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The views in this report are the author's own.

With all reverence mention is made of the fact that the late dr. ir. J. van Veen, who was one of the projectors of the Delta works wrote the foreword to this report, some days before his sudden death.

Moreover the authors are very grateful for the valuable comments made by Mr. J. Th. Verstelle and Mr. D. Eckhart, which have proved of great assistance in writing this report.

FOREWORD

Now that the Netherlands Government has started work on its plan for enclosing the Delta estuaries, viz. the estuaries between the Rotterdam Waterway and the West Scheldt, which is a distance of about 70 kms (42 miles) along the coast as the crow flies, a method of position fixing as accurate as possible has become necessary in this area.

During the period of roughly 25 years, which will be needed to build the dams — four on the sea and three further inland — the erosion and silting up caused by tidal currents will have to be measured continually. It must be possible to take immediate action if erosion threatens. This means, that for taking soundings more accurate position fixing is necessary than is possible by the sextant method; in addition, it must be possible to take soundings and also perhaps carry out other activities in poor visibility and mist.

Besides this work in the estuaries, soundings must be taken regularly, i.e. once a year, in the shoal areas which project about 10 kms. out from the outer coast-line (dunes). Is is expected that the sand masses making up these shoals will alter in shape as a result of variation in the tidal currents. It is likely, for example, that the channels through which the tide now flows towards the estuaries will alter in the future, and that the tendency will be for new channels and shoals to be formed parallel to the continuous coast-line as it will exist about 1980, in accordance with the new tide pattern. This might mean that the shape of the headlands on existing islands would be affected. Taking soundings with position fixing by optical methods for distances of up to 10 kms from the coast would be possible only sporadically, because visibility is rarely good enough.

It was clear from the start, that the sextant method could not be used; recourse would have to be had to modern radio devices. The existing English and German Decca navigation patterns, however were not accurate enough.

It was therefore decided to construct a special Decca system guaranteeing extreme accuracy for the whole Delta area and the adjacent strip of coastal waters. Since this system was required for soundings during the Delta operations, and would also be required for a long time after they had been completed, for the purpose of taking annual soundings of the sea-bed outside the new dams, its construction was one of the first tasks to be carried out. It was used for the first time in January 1958.

The degree of accuracy obtained depends not only on the accuracy of the State triangulation network but also on the accuracy of the apparatus. The layout of the „chain" was decided in consultation with „Decca Navigator". By using more accurate survey decometers, it will probably be possible, early in 1960, to obtain an accuracy of position fixing, expressed as the long axis of the root mean square error ellipse, which will vary from 2 à 4 metres in the centre of the pattern to ± 10 metres in the sea area ± 10 kms out of the coast.

Dr. ir. J. van Veen.

SUMMARY

This report deals with two aspects of the problem of fixing the co-ordinates of points situated on lines which together form the idealized mathematical position pattern, resulting from the erection of the Decca transmitters in the Netherlands for the Delta works. Firstly, the basis and the derivation of the formulae used are discussed; and secondly, attention is given to the sequence of calculations and the way in which they are carried out, with the help of the electronic computer known as Stantec-Zebra.

Fig. 14 shows on a small scale the situation involving a number of lines in this position pattern; the lane numbers corresponding to the lines are given at the sides of the map.

Decca patterns on sea-charts — involving much greater areas than covered by the Delta chain — are always calculated directly in geographical co-ordinates on the ellipsoid and mapped in respect of the network of parallels and meridians appearing on such charts; by this means any desired degree of calculating accuracy can be obtained, although great accuracy is usually not required for the purpose contemplated.

For mapping purposes — and thus also for the Delta chain — it is desirable, also in view of the possibilities of comparison with conventional terrestrial position fixing, to make the Decca calculations in rectangular co-ordinates; for the Delta chain these should of course be the rectangular co-ordinates on the stereographic map projection used in land triangulation (R.D.-co-ordinates).

Direct conversion of geographical co-ordinates of a number of points on Decca lines into rectangular R.D. co-ordinates leads to formulae which are difficult to handle. It is for this reason that the indirect method making use of flat hyperbolae has been followed for the Delta chain, as described in this study; it leads to simple formulae which are very suitable for working out with the help of an electronic computer.

When, as a result of the accuracy of drawing and the map scale used, the differences between Decca lines and flat hyperbolae are sufficiently small, the flat hyperbolic pattern is justified and the map pattern is then constructed from confocal flat hyperbolae. This pattern is discussed in Chapter 3 and can be considered as the approximated pattern for the more accurate Decca pattern. By using fairly simple formulae, based on the distances between a point on the approximated pattern and Master, Slave and the central point situated in Amersfoort, plus several pattern constants and a map projection constant, the approximated pattern is corrected point by point and the Decca pattern is thus obtained. Fig. 8 gives a survey of the size of these corrections, which makes it clear that the corrections should be taken into account for the map scales which are fixed at 1 : 5000 and 1 : 10.000.

However, the problem was whether the pattern to be mapped could not be the flat hyperbolic pattern, so that corrections for map projection could also be included in the correction graph given in Chapter 6; by means of this graph decometer readings are corrected for variations occurring in the uniform propagation speeds of the radio waves which are used in the mathematical pattern.

In order to make sure that deviations from the mathematical pattern which occured in practice are attributed only to physical conditions concerning the pattern actually emitted and to the inaccuracy of the reading devices and not to neglecting the map projection, this solution was not chosen. The investigation of such deviations is consequently simplified. In addition, the fact that the Decca lines should preferably connect with the accurate map content of the land area is also an argument for preferring the Decca pattern to the approximated pattern.

The accuracy of drawing moreover played a part in the method of calculating the points on the pattern which were to be mapped. The pattern is divided into sections, so that the corrections for map projection in the corners of each section are calculated according to the formulae previously referred to, thus making it easier to calculate this correction for the other points in the section, since it can be done linearly from the corrections in the corners.

In order to keep the formulae to be used as simple as possible, the co-ordinates have been

calculated in special systems, with the baseline master-slave as x-axis and the centre of this line as origin. As a consequence of the linear interpolation of the correction for map projection, the dm. in the co-ordinates are taken as computational quantities. If the co-ordinates thus obtained are converted into the R.D. system, the results should be rounded off to halfmetres.

The problem of deriving R.D. co-ordinates for individual points from decometer readings and vice versa is also important in practice. It is dealt with in Chapter 5.

The electronic computer is the best means of computing the co-ordinates of a large number of points (in this case, approx. 300 000). The Zebra installed in the Dr. Neher Laboratory at Leidschendam was kindly made available for this purpose by Dr. W. L. van der Poel, Head of the Mathematical Department of the P.T.T. (Post Office) who originally devised the machine. For expert advice and in order to deal reliably with the peculiarities of electronic computing close collaboration was maintained with Mr. D. Eckhart, who is attached to the I.T.C. at Delft as a mathematician.

The introduction to this report deals with the general operations involved in electronic computing on the Zebra; for its use in calculating the scheme of corrections for map projection and in computing the co-ordinates of the final pattern, see Chapter 4, sections 2 and 4, in which particular attention is paid to the course of the computation, for which a flow diagram, as it is called, is drawn. This flow diagram naturally shows a predominantly cyclic structure, due to the regularly increasing lane numbers in combination with the regularly increasing variables x or y.

The Zebra computes and produces in punched form the co-ordinates of approx. 3000 points per hour. These results are made into typed tables by means of a flexowriter.

SOMMAIRE

L'exposé traite deux aspects du problème de la détermination de l'ensemble des lignes de position qui forment le réseau Decca idéalisé du projet néerlandais „Delta".

1. Présentation du problème et déduction des formules à appliquer;
2. le calcul numérique effectué à l'aide de la calculatrice électronique „Zebra".

Fig. 14 donne une vue d'ensemble à petite échelle du réseau Decca, òu on a représenté un nombre réduit de lignes de position. En bordure de l'esquisse figurent les numéros des „lanes" (lane number), c'est-à-dire les valeurs de la coordonnée elliptique attribuée à chacune des lignes de position.

Les réseaux Decca pour la navigation maritime qui couvrent des surfaces beaucoup plus étendues que le réseau du plan Delta sont calculés directement en coordonnées géographiques et reportés sur les cartes de navigation à l'aide du réseau des méridiens et parallèles. La précision de ces réseaux de navigation n'est généralement pas très poussée quoique l'erreur de ce mode de calcul puisse être réduite à volonté. Par contre les réseaux à des fins géodésiques sont à calculer dans le système de projection cartographique du pays ce qui facilite aussi la comparaison du procédé Decca avec des méthodes géodésiques conventionelles. Pour le réseau du plan Delta c'est donc la projection stéréographique des Pays-Bas qui entre en ligne de compte.

La transformation directe des coordonnés géographiques en coordonnées rectangulaires du pays aboutit à des formules peu maniables. La méthode développée dans la présente publication évite cet inconvenient. En plus les résultats sont obtenus avec un effort minimum et les formules s'adaptent aisément aux procédés de calcul électronique.

Il est évident que le choix d'hyperboles planes ou bien de „lignes Decca", comme lignes de position, dépend de la précision exigée du réseau qui de son côté est fonction de l'échelle de la carte, du but envisagé, de la précision inhérente au procédé Decca et de la méthode de dessin et de reproduction.

Chapitre 3 traite le simple cas du réseau formé par des hyperboles planes à foyer commun. Aux chapitres 4 et 5, le réseau obtenu ainsi est considéré comme solution approximative

du réseau exact formé par les lignes Decca. Les corrections à ajouter aux coordonnées des points du réseau approximatif s'obtiennent à l'aide de formules relativement simples. Fig. 8 représente l'ensemble de ces corrections. On voit que l'importance de ces corrections est telle qu'il faut bien en tenir compte dans les échelles de 1/5000 et de 1/10.000.

Dans la pratique, il est en plus nécessaire d'ajouter au réseau Decca idéalisé (vitesse constante de propagation des ondes) des corrections qui sont fonction de la nature du terrain, qui influence la vitesse de propagation des ondes. Cette question est brièvement discutée au chapitre 6.

Or, il aurait été possible de cumuler toutes sortes de corrections dans un seul graphique de corrections, si on forme le réseau par des hyperboles à foyer commun. Cette solution a été rejetée pour éviter toute confusion entre les deux groupes principaux de corrections, c'est-à-dire corrections purement mathématiques et corrections dues à des influences locales instables (nature du terrain, atmosphère, installations radio).

Le procédé retenu facilite d'ailleurs l'étude prévue des corrections du deuxième groupe et assure l'homogénéité du réseau et du contenu des plans existants 1/5000 et 1/10.000.

Pour le calcul des corrections, le réseau a été subdivisé en secteurs. Les corrections correspondant aux angles des secteurs ont été calculées d'après les formules complètes développées au chapitre 4, tandis que les corrections à l'intérieur des secteurs ont été déterminées par interpolation linéaire en partant des corrections aux angles. Il va de soi que l'étendue des secteurs doit être telle que l'erreur introduite ainsi ne dépasse pas la tolérance.

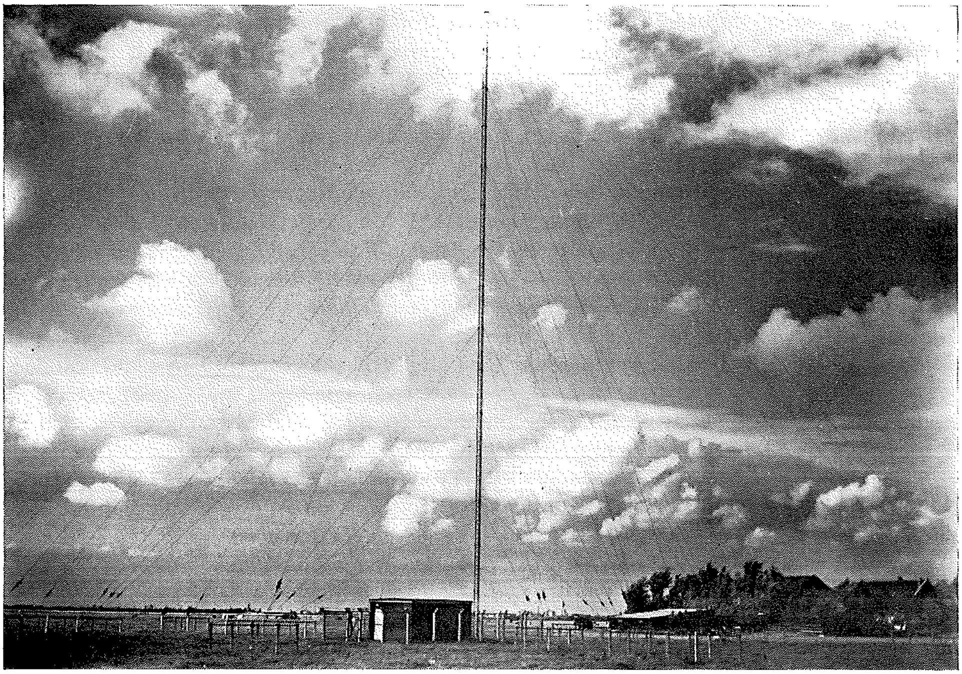
Toutes les formules se réfèrent à un système de coordonnées auxiliaires dont l'origine se situe au point médian de l'émetteur pilote et de l'émetteur secondaire et dont l'axe x coïncide avec la droite passant par les points émetteur. Le choix du système de coordonnées fournit des formules particulièrement simples.

Puisque l'erreur maximum admise dans les coordonnées des points a été fixée à un demi-mètre, les calculs ont été exécutés au décimètre près.

Un problème d'une certaine importance pratique est la détermination des coordonnées d'un point en fonction des lectures du décomètre effectuées sur ce point et inversément. Chapitre 5 traite du sujet.

Le réseau Decca du projet Delta demande la détermination de quelques 300.000 points. Il n'y a que les procédés de calcul électronique pour venir à bout du travail d'une telle envergure. Les calculs ont été exécutés à l'aide de la calculatrice électronique „Zebra" qui est une machine de conception hollandaise. Dans la partie préliminaire de l'exposé, une courte description de la Zebra ainsi qu'une introduction sommaire aux méthodes de calcul électronique et à l'établissement des programmes de calcul est donnée. Cette introduction prépare à la façon plus detaillée du mode de calcul aux par. 2 et 4 au chapitre 4. Les organigrammes démontrent la structure du calcul et font ressortir de manière frappante la caractère cyclique des opérations.

La direction du laboratoire de recherche de P.T.T. à Leidschendam en la personne du docteur W. L. van der Poel a gracieusement mis à la disposition sa calculatrice Zebra, tandis que l'„International Training Centre for Aerial Survey" à Delft représenté par Monsieur D. Eckhart, lic. math., a prêté sa coopération scientifique et technique. Tous les deux ont leur part dans la bonne réussite des travaux exposés dans cet article.



The Purple Slave at Schipluiden

1. INTRODUCTION

1, 1. Posing the problem

A 2-Slave Decca chain has been installed in the Netherlands on behalf of the Delta works; the main transmitter M (Master) is near Rilland Bath and the 2 sub-transmitters are near Sluis (Red Slave) and Schipluiden (Purple Slave) respectively.

Loci with equal decometer readings and at a fixed interval (Decca pattern) were mapped (scale 1 : 5000 and 1 : 10000), according to the stereographic chart projection used in the Netherlands, by the Survey Department of The Ministry of Transport and „Waterstaat". The purpose of this publication is to describe the method used for calculating the co-ordinates of points of these loci (Decca lines).

Similar problems have already been solved by the development and application of the Decca Survey System. The methods applied were described by J. Th. Verstelle in his contributions to the 11th congress of the International Union of Geodesy and Geophysics at Toronto in 1957. „Use of the Decca Navigator Survey System in New Guinea for Hydrography and as a Geodetic Framework" gives a detailed explanation of the method used for a 2-slave chain in New Guinea, which gave patterns for charts (1 : 100 000 scale); and in „Standard Sheets of Hyperbolic Patterns for Survey Use", after a summary of existing methods with bibliographical references, ideas are developed concerning the standardization of 400-lane patterns, based on the urgent need for rapid construction of patterns as an acceptably accurate geodetic framework for parts of the earth's surface which are insufficiently mapped or not mapped at all. Special publication No. 39 of the International Hydrographic Bureau at Monaco, entitled „Radio Aids to Maritime Navigation and Hydrography", also gives a summary of the methods used in various countries for mapping Decca patterns.

