

TU DELFT

MSC THESIS

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# The dynamic behavior of a monopile during the lowering phase through the splash zone utilizing a jack-up vessel

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## Abstract

Rising global CO<sub>2</sub> levels have created a growing need for renewable energy resources. Offshore wind energy is a promising solution, with monopile (MP) foundations preferred for their relatively low cost, provided the site conditions are suitable. Recently, wind turbines have significantly increased in size for greater cost efficiency, leading to the development of 3XL MPs with diameters of 12 meters or more. Depending on the site and marine contractor preferences, a jack-up vessel may be used to install a 3XL MP. Marine contractors typically employ dynamic models to determine suitable weather windows for each installation phase. Lowering the 3XL MP through the splash zone and gripper to just above the seabed is a critical phase. To optimize the dynamic models for this phase, understanding the 3XL MP's behavior during splash zone lowering is essential. This thesis therefore investigates the dynamic behavior of a 3XL MP during lowering phase through the splash zone until just above the seabed using a jack-up vessel.

A 1-DOF model, considering only the rotational DOF of the MP, is established to examine this behavior. Results from this model show that the most probable maximum (MPM) of the MP rotational motion response, evaluated with a dynamic lowering simulation for irregular spread waves, is 75% lower compared to the conventional steady-state simulations at critical MP drafts. Additionally, the dynamic behavior of the MP is highest for increasing peak periods, as these coincide with the natural periods of the 1-DOF model at larger MP drafts where wave-excitation loads are greatest.

Several physical phenomena which are not included in the model are examined to assess their impact. Ambient currents traveling in the same direction as irregular unidirectional waves during a dynamic lowering simulation, reduce the MP motion response by 25%. Vortex shedding frequencies do not coincide with natural periods and are thus neglected. The same goes for the 2nd order difference frequency wave drift forces. Both the dynamic amplification and the mean motion displacement due to these second order wave forces are negligibly small. Lastly, the influence of jack-up motion is negligibly small during peak periods of 8 and 9 seconds but reduces the MP rotational motion during a peak period of 7 seconds.

## **Nomenclature**

|         |                                |
|---------|--------------------------------|
| BC's    | Boundary conditions            |
| COG     | Center of gravity              |
| DOF     | Degree of freedom              |
| EB      | Euler-Bernoulli                |
| EOM     | Equation of motion             |
| FEM     | Finite element method          |
| FFT     | Fast Fourier Transform         |
| IFFT    | Inverse Fast Fourier Transform |
| IVP     | Initial value problem          |
| JONSWAP | Joint North Sea Wave Project   |
| KC      | Keulegan Carpenter             |
| MP      | Monopile                       |
| ODE     | Ordinary Differential Equation |
| PDF     | Probability Density Function   |
| SWL     | Still water level              |
| TS      | Timoshenko                     |

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# 1 Introduction

Amidst the increasing global demand for energy and the concerning rise in CO<sub>2</sub> levels contributing to global warming, there is a significant need for clean and renewable energy sources. One promising solution lies in the utilization of offshore wind turbines to harness wind energy. Research indicates that offshore wind energy extraction offers several advantages over onshore installations, including higher energy production, longer operational lifespan, and reduced visual impact (Randolph, Doan, Tang, Bui, & Dinh, 2019).

The type of foundation for an offshore wind turbine depends on the specific location, where the soil type and water depth play a dominant role. If possible, depending on the location, monopile (MP) foundations are often preferred due to their relatively low installation costs compared to alternatives such as jacket foundations or floating offshore configurations (Prakash, Ankitha, & Arvan, 2023). In 2019, globally 80% of the fixed offshore wind turbines were installed on a monopile foundation (*Offshore Wind Outlook*, 2019). This percentage is only expected to increase as the water depths in which monopiles can be utilized as a foundation for offshore wind turbines have increased. Driven by the economic benefits of larger turbines capturing wind energy more efficiently, the size of wind turbines is growing over time, and this increase is expected to continue (de Lara Garcia, 2013). Hence, the number of MP foundations with a diameter of 12m or higher, which will be referred to as 3XL MP's, is expected to increase as well (GustoMSC, 2023).

Most commonly a 3XL MP is installed using a floating vessel with a dynamic positioning system. A gripper frame is used, through which the MP is lowered such that the MP can be held in place. Preferably, the gripper can partially compensate for the MP motion (GustoMSC, 2023). See Figure 1 for the installation procedure of a 3XL MP, utilizing a floating vessel configuration. The jack-up vessel is another option to install 3XL MP's. Originally used in oil and gas extraction, jack-up vessels possess the capability to navigate to designated sites, penetrate their legs in the seabed to stabilize, and conduct operations above the water surface. This adaptability makes jack-up vessels a practical solution for installing large MP's, since there are, w.r.t. to the floating vessel, significantly less vessel motions, see Figure 2. However, a downside of the jack-up vessel for this operation is the duration of installing a large number of MP's, as the process of penetrating the jack-up legs into the soil is time-consuming and is prone to the right environmental conditions (GustoMSC, 2023). Another downside to the jack-up vessel is that there is a limiting water depth at which these vessels can jack up. Currently, MP's are installed up to water depths of 68 meters, which can be problematic for certain jack-up vessel designs (Elzinga, Stroot, & Kana, 2024).

Especially for relatively small wave periods, the floating vessel configuration for installing a MP is preferred over the same procedure on a jack-up vessel, as the vessel motions are relatively small in this situation. When installing the MP on the leeward side of a floating vessel, the vessel will also cause a shielding effect of the waves. Because of this the MP motions are even lower for short-period waves, i.e. peak periods lower than 11 seconds. For longer wave periods, the vessel motions begin to dominate the response of the structure, due to which the jack-up becomes a more prudent option for installing a MP (Li, Gao, Moan, & Ormberg, 2014).



Figure 1: floating-vessel installing a monopile (Dogger-Bank, 2024)



Figure 2: jack-up vessel installing a monopile (Wind-systems, 2023)

Before installing a MP using either a floating or a jack-up vessel, a downtime analysis for each installation phase is required. The specific phases for installing a MP primarily depend on the vessel type, deck layout, and the preference of the marine contractor. The following phases are commonly observed during the installation procedure using either a floating or a jack-up vessel configuration; upending of the MP, moving the MP in the gripper frame, MP lowering through the splash zone, hammering of the MP in the soil (GustoMSC, 2023). During each phase, engineers set limiting values to specific parameters for which this phase of the installation process can proceed. Some of these limiting parameters are; offlead or side-lead angle of the crane cable, loads in the gripper mechanism, loads on the legs of the jack-up vessel, MP inclination when the MP makes contact with the seabed, distance between the MP and the vessel. These limiting parameters, together with a set of environmental conditions can be used as an input to a dynamic model that evaluates the weather window for a specific phase of the installation procedure, see Figure 3.

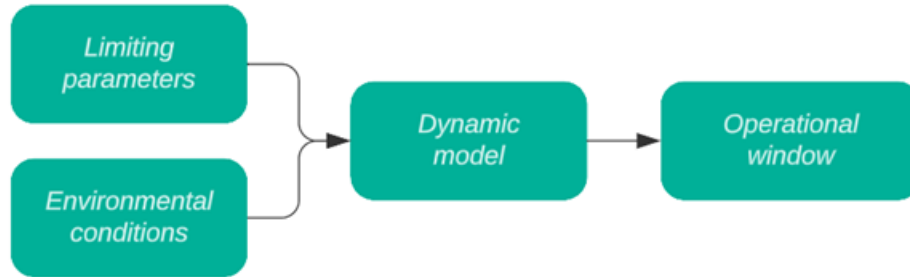


Figure 3: Establishment of a weather window

The lowering phase of the MP through the splash zone until just above the seabed influences the most limiting parameters during the installation procedure (GustoMSC, 2023). Therefore, this phase will be examined during this MSc thesis. Companies intending to install monopiles usually establish their own dynamic model to capture the dynamic behavior of a MP during the splash zone lowering phase. With these models, a downtime analysis can be performed before the installation process, to ensure that the limiting parameters are not exceeded. To properly establish a dynamic model, used for evaluating a weather window for the splash zone lowering phase of the MP, it would be convenient to know what the dynamic behaviour is of the MP during this phase, using either a jack-up vessel or a floating vessel configuration. With this insight, engineers could create more accurate dynamic models, enhancing the operational weather window for the lowering phase of the MP through the splash zone.

In (Li et al., 2014), the dynamic behavior of a MP with a diameter of 5.7m, a length of 60m, and a mass of 500 ton, during the splash zone lowering phase is examined for different sets of environmental conditions. In this paper, the MP is lowered through a gripper utilizing a floating vessel configuration, where the shielding effect of the vessel is considered. The vessel that is considered has a length of 183m, a breadth of 47m, a draught of 12m, and a displacement of 52.000 tons. The hydromechanical loads on the MP are evaluated with the Morison equation, considering the instantaneous position of the MP and the relative velocity and acceleration of the water around the MP. The model used in this paper considers that the MP lowers dynamically in time. The results are compared to the same situation on a jack-up vessel. It is concluded in this paper that for  $T_p$  values below 11 seconds, the MP motion response on a floating vessel is lower as opposed to the same response on a jack-up vessel, due to the shielding effect of the vessel. For  $T_p$  values above 11 seconds, the MP motion response on a floating vessel exceeds those same responses on a jack-up, because the floating vessel is excited closer to its rigid body natural periods.

In (Li, Gao, & Moan, 2016), the dynamic behavior of a MP with a diameter of 5.7m, a length of 60m, and a mass of 500 ton, is analyzed considering a fixed MP draft of 20m. In this paper, the MP is held in place by a gripper frame, where a floating vessel is utilized whilst considering the shielding effect of the vessel. The vessel type and the modeling of the hydromechanical loads on the MP are the same as in (Li et al., 2014). The paper examines the effect of wave spreading on the motion response of the MP. It is concluded that the spreading of waves will yield a higher MP motion response as opposed to unidirectional waves when the vessel shielding effect is considered.

In (Li, Gao, & Moan, 2015) the same process is modeled as in (Li et al., 2014), except that the shielding effect of the vessel is disregarded, and the MP is not lowered through the gripper but only held in place by the crane and crane cable. Furthermore, the hydromechanical model is also the same as in (Li et al., 2014), but now the radiation damping of the MP is added in the time domain through the utilization of the retardation function. A novel method to estimate the radiation-damping forces in the time-domain during the lowering phase of

the MP is brought up in this paper. It is concluded that the addition of radiation damping does yield a different response, and hence is relatively significant, especially for short waves. In this paper, it also becomes apparent that the motion response during a steady-state simulation is much bigger than the motion response during the dynamic lowering simulation, which implies that resonance takes some time to build up.

In (Elzinga et al., 2024) the installation method of a MP with a diameter of 10m, a length of 100m and a mass of 3000 tons on the stern side of a floating vessel is compared to installing it on the leeward side of the vessel. The loads on the MP are evaluated with Morison where the wave-shielding effect of the vessel is considered, the waves are modeled as irregular and unidirectional. In this paper, steady-state simulations at the 3 most critical MP drafts are performed from which operational weather windows are found. It is concluded from this paper that the operational limits are most exceeded for both installation sequences during long waves, i.e.  $T_p$  between 12 and 15 seconds, since these peak periods coincide with the natural period of the pile-cable subsystem and the peak of the motion RAO's of the vessel.

It is common practice to use steady-state simulations at critical MP drafts for establishing weather windows for the splash zone lowering phase of the MP (Elzinga et al., 2024) (GustoMSC, 2023). This is however a rough assumption, since during these steady-state simulation the response has all the time to build up, and will probably be higher, compared to the case where the MP lowers in time through the water, as was concluded from (Li et al., 2015). It might therefore be useful to know what the difference is in motion response of the MP during steady-state simulations at critical MP drafts, and the same response during the dynamic lowering simulation.

The discussed literature mainly examines the dynamic behaviour of 5.7m diameter MP during the lowering phase through the splash zone utilizing a floating vessel. Besides this, the dynamic behaviour of a 2XL MP with a 10m diameter, at fixed critical MP drafts utilizing a floating vessel is also examined in (Elzinga et al., 2024). For situations with larger wave-periods the jack-up vessel has however proved to be a more prudent option, opposed to the floating vessel when lowering a MP through the splash zone (Li et al., 2014). Furthermore, a 3XL MP with a 12m diameter is expected to behave differently in terms of its dynamics, compared to a smaller MP with a diameter of 5.7m, during the splash zone lowering phase. Because of the mentioned arguments, and the fact that there is a clear gap in the literature on the dynamic behaviour of a 3XL MP during the splash zone lowering phase utilizing a jack-up vessel, this thesis aims to answer the following research question.

**What is the dynamic behavior of a 3XL MP during the lowering phase through a gripper and the splash zone until just above the seabed utilizing a crane on a jack-up vessel?**

Analyzing which different physical phenomena influence the dynamics of the MP during the splash zone lowering phase, with an emphasis on the hydromechanics, will help to answer this research question. By answering the research question, companies intending to install MP's with a jack-up vessel can more accurately model the dynamic behaviour of a 3XL MP during the splash zone lowering phase. Subsequently the weather windows, which are now most commonly evaluated based on steady-state simulations at critical MP drafts, are improved such that the MP lowering phase can proceed under more severe environmental conditions.

In this report the methodology that is used to answer the research question will first be elaborated on. Hereafter, it will be explained how the multi-DOF model of the jack-up works, which is an already existing model of the jack-up and MP. Subsequently the same will be done for the 1-DOF model, which is a model that is established in this MSc thesis, build to extract data from time-simulations, that will make it possible to answer the research questions. After this the results from modal analysis on the multi-DOF model will be discussed, which will validate the assumption of using a 1-DOF model for the analysis. Hereafter the results, obtained with the 1-DOF model, and the discussion about the results are presented. And finally there will be a conclusion on the main main answers that are found during this thesis.

## 2 Methodology

In this section the methodology that is used to answer the research question will be explained. First the boundaries in which this research question will be answered will be elaborated on, since the research question on its own is very broad to interpret. Hereafter the procedure for establishing a numerical model that captures the dynamic behavior of the MP during the splash zone lowering phase utilizing a jack-up vessel will be explained. The establishment of the numerical model, and the results that will be extracted from it will make it possible to answer the research question considering the boundaries of this thesis.

### 2.1 Boundaries of this MSc thesis

GustoMSC, an offshore engineering company and industry expert, designed a specialized jack-up vessel capable of installing 3XL diameter MP's, the NG 20.000 x-class. During this MSc thesis, the research question will be answered whilst considering this GustoMSC jack-up vessel design installing MP's with the properties depicted in Table 1. The depth in which to install these MP's is considered to be 50m.

GustoMSC has developed a model that captures the dynamic behavior of the NG 20.000 x-class jack-up vessel and MP during the lowering process of the MP through the splash zone and gripper whilst the jack-up vessel is fully elevated above the sea. This model only considers steady-state simulations at fixed MP drafts, where the hydromechanical loads are evaluated based on irregular waves. The model from GustoMSC is a FEM model which has around 600 degrees of freedom (DOF), and in this thesis, this model will be referred to as the multi-DOF model.

| Parameter | Value | Unit | Description              |
|-----------|-------|------|--------------------------|
| $L_{mp}$  | 115   | m    | Length of the MP         |
| $M_{mp}$  | 3200  | ton  | Mass of the MP           |
| $D_{mp}$  | 12    | m    | Diameter of the MP       |
| $r_2$     | 6     | m    | Outer radius of the MP   |
| $r_1$     | 5.925 | m    | Inner radius of the MP   |
| $t_{mp}$  | 0.15  | m    | Wall thickness of the MP |

Table 1: MP parameters

The environmental conditions that limit the operational weather window depend on many factors such as the magnitude of the significant wave height, the peak period of the irregular sea-state, water depth, and the equipment that is utilized on the jack-up vessel. In general, however, the significant wave height plays a dominant role in establishing the weather window for which the installation of a MP can proceed, as this is a metric for the amount of energy contained in the free-surface waves (Holthuijsen, 2007). The significant wave height that is currently limiting during the installation is around 2.5m (GustoMSC, 2023). It should be noted however that this value can vary significantly depending, amongst other things on, the jack-up design, the equipment used during the installation, and the peak period of the spectrum. As will later in this report become apparent, the most commonly observed peak periods associated with a significant wave height of 2.5m are 7, 8, and 9 seconds. Furthermore, in some cases the current can be as high as  $1.5 \frac{m}{s}$ , this is not a current that is commonly observed, however, it does occur in some extreme situations and it is therefore used as a common condition within the offshore industry under which a MP can be installed (GustoMSC, 2023). Because of these two arguments the environmental conditions as depicted in Table 2 are considered during this MSc thesis. When regular waves are considered in some levels of the numerical model, a wave height of 2.5m is considered with wave periods of 7,8, and 9 seconds.

| Parameter     | Value | Unit          | Description                   |
|---------------|-------|---------------|-------------------------------|
| $H_{sig}$     | 2.5   | m             | Significant wave height       |
| $T_p$         | 7/8/9 | s             | Peak periods of irregular sea |
| $v_{current}$ | 1.5   | $\frac{m}{s}$ | current                       |

Table 2: Environmental conditions considered during this MSc thesis

## 2.2 Development of the numerical model

Later in the report, it will be shown that the first 2 modes of the multi-DOF model from GustoMSC are related to the rotational DOF of the MP in the gripper. The natural periods associated with these 2 modes are closest to the excitation periods, depicted in Table 2. To simplify matters, during this MSc thesis, the multi-DOF model will be reduced to solely the MP rotational DOF in the gripper. In this way, it is easier to keep an overview of what physical phenomena influence the dynamic behavior of the MP during the lowering phase through the gripper and the splash zone until just above the seabed. This assumption is expected to be valid since the rotational DOF is, as will later become clear, the DOF that is closest to the excitation periods from Table 2. This means that during these environmental conditions, the dynamic behavior of the MP during the lowering process through the gripper and the splash zone is governed by the rotation of the MP in the gripper node. The model that will be established in this MSc thesis, which only considers the rotational DOF of the MP in the gripper, will be referred to as the 1-DOF model.

This 1-DOF is depicted in Figure 4, where the MP can rotate around the y-axis in the gripper. The values of the fixed parameters from Figure 4, that will be used throughout this MSc thesis are outlined in Table 3. These values were established in accordance with GustoMSC, since these are the most common values for the NG 20.000 x-class jack-up design layout. Note that the green parameters from Figure 4 are kinematically related to each other since they vary as the cable length increases and the MP lowers through the water, see Equation 1 for these kinematic relations. As the cable length increases, the whole system changes, the mass moment of inertia, stiffness, damping, and wave-excitation forces w.r.t. the coordinate system with its origin in the gripper change. Later on in the report, it will be elaborated more extensively what this implies.

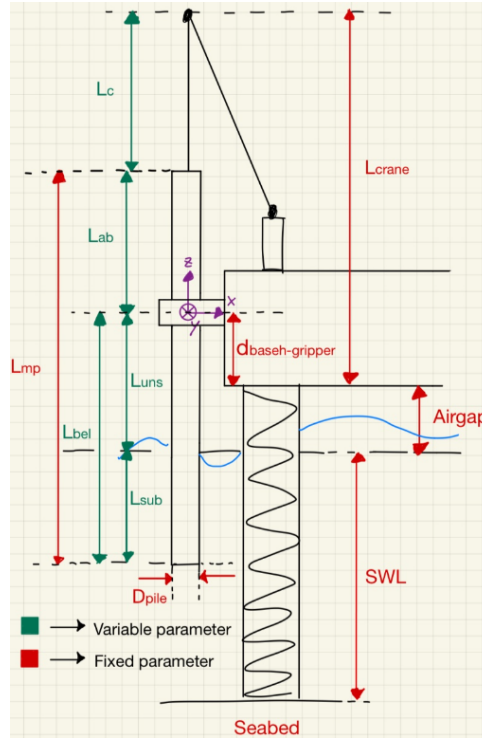


Figure 4: Sketch of the 1-DOF pile-cable model

| Parameter           | Value | Unit | Description                                                               |
|---------------------|-------|------|---------------------------------------------------------------------------|
| $L_{mp}$            | 115   | m    | Length of the MP                                                          |
| $d_{baseh-gripper}$ | 6     | m    | distance between the base of the hull and the gripper.                    |
| $Airgap$            | 10    | m    | Distance between the base of the hull and the SWL                         |
| $D_{mp}$            | 12    | m    | Diameter of the MP                                                        |
| $L_{crane}$         | 200   | m    | Distance between the base of the hull and the z-position of the crane-tip |
| $SWL$               | 50    | m    | Distance between the seabed and the SWL                                   |

Table 3: Values for the fixed parameters of the pile-cable subsystem

$$\begin{aligned}
L_{ab} &= L_{crane} - d_{baseh-gripper} - L_c \\
L_{bel} &= L_{mp} - L_{ab} \\
L_{uns} &= Airgap + d_{baseh-gripper} \\
L_{sub} &= L_{bel} - L_{uns}
\end{aligned} \tag{1}$$

A mechanical model of this 1-DOF model will later on be established. The equation of motion (EOM) related to this 1-DOF model will be solved numerically and primarily in the time-domain, by utilizing the solve IVP package from the Scipy library in Python. Furthermore, this 1-DOF model is incrementally improved, with an emphasis of enhancing the hydromechanical model, such that the motion response in the time domain of the MP during the splash zone lowering phase can be compared between different levels of the model. This ensures that it is more convenient to analyze the impact of different physical phenomena, between the different levels of accuracy of the model.

The 1-DOF model is validated by comparing the output with the output under similar conditions of the multi-DOF model used within GustoMSC. Some physical phenomena or conditions cannot be compared with the multi-DOF model, since these phenomena are simply not included in the multi-DOF model. The levels of the 1-DOF model that have these phenomena implemented are validated with hand calculations, if possible.

With the results of the different increments of the 1-DOF model it is convenient to answer what physical phenomena influence the dynamic behaviour of the MP during the lowering phase through the plash zone utilizing a jack-up vessel. By identifying what physical phenomena influence this dynamic behaviour, an answer will be given to the research question.

### 3 The multi-DOF model

As discussed previously, the combined model that is established by GustoMSC, which captures the dynamic behaviour of the jack-up and monopile (MP) during the MP installation on a jack-up vessel, is modelled in Python. The reason for this is that it is not possible to model different structural or, for that matter, hydromechanical non-linearities within, for example, a program like Orcaflex. The main reason to establish the model in Python, as opposed to orcaflex is that the dynamic lowering simulation with a diffraction model is not possible in orcaflex. In subsection 3.2, it will become apparent why it is necessary to model the hydromechanical loads on the MP with the diffraction model. This is one example of many limitations that could occur when establishing a model in commercial software such as Orcaflex, compared to creating a tailor-made Python code, and hence the software is developed in Python.

#### 3.1 The FEM model

The multi-DOF model from GustoMSC is depicted in Figure 46, it can be seen that this FEM model consists of different components that are linked to each other with nodes. All the different components are modeled with Timoshenko (TS) beam elements. In general, the EOM of this multi-DOF model can be formulated with Equation 2. In which  $\mathbf{M}'_{\text{global}}$  is the global mass matrix,  $\mathbf{A}'_{\text{global}}$  is the global added mass matrix,  $\mathbf{C}'_{\text{global}}$  is the global damping matrix,  $\mathbf{K}'_{\text{global}}$  is the global stiffness matrix,  $\vec{F}'_{\text{ext,global}}$  is the external force-vector, and  $\vec{x}$ ,  $\dot{\vec{x}}$ , and  $\ddot{\vec{x}}$  are respectively the displacement, velocity and acceleration vector containing the motion of each DOF of the model. The prime ' refers to that the global system matrices and vectors are evaluated w.r.t. the global coordinate system, of which the definition will be given later. The structural model has contributions to the mass matrix, the damping matrix, and the stiffness matrix. The hydromechanical model has contributions to the added-mass matrix, the damping matrix, and the external force vector. In reality, of course, the totality of excitation loads consists of both hydromechanical loads and other environmental loads, e.g. wind or seismic loads, if possible. Since these latter two loads are not within the scope of this thesis, only the hydromechanical loads are considered.

In order to solve for the motion velocity or acceleration response in the time-domain, the system properties of the model should be obtained, i.e., the system matrices  $\mathbf{M}'_{\text{global}}$ ,  $\mathbf{A}'_{\text{global}}$ ,  $\mathbf{C}'_{\text{global}}$ ,  $\mathbf{K}'_{\text{global}}$ , and the external force vector  $\vec{F}'_{\text{ext,global}}$ . After the system matrices are obtained, the system can be solved in the time domain. It should be noted that during the steady-state simulation, i.e., fixing the MP at a certain draft, the system matrices are constant over time. During the dynamic-lowering simulation, the system matrices become time-variant since the MP lowers through the water over time due to which the inertia, stiffness, damping, and magnitude of the forces change over time.

In Appendix A the FEM part of the multi-DOF model will be explained in detail, as this is not the main focus of this thesis. In this appendix it is explained how the FEM model is established and how the contribution from the FEM model to the global mass, stiffness and damping matrix are found. As mentioned earlier, during this MSc thesis the NG-20000 X-class GustoMSC design of the jack-up will be considered. The exact dimensions of this jack-up design will not be shared in this thesis because of confidentiality.

This thesis aims to gain insight on the dynamic behaviour of the MP during the splash zone lowering phase using a jack-up vessel, with an emphasis on the hydromechanical model. It is therefore that in this section the hydromechanical model used to evaluate the added-mass, added-damping, and wave-excitation forces for the multi-DOF model will be discussed.

$$(\mathbf{M}'_{\text{global}} + \mathbf{A}'_{\text{global}})\ddot{\vec{x}} + \mathbf{C}'_{\text{global}}\dot{\vec{x}} + \mathbf{K}'_{\text{global}}\vec{x} = \vec{F}'_{\text{ext,global}} \quad (2)$$



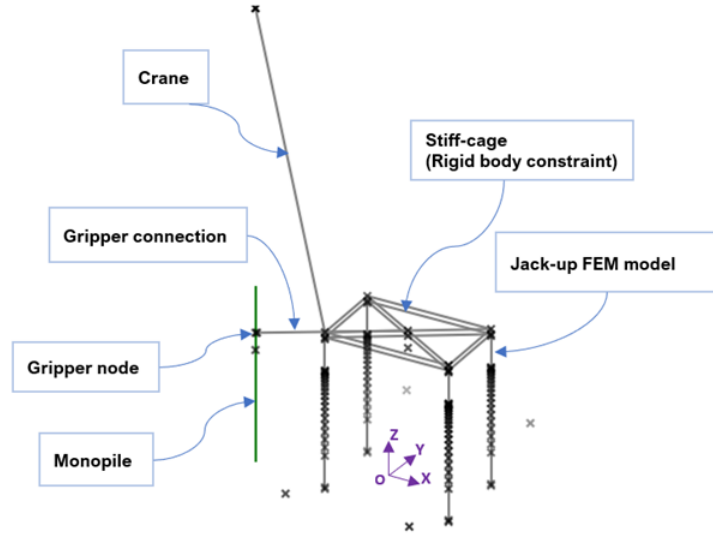


Figure 5: Combined FEM model of the jack-up, gripper, crane

### 3.2 The hydromechanical model

In A the global mass matrix  $\mathbf{M}'_{\text{global}}$ , the global stiffness matrix  $\mathbf{K}'_{\text{global}}$ , and the structural damping matrix  $\mathbf{C}_{\text{structural}}$  have been obtained. As mentioned in the appendix, the damping due to the wave-structure interaction still has to be added to the structural damping matrix to obtain the global damping matrix. Furthermore, when looking at the EOM outlined in Equation 2, the wave-excitation forces and the added mass due to the presence of the water have to be found to solve the EOM for its motion response.

The waves excite the legs of the jack-up and the MP, which is lowered through the gripper and the splash zone. Furthermore, the MP is also moving through the water, generating added damping and added mass in the form of a radiated wave. First, this subsection explains the methods to model the waves that excite both structures. Hereafter, the model used for the hydromechanical forces that act on the jack-up will be elaborated, and subsequently, the model used to evaluate the hydromechanical forces on the MP will be evaluated. Finally, the time simulation in which the EOM will be solved for the motion response will be elaborated on.

#### 3.2.1 Modeling the sea-surface

Within this thesis, two techniques are primarily used to model the sea-surface elevation. First, the most simple form of the sea-surface will be elaborated on, which is a regular wave. The second technique is a bit more sophisticated, namely describing the sea surface with irregular waves.

In Figure 6 a regular wave is depicted. The equation for the sea surface as a function of space and time for a regular wave is outlined in Equation 3. Such a regular wave has an associated wave potential from which the particle velocity and acceleration of the water and the pressure in the water also as a function of time and space can be evaluated (Journée, 2001). See Equation 4 for the formulation of the wave-potential, and see Equation 5, Equation 6, and Equation 7 for the particle-velocity due to a passing regular wave in respectively the x,y, and z direction (Journée, 2001). The acceleration of the water particles due to a passing regular wave can be found by taking the derivative of the particle velocity w.r.t. time. The pressure field due to a passing regular wave can be evaluated with Equation 8, where the first term resembles the hydrostatic pressure, and the second term the dynamic pressure from the undisturbed passing wave. This equation is derived from the linearized unsteady Bernoulli equation (Journée, 2001). In all the discussed equations,  $k$  is the wave-number and can be evaluated with Equation 9, where  $\lambda$  is the wave-length,  $\zeta_a$  is the amplitude of the wave,  $\omega$  is the radial frequency and can be evaluated with Equation 10, with  $T$  being the wave-period,  $\mu$  is the incoming wave-angle w.r.t. the x-axis as defined with Figure 6, and  $d$  is the depth of the water. Note that the purple arrow in the Figure is an example of a wave direction, just to show the meaning of variable  $\mu$ . The dispersion relation is outlined with Equation 11, this equation relates the wave length to the wave period, in simple words the longer the wave, the higher the wave period. The regular wave as depicted in Equation 3, and its corresponding wave-potential, are derived from potential theory.

$$\zeta(x, y, t) = \zeta_a \cos(kx \cos(\mu) + ky \sin(\mu) - \omega t) \quad (3)$$

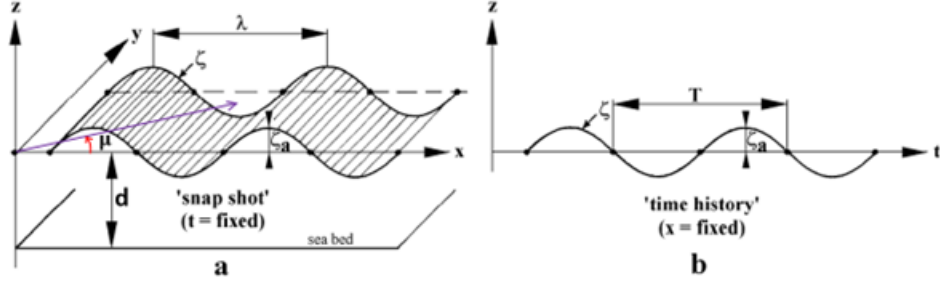


Figure 6: A regular wave (Journée, 2001)

$$\Phi_w(x, y, z, t) = \frac{\zeta_a g}{\omega} \frac{\cosh(k(d+z))}{\cosh(kd)} \cdot \sin(kx \cos(\mu) + ky \sin(\mu) - \omega t) \quad (4)$$

$$u(x, y, z, t) = \frac{\partial \Phi_w}{\partial x} \quad (5)$$

$$v(x, y, z, t) = \frac{\partial \Phi_w}{\partial y} \quad (6)$$

$$w(x, y, z, t) = \frac{\partial \Phi_w}{\partial z} \quad (7)$$

$$p(x, y, z, t) = -\rho g z + \rho g \zeta_a \cdot \frac{\cosh(k(d+z))}{\cosh(kd)} \cdot \cos(kx \cos(\mu) + ky \sin(\mu) - \omega t) \quad (8)$$

$$k = \frac{2\pi}{\lambda} \quad (9)$$

$$\omega = \frac{2\pi}{T} \quad (10)$$

$$\omega^2 = gk \tanh(kd) \approx gk \quad \text{for deep water } \tanh kd \rightarrow 1 \quad (11)$$

A more accurate technique to model free-surface gravity waves is by modeling the sea-surface with irregular waves. Irregular waves are usually represented by the variance density spectrum, i.e. the wave spectrum. In this thesis, the Joint North Sea Wave Project (JONSWAP) spectrum is used to model the wave spectrum, see Equation 12. It can be observed that the JONSWAP spectrum is dependent on the peak period or peak frequency  $T_p, f_p$ , wave frequency  $f$ , the significant wave height  $H_{sig}$ ,  $\alpha, \beta, \sigma$ , and the peak enhancement factor  $\gamma$  (ISO 19901-1, 2015). Values for  $\alpha, \beta, \sigma$ , and  $\gamma$ , can be found with Equations 13, 14, 15, 16, and 19. Note that for a single configuration of the peak period and significant wave height a unique value of the wave spectrum is obtained per wave-frequency. In this way, it is possible to obtain a wave spectrum with a unique shape, where the most energy is contained around the frequencies where the JONSWAP spectrum has the highest value, i.e. at the peak frequency.

If the peak enhancement factor, denoted with  $\gamma$ , is equal to 1, the JONSWAP spectrum will reduce to the PM spectrum, which is also known as a fully developed sea. If the peak enhancement factor is bigger than 1, more energy of the spectrum will be centered around the peak period. The peak enhancement factor is commonly estimated by DNV standards, with equation 19 (DNV RP C205, 2021).

The configuration of significant wave height and peak period differs depending on the location under consideration. However, it is known from within GustoMSC that the current average maximum significant wave height under which jack-ups can install a monopile is around 2.5m (GustoMSC, 2023). Due to physical limitations, the maximum steepness of a wave is defined with equations 17, and 18 (DNV RP C205, 2021). With these equations it can be simply evaluated that with a significant wave height of 2.5m the peak period has a minimum value of 4.899s. It is known that for a fully developed sea with a significant wave height of 2.5m the peak period of the corresponding PM-spectrum will be 7.9242s (Holthuijsen, 2007). In reality the peak period will can be smaller or bigger than a peak period corresponding to a fully developed sea. Therefore peak

periods of 7,8, and 9 seconds are used in this thesis for modeling the sea-surface with irregular waves. In this way waves with shorter, corresponding and longer lengths then wavelengths observed with a fully developed sea will be modeled such that the effect of different wave-periods and wavelengths on the dynamic behaviour of the MP can be observed.

$$E_{\zeta}(f) = \alpha H_{sig}^2 T_p^{-4} f^{-5} \exp(-1.25(T_p f)^{-4}) \gamma^{\beta} \quad (12)$$

$$\alpha = \frac{0.0624}{0.23 + 0.033\gamma - 0.185(1.9 + \gamma)^{-1}} \quad (13)$$

$$\beta = \exp\left(-\frac{(T_p f - 1)^2}{2\sigma^2}\right) \quad (14)$$

$$\sigma = \begin{cases} \sigma_a & \text{when } f \leq f_p \\ \sigma_b & \text{when } f \geq f_p \end{cases} \quad (15)$$

$$\begin{aligned} \sigma_a &= 0.07 \\ \sigma_b &= 0.09 \end{aligned} \quad (16)$$

$$S_p = \frac{2\pi H_s}{g T_p^2} \quad (17)$$

$$S_p = \begin{cases} \frac{1}{15} & \text{for } T_p \leq 8s \\ \frac{1}{25} & \text{for } T_p \geq 15s \end{cases} \quad (18)$$

$$\gamma = \begin{cases} 5 & \text{for } \frac{T_p}{\sqrt{H_{sig}}} \leq 3.6 \\ \exp\left(5.75 - 1.15 \frac{T_p}{\sqrt{H_{sig}}}\right) & \text{for } 3.6 \leq \frac{T_p}{\sqrt{H_{sig}}} \leq 5 \\ 1 & \text{for } 5 \leq \frac{T_p}{\sqrt{H_{sig}}} \end{cases} \quad (19)$$

| Variable        | Value | Unit | Description                                         |
|-----------------|-------|------|-----------------------------------------------------|
| $T_{p,minimum}$ | 5     | s    | Lowest possible $T_p$ value                         |
| $T_{p,PM}$      | 8     | s    | $T_p$ value for a fully developed sea (PM spectrum) |

Table 4: Values for the peak period for a significant wave height of 2.5m

The formulation of Equation 12 comes from (DNV RP C205, 2021), with  $f$  being the frequency in hertz,  $T_p$  being the peak period of the spectrum, and  $H_{sig}$  being the significant wave-height of the spectrum. A wave-spectrum can be expressed in terms of wave frequency  $f$  or in terms of the wave radial frequency  $\omega$ , the conversion from one spectrum to another is outlined in Equation 20. The area under a wave spectrum is equal to the total variance of the sea surface, see Equation 21, where  $m_0$  is the zeroth order spectral moment of the wave spectrum, and  $\sigma_{\zeta}$  is the total variance. The significant wave height of the wave spectrum is related to the variance of the sea surface through Equation 22, where  $\sigma_{\zeta}$  is the standard deviation of the sea surface. For a sea-surface the mean value of that sea-surface w.r.t. the still water level (SWL) is ofcourse 0. The standard deviation of the sea-surface can be interpreted as the deviation the surface elevation has over a long period of time from its mean position, 0, for 68.3% of the cases, see Figure 8 for the gaussian distribution of the surface-elevation with a standard deviation of 1. A high standard deviation of a sea-surface in general means that the normal or gaussian distribution depicted in Figure 8 is broader, implying that the observed individual wave heights over a longer period of time of an irregular sea-state, see Figure 7, are higher.

