006), June 21-23 Leeds, UK. 90-99.

Böcker, L., Dijst, M. & Prillwitz, J., 2013. Impact of Everyday Weather on Individual Daily Travel Behaviours in Perspective: A Literature Review. *Transport Reviews,* 33**,** 71-91.

Boel, R. & Mihaylova, L., 2006. A compositional stochastic model for real time freeway traffic simulation. *Transportation Research Part B: Methodological,* 40**,** 319-334.

Boyles, S., Voruganti, A. & Waller, S., 2011. Quantifying distributions of freeway operational metrics. *Transportation Letters,* 3**,** 21-36.

Bratley, P. & Fox, B. L., 1988. Algorithm 659: Implementing Sobol's quasirandom sequence generator. *ACM Transactions on Mathematical Software (TOMS),* 14**,** 88-100.

Brilon, W., Geistefeldt, J. & Regler, M., 2005. Reliability of freeway traffic flow: a stochastic concept of capacity. Proceedings of the 16th International symposium on transportation and traffic theory.

Brilon, W. & Zurlinden, H., 2003. Breakdown Probability and Traffic Efficiency as Design Criteria for Freeways. *Forschung Strassenbau und Strassenverkehrstechnik,* 870.

Bruneau, M., Chang, S. E., Eguchi, R. T., Lee, G. C., O'rourke, T. D., Reinhorn, A. M., Shinozuka, M., Tierney, K., Wallace, W. A. & Von Winterfeldt, D., 2003. A framework to quantitatively assess and enhance the seismic resilience of communities. *Earthquake spectra,* 19**,** 733-752.

Bruyer, R. & Brysbaert, M., 2011. Combining speed and accuracy in cognitive psychology: is the inverse efficiency score (IES) a better dependent variable than the mean reaction time (RT) and the percentage of errors (PE)? *Psychologica Belgica***,** 5-13.

Buckley, J. J. 2005. *Fuzzy probabilities: new approach and applications,* Springer. 115.

Calvert, S. C. & Minderhoud, M. M., 2012. Generic Freeway Weaving Section Capacity Estimation Through Microsimulation. Transportation Research Board 91st Annual Meeting.

Calvert, S. C. & Snelder, M., 2013a. Influence of rain on motorway road capacity - A data-driven analysis. Intelligent Transportation Systems-(ITSC), 2013 16th International IEEE Conference on. IEEE, 1481-1486.

Calvert, S. C. & Snelder, M., 2013b. Influence of rain on motorway road capacity – a data-driven analysis. Proceedings of the 16th International IEEE Conference on Intelligent Transportation Systems (ITSC 2013), The Hague, The Netherlands, October, The Hague.

Calvert, S. C. & Snelder, M., 2016. Influence of Weather on Traffic Flow: an Extensive Stochastic Multi-effect Capacity and Demand Analysis. *European Transport,* 60(5).

Calvert, S. C., Taale, H. & Hoogendoorn, S. P., 2014a. Quantification of motorway capacity variation: influence of day type specific variation and capacity drop.

Calvert, S. C., Taale, H., Snelder, M. & Hoogendoorn, S. P., 2012. Probability in traffic: a challenge for modelling. Proceedings of the Fourth International Symposium on Dynamic Traffic Assignment (DTA 2012), June 4-6 MA, USA.

Calvert, S. C., Taale, H., Snelder, M. & Hoogendoorn, S. P., 2014b. Application of advanced sampling for efficient probabilistic traffic modelling. *Transportation Research Part C: Emerging Technologies*, 49**,** 87-102.

Calvert, S. C., Taale, H., Snelder, M. & Hoogendoorn, S. P., 2015. Vehicle Specific Behaviour in Macroscopic Traffic Modelling through Stochastic Advection Invariant. *Transportation Research Procedia,* 10**,** 71-81.

Cappé, O., Godsill, S. J. & Moulines, E., 2007. An overview of existing methods and recent advances in sequential Monte Carlo. *Proceedings of the IEEE,* 95**,** 899-924.

Carrion, C. & Levinson, D. M., 2012. A model of bridge choice across the mississippi river in minneapolis. *Network Reliability in Practice***,** 115-129.

Carswell, C. M. & Wickens, C. D., 1990. The perceptual interaction of graphical attributes: Configurality, stimulus homogeneity, and object integration. *Perception & Psychophysics,* 47**,** 157-168.

Chakravarti, I. M. & Laha, R. G. 1967. Handbook of methods of applied statistics. *Handbook of methods of applied statistics.* John Wiley & Sons.

Chang, C., Tung, Y. & Yang, J., 1994. Monte Carlo Simulation for Correlated Variables with Marginal Distributions. *Journal of Hydraulic Engineering,* 120**,** 313-331.

Chen, A., Kasikitwiwat, P. & Yang, C., 2013. Alternate capacity reliability measures for transportation networks. *Journal of Advanced Transportation,* 47**,** 79-104.

Chen, A., Yang, H., Lo, H. K. & Tang, W. H., 1999. A capacity related reliability for transportation networks. *Journal of Advanced Transportation,* 33**,** 183-200.

Chen, A., Yang, H., Lo, H. K. & Tang, W. H., 2002. Capacity reliability of a road network: an assessment methodology and numerical results. *Transportation Research Part B: Methodological,* 36**,** 225-252.

Chen, C., Skabardonis, A. & Varaiya, P., 2003. Travel-time reliability as a measure of service. *Transportation Research Record: Journal of the Transportation Research Board,* 1855**,** 74-79.

Chen, L. & Miller-Hooks, E., 2012. Resilience: an indicator of recovery capability in intermodal freight transport. *Transportation Science,* 46**,** 109-123.

Chen, Y., Bell, M. G. & Kaparias, I. 2007. Reliability analysis of road networks and preplanning of emergency rescue paths. *Critical Infrastructure.* Springer.

Chi, H., Beerli, P., Evans, D. W. & Mascagni, M. 2005. On the scrambled soboĺ sequence. *Computational Science–ICCS 2005.* Springer.

Chow, A. H., Lu, X.-Y. & Qiu, T. Z. 2009. *Empirical analysis of traffic breakdown probability distribution with respect to speed and occupancy,* California PATH Program, Institute of Transportation Studies, University of California at Berkeley.

Chung, E., Ohtani, O., Warita, H., Kuwahara, M. & Morita, H., 2005. Effect of rain on travel demand and traffic accidents. Intelligent Transportation Systems, 2005. Proceedings. 2005 IEEE. IEEE, 1080-1083.

Church, R. & Scaparra, M. P. 2007. Analysis of facility systems' reliability when subject to attack or a natural disaster. *Critical Infrastructure.* Springer.

Clark, S. & Watling, D., 2005. Modelling network travel time reliability under stochastic demand. *Transportation Research Part B: Methodological,* 39**,** 119-140.

Claudel, C. G. & Bayen, A. M., 2010. Lax–Hopf based incorporation of internal boundary conditions into Hamilton–Jacobi equation. Part I: Theory. *Automatic Control, IEEE Transactions on,* 55**,** 1142-1157.

Cools, M., Moons, E. & Wets, G., 2007. Investigating effect of holidays on daily traffic counts: time series approach. *Transportation Research Record: Journal of the Transportation Research Board,* 2019**,** 22-31.

Cools, M., Moons, E. & Wets, G., 2010. Assessing the impact of weather on traffic intensity. *Weather, Climate, and Society,* 2**,** 60-68.

Corthout, R., Himpe, W., Viti, F., Frederix, R. & Tampere, C. M., 2014. Improving the efficiency of repeated dynamic network loading through marginal simulation. *Transportation Research Part C: Emerging Technologies,* 41**,** 90-109.

Corthout, R., Tampère, C. M., Frederix, R. & Immers, L. H., 2011. Marginal dynamic network loading for large-scale simulation-based applications. 90th annual meeting of the Transportation Research Board.

Corthout, R., Tampère, C. M. & Immers, L. H., 2009. Marginal Incident Computation. *Transportation Research Record: Journal of the Transportation Research Board,* 2099**,** 22-29.

Costeseque, G. & Lebacque, J.-P., 2014. A variational formulation for higher order macroscopic traffic flow models: numerical investigation. *Transportation Research Part B: Methodological,* 70**,** 112-133.

Costeseque, G. & Lebacque, J.-P. 2015. Multi-anticipative Car-Following Behaviour: Macroscopic Modeling. *Traffic and Granular Flow'13.* Springer.

Cox, A., Prager, F. & Rose, A., 2011. Transportation security and the role of resilience: A foundation for operational metrics. *Transport Policy,* 18**,** 307-317.

Daganzo, C. F., 1994. The cell transmission model: A dynamic representation of highway traffic consistent with the hydrodynamic theory. *Transportation Research Part B: Methodological,* 28**,** 269-287.

Daganzo, C. F., 1995a. The cell transmission model, part II: network traffic. *Transportation Research Part B: Methodological,* 29**,** 79-93.

Daganzo, C. F., 1995b. Requiem for second-order fluid approximations of traffic flow. *Transportation Research Part B-Methodological,* 29**,** 277-286.

Daganzo, C. F., 2005. A variational formulation of kinematic waves: basic theory and complex boundary conditions. *Transportation Research Part B: Methodological,* 39**,** 187-196.

