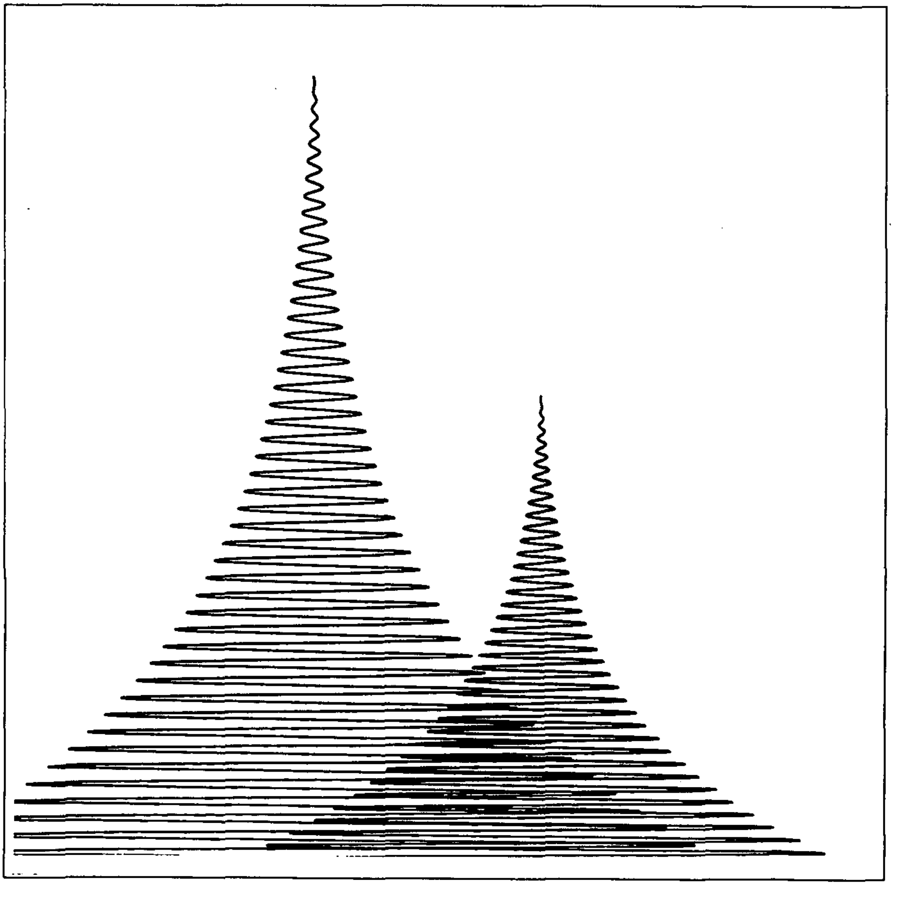
DESIGN OF HIGH-PERFORMANCE NEGATIVE-FEEDBACK OSCILLATORS

C.A.M. Boon



STELLINGEN

behorende bij het proefschrift van C.A.M. Boon

1. Anders dan de definitie doet vermoeden is de verhouding van het draaggolfvermogen tot de vermogensdichtheid van een ruisvloer geen maat voor de frekwentiestabiliteit.

2. Het is geen grapje dat men een versterker moet ontwerpen om een oscillator te maken.

3. De uitdrukking "een niet-lineaire oscillator" bevat een pleonasme.

4. De doorslagspanning van lucht is hoger dan die van lucht gemengd met de damp van transformatorolie.

5. Het trillingsvrij ophangen van bepaalde onderdelen in een apparaat maakt dit apparaat onvervoerbaar.

6. Bij een voortschrijdende kostenverlaging van elektronische basiscomponenten kan kennis over het ontwerpen en toepassen van deze basiscomponenten slechts in een kleine kring gevonden worden.

7. Standaardisatie heeft eerst een versnellende maar vervolgens een vertragende werking op de ontwikkeling van innovatieve producten.

8. In de evolutie van de mens is de proliferatie van de televisie nog belangrijker dan die van de atoombom.

9. Het niet adequaat bestraffen van auto-inbraken is ook een manier om in steden het aantal auto's te beperken.

10. De wet van Murphy berust op selectieve perceptie.

11. Een goede wapensmid zit nooit zonder werk.

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Proefschrift

ter verkrijging van de graad van doctor aan de Technische Universiteit Delft, op gezag van de Rector Magnificus, prof. drs. P.A. Schenck, in het openbaar te verdedigen ten overstaan van een commissie aangewezen door het College van Dekanen op dinsdag 19 september 1989 te 14.00 uur
door

Cornelis Alexander Maria Boon,

geboren te Wormerveer,
elektrotechnisch ingenieur.

Dit proefschrift is goedgekeurd
door de promotor prof. dr. ir. J. Davidse.

Dr. ir. E.H. Nordholt heeft als toegevoegd promotor in hoge mate bijgedragen aan het totstandkomen van het proefschrift.

Gezien of niet de wereld, om het even: het is niets.
wat gij gehoord, gezegd hebt of geschreven: het is niets.
gereisd door de klimaten alle zeven: het is niets.
tot studie en bespiegelen thuis gebleven: het is niets.

(from Rubáiyát of Omar Khayyám, translated by J.H. Leopold)

CONTENTS

PREFACE

Oscillators are indispensable elements in electronic systems. These systems process information in order to optimize its transport, reception, storage, reproduction or display. Electrical signals that represent this information often change during signal processing through the application of oscillators and of other nonlinear elements such as electronic multipliers, switches and combinatorial circuitry. In this way information can be modulated onto a carrier signal, sampled, or synchronized with some reference timing incorporated in the system.

Due to this indispensability the problem of oscillator design was one of the very first to be addressed in electronics [ref.1]. The reader may think that after a history of more than eighty years the average designer has no serious problems with oscillator design. The fact is, however, that in the vast literature on oscillators - for instance Groszowski [ref.2] assembled a bibliography of approximately 1800 articles covering the period 1920-1962 - little can be found on oscillator design. The essential problem of how, starting from a collection of oscillator specifications, such as frequency, frequency stability, amplitude stability, power consumption etc., to arrive at an optimal circuit satisfying these specifications has there been more or less neglected.

Literature on oscillators stresses analysis rather than design. As such literature reflects the practical 'design' process, many 'designers', confronted with the problem of designing an oscillator, scan literature in search of a suitable circuit. In practically all cases their application is so specific that adaptations of this selected circuit are necessary. After an analysis they adapt the circuit by the 'cut and try' method. Especially this last step in the process requires sophisticated hardware experience which looks like magic to outsiders. Because the performance of the final oscillator heavily depends on this available experience, it is frequently not optimal.

The rapid increase in the performance of other electronic, mostly digital functions, clearly reveals the analog bottle-neck in this kind of oscillator design and emphasizes the need for a better method of design. Another aspect influencing design is oscillator

fabrication technology. Nowadays an integrated circuit may incorporate large parts of the oscillator circuit and this integration encumbers the 'cut and try' method which is indispensable in the standard design process.
This work attempts to make a start on a new design method. The development of the design method was largely inspired by the work of Nordholt [ref.3], who treats the corresponding problem in amplifier design. The reader may notice here some resemblance to his work, for the author made good use of the methods he developed.

The oscillator application plays a key role in the design. It is decisive for the selection of the proper oscillator configuration and after this selection, decisive for the elements in the configuration. The design stresses oscillator synthesis from a qualitative point of view. It gives the reader tools to synthesize an optimal oscillator configuration and to estimate ultimate frequency stability, amplitude stability, frequency accuracy etc. as a function of the performance of elements in this configuration.

The first chapter of this work is devoted to general design considerations for high-performance oscillators. A recapitulation of basic oscillator theory will there show that the behavior of these oscillators is preferably weakly nonlinear and that they generate quasi-sinusoidal outputs. Their basic building blocks are a passive resonator with low losses and a nonlinear amplifier. An overview of literature throws light on design difficulties and the negative-feedback oscillator is proposed as a valuable alternative.

The second chapter deals with the dynamic behavior of the negative-feedback oscillator in order to calculate the transfer of small signals, like noise, to the oscillator output, where they manifest themselves as parasitic AM or PM of the oscillator signal. It shows the aliasing of these small signals as a function of the nonlinearity present in the oscillator.

The third chapter gives a classification of the basic configurations of negative-feedback oscillators. The classification will enable the designer to make the proper selection for his specific oscillator application. Useful configurations use LC series and parallel tank resonators and feature intrinsically buffered outputs.

The fourth chapter presents design considerations for optimum stability. It will show that the noise behavior and the power handling in an oscillator can be optimized. The techniques to be used will be borrowed from negative-feedback amplifier theory, adapted for the nonlinear environment in the oscillator. The chapter also deals with the more important effects of a finite power gain that is available from the active elements in the oscillator. Expressions will be discussed that indicate how much power gain must be traded for certain quality aspects of the oscillator.

CHAPTER 1

GENERAL DESIGN CONSIDERATIONS

1.1 Introduction

Oscillator design is a process that produces all the relevant information for the construction of an electronic circuit that delivers an electrical signal with a certain frequency and waveform to a load. The circuit consists of passive and active elements and contains no other sources than that of DC-power.

The oscillator's load is the description of the interface with the rest of the electronic system of which the oscillator forms a part. For optimal behavior of this system the oscillator output signal at the load should meet certain requirements with respect to frequency accuracy, long- and short-term frequency stability, waveform and amplitude stability.

Important as these requirements may be, their fulfillment has not yet earned the epithet 'high performance' to be applied to the oscillator. For this also aspects such as power consumption, available active and passive components, printed circuit board or silicon real estate, or more generally, cost must be taken into account. For high overall performance the oscillator should be optimized for each application, because the relative importance of oscillator requirements changes with the application. The designer will only reach the best compromise available when he has an insight into the interdependence of the various quality aspects of his oscillator.

This chapter starts with a practical example of standard oscillator design in order to acquaint the reader with the design problems. Next a search is made for a class of oscillator circuits that have potentially high-performance qualities. The result of this search is the well-known class of harmonic oscillators, as one might have expected. In this class harmonic oscillators incorporating resonating circuits with high quality factors have turned out to be most promising. Also, among oscillator designers, these are very popular, not in the least because for moderate performance they are indifferent to the active circuitry used. The configurations used for these oscillators are often suboptimal. Especially the implementation of nonlinear circuitry for amplitude stabilization is a neglected aspect: either this stabilization is left to the accidental nonlinearity in the active circuitry or extensive circuits are added in the form of an automatic volume control (AVC) loop.

This work presents an alternative through the application of negative, memoryless or time-invariant feedback in such harmonic oscillators. In cases where more power gain is available from the active oscillator elements than is strictly necessary for oscillation, this oscillation can mainly depend on passive or non-amplifying elements in the feedback path. These elements normally have accurate characteristics and consequently oscillation is accurately controlled. By taking this approach, the oscillator design resembles greatly the design of negative-feedback amplifiers; a reason to call these oscillators "negative-feedback oscillators".

It is through using the concept of negative memoryless feedback that oscillator performance can be described by relatively simple expressions. Though many previous papers, especially those emphasizing mathematical analysis, have described the oscillation process using a simple nonlinear differential equation, their practical value is limited because they do not describe actual oscillator circuits.

A *simple but accurate description of the oscillation process is an* essential condition to the design of high-performance oscillators. Only then is a designer able to see through the various interdependences mentioned and to arrive at an optimal compromise for a specific application. The description is in the form of a simple one-dimensional model and its use will be demonstrated in a calculation of the static frequency error and of the distortion in the output signal of negative-feedback oscillators.

1.2 Practical example

As an illustration of typical problems in oscillator design this section presents a practical example. Of all possible configurations for harmonic oscillators, only very few are used in practice. One of the more popular ones is the "Colpitts" oscillator, especially if also its variants like the "Clapp" oscillator and the "Pierce" oscillator are taken into account [ref.4]. Literature offers a wide variety of implementations and here *a characteristic one [ref.5], depicted in Figure 1.1, will be* discussed. The behavior of the oscillator circuit was simulated using SPICE. The reader may find detailed simulation information in Appendix I. Figure 1.2 shows three relevant oscillator signals: the transistor collector current, collector voltage and emitter

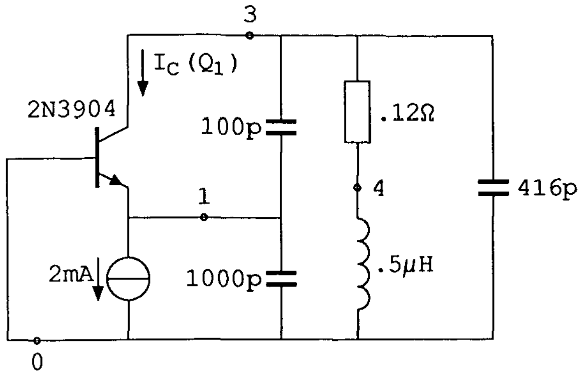

Fig. 1.1: Standard implementation of a Colpitts oscillator.

voltage. The circuit designed to oscillate at 10 Mhz oscillates in simulation at 4% smaller value. The observed frequency deviation is quite large in view of the relatively large Q-factor (ca. 250) of the passive resonator. Transistor capacitances (C_{be} and C_{bc}) directly in parallel with the LC tank circuit and hard nonlinear behavior, as may be concluded from the collector current, cause

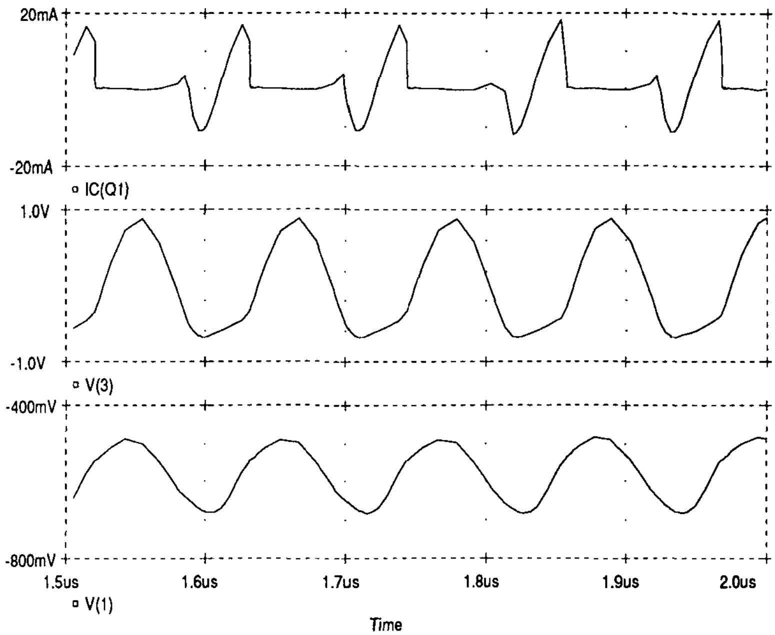

Fig. 1.2: Results of transient simulation of the oscillator circuit from Figure 1.1.

this frequency deviation. Oscillation builds up at emitter voltages below the U_{eb} threshold (ca. -0.7 Volt) and breaks down at collector voltages below the U_{cb} threshold (ca. -0.7 Volt). The resultant oscillation amplitude is the balance of these two processes.

Due to the hard nonlinear behavior, both the static frequency and the static amplitude, the two most important properties of an oscillator, are not simple functions of the elements in the circuit. The circuit's high frequency selectivity guarantees a quasi-sinusoidal output voltage, but an estimation of the distortion in this voltage seems very difficult.

For high performance, special requirements are set on the dynamic or short-term frequency and amplitude of the oscillator. For this purpose a dynamic transistor description is required, which also accurately describes the generated transistor noise. Small-signal transistor models and their corresponding equivalent noise sources are of limited use in this kind of oscillator because the oscillator signals are large and the transistor does not operate exclusively as an amplifier. The absence of a suitable model precludes the dynamic properties of these oscillators from analysis. For many circuits, however, analysis is hardly worthwhile. In the example, for instance, oscillation amplitude is 'stabilized' by the collector-base diode across the LC-tank. The power drawn from and the noise transferred to this tank by this diode are unacceptable in a high-performance oscillator. Design of such oscillators is therefore hardly possible. The last resort to obtain better dynamic properties from these circuits is the 'cut and try' method, which here comes down to trying different capacitor tapping ratios, other transistors or other values for the dc-current source.

The last but not the least detrimental quality of such an oscillator is that connection to a load directly affects the oscillation process. The argument, that for loading, a buffer with a zero input impedance or admittance will be used is not a practical one, because these ideal buffers do not exist.

The only clear advantage of this kind of oscillator is its minimum demand on active circuitry. In the old days of vacuum tubes it was an important one; with today's shrinking costs of active elements this advantage is losing its importance.

1.3 Oscillation condition

In electronics oscillation is a well-known phenomenon. This section recapitulates basic theory in a somewhat unusual way that, however, permits further discussion.

A necessary condition for a circuit that delivers a periodic signal of a certain waveform is that the circuit be instable. Small-signal theory gives a criterion for this instability. Suppose the circuit has a small-signal description in Laplace transformed mesh or nodal equations and let Δ be the corresponding determinant. If the circuit has no inputs, a signal e^{pt} occurs in the circuit with values for the Laplace variable p given by:

$$\Delta = 0$$

For instability, roots of this equation must be found in the right half of the complex plane (RHP). Exact imaginary roots are believed to be unpractical. At instability the circuit incorporates at least one element, let us say with Bode [ref.6], it has a transfer W that is crucial for this instability. Then the circuit becomes stable when this transfer W vanishes. With the help of the determinant Δ^0 of the circuit equations in case $W=0$ and the determinant Δ_{fb}, the cofactor of Δ with respect to the variable W, the expression above can be rewritten to:

$$\Delta^0 + W \Delta_{fb} = 0$$

As Δ^0 has a non-zero value, a zero value for Bode's return difference F is equivalent to the condition for instability:

$$F = 1 + W \frac{\Delta_{fb}}{\Delta^0} = 0$$

The term $-W\Delta_{fb}/\Delta^0$ can be interpreted as the transfer around the loop which is broken as the transfer W vanishes. Thus instability occurs if for RHP-values of Laplace variable p this loop transfer becomes unity. Nyquist showed that this is equivalent to the enclosing of the point (1,0) by the plot of the loop transfer in the complex plane. The plot encloses this point only if the loop transfer has a real value larger than one at a certain frequency.

Instability is an insufficient condition for the generation of periodic signals. Instability as described by RHP-roots of the above equations can not survive in a practical circuit. For this purpose a nonlinear element is present in the loop. In order to not complicate matters too much, let the element with transfer W be nonlinear. Also in this case a loop transfer value of one forms the condition for oscillation. A difficulty arises, though, through this nonlinear element in the loop: the loop transfer is no longer uniquely defined but depends on the place where the actual loop is broken for calculation purposes. Solving the condition for oscillation yields the oscillation signal containing both frequency and amplitude information at the place where the loop has been broken. As no two signals in an oscillator are identical, various loop transfers may be defined.

In our discussion we will assume that the nonlinear element shows a phase shift for the oscillator signal which is independent of the oscillation amplitude. Otherwise highly impractical situations would exist in which the two important oscillator properties, frequency and amplitude, would be related by a nonlinear equation.

The rest of the loop must show some frequency selectivity in the form of a filter. For high performance, first order filters are excluded. These filters must necessarily contain resistors and therefore produce noise and dissipate power, which are both detrimental properties for high-performance oscillators. It is thus implicitly assumed that filters without resistors are available and there is no extreme penalty in cost. Unfortunately this is not always true.

Oscillators that can be modeled in the indicated way, a single loop of a filter of at least second order and a nonlinear real transfer, may have the advantage that the nonlinear element hardly influences the oscillation frequency. Then the oscillator should behave quasi-linearly or weakly nonlinearly and sinusoidal oscillations will result. This behavior is present if, at the frequency of a real, small-signal loop transfer, the loop is capable of transferring a sinusoidal signal of sufficient amplitude without generating substantial harmonic content. Thus for weakly nonlinear behavior the generation of harmonics by the nonlinear element must always be complemented by an adequate amount of filtering in the loop.

The solution of the oscillation condition for weakly nonlinear oscillators, the determination of oscillation amplitude and oscillation frequency, is a classical problem. Probably van der Pol [ref.7] first attacked the problem and many followed afterwards [ref.8,9,10,11]. All approaches have in common that they start with an accurate description of the nonlinear transfer and the filter in the loop. The basic problem in oscillator design is, however, not the solving of the oscillation condition but that, in practice, these accurate transfers are hard to come by.

It is, therefore, because of the above that a very simple approximation of the oscillation condition will be used here. The approximation used is known in literature as the method of equivalent linearization [ref.4,9]. The loop transfer is calculated with respect to the input of the nonlinear element. There the loop is broken for calculation purposes. The transfer of the nonlinear element is described by its complex first harmonic response $FHR(E_i)$ on a sinusoidal input with E_i as amplitude. Because weakly nonlinear behavior is assumed, higher harmonics are neglected.

These approximations greatly simplify the solving of the oscillation condition. With the definition of $G(j\omega)$, the Fourier-transformed transfer of the remaining linear part of the oscillator loop, the oscillation condition becomes:

$$1 - FHR(E_i)\ G(j\omega) = 0$$

This complex equation can be reduced to two equations with real variables, which are known as the Barkhausen conditions [ref.12] for oscillation:

$$\arg\ (FHR(E_i)) + \arg\ (G(j\omega_{osc})) = 0 \qquad \text{'phase balance'}$$

$$|FHR(E_i)|\ |G(j\omega_{osc})| = 1 \qquad \text{'amplitude balance'}$$

From these two equations the oscillation frequency ω_{osc} and the oscillation amplitude E_i at the input of the nonlinear element can be solved without any difficulty.
Subsequent sections discuss the choice of filtering and traditional implementations of the nonlinear element.

1.4 Frequency selectivity

In this section a search is made for filter structures that are best suited for application in high-performance oscillators. The criterion for optimization will be the accuracy of the oscillation frequency.

At this frequency, according to the phase balance in Barkhausen's condition, the net phase shift of the small-signal loop transfer equals zero. The phase shift around the loop will be the resultant phase shift due to both the active and the passive components in the loop circuit. Although in a practical circuit the separate phase shifts are not recognizable because these components have been interwoven, a fictitious separation throws light on the proper choice of filter selectivity [ref.13].

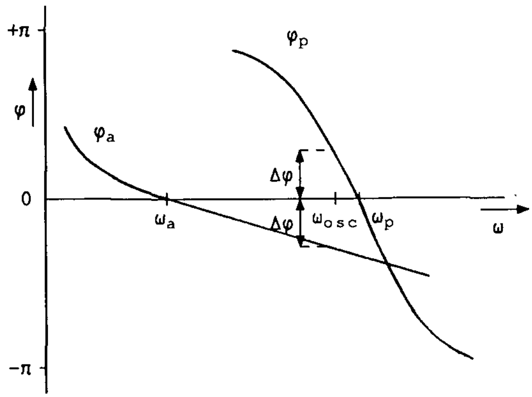

Fig. 1.3: Possible actively and passively generated phase shifts.

Suppose the passively and actively generated phase shift, ϕ_p and ϕ_a respectively as shown in Figure 1.3, can each be approximated for frequencies near the oscillation frequency by the first two terms of a Taylor series:

$$\phi_p(\omega) = \phi(\omega_p) + (\omega - \omega_p) \left.\frac{\delta\phi_p}{\delta\omega}\right|_{\omega_p}$$

$$\phi_a(\omega) = \phi(\omega_a) + (\omega - \omega_a) \left.\frac{\delta\phi_a}{\delta\omega}\right|_{\omega_a}$$

Substitution of the expressions in the phase balance yields the following oscillation frequency ω_{osc}:

$$\omega_{osc} = \omega_a \frac{1}{1 + S} + \omega_p \frac{S}{1 + S}, \quad \text{with} \quad S = \frac{\left.\frac{\delta\phi_p}{\delta\omega}\right|_{\omega_p}}{\left.\frac{\delta\phi_a}{\delta\omega}\right|_{\omega_a}}$$

As active elements are normally much less accurate than passive ones and as also nonlinear elements are less accurate than linear ones, linear passive elements must have a dominating influence on the frequency of oscillation. The best choice is, therefore, a passive filter with a steep phase versus frequency response near zero phase shift and an active circuit with a flat corresponding response. The only filters that show such phase shifts are generally called resonators. Practical examples of these are LC circuits, piëzo-electric crystals and electromagnetic cavities.

A simple second-order differential equation describes any passive filter near resonance. The total transfer of a practical filter may be of a higher order, of course, as long as no other resonance occurs or the high-frequency stability of the oscillator is not jeopardized. Here a second order description suffices for further discussion. Note that more resonances at one single frequency in the oscillator loop are only made possible by resonating filters which have been separated from each other by active circuitry. Design of such oscillators will merely be an extension of the design of oscillators with one resonator. They will be ignored in the following, since the increase in oscillator performance is moderate and the cost of an extra resonator is not often acceptable.

It is agreed that because of the choice of resonators for frequency-determining elements, certain, sometimes valuable, properties of oscillators can not be accomplished, for instance an independent control over oscillation frequency and damping [ref.14], but for high-performance oscillators frequency accuracy outweighs these drawbacks.

Throughout this work the Fourier-transformed transfer of the resonator is described by $H(\omega)$:

$$H(\omega) = \frac{H_o}{1 + jQ\nu}$$

In this expression the variable H_o represents the transfer at resonance. The variable ν represents the "detuning" referenced to the resonance frequency ω_o:

$$\nu = \frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}$$

For small deviations from the resonance frequency the detuning may well be approximated by:

$$\nu \cong \frac{2\ \Delta\omega}{\omega_o}$$

The variable Q represents the well-known quality factor of the resonator. A high quality factor refers to an almost lossless resonator as can be seen from its basic definition [ref.4]:

$$Q = 2\pi \frac{\text{total energy stored}}{\text{energy lost per cycle}}$$

High quality factors are an important criterion in selecting resonators because the factor represents the normalized phase versus frequency slope at resonance:

$$\omega_o \left. \frac{\delta\phi_P}{\delta\omega} \right|_{\omega_o} = 2\ Q$$

A high value for the quality factor and maintaining it in an oscillator circuit (often indicated by 'a high loaded Q-factor') are effective means to reduce the influence of active circuits on the frequency of oscillation. In the pursued situation of high values of the quality factor and relatively small, active phase shifts, the oscillation frequency will be given by:

$$\omega_{osc} \cong \omega_o + \frac{\omega_o}{2Q} \phi_a(\omega_o)$$

For high-performance oscillators, resonators with low losses are the best choice for the filter in the oscillator loop. In order to make full use of their excellent phase-discriminating properties active, nonlinear circuitry, complementing the oscillator loop, must be carefully designed. The following section discusses the various attempts known from literature.

1.5 Standard oscillator amplifiers

Of the four basic functions in an oscillator, viz. filter, amplify, limit and loading [ref.4], only the first three are wanted in the oscillator loop. Elements in this loop define oscillation frequency and amplitude and as the oscillator load is considered inaccurate, certain provisions must isolate it from the loop. Such provisions are dealt with in a following chapter. With a resonator as a filter, the functions of amplification and limiting in the loop remain to be discussed. Amplification is necessary to ensure the circuit is (small-signal) instable, that is to say a real, small-signal loop transfer exists with a value larger than one. And, secondly, limiting is necessary to reduce the effective loop gain of the first harmonic to one. This section gives a survey of known methods to actively close the oscillator loop around resonators.

The first and most propagated method is to close the loop by using an unstabilized amplifier. An example of this method was given in Section 2 of this chapter. As no negative feedback has been applied, transfer parameters of active devices in the amplifier directly influence oscillation. Furthermore the nonlinear behavior of the amplifier has often been designed without deliberation. An attempt was made in some circuits in which the nonlinear processing of the oscillator signal generates an automatic bias at the amplifier input. At larger oscillator signals this bias cuts off the amplification of the circuit ['grid rectification', ref.4]. In all these circuits, however, oscillation is directly influenced by the small- as well as the large-signal characteristics of the active elements used.

A second method is to use amplifiers that are stabilized by linear negative feedback. In principle two variants occur. The feedback is frequency independent [ref.13] or dependent in that it is only absent close to the frequency of oscillation [ref.15]. As long as the amplifier acts quasi-linearly, negative feedback has its normal beneficial effect. As the amplifier can not remain in its linear region, on limiting, the stabilization is lost with unpredictable effects on oscillation. This method can still have far better results than the first one mentioned: linear negative feedback reduces the influence of active elements and enables a restriction of the degree of limiting.

A third method not only stabilizes amplification, but also the limiting action in the loop. The loop around the resonator is closed by a cascade of an accurate amplifier and an accurate limiter [ref.4]. As far as signal processing is concerned the optimum is reached here. There is a problem, however, in implementation. Again negative feedback may stabilize the amplifier transfer. Available limiters will normally not show an accurate transfer when placed between amplifier and resonator. Thus for an accurate loop transfer at least an extra buffer is necessary.

A fourth method overcomes the observed problem in implementation by closing the loop around the resonator with one amplifier stabilized by negative, nonlinear dynamic feedback. Within time intervals comparable to the period of oscillation or shorter, the feedback network has a linear transfer, for the nonlinear properties of the network are designed to be only noticeable on a much larger time scale. This linear transfer is, through the nonlinear dynamic properties, a function of signal magnitude in the network: larger signals cause an increase of negative feedback in the amplifier.

Familiar examples of this method are the Wien-bridge and the Meacham bridge-stabilized oscillators. Temperature-dependent resistors (NTC or PTC) and even tungsten-filament lamps are known as elements in the feedback network that show the required behavior [ref.16]. However the use of these elements cannot be recommended for use in oscillators: they consume considerable power and their production of noise is accordingly greater. Besides, they are technologically unattractive and hard to combine with other components.

Another implementation of the fourth method is the oscillator with an external automatic volume control (AVC) loop. Circuitry in this loop measures the averaged oscillation amplitude, compares it with a reference amplitude and feeds the amplified error back to a controllable element in the afore-mentioned feedback network.

Of all methods, the fourth, in this last implementation, is the most versatile. It enables a high-quality transfer around the oscillator loop and excellent oscillator performance can be the result [ref.17]. In practice, however, the use of oscillators with AVC is limited. The investment in circuitry for the AVC-loop is quite high. Often this loop needs more circuitry than the oscillator loop itself. The design of the AVC-loop is not without problems. Especially the selection of the two nonlinear elements in the loop, the amplitude detector and the controllable element in the feedback network, is critical. Last but not least it must be said that the dynamic behavior of the loop requires attention: for optimal behavior this second-order control loop requires phase compensation.

The next section will show an alternative to the fourth method. There a slight degradation in loop transfer is traded for a very simple feedback network that stabilizes the amplifier transfer.

1.6 Time-invariant nonlinear feedback

For a high-performance oscillator it is necessary to close the loop around the resonator with active circuitry that has an accurate, nonlinear, transfer with negligible phase shift. To be resistant to the influences of active devices, such as of their inaccurate transfer parameters, or of signals like hum and noise, the best configuration for this active circuitry is that of an amplifier with overall negative feedback [ref.3]. In such configurations the transfer of a number of active stages with a large available power gain is stabilized by an accurate feedback path.

The character of the (only) transfer to be stabilized is reciprocal to that of the resonator transfer. By the action of negative feedback the transfer of this resonator will appear largely unaffected in the transfer of the oscillator loop. Thus the strong phase discriminating properties of the resonator are kept in the oscillator circuit.

Because the stabilized transfer must preferably show no phase shift for the oscillator signal, the feedback network must not have such a phase shift. The remaining phase shift will then be from the active elements used: this phase shift is inversely proportional to the amount of negative feedback applied. In addition, this stabilized transfer must also be nonlinear and show a limiting character. Feedback theory teaches that, as an extrapolation of linear feedback, the application of negative, nonlinear feedback is the best way of achieving such a transfer. The previous section dealt with one way to achieve a limiting transfer without considerable phase shift: the application of nonlinear (slow) dynamic feedback in the form of special temperature-dependent resistors or of special, controlled elements (AVC). As was stated before, these approaches are not without problems. The design of such oscillators is complicated and the required investment in hardware for their realization is, in many cases, not cost effective.

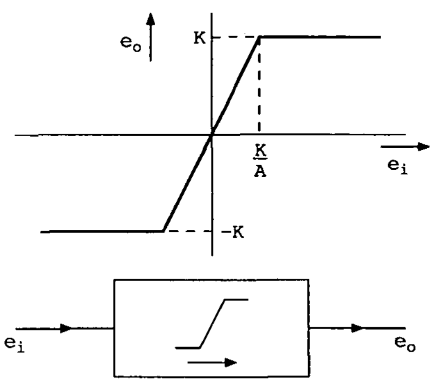

Fig. 1.4: Example of a transfer stabilized by time-invariant feedback.

Here an alternative is presented which has at least the same stabilizing properties. The application of negative, time-invariant nonlinear overall feedback to an amplifier with a large, available *power gain can* result in the *required time-invariant* or memoryless limiting transfer. This time-invariance allows a simple function to describe the stabilized amplifier transfer. Figure 1.4 depicts an example of such a transfer. If an amplifier like this is used to close the loop around a resonator, the small-signal gain of the amplifier guarantees the small-signal instability and its combination with the limiting characteristic enables oscillation. In contrast to the use of nonlinear (slow) dynamic feedback, the oscillator signal is processed in a nonlinear way when observed within one period of oscillation. Consequently the nonlinear amplifier generates harmonics but, as negative feedback controls this generation , the harmonic content can be limited to the amount strictly necessary. In practically all cases, a weakly nonlinear, sinusoidal oscillation becomes possible in this way. In literature on oscillator design this application of time-invariant feedback has been given little attention [ref.18]. For the design of high-performance oscillators this possibility offers some valuable benefits. The most important ones are listed below:

-There is an optimal use of power. The power consumed by the oscillator is the total of power dissipated in the load, the time-invariant feedback network and the resonator, and of power required for the bias of active elements in class A.

-A noise optimization becomes possible if negative feedback is applied to at least two active stages.

-There is a wide variety of feedback elements and feedback structures by means of which oscillator configurations can be optimally designed for a specific application.

-The component count is moderate compared to AVC-oscillators.

-The instantaneous limiting of the oscillator amplitude enables a fast tuning without notorious bouncing effects [ref.16].

-Designability is improved. The description of the active nonlinear transfer is much simpler than that in other oscillators. With all active elements biased in class A, and with the control of the feedback network, all signals in the oscillator can be estimated without sophisticated calculations. Simple relations link the various oscillator quality aspects, enabling a designer to make his compromises in an easier way.

Unfortunately there are also some drawbacks. Short-term frequency stability can be proven to be slightly worse compared to a fictitious linear oscillator and, of course, distortion is also worse. With careful design though, the deterioration can be quite small compared to an AVC-oscillator. Though the ultimate performance of the fictitious linear oscillator can probably best be reached by oscillators with carefully designed *nonlinear dynamic* behavior as in AVC-oscillators, the oscillators with negative time-invariant feedback approximate their performance with a simpler design and with a more cost-effective hardware implementation.

In the continuation of this work an attempt will be made to justify the above claims. Following sections show how relatively simply amplifier characteristics can be defined from the condition for oscillation (phase and amplitude balance) and how this kind of amplifier enables the designer to use a simple model for negative-feedback oscillators.

1.7 Simple model for negative-feedback oscillators

Due the concept of time-invariant nonlinear feedback the transfer of the oscillator loop consists of two accurate transfers: the linear, frequency-selective transfer of the resonator, given by a second-order expression as mentioned in Section 1.4, and the nonlinear amplifier transfer, as mentioned in the previous section. A simple one-dimensional model may therefore describe a negative-feedback oscillator. Figure 1.5 shows this model. As the load had

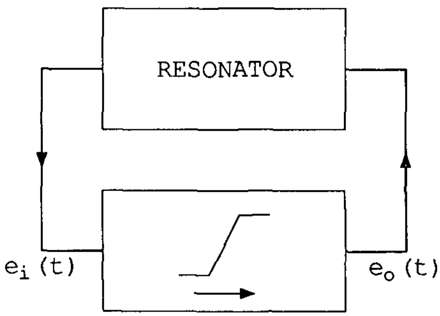

Fig. 1.5: A simple one-dimensional model for a negative-feedback oscillator.

been expelled from the oscillator loop, this model does not yet have a specific output quantity. For the time being both resonator-output and amplifier-output signals are considered. The transfer of one of these signals to the load will be dealt with in a following chapter. This simple model is discussed here by summing up its more important design criteria in a qualitative way.

The choice of a resonator for application in a negative-feedback oscillator is made because of its determining quantities ω_o, Q and H_o. First the resonance frequency ω_o of the resonator must equal the desired oscillation frequency. The stability of this resonance frequency, which may be affected by aging or temperature, must at least be as good as the stability desired from the oscillator. Also provisions must be made to change this frequency (manually or electronically) if there are requirements for tunability. A further selection criterion is the Q-factor. Because the Q-factor directly influences the static frequency accuracy, the short-term frequency stability and the distortion of the oscillator signal, as will be shown in this work, resonators must be selected for the highest possible value of this factor. The third quantity H_o, describing the transfer at resonance, is normally subordinate to the other two. If possible, however, values must be chosen that simplify the design of an amplifier with a transfer of reciprocal value.

In the same way the transfer characteristics of the nonlinear amplifier are discussed. First its transfer must be memoryless in order to avoid the phase shift of the oscillator signal that would cause frequency errors. Subsequently the true small-signal gain A of the amplifier, the small-signal gain with no additional large amplifier input signal, must be chosen large enough to guarantee oscillation. To this end the small-signal loop gain at the resonance frequency, AH_o, must be larger than one. This loop gain, in the following also called the 'excess loop gain', is a measure for the degree of nonlinearity in the oscillator and as such an important design criterion. The excess loop gain AH_o influences amplitude stability, distortion, static frequency errors and short-term frequency stability.

Another parameter in the nonlinear amplifier transfer is the limit level K. It sets the magnitude of oscillator signals and it must therefore be chosen sufficiently large in order to reduce the relative contribution of noise from the circuit. Formally expressed it is the amplitude balance in the condition of oscillation that controls the oscillation amplitude. For the simple oscillator model the following equation expresses this balance:

$$|FHR(E_i)| \; H_o = 1$$

The nonlinear amplifier has a first harmonic response to a sinusoidal input with an amplitude E_i, $|FHR(E_i)|$, as depicted in Figure 1.6. For excess loop gains AH_o larger than one the response is strictly decreasing which is a necessary condition for a stable amplitude. The amplitude E_i of the quasi-sinusoidal amplifier input signal is thus given by:

$$E_i \cong \frac{4\,K}{\pi} H_o$$

The limiting characteristic of the amplifier, as was shown in Figure 1.4, has been formalized from two requirements: first, from the already-discussed, true small-signal gain A and second, from the zero small-signal gain at limiting. When the amplifier reaches limiting its small-signal gain must fall to zero and not to a value

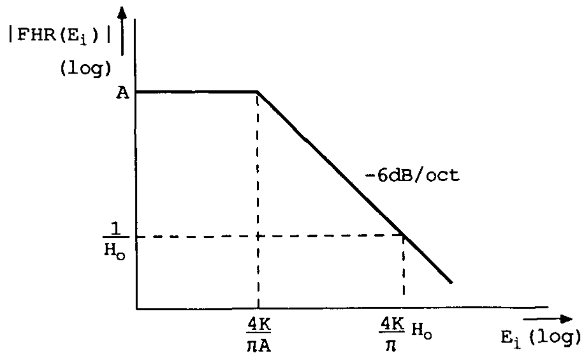

Fig. 1.6: First harmonic response of the nonlinear amplifier on a sinusoidal input.

with a sign opposite to that of the true small-signal gain. Otherwise, temporarily, an extra positive damping in the oscillator is introduced in excess of the Q-related damping from the resonator. The amplifier would then draw power from the resonator. In the active state the amplifier must compensate for this loss and obviously a non-optimum power handling results. Of course transfers of practical amplifiers do not need to correspond with this formalized amplifier transfer exactly. Other time-invariant nonlinear transfers showing the two basic requirements mentioned may be usable as well.

The model here discussed resembles models used in some purely mathematical approaches to the oscillation problem. The next section uses a result of these approaches in order to calculate the effect of the nonlinear amplifier transfer on oscillation frequency.

1.8 Frequency accuracy with nonlinear feedback

The question may arise as to whether the choice of a time-invariant nonlinear amplifier transfer could affect the static oscillation frequency. In this section the method of reactive power balance of harmonics [ref.2,8] is used to examine these frequency errors.