In Figure 9 the wave spectrum at a singular point in an ocean is given, where it is assumed that all the waves are coming from one direction. With the use of an Inverse Fast Fourier Transformation IFFT, and knowing that the phase of each wave component is randomly uniformly distributed (Journée, 2001), the time-signal of the surface elevation can be obtained. The energy content within this time signal should be equivalent to that

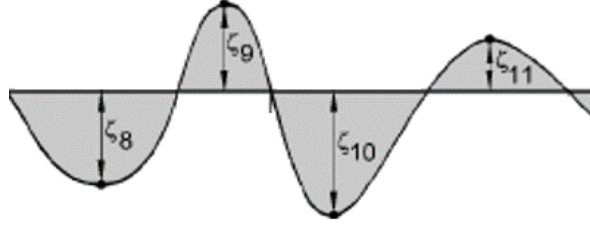


Figure 7: Irregular waves (Journée, 2001)

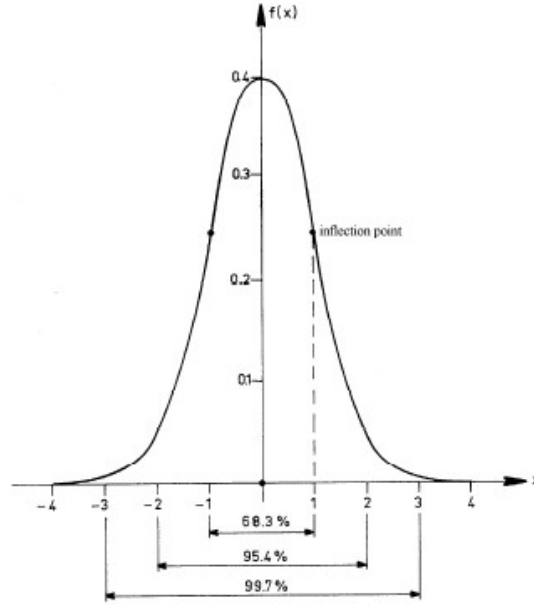


Figure 8: Gaussian distribution (Journée, 2001)

within the wave spectrum. This can be checked by evaluating the standard deviation of the time signal and making sure that it is equal to the standard deviation of the wave spectrum. If this is not the case the spectral density used for the IFFT is probably too high and should be reduced. In other words, the time-step should be decreased and the duration of the time-signal should be increased. To obtain the equivalent wave spectrum from this time signal, the Fast Fourier Transformation (FFT) is applied (Tan & Jiang, 2019).

When considering the spreading of waves, Equation 23 should be applied. When utilizing this equation, multiple wave spectra, each having an individual incoming wave angle and a specific amount of energy content depending on the incoming angle  $\mu$ , are evaluated. In this equation  $\bar{\mu}$  is the main direction of the waves. The time signal of a surface elevation with spread waves can be obtained by applying the IFFT for every individual wave spectrum and adding all the time signals. The energy content of the sum of all the spectra should be equal to that of when the waves come solely from one direction. To check if the time signal for spread waves is obtained correctly, the sum of the energy content contained within all the spread spectra should be equal to the energy content within a spectrum with similar properties where the waves all come from one direction. A measure for the energy content within a spectrum is the standard deviation of the time-signal from which the spectrum is composed, and can be estimated with Equation 22. So by making sure that the standard deviation of the time signal of the sea surface of spread waves is equal to the standard deviation of the time signal of the sea surface of unidirectional waves, with other than spreading similar metocean conditions, one can validate that the modeling of spread waves is executed correctly.

$$E_{\zeta}(\omega) = \frac{1}{2\pi} E_{\zeta}(f) \quad (20)$$

$$m_0 = \int_0^{\infty} E_{\zeta}(\omega) d\omega = \sigma_{\zeta}^2 \quad (21)$$

$$H_{sig} = 4\sigma_{\zeta} = 4\sqrt{m_0} = 4\sqrt{\sigma_{\zeta}^2} \quad (22)$$

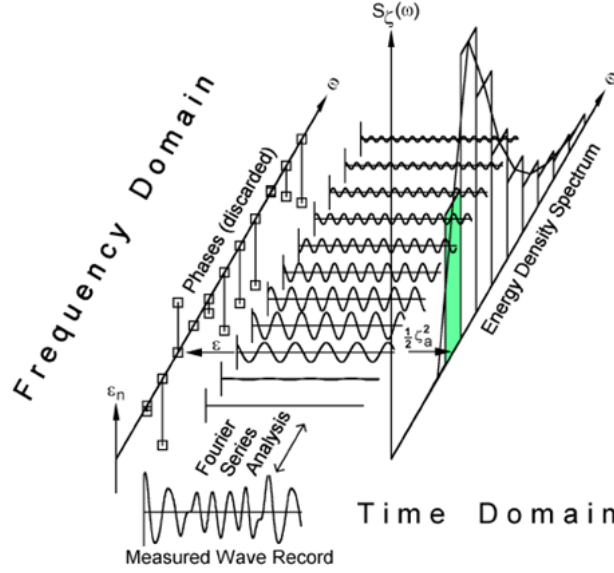


Figure 9: Wave spectrum (variance density spectrum) (Journée, 2001)

$$E_{\zeta}(\omega, \mu) = \frac{2}{\pi} \cos(\mu - \bar{\mu})^2 E_{\zeta}(\omega) \quad (23)$$

### 3.2.2 Hydromechanical forces on the jack-up

In Figure 10 a jack-up that is elevated fully above the sea surface is depicted. It can be seen that the legs are made up of individual cylindrical objects. Since the individual cylindrical objects have a small diameter w.r.t. the wave-length, the hydromechanical loads on these objects are evaluated with the Morison equation. See Equation 24 for the force per meter length of a cylindrical object estimated with the Morison model (Journée, 2001). The Morison equation is valid for cylindrical objects when Equation 25 is satisfied. The reason for this is that the Morison equation evaluates the wave loads, whilst considering the particle velocities and accelerations of a wave w.r.t. the center of the cylindrical object. When Equation 25 is satisfied, the particle velocities are assumed to be constant around the cylinder, see Figure 11. When the  $\frac{D}{\lambda}$  equation is not satisfied, the particle velocities and accelerations vary over the width of the cylinder, as can be seen in Figure 12, due to which the loads would become an overestimation when evaluating them with the Morison equation. With the dispersion relation, from Equation 11, the wave period can be related to the wavelength. Whilst doing this it is found that Equation 25 is valid for most of the energy within a wave spectrum with a significant wave height of 2.5m and a peak period of 7, 8, or 9 seconds. Therefore Morison can be applied to evaluate the hydromechanical forces on the legs of a jack-up vessel.

The drag and inertial coefficient of each cylindrical object of which the jack-up legs are constructed is estimated with the DNV recommended practice (DNV RP C205, 2021). When applying a regular wave the time-signal of the velocity and acceleration of water particles near the legs can be evaluated with potential theory as is just described, see Equation 3, and Equation 4. For an irregular sea-state, the velocity and acceleration spectrum w.r.t. the origin of a specific coordinate system can respectively be obtained with Equation 26, and Equation 27, where  $E_{\zeta}(\omega)$  is the wave spectrum, and  $\omega$  is the radial frequency of a wave (Holthuijsen, 2007). To obtain equivalent spectra at a specific location with coordinates (x,y) w.r.t. the coordinate system for which the wave spectrum was attained, each component of the wave-spectrum should be multiplied with Equation 28. This is the equation for the spatial-phase shift of each wave component. With the dispersion relation, the wave number k can be found for every wave component in a wave spectrum, as the wave frequency of every wave component is already known. When the wave heading of each wave component is known, i.e.  $\mu$ , the spatial phase-shift of every wave component can be obtained. With this information, the equivalent velocity and acceleration spectra can be obtained at every location in the xy-plane.

The time signal of the velocity and acceleration can be evaluated through the utilization of the IFFT (Tan & Jiang, 2019). Multiplying this time signal with the shape function, see Equation 29, will account for the decay of the water particle velocity and acceleration over water depth. Since all the information is now available to estimate the total hydromechanical force with the Morison equation in any location underneath the still water level (SWL), the hydromechanical load per meter length can be evaluated for every individual cylindrical member that together compose the legs. By integrating these values over the length of each member, equivalent lumped loads are found for each member and added to the nodes that model the jack-up legs, see

Figure 48. These separate lumped loads are added into the external force-vector from Equation 2 at the places associated with the correct DOFs.



Figure 10: Jack-up elevated above the sea-surface (Offshore-energy, 2022)

$$F(t) = F_{inertia}(t) + F_{drag}(t) = \frac{\pi}{4}\rho C_m D^2 \dot{u}(t) + \frac{1}{2}\rho C_d D u(t)|u(t)| \quad (24)$$

$$\frac{D}{\lambda} \leq 0.1 \quad (25)$$

$$E_{\dot{\zeta}}(\omega) = \omega^2 E_{\zeta}(\omega) \quad (26)$$

$$E_{\ddot{\zeta}}(\omega) = \omega^4 E_{\zeta}(\omega) \quad (27)$$

$$\epsilon_{spatial} = \exp(ik(x\cos(\mu) + y\sin(\mu))) \quad (28)$$

$$\kappa(k, d, z) = \frac{\cosh(k(d + z))}{\cosh(kd)} \quad (29)$$

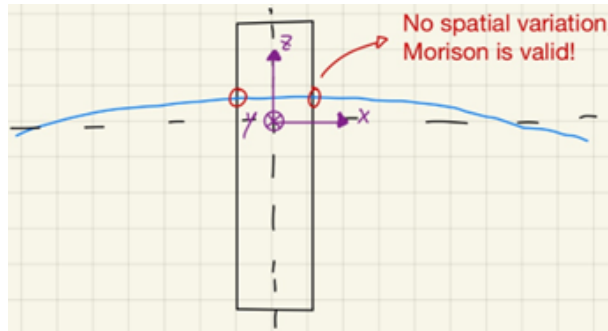


Figure 11: Sketch of a cylinder without spatial variation of the wave

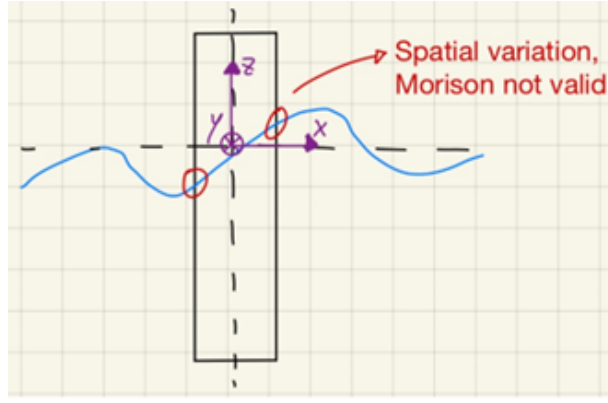


Figure 12: Sketch of a cylinder with spatial variation of the wave

### 3.2.3 Hydromechanical loads on the MP

As discussed in the previous paragraph, using the Morison equation to estimate the hydromechanical loads on a cylindrical object is only valid when the particle velocities and accelerations are constant around the diameter of the MP. As discussed in the introduction, this MSc thesis focuses on the MP installation on a jack-up vessel at water depths of 50m. The MP is assumed to be a rigid body. Assuming deep water conditions, the wavelength of a regular wave can, through the dispersion relation, be estimated with Equation 30. Water is assumed to be 'deep' when Equation 31 is satisfied (Holthuijsen, 2007). For a 7,8, and 9-period regular wave the wavelength, and the  $kd$ -values are evaluated and outlined in Table 5, assuming deep water conditions. It can be seen that the deep water condition is satisfied for a regular wave with a period of 7 and 8 seconds, but for a 9-second period wave the water is in intermediate depth conditions. Therefore the wavelength of this 9-second period wave is evaluated with Equation 32, which is essentially the dispersion relation for intermediate water depths. Note that this equation is solved for the wavelength iteratively.

The  $\frac{D}{\lambda}$  values for a 7,8 and 9-second period regular wave are outlined in Table 6. It can be seen that solely for a 9-second period regular wave the Morison equation would be valid. For the 7 and 8-second period waves the particle velocity and acceleration would vary too much across the MP, see Figure 12, due to which the hydromechanical load evaluated with Morison would be an overestimation. Since in this thesis regular waves with a period of 7,8 and 9 seconds and irregular waves with a peak period of 7, 8, and 9 seconds will be considered, the Morison equation to model the hydromechanical load on the MP is not a valid option. In a spectrum with a peak period of 9 seconds, there will be wave periods present for which Morison would be a valid equation since most of the energy in this wave spectrum is located around a wave period of 9 seconds. However, there is also a big part of the energy contained within the spectrum for which the Morison assumption is not met, i.e., wave periods lower than 8-9 seconds. For these periods, the Morison equation would yield an overestimation of the actual hydromechanical load. For this reason, the Morison equation is also not utilized for irregular waves with a peak period of 9 seconds or lower.

$$\lambda = \frac{gT^2}{2\pi} \quad (30)$$

$$kd > 3 \quad (31)$$

$$\lambda = \frac{gT^2}{2\pi} \tanh(kd) \quad (32)$$

| Parameter        | Value  | Unit | Description                            |
|------------------|--------|------|----------------------------------------|
| $\lambda_{T=7s}$ | 76.504 | m    | Wave-length for a 7 second period wave |
| $\lambda_{T=8s}$ | 99.92  | m    | Wave-length for a 8 second period wave |
| $\lambda_{T=9s}$ | 124.82 | m    | Wave-length for a 9 second period wave |
| $(kd)_{T=7s}$    | 4.10   | -    | kd for a 7 second period wave          |
| $(kd)_{T=8s}$    | 3.14   | -    | kd for a 8 second period wave          |
| $(kd)_{T=9s}$    | 2.48   | -    | kd for a 9 second period wave          |

Table 5: Wavelengths and kd-values for a 7,8 and 9 period regular wave

| Parameter          | Value | Unit | Description                            |
|--------------------|-------|------|----------------------------------------|
| $D/\lambda_{T=7s}$ | 0.156 | -    | $D/\lambda$ for a 7 second period wave |
| $D/\lambda_{T=8s}$ | 0.120 | -    | $D/\lambda$ for a 8 second period wave |
| $D/\lambda_{T=9s}$ | 0.090 | -    | $D/\lambda$ for a 9 second period wave |

Table 6:  $\frac{D}{\lambda}$  values for a 7,8 and 9 second period regular wave

Since the Morison equation is not a valid option to evaluate the hydromechanical load on the MP, the diffraction model is utilized. Some of the key takeaways of the diffraction model are:

1. Fluid is assumed incompressible, irrotational, and inviscid.
2. Forces and added properties are evaluated w.r.t. the mean position of the object.
3. The diffraction model assumes that the total force on a structure in a fluid subject to wave is evaluated by integrating the pressure from the following 3 wave potentials over the mean wetted surface area:
  - (a) The undisturbed wave-potential, denoted with  $\Phi_w$
  - (b) The diffracted wave-potential, denoted with  $\Phi_d$
  - (c) The radiated wave-potential, denoted with  $\Phi_{r,j}$
4. Hydromechanical loads are wave-frequency dependent. This is because it is assumed that the frequency of motion of the object subject to waves is equal to the wave frequency, due to which the hydromechanical loads are assumed to be wave frequency dependent. In reality, these loads are however motion frequency dependent.

The wave-forces with the diffraction model are extracted from WAMIT of which the details will be given later on in this chapter. As mentioned in the list above, the diffraction model complete neglects the viscous drag forces, and only considers that the wave induced forces come from the undisturbed wave and the diffracted wave. In order to validate the assumption to use the diffraction model over the Morison equation for evaluating the wave-induced forces, the wave-induced pitch moment w.r.t. the gripper, see Figure 4, is evaluated with the diffraction model and the Morison model, where both models are afterwards compared. This is done for the situation where a MP is fixed at a draft of 20m with a regular wave with a height of 2.5m, wave periods of 5,6,7,8, and 9 seconds, and without an ambient current. It is assumed that the MP cannot move, and the drag and mass coefficient for the Morison equation, see Equation 24, are evaluated with the recommended practice from the DNV standards (DNV RP C205, 2021). See Figure 13 for the different wave-induced forces for a wave-period of 6 seconds. It can be seen that the viscous drag force is relatively small and negligible compared to the inertia forces, i.e. the undisturbed and diffracted wave-force. It is also noticed that the inertia forces evaluated with the Morison equation are bigger than the inertia forces evaluated with the diffraction model. This is because the Morison equation evaluates the wave-excitation forces w.r.t. the center of the cylinder, assuming that the wave particle velocities and accelerations are constant over the diameter of the MP. Since for a relatively short, 6 second period wave this is not the case, the wave-induced inertia force evaluated with the Morison equation is an overestimation w.r.t. the wave-induced inertia force evaluated with the diffraction model. The inertia force evaluated with Morison is 10 % larger than the inertia force evaluated with the diffraction model, see Figure 13. Because for larger wave-periods the wave profile will be more constant over the diameter of the MP, this difference will decrease for increasing wave-period, see the results in Table 7 for different wave-periods under the exact same circumstances used in Figure 13. It can be seen that for 5, 6 and 7 second period waves the wave-induced inertia forces evaluated with Morison overestimates the inertia force significantly w.r.t. the diffraction model. What can also be seen from the table is that the wave-induced viscous drag forces are almost negligible compared to the inertia forces, due to which it is valid to disregard these wave-induced viscous drag forces.



Since for the modeling of the environmental conditions in this thesis peak periods of 7,8, and 9 seconds are used, the diffraction model is not deemed to be an appropriate choice for evaluating the hydromechanical forces on the MP. Especially for peak periods of 7 and 8 seconds there is plenty of energy in the JONSWAP spectrum associated with wave periods of 5 and 6 seconds or even lower, for which the Morison equation will be an overestimation of the wave-induced inertia force. This reason, together with the negligible viscous drag force is why the diffraction model is utilized in order to establish the hydromechanical forces on the MP, as this model will, within linear diffraction theory, not likely yield an overestimation of the wave-induced inertia forces.

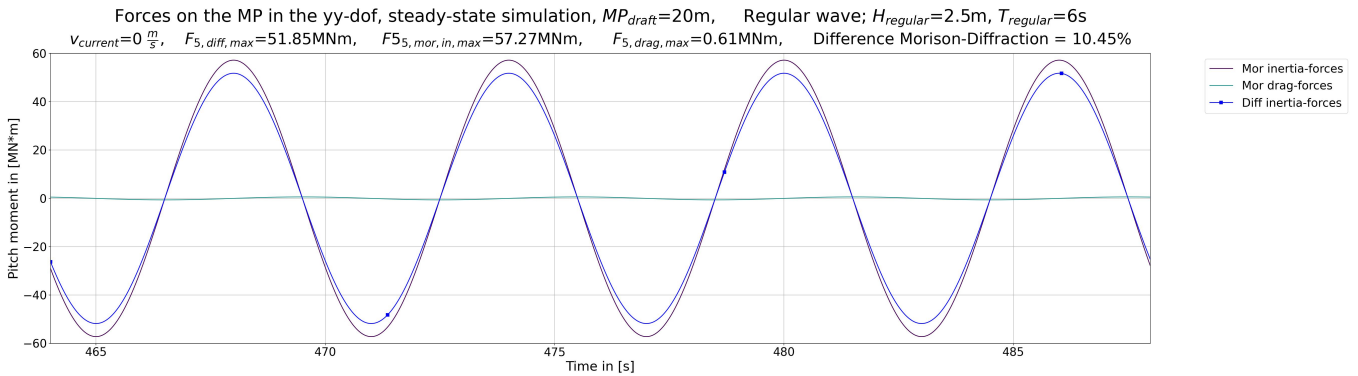


Figure 13: Wave-induced inertia and drag forces at a fixed MP draft of 20m

| $T_{regular}$ [s] | $F_{5,in,diff,max}$ [MNm] | $F_{5,in,mor,max}$ [MNm] | $F_{5,drag,mor,max}$ [MNm] | $F_{in}$ difference [%] |
|-------------------|---------------------------|--------------------------|----------------------------|-------------------------|
| 5                 | 41.33                     | 58.36                    | 0.58                       | 41.19                   |
| 6                 | 51.58                     | 57.27                    | 0.61                       | 10.45                   |
| 7                 | 52.60                     | 53.62                    | 0.63                       | 1.94                    |
| 8                 | 48.86                     | 48.76                    | 0.63                       | 0.21                    |
| 9                 | 43.72                     | 43.65                    | 0.61                       | 0.14                    |

Table 7: Different wave-induced forces on a MP with a draft of 20m.

Looking at the list with key takeaways of the diffraction model, the diffracted wave is the wave that is caused by the scattering of the wave due to the presence of the object subject to waves, where it is assumed that the object does not move. The radiated wave is a wave that is caused by the object it's motion, under the assumption that the water is initially at rest, the subscript j indicates the DOF of motion of the object subject to waves. The undisturbed wave is simply the passing regular wave, where it is assumed that the wave can pass through the structure and will not be disturbed by its presence.

WAMIT is the panel method program that is utilized to obtain the hydromechanical forces by means of the diffraction model. In this program a panel mesh of the MP is imported, see Figure 14. After implementing this panel mesh, the added mass, added damping and wave-excitation forces are obtained for a set of regular waves with varying periods and incoming wave angles  $\mu$ , given a certain MP draft. Since the cylinder is symmetric w.r.t. the xz and yz-plane the values of the added mass, added damping, and wave-excitation forces are equal for every incident wave angle. In order to obtain added mass, added damping, and wave-excitation values at specific MP drafts and wave frequencies, the extracted values from WAMIT are interpolated between the desired MP drafts and wave frequencies.

For every wave period and MP draft, there are 36 unique added mass and added damping terms, since the MP has 6 DOFs, and the velocity or acceleration of the MP can cause a force in 6 DOFs. The force due to added mass and the force due to added damping are outlined with Equations 33, and 34. It can be seen from these equations that the added mass and added damping forces are related to respectively the acceleration and velocity of the MP in the 6 DOFs. Note that the diagonal terms of both matrices are the terms that cause a force in the direction of movement of the MP, and that the off-diagonal terms are terms that cause a force in one direction, due to a movement of the MP in another direction, i.e. the  $a_{15}$  term causes a force in the x-direction due to a pitch acceleration. The values of the rows and columns associated with the heave and yaw DOF are all set at zeros for both the added mass and the added damping matrix. The reason for this is that the accelerations of the pile-cable subsystem in the heave and yaw DOF are assumed to be zero.

Furthermore, the added mass from WAMIT only considers the added mass outside of the MP. However, since the MP is open on both ends, there will be water entrapped inside the MP. The mass of this entrapped is added to the added mass assuming that the water is frozen inside the cylinder, because in the multi-DOF model the accelerations of the MP and the water in the z-direction are neglected. Because of this the additional inertia

effects due to accelerations of water in the z-direction is fully neglected. Furthermore, if the accelerations in the z-direction were not neglected this effect would probably be insignificant since there is no area normal to the z-axis in the cylinder that can be excited by acceleration of the water in the z-direction. The mass moment of inertia of the entrapped water can be evaluated with Equation 35, w.r.t. the COG of the mass of this entrapped water. See Figure 15 for the definitions of the values used in this equation. A mass matrix of the entrapped water is obtained w.r.t. the COG of this entrapped water, with solely values on the diagonal, where the added mass due to the entrapped water is zero in the heave and yaw DOF. This added mass matrix is hereafter translated with a transformation matrix to the SWL and added to the added mass matrix of WAMIT since all the values from WAMIT are evaluated w.r.t. the SWL.

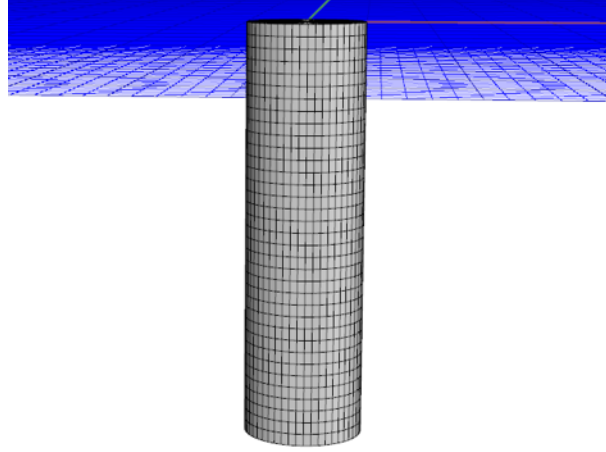


Figure 14: MP subdivided into a mesh of panels in WAMIT

$$\mathbf{A}\vec{x} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & a_{56} \\ a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & a_{66} \end{bmatrix} \begin{bmatrix} \ddot{\zeta}_{a,1} \\ \ddot{\zeta}_{a,2} \\ \ddot{\zeta}_{a,3} \\ \ddot{\zeta}_{a,4} \\ \ddot{\zeta}_{a,5} \\ \ddot{\zeta}_{a,6} \end{bmatrix} = F_{added-mass} \quad (33)$$

$$\mathbf{C}\vec{\dot{x}} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\ c_{21} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\ c_{31} & c_{32} & c_{33} & c_{34} & c_{35} & c_{36} \\ c_{41} & c_{42} & c_{43} & c_{44} & c_{45} & c_{46} \\ c_{51} & c_{52} & c_{53} & c_{54} & c_{55} & c_{56} \\ c_{61} & c_{62} & c_{63} & c_{64} & c_{65} & c_{66} \end{bmatrix} \begin{bmatrix} \dot{\zeta}_{a,1} \\ \dot{\zeta}_{a,2} \\ \dot{\zeta}_{a,3} \\ \dot{\zeta}_{a,4} \\ \dot{\zeta}_{a,5} \\ \dot{\zeta}_{a,6} \end{bmatrix} = F_{added-damping} \quad (34)$$

$$I_x = I_y = \frac{1}{12}m(3r^2 + h^2) \quad (35)$$

The excitation force that is obtained from WAMIT is presented in the form of a force response amplitude operator (RAO). This complex valued force-RAO is obtained by rewriting the EOM of the rigid-body MP in Euler-notation and solving for the complex-valued force amplitude divided by the wave-amplitude, see Equations 36, 37, and 38. In these equations,  $\vec{F}_{a,j}(\omega)$  is the complex-valued force-vector where the value for each DOF is defined with Equation 39, and  $\vec{x}_a$  is the complex-valued motion amplitude vector, where the value each DOF is defined with Equation 40, in which  $\epsilon_{x_j,\zeta}$  is the phase-shift between the motion and the wave. The phase-shift between the wave and the excitation force can be found with Equation 41. The force amplitude, which is wave-frequency dependent, can be found for a given DOF and wave amplitude with Equation 42. In all these equations the subscript j indicates which DOF is considered. The force in a specific DOF for a given wave frequency as a function of time can be found with Equation 43. Note that the force-RAO is wave-frequency

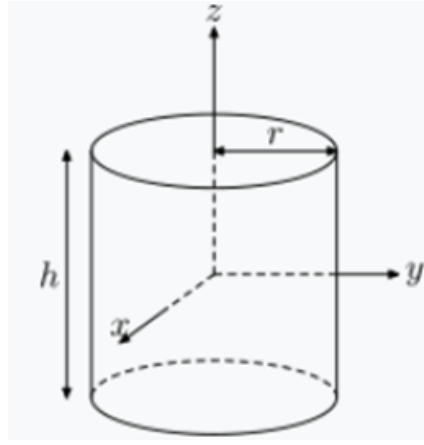


Figure 15: Sketch to evaluate the mass moment of inertia of the entrapped water

dependent and that there are unique RAO values for every DOF and wave-frequency.

Because WAMIT obtains the values for the added mass, added damping, and wave-excitation forces w.r.t. the SWL, the obtained matrices and force amplitude vector has to be transformed to the gripper node. See Figure 16 for the axis system located at the SWL, and the axis system located at the gripper node. In order to transform a system matrix from the coordinate system located at the SWL to the coordinate system located at the gripper node, transformation matrices  $\mathbf{T}_u$  and  $\mathbf{T}_f$  are utilized, see Equations 46, 47, and 48. The same can be done for the damping matrix. The force vector for a given wave-frequency w.r.t. the gripper node can simply be obtained with Equation 46.

Now that the force-amplitude vector and added-mass and added damping matrices are transformed to the gripper node, the force can be found as a function of time at the gripper node with Equation 43. The added mass and added damping values can be added to the global mass and structural damping matrix at the indexes with the associated gripper DOFs, to find the complete mass and damping matrix of the combined system of the jack-up and pile-cable model. Since the force as a function of time for a regular wave is also obtained w.r.t. the gripper node, the EOM in Equation 2 can be solved in the time-domain with a numerical time-integration scheme for a fixed MP draft.

$$\Re \left( \left( -\omega^2(\mathbf{M} + \mathbf{A})\vec{x}_a - i\omega\mathbf{B}\vec{x}_a + \mathbf{C}\vec{x}_a \right) e^{i\omega t} \right) = \Re \left( \vec{F}_a e^{i\omega t} \right) \quad (36)$$

$$\left( -\omega^2(\mathbf{M} + \mathbf{A}) - i\omega\mathbf{B} + \mathbf{C} \right) \vec{x}_a = \vec{F}_a \quad (37)$$

$$RAO_{force,j}(\omega) = \frac{\hat{F}_{a,j}}{\zeta_a} = \left( -\omega^2(\mathbf{M} + \mathbf{A}) - i\omega\mathbf{B} + \mathbf{C} \right) \frac{\hat{x}_a}{\zeta_a} \quad (38)$$

$$\hat{F}_{a,j}(\omega) = F_{a,j} \exp(i\epsilon_{F,j,\zeta}) \quad (39)$$

$$\hat{x}_{a,j} = x_{a,j} \exp(i\epsilon_{x,j,\zeta}) \quad (40)$$

$$\epsilon_{F,j,\zeta}(\omega) = \tanh \left( \frac{\Im(RAO_{force,j})}{\Re(RAO_{force,j})} \right) \quad (41)$$

$$F_{a,j}(\omega) = |RAO_{force,j}(\omega)| \zeta_a \quad (42)$$

$$F_j(t, \omega) = \Re(F_{a,j}(\omega) \exp(i\omega t)) \quad (43)$$

$$\vec{u}' = \mathbf{T}_u \vec{u} \quad (44)$$

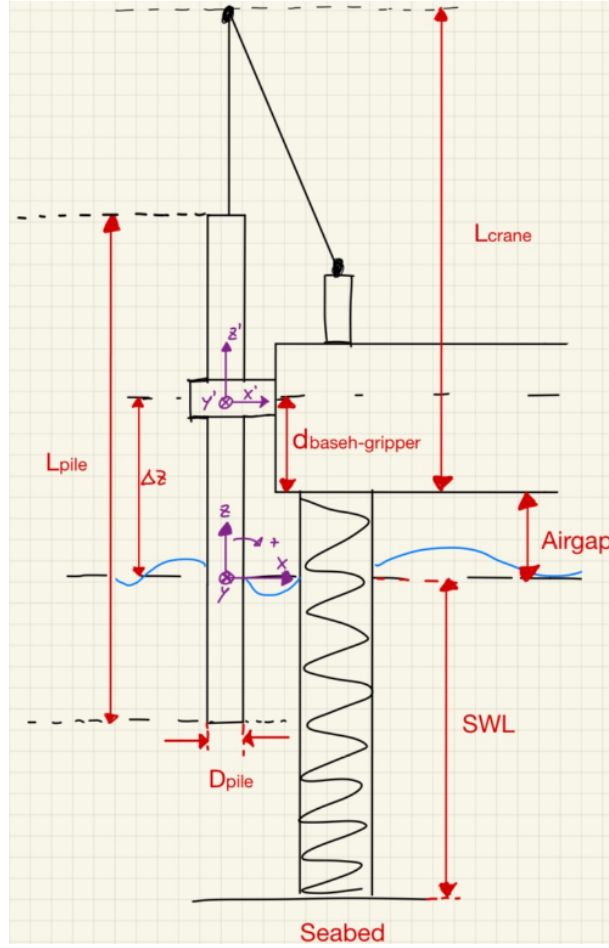


Figure 16: Sketch of the MP and the jack-up

$$\vec{F}' = \mathbf{T}_f \vec{F} \quad (45)$$

$$\mathbf{T}_f = \mathbf{T}_u^{-T} \quad (46)$$

$$\mathbf{A}' = \mathbf{T}_u^{-T} \mathbf{A} \mathbf{T}_u \quad (47)$$

$$\mathbf{T}_u = \begin{bmatrix} 1 & 0 & 0 & 0 & \Delta z & 0 \\ 0 & 1 & 0 & -\Delta z & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (48)$$

For an irregular sea state, the force spectrum for a given DOF at a fixed MP draft is obtained with Equation 49 (Journée, 2001), in which  $E_\zeta(\omega)$  is the value of the wave spectrum for a given wave-frequency. Note that because the force RAOs were obtained w.r.t. the axis system located at SWL as depicted in Figure 16, which is already located at the center of the cylinder, each wave component of Equation 49 does not have to be multiplied with the associated phase-shift, because the x and y coordinate in Equation 28 are both zero, resulting in a multiplication with 1,  $\exp(0) = 1$ . The relation between the force-spectrum and the force Fourier amplitudes for a given DOF is outlined with Equation 50. The phase-shift of all the waves in an irregular sea state are random and uniformly distributed between 0 and  $2\pi$ . There also exists a phase shift between the force in a specific DOF and the wave that causes this force. Both these phase shifts have to be added to the force Fourier amplitudes in order to obtain the complex-valued force Fourier amplitudes with the proper phases,

see Equation 51. Hereafter the time signal of the force due to an irregular sea state can be obtained by applying the IFFT. This force signal as a function of time can be obtained for every DOF of the MP. Hereafter, the force as a function of time can be added to the EOM outlined in Equation 2 at the indexes associated with the gripper DOFs, after which the EOM can be solved with a numerical time-integration scheme.

The spreading of irregular waves can easily be incorporated into the model now that it is known how to evaluate the force signal, from the RAO function and the wave spectrum. The spreading function outlined in Equation 23 can be utilized to obtain a wave spectrum for different headings, every wave spectrum having a certain amount of energy content. In general, the energy content decreases when the  $\mu - \bar{\mu}$  term approaches  $\frac{\pi}{2}$ , and the energy content becomes bigger when the  $\mu - \bar{\mu}$  term approaches zero. The total energy contained in all the spectra, each having a different heading, should be equal to the energy that is contained within the spectrum in the situation where all the waves were coming from one direction. A measure for the energy content within a spectrum would be the area underneath the graph, or for that matter the significant wave height. After making sure that no energy is added or dissipated, after establishing the different spectra with different headings, the force-spectrum for every spectrum can be obtained in the same manner as for uni-directional waves. From this force spectrum, the force as a function of time for every spectrum can also be found in the same manner, as previously discussed. To obtain the total force-signal as a function of time, all the individual force-signals can be added together, whilst considering their heading w.r.t. trigonometric functions. For example, to obtain the contribution of a force signal in the x-direction for a specific force spectrum which makes a  $\frac{\pi}{4}$  rad angle w.r.t. the x-axis, the obtained force-signal should be multiplied with  $\cos(\frac{\pi}{4})$ . The same can be done for obtaining the force as a function of time along the y-axis, however, now the force signal should be multiplied with  $\sin(\frac{\pi}{4})$ . Doing this for every force signal and every axis, and adding up all the force signals associated with one particular axis, will yield the total force signal as a function of time along that specific axis. It should be noted that the standard deviation of this force signal along for example the x-axis is lower, as opposed to the standard deviation of the force signal for uni-directional waves traveling in the positive x-direction, since energy is more spread along different axes when including the spreading of waves.

$$E_{F,j}(\omega) = E_{\zeta}(\omega) |RAO_{force,j}(\omega)|^2 \quad (49)$$

$$E_{F_j}(\omega) = \frac{1}{2} F_{j,fourier}^2(\omega) \rightarrow F_{j,fourier}(\omega) = \sqrt{2E_{F_j}(\omega)} \quad (50)$$

$$F_{j,fourier,phase}(\omega) = F_{j,fourier}(\omega) \exp(i\epsilon_{uniform}) \exp(1\epsilon_{F_j,\zeta}) = \left( \sqrt{2E_{F_j}(\omega)} \right) \exp(i\epsilon_{uniform}) \exp(1\epsilon_{F_j,\zeta}) \quad (51)$$

There is one remaining contribution to the hydromechanical load on the MP for the multi-DOF model utilized within GustoMSC. This is the viscous drag due to the current, if present. This force is considered by adding the second term of the Morison equation to the total force in the direction of the current, see Equation 24. The drag coefficient in this 2nd term of the Morison equation is estimated with the DNV recommended practice (DNV RP C205, 2021). The current profile along the water depth can be obtained with Equation 52. With this information, the drag force per meter length can be evaluated along the draft of the MP with the second term in Equation 24. Integrating this force per meter length over the draft of the MP will yield the drag force in the direction of the current. The moment w.r.t. the gripper node due to the drag force can be found by multiplying the drag force per meter length with the distance from the gripper node and integrating all these values along the MP draft. The obtained values of the force due to the current can be added to the global force vector from Equation 2 at the associated gripper node DOFs.

$$v_c = v_{c,surface} \left( \frac{(SWL - z)}{SWL} \right)^{\frac{1}{7}} \quad (52)$$

### 3.3 Time-simulation

Now that for a regular and irregular sea-state at a fixed MP draft all the terms in Equation 2 are obtained, the EOM can be solved in the time domain. The EOM is solved in the time domain, as opposed to the frequency domain because of the addition of the non-linear viscous drag force due to the current. The EOM of the combined jack-up and pile cable model is solved with the Newmark numerical time-integration scheme. This scheme is unconditionally stable and hence, it is used to solve the EOM, as the solution will not explode to infinity (Vuik, Vermolen, & van Gijzen, 2023).

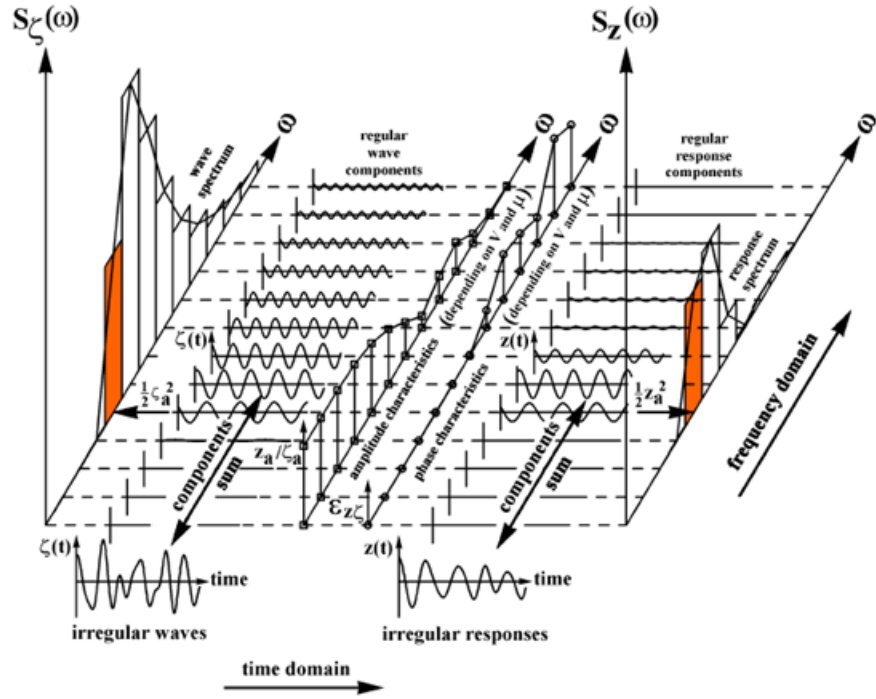


Figure 17: Visualization of the wave and response spectrum (Journée, 2001)

The time simulation that can solve for the motion, velocity, and acceleration of each node of the combined system is only considered for the case where the MP has a fixed draft. This is also referred to as the steady-state simulation. In reality, the MP lowers through the gripper and splashzone onto the seabed, due to which a different motion response is expected. For this simulation, the stiffness, and the mass moment of inertia of the pile-cable subsystem need to be updated as they change when the MP is lowered. The same goes for the added mass, added damping, and wave excitation forces. This simulation is also referred to as the dynamic-lowering simulation. The multi-DOF model which is utilized within GustoMSC, can only solve the steady-state simulation, the dynamic lowering simulation is one of the improvements that will be endeavored when building the 1-DOF pile-cable model.