Daganzo, C. F., Cassidy, M. J. & Bertini, R. L., 1999. Possible explanations of phase transitions in highway traffic. *Transportation Research Part A: Policy and Practice,* 33**,** 365-379.

Davidson, P., 2011. A new approach to transport modelling-the Stochastic Segmented Slice Simulation (4S) model and its recent applications. In *Australian Transport Research Forum.*

Dror, M., L'ecuyer, P. & Szidarovszky, F. 2002. *Modeling Uncertainty: An Examination of Stochastic Theory, Methods, and Applications,* Springer. 46.

Ecola, L. & Wachs, M., 2012. Exploring the relationship between travel demand and economic growth. *Federal Highway Administration. The RAND Corporation, Santa Monica.*

El-Rashidy, R. A. & Grant-Muller, S. M., 2014. An assessment method for highway network vulnerability. *Journal of Transport Geography,* 34**,** 34-43.

Elefteriadou, L. & Heaslip, K., 2008. Field Data Collection and Analysis for Freeway Work Zone Capacity Estimation. *Transportation Research Center, University of Florida*.

Elefteriadou, L. & Lertworawanich, P., 2003. Defining, measuring and estimating freeway capacity. 82nd Annual Meeting of the Transportation Research Board, Washington, DC.

Elefteriadou, L., Roess, R. & Mcshane, W., 1995. Probabilistic nature of breakdown at freeway merge junctions. *Transportation Research Record***,** 80-89.

Ernst, J. M., Day, C. M., Krigmeier, J. V. & Bullock, D. M., 2012. Probe Data Sampling Guidelines for Characterizing Arterial Travel Time. 91th meeting of the Transportation Research Board, Washington DC.

Farrell, P. S., 1999. The hysteresis effect. *Human Factors: The Journal of the Human Factors and Ergonomics Society,* 41**,** 226-240.

Faturechi, R. & Miller-Hooks, E., 2014. Travel time resilience of roadway networks under disaster. *Transportation Research Part B: Methodological,* 70**,** 47-64.

Frederix, R., Viti, F. & Tampère, C. M., 2011. Dynamic origin–destination estimation in congested networks: theoretical findings and implications in practice. *Transportmetrica***,** 1-20.

Fuller, R., 2005. Towards a general theory of driver behaviour. *Accident analysis & prevention,* 37**,** 461-472.

Gao, S., Frejinger, E. & Ben-Akiva, M., 2011. Cognitive cost in route choice with real-time information: an exploratory analysis. *Procedia-Social and Behavioral Sciences,* 17**,** 136-149.

Geistefeldt, J. & Brilon, W., 2009. A comparative assessment of stochastic capacity estimation methods. *Transportation and Traffic Theory 2009: Golden Jubilee***,** 583-602.

Godunov, S. K., 1959. A difference method for numerical calculation of discontinuous solutions of the equations of hydrodynamics. *Matematicheskii Sbornik*, 89**,** 271-306.

Goldberg, M. A., 1975. On the inefficiency of being efficient. *Environment and Planning A,* 7**,** 921-939.

Govindarajulu, Z., 1999. Elements of sampling theory and methods. *Recherche,* 67**,** 02.

Graham, D. J. & Glaister, S., 2004. Road traffic demand elasticity estimates: a review. *Transport Reviews,* 24**,** 261-274.

Grubesic, T. H., Murray, A. T. & Mefford, J. N., 2007. Continuity in critical network infrastructures: accounting for nodal disruptions. *Critical Infrastructure***,** 197-220.

Guo, X., Yang, H. & Liu, T.-L., 2010. Bounding the inefficiency of logit-based stochastic user equilibrium. *European Journal of Operational Research,* 201**,** 463-469.

Hall, F. L. & Agyemang-Duah, K., 1991. Freeway capacity drop and the definition of capacity. *Transportation Research Record,* 1320.

Hanbali, R. M. & Kuemmel, D. A., 1993. Traffic volume reductions due to winter storm conditions. *Transportation Research Record*.

Helbing, D., Hennecke, A., Shvetsov, V. & Treiber, M., 2001. MASTER: macroscopic traffic simulation based on a gas-kinetic, non-local traffic model. *Transportation Research Part B: Methodological,* 35**,** 183-211.

Helbing, D. & Treiber, M., 1999. Numerical simulation of macroscopic traffic equations. *Computing in Science & Engineering,* 1**,** 89-98.

Heritage, A. 2011. *Dictionary of the English Language*, Houghton Mifflin Harcourt Publishing Company. Fifth Edition.

Herrera, J. C. & Bayen, A. M., 2010. Incorporation of Lagrangian measurements in freeway traffic state estimation. *Transportation Research Part B: Methodological,* 44**,** 460-481.

Hess, S., Train, K. E. & Polak, J. W., 2006. On the use of a Modified Latin Hypercube Sampling (MLHS) method in the estimation of a Mixed Logit model for vehicle choice. *Transportation Research Part B: Methodological,* 40**,** 147-163.

Hilbers, H., Van Eck, J. R. R., Snellen, D. & Langeweg, S. J. 2004. *Behalve de dagelijkse files: over betrouwbaarheid van reistijd*, NAi Uitgevers.

Hogema, J. 1996. Effects of rain on daily traffic volume and on driving behaviour. TNO rapport. TNO-TM-96-8019

Hoogendoorn, S., Landman, R., Van Kooten, J., Schreuder, M. & Adams, R., 2015. Design and Implementation of an Integrated Network Management Methodology in a Regional Network. Transportation Research Board 94th Annual Meeting.

Hoogendoorn, S., Landman, R., Van Kooten, J., Taale, H. & Schreuder, M., 2014. Integrated network management amsterdam: Towards a field operational test. Transportation Research Board 93rd Annual Meeting.

Hoogendoorn, S. P. & Bovy, P. H. L., 1998. Modeling multiple user-class traffic. *Transportation Research Record: Journal of the Transportation Research Board,* 1644**,** 57-69.

Hoogendoorn, S. P. & Bovy, P. H. L., 2001. State-of-the-art of vehicular traffic flow modelling. *Proceedings of the Institution of Mechanical Engineers Part I-Journal of Systems and Control Engineering,* 215**,** 283-303.

Hoogendoorn, S. P., Van Lint, H. & Knoop, V. L., 2008. Macroscopic modeling framework unifying kinematic wave modeling and three-phase traffic theory. *Transportation Research Record: Journal of the Transportation Research Board,* 2088**,** 102-108.

Hranac, R., Sterzin, E., Krechmer, D., Rakha, H. & Farzaneh, M. 2006. Empirical studies on traffic flow in inclement weather. Publication No. FHWA-HOP-07-073. Federal Highway Administration, Washington, DC.

Husdal, J., 2004. Reliability/vulnerability versus cost/benefit.

Iman, R. L. & Conover, W., 1980. Small sample sensitivity analysis techniques for computer models. with an application to risk assessment. *Communications in statistics-theory and methods,* 9**,** 1749-1842.

Ishfaq, R., 2012. Resilience through flexibility in transportation operations. *International Journal of Logistics Research and Applications,* 15**,** 215-229.

Jabari, S. E. & Liu, H. X., 2012. A stochastic model of traffic flow: Theoretical foundations. *Transportation Research Part B: Methodological,* 46**,** 156-174.

Jenelius, E., Petersen, T. & Mattsson, L.-G., 2006. Importance and exposure in road network vulnerability analysis. *Transportation Research Part A: Policy and Practice,* 40**,** 537-560.

Jia, A., Williams, B. M. & Rouphail, N. M., 2010. Identification and Calibration of Site-Specific Stochastic Freeway Breakdown and Queue Discharge. *Transportation Research Record: Journal of the Transportation Research Board,* 2188**,** 148-155.

Jin, P. J., Han, K. & Ran, B., 2014. Some Theoretical and Practical Perspectives of the Travel Time Kinematic Wave Model: Generalized Solution, Applications, and Limitations. Transportation Research Board 93rd Annual Meeting.

Joe, S. & Kuo, F. Y., 2003. Remark on algorithm 659: Implementing Sobol's quasirandom sequence generator. *ACM Transactions on Mathematical Software (TOMS),* 29**,** 49-57.

Joe, S. & Kuo, F. Y., 2008a. Constructing Sobol sequences with better two-dimensional projections. *SIAM Journal on Scientific Computing,* 30**,** 2635-2654.

Joe, S. & Kuo, F. Y., 2008b. Notes on generating Sobol sequences. *ACM Transactions on Mathematical Software (TOMS), 29(1), 49-57.*

Jonnalagadda, N., Freedman, J., Davidson, W. A. & Hunt, J. D., 2001. Development of microsimulation activity-based model for San Francisco: destination and mode choice models. *Transportation Research Record: Journal of the Transportation Research Board,* 1777**,** 25-35.

Kaplan, E. L. & Meier, P., 1958. Nonparametric estimation from incomplete observations. *Journal of the American statistical association,* 53**,** 457-481.

Keay, K. & Simmonds, I., 2005. The association of rainfall and other weather variables with road traffic volume in Melbourne, Australia. *Accident analysis & prevention,* 37**,** 109-124.

Kerner, B. S., 2000. Phase transitions in traffic flow. Traffic and Granular Flow. 253-283.