In the simple model for the negative-feedback oscillator, Fourier series may describe both the resonator output $e_i(t)$ and the the amplifier output $e_o(t)$ with corresponding coefficients e_{in} and e_{on}:

$$e_i(t) = \sum_{n=-\infty}^{\infty} e_{in}\, e^{jn\omega_o t}$$

$$e_o(t) = \sum_{n=-\infty}^{\infty} e_{on}\, e^{jn\omega_o t}$$

The method of balance of harmonic power is based on the fact that for a time-invariant nonlinear transfer ($e_i \rightarrow e_o$) the following equation is valid:

$$\oint e_o \; de_i = 0$$

Substituting both Fourier series into this equation and noting * indicates a conjugated value, yields:

$$\sum_n n \,\mathrm{Im}\,[\, e_{on} \; e_{in}^{*} \,] = 0$$

In the evaluation of this equation only odd coefficients are to be examined because of the odd nonlinearity of the amplifier. As the resonator transfer $H(j\omega)$ interrelates both Fourier series, a following approximation links e_{in} to e_{on}:

$$e_{i1} = e_{o1} \frac{H_o}{1 + jQv}$$

$$e_{in} \simeq e_{on} \frac{H_o}{jnQ} \qquad \text{for} \quad n \neq \pm 1$$

Substituting these two equations again into the condition for harmonic balance yields, for values of the Q-factor that are not extremely low, the oscillator detuning ν from resonance:

$$\nu = -\frac{1}{Q^2} \sum_{n \neq \pm 1} \left[\frac{e_{on}}{e_{o1}}\right]^2$$

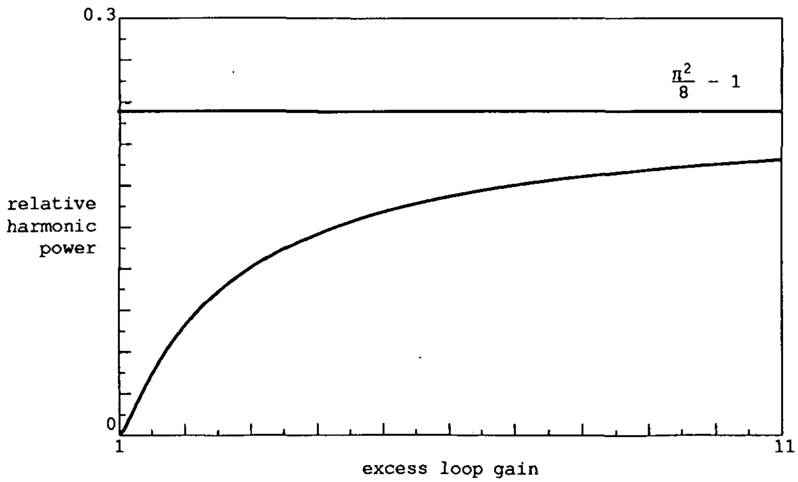

Fig. 1.7: The relative harmonic power as a function of the excess loop gain.

As could be predicted, a large Q-factor is an effective means to ensure an accurate oscillation frequency. The sum in the expression stands for the total of (high) harmonic power relative to the power at the fundamental at the output of the nonlinear amplifier. The previous section showed that the oscillator signals are rather simple: the output of the resonator is approximately sinusoidal and the output of the nonlinear amplifier is a clipped sine wave. Therefore this relative harmonic power can be calculated for the negative-feedback oscillator as a function of the excess loop gain AH_o, the degree of nonlinearity. Figure 1.7 shows this relative power. With large excess loop gains the relative harmonic power reaches its maximum value: the harmonic content of a square wave. The frequency deviation is then given by:

$$v_{max} = - \frac{1}{Q^2} \left(\frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \ \ldots \right) = - \frac{1}{Q^2} \left(\frac{\pi^2}{8} - 1\right) \simeq - \frac{1}{4Q^2}$$

or:

$$\Delta\omega_{max} \simeq \frac{1}{8Q^2} \omega_o$$

Fortunately this deviation is in most cases small enough. As far as frequency accuracy is concerned there is no reason to forbid this kind of nonlinearity.

1.9 Distortion with nonlinear feedback

For some applications sinusoidal signals with a low harmonic content are indispensable. Such signals are in principle best generated by oscillators that use nonlinear dynamic elements with within one period of oscillation, a linear transfer. However, perhaps somewhat in contrast with normal expectation, also negative-feedback oscillators that use time-invariant nonlinear elements may have fairly good distortion figures. There the control by negative feedback may restrict the generation of harmonics to the absolute minimum that is compatible with the condition for oscillation and may leave the resonator transfer unaffected. This section deals with the distortion in these oscillators, modeled in the simple way as shown in Section 1.7. The distortion is calculated as a function of AH_o, the degree of nonlinearity applied. As a secondary result the calculated distortion provides a criterion as to whether the condition of 'weakly nonlinear oscillation' is violated. This condition is that the large-signal loop transfer for harmonics is much smaller than one and it is therefore directly related to the distortion.

The distortion of a sine wave is given, by definition, by 100% times the square root of the relative (high) harmonic power. Thus with use of the Fourier series from Section 1.8, which describes the amplifier output signal $e_o(t)$, the distortion d_{lim} of the amplifier output follows from:

$$d_{lim} = 100\% \sqrt{\sum_{n \neq \pm 1} \left[\frac{e_{on}}{e_{o1}}\right]^2}$$

This distortion can be calculated as a function of the excess loop gain AH_o in a way comparable to that in Section 1.8. Figure 1.8 shows the mentioned dependency. Of course distortion of the

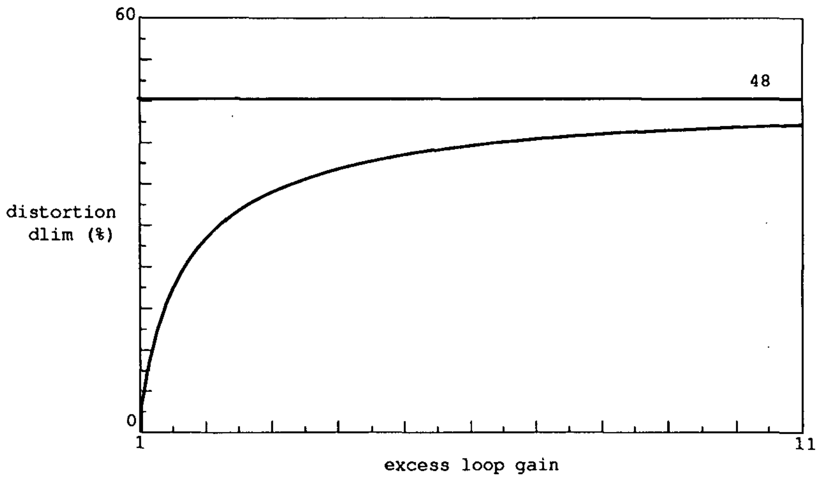

Fig. 1.8: Percentual distortion of the amplifier output signal.

amplifier output is unacceptably high. For these typical applications the output of the resonator must be used. The distortion in this output, d_{res}, follows in the same way from:

$$d_{res} = 100\% \sqrt{\sum_{n \neq \pm 1} \left[\frac{e_{in}}{e_{i1}}\right]^2}$$

The exact calculation of this distortion as a function of the excess loop gain AH_o is somewhat long. For application purposes here it suffices to use the approximation:

$$d_{res} \cong \frac{1}{3\,Q}\, d_{lim}$$

A high value of the Q-factor is therefore essential for an oscillator signal with low distortion. The use of these formulae for distortion is best illustrated with an example. Suppose 5% distortion in the resonator is the maximum allowed. If an excess loop gain AH_o of 2 is necessary to guarantee oscillation with a stable amplitude, the resonator must have at least a Q-factor of 2.

Maximum distortion occurs with maximum nonlinear behavior, which corresponds in a negative-feedback oscillator with large values of the excess loop gain AH_o. The distortion figures in that situation are:

$$d_{lim.max} = 100\% \sqrt{(\frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \ldots)} = 100\% \sqrt{(\frac{\pi^2}{8} - 1)} \approx 48\%$$

$$d_{res.max} = \frac{100\%}{Q} \sqrt{(\frac{1}{3^4} + \frac{1}{5^4} + \frac{1}{7^4} + \ldots)} = \frac{100\%}{Q} \sqrt{(\frac{\pi^4}{96} - 1)} \approx \frac{12\%}{Q}$$

The above shows that even with moderate Q-factors reasonably clean sine waves can be generated. Moreover it shows that no values of the excess loop gain AH_o can violate the condition of quasi-linear or weakly nonlinear behavior, if at least moderate Q-factors are available.

1.10 Discussion

This chapter introduced the negative-feedback oscillator as an oscillator in which negative, time-invariant nonlinear feedback has been applied. To demonstrate the desirability of such oscillators it started with a typical example of an ordinary harmonic oscillator, known as a 'Colpitts' oscillator. The performance of this oscillator turned out to be largely influenced by the dynamic, large-signal characteristics of the active element in the circuit. The inaccuracy and the complicated description of these characteristics make the design of a high-performance oscillator in such standard configurations virtually impossible.

Next, basic oscillator theory was reviewed showing that an oscillator must necessarily consist of a loop of a nonlinear element and a filtering element. For high-performance oscillators the nonlinear element preferably has a transfer with no phase shift and the filter element is preferably of second order or higher, enabling quasi-linear or weakly nonlinear oscillation.

It was shown that such oscillators can, viewed electronically, effectively be described in a simple way by the equivalent linearization of the nonlinear element. This description reduces the complicated condition for oscillation, normally a nonlinear differential equation of second order, to two simple equations. These equations have straightforward solutions for the oscillation frequency and the oscillation amplitude and are known as the Barkhausen conditions.

One of them, the phase balance, determines the oscillation frequency. As high performance forbids the influence of active elements on this frequency, only accurate passive elements may filter and introduce phase shift in the oscillator loop.

Further analysis showed that these elements must be resonators with low losses because of their excellent phase discriminating properties.

For the second equation, the amplitude balance, the first harmonic response of the nonlinear amplifier is required. For an evaluation of this equation an overview was given of known nonlinear amplifiers that have been employed to close the oscillator loop around resonators. For potential high performance only amplifiers with nonlinear dynamic feedback are available. The limited application of this kind of feedback, caused by the necessity of exotic nonlinear devices (NTC, PTC etc.) or of extensive electronic circuitry (AVC), urges for a more versatile kind of feedback.

This versatility was found in the application of negative time-invariant nonlinear feedback. This valuable form of feedback proved to have many potential benefits. For instance: the control of oscillation through only accurate passive elements and the rather simple implementation. Other advantageous properties are the possibility of a noise optimization, a wide variety of possible feedback elements and the absence of the bouncing phenomenon. The most obvious benefit however was the fact that it simplifies the description of the transfer around the loop. A time-invariant nonlinear amplifier transfer and a resonator transfer fully characterize this transfer. This feature enables the modeling of the negative-feedback oscillator in a simple one-dimensional way.

Two possible drawbacks of the time-invariant nonlinear feedback, the reduced frequency accuracy and the increased distortion of the oscillator output, were analyzed with the aid of this model. The results of this analysis showed that these oscillators do not reach the ultimate performance possible with nonlinear dynamic feedback, but that they can have a far better performance than that which can be expected from other oscillators.

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CHAPTER 2

SMALL-SIGNAL TRANSFER OF NEGATIVE-FEEDBACK OSCILLATORS

2.1 Introduction

The presence of noise in electronic oscillators hampers the designer in realizing an accurate short-term or dynamic frequency and waveform of the oscillator output signal. The losses of the resonator and the active components, required in the rest of the oscillator to compensate for these losses, are afflicted with noise. This noise is transferred to the oscillator output where it manifests itself as parasitic phase and amplitude modulation. For a high performance of the oscillator in an electronic system a low parasitic modulation is essential. For example, in an application of a clock oscillator in digital circuitry this modulation may result in timing errors. Other examples are electronic mixers in Doppler radar and in communication systems in which this parasitic modulation limits sensitivity by 'reciprocal mixing' [ref.1].

This chapter treats the influence of internal noise on the phase and amplitude of the output of negative-feedback oscillators. For this purpose the small-signal transfer of these oscillators will be calculated. In contrast with many preceding attempts [ref.2,3,4] here calculation will take full notice of the nonlinear element in the oscillator. If the loop around the resonator is closed by an element with a time-invariant transfer there is a simple description of its small-signal transfer. With this description it becomes possible to express the transfer of small, independent signals in the oscillator to phase and amplitude variations in the output. This transfer will be a function of the specific time-invariant nonlinear transfer applied.

This expression will show which time-invariant nonlinear transfers favor high performance. Of all possible time-invariant nonlinear transfers, the linear transfer and the limiting transfer in the negative-feedback oscillator will be evaluated. This evaluation will yield simple design criteria for the degree of nonlinearity to be applied in the oscillator: transfers to amplitude or phase noise will be given as a function of the excess loop gain. The designer will have to find the compromise between acceptable levels of transfer to phase and to amplitude by choosing a proper excess loop gain.

2.2 Small-signal transfer of memoryless nonlinear elements

In the oscillator a distinction can be made between the oscillator signal and the noise sources related to passive resistances and active components. Throughout this work it is assumed that such noise is the result of stationary Gaussian random processes. These noise sources form the input of the oscillator. The difference between the oscillator signal and the oscillator signal that would result from the noise-free oscillator, forms the output noise. The subject of this chapter is the transfer of input noise to output noise.
As long as both input and output noise are sufficiently small compared to the actual oscillator signal, a small-signal approximation can lead to this transfer. However, due to the large amplification at frequencies very close to the oscillation frequency the transfer of noise components with these frequencies does not follow from such an approximation. In practice this gives no serious problems. The output noise that is much smaller than the oscillator signal (let us say 40 dB) determines the 'short-term' stability of the oscillator. Other components are normally less important. They do not determine the 'long-term' stability. The temperature dependence or the aging of the resonator usually has a much larger effect on this stability.

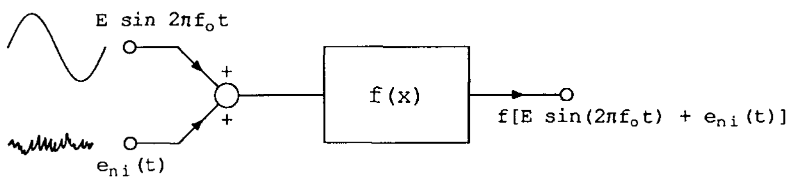

Fig. 2.1: Inputs of a time-invariant nonlinear element.

In order to find the small-signal transfer of the oscillator, first the corresponding transfer of a separate time-invariant nonlinear element is analyzed. This transfer depends on the element itself but also on the large signal that accompanies the small signal [ref.5,6]. In a negative-feedback oscillator this large input signal will be sinusoidal. Figure 2.1 depicts this situation. An element with a memoryless transfer f transfers an input of a small signal $e_{ni}(t)$ and a large sinusoidal signal with amplitude E. The output can very well be approximated by the first two terms of a Taylor expansion:

$$f\,[E\sin(2\pi f_o t)+e_{ni}(t)] = f\,[E\sin(2\pi f_o t)] + \left.\frac{df}{dx}\right|_{x=E\sin(2\pi f_o t)} e_{ni}(t)$$

The output consists of harmonics of the sinusoidal input and of the transferred small signal. Because of the periodicity of the large signal, the small-signal transfer is periodic too and so the output signal $e_{no}(t)$ may be expressed, with the help of a Fourier series with coefficients c_n, as :

$$e_{no}(t) = e_{ni}(t) \sum_n c_n e^{jn2\pi f_o t}$$

Or after Fourier transformation as:

$$E_{no}(f) = \sum_n c_n E_{ni}(f-nf_o) \quad \text{with the Fourier pairs:} \quad e_{ni}(t) \Leftrightarrow E_{ni}(f), \quad e_{no}(t) \Leftrightarrow E_{no}(f)$$

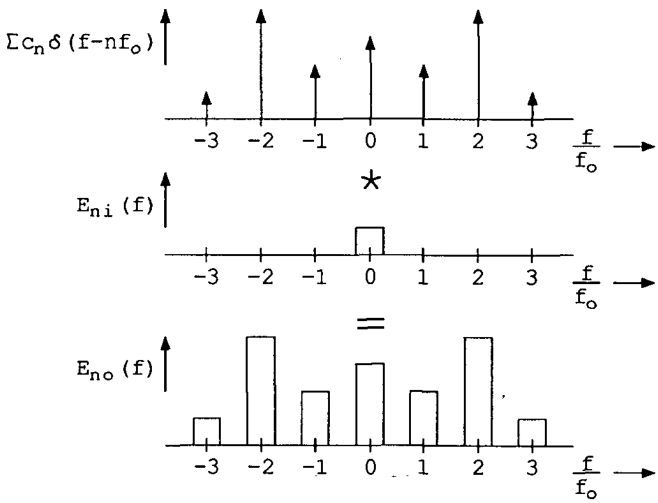

Fig. 2.2: Small-signal transfer in the frequency domain.

Figure 2.2 shows in the frequency domain how the small signal is transferred. The output signal $E_{no}(f)$ is the result of the convolution of the input signal $E_{ni}(f)$ and a series of dirac pulses in the frequency domain at multiples of f_o, that describes the time-dependent transfer. As the small signals are normally formed by noise, expressions with input and output power spectra, like S_{ni} and S_{no}, are more relevant:

$$S_{no}(f) = \sum_n |c_n|^2 S_{ni}(f-nf_o) \quad \text{with} \quad S_{ni} = 2 E_{ni}^* E_{ni}, \quad S_{no} = 2 E_{no}^* E_{no}$$

Evaluating the small-signal transfer of a time-invariant nonlinear element is not possible without knowledge of the coefficients c_n. These coefficients only depend on the amplitude of the input sine and the specific nonlinear transfer. In the next section the expressions above will be evaluated for the limiting transfer suggested for application in negative-feedback oscillators.

2.3 Small-signal transfer of a sine-driven limiter

The simplicity of the time-invariant nonlinear transfer in the negative-feedback oscillator can also be recognized in the small-signal transfer of the nonlinear element. Figure 2.3 shows again this time-invariant transfer of the nonlinear element, in the following also indicated as 'limiter'. Due to the simple characteristic its small-signal transfer has a constant value A as long as no limiting occurs. At limiting, this small-signal transfer is reduced to zero. Due to a large sinusoidal input of this limiter

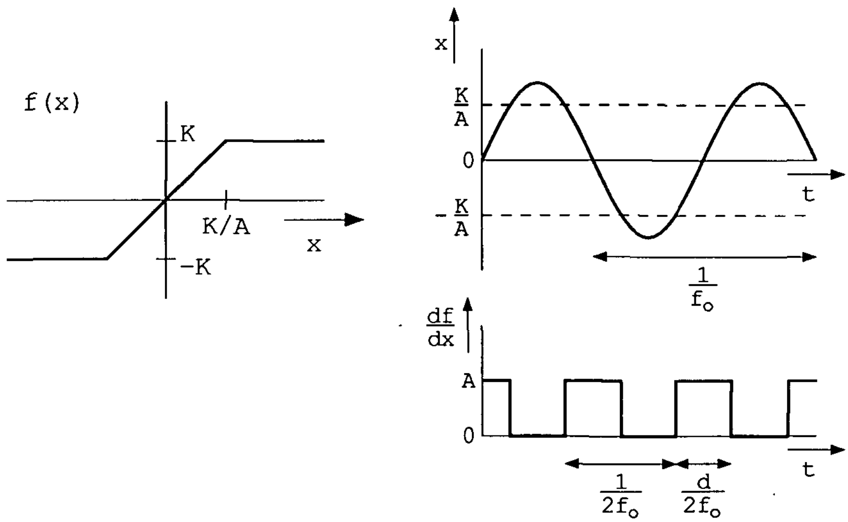

Fig. 2.3: Time dependent small-signal transfer of a sine-driven limiter.

with a frequency and an amplitude of f_o and E respectively, comparable to the situation in the negative-feedback oscillator, the small-signal transfer varies with time like a square wave. Frequency and a (peak-to-peak) amplitude of this wave are $2f_o$ and A respectively. This is illustrated in Figure 2.3. The time that the limiter behaves as a linear amplifier is given by the duty cycle d of this square wave:

$$d = \frac{2}{\pi} \arcsin\left(\frac{K}{A\,E}\right)$$

As mentioned in the previous section, a Fourier series may describe this time-variant small-signal transfer. The coefficients c_n for this transfer are:

$$c_n = A\,d\,\frac{\sin(\frac{n\pi d}{2})}{\frac{n\pi d}{2}} \qquad \text{for} \quad n = \text{even}$$

$$c_n = 0 \qquad \text{for} \quad n = \text{odd}$$

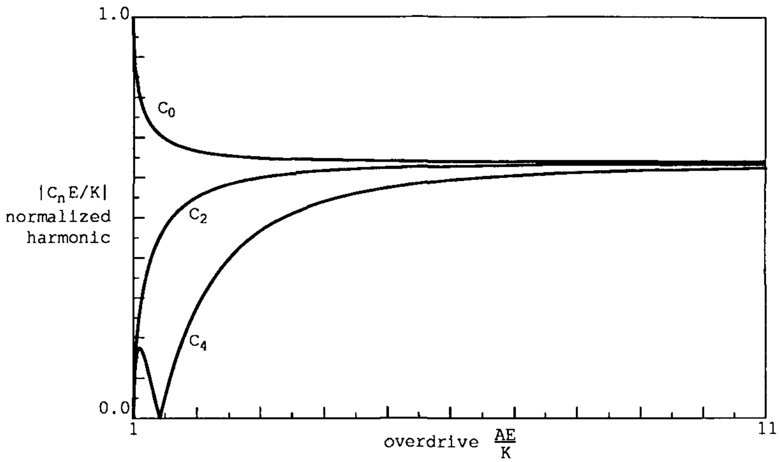

Fig. 2.4: Conversion factors of a sine-driven limiter as a function of the overdrive.

With the previous expression for the duty cycle d it becomes possible to plot the value of these coefficients as a function of the factor AE/K, which factor here is called 'the overdrive' of the limiter. Figure 2.4 depicts the first three coefficients as a function of this overdrive. For very small values of the overdrive the linear approximation still holds:

$$c_o = \frac{K}{E} \qquad \text{and} \qquad c_n = 0 \quad \text{for} \quad n \neq 0$$

For large values of the overdrive the coefficients are given by:

$$c_n = \frac{2}{\pi}\frac{K}{E} \qquad \text{for} \quad n = \text{even}$$

For a given value of the overdrive, coefficients with increasing values of n will decrease. The envelope of the Fourier transformed time-varying small-signal transfer, in case of overdrives of 2 and 5, is shown in Figure 2.5 for positive frequencies only.

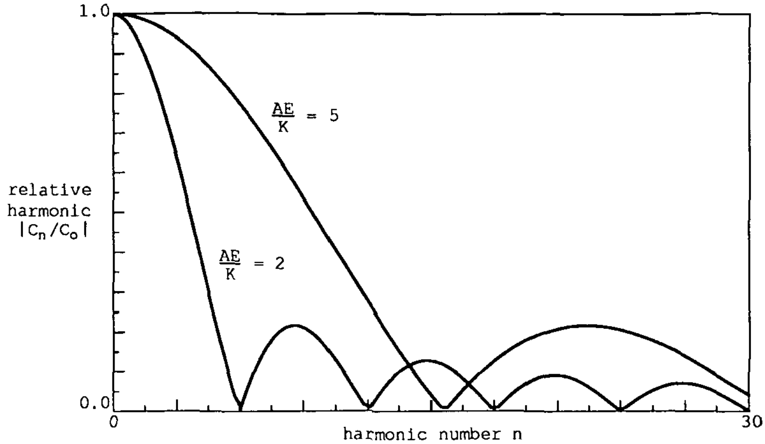

Fig. 2.5: Envelope of conversion factors for overdrives of 2 and 5.

Larger values for the overdrive possibly result in more conversion products. These products may reduce the spectral purity at the output of the limiter. For instance, there a higher noise floor, relative to the carrier power, the power of the fundamental at f_o, is found than at the input of the limiter. At a large overdrive the output spectrum S_{no} due to a white input noise [ref.7] with a spectrum of S_{ni}, is given by:

$$S_{no}(f) = A^2 \; S_{ni}(f) \; \left[d \right]_{\frac{AE}{K} \gg 1} \cong \frac{2\,A\,K}{\pi\,E} \; S_{ni}(f)$$

In this case a comparison between the carrier-to-noise at the input, CNR_i, and at the output, CNR_o, shows the a degradation of spectral purity:

$$CNR_o = CNR_i - 10 \log \left(\frac{A\,E\,\pi}{8\,K}\right)$$

The decrease in CNR is directly related to the overdrive AE/K. Following sections will use the description here derived of the time-varying small-signal transfer of time-invariant nonlinear elements. There the small-signal transfer of the negative-feedback oscillator will be calculated. For this calculation other descriptions are necessary, namely the relations between the oscillator output spectrum and the amplitude and frequency stability of an oscillator. The next section enumerates them.

2.4 Short-term amplitude and frequency stability

This section deals with the influence of a disturbing signal in the oscillator output on the short-term frequency and amplitude stability of the oscillator. The stability is described in the frequency domain by the power spectral densities of phase and amplitude variations in agreement with definitions from literature [ref.8,9]. These spectral descriptions are believed to be the most universal: various other parameters such as jitter, effective frequency error etc. can be derived from them [ref.10].
In characterizing the amplitude or frequency stability of a signal it is desirable that this signal only consists of one large sinusoidal signal (carrier), with an amplitude E and a frequency f_o, and a much smaller signal $e_{no}(t)$ limited to a bandwidth $2f_o$. Otherwise the nonlinear process of determining oscillator amplitude or frequency and the presence of high-frequency components cause various conversion products that complicate the description of these oscillator properties. Fortunately, the disturbing signals in the oscillator output may be limited to a very small bandwidth centered around the oscillation frequency f_o, except perhaps for some additive white noise. The influence of this noise on stability will be discussed in a following section and will here be ignored. The oscillator output signal o(t) can with this assumption be given by:

$$o(t) = E\ [1 + a(t)]\ \sin\ [2\pi f_o t + \phi(t)]$$

The functions a(t) and $\phi(t)$ describe the low-frequency amplitude and phase fluctuations in the oscillator signal. They vary at random, have a zero mean, and are effectively described by their power densities Sa and $S\phi$ respectively. These power spectra are a measure for amplitude and frequency stability: for high performance very low values are essential.

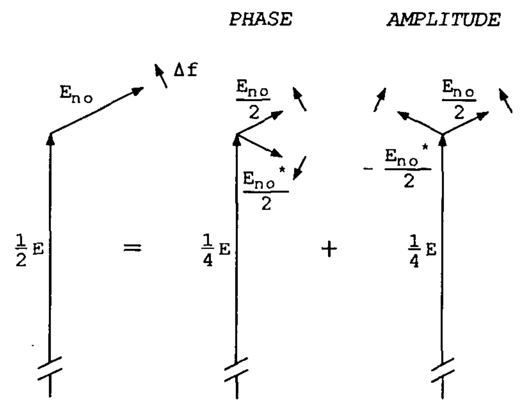

Fig. 2.6: Resolution of a sideband into equivalent amplitude- and phase-related sidebands.

The disturbing signal $e_{n_o}(t)$ in the oscillator output is related to the power spectra Sa and $S\phi$. In order to find this relation, the signal, Fourier transformed to $E_{n_o}(f)$, is split up in the phase modulating component $E_{n_o}\phi(f)$ and the amplitude modulating component $E_{n_o}a(f)$:

$$E_{n_o}\phi\ (f_o+\Delta f) = \frac{E_{n_o}(f_o+\Delta f) + E_{n_o}^{*}(f_o-\Delta f)}{2}$$

$$E_{n_o}a\ (f_o+\Delta f) = \frac{E_{n_o}(f_o+\Delta f) - E_{n_o}^{*}(f_o-\Delta f)}{2}$$

The familiar phasor diagram in Figure 2.6 illustrates this split.

In the same way S_{n_o}, the power spectrum of the disturbing signal $e_{n_o}(t)$, can be divided in a phase-related spectrum $S_{n_o}\phi$ and an amplitude-related spectrum $S_{n_o}a$. Modulation theory shows that these two spectra, when normalized to the carrier power, represent the power spectra of the amplitude and phase fluctuations:

$$S\phi\ (\Delta f) = \frac{4\ S_{n_o}\phi(f_o+\Delta f)}{E^2}$$

$$Sa\ (\Delta f) = \frac{4\ S_{n_o}a(f_o+\Delta f)}{E^2}$$

Literature also gives some related measures for frequency stability [ref.1,8]. These are given here for reference. The 'script $\mathscr{L}(\Delta f)$' is defined as:

$$\mathscr{L}(\Delta f) = \frac{\text{power density (one phase-modulation sideband)}}{\text{power (total signal)}}$$

Another measure is the carrier-to-noise ratio (CNR), which is defined as:

$$CNR(\Delta f) = 10 \log \frac{\text{carrier power}}{\text{power density (one modulation sideband)}}$$

There are various other measures [ref.11]. However the measures $S\phi(\Delta f)$, $\mathscr{L}(\Delta)$ and $CNR(\Delta f)$ are the most common. They are more or less equivalent, for according to these definitions:

$$S\phi\ (\Delta f) = 2\ \mathscr{L}(\Delta f)$$

And if all noise is considered to be phase noise:

$$CNR(\Delta f) = -\ 10 \log \mathscr{L}(\Delta f)$$

The next section uses the here-mentioned split of phase-and amplitude-related components. There a general expression for the influence of a small input signal of oscillator on the phase and the amplitude will be derived.

2.5 General expression for oscillator small-signal transfer

For an accurate expression that indicates how a small, independent signal in an oscillator influences oscillator phase and oscillator amplitude, an adequate model of this oscillator is necessary. The previous chapter showed that the application of negative feedback in an oscillator enables simple modeling. There the oscillator model consisted of a resonator and a time-invariant limiting amplifier. Here the same model will be used in a more general way: the time-invariant nonlinear transfer will not be made explicit. Fourier coefficients c_n will describe this transfer (cf. Section 2.2) with the objective of afterwards selecting the nonlinear transfer optimal for short-term frequency and amplitude stability.
The small-signal transfer of the thus-modeled oscillator has to be defined from a certain input to a certain output. The input must be the input of the time-invariant nonlinear amplifier in order to take nonlinear effects fully into account. The output must be the resonator output, for this output exhibits the band limited character necessary for unambiguous definition of oscillator amplitude and frequency.

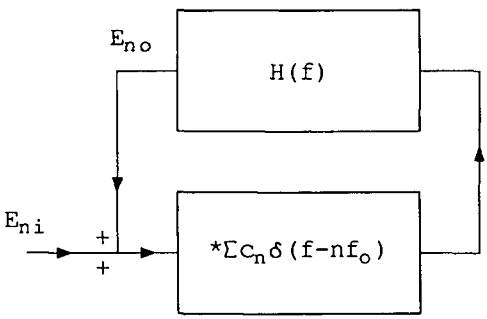

Fig. 2.7: Block diagram for calculation of the oscillator small-signal transfer.

Figure 2.7 shows the block diagram of the oscillator of which the small-signal transfer will be derived. A small, broad band signal $e_{ni}(t)$ with a power spectrum $S_{ni}(f)$, is transferred to the small output signal $e_{no}(t)$ having a power spectrum $S_{no}(f)$. In the frequency domain the resonator transfer is given by the transfer H(f) and the small-signal transfer of the nonlinear amplifier by a convolution with a series of dirac pulses. The small-signal transfer of an oscillator, modeled in this way, is implicitly given by the following relation:

$$E_{no}(f) = H(f) \left(E_{ni}(f) * \sum_n c_n \, \delta(f-nf_o) \right) + H(f) \left(E_{no}(f) * \sum_n c_n \, \delta(f-nf_o) \right)$$

This small-signal transfer is made explicit for the transfer to phase and amplitude variations with help of the following approximations:

$$f_{osc} \cong f_o$$

$$H(f_o+\Delta f) \cong H^*(f_o-\Delta f)$$

$$H(f) \left(E_{no}(f) * \sum_n c_n \, \delta(f-nf_o) \right) \cong$$

$$H(f) \left(c_o \, E_{no}(f) + c_2 \, E_{no}(f-2f_o) + c_{-2} \, E_{no}(f+2f_o) \right)$$

The last approximation made is that the earlier assumption (cf. Section 1.9) of a sinusoidal output of the resonator implies only spectral values of $E_{no}(f)$ close to the oscillation frequency are to be considered. Appendix II shows how these approximations can make the oscillator small-signal transfer explicit. The transfer to phase-related sidebands and to amplitude-related sidebands are expressed by the following relations:

$$E_{no}\phi(f_o+\Delta f) = \frac{H_o}{j2Q\Delta f/f_o} \sum_p \frac{c_{-p+1} + c_{-p-1}}{2} E_{ni}(pf_o+\Delta f)$$

$$E_{no}a(f_o+\Delta f) = \frac{H_o}{1-(c_o-c_2)H_o + j2Q\Delta f/f_o} \sum_p \frac{c_{-p+1} - c_{-p-1}}{2} E_{ni}(pf_o+\Delta f)$$

As the small signals are normally formed by noise, expressions with input and output power spectra, S_{ni} and S_{no} respectively, are more relevant. With the allowed approximation that the input power spectrum S_{ni} close to a multiple of the oscillation frequency is symmetrical with respect to this multiple, the phase-related power spectrum $S_{no}\phi$ and the amplitude-related power spectrum $S_{no}a$ become:

$$S_{no}\phi \, (f_o+\Delta f) = \left| \frac{H_o}{j2Q\Delta f/f_o} \right|^2 \sum_{p=1}^{\infty} \left| c_{p-1} + c_{p+1} \right|^2 \frac{1}{2} S_{ni}(pf_o+\Delta f)$$

$$S_{no}a\ (f_o+\Delta f) = \left|\frac{H_o}{1-(c_o-c_2)H_o+j2Q\Delta f/f_o}\right|^2 \left[c_1^2\ S_{ni}(\Delta f) + \sum_{p=1}^{\infty} \left| c_{p-1} - c_{p+1} \right|^2 \frac{1}{2} S_{ni}(pf_o+\Delta f) \right]$$

All small input signals with frequencies close to multiples of the oscillation frequency influence amplitude or phase stability of the oscillator. This influence can only be decreased by minimizing the coefficients c_n. This implies that the transfer should not be more nonlinear than is strictly necessary. The nonlinear behavior is necessary for limiting the first harmonic response in the oscillator loop. An odd nonlinear transfer is sufficient for this purpose [ref.12]. Other nonlinear transfers introduce more non-zero coefficients c_n resulting in a stability degraded by extra conversion products.

Following sections will apply the here-derived small-signal transfer to oscillators with some typical time-invariant nonlinear amplifiers.

2.6 Linear oscillator

The term linear oscillator may be a bit confusing, for a true oscillator is always nonlinear and a linear oscillator is therefore a 'contradictio in terminis'. The reason for the use of this term here is that it describes the interesting highly band-selective, active filter and not a (nonlinear) oscillator. It is, in fact, the situation that a long series of authors have assumed to be present in a real oscillator [ref.2,4]. From previous sections it may be clear that the nonlinear character may seriously influence oscillator stability. By comparing the small-signal transfer of the imaginative linear oscillator to that of the nonlinear oscillator this influence can be made more explicit.

For the linear oscillator the small-signal transfer of the amplifier is not time dependent, of course, and all coefficients c_n are zero except for c_o:

$$c_o = \frac{1}{H_o}$$

The active part in this case is a linear amplifier with a gain such that at resonance a loop transfer of exactly one is realized. Expressions for amplitude and phase noise become very simple now:

$$S_{no}\phi\ (f_o+\Delta f) = \left[\frac{1}{2Q\Delta f/f_o}\right]^2 \frac{1}{2} S_{ni}(f_o+\Delta f)$$

$$S_{no}a\ (f_o+\Delta f) = \left[\frac{1}{2Q\Delta f/f_o}\right]^2 \frac{1}{2}\ S_{ni}(f_o+\Delta f)$$

Half 'oscillator' sideband power is related to phase noise, the other half to amplitude noise. Sideband noise only originates from noise components with frequencies close to the oscillation frequency, as may be expected from a purely linear circuit.

2.7 Negative-feedback oscillator

These following sections convert the general expressions for the small-signal transfer to expressions that are specific for the negative-feedback oscillator. First the Fourier coefficients that describe the small-signal transfer of the nonlinear amplifier are derived. Thereupon phase and amplitude conversion in this oscillator are treated.

2.7.1 Small-signal transfer of a limiting amplifier

From the general expression for the small-signal transfer of an oscillator it can concluded that an amplifier with a time-invariant limiting transfer is an appropriate choice in a negative-feedback

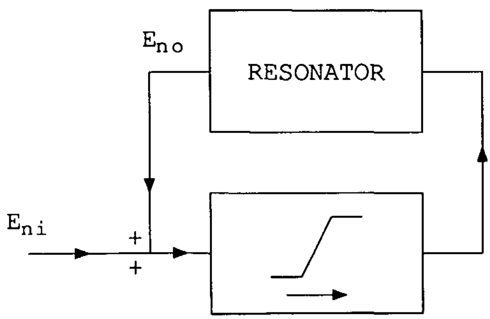

Fig. 2.8: The simple model for a negative-feedback oscillator.

oscillator. It has the preferred odd characteristic and the degree of nonlinearity can be adjusted with its true small-signal gain A. Figure 2.8 depicts again the simple model of the negative-feedback oscillator with this kind of amplifier and a resonator.
The two parameters that must be designed in this amplifier are the limit level K and the true small-signal gain A. For a high stability, large values for this limit level K are obviously

favorable for they result in a large carrier power. The distance between the carrier power and noise power in oscillator sidebands (cf. Section 2.4), which is a measure for stability, is directly influenced by this limit level. Influence of the true small-signal gain A on stability can not be estimated so easily. This and following sections deal with this influence.
A Fourier series may describe the time-varying small-signal transfer of the limiting amplifier in a negative-feedback oscillator in the same way as it described that of a separate limiter in Section 2.3. The coefficients c_n are once again given by:

$$c_n = A\, d \frac{\sin(\frac{n\pi d}{2})}{\frac{n\pi d}{2}} \qquad \text{for} \qquad n = \text{even}$$

$$c_n = 0 \qquad \text{for} \qquad n = \text{odd}$$

The duty cycle d is also here a function of the degree of nonlinearity. In a negative-feedback oscillator the true small-signal loop gain at the resonance, having a value AH_o, is a measure for this degree. Appendix III shows the derivation of this relation between duty cycle d and the excess loop gain AH_o. The relation is given by:

$$d\left(1 + \frac{\sin(\pi d)}{\pi d}\right) = \frac{1}{A H_o}$$

The relation between excess loop gain and duty cycle is depicted in Figure 2.9. For soft nonlinear behavior, values of AH_o slightly

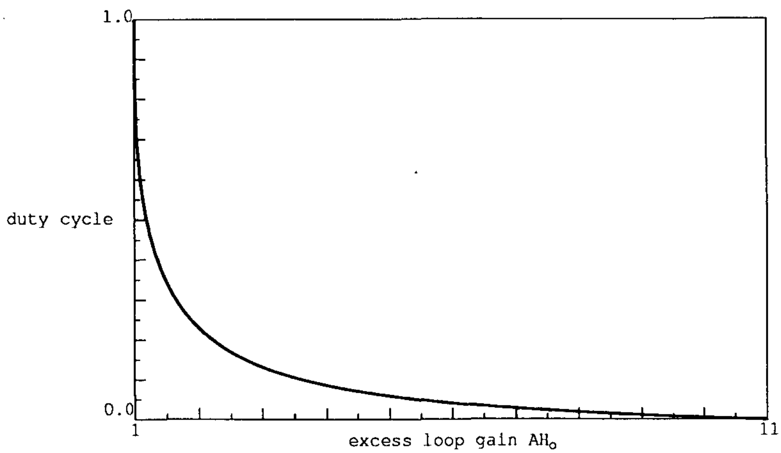

Fig. 2.9: Dependency of the duty cycle on the excess loop gain AH_o.

larger than 1 are found. The duty cycle d then almost equals 1: the amplifier only limits during a very small part of the oscillation period. Large values of the excess loop gain ($AH_o >> 1$) indicate hard nonlinear behavior. In this case a simple approximation for the duty cycle d exists:

$$d \cong \frac{1}{2 A H_o} \qquad \text{for} \quad AH_o \gg 1$$

This approximation gives an error smaller than 1.5 % for $AH_o > 2$. The above relations link the Fourier coefficients c_n to the excess loop gain AH_o. These coefficients c_n vary almost in the same way with the excess loop gain as those of a separate, sine-driven limiter do with the overdrive (cf. Figure 2.4 in Section 2.3). As it is known from Section 2.5 how these coefficients influence amplitude and frequency stability, it becomes possible to find the relation between the excess loop gain AH_o and the oscillator stability.