## 4 The 1-DOF model

To analyze how certain physical phenomena influence the dynamic behavior of the MP during the lowering process through the gripper and the splash zone in a structured manner, it is convenient to reduce the combined jack-up and pile-cable model from GustoMSC to a 1-DOF pile-cable model. Also, as will be shown in the results from modal analysis of the multi-DOF model, the natural periods of the first 2 modes of the multi-DOF model, which are related to the MP rotation in the gripper, are closest to the excitation periods of the environmental conditions considered in this MSc thesis, see Table 2. This implies that for the considered environmental conditions the dynamic behavior of the MP during the lowering process through the gripper and the splash zone is governed by the rotational DOF of the MP in the gripper. Hence the reduction from the multi-DOF model to the 1-DOF pitch rotation of the MP in the gripper node is a valid assumption when analyzing the dynamic behavior of the MP during the discussed process.

The rotational DOF around the y-axis is considered during this MSc with waves solely traveling in the positive x-direction, as the rotational DOF around the y-axis is the most excited DOF when waves travel in the positive x-direction. The layout of the 1-DOF pile-cable subsystem is depicted in Figure 4. Note that all the green parameters are variable and all the red parameters are fixed. The kinematic relations for different variable parameters are outlined in Equation 1. To simplify matters, in this MSc thesis, the fixed parameters will not vary and are set at the values outlined in Table 3. The reason for this is that these values are the values that are most commonly used for an NG-20000 F-class GustoMSC jack-up vessel design.

### 4.1 Assumptions of the 1-DOF pile-cable model

The assumptions of the 1-DOF pile-cable model will be the same as for the pile-cable subsystem in the combined multi-DOF model used within GustoMSC, see Appendix A. These assumptions are as follows:

1. The MP is a rigid body
2. The moments that arise due to the rotation of the MP are linearly related to the angle  $\theta$ , so the trigonometric functions are linearized.
3. The mass of the steel of the MP is assumed to be uniformly distributed, i.e., the non-uniform distribution of the mass due too a possible tapered end of the MP is not considered.
4. The translational and rotational accelerations, in the direction of, and around, the z-axis are neglected
5. Structural damping is neglected
6. The MP is free to rotate around the gripper node
7. The cable is free to elongate due to a rotation  $\theta$ , i.e. the tension in the cable is constant.

See Figure 18 for a sketch with an overview of the dimensions of the pile-cable subsystem. It can be seen from figures 19, and 20, that when the MP is given a certain angle theta, two forces arise which cause two moments w.r.t the gripper node:

1. A moment due to the tension in the cable:  $M_t$
2. A moment due to the weight of the MP:  $M_w$

When linearizing these two moments w.r.t. their trigonometric functions, the stiffness values of the pile-cable subsystem can be found, since the above two moments are caused by a (static) displacement. The values of  $\tan(\alpha)$ , and the vertical and horizontal force due to the tension in the cable can be found respectively with equations 53, 54, and 55. With these equations, the moment due to the tension in the cable for a given MP angle  $\theta$  can be found with Equation 56. The linearized form of the 3 equations is outlined after the approximate sign ( $\approx$ ). The linearized form of the 3 equations assumes small rotations, in subsection 5.3, it will be verified up until what rotation this assumption is valid. The exact same can be done for the moment that arises from a combination of the weight of the MP and a given rotation  $\theta$ , see Equation 57. After adding up these two moments the stiffness term due to a rotation  $\theta$  can be found w.r.t. the gripper node for the 1-DOF model.

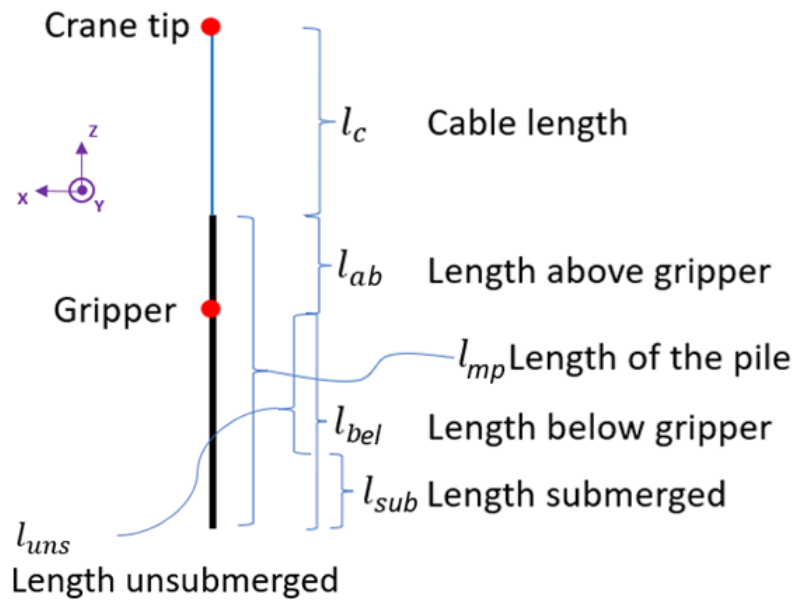


Figure 18: Sketch and dimensions of the pile-cable subsystem

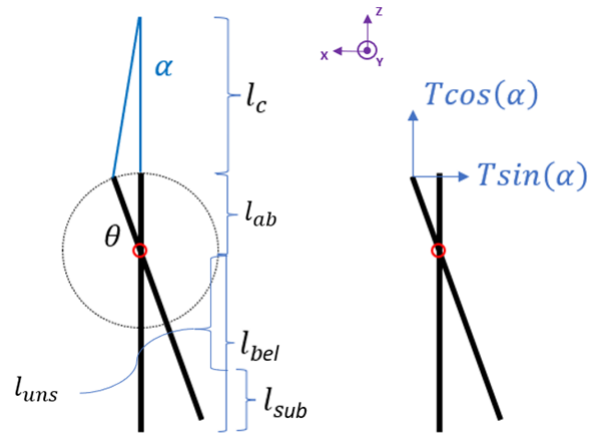


Figure 19: Moment due to the tension in the cable of the MP w.r.t. the gripper node

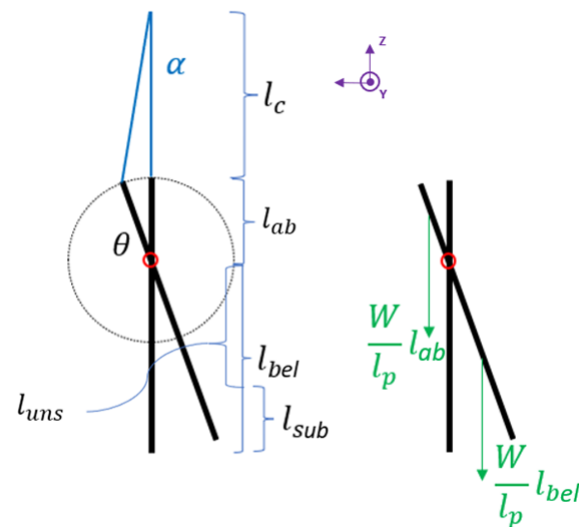


Figure 20: Moment due to the weight of the MP w.r.t. the gripper node



$$\tan(\alpha) = \frac{\tan(\theta)l_{ab} \cos(\theta)}{l_c + l_{ab}(1 - \cos(\theta))} \approx \frac{l_{ab}}{l_c} \theta \quad (53)$$

$$F_{v,t} = W = T \cos(\alpha) \approx T \quad (54)$$

$$F_{h,t} = T \sin(\alpha) = W \tan(\alpha) \approx W\alpha \quad (55)$$

$$M_t = -W \frac{\tan(\theta)l_{ab}^2 \cos(\theta)}{l_c + l_{ab}(1 - \cos(\theta))} - T \cos(\alpha)l_{ab} \sin(\theta) \approx -W \frac{l_{ab}^2}{l_c} \theta - Wl_{ab} \theta \quad (56)$$

$$M_w = \frac{Wl_{ab}}{l_{mp}} \frac{l_{ab}}{2} \sin(\theta) - \frac{W}{l_{mp}} l_{bel} \frac{l_{bel}}{2} \sin(\theta) = -\frac{W}{l_{mp}} \frac{l_{bel}^2 - l_{ab}^2}{2} \sin(\theta) \approx -\frac{W}{l_{mp}} \frac{l_{bel}^2 - l_{ab}^2}{2} \theta \quad (57)$$

As mentioned previously, the assumptions and mechanics of the 1-DOF pile-cable model are similar to the pile-cable subsystem used in the multi-DOF model of GustoMSC, see subsubsection A.1.3 for the pile-cable subsystem of the multi-DOF model. The only difference is that this 1-DOF pile-cable model only considers the pitch DOF at the gripper node, i.e. the rotation around the y-axis at the gripper node, see Figure 4. Because of this the stiffness, mass moment of inertia, added damping, and wave-excitation force values associated with solely the pitch DOF at the gripper node can be extracted from the multi-DOF model. The resulting EOM w.r.t the gripper node is outlined in Equation 58. In this equation,  $I_{m,55,tot}$  consists of the mass moment of inertia of the MP its own mass, the added mass obtained from WAMIT, i.e., the diffraction model, and the added mass from the entrapped water inside the MP. The value of  $C_{55}$  is the added damping from WAMIT, and the value of  $K_{55}$  is the stiffness in solely the pitch DOF of the gripper node, which arises from the moment due to the weight of the MP and the tension in the cable, see Equation 56, and Equation 57. The wave excitation forces can also be found w.r.t the gripper node for solely the pitch DOF. This is done in the same manner as for the multi-DOF model used within GustoMSC, except that now only the force in the pitch DOF at the gripper node is considered. Depending on the increment of the 1-DOF model, forces due to regular waves, irregular waves, or irregular waves with spreading can be obtained.

Note that the 6x6 system matrices, and force vector of the pile-cable subsystem, from the multi-DOF model, are all first moved to the gripper node. Hereafter the value associated solely with the pitch DOF is obtained, i.e. the element on row 5, column 5 for a system matrix, and the 5th element of the force vector.

$$I_{m,55,tot} \ddot{x}_5 + C_{55} \dot{x}_5 + K_{55} x_5 = F_{5,wave-excit} \quad (58)$$

$$I_{m,55,tot} = I_{m,55,MP-weight} + I_{m,55,AM-WAMIT} + I_{m,55,EW} \quad (59)$$

## 4.2 Viscous drag

In the multi-DOF model the viscous drag due to the current is added through the second term of the Morison equation, see Equation 24. Later in certain levels of accuracy of the 1-DOF model, the viscous drag due to the relative velocity of water w.r.t. the MP will be added to the model. To estimate this viscous dragforce due to the relative velocity of the MP w.r.t. the water, the Morison equation is utilized. In reality, this relative velocity consists of the current, the velocity of the MP through the water, and the wave-particle velocity. However, since the wave-particle velocity around a MP with a diameter of 12m, whilst considering the wave-lengths as discussed in subsubsection 3.2.1, will experience variation along the width of the MP, see Figure 12, the inclusion of the wave-particle velocity with Morison would yield an overestimation of the viscous drag. Because the viscous drag due to the relative velocity of the MP through the water will in principle cause the motion response to be damped, the particle velocity of the waves is not considered since this causes an overestimation of the damping when the MP moves opposite to the wave-particle velocities

To add the viscous drag force due to the current and the velocity of the MP through the water, the 2nd term of the Morison equation would in principle be utilized, see Equation 60. However, since there is a relatively rapid change in the velocity field around a large MP, the eddies behind the MP, which govern the drag force, have almost no time to build up, due to which the considered equation would also yield an overestimation of the viscous drag force (Moe & Verley, 1980). The region for which the dragforce per meter length obtained with the relative velocity form of the Morison equation, see Equation 60, is marked grey in Figure 21. In

this figure, on the y-axis, the values of the KC-number, and on the x-axis values for the reduced velocity are plotted. The reduced velocity parameter is evaluated with Equation 62, in which  $V_0$  is the velocity amplitude of a 2-dimensional oscillating flow, and  $T_0$  is the MP period of oscillation, see Equation 63 (Laya & Connor, 1984). In this latter equation,  $v_{current}$  is the current velocity, and  $v_{tangential}$  is the tangential velocity of the MP through the still water due to a rotation of the MP around the gripper node. Assuming, a unrealistically high but therefore also conservative, maximum rotational velocity of  $15 \frac{deg}{s}$  w.r.t. the gripper node, at a MP draft of 20m will yield a maximum tangential velocity at the lowest part of the MP of  $9.4 \frac{m}{s}$ . Adding to this velocity the current speed, to obtain the maximum velocity amplitude of the MP through the water yields a  $V_0$  of  $10.92 \frac{m}{s}$ . Utilizing Equation 62, a maximum reduced velocity of 8.1935 is obtained when assuming that the period of oscillation,  $T_0$  is 9 seconds, which means that the relative flow model is not valid even for these unrealistically high velocities, see Figure 21 (Laya & Connor, 1984). In subsubsection B.4.2, it will be shown that for a viscous undamped case, the maximum rotational velocity of the MP w.r.t. the gripper node at a draft of 15m during a regular wave of 2.5m high and a period of 9 seconds is around  $9.5 \frac{deg}{s}$ . To be conservative, the reduced velocity parameter is therefore estimated assuming a MP draft of 20m during a maximum rotational velocity w.r.t the gripper node of  $15 \frac{deg}{s}$  with a pile period of motion of 9 seconds. Even in this unrealistically extreme situation, the relative velocity flow form of the Morison equation is not valid.

An alternative to the relative velocity flow form of the Morison equation is proposed by (Moe & Verley, 1980), namely the viscous dragforce per meter length with the separate flow model, see Equation 61. This equation was originally constructed for a situation with an oscillating cylinder in a steady current. Later it will be shown that this equation is more conservative than Equation 60. Therefore, this separate flow form of Morison will be utilized to evaluate the dragforce per meter length due to the velocity of the MP in the water, as it is the most conservative choice to gain damping.

The current profile over the water depth can be evaluated with Equation 52 (DNV RP C205, 2021), in which  $z$  is the vertical position w.r.t. the SWL. This current profile yields different current velocities depending on the  $z$  position w.r.t the SWL. Together with the rotation of the MP around the gripper node, Equation 61 can be utilized to obtain the dragforce per meter length. In this equation, the drag coefficient  $C_d$  is set at 0.65 in accordance with (DNV RP C205, 2021).

The drag coefficient can more accurately be estimated when considering that this coefficient is dependent on the Keulegan Carpenter (KC) number (Journée, 2001). From the well known KC /  $C_d$  distributions the drag coefficient can be obtained. The KC number varies over the MP draft and over time because the tangential velocity of the MP w.r.t. the water changes over time and MP draft, see Equation 64. In this equation  $U_a$  is the velocity amplitude,  $T$  is the period of oscillation of the velocity, and  $D$  is the diameter of the cylinder. This implies that the dragcoefficient is also time and MP draft dependend. It can be seen that Equation 61 has two separate dragforce coefficients, one for the term that is associated with the velocity of the water  $C_{d,v}$ , and one for the term that is related to the MP velocity  $C_{d,\dot{x}}$ . Both drag coefficients can be evaluated based on their individual KC numbers. Properly assessing these drag coefficients is, however, a study on its own and is not the focus of this MSc thesis, therefore both drag coefficients will be fixed at 0.65 during this thesis.

$$f_{drag,relative} = \frac{1}{2} \rho_w C_d D_{pile} (v_{current} - \dot{x}_{tangential}) |v_{current} - \dot{x}_{tangential}| \quad (60)$$

$$f_{drag,separate} = \frac{1}{2} \rho_w C_{d,v} D_{pile} v_{current} |v_{current}| + \frac{1}{2} \rho_w C_{d,\dot{x}} D_{pile} \dot{x}_{tangential} |\dot{x}_{tangential}| \quad (61)$$

$$V_r = \frac{V_0 T_0}{D} \quad (62)$$

$$V_0 = v_{current} + v_{tangential} \quad (63)$$

$$KC = \frac{U_a T}{D} \quad (64)$$

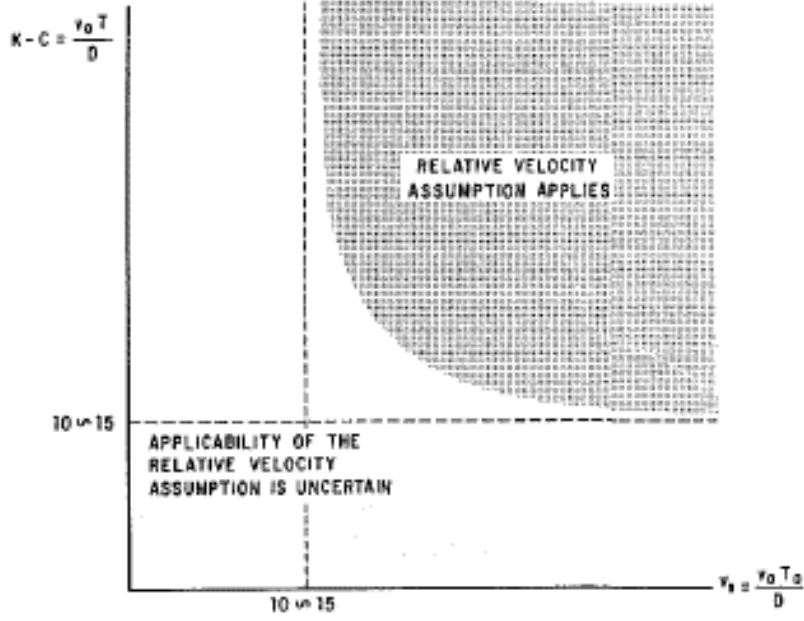


Figure 21: Relative velocity region of applicability (Laya & Connor, 1984)

### 4.3 Time-simulations

For different increments of the hydromechanical load, Equation 58 will be solved in the time domain. The EOM is first rewritten in a set of two first-order ordinary differential equations (ODEs). Hereafter, the system of first-order ODEs is solved in Python, by utilizing the 'Solve IVP' package from Scipy. The numerical time-integration scheme used in this package is the 'Radau' method. This time-integration scheme is an Implicit Runge-Kutta method of the Radau 2A family of order 5. This numerical time-integration scheme is unconditionally stable, because of which the solution will never explode to infinity (Vuik et al., 2023).

Solving Equation 58, during a steady-state simulation, i.e., at a fixed MP draft, is rather convenient. During this simulation, the system properties of the 1-DOF pile-cable model used in the EOM,  $I_{m,55,tot}$ ,  $C_{55}$ , and  $K_{55}$ , are all constant in time, only  $F_{5,wave-excit}$  varies over time, the magnitude of this signal stays the same. Because of this, the EOM for the steady-state simulation can be solved by rewriting the 2nd-order ODE into a system of two 1st-order ODEs and solving the system of equations with the 'Solve IVP' function from the Scipy library in Python, which utilizes the Radau time-integration scheme.

Solving Equation 58 during the dynamic lowering simulation is not so straightforward as solving the EOM for the steady-state simulation. During the dynamic lowering simulation the system properties,  $I_{m,55,tot}$ ,  $C_{55}$ , and  $K_{55}$ , are all varying over time since they change as the MP lowers through the water. Furthermore, the magnitude of the time-varying wave excitation force,  $F_{5,wave-excit}$  also changes over time as the MP lowers through the water since the wetted area of the MP increases. To solve this time-varying EOM, the EOM in Equation 58 is solved at discrete values of the MP draft, starting at a MP draft of 0.1m. Depending on the winch speed of the crane cable, and the distance between two discrete MP drafts, the specific time for which Equation 58 has to be solved at a fixed draft is specified, this time will be referred to as  $t_{fixed,MP-draft}$ . After Equation 58 is solved at a MP draft of 0.1m, with the system properties  $I_{m,55,tot}$ ,  $C_{55}$ , and  $K_{55}$ , evaluated at this draft, during the time interval  $0 < t < t_{fixed,MP-draft}$ , the MP is lowered one discrete length of MP draft. New system properties of the EOM are found at this draft, and by using the velocity and position of the MP at the last time-step of the previous MP draft as the IC's for the new EOM, Equation 58 can be solved for this new MP draft during time interval  $t_{fixed,MP-draft} < t < 2t_{fixed,MP-draft}$ . The magnitude of the wave-excitation force,  $F_{5,wave-excit}$  is also updated at every MP draft. Note that updating the magnitude of a regular wave force for every MP draft is relatively straightforward, as only the amplitude changes. When wave forces due to an irregular sea state are considered, the irregular wave force should be evaluated for every MP draft using the same wave seed for every MP draft to ensure a continuous force signal. The solutions of Equation 58 at different MP drafts can be concatenated to obtain the solution of the time-varying EOM, in this way the solution for the dynamic lowering simulation is found.

The EOM for a steady-state simulation is found by taking the derivative of the impulse of the system w.r.t. time, see Equation 65. For a time-variant system, the mass moment of inertia of the system changes over time

and is therefore time-dependent. Because of this the EOM of a system with a time-dependent mass moment of inertia is found with Equation 66. This means that the term  $\frac{I_{m,55,tot}(t)}{dt} \cdot \dot{\theta}(t)$  has to be added on the left hand side of the EOM depicted in Equation 58 when solving the dynamic lowering simulation. Although this term is not explicitly added in this EOM, it is added implicitly. As mentioned previously, the initial conditions at the first time-step of a new discrete MP draft are equal to the MP motion and velocity of the last time-step of the previous MP draft. This means that an extra force is added to the system, namely the term  $\frac{I_{m,55,tot}(t)}{dt} \cdot \dot{\theta}(t)$ . In this way the second term in Equation 66 is already included in the dynamic lowering simulation.

$$F_{\theta} = I_{m,55,tot} \cdot \dot{\theta}(t) \frac{d}{dt} = I_{m,55,tot} \cdot \ddot{\theta}(t) \quad (65)$$

$$F_{\theta} = I_{m,55,tot}(t) \cdot \dot{\theta}(t) \frac{d}{dt} = I_{m,55,tot}(t) \cdot \ddot{\theta}(t) + \frac{I_{m,55,tot}(t)}{dt} \cdot \dot{\theta}(t) \quad (66)$$

## 5 Modal analysis

Before starting with the time simulations, it would be convenient to gain a preliminary insight into the dynamic behavior of the multi-DOF model and the 1-DOF model. Therefore, the generalized modal analysis is utilized, to extract the natural periods, mode shapes, and damping ratio values of the multi-DOF model. Hereafter the natural periods and damping ratio's are also evaluated for the 1-DOF model. This information yields insight into how both systems would respond to a certain frequency of excitation. Note that the results of the natural periods and damping ratios are obtained for different drafts of the MP, as the system properties at different MP drafts will change. Note that the damping ratios are obtained whilst only considering structural damping from the FEM model and added damping on the MP from the diffraction model. First, the results of the multi-DOF model will be presented, hereafter the results of the 1-DOF model will be outlined. After the results of the 1-DOF pile-cable model are discussed, some of the assumptions regarding this model will be validated and elaborated on.

### 5.1 Multi-DOF model

The EOM of the multi-DOF model from GustoMSC is outlined in Equation 67. To apply the generalized modal analysis, this EOM can be rewritten into the eigenvalue problem outlined in Equation 76, by the utilization of Equations 69, 70, 71, 73, 74, and 75 (Brincker & Ventura, 2015). Solving this system for  $\vec{\phi}_n$  and  $\lambda_n$  will yield the eigenvectors and eigenvalues of the generalized modal analysis problem. Considering that the system has  $N$  DOFs, there will be  $2N$  eigenvalues and mode shapes. For underdamped systems the  $2N$  eigenvalues and mode shapes will appear in complex conjugate pairs, resulting effectively in  $N$  unique mode shapes. Furthermore, the first half of vector  $\vec{\phi}_n$  is associated with the mode shape velocities, whilst the second half of the vector contains all the nodal coordinates of the  $n$ th mode shape displacements, see Equation 77. The complex eigenvalue associated with mode shape  $n$  can be broken down into a real and imaginary part, where the imaginary part is the damped natural frequency, see Equation 78. The undamped natural frequency can be found with Equation 79, and the damping ratio in terms of the critical damping can be found with Equation 80 (Brincker & Ventura, 2015). For more information on the theoretical background of the generalized modal analysis, see (Brincker & Ventura, 2015).

$$(\mathbf{M}_{\text{global}} + \mathbf{A}'_{\text{global}})\ddot{\mathbf{x}}(t) + \mathbf{C}'_{\text{global}}\dot{\mathbf{x}}(t) + \mathbf{K}'_{\text{global}}\mathbf{x}(t) = \mathbf{F}(t) \quad (67)$$

$$\mathbf{A}\dot{\mathbf{x}}(t) + \mathbf{B}\mathbf{x}(t) = \mathbf{f}(t) \quad (68)$$

$$\mathbf{A}\dot{\mathbf{x}}(t) + \mathbf{B}\mathbf{x}(t) = 0 \quad (69)$$

$$\mathbf{A} = \begin{bmatrix} 0 & \mathbf{M}_{\text{global}} + \mathbf{A}'_{\text{global}} \\ \mathbf{M}_{\text{global}} + \mathbf{A}'_{\text{global}} & \mathbf{C}'_{\text{global}} \end{bmatrix} \quad (70)$$

$$\mathbf{B} = \begin{bmatrix} \mathbf{M}_{\text{global}} + \mathbf{A}'_{\text{global}} & 0 \\ 0 & \mathbf{K}'_{\text{global}} \end{bmatrix} \quad (71)$$

$$\mathbf{f}(t) = \begin{bmatrix} \vec{0} \\ \mathbf{F}(t) \end{bmatrix} \quad (72)$$

$$\mathbf{x}(t) = \begin{bmatrix} \vec{x}(t) \\ \mathbf{x}(t) \end{bmatrix} \quad (73)$$

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} \vec{\dot{x}}(t) \\ \dot{\mathbf{x}}(t) \end{bmatrix} \quad (74)$$

$$\mathbf{x}(t) = \vec{\phi} e^{\lambda \cdot t} \quad (75)$$

$$\lambda \mathbf{A} \vec{\phi} + \mathbf{B} \vec{\phi} = 0 \quad (76)$$

$$\vec{\phi}_n = \begin{bmatrix} \lambda_n \vec{b}_n \\ \vec{b}_n \end{bmatrix} \quad (77)$$

$$\left. \begin{matrix} \lambda_n \\ \lambda_n^* \end{matrix} \right\} = \sigma_n \pm i\omega_{dn} \quad (78)$$

$$\omega_{0n} = \sqrt{\lambda_n \cdot \lambda_n^*} = \sqrt{\sigma^2 + \omega_{dn}^2} \quad (79)$$

$$\zeta_n = \frac{-\sigma_n}{\omega_{0n}} \quad (80)$$

Because the multi-DOF model used within GustoMSC has approximately 600 DOFs, there will subsequently be 600 unique mode shapes, natural periods, and damping ratios, for every MP draft and assumed pile-period of motion. As discussed earlier, the lowering of the MP will change the stiffness, and mass moment of inertia properties of the pile-cable subsystem w.r.t. the gripper node, and hence it will affect the outcome of the modal analysis. Also, because the added mass surrounding the MP is estimated with WAMIT, and through diffraction theory motion-period dependent, the assumed pile-period of motion will also affect the outcome of the modal analysis. Usually, the mode shapes associated with the highest natural periods will affect the dynamic behavior of a system the most since these natural periods are closest to the periods of excitation. For this reason, the generalized modal analysis problem will be solved for the first 5 mode shapes for different MP drafts and different assumed pile periods of motion.

### 5.1.1 Results of the modal analysis

In Figure 22 the first 5 mode shapes of the combined jack-up and pile-cable model are depicted. From these figures, it can be seen that the first 2 mode shapes are solely related to the pile-cable deformation. Mode shape 1 is associated with the rotation in the gripper node perpendicular to the gripper connection, and mode shape 2 is associated with the rotation in the gripper node in line with the gripper connection. Modeshapes 3, 4, and 5 are combinations of the pile-cable and jack-up deformations. In Figures 23, 24, 25, 26, 27, the natural periods associated with respectively mode 1,2,3,4, and 5 are depicted for different MP drafts, and different assumed pile periods of motion. Note that in the mentioned graphs, the values of the natural periods change for different MP drafts, which is expected because the mechanical system changes as the MP lowers through the water. The values of the natural periods also change for different assumed pile periods of motion because the values of the added damping and added mass are MP motion period dependent.

Note that the results and the discussion on these results for the damping ratios of the multi-DOF model are put in the appendix, see subsection A.2.

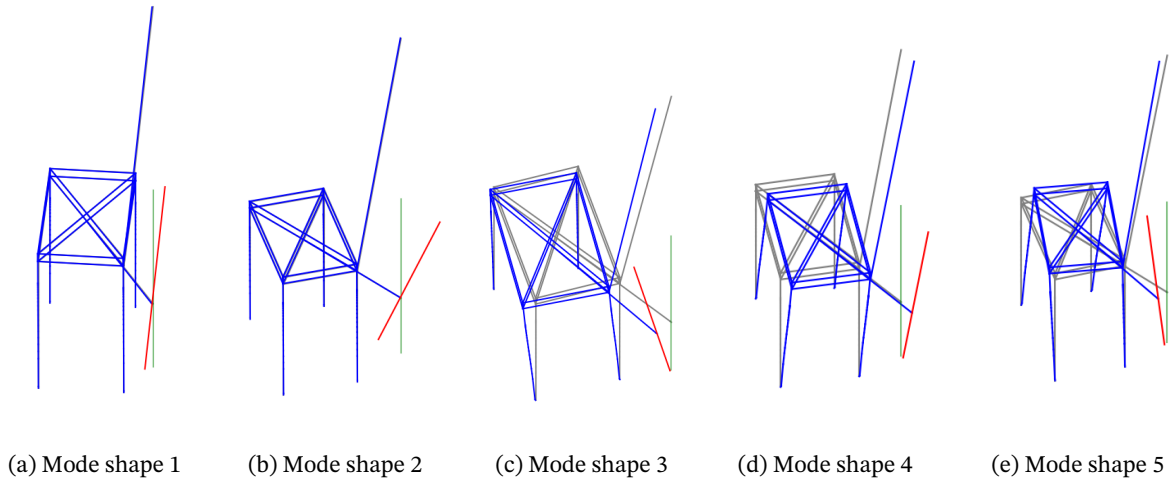


Figure 22: Mode shapes of the multi-DOF model

### Natural periods for modeshape 1 of the combined system; jack-up + pile-cable model

Base-configuration:  $L_{pile} = 115$  m,  $d_{baseh - grip} = 6$  m,  $L_{crane} = 200$  m,  $D_{pile} = 12$  m

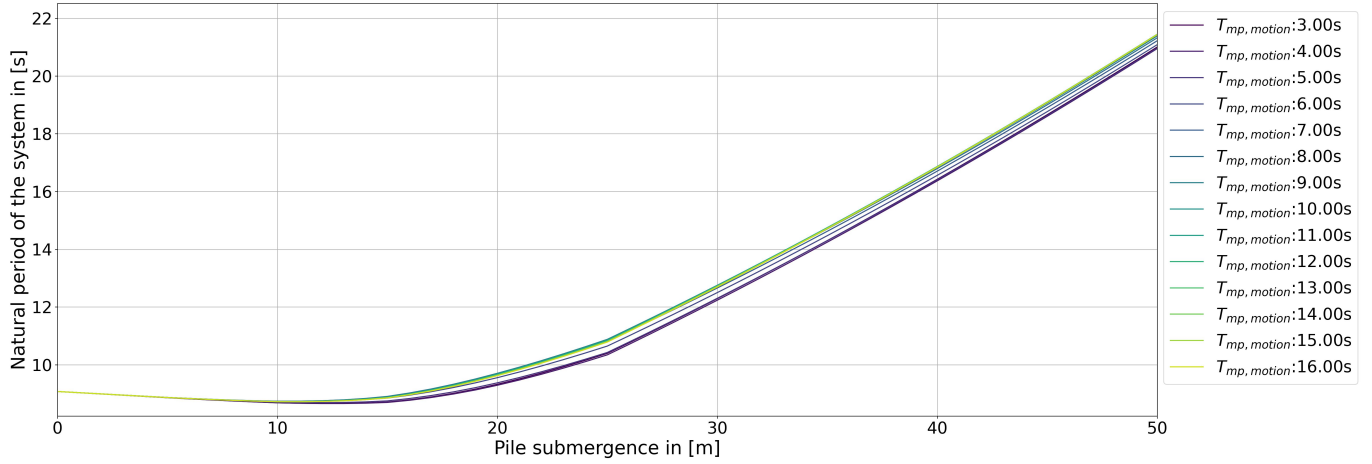


Figure 23: Natural periods of mode-shape 1

### Natural periods for modeshape 2 of the combined system; jack-up + pile-cable model

Base-configuration:  $L_{pile} = 115$  m,  $d_{baseh - grip} = 6$  m,  $L_{crane} = 200$  m,  $D_{pile} = 12$  m

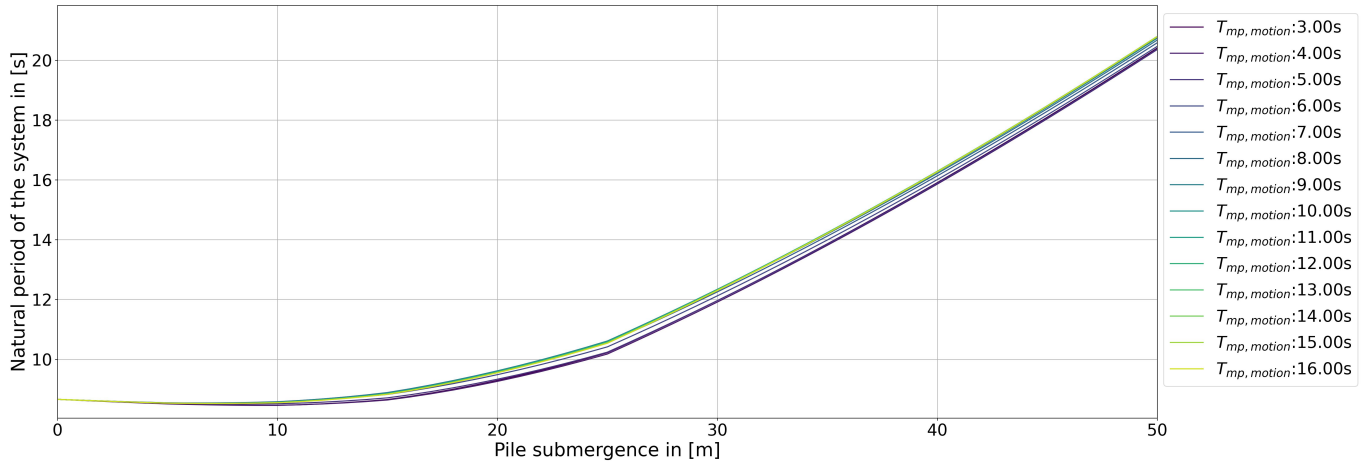


Figure 24: Natural periods of mode-shape 2

### Natural periods for modeshape 3 of the combined system; jack-up + pile-cable model

Base-configuration:  $L_{pile} = 115$  m,  $d_{baseh - grip} = 6$  m,  $L_{crane} = 200$  m,  $D_{pile} = 12$  m

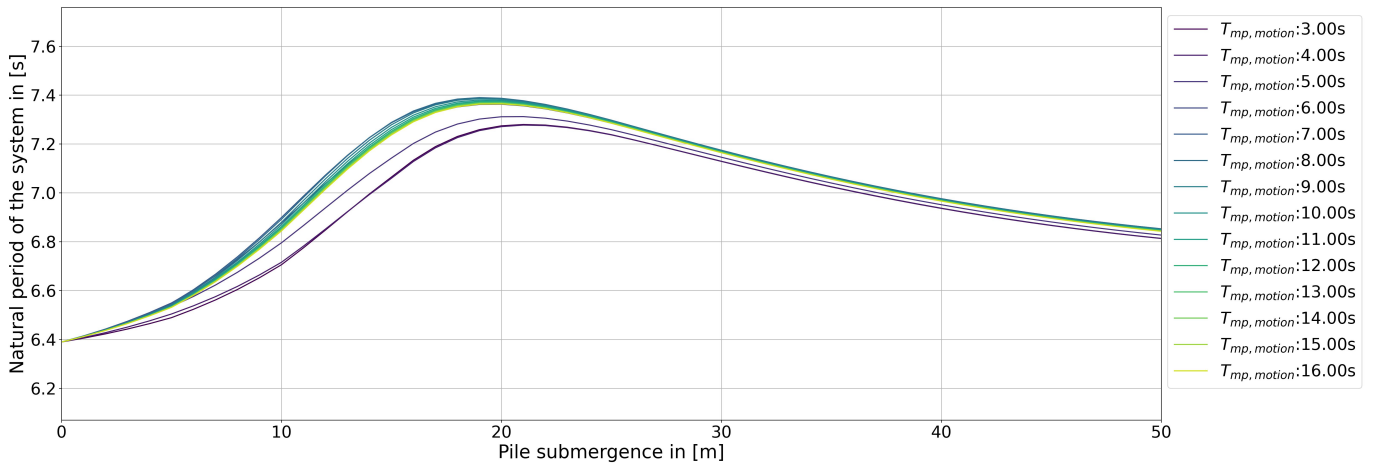


Figure 25: Natural periods of mode-shape 3

### Natural periods for modeshape 4 of the combined system; jack-up + pile-cable model

Base-configuration:  $L_{pile} = 115$  m,  $d_{baseh-grip} = 6$  m,  $L_{crane} = 200$  m,  $D_{pile} = 12$  m

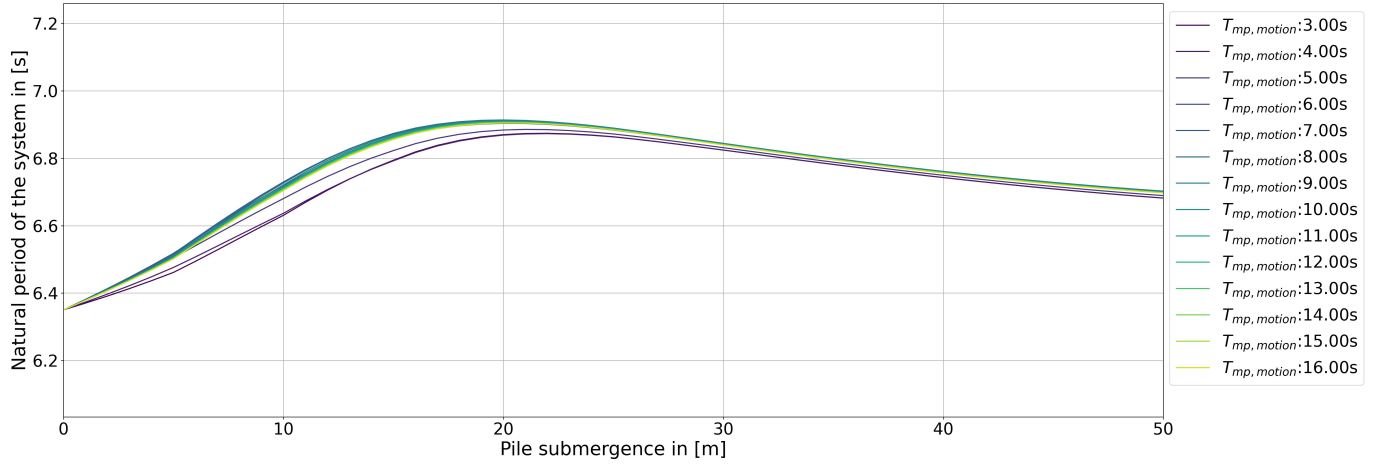


Figure 26: Natural periods of mode-shape 4

### Natural periods for modeshape 5 of the combined system; jack-up + pile-cable model

Base-configuration:  $L_{pile} = 115$  m,  $d_{baseh-grip} = 6$  m,  $L_{crane} = 200$  m,  $D_{pile} = 12$  m

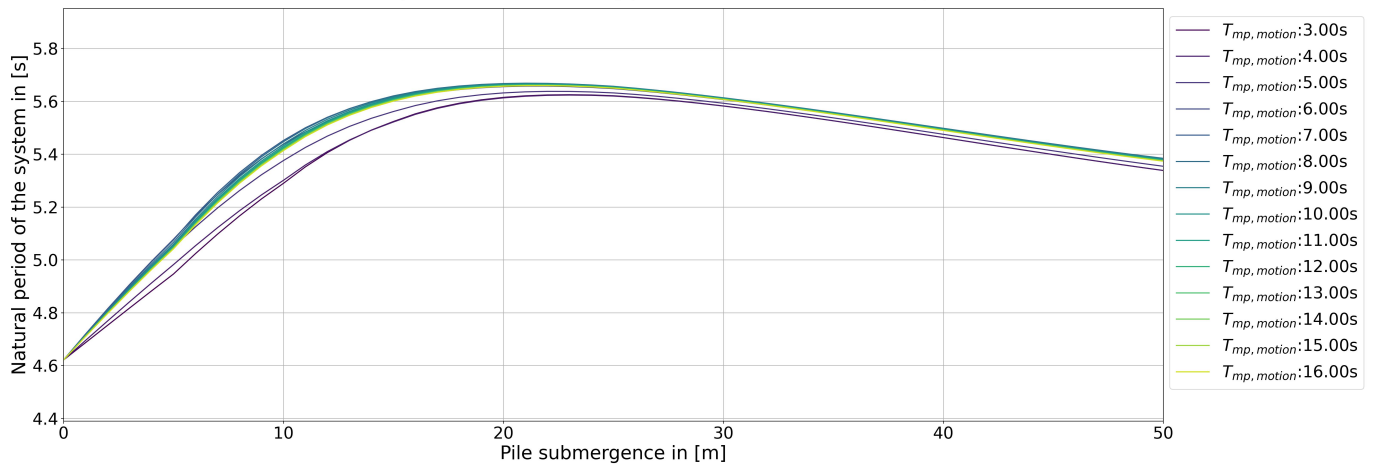


Figure 27: Natural periods of mode-shape 5



### 5.1.2 Discussion of the modal analysis results

As mentioned earlier, the first 2 mode shapes are associated with the rotational DOFs of the MP at the gripper node. It can be seen that the excitation periods of the environmental conditions considered during this MSc thesis, see Table 2, are closest to the natural periods of mode shapes 1 and 2 until a MP draft of around 25m, see Figures 22a and 22b. Because of this, it is reasonable to assume that the dynamic behavior of the MP is governed by the rotational MP DOF in the gripper, as this DOF will be excited the most for the considered environmental conditions. This is one of the reasons that the multi-DOF model is reduced to solely the pitch DOF of the MP in the gripper. The other argument is that it is easier to analyze what specific physical phenomenon influence the dynamic behavior of the MP during the discussed lowering process, when a 1-DOF system is considered. In a 1-DOF system there is no interference from possible jack-up excitations. In the multi-DOF model some excitation frequencies might excite the jack-up, which in return excite the MP, due to which it is difficult to separate what the exact phenomena are that influence the dynamic behavior of the MP. Because of these two arguments, the multi-DOF is reduced to a 1-DOF pile-cable model that can rotate in the pitch DOF in the gripper.

