Velocity Analysis by Subtraction of Zero-Offset Traveltime

Milos Savic, Anton M. Ziolkowski, and Jacob T. Fokkema,
Delft University of Technology, Netherlands

SUMMARY

An algorithm for velocity analysis is presented which utilizes the nearly linear relation of offset (X) and quantity Δ, which has the dimension of time. The quantity Δ is calculated as the square root of the difference between the square of the measured traveltime and the square of the vertical (zero-offset) traveltime, namely: $[T_n^2(X) - T_n^2(0)]^{1/2} = \frac{X}{V_{rms}}$. The nearly hyperbolic reflection in X-T domain becomes a nearly linear feature in the X-Δ domain. The "line" is constrained to pass through the origin (X=0 , Δ=0), and at the origin its slope is exactly $1/V_{rms}$ The algorithm for the velocity analysis performs in a time stripping manner which makes it efficient since the volume of the data is reduced as the vertical time increases. However, by increasing the zero-offset traveltime the stretch at the beginning of the Δ trace is increased which lowers the resolution of velocities at the later traveltimes.

INTRODUCTION

Taner and Koehler (1969) showed that the square of the reflection time is related to the offset distance, assuming horizontally layered earth, in form of an infinite Taylor series:

$$T_n^2(X) = T_n^2(0) + \frac{X^2}{V_{rms}^2} + C_3X^4 + C_4X^6 + .. \qquad (1)$$

where

$T_n(X)$ - two-way traveltime for the n^{th} reflector,
X - source receiver separation,
C_j - coefficients of the series which are functions of thicknes and velocity, and
V_{rms} - rms velocity along the normal incidence (zero-offset) trajectory.

The first derivative of the series given by equation (1) with respect to X^2 , at $X^2=0$, is given by:

$$\frac{d[T_n^2(X)]}{d[X^2]} = \frac{1}{V_{rms}^2}, \qquad (2)$$

where V_{rms} is the inverse slope of the tangent to the $T^2(X) - X^2$ plot at $X^2=0$ (Figure 1 from Al-Chalabi, 1979).

The series given by equation (1), if truncated to two terms, gives the well-known formula used for obtaining velocities from multifold reflection data.

It was shown earlier (Savic et al, 1987) that the formula (1) can be rearanged in a following way:

$$[T_n^2(X) - T_n^2(0)]^{1/2} = X[\frac{1}{V_{rms}^2} + C_3X^2 + C_4X^4 + ..]^{1/2} \qquad (3)$$

The derivative with respect to X and for X=0, is equal to:

$$\frac{d\{[T_n^2(X) - T_n^2(0)]^{1/2}\}}{d[X]} = \frac{1}{V_{rms}} \qquad (4)$$

This means that the slope of the Δ versus X curve at X=0 and Δ=0 is equal to the inverse of the rms velocity.

Usually, the higher order terms of the expansion series are neglected. This approximation provides a safisfactory accuracy (Dix, 1955), while inclusion of the higher order terms introduces instability in velocity calculation (Al-Chalabi, 1973). After truncation, the series given by (3) becomes:

$$[T_n^2(X)-T_n^2(0)]^{1/2}=\frac{X}{V_{rms}}. \qquad (5)$$

The algorithm for the velocity analysis that will be described in this paper is based on the formula (5).

DESCRIPTION OF THE ALGORITHM

The theoretical, true, hyperbola is shown in Figure 2. The curve corresponds to the reflection from a single, horizontal, isovelocity layer. The zero-offset traveltime is equal to 2.210 s . The range of vertical two-way traveltimes between 2.205 and 2.214 s is chosen to search for the zero-offset traveltime that corresponds to the reflection simulation shown in Figure 2. Using equation (5), a family of Δ curves is calculated each corresponding to zero-offset traveltimes between 2.205 and 2.214 s with an increment of 1 ms. The family of curves is displayed in the X-Δ plot shown in Figure 3. Obviously the line from the family of curves that corresponds to the zero-offset time of 2.210 s passes through the origin of the plot with a slope which is the inverse of the V_{rms}.

The velocity analysis algorithm based on the formula (5) has been tested on the numerical model data shown in Figure 4. The program "slides" along the time axis and creates a X-Δ matrix for every value of T(0). During calculation $Tn(x) \geq Tn(0)$. The family of lines $\Delta_i = X_i/V_j$ is fitted to the data in X-Δ matrix and the measure of coherence (for instance, semblance) is calculated. The offset ranges from X_{min} to X_{max} and velocity ranges from V_{min} to V_{max}.

The X-Δ matrix for T(0)=0.4 s is shown in Figure 5. The slope of the line corresponds to velocity of 1500 m/s, which is the first velocity of the subsurface model.

Mapping of the traces from the X-T to the X-Δ space requires interpolation. In this case the interpolation has been carried out by use of a piecewise continuous cubic polynomial.

As the zero-offset traveltime increases, the stretch at the beginning of the Δ trace becomes larger. This is obvious from Figure 6a which represents the X-Δ matrix for the T(0)=1.880 s. The presence of the stretch lowers the resolution of velocities at the later traveltimes. The problem of stretch can be partly handled by use of a constant applied to the Δ values. The constant is calculated in a way such that frequency content of the recovered data is not increased. In other words the last sample spacing of the recovered data is set equal to the sample interval of the original data. In such a way the sample interval of the recovered data is greater or equal to the sample interval of the original data.

Figure 6b shows the same X-Δ panel shown in Figure 6a after application of a constant.

ACKNOWLEDGEMENT

The part of this research has been sponsored by the Seismic Acoustics Laboratory at the University of Houston.