The method used for the Netherlands Delta plan can be described as a modification of that used for New Guinea; this was considered desirable because of the map scales used here (1 : 5 000 and 1 : 10 000), so as to link up the maps to be made with the land areas which have already been accurately mapped.

An explanation of the operation of the Decca System is not necessary, since handbooks and the literature referred to by Verstelle can be consulted. Wherever it is thought worthwhile, the emphasis will however again be laid on the elementary principles.

The formulae which are the basis of the method developed appeared suitable for working out on electronic computers, which meant that the result could be obtained in the quickest possible way.

This computation of the Decca pattern for the Delta works was the first experience of the Survey Department of the Ministry of Transport and „Waterstaat" in the field of practical electronic computing and involved the use of the recently built „Zebra". The general lines of the computing method on „Zebra" are interesting and not widely known, and hence an explanation of them and of their application to the formulae referred to has been included in this publication. The general lines are set out in the next section.

1, 2. The electronic programme controlled computer Stantec Zebra

The *Stan*dard *Te*lephones and *C*ables, *Z*eer *E*envoudige *B*inaire *R*eken *A*utomaat (Very simple binary calculating machine), which was built in England but designed in the Netherlands and is referred to in this publication as „Zebra", is a universal computer, viz. it can be used for all types of computation.

The machine consists essentially of 5 main parts, which are called as follows:

1. The *input,* i.e. the part by means of which the machine is supplied with „information" about the instructions to be carried out and about numerical data. Input in the Zebra consists first of all of a high-speed (100 symbols per second) punched tape-reader since the information is recorded on a punched tape in code, which is stepped through the reading apparatus, so that the symbols are read one after the other with the aid of photo-cells. There are 32 different symbols on a tape, since there are 5 positions ($32 = 2^5$).
   N.B. The tape runs in one direction only, and hence a part which has been read can not reappear.
   In addition, information of a special nature can be given to the machine by means of switches or with the help of a dial e.g. the information "start". Thus this information does not reach the machine via the unit described as "input".
2. The function of the *output* is to give a readable, workable and durable form to the results provided by the machine in electrical and magnetic form. This is done in the Zebra by means of a teleprinter and/or tapepunch.
3. The *arithmetic part,* which is a complex of electronic switches and counting registers, carries out the computations required.
4. The *memory* in the Zebra consists of a rapidly rotating metal drum, on which the information "to be remembered" is „written" in the form of magnetized dots by means of a series of writing heads. These heads are also used for "reading" this information.
5. The part known as the *control* co-ordinates the activities of the other parts. The Zebra counts internally in the binary system, which is built up on the 2 numerical symbols 0 en 1. The machine handles binary numbers corresponding to 9 significant decimal places.

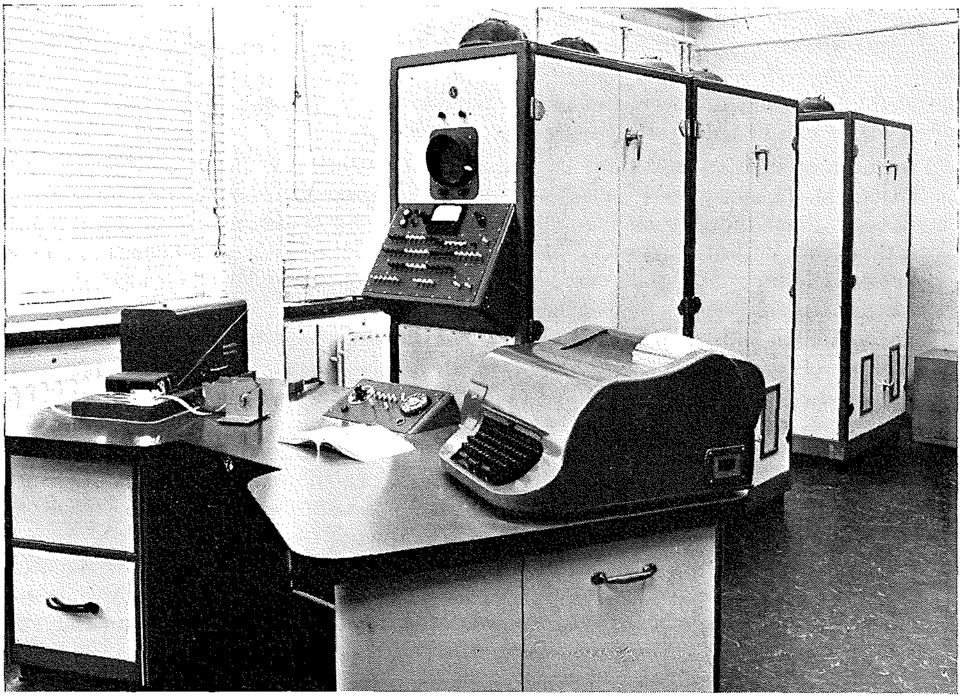
The arithmetic part can carry out only the most simple basic operations, just as, for example, a desk computer.

For complicated calculations, the calculator will draw up a computational scheme when operating a desk computer. When similar calculating problems occur again and again he will devise a formular, which saves him a certain amount of the repetitive mental work involved in such problems. All he then has to do is to fill in formulars. If the formular is a good one, the calculating work can be done by a conscientious assistant. A modern computer can very well be compared to a completely unintelligent but extremely methodical and incredibly industrious assistant.

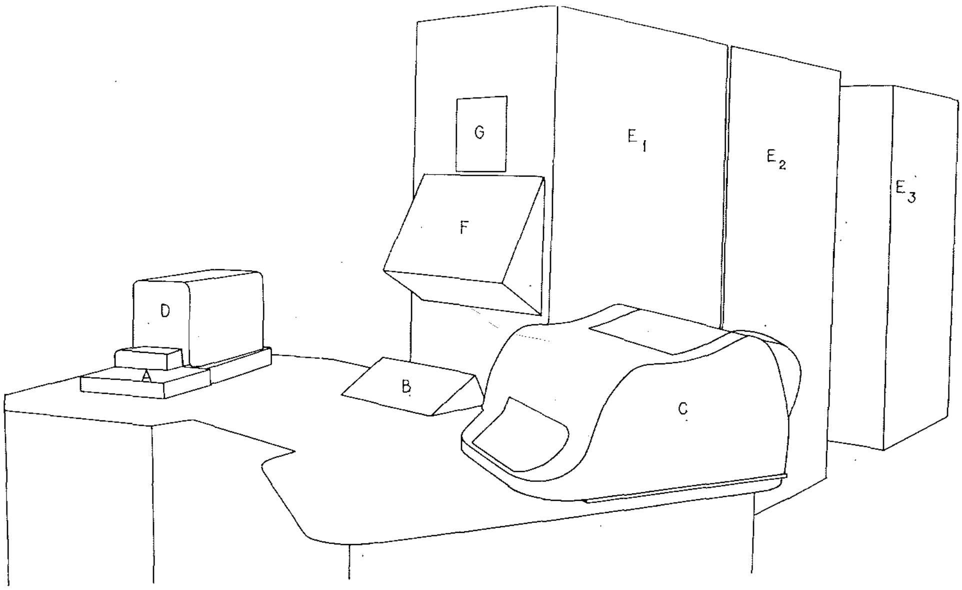
Programming can thus be considered as designing a calculating formular for a calculator of this type.

The programme consists of a series of instructions which enable the computer to carry out the operations described above. These instructions are put in coded form on to a punched tape, which is then inserted into the reading apparatus of the Zebra. This programme is read, stored in the memory, carried out and the results are typed or punched.

Since the programme lays down all the elementary operations for the machine, it is plain that each programming process is preceded by a full analysis of the formulae involved and of the representational aspects of the result.



The Zebra installation in the Dr. Neher Laboratory at Leidschendam photo P.T.T.



A. High speed photo electric tape reader; B. Control panel with manual switches and telephone dial; C. Teleprinter equipment; D. High speed tape punch; $E_{1,2}$. Computer cubicle; E_3. Power cubicle; F. Control panel of the computer for test and maintenance purposes; G. Cathode ray tube.

This analysis concerns mathematical questions such as; elucidation of the formulae, limiting values occurring, internal checks, tolerances, range of variables, any intervals in a tabular result, methods of solving the problem by direct iteration or interpolation. The representational aspects which must be analysed concern the form of the results obtained via the output, such as full definition of the position in a lay-out of each number to be obtained, the number of places before and after the decimal point, the possible omission of decimal points if desired, the maximum size of the tables, deciding whether the results will be typed directly or punched on tape. A way of interrupting the computation must also be stated. A reliable programme test must be determined.

The programme as set up by using the simple Zebra code can now be specified in more detail. It must first be pointed out that the memory can be subdivided into an instruction memory and a number memory. The latter is divided into 1400 "cells". Each cell bears a fixed identification number, the "address" as it is called. By means of these addresses a stored number is permanently at the disposal of the machine. The instruction memory also has 1400 available cells. However these cells are not provided with a fixed address, since normally the instructions are carried out in the order in which they have been stored in the memory. Nevertheless, individual cells can be made available arbitrarily as required, by adding a characterizing number to them; this number is called a "parameter" Ninety-nine parameters are available.

The notion "instruction" must now be explained. An instruction consists of two parts, viz. the operative part and the addressing. As examples, here are two simple instructions: D 235 means — divide the number occurring in the counting register by the number occurring in cell no. 235 of the number memory, leave the result in the counting register and proceed to carry out the next instruction; the number in cell 235 remains unaltered: T 702 means — store the number occurring in the counting register in cell 702 of the number memory, at the same time clearing the counting register, i.e. put the number 0 in the counting register and proceed to carry out the next instruction; the number which was in cell 702 is irrecoverably lost. The first instruction is arithmetical, the second more clerical in nature.

A small but important group of „input indications" on the punched tape is not stored in the instruction memory, but on being read they are immediately transferred to the control and carried out at once.

Thus at the beginning of each programme there is the special input indication Y. When the input reads this symbol, the machine is immediately brought into the correct position for reading a new programme.

Labelling an instruction in the instruction memory, i.e. giving it a parameter, is also done by means of an input indication, expressed by the symbol Q. If for example the input reads Q 68, the next instruction read is given parameter 68. The desirability of such parameters in the organization of a computing programme is explained later.

When the reading of the instructions belonging to the programme has been completed, this must obviously be brought to the notice of the machine. This is done by means of the input indication Yp Yoo, which stops further reading of the tape and starts the machine carrying out the programme which has just been read at the instruction which has previously been given the label p.

The instruction Z is often found in the parameter address p referred to above; this is the stop instruction. While the machine waits, the number tape, which contains the required starting values and computational constants in the order fixed by the programme, is placed in the reading apparatus.

After the starting button has been pressed the machine proceeds to carry out the next instruction and so on; as soon as it encounters a reading instruction it proceeds to read the appropriate numbers from the number tape and store them in the number memory; it uses them in the calculations after having obtained computing instructions, and finally produces the results via the output instructions.

The programme gives the stop instruction at the end, the machine stops and, if required, new numbers can be put into the punched tape-reader and the calculation can be carried out with new numerical values.

It could be said that the Zebra, by virtue of the programme existing in the instruction memory, has become a special computer for just the calculating work contained in this programme.

Successive instructions regarding operations for calculating a function for different values of the variables can be thought of as linked together to form a chain. This chain, which may be long even for simple calculations, will have identical parts. The construction of the Zebra is such that an identically recurring part need to be programmed only once. In the programme this part is flanked by 2 special count instructions as they are called; this programme part is carried out the specified number of times in succession. A programme part such as this, together with the flanking count instructions, is called a cycle. A single completion of the instructions lying in between the two count instructions is called a loop. The two count instructions make sure that the cycle is performed for a specified number of loops. Of course when programming, care must be taken to see that the correct numerical data are inserted into the arithmetic part for each loop of the cycle. A cycle can itself also be a subdivision of an other more extensive cycle, the internal and the external cycle respectively. Programmes often contain whole hierarchies of such cycles.

Whereas the instructions are normally carried out in order, a varying order of operations is made possible by introducing a jump instruction, as it is called, into the programme. The parts of the programme which are the object of a jump instruction must be labelled with a parameter p. Carrying out a jump instruction is often made conditional and dependent on the sign of the number which occurs in the counting register. The machine is in this way enabled to make the correct choice from 2 or more possible calculations or parts of calculations, which are or are not to be carried out, depending on certain quantities.

This jump instruction is also used if identical programme parts are repeated irregularly. This identical programme part is called a sub-routine. An automatic return to the instruction which follows the jump instruction is thus required. Programme parts are also sometimes put in sub-routine form, in order to divide long and inconveniently arranged programmes into more easily handled units. If programme parts correspond to previously used sub-routines, the latter can easily be incorporated in the form of sub-programmes.

2. THE SPHERICAL HYPERBOLIC PATTERN

From the formulation of the problem, given in the Introduction it follows, that first of all the geometrical properties of the locus of points, at which a decometer registers the same readings, should be examined.

Decometers react to electro-magnetic phenomena, that is, they show the difference in phase between the radio-wave pattern received directly from the main transmitter M and that received via the sub-transmitter S1. The arrangement of the reading devices is immaterial to our problem.

These phase differences are converted into distance differences by means of the following well-known formula, based on the usual synchronisation, that the phase difference = 0 for the extension of the base-line on the Master side, $\frac{\Delta\phi_{M-S1}}{360^\circ} = \frac{b + l_M - l_{S1}}{\lambda}$, where ϕ is the phase, $\Delta\phi$ the phase difference, b the distance covered by the radio-waves from M to S1, l_M that from M to the observation point S, l_{S1} that form S1 to S, and λ the wavelength of the observed waves.

Geometrically, this formula means that the distance $b + l_M - l_{S1}$ is measured with a unit of measurement λ, for the ratio $\frac{\Delta\phi_{M-S1}}{360^\circ}$ shows how many times λ has been included in the distance $b + l_M - l_{S1}$ covered by the radio-waves. The physical model of the points with equal decometer readings is thus defined as the locus of points with $\frac{\Delta\phi_{M-S1}}{360^\circ}$ is constant.

A surveyboat for the Delta area

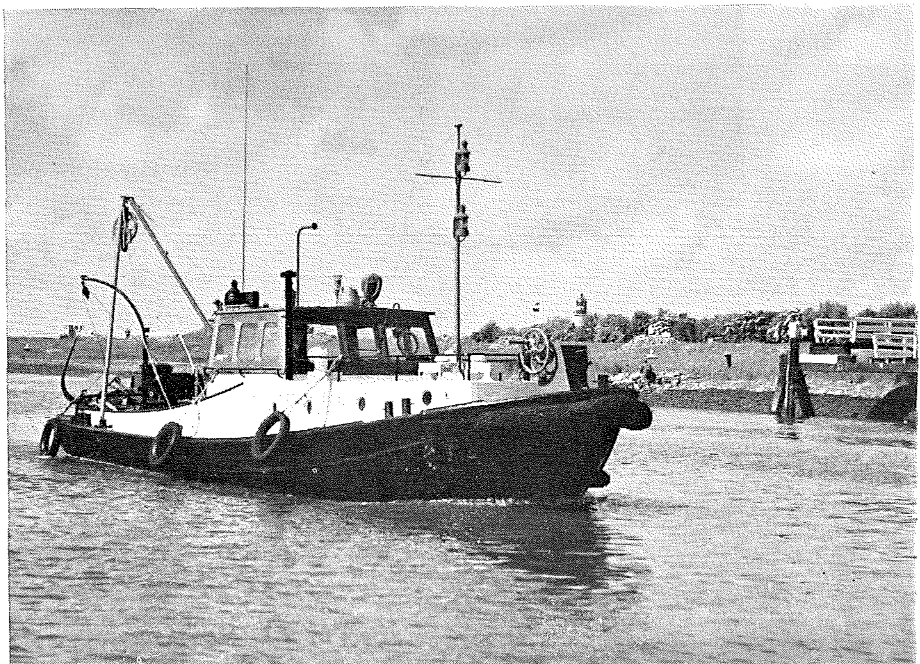


photo Stuvel

Decometer equipment on board

These points have the geometrical property, that $\frac{b + l_M - l_{SI}}{\lambda}$ is constant.

The transmitted waves appear to follow paths which roughly follow the curvature of the earth, so that the distances $b + l_M - l_{SI}$ can be measured according to the distances over the surface of the earth, idealised in the shape of an ellipsoid of revolution, or still further idealised in the shape of a sphere, at least in so far as the differences in altitude between observation point and the transmitters have practically no effect on these distances. For the Netherlands chain these effects are in fact negligeable, for, in addition to the fact that, because of the occurrence of induction, the chain can be used only at a minimum distance of 4 kms. from the transmitters, the differences in altitude are sufficiently small in the Netherlands.