Throughout the lowering process of the MP, the stiffness of the pile-cable subsystem constantly decreases, see Equation 111, and Equation 112. The mass moment of inertia of the pile-cable subsystem w.r.t. the gripper node will decrease between the lowering of the MP draft from 0 to 15m, and increase between the lowering of the MP draft from 20m to 50m. The reason for this is that at first a large part of mass of the steel of the MP will be above the gripper node, due to which the mass moment of inertia will decrease as the MP lowers through the gripper. When the MP lowers further through the gripper the mass moment of inertia will start increasing again as more MP mass is moving farther away from the lower side of the gripper and the effect of the added mass from the water becomes more significant. Later on in this section, this effect will be shown with a plot. Because of these two arguments, the natural periods for mode shapes 1 and 2 are roughly expected to be constant between a MP draft of 0 and 15m and start to increase after MP drafts higher than 15m. This behavior can also be seen in Figure 23, and Figure 24.

Modeshapes 3,4 and 5 are combinations of the first 3 jack-up modeshapes and the pile-cable modes but are primarily related to the first 3 jack-up modes. Mode shape 3 is associated with the surge jack-up mode, mode shape 4 is associated with the sway jack-up mode and mode shape 5 is associated with the yaw jack-up mode. The natural periods of these first 3 jack-up modes are outlined in Table 8 (GustoMSC, 2023). Figures 25, 26, 27 are the natural periods associated with the combined mode-shape of the first 3 jack-up modes and the pile-cable modes. It can be seen from this figure that since the mass moment of inertia decreases between a MP draft of 0 and 15m the natural period of the system increases in this region. As discussed previously, the mass moment of inertia will increase from MP drafts of 15m and higher, due to which the natural periods of the system increase in this region. Note that the natural periods of mode shapes 3,4, and 5 start around respectively the natural periods of the first 3 jack-up modes, which is also expected.

Overall the natural periods of the first 5 combined jack-up and pile-cable model mode shapes show a slight variety depending on the assumed pile-period of motion. This difference is however not too big, since it only has an effect on the added mass outside the MP.

As mentioned previously, the multi-DOF model is reduced to solely the pitch DOF of the MP in the gripper. It can be seen from Figures 25, 26, 27 that wave periods between 4.5–7.5s will excite the jack-up. For earlier given arguments the motion of the jack-up is completely neglected. However for the considered environmental conditions with peak periods of the JONSWAP spectrum of 7,8, and 9 seconds, there is a significant part of these spectra that excite the jack-up, especially when a peak period of 7 seconds is considered. In reality, the jack-up motion might therefore influence the dynamic behavior of the MP during the lowering phase through the splash zone. In subsection 6.6, the error that has come with the assumption of solely considering the pitch DOF of the MP in the gripper will be quantified for different peak periods of the JONSWAP spectrum.

| Parameter         | Value | Unit | Description                                |
|-------------------|-------|------|--------------------------------------------|
| $T_{n,jack-up,1}$ | 6.49  | s    | Natural period jack-up mode 1 (sway mode)  |
| $T_{n,jack-up,2}$ | 6.45  | s    | Natural period jack-up mode 2 (surge mode) |
| $T_{n,jack-up,3}$ | 4.53  | s    | Natural period jack-up mode 3 (yaw mode)   |

Table 8: Jack-up first 3 natural periods

## 5.2 1-DOF model

In the previous subsection, it was mentioned that the mass moment of inertia of the pile-cable model w.r.t. the gripper node would decrease when the MP is being lowered from 0 to around 15 m of MP draft. Increasing the MP draft beyond 15m would only increase the mass moment inertia of the pile-cable model w.r.t. the gripper node, as the effect of the water in and outside the MP starts to become more significant, and the mass contribution from the steel of the MP moves further down the gripper. In Figure 28, the mass moment of inertia of the 1-DOF pile-cable model is depicted as a function of the MP draft, for different assumed pile-periods of motion. The discussed effect can clearly be seen in this graph. This graph would serve as a convenient substantiation for the reason why the mass moment of inertia first decreases and hereafter increases as was also discussed in subsubsection 5.1.2.

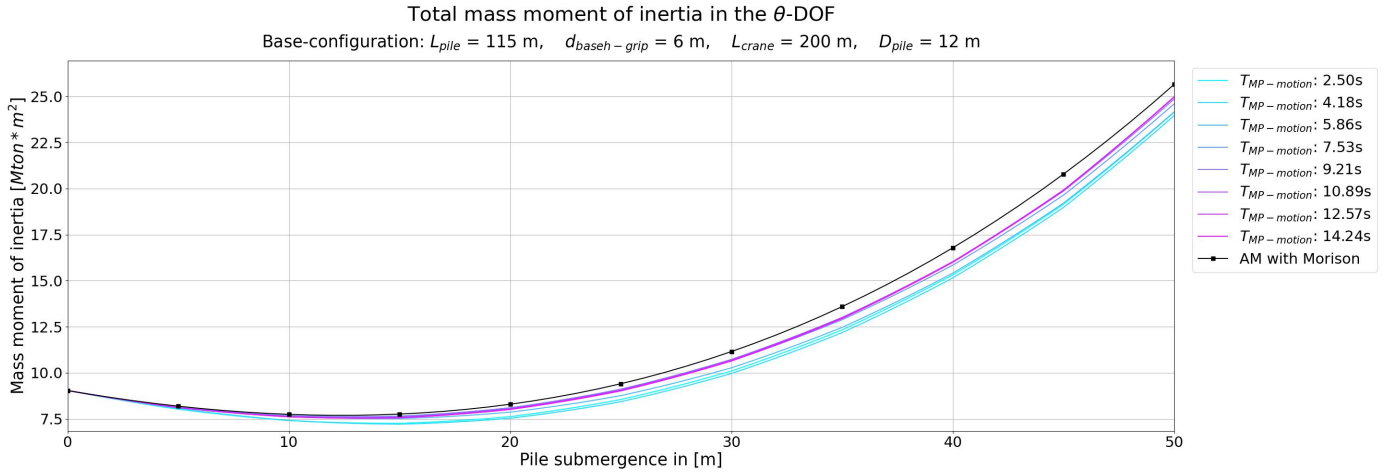


Figure 28: Total mass moment of inertia of the 1-DOF model

The natural frequency of a 1-DOF system is found with Equation 81. The damping ratio can be obtained with Equations 82, and 83. Since the values of the stiffness,  $k_{55}$ , mass moment of inertia  $I_{m,55,tot}$ , and the damping  $C_{55,tot}$  of the 1-DOF pile cable model can be obtained for different MP drafts, and different assumed pile periods of motion, the natural periods and damping ratio of the pile-cable subsystem can be obtained.

$$\omega_n = \sqrt{\frac{k}{m}} \quad (81)$$

$$\zeta = \frac{C}{C_{crit}} \quad (82)$$

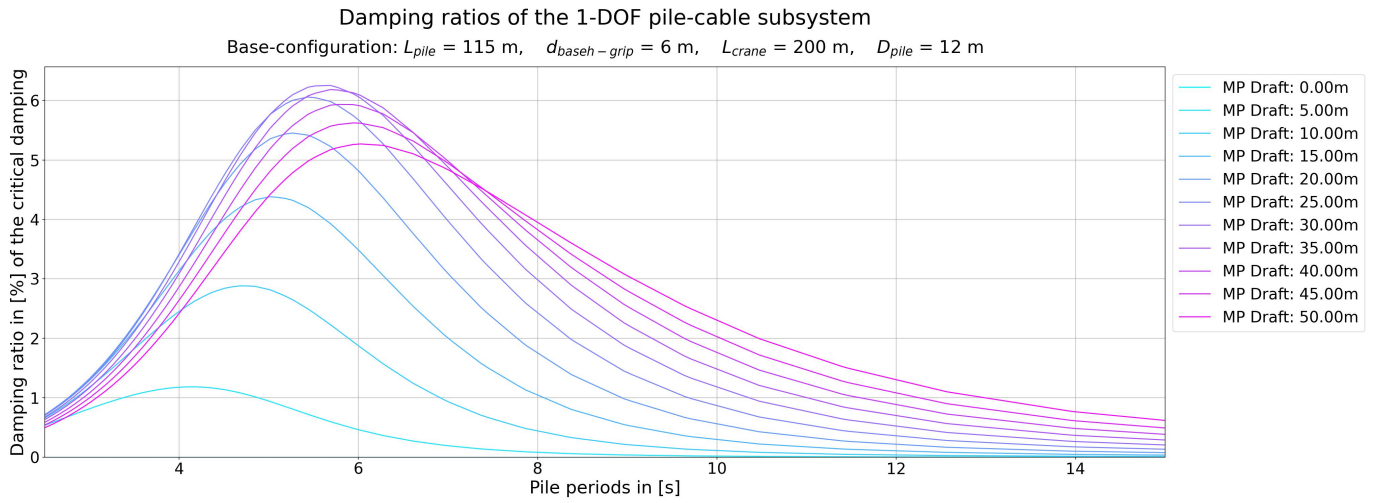
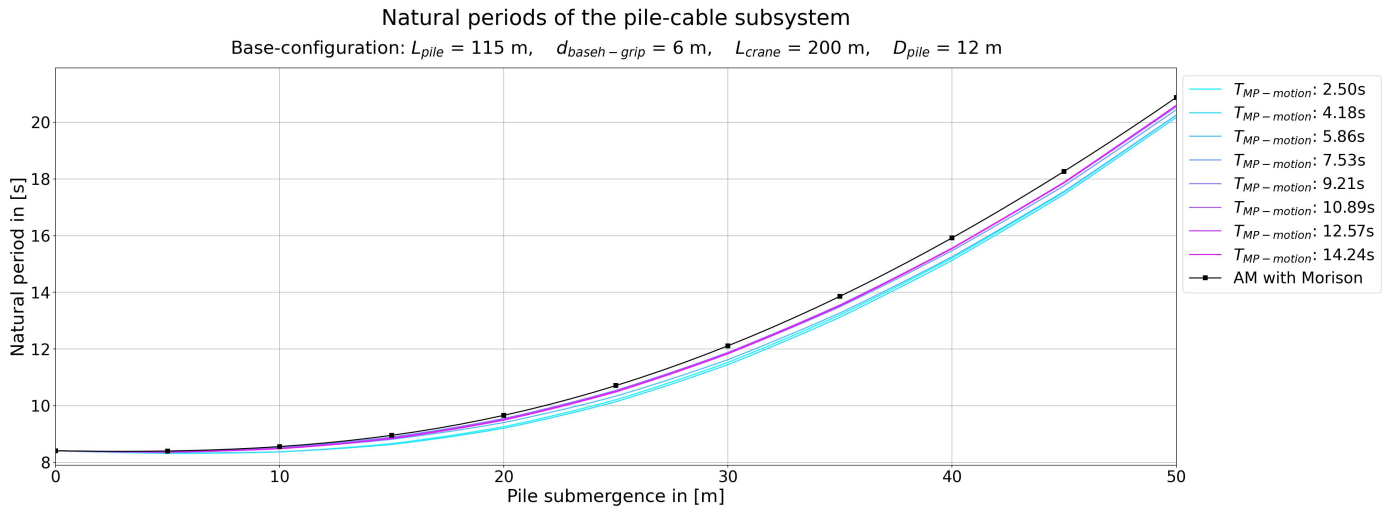
$$C_{crit} = 2 \cdot m \sqrt{\frac{k}{m}} = 2 \cdot m \omega_n \quad (83)$$

### 5.2.1 Results of the modal analysis

The mode shape of the 1-DOF model would only show a rotational displacement at the gripper node, since that is the only mode in which the structure can deform. With the equations as discussed in the previous paragraph, the results for the natural periods and damping ratios of the 1-DOF pile-cable model for different MP drafts and assumed pile-periods of motion are found and depicted in Figures 29, and 30. For convenience the added-mass in Figure 29 is also evaluated with the added-mass Morison assumption, see the black squared graph.

### 5.2.2 Discussion of the modal analysis results

The natural period plot of the 1-DOF model is very similar to the natural period plot of modes 1 and 2 of the multi-DOF model, see Figures 29, 23, and 24. This is also expected as the first 2 modes of the multi-DOF model are primarily associated with the pile-cable rotational DOF in the gripper. The natural period plot of the 1-DOF pile-cable model can therefore also be explained by the same substantiations as was done for the first 2 modes of the multi-DOF model, and will not again be done in this paragraph. The same conclusion can be drawn for the plot of the damping ratios of the 1-DOF pile-cable model, see Figures 30, 61, and 62.



### 5.3 Validation of the 1-DOF mechanical model

In section 4, the assumptions of the 1-DOF pile-cable model are discussed. In the appendix the following assumptions w.r.t. the 1-DOF pile-cable model will be validated, to make sure that they make sense, see subsection B.2:

1. The moments that arise due to the rotation of the MP are linearly related to the angle  $\theta$ , so the trigonometric functions are linearized.
2. The mass of the steel of the MP is assumed to be uniformly distributed, i.e., the non-uniform distribution of the mass due to a possible tapered end of the MP is not considered.

Furthermore, the effect of different layouts of the jack-up are also discussed in the appendix, see subsection B.3,

## 6 Results & Discussion

As is discussed in section 4, the EOM of the pile-cable model that only considers the pitch-DOF w.r.t. the gripper node, depicted in Equation 58, will be solved in the time-domain, except for the results of level 1 which are solved in the frequency domain. The results of, mainly the time domain simulations, will be presented and discussed in this section.

In section 4, it is discussed how the values for the stiffness, added damping, mass moment of inertia, and wave excitation forces of the 1-DOF model can be found for solely the pitch DOF, see Equations 58, and 59. Note that these values change when the MP is lowered in the water, taking into account the kinematic relations defined in Equation 1, and Figure 4.

At each level of accuracy, some physical phenomena will be added to the model, w.r.t. the previous level, so that its effect will become apparent. These physical phenomena primarily have to do with the modeling of the wave-excitation loads, the modeling of the added mass and damping values, and the modeling of the time domain simulation, i.e., either a steady-state simulation or a dynamic lowering simulation. Specific parts of these results, obtained in each increment will be compared to either, hand calculations, similar output from the multi-DOF model, the previous increment, or a combination of these 3. In each subsection of this section, a level of accuracy of the 1-DOF model is discussed, the following structure within each subsection will be used:

1. Explanation of the hydromechanical model of that level
2. Results of the time-simulation of that level
3. Discussion about the results of that level

Within certain levels, some of the included physical phenomena of that level will be validated, in order to make sure that this phenomena is implemented correctly in the 1-DOF model. In order to keep this section concise, not every validation of such a physical phenomena will be discussed in the associated subsection. The conclusion of some of these validations will be shortly discussed in the associated subsection, however the actual plots and full detailed explanation will be put in the appendix, see Appendix B. The same is done for the plots used to explain the results, not all of the plots will be shown in each level, for every plot see Appendix C.

There are multiple disregarded physical phenomena that are not included in any level of the 1-DOF model. At the end of this section these physical phenomena, and the reason why they are disregarded will be discussed, based on several arguments and plots.

## 6.1 Level 1: Motion RAO's

In this level, the motion RAO's of the pitch DOF will be evaluated. This is done by evaluating the maximum motion response of the 1-DOF system during steady state simulations at MP drafts of 5, 10, 15, 20, 25 and 30m for discretized wave-periods ranging between 6-15 seconds with a wave height of 2.5m and whilst setting the ambient current at  $0 \frac{m}{s}$ . See Table 9 for the environmental conditions considered at this level. The maximum motion response of each time-simulation in a steady state situation is divided with the wave amplitude used in this simulation such that for different wave-periods and MP drafts a motion response amplitude operator, or motion RAO, is obtained. In this way the EOM depicted in Equation 58 is solved in the frequency domain with 2 different models:

- Model 1: this model does not consider damping due to the viscous drag.
- Model 2: this model does consider damping due to the viscous drag.

The wave-excitation forces, added mass and added damping outside of the MP for both models are evaluated with WAMIT. Furthermore, the total mass moment of inertia and the stiffness of the EOM are evaluated in the same way as was discussed in section 4. For model 2, the viscous drag is evaluated with the 2th term of the separate flow form of the Morison equation depicted in Equation 61, see subsection 4.2 for the full details on how this viscous drag force is evaluated.

| Parameter     | Value | Unit | Description                |
|---------------|-------|------|----------------------------|
| $H_{regular}$ | 2.5   | m    | Height of the regular wave |
| $T_{regular}$ | 6-15  | s    | Period of the regular wave |
| $V_{current}$ | 0     | m/s  | Velocity of the current    |

Table 9: Level 1: Environmental conditions

To make sure that the correct wave-forces are obtained for regular waves, the regular wave forces evaluated for the 1-DOF model are compared with the regular wave forces under similar conditions in the multi-DOF model. The results are showing to be near coincident, indicating that the regular wave forces are evaluated correctly, see subsubsection B.4.1. The viscous drag force due to the current and the MP motion is also validated with a hand calculation. These hand calculations show that the viscous drag force due to the current and the MP is evaluated correctly in the Python model, see subsubsection B.4.2. Note that the current is not considered during this level of the 1-DOF model, this phenomenon will however be added in future levels and hence it is already validated at this level. Furthermore, proof is established that the 2nd term in the separate flow form of the Morison equation is indeed more conservative w.r.t. the relative flow form of the Morison equation, see subsubsection B.4.2.

### 6.1.1 Results

The results of the motion RAO's in the pitch DOF of the 1-DOF system under the mentioned conditions for different MP drafts is shown in Figure 31. In this figure, the y-axis represents the motion RAO in the pitch DOF in  $[\frac{deg}{m}]$ , and the x-axis is the wave period in  $[s]$ . In this plot the continuous line is the model that does not consider the damping due to the viscous drag, evaluated with Equation 61. The dashed line in the plot is the model which does consider the damping due to viscous drag, evaluated with Equation 60. As mentioned earlier the motion RAO plots are evaluated for different MP drafts, since the components that make up the EOM, see Equation 58, is different depending on the MP draft.

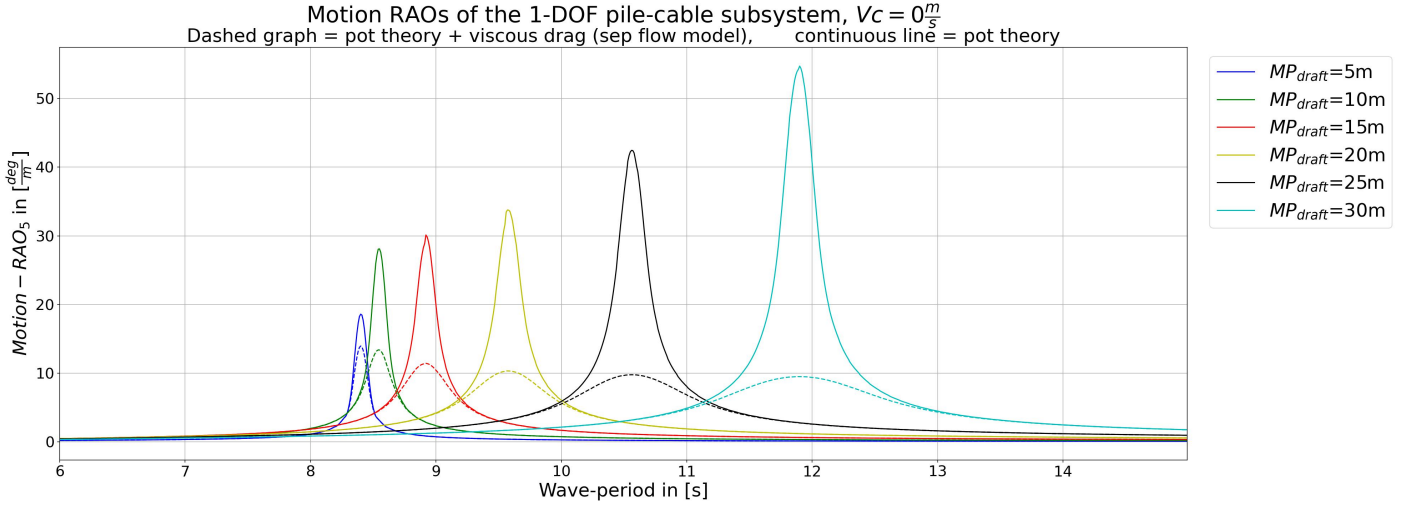


Figure 31: Motion RAOs of the 1-DOF pile-cable model

### 6.1.2 Discussion about the results

The peaks of the motion RAOs in Figure 31 coincide with the natural periods of the 1-DOF pile-cable model, see Figure 29, which was also expected. Furthermore, there is a relatively large difference between the viscous damped and viscous undamped motion RAOs at larger MP drafts. This is also expected because at these larger MP drafts the damping due to the viscous drag is larger since there is more MP in the water. At smaller MP drafts there is a smaller difference between the viscous damped RAO and the RAO that does not consider the damping due to viscous drag. The reason for this is that the MP cannot gain much damping in this region as there is not much length of the MP in the water. The natural periods of the MP at these lower MP drafts are close to the most expected wave periods for the considered environmental conditions, see Table 2. These are worrying MP drafts, since at resonance the motion response of a system is governed by damping, of which there is not much at these drafts. Therefore the motion response during a steady-state simulation at lower drafts is expected to be relatively high.

## 6.2 Level 2: Dynamic lowering simulation, during regular waves

In this level the MP motion will be evaluated whilst lowering the MP through the splash zone over time. Regular waves and current are considered with values for the waveheight, wave period and current intensity as shown in Table 10. The added mass, added damping, and wave-excitation forces are evaluated with WAMIT, and the viscous drag due to the current and the MP motion is evaluated as is discussed in subsection 4.2. Furthermore, the stiffness and the total mass moment of inertia are evaluated in the same way as was discussed in section 4. In subsection 4.3 it is explained how the time-variant EOM, see Equation 58, is evaluated in the dynamic lowering simulation. The winch speed of the crane cable is set at  $0.0833 \frac{m}{s}$ , in accordance with GustoMSC (GustoMSC, 2023). Because the MP is lowered through the water the following components of Equation 58 also change over time:

1. The total mass moment of inertia,  $I_{m,55,tot}$ , because the following contributions of this mass moment of inertia change over time:
  - (a) Added mass outside of the MP, evaluated with WAMIT
  - (b) Added mass due to the entrapped water in the MP
  - (c) Mass from the steel of the MP
2. The added damping  $C_{55}$ .
3. The stiffness of the MP  $K_{55}$ .
4. The magnitude of the wave excitation forces from WAMIT, due to a regular wave,  $F_{5,wave-excit}$ .
5. The magnitude of the viscous dragforce

| Parameter     | Value | Unit | Description                |
|---------------|-------|------|----------------------------|
| $H_{regular}$ | 2.5   | m    | Height of the regular wave |
| $T_{regular}$ | 7/8/9 | s    | Period of the regular wave |
| $V_{current}$ | 1.5   | m/s  | Velocity of the current    |

Table 10: Level 2: Environmental conditions

### 6.2.1 Results

In Figure 32, 33, 34 the motion response of the dynamic lowering simulations are depicted for a regular wave with a height of  $2.5m$  and respectively a period of 7,8 and 9 seconds. In all 3 figures the left y-axis indicates the pitch motion of the MP w.r.t. the gripper in [deg], the right side of the y-axis indicates the MP draft in [m], and the x-axis shows the time in [s]. The purple graph is the MP motion evaluated with the model that does not consider the viscous drag, and the cyan graph is the MP motion evaluated with the model which does consider the viscous drag. The maximum and minimum motion response of each graph, considering both models is shown in the subtitle of each plot. Note that because the viscous drag does not dampen the motion significantly enough in Figures 32, and 33, it looks like as if only the model without viscous drag is plotted. The motion response of both models is however plotted but since they are almost exactly similar the second model is not observable. The difference between the two models can be observed in Figure 34, because the damping due to viscous drag is more dominant in this plot. Finally, the mean displacement due to the current is also clearly noticeable at larger MP drafts, see Figures 32, 33, and 34.

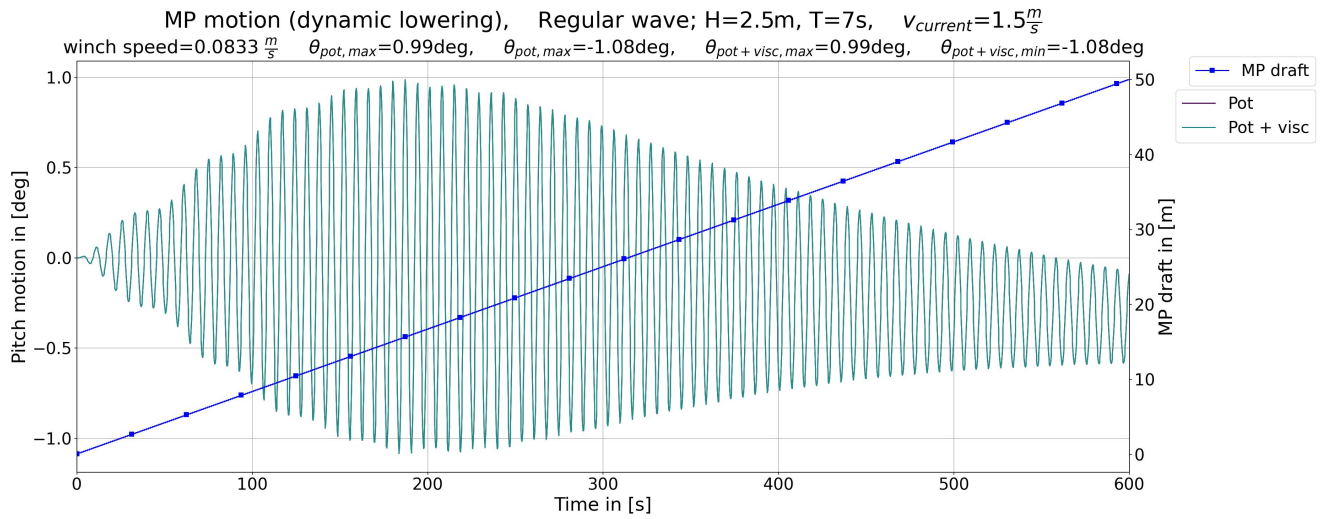


Figure 32: Motion response dynamic lowering simulation, 1-DOF model:  $H=2.5\text{m}$ ,  $T=7\text{s}$ ,  $V_c = 1.5\text{ m/s}$ , potential theory + viscous drag

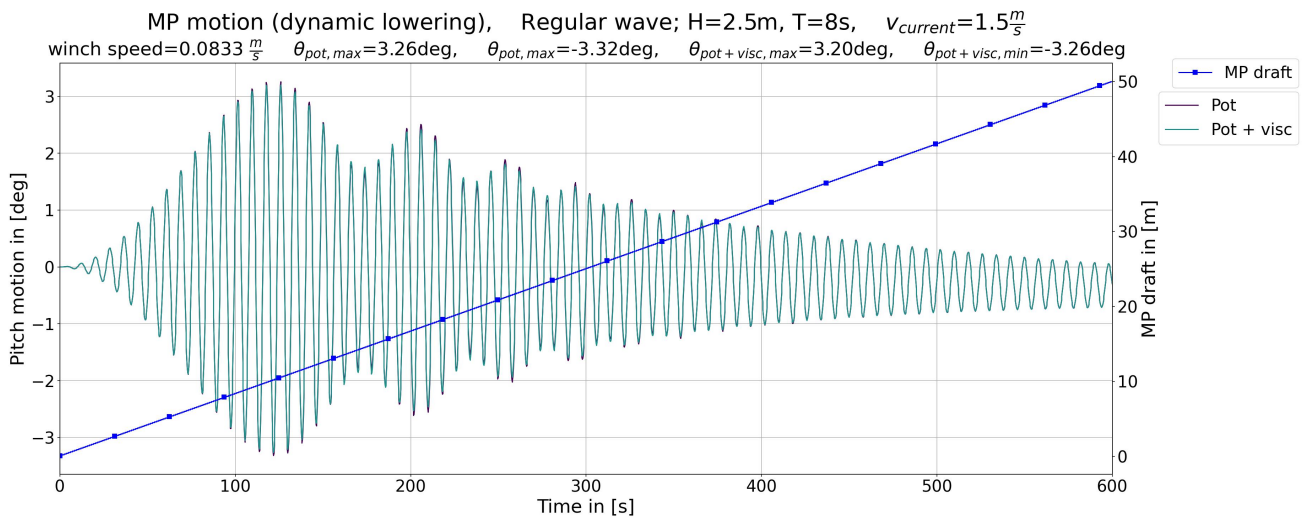


Figure 33: Motion response dynamic lowering simulation, 1-DOF model:  $H=2.5\text{m}$ ,  $T=8\text{s}$ ,  $V_c = 1.5\text{ m/s}$ , potential theory + viscous drag

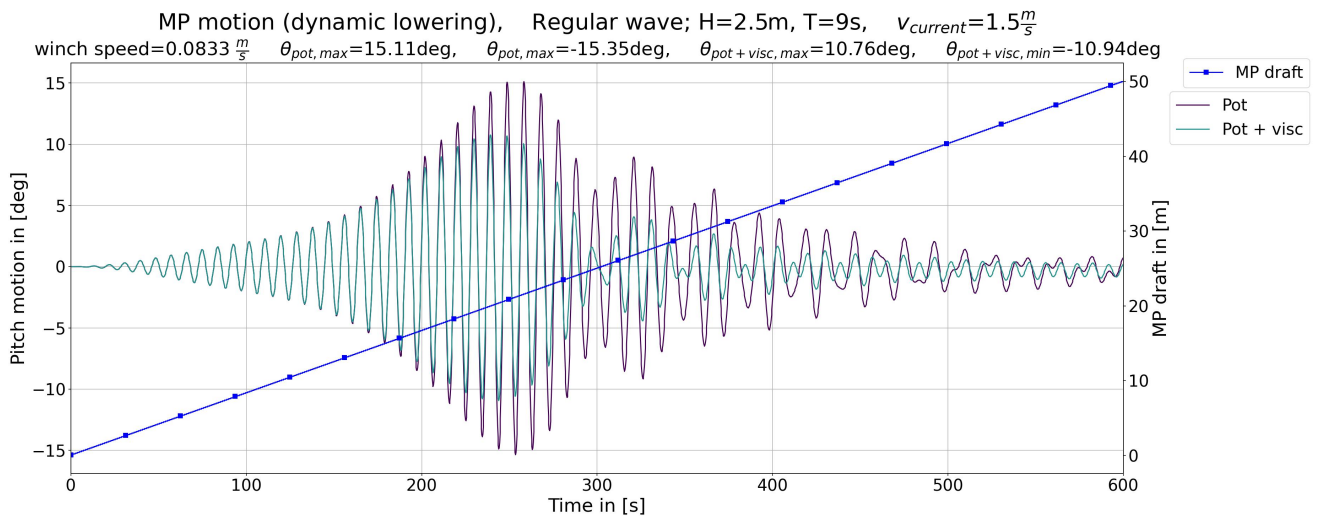


Figure 34: Motion response dynamic lowering simulation, 1-DOF model:  $H=2.5\text{m}$ ,  $T=9\text{s}$ ,  $V_c = 1.5\text{ m/s}$ , potential theory + viscous drag



### 6.2.2 Discussion about the results

It can be seen that the response is the lowest for a 7-second period regular wave, see Figure 32. This is because the natural period of the 1-DOF pile-cable subsystem starts around 8.5 seconds, and increases as the MP is lowered through the water, see Figure 29. Resonance does therefore not occur at any MP draft as the natural period of the system will never pass through 7 seconds. The motion response of the system during an 8-second regular wave is already a bit higher, but the motion response of the system during a 9-second regular wave is by far the biggest as in this simulation the system passes fully through resonance. Not only is the peak of the motion response higher for the 9 second regular wave, it also occurs later in time, i.e. at larger MP drafts. Because the motion amplitudes are largest for a 9 second period wave, the viscous effects are also more dominant for this wave period. See Figure 34 for the difference between the model that does consider damping due to viscous drag, and the model that does not consider damping due to viscous drag.

Another important observation from Figure 34 is that the peak does not occur exactly when the system crosses through resonance, see Figure 29, but slightly after this instant. The reason for this is that the increase of the response is highest at resonance, but because the motion takes time to build up it will not peak at this exact instant. As the MP lowers further through the water, the natural period of the system moves away from the excitation periods, however, the magnitude of the excitation loads is still increasing whilst the natural period of the system is not at the excitation period but still relatively close to the natural period. This means that the response still increases but with a smaller slope when comparing it with the instant when resonance occurs. When further lowering the MP through the water, the motion response starts to decrease as the gap between the natural period and the excitation periods, and the damping both increase further.