Kerner, B. S., 2013. Criticism of generally accepted fundamentals and methodologies of traffic and transportation theory: A brief review. *Physica A: Statistical Mechanics and its Applications,* 392**,** 5261-5282.

Kerner, B. S. & Rehborn, H., 1997. Experimental properties of phase transitions in traffic flow. *Physical Review Letters,* 79**,** 4030.

Kesting, A. & Treiber, M., 2008. How reaction time, update time, and adaptation time influence the stability of traffic flow. *Computer‐Aided Civil and Infrastructure Engineering,* 23**,** 125-137.

Khoshyaran, M. M. & Lebacque, J.-P. 2009. A stochastic macroscopic traffic model devoid of diffusion. *Traffic and Granular Flow’07.* Springer.

Kim, S. & Coifman, B., 2013. Freeway On-ramp Bottleneck Activation, Capacity and the Fundamental Relationship. *Procedia-Social and Behavioral Sciences,* 80**,** 698-716.

Kim, T. & Zhang, H., 2008. A stochastic wave propagation model. *Transportation Research Part B: Methodological,* 42**,** 619-634.

Knmi. 2014. *Meteological Data* [Online]. Available: http://www.knmi.nl/klimatologie/daggegevens/uitleg.html; http://www.knmi.nl/klimatologie/metadata/rotterdam.html.

Knoop, V. L., Snelder, M., Van Zuylen, H. J. & Hoogendoorn, S. P., 2012. Link-level vulnerability indicators for real-world networks. *Transportation Research Part A: Policy and Practice,* 46**,** 843-854.

Knospe, W., Santen, L., Schadschneider, A. & Schreckenberg, M., 2004. Empirical test for cellular automaton models of traffic flow. *Physical Review E,* 70**,** 016115.

Knottnerus, P. 2003. *Sample survey theory: some Pythagorean perspectives*, Springer Verlag.

Kondyli, A., Elefteriadou, L., Brilon, W., Hall, F. L., Persaud, B. & Washburn, S., 2013. Development and evaluation of methods for constructing breakdown probability models. *Journal of Transportation Engineering,* 139**,** 931-940.

Kroese, D. P., Taimre, T. & Botev, Z. I. 2011. *Handbook of Monte Carlo Methods*, Wiley. 706.

Kwon, T. J., Fu, L. & Jiang, C., 2013. Effect of winter weather and road surface conditions on macroscopic traffic parameters. *Transportation Research Record: Journal of the Transportation Research Board,* 2329**,** 54-62.

Landman, R., Hoogendoorn, S., Westerman, M., Hoogendoorn-Lanser, S. & Van Kooten, J., 2010. Design and implementation of integrated network management in the Netherlands. 89th Annual Meeting of the Transportation Research Board, Washington, DC.

Larson, V. E., Golaz, J. C., Jiang, H. & Cotton, W. R., 2005. Supplying local microphysics parameterizations with information about subgrid variability: Latin hypercube sampling. *Journal of the atmospheric sciences,* 62**,** 4010-4026.

Laval, J. A. 2004. *Hybrid models of traffic flow: impacts of bounded vehicle accelerations.* PhD Thesis, University of California, Berkeley.

Laval, J. A., 2006. Stochastic processes of moving bottlenecks: Approximate formulas for highway capacity. *Transportation Research Record: Journal of the Transportation Research Board,* 1988**,** 86-91.

Laval, J. A. & Leclercq, L., 2013. The Hamilton–Jacobi partial differential equation and the three representations of traffic flow. *Transportation Research Part B: Methodological,* 52**,** 17-30.

Lebacque, J.-P., 1996a. The Godunov scheme and what it means for first order traffic flow models. Internaional symposium on transportation and traffic theory. 647-677.

Lebacque, J.-P. & Khoshyaran, M. M., 2013. A variational formulation for higher order macroscopic traffic flow models of the GSOM family. *Transportation Research Part B: Methodological,* 57**,** 245-265.

Lebacque, J.-P., Mammar, S. & Salem, H. H., 2007a. Generic second order traffic flow modelling. Transportation and Traffic Theory 2007. Papers Selected for Presentation at ISTTT17.

Lebacque, J., 1996b. The Godunov scheme and what it means for first order traffic flow models. Internaional symposium on transportation and traffic theory. 647-677.

Lebacque, J., 2003. Two-phase bounded-acceleration traffic flow model: analytical solutions and applications. *Transportation Research Record: Journal of the Transportation Research Board,* 1852**,** 220-230.

Lebacque, J. 2005. Intersection modeling, application to macroscopic network traffic flow models and traffic management. *Traffic and Granular Flow'03.* Springer.

Lebacque, J. P., Mammar, S. & Salem, H. H., 2007b. Generic second order traffic flow modelling. Transportation and Traffic Theory: Proceedings of the 17th International Symposium on Transportation and Traffic Theory, London, UK.

Leclercq, L., 2007a. Bounded acceleration close to fixed and moving bottlenecks. *Transportation Research Part B: Methodological,* 41**,** 309-319.

Leclercq, L., 2007b. A new numerical scheme for bounding acceleration in the LWR model. 4th IMA International Conference on Mathematics in Transport.

Leclercq, L., Laval, J. A. & Chevallier, E., 2007. The Lagrangian coordinates and what it means for first order traffic flow models. Transportation and Traffic Theory 2007. Papers Selected for Presentation at ISTTT17.

Leclercq, L., Laval, J. A. & Chiabaut, N., 2011. Capacity drops at merges: An endogenous model. *Transportation Research Part B: Methodological,* 45**,** 1302-1313.

Li, J., Chen, Q. Y., Wang, H. & Ni, D. 2009. Analysis of LWR model with fundamental diagram subject to uncertainties. *The 88th Transportation Research Board Annual Meeting.* Washington DC.

Li, M. 2008. Combining DTA Approaches for Studying Road Network Robustness. TRAIL Thesis Series.

Lighthill, M. J. & Whitham, G. B., 1955. On Kinematic Waves .2. A Theory of Traffic Flow on Long Crowded Roads. *Proceedings of the Royal Society of London Series a-Mathematical and Physical Sciences,* 229**,** 317-345.

Lin, W.-H., 2001. A Gaussian maximum likelihood formulation for short-term forecasting of traffic flow. Intelligent Transportation Systems, 2001. Proceedings. 2001 IEEE. IEEE, 150-155.

Lo, H. K. & Tung, Y.-K., 2003. Network with degradable links: capacity analysis and design. *Transportation Research Part B: Methodological,* 37, 345-363.

Lorenz, M. R. & Elefteriadou, L. 2000. A probabilistic approach to defining freeway capacity and breakdown. *In Proceedings of the 4th International Symposium on Highway Capacity (pp. 84-95). Washington, DC, USA: Transportation Research Board.*

Lorenz, M. R. & Elefteriadou, L., 2001. Defining freeway capacity as function of breakdown probability. *Transportation Research Record: Journal of the Transportation Research Board,* 1776, 43-51.

Maceachren, A. M., Roth, R. E., O'brien, J., Li, B., Swingley, D. & Gahegan, M., 2012. Visual Semiotics & Uncertainty Visualization: An Empirical Study. *Visualization and Computer Graphics, IEEE Transactions on,* 18, 2496-2505.

Mahmassani, H. S., Hou, T. & Dong, J., 2012. Characterizing Travel Time Variability in Vehicular Traffic Networks. *Transportation Research Record: Journal of the Transportation Research Board,* 2315, 141-152.

Makigami, Y., Newell, G. F. & Rothery, R., 1971. Three-dimensional representation of traffic flow. *Transportation Science,* 5, 302-313.

Mallikarjuna, C. & Rao, K. R., 2009. Cellular automata model for heterogeneous traffic. *Journal of Advanced Transportation,* 43, 321-345.

Maze, T. H., Agarwai, M. & Burchett, G., 2006. Whether weather matters to traffic demand, traffic safety, and traffic operations and flow. *Transportation Research Record: Journal of the Transportation Research Board,* 1948, 170-176.

Mckay, M. D., Beckman, R. J. & Conover, W., 1979. A comparison of three methods for selecting values of input variables in the analysis of output from a computer code. *Technometrics*, 239-245.

Metropolis, N. & Ulam, S., 1949. The monte carlo method. *Journal of the American statistical association,* 44, 335-341.

Minasny, B. & Mcbratney, A. B., 2006. A conditioned Latin hypercube method for sampling in the presence of ancillary information. *Computers & geosciences,* 32, 1378-1388.

Minderhoud, M. M., Botma, H. & Bovy, P. H., 1997. Assessment of roadway capacity estimation methods. *Transportation Research Record: Journal of the Transportation Research Board,* 1572, 59-67.

Mirchandani, P. & Soroush, H., 1987. Generalized traffic equilibrium with probabilistic travel times and perceptions. *Transportation Science,* 21**,** 133-152.

Monsell, S., 2003. Task switching. *Trends in cognitive sciences,* 7**,** 134-140.

Moskowitz, K., 1965. Discussion of ‘freeway level of service as influenced by volume and capacity characteristics’ by DR Drew and CJ Keese. *Highway Research Record,* 99**,** 43-44.