A second idealisation concerns the standard measure λ. This matter is dealt with in detail by Verstelle. Generally, if the wavelength $\lambda = \frac{v}{f}$, at unvarying frequency f, λ varies proportionately to the propagation speed v, and this last quantity v is not stable, but varies according to the electro-magnetic conductivity of the terrain over which the waves pass. Thus it also makes a difference whether the paths run, for example, over land or over water. Thus an unstable λ will also occur at times, if atmospheric conditions affect the v, or if the frequency f is not absolutely stable. If however we idealise the λ, which varies, as we have seen, in practice, to a constant Λ, the geometrical properties of the locus will be simpler, and simpler formulae for the Decca lines can be established. Whereas the previous idealisations were justified in practice, the idealisation now under consideration cannot be passed over without further discussion. Empirically established deviations of the real pattern in respect of the mathematically fixed pattern with a constant Λ are ascribed to this idealisation.

In practice the decometer readings observed are corrected according to previously drawn up tables, in order that the mathematically fixed pattern shall hold good. These corrections thus have essentially a physical origin, which is generally related to a deviating ratio of, for example, water/land on the paths from S to Master and Slave as compared with that for which the Λ used holds good as an average for the path from Master to Slave.

Replacing our physical model of the points with equal decometer readings by a geometrical one, the locus of points is defined by the formula $\frac{s_{MSI} + s_{MS} - s_{SIS}}{\Lambda}$ is constant. This notation means that the distances are measured according to the geodesics on ellipsoid or sphere or, since s_{MSI} is a constant, the locus becomes that for which $s_{MS} - s_{SIS}$ is constant. This line is consequently hyperbolic in nature and a hyperbolic pattern will be formed when the constant $s_{MS} - s_{SIS}$ is made to vary according to a fixed interval, in formula according to a fixed parameter.

In the publication of the Ministry of Marine entitled "Hydrografisch Opnemen" (Hydrographic Surveys) the values s_{MS} and s_{SIS} are derived for the ellipsoid, in which the points involved are naturally defined by latitude and longitude, φ and λ. For the Netherlands a simpler plan can be made. The Netherlands charts are in fact based on a triangulation which was projected conformally from the ellipsoid of Bessel on to a sphere having as its radius the average radius of curvature of the ellipsoid at the central point Amersfoort, before being projected on to the plane of projection. This conformal projection was done, so we read in the publication of the "Rijkscommissie voor Graadmeting en Waterpassing" (State Commission for primary triangulation and levelling) entitled „De stereografische kaartprojectie in hare toepassing bij de Rijksdriehoeksmeting" (Stereographic map projection in its application to State triangulation) with an enlargement $\underline{m}$, which is dependent on the difference of latitude compared with Amersfoort; $\underline{m} = 1$ for the parallel passing through Amersfoort while for the greatest difference in latitude compared with Amersfoort reached in the Netherlands, namely 80'; $\log \underline{m} = 0.12 \times 10^{-7}$, i.e. $\underline{m} = 1.0000000276$, which is a correction of 5.5 cms. for 200 kms.

This $\underline{m}$ will have no influence in practice on the distances and distance differences in question, and so the Decca pattern can be considered as lying on a sphere, whose radius is equal to the average radius of curvature at Amersfoort according to the ellipsoid of Bessel; in other words, for our Decca pattern we may idealise the earth in spherical form.

Such a locus of equal spherical distance difference with respect to 2 given points can now be simply expressed in formulae. If we define the points on the sphere, M (Master), R (red Slave), P (purple Slave) and S (observation point), by using their geographical co-ordinates φ and λ, then:

$$\cos s_{MS} = \cos \varphi_M \cos \varphi_S \cos (\lambda_S - \lambda_M) + \sin \varphi_M \sin \varphi_S$$

$$\cos s_{RS} = \cos \varphi_R \cos \varphi_S \cos (\lambda_S - \lambda_R) + \sin \varphi_R \sin \varphi_S$$

$$\cos s_{PS} = \cos \varphi_P \cos \varphi_S \cos (\lambda_S - \lambda_P) + \sin \varphi_P \sin \varphi_S$$

from which:

$$s_{MS} - s_{RS} = r \arccos (\cos \varphi_M \cos \lambda_M \cos \varphi_S \cos \lambda_S + \cos \varphi_M \sin \lambda_M \cos \varphi_S \sin \lambda_S + \sin \varphi_M \sin \varphi_S) - r \arccos (\cos \varphi_R \cos \lambda_R \cos \varphi_S \cos \lambda_S + \cos \varphi_R \sin \lambda_R \cos \varphi_S \sin \lambda_S + \sin \varphi_R \sin \varphi_S)$$

and :

$$s_{MS} - s_{PS} = r \arccos (\cos\varphi_M \cos \lambda_M \cos \varphi_S \cos \lambda_S + \cos \varphi_M \sin \lambda_M \cos \varphi_S \sin \lambda_S + \sin \varphi_M \sin \varphi_S) - r \arccos (\cos \varphi_P \cos \lambda_P \cos \varphi_S \cos \lambda_S + \cos \varphi_P \sin \lambda_P \cos \varphi_S \sin \lambda_S + \sin \varphi_P \sin \varphi_S)$$

in which the arcs are measured in radians.

The first equation is that of the spherical hyperbola, whose points show equal spherical distance differences $s_{MS} - s_{RS}$ with respect to the foci M and R, the second represents the spherical hyperbola, whose points show equal spherical distance differences $s_{MS} - s_{PS}$ with respect to the foci M and P.

If these spherical distance differences are measured via the corrected decometer readings referred to, then by solving the unknowns φ_S and λ_S from both equations the position of the observation point can be determined by calculation.

The spherical hyperbolic pattern arises through making $s_{MR} + s_{MS} - s_{RS}$ equal to $L_R \times \Lambda_R$ and accordingly $s_{MP} + s_{MS} - s_{PS}$ equal to $L_P \times \Lambda_P$, in which L thus stands for a parameter which also defines the so-called "lane-number". This lane-number is also given by the number of cycles included in the phase difference, i.e. $L = \dfrac{\Delta\phi_{M-Sl}}{360°}$.

The equation of these patterns is then of course in φ and λ, but offers no practical prospects. Projection on to the chart would in fact have to be done again in φ and λ via the graticule, whereas it is desirable to use a method based on Cartesian co-ordinates, in view of the use of the co-ordinate plotter.

3. THE FLAT HYPERBOLIC PATTERN (APPROXIMATED PATTERN)

3, 1. Introduction

The number of curves making up the spherical hyperbolic pattern is dependent in the first place on the assumed interval Δ L of the parameter L. First of all we again consider this parameter, the lane-number L, the formula for which reads: $L = \dfrac{s_{MSI} + s_{MS} - s_{SIS}}{\Lambda}$

L is minimum for $s_{MS} - s_{SIS} = - s_{MSI}$ that is, for points on the extension of the base-line on the side of the Master. Accordingly, the synchronisation assumed and used in practice is then $L = 0$ and thus $\Delta\phi_{M-SI} = 0$. L is maximum for $s_{MS} - s_{SIS} = + s_{MSI}$, that is, for points on the extension of the base-line on the side of the Slave. Therefore becomes $\dfrac{2\ s_{MSI}}{\Lambda} = \dfrac{s_{MSI}}{\frac{1}{2}\ \Lambda} = n$, and in this case is thus $\Delta\phi_{M-SI} = n \times 360°$.

This n can therefore be easily fixed by means of a decometer reading at a point on the extension of the base-line on the side of the Slave. The wavelength Λ also follows from this, at a known distance s_{MSI}; $\Lambda = \dfrac{2\ s_{MSI}}{n}$.

The number of curves in our pattern is thus also dependent on this n, the „lanecount". If we now take Δ L = 1, the constant spherical distance differences $s_{MS} - s_{SIS}$, which are valid for points on the same curve will vary as compared with those for points on succeeding curves by Λ; if we take Δ L = 0.1, then this variation is 0.1 Λ.

If the number of curves per pattern were standardised at 400, as suggested by Verstelle, this variation would become $\dfrac{s_{MSI}}{200}$, or, expressed in Λ, $\dfrac{n \times \Lambda}{400}$. Thus the curves divide the base-line into 400 equal parts. Provision would preferably be made in the decometers for certain accelerations in the existing reading devices, by means of which the lane-number 400 on the extension of the base-line on the side of the Slave could be read and after which each lane-number as required could be read directly. These instruments, as Verstelle has said, have not yet been perfected, and therefore only the current pattern is considered in what follows.

The central projection of our spherical hyperbolic pattern according to the Netherlands stereographic chart projection on to the plane of projection, on which the position of the points is given according to the R.D. system of co-ordinates,*) will yield the Decca pattern, which must be mapped. The central projection of a spherical hyperbola is not a flat hyperbola, nor is it another simple curve, so that it will hardly be possible to obtain a direct graphical construction of the Decca lines, even when leaving out of account the insufficient accuracy which can be obtained by using a construction method. The calculation in co-ordinates of a sufficient number of points and the mapping and linking of them is the obvious method of depicting the Decca lines on the map.

*) **Note:** The R.D. system of XY-co-ordinates is the accepted system in the Netherlands, with Amersfoort as origin and the Y-axis in the direction of North.

For this purpose the Decca line is taken as a deformed branch of a flat hyperbola and is calculated by correcting the latter in order to obtain the former. Consequently, a flat hyperbolic pattern must first be calculated as an approximated pattern, and then corrected to give the Decca pattern. For this purpose it is supposed that the main and sub transmitters lie in the plane of projection of the State Triangulation grid, as the position in R.D. co-ordinates X en Y is fixed by field survey, and also that the variable observation point S lies in this plane of projection. This is the situation in which a flat hyperbolic position pattern would occur. The lane-number n on the extension of the baseline on the side of the Slave remains unaltered, the baseline s_{MS1} is altered into MS1, the relation between the two distances still comes into consideration. Because of this the wavelength Λ also changes into Λ_v.

Both patterns are therefore built up according to the same system based on the lane-number L, although the curves with corresponding lane-numbers are not the projections of one another. In the next chapter the corrections are derived, according to which a point on the approximated pattern must be shifted to a point on the projection of the spherical hyperbola with the same lane-number, that is to a point on the corresponding Decca line. The formulae for the flat hyperbolic pattern are derived in the next section.

3, 2. **Formulae**

The flat pattern is expressed in respect of a special co-ordinate system x, y with the origin 0 in the centre of the base-line MS1, the positive x-axis in the direction 0.M. With the help of the RD co-ordinates of M and S1, special system and RD system can be converted into each other, as special systems for the red and purple pattern reciprocally.

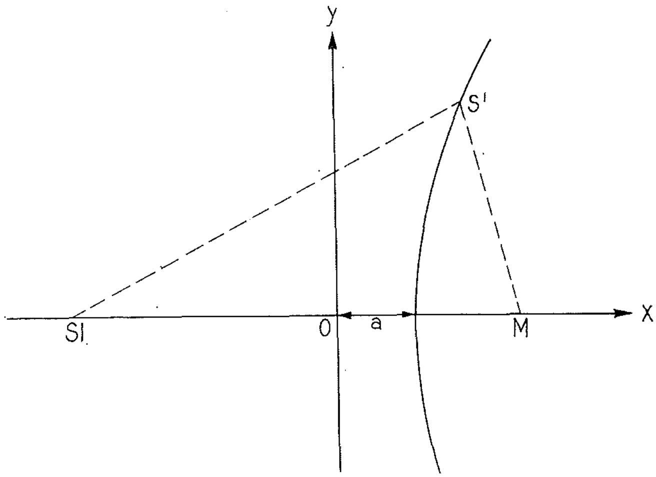


Fig. 1

The equation of the hyperbola branch trough S′ (fig. 1) can be written as: MS′ — S1S′ = $L\Lambda_v$ — MS1, or expressed in the x and y co-ordinates:

$L\Lambda_v - MS1 = \sqrt{(x_M - x)^2 + y^2} - \sqrt{(x_{S1} - x)^2 + y^2}$ or, since $x_{S1} = - x_M$:

$L\Lambda_v - MS1 = \sqrt{(x_M - x)^2 + y^2} - \sqrt{(x_M + x)^2 + y^2}$ where $MS1 = 2\, x_M$,

so that the pattern formula, explicitly expressed in the parameter L, becomes:

$$L = \frac{2x_M + \sqrt{(x_M - x)^2 + y^2} - \sqrt{(x_M + x)^2 + y^2}}{\Lambda_v} \qquad 1)$$

For a given pattern X_M and Λ_v are constants.
In place of the parameter L, an auxiliary parameter a can be introduced, a being the x-co-ordinate of the point of the hyperbolic branch for which y is 0. As a result formula 1) becomes:

$$L = \frac{2\,x_M + x_M - a - x_M - a}{\Lambda_v} = \frac{2(x_M - a)}{\Lambda_v}$$

from which: $a = x_M - \frac{L\Lambda_v}{2}$. 1a)

The pattern formula, explicitly expressed in the auxiliary parameter a, then becomes:

$$a = \frac{\sqrt{(x_M + x)^2 + y^2} - \sqrt{(x_M - x)^2 + y^2}}{2} \qquad 2)$$

We can now construct the pattern formulae, explicit in x and y, and obtain for them:

$$x = + a \sqrt{1 + \frac{y^2}{x^2_M - a^2}} \qquad 3)$$

and: $$y \pm \frac{1}{a}\sqrt{(x^2_M - a^2)(x^2 - a^2)} \qquad 4)$$

With the help of formulae 1) and 2) the lane-number L or the distance a on the base-line can be computed, for given x and y. Formulae 3) and 4) are of practical importance for mapping the pattern. If a is varied according to formula 1a, formula 3) gives, with a constant y, the x for the points on the successive lanes; geometrically speaking we thus fix the points of intersection of a line parallel to the x-axis with the pattern. Via formula 1a), formula 3) gives, with a constant x, the y for the points on the successive lanes; geometrically we now fix the points of intersection of a line parallel to the y-axis with the pattern. Variation of y and/or x as necessary supplies the number of points required for making it possible to map the pattern.

In the first place, it appears from this that plotting co-ordinates in the special x, y system according to the indicated uses of formulae 3) and 4) makes the employment of the co-ordinate plotter particularly practical. Further, these formulae are thus extremely suitable for working out with the Zebra. However, care must be taken to obtain a good distribution of points lying on the same hyperbola branch, since these points must be linked with one another. The distribution must be such that, on the chart scale, connection by straight lines is permissible, bearing in mind the accuracy of drawing or connection by curved lines by means of a spline. The distribution will thus be fixed by the curvature of the hyperbola branch. This is strongest close to the base-line and decreases with the distance to the point intersection with the base-line. Moreover the curvature varies according to the hyperbola branch; hyperbola branches close to the transmitters are the most strongly curved. The number of points which has to be computed thus varies according to the place in the pattern. This consideration leads to dividing the pattern into sections, and to deciding whether formula 3) or formula 4) shall be used per section, during determination of a constant interval Δ y and Δ x, according to which y and x must be varied. Δ y and Δ x are easily determined with the help of hyperbolic grid tables.

The division into sections is however decided in particular by considerations which will be dealt with in Chapter 4. Mention must still be made here of division of the pattern into zones, which will determine the use of either formula 3) or formula 4). It is obvious that formula 3) will be preferred, if the tangents to the hyperbola branches have an azimuth in respect of the y-axis of between 350^g and 50^g or between 150^g and 250^g, and formula 4) if these azimuths lie between 50^g and 150^g or between 250^g and 350^g. Thus for the ideal zone limit $\frac{dy}{dx} = \pm 1$ applies.

Differentiation of formula 4) will provide the desired function of the zone limit:

$$\frac{dy}{dx} = \pm 1 = \frac{(x^2_M - a^2)\, 2\, x}{2a \sqrt{(x^2 - a^2)\,(x^2_M - a^2)}} = \frac{x\,(x^2_M - a^2)}{ya^2}$$

or $a^4y^2 = x^2(x^2_M - a^2)^2$

whereas according to formula 4): $a^2y^2 = (x^2 - a^2)\,(x^2_M - a^2)$

Elimination of a^2 from both formulae gives the required function:

$$y^2 = x^2 - x^2_M \qquad 5),$$

the equation of a parabola through M, and a parabola through S1.