For a 9 second period wave, the natural period of the 1-DOF system crosses through resonance. Because the damping is relatively low, between 0 and 200 seconds, the motion response of the MP for the model that does not consider damping due to viscous drag, has the same period of motion as the wave in this region (Rao, 2011). After the system has passed through resonance it can be seen that the period of motion of the MP increases, because the absolute value of the added damping increases as the MP is deeper in the water and the velocities are relatively high. At around 250 seconds, the natural period of the system is moving further away from the wave period due to which the motion response starts decreasing. As the added damping increases only more when the MP is lowered further through the water, the period of motion of the MP also increases more. At a certain point the regular wave starts acting in the same direction as the MP motion, because the period of motion of the MP is so slow and the wave is catching up with the MP motion. Because of this a second peak at 330 seconds and the third peak at 360 seconds, and the non-linear behaviour between 450 and 600 seconds is observed for the model that does not consider the damping due to viscous drag. This effect is even more apparent for the model that also considers the damping due to viscous drag. Note that this effect is not apparent at a 7 second period wave, because the motion response of the MP is relatively low in this situation, due to which the damping is also low. For the 8 second wave, the added damping already increases, since the MP motion increases, and hence the second and third peak of the motion response can be observed at 200 and 250 seconds. As the motion response quickly decreases after this, the added damping also decreases, and hence in this situation there is no non-linear behaviour of the MP motion between 400 and 600 seconds, as opposed to the situation with a 9 second period wave.

Finally, the mean displacement due to the current is also clearly noticeable at larger MP drafts, see Figures 32, 33, and 34.

### 6.3 Level 3: Steady-state simulation during irregular spread waves

In this level the steady-state simulation is evaluated for irregular spread waves with the environmental conditions shown in Table 11 at critical MP drafts. The critical MP drafts are chosen at the draft where the peak period is very close to the natural period of the 1-DOF system, and where the biggest part of the MP is in the water. The critical MP draft for irregular waves with a peak period of 7 seconds is at 10m MP draft, for a 8 second peak period the critical MP draft is located at 15m, and for a 9 second peak period the critical MP draft is at 20m. These locations were established by finding the MP draft where the standard deviation of the MP motion response, i.e. energy content of the MP motion response, was the largest for that peak period. The EOM, depicted in Equation 58 is evaluated for these MP drafts in the steady state situation, where the mass moment of inertia, stiffness added damping and wave-excitation forces due to spread irregular waves are evaluated in the procedure as is explained in section 4. Note that the added damping, added mass outside of the MP and the wave-excitation forces are evaluated with diffraction theory, through the utilization of WAMIT. The added mass and added damping are MP motion period dependent, and these values are therefore evaluated assuming that the motion period of the MP is equal to the value of the peak period of the JONSWAP spectrum. The motion response is evaluated with 2 models, 1 model that does not consider the damping due to viscous drag, and a second model that does consider the damping due to viscous drag. The viscous drag is evaluated as described in subsection 4.2.

As discussed in subsubsection 3.2.1 the standard deviation for an irregular sea-state serves as a metric for the energy contained within the sea. Since the irregular waves produce an irregular force signal, and subsequently an irregular motion response signal, the standard deviation for these latter two signals also serves as a metric for the energy contained in the force or motion signal. With the standard deviation the most probable maximum (MPM) of the pitch motion can be evaluated, see Equation 84 (Journée, 2001). In this equation,  $m_{0,\theta}$  is the zeroth order spectral moment of the response spectrum, i.e. the total variance of the response signal, and  $N$  is the amount of cycles that is considered.  $N$  can be evaluated with Equation 85, where  $T_r$  is the return period over which the MPM needs to be evaluated, and  $T_{2,\theta}$  is the zero up-crossing period of the MP pitch motion. The return period is established at 600s, since the MPM of the steady-state simulation will in level 4 be compared with an estimate of the MPM of the MP motion during the dynamic lowering simulation under irregular waves, which takes 600 seconds.

Note that the maximum motion response during irregular waves for different wave seeds under statistically equivalent environmental conditions is Rayleigh distributed (Journée, 2001). The MPM is defined as the position where this Rayleigh PDF of the maximum motion response peaks, i.e. the mode, hence the name 'most probable'. It should be noted that the peak of the PDF of a Rayleigh distribution is exceeded in 60.6% of the cases, and hence the MPM is exceeded in 60.6% of the simulations (Dekking, Kraaikamp, Lophuaä, & Meester, 2005). Within the offshore industry the MPM is used as an indication of what maximum motion response value to expect for a given sea-state during  $N$  cycles, see Equation 85.

| Parameter     | Value | Unit | Description                 |
|---------------|-------|------|-----------------------------|
| $H_{sig}$     | 2.5   | m    | Significant wave height     |
| $T_{peak}$    | 7/8/9 | s    | Peak period of the spectrum |
| $V_{current}$ | 1.5   | m/s  | Velocity of the current     |

Table 11: Level 3: Environmental conditions

$$MPM(\theta) = \sqrt{2 \cdot m_{0,\theta} \cdot \ln(N)} \quad (84)$$

$$N = \frac{T_r}{T_{2,\theta}} \quad (85)$$

To make sure that the forces on the MP due to irregular waves are evaluated correctly, the forces on the MP for irregular waves at certain MP drafts on the 1-DOF model are evaluated and compared to the same results of the multi-DOF model. See subsubsection B.4.3 for the full details of these results. The main conclusion is that the wave-excitation forces due to irregular unidirectional or spread waves are evaluated correctly.

### 6.3.1 Results

The time domain plots of the steady-state simulations at the discussed critical drafts for the environmental conditions shown in Table 11 are depicted in Appendix C. The main results of these plots are depicted in Table 12. As discussed, the assumed return period is set at 600s, because the values of the MPM found in this level will be compared to an estimate of the MPM during the dynamic lowering simulation, which takes 600 seconds. Note that the results in Table 12 are related to the model that considers damping due to viscous drag.

| $H_{sig}$ [m] | $T_p$ [s] | $v_c \frac{m}{s}$ | $MP_{draft}$ [m] | $\sigma_{\theta, visc}$ [deg] | $T_{2, \theta, visc}$ [s] | $T_r$ [s] | $MPM_{\theta, visc}$ [deg] |
|---------------|-----------|-------------------|------------------|-------------------------------|---------------------------|-----------|----------------------------|
| 2.5           | 7         | 1.5               | 10               | 0.96                          | 8.45                      | 600       | 2.80                       |
| 2.5           | 8         | 1.5               | 15               | 1.71                          | 8.83                      | 600       | 4.98                       |
| 2.5           | 9         | 1.5               | 20               | 2.33                          | 9.47                      | 600       | 6.72                       |

Table 12: Main results of level 3

### 6.3.2 Discussion about the results

What can be seen from the figures in the appendix, see subsection C.1, is that the response of the model that considers damping due to viscous drag is the lowest. Furthermore, from Table 12 it can be seen that the standard deviation is higher for environmental conditions with higher peak periods. This is because the critical MP draft is larger for increasing peak periods, as the natural period of the system increases for larger MP drafts. Therefore, the wave-excitation loads are higher at these drafts, since more MP is in the water, and hence for these environmental conditions, the motion response is also the largest. Subsequently, the MPM for higher peak periods is also larger for the considered return period, see Table 12, and Equation 84.

## 6.4 Level 4: Dynamic lowering simulation during irregular spread waves

At this level, the dynamic lowering simulation will be modeled, during irregular spread waves, see Table 13 for the environmental conditions considered at this level. The hydromechanical forces are evaluated based on the diffraction theory, for which WAMIT is utilized. The force signal from the diffraction theory is evaluated in the same way as was done in level 3, but now for a dynamic lowering simulation. Furthermore, the added mass and added damping values from WAMIT are evaluated assuming that the assumed period of MP motion is equal to the peak period of the wave spectrum. The MP motion is evaluated with 2 models, one model that does not consider the damping due to the viscous drag, and another model that does consider the damping due to the viscous drag. The viscous drag is evaluated in the procedure as explained in subsection 4.2. The winch speed of the crane cable is set at  $0.0833 \frac{m}{s}$ , in accordance with GustoMSC (GustoMSC, 2023). Because the MP is lowered through the water the following components of Equation 58 also change over time:

1. The total mass moment of inertia,  $I_{m,55,tot}$ , because the following contributions of this mass moment of inertia change over time:
  - (a) Added mass outside of the MP, evaluated with WAMIT
  - (b) Added mass due to the entrapped water in the MP
  - (c) Mass from the steel of the MP
2. The added damping  $C_{55}$ .
3. The stiffness of the MP  $K_{55}$ .
4. The magnitude of the wave excitation forces from WAMIT, due to a regular wave,  $F_{5,wave-excit}$ .
5. The magnitude of the viscous dragforce

| Parameter     | Value | Unit | Description                 |
|---------------|-------|------|-----------------------------|
| $H_{sig}$     | 2.5   | m    | Significant wave height     |
| $T_{peak}$    | 7/8/9 | s    | Peak period of the spectrum |
| $V_{current}$ | 1.5   | m/s  | Velocity of the current     |

Table 13: Level 4: Environmental conditions

In level 3 the standard deviation of the MP motion response in steady-state under irregular spread waves at critical MP drafts was used to establish a value for the MPM during a 600-second simulation. Because during the dynamic lowering simulation, the EOM of the 1-DOF system is time-variant, the standard deviation cannot be used anymore to establish the MPM, since the system changes over time due to which the standard deviation changes over time. To estimate the MPM during the dynamic lowering simulation, several simulations with unique wave seeds for a specific set of environmental conditions are evaluated. The maximum MP motion response of every simulation is extracted, from which a bin plot is established that serves as a probability distribution of the maximum MP motion responses, for a specific set of environmental conditions. These bin plots were found to be Rayleigh distributed, and therefore the data points of the maximum MP motion response under a specific set of environmental conditions were fitted with a Rayleigh PDF. The pitch motion where this Rayleigh PDF is maximum is defined as the MPM. In this way an estimate of the MPM for the dynamic lowering simulation under irregular waves is established. Note that this is only done for the model that does consider the damping due to the viscous drag.

### 6.4.1 Results

In the appendix, see subsection C.2, the MP motion response in the time-domain for the dynamic lowering simulation and the mentioned environmental conditions are plotted for a single wave-seed, see Figures 93, 94, 95. As mentioned before these 3 plots are unique for every wave seed, due to the stochastic behavior of irregular waves, and therefore it is not possible to conclude something from a single simulation. Because of this, an estimate of the MPM is evaluated in this level, by establishing a Rayleigh distribution of the maximum MP motion responses which are extracted from the 220 wave seeds for each set of environmental conditions. See Figures 35, 36, and 37 for the bins and the fitted Rayleigh PDF that were established from the maximum MP motion response of every wave-seed, for each set of environmental conditions. Note that for every set of environmental conditions, 220 simulations were run in order to extract the maximum MP motion response, from which the bins plot and the Rayleigh PDF fit was established. The most important results are depicted in Table 14. The subscript 'dl-simu' refers to the estimate of the MPM during the dynamic lowering simulation in the pitch DOF for the model that does consider damping due to viscous drag. The subscript 'ss-simu' refers to the MPM from the steady-state simulation, obtained in level 3.

| $H_{sig}$ [m] | $T_p$ [s] | $v_c$ [ $\frac{m}{s}$ ] | $MPM_{dl-simu}$ [deg] | $N_{seeds}$ [-] | $MPM_{dl-simu}$ in % of the $MPM_{ss-simu}$ |
|---------------|-----------|-------------------------|-----------------------|-----------------|---------------------------------------------|
| 2.5           | 7         | 1.5                     | 0.59                  | 220             | 21.07                                       |
| 2.5           | 8         | 1.5                     | 1.12                  | 220             | 22.49                                       |
| 2.5           | 9         | 1.5                     | 2.07                  | 220             | 30.80                                       |

Table 14: Main results of level 4

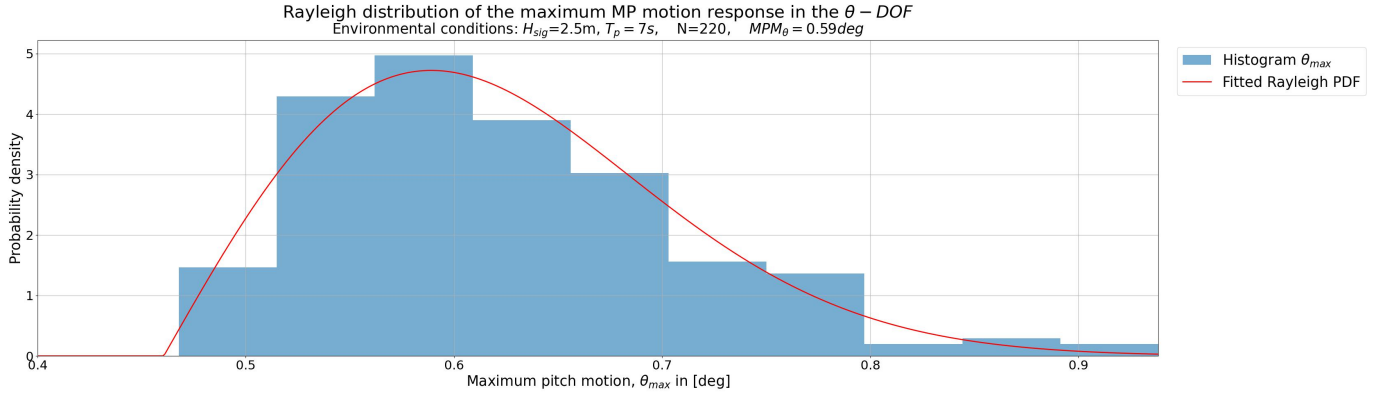


Figure 35: Fitted Rayleigh distribution for the  $\theta_{max}$  during the dynamic lowering simulation in irregular spread waves,  $H_{sig} = 2.5m$ ,  $T_p = 7s$

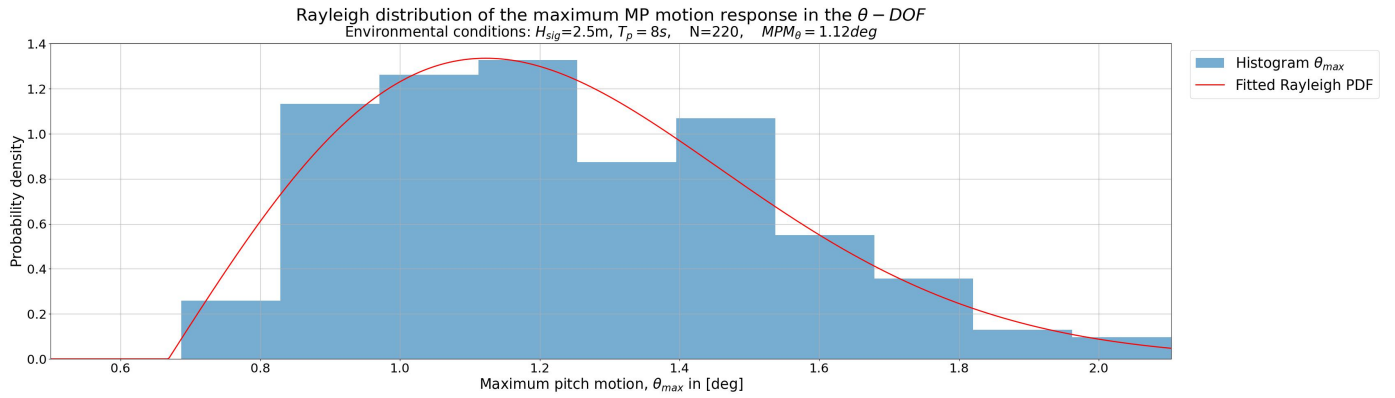


Figure 36: Fitted Rayleigh distribution for the  $\theta_{max}$  during the dynamic lowering simulation in irregular spread waves,  $H_{sig} = 2.5m$ ,  $T_p = 8s$

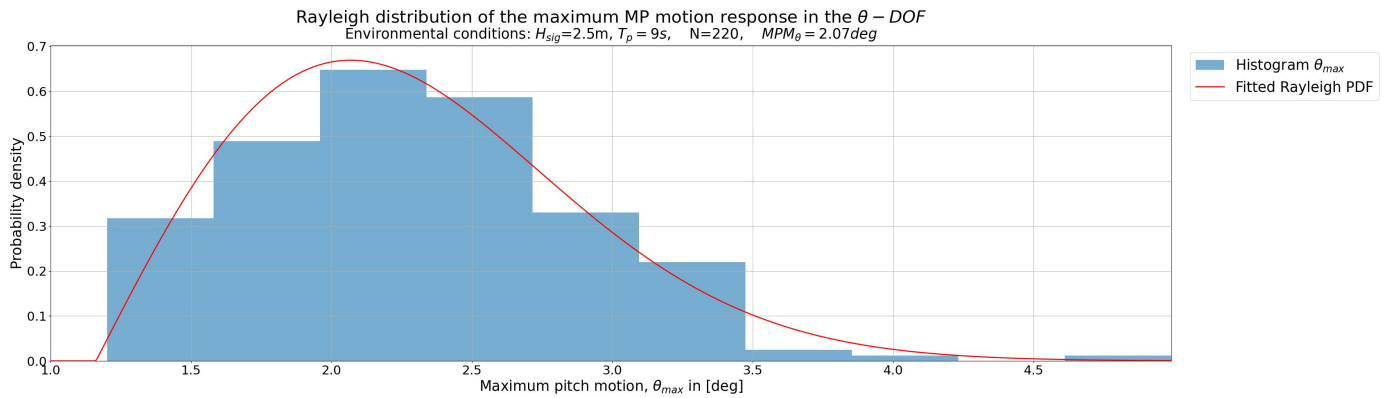


Figure 37: Fitted Rayleigh distribution for the  $\theta_{max}$  during the dynamic lowering simulation in irregular spread waves,  $H_{sig} = 2.5m$ ,  $T_p = 9s$

#### 6.4.2 Discussion about the results

It can be seen from Table 14 that the MPM during the dynamic lowering simulation is for all 3 cases approximately only 25% of the MPM during the steady-state simulations at critical MP drafts. In other words, the MPM is reduced by approximately 75% when evaluating it with the dynamic lowering simulation, compared to the MPM with the steady-state simulation.

As was already concluded, the MP motion response is largest at higher peak periods, especially during the steady-state simulations, since the critical MP drafts are established at the drafts where the peak periods coincide with the natural period. Because of this the viscous effects become more dominant at these peak periods during the steady-state simulations, due to which the motion response in this simulation, and hence the MPM, is significantly reduced. This is the reason that the difference in MPM between the steady-state simulation and the dynamic lowering simulation is smallest for higher peak periods, see Table 14.

Due to time constraints, only 220 simulations with unique wave seeds are used to establish a Rayleigh PDF plot for every set of environmental conditions. In order to improve the accuracy of this PDF a convergence test has to be done, such that it can be determined after how many simulations an accurate estimate of the maximum pitch motion distribution is obtained. This is not done during this thesis, due to time constraints. Therefore, it is not known how accurate the MPM is for the dynamic lowering simulation. However from the shape of the bins in Figures 35, 36, and 37 the well known Rayleigh distribution is observed, from which it can be concluded that the Rayleigh fit from 220 wave seeds is most likely a reasonable estimate of the distribution of the maximum pitch motion.

The decrease of the MPM in the dynamic lowering simulation is significant. However it should be noted that to use this value to establish a weather window, there has to be a certainty that the MP will not stop with lowering during the lowering phase. Because if this would happen, the MP motion will go into a steady-state situation, due to which the motion response would drastically increase. In the recommendations of this report a practical solution will be discussed to ensure that the lowering of the MP will not be interrupted.

## 6.5 Disregarded physical phenomena

Including as many physical phenomena as possible in the 1-DOF pile-cable model, would in theory be possible. However, due to time constraints during this MSc thesis, and limitations in computational power, some of these physical phenomena are disregarded, and not included in the 1-DOF pile-cable model. This subsection will discuss the relevance of certain physical phenomena and why they are not implemented in the 1-DOF pile-cable model. These phenomena include:

1. The effect of ambient current
2. Vortex shredding frequencies
3. 2nd order difference frequency wave drift forces
4. Hydromechanical reaction forces in the time-domain
5. Jack-up motion

The environmental conditions for which the relevance of the above 3 physical phenomena will be scrutinized are depicted in Table 15.

| Parameter     | Value | Unit | Description                 |
|---------------|-------|------|-----------------------------|
| $H_{sig}$     | 2.5   | m    | Significant wave height     |
| $T_{peak}$    | 7/8/9 | s    | Peak period of the spectrum |
| $V_{current}$ | 1.5   | m/s  | Velocity of the current     |

Table 15: Environmental conditions for the disregarded physical phenomena

In each subsubsection of this subsection, the theory of the physical phenomena will first be explained. Hereafter results will be presented for which the considered phenomena is implemented. The subsection will finish with a short discussion about the obtained results.

### 6.5.1 The effect of ambient current

In this subsection the effect of the ambient current on the JONSWAP spectrum will be discussed. For an irregular sea-state with an ambient current, there will be an energy transfer between waves and current. This energy transfer might affect the JONSWAP spectrum which has to be modified to incorporate this effect. For an irregular sea state, the following wave parameters will change during the presence of an ambient current (Holthuijsen, 2007):

1. Amplitude
2. Frequency
3. Direction

The frequency of the wave for a fixed observer is called the apparent wave frequency and can be described with Equation 86, where  $k$  is the wave number,  $U$  is the current, and  $\omega_{int}$  is the intrinsic frequency of the wave, meaning the frequency which an observer traveling with the same speed as the current would measure. Equation 86 is derived from the fact that the wavelength with and without ambient current is the same, however, the speed at which the wave passes a fixed observer is affected by an ambient current. Because of this the frequency of the wave for a fixed observer, i.e. an observer that is not moving with the speed of the current, is different with ambient current as opposed to without ambient current. Note that the apparent wave frequency without current is simply equal to the intrinsic wave frequency, see Equation 86.

$$\omega_{app} = kU + \omega_{int} \quad (86)$$

The wave amplitude is affected because the wave-action has to be conserved between the situation with and without ambient current (Holthuijsen, 2007). Because of this, the wave amplitude with ambient current traveling in the direction of the wave is smaller compared to the situation without current. The opposite happens with an ambient current traveling in the opposite direction as the wave, in this situation the wave amplitude increases compared to the situation without a current. Because wave-action between the situation with and without current is conserved, there is an energy transfer between the wave and the current when ambient current is present (Holthuijsen, 2007).

In (Huang, 1972) a formulation for a wave-spectrum with ambient current is proposed, see Equation 87. In this equation,  $\omega_{app}$  is the apparent frequency,  $U$  is the current,  $g$  is the gravitational acceleration, and  $S(\omega_{app})$

is the wave spectrum without an ambient current. This formulation is proposed for deep water conditions, with a constant current over the full water depth. During this level the environmental conditions in Table 15 are considered. It is known from the dispersion relation, see Equation 11 that deep water cannot be assumed for a large part of the spectra when considering peak periods of 8 and 9 seconds, since the waves will interact with the bottom for the wave-frequencies containing the majority of the energy of these wave spectra. This interaction of the waves with the seabed is however assumed to be small for these peak periods, and therefore the water is assumed to be deep for the considered environmental conditions during this level. It was mentioned in subsubsection 3.2.3 that the current profile along water depth is modeled with Equation 52. For a current at the SWL of  $1 \frac{m}{s}$  the current at a draft of 42m with the assumed water depth of 50m during this thesis, is still  $0.82 \frac{m}{s}$ . Because of this small discrepancy with the  $1 \frac{m}{s}$  current at the SWL, the current during this level is also assumed to be constant along the water depth. Because of these two arguments, Equation 87 can be utilized.

$$S(\omega_{app}, U) = S(\omega_{app}) \frac{4}{\left(1 + \sqrt{1 + \frac{4U\omega_{app}}{g}}\right)^2 \sqrt{1 + \frac{4U\omega_{app}}{g}}} \quad (87)$$

Equation 87 is only valid when Equation 88 is satisfied. If this is not the case Equation 87 would go to infinity since the denominator of this equation would go to 0. Furthermore, in (Hedges, 1981), Equation 87 was modified to account for a wave-breaking limit. Especially when current travels in the opposite direction as the wave, the waves tend to break which has to be accounted for in the spectrum. Equation 89 accounts for this wave-breaking limit, where 'ER' refers to the equilibrium range and  $\alpha$  is a numerical constant. When the value of this equation is smaller than the value found with Equation 87, Equation 89 should be utilized to model the wave spectrum. When the value found with Equation 89 is bigger than the value found with Equation 87, the value found with Equation 87 should be used. When setting the coefficient  $\alpha$  of this equation to 0.008, the tail of the PM spectrum is modeled, therefore  $\alpha$  is set at 0.008.

$$1 + \frac{4U\omega_{app}}{g} \quad (88)$$

$$S_{ER}(\omega_{app}) = \frac{\alpha g^2}{(\omega_{app} - kU)^5} \frac{1}{1 + \frac{2U(\omega_{app} - kU)}{g}} \quad (89)$$

In (Soares, 2005), experimental tests were performed to validate Equation 87. In an open basin, irregular waves were modeled with a wavemaker where for each configuration of a significant wave height and peak period the waves were measured in 3 situations:

1. Opposing current.
2. No current.
3. Current flowing in the same direction as the propagation of the waves.

For every set of environmental conditions, the measured surface elevation was transformed into a wave-spectrum utilizing the FFT. The following main conclusions were drawn from these experiments:

1. The position of the peak of the wave spectrum is conserved for the same significant wave height and peak period, between the situations with opposing current, no current, and current traveling in the same direction as the waves.
2. For equivalent significant wave height and peak period, the energy in the wave spectrum under opposing current is the highest, followed by the situation without current. The situation with current traveling in the same direction as the waves contained the least amount of energy in the spectrum.

Hereafter the experimental results were compared with the wave spectrum obtained with Equation 87, where  $S(\omega_{app})$  was modeled with the JONSWAP spectrum, assuming a peak enhancement factor of 2. It turned out that Equation 87 did not converge with the results found in the experiments when the value of the current,  $U$ , in the equation was set to the value measured during the experiments. The spectra modeled with Equation 87 did however converge when increasing or decreasing  $U$  used in this equation a little bit, depending on the significant wave height and peak period of that experiment. This modified value of  $U$  is called the converged velocity.



In the paper, it also became apparent that for most configurations of significant wave height, peak period, and current intensity, only the situation with current traveling in the same direction as the waves could be modeled with Equation 87. The reason for this is that for most configurations of significant wave height, peak period, and current intensity, with current traveling in the opposite direction as the waves, the wave-breaking limit would be reached. For most cases, the modification with the wave-breaking limit, see Equation 89, would prove not to model the wave spectrum correctly. This is because especially for situations with short waves, i.e. low peak periods, and high opposing currents, the waves become very non-linear (Holthuijsen, 2007), which is difficult to model with linear wave theory. Because of this, only the situation with current traveling in the same direction as the waves will be considered during this MSc thesis.

In (Soares, 2005) several experiments have been conducted for which wave spectra are found. The converged velocity for which Equation 87 would model the same spectrum as the experiments, is evaluated for each experiment. Since the significant wave height, peak period, measured current, and converged velocity are known for several tests, the results are interpolated such that a converged velocity is found for the environmental conditions in Table 15. Using these converged velocities, and the JONSWAP spectrum as described in subsubsection 3.2.1, the JONSWAP spectra during a positive, negative and no current are evaluated, see in subsection C.3 Figures 96, 97, and 98. In these figures, the equivalent significant wave heights are also depicted to see how much energy transfer there is between waves and current. It can be seen from this figure that the equivalent significant wave height of the JONSWAP spectrum with the following current is much lower compared to the situation without current, this was also expected.

Note that from these figures it becomes apparent that the JONSWAP spectrum with opposing current does not make any sense at all, since the wave-breaking limit is already reached before the peak of the spectrum, hence the weird shape of these spectra. It is for this reason that only the situation with current traveling in the same direction as the waves is considered during this MSc thesis. These are the JONSWAP spectra that will be considered when modeling the wave-excitation loads for the dynamic lowering simulation.

Apart from this modification of the JONSWAP spectrum, which only influences the magnitude of the wave-excitation forces, and apart from the fact that the spreading of waves is not included, the dynamic lowering simulation is modeled in the same way as was done in level 4 and will therefore not again be discussed here.

## Results

To establish the impact of ambient current on the MP motion response, the time-variant version of the EOM in Equation 58, i.e. the dynamic lowering simulation, is solved for unidirectional irregular waves and with current, see Table 15 for the considered environmental conditions. As discussed, for every set of environmental conditions, the time-variant EOM is solved when considering the conventional, i.e. non-modified, JONSWAP spectrum, and the modified JONSWAP spectrum due to the ambient current flowing in the same direction as the propagation of the waves. For every specific set of environmental conditions, the same wave seed is used in both models so that the maximum pitch motion of the MP can be used as a metric to establish the significance of the ambient current effect. Furthermore, the EOM is established and solved in the same way as level 4 of the results, except that in this simulation unidirectional irregular waves are considered, and ofcourse in one model the effect of the ambient current is implemented. Both models consider damping due to the viscous drag.

The plots of the time domain simulations with the 2 models for the environmental conditions described in Table 15 are shown in subsection C.4, see Figures 99, 100, 101, 102, 103, and 104, the most important results from these plots are however shown in Table 16. The subscript 'conv' refers to the model that utilizes the conventional JONSWAP spectrum, and the subscript 'ambcurrent' refers to the model that utilizes the modified JONSWAP spectrum.

| $T_p$ [s] | $v_c \frac{m}{s}$ | $H_{sig}$ [m] | $H_{sig,eq}$ [m] | $\theta_{max,conv}$ [deg] | $\theta_{max,ambcurrent}$ [deg] | $\theta_{max,ambcurrent}$ in % of $\theta_{max,conv}$ |
|-----------|-------------------|---------------|------------------|---------------------------|---------------------------------|-------------------------------------------------------|
| 7         | 1.5               | 2.5           | 1.9              | 0.72                      | 0.58                            | 80.55                                                 |
| 8         | 1.5               | 2.5           | 2.0              | 1.15                      | 0.98                            | 83.00                                                 |
| 9         | 1.5               | 2.5           | 2.1              | 3.18                      | 2.48                            | 70.00                                                 |

Table 16: Impact of the ambient current

## Discussion about the results

From the results in the table it becomes apparent that the maximum motion response is significantly reduced when considering the ambient current effect for current flowing in the same direction as the wave. For all 3 the sets of environmental conditions the maximum response is reduced by around 25%. The relative reduction in the maximum MP motion response is larger during higher peak periods.

It should be noted that the assumption of the steady current along the full water depth and the assumption

that all the waves propagate in deep water is for the considered environmental conditions not true. This assumption might cause the reduction in MP motion response when considering the ambient current effect, to be overestimated. Furthermore, it is also possible that the converged velocity, which is found by interpolation of the experimental data from (Soares, 2005), might be different for the considered environmental conditions during this MSc thesis. To be certain what the converged velocity would be, a wave-basin test should be performed for the considered environmental conditions which will not be done during this study.

It should also be noted that with Equation 87, and considering the environmental conditions depicted in Table 15 it is not possible to model a correct modified JONSWAP spectrum for the situation with opposing current. The amplification of the energy in the spectrum for this situation might yield the highest MP motion response, due to which it would be conservative to analyze this situation. It is therefore suggested to try to model this effect by utilizing for example the SWAN model, which is capable of modeling non-linear ocean waves (Holthuijsen, 2007). This will however not be done during this thesis as this is outside the scope of this study.

Furthermore, this model only considers the irregular waves coming from one direction. To correctly implement the ambient current effect, the modified JONSWAP spectrum should be utilized for spread waves. In this situation the ambient current effect is expected to be less significant, compared to the unidirectional irregular waves since for some of the 'spread' waves, the current has a less severe effect on the energy transfer from wave to current.

### 6.5.2 Vortex shedding frequencies

The drag force on a cylinder in a fluid with a velocity  $U$  is governed by the shedding of vortices, see Figure 38. As these vortices shed from the cylinder, a lift force, perpendicular to the drag force occurs, see Figure 38. The frequency at which these vortices shed is called the vortex shedding frequency. If the vortex shedding frequency coincides with the natural frequency of the 1-DOF pile cable subsystem, resonance occurs, resulting in an amplification of the motion response, which is undesirable. It is therefore important to check if these frequencies are close to the natural frequencies of the 1-DOF pile cable model.

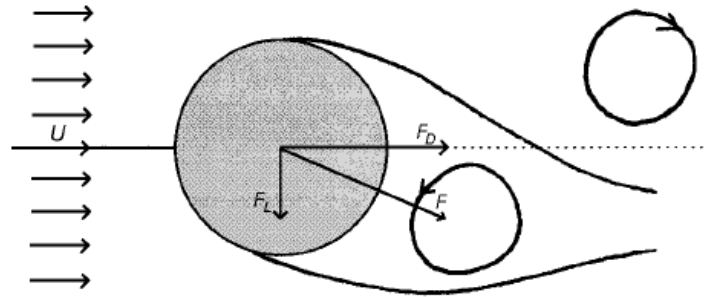


Figure 38: Lift force due to vortex shedding on a cylinder (Journée, 2001)

Given a vortex-shedding frequency, the dimensionless Strouhal number is defined with Equation 90. From (Journée, 2001) it is known that the Strouhal number is related to the Reynolds number as depicted in Equation 91, with the Reynolds number for flow around a cylinder defined with Equation 92. In these equations  $f_v$  is the vortex shedding frequency,  $T_v$  is the vortex shedding period,  $D$  is the diameter of the cylinder,  $U$  is the uniform flow of the fluid,  $\nu$  is the kinematic viscosity of water, which is  $10^{-6} \frac{m^2}{s}$  at 20 deg. The velocity of the water around the MP is time and MP draft-dependent since the velocity of fluid around the MP is a superposition of the constant current and the relative velocity of the MP through the still water. In this analysis, the wave-particle velocities are disregarded. Furthermore, the vortex shedding periods will be evaluated during the dynamic lowering simulation with irregular waves whilst including the spreading of waves, and the viscous drag, and whilst excluding the ambient current effect, i.e. the same simulation as considered in level 4 is used in this analysis. Since the velocity changes over time and MP draft, the vortex shedding periods also vary over the MP draft and time.

To check whether the vortex shedding periods coincide with the natural periods of the 1-DOF pile cable model, only the vortex shedding period at the location with the maximum velocity of the fluid and the location with the minimum velocity of the fluid are of interest. In other words, the vortex-shedding periods will only be evaluated at the open bottom end of the MP, and at the location where the MP just touches the water, i.e., at SWL.

$$St = \frac{f_v D}{U} = \frac{\frac{1}{T_v} D}{U} \quad (90)$$

$$\begin{aligned} St \approx 0.2 & \rightarrow 10^2 < Re < 2 \cdot 10^5 \\ St \approx 0.3 & \rightarrow Re > 2 \cdot 10^5 \end{aligned} \quad (91)$$

$$Re = \frac{UD}{\nu} \quad (92)$$

## Results

The results of the vortex shedding periods with the mentioned modeling techniques, and for the environmental conditions depicted in Table 15, are shown in Figures 39, 40, and 41. In these figures the y-axis on the left side is the natural period or vortex shedding period in [s], the y-axis on the right hand side is the MP draft in [m], and the x-axis is the time in [s]. Furthermore, the purple graph is the natural period of the 1-DOF system, the dark blue graph is the MP draft, the green graph is the vortex shedding period at the SWL of the MP, and the light blue graph is the vortex shedding period at the bottom of the MP.