Muñoz, L., Sun, X., Horowitz, R. & Alvarez, L., 2003. Traffic density estimation with the cell transmission model. IEEE, 3750-3755 vol. 5.

Murray-Tuite, P. M., 2006. A comparison of transportation network resilience under simulated system optimum and user equilibrium conditions. Simulation Conference, 2006. WSC 06. Proceedings of the Winter. IEEE, 1398-1405.

Murray-Tuite, P. M. & Mahmassani, H. S., 2004. Methodology for determining vulnerable links in a transportation network. *Transportation Research Record: Journal of the Transportation Research Board,* 1882**,** 88-96.

Newell, G. F., 1993. A simplified theory of kinematic waves in highway traffic, Part I: General theory. *Transportation Research Part B: Methodological,* 27**,** 281-287.

Ng, M. & Waller, S. T., 2012. A dynamic route choice model considering uncertain capacities. *Computer‐Aided Civil and Infrastructure Engineering,* 27**,** 231-243.

Nicholson, A., 2007. Road network unreliability: impact assessment and mitigation. *International journal of critical infrastructures,* 3**,** 346-375.

Nicholson, A., Schmöcker, J., Bell, M. & Iida, Y., 2003. Assessing transport reliability: malevolence and user knowledge. Network Reliability of Transport. Proceedings of the 1st International Symposium on Transportation Network Reliability (INSTR).

O’kelly, M. E. & Kim, H. 2007. Survivability of commercial backbones with peering: a case study of Korean networks. *Critical Infrastructure.* Springer.

Omnitrans. 2015. http://www.omnitrans-international.com/. 2015].

Papageorgiou, M., 1998. Some remarks on macroscopic traffic flow modelling. *Transportation Research Part a-Policy and Practice,* 32**,** 323-329.

Payne, H. J., 1971. Models of freeway traffic and control. *Mathematical models of public systems*.

Peeta, S. & Yu, J. W., 2005. A hybrid model for driver route choice incorporating en-route attributes and real-time information effects. *Networks and Spatial Economics,* 5**,** 21-40.

Persaud, B., Yagar, S. & Brownlee, R., 1998. Exploration of the breakdown phenomenon in freeway traffic. *Transportation Research Record: Journal of the Transportation Research Board,* 1634**,** 64-69.

Polus, A. & Pollatschek, M. A., 2002. Stochastic nature of freeway capacity and its estimation. *Canadian Journal of Civil Engineering,* 29**,** 842-852.

Potter, K., Rosen, P. & Johnson, C. R. 2012. From quantification to visualization: A taxonomy of uncertainty visualization approaches. *Uncertainty Quantification in Scientific Computing.* Springer.

Ravden, S. & Johnson, G. 1989. *Evaluating usability of human-computer interfaces: a practical method,* Halsted Press.

Reggiani, A., 2013. Network resilience for transport security: Some methodological considerations. *Transport Policy,* 28**,** 63-68.

Reggiani, A., De Graaff, T. & Nijkamp, P., 2002. Resilience: an evolutionary approach to spatial economic systems. *Networks and Spatial Economics,* 2**,** 211-229.

Richards, P. I., 1956. Shock waves on the highway. *Operations research,* 4**,** 42-51.

Rijkswaterstaat 2003. Handbook Sustainable Traffic Management - A guide for users. . Rotterdam, The Netherlands.: AVV Transport Research Centre. .

Rijkswaterstaat 2015. Autosnelwegen Capaciteitswaarden Infrastructuur (CIA), Handboek Versie 4. Rijkswaterstaat, The Netherlands.

Roess, R. P. & Ulerio, J. M., 2009. Level of service analysis of freeway weaving segments. *Transportation Research Record: Journal of the Transportation Research Board,* 2130**,** 25-33.

Rogers, R. D. & Monsell, S., 1995. Costs of a predictible switch between simple cognitive tasks. *Journal of experimental psychology: General,* 124**,** 207.

Rose, A. Z., 2009. Economic resilience to disasters. *Community and Regional Resilience Institute Report No. 8.*

Ross, S. M. 2009. *Introduction to probability and statistics for engineers and scientists,* Academic Press.

Rubinstein, R. Y., 1981. Simulation and the Monte Carlo Method. John Wiley&Sons. *New York*.

Ryu, S., Chen, A. & Choi, K., 2015. Solving the stochastic multi-class traffic assignment problem with asymmetric interactions, route overlapping, and vehicle restrictions. *Journal of Advanced Transportation*.

Salling, K. B. & Leleur, S., 2011. Transport appraisal and Monte Carlo simulation by use of the CBA-DK model. *Transport Policy,* 18**,** 236-245.

Sanyal, J., Zhang, S., Bhattacharya, G., Amburn, P. & Moorhead, R. J., 2009. A user study to compare four uncertainty visualization methods for 1d and 2d datasets. *Visualization and Computer Graphics, IEEE Transactions on,* 15**,** 1209-1218.

Sarewitz, D., Pielke, R. & Keykhah, M., 2003. Vulnerability and risk: some thoughts from a political and policy perspective. *Risk analysis,* 23**,** 805-810.

Sarvi, M., 2013. Freeway weaving phenomena observed during congested traffic. *Transportmetrica A: Transport Science,* 9**,** 299-315.

Scott, D. M., Novak, D. C., Aultman-Hall, L. & Guo, F., 2006. Network robustness index: a new method for identifying critical links and evaluating the performance of transportation networks. *Journal of Transport Geography,* 14**,** 215-227.

Shawky, M. & Nakamura, H., 2007. Characteristics of breakdown phenomenon in merging sections of urban expressways in Japan. *Transportation Research Record: Journal of the Transportation Research Board,* 2012**,** 11-19.

Smith, B. L., Byrne, K. G., Copperman, R. B., Hennessy, S. M. & Goodall, N. J., 2004. An investigation into the impact of rainfall on freeway traffic flow. 83rd annual meeting of the Transportation Research Board, Washington DC.

Smith, P. J., Shafi, M. & Gao, H., 1997. Quick simulation: A review of importance sampling techniques in communications systems. *Selected Areas in Communications, IEEE Journal on,* 15**,** 597-613.

Snelder, M. & Calvert, S. C., 2016. Quantifying the impact of adverse weather conditions on a road network performance. *European Journal of Transport and Infrastructure Research,* 16(1), 128-149.

Snelder, M., Van Zuylen, H. & Immers, L., 2012. A framework for robustness analysis of road networks for short term variations in supply. *Transportation Research Part A: Policy and Practice,* 46**,** 828-842.

Sobol, I. M., 1967. On the distribution of points in a cube and the approximate evaluation of integrals. *USSR Computational Mathematics and Mathematical Physics,* 7**,** 86-112.

Srinivasan, K., 2002. Transportation Network Vulnerability Assessment: A Quantative Framework. *SECURITY PAPERS*, 60.

Srivastava, A. & Geroliminis, N., 2013. Empirical observations of capacity drop in freeway merges with ramp control and integration in a first-order model. *Transportation Research Part C: Emerging Technologies,* 30**,** 161-177.

Sullivan, J., Novak, D., Aultman-Hall, L. & Scott, D. M., 2010. Identifying critical road segments and measuring system-wide robustness in transportation networks with isolating links: A link-based capacity-reduction approach. *Transportation Research Part A: Policy and Practice,* 44**,** 323-336.

Sumalee, A., Zhong, R., Pan, T. & Szeto, W., 2011. Stochastic cell transmission model (SCTM): A stochastic dynamic traffic model for traffic state surveillance and assignment. *Transportation Research Part B: Methodological,* 45**,** 507-533.

Sun, X., Muñoz, L. & Horowitz, R., 2003. Highway traffic state estimation using improved mixture Kalman filters for effective ramp metering control. IEEE, 6333-6338 Vol. 6.

Szeto, W., Jiang, Y. & Sumalee, A., 2011. A Cell-Based Model for Multi-class Doubly Stochastic Dynamic Traffic Assignment. *Computer-Aided Civil and Infrastructure Engineering,* 26**,** 595-611.

Tamminga, G., Maton, J., Poorteman, R. & Zee, J., 2005. De Robuustheidscanner-Robuustheid van netwerken: een modelmatige verkenning. *Grontmij Nederland, iov Adviesdienst Verkeer & Vervoer*.

Tampère, C. & Viti, F., 2010. Dynamic Traffic Assignment under Equilibrium and Non-equilibrium: Do We Need a Paradigm Shift? 3rd International Symposium on Dynamic Traffic Assignment.

Tampere, C. M., Stada, J., Immers, B., Peetermans, E. & Organe, K., 2007a. Methodology for identifying vulnerable sections in a national road network. *Transportation Research Record: Journal of the Transportation Research Board,* 2012**,** 1-10.

Tampère, C. M. J., Corthout, R., Cattrysse, D. & Immers, L. H., 2011. A generic class of first order node models for dynamic macroscopic simulation of traffic flows. *Transportation Research Part B: Methodological,* 45**,** 289-309.

Tampere, C. M. J., Stada, J., Immers, B., Peetermans, E. & Organe, K., 2007b. Methodology for identifying vulnerable sections in a national road network. *Transportation Research Record: Journal of the Transportation Research Board*, 2012**,** 1-10.