Thus the pattern is divided into 3 zones as shown in fig. 2, on which the applicable formulae 3) or 4) are also indicated. These parabolas are for practical reasons replaced by the hyperbola branches, which fit in best.

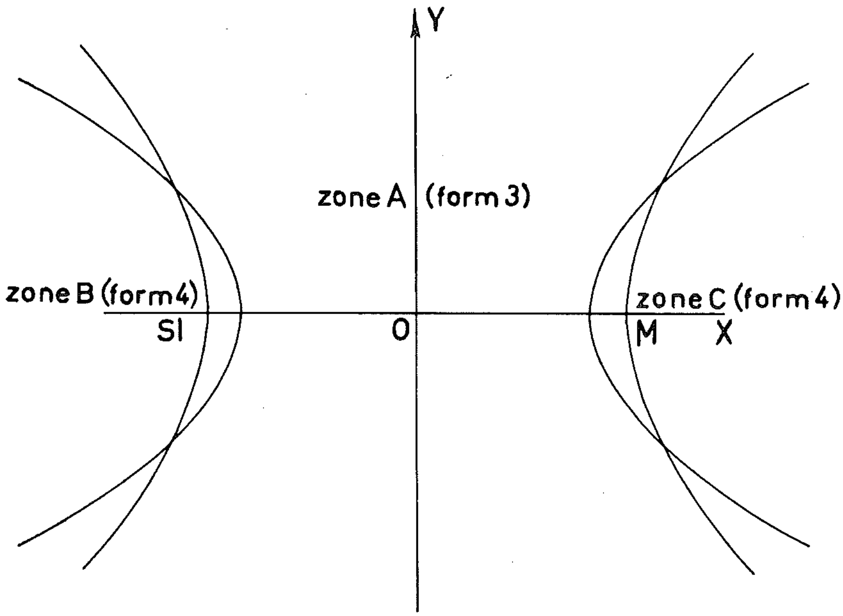


Fig. 2

4. THE DECCA PATTERN

4. 1. The theoretical correction formulae for obtaining the Decca pattern from the approximated pattern

Formula 3) in combination with formula 1a) provided us with the co-ordinate x_i for a point S′ on the approximated pattern with assumed y_i and L_i. In order to maintain the advantage of plotting on one axis of the co-ordinate plotter, it is plain that a point calculated in this way must be corrected in x-direction by c_{x_i}, in such a way that the definitive point S with co-ordinates $x_i + c_{x_i}$, y_i is found on the Decca line with the same lane-number L_i.

If however the computation of a point S′ on the approximated pattern is done with the help of formula 4) in combination with formula 1a), that is x_i and L_i are assumed and y_i is calculated, then a correction in the y-direction c_{y_i} must be made, and in such a way that the definitive point S on the Decca line with the same lane-number L_i has as co-ordinates x_i, $y_i + c_{y_i}$.

The Zebra can tabulate the result $x_i + c_{x_i}$, with reference to y and L, according to assumed intervals Δ y and Δ L; or the result $y_i + c_{y_i}$, with reference to x and L, according to the assumed intervals Δ x and Δ L.

For an uncorrected point S′, $L = \dfrac{MS1 + MS' - S1S'}{\Lambda_v}$

For a corrected point S, $L = \dfrac{s_{MS1} + s_{MS} - s_{S1S}}{\Lambda}$. By substitution of $\Lambda_v = \dfrac{2\,MS1}{n}$ and $\Lambda = \dfrac{2s_{MS1}}{n}$ and equalization of L, the relation between uncorrected and corrected point becomes:

$$\frac{MS1 + MS' - S1S'}{\dfrac{2\,MS1}{n}} = \frac{s_{MS1} + s_{MS} - s_{S1S}}{\dfrac{2\,s_{MS1}}{n}}$$

or $$\frac{MS1 + MS' - S1S'}{MS1} = \frac{s_{MS1} + s_{MS} - s_{S1S}}{s_{MS1}}$$

or $$\frac{MS' - S1S'}{MS1} = \frac{s_{MS} - s_{S1S}}{s_{MS1}}$$

or $$s_{MS} - s_{S1S} = \frac{s_{MS1}}{MS1} \times (MS' - S1S').$$

From the relationship between the spherical distance s between 2 points and the corresponding distance k′ in stereographic projection, the spherical distances s_{MS1}, $s_{MS'}$ and $s_{S1S'}$ can be determined and from this the spherical distance difference $s_{MS'}$ — $s_{S1S'}$ for S′ can be deduced. The relationship in question will be given in formula form later.

Generally this spherical distance difference $s_{MS'}$ — $s_{S1S'}$ is not equal to $\frac{s_{MS1}}{MS1}$ × (MS′ — S1S′). Point S must now be looked for by translation of co-ordinates by c_x or c_y from S′, for which the value for the spherical distance difference $s_{MS} - s_{S1S} = \frac{s_{MS1}}{MS1} \times$ (MS′ — S1S′), that is, the spherical distance difference $s_{MS'} - s_{S1S'}$ thereby undergoes a variation from: $\frac{s_{MS1}}{MS1} \times (MS' - S1S') - (s_{MS'} - s_{S1S'})$.

If this variation is fixed for a translation of co-ordinates c_{x_1} or c_{y_1} of 1 M., the required translation c_x or c_y expressed in meters can be deduced, assuming that for a restricted field this variation is linear.

On pages 22 and 23 of the publication of the „Rijkscommissie voor Graadmeting en Waterpassing" (State Commission for primary Triangulation and Levelling) entitled „De stereografische Kaartprojectie in hare toepassing bij de Rijksdriehoeksmeting" (Stereographic map projection in its application to the State triangulation), the relationship $k' = k \sqrt{m_1 m_2}$ is derived, in which k′ is the distance between 2 points in the stereographic projection, k is the corresponding chord of the sphere, m_1 is the enlargement at point 1 as limit of $\frac{k'}{k}$, if point 2 approaches point 1, m_2 is the enlargement at point 2 as limit of $\frac{k'}{k}$, if 1 approaches point 2. For the enlargement at a point T in the plane of projection the formula $m_T = m_o + \frac{TA^2}{4r^2 m_o}$ applies, in which m_o is the known enlargement at Amersfoort, TA is the distance from T to the central point Amersfoort, r is the radius of the earth's sphere. Thus the enlargement at the Master will be $m_M = m_o + \frac{MA^2}{4r^2 m_o}$, at the Slave $m_{S1} = m_o + \frac{S1A^2}{4r^2 m_o}$ and at point $S' = m_o + \frac{S'A^2}{4r^2 m_o}$.

For determining the distances MA, S1A and S′A it will be necessary to know the co-ordinates of A for which X = 0 and Y = 0 in the special system x, y; which can easily be done with the help of the transformation formulae of the RD system for obtaining the special system.

The ratio of the chords $\frac{k'}{k} = \sqrt{m_1 m_2}$ can be replaced in practice by $\frac{1}{2} m_1 + \frac{1}{2} m_2$, through which $k - k' = k(1 - \frac{1}{2} m_1 - \frac{1}{2} m_2)$ or approximately $k' (1 - \frac{1}{2} m_1 - \frac{1}{2} m_2)$, the correction of the distance in the plane of projection for obtaining the chord of the sphere. For an extreme situation in the Delta area of k′ = 150 km., $m_1 = 1$, $m_2 =$

1.000115 according to the approximate formula k — k′ = — 8.625 m; according to the rigorous formula it becomes k — k′ = —8.624 m. The approximate formula has thus been positively justified.

In order to determine the spherical distance s starting from k, k must be corrected by $+\dfrac{k^3}{24r^2}$, which is in practice equal to $\dfrac{k'^3}{24r^2m_o}$, (correction terms of higher order have an effect on the maximum distances considered here which is less than 1 cm., and they can therefore be disregarded), so that the correction of the projection k′ into the spherical distances will amount to:

$$s - k' = k' \left(1 - \tfrac{1}{2} m_1 - \tfrac{1}{2} m_2 + \frac{k'^2}{24\ r^2\ m_o}\right).$$

Consequently:

$$s_{MS1} - MS1 = MS1 \left(1 - \frac{m_o}{2} - \frac{MA^2}{8r^2\,m_o} - \frac{m_o}{2} - \frac{S1A^2}{8r^2\,m_o} + \frac{MS1^2}{24r^2\,m_o}\right) =$$

$$= MS1 \left\{ \frac{MS1^2 - 3MA^2 - 3\ S1A^2 + 24\ r^2\ m_o\ (1 - m_o)}{24\ r^2\ m_o} \right\}$$

$$s_{MS'} - MS' = MS' \left\{ \frac{MS'^2 - 3MA^2 - 3S'A^2 + 24\ r^2\ m_o\ (1 - m_o)}{24\ r^2\ m_o} \right\}$$

$$s_{S1S'} - S1S' = S1S' \left\{ \frac{S1S'^2 - 3S1A^2 - 3\ S'A^2 + 24\ r^2 m_o\ (1 - m_o)}{24\ r^2\ m_o} \right\}$$

The variation in $s_{MS'}$ — $s_{S1S'}$, which must be found, amounted to:

$\dfrac{s_{MS1}}{MS1} \times (MS' - S1S') - (s_{MS'} - s_{S1S'})$, which can be written as:

$$\left\{ 1 + \frac{MS1^2 - 3MA^2 - 3S1A^2 + 24\ r^2 m_o\ (1 - m_o)}{24\ r^2 m_o} \right\} \times (MS' - S1S') - (s_{MS'} - s_{S1S'}) = (s_{S1S'} - S1S') - (s_{MS'} - MS') + \frac{MS1^2 - 3MA^2 - 3S1A^2 + 24r^2 m_o\ (1 - m_o)}{24r^2 m_o} \times (MS' - S1S')$$

$$= S1S' \left\{ \frac{S1S'^2 - 3\ S1A^2 - 3S'A^2 + 24\ r^2 m_o\ (1 - m_o)}{24r^2 m_o} - \frac{MS1^2 - 3MA^2 - 3S1A^2 + 24r^2 m_o\ (1 - m_o)}{24\ r^2 m_o} \right\}$$

$$- MS' \left\{ \frac{MS'^2 - 3MA^2 - 3S'A^2 + 24\ r^2 m_o\ (1 - m_o)}{24\ r^2 m_o} - \frac{MS1^2 - 3MA^2 - 3S1A^2 + 24r^2 m_o\ (1 - m_o)}{24\ r^2 m_o} \right\}$$

$$= \frac{S1S'\ (S1S'^2 - 3S'A^2 - MS1^2 + 3MA^2) - MS'\ (MS'^2 - 3\ S'A^2 - MS1^2 + 3\ S1A^2)}{24\ r^2 m_o}$$

In this formula $-MS1^2 + 3MA^2 = K_1$ and $-MS1^2 + 3S1A^2 = K_2$ are constants per pattern, whereas $24\, r^2 m_0 = K_3$ is constant for the stereographic projection, so that the required variation can be written as:

$$\frac{S1S'\,(S1S'^2 - 3S'A^2 + K_1) - MS'\,(MS'^2 - 3S'A^2 + K_2)}{K_3}.$$

The distances MS′, S1S′ and S′A can again be derived directly from the differences in co-ordinates.

The question now is, how great is the variation in $s_{MS'} - s_{S1S'}$, if we shift S′ over 1 M, in the x and y direction respectively? This variation is made equal to that in MS′ —S1S′, which is justified for this limited shift.

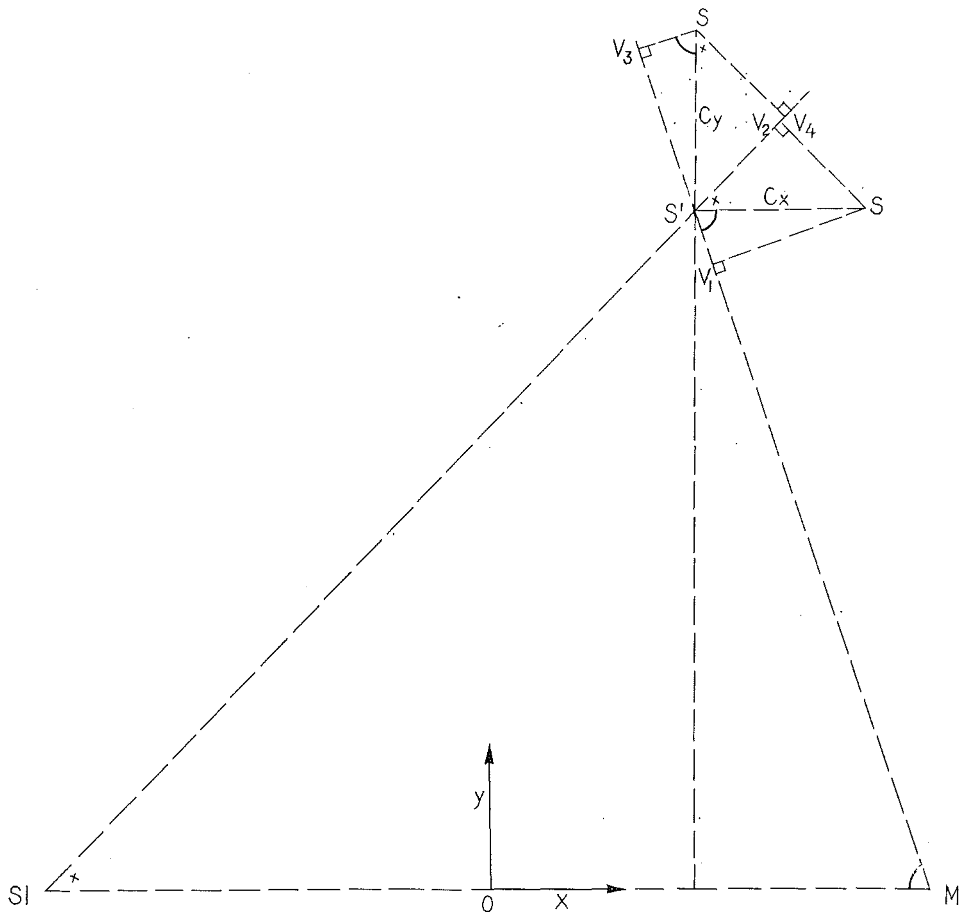


Fig. 3

The value required is thus (see fig. 3):
(MS — S1S) — (MS′ — S1S′) = (MS — MS′) — (S1S — S1S′). For a shift c_x this is practically $- S'V_1 - S'V_2 = - \frac{c_x (x_M - x_{S'})}{MS'} - \frac{c_x (x_{S'} - x_{S1})}{S1S'}$ that is, for a shift of 1 M, and introduced for $x_{S1} = -x_M = \frac{x_{S'} - x_M}{MS'} - \frac{x_{S'} + x_M}{S1S'}$.