2.7.2 Conversion to phase fluctuations

In the small-signal transfer to phase fluctuations the sum of two coefficients c_n with subsequent odd values for n determine the conversion from a frequency band close to a multiple of the oscillation frequency. For this purpose the phase-conversion factor PM_n is defined as:

$$PM_n = H_o \left| c_{n-1} + c_{n+1} \right|$$

It describes the conversion from frequency band close to nf_o. This definition simplifies the appearance of the small-signal transfer. A small oscillator input signal, with a power spectrum S_{ni}, is transferred to a the phase-related sideband spectrum $S_{no}\phi(f_o \mp \Delta f)$ at the oscillator output, given by:

$$S_{no}\phi \; (f_o+\Delta f) = \left[\frac{1}{2Q\Delta f/f_o}\right]^2 \sum_{n=1}^{\infty} PM_n^{\;2} \; \frac{1}{2} \; S_{ni}(nf_o+\Delta f)$$

Evaluation of the phase-conversion factors PM_n yields that all even numbered factors are zero because of the odd nonlinearity in the limiting transfer. Further the amplitude balance in the condition for oscillation yields a constant value for PM_1:

$$PM_1 = 1$$

Other odd-numbered coefficients PM_n vary with the excess loop gain. With the equations given, they can all be found as a function of AH_o. Unfortunately explicit relations do not seem possible due to

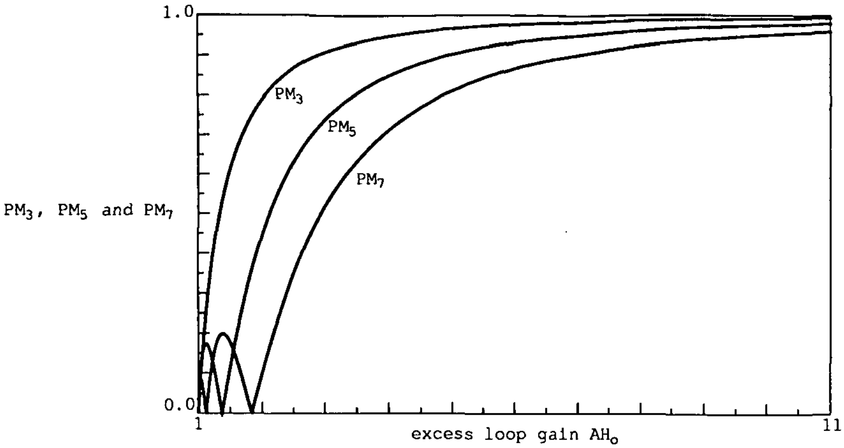

Fig. 2.10: Phase-conversion factors PM_3, PM_5, PM_7 as a function of the excess loop gain AH_o.

the implicit relation between the duty cycle d and the excess loop gain AH_o (cf. Section 2.7.1). Figure 2.10 depicts the dependency of the phase-conversion factors PM_3, PM_5, and PM_7 on the the excess loop gain AH_o.
Of the infinite sum of conversion products that form the resulting phase-related sideband in the oscillator output, only a limited number of products contributes substantially. This is demonstrated in Figure 2.11, where the envelope of the coefficients PM_n is

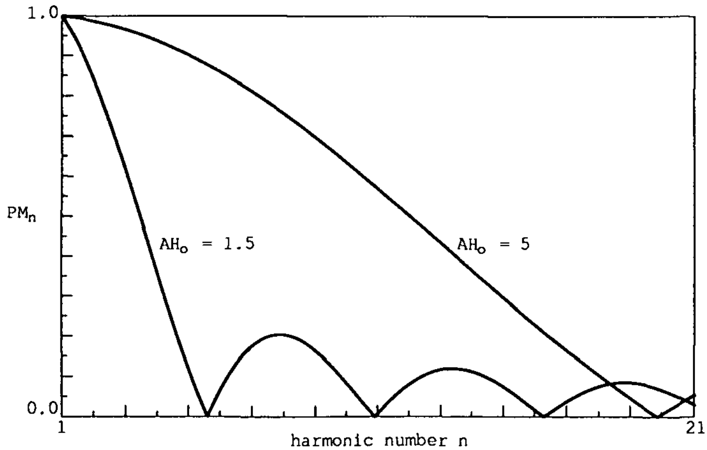

Fig. 2.11: Envelope of conversion factors PM_n.

depicted for two values of the excess loop gain AH_o. Conversion products from the frequency region around higher multiples of the oscillation frequency contribute less to the total phase-related sideband. Moreover their contribution decreases with decreasing excess loop gain.
It can be concluded that for larger excess loop gains or, expressed alternatively, for harder nonlinear behavior of the oscillator, more conversion products must be taken into account. A reduction of the phase-related sidebands is made possible by a design that allows only soft nonlinear behavior of the oscillator.

2.7.3 Conversion to amplitude fluctuations

In a comparable way, amplitude-conversion factors are defined. The amplitude-conversion factor AM_n describes the conversion from the frequency band around nf_o to the amplitude-related sideband in the oscillator output. The amplitude-conversion factor is defined as:

$$AM_n = H_o \left| c_{n-1} - c_{n+1} \right|$$

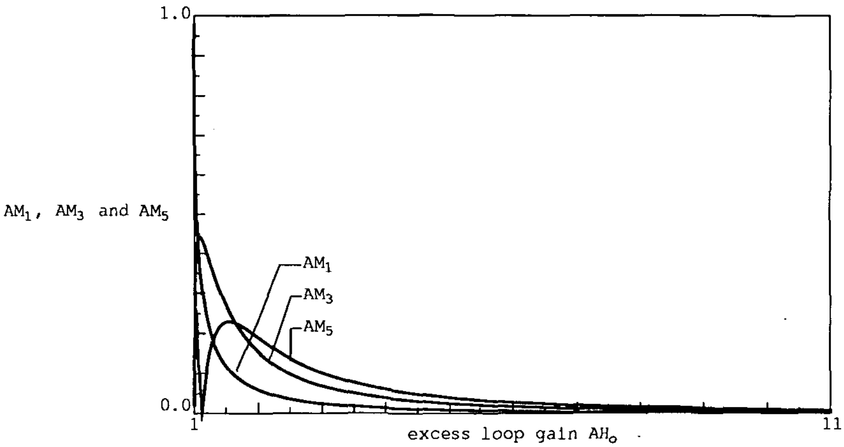

Fig. 2.12: Amplitude-conversion factors AM_1, AM_3, AM_5 as a function of the excess loop gain AHo.

With this definition and the notion that all odd-numbered coefficients c_n are zero in a negative-feedback oscillator, the transfer to the amplitude-related sideband $S_{no}a$ is simplified to:

$$S_{no}a\ (f_o+\Delta f) = \left| \frac{1}{1 - AM_1 + j2Q\Delta f/f_o} \right|^2 \sum_{n=1}^{\infty} AM_n^{\ 2}\ \frac{1}{2}\ S_{ni}(nf_o+\Delta f)$$

Because the amplitude-conversion factors AM_n are a function of AH_o, here the degree of nonlinear behavior in the oscillator influences not only the amplitude noise conversion products but also the way these conversion products are filtered before they show up in the oscillator output. First the magnitude of the conversion products will be considered and subsequently the filtering of these products.
Of the amplitude-conversion factors the first three are shown in Figure 2.12 as a function of the excess loop gain. The factors are small or they rapidly decrease with increasing excess loop gain.
The amplitude conversion products appear in the oscillator output as filtered by a second-order bandpass filter with a center frequency f_o and a -3 dB bandwidth of $2\Delta f_{AM}$. Half this bandwidth is given by the equation:

$$\Delta f_{AM} = (1 - AM_1)\ \frac{f_o}{2Q}$$

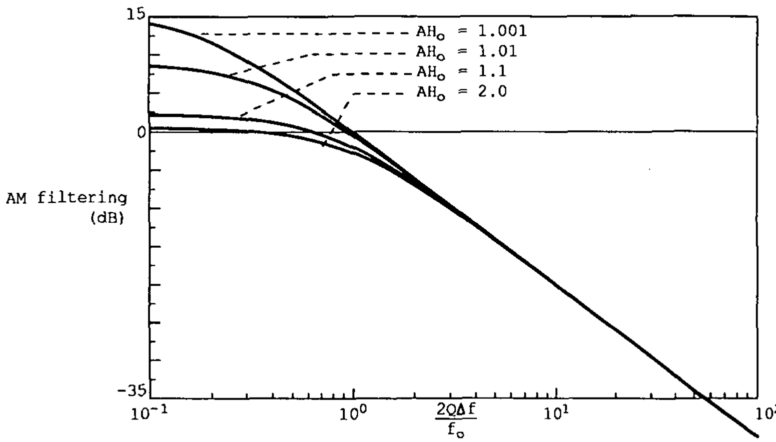

Fig. 2.13: Filtering of amplitude-conversion products at different excess loop gains.

As the conversion factor AM_1 decreases very rapidly with the excess loop gain (cf. Figure 2.12), also the filtering of the amplitude conversion products changes rapidly with a change in the degree of the nonlinear behavior of the oscillator. Figure 2.13 shows the filter characteristics for various excess loop gains. In the case of very hard nonlinear behavior a filtering bandwidth of f_o/Q results. In this case, however, there is hardly any amplitude conversion. The change in filtering goes rapidly with increasing excess loop gains as may be illustrated by the fact that an excess loop gain of only 2 results in a filter bandwidth which is within 6% of f_o/Q. These quantitative results are in good agreement with the more qualitative results known in literature [ref.13].

2.8 The degree of nonlinearity

With the knowledge about the conversion process, described by the factors AM_n and PM_n and by the corresponding filtering of the conversion products, a complete small-signal analysis exists for an oscillator that can be modeled in the simple way by a resonator and a time-invariant limiting amplifier. For all possible small-signal inputs of the oscillator, the output spectrum, divided into amplitude- and phase-related sidebands, can be calculated.
However, the interest is not primarily in analysis but in the design of oscillators. During the design of a negative-feedback oscillator the question arises as to how hard the nonlinear behavior must be or how much excess loop gain must be incorporated for a specific application.
The first criterion for a choice of the excess loop gain is, of course, that oscillation is guaranteed under various conditions. For in practice, the resonator transfer H_o and the true small-signal transfer A of the limiting amplifier will show some inaccuracy. The result of such inaccuracy must be, for instance, that the worst case excess loop gain is slightly larger than one while the typical excess loop gain can be much larger.
As far as the filtering of the conversion products is concerned, only very low values of the excess loop gain change this filtering (cf. Figure 2.13). For values 1.1 or larger its influence may be ignored.
Which conversion factors are acceptable in the oscillator depends on the noise present at the oscillator input. This differs with the application. However two typical forms of noise will often be present. First the noise that has been limited in bandwidth by resonator filtering. It contains only components with frequencies close to f_o. White noise forms the second typical input. The following sections discuss how the excess loop gain modifies the influence of these two inputs on oscillator stability.

2.8.1 Band-limited input noise

If the oscillator input noise, with a power spectrum S_{ni}, has been limited to a bandwidth of $2f_o$, the only important conversion factors are AM_1 and PM_1. The phase-related output spectrum $S_{no}\phi$ and the amplitude-related output spectrum $S_{no}a$ are with this input noise given by:

$$S_{no}\phi\ (f_o+\Delta f) = \left|\frac{1}{j2Q\Delta f/f_o}\right|^2 \frac{1}{2}\ PM_1^{\ 2}\ S_{ni}(f_o+\Delta f)$$

$$S_{no}a\ (f_o+\Delta f) = \left|\frac{1}{1-AM_1 + j2Q\Delta f/f_o}\right|^2 \frac{1}{2}\ AM_1^{\ 2}\ S_{ni}(f_o+\Delta f)$$

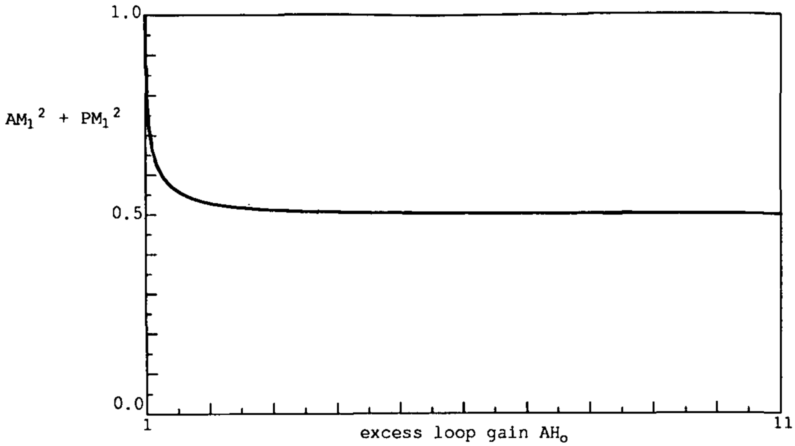

Fig. 2.14: The effect of nonlinear behavior on oscillator output noise in the case of band-limited input noise.

Conversion factors AM_1 and PM_1 differ by the influence of the excess loop gain AH_o. This difference is implicitly shown in Figure 2.14. The sum of their squares rapidly halves as a function of the excess loop gain. This corresponds to the effective removal of amplitude-related noise. For this removal no large excess loop gain is necessary, for with an excess loop gain of 2 all amplitude noise is removed.

2.8.2 White input noise

When the oscillator input consists of white noise with a power spectrum S_{ni}, all conversion factors AM_n and PM_n must be considered. With this kind of input noise the phase-related output spectrum $S_{no}\phi$ and the amplitude-related output spectrum $S_{no}a$ are given by:

$$S_{no}\phi\ (f_o+\Delta f) = \left|\frac{1}{j2Q\Delta f/f_o}\right|^2 \frac{1}{2} S_{ni}(f) \sum_{n=1}^{\infty} PM_n{}^2$$

$$S_{no}a\ (f_o+\Delta f) = \left|\frac{1}{1-AM_1 + j2Q\Delta f/f_o}\right|^2 \frac{1}{2} S_{ni}(f) \sum_{n=1}^{\infty} AM_n{}^2$$

For low excess loop gains ($AH_o \cong 1$) no difference occurs in comparison with a band-limited input. For larger loop gains ($AH_o \geq 2$) there is mainly phase conversion, as Sections 2.7.2 and 2.7.3 demonstrated for these loop gains the following relation holds:

$$\sum_{n=1}^{\infty} PM_n^{\ 2} \quad \gg \sum_{n=1}^{\infty} AM_n^{\ 2}$$

As hardly any amplitude-related components occur the phase-related output spectrum $S_{no}\phi$ may be very well approximated by:

$$S_{no}\phi\ (f_o+\Delta f) = \left[\frac{1}{2Q\Delta f/f_o}\right]^2 \frac{1}{2}\ S_{ni}(f) \sum_{n=1}^{\infty} AM_n^{\ 2} + PM_n^{\ 2}$$

As previous sections revealed that:

$$\sum_{n=1}^{\infty} PM_n^{\ 2} + AM_n^{\ 2} = 2\ H_o^{\ 2} \sum_{n} |c_n|^2 = 2\ d\ A^2\ H_o^{\ 2} \cong A\ H_o$$

the phase-related output spectrum $S_{no}\phi$ is given for larger excess loop gains ($AH_o \geq 2$) by the simple expression:

$$S_{no}\phi\ (f_o+\Delta f) = \left[\frac{1}{2Q\Delta f/f_o}\right]^2 \frac{1}{2}\ A\ H_o\ \ S_{ni}(f)$$

This result shows that the phase-related output power is a factor AH_o, the value of the excess loop gain, larger than the phase-related output power in a 'linear' oscillator (cf. Section 2.6). Thus a frequently acceptable compromise between amplitude and frequency stability is possible in negative-feedback oscillators. For oscillators with a good frequency stability the excess loop gain should as far as possible approach the value of one. A choice for an excess loop gain of 2, however, practically removes all amplitude-related noise and only doubles the phase-noise power.

2.9 Oscillator noise floor

So far only band-limited oscillator outputs have been discussed. In practice however some additive white noise is always present at the output. This noise influences the oscillator stability. This section shows how stability is degraded and what measures can be taken to ensure a high stability.
In the simple oscillator model the resonator output is band limited. However, if the input noise of the oscillator is white, the other output of the oscillator, the output of the nonlinear amplifier, will show a noise floor. The influence of this noise floor is determined by the character of the oscillator load. If the load handles the oscillator signal at this output linearly, the spectral value of this floor is decisive for performance. As the

small-signal transfer of the nonlinear amplifier is known , this value can be calculated. Section 2.3 gave an example of such a calculation.
However in cases where frequency stability is important, the load, in the form of an electronic mixer or counter, handles the signal in a nonlinear way. If the load treats the signal in a time-invariant nonlinear way, conversion of white noise degrades the stability at the load. It is here assumed that this conversion is not present in this load nor in any other phase-sensitive device. For high-performance applications the load is preferably designed to be nonlinearly dynamic in such a way that it filters out all small, high-frequency inputs. Appendix IV gives an example of this nonlinear filtering. With these loads only noise components with frequencies close to f_o influence stability. The following shows the influence of a white noise source in the oscillator with a power spectrum S_{ni} and the influence of a white noise source with a spectrum S_{nl} which represents the equivalent noise of the load.
Figure 2.15 depicts how the broad band oscillator output is delivered to a load. In order to demonstrate the influence of two white noise sources the power spectrum of the phase fluctuations

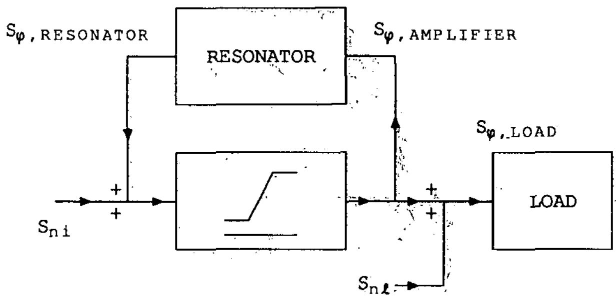

Fig. 2.15: White noise sources in an oscillator-load combination

$S\phi(\Delta f)$ is drawn up at the band-limited resonator output, at the broad-band amplifier output and at the load. According to the definition from Section 2.4 spectral phase fluctuations are given by:

$$S\phi_{resonator}(\Delta f) = \frac{4}{E^2} S_{no}\phi(f_o+\Delta f)$$

With a white input spectrum S_{ni} these fluctuations follow from (cf. Section 2.7.3):

$$S\phi_{resonator}(\Delta f) = \frac{4}{E^2} \left[\frac{1}{[2Q\Delta f/f_o]^2} \frac{1}{2} AH_o S_{ni}\right]$$

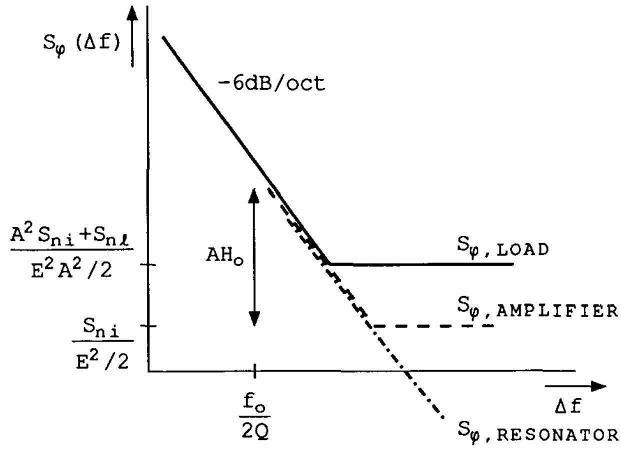

Fig. 2.16: Influence of internal and external white noise sources on the spectra of phase fluctuations.

At the amplifier output the same phase fluctuations occur as at the amplifier input because the amplifier has a time-invariant limiting transfer:

$$S\phi_{amplifier}(\Delta f) = \frac{4}{E^2}\left[\frac{1}{[2Q\Delta f/f_o]^2}\ \frac{1}{2}\ AH_o\ S_{ni} + \frac{1}{2}\ S_{ni}\right]$$

As a well-designed nonlinear load will reach some form of limiting before limiting occurs in the oscillator, the amplifier acts linearly with respect to the influence of the equivalent load noise. At the load the phase fluctuations are then given by:

$$S\phi_{load}\ (\Delta f) = \frac{4}{E^2}\left[\frac{1}{[2Q\Delta f/f]_o^2}\ \frac{1}{2}\ AH_o\ S_{ni} + \frac{1}{2}\ S_{ni} + \frac{1}{2}\ \frac{S_{nl}}{A^2}\right]$$

Figure 2.16 shows the three power spectra of phase fluctuations as a function of the offset frequency Δf. The white noise sources reduce frequency stability for large offset frequencies.
High values of the excess loop gain suppress the influence of load noise. However such values cause considerable phase conversion in the oscillator itself and are often not allowed. Then the presence of high load noise necessitates a high carrier power ($E^2/2$) or an extra low noise buffer in front of the load.

2.10 Discussion

This chapter focused on the small-signal transfer of negative-feedback oscillators. For this purpose the oscillator was treated like some kind of amplifier with an input and an output. As an oscillator has a very large amplification close to the oscillation frequency, the only practical inputs are small signals such as noise. The resulting outputs were components modulating the oscillator frequency and amplitude.

Also in this chapter a loop of a time-invariant nonlinear amplifier and a resonator formed the oscillator model. The accurate nonlinear transfer of the amplifier and the accurate large sinusoidal input ensured an accurate small-signal transfer of this amplifier. Due to the large periodic input signal this small-signal transfer is periodic too. Spectrally this results in a conversion process. The small-signal output of the amplifier consists of an accurate set of small input signals, each shifted in frequency over a multiple of the large-signal's frequency.

With the accurate description of the small-signal transfer of the nonlinear amplifier it was thereupon possible to derive the small-signal transfer of the oscillator itself. This chapter showed that there are simple, surveyable expressions for the transfer of small independent signals in the oscillator to phase- or amplitude-modulating components in the oscillator output. The general character of these expressions enabled the design of the time-invariant nonlinear transfer in the oscillator. It could be concluded that, as was often suggested in literature in a qualitative way, soft, odd-nonlinear transfers are to be preferred because they limit the number of conversion products. Moreover it showed how the degree of nonlinear oscillator behavior (soft or hard) influences stability in a quantitative way.

The spectral form of the output signals turned out to be practically independent of the nonlinearity applied. Both phase- and amplitude-disturbing components are filtered by second-order band pass filters. These filters have both the oscillation frequency as center frequency and have identical transfers for large offset frequencies. For phase-related components the filter bandwidth can be considered as infinitely small and for amplitude-related components this bandwidth equals that of the resonator.

The small-signal transfer was subsequently evaluated for the special case of the negative-feedback oscillator. The transfer to phase- as well as to amplitude-modulating components, described by conversion factors, were shown to be simple functions of the excess loop gain in the oscillator. This excess loop gain proved to be a valuable measure for the nonlinear behavior of the oscillator. Very small values (<1.1) indicated very soft nonlinear behavior with a corresponding small-signal transfer that resembled that of a highly selective filter. For a larger value (≥2) of the excess loop gain amplitude conversion became negligible, while this value indicated the number of phase-conversion factors that must be taken into account.

With a quantitative description of the conversion process in a negative-feedback oscillator the design of the nonlinear behavior became possible by the selection of an excess loop gain. Apart from various component tolerances in the oscillator, an excess loop gain of 2 was proven to be sufficient for a stable amplitude, while a white noise source degraded the frequency stability no more than 3 dB extra. By this feature, an optimal amplitude stability and a frequency stability very close to the optimum, negative-feedback *oscillators compare favorably with* AVC-*oscillators*. The *slight* degradation in frequency stability will often be negotiable for the simpler hardware implementation.

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CHAPTER 3

CONFIGURATIONS FOR NEGATIVE-FEEDBACK OSCILLATORS

3.1 Introduction

In the first chapter we defined the simple oscillator model for negative-feedback oscillators. The simple structure of the model allowed us to calculate various oscillator quality aspects such as waveform, frequency and frequency stability. This chapter shows the correctness of this simple modeling by deriving the appropriate oscillator configurations. The resonators used in our negative-feedback oscillators will be rather simple: they are formed by the well-known LC series and LC parallel resonator circuits. These oscillators will belong, therefore, to the familiar class of LC-oscillators. Some surveys on LC-oscillators configurations are known from literature [ref.1] and here adding new material to this collection runs the risk of summarizing previous work. However, there is latitude for new investigations, because in literature various important aspects of the oscillator configuration are veiled.

For instance, the total of LC-oscillator configurations does not show a systematic structure in which all configurations take their place. Such a structure would greatly simplify the selection of a configuration for a specific application. The reader will acquire a feeling for other omissions in literature by asking how familiar oscillators should be loaded, other than by an ideal buffer. Or perhaps by looking for equivalencies in and differences between two- and three-port LC-resonators. Another 'notorious' topic is the choice of the ground terminal in the oscillator configuration.

The systematic structure mentioned is here set up by classifying the possible negative-feedback oscillators. The basic properties of these oscillators are an appropriate termination of the resonator to enable oscillation and a unilateral transfer of the oscillation signal to the load in order to exclude it from the oscillation process. It will be shown that such properties imply that the oscillator consists of a resonator and an amplifier placed between this resonator and the load. Moreover it turns out, that, with one type of resonator, there are four basic oscillator configurations. As these configurations all have different kinds of output signals, each of them is suited for a specific group of applications.

After classifying the basic oscillator configurations this chapter will deal with their implementation. It will show that the implementation of negative-feedback oscillators much resembles that of negative-feedback amplifiers. There is, however, an obvious difference: from those amplifiers a linear transfer is desired whereas in these oscillators nonlinear behavior is required. Therefore a search will be made for suitable time-invariant nonlinear elements for application in a feedback network. Though time-invariant nonlinear elements are scarce, the few existing approximations have versatile potentials, as will be illustrated in various implementations.

3.2 The simple oscillator model

In this section we will investigate the two possible LC-resonators, the series and the parallel LC-tank circuits and the nonlinear terminations needed to form an oscillator with these resonators.

3.2.1 The resonator: series or parallel LC-circuit

From the first chapter we recall that in negative-feedback oscillators the oscillator frequency is determined by a passive, highly selective filter in the form of a resonator with a transfer H(f) following from:

$$H(f) = \frac{H_o}{1 + j\,Q\,\left(\frac{f}{f_o} - \frac{f_o}{f}\right)}$$

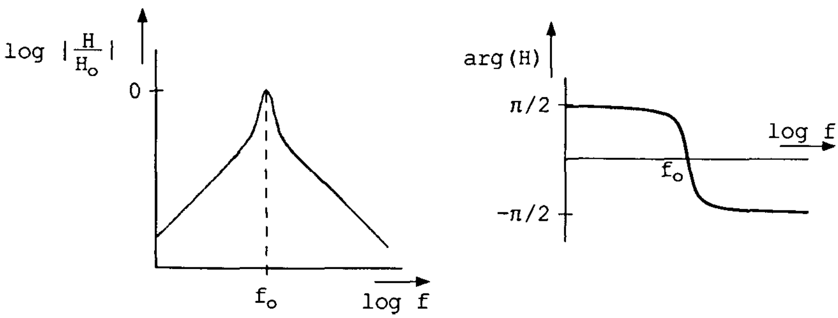

Fig. 3.1: Amplitude and phase transfer of a resonator.

Figure 3.1 depicts the well-known amplitude and phase transfer of such a resonator as a function of frequency. There are two types of resonators with practical capacitors and coils: the series and the

parallel LC-circuit. The equivalent electric diagram of these resonators, which is depicted in Figure 3.2, consists of an ideal capacitor, an ideal inductor and a resistor. This resistor represents the losses of the practical elements in a lumped form.
The resonators can be considered to be each other's dual. By a description of these resonators justifying this duality, as well as by a comparable description of their nonlinear terminations, only classification on the basis of one resonator would be sufficient. However to ease the introduction of the classification, both tank circuits are dealt with.

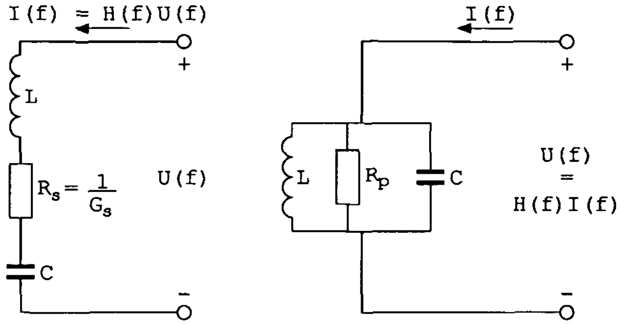

Fig. 3.2: Equivalent electric diagrams of series and parallel LC-circuits.

For the series resonator, that has a resonant voltage-to-current transfer, the relevant parameters are:

$$H_o = G_S = \frac{1}{R_S} \quad , \quad Q = \frac{2\pi f_o L}{R_S} \quad , \quad f_o^2 = \frac{1}{2\pi LC}$$

For the parallel resonator, that has a resonant current-to-voltage transfer, the relevant parameters are:

$$H_o = R_P \quad , \quad Q = \frac{R_P}{2\pi f_o L} \quad , \quad f_o^2 = \frac{1}{2\pi LC}$$

In many LC-circuits losses are concentrated in the coil and, in contrast with our assumption, are not lumped. In a first-order approximation the coil is represented by a lumped resistor in series with an ideal inductor. In the series resonator this corresponds to the proposed equivalent circuit. A parallel resonator with this representation of the losses has a transfer:

$$H(f) = \frac{Q^2 R_S}{1 + j Q \left(\frac{f}{f_o} - \frac{f_o}{f}\right)} \left(1 + \frac{f_o}{j f Q}\right) \qquad \text{with:} \quad Q = \frac{2\pi f_o L}{R_S}$$

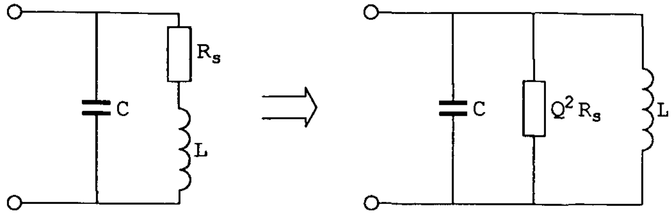

Fig. 3.3: Transformation of series losses to parallel losses.

Close to resonance and with relatively high Q-factors, this series circuit in a parallel resonator may well be substituted by a resistor-inductor parallel circuit. Figure 3.3 demonstrates this substitution. By this approximation both resonators also show their duality in the representation of their losses.

3.2.2 Resonator terminations

From our simple oscillator model we will now derive the nonlinear terminations that complement the resonators such that oscillation becomes possible.
The resonant voltage-to-current transfer of the series resonator implies that for oscillation the termination of this resonator must be a two-pole with a memoryless limiting, negative current-to-voltage transfer. Figure 3.4 shows the required i-u characteristic. These two-poles are known in literature as current controlled, or 'open stable', negative resistances [ref.1,2].

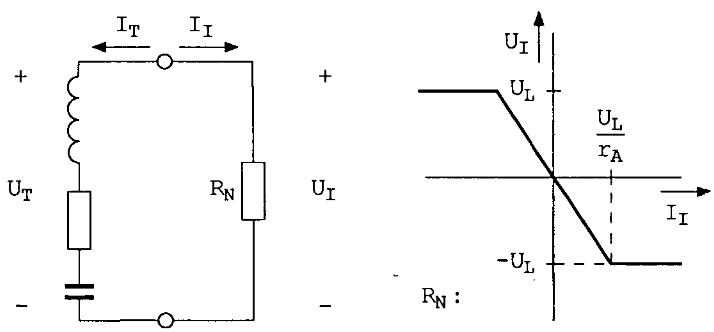

Fig. 3.4: The termination of a series resonator.

The termination has a maximum (output) voltage U_L and a small-signal resistance at zero input, or a true small-signal resistance, $-r_A$. Acceptable values for this resistance follow from the condition for oscillation:

$r_A \, G_S \quad > \quad 1$

The resonator terminated with this two-pole forms an oscillator. Figure 3.5 shows both the resonator current and the resonator voltage in this oscillator. With reasonable values for the quality factor Q the resonator (output) current is approximately sinusoidal, while the resonator (input) voltage is a clipped sine wave or a square wave.

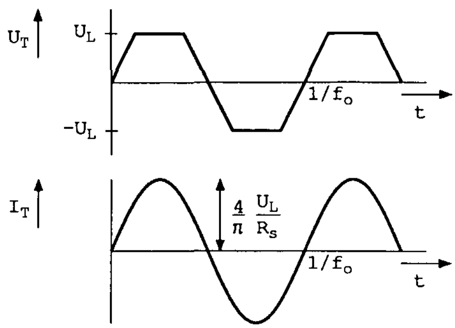

Fig. 3.5: Voltage and current in the negative resistance oscillator using a series LC-circuit.

It goes without saying that a dual oscillator structure, shown in Figure 3.6, is synthesized by the combination of a parallel resonator and a termination with a negative, time-invariant limiting voltage-to-current transfer. This two-pole is known as voltage controlled, or 'short-circuited stable' negative conductance.

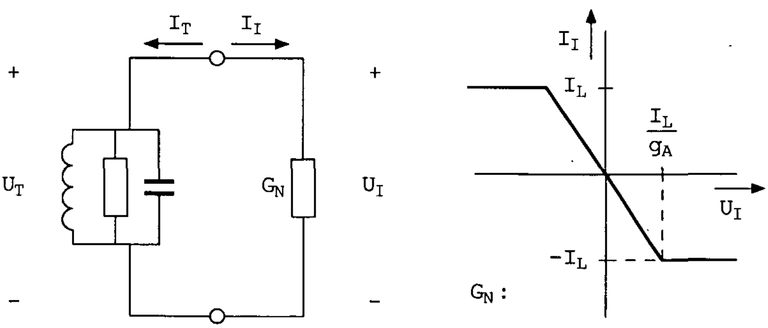

Fig. 3.6: The termination of a parallel resonator.

The (output) current is limited to I_L and the true small-signal conductance is defined as $-g_A$. For oscillation this last value follows from:

$$g_A \, R_P \quad > \quad 1$$

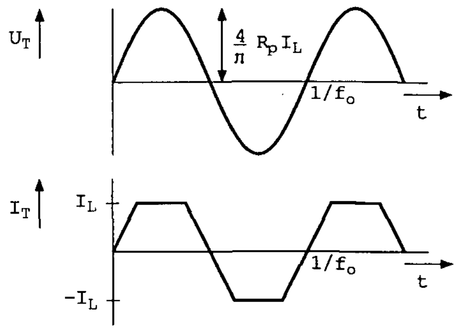

Fig. 3.7: Voltage and current in the negative resistance oscillator using a parallel LC-circuit.

Signals in an oscillator formed out of a parallel LC-resonator and this negative conductance are completely dual to the previous oscillator with the series oscillator. They are depicted for the sake of completeness in Figure in 3.7.
Up to now we have only looked for simple structures that will oscillate. Oscillation is important, but the possibility of loading the oscillator is at least equally important. The structures shown do not have special outputs. Therefore we will investigate more extensive amplifier structures in the next sections.

3.3 Oscillator loading

Designing a high-performance oscillator implies that the oscillator is optimized for its application. Special attention must therefore be paid to the oscillator load. Here we look at two topics concerning the load: the oscillator output quantity and the oscillator ground terminal.

3.3.1 Oscillator signal

Traditionally, only sinusoidal waveforms were expected from LC-oscillators and those preferably in form of voltages. The suspicion may arise that stepmotherly treatment of the oscillator load is the cause of this tradition, for nonlinear loads may behave well, driven by non-sinusoidal oscillator signals. Such nonlinear loads may consist of Schmitt-triggers, mixers, comparators etc.

Here we strive for maximum flexibility. The two waveforms that occur in the oscillator, the sinusoidal and the square waveform, may each be valuable for a certain application. Sinusoidal waveforms may be particularly suited for measurement applications or for transmitting purposes, while square waveforms are more suited for driving nonlinear loads, as has been mentioned. We want to be able to transfer both kind of waveforms to the load. Moreover we want to be able to drive the load with a current as well as with a voltage. Most loads are preferably voltage driven but it is not believed to be a universal quality of oscillator loads.

3.3.2 Oscillator grounding

As in any other electronic circuit, some terminal of an electronic oscillator must be connected to ground. In principle any terminal will do the job; the oscillator will remain oscillating. Only the influence of parasitics changes (sometimes quite dramatically though) and, in fact, the oscillator configuration is not changed by grounding a terminal. However, with another ground terminal in the oscillator, the oscillator circuit can be redrawn and through the requirement for a completely different DC-biasing circuit the oscillator circuit seems to stem from a completely different configuration. In literature this effect has been signaled [ref.3], but no clear conclusion has been drawn. Figure 3.8 shows the

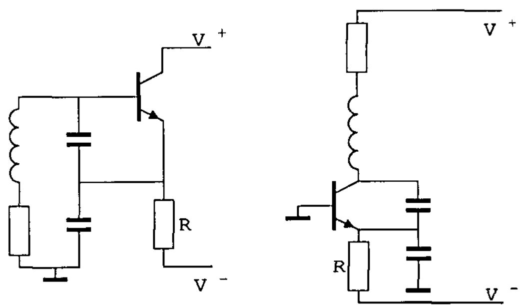

Fig. 3.8: Two different choices for a ground terminal in a one-transistor Colpitts oscillator.

problem: two variants of the well-known Colpitts oscillator. The only differences are the choice of the ground terminal and the influence of bias-circuitry (resistor R). Comparing and classifying oscillator circuits can therefore only be done with an unambiguous specification of the ground terminal. A better way of specifying the oscillator ground terminal is to start from the load. An oscillator is often a small electronic circuit in a large electronic system and to this system it should be adapted. This system practically dictates the relation between oscillator load

and ground terminal. In this work we will only consider loads that can be modeled as a one-port with one terminal grounded.
If possible, the resonator also is grounded. The resonator acts as a frequency reference and grounded this reference is minimally afflicted by parasitics. Otherwise, especially in the case of tunable resonators (mechanically by hand or electronically by varactors) these parasitics become severe.

3.4 Classification of LC-oscillators

As we learned from previous sections we can compensate for resonator losses by terminating the resonator with a suitable negative resistance. Furthermore we demand that the oscillator signal will be applied to a load in such a way that this load has a minimum influence on the oscillator signal. The situation is perhaps best demonstrated in Figure 3.9. The active structure

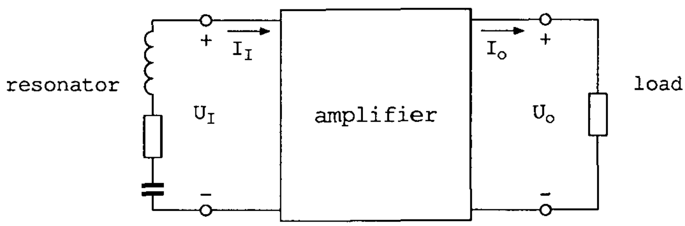

Fig. 3.9: The amplifier between resonator and load.

between resonator and load terminates the resonator with a negative resistance and transfers one of the resonator quantities, its current or its voltage, upon the load. In the optimal form these structures are amplifiers with an accurate input impedance and a current or a voltage output (having a infinite or zero output impedance respectively). As such they are known and have been classified in literature [ref.4,5]. In this classification the amplifiers have a linear transfer and a linear input impedance. Their properties are fruitfully described by 'transmission parameters'. With these parameters the input quantities of the amplifier are expressed in the frequency domain as a function of its output quantities:

$$U_i = A\,U_o + B\,I_o$$

$$I_i = C\,U_o + D\,I_o$$

In principle every transmission parameter can be stabilized by its own negative-feedback loop around an active part with large available power gain. Stabilizing two parameters, either A and C or B and D, by two separate negative-feedback loops around such an active part, gives the amplifier the properties mentioned.

Here comparable transmission parameters are defined for the transfer of the amplifier between the resonator and the load. The termination of the resonator requires, however, an accurate nonlinear, negative resistance or conductance, as shown in Figure 3.10. The required time-invariant nonlinear behavior limits the use of transmission parameters to the description of the small-signal

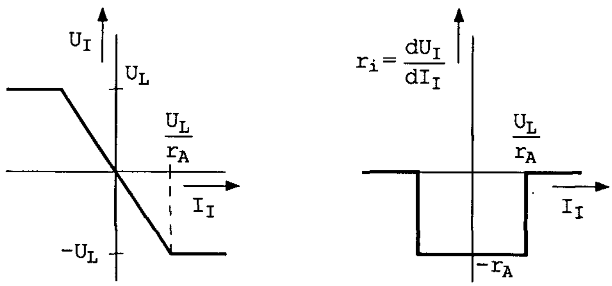

Fig. 3.10: Required large- and small-signal characteristics of amplifier input resistance.

transfer of the amplifier in the time domain. The transmission parameters are in our nonlinear amplifier no longer constants but depend on the corresponding, large-signal, output quantity:

$$u_i(t) = A(U_O)\, u_o(t) + B(I_O)\, i_o(t)$$

$$i_i(t) = C(U_O)\, u_o(t) + D(I_O)\, i_o(t)$$

The transmission parameters are defined in the usual way, by the reciprocals of the small-signal transfer parameters in the time domain:

$$\frac{1}{A(U_O)} = \left.\frac{dU_O}{dU_I}\right|_{I_O=0} \qquad \frac{1}{B(I_O)} = \left.\frac{dI_O}{dU_I}\right|_{U_O=0}$$

$$\frac{1}{C(U_O)} = \left.\frac{dU_O}{dI_I}\right|_{I_O=0} \qquad \frac{1}{D(I_O)} = \left.\frac{dI_O}{dI_I}\right|_{U_O=0}$$

From these small-signal transmission parameters a small-signal input resistance r_i or conductance g_i should result that meets the small-signal requirements of the resonator termination:

$$r_i = \frac{A(U_O)\, u_o + B(I_O)\, i_o}{C(U_O)\, u_o + D(I_O)\, i_o}$$

$$g_i = \frac{C(U_O)\, u_o + D(I_O)\, i_o}{A(U_O)\, u_o + B(I_O)\, i_o}$$

These small-signal requirements are also depicted in Figure 3.10. The selection of the proper small-signal transmission parameters of the amplifier is done on the following arguments:
First the small-signal input resistance or conductance must be accurate. Thus, either parameters A and C or parameters B and D are chosen zero in order to avoid the influence of the load on the oscillation process.
And secondly, of the two remaining parameters that are to be stabilized, one must be constant to ensure a linear transfer of a resonator quantity, its current or its voltage, to the load. The other parameter is a time-invariant function of the large-signal output of the amplifier in order to reduce the small-signal input resistance or conductance to zero. Depending on the parameter chosen its value approximates zero or infinity for large amplifier outputs. Requirements for oscillation do not dictate which parameter has to be constant or varying with the amplifier output signal: so also here there are two possibilities.

parameters A B C D	input resis.	feedback	quality		output
		configuration I			
A 0 C 0	$\frac{A}{C}$	A:positive C:negative	varying constant	$(A \rightarrow 0)$	sine voltage
		configuration II			
A 0 C 0	$\frac{A}{C}$	A:positive C:negative	constant varying	$(C \rightarrow \infty)$	square voltage
		configuration III			
0 B 0 D	$\frac{B}{D}$	B:positive D:negative	varying constant	$(B \rightarrow 0)$	sine current
		configuration IV			
0 B 0 D	$\frac{B}{D}$	B:positive D:negative	constant varying	$(D \rightarrow \infty)$	square current

Table I: Basic configurations for oscillators with a series resonator.