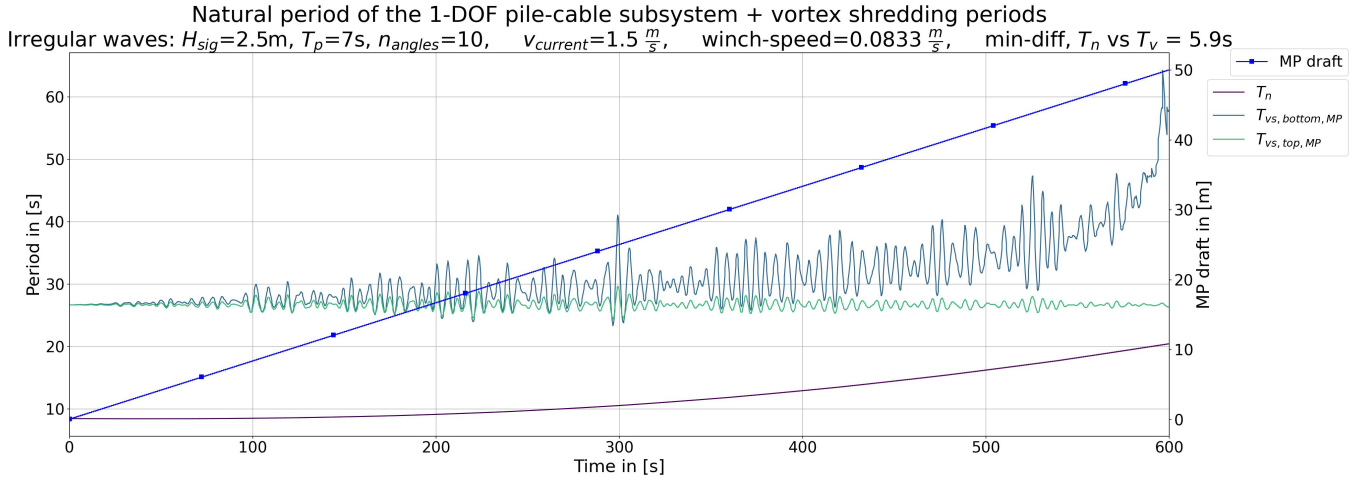


Figure 39: Vortex shedding periods during the dynamic lowering simulation,  $H_{sig} = 2.5\text{m}$ ,  $T_p = 7\text{s}$ ,  $v_{current} = 1.5 \frac{\text{m}}{\text{s}}$

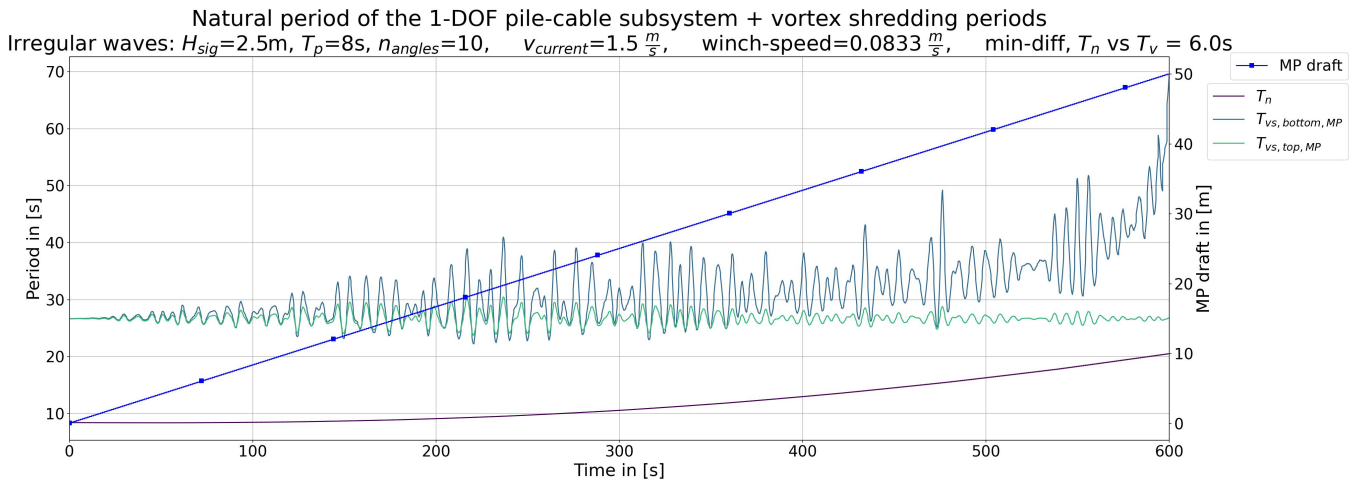


Figure 40: Vortex shedding periods during the dynamic lowering simulation,  $H_{sig} = 2.5\text{m}$ ,  $T_p = 8\text{s}$ ,  $v_{current} = 1.5 \frac{\text{m}}{\text{s}}$

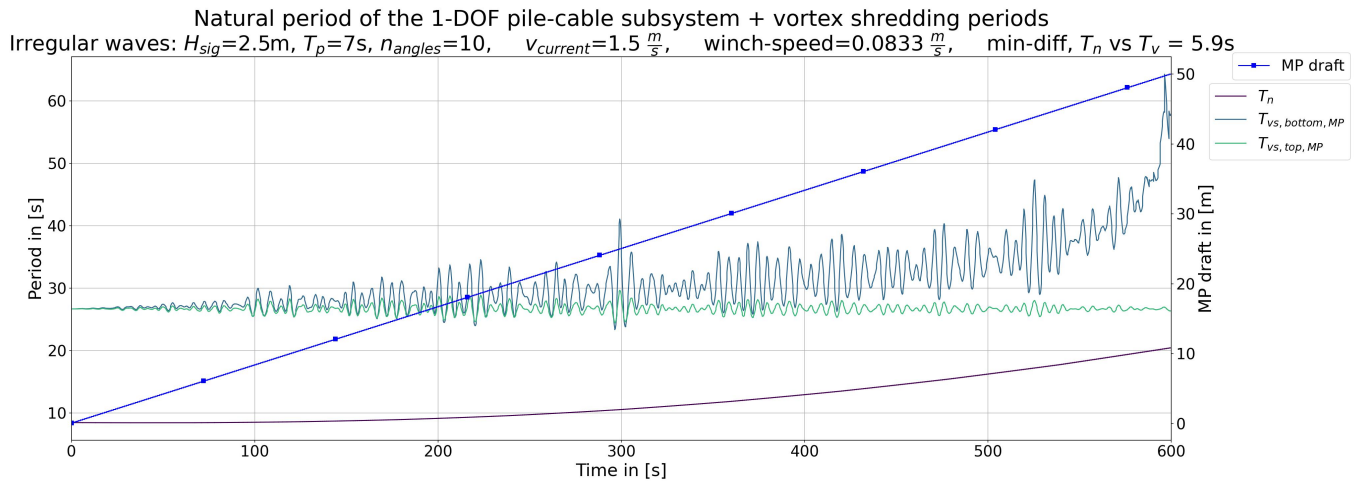


Figure 41: Vortex shedding periods during the dynamic lowering simulation,  $H_{sig} = 2.5m$ ,  $T_p = 9s$ ,  $v_{current} = 1.5 \frac{m}{s}$

### Discussion about the results

It can be seen from all 3 graphs that the vortex-shedding period of the bottom part of the MP is closest to the natural period of the 1-DOF pile-cable model. For all 3 environmental conditions, the minimum difference between the natural period of the 1-DOF model and the vortex shedding period on the bottom part is around 6 seconds, see the subtitle in the 3 mentioned graphs. Because the vortex shedding periods vary over the MP height and will contain a value in between the vortex shedding periods at the bottom of the MP and at the SWL, it can be concluded that over the full MP height resonance due to the liftforce is not likely to occur. Because of this, the effect on the dynamic behaviour of the vortex shedding is expected to be negligible

### 6.5.3 2nd order difference frequency wave drift forces

From Figure 29 it can be seen that the natural periods are relatively high at larger MP drafts. Since the second-order wave forces are not considered in the 1-DOF pile-cable model it would be convenient to check the magnitude and frequencies of the 2nd-order difference frequency drift forces on the MP. If there are large 2nd-order drift forces with periods close to the natural period of the 1-DOF pile cable model at large MP drafts, resonance would occur, yielding an amplification in the MP motion response, which is undesirable. In this study, only the difference frequency drift forces will be checked at large MP drafts, since the natural periods are the highest at large MP drafts and are therefore most likely to coincide with the 2nd order difference frequencies at these drafts. The sum frequencies are assumed to be far away from the natural periods of the 1-DOF pile-cable model for all the MP drafts and are therefore not checked.

The rigorous procedure to obtain the full 2nd order wave forces would be through utilizing the full QTF matrix. This is however quite a study on its own and will therefore not be done during this MSc thesis. In (Journée, 2001) another method is proposed to evaluate the 2nd-order difference frequency drift force on a cylinder for irregular waves. This is done by establishing a wave envelope from an irregular wave train and treating the absolute value of each peak or trough as an amplitude from an individual regular wave that contributes to the drift force through Equation 93. In this equation,  $R$  is the radius of the cylinder,  $C_{refl}$  is the reflection coefficient, which is assumed to be 1, and  $\zeta_a$  is the wave amplitude. See Figure 42 for the irregular wave-train where for each peak or trough the drift force can be found with Equation 93.

This same procedure is also followed for finding the difference frequency drift force for the 1-DOF pile cable model at a MP draft of 50m. Hereafter this drift force signal as a function of time is transformed to the frequency domain utilizing the FFT, to see which frequencies contain the most energy in terms of difference frequency drift forces.

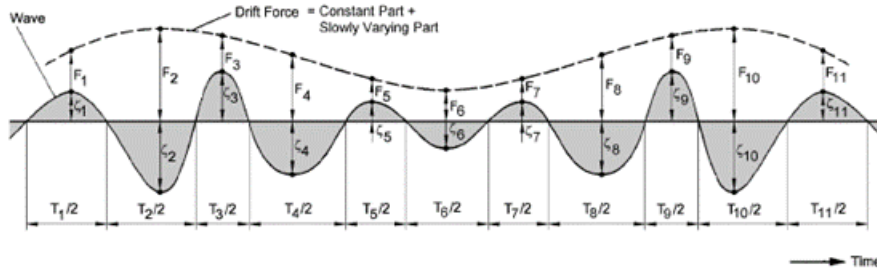


Figure 42: Drift force due to an irregular wave-train (Journée, 2001)

$$F_{drift}^{(2)} = \frac{1}{2} \rho_{sw} g R C_{refl}(\omega) \zeta_a^2 \quad (93)$$

### Results

First, the wave envelope for an irregular wave train is established, see Figure 43. The peaks and troughs of this wave envelope are used as the individual wave amplitudes to establish a 2nd order wave drift force signal as a function of time with Equation 93, see Figure 44. To check if this 2nd order difference frequency wave drift force signal is obtained correctly, the mean is checked with the mean wave drift-force on a cylinder, which can be evaluated simply with Equation 94 (Journée, 2001). It turns out that both these mean wave drift forces are equal, see the subtitle in Figure 44, hence the obtained 2nd order difference frequency wave drift force signal is assumed to be correct. This force signal is hereafter transformed to the frequency domain, utilizing the FFT, see Figure 45. The same is done for a peak period of 7, and 8 seconds, see subsection C.5 for Figures 105, and 106.

$$\overline{F_{drift}^{(2)}} = \frac{1}{12} \rho_{sw} g R C_{refl}(\omega) H_{sig}^2 \quad (94)$$

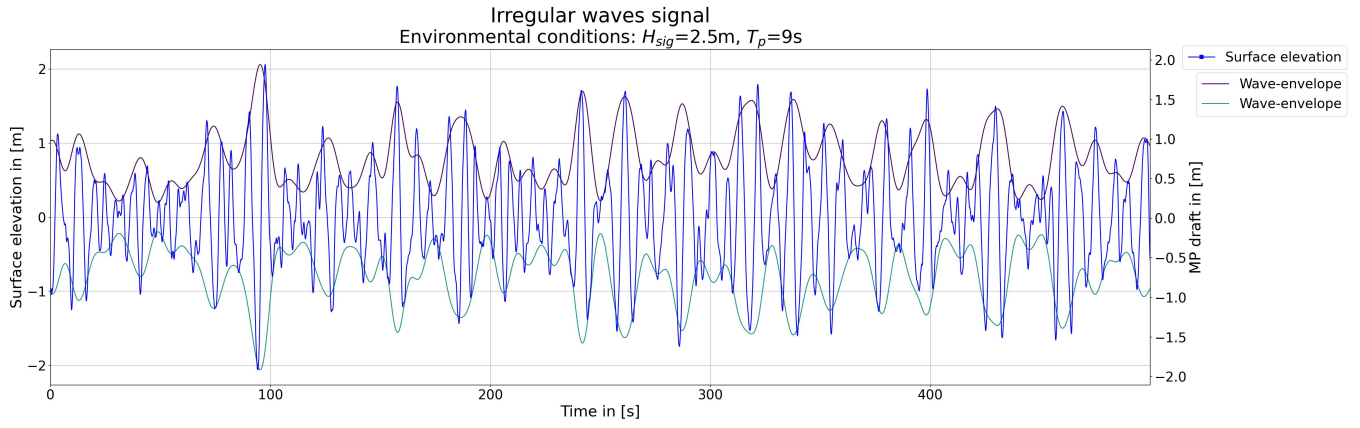


Figure 43: Wave-envelope for an irregular wave-train  $H_{sig} = 2.5m$ ,  $T_p = 9s$

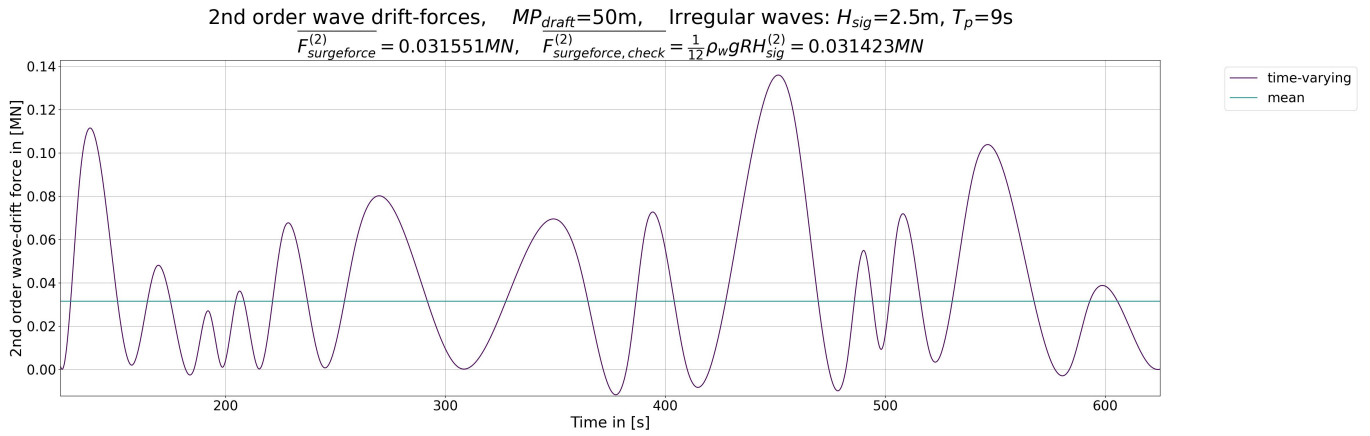


Figure 44: 2nd order difference frequency wave drift-force signal  $H_{sig} = 2.5m$ ,  $T_p = 9s$

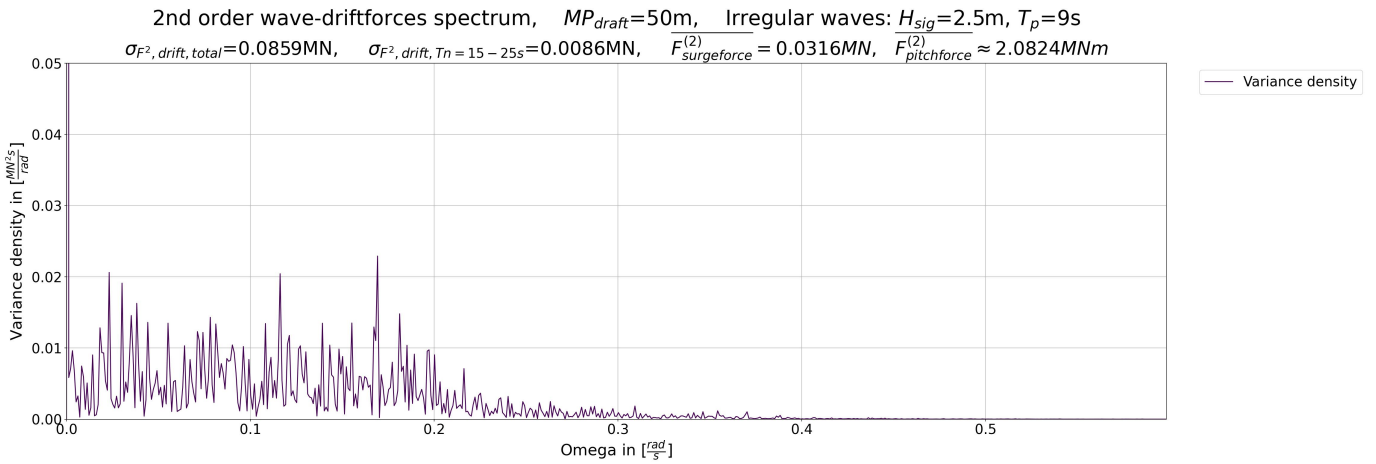


Figure 45: 2nd order difference frequency wave drift-force spectrum,  $H_{sig} = 2.5m$ ,  $T_p = 9s$

| $H_{sig}$ [m] | $T_p$ [s] | $v_c[\frac{m}{s}]$ | $MP_{draft}$ [m] | $\sigma_{F2,drift,total}[MN]$ | $\sigma_{F2,drift,Tn=15-25s}[MN]$ | $\overline{F_{\theta}^{(2)}} [MNm]$ |
|---------------|-----------|--------------------|------------------|-------------------------------|-----------------------------------|-------------------------------------|
| 2.5           | 7         | 0                  | 50               | 0.0856                        | 0.0078                            | $\approx 2.0532$                    |
| 2.5           | 8         | 0                  | 50               | 0.0876                        | 0.0087                            | $\approx 2.0908$                    |
| 2.5           | 9         | 0                  | 50               | 0.0859                        | 0.0086                            | $\approx 2.0824$                    |

Table 17: Main results on the impact of the 2nd order difference frequency wave-drift forces

### discussion about the results

The results of the second order wave drift forces are almost equal between the 3 different configurations of the environmental conditions, see Table 17, therefore during the discussion there will be no distinction between

results found for different peak periods.

To obtain the pitch moment, from the drift forces, it is assumed that these forces act at half the MP draft, which is conservative considering the exponential decay of the dynamic pressure. The standard deviation of the 2nd order difference frequency pitch moment under equal environmental conditions would be approximately  $3.5MNm$ , i.e.  $(Airgap + d_{baseh-gripper} + \frac{MPdraft}{2}) \cdot \sigma_{F^2,drift,total}$ . Only  $\frac{1}{10}$  of this pitch moment standard deviation, caused by the 2nd order difference frequency drift forces, is located around the natural period of the 1-DOF pile cable model, i.e. periods between 15 and 25 seconds, see Table 17. Because of this, it is fair to conclude that the 2nd-order drift forces would only cause a minor mean displacement and not a dynamic MP motion amplification, since only a minor part of the energy contained in the 2nd-order difference frequency wave drift force spectrum is located near the natural period of the 1-DOF model at a MP draft of 50m. See Figure 88 for the order of magnitude of 1st order wave forces at a MP draft of 20m. Hence, for all the environmental conditions shown in Table 15, the 2nd order wave drift forces in the time domain are assumed to have an irrelevant contribution to the dynamic behavior of the MP.

The mean wave drift force in the pitch DOF is about  $2MNm$  for all 3 configurations of the environmental conditions, see Table 17. The constant viscous dragforce, coming from the current, is  $13.42 MNm$  at a MP draft of 50m. This dragforce causes a mean pitch displacement of around  $-0.25$  deg, see Figure 32. This means that the mean wave drift force would cause a mean pitch displacement of the MP at 50m draft of  $-0.25 \cdot \frac{2}{13.42} = -0.037deg$ , and hence it can be concluded that the mean wave drift force can also be neglected.

#### 6.5.4 Hydromechanical reaction forces in the time-domain

In levels 3 and 4 of the result of the 1-DOF pile cable model, the added damping and added mass were set at the MP period of motion which is equal to the peak period of the irregular sea surface. This is a rough assumption because the added damping and added mass are MP motion period dependent, and it is not always true that this period is equal to the peak period of the JONSWAP spectrum.

From Figure 29 it can be seen that a different assumed pile-period of motion does not result in a different natural period of the 1-DOF MP. Therefore, it is prudent to conclude that the discussed assumption does not influence the dynamic behavior of the 1-DOF pile cable model significantly through the error in the added mass. It can be observed from Equation 82 however that the assumed pile period of motion has a very significant impact on the added damping of the 1-DOF pile-cable model. Although slightly depending on the MP draft, the added damping in terms of the critical damping in general peaks around 6 seconds and quite drastically decreases when moving away from this pile period of motion.

#### Results

To check what the influence is of the discussed assumption, pile periods of motion of 6, 9, and 12 seconds are used for evaluating the added damping during the dynamic lowering simulation for irregular unidirectional waves, whilst including viscous drag and current under the environmental conditions depicted in Table 15, see subsection C.6 for Figures 108, 109, 110. The winch speed during these simulations is set at  $0.0833 \frac{m}{s}$ . Note that the same wave seed is used for every configuration of environmental conditions, such that the absolute maximum pitch motion can be used as a metric for the magnitude of the motion response. The main results from these plots are shown in Table 18. It can be seen from all 3 figures that the assumption of the pile period of motion has a significant influence on the motion response of the MP. The difference between an assumed pile period of motion of 6 and 9 seconds is the greatest, which is also expected since the added damping in terms of the critical damping peaks at a pile period of motion of 6 seconds, see Figure 30.

| $H_{sig}$ [m] | $T_p$ [s] | $\theta_{max, Tmp=6s}$ [deg] | $\theta_{max, Tmp=9s}$ [deg] | $\theta_{max, Tmp=12s}$ [deg] | Max error in % of $\theta_{max, Tmp=9s}$ |
|---------------|-----------|------------------------------|------------------------------|-------------------------------|------------------------------------------|
| 2.5           | 7         | 0.67                         | 0.81                         | 1.00                          | 40.7                                     |
| 2.5           | 8         | 1.39                         | 1.48                         | 1.55                          | 10.8                                     |
| 2.5           | 9         | 1.66                         | 2.08                         | 2.15                          | 23.5                                     |

Table 18: Impact of added damping through different MP motion periods

#### Discussion about the results

From the main results shown in Table 18, it can be seen that the assumption of the pile-period of motion has a significant influence on the motion response of the MP. In general the MP motion response is lowest for the assumed MP period of motion of 6 seconds, followed by the response associated with an assumed MP period of motion of 9 seconds, and finally the highest response is found at an assumed MP period of motion of 12 seconds. The maximum possible error w.r.t. the maximum pitch motion assuming a 9 second period MP motion can be as high as 40%. It can be observed from the table that the biggest step in difference in the MP motion response is between the assumed MP motion period of 6 and 9 seconds, as opposed to the assumed MP motion period of 9 and 12 seconds. This is in line with the expectations, considering Figure 30.

From these results, it can be concluded that assuming that the pile period of motion is equal to the peak period of the JONSWAP spectrum is not valid, as the motion response is very different for various pile periods of motion. It is however relatively conservative to assume that the MP motion period is equal to the peak period of the spectrum. The reason for this is that the highest motion response in level 4 is found at higher peak periods. Therefore in these simulations the added damping will most likely not be much overestimated. Furthermore, from the plots of level 4, see Figures 93, 94, and 95, it can be concluded that the MP motion period is relatively low between 0 and 300 seconds. After 300 seconds the motion period of the MP in general starts increasing. The highest response in the dynamic lowering simulation is in general observed slightly after the natural period has crossed the peak period of the spectrum, somewhere between 150 and 300 seconds depending on the peak period of the spectrum. Therefore with the assumption that the MP motion period is equal to the peak period the added damping is not overestimated in this region, which is the most critical region to not overestimate damping.

In order to properly establish the hydromechanical reaction forces coming from the added damping and added mass, in the time-domain, the impulse response function has to be utilized. See Equation 95 for the hydromechanical reaction forces as a function of time. The first term in Equation 95, is associated with the added mass force, in which  $\mathbf{A}(\infty)$  is the 6x6 added mass matrix at infinite frequency. Because the radiation forces in time are evaluated based on multiple discrete impulses, the frequency at which the added mass should be evaluated



is at infinity. The 2nd term in Equation 95 is associated with the added damping force in the time domain, in which  $\mathbf{IRF}(\tau)$  is the impulse response function, defined with Equation 96, and  $\tau$  is the time lag integration variable (Journée, 2001). For each time step the convolution integral in Equation 95, considers the impulse response function, and the velocity of the vessel from all the previous timesteps, see the right-hand side of Equation 97 for the radiation damping force in the time-domain. By utilizing the convolution integral, the vessel motion in the past is convoluted with the impulse response function such that the radiation forces at the current time-step are obtained. By doing this the system has a form of 'memory' of the ship's past velocities in which the radiation forces at the current timestep can be obtained. In Equation 96, the matrix  $\mathbf{B}(\omega)$  is the frequency-dependent 6x6 added damping matrix. The impulse response function is extracted from WAMIT (*Wamit User Manual V7.5*, 2023).

This implementation of the impulse response function to obtain the hydromechanical reaction forces as a function of time is not done in this MSc thesis, due to time constraints. It is however suggested for future research to implement this feature as it will give real insight into the influence of the implementation of hydromechanical reaction forces in the time domain. Although this feature is not implemented and the exact improvement is not known, there is however a clear indication that it might improve the accuracy of the MP motion response significantly, see Table 18.

$$F_{hydro, reaction}(t) = -\mathbf{A}(\infty)\ddot{\xi}_{j,a}(t) - \int_{\tau=0}^{\infty} \mathbf{IRF}(\tau)\dot{\xi}_{j,a}(t-\tau)d\tau \quad (95)$$

$$\mathbf{IRF}(\tau) = \frac{2}{\pi} \int_0^{\infty} \mathbf{B}(\omega) \cos(\omega\tau) d\omega \quad (96)$$

$$\int_{\tau=0}^{\infty} \mathbf{IRF}(\tau)\dot{\xi}_{j,a}(t-\tau)d\tau = \int_{-\infty}^t \mathbf{IRF}(t-\tau)\dot{\xi}_{j,a}(\tau)d\tau \quad (97)$$

## 6.6 Jack-up motion

From modal analysis of the multi-DOF model, the assumption was made to reduce the multi-DOF model to a 1-DOF model, to model the dynamic behavior of the MP during the splash zone lowering phase. Considering the environmental conditions that are used in this thesis, see Table 2, together with the sway and surge modes, see Figures 22c, and 22d, it can be concluded that during environmental conditions with a peak period of 7 seconds, the jack-up will significantly be excited in these modes. To establish how significant this effect is, steady-state simulations of the multi-DOF model and the 1-DOF model at critical MP drafts, as defined in subsection 6.3, and considering the environmental conditions shown in Table 15 will be compared. Both models consider wave-excitation forces on the MP from irregular spread waves, evaluated through the utilization of WAMIT, where the main wave direction is in the positive x-axis. Furthermore, the damping due to the viscous drag of the MP motion in the water is neglected in both models as this feature is not yet implemented in the multi-DOF model. The damping due to the viscous drag of the current on the MP is included in both models. The wave-excitation forces on the legs of the multi-DOF model are evaluated in the procedure as was discussed in subsubsection 3.2.2. In the multi-DOF model, the rotational motion around the x and y-axis of the MP in the gripper, and the translational motion of the gripper along the x and y-axis will be extracted, see Figure 4 for the definition of the axis system.

## Results

The plots of the steady-state simulations for the mentioned conditions of the 1-DOF and the multi-DOF model are shown in the appendix, see subsection C.1, and subsection C.7. In Figures 90, 91, and 92 the results of the 1-DOF model are depicted, in Figures 111, 112, 113, 114, 115, and 116 the results of the multi-DOF model are shown. The most important results from these steady state simulations are shown in Table 19.

| $H_s$ [m] | $T_p$ [s] | MP <sub>draft</sub> [m] | $\sigma_{yy,Mdof}$ [deg] | $\sigma_{yy,1dof}$ [deg] | $\sigma_{xx,Mdof}$ [deg] | $\sigma_{x,Mdof}$ [m] | $\sigma_{y,Mdof}$ [m] | $\sigma_{yy,1dof}$ in % of $\sigma_{yy,Mdof}$ |
|-----------|-----------|-------------------------|--------------------------|--------------------------|--------------------------|-----------------------|-----------------------|-----------------------------------------------|
| 2.5       | 7         | 10                      | 0.633                    | 1.00                     | 0.420                    | 0.141                 | 0.141                 | 158                                           |
| 2.5       | 8         | 15                      | 1.929                    | 1.91                     | 1.246                    | 0.139                 | 0.173                 | 99                                            |
| 2.5       | 9         | 20                      | 3.041                    | 3.05                     | 1.613                    | 0.113                 | 0.133                 | 100.3                                         |

Table 19: Impact of the jack-up motion

## Discussion about the results

From Table 19 it can be seen that the standard deviation of the MP motion in the pitch DOF, i.e. the rotation around the y-axis in the gripper indicated with subscript yy, of the 1-DOF and the multi-DOF model are almost equal for peak periods of 8 and 9 seconds. During a peak period of 7 seconds the MP motion in the pitch DOF is bigger in the 1-DOF model, compared to the multi-DOF model. The MP motion in the pitch DOF of the 1-DOF model is about 158% of the MP motion in the pitch DOF of the multi-DOF model during this peak period.

The reason that the MP motion response in the pitch DOF between both models is equal during an 8 and 9 second peak period is because almost no energy in the wave spectrum during these conditions is exciting mode 4 of the jack-up, since the natural periods of this jack-up mode is about 7 seconds in this situation. Therefore it can be concluded that for higher peak periods, i.e. 8, 9 seconds, or even higher, the MP motion response in the 1-DOF model is almost identical to the MP motion response on the multi-DOF model, because the jack-up is not excited as much in these conditions. This is convenient since from the results of level 3 and 4 it has become apparent that the situations with higher peak periods yield larger MP motion responses, and are hence more critical to model as accurate as possible.

It can be seen from Table 19 as well that the standard deviation in the x direction of the gripper is also the lowest at peak periods of 8 and 9 seconds, for the mentioned reasons. In line with the expectations from the results of modeshape 3, see Figure 22c, the motion response of the jack-up in the y-direction is the highest at a peak period of 8 seconds. Please note how much the jack-up is excited in the y-direction and the MP rotation around the x-axis, due to the spreading of waves.

During a peak period of 7 seconds, the MP motion response in the pitch DOF is highly influenced by the motion of the jack-up. It can be seen from Table 19 that the MP motion in the pitch DOF in the gripper is much lower in the multi-DOF model than in the 1-DOF model. It is expected that this phenomena is caused because the MP is excited by the jack-up with periods close to 7 seconds, due to which the period of motion of the MP is moving further away from the natural periods of the MP, see Figure 29. Because of this the jack-up motion is said to be counteracting the MP motion in some degree due to which the motion response of the MP is lower in the multi-DOF model compared to the 1-DOF model.

In hindsight the 1-DOF model could be improved significantly by adding a translational DOF in the x-direction. The motion of the jack-up due to irregular waves in the x-direction could be added to this DOF, in order to improve the accuracy of the MP motion response estimate in a fairly simple way. Especially during peak periods of 7 seconds this effect would become apparent.

## 7 Conclusion

Due to rising global CO<sub>2</sub> levels, there is a growing need for renewable energy resources. Offshore wind energy is a promising solution, with the MP foundation being preferred for its relatively low cost, provided the site conditions are suitable. In recent years, the size of wind turbines has increased significantly, as these have proved to be more cost efficient. Subsequently the size of the MP's has increased to 3XL MP's, which are MP's with a diameter of 12m or higher. Depending on the site and the marine contractor's preferences, a jack-up vessel may be chosen to install a 3XL MP.

Marine contractors typically use dynamic models to determine suitable weather windows for each installation phase. A critical phase is lowering the MP through the splash zone and the gripper until just above the seabed. This phase is often evaluated using steady-state simulations to assess the MPM value of the MP's motion response, at critical drafts where the natural periods of the MP align with peak wave periods. To optimize the dynamic models used to evaluate the weather window for this phase, it is essential to understand the MP's dynamic behavior during the splash zone lowering phase. This information can enhance the accuracy of the dynamic model used to evaluate weather windows. This study therefore answers the following research question:

**What is the dynamic behavior of a 3XL MP during the lowering phase through a gripper and the splash zone until just above the seabed utilizing a crane on a jack-up vessel?**

To answer this research question a 1-DOF model was established to model the rotational DOF of the MP w.r.t. the gripper. The wave-excitation forces on the MP were evaluated based on diffraction theory through the program WAMIT. The dynamic behavior of the MP is examined through time-domain simulations, whilst considering the environmental conditions shown in Table 2.

From the results that are obtained with this model, 2 main conclusions arise. The first one is that the MPM of the MP rotation, evaluated with the dynamic lowering simulation during irregular spread waves will be around 75% lower, compared to evaluating the MPM of the MP rotation during the same conditions with the common used steady-state simulations at critical MP drafts. The second main conclusion is that the MP's dynamic behavior during the splash zone lowering phase is more severe during longer wave periods, primarily due to resonance at larger MP drafts where wave excitation forces are stronger.

Furthermore this thesis also examined the influence of various physical phenomena on the dynamic behavior of the MP during the splash zone lowering phase, which can be summarized in the following 8 conclusions:

1. The damping due to viscous drag of the MP motion and current is significant for large MP motions.
2. The period of the rotational MP motion increases for larger wave periods and MP drafts.
3. The ambient current effect, for current flowing in the same direction as the irregular unidirectional waves, decreases the MP motion response by 25%.
4. Vortex shedding periods due to viscous drag do not coincide with the natural periods of the 1-DOF model. Hence, they are expected not to dynamically amplify the MP motion response.
5. 2nd order difference frequency drift forces do not dynamically amplify the MP motion response at large MP drafts.
6. The mean MP rotation induced by the mean wave drift force of the 2nd order wave forces is negligibly small.
7. During irregular wave analysis, the assumed MP motion period is affecting the MP motion response significantly through the change in added damping.
8. Because of the jack-up motion, a lower MP motion response is found during a peak period of 7 seconds. For peak periods of 8 and 9 seconds, the MP motion response is near identical without the motion of the jack-up.

One of the biggest limitations of this study is that the model that is used for extracting results in the time-domain, only considers the MP rotational DOF w.r.t. the gripper. This limits the accuracy of the model, since the jack-up motion is completely neglected. Furthermore, it is also not possible with this model to evaluate what the rotational motion of the MP in the gripper along the other horizontal axis is, due to for example the spreading of waves. Adding a translational DOF of the MP along the x-axis in the gripper and assigning jack-up motion to this DOF, obtained from the model of the jack-up, would improve the accuracy of this model. An even better approach would be to extend the current model to 6 DOF's of the MP in the gripper, and integrating this model into the jack-up multi-DOF model.

Assuming that the MP motion period is equal to the peak period also limits the accuracy of the model, as different assumed MP motion periods significantly affect the dynamic behavior of the MP. Implementing the

retardation function, would allow proper a proper evaluation of the hydromechanical reaction forces in the time-domain.

The MPM of the MP rotation in level 4 of the results is evaluated based on 220 dynamic lowering simulations. To determine the required number of simulations for an accurate MPM estimate, a convergence study is suggested to identify the point at which the Rayleigh distribution shape stabilizes.

To ensure the 75% reduction in MPM through the utilization of the dynamic lowering simulation, the MP must continuously lower. In practice, this is challenging due to potential crane cable, winch, or system defaults. A proposed solution involves equipment with pulleys, extra crane cable, and a winch, attached between the crane-tip and the MP. If the MP stops lowering, this equipment would immediately deploy extra crane cable with a winch speed that is used in the dynamic lowering simulation, ensuring continuous lowering even if a system defaults.

## A Multi-DOF FEM model

In this appendix the FEM model of the multi-DOF of the NG 20.000 x-class GustoMSC jack-up design will be discussed. The establishment of this FEM model and the corresponding global mass, stiffness and structural damping matrix are explained.

### A.1 FEM model

The FEM model consists of the jack-up, stiff-cage + gripper-connection + gripper node, crane, and pile-cable subsystems, see Figure 46 for the FEM model of this combined system. As can be seen from the figure, the global coordinate system of this FEM model is located at the lowest leg nodes at the seabed level and in the center of the hull.

The stiff-cage, together with the gripper connection, ensures that the horizontal forces on the gripper node are equally distributed among the 4 legs. This structure mimics the rigid body constraint. The pile-cable subsystem is a system that provides stiffness to the MP at the gripper node due to the attachment of the cable to the MP. This stiffness does not come from a physical element in the FEM model, hence, this pile-cable subsystem cannot be seen in Figure 46. Later in this report, it will be explained where this stiffness of the pile-cable subsystem comes from. It can be seen from this figure that the origin of the coordinate system is located at the lowest jack-up leg nodes in the center of the hull.

To find the global stiffness matrix of the combined system as outlined in Figure 46, the individual system matrices associated with the mentioned subsystems have to be found first. When these individual system matrices are obtained, they can be added in a specific way, such that the global stiffness matrix from Equation 2 can be constructed.

All the previously mentioned subsystems, except for the pile-cable subsystem, have an individual stiffness matrix. Each subsystem is made up of individual elements that are connected through nodes. All of the elements used within every structural subsystem are made out of Timoshenko (TS) beam elements. Each node of a TS beam can deform in 6 DOF's, 3 translational and 3 rotational, see Figure 47 (Heik & Panzer, 2009). The elemental matrix of a TS beam is given in Equation 98, with the values of  $S_1$  until  $S_{10}$  as outlined below the matrix. Since the jack-up, its legs are relatively long, and the jack-up might experience combined load cases of horizontal and vertical loads, the big p-delta, i.e.  $P - \Delta$ , the effect should be considered (Powell, 1937). The  $P - \Delta$  effect is included through the addition of the  $-\frac{N}{L}$  term in the variables  $S_1$  and  $S_2$  (Bordes, 1990), that make up the values for some of the elements in the TS-beam elemental matrix.

Each row and column of the elemental stiffness matrix is associated with one of the 6 DOF's, such that the internal forces associated with a specific TS beam element can be found with Equation 100, see Equation 101 for the result. The values in the displacement vector  $\vec{u}$ , are associated with the displacement of the TS beam element that is expressed in the local coordinate system as defined in Figure 47. In this vector,  $u$ ,  $v$ , and  $w$  are respectively the deformations of a node along the x, y, and z-axes, and  $\phi$ ,  $\theta$ , and  $\psi$  are the rotations over the x, y, and z axes, where the subscript 1 or 2 denotes the nodes associated with these displacements.