For a shift c_y, (MS — MS′) — (S1S — S1S′) is practically $S'V_3 - S'V_4 = \frac{c_y\, y_{S'}}{MS'} - \frac{c_y\, y_{S'}}{S1S'}$, that is for a shift of 1 M: $\frac{y_{S'}}{M_{S'}} - \frac{y_{S'}}{S1S'}$.

$$\text{Therefore: } c_x = \frac{\dfrac{S1S'\,(S1S'^2 - 3S'A^2 + K_1) - MS'\,(MS'^2 - 3S'A^2 + K_2)}{K_3}}{\dfrac{x_{S'} - x_M}{MS'} - \dfrac{x_{S'} + x_M}{S1S'}}.$$

This formula is suitable for computing on the electric computer; it is worked out for the Zebra into:

$$c_x = \frac{MS' \times S1S' \times [S1S'\,(S1S'^2 - 3\,S'A^2 + K_1) - MS'\,(MS'^2 - 3S'A^2 + K_2)]}{K_3\,[S1S'\,(x_{S'} - x_M) - MS'\,(x_{S'} + x_M)]} \quad 6)$$

The corresponding formulae for c_y are:

$$c_y = \frac{\dfrac{S1S'\,(S1S'^2 - 3S'A^2 + K_1) - MS'\,(MS'^2 - 3S'A^2 + K_2)}{K_3}}{\dfrac{y_{S'}}{MS'} - \dfrac{y_{S'}}{S1S'}}$$

$$\text{and } c_y = \frac{MS' \times S1S' \times [S1S'\,(S1S'^2 - 3S'A^2 + K_1) - MS'\,(MS'^2 - 3S'A^2 + K_2)]}{y_{S'}\,K_3\,(S1S' - MS')} \quad 7)$$

The formulae for c_x and c_y also lead to a division of the pattern into zones. However the proviso should be made that the values c_x and c_y are small. If $c_x < c_y$, then a shift according to c_x is preferable; if $c_y < c_x$ then a shift according to c_y is to be preferred. The ideal zone limit thus has as equation: $c_x = c_y$, that is:

$$\frac{x_{S'} - x_M}{MS'} - \frac{x_{S'} + x_M}{S1S'} = \frac{y_{S'}}{MS'} - \frac{y_{S'}}{S1S'}$$

$$\text{or: } \frac{x - x_M}{\sqrt{(x - x_M)^2 + y^2}} - \frac{x + x_M}{\sqrt{(x + x_M)^2 + y^2}} = \frac{y}{\sqrt{(x - x_M)^2 + y^2}} - \frac{y}{\sqrt{(x + x_M)^2 + y^2}}$$

$$\text{or: } \frac{x - x_M - y}{\sqrt{(x - x_M)^2 + y^2}} = \frac{x + x_M - y}{\sqrt{(x + x_M)^2 + y^2}}$$

or: $$\frac{(x - x_M - y)^2}{(x - x_M)^2 + y^2} = \frac{(x + x_M - y)^2}{(x + x_M)^2 + y^2}$$

or: $$\frac{(x - x_M)^2 - 2y(x - x_M) + y^2}{(x - x_M)^2 + y^2} = \frac{(x + x_M)^2 - 2y(x + x_M) + y^2}{(x + x_M)^2 + y^2}$$

or: $$1 - \frac{2y(x - x_M)}{(x - x_M)^2 + y^2} = 1 - \frac{2y(x + x_M)}{(x + x_M)^2 + y^2}$$

or: $$(x + x_M)(x^2 - x^2_M) + y^2(x - x_M) = (x - x_M)(x^2 - x^2_M) + y^2(x + x_M)$$

or: $$2x(x^2 - x^2_M) = 2xy^2$$

or: $y^2 = x^2 - x^2_M$, which is formula 5).

Our zone limits have thus, in view of the corrections to be made to the approximated pattern, come at the ideal positions.

In order to reduce the amount of computing work, formulae 6) and 7) are not applied to every point in the approximated pattern. The variations in the corrections c_x and c_y in fact occur regularly throughout the pattern, and so an interpolation method appears to be justified for the computation. Practical correction formulae will therefore be drawn up, about which more will be said in section 3 of this chapter.

After the pattern has been divided into sections, formulae 6) and 7) are applied only to the corners of the section. Within the section the corrections will be calculated by means of linear interpolation. The criterion to be applied for the division into sections is, in the first place, the admissibility of the fixing of the corrections according to the linear interpolation referred to above. The strongest curvature which thereby occurs in the lines of the section fixes the value of Δ y or Δ x, as was explained in chapter 3, section 2.

Further splitting of a section, in order to make it possible to apply a larger Δ y or Δ x to a part where the curvature of the lines is less strong, so as thus to reduce the amount of computation and charting work must be considered.

In order to establish the section limits, an idea of the variation of c_x and c_y over the whole pattern must be obtained, and for this purpose a number of broadly distributed points are calculated according to formulae 6) and 7), as will be explained in the next section with reference to the Zebra. The scheme of corrections which is thus constructed is therefore used for determining the section limits.

4, 2. **Computation of the scheme of corrections**

The scheme of corrections for zone A will be computed from formula 3) and for zone B and C from formula 4). We give here a description of the computation for zone A only and then indicate the modifications which are involved in the computation for zone B and C.

The computation for zone A is thus done by using the following formulae:

$$x = a\sqrt{1 + \frac{y^2}{x^2_M - a^2}} \qquad 3), \text{ with } a = x_M - \frac{L\Lambda_v}{2} \qquad 1a)$$

$$\text{and } c_x = \frac{MS' \times S1S' \times [S1S' (S1S'^2 - 3S'A^2 + K_1) - MS' (MS'^2 - 3S'A^2 + K_2)]}{K_3 \times [S1S' (x_{S'} - x_M) - MS' (x_{S'} + x_M)]} \quad 6)$$

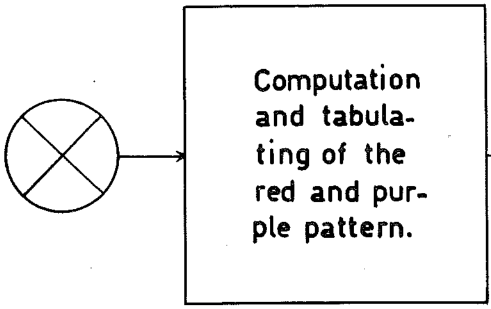
and for regularly increasing values of the independent variables y and L, taking y from y_o to y_e with an interval of Δ y and taking L from L_o to L_e with an interval of Δ L. The lay-out of the results in tabular form is shown in the following example.

y = + 90000

L	x	c_x
13	+ 121548.8	+ 5.7
21	+ 84465.9	+ 6.4
29	+ 61406.6	+ 6.3
37	+ 44160.2	+ 6.2
45	+ 29844.1	+ 6.3
53	+ 17101.0	+ 6.4
61	+ 5142.2	+ 6.7
69	— 6595.5	+ 7.2
77	— 18618.0	+ 7.8
85	— 31505.4	+ 8.8
93	— 46092.2	+ 10.4
101	— 63849.7	+ 13.0
109	— 87996.7	+ 18.2
117	— 128167.0	+ 32.1

y = + 96000

L	x	c_x
13	+ 129372.2	+ 6.5
21	+ 89809.6	+ 7.2
29	+ 65237.0	+ 7.1
37	+ 46884.4	+ 7.0
45	+ 31670.0	+ 7.1
53	+ 18141.7	+ 7.2
61	+ 5454.3	+ 7.6
69	— 6995.9	+ 8.1
77	— 19751.6	+ 8.8
85	— 33434.7	+ 9.9
93	— 48938.9	+ 11.7
101	— 67838.7	+ 14.7
109	— 93574.7	+ 20.7
117	— 136436.0	+ 36.4



simple flow diagram

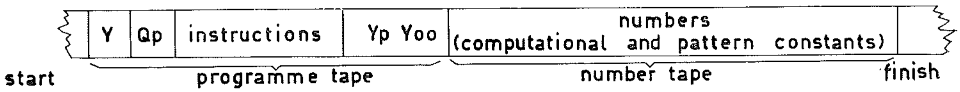
Fig. 4

The computation for both patterns (red and purple) must be done in one uninterrupted operation; the results must be printed bij the machine.
Fig. 4 shows this operation clearly. This type of representation of the computing operation is called a flow diagram. The left symbol comprises the following stages: laying the punched tape in the reading apparatus (input), pressing in the Zebra's starting key, reading of the programme by the input and the input-indication for starting to carry out the programme which has now been stored.

This external part of the flow diagram thus contains the actions which are not carried out by means of instructions from the instruction memory. The latter are considered to be the internal part of the flow diagram.

The number tape which contains the pattern and computational constants must be read in conjunction on the programme tape. If the instructions and number data are punched on a single tape, the tape is made up as shown in fig. 5.

If the instructions and number data appear on separate punched tapes, the instruction labelled p must be a stop instruction, so as to provide an opportunity for laying the number tape in the reading apparatus (see section 2 of the Introduction).



Example of the composition of tape symbols

Fig. 5

The arrangement shown in fig. 5 is used in the following programme. The configuration of the points to be computed is given in fig. 6.

The computation runs as follows:

For the first value y_o of y in the series y_o ....(Δy) ....y_e, the values belonging to it in the series L_o ....(ΔL) ....L_e are computed i.e. $x_{o.o}$, $c_{x_{o.o}}$; $x_{o.1}$, $c_{x_{o.1}}$; ....; $x_{o.e}$ $c_{x_{o.e}}$.

The second value $y_i = y_o + \Delta y$ is then taken and the values $x_{1.o}$, $c_{x_{1.o}}$; $x_{1.1}$, $c_{x_{1.1}}$; ....; $x_{1.e}$, $c_{x_{1.e}}$ are computed in succession.

After y_3 etc., y_e is taken as the final value and the computation of the values $x_{e.o}$, $c_{x_{e.o}}$; $x_{e.1}$, $c_{x_{e.1}}$; .... $x_{e.e}$, $c_{x_{e.e}}$, is then carried out.

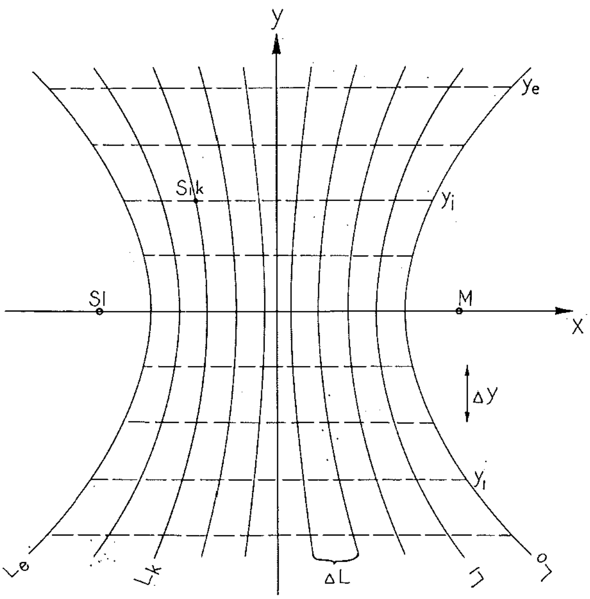


Fig. 6

This computation process leads to the drawing up of 3 main cycles.

1. *The pattern cycle.* Starting from the constants belonging to each pattern, the same series of calculations must be carried out for each pattern. This cycle must be performed in two loops, viz. one loop for the red pattern and one loop for the purple pattern;
2. *the y-cycle.* This lies within the pattern cycle. For each y-value in the series y_o ....(Δy) ....y_e the values x and c_x must be computed in the same way in the sequence L_o ....(ΔL) ....L_e. The number of loops in which the y-cycle must be carried out is the number of terms in the series y_o ....(Δy) ....y_e, called below n_y;
3. *the L-cycle.* This lies within the y-cycle. For each L-value in the series L_o .... (ΔL) ....L_e the x and c_x belonging to it must be calculated in the same way. The number of terms in the L series is the number of loops in which the L-cycle must be carried out, called below n_L.

A summary of the successive operations is given on page 30 which shows how the machine goes through the operations, reading line by line, from left to right.

The internal part of the flow diagram in fig. 4 is elaborated in fig. 7. The three cycles mentioned are obvious. The different boxes in the diagram are indicated by letters. The explanation which follows is in alphabetical order.

a. The pattern cycle is set, which means that the computational constants are inserted in certain count registers recording the number of loops making up the cycle in question;

b. since a new cycle, an inner cycle of the pattern cycle, is introduced in d) and with it new number data enter the auxiliary registers mentioned under a), the computational constants of the pattern cycle must be taken out of the count registers and put in a safe place. The instruction under b) puts these constants into the cell

<table>
<tr><th>pattern cycle</th><th>y-cycle</th><th>L-cycle</th><th colspan="3">Operation</th></tr>
<tr><th>loop</th><th>loop</th><th>loop</th><th>substitution constants</th><th>compute</th><th>print</th></tr>
<tr><td>R(ed)</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td>R_1</td><td>
$R_{1.1}$
$R_{1.2}$
|
|
|
$R_{1.n_L}$</td><td>y_o
L_o
L_1
|
|
|
L_e</td><td>
$x_{o.o}$ $c_{x_{o.o}}$
$x_{o.1}$ $c_{x_{o.1}}$
| |
| |
| |
$x_{o.e}$ $c_{x_{o.e}}$</td><td>y_o
L_o $x_{o.o}$ $c_{x_{o.o}}$
L_1 $x_{o.1}$ $c_{x_{o.1}}$
| | |
| | |
| | |
L_e $x_{o.o}$ $c_{x_{o.e}}$</td></tr>
<tr><td></td><td>R_2</td><td>
$R_{2.1}$
$R_{2.2}$

$R_{2.n_L}$</td><td>y_1
L_o
L_1

L_e</td><td>
$x_{1.o}$ $c_{x_{1.o}}$
$x_{1.1}$ $c_{x_{1.1}}$

$x_{1.e}$ $c_{x_{1.e}}$</td><td>y_1
L_o $x_{1.o}$ $c_{x_{1.o}}$
L_1 $x_{1.1}$ $c_{x_{1.1}}$

L_e $x_{1.e}$ $c_{x_{1.e}}$</td></tr>
<tr><td></td><td>etc.</td><td>etc.</td><td>etc.</td><td>etc.</td><td>etc.</td></tr>
<tr><td></td><td>R_{n_y}</td><td>
$R_{n_y.1}$
$R_{n_y.2}$

$R_{n_y.n_L}$</td><td>y_e
L_o
L_1

L_e</td><td>
$x_{e.o}$ $c_{x_{e.o}}$
$x_{e.1}$ $c_{x_{e.1}}$

$x_{e.e}$ $c_{x_{e.e}}$</td><td>y_e
L_o $x_{e.o}$ $c_{x_{e.o}}$
L_1 $x_{e.1}$ $c_{x_{e.1}}$

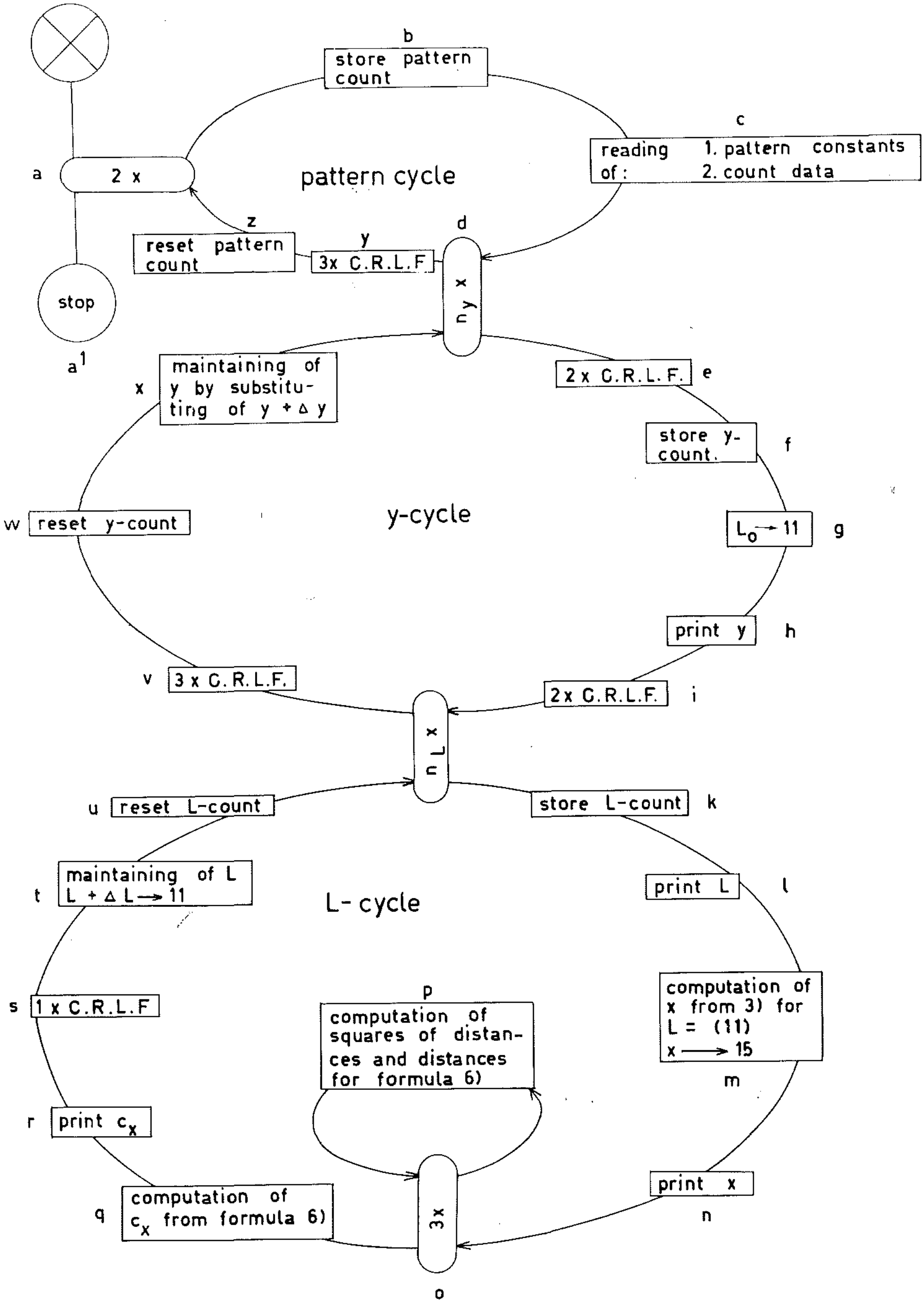
L_e $x_{e.e}$ $c_{x_{e.e}}$</td></tr>
<tr><td>P(urple)</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td>P_1
|
|
|
|
|
etc.</td><td>
$P_{1.1}$

