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A Symmetric Interior Penalty Discontinuous Galerkin Method with Local Time Stepping for Anisotropic Elasticity Problems

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ABSTRACT

We present and analyse a Symmetric Interior Penalty Discontinuous Galerkin (SIPDG) method combined with an explicit hybrid Local Time Stepping (LTS) scheme for solving seismic wave problems in heterogeneous anisotropic elastic media. The discontinuous Galerkin method allows for local adjustment of the mesh size and approximation order, and can deal with unstructured non-conforming meshes of mixed types. Furthermore, when using orthogonal basis functions and an explicit time integration scheme, the method becomes fully explicit and inherently parallel, making it an attractive method for seismic applications involving large three-dimensional domains, sharp material contrasts, and detailed internal structures.

We provide fast algorithms for computing sufficient lower bounds for the penalty term as well as upper bounds for the largest eigenvalue of the stiffness matrix, ensuring stability of the numerical scheme. We also enhance the efficiency of the SIPDG method by developing a time integration method that allows time step refinement on multiple levels, such that local mesh refinement and varying slowest/fastest wave-velocity ratios do not impose unnecessary time step restrictions globally. In the absence of elastic attenuation we use a symmetric forward-backward Taylor approach, which requires about half the number of computations compared to forward Taylor approximations or Runge-Kutta methods, without any additional memory costs. The forward-backward Taylor approach is also energy conserving and can be extended to have an arbitrary order of accuracy. However, in the presence of damping, for example at absorbing boundaries or perfectly matched layers, this scheme loses its stability, so there we use a standard Taylor approach instead. The two methods are combined and extended into a multilevel local time stepping method, with only small additional costs in memory at the interface of domains that have a different time step size or Taylor approach. A variety of numerical tests illustrate the effectiveness and versatility of the proposed method.