Taylor, M. A. & D'este, G., 2003. Concepts of network vulnerability and applications to the identification of critical elements of transport infrastructure. New Zealand Transport Research Forum.

Taylor, M. A. & D’este, G. M. 2007. *Transport network vulnerability: a method for diagnosis of critical locations in transport infrastructure systems*, Springer.

Theis, S. W. 2011. *Comparing efficiency and cognitive load of uncertain data visualizations,* CWI. Information Systems [INS].

Thomas, T., Weijermars, W. & Berkum, E., 2008. Variations in urban traffic volumes. *European Journal of Transport and Infrastructure Research,* 8**,** 251-263.

Thomson, J., Hetzler, E., Maceachren, A., Gahegan, M. & Pavel, M., 2005. A typology for visualizing uncertainty. Electronic Imaging 2005. International Society for Optics and Photonics, 146-157.

Tno. 2014. *CommonSense* [Online]. Available: http://tnopresenter.nl/commonsense/.

Toledo, T., 2007. Driving behaviour: models and challenges. *Transport Reviews,* 27**,** 65-84.

Treiber, M., Kesting, A. & Helbing, D., 2006. Delays, inaccuracies and anticipation in microscopic traffic models. *Physica A: Statistical Mechanics and its Applications,* 360**,** 71-88.

Tu, H., Li, H., Van Lint, H. & Van Zuylen, H., 2012. Modeling travel time reliability of freeways using risk assessment techniques. *Transportation Research Part A: Policy and Practice,* 46**,** 1528-1540.

Tu, H., Van Rij, M., Henkens, N. & Heikoop, H., 2010. Empirical investigation on stochastic Dutch freeway capacity. Intelligent Transportation Systems (ITSC), 2010 13th International IEEE Conference on. IEEE, 825-830.

Tu, H. Z., Van Lint, J. W. C. & Van Zuylen, H. J., 2007. Impact of traffic flow on travel time variability of freeway corridors. *Transportation Research Record***,** 59-66.

Tufte, E. R. & Graves-Morris, P. 1983. *The visual display of quantitative information*, Graphics press Cheshire, CT. 2.

Van Arem, B. & Van Der Vlist, M. J. 1992. An on-line procedure for estimating current capacity. Rapport. No. 92/NV/141.

Van Lint, J. W. C., Hoogendoorn, S. P. & Schreuder, M., 2008. FASTLANE New Multiclass First-Order Traffic Flow Model. *Transportation Research Record***,** 177-187.

Van Lint, J. W. C., Miete, O., Taale, H. & Hoogendoorn, S. P., 2012. A systematic framework for the assessment of traffic measures and policies on the reliability of traffic operations and travel time. *91th meeting of the Transportation Research Board*.

Van Stralen, W., Calvert, S. C. & Molin, E., 2014. The influence of adverse weather conditions on the probability of congestion on Dutch motorways. the 91st Annual Meeting of the TRB, January Washington DC.

Van Stralen, W., Calvert, S. C. & Molin, E. J. E., 2015. The influence of adverse weather conditions on probability of congestion on Dutch motorways. *European Journal of Transport and Infrastructure Research* (accepted for publication).

Van Wageningen-Kessels, F., Van Lint, H., Hoogendoorn, S. P. & Vuik, K., 2010. Lagrangian formulation of multiclass kinematic wave model. *Transportation Research Record: Journal of the Transportation Research Board,* 2188**,** 29-36.

Van Wageningen-Kessels, F., Van Lint, H., Hoogendoorn, S. P. & Vuik, K., 2014. New Generic Multiclass Kinematic Wave Traffic Flow Model. *Transportation Research Record: Journal of the Transportation Research Board,* 2422**,** 50-60.

Van Wageningen-Kessels, F., Van Lint, J., Hoogendoorn, S. & Vuik, C., 2009. Implicit time stepping schemes applied to the kinematic wave model in Lagrangian coordinates. Proceedings of traffic and granular flow.

Van Wageningen-Kessels, F., Yuan, Y., Hoogendoorn, S. P., Van Lint, H. & Vuik, K., 2013. Discontinuities in the Lagrangian formulation of the kinematic wave model. *Transportation Research Part C: Emerging Technologies,* 34**,** 148-161.

Vlahogianni, E. I., Karlaftis, M. G. & Golias, J. C., 2005. Optimized and meta-optimized neural networks for short-term traffic flow prediction: A genetic approach. *Transportation Research Part C: Emerging Technologies,* 13**,** 211-234.

Vukovic, D., Adler, M. W. & Vonk, T. 2013. Neerslag en Verkeer. TNO-rapport TNO-060-DTM-2013-0023102.

Wagner, P., 2012. Analyzing fluctuations in car-following. *Transportation Research Part B: Methodological,* 46**,** 1384-1392.

Wakabayashi, H. & Ilda, Y., 1992. Upper and lower bounds of terminal reliability of road networks: an efficient method with Boolean algebra. *Journal of Natural Disaster Science,* 14**,** 29-44.

Wang, Z., Chan, A. P., Yuan, J., Xia, B., Skitmore, M. & Li, Q., 2014. Recent Advances in Modeling the Vulnerability of Transportation Networks. *Journal of Infrastructure Systems,* 21(2), 06014002.

Ware, C. 2012. *Information visualization: perception for design*, Elsevier.

Weng, J. & Yan, X., 2015. Probability distribution-based model for work zone capacity prediction. *Journal of Advanced Transportation*.

Wieland, A. & Wallenburg, C. M., 2012. Dealing with supply chain risks: Linking risk management practices and strategies to performance. *International Journal of Physical Distribution & Logistics Management,* 42**,** 887-905.

Wolfe, J. M. & Horowitz, T. S., 2004. What attributes guide the deployment of visual attention and how do they do it? *Nature Reviews Neuroscience,* 5**,** 495-501.

Wu, N., 2013. A Stochastic Model for Reliability Analysis in Freeway Networks. *Procedia-Social and Behavioral Sciences,* 96**,** 2823-2834.

Yeon, J., Hernandez, S. & Elefteriadou, L., 2009. Differences in freeway capacity by day of the week, time of day, and segment type. *Journal of Transportation Engineering,* 135**,** 416-426.

Yin, Y. & Ieda, H., 2001. Assessing performance reliability of road networks under nonrecurrent congestion. *Transportation Research Record: Journal of the Transportation Research Board,* 1771**,** 148-155.

Yperman, I., 2007. The Link Transmission Model for dynamic network loading. *Ph.D. Thesis, Katholieke Universiteit Leuven.*

Zahavi, Y., Beckmann, M. J. & Golob, T. F. 1981. The UMOT/urban interactions.

Zhang, H. M., 2002. A non-equilibrium traffic model devoid of gas-like behavior. *Transportation Research Part B: Methodological,* 36**,** 275-290.

Zhang, K., Mahmassani, H. S. & Lu, C. C., 2008. Probit-based time-dependent stochastic user equilibrium traffic assignment model. *Transportation Research Record: Journal of the Transportation Research Board,* 2085**,** 86-94.

Zhang, L. & Levinson, D. M., 2003. Ramp metering and the capacity of active freeway bottlenecks. *Transportation Research: A Policy and Practice,,* 44**,** 218-235.

Zuk, T. & Carpendale, S., 2006. Theoretical analysis of uncertainty visualizations. Electronic Imaging 2006. International Society for Optics and Photonics, 606007-606007-14.

Summary

When congestion becomes a problem on a road or road network, there are generally three main solution areas available to tackle it: construction, pricing or traffic management. Traffic management became an increasingly preferred option towards the end of the twentieth century as an alternative to construction in many cases. Traffic management proves a more efficient alternative and focusses on influencing traffic flows such that the existing road and network capacity is more effectively utilised resulting in a reduction in congestion. The effectiveness of traffic management is dependent on the ability to influence traffic flow. However, traffic contains a relatively large amount of stochastic behaviour, which is connected to human driving behaviour. The fluctuations that occur in traffic flow due to this stochastic behaviour have a large effect on the effectiveness of traffic management. Furthermore, uncertainty between time dependant scenarios has also shown to have a large influence on the outcome of the analysis of traffic management measures. In the past, little attention has been paid to these effects. Therefore, the main objective of this thesis is to give insight into the stochastic fluctuations and uncertainty in traffic flow for the application of traffic management measures and to propose tools that allow these effects to be analysed and subsequently modelled in aggregated macroscopic flows. In doing this, the necessity to consider uncertainty and fluctuations for traffic management is also demonstrated. Stochastic processes are considered as *uncertainty*, which describes day-to-day uncertainties *between* traffic flows, and *fluctuations*, which describes microscopic variability *in* the traffic flow. Three main areas are focussed on: the analysis of variations in traffic, modelling fluctuations and uncertainty in traffic, and the visual communication of uncertainty from traffic models.

Analysis

Analysis of stochastic characteristics of the variables that influence traffic flow is necessary in order to understand and model uncertainty and fluctuations. Both traffic demand and capacity are directly affected by such variables. Two methodological frameworks are presented for stochastic analysis of traffic; one based on stochastic capacity and the other on combined stochastic demand and capacity. The first methodological framework is a conceptual model for practical stochastic capacity estimation that allows the stochastic nature of capacities to be captured and quantified based on the adapted Product Limit Method (PLM) and quantified as a distribution. The second framework considers the joint stochastic effect of demand and capacity on traffic flow. The methodology applied the same capacity approach as the first framework and derived demand distributions through an empirical process of cordoned

observations. Considering demand and capacity together is found to give a deeper and holistic understanding of the effect of variation on the traffic flow system.