etc.</td><td>y_o
L_o

etc.</td><td>
$x_{o.o}$ $c_{x_{o.o}}$

etc. etc.</td><td>y_o
L_o $x_{o.o}$ $c_{x_{o.o}}$

etc. etc. etc.</td></tr>
</table>



Flow diagram for the computation of the scheme of corrections

Fig. 7

of the number memory reserved for them. We used cell 9 for this purpose. In short, we can also say that the pattern count is preserved;

c. the constants for the pattern co-ordinates of the transmitters, in addition to y_o, Δy, n_y, L_o, ΔL, n_L etc. are read from the number tape and put in the number memory;

d. the pattern cycle is temporarily abandoned and the y-cycle entered. The instructions under d) set this cycle;

e. the instruction C.R.L.F. means: carriage return, line feed; thus a new line is started in the table which is drawn up. 2 × C.R.L.F. produces an interval of one blank line. A similar interval is appropriate at the beginning of the table;

f. the y-count is put into the number memory, in this case in cell 10 (see also b);

g. the initial value L_0 of the L-series is put, for example, in cell 11 of the number memory, in preparation for the L-cycle. L_0 is constantly required at the start of each loop of the L-cycle;

h. the y which is put into the number memory, that is y_o, is printed in the form ± xxxxxx.x;

i. one blank line is inserted between the y just printed and the subsequent table part;

j. the L-cycle is set;

k. since another small auxiliary cycle must be inserted into the L-cycle, the L-count is put into the number memory, in this case into cell 16;

l. the L, that is at the beginning L_o, which has been put into cell 11 of the number memory is printed in the form x x x. L is a whole number and is printed as such;

m. the x is computed from formula 3) and put into cell 15 of the number memory, since this value must be used again for the computation of c_x;

n. x is printed in the form ± xxxxxx.x;

o. and p. with the help of an auxiliary cycle the distances and squares of distances required in formula 6) are calculated;

q. c_x is calculated from formula 6);

r. c_x is printed in the form ± xx.x and this result is rounded off;

s. carriage return, line feed;

t. the L which was put into cell 11 of the number memory is increased by ΔL and maintained. Thus after t) the L which is to be used for the next loop of the L-cycle comes immediately into cell 11 of the number memory;

u. the computational data of the L-cycle which have been put into cell 16 must be put back into the count registers, in order to be able to maintain the count of the number of loops completed. In the subsequent link j the first loop of the L-cycle is closed and counted, after which a new loop of the L-cycle starts. In other words, the L-count is reset. After n_L loops have taken place, the y-cycle is started;

v. two blank lines are added at the end of the part of the table referring to y_o;

w. the y-count is reset;

x. Δy is added to y (now y_o). The y for the next loop of the y-cycle is therefore in the number memory. Link d is now reached again, one loop of the y-cycle is thereby completed and another starts. It must be remembered that in a y-loop n_L loops of the L-cycle occur. After n_y loops of the y-cycle the pattern cycle is restarted. The table for area A of the first pattern has been produced;

y. the table for the first pattern is closed with 2 blank lines;

z. the pattern count is reset.

The first loop of the pattern cycle is closed by a) and the second loop starts. The operations referred to above are reset with the new pattern constants introduced under c).

After the second loop of the pattern cycle, which therefore comprises n_y loops of the y-cycle and $n_y \times n_L$ loops of the L-cycle, the computation is finished and the pattern cycle is abandoned.

a[1]). The computer stops and the tables can be taken out of the teleprinter.

An analogous working out of area B would produce only that part of the pattern for which y is positive. For the part with negative y the root in formula 4) must be multiplied by —1.

In order to achieve this, we insert a σ cycle into the programme for area B and C between the pattern cycle and the inner cycles; this σ cycle must be completed in two loops, for the first of which $\sigma = +1$ and for the second of which $\sigma = -1$ is put into the number memory. The values calculated for y in the L-cycle are then

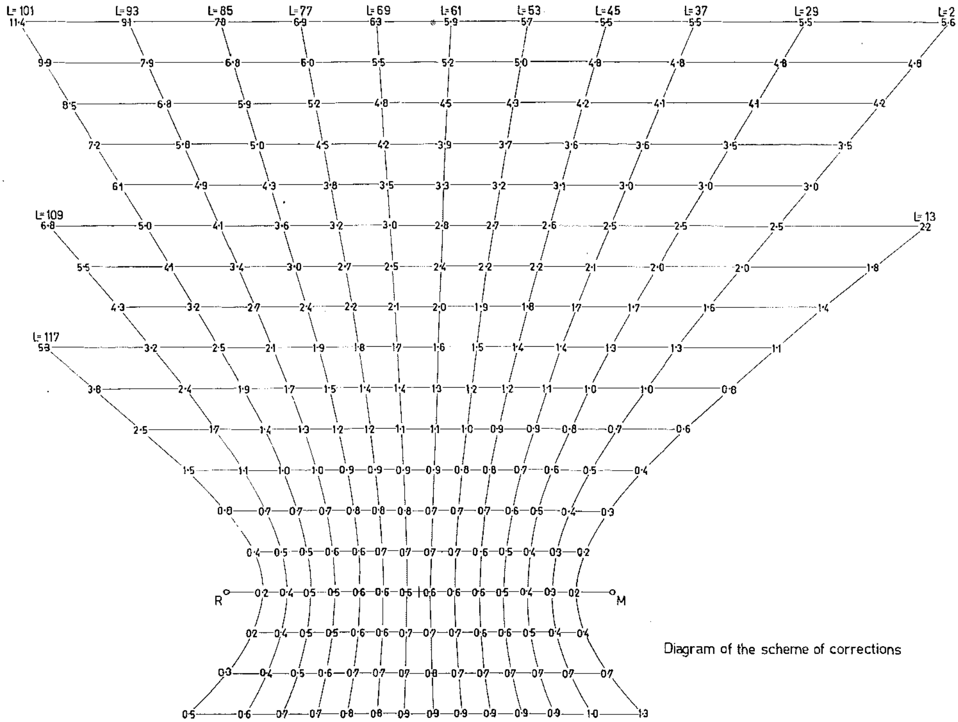


Diagram of the scheme of corrections

Fig. 8

multiplied by σ for each further application. In this way, when computing zone B, first the positive and then the negative parts of the pattern are calculated. When computing zone C, where $a = x_M - \dfrac{L\Lambda_v}{2}$ is negative, formula 1a), the negative part is computed in the first loop of the σ cycle and the positive part in the second loop.

4, 3. The practical correction formulae for obtaining the Decca pattern from the approximated pattern

Fig. 8 shows the scheme of corrections for zone A of the red pattern of the Delta chain, calculated as in section 2 of this chapter. All corrections are positive and are given in metres. Thus the aim is to investigate the possibility of linear interpolation between accurately calculated corrections for corners of a section. For a section containing 10.000 points the saving is approximately 384.000 instructions, which represents a Zebra time of 384.000 × 30 milliseconds, i.e. at least 3 hours, assuming that the average time required for carrying out an instruction in a programme is 30 milliseconds. For the differences between interpolated and accurately calculated corrections we shall have to fix a tolerance, by means of which the section limits can be derived from the scheme of corrections.

The interpolation formulae are established as follows. A section of zone A (fig. 9) is dealt with here; it is limited by the grid lines y_o and y_e and the lines of lanes L_o and L_e.

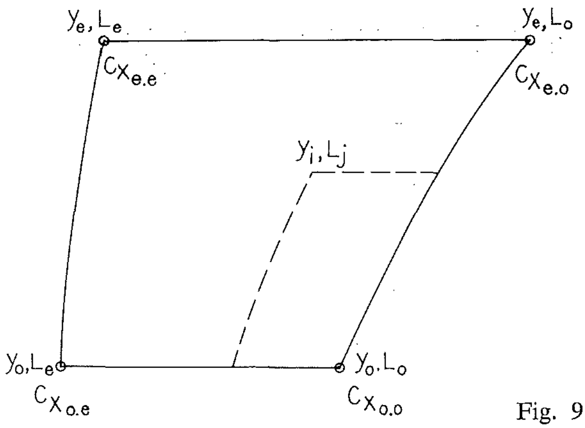


Fig. 9

The c_x corrections are calculated from formula 6) at the corners, indicated by $c_{x_{o.o}}$, $c_{x_{o.e}}$, $c_{x_{e.o}}$ and $c_{x_{e.e}}$.

The x-co-ordinates of the points on the approximated pattern are calculated per section, first of all for $y = y_o$ and for L varying from L_o to L_e with an interval of ΔL, for which 0.1 is taken.

The c_{x_o} correction is interpolated linearly between $c_{x_{o.o}}$ and $c_{x_{o.e}}$ according to the formula:

$$c_{x_{o.j}} = p_o L_j + q_o.$$

Thus for point y_o, L_o, $c_{x_{o.o}} = p_o L_o + q_o$
and for point y_o, L_e, $c_{x_{o.e}} = p_o L_e + q_o$

$$\text{from which: } p_o = \frac{c_{x_{o.e}} - c_{x_{o.o}}}{L_e - L_o} \quad \text{and } q_o = \frac{c_{x_{o.o}} L_e - c_{x_{o.e}} L_o}{L_e - L_o}$$

In the next loop y has become $y_o + \Delta y = y_1$.

The x is now corrected according to the formula:

$$c_{x1.j} = p_1 L_j + q_1.$$

Generally, when y has become y_i, the correction formula becomes:

$$c_{xi.j} = p_i L_j + q_i$$

For the line $y = y_e$ the following holds good:

$$c_{xe.j} = p_e L_j + q_e.$$

As a result, for the point y_e, L_o:

$$c_{xe.o} = p_e L_o + q_e$$

and for the point y_e, L_e: $c_{x_{e.e}} = p_e L_e + q_e$

from which $p_e = \dfrac{c_{x_{e.e}} - c_{x_{e.o}}}{L_e - L_o}$ and $q_e = \dfrac{c_{x_{e.o}} L_e - c_{x_{e.e}} L_o}{L_e - L_o}$

The p and q coefficients of the interpolation formulae for the intermediate lines of constant y are found by linear interpolation between p_e and p_o and between respectively q_e and q_o, as a result of which the differences $p_{i+1} - p_i = \Delta p$ and

$q_{i+1} - q_i = \Delta q$ are constant. Thus $\Delta p = \dfrac{p_e - p_o}{y_e - y_o} \Delta y$ and $\Delta q = \dfrac{q_e - q_o}{y_e - y_o} \Delta y$.

By this means p_{i+1} can easily be derived from: $p_i + \Delta p = p_{i+1}$ and similarly $q_i + \Delta q = q_{i+1}$.

As a general formula this becomes:

$$p_i = p_o + i \frac{p_e - p_o}{y_e - y_o} \Delta y = p_o + \frac{(p_e - p_o)(y_i - y_o)}{y_e - y_o}$$

and:

$$q_i = q_o + i \frac{q_e - q_o}{y_e - y_o} \Delta y = q_o + \frac{(q_e - q_o)(y_i - y_o)}{y_e - y_o}$$

as $i = \dfrac{y_i - y_o}{\Delta y}$.

With omission of the indices i, j becomes

$$c_x = \left\{ p_o + \frac{(p_e - p_o)(y - y_o)}{y_e - y_o} \right\} L + \left\{ q_o + \frac{(q_e - q_o)(y - y_o)}{y_e - y_o} \right\}$$

or, changing over to the general constants P, Q, R and S

$$c_x = (Py + Q) L + (Ry + S).$$

This general formula implies linear interpolation both for the y constant and for the L constant. The next task is to find out for a part of the scheme of corrections the differences between the accurately calculated c_x values and the values obtained by using the above interpolation method. These differences are listed on page 36 they refer to an area lying between L = 45 and L = 85, and y = 60 kms and y = 102 kms. If these differences exceed the tolerance, taken as o. 25 m, the section chosen is too large. This is the case in the example given; the section must therefore be reduced in size. In addition the contents of 3.2 are also taken into account. Reduction of the size of the sections makes it possible to reduce the number of Δy and Δx intervals.

The tolerance for the above differences refers to the accuracy of the co-ordinates to be mapped. This must correspond to the accuracy with which the Decca lines are mapped. The latter are created by connection of the calculated points set out with a co-ordinate plotter. Connection by means of straight lines has been aimed at. If the accuracy of the mapping is fixed at 0.1 mm, corresponding on the scale 1 : 5000 to

	L = 85		L = 77		L = 69		L = 61		L = 53		L = 45		
y = 102 km	11.1	0	10.5	0.6	9.8	0.8	9.2	0.7	8.5	0.4	7.9	0	c_x(interp.)
	11.1		9.9		9.0		8.5		8.1		7.9		c_x(exact)
96 km	10.1	0.2	9.5	0.7	9.0	0.9	8.4	0.8	7.8	0.6	7.2	0.1	
	9.9		8.8		8.1		7.6		7.2		7.1		
90 km	9.2	0.4	8.6	0.8	8.1	0.9	7.6	0.9	7.1	0.7	6.5	0.2	
	8.8		7.8		7.2		6.7		6.4		6.3		
84 km	8.2	0.4	7.7	0.8	7.2	0.9	6.8	0.9	6.3	0.6	5.8	0.3	
	7.8		6.9		6.3		5.9		5.7		5.5		
78 km	7.2	0.4	6.8	0.8	6.4	0.9	6.0	0.8	5.6	0.6	5.2	0.4	
	6.8		6.0		5.5		5.2		5.0		4.8		
72 km	6.2	0.3	5.9	0.7	5.5	0.7	5.2	0.7	4.8	0.5	4.5	0.3	
	5.9		5.2		4.8		4.5		4.3		4.2		
66 km	5.3	0.3	5.0	0.5	4.7	0.5	4.4	0.5	4.1	0.4	3.8	0.2	
	5.0		4.5		4.2		3.9		3.7		3.6		
60 km	4.3	0	4.1	0.3	3.8	0.3	3.6	0.3	3.3	0.1	3.1	0	
	4.3		3.8		3.5		3.3		3.2		3.1		

0.50 m in reality, it is safe to permit a tolerance of 0.25 m for the co-ordinates and a tolerance of 0.25 m for the middle of the connection lines between 2 calculated points. With the help of the latter tolerance, Δy and Δx can be fixed for each section.

These tolerances could be 0.50 m for a map scale of 1 : 10 000 and 2.50 m for a scale of 1 : 50 000. Therefore for smaller scales the sections and also the Δy and Δx intervals can be larger.

4, 4. The programmes for computing the Decca pattern

Just as for the scheme of corrections, zone A on the one hand and zones B and C on the other are computed with the help of separate programmes. For zone

A the x-co-ordinates of the points in the Decca pattern corresponding to an arithmetical series of values running from y_o ....(Δy) ....y_e and from L_o ....($\Delta L = 0.1$) ....L_e must be computed in sections, whereas for zones B and C the y-co-ordinates of the points in the Decca pattern corresponding to the values x_o ....(Δx) .... x_e and L_o .... ($\Delta L = 0.1$) ....L_e must be computed.

The following organisational requirements are drawn up in view of the great extent of the computational work.

a. the computer must be capable of working discontinuously by day and continuously by night for this computational work;

b. after the production of one section the calculations must, if necessary, be capable of being interrupted;

c. the machine must be in a position to choose on its own from the number memory the pattern constants corresponding to a section;

d. during the day work the calculations must be capable of being interrupted after the production of any particular line. On resumption of the computational work, it must be possible to choose from among the 3 possibilities given below, with the help of the dial.

1. The computation must, beginning with any page of the table, be started and carried on normally. The input of the y or, as the case may be x co-ordinate corresponding to this page is done by means of the dial.
2. The computation of the partly completed page must be restarted at the beginning of this page.
3. The machine must be in a position to start computing a new section.