Furthermore the input resistance or conductance should be negative. Thus the two small-signal transmission parameters must have an opposite sign. Of the two feedback loops that stabilize these two transmission parameters, one loop should result in negative and the other in positive feedback. As the large input current should control the to-zero-falling small-signal input resistance or the large-signal input voltage the corresponding input conductance, the parameters describing the transfer of these controlling inputs to the output are to be stabilized by negative feedback.

These considerations lead to four basic configurations for the series LC-resonator and four dual basic configurations for the parallel LC-resonator. Table I summarizes the information on configurations for the series circuit and Table II does the same for the parallel circuit.

parameters A B C D	input condu.	feedback	quality		output
		configuration I			
A 0 C 0	$\frac{C}{A}$	C:positive A:negative	varying constant	(C→ 0)	sine voltage
		configuration II			
A 0 C 0	$\frac{C}{A}$	C:positive A:negative	constant varying	(A→ ∞)	square voltage
		configuration III			
0 B 0 D	$\frac{D}{B}$	D:positive B:negative	varying constant	(D→ 0)	sine current
		configuration IV			
0 B 0 D	$\frac{D}{B}$	D:positive B:negative	constant varying	(B→ ∞)	square current

Table II: Basic configurations for oscillators with a parallel resonator.

The classification given here is believed to be complete. As will be more clearly visible in the practical oscillator implementations it comprises not only negative-feedback versions of well-known LC-oscillators but also some interesting new types.

3.5 Implementations of negative-feedback LC-oscillators

Stabilizing the transmission parameters of the amplifier, placed between resonator and load, can be done in various ways. Here we apply negative feedback for this purpose. Then the amplifier consists of an active structure with a large available power gain with a suitable feedback network that stabilizes the transfer of the amplifier. As the construction of such an active part has been dealt with elsewhere [ref.5], we assume this active part is readily available. Moreover its power gain is assumed to be so large, that a nullor is an adequate abstraction of this active part, as far as it concerns this aspect. A following chapter will treat some of the errors made in this approximation. Of the two possible forms of this nullor, the three- and the four-terminal active part, the first is to be preferred, for it usually is less sensitive to parasitics. The symbols used for these active parts are both shown in Figure 3.11. Problems in implementation of negative-feedback oscillators are then restricted to finding suitable feedback networks. Primarily the feedback network must able to stabilize the

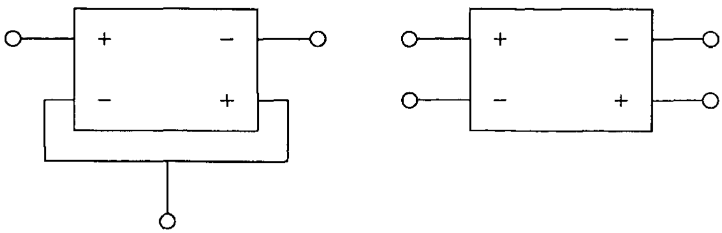

Fig. 3.11: The symbols for the active part.

amplifier in the time-invariant nonlinear way laid down in the previous section. However, the network must also contribute to the oscillator performance in that it does not produce relevant noise or enlarge the noise from the active part. Also it preferably allows that all power from the active part is available for the resonator or the load in order to make the oscillator power efficient. Starting from ideal hypothetical feedback networks we will arrive at more practical ones in the following sections.

3.5.1 Gyrators and transformer feedback

Literature has shown that neither loss in noise performance nor loss in power efficiency occurs in negative-feedback amplifiers, if non-energic feedback networks are used [ref.5]. As the independent noise sources in the oscillator and the power available for the carrier (cf. Sections 2.4 and 2.5) determine the short-term frequency stability, the use of these feedback networks is also recommended in oscillators. Non-energic feedback networks necessarily consist of ideal transformers and gyrators. The transformer enables one to convert a voltage into a voltage or current into a current, while with a gyrator a current can be converted into a voltage or a voltage into a current. In principle these feedback elements can stabilize all transmission parameters of an amplifier.
Thus with one ideal transformer, one ideal gyrator, a three-terminal active part and an LC-resonator, all configurations classified in Section 3.4 may be drawn up. As the transformer feedback must be of an opposite sign compared to the feedback from the gyrator, one of the two feedback elements has to invert in the feedback path. Moreover, in order to acquire the nonlinear behavior needed, the small-signal transfer of one of these feedback elements has to depend in a memoryless way on the momentaneous value of the sensed amplifier output. Of course it is purely hypothetical to suggest that the turns ration of a transformer or the gyration of a gyrator behaves in such a way, but it reveals the basic requirements for feedback networks in these oscillators.

An example of this feedback, applied in configuration II from Table I, is given in Figure 3.12. The feedback amplifier undamps a series LC-resonator and delivers a square voltage to the load of the oscillator. The transfer of the gyrator is here nonlinear. Its small-signal gyration g increases with increasing amplifier output voltages.

configuration II

A: constant, C: varying (→ ∞)

parameters:	input resistance:	output quantity:
$A = \frac{1}{n}$	$r_i = \frac{-1}{n\,g}$	$U_o = n\,U_T$
$C = -g$		(square)

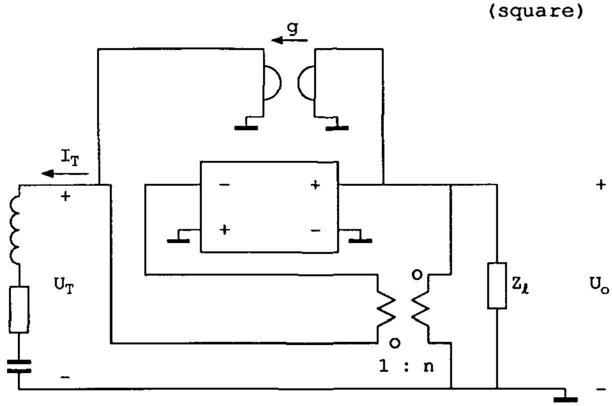

Fig. 3.12: Stabilizing the parameters A and C in an oscillator by means of a gyrator and a transformer.

In the same way one may derive the other three configurations or the four configurations with the parallel LC-resonator. They have been omitted here for brevity, the more their practical value is limited.

In the real world passive, accurate, nonlinear gyrators and transformers do not exist. Therefore we will look for alternatives for these elements. Of course with these alternatives the non-energicness of gyrators and transformers is lost and as a consequence the noise performance and the power efficiency of the oscillator will be deteriorated.

3.5.2 Transformer feedback

Of both non-energic feedback elements, the gyrator and the transformer, only for the transformer is there a good practical approximation. A first approximation of the gyrator is the passive

resistor. Because the practical transformer behaves almost linearly, these resistors have to be nonlinear. We have to distinguish two possible nonlinear resistances for application in the feedback path of an oscillator:
-a voltage-limiting device in the form of a current-controlled, nonlinear resistor R_V. For application in oscillators its small-signal resistance r_v falls to zero for large currents in this resistor.
-a current-limiting device in the form of a voltage-controlled, nonlinear conductor G_c. Its small-signal conductance g_c falls to zero for large voltages across this conductance.

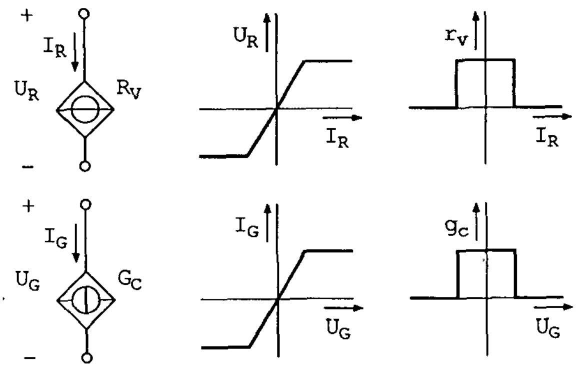

Fig. 3.13: Large- and small-signal characteristics of the nonlinear resistor R_V and conductor G_C.

Figure 3.13 shows the characteristics of these elements. Their frequent appearance in this text justifies the introduction of dedicated symbols for these resistances. The symbols used, also depicted in Figure 3.13, are abstracted forms of some practical implementations that will be discussed in a following section. With nonlinear resistors and a (linear) transformer only feedback networks are found for configurations with varying parameters B and C. Figure 3.14 and 3.15 show these configurations using a series resonator. The dual procedure yields configurations using the parallel resonator. They are depicted in Figure 3.16 and 3.17.
Because the feedback loops must have opposite signs and because passive resistances do not invert, the transformer is the inverting element in the feedback path. In many cases the transformer is a less desirable component in an oscillator. As tapped resonators show transformer-like properties, the here-discussed configurations can be built by tapping the resonator and omitting the transformer. A future section will deal with these variants. The next section will show yet another way to implement this inversion.

configuration II

A: constant, C: varying (→ ∞)

parameters:	input resistance:	output quantity:
$A = \frac{1}{n}$	$r_i = -\frac{r_v}{n}$	$U_0 = n\ U_T$
$C = -\frac{1}{r_v}$		(square)

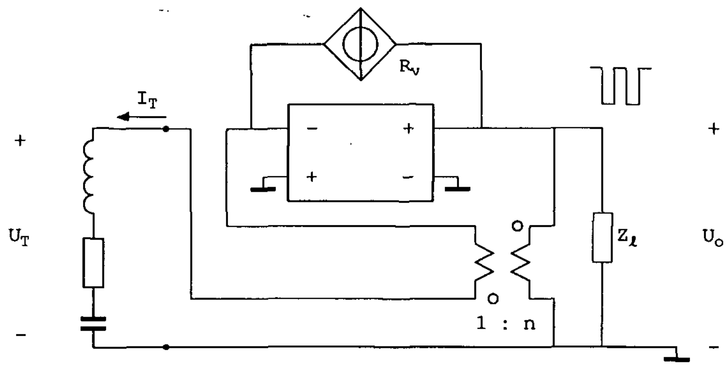

Fig. 3.14: Stabilizing the parameters A and C in an oscillator with a series resonator.

configuration III

D: constant, B: varying (→ 0)

parameters:	input resistance:	output quantity:
$B = -r_v$	$r_i = -n\ r_v$	$I_0 = -n\ I_T$
$D = \frac{1}{n}$		(sine)

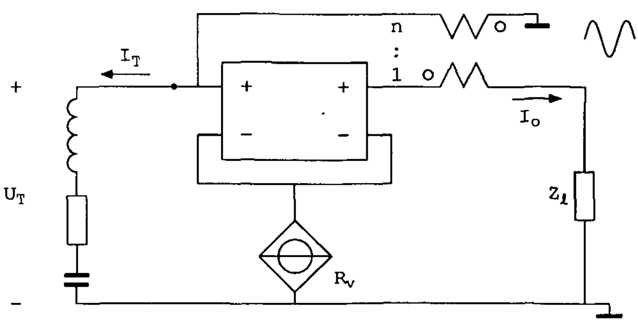

Fig. 3.15: Stabilizing the parameters B and D in an oscillator with a series resonator.

configuration I

A: constant, C: varying (→ 0)

parameters:	input conductance:	output quantity:
$A = \frac{1}{n}$	$g_i = -n\, g_c$	$U_O = n\, U_T$
$C = -g_c$		(sine)

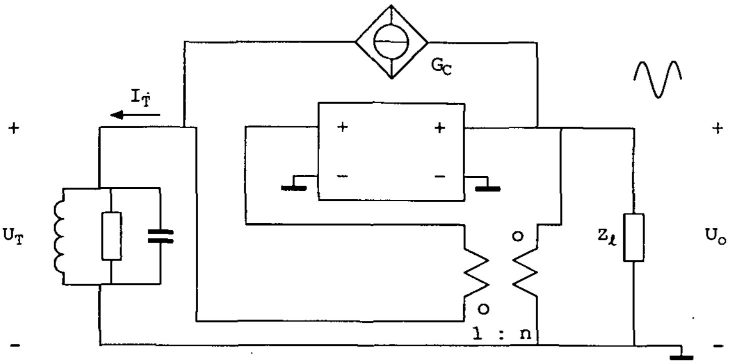

Fig. 3.16: Stabilizing the parameters A and C in an oscillator with a parallel resonator.

configuration IV

D: constant, B: varying (→ ∞)

parameters:	input conductance:	output quantity:
$B = -\frac{1}{g_c}$	$g_i = -\frac{g_c}{n}$	$I_O = -n\, I_T$
$D = \frac{1}{n}$		(square)

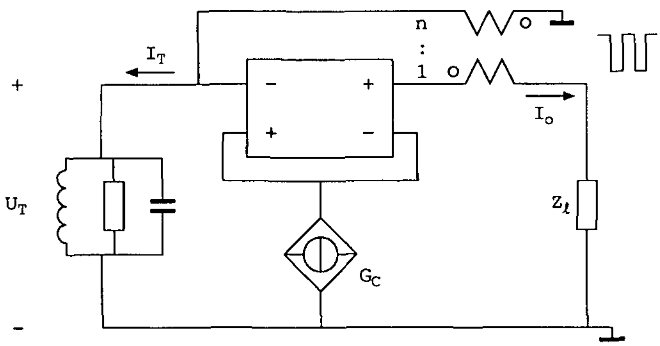

Fig. 3.17: Stabilizing the parameters B and D in an oscillator with a parallel resonator.

3.5.3 Resistive feedback

The function the transformer in the feedback path, viz. the scaling of voltages or currents, can also be performed by two impedances with a constant, frequency-independent ratio. However the transformer also provides the necessary inversion in the feedback network. Without the transformer, passive feedback elements around a three-terminal nullor cannot create two feedback loops with opposite sign. With these feedback elements a four-terminal nullor, with fully floating input and output terminals, is required. Around such an active part oscillator configurations can be synthesized in which transmission parameters B and C vary with time. As such they are directly comparable to the configurations discussed in the previous section. They all are variants of the well-known Meacham oscillator [ref.1,6]. Only the nonlinear dynamic

configuration II

A: constant, C: varying $(\rightarrow \infty)$

parameters:

$$A = \frac{Z_2}{Z_1 + Z_2}$$

$$C = -\frac{Z_1}{r_v(Z_1+Z_2)}$$

input resistance:

$$r_i = -\frac{Z_2}{Z_1} r_v$$

output quantity:

$$U_o = \frac{Z_1+Z_2}{Z_2} U_T$$

(square)

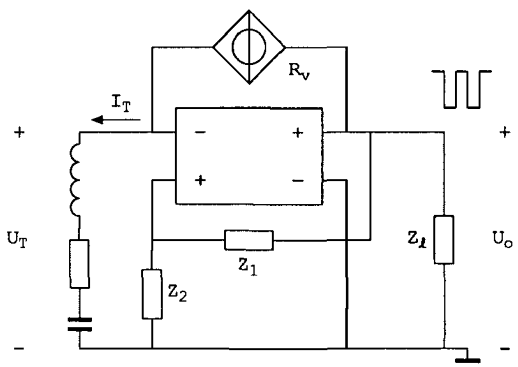

Fig. 3.18: Stabilizing the parameters A and C in an oscillator with a series resonator by using passive resistors.

element has been replaced by a time-invariant nonlinear one. Figures 3.18 up to 3.21 show the various possibilities.

So far only parameters B and C have been chosen time variant. Here, however, the use of nonlinear resistors seems also to enable the synthesis of configurations in which these parameters are constant and parameters A and D vary with time. Then feedback networks would exist for the other configurations from Table I and Table II (cf. Section 3.4). As a ratio of two resistors can stabilize the parameters A and D, two solutions exist: the voltage or the current limiter can be used. In order to minimize the number of accurate configuration III

D: constant, B: varying (→ 0)

parameters:

$$B = - r_v \frac{Z_1}{Z_1+Z_2}$$

$$D = \frac{Z_2}{Z_1 + Z_2}$$

input resistance:

$$r_i = - \frac{Z_1}{Z_2} r_v$$

output quantity:

$$I_o = - \frac{Z_1+Z_2}{Z_2} I_T$$

(sine)

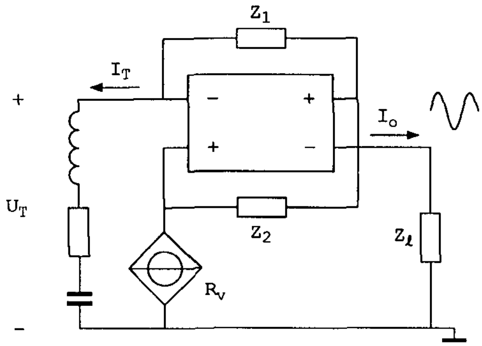

Fig. 3.19: Stabilizing the parameters B and D in an oscillator with a series resonator by means of passive resistors.

configuration I

A: constant, C: varying (→ 0)

parameters:

$$A = \frac{Z_2}{Z_1 + Z_2}$$

$$C = - g_c \frac{Z_1}{Z_1+Z_2}$$

input conductance:

$$g_i = - \frac{Z_1}{Z_2} g_c$$

output quantity:

$$U_o = \frac{Z_1+Z_2}{Z_2} U_T$$

(sine)

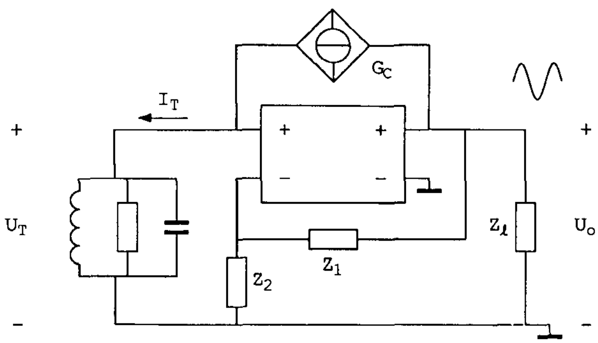

Fig. 3.20: Stabilizing the parameters A and C in an oscillator with a parallel resonator by means of passive resistors.

configuration IV

D: constant, B: varying ($\to \infty$)

parameters:

$$D = \frac{Z_2}{Z_1 + Z_2}$$

$$B = -\frac{1}{g_c}\frac{Z_1}{Z_1+Z_2}$$

input conductance:

$$g_i = -\frac{Z_2}{Z_1} g_c$$

output quantity:

$$I_o = -\frac{Z_1+Z_2}{Z_2} I_T$$

(square)

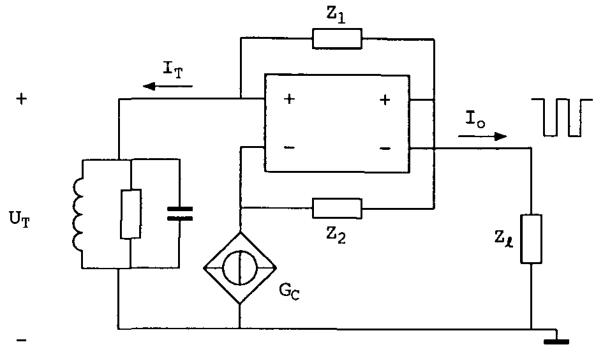

Fig. 3.21: Stabilizing the parameters B and D in an oscillator with a parallel resonator by means of passive resistors.

configuration I

C: constant, A: varying ($\to 0$)

parameters:

$$A = \frac{r_v}{r_v + Z_1}$$

$$C = -\frac{1}{Z_2}\frac{Z_1}{r_v+Z_1}$$

input resistance:

$$r_i = -\frac{Z_2}{Z_1} r_v$$

output quantity:

$$U_o = U_T + Z_2 I_T$$

(square + sine)

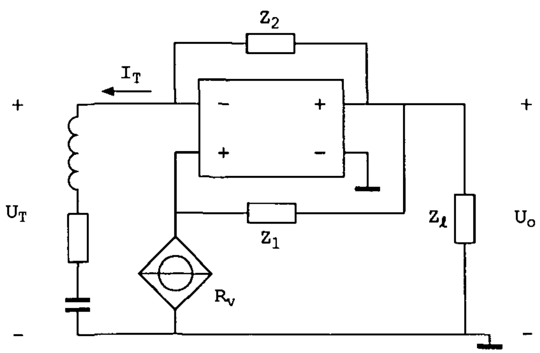

Fig. 3.22: Stabilizing the parameters A and C in an oscillator with a series resonator by means of passive resistors.

resistors needed, preference goes to types with the same kind of nonlinearity as is required in the resonator termination. We choose voltage limiters for configurations with the series resonator and current limiters for configurations with the parallel resonator. The feedback network with varying parameters A or D then consists of a nonlinear resistor or conductor and two impedances with a linear, frequency-independent ratio.
Investigation shows, however, that it is no longer possible to stabilize one parameter to a constant and another to a time-varying value. The result is that the required accurate, negative resistance can be synthesized to undamp the resonator, but that the oscillator output signal is a mix of the sine and the square signal at the resonator. Thus only approximating implementations exist for these configurations. Figure 3.22 shows an example of such an approximation in a configuration with a series resonator. The synthesis of other comparable configurations have been omitted here for the sake of brevity. The oscillator load determines whether these approximations are useful or not. If it handles the oscillator signal in a nonlinear way, the mix of sine and square signals in the output will often be acceptable.

3.5.4 Practical nonlinear resistors

From the abundance of nonlinear elements that modern electronics offers, we will look in this section for practical implementations of the nonlinear resistors defined in the previous section. Selection criteria for these one-port elements are summarized as follows:

-They show a time-invariant nonlinear behavior.
-They have a limiting characteristic that is either current or voltage controlled.
-The nonlinear behavior is accessible for design. Preferably both the true small-signal behavior and the limit level can be adapted.
-The i-u or u-i characteristic is odd symmetrical. Or more precisely, it has an even first derivative.

Most nonlinear elements do not satisfy these requirements: only a very few show usable features. Many elements have a transfer that has in some way a limiting character. Suitable elements will only have memory-effects within a small part of the oscillation period. The nonlinear behavior will practically always be harder than is required in the oscillator. In that case resistors, with current limiters in series or with voltage limiters in parallel, may change the true small-signal resistance or conductance up to the necessary value. Limit levels are not so easily modified. These levels may prevent the application of the nonlinear element, for they lead to impracticably large or small oscillator signals. Such elements can only be applied in oscillator configurations in which the limit level can effectively be scaled down by resonator tapping (cf. Section 3.7), by a transformer or by impedances in the feedback

network. The last requirement, the odd symmetrical character usually gives no problems. Two suitable elements in a symmetrical structure, in series or in parallel, produce the desired characteristic.
We found that diodes are fast, versatile, nonlinear components. Several interesting nonlinear resistances can be synthesized with these elements [ref.7,8]. If the diode could behave as a ideal switch the necessary nonlinear resistors could be simply synthesized by taking a DC voltage or DC current source into a Graetz-bridge of such diodes. Figure 3.23 shows the intended structures. The value of the DC source determines the limit level,

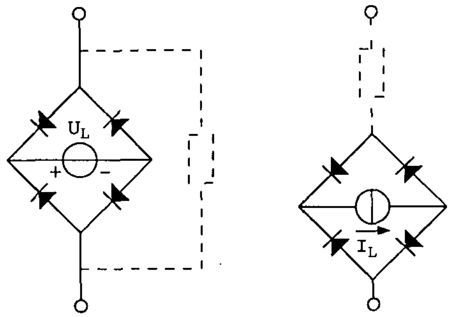

Fig. 3.23: Nonlinear resistors made up from DC-source and diode bridge.

while the small-signal resistance or conductance can be set to the required value by linear resistors in series or in parallel. The need for floating DC sources may be overcome by single-ended versions like the ones depicted in Figure 3.24. Of these latter versions the i-u or u-i characteristic is not completely odd, but the important first derivative is even. The single-ended current

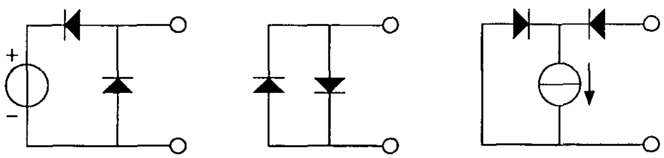

Fig. 3.24: Single-ended nonlinear resistors.

limiter is the most versatile: its true small-signal conductance is sufficiently large for many applications and its limit level can be adapted over a large range of currents. Appendix V shows the versatility of this voltage-controlled resistor more clearly. There

a special implementation is given and its application in various oscillator configurations with resistive feedback is discussed.
The implementation of the voltage limiter has some drawbacks. The finite threshold voltage of the diode switch sets bounds to the minimum limit level of the voltage-limiting resistor. The simple voltage limiter consisting of two anti-paralleled diodes has the minimum limit level. Even this limit level is too large for a large number of applications.
Important benefits of these diode structures are the almost memoryless behavior, especially in the case of Schottky-diodes, and the rather good compatibility with other oscillator elements such as transistors, resistors etc.

3.5.5 Feedback by nonlinear transactances

Apart from passive elements we can also think of active elements as feedback elements. The nonlinear character of and the possible signal inversion by these elements can be useful properties in the feedback network of an oscillator. Unfortunately there are also disadvantages in the use of these elements. Normally a feedback network with active elements produces more noise and consumes more power than a passive network. Consequently some loss in performance must be accepted.
First we will assume that all necessary, active transfers are available from ideal, fully-floating, accurate, time-invariant nonlinear transactances. This is of course in strong contradiction with common practice, but the assumption is made here in order to be able to draw up the complete list of oscillator configurations, independent of the technology used for realization.
As two different kinds of varying transmission parameters are required, for a parameter must approximate zero or infinity at large amplifier outputs, also two kinds of nonlinear transfers must be available in the ideal transactances. A limiting transfer, having a decreasing small-signal transfer, and a widening transfer, having a increasing small-signal transfer both at increasing (positive or negative) input signals. Feedback by a limiting transactance may reduce a (small-signal) transmission parameter to zero, while feedback by a widening transactance increases a transmission parameter to values that approximate infinity in the oscillator. As can be concluded from Table I, the feedback network for each of the four basic oscillators with a series resonator requires a different nonlinear transactance. We require a limiting voltage-to-voltage and limiting current-to-voltage transfer with small-signal transfers μ and ζ respectively. And further two transactances with a widening transfer: a voltage-to-current and a current-to-current transactance, with small-signal transfers γ and α respectively. The four oscillator configurations with active feedback are summarized in Table III. Figure 3.25 shows the corresponding configurations. With the examples of active feedback in oscillator configurations using a series resonator it is not

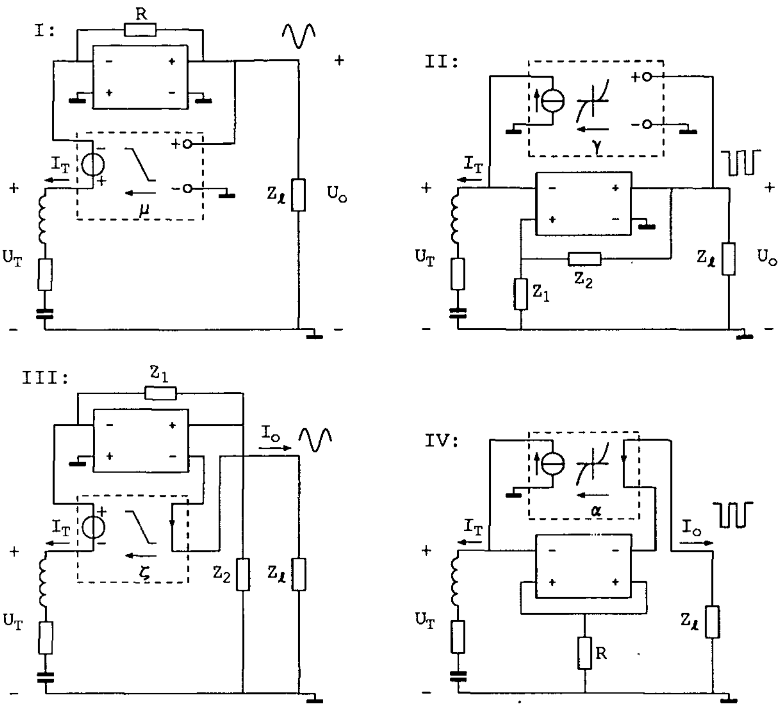

Fig. 3.25: Oscillator configurations with a series resonator using feedback by ideal, nonlinear transactances.

parameters: A	B	C	D	input resistance:	output signal:
				configuration I	
μ	0	$-\frac{1}{R}$	0	$-\mu R$	$U_O = R\, I_T$ (sine)
				configuration II	
$\frac{Z_1}{Z_1+Z_2}$	0	$-\gamma$	0	$-\frac{1}{\gamma}\frac{Z_1}{Z_1+Z_2}$	$U_O = \frac{Z_1+Z_2}{Z_1} U_T$ (square)
				configuration III	
0	$-\zeta$	0	$\frac{Z_2}{Z_1+Z_2}$	$-\zeta\frac{Z_1+Z_2}{Z_2}$	$I_O = -\frac{Z_1+Z_2}{Z_2} I_T$ (sine)
				configuration IV	
0	$-R$	0	α	$-\frac{R}{\alpha}$	$I_O = -\frac{1}{R} U_T$ (square)

Table III: Properties of oscillator configurations with active feedback (series resonator).

difficult to derive the four dual configurations around a parallel resonator. They are omitted here for reasons of brevity. In the next section we will look for the interesting practical implementations of the here-assumed transactances.

3.5.6 Feedback by practical transactances

For the implementation of the transactances of the preceding section we restrict ourselves to bipolar transistors. The only accurate, nonlinear transfer of the transistor, which also approximates the required memorylessness, is its voltage-to-current transfer. Non-idealities, such as a non-zero base-resistance (r_b) and a non-zero feedback capacitance (C_{bc}) disturb the memorylessness, but for a large range of frequencies these effects can be left out of consideration.

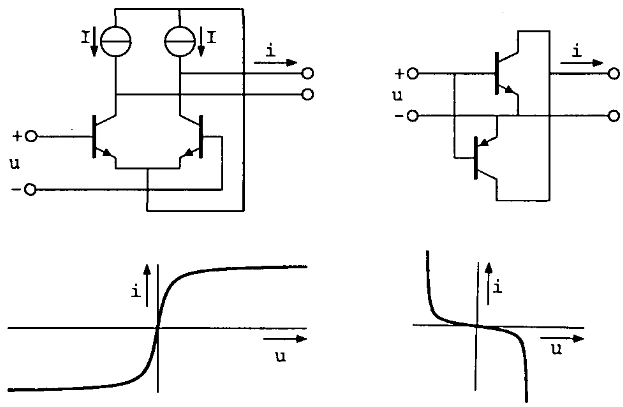

Fig. 3.26: Symmetrical bipolar voltage-to-current converters with limiting and widening transfers.

Connecting transistors in series or in parallel yields the necessary symmetry in the characteristic. The series connection forms the well-known differential pair with an accurate, in good approximation memoryless, limiting, voltage-to-current transfer. Other limiting transfers can be implemented by a differential pair and additional resistors.
Two complementary bipolar transistors in parallel have an accurate, memoryless, widening, voltage-to-current transfer. Other widening transfers required in the oscillator, can again be acquired in combination with additional resistors. The so-formed converter is not fully floating, for input and output have one common terminal. This causes oscillator configurations using this converter to differ somewhat from the ones discussed in the previous section. Figure 3.26 depicts the two relevant transistors structures augmented with their u-i characteristic. Negative feedback, applied to the differential pair itself, may reduce the transconductance.

This is important for the design of the excess loop gain: for optimal stability this loop gain is not larger than is strictly necessary (cf. Section 2.8.2). Unfortunately the transconductance of the two complementary transistors in parallel cannot be modified so simply. Preferably the circuit incorporating these transistors is such that the transistor's transconductance determines the signal transfer. This implies that the rest of the oscillator circuit must drive and load the transistors at a relatively low

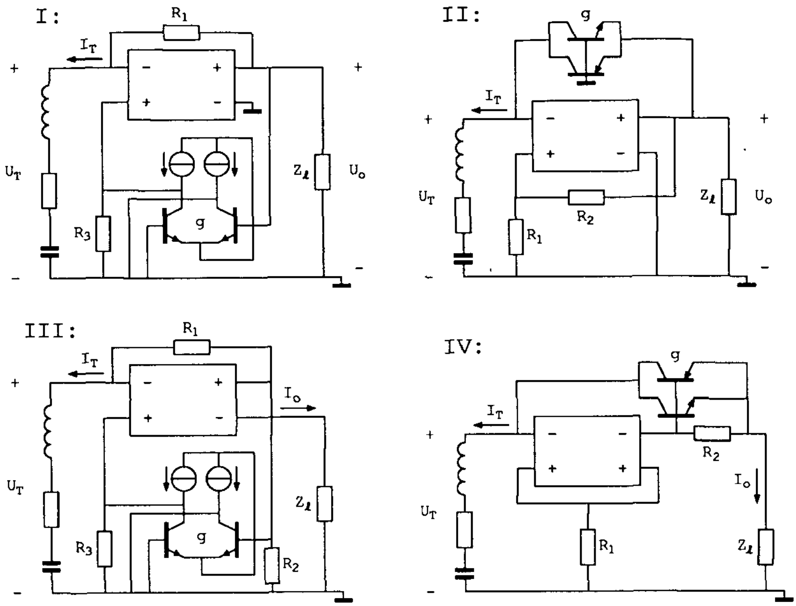

Fig. 3.27: Oscillator configurations with active feedback using a series resonator.

configuration:	input resistance:	output signal:
I	$-\dfrac{g\,R_1R_3}{1-g\,R_3}$	$U_O = R_1I_T + U_T$
II	$-\dfrac{1}{g}\;\dfrac{R_1}{R_1+R_2}$	$U_O = \dfrac{R_1+R_2}{R_1}\,U_T$
III	$-\dfrac{g\,R_1R_3}{1-g\,R_3}$	$I_O = -\dfrac{R_1+R_2}{R_2}\,I_T - \dfrac{1}{R_2}\,U_T$
IV	$-\dfrac{1}{g}\;\dfrac{R_1}{R_2}$	$I_O = -\dfrac{1}{R_1}\,U_T - I_T$

Table IV: Properties of oscillator configurations with active feedback (series resonator).

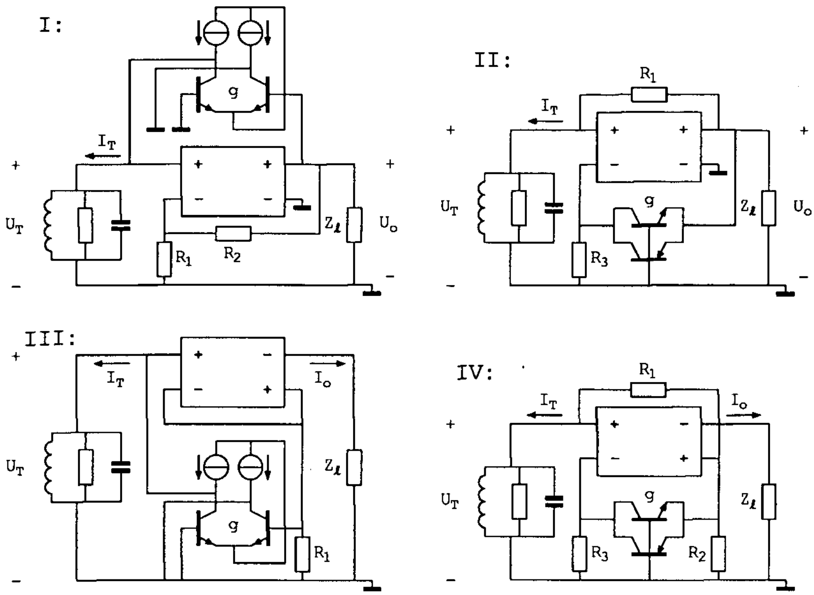

Fig. 3.28: Oscillator configurations with active feedback using a parallel resonator.

configuration:	input conductance:	output signal:
I	$-\ g\ \dfrac{R_1 + R_2}{R_1}$	$U_O = \dfrac{R_1 + R_2}{R_1}\ U_T$
II	$-\dfrac{1}{g\ R_1 R_3}\ (1-gR_3)$	$U_O = R_1 I_T + U_T$
III	$-\ g$	$I_O = -\dfrac{1}{R_1}\ U_T$
IV	$-\dfrac{1}{g\ R_1 R_3}\ (1-gR_3)$	$I_O = -\dfrac{R_1+R_2}{R_2}\ I_T - \dfrac{R_3+R_2}{R_3 R_2}\ U_T$

Table V: Properties of oscillator configurations with active feedback (parallel resonator).

impedance level. For proper behavior the transistor must be able to work in the active mode when the transistor input voltage has the required value. An extra bias circuit should therefore guarantee the collector-base junction is blocked during operation. To avoid concealing the essential oscillator configurations too much, these DC-bias arrangements are omitted, although they are indispensable in practical circuits.

The use of these u-i converters in the feedback path yields implementations of the four basic configurations. Figure 3.27 shows them with a series resonator. The small-signal resonator terminations and the output signals of these oscillators are summed up in Table IV. The variable g indicates here the small-signal transconductance, varying to very small or to very large values according to the nonlinear transfer applied. We have slightly changed the configurations in comparison with those from the previous section. In this way full use has been made of the buffering capabilities of the active part and also transactances which were not fully floating could be placed in the feedback path. As a consequence, however, it is not always the case that one of the two stabilized parameters is constant and the other time varying. Therefore most configurations show outputs consisting of both square and sinusoidal signals. The same procedure yields configurations using the parallel resonator. Figure 3.28 depicts the four configurations and their properties are collected in Table V. Again the configurations have been slightly changed. The comment just made on configurations with the series resonator applies also here.
The differential pair as a feedback element enables one to stabilize the magnitude of the oscillator signal to values highly irrespective of the resonator impedance. Appendix V shows an elaboration of this property in oscillator configuration I with a parallel resonator.

3.6 Forward stabilization of the nonlinear transfer

In previous sections we dealt with how the feedback network stabilized two small-signal transmission parameters of the amplifier between resonator and load. This arrangement aims at ultimate performance and must be preferred whenever possible. However, in exchange for a slight degradation in performance, there are also attractive oscillator configurations in which the feedback network stabilizes only one parameter of the amplifier. The other parameter of the amplifier should then be intrinsically accurate. This other parameter is usually not accurate in the sense that it is constant. A constant small-signal transfer is not intrinsic, but indicates that negative feedback is present. On the other hand accurate varying transmission parameters can occur with accurate nonlinear elements in the forward path of the amplifier.
The name 'negative-feedback oscillators' does not seem particularly suited for these oscillators. However, as we will see in the next section, such accurate nonlinear transfers are only acquired by the presence of buffers, which are stabilized by negative feedback. Negative feedback is 'local' in these oscillators instead of 'overall' as in previously discussed types.
Investigation shows that only transactances with an accurate limiting characteristic and consequently a falling ($\rightarrow 0$) small-signal transfer are useful in such configurations. From tables I

and II (cf. Section 3.4) it can be concluded that in oscillators with a series resonator a limiting current-to-voltage or a limiting current-to-current transactance is needed. In configurations with a parallel resonator we need a dual limiting voltage-to-voltage or limiting voltage-to-current transactance. Figure 3.29 depicts an example with a parallel resonator. The amplifier has an intrinsic, limiting voltage-to-voltage transfer, with a small-signal transfer μ. The other parameter has been stabilized by feedback.

configuration II

parameters:	input conductance:	output quantity:
$A = \frac{1}{\mu}$	$g_i = -\frac{1}{R}(\mu - 1)$	$U_O = U_T + R\, I_T$
$C = -\frac{1}{R}\frac{\mu - 1}{\mu}$		(square + sine)

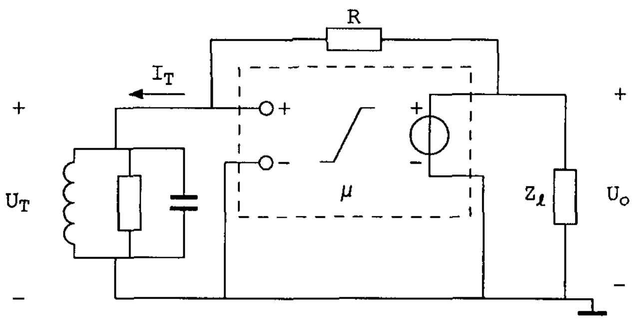

Fig. 3.29: Stabilization by an accurate forward limiter.

In some configurations, such as in the example, the feedback network needs an additional (linear) resistor. Without extra buffering this resistor prevents a zero small-signal input resistance or conductance of the amplifier and a less optimal termination of the resonator results.
In the next section we will look for implementations of these accurate forward limiters and the corresponding oscillator configurations.