Models

The developed models consider the distinction between uncertainty and fluctuations, which is necessary as both have inherently different consequences and require different modelling approaches, even if the source of the stochasticity is the same. Two types of uncertainty modelling are presented in this thesis: *Monte Carlo modelling* and *'one shot' analytical modelling*. The first type considers the application of Advanced Monte Carlo simulation to include uncertainty, which makes use of algorithms to spread samples of variables and requires fewer samples to give a representative distribution. These techniques were investigated for their ability to reduce the computational load in uncertainty scenario modelling. Three approaches were analysed: *Latin Hypercube Sampling*, *Importance Sampling*, and *Sobol Quasi-random Sequencing*. The techniques were shown to be stable and consistently able to improve the convergence to a true distribution allowing for a reduction in computational load and to make Monte Carlo simulation in traffic modelling more applicable and efficient. These techniques had not previously been demonstrated for traffic modelling. Of the analysed techniques, Sobol Quasi-random Sequences are shown to be the most effective.

The second type of uncertainty modelling considered a new modelling framework: The Core Probability Framework (CPF), which is a probabilistic framework for modelling multi-dimensional variations in capacity and traffic demand in dynamic macroscopic traffic flow. The CPF makes use of a propagation model, the *Discrete-Element Core Probability Model (DE-CPM)* that extends a base model, such as the Cell Transmission Model (CTM), by considering each traffic variable as a stochastic variable denoted as a probability distribution of the chance of values for each traffic variable. The CPF and DE-CPM extend current deterministic traffic flow models by redefining traffic variables in the core of the model as discrete distribution vectors of probable values for each traffic variable. Important issues facing stochastic traffic flow modelling: *computational efficiency*, *spatiotemporal dependency*, *stochastic propagation of probability*, and *stochastic generality*, were shown to be tackled by the CPF. The outcome of the calculation time tests on simple networks, compared to a CTM Monte Carlo model, showed that the DE-CPM has great potential to reduce computation times, especially for larger networks and for greater levels of stochasticity.

Modelling of fluctuations in traffic flow is considered in the developed First Order Model with Stochastic Advection (FOMSA). This is a first order macroscopic kinematic wave model in a platoon-based Lagrangian coordinate system. Capturing micro-stochastic driving behaviour in a macroscopic model is important to accurately describe traffic flow phenomena on a macroscopic level. The proposed model makes use of first order traffic flow theory in conjunction with an additional invariant term, the vehicle specific invariant, which describes the heterogeneous effect of vehicle behaviour and the level of aggressiveness of drivers and represents the vehicle specific change to a deterministic density value. To include the effects of the capacity drop, further analysis was performed through two different approaches: bounded acceleration and driver reaction times. The model offers the advantages of including vehicle behaviour with an increased accuracy due to reduced diffusion effects, while doing this in a first order setting and therefore avoiding some of the complexity involved in second order model that are often applied to incorporate vehicle behaviour in macroscopic modelling.

To detect road sections requiring attention to improve throughput, a methodology is presented that evaluates the resilience level of road sections based on traffic flow stochastics. The methodology applies the developed *Link Performance Index for Resilience (LPIR)*, which evaluates the resilience level of individual road sections in relation to a wider road network. The focus of the methodology is on resilience and makes a distinction between a resistance part and a recovery part as part of the entire methodology with a focus on homogenous and volatile traffic. The resilience is calculated in relation to the traffic flow characteristics at a flow level and the ability of road sections to maintain their predefined purpose to serve vehicles without overly experiencing congestion. The method is explicitly aimed at capturing the level of traffic heterogeneity. A demonstration of the entire model suite is given in a comprehensive case study for a real network and problem case of the A20 North ring road of Rotterdam and was performed for the application of stochastic effects in traffic modelling to aid the application of traffic management. In this, the LPIR, Advanced Monte Carlo simulation, and FOMSA models were all applied. The models showed they are able to perform well and demonstrated their value for their specific purposes and their ability to a-priori evaluate potential traffic management measures for vulnerable road sections and carriageways. The importance of consideration of the stochastic influence of traffic was further demonstrated in the case for both day-to-day variations as well as intraday and inter-vehicle stochastics for the outcome of studies.

Visual communication

Communication of results based on uncertainty also requires attention and is considered. This is performed through an investigation of different methods to visualise uncertainty in traffic for static representations of macroscopic stochastic traffic model predictions on road networks. Several graphical uncertainty representations were analysed in a cognitive alternative task-switching experiment. Although the actual choice of a graphical uncertainty representation strongly depends on the desired type, speed and level of information disclosure it was possible to find appropriate representations and it was shown that boxplot and line-style representations of uncertainty were generally favourable additions to current macroscopic model visualisations. Further analysis on designing model visualisations showed that three main considerations should be taken into account. Firstly, it is important to avoid interference between the visual cues on different dimensions, secondly, to apply cues that allow explicit comparison or quantification where required, and thirdly, to be aware of the detrimental effects of clutter or overloading visualisations with too much information.

Implications

This research has strong implications for theory, but all the more for practice. The developed frameworks and methodologies allow the effects of traffic management, but also for other traffic analyses, to be evaluated to a much greater degree of accuracy prior to implementation, much more than what is current practice. Correct a-priori analysis should allow more extensive and specifically tuned measures to be analysed and applied to further utilise road capacity, improve traffic flow and ultimately reduce delays and congestion. It is therefore highly recommended that uncertainty and fluctuations in traffic are considered when planning for traffic management.

Samenvatting

Als congestie een probleem wordt op een weg of in een verkeersnetwerk zijn er in het algemeen drie belangrijke oplossingsrichtingen: weguitbreidingen, beprijzen of verkeersmanagement. Aan het einde van de twintigste eeuw begon verkeersmanagement in toenemende mate de voorkeur te krijgen boven weguitbreiding, omdat verkeersmanagement vaak een efficiëntere optie bleek te zijn. Verkeersmanagement richt zich op het beïnvloeden van verkeersstromen om de bestaande infrastructuur beter te benutten, wat vaak resulteert in minder filevorming. De effectiviteit van verkeersmanagement is afhankelijk van het vermogen om verkeer te beïnvloeden. Verkeer bevat echter een grote mate van stochasticiteit afkomstig van menselijk rijgedrag. De fluctuatie in het verkeer als gevolg van dit stochastische rijgedrag heeft een groot effect op de effectiviteit van verkeersmanagementmaatregelen. In het verleden werd hier weinig aandacht aan besteed.

Het hoofddoel van deze dissertatie is het geven van inzicht in stochastische fluctuaties en onzekerheden in verkeersstromen om de effectiviteit van verkeersmanagement te vergroten. Nieuwe methodieken worden geïntroduceerd om deze effecten in geaggregeerde macroscopische verkeersstomen te analyseren en te modelleren. Hierbij wordt de noodzaak om rekening te houden met onzekerheid en fluctuaties ook aangetoond. Twee stochastische processen worden onderscheiden: *onzekerheden* beschrijven dag-specifieke variaties *tussen* verkeerstromen en *fluctuaties* beschrijven microscopische variaties *in* verkeer. Deze dissertatie bestaat uit drie onderdelen: analyse van variaties in verkeer, het modelleren van fluctuaties en onzekerheden in verkeer, en visuele communicatie van onzekerheid in verkeersmodellen.

Analyse

Analyse van de stochastische karakteristieken van variabelen, die invloed hebben op verkeersstromen, is noodzakelijk om onzekerheden en fluctuaties in modellen te begrijpen. De verkeersvraag en de wegcapaciteit worden beïnvloed door dergelijke variabelen. Twee methodologische raamwerken zijn gepresenteerd voor de stochastische analyse van verkeer. Het eerste methodologische raamwerk betreft een conceptuele model voor het schatten van stochastische capaciteiten in praktijk. De stochastische eigenschappen van capaciteiten worden geschat met behulp van een aangepaste Product Limit Method (PLM) en gekwantificeerd als een verdeling. Het tweede raamwerk betreft het gezamenlijke effect van vraag en capaciteit op verkeersstromen. Deze methodologie gebruik dezelfde capaciteitsaanpak als de eerste methodologie en voegt afgeleide vraagverdelingen hieraan toe door een empirisch proces van afgebakende vraagwaarnemingen. Het gezamenlijk bekijken van vraag en capaciteit blijkt een dieper en universeler begrip mogelijk te maken van het effect van variatie op verkeerssystemen.

Modellen

Voor het modelleren van onzekerheid en verkeersfluctuaties zijn verschillende modelaanpakken ontwikkeld. Dit is noodzakelijk gebleken omdat beide vormen van variaties inherent verschillend zijn, zelfs al is de bron van stochasticiteit hetzelfde. Twee type onzekerheidsmodellen zijn beschouwd: *Monte Carlo modellen* en *'one shot' analytische modellen*. Voor de modellering van verkeersfluctuaties is een nieuw model ontwikkeld.