It can be observed from these equations that the off-diagonal terms in the stiffness matrix represent the coupling between different DOFs, for example, a translation in the z-direction at node i+1 induces to some extent a shear-stress in the xz-direction at node i, i.e.  $\tau_{xz}$ , see element  $K_{2,8} = -S_2$  in the elemental stiffness matrix outlined in Equation 98.

$$\mathbf{K}_{el} = \begin{bmatrix} S_1 & 0 & 0 & 0 & 0 & 0 & -S_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & S_2 & 0 & 0 & 0 & S_3 & 0 & -S_2 & 0 & 0 & 0 & S_3 \\ 0 & 0 & S_4 & 0 & -S_5 & 0 & 0 & 0 & -S_4 & 0 & -S_5 & 0 \\ 0 & 0 & 0 & S_6 & 0 & 0 & 0 & 0 & 0 & -S_6 & 0 & 0 \\ 0 & 0 & -S_5 & 0 & S_9 & 0 & 0 & 0 & S_5 & 0 & S_7 & 0 \\ 0 & S_3 & 0 & 0 & 0 & S_{10} & 0 & -S_3 & 0 & 0 & 0 & S_8 \\ -S_1 & 0 & 0 & 0 & 0 & 0 & S_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -S_2 & 0 & 0 & 0 & -S_3 & 0 & S_2 & 0 & 0 & 0 & -S_3 \\ 0 & 0 & -S_4 & 0 & S_5 & 0 & 0 & 0 & S_4 & 0 & S_5 & 0 \\ 0 & 0 & 0 & -S_6 & 0 & 0 & 0 & 0 & 0 & S_6 & 0 & 0 \\ 0 & 0 & -S_5 & 0 & S_7 & 0 & 0 & 0 & S_5 & 0 & S_9 & 0 \\ 0 & S_3 & 0 & 0 & 0 & S_8 & 0 & -S_3 & 0 & 0 & 0 & S_{10} \end{bmatrix} \quad (98)$$

$$\vec{u} = [ u_1 \quad v_1 \quad w_1 \quad \phi_1 \quad \theta_1 \quad \psi_1 \quad u_2 \quad v_2 \quad w_2 \quad \phi_2 \quad \theta_2 \quad \psi_2 ] \quad (99)$$

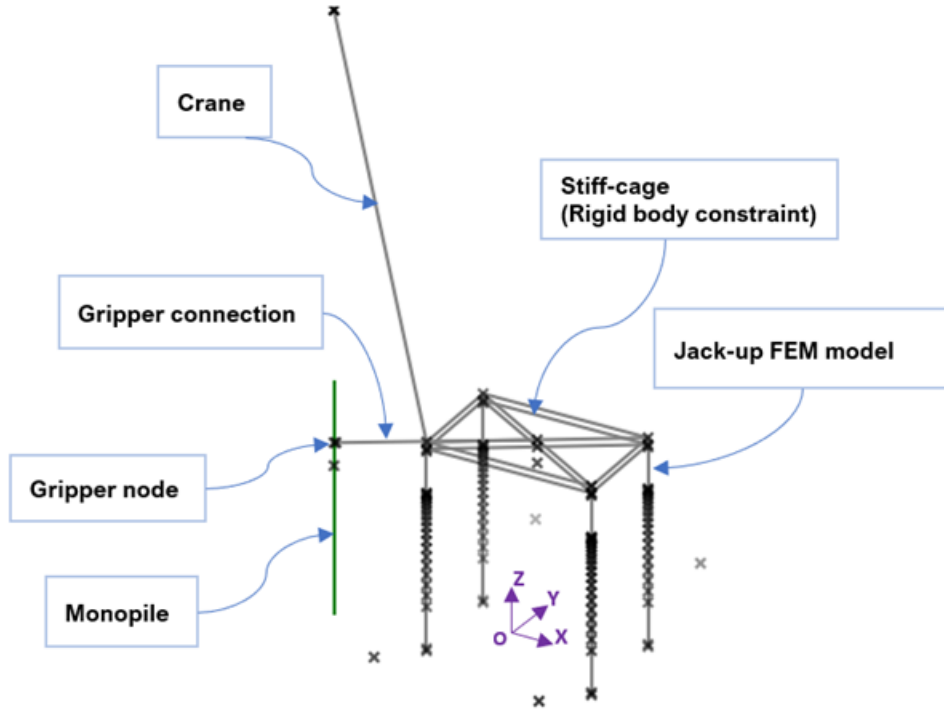


Figure 46: Combined FEM model of the jack-up, gripper, crane

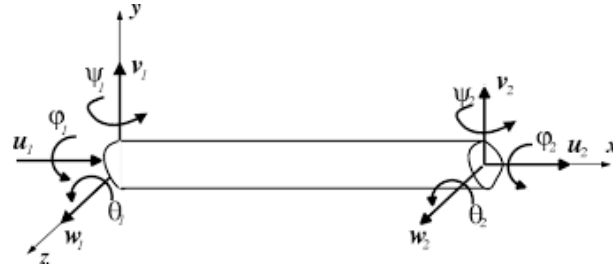


Figure 47: Element of a TS beam (Heik & Panzer, 2009)

$$F_{internal} = \mathbf{K}_{el} \vec{u} \quad (100)$$

$$F_{internal}^{\rightarrow} = \begin{bmatrix} \sigma_{xx,1} & \tau_{xz,1} & \tau_{xy,1} & \sigma_{bend,\phi,1} & \sigma_{bend,\theta,1} & \sigma_{bend,\psi,1} & \sigma_{xx,2} & \tau_{xz,2} & \tau_{xy,2} & \sigma_{bend,\phi,2} & \sigma_{bend,\theta,2} & \sigma_{bend,\psi,2} \end{bmatrix} \quad (101)$$

$$\begin{aligned} S_1 &= \frac{EA_{xx}}{L} & S_6 &= \frac{GI_{xx}}{L} \\ S_2 &= \frac{12EI_{zz}}{(1+a_y)L^3} - \frac{N}{L} & S_7 &= (2-a_z) \frac{EI_{yy}}{(1+a_z)L} \\ S_3 &= \frac{6EI_{zz}}{(1+a_y)L^2} & S_8 &= (2-a_y) \frac{EI_{zz}}{(1+a_y)L} \\ S_4 &= \frac{12EI_{yy}}{(1+a_z)L^3} - \frac{N}{L} & S_9 &= (4+a_z) \frac{EI_{yy}}{(1+a_z)L} \\ S_5 &= \frac{6EI_{yy}}{(1+a_z)L^2} & S_{10} &= (4+a_y) \frac{EI_{zz}}{(1+a_y)L} \\ a_z &= \frac{12EI_{yy}}{A_{zz}GL^2} & a_y &= \frac{12EI_{zz}}{A_{yy}GL^2} \end{aligned}$$

### A.1.1 Jack-up

As discussed previously, every structural subsystem is made out of TS beam elements, therefore, the jack-up is also made out of TS beam elements. The TS-beam is used for cases where there is significant shear deformation (Ahmed, 2021), which is the case for a jack-up, as will later in this report be shown. In Figure 48 the FEM model of the jack-up with the stiff-cage on top is depicted. This FEM model of the jack-up is widely used within GustoMSC to model the structure of the hull and the legs.

Each cross in the figure resembles a node of a TS-beam element. All of the elements are connected via these nodes. Every individual element has a stiffness matrix that is associated with the local coordinate system of that element. The definition of such a local coordinate system is outlined in Figure 47, and depends on the orientation of that element. The transformation of a displacement vector and a force vector from the local to the global coordinate system are respectively outlined with equations 102, and 103. With this, the transformation matrix of a stiffness matrix can be found with equation 104. For every element, a transformation matrix  $\mathbf{T}_u$ , can be found such that each element can be expressed into global coordinates with equation 104. In these equations,  $\vec{u}'$ ,  $\vec{F}'$ , and  $\mathbf{K}'$  are the vectors and matrix w.r.t. global coordinates, and  $\vec{u}$ ,  $\vec{F}$ , and  $\mathbf{K}$ , are the vectors and matrix w.r.t. local coordinates. After transforming every stiffness matrix associated with an individual element from local to global coordinates, the individual stiffness matrices can be added into 1 global coordinate system. The global coordinate system is located at the lowest leg nodes, at seabed level, in the centre of the hull of the jack-up; see Figure 48.

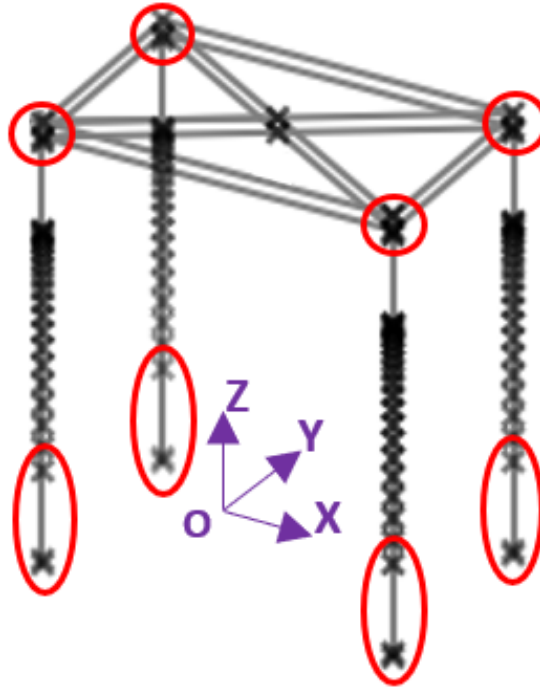


Figure 48: Jack-up FEM model + stiff-cage

$$\vec{u}' = \mathbf{T}_u \vec{u} \quad (102)$$

$$\vec{F}' = \mathbf{T}_f \vec{F} \quad (103)$$

$$\mathbf{K}'_{el} = \mathbf{T}_u^{-T} \mathbf{K}_{el} \mathbf{T}_u \quad (104)$$

To find the stiffness matrix of the jack-up FEM model expressed in global coordinates, all the individual stiffness matrices of every TS beam element in Figure 48 that are expressed in global coordinates have to be added appropriately into the jack-up stiffness matrix. Since each component of an individual stiffness matrix is associated with a specific DOF, each component of the individual stiffness matrix has to be added to the jack-up stiffness matrix at the corresponding indexes. Because some nodes have multiple elements connected to them, there will be components in the jack-up stiffness matrix that have overlapping stiffness values, i.e., multiple

elements contributing to a specific component in the jack-up stiffness matrix. These values should be added together to obtain the appropriate value in the jack-up stiffness matrix at that specific component. See Equation 105 for the relation between the jack-up stiffness matrix and the jack-up displacement vector. Where the size of the jack-up stiffness matrix is  $N \times N$  and the size of the displacement or force vector is  $N$ , with  $N$  = amount of DOF's. It can be observed that the columns and rows of the jack-up stiffness matrix are associated with the DOF's outlined in Equation 106.

$$F'_{jack-up} \vec{u}_{jack-up} = \mathbf{K}'_{jack-up} \vec{u}_{jack-up} \quad (105)$$

$$\vec{u}_{jack-up} = \begin{bmatrix} u_1 & v_1 & w_1 & \phi_1 & \theta_1 & \psi_1 & u_2 & v_2 & w_2 & \phi_2 & \theta_2 & \psi_2 & \dots \\ & u_{N-1} & v_{N-1} & w_{N-1} & \phi_{N-1} & \theta_{N-1} & \psi_{N-1} & u_N & v_N & w_N & \phi_N & \theta_N & \psi_N \end{bmatrix} \quad (106)$$

$$F_{jack-up} \vec{u}_{jack-up} = \begin{bmatrix} \sigma_{xx,1} & \tau_{xz,1} & \tau_{xy,1} & \sigma_{bend,\phi,1} & \sigma_{bend,\theta,1} & \sigma_{bend,\psi,1} & \sigma_{xx,2} & \tau_{xz,2} & \tau_{xy,2} \\ \sigma_{bend,\phi,2} & \sigma_{bend,\theta,2} & \sigma_{bend,\psi,2} & \dots & & & & & \\ \sigma_{xx,N-1} & \tau_{xz,N-1} & \tau_{xy,N-1} & \sigma_{bend,\phi,N-1} & \sigma_{bend,\theta,N-1} & \sigma_{bend,\psi,N-1} & \sigma_{xx,N} & \tau_{xz,N} & \tau_{xy,N} \\ \sigma_{bend,\phi,N} & \sigma_{bend,\theta,N} & \sigma_{bend,\psi,N} & & & & & & \end{bmatrix} \quad (107)$$

These so-called overlapping nodes and their associated elemental values have to be summed to obtain the corresponding values in the global jack-up stiffness matrix. See, for example, the global mass and stiffness matrix assembly of a random combination of elements depicted in Figure 49. It can be seen from Figure 50 that the elemental mass and stiffness matrix of the red element are first transformed from local to global coordinates, as was also done for the jack-up FEM model. Hereafter, the elemental mass and stiffness matrices are added to the indexes associated with the corresponding DOF's. The exact same is done for the yellow and brown elements, as can be seen in Figures 51, and 52. In these figures,  $\ddot{\mathbf{u}}^i$ , and  $\mathbf{u}^i$  are respectively the acceleration and motion vectors of node  $i$ . It can be seen that there are 4 nodes; therefore, the size of the global mass and stiffness matrices are  $4 \times N_{dof,node}$ , where  $N_{dof,node}$  is the number of DOFs per node.

It can also be observed that the yellow and red elements have one overlapping node, namely node 1, see Figure 51. The values in the individual stiffness matrices of elements 1 and 2, which solely correspond to the DOF's at node 1, have to be added to the global stiffness matrix such that the correct values of the global stiffness matrix at the DOF's of node 1 are obtained. The global mass matrix is found in the same procedure as the global stiffness matrix.

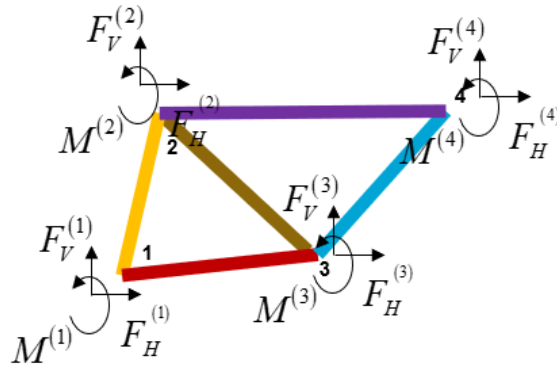


Figure 49: Five arbitrarily oriented elements (Colomés, 2023)



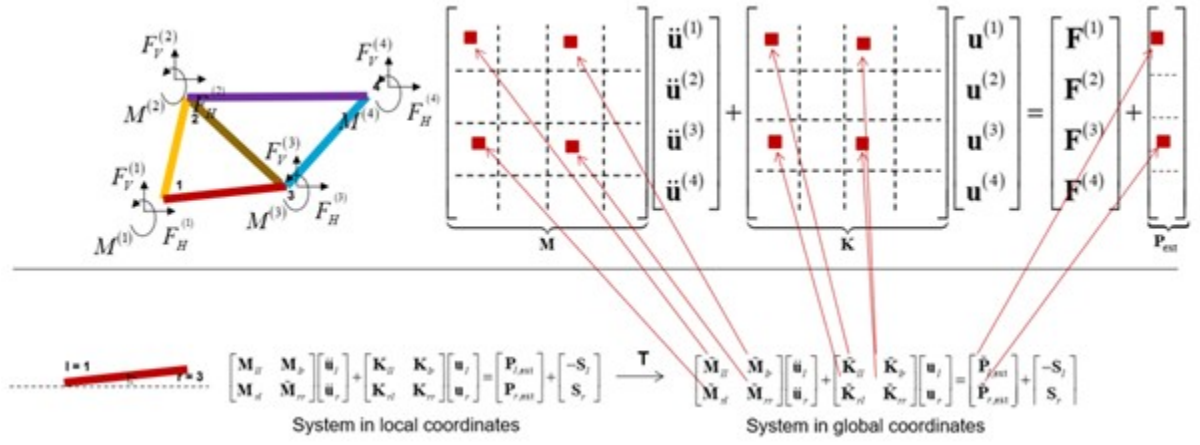


Figure 50: Global stiffness matrix assembly red element (Colomés, 2023)

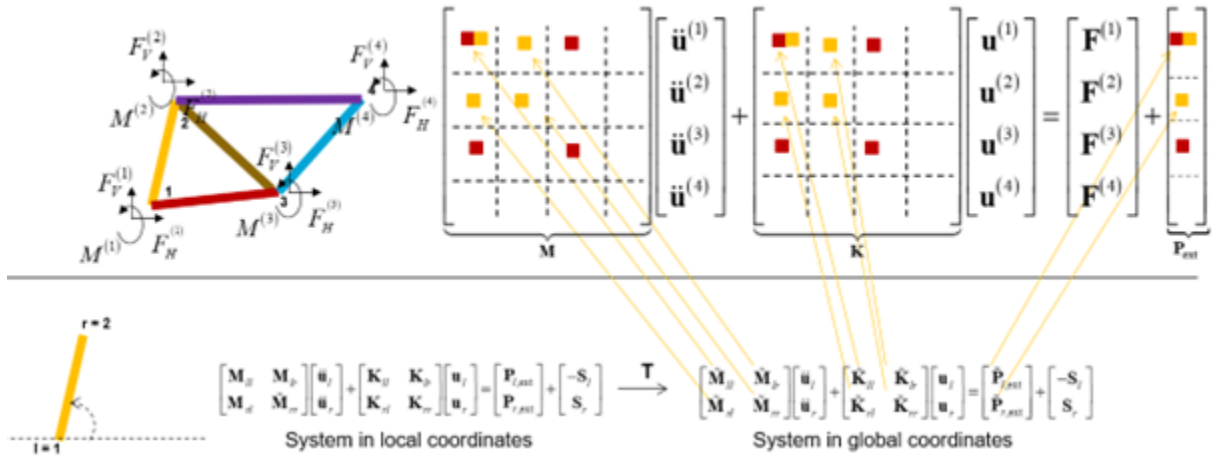


Figure 51: Global stiffness matrix assembly yellow element (Colomés, 2023)

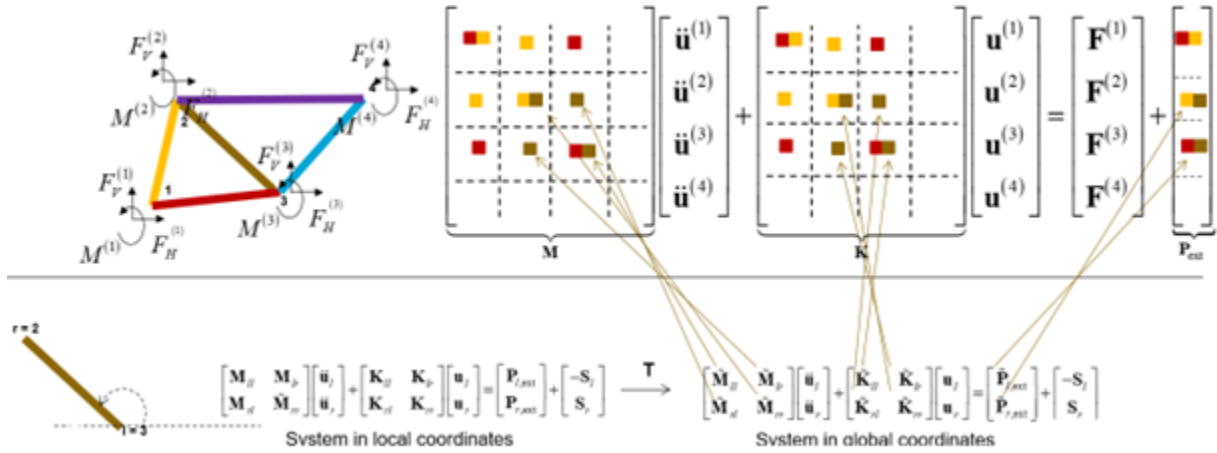


Figure 52: Global stiffness matrix assembly brown element (Colomés, 2023)

The assembly of the global stiffness matrix of the jack-up is executed in the same way as is described with the arbitrarily oriented elements of Figure 49. The values for the components of the individual stiffness matrices of the TS-beam elements, i.e., S1 until S10,  $a_z$  and  $a_y$ , vary depending on the element; think of, for example, the TS-beam element of the hull versus that of a leg. These specific values are established and well thought out within GustoMSC. In practice, it turns out that these values are very properly chosen. These specific values will not be shared within this report, because of confidentiality reasons.

The jack-up vessel is connected with the seabed trough linear spring elements that model the soil-structure interaction, see the lowest red encircled elements in Figure 48. The lowest node of the spring elements at the seabed is fixed in all 6-DOFs, from which the loads that act upon the jack-up will be transferred into the

seabed. The top red encircled elements in the figure are also linear spring elements; these elements model the stiffness of the interface between the legs and the hull of the jack-up. Depending on the site where the jack-up is located and depending on which jack-up and jacking system is used, specific values can be set to the spring stiffness in 6-DOF's for all 8 spring elements. This is, however, not discussed within this thesis. Note that a spring element is simply an element with 2 nodes, where each node has 6 DOF's. Such a spring element has an elemental stiffness matrix with solely diagonal terms, i.e., no coupling between DOF's. The stiffness values of these spring elements associated with a certain DOF can be added to the corresponding indexes in the global jack-up mass matrix in the same procedure as was discussed in the example of the 5 arbitrarily oriented elements in Figure 49. After adding these stiffness values to the global jack-up stiffness matrix, the complete global jack-up stiffness matrix w.r.t. the global coordinate system is obtained. The global jack-up mass matrix is obtained in a slightly different way, as opposed to the global jack-up stiffness matrix. The mass of the jack-up and its legs are modeled as lumped masses per individual node, i.e., there is no coupling of mass terms in the global jack-up mass matrix as there are only diagonal terms. In general, 50% of the mass of the jack-up and the legs is distributed among the 4 corners, and the other 50% of the mass is located in the center of the hull, i.e., at the intersection of the cross, see Figure 48. It is common within GustoMSC to model the mass matrix of the jack-up in this way.

### A.1.2 Stiff cage + gripper

In order to transfer the horizontal loads from the gripper, which come from the MP response, equally among the 4 legs of the jack-up, a so-called stiff-cage is established. In Figure 53, the FEM model of the stiff-cage is depicted, where the red encircled node is the gripper node. This node is attached to a spring element which solely has a translational stiffness contribution in the xy-plane. The stiff-cage is assumed to have no shear deformation, only axial and bending. The axial and bending stiffness of the elements used for the stiff-cage are set at near-infinite values, such that the horizontal loads coming from the gripper are equally distributed among the 4 legs. In this way, the stiff-cage mimics the rigid body constraint. The global stiffness matrix of the stiff cage, including the spring element at the gripper node, can be found in the same way as was done for the jack-up model. The stiff-cage has no mass matrix.

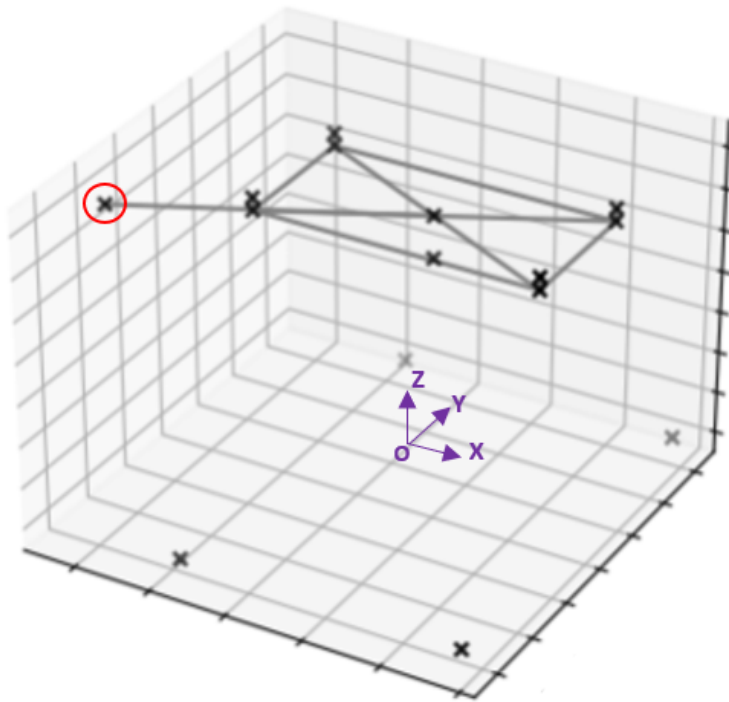


Figure 53: Stiff cage

### A.1.3 Pile-cable subsystem

The pile-cable subsystem is a system that models the dynamics of the pile-cable interaction. For the modelling of the pile-cable subsystem, a couple of assumptions are made, which will later be verified in this report. Assumptions regarding the pile-cable subsystem:

1. The MP is a rigid body
2. The moments that arise due to the rotation of the MP are linearly related to the angle  $\theta$ , so the trigonometric functions are linearized.
3. The MP's weight is assumed to be uniformly distributed, i.e., the non-uniform distribution of the weight due to a possible tapered end of the MP is not considered.
4. The translational and rotational accelerations, in the direction of, and around, the z-axis are neglected
5. Structural damping is neglected
6. The MP is free to rotate around the gripper node
7. The cable is free to elongate due to a rotation  $\theta$ , i.e. the tension in the cable is constant.

Note that the linear relation between the rotation of the MP and the associated moments that arise due to the weight and tension in the cable, already implicitly states that the tension in the cable is constant and the cable is free to elongate. For the sake of clarity, however, all 3 assumptions are mentioned. Considering displacements and rotations of the pile and the top of the crane tip, certain stiffness values can be found, which linearly relate the displacement to the reaction forces of the pile cable subsystem given a certain displacement, either translational or rotational. These stiffness values will later be added to the global stiffness matrix at the corresponding indexes that are associated with the DOF's of the gripper and crane-tip node. See Figure 54 for a sketch with an overview of the dimensions of the pile-cable subsystem.

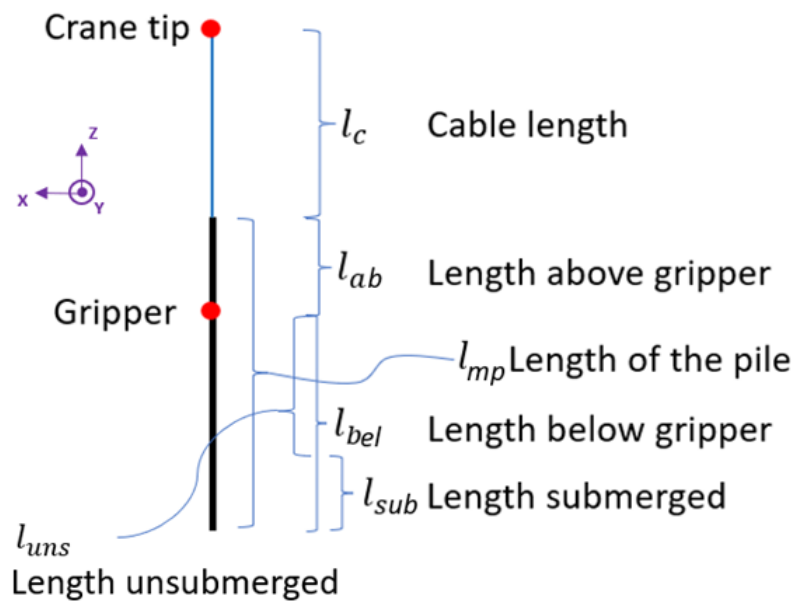


Figure 54: Sketch and dimensions of the pile-cable subsystem

It can be seen from figures 55, and 56, that when the MP is given a certain angle  $\theta$ , two forces arise which cause two moments w.r.t the gripper node:

1. A moment due to the tension in the cable:  $M_t$
2. A moment due to the weight of the MP:  $M_w$

When linearizing these two moments w.r.t. their trigonometric functions, the stiffness values of the pile-cable subsystem can be found, since the above two moments are caused by a (static) displacement. The values of  $\tan(\alpha)$ , and the vertical and horizontal force due to the tension in the cable can be found respectively with equations 108, 109, and 110. With these equations, the moment due to the tension in the cable for a given MP angle  $\theta$  can be found with Equation 111. The linearized form of the 3 equations is outlined after the approximate sign ( $\approx$ ). The linearized form of the 3 equations assumes small rotations, in subsection 5.3, it is verified

up until what rotation this assumption is valid. The exact same can be done for the moment that arises from a combination of the weight of the MP and a given rotation  $\theta$ , see Equation 112.

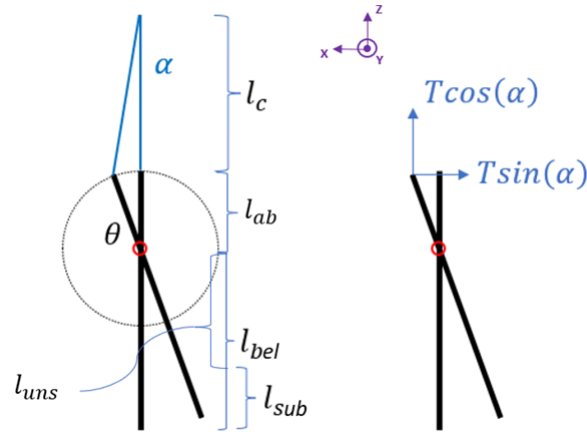


Figure 55: Moment due to the tension in the cable of the MP w.r.t. the gripper node

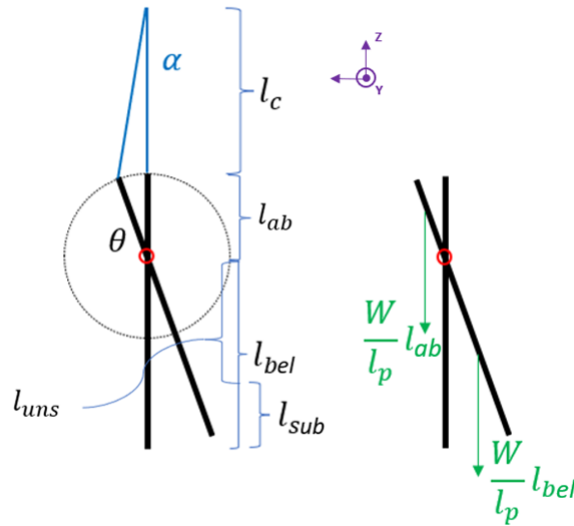


Figure 56: Moment due to the weight of the MP w.r.t. the gripper node

$$\tan(\alpha) = \frac{\tan(\theta)l_{ab} \cos(\theta)}{l_c + l_{ab}(1 - \cos(\theta))} \approx \frac{l_{ab}}{l_c} \theta \quad (108)$$

$$F_{v,t} = W = T \cos(\alpha) \approx T \quad (109)$$

$$F_{h,t} = T \sin(\alpha) = W \tan(\alpha) \approx W \alpha \quad (110)$$

$$M_t = -W \frac{\tan(\theta)l_{ab}^2 \cos(\theta)}{l_c + l_{ab}(1 - \cos(\theta))} - T \cos(\alpha)l_{ab} \sin(\theta) \approx -W \frac{l_{ab}^2}{l_c} \theta - W l_{ab} \theta \quad (111)$$

$$M_w = \frac{W l_{ab}}{l_{mp}} \frac{l_{ab}}{2} \sin(\theta) - \frac{W}{l_{mp}} l_{bel} \frac{l_{bel}}{2} \sin(\theta) = -\frac{W}{l_{mp}} \frac{l_{bel}^2 - l_{ab}^2}{2} \sin(\theta) \approx -\frac{W}{l_{mp}} \frac{l_{bel}^2 - l_{ab}^2}{2} \theta \quad (112)$$

If a MP translation is considered along the x-axis, forces are induced in the gripper node and crane-tip, as can be seen in Figure 57. In this figure the gripper node is indicated with 'u', and the crane-tip node is indicated with 'v'. The resulting horizontal force along the x-axis and a moment around the y-axis are outlined

with Equation 113, and Equation 114. The resulting horizontal force in the crane tip can be described with Equation 115. Within these equations, the first subscript notation indicates the location and direction of the reaction forces, and the second subscript notation indicates due to what displacement this reaction force is caused. For example, the force  $F_{uyy,ux}$  would be a reaction moment over the y-axis, w.r.t point u, due to a displacement in the x-direction of point u.

Because of symmetry the equations for the forces that are present in the gripper and the crane-tip due to a translation of the MP in the  $'u_y'$  direction are of similar form as the previously discussed equations. See equations 116, 117, and 118 for respectively the force in the y-direction and moment around the x-axis in the gripper node, and the force in the y-direction in the crane-tip due too a translation in the y-direction of the MP, i.e.,  $'u_y'$ .

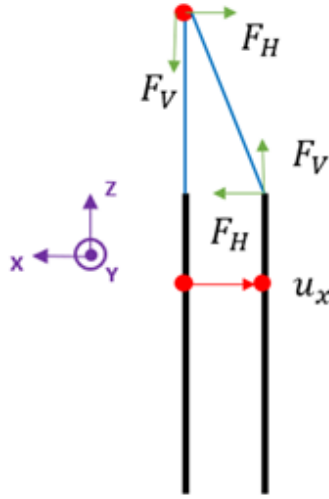


Figure 57: Translation of the MP in the x-direction

$$F_{ux,ux} = F_h = -W \tan(\alpha) \approx -W\alpha = -W \frac{u_x}{l_c} \quad (113)$$

$$F_{uyy,ux} = -F_{ux,ux} l_{ab} \approx -W \frac{u_x}{l_c} l_{ab} \quad (114)$$

$$F_{vx,ux} \approx \frac{W}{l_c} u_x \quad (115)$$

$$F_{uy,uy} = F_h = -W \tan(\alpha) \approx -W\alpha = -W \frac{u_y}{l_c} \quad (116)$$

$$F_{uxx,uy} = -F_{uy,uy} l_{ab} \approx -W \frac{u_y}{l_c} l_{ab} \quad (117)$$

$$F_{vy,uy} \approx \frac{W}{l_c} u_y \quad (118)$$

If a MP rotation around the x-axis is considered, the forces as depicted in Figure 58 are induced. These resulting forces in the gripper node and the crane tip can be evaluated with Equation 119, Equation 120, and Equation 121. Because of symmetry, similar-looking equations can be obtained when the MP rotates around the y-axis, see Equation 122, Equation 123, and Equation 124. Note that the terms in these equations are a combination of the moment coming from the tension in the cable and the weight of the MP as defined with Equation 111, and Equation 112.

$$F_{uy,uxx} = F_h = W \tan(\alpha) \approx W \frac{l_{ab}}{l_c} u_{xx} \quad (119)$$

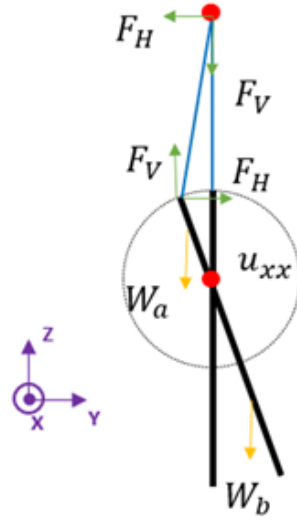


Figure 58: Rotation of the MP around the x-axis

$$F_{u_{xx},u_{xx}} = -Wl_{ab} \sin(u_{xx}) - W \tan(\alpha)l_{ab} \cos(u_{xx}) + W \frac{l_{ab}}{l_{mp}} \frac{l_{ab}}{2} \sin(u_{xx}) - W \frac{l_{bel}}{l_{mp}} \frac{l_{bel}}{2} \sin(u_{xx}) \approx W \left( -l_{ab} - \frac{l_{ab}^2}{l_c} + \frac{l_{ab}^2}{2l_{mp}} - \frac{l_{bel}^2}{2l_{mp}} \right) u_{xx} \quad (120)$$

$$F_{vy,u_{xx}} = -W \frac{l_{ab} \cos(u_{xx})}{l_c + l_{ab}(1 - \cos(u_{xx}))} \tan(u_{xx}) \approx -W \frac{l_{ab}}{l_c} u_{xx} \quad (121)$$

$$F_{ux,u_{yy}} = F_h = W \tan(\alpha) \approx W \frac{l_{ab}}{l_c} u_{yy} \quad (122)$$

$$F_{u_{yy},u_{yy}} = -Wl_{ab} \sin(u_{yy}) - W \tan(\alpha)l_{ab} \cos(u_{yy}) + W \frac{l_{ab}}{l_{mp}} \frac{l_{ab}}{2} \sin(u_{yy}) - W \frac{l_{bel}}{l_{mp}} \frac{l_{bel}}{2} \sin(u_{yy}) \approx W \left( -l_{ab} - \frac{l_{ab}^2}{l_c} + \frac{l_{ab}^2}{2l_{mp}} - \frac{l_{bel}^2}{2l_{mp}} \right) u_{yy} \quad (123)$$

$$F_{vx,u_{yy}} = -W \frac{l_{ab} \cos(u_{yy})}{l_c + l_{ab}(1 - \cos(u_{yy}))} \tan(u_{yy}) \approx -W \frac{l_{ab}}{l_c} u_{yy} \quad (124)$$

If a crane tip motion in the x-direction is considered, the resulting forces on the MP node and in the crane tip as depicted in Figure 59 arise. These forces can be evaluated with Equation 125, Equation 126, and Equation 127. Because of symmetry, similar forces arise when the crane-tip moves in the y-direction, see Equation 128, Equation 129, and Equation 130 for the resulting forces on the MP node and in the crane tip.

$$F_{ux,vx} = W \tan(\alpha) = \frac{W}{l_c} v_x \quad (125)$$

$$F_{u_{yy},vx} = \frac{W}{l_c} l_{ab} v_x \quad (126)$$

$$F_{vx,vx} = -W \tan(\alpha) = -\frac{W}{l_c} v_x \quad (127)$$

$$F_{uy,vy} = \frac{W}{l_c} v_y \quad (128)$$

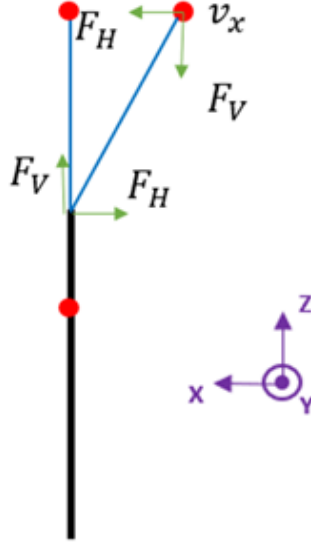


Figure 59: Translation of the crane-tip in the x-direction

$$F_{uxx,vy} = \frac{W}{l_c} l_a v_y \quad (129)$$

$$F_{vy,vy} = -\frac{W}{l_c} v_y \quad (130)$$

As discussed previously, the forces and accelerations around and along the z-axis are not considered within the pile-cable subsystem. Because of this, and since the forces due to translations and rotations around and along the x and y axes for both the crane-tip and gripper node are known, it is possible to construct a stiffness matrix of the pile-cable subsystem. This stiffness matrix linearly relates the described motions to resulting forces, see Equation 131. Equation 132 is the displacement vector associated with the stiffness matrix of the pile-cable subsystem. Note that it is assumed that the cable can rotate freely in the crane-tip around the x and y axes, and that by setting  $k_c$  to infinite, the pile-cable subsystem will not rotate or translate over and along the z-axis. It should be noted from the discussed stiffness matrix, that the values in this matrix vary, depending on the position of the MP. As the MP lowers through the gripper and the splash zone, the values of  $l_{ab}$  and  $l_c$  will change over time. Therefore the stiffness matrix is said to be time-variant.

$$\mathbf{K}_{\text{pile-cable}} = \begin{bmatrix} -k_1 & 0 & 0 & 0 & k_2 & 0 & k_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -k_1 & 0 & k_2 & 0 & 0 & 0 & k_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & k_c & 0 & 0 & 0 & 0 & 0 & -k_c & 0 & 0 & 0 \\ 0 & -k_2 & 0 & -k_3 & 0 & 0 & 0 & k_2 & 0 & 0 & 0 & 0 \\ -k_2 & 0 & 0 & 0 & -k_3 & 0 & k_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & k_c & 0 & 0 & 0 & 0 & 0 & k_c \\ k_1 & 0 & 0 & 0 & k_2 & 0 & -k_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & k_1 & 0 & k_2 & 0 & 0 & 0 & -k_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -k_c & 0 & 0 & 0 & 0 & 0 & k_c & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -k_c & 0 & 0 & 0 & 0 & 0 & k_c \end{bmatrix} \quad (131)$$

$$\vec{x} = [ u_x \quad u_y \quad u_z \quad u_{xx} \quad u_{yy} \quad u_{zz} \quad v_x \quad v_y \quad v_z \quad v_{xx} \quad v_{yy} \quad v_{zz} ] \quad (132)$$



$$\begin{aligned}
k_1 &= \frac{W}{l_c} \\
k_2 &= \frac{W l_{ab}}{l_c} \\
k_3 &= W \left( l_a + \frac{l_a^2}{l_c} - \frac{l_a^2}{2l_{mp}} + \frac{l_b^2}{2l_{mp}} \right) \\
k_c &= \infty
\end{aligned}$$

The mass matrix of the MP its own weight, w.r.t. the center of gravity (COG) of the MP can be found with Equation 134. In this equation,  $M_{MP}$  is the mass of the MP, and  $I_{MP,xx}$ , and  $I_{MP,yy}$  are the mass moments of inertia of the MP around respectively the x and y-axis, w.r.t. the COG of the MP, and can be obtained with Equation 133. Note that the mass in the z-direction and the mass moment of inertia around the z-axis are neglected since it is assumed that the translational and rotational accelerations along and around the z-axis are 0. The parameters of the MP that is considered in this MSc thesis are outlined in Table 1. Note that the mass of the cable is neglected when evaluating the mass matrix of the pile-cable subsystem.