3.6.1 Forward stabilization by practical transactances

The only practical limiting transactance we have at our disposal in bipolar technology is the well-known differential pair. This pair may be considered as a good approximation of the limiting u-i transactance, as was already discussed in Section 3.5.6. Other transactances are formed from this pair and additional resistors. The differential pair with these resistors is a poor approximation

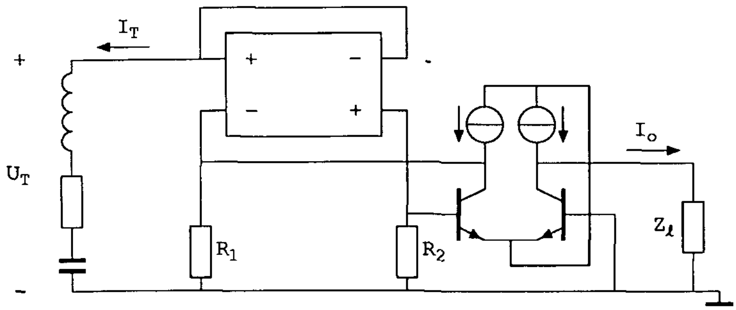

Fig. 3.30: Implementation of accurate forward limiter in an oscillator with a series resonator.

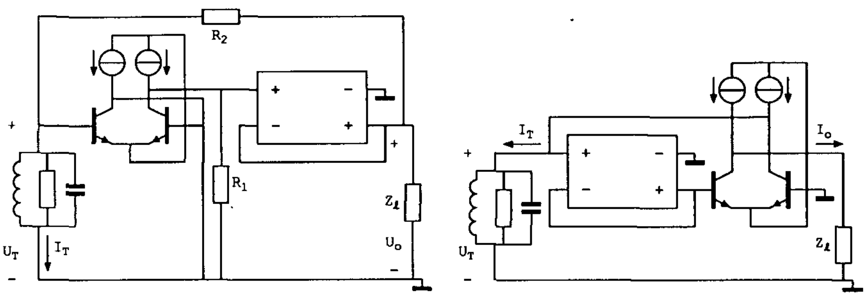

Fig. 3.31: Implementation of accurate forward limiting in an oscillator with a parallel resonator.

of the transactance required in the oscillator. To realize the proper impedance levels, additional buffering is required. As these buffers must be accurate too, (local) negative feedback must be applied to them. Moreover the use of these buffers is restricted by the high-frequency stability (cf. Section 4.5.4) of the oscillator. Therefore only one buffer will be allowed. A current-to-voltage limiter built around a differential pair is for these reasons less attractive. A current-to-current limiter requires only one buffer and one resistor, and is more versatile. Figure 3.30 shows it in a configuration with a series resonator. Here the transmission parameters B and D of the amplifier have been stabilized. This configuration has proven to be particularly useful in a crystal oscillator. Appendix V shows the oscillator circuit for this application and gives its (simulated) output signals. For configurations with a parallel resonator, a voltage-to-current limiter and a voltage-to-voltage limiter are necessary. They are formed from a differential pair and a buffer, and a differential pair, a buffer and a resistor, respectively. Figure 3.31 depicts them in the corresponding oscillator configurations.

3.7 Variants with tapped resonators

It is a well-known fact that tapped resonators may show transformer-like properties in the frequency region near resonance. These properties are often useful in the sense that scaling or even inversion in oscillators is possible without the actual transformer or a four-terminal nullor. Both series and parallel resonators can be tapped. Moreover there are two ways of tapping: capacitively or inductively. In the latter case the two coils at the tap may even be inductively coupled; in fact such coupling ensures the transformer-like properties.

resonator-transformer	tapped resonator
$A = -\frac{1}{n}$	$A = -\frac{1}{n}\left(\frac{Q - jn}{Q + j}\right)$
$B = 0$	$B = \frac{j R_P}{Q + j}$
$C = -\frac{1}{R_P}\frac{(n+1)^2}{n}$	$C = -\frac{1}{R_P}\frac{(n+1)^2}{n}\frac{Q}{Q + j}$
$D = -n$	$D = -n\left(\frac{Q - j/n}{Q + j}\right)$

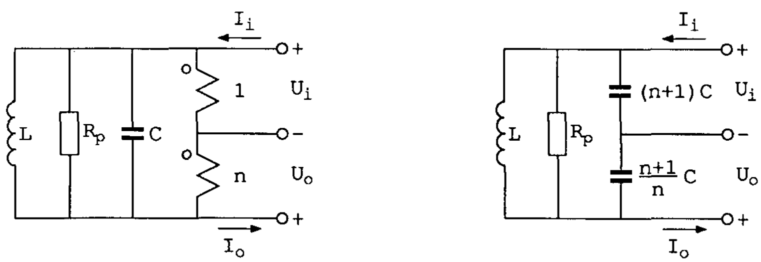

Fig. 3.32: Equivalence between transformer and capacitively tapped parallel resonator.

As an example we look at the capacitive tapping of a parallel resonator. The interest is in the degree to which such tapping yields properties equal to the ideal transformer tapping. Here we compare the two circuits from Figure 3.32 with respect to their transmission parameters at resonance. The parameters show that as long as the quality factor of the resonator is much larger than the tapping ratio, the tapped resonator has transformer-like

Fig. 3.33: Equivalence between transformer and inductively tapped series resonator.

properties. Some differences exist for parameter B, but this parameter is irrelevant for the tapped parallel resonator: with low impedances at both ports the selectivity of this resonator disappears. Of course tapping is also possible with the series resonator. The transmission parameters of the circuits, depicted in Figure 3.33, approximate one another under the conditions just described.

3.7.1 Scaling the resonator impedance

One way of using the transformer-like properties of a tapped resonator is an impedance transformation of the resonator. Reasons for this impedance transformation can be:

-A better noise match between the resonator and its active termination (cf. Chapter IV). Very high or very low impedances are difficult to match.

-A better power match between resonator and its termination, as the limit level in the resonator termination and the resonator impedance determine the resonator dissipation.

In tapped resonators, however, there is more than one resonant mode and therefore attention must be paid to the negative resistance or conductance terminating the resonator, in order to select the proper one for oscillation. This causes no serious difficulties because the series and the parallel resonant mode require widely different terminations for oscillation: differences in the small-signal resistance or conductance and in the nonlinear character. Figure 3.34 shows the case of capacitive tapping for both the parallel and the series resonator. The resultant resonator impedance Z_{res} or admittance Y_{res} is given by:

$$Z_{res}(f) \cong \left(\frac{C_2}{C_1 + C_2}\right)^2 \frac{R_P}{1 + jQ\left(\frac{f}{f_o} - \frac{f_o}{f}\right)} \qquad \text{with} \quad f_o^{\,2} = \frac{C_1 + C_2}{4\pi^2 L\, C_1 C_2}$$

$$Y_{res}(f) \cong \left(\frac{C_1}{C_1 + C_2}\right)^2 \frac{G_S}{1 + jQ\left(\frac{f}{f_o} - \frac{f_o}{f}\right)} \qquad \text{with} \quad f_o^{\,2} = \frac{1}{4\pi^2 L (C_1 + C_2)}$$

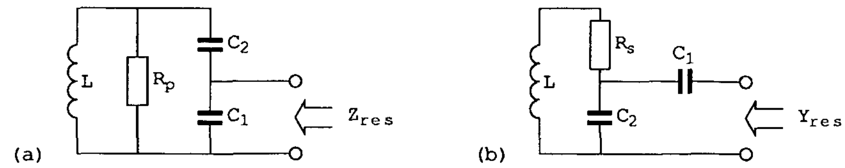

Fig. 3.34: Scaling of impedance with parallel (a) and series (b) resonator.

The tapping variants are especially valuable in cases of high quality factors, for then large tapping ratios can be realized. In ordinary LC resonators quality factors are moderate and in the range of 50 - 200. Tapping ratios are likewise in the range of, let us say, 5-20. Much larger tapping ratios are possible in oscillators with a piezoelectric crystal as a resonator. Close to resonance this crystal can be modeled as an LC series resonator with an extremely high quality factor (10^4-10^5), and across the terminals of this LC series resonator a capacitor that represents first-order parasitics. From Figure 3.34 we conclude the only possible tapping with the crystal resonator is parallel. Oscillation will occur at a frequency slightly higher than the (series) resonance frequency of the crystal.

3.7.2 Scaling the parallel or series feedback

The scaling of resonator impedance by tapping as discussed in the previous section can be considered as tapping both series and parallel feedback at the resonator in the same way. From this point of view it is also possible to tap either the series or the

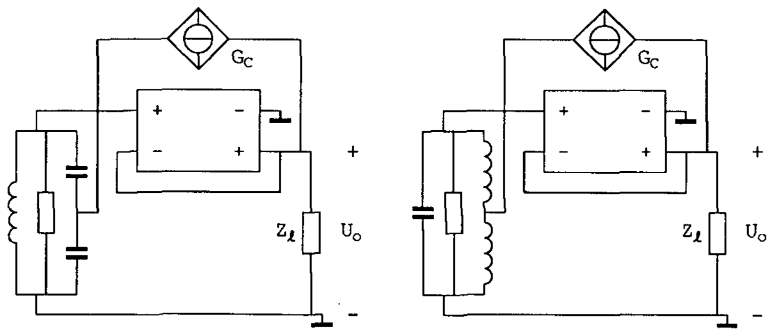

Fig. 3.35: Negative-feedback versions of Colpitts and Hartley oscillators with sinusoidal output voltage.

parallel feedback at the resonator. This possibility exists for all oscillator configurations dealt with. In some cases, however, interesting simplifications in the feedback network become feasible with this tapping. The transformer in this network or the fully floating, four-terminal active part are then no longer necessary elements in the configuration.
Tapping the parallel resonator in this way yields the possibility of stabilizing the transmission parameters A and C to the required values with only a three-terminal active part and a passive, voltage-controlled resistor (configuration I in Table II). Figure 3.35 shows this configuration for both capacitive and inductive tapping. The feedback controls the output of these oscillators to an accurate sinusoidal output voltage. Both configurations are negative-feedback versions of the well-known Colpitts and Hartley oscillator. The original types are found by leaving out the oscillator load and by replacing the current limiter by a short circuit and the active part by a single transistor (tube).

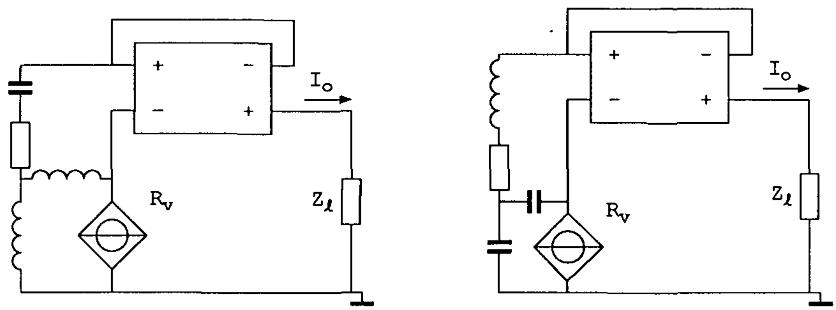

Fig. 3.36: Duals of negative-feedback Colpitts and Hartley oscillators with sinusoidal output currents.

Application of this tapping to oscillators with a series resonator leads to the configurations in which amplifier parameters B and D are stabilized (configuration III in Table I). Their output is an accurate sinusoidal current. They can be considered as the duals of the negative-feedback Colpitts and Hartley oscillator. Figure 3.36 depicts them both.

3.7.3 Inversion in the resonator

Apart from the impedance scaling and the scaling of series or parallel feedback there is yet a third possibility to use resonator tapping in an oscillator configuration. With the scaling of resonator impedance, series and parallel feedback paths have two resonator terminals in common, one of which is, of course, the resonator tap. In other uses of tapping these feedback paths have one resonator terminal in common. This terminal is either the resonator tap or not. The previous section showed configurations with this last option. Here we deal with configurations using the first.

The fact that the resonator tap is the only terminal common to the two feedback paths can be interpreted as an inversion in one of these paths. As resonator terminations must incorporate some kind of inversion to realize the negative resistance or conductance, also this use of tapping simplifies oscillator configurations. Tapping the parallel resonator in this way yields simple configurations in which the amplifier parameters B and D have been stabilized (configuration IV in Table II). Figure 3.37 shows them with capacitive and inductive tapping. They both have accurate

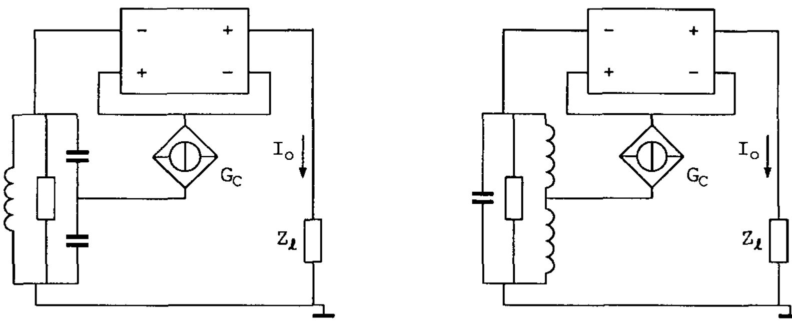

Fig. 3.37: Negative-feedback versions of the Colpitts and Hartley oscillators with square output currents.

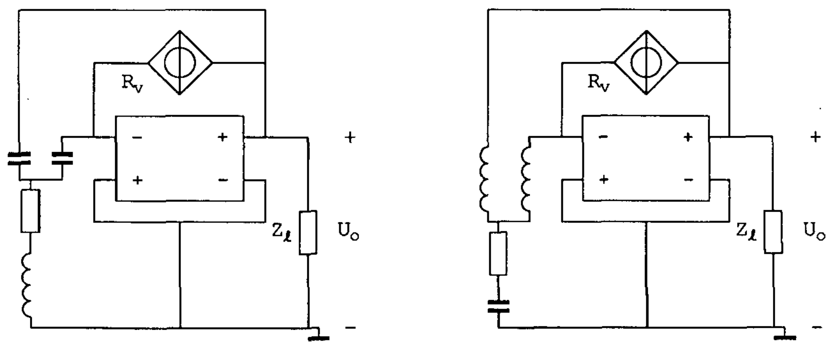

Fig. 3.38: Duals of the negative-feedback Colpitts and Hartley oscillator with square output voltages.

output currents of a square waveform. Also these configurations are negative-feedback versions of the Colpitts and Hartley oscillator. The only difference from the oscillator configurations from the previous section is the connection of the load. In the same way, tapping the series resonator can give an inversion in the resonator network. Again the duals of the negative-feedback Colpitts and Hartley oscillator are so synthesized. They are shown in Figure 3.38. The internal amplifier has accurate parameters A and C, stabilized by feedback (configuration II in Table I). They both exhibit accurate output voltages of a square waveform.

3.8 Discussion

In this chapter we dealt with configurations for negative-feedback oscillators. These configurations consist of an active structure that compensates for the losses in the resonator and transfers the oscillation signal from the undamped resonator to the load. The assumption in this chapter that resonator and load are both one-port elements implies the active structure in the oscillator is in fact an amplifier. For oscillation, the termination of the resonator, formed by the amplifier input, necessarily behaves as an negative resistor, either voltage or current controlled. For accurate oscillation, the output of the amplifier is a true current or voltage source driving the load with a signal that represents the resonator voltage or current.

Of the four small-signal transmission parameters that describe this amplifier in the oscillator, two have to be zero and two have to be stabilized to accurate values for the required behavior. It was shown that the two stabilized parameters must be frequency independent. Moreover one is constant to guarantee a linear transfer of the resonator current or voltage to the output. The other parameter is to be stabilized to a value time-invariant dependent on the large-signal amplifier output. It appeared that for each resonator there are four fundamentally different sets of accurate parameters. This resulted in four basic oscillator configurations for each LC-resonator (series or parallel).

The remainder of this chapter mapped out the various implementations of these eight basic oscillator configurations. First configurations with overall feedback were examined. On the assumption that an active part could be used with a large available power gain we looked for suitable feedback networks. For high performance, non-energic networks are preferred, but for oscillators they do not exist. Starting from these ideal feedback networks, we derived practical ones with transformers, nonlinear resistors and nonlinear transactances. As the nonlinear elements in this network must be time-invariant, difficulties in the selection of these elements were discussed. Some practical nonlinear circuits built from diodes and bipolar transistors were given. Some of the oscillator configurations presented are known in literature: they can be considered to be derived from the well-known Meacham, Colpitts and Hartley oscillators [ref.1]. Others are believed to be 'new'. The reason they have been under-exposed up to now is probably that without negative feedback most of them have no ground for existence.

The classification of basic oscillator configurations also showed other possibilities which exclude overall negative feedback. Naturally some degradation in performance must be accepted but in return some attractive implementations are given. In two of the four basic oscillator configurations it is possible to replace overall negative feedback by the accurate forward transfer of the amplifier itself. This amplifier should have an accurate limiting transfer for that purpose. We have shown in this chapter that such amplifiers can be built around the versatile differential pair. However, also here negative feedback proves to be indispensable. In an oscillator configuration synthesized in this way a buffer, that has been stabilized by negative feedback, is still necessary in order to attain the required accuracy.

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CHAPTER 4

DESIGN CONSIDERATIONS FOR OSCILLATORS WITH OPTIMAL STABILITY

4.1 Introduction

One of the most important properties of an oscillator is that the amplitude and the frequency of its output are stable. In oscillator applications residual amplitude and frequency fluctuations are unwanted and are considered to be oscillator noise. The oscillator output spectrum shows these fluctuations as parasitic amplitude and phase modulation sidebands added to the spectrum of the undisturbed oscillator signal.

As the relative height of these sidebands is a direct measure for oscillator instability, the design is focused on maximizing the magnitude of the oscillator signal and minimizing oscillator sidebands. Chapter II showed how the stability of oscillators, that can be modeled as a control diagram, depends on various parameters like resonator quality factor, degree of applied nonlinearity, input noise etc. In practice, however, oscillators often do not have an accurate amplitude stabilization and an accurate small-signal transfer. Such oscillators are therefore badly modeled by a simple control model, and results from calculations must often be corrected. On the other hand, as discussed in previous chapters, the application of negative feedback in oscillators enables the designer to model his oscillator more accurately. In these oscillators the stability can be calculated by taking some of the major nonlinear effects into account.

In this chapter we will apply the results of the formalized stability calculation from Chapter II in the oscillator configurations as derived in Chapter III. A parallel or series LC circuit forms the resonator and the amplifier between this resonator and the oscillator load forms the active resonator termination. The amplifier input behaves as a negative, nonlinear resistor. The stability of this negative-feedback oscillator is then given by the magnitude of the oscillator signal limited by this nonlinear resistor and by the oscillator sidebands. These sidebands are the result of the oscillator small-signal transfer, as derived in Chapter II, of the equivalent oscillator input noise.

First a general expression will be given for stability in negative-resistance oscillators, followed by more specific results for the 'linear' oscillator and the negative-feedback oscillator.
Next a search will be made for the various contributions to the equivalent oscillator input noise, such as noise from the resonator, the feedback network and the input stage of the active part. An inventory of these contributions will indicate how this input noise can be minimized. Further, for an optimum stability the oscillator signal must have a maximum value. This chapter will show how the limited output power from the active part can be optimally employed for this purpose and how consequently the oscillator's power efficiency is optimized.
Finally we will discuss the necessity of an active part with a large available power gain for an oscillator with optimum stability. The assumption that a negative-feedback oscillator is built around an active part with an infinite available power gain is obviously not true. In this chapter we will deal with the more detrimental effects of this finite power gain on the stability of the oscillator. A finite power gain will be shown to enlarge the transfer of equivalent input noise to the output and to reduce the power efficiency of the oscillator.

4.2 Frequency stability of negative-resistance oscillators

In this section we discuss the procedure for calculating the frequency stability in negative-resistance oscillators. The resonators will be terminated by a suitable negative resistance formed by an amplifier between this resonator and the oscillator load, as was discussed in the previous chapter.

Interest is naturally taken in the frequency stability of the oscillator signal at the load. Here, however, we choose to calculate the stability at a different location in accordance with linear amplifier theory where it has been shown that the calculation of amplifier noise is best done at the amplifier input. For oscillators we make a comparable choice. If all noise in the oscillator can be transformed to sources in front of the amplifier between the resonator and the load, this amplifier can be modeled noise free. The frequency stability of the amplifier output at the load terminals is then equal to the frequency stability of the amplifier input. This is due to the fact its input current and its input voltage have identical stability and one of the two stabilized small-signal-transmission parameters has a constant value (cf. Chapter III). Consequently the stability at the load can be calculated at the amplifier input with this modeling.
Just as the noise of any amplifier, the noise of the amplifier between resonator and load can be represented by two noise sources at the amplifier input. A noise voltage $u_n(t)$, with a power spectrum Su_n, in series with and a noise current $i_n(t)$, with a spectrum Si_n, in parallel with the amplifier input terminals.

Figure 4.1 shows this noise modeling in oscillators with the series or the parallel resonator. For the time being we consider this noise to be a result of a stationary Gaussian random process. Though this is not completely true for an amplifier with nonlinear feedback elements, it aids the intuitive understanding of noise filtering in the oscillator. Future sections will deal with exceptions to this approximation.
Besides the amplifier noise also the resonator noise must be taken into account. This noise is thermal and is related to the resonator losses. The white noise current $i_{rn}(t)$ in parallel with the resonator represents the noise of the parallel resonator, while the white noise voltage $u_{rn}(t)$ in series with the resonator represents the noise of the series resonator.

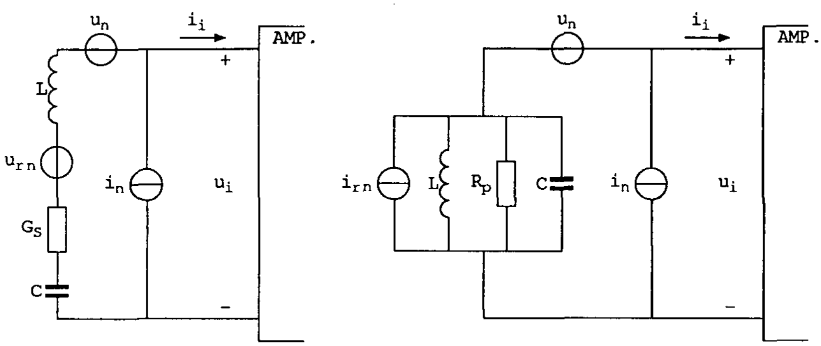

Fig. 4.1: Noise modeling in negative-resistance oscillators.

For the calculation of frequency stability we have a choice between the input current or the input voltage of the amplifier. As Chapter II indicated that frequency stability can be unambiguously defined for band limited signals with some additive noise, for calculation purposes the input current must be preferred in the oscillator with the series resonator and the input voltage in the one with the parallel resonator.
Actual stability calculations in this chapter will be restricted to oscillators with the series resonator. Duality between oscillators with series and oscillators with parallel resonators yields comparable results for the other oscillators. For these results r_i, G_S, y, u_i, i_i, u_n, i_n, u_{rn}, etc. must be replaced by g_i, R_P, z, i_i, u_i, i_n, u_n, i_{rn}, etc.
Further we want to make full use of calculations made in Chapter II. Therefore an equivalent input noise is chosen at the input of the nonlinear element. In the oscillator with the series resonator the negative resistor is current controlled and so the equivalent noise must be a current $i_{eq}(t)$, with a power spectrum Si_{eq}, following from:

$$i_{eq}(t) \quad = \quad i_n(t) + y(t) * (u_{rn}(t)+u_n(t))$$

In this equation y represents the impulse response of the resonator transconductance. Figure 4.2 depicts the intended transformation to the equivalent noise in an oscillator with a series resonator.

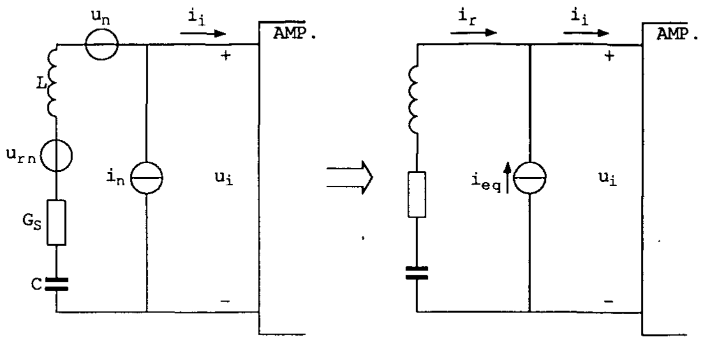

Fig. 4.2: Transformation of noise to equivalent input noise.

In the following we will derive the influence of this equivalent noise current $i_{eq}(t)$ on oscillator stability. The nonlinear resistor that terminates the resonator can be modeled for noise calculations as a linear time-varying resistance $r_i(t)$. As Chapter II showed, a Fourier series is an adequate description of this resistance:

$$r_i(t) = \sum_{n=-\infty}^{\infty} r_n \; e^{jn2\pi f_o t} \qquad (r_n : \text{real})$$

The resonator current $i_r(t)$ follows from an implicit equation that is similar to the implicit one in Section 2.5:

$$i_r(t) = - y(t) * [r_i(t) (i_{eq}(t) + i_r(t))]$$

Results in Chapter II also yield the phase-related sidebands $Si_r\phi$ in this resonator current:

$$Si_r\phi \; (f_o+\Delta f) = \left[\frac{1}{2Q\Delta f/f_o}\right]^2 G_S{}^2 \sum_{p=1}^{\infty} \left| r_{p-1} + r_{p+1} \right|^2 \frac{1}{2} \, Si_{eq}(pf_o+\Delta f)$$

As mentioned before, here we want to calculate the stability of the input current of the amplifier between resonator and load. This input current $i_i(t)$ consists of the resonator current $i_r(t)$ and additive noise in the form of the equivalent noise current $i_{eq}(t)$. Of this additive noise only components with frequencies near f_o affect stability, as suitable loads will filter out other components (cf. Section 2.9). Moreover amplitude-related components in this additive noise, represented by half its power, do not degrade stability. So the phase-related sidebands $Si_i\phi$ in the input current i_i follow from:

$$Si_i\phi\ (f_o+\Delta f) = \left[\frac{1}{2Q\Delta f/f_o}\right]^2 G_S{}^2 \sum_{p=1}^{\infty} \left| r_{p-1} + r_{p+1} \right|^2 \frac{1}{2} Si_{eq}(pf_o+\Delta f)$$

$$+ \frac{1}{2} Si_{eq}(f_o+\Delta f)$$

Finally, as a measure for frequency stability, the spectral phase fluctuations $S\phi$ are found in twice the ratio of the this phase-related sideband $Si_i\phi$ to the squared effective input current (cf. Section 2.4). This squared effective current is defined as $G_s P_{res}$, in which P_{res} is the resonator dissipation. The spectral phase fluctuations are then given by:

$$S\phi(\Delta f) = \left[\frac{1}{2Q\Delta f/f_o}\right]^2 \frac{G_S}{P_{res}} \sum_{p=1}^{\infty} \left| r_{p-1} + r_{p+1} \right|^2 Si_{eq}(pf_o+\Delta f)$$

$$+ \frac{1}{G_S\ P_{res}} Si_{eq}(f_o+\Delta f)$$

The well-known conclusion can be derived that for stable oscillators equivalent noise must be minimized and Q^2P_{res}, the reactive power in the resonator must be optimized. The following sections will discuss the effect of the oscillator's nonlinearity by comparing the 'linear' oscillator with the negative-feedback oscillator.

4.3 Frequency stability of a 'linear' oscillator

In this section we assume that the oscillator is linear. Though real oscillators are nonlinear there are two important reasons to study the linear situation. First this assumption greatly simplifies expressions for oscillator stability and shows the relative importance of parameters that influence stability. Secondly the linear situation is the situation that must be approximated in the negative-feedback oscillator for an optimal stability, as it was concluded in Chapter II.

In a linear oscillator an ideal, constant, negative resistance r_i will undamp a series resonator. Figure 4.3 shows this linear undamping with the relevant noise sources. As the resistance r_i is constant, it is described by one single term in the Fourier series:

$$r_o = - G_s^{-1} \quad \text{and} \quad r_n = 0 \quad \text{for } n \neq 0$$

Substituting the spectrum of the equivalent input noise current i_{eq} given by:

$$Si_{eq} = Si_n + \frac{4kTG_S + G_S{}^2\ Su_n}{1 + [2Q\Delta f/f_o]^2}$$

in the expression, for the phase-related sideband $Si_i\phi$ in the amplifier input current (cf. Section 4.2) yields:

$$Si_i\phi\ (f_o+\Delta f) = \left[\frac{1}{2Q\Delta f/f_o}\right]^2 \frac{1}{2}\ (Si_n(f_o) + G_S{}^2 Su_n(f_o) + 4kTG_S) + \frac{1}{2}\ Si_n(f_o)$$

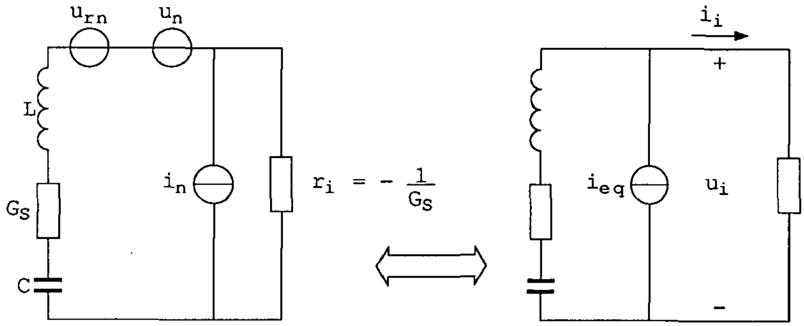

Fig. 4.3: Ideal linear undamping of a series resonator.

The important spectrum for low-offset frequencies Δf is influenced by all three noise sources. As is common in calculations of amplifier noise, also here a noise figure F may be defined as the factor that the total noise exceeds the resonator noise:

$$F = 1 + \frac{Su_n G_s + Si_n/G_s}{4kT}$$

Due to the possibly high conductance G_S of the LC-circuit at resonance the noise voltage u_n may dominate the equivalent oscillator input noise. Fortunately the transfer of the resonator can be changed by a transformer or by resonator tapping (cf. Chapter III). Another possibility is that the lower noise voltage u_n can be traded for higher noise current i_n in the amplifier [ref.1]. In such situations the lowest possible sideband noise is reached with a noise match formulated by:

$$G_s{}^2\ Su_n = Si_n$$

Note that the noise match only minimizes oscillator sidebands at low-offset frequencies. At the noise match the noise floor, caused here by the noise current i_n, may be unacceptably high.

Again the spectral phase fluctuations $S\phi$ can be calculated in the way described in the previous section. For the linear oscillator these fluctuations are given by:

$$S\phi(\Delta f) = \frac{Si_n(f_o)}{P_{res}G_s} + \frac{FkT}{P_{res}\,Q^2}\left[\frac{f_o}{\Delta f}\right]^2$$

floor + sideband

This result is comparable to results found by others [ref.2,3,4]. They often use in their expressions the resonator power available to the amplifier input. With a one-port resonator termination this terminology is somewhat misleading, for power is delivered to the resonator instead of the reverse. Therefore the less universal but more direct resonator dissipation P_{res} is used.

4.3.1 Degradation of stability by linear feedback elements

From linear amplifier theory we can also derive some considerations on the design of the feedback network in the oscillator. This feedback network stabilizes the transfer and the input impedance of the amplifier between the resonator and the load. For the network the non-energic transformer and gyrator are ideal elements for they do not deteriorate the noise performance nor the power efficiency of the oscillator [ref.1]. For a given resonator the noise performance and the output capabilities of the active part determine the stability of an oscillator equipped with these ideal-feedback elements.
As no non-energic approximation is available for the gyrator, a feedback network that stabilizes an impedance to a real value, must necessarily contain some noisy, power consuming i-u or u-i conversion. Often resistors will be used in this feedback network and in this case an interesting trade of power consumption for noise contribution can be made. In this section we will discuss this trade in a linear oscillator. Conclusions are then rather simple and in the more practical nonlinear oscillators their qualitative content is still valid.
We assume the transformer is an accepted element in the feedback network. The design considerations that are here derived are also applicable to the situation in which a four-terminal active part and an extended feedback network make this transformer redundant (cf. Section 3.5.3).
For accurate input impedance of a feedback amplifier, the transconductance or the transimpedance of this amplifier must be stabilized (parameters B or C, cf. Chapter III). If for this purpose feedback in series or in parallel with the input is applied by resistors, the equivalent noise voltage or noise current of the amplifier is enlarged by the thermal noise of these resistors. As for high stability the equivalent oscillator input noise must be minimized, series feedback by small and parallel feedback by large resistors seem beneficial. However in an oscillator this choice leads to a suboptimal stability, because such resistors consume

considerable power. For a high stability not only the equivalent input noise must be minimized, but also Q^2P_{res}, the reactive power in the resonator, must be optimized (cf. Section 4.2). Thus preferably all power available from the active part is dissipated in the resonator. This is the case if parallel feedback resistors are very small and series feedback resistors are large. Obviously a trade-off exists in the choice of the feedback resistor.

Suppose that the noise from the feedback resistor and from the resonator is the only relevant noise in the oscillator. Then a series or parallel feedback resistor being n times smaller or larger respectively than the resonator impedance enlarges the total noise power by a factor 1+1/n. Because the voltage across a series feedback resistor equals the resonator voltage, or because through the parallel feedback resistor the corresponding current flows, the power available to the resonator reduces by a factor 1+n in the assumption of a constant output power of the active part. Simple *calculation shows that here the optimal stability is found at n=1.* Consequently at least twice the resonator noise is present and only half the power delivered to the feedback network is dissipated in the resonator. In comparison to the non-energic feedback the resistive feedback reduces the attainable CNR with at least 6 dB. An application of this result is that for minimum degradation in stability by the feedback network the tapping ratio in the well-known Colpitts and Hartley oscillator must 1:1 (equal tapping capacitors and coils).

With transactances in the feedback network the strict relation between noise production and power consumption no longer exists. The i-u or u-i transactances can replace the noisy and power-consuming resistors, but generally no better stability is to be expected. Practical elements approximating the necessary transactances produce more noise and consume considerably more power. Power consumed by the these elements can be used more effectively if the feedback network also delivers the power to the resonator (cf. Section 3.5.6). Consequently lower demands are made on the output capabilities of the active part. However power gain and accuracy are contradictory demands on the feedback network. With these feedback networks some accuracy or long-term stability is lost in favor of an increased short-term stability.

Non-energic feedback also enlarges the influence of noise from the active part. In a linear oscillator this does not differ fundamentally from the situation in a negative-feedback amplifier [ref.1]. Extensive treatment of this effect has therefore been omitted. In case of a nonlinear or time-varying feedback network it will be examined more closely.

4.4 Sidebands in negative-feedback oscillators

In a negative-feedback oscillator the resonator is terminated by an amplifier with a negative input resistance that is optimally suited for undamping. From Chapter III (Figure 3.4) we recall that the preferred undamping of a series resonator is by a current controlled, negative resistor, with a true small-signal resistance $-r_A$ and a symmetrical limit level U_L.
For the calculation of the frequency stability in these oscillators the small-signal description of the resonator termination, the resonator dissipation and the equivalent oscillator input noise are required (cf. Section 4.2). In this section we deal with the first two.
The resonator dissipation is stabilized by the voltage-limiting i-u characteristic of the negative resistor. A limit level U_L stabilizes this dissipation P_{res} to:

$$P_{res} \simeq \frac{1}{2}\left(\frac{4}{\pi}U_L\right)^2 G_S$$

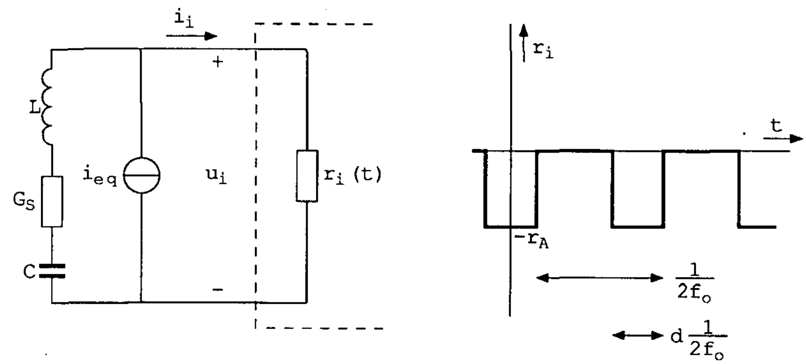

Fig. 4.4: The time-varying input resistance of the amplifier.

The large oscillation signal modulates the small-signal input resistance r_i of the amplifier. Figure 4.4 shows this time-varying resistance. From Section 2.7.1 we conclude this resistance is well described by a Fourier series with coefficients r_n given by:

$$r_n = -r_A\, d\,\frac{\sin(n\pi d/2)}{n\pi d/2} \qquad n : \text{even}$$

$$r_n = 0 \qquad n : \text{odd}$$

In this equation d represents the 'duty cycle' and is a function of the excess loop gain $r_A G_S$, the true small-signal loop gain at resonance:

$$d \simeq \frac{1}{2\, r_A\, G_s} \qquad \text{for} \quad r_A G_S > 2$$

These coefficients r_n and resonator data like Q and G_S determine the transfer of an equivalent input noise current i_{eq}, as it is depicted in Figure 4.4, to the amplifier input current i_i. The description of this transfer is somewhat complicated by the infinite sum of conversion products. In Chapter II it was shown, however, that the expression can well be simplified for two important types of equivalent input noise. Where this input noise is Gausssian and has been limited in bandwidth to $2f_o$, the transfer to a phase-related input current does not differ from the one in a 'linear' oscillator.. The spectrum $Si_i\phi$ of the phase-related input current then follows from:

$$Si_i\phi\ (f_o+\Delta f) \;=\; \left[\frac{1}{2Q\Delta f/f_o}\right]^2 \frac{1}{2}\, Si_{eq}(f_o+\Delta f) + \frac{1}{2}\, Si_{eq}(f_o+\Delta f)$$

The other important type of equivalent input noise is white noise. This input noise results in a spectrum $Si_i\phi$ of the phase-related amplifier input current, given by:

$$Si_i\phi\ (f_o+\Delta f) \;=\; \left[\frac{1}{2Q\Delta f/f_o}\right]^2 \frac{1}{2}\, r_A G_S\, Si_{eq} + \frac{1}{2}\, Si_{eq}$$

This spectrum is, apart from the noise floor, a factor $r_A G_S$, equal to the excess loop gain, higher than in the case of the previous band-limited equivalent input noise.
The determination of the equivalent input noise of a negative-feedback oscillator is, in principle, identical to the determination of the equivalent noise of a negative-feedback amplifier [ref.1]. The use of appropriate transformation techniques like the Blakesley transformation or e-shift, its dual the 'current split method' and the Norton-Thevenin conversion allows the transformation of every noise source in a linear amplifier into an equivalent noise voltage and an equivalent noise current at its input. From these two equivalent noise sources the equivalent oscillator input noise can found by a subsequent Norton-Thevenin transformation at the resonator, as was already indicated in Figure 4.2. In a negative-feedback oscillator the amplifier between resonator and load has a linear transfer for the small noise signals under consideration. However ,in contrast with more common negative-feedback amplifiers, this transfer is time variant. Because of this time dependency the necessary transformations are somewhat unusual. In the following sections we deal with the important differences from transformations in linear time-invariant circuits. There the equivalent input noise of some typical oscillator circuits and the phase-related signals at the amplifier input, which this noise induces, will be determined.

4.4.1 Noise from linear active elements

In this section we will show how noise from linear elements can be transformed to the equivalent input noise of a negative-feedback oscillator. Contributions of nonlinear elements will be dealt with in a following section.

From linear theory it is known that the application of energic feedback to an amplifier increases the contribution of internal amplifier noise to the total noise. In the oscillator this also applies to the time-varying feedback applied to the amplifier between the resonator and the load.
First, using standard techniques, we transform all noise from linear amplifier elements into a noise voltage in series and a noise current in parallel with the input terminals of the active part. Subsequently these noise sources are transformed to an equivalent oscillator input noise, from which finally the oscillator sideband is calculated.