Advanced Monte Carlo simulatiemodellen trekken herhaaldelijk invoervariabelen uit kansverdelingen en voeren met die variabelen een simulatie uit. Door gebruik te maken van algoritmes om lotingen beter te spreiden is een kleinere steekproefomvang vereist om een representatieve verdeling op te bouwen. Drie lotingstechnieken zijn beoordeeld op hun vermogen om de benodigde rekentijd te reduceren: *Latin Hypercube Sampling*, *Importance Sampling*, en *Sobol Quasi-random Getallen.* De technieken bleken stabiel en consistent in staat om convergentie naar de werkelijke verdeling te vergroten, en daarmee de benodigde rekentijd te reduceren en de toepasbaarheid en efficiëntie van Monte Carlo simulatie te vergroten. De technieken zijn niet eerder gebruikt voor verkeersmodellering. Sobol Quasi-random Getallen bleek het meest effectief te zijn.

In deze dissertatie is daarnaast een nieuw analytisch raamwerk ontwikkeld waarmee via één ('one-shot') modelrun onzekerheid kan worden gemodelleerd: het 'Core Probability Framework' (CPF). Het CPF is een probabilistisch raamwerk voor het modelleren van multi-dimensionele variaties in capaciteit en verkeersvraag in een dynamische macroscopische verkeersstroom. Het CPF maakt gebruik van een nieuw propagatiemodel: het 'Discrete-Element Core Probability Model' (DE-CPM). Het DE-CPM maakt gebruik van een bestaand basismodel, zoals het Cell Transmission Model (CTM) en beschouwt elke verkeersvariabel als een kansverdeling. Het CPF en DE-CPM bouwen voort op bestaande deterministische verkeersstroommodellen door de variabelen in de kern van het model te definiëren als vectoren van waarschijnlijke waarden voor elke invoervariabele. Het CPF pakt belangrijke uitdagingen voor stochastische verkeersmodellering aan, zoals *rekentijd*, *ruimtelijke afhankelijkheid*, *stochastische propagatie van verdelingen* en *stochastische geldigheid*. Uit rekentijdexperimenten op simpele netwerken is gebleken dat het DE-CPM in vergelijking met CTM Monte Carlo in staat is rekentijden (fors) te reduceren. Voor grotere netwerken en een groter mate van stochasticiteit bleek de rekentijd besparing nog groter te zijn.

Voor het modelleren van verkeersfluctuaties in verkeer is het 'First Order Model with Stochastic Advection' (FOMSA) ontwikkeld. Dit is een eerste orde macroscopische kinematische golf model in een Lagrangiaans coördinatenstelsel. Om verkeersfenomenen nauwkeurig te beschrijven op macroscopisch niveau is het van belang om micro-stochastisch rijgedrag aan macroscopische modellen toe te voegen. Het voorgestelde model maakt gebruik van eerste order verkeersstroomtheorie in combinatie met een extra invariant term, de voertuig specifieke invariant, om de heterogeniteit van rijgedrag in verkeer en een mate van agressiviteit van bestuurders te beschrijven door de deterministische dichtheid van het verkeer aan te passen. De capaciteitsval is op twee manieren meegenomen: via acceleratiebeperking en door rekening te houden met reactietijden van bestuurders. Het model maakt het mogelijk om rijgedrag mee te nemen samen met een verhoogde mate van nauwkeurigheid door een afname van de diffusie-effecten in het model. Dit wordt bovendien gedaan in een eerste orde macroscopische beschrijving, waardoor tweede orde processen worden vermeden, die vaak leiden tot extra complexiteit.

Tot slot is een methode ontwikkeld voor het identificeren van wegvakken waar de doorstroming naar verwachting het meest kan worden verbeterd door inzet van verkeersmanagement of andere maatregelen. Deze methode evalueert de mate van veerkracht van een wegvak ten opzichte van het omliggende netwerk via de ontwikkelde *Link Performance Index for Resilience (LPIR)* toe. De focus van de methode richt zich op veerkracht en maakt onderscheid tussen een weerstandsdeel en een hersteldeel met een focus op de homogeniteit en volatiliteit van het verkeer. De weerstand is berekend op basis van de verkeersstroomkarakteristieken en het vermogen van wegvakken om hun gespecificeerde afwikkelingsniveau te faciliteren zonder overtallig filevorming. De methode richt zich explicit op het afleiden van de verkeersheterogeniteit.

In een uitgebreide case-studie voor het verkeersnetwerk van de A20 ring Rotterdam Noord zijn de effecten van verschillende verkeersmanagementmaatregelen berekend. Hierin zijn de LPIR, Advanced Monte Carlo simulatie, en FOMSA ingezet. De case-studie heeft aangetoond dat de modellen goed werken. Tevens is de meerwaarde van de afzonderlijke modellen aangetoond om de potentie van verkeersmanagement a-priori te evalueren. Het belang van het beschouwen van stochasticiteit werd hierin verder aangetoond voor zowel het aspect onzekerheid als voor verkeersfluctuaties tussen voertuigen.

Visuele communicatie van onzekerheid

Communicatie van resultaten van onzekerheden vergt ook aandacht. Meerdere grafische weergaves van onzekerheid in verkeer zijn geanalyseerd met een zogenaamd cognitief 'task-switching' experiment voor een statische weergave van resultaten van een macroscopische verkeersmodel. De keuze voor een weergavevorm voor onzekerheid hangt af van de gewenste de snelheid en het niveau en type van informatie-uitwisseling. Over het algemeen is de 'boxplot' en 'lijnstijl' het meest geschikt gebleken om onzekerheden in verkeer te communiceren. Verdere analyse van ontwerpaspecten voor modelvisualisaties lieten een drietal aandachtpunten zien. Ten eerste dient interferentie tussen 'visual cues' van verschillende dimensies te worden vermeden. Ten tweede moeten 'visual cues' toegepast worden die een bepaalde mate van kwantificatie toestaan. En ten derde moet worden gewaakt om niet te veel informatie in een visualisatie op te nemen.

Implicaties

De ontwikkelde raamwerken en methodieken maken het mogelijk om de effecten van verkeersmanagement, maar ook andere verkeerkundige maatregelen, vooraf met een hoger mate van nauwkeurigheid te evalueren. Hierdoor kunnen verkeersmanagementmaatregelen extensiever en doelgerichter worden ingezet waardoor wegen beter kunnen worden benut, en minder reistijdverlies en congestie ontstaat. Daarom wordt het sterk aanbevolen om onzekerheden en verkeersfluctuaties mee te nemen wanneer nieuwe verkeersmanagement maatregelen worden gepland.

About the Author

Simeon Calvert was born in Edinburgh, Scotland, on the 28th of July 1983. He followed his primary education in Glasgow and moved to The Netherlands with his family at the age of 12. He completed his secondary education in the city of Rotterdam before moving to Delft to study. In Delft, Simeon obtained his Bachelor of Science and Master of Science in Civil Engineering at the Delft University of the Technology. During his studies, he undertook internships at highway agency Rijkswaterstaat and consultant Goudappel Coffeng. In his thesis work for his Masters, he performed research on 'the modelling of traffic in roadwork zones' under the supervision of Hans van Lint and Serge Hoogendoorn. After completing his education in Delft, Simeon worked for a short time as a research assistant at the university and in 2010 joined the research institute TNO, The Netherlands Organisation for Applied Scientific Research, as a research scientist. Simeon continues to work at TNO where his main activities involve applied scientific research on topics relating to traffic flows, traffic flow modelling, traffic management, network analysis and the impact of automated and cooperative driving.

In July 2011, Simeon commenced his PhD research in part-time at the Delft University of Technology, while continuing to work at TNO. The topic of this research focussed on the analysis and modelling of stochasticity in traffic flow to improve evaluation and implementation of traffic management on high level roads. The research was performed under the supervision of Serge Hoogendoorn, Henk Taale and Maaike Snelder, and was co-financed and supported by his employer, TNO, and TrafficQuest, a joint collaboration between TNO, Delft University of Technology and Rijkswaterstaat, highway agency of the Dutch Ministry of Infrastructure and the Environment. Simeon maintains a broad interest in a vast number of traffic flow related topics and has a particular interest in traffic management and the analysis of traffic flow, as well as the impact of automated and cooperative driving towards the future.

Authors Publications

Journal articles

Calvert, S. C. & Snelder, M. 2016. Influence of Weather on Traffic Flow: an Extensive Stochastic Multi-effect Capacity and Demand Analysis. *European Transport*, 60 (5).

Snelder, M. & Calvert, S. C. 2016. Quantifying the impact of adverse weather conditions on road network performance. *European Journal of Transport and Infrastructure Research*, 16, 128-149.

Calvert, S. C., Rypkema, J., Holleman, B., Azulay, D. & De Jong, A. 2015a. Visualisation of uncertainty in probabilistic traffic models for policy and operations. *Transportation*, 1-29.

Calvert, S. C., Snelder, M., Bakri, T., Heijligers, B. & Knoop, V. L. 2015b. Real-Time Travel Time Prediction Framework for Departure Time and Route Advice. *Transportation Research Record: Journal of the Transportation Research Board*, 2490, 56-64.