In order to keep the description as simple as possible, we separate the day programme from the night programme. The directions b and c must be built into the night programme and the directions b-c and d into the day programme. According to the day programme the machine stops after producing a section. The general structure of the night programme for producing the tables for zone A is shown in figure 10.

Less details of an arithmetical and organisational nature are given in it thans in fig. 7 in order to stress both the course of the computation and, which is new, the very characteristic branches which occur in such a programme. After examination of this programme, we shall deal with the realisation of direction d in the day programme.

In the flow diagram as shown in fig. 10 the internal computing process is indicated by ordinary lines while the dotted lines indicate the external acts.

After the starting button has been pressed, the programme is read in and the instruction for carrying out the programme which has just been read in is given (symbol A); in block B preparatory calculations, which are independent of the section, are carried out and the general computational constants and pattern constants are read in.

The machine now stops according to C, so as to enable the operator to place a separate tape with punched number data (section constants) in the reading apparatus and start the machine again. If the number data and the instructions have been punched on one tape, the operator simply starts the machine again. Phase D is thus reached, in which the section constants are read in.

F. The pattern constants were put into a permanent place in the number memory in phase B. These permanent places can be considered as the storage space in the number memory for the various constants. The computer transfers these data to other per-

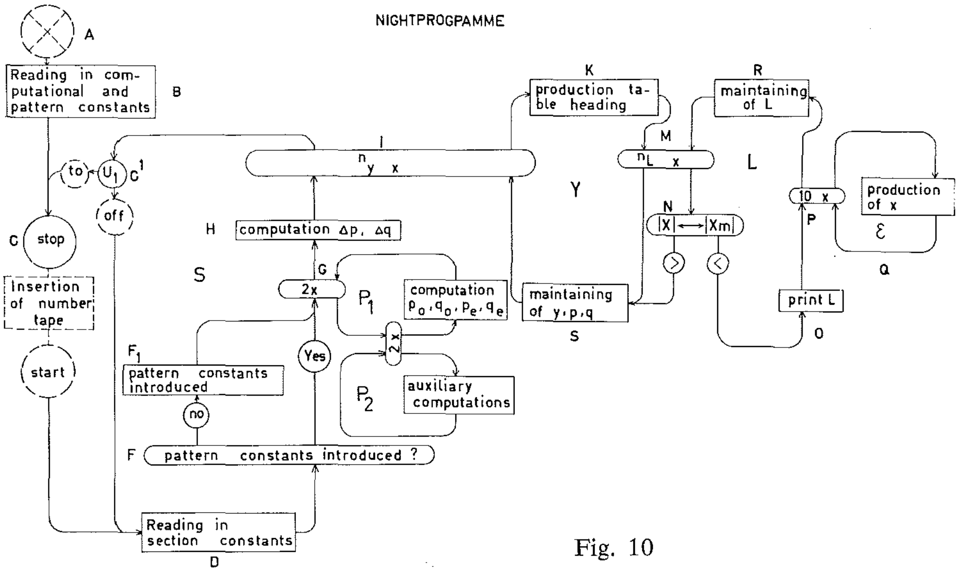


Fig. 10

manent places in the number memory on behalf of the section computations; these memory places are called working registers. Transferring the pattern constants to the working registers must however be done only if this has not already occurred in connection with a previous section calculation. F now applies a test, by which an answer is given to the question: are the pattern constants belonging to the section which is to be calculated in their working register or not? If they are, continue the calculations with G; if they are not, first put the pattern constants into their working registers (F_1). In G the alternative paths from test F are reunited.

In cycle P_1 the starting values p_0 and q_0 of the correcting polynomial are then calculated via the auxiliary cycle P_2 for the auxiliary calculations in the first loop, and are stored in the memory; in the second loop, the end values p_e and q_e are calculated.

In H Δ p and Δ q are now calculated and transferred to the memory. The stage has now been reached at which the y-cycle is inserted (I), which will produce a whole page of the table in one loop.

The headline of the table is reproduced in K. The L-cycle is then gone through in M.

For each loop of the L-cycle a test occurs in phase N. The production of the tables is in fact made dependent on the following limitation: in order to avoid excessive computing work at the sides of the pattern, the possibility of limiting a section according to co-ordinate values (in our case a line of the x-constant) must be included in the programme; in other words it must be possible to avoid the computation of points in the pattern having co-ordinates which exceed a previously established limit. Fig. 11 shows a case of this type.

The computation of the arc part of the section is not worthwhile and is prevented by means of N. The value $x_{max.}$ is one of the section constants which were read in by D.

In N the problem is to find out whether the x-co-ordinate computed at the end of

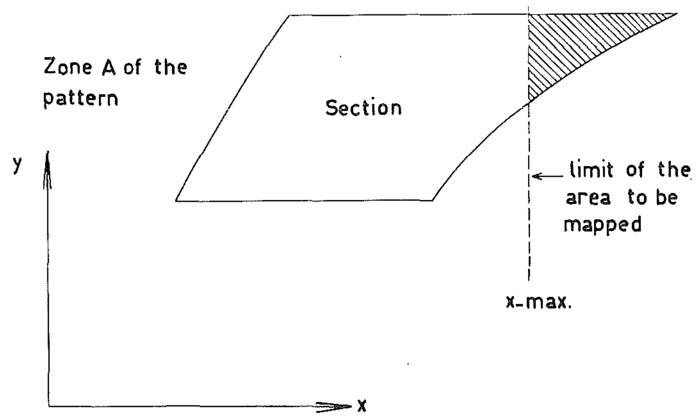


Fig. 11

the previous loop of the L-cycle exceeds the limiting value $x_{max.}$ (in absolute value). If it does, the production of the page of the table for the y-value printed in K is completed; the computing procedure reverts within the y-cycle to S, where the count of y, p and q is maintained. If it does not, the L which has to be placed at the beginning of each line is printed in 0. The proper line of the table is produced in an ε-auxiliary cycle, which is set in P. During each loop the values x and c_x are calculated in Q, added up and printed. The L is increased by 0.1 each time, so that after 10 loops the ε-cycle has produced a line of the table. The calculating operation now reverts to the L-cycle and we come to phase R, where the L-count is maintained.

After n_L loops of the L-cycle (except for the interception in phase N) in which 10 n_L loops of the ε-cycle are completed, the y-cycle is again reached, and y and the co-efficients p and q of the correcting polynomial are maintained in S. After n_y loops of the y-cycle the production of the tables for the desired section is completed, and the calculating operation turns to phase C^1, in which the position of the switch U_1 is examined. If it is on, then we stop, in accordance with C. Only one section then had to be computed. A new section can then be started, in the way already described. If the switch is off, a new section is automatically started. The section constants to be used for it must then be punched with the instructions on one tape, so as to be read in by phase D. All the phases mentioned are now gone through again.

Most of the stages in the day programme (fig. 12) are the same as in the night programme. After the section constants have been read in by phase D, a bifurcation E occurs, which is dependent on the position of the switch U_1. The position of this switch also plays a part in stage T. If the switch is off, the whole computation proceeds exactly as for the night programme; however it is stopped at the end of each section; the computation of a new section does not follow automatically. If the switch is on, the stop instruction is given in V via E or T. The machine now waits for the insertion of the number 1, 2 or 3 by means of the dial; this number corresponds to the calculation procedure to be followed, as described under instruction d.

1. The machine stops again and waits for y to be inserted by means of the dial, shown in the flow diagram bij $y \rightarrow \alpha$.

After this the machine starts automatically and the starting values for the page of the table which must be begun are calculated in W. The calculation then continues with F.

2. Since the maintaining of y in T has not yet occurred, the page of the table which has been started may again be produced, because a jump to F is performed. This jump is prepared in Z.
3. The calculating operation will simply switch over to the beginning of the section cycle S. The first instruction in this cycle is the stop instruction C. The number tape with the constants of the section to be calculated can be inserted. After pressing the starting key the production of a new section is started.

Logically, a discussion of the transference of figs. 10 and 12 into the Zebra code should now be given. This however falls outside the scope of this report, in which we are concerned only with the general lines of electronic computing.

The diagrammatic structure of the programme tape is also shown for the day programme; fig. 13 refers to the letters in fig. 12.

As soon as the input reads the input indication Y, the machine is brought into readiness for reading in the programme. The instructions enter the instruction memory in the order in which they have been punched on the programme tape.

The first signal after Y is the input indication Q 1, which means: the following instruction has the parameter label 1. In this case it is the first instruction in the groups D and F-Z.

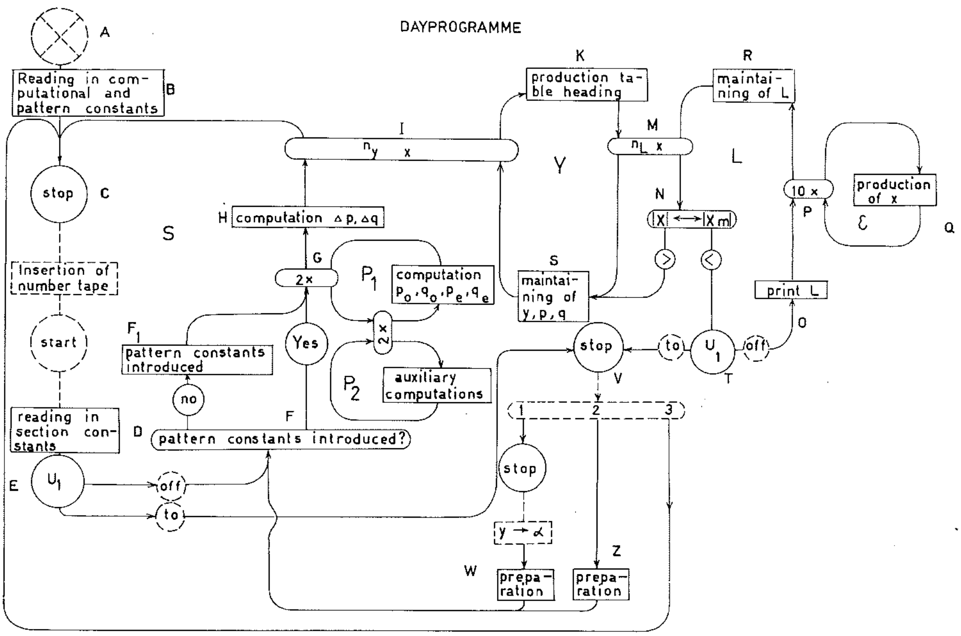
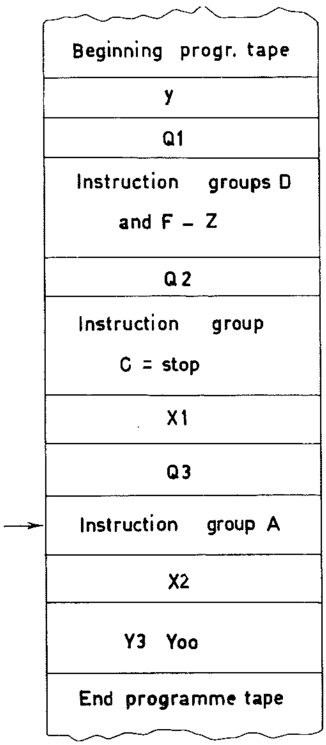


Fig. 12



The programme tape
Fig. 13

In the same way the stop instruction in C gets the parameter label 2 and the first instruction in group A the label 3.

At the end of the programme tape the input reads the input indication Y3 Y00, which means: start carrying out the programme with the instruction bearing the parameter label 3. This is the first instruction in the instruction group A (see arrow in fig. 13) which follows the symbol Q3. The A instructions are then carried out in the order in which they were read in. The computing operation now comes to the unconditional jump instruction X2. This tells the machine: continue with the instruction bearing the parameter label 2. This is stop instruction C. The machine stops and the section tape can be inserted. After pressing the starting key the machine proceeds to the next instruction. This is also an unconditional jump instruction X1, which tells the machine to proceed to carry out the instruction bearing the parameter label 1. This is the first instruction in the groups D and F-Z which, as we know, actually produce the tables. The instruction which follows the last one in this group is again the stop instruction C, and so the next section tape can be inserted, after which the production cycle for the next section is started by pressing the starting key.

Page 42 shows an example of a table. The number —41 000 is the value for y. The number 2.55 refers to the section indication, which in fact means pattern 2 (purple), section No. 55. At the beginning of each line there is the full lane number and at the top of each column the sub-division of it. The x-co-ordinates of the Decca lines have thus been put in table form. Thus for lane 66.2, for example, the point x = 12.192.5 y = —41.000 has been calculated. The programmes for calculating the tables for zones B and C have essentially the same structure as that illustrated in figs. 10 and 12. The difference in the basic formulae must of course be taken into account.

Since, as we have already said, in the approximated pattern one x-value corresponds to two symmetrical + y and — y values, additional information must be included in the computing scheme, which instructs the machine, for the positive or negative value of the y-co-ordinate to be chosen, depending on where the section is situated. This is done in the programmes for zones B and C by inserting new section constant σ. If the section is situated in quadrants with a positive y, the section constant $\sigma = +1$ is inserted; in the other case $\sigma = -1$ is taken. Since however the co-efficient a in formula 1a is also positive or negative, according to whether the section is situated in zone B or zone C, we replace the y-value after calculating it by the absolute amount. The sign of $y_i + c_{y_i}$ thus depends only on the choice of the section constant σ.

Thus the single principle alteration required in the programme for zone A in order

— 41000 2.55

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
46	26015.9	25939.0	25862.2	25785.6	25709.1	25632.7	25556.4	25480.2	25404.1	25328.2
47	25252.4	25176.7	25101.1	25025.6	24950.2	24875.0	24799.8	24724.8	24649.9	24575.0
48	24500.3	24425.7	24351.2	24276.8	24202.6	24128.4	24054.3	23980.3	23906.5	23832.7
49	23759.0	23685.5	23612.0	23538.6	23465.4	23392.2	23319.1	23246.2	23173.3	23100.5
50	23027.9	22955.3	22882.8	22810.4	22738.1	22665.9	22593.8	22521.7	22449.8	22378.0
51	22306.2	22234.5	22163.0	22091.5	22020.1	21948.8	21877.6	21806.4	21735.4	21664.4
52	21593.5	21522.7	21452.0	21381.4	21310.9	21240.4	21170.0	21099.7	21029.5	20959.4
53	20889.3	20819.3	20749.5	20679.6	20609.9	20540.2	20470.7	20401.1	20331.7	20262.4
54	20193.1	20123.9	20054.8	19985.7	19916.7	19847.8	19779.0	19710.2	19641.5	19572.9
55	19504.4	19435.9	19367.5	19299.2	19230.9	19162.7	19094.6	19026.5	18958.5	18890.6
56	18822.8	18755.0	18687.3	18619.6	18552.0	18484.5	18417.0	18349.6	18282.3	18215.0
57	18147.8	18080.7	18013.6	17946.6	17879.7	17812.8	17745.9	17679.2	17612.5	17545.8
58	17479.2	17412.7	17346.2	17279.8	17213.5	17147.2	17080.9	17014.8	16948.6	16882.6
59	16816.6	16750.6	16684.7	16618.9	16553.1	16487.4	16421.7	16356.1	16290.5	16225.0
60	16159.5	16094.1	16028.7	15963.4	15898.2	15833.0	15767.8	15702.7	15637.7	15572.7
61	15507.7	15442.8	15378.0	15313.2	15248.5	15183.7	15119.1	15054.5	14989.9	14925.4
62	14861.0	14796.5	14732.2	14667.9	14603.6	14539.3	14475.2	14411.0	14346.9	14282.9
63	14218.9	14154.9	14091.0	14027.1	13963.3	13899.5	13835.8	13772.0	13708.4	13644.8
64	13581.2	13517.7	13454.2	13390.7	13327.3	13263.9	13200.6	13137.3	13074.0	13010.8
65	12947.7	12884.5	12821.4	12758.4	12695.4	12632.4	12569.4	12506.5	12443.6	12380.8
66	12318.0	12255.3	12192.5	12129.9	12067.2	12004.6	11942.0	11879.5	11816.9	11754.5
67	11692.0	11629.6	11567.3	11504.9	11442.6	11380.3	11318.1	11255.9	11193.7	11131.6
68	11069.5	11007.4	10945.3	10883.3	10821.3	10759.4	10697.5	10635.6	10573.7	10511.9
69	10450.1	10388.3	10326.6	10264.9	10203.2	10141.5	10079.9	10018.3	9956.7	9895.2
70	9833.7	9772.2	9710.7	9649.3	9587.9	9526.5	9465.2	9403.9	9342.6	9281.3
71	9220.1	9158.8	9097.6	9036.5	8975.3	8914.2	8853.1	8792.1	8731.0	8670.0
72	8609.0	8548.0	8487.1	8426.1	8365.2	8304.4	8243.5	8182.7	8121.9	8061.1
73	8000.3	7939.6	7878.8	7818.1	7757.5	7696.8	7636.2	7575.5	7514.9	7454.4
74	7393.8	7333.3	7272.8	7212.3	7151.8	7091.3	7030.9	6970.5	6910.1	6849.7
75	6789.3	6729.0	6668.6	6608.3	6548.0	6487.8	6427.5	6367.3	6307.0	6246.8

to obtain the programme for zones B and C is that the following instructions are incorporated into the ε-cycle:

replace y by $|y|$
multiply $|y|$ by σ

$\sigma = +1$ for a quadrant with positive y.
$\sigma = -1$ for a quadrant with negative y.
σ is introduced into the machine as section constant.