In negative-feedback oscillators there are two principally different energic-feedback arrangements: a nonlinear resistor realizes either series or parallel feedback at the input in order to stabilize the small-signal transmission parameter B or C.
Each arrangement affects the noise from linear amplifier elements in a different way. In this section we discuss both ways in an oscillator with a series resonator.
Figure 4.5 shows the series resonator terminated by an amplifier of which the transmission parameters A and C are stabilized by a current-controlled nonlinear resistor and a transformer. Already all noise of linear amplifier elements has been transformed into a noise voltage u_n and a noise current i_n. The subsequent transformation into an equivalent oscillator input noise current i_{eq} is illustrated step by step in Figure 4.5. The resulting current follows from:

$$i_{eq}(t) = u_n(t) * y(t) - \frac{u_n(t)}{n\ r_i(t)} + i_n(t)$$

The presence of the feedback network does not influence the noise current. The influence of the noise voltage is however enlarged, as the input resistance $r_i(t)$ is negative for undamping. For the input current i_i of the amplifier only surveyable expressions exist if the current i_n consists of white noise. In that case calculation shows the spectrum $Si_i\phi$ of the phase-related input current i_i equals:

$$Si_i\phi(f_o+\Delta f) = \frac{1}{2}\left[\frac{1}{2Q\Delta f/f_o}\right]^2 \left[\ (1+\frac{1}{n})^2 G_S{}^2 Su_n(f_o) + r_A G_s Si_n\ \right] + \frac{1}{2}\left[\frac{1}{n^2 r_A{}^2} Su_n(f_o) + Si_n\right]$$

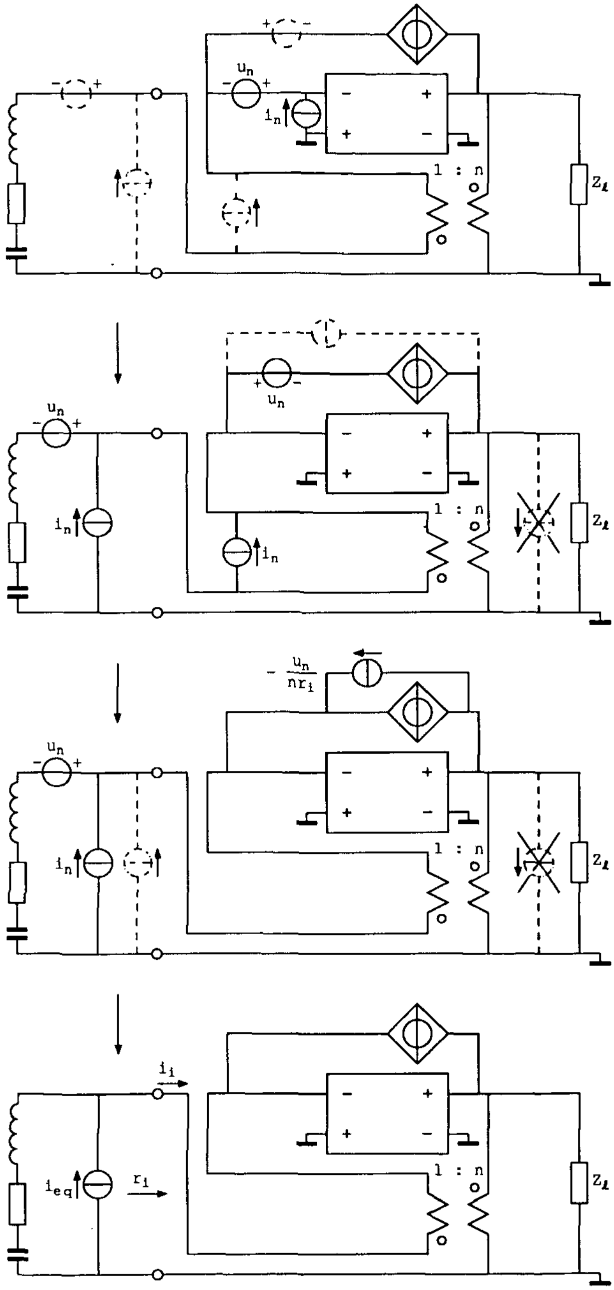

Fig. 4.5: Noise transformations in an oscillator with an accurate output voltage.

For high stability, the excess loop gain must not be larger than is strictly necessary for amplitude stabilization ($r_A G_S \cong 2$). Large values for the transformer ration n reduce some noise contributions but are not recommended for high stability, as with these values only a small part of the total power is dissipated in the resonator. More favorable values ($n \cong 1$) double the influence of the noise voltage u_n.
If the current-controlled, nonlinear resistor realizes series feedback at the amplifier input, oscillator noise is influenced in a comparable way. Figure 4.6 depicts an oscillator with this type of feedback. The internal amplifier has stabilized transmission parameters B and D. In the transformation of the noise sources u_n and i_n into an equivalent noise current i_{eq}, the voltage u_n can be shifted in series with resonator and transformed into a current $y{*}u_n$ by a Thevenin-Norton transformation at the resonator. The noise current i_n is split between the resonator and the feedback resistor. A Norton-Thevenin transformation at this resistor yields a voltage $i_n r_i/n$, that is shifted in series with the resonator and thereupon converted to a current. The equivalent noise current i_{eq} that results from these transformations is given by:

$$i_{eq}(t) = i_n(t) + u_n(t) * y(t) - \frac{i_n(t)\ r_i(t)}{n} * y(t)$$

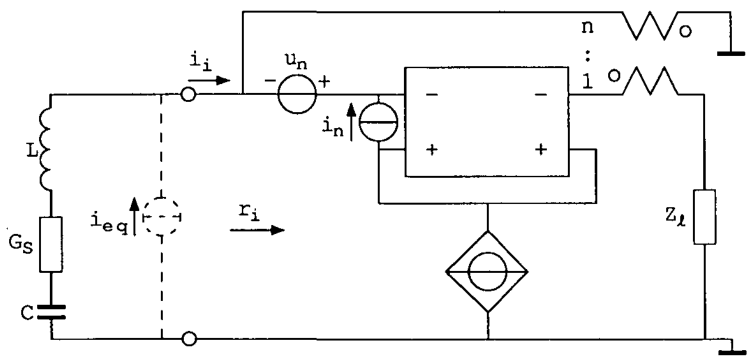

Fig. 4.6: Noise transformations in an oscillator with an accurate output current.

This current induces an amplifier input current i_i. Again with the assumption the current i_n consists of white noise, the spectrum $Si_i\phi$ of the phase-related components in the input current i_i follows from:

$$Si_i\phi(f_o+\Delta f) = \frac{1}{2}\left[\frac{1}{2Q\Delta f/f_o}\right]^2 \left[\ (1+\frac{1}{n})^2 r_A G_S Si_n + G_S{}^2 Su_n(f_o)\ \right] + \frac{1}{2}\ Si_n$$

Here, in the case of energic series feedback, the same remarks can be made as for the corresponding parallel feedback. The series feedback resistor increases the influence of the noise current i_n.

4.4.2 Noise from nonlinear feedback elements

Besides the noise from linear oscillator elements the single nonlinear element in the oscillator also produces noise. In this section we show how noise from this somewhat exotic origin can be handled and what sidebands result from this noise.
In the feedback path of negative-feedback oscillators we use two types of nonlinear resistors: the current-controlled resistor that limits its voltage and the voltage-controlled resistor that limits its current (cf. Section 3.5.2). Instead of these resistors also nonlinear transactances can placed in the feedback network (cf. Section 3.5.6). In all these nonlinear elements two regions of operation can be distinguished: operation close to zero controlling inputs and operation with inputs so large that the resonator termination in an oscillator, equipped with these elements, shows the required limiting characteristics. Normally noise production by the nonlinear element will only be relevant in the first mode of operation. This can be illustrated for the nonlinear resistances: for large inputs the resistor behaves as an open or a short circuit with no relevant noise production.
Just as in linear elements there are several representations for the noise of these elements. In a linear element they are simply related by its constant small-signal transfer. In nonlinear

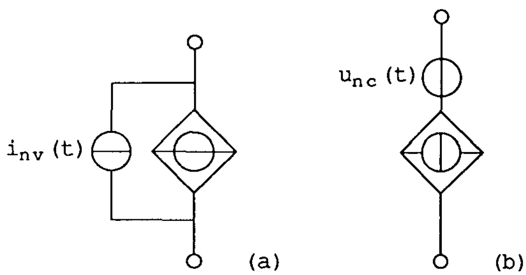

Fig. 4.7: Noise representation in current controlled resistor (a) and in a voltage controlled resistor (b).

elements, however, this transfer is not necessarily constant; therefore preference is given to those representations of the noise that are highly independent of the momentary small-signal transfer of the nonlinear element. Generally these are representations at the controlling inputs of these elements. Consequently a current

$i_{nv}(t)$ in parallel represents the noise in a current-controlled resistor and a voltage $u_{nc}(t)$ in series represents the noise in a voltage-controlled resistor. Figure 4.7 depicts both types of nonlinear resistors and the equivalent noise representation.

We will relate the equivalent noise of nonlinear resistors to the thermal noise of their true small-signal resistance: their noise (power) will be assumed to be a factor χ larger. An exact description of the noise in various nonlinear resistances is believed to be beyond the scope this treatise. The reader may find a starting point in literature [ref.5].
With these definitions we are able to analyze the influence of the noise of nonlinear elements on the stability of a negative-feedback oscillator. First we deal with noise from nonlinear resistors. As in oscillators with a series resonator this resistor realizes either series or parallel feedback at the input, stability can be influenced in two different ways. Figure 4.8 shows the first way in an oscillator configuration with resistive parallel feedback. The equivalent current noise of this nonlinear resistor is related to the true small-signal resistance nr_A of this resistor. The spectrum of this equivalent current noise follows from:

$$\overline{i_{nv}(t)^2} = \chi \frac{4kT}{nr_A}$$

Transformation of this current to the equivalent input noise current of this oscillator is rather simple for:

$$i_{eq}(t) = i_{nv}(t)$$

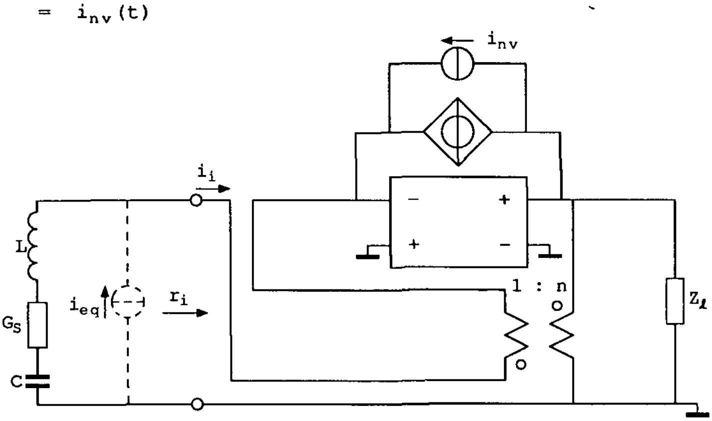

Fig. 4.8: Transformation of noise from a parallel-connected, current-controlled resistor.

This equivalent input noise current induces an current i_i at the input of the amplifier in the oscillator. The spectrum $S_{i_i\phi}$ of the phase-related components in this current is given by:

$$Si_i\phi(f_o+\Delta f) \quad = \quad \left[\frac{1}{2Q\Delta f/f_o}\right]^2 r_A G_S \, \chi \, \frac{2kT}{nr_A} \; + \chi \, \frac{2kT}{nr_A}$$

Note that only the noise floor in this input current is a function of the excess loop gain. As discussed in Section 4.3.1 transformer ratios near unity must be chosen for optimum stability.
A comparable situation exists for an oscillator built from a series resonator and an amplifier in which transformer feedback and resistive series feedback stabilize the parameters B and D, as depicted in Figure 4.9. Again the current $i_{nv}(t)$ representing the noise of the current-controlled resistor is related to the true small-signal resistance r_A/n of this resistor:

$$\overline{i_{nv}(t)^2} \quad = \quad \chi \, n \, \frac{4kT}{r_A}$$

A Norton-Thevenin transformation at this resistor, a subsequent Blakesley transformation and a Thevenin-Norton transformation at the resonator convert this current to an equivalent input noise current $i_{eq}(t)$:

$$i_{eq}(t) \quad = \quad y(t) \, * \, (\, i_{nv}(t) \, \frac{r_i(t)}{n})$$

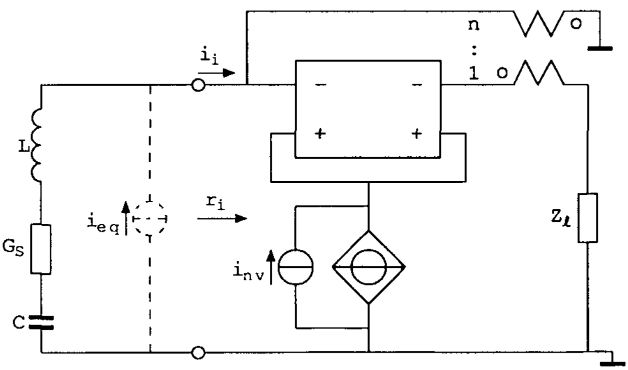

Fig. 4.9: Transformation of noise from a series-connected, current-controlled resistor.

The spectrum $Si_i\phi$ of the phase-related components in the input current i_i, induced by this equivalent input noise is given by:

$$Si_i\phi(f_o+\Delta f) \quad = \quad \left[\frac{1}{2Q\Delta f/f_o}\right]^2 r_A G_S \, \chi \, \frac{2kT}{nr_A}$$

The spectrum largely coincides with that in the case of parallel resistive feedback. With series feedback only the noise floor is absent.

Instead of the nonlinear resistors also a nonlinear transactance can be used as a feedback element. We discuss the degradation of oscillator stability by noise from such a transactance in an oscillator with a series resonator and an accurate output voltage. Figure 4.10 depicts the relevant configuration. In this oscillator feedback by a limiting voltage-to-voltage transactance has been applied (cf. Section 3.5.5). The equivalent noise of this transactance is assumed to be a white noise voltage u_n at the input of this transactance. This noise voltage can be transformed to an equivalent input noise current $i_{eq}(t)$ equal to:

$$i_{eq}(t) = y(t) * \left(u_n(t) \frac{r_i(t)}{R} \right)$$

The corresponding spectrum $Si_i\phi$ of the phase-related input current i_i follows from:

$$Si_i\phi(f_o+\Delta f) = \left[\frac{1}{2Q\Delta f/f_o}\right]^2 \frac{1}{2} r_A G_S \frac{Su_n}{R^2}$$

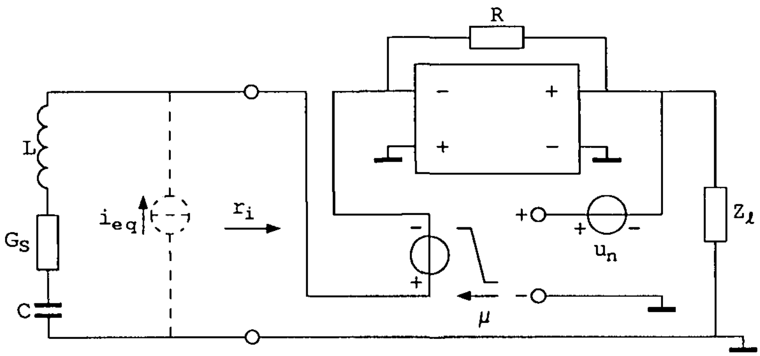

Fig. 4.10: Transformation of noise from a feedback transactance.

For a low input noise current i_i, large values of the feedback resistor R are beneficial, but they are not recommended for high stability. With such values the resonator dissipation is only a small portion of the total power dissipated in the feedback network and such values result in a suboptimal stability. Section 4.3.1 showed that values near $1/G_S$ are optimum.

4.5 Effects of a finite available power gain

Up till now we have calculated the frequency stability of the oscillator in the assumption that the active part around which the oscillator has been built incorporates an infinite available power gain. In practice, however, this gain is is limited, of course, and deviations from results in previous sections will occur. The following will deal with these deviations and will show that it is essential to maximize the available power gain for a high frequency stability.

The finite available power gain impairs the resonator termination in all its important properties. These properties are the true small-signal impedance, the limit level and the impedance during limiting. In the following we derive expressions for these aspects of the resonator termination. Interest is in the effect of the changes in the resonator terminations on frequency stability. Exact calculation of these effects requires the handling of expressions describing a nonlinear dynamic two-pole. Such expressions tend to be rather complicated and are less suited for design purposes. Therefore these effects are here discussed qualitatively and only exactly described in oscillators that show a soft nonlinear behavior. Then the results take on a simple form and design criteria can be formulated.

As in previous sections we will also restrict ourselves to oscillators with series resonators. We believe that the effect of a finite power gain on frequency stability in oscillators with a parallel resonator follows from dual considerations.

4.5.1 Condition for oscillation

In an oscillator built around an active part with an infinite available power gain, solving the equations that represent the condition for oscillation is almost simplicity itself. If we want to solve these equations in the case of a power gain of a practical active part the situation is complicated by the frequency dependency of this gain. However, here we hold to the description of the oscillation process used in previous sections: the resonator and its termination. In the case of a frequency-dependent available power gain, the resonator termination is not purely resistive. Though memoryless feedback around the active part tends to stabilize the resonator termination to a resistive value, some of the frequency dependency of the active part trickles in. This effect is here described by the (true small-signal) impedance Z_a, the input impedance of the amplifier in the active mode. Blackman [ref.6] has given an expression for an active impedance of this kind, using two different loop gains in the active circuit in which an impedance is to be calculated: a loop gain $A\beta_o$ with the impedance terminals open and a loop gain $A\beta_{sc}$ with these terminals short circuited. Both loop gains can be considered as the product of the gain A of an internal amplifier and a feedback factor β. Blackman's expression for an active impedance Z_a reads:

$$Z_a = \rho \frac{1 - A\beta_{sc}}{1 - A\beta_o}$$

In this expression the impedance ρ represents the value of Z_a when both loops are broken. For our purposes we use a comparable expression that stresses the action of feedback more explicitly. It is derived from the 'asymptotic-gain model', a model suited to describe the transfer of a negative-feedback amplifier [ref.1]. It appears that by defining:

$$Z_{a,\infty} = \rho \frac{\beta_{sc}}{\beta_o}$$

the active impedance can be rewritten as:

$$Z_a = Z_{a,\infty} \frac{1 - A\beta_{sc}}{-A\beta_{sc}} \frac{-A\beta_o}{1 - A\beta_o}$$

If a large power gain is available from the active part, both loop gains are large (>>1) and the active impedance has the value which is anchored in the feedback network. For instance, in the case of a termination of a series resonator we choose:

$$Z_{a,\infty} = - r_{A,\infty}$$

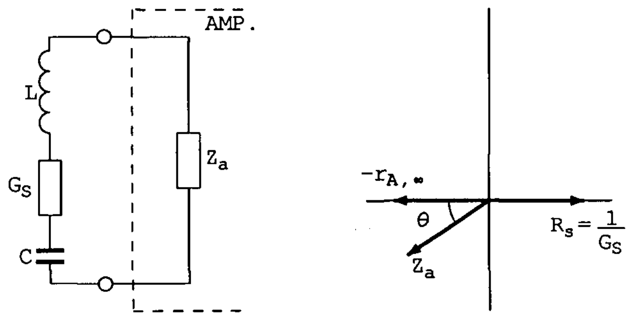

Fig. 4.11: The non-resistive undamping of the resonator and the phasor diagram of relevant impedances.

For smaller power gains the impedance also becomes a function of the active part in the oscillator and it must be corrected with the two given factors. The impedance Z_a is capable of undamping a series resonator in an oscillator, as depicted in Figure 4.11, if it satisfies the condition for oscillation. This condition reads in this case:

$$- Z_a \frac{G_S}{1 + jQ\nu} = 1$$

For compact notations in the following we introduce ϑ as:

$$\vartheta = \arctan \frac{\mathrm{Im}(Z_a)}{\mathrm{Re}(Z_a)} \bigg|_{f=f_{osc}}$$

The condition for oscillation can be expressed with the help of the equations above as:

$$f_{osc} = f_o + \frac{f_o}{2Q} \tan \vartheta$$

$$r_{A,\infty} G_S \left| \frac{1 - A\beta_{sc}}{-A\beta_{sc}} \frac{-A\beta_o}{1 - A\beta_o} \right|_{f=f_{osc}} \cos \vartheta = 1$$

The most obvious conclusion is that the phase shift in the undamping impedance must be compensated by an off-resonance operation in order to restore the phase balance in the loop. In practical amplifiers only negative values for ϑ are expected and so the oscillation frequency is always smaller than the resonance frequency. For an accurate oscillation frequency, a quasi-resistive termination, which corresponds to a small phase shift ϑ, and a high value of the quality factor Q of the resonator are required.

The second equation expresses the amplitude balance. It describes the balance between the resonator loss resistance and the negative, real part of the impedance Z_a. From the alternative reading of Blackman's formula we conclude that this real part has a smaller (absolute) value than $r_{A,\infty}$. The feedback network aims at this value and it is only attained at an infinite available power gain. With a certain amount of power gain the feedback network must be designed in such a way that this value is large enough to guarantee oscillation. The design of this feedback network follows from the value of $r_{A,\infty}$ given by the equation for the amplitude balance. Of course, a change in the feedback network also influences the loop gains $A\beta_{sc}$ and $A\beta_o$. However the changes in these loop gains only have a minor effect on the condition for oscillation; generally a change in the feedback network has a much greater effect on the condition for oscillation by the changed value of $r_{A,\infty}$.

In the following we will apply these results in the derivation of the small-signal transfer of the oscillator and of the reactive resonator power in the case of a finite available power gain.

4.5.2 Small-signal transfer at off-resonance operation

In the previous section we saw how the use of a frequency-dependent active part causes the oscillator to operate in an off-resonance mode. In this and the following section we show how, in addition to the frequency accuracy, also the frequency stability is reduced in this mode.

The use of a frequency-dependent active part can have consequences for the transfer of a small, independent disturbing signal in the oscillator to the oscillator output. Here we discuss these consequences in an oscillator with a series resonator. Figure 4.12 depicts the situation under consideration. The series resonator is terminated by the input impedance Z_a of a negative-feedback amplifier. The current I_{eq} represents the disturbing signal in the frequency domain. The small-signal transfer of this current to the oscillator output can very well be replaced by the transfer to the input current I_i of the amplifier, as was demonstrated earlier in this chapter.

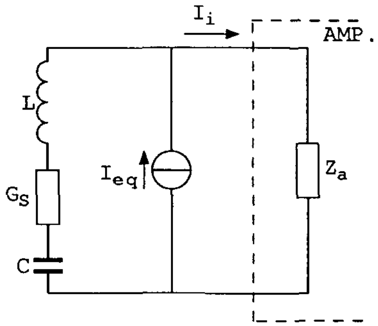

Fig. 4.12: Model for calculation of the oscillator small-signal transfer.

Not only for the sake of simplicity but also because the situation corresponds to one with optimal stability, we assume almost linear oscillator behavior. The transfer of the current I_{eq} to the amplifier input current I_i is in this case simply given by the following equation:

$$I_i = \frac{-Z_a}{Z_a + R_S(1+jQ\nu)} I_{eq} + I_{eq}$$

Fourier pairs:

$i_{eq}(t) \Leftrightarrow I_{eq}(f)$

$i_i(t) \Leftrightarrow I_i(f)$

Using both the amplitude and phase balance, as derived from the condition for oscillation in the previous section, we can rewrite this equation to:

$$I_i = \frac{1 + j\tan\vartheta}{jQ(\nu - (\tan\vartheta)/Q)} I_{eq} + I_{eq}$$

For stability calculations, the interest is in the power density $Si_i\phi$ of the phase-related input current as a result of the equivalent oscillator input noise with a spectrum Si_{eq}. As in the assumed linear case this spectrum will be half the power density of the input current i_i, the spectrum $Si_i\phi$ follows from:

$$Si_i\phi(f_{osc}+\Delta f) = \frac{1}{2}\left[\frac{1}{2Q\Delta f/f_{osc}}\right]^2 \frac{1}{\cos^2\vartheta}\; Si_{eq}(f_{osc}+\Delta f) + \frac{1}{2}\, Si_{eq}(f_{osc}+\Delta f)$$

From this expression we conclude that off-resonance operation enlarges the transfer of equivalent input noise to the oscillator output by a factor $1/\cos^2\vartheta$. Thus for a high frequency stability a low phase shift ϑ or expressed alternatively a quasi-resistive termination of the resonator is required. This result has also been derived in literature [ref.4]. The derivation given above is, however, believed to be of a simpler form and more suited for design purposes. This may be illustrated by deriving the transfer of the thermal resonator noise to the oscillator output. A discussion, comparable to the above, shows that off-resonance operation does not enlarge this transfer.

4.5.3 Reactive resonator power

The finiteness of the available power gain not only enlarges the transfer of noise to the oscillator output, but it also affects the reactive power in the resonator. As a high reactive resonator power is essential for a high stability, we discuss this relation between power gain and resonator power in the oscillator. Just as in the previous section this relation is only derived for an almost linear oscillator with a series resonator.
Because the Q-factor of the resonator links the reactive resonator power to the resonator dissipation, it suffices to deal with this dissipation only. The resonator dissipation P_{res} in an oscillator with a series resonator is here expressed by:

$$P_{res} = \frac{1}{2}\, I_{res}^2\, R_S$$

In this equation I_{res} is the amplitude of the sinusoidal resonator current. Both the limit level U_L and the impedance Z_a determine this amplitude, for in the almost linear case the following equation holds:

$$I_{res} = \frac{U_L}{|Z_a|}$$

Because of their common origin the limit level U_L as well as the impedance Z_a depend on the available power gain in practically the same way, for we can write:

$$U_L = U_{L,\infty} \left| \frac{1 - A\beta_{sc}}{-A\beta_{sc}}\, \frac{-A\beta_o}{1 - A\beta_o} \right|_{f=f_{osc}}$$

$$Z_a = -r_{A,\infty}\left[\frac{1-A\beta_{sc}}{-A\beta_{sc}}\,\frac{-A\beta_o}{1-A\beta_o}\right]_{f=f_{osc}}$$

From these last three equations and the amplitude balance in the condition for oscillation it can be deduced that:

$$I_{res} = G_S\,U_{L,\infty}\left|\frac{1-A\beta_{sc}}{-A\beta_{sc}}\,\frac{-A\beta_o}{1-A\beta_o}\right|_{f=f_{osc}}\cos\vartheta$$

From this current the resonator dissipation P_{res} is found to be given by:

$$P_{res} = \frac{1}{2}\,G_S\,U_{L,\infty}^2\left|\frac{1-A\beta_{sc}}{-A\beta_{sc}}\,\frac{-A\beta_o}{1-A\beta_o}\right|^2_{f=f_{osc}}\cos^2\vartheta$$

Thus high values for both the loop gains $A\beta_{sc}$ and $A\beta_o$ are essential for a high resonator dissipation, for a high reactive resonator power and for a high frequency stability. At a low available power gain one might suggest adapting the feedback network by a choice for a higher value for $U_{L,\infty}$ in order to compensate for the reduction in resonator dissipation. Although this dissipation can be held constant in this way, overall performance is not increased. The finiteness of the available power gain deteriorates the power efficiency of the oscillator and consequently for a certain resonator dissipation the output capabilities of the active part must increase with a decreasing power gain.

4.5.4 High-frequency behavior

In order to attain a high frequency accuracy and a high frequency stability it is necessary to maximize the available power gain in the active part of the oscillator. With a predetermined resonator, feedback network and oscillator load we can also maximize the two loop gains $A\beta_{sc}$ and $A\beta_o$, over a frequency range, centered around f_o, as large as possible. In the feedback loops the number of active stages is limited by the requirements with respect to the high-frequency stability. As the termination of a resonator is either voltage or current controlled, it must be stable at short-circuited or at open terminals. An oscillator with a reduced high-frequency stability is sensitive to noise at frequencies much higher than the oscillator frequency and is likely to oscillate parasitically. For a high performance of the oscillator an optimal use of the limited number of active stages is essential. Negative feedback theory gives the design considerations for such optimal use [ref.1]. Firstly active elements should be selected with high values for their transfer parameters. Secondly these elements should preferably be used in configurations without local negative feedback. And thirdly signal loss in the feedback loop as a result of unnecessary series or parallel impedances should be avoided.

The effect of finite loop gains $A\beta_o$ and $A\beta_{sc}$ is here dealt with for each loop gain separately. Though the loop gains are related to one another, as most active elements form part of both loops, the relation differs with the oscillator application. Separate treatment can suffice in practice, the more since often the effect of one loop dominates and since both loops tend to influence oscillation in the same way.
Also here we restrict ourselves to oscillators with a series resonator. The active termination of such a resonator is also here described by the alternative notation of Blackman's formula:

$$Z_a(p) = -r_{A,\infty} \frac{1 - A\beta_{sc}}{-A\beta_{sc}} \frac{-A\beta_o}{1 - A\beta_o}$$

In the evaluation of this expression we assume that both loop gains show no bandwidth limitation for frequencies smaller than the resonance frequency. This can be achieved by the use of a DC-coupled active part. The choice of such an active part avoids possible low-frequency instability and reduces the phase shift at the intended oscillation frequency. The remaining frequency dependency of the two loop gains can then accurately be described by the poles in their transfer function. The two loop gains are 'all-pole' functions that can be written as:

$$A\beta_o(p) = A_o\beta_{oo} \frac{1}{(1-p/p_1)(1-p/p_2)\cdots(1-p/p_k)} \qquad A_o\beta_{oo} < 0, \quad Re(p_n) < 0$$

$$A\beta_{sc}(p) = A_o\beta_{sco} \frac{1}{(1-p/p_1{}')(1-p/p_2{}')\cdots(1-p/p_k{}')} \qquad A_o\beta_{sco} > 0, \quad Re(p_n{}') < 0$$

A valuable figure of merit for these loop gains is the so-called loop-gain-poles product (L-P product) [ref.1]. For both loop gains these figures are defined by:

$$\omega_{no}{}^k = \left|(1-A_o\beta_{oo})p_1p_2\cdots p_k\right|$$

$$\omega_{nsc}{}^k = \left|(1-A_o\beta_{sco})p_1{}'p_2{}'\cdots p_k{}'\right|$$

This characterization of both loop gains enables an evaluation of their effect on oscillation. We examine loop gains with one or two dominant poles in their transfer; other loop gains are believed to be less practical.
The effect of a loop gain $A\beta_o$ with one dominant pole on oscillation is described by its factor in the impedance Z_a, that terminates the resonator:

$$\frac{-A\beta_o}{1 - A\beta_o} = \frac{-A_o\beta_{oo}}{1 - A_o\beta_{oo}} \frac{\omega_{no}}{\omega_{no} + p}$$

Such a loop gain causes a reduction in resonator dissipation by the finite value of $A_o\beta_{oo}$. The pole in the left half of the complex plane (LHP) at $-\omega_{no}$ is responsible for a phase shift. As we learned from previous sections a frequency error, an increased transfer of equivalent noise and another reduction in resonator dissipation are all a result of such a phase shift. For a high performance a large DC loop gain and a large L-P product are required.
In the case of a loop gain $A\beta_o$ with two dominant poles this factor resolves into:

$$\frac{-A\beta_o}{1 - A\beta_o} = \frac{-A_o\beta_{oo}}{1 - A_o\beta_{oo}} \frac{\omega_{no}^2}{\omega_{no}^2 + 2\zeta\omega_{no}p + p^2}$$

Here the same remarks with respect to $A_o\beta_{oo}$ and ω_{no} can be made. In this case, however, the damping ζ of the two closed loop poles is still a free parameter. Very low values for this damping favor an accurate resistive resonator termination. On the other hand such values reduce the high-frequency stability of the oscillator, for the termination can no longer be considered as 'open stable'. In practice some compromise must be made between the frequency error caused by a non-resistive termination and the high-frequency stability. A somewhat conservative but very practical compromise is to position the poles such that:

$$\zeta = \frac{1}{2}\sqrt{2}$$

Then a second-order maximally flat magnitude characteristic ('Butterworth') is formed. Preferably 'phantom zeros' position the poles [ref.1,7]. These zeros have the great advantage that a frequency compensation can be implemented without serious degradation of the L-P product. They are to be implemented in that part of the feedback network that determines the feedback factor β_o.
The influence of the loop gain $A\beta_{sc}$ follows from a comparable treatment. A loop gain $A\beta_{sc}$ with one dominant pole influences the impedance Z_a by the following factor:

$$\frac{1 - A\beta_{sc}}{-A\beta_{sc}} = \frac{1 - A_o\beta_{sco}}{-A_o\beta_{sco}} \frac{\omega_{nsc} - p}{\omega_{nsc}}$$

Also this loop gain causes a reduction in resonator dissipation by the finite value of $A_o\beta_{sco}$. The zero in the RHP at ω_{nsc} is responsible for a phase shift in the same direction as the shift by the closed loop poles in $A\beta_o$. High values for both the DC loop gain and the L-P product are required for an accurate oscillation.
For a loop gain $A\beta_{sc}$ with two dominant poles the same factor is given by:

$$\frac{1 - A\beta_{sc}}{-A\beta_{sc}} = \frac{1 - A_o\beta_{sco}}{-A_o\beta_{sco}} \frac{\omega_{nsc}^2 - 2\zeta\omega_{nsc}p - p^2}{\omega_{nsc}^2}$$

According to this factor, phase shift originates from two zeros, one in the LHP and one in the RHP, that both correspond to the closed loop poles of $A\beta_{sc}$. Also here a zero value for the damping ζ minimizes the phase shift. Then the closed loop poles are positioned symmetrically in the complex plane. Normally this zero damping requires some form of high frequency compensation. Preferably a phantom zero is used for this compensation. This zero, for instance at n_1, must be implemented in that part of the feedback network that determines the feedback factor β_{sc}, in such a way that:

$$Z_{A,\infty} = -r_{A,\infty}(1 - p/n_1) \qquad n_1 < 0$$

Consequently the loop gain $A\beta_{sc}$ has been changed to:

$$A\beta_{sc}(p) = A_o\beta_{sco}\frac{1 - p/n_1}{(1-p/p_1')(1-p/p_2')}$$

With this loop gain the condition for a symmetrical pole pattern reads:

$$p_1' + p_2' - \frac{A_o\beta_{sco}\, p_1'p_2'}{n_1} = 0$$

Such a compensation is demonstrated for the case that:

$$A_o\beta_{sco} = 20 \qquad p_1' = -1$$
$$n_1 = -18 \qquad p_2' = -5$$

Figure 4.13 shows the root locus without and with the compensation by a phantom zero. The effect of the compensation is best noticeable in the argument of the impedance Z_a; Figure 4.14 depicts this argument as function of frequency with and without the compensating zero.

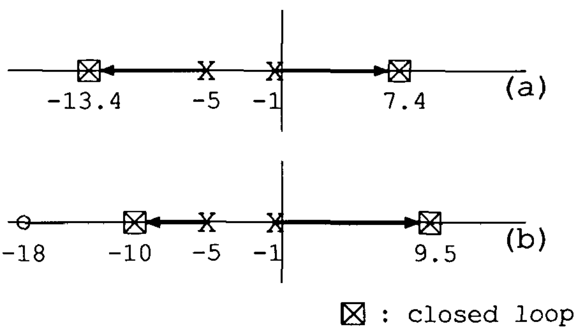

Fig. 4.13: Root loci of a second-order system with positive feedback (a: without, b: with compensation).

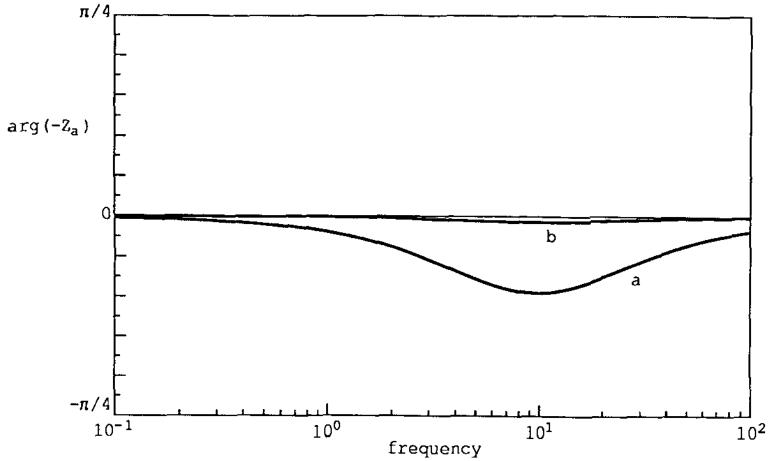

Fig. 4.14: Phase error caused by the loop gain $A\beta_{sc}$ (a: without, b: with compensation).

From this discussion we conclude that for a high performance both loop gains, $A\beta_0$ and $A\beta_{sc}$, must be optimized by large DC loop gains and large L-P products. Further that phantom zeros are to be preferred to ensure high-frequency stability and to minimize phase shift.

4.5.5 Limiting characteristics

With an infinite available power gain the limiting characteristics of the resonator terminations are ideal: during limiting the series resonator is terminated by a short circuit and the parallel resonator by an open circuit. Such ideal terminations no longer exist with a finite available power gain: during limiting the resonator is terminated by some remnant impedance.

Principally these impedances have a similar effect on oscillation as the impedance of the resonator termination in the active mode. Some remnant resistive component can cause an additional loss of power during limiting. Or during limiting a remnant undamping is still present which increases the oscillation amplitude and which causes a suboptimal use of the output capabilities of the active part. A reactive component also causes here a frequency error and consequently a reduction of frequency stability. Moreover also during limiting feedback controls the impedance of the termination and requirements with respect to high-frequency stability must still be taken into account.

The application of Blackman's formula may also yield the remnant impedance or admittance of the resonator termination during limiting. The then necessary values are to be computed by means of a small-signal analysis in the case where the resonator termination is biased in a point that corresponds to limiting operation. As an example we consider a termination of a series resonator. According to Blackman's formula the impedance Z_l of the termination during limiting is given by:

$$Z_l = \rho_l \frac{1 - A\beta_{sc,l}}{1 - A\beta_{o,l}}$$

Necessary variables are discriminated by the index l from previous ones that correspond to bias points in the active mode. Fortunately this expression can often be simplified. In the case of an infinite available power gain this value is given by $Z_{l,\infty}$. According to the previous definition (cf. Section 4.5.1) this value follows from:

$$Z_{l,\infty} = \rho_l \frac{\beta_{sc,l}}{\beta_{o,l}}$$

But in this ideal case a short circuit terminates the resonator, which corresponds to a value of $Z_{l,\infty}$ that approximates zero. The feedback network is arranged in such a way either $\beta_{sc,l}$ approximates zero or $\beta_{o,l}$ approximates infinity. A zero feedback factor is only possible with a limiter in the feedback network. Substituting this value for $\beta_{sc,l}$ in the expression for the impedance Z_l yields:

$$Z_l = \rho_l \frac{1}{1 - A\beta_{o,l}}$$

This result is identical to the result from a calculation of an impedance that has actively been lowered by the application of parallel negative feedback. Using standard negative-feedback theory one can evaluate the impedance Z_l and its consequences for the oscillation process.

In the second case the feedback factor $\beta_{o,L}$ approximates infinity. Such feedback factors are only found in oscillator configurations with active feedback by amplifiers with a widening transfer (cf. Section 3.5.5). Here a comparable situation exists. In both cases the impedance Z_l in the limiting mode is lowered by the negative loop gain $A\beta_{o,l}$. Degradation by non-ideal limiting must be minimized by optimizing the DC loop gain and the L-P product as was pointed out in the previous section.

Figure 4.15 shows a possible i-u characteristic of a series resonator termination with the discussed effect. A finite and frequency independent power gain of an active part results here in an additional damping Z_l in the limiting mode.

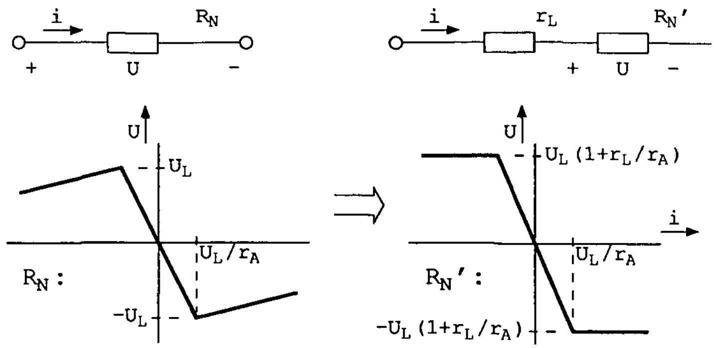

Fig. 4.15: A non-ideal termination of a series resonator and its transformation.

This non-ideal termination R_N can be thought of a series connection of a linear resistance with value r_L and an nonlinear resistance R_N' with ideal limiting characteristics. Figure 4.15 also depicts this transformation. It increases both limit level and true small-signal transfer of the nonlinear resistance with a factor $1+r_L/r_A$.

4.6 Discussion

This chapter dealt with design considerations for oscillators with an optimal stability. The calculation of frequency stability, made in Chapter II, was here applied to the oscillator configurations, which were derived in Chapter III. The calculation of frequency stability was formalized by transforming all independent noise sources in the oscillator to either noise voltage in series or a noise current in parallel with the two resonator terminals in the form of an equivalent oscillator input noise. After this transformation the stability of the oscillator output signal at the load can be calculated at the resonator. It can be calculated in the simple structure of a resonator and its corresponding, undamping two-pole and is independent of the actual active configuration between resonator and load.

From a general expression for frequency stability in these oscillators we concluded that stability is proportional to the reactive power that circulates in the resonator and is inversely proportional to the equivalent oscillator input noise. Moreover stability decreases with harder nonlinear behavior of the oscillator, since high-frequency components in the equivalent input noise are converted to phase variations in the oscillator signal.

In the case of a linear oscillator we derived that for a high stability, noise from the active components must be sufficiently low compared to the noise of the resonator. In a linear oscillator this noise can be minimized by a noise match of the undamping two-pole to the real impedance of the resonator. We revealed that in such an oscillator, minimizing the noise of the resistive feedback network is not possible without serious decrease in the reactive power of the resonator. It was shown that at a power match this feedback network minimally deteriorates stability. This network then produces the same amount of noise and dissipates the same amount of power as the resonator itself.