REFERENCES

Al-Chalabi, M., 1973, Series approximations in velocity and traveltime computations: Geophysical Prospecting, 21, 783-795.

Al-Chalabi, M., 1979, Velocity determination from seismic reflection data, in Developments in Geophysical Exploration Methods - 1: Applied Science Publishers Ltd., London, 311.

Dix, C.H., 1955, Seismic velocities from surface measurements: Geophysics, 20, 68-86.

Savic, M., McDonald, J.A., Gardner, G.H.F., and Simpson, A.L., 1987, Minimum error zero offset traveltime method of rms velocity estimate: presented at 49 EAEG meeting.

Taner, M.T., and Koehler, F., 1969, Velocity spectra- digital computer derivation and applications of velocity functions: Geophysics, 34, 895-881.

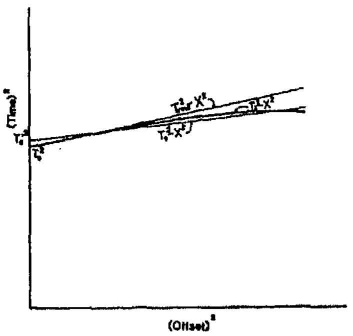

FIG. 1. (T^2-X^2) plot showing relation between traveltime curve (T_n^2-X^2), straight line best fitting curve (T_s^2-X^2), and tangent to curve at $X^2 = 0$. Note difference between $T'(0)$ and $T(0)$ (modified from Al-Chalabi, 1979).

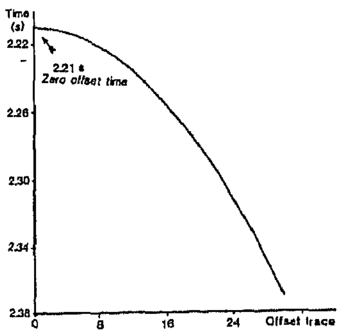

FIG. 2. Reflection hyperbola for a single horizontal layer of constant velocity. This is numerical simulation that includes zero-offset reflection time.

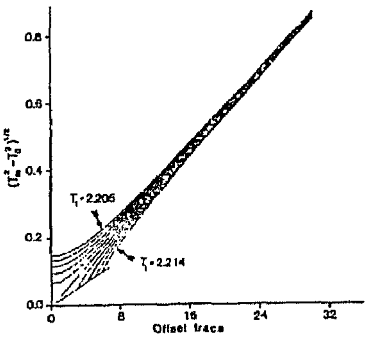

FIG. 3. Family of curves in X-Δ space generated for a range of zero-offset traveltimes between 2.205 and 2.214 s. Note straight line for correct zero-offset traveltime.

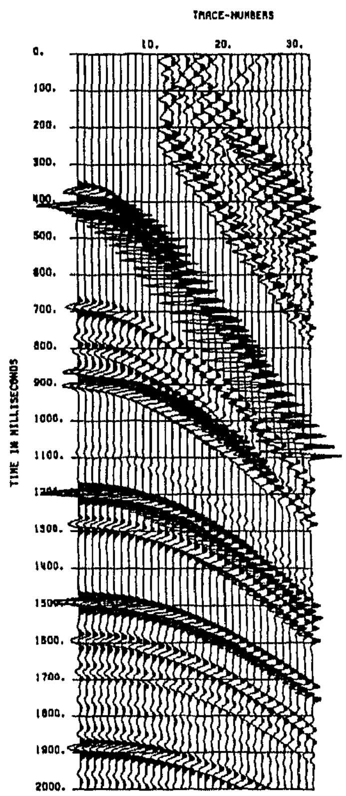

FIG. 4. Numerical simulation used as input for velocity analysis.

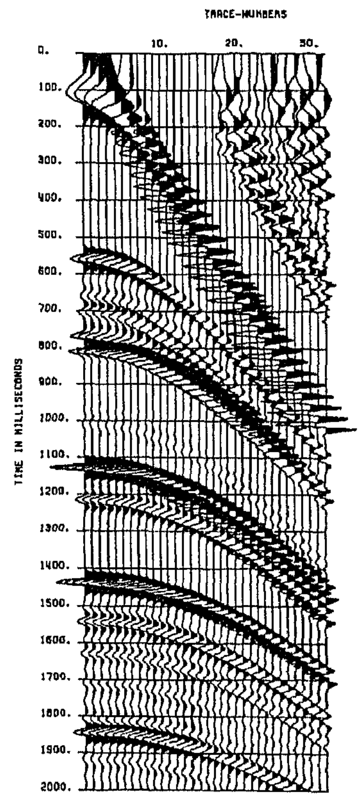

FIG. 5. X-Δ matrix calculated for $T(0)=0.4$ s. Note linear feature passing through origin.

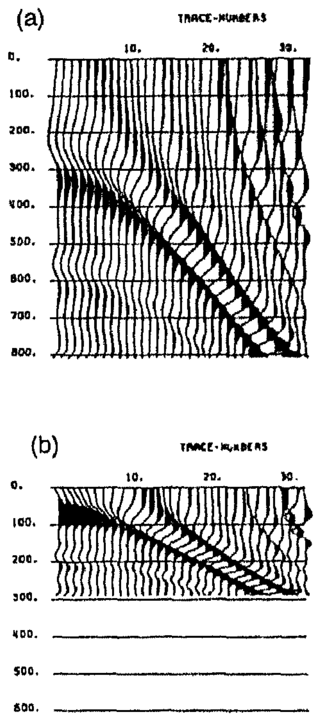

FIG. 6. (a) X-Δ matrix for $T(0)=1{,}880$ s. Note stretch. (b) Same matrix from (a) with scalar applied to Δ values.