Finally attention is drawn to the small amount of "calculations" involved in the programmes discussed the work consists mainly of organisational processes.

The opportunities for making errors when drawing up a programme are so numerous that it is practically impossible to obtain an errorfree programme at once. We cannot here go into the various types of errors and how to trace them. It is therefore always essential to test a programme which has been drawn up very carefully.

Testing the programmes mentioned was done in the following way. The Zebra was allowed to print the intermediate results obtained at particularly critical points in the programme. These intermediate results were compared with the results obtained independently on the electric computer. Errors whose origin must be attributed to the programme can in this way quickly be discovered and corrected.

5. COMPUTATION OF DECCA CO-ORDINATES FROM RD CO-ORDINATES AND VICE VERSA

Both the problems given in the title may arise. In practice a sufficiently accurate solution can be obtained via the mapped Decca pattern; the larger the map scale the more accurate the solution. The arithmetical solution is set out below.

Given: the RD co-ordinates X_S, Y_S of point S.

Required: the lane numbers L_R and L_P of this point.

Solution: Calculate:

$$s_{MS} = MS + MS \times \frac{MS^2 - 3\ MA^2 - 3\ SA^2 + 24\ r^2 m_o\ (1 - m_o)}{24\ r^2 m_o}$$

$$s_{RS} = RS + RS \times \frac{RS^2 - 3\ RA^2 - 3\ SA^2 + 24\ r^2 m_o\ (1 - m_o)}{24\ r^2 m_o}$$

$$s_{PS} = PS + PS \times \frac{PS^2 - 3\ PA^2 - 3\ SA^2 + 24\ r^2 m_o\ (1 - m_o)}{24\ r^2 m_o}$$

Then: $L_R = \dfrac{s_{MR} + s_{MS} - s_{RS}}{\Lambda_R}$

and $L_P = \dfrac{s_{MP} + s_{MS} - s_{PS}}{\Lambda_P}$

If the result need not be so accurate, the Decca co-ordinates can be calculated from the flat approximated pattern and the process is then much simpler:

$$L_{Rv} = \frac{MR + MS - RS}{\Lambda_{Rv}}$$

and $L_{Pv} = \dfrac{MP + MS - PS}{\Lambda_{Pv}}$

The reverse problem is arithmetically more complicated:

Given: the lane numbers L_R and L_P of the point S.

Required: the RD co-ordinates X_S and Y_S of the point S.

Solution: The required co-ordinates can be estimated graphically via the map, thus giving $X_{S'}$ and $Y_{S'}$, so that the corrections $\Delta X_{S'}$ and $\Delta Y_{S'}$ have to be calculated, since $X_S = X_{S'} + \Delta X_{S'}$ and $Y_S = Y_{S'} + \Delta Y_{S'}$.

It follows from the given L_R and L_P that:

$$\begin{cases} s_{MS} - s_{RS} = L_R\ \Lambda_R - s_{MR} \\ s_{MS} - s_{PS} = L_P\ \Lambda_P - s_{MP} \end{cases}$$

For the estimated point S′ with co-ordinates $X_{S'}$, and $Y_{S'}$ the following applies:

$$s_{MS'} - s_{RS'} = (MS' - RS') + \frac{MS'\ (MS'^2 - 3\ S'A^2 + C_1) - RS'\ (RS'^2 - 3\ S'A^2 + C_2)}{C_3}$$

$$s_{MS'} - s_{PS'} - (MS' - PS') + \frac{MS'\ (MS'^2 - 3\ S'A^2 + C_1) - PS'\ (PS'^2 - 3\ S'A^2 + C_2)}{C_3}$$

in which
$$C_1 = -3\ MA^2 + 24\ r^2 m_o\ (1 - m_o)$$
$$C_2 = -3\ SlA^2 + 24\ r^2 m_o\ (1 - m_o)$$
$$C_3 = 24\ r^2 m_o$$

From this $(s_{MS} - s_{RS}) - (s_{MS'} - s_{RS'})$ and $(s_{MS} - s_{PS}) - (s_{MS'} - s_{PS'})$, can be determined, these being the corrections to be made to the spherical distance differences $s_{MS'} - s_{RS'}$ and $s_{MS'} - s_{PS'}$ in order to arrive at those for S.

The variations for these spherical distance differences can, for a limited area, again be taken as equal to those of the distance differences for the plane of projection, so that in practice the following can be established:

$$(s_{MS} - s_{RS}) - (s_{MS'} - s_{RS'}) = (MS - RS) - (MS' - RS')$$

and
$$(s_{MS} - s_{PS}) - (s_{MS'} - s_{PS'}) = (MS - PS) - (MS' - PS')$$

The values $\Delta X_{S'}$ and $\Delta Y_{S'}$, thus have to be found, for which:

$$(MS - RS) - (MS' - RS') = r \qquad \text{(given)} = f\ (\Delta X_{S'}\ ,\ \Delta Y_{S'})$$
$$(MS - PS) - (MS' - PS') = p \qquad \text{(given)} = g\ (\Delta X_{S'}\ ,\ \Delta Y_{S'})$$

If we indicate a point S'_1, with co-ordinates $(X' + 10\text{ m}),\ Y'$

$$\left.\begin{array}{ll}\text{then:} & (MS'_1 - RS'_1) - (MS' - RS') = r_1 \\ \text{and:} & (MS'_1 - PS'_1) - (MS' - PS') = r_2\end{array}\right\}$$

Similarly, for a point S'_2, with co-ordinates $X',\ (Y' + 10\text{m})$

$$\left.\begin{array}{ll} & (MS'_2 - RS'_2) - (MS' - RS') = p_1 \\ \text{and:} & (MS'_2 - PS'_2) - (MS' - PS') = p_2\end{array}\right\}$$

Then for a limited area the following holds good:

$$\left.\begin{array}{l} r = r_1\ \Delta X + p_1 \Delta Y \\ p = r_2\ \Delta X + p_2 \Delta Y\end{array}\right\} \text{ in which } \Delta X \text{ and } \Delta Y \text{are expressed in units of 10 m.}$$

$$\text{or } \Delta Y = \frac{pp_1 - rp_2}{r_2 p_1 - r_1 p_2}\ , \text{ or in metres: } \Delta X = 10 \times \frac{pp_1 - rp_2}{r_2 p_1 - r_1 p_2}$$

$$\text{and } \Delta Y = \frac{r_2 r - r_1 p}{r_2 p_1 - r_1 p_2}\ , \text{ or in metres: } \Delta Y = 10 \times \frac{r_2 r - r_1 p}{r_2 p_1 - r_1 p_2}$$

A check can be obtained from the solution to the first problem. For a less accurate result the flat approximated pattern can again be used. The following differences are to be ascertained:

$$\left.\begin{array}{l} MS - RS = L_R \Lambda_{RV} - MR \\ MS - PS = L_P \Lambda_{PV} - MP \end{array}\right\}$$

From the estimated point S′, MS′ — RS′ and MS′ — PS′ follow.

The values $\Delta X_{S'}$ and $\Delta Y_{S'}$ have to be found, for which:

$(MS - RS) - (MS' - RS') = r$ (given)

$(MS - PS) - (MS' - PS') = p$ (given)

The rest of the calculation is done in a similar way to that for the previous method.

6. THE RELATIONSHIP BETWEEN THE WAVELENGHTS OF THE MATHEMATICAL PATTERN AND THE RADIATED PATTERN

In Chapter 2 the idealisation of the radiated pattern into the mathematical pattern in respect of the wavelength was discussed. A fixed wavelength Λ_R and Λ_P, or Λ_{Rv} and Λ_{Pv} is derived from the lane count for the mathematical pattern. Because of the difference in electromagnetic conductivity of the areas over which the radiated waves pass, the propagation speed, and thus the wavelength, which has to be taken geodesically as standard measure, are not constant in the radiated pattern.

If we reinterpret the considerations made by Verstelle in his Toronto publications for this standard measure and if we restrict ourselves to three types of terrain, viz. sea, land except sandy soil and sandy soil principally dunes, then three different wavelengths must be distinguished, viz. Λ_z, Λ_l and Λ_d respectively. Thus, for example, the radiated red pattern can be formulated as:

$$L'_R = \frac{s_{MR}}{\Lambda_R} + \frac{s_{MS_z}}{\Lambda_{z_R}} + \frac{s_{MS_1}}{\Lambda_{1_R}} + \frac{s_{MS_d}}{\Lambda_{d_R}} - \frac{s_{RS_z}}{\Lambda_{z_R}} - \frac{s_{RS_1}}{\Lambda_{1_R}} - \frac{s_{RS_d}}{\Lambda_{d_R}}$$

whereas the map pattern is:

$$L_R = \frac{s_{MR} + s_{MS} - s_{RS}}{\Lambda_R} = \frac{s_{MR}}{\Lambda_R} + \frac{s_{MS_z} + s_{MS_1} + s_{MS_d} - s_{RS_z} - s_{RS_1} - s_{RS_d}}{\Lambda_R}$$

The map pattern must therefore be corrected by $L'_R - L_R$, or the decometer reading can be made to agree with the map pattern by correcting L_R by $L_R - L'_R$.

These corrections thus amount to:

$$L_R - L'_R = \left(\frac{1}{\Lambda_R} - \frac{1}{\Lambda_{z_R}}\right)(s_{MS_z} - s_{RS_z}) + \left(\frac{1}{\Lambda_R} - \frac{1}{\Lambda_{1_R}}\right)$$

$$(s_{MS_1} - s_{RS_1}) + \left(\frac{1}{\Lambda_R} - \frac{1}{\Lambda_{d_R}}\right)(s_{MS_d} - s_{RS_d})$$

$$= C_1 (s_{MS_z} - s_{RS_z}) + C_2 (s_{MS_1} - s_{RS_1}) + C_3 (s_{MS_d} - s_{RS_d})$$

in which s_{MS_z} and s_{RS_z} represent the distances over sea, included in s_{MS} and s_{RS}; s_{MS_1} and s_{RS_1} the distances over the second type of terrain and s_{MS_d} and s_{RS_d} the distances over the last type of terrain. Accordingly, the correction in the purple pattern is:

$$L_P - L'_P = \left(\frac{1}{\Lambda_P} - \frac{1}{\Lambda_{z_P}}\right)(s_{MS_z} - s_{PS_z}) + \left(\frac{1}{\Lambda_P} - \frac{1}{\Lambda_{1_P}}\right)$$

$$(s_{MS_1} - s_{PS_1}) + \left(\frac{1}{\Lambda_P} - \frac{1}{\Lambda_{d_P}}\right)(s_{MS_d} - s_{PS_d})$$

$$= C_4 (s_{MS_z} - s_{PS_z}) + C_5 (s_{MS_1} - s_{PS_1}) + C_6 (s_{MS_d} - s_{PS_d})$$

Mr. Verstelle wished to point out in this connection that these formulae are correct only within acceptable limits if the assumed division into 3 types of terrain is an adequate approximation of the actual electromagnetic conductivity of the soil. It is expected that this will be sufficiently the case for the Delta area; elsewhere however areas are known where the conductivity is so low that the pattern corrections must be calculated by a much more complicated method.

The coefficients C_1 — C_6 can be calculated if Λ is known for the different types of terrain; in other words, the different propagation speeds must be known; this is sufficiently the case for the Netherlands. The pattern corrections for regularly distributed points are then calculated, for which the distances over the different types of terrain from S to M, R and P can be taken from the map. This correction table can be converted into a correction graph, by means of which the corrections to be made to the decometer readings can easily be determined in practice.

In order to check the pattern corrections calculated in this way, they will be determined for a restricted number of points by comparison between directly observed Decca readings and the calculated pattern values; of course the observation point must be very accurately fixed.

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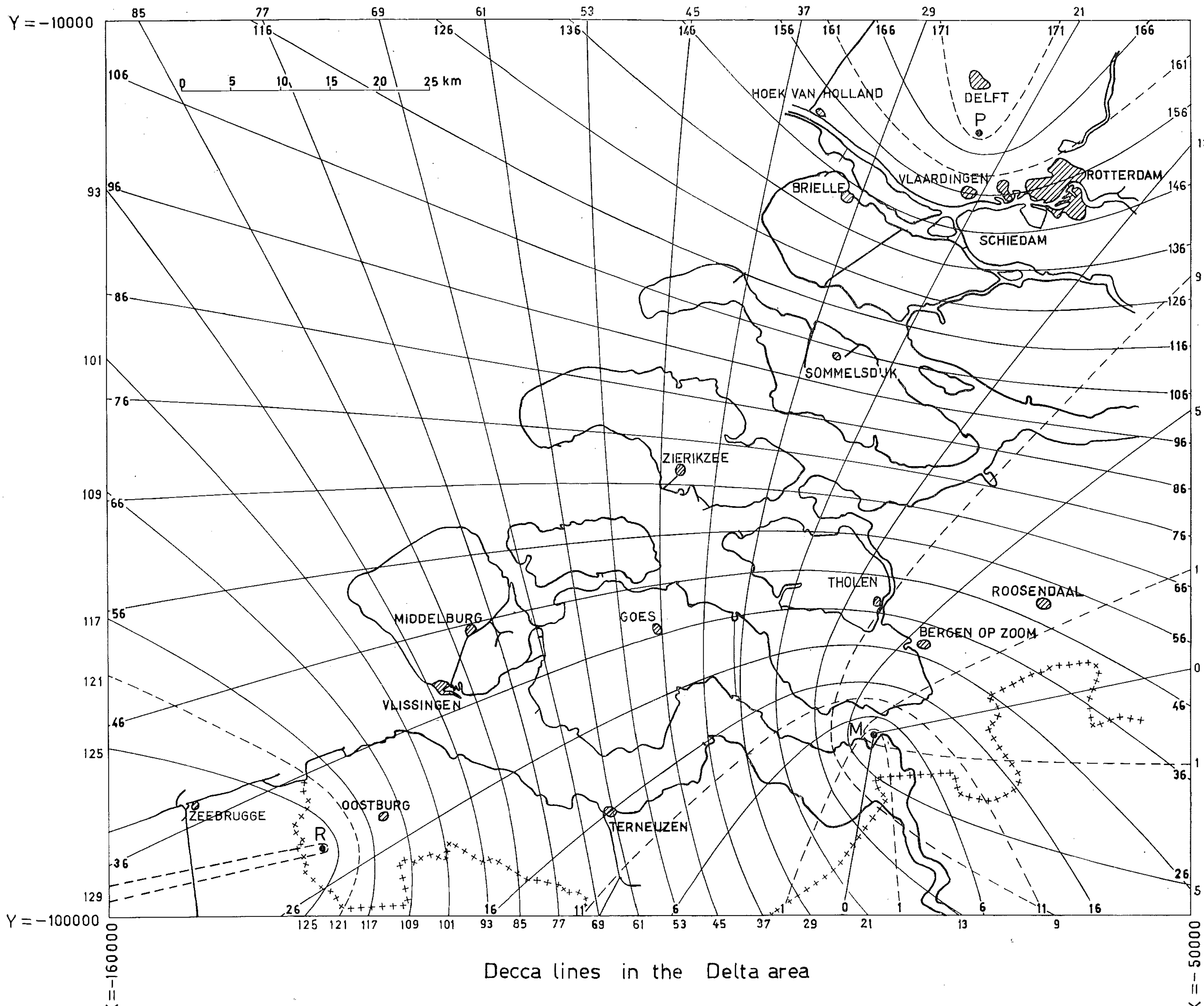

Decca lines in the Delta area