For negative-feedback oscillators conclusions are essentially the same. However, if the equivalent oscillator input noise partly consists of white noise, this part will have a larger contribution to the instability of the oscillator than in a linear oscillator. On a power basis the increase is equal to the excess loop gain, the true small-signal loop gain at resonance. Preferably this loop gain is as low as is compatible with component tolerances and with required suppression of amplitude noise. For low excess loop gains (<2) differences from the linear oscillator are negligible. For larger loop gains the noise match is influenced, as out of the two equivalent noise sources of the active part one is always subject to the conversion process.

The design considerations summarized above apply in the assumption that the negative-feedback oscillator is built around an active part with a very large, available power gain. These considerations aim at the ultimate accuracy and stability. Practical frequency-dependent active parts only have a limited power gain. It was shown that with these active parts the oscillation frequency differs from the resonance frequency. Moreover frequency stability is reduced by an enlarged transfer of equivalent oscillator input noise to the oscillator output and by a decreased reactive power in the resonator. As requirements with respect to high-frequency stability limit the number of stages in the active part, these stages must be optimally employed. Analysis showed oscillator performance is optimized by maximizing the feedback in the two loops that control the resonator termination: each loop should have a large DC loop gain and a large loop-gain-poles product. The value of high-frequency compensation by phantom zeros was demonstrated for oscillators that incorporate an active part with a frequency dependency of second order. These zeros were shown to be an effective means in minimizing phase errors or in optimizing the high-frequency stability of the oscillator.

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APPENDIX I

SPICE SIMULATION OF A PRACTICAL EXAMPLE

```
simulation class-C Colpitts oscillator
*ref. Introduction to Radio Frequency Design
*     W.H. Hayward
*     Prentice-Hall, 1982, pp 265-279
Q1  3 0 1   N3904
IDC 1 0     2E-3
C1  3 1     100E-12
C2  1 0     1000E-12
C3  3 0     416E-12 IC=2
RL  3 4     0.12
L1  4 0     0.5E-6
.MODEL N3904 NPN (BF=180 IS=3E-15 RE=1 RB=53 RC=10 VAF=135
+TF=300P CJE=5P VJE=1.5 MJE=0.45 CJC=3.7P VJC=0.5 MJC=0.24)
.TRAN 2.5NS 2000N 1500N UIC
*.TRAN .5NS 5300N 5000N UIC
.PROBE
.END
```

APPENDIX II

OSCILLATOR SMALL-SIGNAL TRANSFER

First the equation which truly describes relation between the in- and output signals in the block diagram of the oscillator:

$$E_{no}(f) = H(f) \left(E_{ni}(f) * \sum_n c_n \, \delta(f-nf_o) \right) + H(f) \left(E_{no}(f) * \sum_n c_n \, \delta(f-nf_o) \right)$$

An approximated version of this equation (resonator output has a band-limited character) reads:

$$E_{no}(f) = H(f) \left(E_{ni}(f) * \sum_n c_n \, \delta(f-nf_o) \right) + H(f) \left(c_o E_{no}(f) + c_2 E_{no}(f-2f_o) + c_{-2} E_{no}(f+2f_o) \right)$$

for the upper sideband:

$$E_{no}(f_o+\Delta f) = H(f_o+\Delta f) \left(\sum_n c_n E_{ni}(f_o+\Delta f-nf_o) \right) + H(f_o+\Delta f) \left(c_o E_{no}(f_o+\Delta f) + c_2 E_{no}(-f_o+\Delta f) \right)$$

for the lower sideband:

$$E_{no}(f_o-\Delta f) = H(f_o-\Delta f) \left(\sum_n c_n E_{ni}(f_o-\Delta f-nf_o) \right) + H(f_o-\Delta f) \left(c_o E_{no}(f_o-\Delta f) + c_2 E_{no}(-f_o-\Delta f) \right)$$

or rewritten for the upper sideband:

$$E_{no}(f_o+\Delta f) = H(f_o+\Delta f) \left(\sum_p c_{1-p} E_{ni}(pf_o+\Delta f) \right) + H(f_o+\Delta f) \left(c_o E_{no}(f_o+\Delta f) + c_2 E^*_{no}(f_o-\Delta f) \right)$$

or rewritten for the lower sideband:

$$E_{no}(f_o-\Delta f) = H(f_o-\Delta f) \left(\sum_p c_{1-p} E_{ni}(pf_o-\Delta f) \right) + H(f_o-\Delta f) \left(c_o E_{no}(f_o-\Delta f) + c_2 E^*_{no}(f_o+\Delta f) \right)$$

According to definitions made in Section 2.4 :

$$E_{no}\phi(f_o+\Delta f) = \frac{E_{no}(f_o+\Delta f) + E_{no}^*(f_o-\Delta f)}{2}$$

$$E_{no}a(f_o+\Delta f) = \frac{E_{no}(f_o+\Delta f) - E_{no}^*(f_o-\Delta f)}{2}$$

By manipulating the two given expressions for upper and lower sidebands we are able to find amplitude- and phase-related sidebands. The following equations are necessary for this manipulating process:

$$H(f_o+\Delta f) \cong H^*(f_o-\Delta f) \qquad \text{(see Section 2.5)}$$

$$c_o + c_2 = \frac{1}{H_o} \qquad \text{(condition for oscillation)}$$

The results of such an exercise are:

$$E_{no}\phi(f_o+\Delta f) = \frac{H_o}{j2Q\Delta f/f_o} \sum_p \frac{c_{-p-1} + c_{-p+1}}{2} E_{ni}(pf_o+\Delta f)$$

$$E_{no}a(f_o+\Delta f) = \frac{H_o}{1-(c_o-c_2)H_o + j2Q\Delta f/f_0} \sum_p \frac{c_{-p-1} - c_{-p+1}}{2} E_{ni}(pf_o+\Delta f)$$

We want to use the same kind of expressions for noise signals, but for these signals only power densities are known. We rewrite them for this purpose. We define therefore the power densities:

$$S_{ni}(f) = 2\, E_{ni}(f)\, E_{ni}^*(f)$$

$$S_{no}(f) = 2\, E_{no}(f)\, E_{no}^*(f)$$

Power densities for phase- and amplitude-related sidebands are given by the following equations:

$$S_{no}\phi(f_o+\Delta f) = \left|\frac{H_o}{j2Q\Delta f/f_o}\right|^2 \sum_p \left|\frac{c_{-p-1} + c_{-p+1}}{2}\right|^2 S_{ni}(pf_o+\Delta f)$$

$$S_{no}a(f_o+\Delta f) = \left|\frac{H_o}{1-(c_o-c_2)H_o+j2Q\Delta f/f_o}\right|^2 \sum_p \left|\frac{c_{-p-1} - c_{-p+1}}{2}\right|^2 S_{ni}(pf_o+\Delta f)$$

Considering that,

$$\left|c_{-p+1} + c_{-p-1}\right| = \left|c_{p-1} + c_{p+1}\right| \quad \text{and}$$

$$\left|c_{-p+1} - c_{-p-1}\right| = \left|c_{p-1} - c_{p+1}\right|$$

because even coefficients are real and odd ones are imaginary and considering also that the spectrum of the input noise will be approximately symmetrical with respect to each harmonic, as was mentioned in Section 2.5 and is expressed by:

$$S_{ni}(pf_o+\Delta f) \cong S_{ni}(pf_o-\Delta f)$$

the power densities of both types of sidebands can be reduced to:

$$S_{no}\phi\ (f_o+\Delta f) = \left|\frac{H_o}{j2Q\Delta f/f_o}\right|^2 \sum_{p=1}^{\infty} \left| c_{p-1} + c_{p+1}\right|^2 \frac{1}{2} S_{ni}(pf_o+\Delta f)$$

$$S_{no}a\ (f_o+\Delta f) = \left|\frac{H_o}{1-(c_o-c_2)H_o+j2Q\Delta f/f_o}\right|^2 \left[c_1^2\, S_{ni}(\Delta f) + \sum_{p=1}^{\infty} \left| c_{p-1} - c_{p+1}\right|^2 \frac{1}{2} S_{ni}(pf_o+\Delta f)\right]$$

APPENDIX III

RELATION DUTY CYCLE AND EXCESS LOOP GAIN

First we assume the limiter input, formed by the resonator output, is equal to: $E\sin(2\pi f_o t)$. The output of the limiter is a clipped sine wave. The limiter linearly amplifies its input with a factor A during a time d/(2fo) around each zero crossing. During the rest of the oscillation period its output is clipped at the limit level K. For the relation here sought for, the output at the first harmonic is important. As the Fourier coefficients a_n may describe the complete output of the limiter, the output at the first harmonic is given by $2a_1$. The relevant Fourier integral, here drawn up for only one quarter of the oscillation period, reads:

$$a_1 = 4f_o \left[\int_0^{\frac{d}{4f_o}} AE \sin(2\pi f_o t) \sin(2\pi f_o t)\, dt + \int_{\frac{d}{4f_o}}^{\frac{1}{4f_o}} K \sin(2\pi f_o t)\, dt \right]$$

Evaluating these integrals yields:

$$2\, a_1 = A\, E\, d \left[1 - \frac{\sin(\pi d)}{(\pi d)} \right] + \frac{4\, K}{\pi} \cos(\frac{\pi d}{2})$$

For oscillation the product of the first harmonic response of the limiter and the resonator transfer at resonance equals one:

$$\frac{2\, a_1}{E} H_o = 1$$

The boundary between linear amplification and clipping yields another equation:

$$2\, a_1\, A\, H_o \sin(\frac{\pi d}{2}) = K$$

Eliminating a_1 and K from the evaluated expression for a_1 with these two last equations yields the relation we were looking for:

$$d\, (1 + \frac{\sin(\pi d)}{\pi d}) = \frac{1}{A\, H_o}$$

APPENDIX IV

SMALL-SIGNAL TRANSFER OF NON-MEMORYLESS LIMITERS

As an example we will derive the non-memoryless current-to-current transfer of a bipolar differential pair which is used as a limiter. During the time-slot around each zero crossing of the large input signal, the differential pair is active and transfers small signals from the input to the output. Here we will only consider the small-signal current gain assuming that in those time-slots the limiter is current driven and loaded by a short circuit.

In order not to complicate the derivation too much we make the following simplifications:

1- The low frequency current gain i_c/i_b is infinitely high.

2- The current gain of the differential pair is approximated by the one of a single switched transistor.

3- The only parameters that are affected by the switching of the transistor current are the transconductance g and the diffusion capacitance C_{be}. Their ratio will be defined by a constant ω_T':

$$\omega_T' = \frac{g}{C_{be}} = \frac{C_{be} + C_{bc}}{C_{be}} \omega_T$$

4- The transistor bias current is non zero only during the active time-slot.

The time-dependent small-signal transfer $i_i(t) \rightarrow i_o(t)$ is given by the differential equation:

$$i_i = \frac{1}{\omega_T'} \frac{d\, i_o}{dt} + C_{bc} \frac{d}{dt} \left(\frac{i_o}{g}\right)$$

After an integration with respect to the time variable, this equation can be rewritten to:

$$\omega_T' \int_{-\infty}^{t} i_i \, dt = i_o + \frac{\omega_T' C_{bc}}{g} i_o$$

After Fourier transformation $[x \rightarrow \mathcal{F}(x)]$ it follows that:

$$\frac{\omega_T'}{j\omega} I_i \quad * \quad \mathcal{F}\left(\frac{g}{g + \omega_T' C_{bc}}\right) \quad = \quad I_o$$

The input current i_i is first amplified, and thereby low pass filtered. Subsequently it is transferred by a memoryless limiter to form the output current i_o. The memoryless limiter has a time-dependent small-signal transfer, which is dominated by the transconductance g, varied by the switching transistor current.

Let, for instance, outside the active time-slots g=0. The large input signal together with the large-signal gain of the differential pair will set the time-slot to a value $d/(2f_o)$, in which d is the duty-cycle and $1/(2f_o)$ the period between two subsequent zero crossings. With this information we are capable of finding the following Fourier transform of the time-dependent small-signal transfer of the mentioned memoryless limiter:

$$\mathcal{F}\left(\frac{g}{g + \omega_T' C_{bc}}\right) = \frac{C_{be}}{C_{be} + C_{bc}} \; d \sum_{\substack{n=-\infty \\ \text{even}}}^{\infty} \frac{\sin(n\pi d/2)}{n\pi d/2} \; \delta(f - nf_o)$$

In such a case a non-memoryless limiter behaves for small signals as a cascade of a low pass filter and a memoryless limiter with a duty cycle determined by the large signal characteristics of the non-memoryless limiter. The complete transfer in this example is given by:

$$\frac{\omega_T'}{j\omega} I_i \quad * \quad \frac{C_{be}}{C_{be} + C_{bc}} \; d \sum_{\substack{n=-\infty \\ \text{even}}}^{\infty} \frac{\sin(n\pi d/2)}{n\pi d/2} \; \delta(f - nf_o) \quad = \quad I_o$$

SOME OSCILLATOR IMPLEMENTATIONS

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(71) Applicant: N.V. Philips' Gloeilampenfabrieken
Groenewoudseweg 1
NL-5621 BA Eindhoven(NL)

(72) Inventor: Boon, Cornelis Alexander Maria
c/o INT. OCTROOIBUREAU B.V. Prof. Holstlaan 6
NL-5656 AA Eindhoven(NL)
Inventor: Nordholt, Ernst Hugo
c/o INT. OCTROOIBUREAU B.V. Prof. Holstlaan 6
NL-5656 AA Eindhoven(NL)

(74) Representative: Minczeles, Roger et al
INTERNATIONAAL OCTROOIBUREAU B.V.
Prof. Holstlaan 6
NL-5656 AA Eindhoven(NL)

(54) Oscillator circuit.

(57) An oscillator circuit comprising a differential amplifier (1) as well as a negative feedback network (6-11) and a positive feedback network (2-5) which are connected to the amplifier. One of these networks determines the frequency of the generated oscillation. For obtaining a constant amplitude for the oscillation the negative feedback network comprises a current limiter (7-11) active as a controllable one-port network which is connected between the negative feedback input of the differential amplifier and a fixed potential.

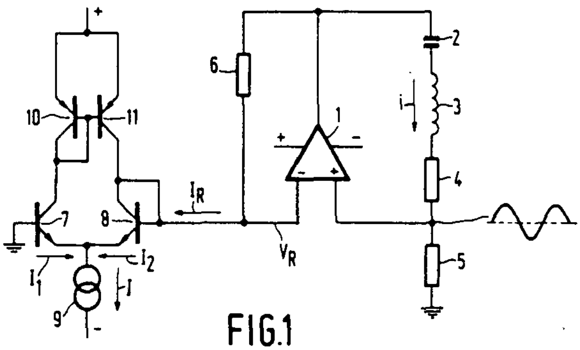

FIG.1

EP 0 261 714 A1

Oscillator circuit

The invention relates to an oscillator circuit comprising a differential amplifier having a negative feedback input, a positive feedback input and an output, a first feedback network connected between the output and one of the inputs, which network determines the frequency of the generated oscillation, a second feedback network connected between the output and the other input, and a limiter element forming part of one of the feedback networks.

An oscillator circuit of this type is known from French Patent Specification 2,044,275. For generating an oscillation at a constant amplitude a part of the voltage at the output of the differential amplifier in this known circuit is fed back to the non-inverting (the positive feedback) input of the amplifier. The frequency-determining network is a rejection circuit which forms part of a feedback network connected to the inverting (the negative feedback) input of the differential amplifier. At the resonant frequency of the rejection circuit the negative feedback is at a minimum, thus oscillation is produced.

In this circuit the positively fed-back voltage is limited by means of Zener diodes. The voltages across these diodes must be equal and must remain equal in spite of possible variations which may be caused by tolerances, ageing, temperature fluctuations, etc. Moreover, these voltages are not easily controllable.

It is an object of the invention to provide an oscillator circuit of the type described above in which the limiter is formed in a simple manner as an accurately controllable element. To this end an oscillator circuit according to the invention is characterized in that the feedback network of which the limiter element is part is a negative feedback network, the limiter element being a controllable current limiter active as a controllable one-port network which is connected between the negative feedback input of the differential amplifier and a fixed potential.

The invention is based on the recognition that the limit value of the current can be adjusted in a simple and accurate manner by means of such a current-limiting element, whilst the one-port network according to the invention can be incorporated in an integrated circuit.

The limiter element may be a non-linear resistor through which a current flows in opposite directions at substantially the same intensity during the two halves of the period of the generated oscillation. The current through the non-linear resistor may be adjustable for adjusting the amplitude of the generated oscillation.

In a simple embodiment an oscillator circuit according to the invention is characterized in that the non-linear resistor is in the form of a first and a second transistor whose emitters are coupled together and to an adjustable current source, whilst the base of the first transistor is connected to the fixed potential and the base and the collector of the second transistor are coupled together and to the negative feedback input of the differential amplifier, whilst the two collectors are connected to a current mirror circuit. It is to be noted that such a configuration is known per se from United States Patent 3,761,741, but not as part of an oscillator.

In a first embodiment the frequency-determining network comprises a series resonant network connected between the output and the positive feedback input of the differential amplifier and a first resistor connected between the same input and the fixed potential, whilst the second feedback network comprises a second resistor connected between the output and the negative feedback input of the differential amplifier. In this case the sinusoidal current flowing through the series resonant network is the output signal of the circuit.

In a second embodiment the frequency-determining network comprises a parallel resonant network connected between the positive feedback input of the differential amplifier and the fixed potential, whilst the series arrangement of two impedances is connected between the two inputs of the differential amplifier and a load is provided between the output of the differential amplifier and the junction point of the impedances. In this case the current through the load may be the output signal of the circuit.

In a third embodiment of the frequency-determining network comprises a parallel resonant network connected between the output and the negative feedback input of the differential amplifier, whilst the second feedback network comprises a first impedance connected between the output and the positive feedback input of the differential amplifier and a second impedance connected between the same input and the fixed potential. In this case the voltage prevailing between the output of the differential amplifier and the fixed potential is the output signal of the circuit.

An oscillator circuit according to the invention being in the form of a semiconductor body is preferably characterized in that all said elements of the circuit, except for reactive elements which form part of the frequency-determining network, are integrated in the semiconductor body. As a rule, the frequency-determining network comprises inductances and capacitors , but all other components of the oscillator circuit are easily integratable.

The invention will be described in greater detail by way of example with reference to the accompanying drawings. In these drawings:

Fig. 1 shows a first embodiment of an oscillator circuit according to the invention:

Fig. 2 shows the characteristic curve of a non-linear resistor forming part of the circuit of Fig. 1 and

Figs. 3 and 4 show second and third embodiments of an oscillator circuit according to the invention with the said non-linear resistor forming part thereof.

In Fig. 1 the reference numeral 1 denotes a differential amplifier having a high gain factor. The series arrangement of a capacitor 2, an inductance 3 and a resistor 4 is connected between a non-inverting input and an output amplifier 1, whilst resistor 4 comprises the losses of capacitor 2 and inductance 3. A resistor 5 is connected between the same input and a fixed potential, for example, ground and a resistor 6 is disposed between the inverting input and the output of amplifier 1. Finally, a circuit to be described hereinafter is connected between the inverting input and ground, which circuit will be tentatively denoted as a resistor R. For its supply amplifier 1 is connected to a positive and a negative voltage.

It is apparent from the foregoing that during operation a part of the voltage at the output of amplifier 1 is negatively fed back to the inverting input, which part depends on the values of resistors 6 and R and that another part of the output voltage is positively fed back to the non-inverting input, which other part depends on the impedance of the series arrangement 2, 3, 4 and on the value of resistor 5. The (small) difference between the positively fed-back voltage and the negatively fed-back voltage is amplified by amplifier 1 for supplying the output voltage. At the series resonant frequency of the series network 2, 3 the value of the positively fed-back voltage is highest. It will be evident that in a suitable design in which the gain factor of amplifier 1 is also one of the parameters the described circuit is an oscillator. A sinusoidal current at the said frequency flows through the series arrangement 2, 3, 4 and due to the high gain of the amplifier 1 substantially the same current flows through resistor 5, whereas substantially no current flows to the non-inverting input.

The properties of the oscillator are determined to a great extent by the passive circuit elements which must therefore be accurate. The active part of the circuit, i.e. amplifier 1, may be inaccurate but it must have much power amplification for generating a considerable current through the series arrangement. For satisfactorily fixing the amplitude of this current, resistor R must be an accurate non-linear element which thus functions as a limiter. In order not to influence the frequency of the oscillator this element must also be frequency-independent, in other words, it must indeed behave as a resistor.

For realising resistor R, the emitters of two non-transistors 7 and 8 are connected together and to a current source 9 which is connected to a negative supply voltage at the other end. The base of transistor 7 is connected to ground and that of transistor 8 is connected to the collector and to the junction point of resistor 6 and the inverting input of amplifier 1. The collector of transistor 7 is connected to the base and to the collector of a pnp-transistor 10 whose emitter is connected to a positive supply voltage. The emitter of a further pnp-transistor 11 is connected to the said positive voltage whilst the base is connected to the base and the collector of transistor 10 and the collector is connected to the base and the collector of transistor 8.

The current I of source 9 is equal to the sum of the emitter current I_1 of transistor 7 and the emitter current I_2 of transistor 8. Transistors 10 and 11 constitute a current mirror circuit so that the collector current of transistor 11 is substantially equal to current I_1. Thus a current $I_R = I_2 - I_1$ flows to the base of transistor 8. Due to the high gain factor of amplifier 1 the two input voltages thereof are little different. The voltage V_R at the inverting input thus has a shape which is the same as that of the current i through the resonant network 2, 3, 4, i.e. sinusoidal. During the positive half of voltage $V_R$transistors 7, 10 and 11 do not convey current and current I_1 is zero so that $I_2 = I$. Current I_R is equal to I and flows through the collector-emitter path of transistor 8 to source 9. During the negative half of voltage $V_R$transistor 8 does not convey current and current I_2 is zero so that $I_1 = I$. Current I_R is equal to -I and flows from the collector of transistor 11 to the junction point of resistor 6 and the inverting input of amplifier 1. It is evident therefrom that the circuit with elements 7 to 11 behaves as a non-constant resistor

$$R = \frac{V_R}{I_R}$$

between the said input and ground, the potential level of source 9 being of no importance in view of the high impedence of this source. A square-shaped current whose intensity is I flows through resistor R. It flows in the given direction when current i flows to ground and it flows in the opposite direction when current i flows to the output of amplifier 1, even if voltage V_R varies sinusoidally. Fig. 2 shows the current-voltage characteristic curve of resistor R in this embodiment by means of a solid line. Transistors 7 and 8 constitute a switching differential amplifier so that the voltage range of the characteristic curve in which I_R is not equal to I and -I is very narrow.

Due to the high gain of amplifier 1 substantially the same current as through resistor R flows through resistor 6. The voltage across this resistor is thus square-shaped at the same frequency as current i. It is apparent therefrom that the voltage across the series arrangement 2, 3, 4 is also square-shaped. The amplitude of this voltage is proportional to current I and to the value of resistor 6. The same applies to the amplitude of current i. By adjusting the value of current I, for example, because source 9 is formed with a transistor whose base voltage is adjustable, the amplitude of current i can therefore be adjusted to a substantially constant value. This amplitude may be rendered variable in a desired manner in that current I undergoes the same variation. Thus, current i can be amplitude-modulated, the envelope having a lower frequency than the frequency of the oscillator described because current I is varied at this lower frequency.

The foregoing implies that the sinusoidal current i is the output signal of the oscillator. By suitable design a large power can be obtained with the circuit described. For the output signal of the oscillator it is also possible to choose the sinusoidal voltage which is present across resistor 5 or the square-shaped voltage which is present across a winding coupled to inductance 3, provided, however, that a load connected to resistor 5 or to the said winding, respectively, does not adversely affect or hardly affects the value of the resonant frequency and/or the constancy shape of the amplitude.

Fig. 3 shows a modification of the circuit of Fig. 1 with the same elements 1,2 and 3 as in Fig. 1. In Fig. 3 inductance 3 and capacitor 2 constitute a parallel resonant circuit with a resistor 4' comprising the losses of the circuit connected parallel thereto. Elements 2, 3 and 4' are connected between the non-inverting input of amplifier 1 and ground. The circuit formed by elements 7 to 11 and designated by a resistor R in Fig. 3 is connected between the inverting input of amplifier 1 and ground. Furthermore, an impedance 12 is connected to the inverting input and an impedance 13 is connected to the non-inverting input. Impedances 12 and 13 are connected together at the other end and to an impedance 14 which is connected at the other end to the output of amplifier 1.

During operation a part of the output voltage of amplifier 1 is positively fed back in the circuit of Fig. 3, which part is at a maximum at the parallel resonant frequency of the circuit 2, 3. In the case of a suitable design the circuit oscillates, a sinusoidal voltage at the said frequency being present at the non-inverting input of the amplifier. A voltage which is substantially equal thereto is thus present at the other input. A current I_R flows through resistor R and also through impedance 12 and has the value I determined by source 9 during the positive half cycles of the sine waveform and the value -I during the negative half cycles. Under these circumstances the voltage across impedance 12 and hence the voltage across impedance 13 is square-shaped at the frequency of the sine waveform. Square-shaped currents flow through impedances 12 and 13, the current through impedance 13 being equal to the current through impedance 12 multiplied by the ratio

$$\frac{Z_{12}}{Z_{13}}$$

of the respective values of impedances 12 and 13. A square-wave current at the same frequency as the sine waveform and at an amplitude which is equal to the amplitude of the current through impedance 12 multiplied by a factor

$$1 + \frac{Z_{12}}{Z_{13}}$$

flows through impedance 14. Impedance 14 may be a load : the square-shaped current therethrough is the output signal of the oscillator. Possible variations of the load do not affect this current. It must hold for the circuit of Fig. 3 that the ratio

$$\frac{Z_{12}}{Z_{13}}$$

is independent of the frequency. In the simplest case the reference numerals 12 and 13 designate resistors.

In the modification of Fig. 4 the parallel circuit 2, 3, 4' is disposed between the inverting input and the output of amplifier 1. Resistor R is connected between the same input and ground and an impedance 15 is connected between the non-inverting input and ground. A further impedance 16 is disposed between the non-inverting input and the output to which output the load 14 is connected. Load 14 is connected to ground at the other end. In

Fig. 4 the circuit oscillates because the negative feedback is at a minimum at the parallel resonant frequency of the circuit 2, 3. Sinusoidal voltages having the said frequency prevail across the circuit and hence across impedance 16 and consequently across impedance 15 and across resistor R, the voltage across impedance 15 being equal to the voltage across impedance 16 multiplied by the ratio

$$\frac{Z_{15}}{Z_{16}}$$

of the respective values of these impedances. The voltage at the output of amplifier 1 is also sinusoidal with an amplitude which is equal to the amplitude of the voltage across impedance 16 multiplied by a factor

$$1 + \frac{Z_{15}}{Z_{16}}$$

and is thus proportional to the current I of source 9 and to the value of resistor 4'. The voltage at the said output is the output signal of the circuit of Fig. 4 and is independent of possible variations of the value of the impedance 14 connected thereto. For this circuit it must hold that the ratio

$$\frac{Z_{15}}{Z_{16}}$$

is independent of the frequency. In the simple case in which impedances 12 and 13 (Fig. 3) and 15 and 16 (Fig. 4) are resistors, all elements of the circuit, with the exception of inductance 3 and capacitor 2 and possibly load 14, can be integrated in a semiconductor body in which the envisaged accuracy of the elements of the circuit can be achieved.

The current-limiting characteristic curve of Fig. 2 is obtained with the construction of resistor R by means of the circuit formed by elements 7 to 11, whilst the value of I is important, i.e. the value of current I_R which corresponds to a voltage V_R which is higher than a given value. It will be evident that circuits having different characteristic curves can be designed. In Fig. 2 such a characteristic curve is shown in a broken line in which the limit value I of current I_R is reached for a given value of voltage V_R and in which current I_R has a value which is lower than I for higher values of voltage V_R. It is important that the characteristic curve is odd in order to realize a symmetrical drive of the active part 1 of the circuit.

Claims

1. An oscillator circuit comprising a differential amplifier having a negative feedback input, a positive feedback input and an output, a first feedback network connected between the output and one of the inputs, which network determines the frequency of the generated oscillation, a second feedback network connected between the output and the other input, and a limiter element forming part of one of the feedback networks, characterized in that the feedback network of which the limiter element is part is a negative feedback network, the limiter element being a current limiter active as a controllable one-port network which is connected between the negative feedback input of the differential amplifier and a fixed potential.

2. An oscillator circuit as claimed in Claim 1, characterized in that the limiter element is a non-linear resistor through which a current flows in opposite directions at substantially the same intensity during the two halves of the period of the generated oscillation.

3. An oscillator circuit as claimed in Claim 2, characterized in that the current through the non-linear resistor is adjustable for adjusting the amplitude of the generated oscillation.

4. An oscillator circuit as claimed in Claim 3, characterized in that the current through the non-linear resistor undergoes a predetermined variation for modulating the amplitude of the generated oscillation.

5. An oscillator circuit as claimed in Claim 2, characterized in that the non-linear resistor is in the form of a first and a second transistor whose emitters are coupled together and to an adjustable current source, whilst the base of the first transistor is connected to the fixed potential and the base and the collector of the second transistor are coupled together and to the negative feedback input of the differential amplifier, whilst the two collectors are connected to a current mirror circuit.

6. An oscillator circuit as claimed in any one of the preceding Claims, characterized in that the frequency-determining network comprises a series resonant network connected between the output and the positive feedback input of the differential amplifier and a first resistor connected between the same input and the fixed potential, whilst the second feedback network comprises a second resistor connected between the output and the negative feedback input of the differential amplifier.

7. An oscillator circuit as claimed in Claim 6, characterized in that the sinusoidal current flowing through the series resonant network is the output signal of the circuit.

8. An oscillator circuit as claimed in any one of Claims 1 to 5, characterized in that the frequency-determining network comprises a parallel resonant network connected between the positive feedback input of the differential amplifier and the fixed potential, whilst the series arrangement of two impedances is connected between the two inputs of the differential amplifier and a load is provided between the output of the differential amplifier and the junction point of the impedances.

9. An oscillator circuit as claimed in any one of Claims 1 to 5, characterized in that the frequency-determining network comprises a parallel resonant network connected between the output and the negative feedback input of the differential amplifier, whilst the second feedback network comprises a first impedance connected between the output and the positive feedback input of the differential amplifier and a second impedance connected between the same input and the fixed potential.

10. An oscillator circuit as claimed in Claim 9, characterized in that the voltage prevailing between the output of the differential amplifier and the fixed potential is the output signal of the circuit.

11. An oscillator circuit as claimed in any one of Claims 8 and 9, characterized in that the ratio of the values of the first and the second impedance is frequency-independent.

12. An oscillator circuit in the form of a semiconductor body, the oscillator circuit being as claimed in any one of the preceding Claims, characterized in that all said elements of the circuit, except for reactive elements which form part of the frequency-determining network, are integrated in the semiconductor body.

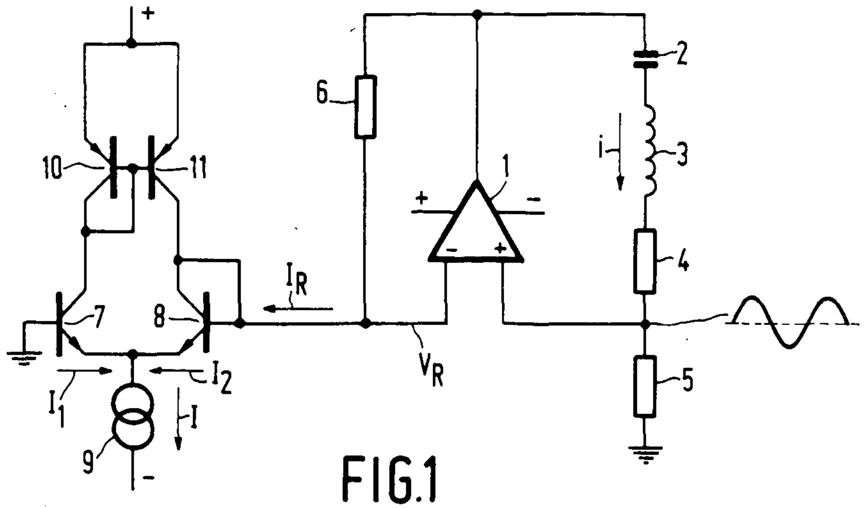

FIG.1

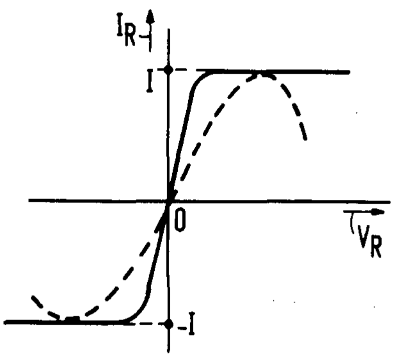

FIG.2

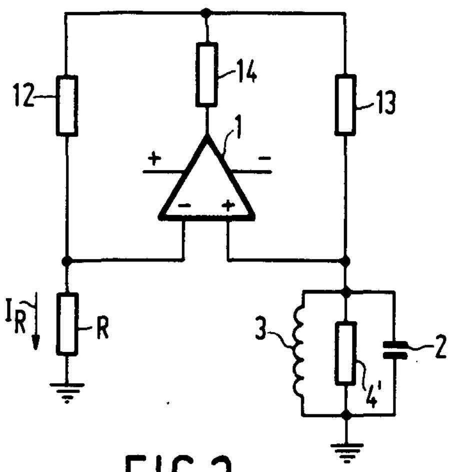

FIG.3

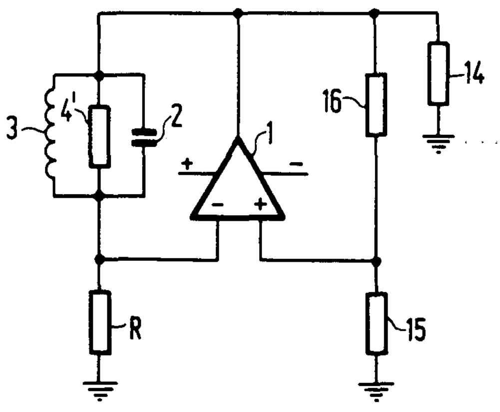

FIG.4

United States Patent [19]

Nordholt et al.

[11] **Patent Number: 4,608,544**

[45] **Date of Patent: Aug. 26, 1986**

[54] **OSCILLATOR CIRCUIT**

[75] Inventors: **Ernst H. Nordholt,** Berkel en Rodenrijs; **Cornelis A. M. Boon,** Eindhoven, both of Netherlands

[73] Assignee: **U.S. Philips Corporation,** New York, N.Y.

[21] Appl. No.: **675,110**

[22] Filed: **Nov. 27, 1984**

[30] **Foreign Application Priority Data**

Nov. 29, 1983 [NL] Netherlands 8304085

[51] **Int. Cl.**[4] **H03L 5/00**

[52] **U.S. Cl.** .... **331/109;** 331/117 R; 331/183

[58] **Field of Search** .... 331/109, 117 R, 117 FE, 331/183

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Primary Examiner—Eugene R. LaRoche

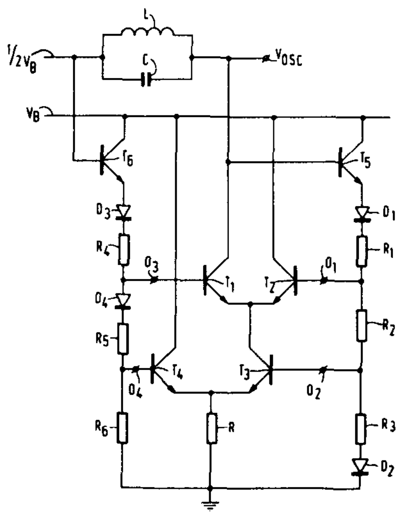
Assistant Examiner—Robert J. Pascal

Attorney, Agent, or Firm—Thomas A. Briody; William J. Streeter; Marianne R. Rich

[57] **ABSTRACT**

Oscillator circuit has a first and second transistors whose emitters are intercoupled and jointly connected to a variable current source, the first transistor being connected in a grounded-base connection. The oscillator current also has a parallel resonant circuit coupled to the collector of the first transistor, this collector being coupled for regenerative feedback of the circuit voltage to the base of the second transistor. Stabilization of the circuit voltage is accomplished by coupling the last-mentioned collector also to a control input of a variable current source, so that the collector current applied to the circuit is modulated in anti-phase with the circuit voltage, that is to say it instantaneously decreases at an increase of the circuit voltage and vice versa.

3 Claims, 8 Drawing Figures



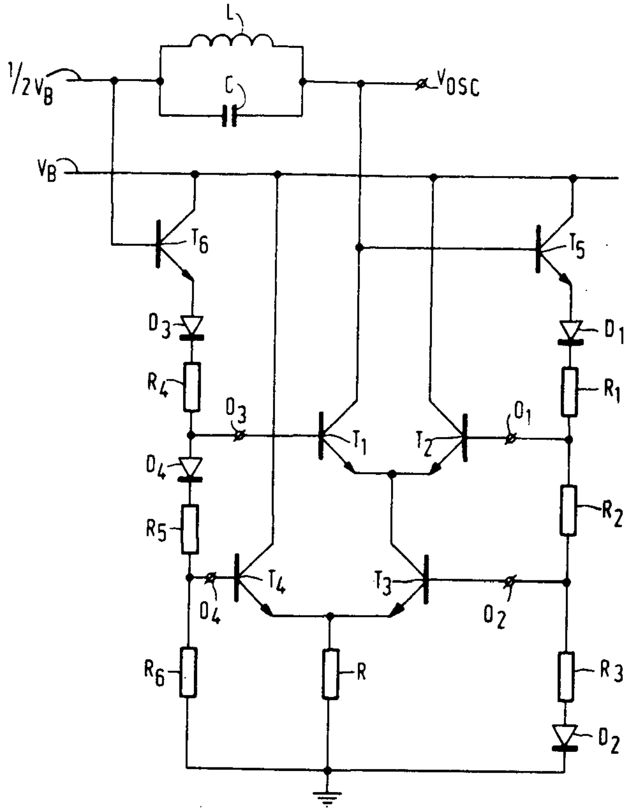

L
C
1/2 VB
VOSC
VB
T6
T5
D3
D1
R4
R1
O3
O1
T1
T2
D4
R2
R5
T4
T3
O4
O2
R6
R
R3
D2

FIG.1

OSCILLATOR CIRCUIT

BACKGROUND OF THE INVENTION

1. Field of the Invention

The invention relates to an oscillator circuit comprising first and second transistors, whose emitters are intercoupled and jointly connected to a variable current source, the first transistor being connected in grounded-base connection, also comprising a parallel resonant circuit coupled to the collector of the first transistor, the collector being coupled to the base of the second transistor for a regenerative feedback of the circuit voltage.

2. Description of the Prior Art

Such an oscillator circuit is described in the published Netherlands Patent Application No. 147294.

When the prior art oscillator circuits are produced in larger quantities, the individual circuits may mutually show spread in the amplitude of the oscillator output signal, for example due to inaccuracies in the values of the circuit elements. In addition, the oscillator amplitude may vary due to inter alia temperature fluctuations or ageing.

The influence of such factors on the oscillator amplitude can be reduced by amplitude detection of the oscillator signal and by negative feedback of the d.c. voltage signal thus obtained to the variable current source. Such an automatic gain control (AGC) for the IF-signal in AM-receivers is known from, for example, the U.S. Pat. No. 3,284,713 and is not so suitable for integration because of the comparatively large capacitance required for amplitude detection. In addition the amplitude detection when used in an oscillator circuit introduces a certain degree of control inertia, which may result in unwanted transients.

The invention has for its object to provide an oscillator circuit comprising a simple stabilizing circuit, which can be easily integrated and stabilizes the oscillator amplitude at a predetermined value without any noticeable delay and highly independent of the spread in values of the circuit elements.

According to the invention, an oscillator circuit of the type described in the opening paragraph is characterized in that said collector is also coupled to a control input of the variable current source for an instantaneous variation of the current of the variable current source in dependence on the circuit voltage.

The invention is based on the recognition that stabilizing the oscillator amplitude is possible without amplitude detection by an instantaneous, negative feedback of the circuit voltage.

Such an instantaneous negative feedback is realized by applying the measure according to the invention. Then an increase or decrease, respectively of the emitter current produced by the variable current source instantaneously coincides with a decrease or increase, respectively in the conduction of the first transistor. If the oscillator amplitude or circuit voltage variation increases, for example as a result of the above-mentioned ambient factors, then also the emitter current variations increases, which for a suitably chosen transistor setting results in an instantaneous decrease of the collector current of the first transistor applied to the resonant circuit. The reverse holds for the case in which the oscillator amplitude decreases.

Although the collector current of the second transistor varies in the same phase as the oscillator signal this does not have any effect on the circuit voltage as the second transistor only functions as an emitter follower and its collector current flows to ground without passing through the resonant circuit.

A preferred embodiment of the oscillator circuit according to the invention is characterized in that the variable current source comprises a third transistor, whose collector is connected to the intercoupled emitters of the two first and second transistors and that the collector of the first transistor is coupled via an emitter follower to a first voltage divider, a first output of which is connected to the base of the second transistor and a second output to the base of the third transistor.