Calvert, S. C., Taale, H. & Hoogendoorn, S. P. 2015c. Quantification of motorway capacity variation: influence of day type specific variation and capacity drop. *Journal of Advanced Transportation*. 10.1002/atr.1361.

Calvert, S. C., Taale, H., Snelder, M. & Hoogendoorn, S. P. 2015d. Vehicle Specific Behaviour in Macroscopic Traffic Modelling through Stochastic Advection Invariant. *Transportation Research Procedia*, 10, 71-81.

Mouter, N., Holleman, M., Calvert, S. C. & Annema, J. A. 2015. Towards improved handling of uncertainty in cost-benefit analysis: addressing the 'price-quality' and 'communication' dilemmas. *European Journal of Transport and Infrastructure Research*, 15 (3), 2015.

Van Stralen, W., Calvert, S. C. & Molin, E. J. E. 2015. The influence of adverse weather conditions on probability of congestion on Dutch motorways. *European Journal of Transport and Infrastructure Research*, 15(4), pp. 482-500.

Calvert, S. C. & Taale, H. 2014. De noodzaak van probabilistisch modelleren voor tactische en operationele analyses. *Tijdschrift Vervoerswetenschap*, 50 (2), 2014.

Calvert, S. C., Taale, H., Snelder, M. & Hoogendoorn, S. P. 2014. Application of advanced sampling for efficient probabilistic traffic modelling. *Transportation Research Part C: Emerging Technologies*, 49, 87-102.

The following articles are under review:

Calvert, S. C. & Snelder, M., 2016. A methodology for road traffic resilience analysis and review of related concepts. *Transportmetrica part A: Special issue on reliability and resilience.*

Calvert, S. C., Taale, H., Snelder, M., & Hoogendoorn, S. P., 2016. Improving traffic management through consideration of uncertainty and stochastics in traffic flow. In *Case-studies in Transport policy.*

Peer reviewed conference papers

Calvert, S. C. & Snelder, M. 2015. A methodology for road traffic resilience analysis and review of related concepts. 6th International Symposium on Transportation Network Reliability (INSTR). Nara, Japan.

Calvert, S. C., Snelder, M., Bakri, T., Heijligers, B. & Knoop, V. L. Real-Time Travel Time Prediction Framework for Departure Time and Route Advice. Transportation Research Board 94th Annual Meeting, 2015a.

Calvert, S. C., Snelder, M., Taale, H., Wageningen-Kessels, V., Femke, L. & Hoogendoorn, S. P. Bounded acceleration capacity drop in a Lagrangian formulation of the kinematic wave model with vehicle characteristics and unconstrained overtaking. Intelligent Transportation Systems (ITSC), 2015 IEEE 18th International Conference on, 2015b. IEEE, 1371-1376.

Calvert, S. C., Taale, H. & Hoogendoorn, S. P. Introducing the Core Probability Framework and Discrete-Element Core Probability Model for efficient stochastic macroscopic modelling. DTA 2014: 5th International Symposium on Dynamic Traffic Assignment, Salerno, Italy, 17-19 June 2014, 2014.

Van Stralen, W., Calvert, S. C. & Molin, E. The influence of adverse weather conditions on the probability of congestion on Dutch motorways. Transportation Research Board 93rd Annual Meeting, January 2014 Washington DC.

Calvert, S. C. & Snelder, M. Influence of rain on motorway road capacity - A data-driven analysis. Intelligent Transportation Systems-(ITSC), 2013 16th International IEEE Conference on, 2013a. IEEE, 1481-1486.

Calvert, S. C. & Snelder, M. Influence of rain on motorway road capacity – a data-driven analysis. Proceedings of the 16th International IEEE Conference on Intelligent Transportation Systems (ITSC 2013), The Hague, The Netherlands, October, 2013b The Hague.

Mouter, N., Holleman, M., Calvert, S. & Annema, J. Communicating uncertainty in cost-benefit analysis: A cognitive psychological perspective. 1st International Conference on Uncertainties in Transport Infrastructure Evaluation (UNITE), Copenhagen, Denmark, 16-18 September 2013, 2013.

Calvert, S. C. & Minderhoud, M. M. Generic Freeway Weaving Section Capacity Estimation Through Microsimulation. Transportation Research Board 91st Annual Meeting, 2012.

Calvert, S. C., Taale, H., Snelder, M. & Hoogendoorn, S. P. Probability in traffic: a challenge for modelling. DTA2012: 4th International Symposium on Dynamic Traffic Assignment, Martha's Vineyard, USA, 4-6 June 2012; Authors version, 2012a.

Calvert, S. C., Taale, H., Snelder, M. & Hoogendoorn, S. P. Probability in traffic: a challenge for modelling. Proceedings of the Fourth International Symposium on Dynamic Traffic Assignment (DTA 2012), June 4-6 2012b MA, USA.

Calvert, S. C., Van Den Broek, T. H. A. & Van Noort, M. Cooperative driving in mixed traffic networks—Optimizing for performance. Intelligent Vehicles Symposium (IV), 2012 IEEE, 2012c. IEEE, 861-866.

Calvert, S. C., Van Den Broek, T. H. A. & Van Noort, M. Modelling cooperative driving in congestion shockwaves on a freeway network. Intelligent Transportation Systems (ITSC), 2011 14th International IEEE Conference on, 2011. IEEE, 614-619.

Calvert, S. C., Van Lint, J. & Hoogendoorn, S. A hybrid travel time prediction framework for planned motorway roadworks. Intelligent Transportation Systems (ITSC), 2010 13th International IEEE Conference on, 2010. IEEE, 1770-1776.

Non peer reviewed conference papers or extended abstracts

Calvert, S. C. Communicatie van model onzekerheden: meer of minder? Colloquium Vervoersplanologisch Speurwerk 2014, Eindhoven, The Netherlands, 20-21 november 2014, 2014. Stichting CVS.

Calvert, S. C. 2013. Modelleren van onzekerheid, met zekerheid! Vergroting van betrouwbaarheid in verkeersmodellen in een onbetrouwbaar systeem. PLATOS Colloquium 2013. Utrecht, the Netherlands.

Calvert, S. C., Van Stralen, W. & Molin, E. J. Kwantificatie van de invloed van regen op de verkeerdoorstroming. Colloquium Vervoersplanologisch Speurwerk 2013, Rotterdam, 21-22 november 2013. Stichting Colloquium Vervoersplanologisch Speurwerk (CVS).

Snelder, M., Minderhoud, M. & Calvert, S. 2013. Op de Amsterdamse grachten, hebben wij nu de drukte in de hand, Amsterdam. Bijdrage aan het Colloquium Vervoersplanologisch Speurwerk, 21 en 22 November 2013, Rotterdam, 1-11.

Calvert, S., Taale, H., Snelder, M. & Hoogendoorn, S. Improving probabilistic traffic modelling through advanced sampling. Proceedings LATSIS symposium 2012, 1-5, 2012.

Calvert, S. C. & Taale, H. Verschuiving naar Probabilistische Modellering is geen keuze. Colloquium Vervoersplanologisch Speurwerk, Amsterdam, 22-23 November, 2012, 2012.

Duijnisveld, M. a. G., Peijs, L. & Calvert, S. C. 2012. Sturen op verkeershinder op het tracé Schiphol-Amsterdam-Almere. CROW Infradagen. Thema 1: Aanbesteding en contractvormen, 1-12.

Selected technical reports and other publications

This is a limited selection of other reports and articles by the author.

Van Erp, J. B. F., Van Maanen, P. P., Venrooij, W., Calvert, S. C. & Van Beurden, M. H. P. H. 2014. ETP Modelling. Intuitive User interaction. Overview 2011-2014. TNO.

Snelder, M. & S.C. Calvert ,2014, Modeling infrastructure robustness and network failure – assessing climate risks. TNO-rapport TNO 2014 R11925, Kennis voor Klimaat – INCAH project. December 2014

Calvert, S.C., Snelder, M., Minderhoud, M.M., & Bakri, T., 2012. Analysis of the robustness of Dutch highways (translation) , for Rijkswaterstaat, Dienst Verkeers en Scheepvaart, November 2012.

Noort, M. van, S.C. Calvert, T.H.A. van den Broek, 2012. Modelling Cooperative Driving in Congestion Shockwaves on a Freeway Network, Paramics Newsletter, 2012.

Calvert, S.C., Keijzer, E.E., Giezen, C., 2012. CBA analysis tool Merwede bridge A27, TNO-rapport for InfraQuest, July 2012.

Snelder, M., T. Bakri, S. Calvert, M. Minderhoud, H. Drolenga, G. Tamminga 2011, Operationalisation of Robustness (translation), for Rijkswaterstaat, Dienst Verkeer en Scheepvaart.

Noort, M. van, S.C. Calvert, T.H.A. van den Broek, 2012. Modelling Cooperative Driving in Congestion Shockwaves on a Freeway Network, Paramics Newsletter, 2012.

Duijnisveld, M., M. Snelder, M. Minderhoud, S.C. Calvert 2010, Calibration of traffic model for traffic disturbance on A10-A1 (translation), TNO-rapport, Delft, the Netherlands.

Thesis

Calvert, S. C. 2010. A-Priori Travel Time Predictor for Long Term Roadworks on Motorways. TU Delft, Delft University of Technology.

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