When this measure is applied, an adequate setting of on the one hand a sufficiently strong positive feedback to maintain an oscillation and on the other hand a sufficiently strong negative feedback for an acceptable stabilization of the oscillator amplitude can be obtained in a simple way.

A further preferred embodiment of such an oscillator circuit is characterized in that the current source comprises a fourth transistor, whose base is connected to a fixed base-biassing voltage and whose emitter is coupled to the emitter of the third transistor, both emitters being jointly connected to a constant current source, the current thereof at the biassing voltage of the third and fourth transistors flowing to a sufficient extent through the third transistor for amplifying the circuit voltage in the first transistor, such that regeneration of the circuit voltage occurs.

When this measure is applied, the emitter currents of the first and second transistors vary more than linear, with the circuit voltage which is fed back to the base of the third transistor, as a result of which the influence of the circuit quality on the amplitude of the circuit voltage decreases still further.

A simple base voltage bias of the first and fourth transistors is obtained in a still further preferred embodiment, in which a second voltage divider is arranged between a supply voltage and ground, a first output thereof being coupled to the base of the first transistor and a second output to the base of the fourth transistor.

The invention will now be described in greater detail by way of example with reference to the Figures in the accompanying drawing.

BRIEF DESCRIPTION OF THE DRAWING

FIG. 1 shows an oscillator circuit according to the invention;

FIGS. 2A-2D show the voltage and time-dependent variations, respectively of the current of the variable current source used in the oscillator circuit of FIG. 1 and the voltage and time-dependent variations of the current applied to the resonant circuit;

FIGS. 3A-3C show the voltage and time-dependent variations, respectively of the current of a different embodiment of the variable-current source and that of the current applied to the resonant circuit.

DESCRIPTION OF THE PREFERRED EMBODIMENTS

FIG. 1 shows an oscillator circuit according to the invention comprising first and second emitter-coupled transistors T_1 and T_2, respectively with a variable current source T_3, T_4, R included in a common emitter path. The current source T_3, T_4, R comprises third and fourth emitter-coupled transistors T_3 and T_4, respectively with a high ohmic resistor R included in a com-

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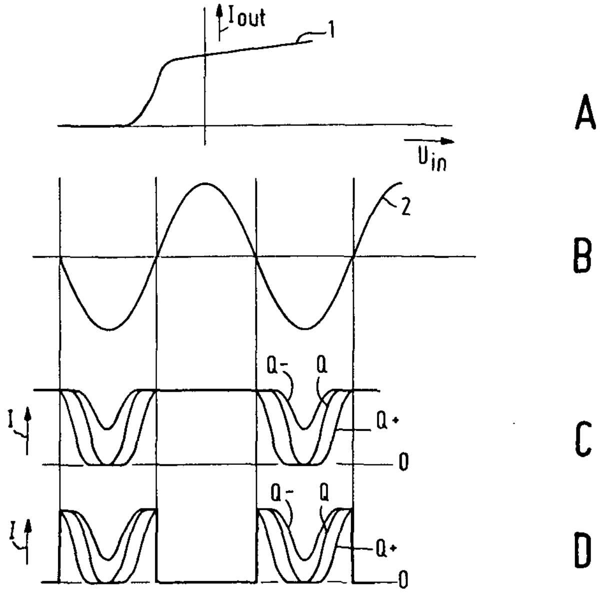

FIG.2

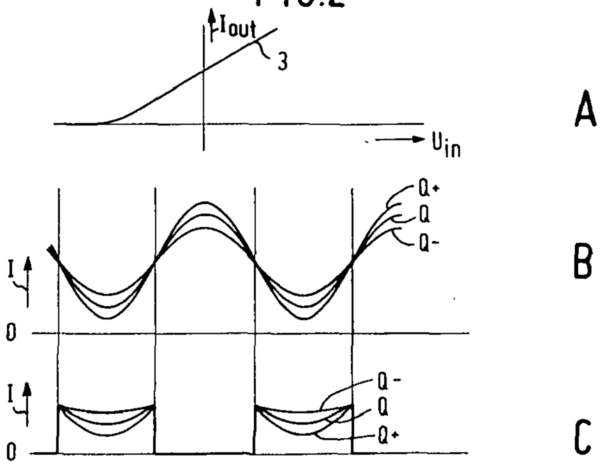

FIG.3

mon emitter path and acting as a constant-current source. The collector of the third transistor T_3 is connected to the coupled emitters of the first and second transistors T_1 and T_2. The base of this third transistor T_3 constitutes a control input of the variable current source T_3, T_4, R.

The base biasing voltages of said transistors T_1–T_4 are obtained by d.c. coupling from a supply voltage $\frac{1}{2}V_B$ via emitter followers T_5 and T_6 and first and second voltage dividers D_1, D_2, R_1–R_3 and D_3, D_4, R_4–R_6. In this situation the collectors of the two emitter followers T_5 and T_6 are connected to a supply voltage V_B. The first voltage divider D_1, D_2, R_1–R_3 comprises, arranged between the emitter output of the emitter follower T_5 and ground, a series arrangement of, in succession, a diode D_1, three resistors R_1–R_3 and a diode D_2, which has a first output O_1 located at the junction between the resistors R_1 and R_2 and connected to the base of the second transistor T_2 and a second output O_2 located at the junction between the resistors R_2 and R_3 and connected to the base of the third transistor T_3. The second voltage divider D_3, D_4, R_4–R_6 comprises, arranged between the emitter output of the emitter follower T_6 and ground, a series arrangement of, in succession, a diode D_3, a resistor R_4, a diode D_4 and two resistors R_5 and R_6, which has a first output O_3 located at the junction between the resistor R_4 and the diode D_4 and connected to the base of the first transistor T_1 and a second output O_4 at the junction between the resistors R_5 and R_6 and connected to the base of the fourth transistor T_4.

The resistors R_1 and R_4; R_2 and R_5; R_3 and R_6 have mutually identical values, as have also the diodes D_1–D_4, so that the base biasing voltages of the first and second transistors T_1 and T_2 are mutually the same, while the base biasing voltage of the third transistor T_3 is located one diode voltage step higher than the base biasing voltage of the fourth transistor T_4. As a result thereof, in the quiescent state, the third transistor T_3 is in a conducting state and the fourth transistor T_4 is non-conductive, while the first and second transistors T_1 and T_2 are mutually in balance. The collectors of the second and fourth transistors T_2 and T_4 are connected to the supply voltage V_B and therewith connected to earth for RF, so that these transistors are arranged in a grounded-collector connection. The base of the emitter follower T_6 is connected to the supply voltage $\frac{1}{2}V_B$ and therewith connected to earth for RF, so that the first transistor T_1 is arranged in a grounded-base connection. The collector of the first transistor T_1 is connected via a parallel LC-resonant circuit to the supply voltage $\frac{1}{2}V_B$ and constitutes an output V_{OSC} of the oscillator circuit. This collector is connected to the base of the emitter follower T_5, as a result of which the voltage across the resonant circuit LC or so-called circuit voltage is positively fed back to the base of the second transistor T_2 via this emitter follower T_5 and the first voltage divider D_1, D_2, R_1–R_3. The second transistor T_2 forms together with the first transistor T_1 a switching differential amplifier T_1, T_2 whose output voltage at the collector of T_1 is in phase with the input voltage at the base of T_2. This output voltage is at its maximum at the resonant frequency of the LC-resonant circuit, so that in a manner which is known per se at an adequate gain of the differential amplifier T_1, T_2, that is to say at a sufficiently large collector current of T_3 a loop gain equal to unity for said resonant frequency is reached and a spontaneous oscillator is effected from a quiescent or initial condition.

The amplitude of the circuit voltage, i.e. the oscillator amplitude depends on the quality factor of the LC-resonant circuit and the magnitude of the collector current of T_1 applied to this circuit. An instantaneous stabilization of the circuit voltage is obtained by modulating the collector current in anti-phase by the circuit voltage, that is to say by reducing the collector current instantaneously for an increasing circuit voltage and vice versa. By increasing the circuit voltage-dependent collector current variation the stabilization can be improved.

In the embodiment shown such a modulation of the collector current of T_1 is obtained because that portion of the circuit voltage being applied to the base of T_3 is in-phase with the circuit voltage portion applied to the base of T_2. As, in addition, the transistors T_1 and T_2 switch alternately and in mutual phase opposition from the conductive to the non-conductive state, the collector current of the first transistor T_1, when conducting, corresponds to the collector current of the third transistor T_3.

The third transistor T_3 forms, together with the fourth transistor T_4 a differential amplifier T_3, T_4, which in certain circumstances acts as a current limiter and whose output current varies in a highly non-linear way with the input voltage. The base biasing voltage of T_3 is one diode voltage step higher than the base biasing voltage of T_4, as a result of which on the one hand, as mentioned above, a spontaneous oscillation from the quiescent or initial condition is possible and on the other hand the range in which the collector current of T_3 has its maximum variation versus the base voltage and in which circuit voltage amplitude deviations are negatively fed back to the highest degree, has been set advantageously for the chosen use of the oscillator circuit.

Then the collector current of the third transistor T_3 varies in dependence on the base input voltage as illustrated in FIG. 2*a* by means of curve 1. Starting from a certain standardized value Q for the quality factor of the LC-resonant circuit, a sinusoidal circuit voltage having a given desired amplitude whose variation is illustrated in FIG. 2B by means of curve 2, was obtained. The variation of the collector current of the third transistor T_3 and the variation of the first transistor T_1 are illustrated by means of the respective curves Q in FIGS. 2C and 2D, respectively. A reduction of the circuit voltage, for example due to a variation of the tuning of the LC-resonant circuit because of the use of an LC-resonant circuit having a lower quality factor Q_- or due to ambient factors result in variations of the collector currents of T_3 and T_1 as illustrated by the respective curves Q_- in FIG. 2B and FIG. 2C, respectively. In response thereto the average collector current of T_1 increases instantaneously, so that the circuit voltage also increases instantaneously to the desired standardized value. The opposite situation is obtained when the circuit voltage is increased; the collector currents of T_3 and T_1, respectively varies as illustrated by curves Q_+ in FIGS. 2B and 2C, respectively. Then the average collector current from T_1 to the circuit decreases instantaneously, so that also the circuit voltage decreases instantaneously to the desired standardized value. In said practical embodiment the resistors had the following values:

R_1 and R_4: 200Ω

R_2 and R_5: 700Ω

R_3, R_6 and R: 1 KΩ.

The LC-resonant circuit was tunable between 10 and 17 MHz and had a quality factor varying between 30 and 50. Also larger variations of the quality factor, for example between 10 and 130 had hardly any noticeable influence on the oscillator amplitude, thanks to the stabilizing action of the oscillator circuit according to the invention.

It will be obvious that the invention is not limited to the embodiment shown. Thus, it is alternatively possible to omit the fourth transistor T_4 and to use only the third transistor T_3 with the resistor R as the variable current source for the differential amplifier T_1, T_2. The collector current of the third transistor T_3 then varies linearly versus the base voltage, as illustrated in FIG. 3A by means of curve 3. Starting from a circuit voltage varying as shown in FIG. 2B by means of curve 2, the collector current of T_3 and T_1, respectively varies as shown in the respective FIGS. 3B and 3C for the desired standardized value of the circuit voltage amplitude in accordance with curve Q for a certain increase in the circuit voltage amplitude in accordance with the curve Q_+ and for a certain decrease in accordance with the curve Q_-.

It should be noted that for a person skilled in the art it will not be difficult to apply the inventive idea to circuits of different types, which in essence function in the same way as the circuit shown and by using, for example, a non-switching differential amplifier T_1, T_2. It will further be obvious that the word "transistor" also includes field effect transistors, of which, as is known, the gate, source and drain electrodes, respectively correspond to the base, collector and emitter electrodes of bipolar transistors, used in this description.

What is claimed is:

1. An oscillator circuit comprising first and second transistors whose emitters are intercoupled and jointly connected to a variable current source, the first transistor being connected in a grounded-base connection, also comprising a parallel resonant circuit coupled to the collector of the first transistor, this collector being coupled to the base of the second transistor for a regenerative feedback of the circuit voltage, characterized in that said collector is also coupled to a control unit of the variable current source for an instantaneous variation of the current of the variable current source in dependence on the circuit voltage; and

wherein said variable current source comprises a third transistor whose collector is connected to the intercoupled emitters of the two first and second transistors and that the collector of the first transistor is coupled via an emitter follower to a first voltage divider a first output of which is connected to the base of the second transistor and a second output to the base of the third transistor.

2. An oscillator circuit as claimed in claim 1, characterized in that the current source further comprises a fourth transistor whose base is connected to a fixed base biasing voltage and whose emitter is coupled to the emitter of the third transistor, both emitters being jointly connected to a constant current source.

3. An oscillator circuit as claimed in claim 2, characterized in that a second voltage divider is arranged between a supply voltage and ground a first output thereof being coupled to the base of the first transistor and a second output to the base of the fourth transistor.

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A SYSTEMATIC APPROACH TO THE DESIGN OF SINGLE-PIN INTEGRATED CRYSTAL OSCILLATORS

E.H. Nordholt* and C.A.M. Boon**

Delft University of Technology
Dept. of Electrical Engineering
Mekelweg 4, 2628 CD DELFT, The Netherlands

ABSTRACT

Oscillator configurations for integrated-circuit applications where the quartz resonator can be connected between one pin of the IC and ground are presented. Excellent performance and good designability are obtained by applying negative feedback around a high-gain amplifier circuit and by including a symmetrical limiter in the feedback loop. The feedback mechanism provides an inherent buffering against load variations. A simple model for the negative impedance at the crystal port reveals the methods for avoiding parasitic oscillations due to the shunt capacitance of the crystal. The performance of one semicustom integrated oscillator in bipolar technology is simulated and measured.

1. INTRODUCTION

This paper presents a systematic and hierarchical approach to the design of reliable, stable and accurate single-pin crystal oscillators for integrated circuit applications. This design approach is based on the simultaneous application of both positive and negative feedback loops around a high-gain amplifier circuit. The feedback mechanism fixes the negative input resistance at the crystal port and provides an inherent buffering action against load variations.

It will be shown that four fundamentally different basic configurations exist. In these configurations, various methods can be used for defining the amplitude of the output signal. A very simple technique is introduced here using a symmetrical voltage or current limiter in the feedback network. Though this technique may give some degradation in noise performance, it highly favors the designability.

One oscillator type which has been integrated on a semicustom 400 MHz bipolar array will be discussed in some detail including the method that can be used for avoiding parasitic oscillations due to the shunt capacitance of the crystal.

2. BASIC CONFIGURATIONS FOR SINGLE-PIN CRYSTAL OSCILLATORS

An accurate negative resistance that compensates for the losses in a quartz resonator can be realized by applying both negative and positive feedback around a high-gain amplifier circuit. For finding suitable negative-resistance configurations we will model the amplifier circuit as a nullor, similar to negative-feedback amplifier modeling [1]. The transfer properties of a nullor are characterized by a transmission matrix with a zero value:

$$\begin{pmatrix} U_i \\ I_i \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} U_o \\ I_o \end{pmatrix} = 0.$$

By using dual-loop feedback around a nullor two out of four transmission parameters -and as a result one of the port impedances- can be fixed by the elements of the feedback network exclusively.

After making an inventory of all possible combinations with two feedback loops, we find six fundamentally different basic configurations. Table 1 lists their properties.

Configuration	Transmission parameters				Z_i	Z_o
A	A	0	C	0	A/C	0
B	0	0	C	D	0	D/C
C	A	0	0	D	AZ_ℓ/D	DZ_s/A
D	0	B	C	0	B/CZ_ℓ	B/CZ_s
E	A	B	0	0	∞	B/A
F	0	B	0	D	B/D	∞

Table 1 Transmission parameters and port impedances of dual-feedback-loop configurations. Z_ℓ and Z_s are the load- and source impedances, respectively.

In order to realize negative resistances dependent exclusively on the transmission parameters of the configuration, we obviously can use configurations A, B, E and F. One of the transmission parameters should have a negative sign and the other a positive sign.

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*E.H. Nordholt is also with
CATENA Microelectronics, Delft, The Netherlands

**C.A.M. Boon is also with
Van der Heem Electronics, Den Haag, The Netherlands

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Since for optimum accuracy the crystal should be used at its series resonance, a voltage should be forced to it and its current should be sensed and amplified. This requires negative shunt feedback at the port where the quartz resonator is to be connected. A positive feedback loop then realizes the required negative resistance.

For practical reasons only feedback networks with impedances and active devices will be considered. It should be noted that theoretically the best performance can be realized by employing nonenergic feedback networks, similar to negative-feedback amplifiers.

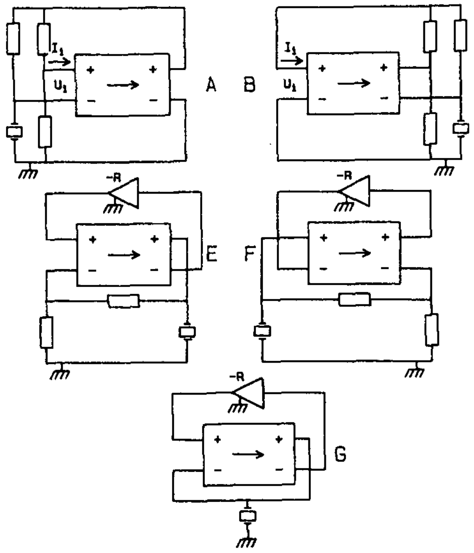

Fig. 1 Basic single-pin crystal oscillators

Figure 1 shows the four basic configurations with a one-side grounded crystal that meet the above requirements. The arrows within the nullor symbols indicate the direction of the signal transmission. Two configurations need an inverting transimpedance (labeled in the figure as -R) in one loop in order to realize the required positive feedback. Only in the first configuration the output of the nullor is grounded at one side. This means that a one-side grounded load can be connected in parallel or in series with these output terminals without any influence on the oscillation conditions. When practical amplifier circuits are used instead of the nullor, an inherent buffering is obtained. The other configurations do not have this property. The buffering action can only be obtained for floating loads.

A fifth configuration G can be derived from either configuration E or F. The negative feedback loop that fixes the voltage-gain factor and the current-gain factor, respectively, is replaced by a unity feedback loop, thereby converting -dependent on the initial point of view- either of these factors to unity.

Amplitude stabilization can be realized simply in these oscillators by using a memoryless symmetrical limiting device in one of the branches of the feedback network, thereby providing the possibility of driving a load with sine-wave or square-wave voltages or currents. Figure 2 shows some one-port and two-port implementations of symmetrical limiters together with their transfer characteristics. These limiters of course have to be inserted at the proper places in the oscillators.

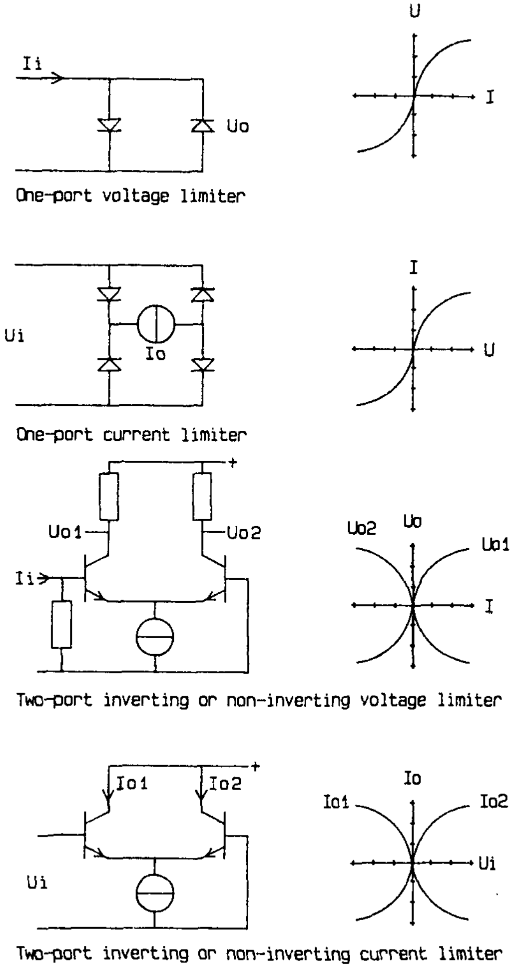

Fig. 2 Some implementations of symmetrical limiters and their transfer characteristics

The above considerations deal with the first steps in the design of suitable oscillators and form the basis for the design at the device level independent of the implementation technology. The reason why oscillators based on this concept can offer a very good performance is that the amplifier circuit can continuously operate in its linear region. This means that the crystal is always terminated by a well controlled impedance, unlike oscillators where the amplitude limiting is determined in the amplifier circuit itself.

3. DESIGN EXAMPLE OF A BIPOLAR INTEGRATED SINGLE-PIN CRYSTAL OSCILLATOR

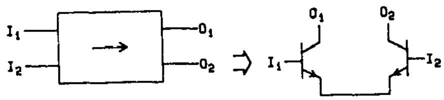

Fig. 3 Differential pair as a simple approximation of a nullor

In the implementation of high-performance oscillators, the use of lateral PNP transistors in the signal path, because of their inferior transfer properties, should be avoided. The most simple implementation of the amplifier with floating input and output ports as represented by the nullor symbol in the previous section is the differential pair as depicted in Figure 3. An even simpler alternative, however, can be used for the implementation of configuration G. Since the amplifier circuit has two terminals in common, a single transistor will suffice. With such simple amplifier circuits substantial deviations of the ideal nullor behavior will occur resulting in non-zero voltages and currents at the input of the amplifier circuit. However, oscillators with these simple amplifiers will not easily show parasitic oscillations and will offer acceptable solutions in many situations. In more critical situations two-stage amplifiers may perform better, all the more so because a noise match to the crystal can then be made virtually independent of the oscillator power.

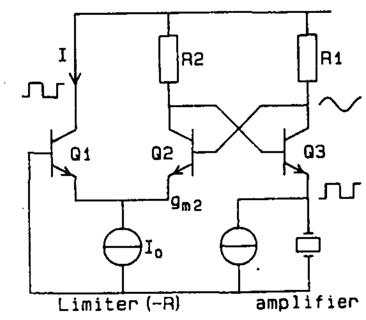

Fig. 4 Basic circuit diagram of an oscillator based on configuration G

Figure 4 shows the basic circuit diagram of an oscillator based on configuration G. The amplifier circuit consists of a single transistor and the limiter is a differential pair. A square-wave voltage at the output of the differential pair is forced on the crystal. Due to the crystal selectivity, the current in response to this voltage is a sine wave, which is converted into a sine-wave voltage at the collector of Q_3. This voltage drives the limiting differential pair.

A square-wave output current is available at the collector of Q_1. Inserting a load in this collector, hardly influences the oscillator loop and therefore the output signal is well buffered even though it is not taken from the output of the amplifier circuit.

The circuit is designed such that Q_3 operates in its linear region for values of the crystal series resistance between 50 and 100 Ω. The negative resistance at the resonator port has a value of about -200 Ω.

For the purpose of investigating the frequency behavior and the possibility of unstable behavior of the negative impedance we can advantageously use an alternative reading of Blackmans' formula [1]. It presents a driving-point impedance in terms of an ideal impedance $Z_{i\infty}$ (amplifier is a nullor) and two loop transfer functions, one with a short circuited ($A\beta_{sc}$) and one with an open input port ($A\beta_o$).

$$Z_i = Z_{i\infty} \frac{-A\beta_o}{1 - A\beta_o} \frac{1 - A\beta_{sc}}{-A\beta_{sc}} .$$

Evaluation of the loop transfer functions shows that the influence on the port impedance of the factor containing $A\beta_{sc}$ can normally be disregarded. Taking into account only the most dominant pole in the loop transfer function $A\beta_o$, we can easily estimate the phase shift in the negative impedance and determine the frequency error of the oscillator.

The parasitic capacitance of the crystal introduces a pole and a zero in the dominant pole patterns of $A\beta_o$. The positive feedback may then force the poles of the resulting impedance function into the right half plane, thereby causing parasitic oscillations that may take the place of the intended oscillation. The damping of the impedance function can be increased by increasing the capacitive load of transistor Q_3 and by selecting a low-r_b device for Q_3.

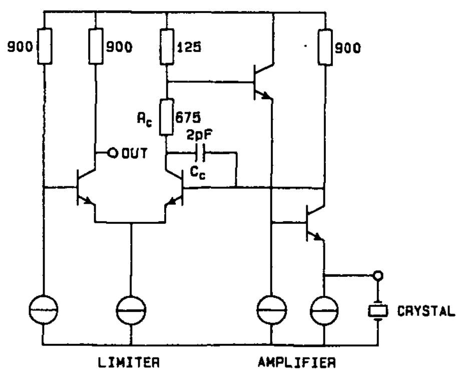

Fig. 5 Integrated single-pin crystal oscillator

The circuit of Figure 4 has been integrated with a slight modification as shown in Figure 5. It uses an additional emitter follower in the amplifier circuit in order to increase the linear output range of Q_3. In order to adequately damp the poles of the negative-impedance function in the presence of a shunt capacitance, the capacitive load of Q_3 has been increased by inserting the network C_c, R_c. In this case perfectly stable oscillations are observed with a crystal shunt capacitance up to about 30 pF.

With a value of 70 pF the circuit is on the edge of parasitic instability as may be clear from the simulated start-up behavior shown in Figure 6. In order to restrict the simulation time, an LCR circuit with a Q of 50 and a series resistance of 50 Ω has been inserted instead of a crystal.

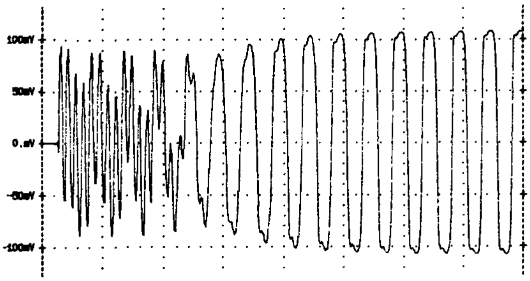

Fig. 6 Start up behavior of the oscillator of Fig. 5 with a capacitance of 70 pF in parallel with the crystal

CONCLUSIONS

We have presented four fundamentally different basic configurations of high-performance single-pin dual-feedback-loop crystal oscillators. Depending on the type of limiter used in the feedback network, they can generate sine-wave or square-wave voltages and currents. A simple model for the negative impedance at the resonator port reveals methods for avoiding instability due to parasitic shunt capacitance of the crystal. One particularly simple integrated oscillator has been discussed in some detail. Parasitic oscillations are effectively eliminated for shunt capacitances of more than 30 pF.

REFERENCES

[1] E.H. Nordholt. Design of high-performance negative-feedback amplifiers. Elsevier, Amsterdam, 1983.

SUMMARY

This dissertation describes the design of high-performance negative-feedback oscillators. These oscillators deliver a signal with an accurate frequency, waveform and amplitude to an inaccurate load. Basic theory shows that for high performance these oscillators necessarily contain a loop consisting of a passive resonating filter of which the resonance frequency acts as a reference for the frequency of oscillation, an amplifier, in order to guarantee the small-signal instability and a nonlinear element that stabilizes the oscillation amplitude.
In practical elaborations of this concept the implementation of the nonlinear element plays a key role in the realization of a certain oscillator performance. Today, in many oscillators the required nonlinearity is not deliberately designed but is left to the accidental nonlinear characteristics of the amplifying elements in the loop, which usually results in poor performance. For high performance the application of negative feedback to the amplifier in the oscillator loop is essential. So far, however, only configurations that apply slow nonlinear dynamic feedback are in use. Such feedback requires the use of special nonlinear elements, like NTC or PTC resistors, or an amplitude volume control (AVC) loop. These configurations have not proven to be very popular. Exotic nonlinear resistors or hardware-extensive AVC loops with possible dynamic instability discourage many a designer.

In this work the way in which the feedback stabilizes the transfer of the amplifier in a non-traditional way to that of a memoryless limiter is described. As the feedback isolates the inaccurate load from the oscillator loop, oscillation can be described by the transfer of the filter, which is preferably that of a resonator with low losses, and the memoryless transfer of the amplifier. This amplifier transfer avoids off-resonance operation. Moreover its small-signal transfer is chosen sufficiently large to guarantee oscillation and its limit level equals the intended amplitude of oscillation. With such a feedback arrangement all active elements can operate in class A. This enables a simple modeling of the oscillator. All signals can be calculated as though they would occur in a negative-feedback amplifier. In addition the amplitude stabilization is truly simple. A further benefit is that with the instantaneous limiting a fast tunability is possible without the

'bouncing phenomenon'. The static frequency error and distortion are obviously not minimal in this kind of nonlinear amplifier transfer. Simple modeling of these oscillators, however, shows that degradation is relatively low.

The limiter in the oscillator loop influences the transfer of independent small signals, such as noise present in the oscillator, to amplitude and phase variations in the output signal in a simple way. This transfer depends not only on the resonator characteristics but also on the excess loop gain, defined as the true small-signal loop gain at resonance controlling the increment of oscillation during startup. Compared to the linear situation with an excess loop gain of exactly one, slightly larger values only effectively decrease the transfer to amplitude variations. Much larger values must be avoided, for they cause the fold-back of high-frequency signals in the oscillator to the frequency region of oscillation, where these signals degrade frequency stability. A rule of thumb here is that for a white noise source at the input of the amplifier, the transfer to phase noise power is a factor equal to the excess loop gain higher than this transfer in the linear situation. In most cases an excess loop gain of 2 is a good compromise. The more the resonator limits the relevant noise in bandwidth, the higher are the acceptable values for the excess loop gain. With these choices of the excess loop gain the oscillator acts as though it is controlled by an almost ideal AVC; amplitude noise is strongly suppressed and phase noise is only caused by noise originated from the frequency region close to the oscillator frequency.

In negative-feedback oscillators the resonator is a simple LC circuit. In the proposed concept a series LC circuit is terminated by a current controlled negative resistance that has a limiting current-to-voltage characteristic. A voltage controlled negative conductance with a limiting voltage-to-current characteristic functions dually to terminate a parallel LC circuit. The oscillator consists of a resonator and an amplifier placed between a two-pole resonator and a two-pole load. The amplifier input acts as the required resonator termination, while the amplifier itself unilaterally transfers the resonator current or voltage to the load in the form of a voltage or current. Here four basically different configurations can be distinguished for each type of LC resonator. Two overall feedback loops around an active part with a large available power gain are able to give the amplifier the necessary qualities: feedback in series as well as in parallel with the input stabilizes the amplifier input to the required resonator termination, and series or parallel feedback at the output results in the current- or voltage-drive of the load. The nonlinear character of the resonator termination is acquired by taking an accurate memoryless nonlinear element in one of the two feedback paths. For all eight basic oscillator configurations a classification is made of the possible implementations of the feedback network. The classification shows variants of the well-known

Meacham oscillator, negative feedback versions of the Colpitts and Hartley oscillator and many other less familiar oscillators. The fewer non-energic elements the feedback network contains, the better the oscillator performance to be expected. Therefore in the feedback path the use of transformers or tapped resonators is recommended.

The calculation of the frequency stability of these oscillators is almost analogous to that of the signal-to-noise ratio in negative-feedback amplifiers. After a transformation of all independent noise sources in the oscillator to an equivalent noise source that adds up to the filtered resonator output, the input signal of the amplifier is calculated irrespective of the actual oscillator configuration. In this signal the ratio between the total power and the noise power at a certain frequency distance from the oscillation frequency defines the stability. By considering the nonlinear element in the feedback network as a linear time-varying element in these noise calculations, transformation techniques common in the design of negative-feedback amplifiers are also applicable in the design of negative-feedback oscillators. Analysis shows that both a noise match and power match are essential for an optimal stability.
Just as in a negative-feedback amplifier, the amount of power gain available from the active part sets bounds to the attainable oscillator performance. This dissertation gives expressions from which the static frequency error as well as the frequency stability can be calculated as a function of the gain in the two feedback loops. The larger these two loop gains are, the more the oscillator performance is stabilized by the accurate elements in the feedback paths.

SAMENVATTING

Dit proefschrift beschrijft het ontwerp van negatief teruggekoppelde oscillatoren van hoge kwaliteit. Deze oscillatoren leveren een signaal met een nauwkeurige frekwentie, golfvorm en amplitude aan een onnauwkeurige belasting. De basistheorie voor oscillatoren leert dat voor een hoge kwaliteit deze oscillatoren noodzakelijkerwijs een lus bevatten van een passief resonerend filter, waarvan de resonantiefrekwentie de referentie voor de oscillatiefrekwentie vormt, een versterker om de klein-signaal instabiliteit te garanderen en een niet-lineair element dat de oscillatie-amplitude stabiliseert.
In praktische uitwerkingen van dit concept speelt de implementatie van het niet-lineaire element een sleutelrol bij de realisatie van een bepaalde oscillatorkwaliteit. Tegenwoordig is de vereiste niet-lineariteit in vele oscillatoren niet speciaal ontworpen, maar wordt deze overgelaten aan de toevallig aanwezige niet-lineariteit in de versterker in de oscillatorlus, hetgeen zich doorgaans uit in een lage kwaliteit. Voor een hoge kwaliteit is de toepassing van negatieve terugkoppeling in deze versterker essentieel. Tot nu toe echter zijn slechts configuraties in gebruik die trage, dynamisch niet-lineaire terugkoppeling toepassen. Deze soort terugkoppeling vereist speciale niet-lineaire elementen, zoals NTC- of PTC-weerstanden of een amplitude regellus (AVR). Deze configuraties zijn niet bijzonder populair gebleken. Exotische niet-lineaire weerstanden of een uitgebreide AVR-lus met mogelijke instabiliteitsproblemen ontmoedigen menig ontwerper.

Hier stabiliseert de terugkoppeling de overdracht van de versterker op een niet-traditionele manier tot die van een geheugenloze begrenzer. Omdat de terugkoppeling de onnauwkeurige belasting isoleert van de oscillatorlus, kan de oscillatie beschreven worden met de filteroverdracht, die bij voorkeur wordt gegeven door die van een resonator met kleine verliezen, en de geheugenloze overdracht van de versterker. De geheugenloosheid van de versterker beoogt oscillatie op de resonantiefrekentie. Bovendien wordt zijn klein-signaal versterking voldoende groot gekozen om oscillatie te verzekeren en is zijn begrenzingsnivo gelijk aan de gewenste oscillatie-amplitude. Met een dergelijke terugkoppeling kunnen alle actieve elementen in klasse A bedrijf werken. Dit maakt een eenvoudige beschrijving mogelijk. Alle signalen kunnen worden

berekend alsof zij voor zouden komen in een negatief teruggekoppelde versterker. Verder is ook de amplitudestabilisatie eenvoudig. Een bijkomend voordeel is dat met instantane begrenzing een snelle oscillatorverstemming mogelijk is zonder amplitude-instabiliteit ('bouncing'). De statische frekwentiefout en de vervorming zijn met deze niet-lineaire versterkeroverdracht duidelijk niet minimaal. Aan de hand van een eenvoudig model kan echter worden aangetoond dat de verslechtering relatief klein is.

De begrenzer in de oscillatorlus beinvloedt de overdracht van onafhankelijke, kleine signalen, zoals de in de oscillator aanwezige ruis, naar amplitude- en fasevariaties in het uitgangsignaal op een eenvoudige wijze. Deze overdracht hangt niet alleen af van resonatorgrootheden, maar ook van de klein-signaal lusversterking, die het aangroeien van de oscillatie tijdens opstarten bepaalt. Vergeleken met de lineaire situatie, waarin deze lusversterking precies gelijk aan 1 is, verminderen enigzins hogere waarden op effectieve wijze alleen de overdracht naar amplitude-variaties. Veel hogere waarden moeten worden vermeden, omdat zij het terugvouwen veroorzaken van hoogfrekwente componenten naar het gebied rond de oscillatorfrekwentie, waar deze componenten de frekwentiestabiliteit verslechteren. Een vuistregel is hier dat met een witte ruisbron aan de ingang van de begrenzende versterker de overdracht naar het spectrum van de fasevariaties een factor gelijk aan de genoemde lusversterking hoger is dan in het lineaire geval. Meestal is een klein-signaal lusversterking van 2 een goed compromis. Hoe meer de resonator de aanwezige ruis in bandbreedte beperkt, hoe hoger de lusversterking gekozen kan worden. Met deze keuze voor de lusversterking gedraagt de oscillator zich alsof zij geregeld wordt door een ideale AVR; amplituderuis wordt sterk onderdrukt en faseruis wordt alleen veroorzaakt door ruiscomponenten met frekwenties die niet veel verschillen van de oscillatorfrekwentie.

In negatief teruggekoppelde oscillatoren wordt de resonator gevormd door een eenvoudige LC-kring. In de voorgestelde opzet wordt een seriekring afgesloten door een stroomgestuurde negatieve weerstand die een begrenzende stroom-spanningsoverdracht bezit. Duaal fungeert een spanningsgestuurde negatieve geleiding met een begrenzende spanning-stroomoverdracht als een afsluiting van een parallelkring. De oscillator bestaat uit een resonator en een versterker die tussen de tweepolige resonator en de tweepolige belasting wordt geplaatst. De ingang van de versterker vormt de vereiste resonatorafsluiting, terwijl de versterker zelf op unilaterale wijze de resonatorstroom of -spanning overdraagt naar de belasting in de vorm van een spanning of een stroom. Vier fundamenteel verschillende configuraties kunnen hier worden onderscheiden voor elk type resonator. Twee terugkoppellingen over een actief deel met een hoge beschikbare vermogensversterking geven een versterker de benodigde eigenschappen: één in serie met en één parallel aan de ingang stabiliseren de versterkeringang en beide in serie met of parallel aan de uitgang van de versterker verzorgen de stroom- of spanningsturing van de belasting. De niet-lineariteit in de resonatorafsluiting wordt verkregen door een nauwkeurig, geheugenloos niet-lineair element in één van beide terugkoppelwegen op te nemen. Voor alle acht fundamenteel verschillende configuraties wordt een classificatie gegeven van de mogelijke implementaties van het terugkoppelnetwerk. De classificatie bevat varianten van de bekende Meacham oscillator, negatief teruggekoppelde versies van de Colpitts en Hartley oscillator en vele andere minder bekende oscillatoren. Hoe meer het terugkoppelnetwerk slechts niet-energetische elementen bevat, hoe hoger in theorie de oscillatorkwaliteit kan zijn. Het gebruik van transformatoren of van resonante stroom- of spanningsdeling in het terukoppelnetwerk wordt daarom aanbevolen.

De berekening van de frekwentiestabiliteit van deze oscillatoren verloopt praktisch analoog aan die van de signaal-ruis verhouding in negatief teruggekoppelde versterkers. Na een transformatie van alle onafhankelijke ruisbronnen in de oscillator naar een equivalente ruisbron die optelt bij het gefilterde resonatorsignaal, kan het ingangsignaal van de versterker worden berekend zonder rekening te houden met de opbouw van de oscillator. In dit ingangssignaal bepaalt de verhouding tussen het totaal vermogen en het ruisvermogen op een zekere afstand van de oscillatorfrekwentie de stabiliteit. Door het niet-lineaire terugkoppelelement in deze ruisberekeningen te beschouwen als een tijdvarierend, lineair element , zijn transformatietechnieken die bekend zijn uit de ontwerptheorie voor versterkers ook toepasbaar bij het ontwerpen van oscillatoren. Analyse laat zien dat voor een optimale stabiliteit zowel een ruis- als een vermogensaanpassing op de resonator nodig is.
Evenals in een negatief teruggekoppelde versterker stelt de beschikbare vermogensversterking een beperking aan de bereikbare oscillatorkwaliteit. Dit proefschrift geeft uitdrukkingen met behulp waarvan de statische frekwentiefout en de frekwentiestabiliteit kunnen worden berekend als functie van de versterkingen in de beide terugkoppellussen. Hoe hoger deze lusversterkingen zijn deste beter worden oscillatoreigenschappen vastgelegd door de nauwkeurige elementen in het terugkoppelnetwerk.

ACKNOWLEDGEMENTS

When someone forms the intention of writing a dissertation, he is usually unaware of all it involves. There is nothing seriously wrong in this, for otherwise he would not start writing at all. However, through the process of writing he begins to formulate his theories and all the various difficulties begin to accumulate. I consider myself fortunate in that during the preparation of this thesis I was surrounded by people who were willing to endure my headstrongness, discuss my problems with me and suggest solutions to them. It is due to their support, and sometimes even in spite of my own efforts, that this work could be completed.

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ABOUT THE AUTHOR

Corlex Boon was born in Wormerveer, the Netherlands, on the 18th July, 1954. His initial introduction to electrical engineering took place during his childhood and had to do with the detailed analysis of a pocket torch. No other serious signs of an incipient technical career were then to be perceived other than his penchant for dismantling various pieces of equipment in order to examine their interiors. At secondary school he liked maths, physics and chemistry and he soon became fascinated by the mysteries of the world of electronics. He decided to study electrical engineering at the Delft University of Technology. During the course of his studies he found that many of the mysteries had been clarified, however, especially in the field of analog electronics, there were still many available to investigate. After his graduation in 1978 he became a research assistant at the Electronics Research Laboratory of the same university. His research then done on electronic oscillators is the basis of this thesis. In 1984 he joined the Philips group. Currently he is employed by Hollandse Signaalapparaten. There his main interests lie in the design of analog circuitry for customized power supplies and sonar equipment.