To evaluate the mass matrix of the MP w.r.t. the gripper node 'u', Equation 135, and 137 should be utilized. In this latter equation,  $\mathbf{T}_u$  is the transformation matrix that can transform a state vector from the COG of the MP to the gripper node. Together with the transformation matrix for a force, see Equation 136, the transformation of a system matrix can be found, as can be seen in Equation 135. Since the MP lowers through the gripper and the splash zone over time, the values in the transformation matrix  $\mathbf{T}_u$  also change. Therefore the mass matrix of the MP w.r.t. the gripper node should be evaluated at every time step since the MP position w.r.t. the gripper node changes in time.

$$I_{MP,xx,cog} = I_{MP,yy,cog} = \frac{1}{12} M_{mp} (3(r_2^2 + r_1^2) + L_{mp}^2) \quad (133)$$

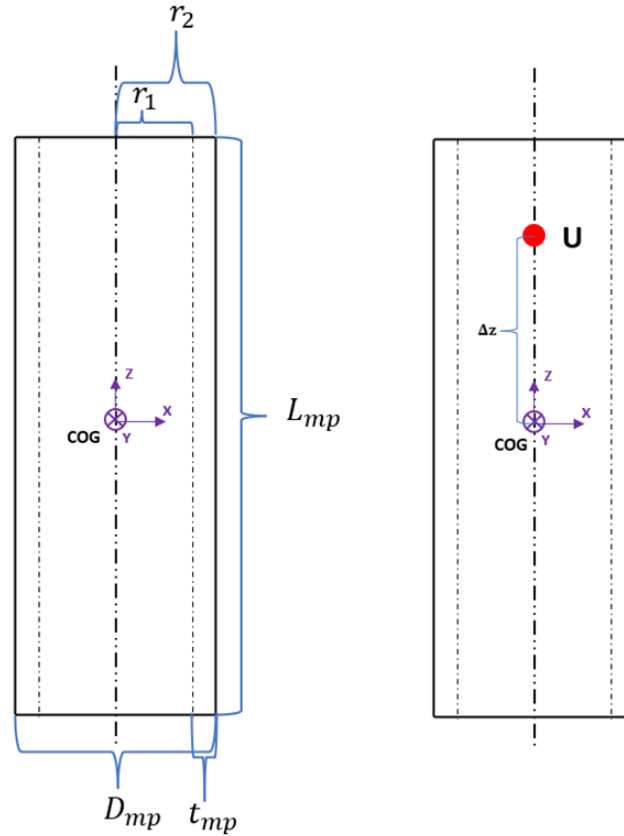


Figure 60: MP sketch with uniformly distributed mass

$$\mathbf{M}_{MP,cog} = \begin{bmatrix} M_{mp} & 0 & 0 & 0 & 0 & 0 \\ 0 & M_{mp} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I_{MP,xx,cog} & 0 & 0 \\ 0 & 0 & 0 & 0 & I_{MP,yy,cog} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (134)$$

$$\mathbf{M}_{MP,gripper-node} = \mathbf{T}_u^{-T} \mathbf{M}_{MP,cog} \mathbf{T}_u \quad (135)$$

$$\mathbf{T}_f = \mathbf{T}_u^{-T} \quad (136)$$

$$\mathbf{T}_u = \begin{bmatrix} 1 & 0 & 0 & 0 & \Delta z & 0 \\ 0 & 1 & 0 & -\Delta z & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (137)$$

#### A.1.4 Combining all the subsystems

Because the crane is modeled as an infinitely stiff rod, and all the other subsystems their mass and stiffness matrices are known, i.e., the jack-up, stiff-cage, and pile-cable subsystem, the global mass and stiffness matrix can be obtained. The size of the global mass and stiffness matrix is equal to;  $(N_{global,dofs} \times N_{global,dofs})$ , where  $N_{global,dofs}$  is the amount of DOF's of the combined system, where the size of a state-vector is  $N_{global,dofs}$ . After properly transforming the mass and stiffness matrices of all the individual subsystems to the global coordinate system, which is located at the seabed, see Figure 46, the global mass and stiffness matrix can be assembled. The assembly of both the global mass and stiffness matrix will be done in the exact same procedure as the example of the 5 arbitrarily oriented elements, see Figure 49. Note that the stiffness matrix of the pile-cable subsystem has contributions to the DOF's located at the gripper and the crane-tip node and that the mass matrix of the pile-cable subsystem solely has a contribution to the DOF's at the gripper node. The results of the combined FEM model with the associated nodes and beams can be seen in Figure 46.

$$\mathbf{F}'_{internal,global} = \mathbf{K}'_{global} \mathbf{u}'_{global} \quad (138)$$

#### A.1.5 Structural damping

With Equation 139 the modal damping matrix can be found, where  $\zeta$  is a diagonal matrix with the damping ratio of each modeshape on the diagonal,  $\mathbf{M}_{m,global}$  is the modal global mass matrix,  $\mathbf{K}_{m,global}$  is the modal global stiffness matrix, and  $\sqrt{\mathbf{M}_{m,global} \mathbf{K}_{m,global}}$  is the modal critical damping. The modal mass, modal damping, and modal stiffness matrices are defined with Equations 140, 141, and 142, where  $\Phi$  is the modeshape matrix where the modeshapes are placed in the columns in descending order. With the equation for the critical damping matrix, the modal mass, modal damping, and modal stiffness matrix, and with the relations outlined in Equation 143, the structural damping matrix can be obtained with Equation 144.

In the damping ratio matrix  $\zeta$ , the damping ratio is set at 5% of the critical damping for all modes. All the matrices in these equations are of size  $(N_{global,dofs} \times N_{global,dofs})$ , since there are an equal amount of modeshapes and natural frequencies as there are DOFs.

$$\mathbf{C}'_{m,structural} = 2\zeta \sqrt{\mathbf{M}'_{m,global} \mathbf{K}'_{m,global}} \quad (139)$$

$$\mathbf{M}'_{m,global} = \Phi^T \mathbf{M}'_{global} \Phi \quad (140)$$

$$\mathbf{C}'_{m,global} = \Phi^T \mathbf{C}'_{global} \Phi \quad (141)$$

$$\mathbf{K}'_{m,global} = \Phi^T \mathbf{K}'_{global} \Phi \quad (142)$$

$$\Phi^T \mathbf{M}'_{\text{global}} \Phi = \mathbf{I} \rightarrow (\Phi^{-T}) = \mathbf{M}'_{\text{global}} \Phi \quad \& \quad (\Phi^{-1}) = \Phi^T \mathbf{M}'_{\text{global}} \quad (143)$$

$$\mathbf{C}'_{\text{structural}} = \mathbf{M}'_{\text{global}} \Phi \mathbf{C}'_{\text{m,global}} \Phi^T \mathbf{M}'_{\text{global}} = \mathbf{M}'_{\text{global}} \Phi \left( 2\zeta \sqrt{\mathbf{M}'_{\text{m,global}} \mathbf{K}'_{\text{m,global}}} \right) \Phi^T \mathbf{M}'_{\text{global}} \quad (144)$$

### A.1.6 Global system matrices

Taking a look back at the general EOM from Equation 2, the global mass matrix  $\mathbf{M}'_{\text{global}}$ , the global stiffness matrix  $\mathbf{K}'_{\text{global}}$ , and part of the global damping matrix  $\mathbf{C}'_{\text{global}}$  of the full model are now obtained. With the 'full model', a combined model of the following subsystems is meant: jack-up, crane, stiff-cage, pile-cable subsystem, and gripper, see Figure 46. As mentioned, not the full damping matrix has been obtained yet, since the obtained damping matrix from Equation 144 is solely the damping matrix due to structural damping. The damping values due to the wave-MP interaction will, later on, have to be obtained and added to the structural damping matrix to find the global damping matrix.

## A.2 Damping ratios

In Figures 61, 62, 63, 64, 65 the damping ratios as a percentage of the critical damping of the multi-DOF model are depicted. The plots for the damping ratio as a percentage of the critical damping for the first 2 modes, i.e. the pile-cable modes, are depicted in Figures 61, and 62. The highest damping ratio is found around a MP draft of 25m and a pile period of motion of 5.5 seconds. It can be seen from both figures that the peak of the damping ratio shifts to higher assumed pile periods of motion for increasing MP drafts. This is beneficial since, for the first 2 modes the natural periods increase for increasing MP draft, it is at the natural periods that damping is convenient. One thing that is worrying to see is that there is almost no added damping for MP drafts of 5, 10, and 15m at pile periods of motion between 8-10 seconds, which is the natural period of mode shapes 1 and 2 at these MP drafts, see Figures 23, and 24. Since at resonance the response is governed by damping, this is a very worrying outcome, w.r.t. design perspectives.

The damping ratios for modes 3,4, and 5 are depicted in Figures 63, 64, and 65. It can be seen from these figures that the 5% structural damping is present through the mean shift of the damping ratio as opposed to the damping ratio of modes 1 and 2. This mean is not exactly 5% of the critical damping, because the damping ratios in Figures 63, 64, and 65, are for the combined pile-cable and jack-up model. Because this 5% of the structural damping was only added to the FEM model, the combined system its mean damping ratio is a bit below this 5%. Furthermore, a peak of the damping ratio can be observed around pile periods of motion of 5 seconds, which was expected, since these peaks of the damping ratio are also observed in the pile-cable modes, i.e. modes 1 and 2.

Because the damping ratios are relatively small for all modes, the natural frequencies are assumed to be equal to the damped natural frequencies (Brincker & Ventura, 2015). See Equation 145 for the damped natural frequency of a 1-DOF system, when  $\zeta$  is a small figure, say 0.08, the natural frequency is almost equal to the undamped natural frequency.

Damping ratios for modeshape 1 of the combined system; jack-up + pile-cable subsystem

Base config:  $L_{\text{pile}} = 115 \text{ m}$ ,  $d_{\text{baseh-grip}} = 6 \text{ m}$ ,  $L_{\text{crane}} = 200 \text{ m}$ ,  $D_{\text{pile}} = 12 \text{ m}$

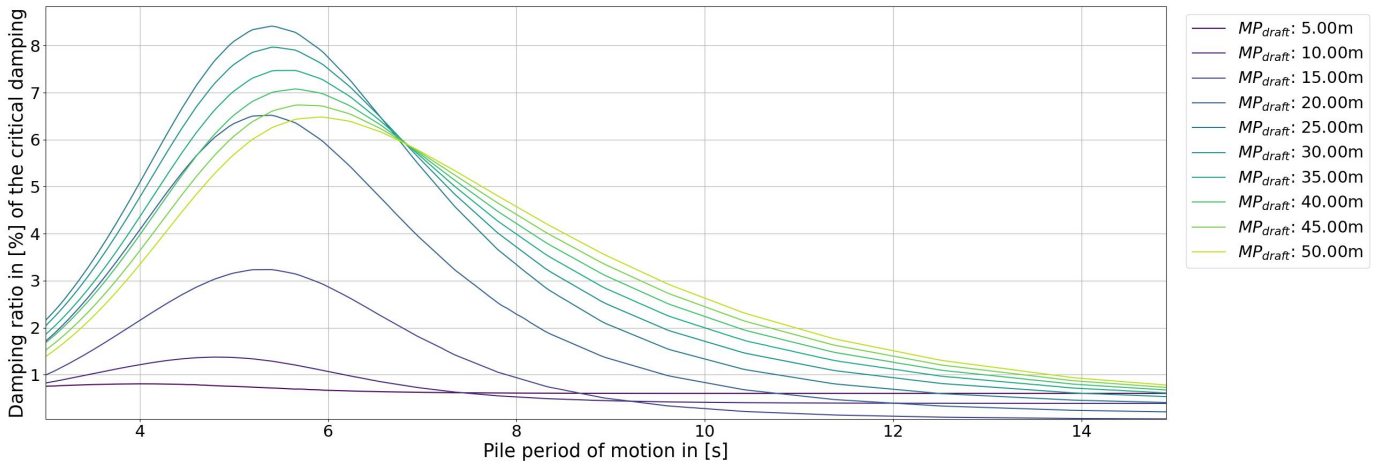


Figure 61: Damping ratios of mode-shape 1

### Damping ratios for modeshape 2 of the combined system; jack-up + pile-cable subsystem

Base config:  $L_{pile} = 115$  m,  $d_{baseh-grip} = 6$  m,  $L_{crane} = 200$  m,  $D_{pile} = 12$  m

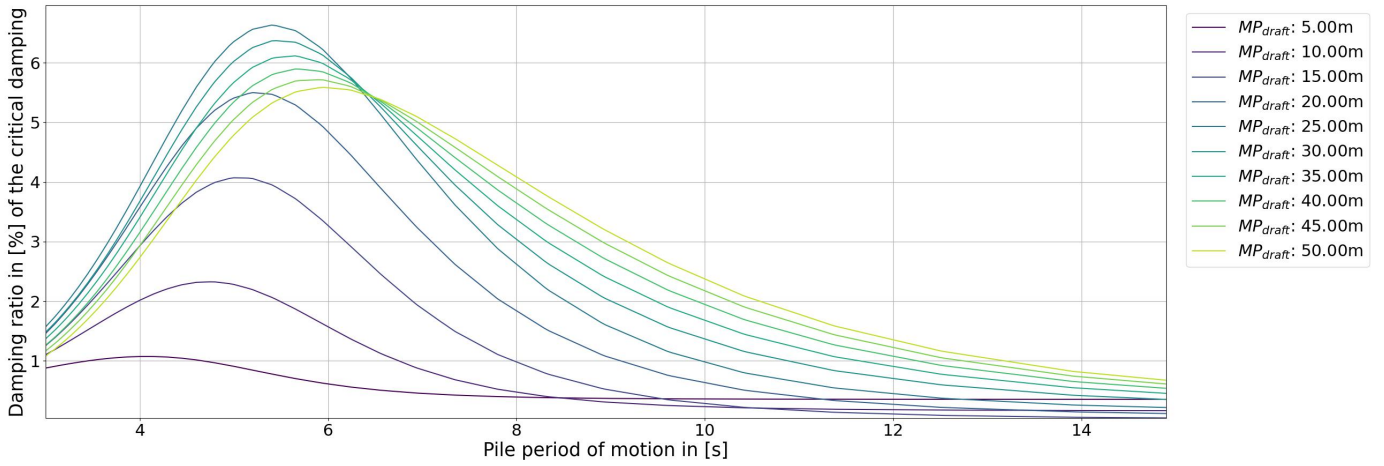


Figure 62: Damping ratios of mode-shape 2

### Damping ratios for modeshape 3 of the combined system; jack-up + pile-cable subsystem

Base config:  $L_{pile} = 115$  m,  $d_{baseh-grip} = 6$  m,  $L_{crane} = 200$  m,  $D_{pile} = 12$  m

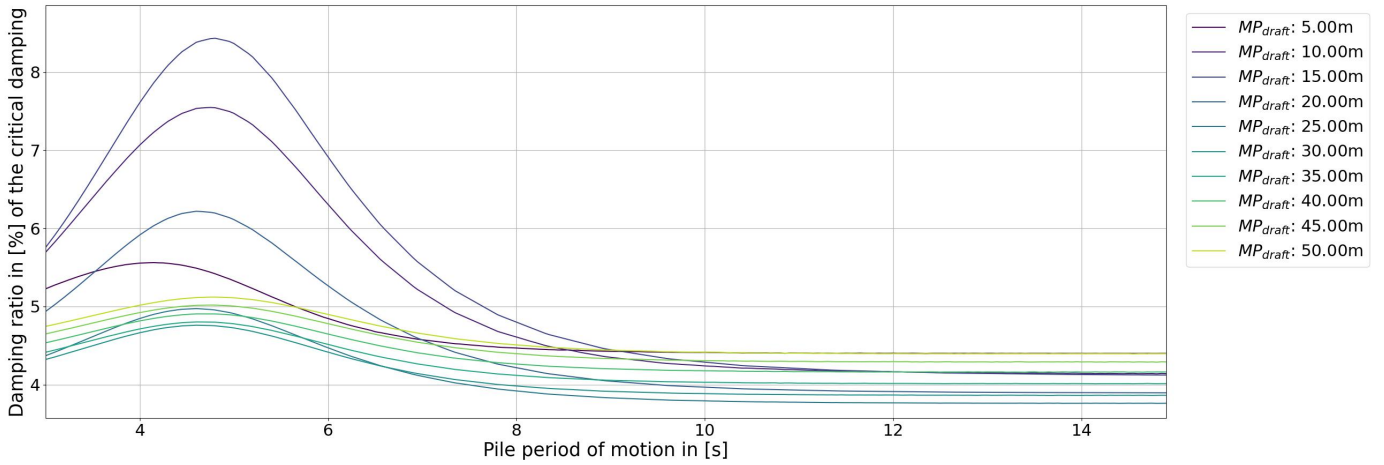


Figure 63: Damping ratios of mode-shape 3

### Damping ratios for modeshape 4 of the combined system; jack-up + pile-cable subsystem

Base config:  $L_{pile} = 115$  m,  $d_{baseh-grip} = 6$  m,  $L_{crane} = 200$  m,  $D_{pile} = 12$  m

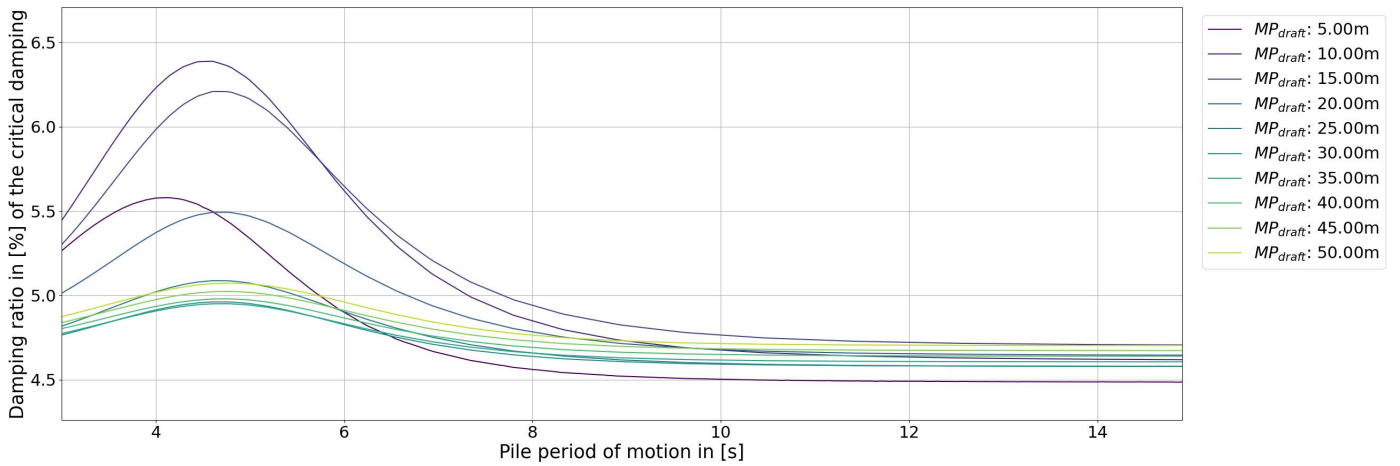


Figure 64: Damping ratios of mode-shape 4

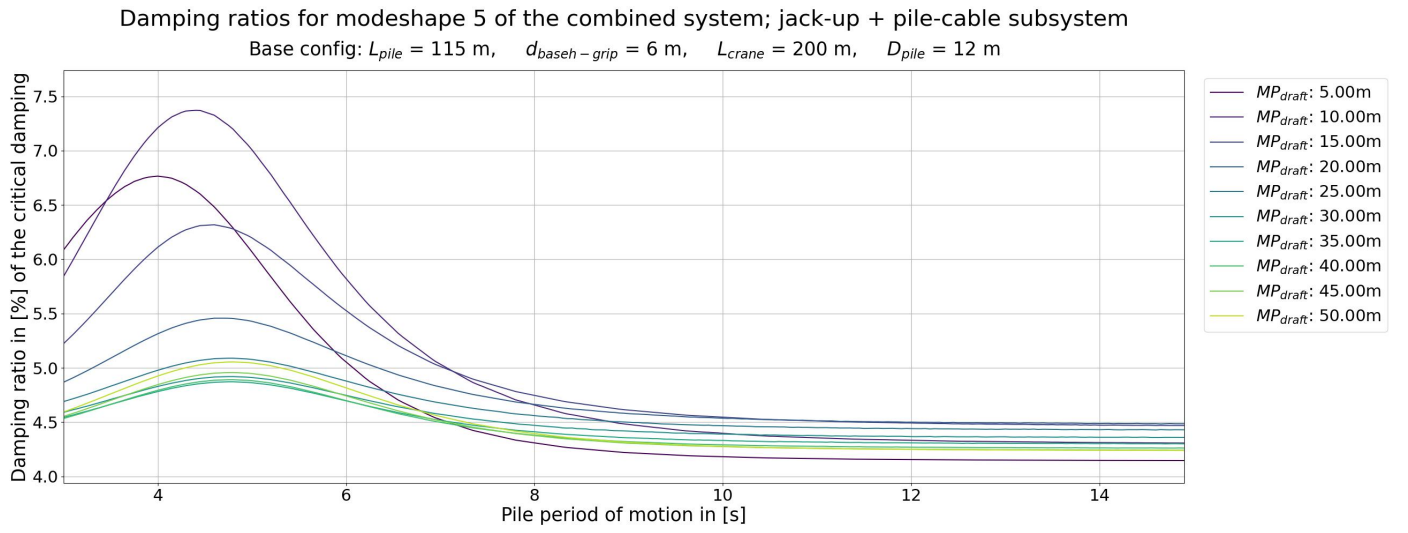


Figure 65: Damping ratios of mode-shape 5

$$\omega_d = \omega_n \sqrt{1 - \zeta^2} \quad (145)$$

## B Validations

In this appendix different validations will be discussed. Validations of the multi-DOF FEM model, the 1-DOF mechanical model and the hydromechanical loads used in the 1-DOF model will be elaborated on.

### B.1 Validation of the multi-DOF FEM model

Since the combined FEM model has a lot of DOFs, it is difficult to get an intuitive grasp of what would happen if an external load was applied. To understand more thoroughly how the jack-up FEM model would respond to a given external loading and to validate the assumption of using a TS beam element, the multi-DOF model will be reduced to a 1-DOF TS beam to see if the response of this beam coincides with the response of the jack-up for a similar load configuration.

A horizontal load of 100MN will be applied to the jack-up at the node in the middle of the cross, as can be seen in figure Figure 66. The jack-up model will be mimicked by creating a 1-DOF model of a leg and assuming that the force that acts in the cross of the jack-up will be equally distributed along the 4 legs. Therefore, this 1d model only has a horizontal load acting on the top of the leg, i.e.  $F_{1dof,leg}$ , see Equation 146 for the force on the leg, w.r.t. the force on the jack-up, i.e.,  $F$ . See Figure 67 for the sketch of the 1d leg model. Note that in this model, there is 1 translational and 1 rotational spring at the base, modeling the response of the soil. At the top of the beam, there are 2 rotational springs in series, where 1 spring models the interface between the leg and the hull of the jack-up, and the second rotational spring models the bending stiffness of the hull. The value for the rotational stiffness resembling the interface between the leg and the hull of the jack-up is known within GustoMSC. The rotational stiffness resembling the bending stiffness of the hull can be evaluated by assuming that the jack-up will deform in the mode as outlined in Figure 68. By assuming this 1-node bending mode, it is known that the internal bending moment is 0 at half the hull length, therefore, a boundary condition can be formulated at half of the hull length, denoted with  $\frac{B}{2}$ , as is done in Figure 68. From (Hibbeler, 2011) the equation for the rotation at an Euler-Bernoullie (EB) beam-element, given an applied moment is known, with which the rotational stiffness can be found, see Figure 68. Where  $E$  and  $I$  are respectively the Youngs modulus and the 2nd area moment of inertia of the jack-up hull. The equivalent rotational spring stiffness at the top of the leg is obtained with Equation 150.

The 2nd area moment of inertia around the x and y- axes, effective shear area, and length of the leg of the 1d-leg model are denoted with, respectively:  $I_{xx,leg}$ ,  $I_{yy,leg}$ ,  $A_{eff}$ , and  $L_{leg}$ . The 4 boundary conditions (BC's) of this single DOF leg are denoted with Equation 153. In the formulation of these BC's,  $w(z)$  is the deflection of the beam,  $V(z)$  is the internal shear stress of the beam,  $M(z)$  is the internal bending moment in the beam, and  $\phi(z)$  is the internal angle due to bending deformation, which for a TS beam can be evaluated with Equation 151, and for an EB-beam can be evaluated with Equation 152 (Ahmed, 2021). The formulations of the bending moment, shear, and internal angle due to bending deformation are evaluated as outlined in (Ahmed, 2021). For a TS beam, the deflection can be evaluated by solving the differential equations outlined with Equation 148, and Equation 149, which is possible now that 4 BC's have been established (Ahmed, 2021). The same can be done for an EB-beam with equation Equation 147.

In Table 20 the results of the 1-DOF EB-beam model, the 1-DOF TS-beam model, and the multi-DOF FEM model of the jack-up are outlined for an equivalent load case. The 1-DOF results of the 1-DOF leg model are found with the equations that have just been elaborated. The results of the multi-DOF model are extracted from the Python model used within GustoMSC.

It can be seen from the results that the 1-DOF TS-beam model shows results that are very close to the results of the multi-DOF FEM model of the jack-up. The results of the 1-DOF EB-beam vary significantly w.r.t. the results found for the multi-DOF FEM model of the jack-up, due to which it can be concluded that the EB-beam model for the leg would not be an appropriate choice as the shear-deformation proves to have a relevant contribution to the deflection of the beam.

The difference between the deflection of the multi-DOF jack-up model and the 1-DOF TS-beam model can be explained by the fact that the equivalent rotational stiffness might be overestimated. Another phenomenon that might contribute to the discrepancy is that in the multi-DOF jack-up model, the  $P - \Delta$  effect is included. When including this phenomenon, the deflection might be higher during a combination of a horizontal and vertical load, as opposed to when not including this phenomenon (Powell, 1937), and (Bordes, 1990). Since the weight of the jack-up is included in the multi-DOF model of GustoMSC, the deflection of the jack-up is expected to be higher in this model as opposed to the deflection of the 1-DOF model, where the  $P - \Delta$  is not included.

$$F_{1dof,leg} = \frac{F}{4} \quad (146)$$

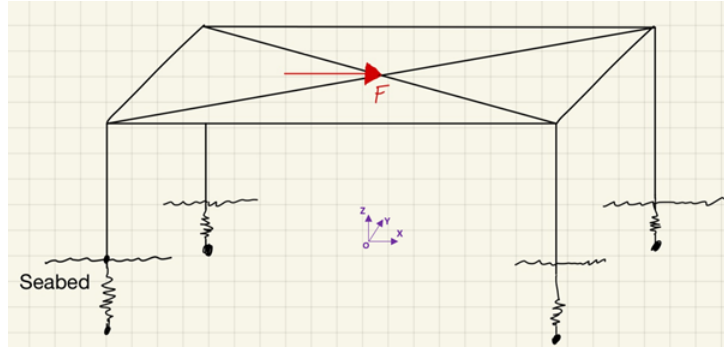


Figure 66: Jack-up with a load applied on the node at the cross

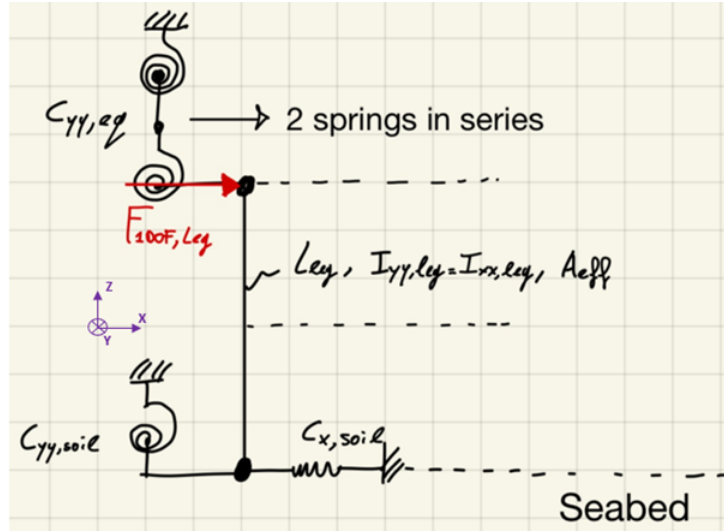


Figure 67: 1D model of the leg of a jack-up

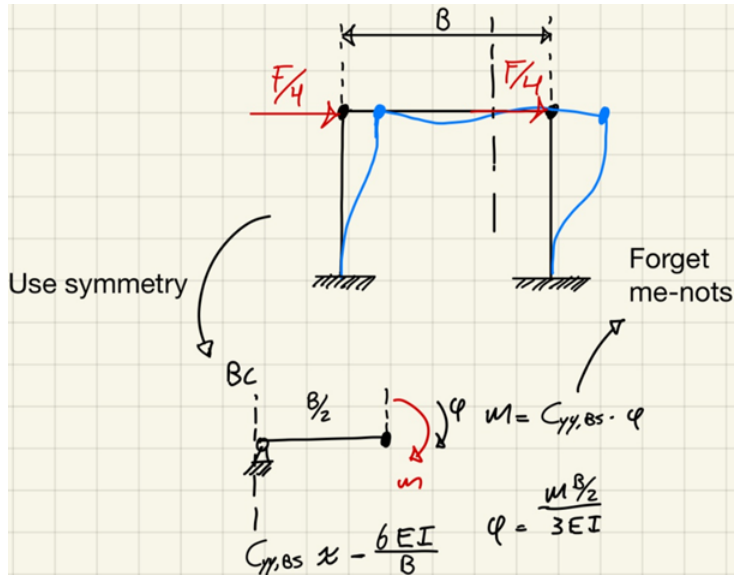


Figure 68: Sketch for obtaining the equivalent bending stiffness of the hull

$$0 = EI \frac{\partial^4 w(z)}{\partial z^4} \quad (147)$$

$$0 = GA_{eff} \left( \frac{\partial^2 w(z)}{\partial z^2} + \frac{\partial \phi(z)}{\partial z} \right) \quad (148)$$

$$0 = EI \frac{\partial^2 \phi(z)}{\partial z^2} - GA_{eff} \left( \frac{\partial w(z)}{\partial z} + \phi(z) \right) \quad (149)$$

$$C_{eq} = \frac{1}{C_{rot,1} + C_{rot,2}} \quad (150)$$

$$\frac{\partial w(x)}{\partial x} = \gamma(x) - \phi(x) \quad (151)$$

$$\frac{\partial w(x)}{\partial x} = -\phi(x) \quad (152)$$

$$\begin{aligned} \text{BC1 at } z = -\frac{L}{2} : C_{x1} w(z = -\frac{L}{2}) &= -V(z = -\frac{L}{2}) \\ \text{BC2 at } z = -\frac{L}{2} : -\phi(z = -\frac{L}{2}) C_{yy1} &= M(z = -\frac{L}{2}) \\ \text{BC3 at } z = \frac{L}{2} : F_{1d,leg} &= V(z = \frac{L}{2}) \\ \text{BC4 at } z = \frac{L}{2} : \phi(z = \frac{L}{2}) C_{yy,eq} &= M(z = \frac{L}{2}) \end{aligned} \quad (153)$$

| Parameter                      | Value | Unit | Description                                           |
|--------------------------------|-------|------|-------------------------------------------------------|
| $F$                            | 100   | MN   | Horizontal force acting on the jack-up                |
| $F_{1dof,leg}$                 | 25    | MN   | Equivalent force acting on the 1-DOF leg model        |
| $w_{1dof,EB-beam}(z = L/2)$    | 1.468 | m    | Beam deflection with the 1-DOF EB-beam model          |
| $w_{1dof,TS-beam}(z = L/2)$    | 1.826 | m    | Beam deflection with the 1-DOF TS-beam model          |
| $w_{multi-dof,model}(z = L/2)$ | 1.917 | m    | Horizontal translation of the multi-DOF jack-up model |

Table 20: Results of the displacement due to an external static load on the jack-up

## B.2 Validation of the 1-DOF mechanical model

In section 4, the assumptions of the 1-DOF pile-cable model are discussed. In this subsection the following assumptions w.r.t. the 1-DOF pile-cable model will be validated, to make sure that they make sense:

1. The moments that arise due to the rotation of the MP are linearly related to the angle  $\theta$ , so the trigonometric functions are linearized.
2. The mass of the steel of the MP is assumed to be uniformly distributed, i.e., the non-uniform distribution of the mass due too a possible tapered end of the MP is not considered.

### B.2.1 Linearization of the trigonometric functions

As discussed in subsubsection A.1.3, the moments that arise at the gripper node due to the rotation of the MP are linearly related to the angle  $\theta$ . In reality, the moments are related to  $\theta$  through trigonometric functions. See Equation 155 for the moments that occur due to a rotation of the MP in the non-linear form, and see Equation 156 for the moments that occur due to a rotation of the MP in the linearized form. With these equations of the moments that arise due to a rotation of the MP, and a given set of initial conditions, the EOM of the undamped 1-DOF pile-cable model without excitation forces can be solved, see Equation 154. As discussed in previous sections, the mass moment of inertia consists of the following 3 components: the mass of the steel of the MP, the added mass outside of the MP, and the added mass due to the entrapped water. The added mass component outside of the MP, used in this EOM is solved with the diffraction model assuming a pile-period of motion of 8 seconds. The solutions of both the linear and non-linear EOM, given an initial displacement of 21.5 deg and an initial velocity of  $0 \frac{\text{deg}}{\text{s}}$  are depicted in Figure 69. It can be seen that the natural period of the non-linear form of the 1-DOF pile-cable model, is slightly higher than the natural period of the linearized form of the 1-DOF pile-cable model. Note that this plot shows the natural period, since the EOM is solved without damping and excitation force, which yields the same results as the modal analysis.



$$I_{m,55,tot}\ddot{\theta} = M_w + M_t \quad (154)$$

$$M_t + M_w = -W \frac{\tan(\theta) l_{ab}^2 \cos(\theta)}{l_c + l_{ab}(1 - \cos(\theta))} - T \cos(\alpha) l_{ab} \sin(\theta) + \frac{W l_{ab}}{l_{mp}} \frac{l_{ab}}{2} \sin(\theta) - \frac{W}{l_{mp}} l_{bel} \frac{l_{bel}}{2} \sin(\theta) \quad (155)$$

$$M_t + M_w \approx -W \frac{l_{ab}^2}{l_c} \theta - W l_{ab} \theta - \frac{W}{l_{mp}} \frac{l_{bel}^2 - l_{ab}^2}{2} \theta \quad (156)$$

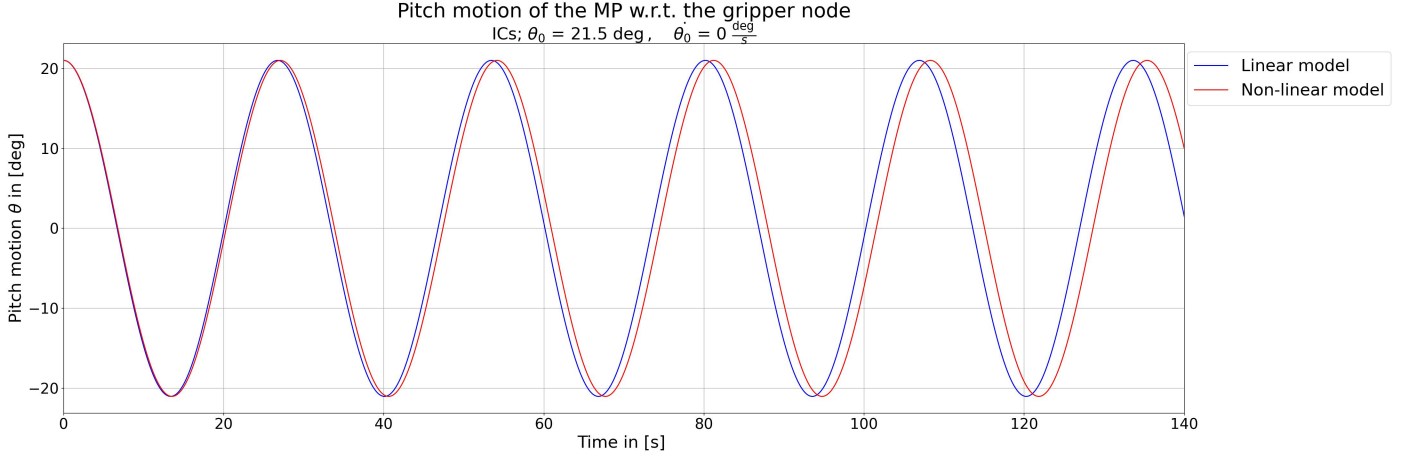


Figure 69: Pitch motion w.r.t. the gripper node for the linear and non-linear EOM

This error between the natural period of the linear and non-linear EOM varies depending on the initial conditions and MP drafts. See Figure 70 for the absolute error, and see Figure 71 for the relative error between the natural periods of the linear and non-linearized form of Equation 154. From these figures, it can be seen that the error increases when increasing the initial displacement. Furthermore, there is also a slight variation in the error, depending on the MP draft. It can be concluded from these figures that until 15 deg of MP displacement, the linearized form of the moments that arise due to a pitch displacement will suffice. At 15 deg of MP motion a maximum of a 2% error w.r.t. the non-linear natural period occurs, which is tolerable (GustoMSC, 2023), see Figure 71.

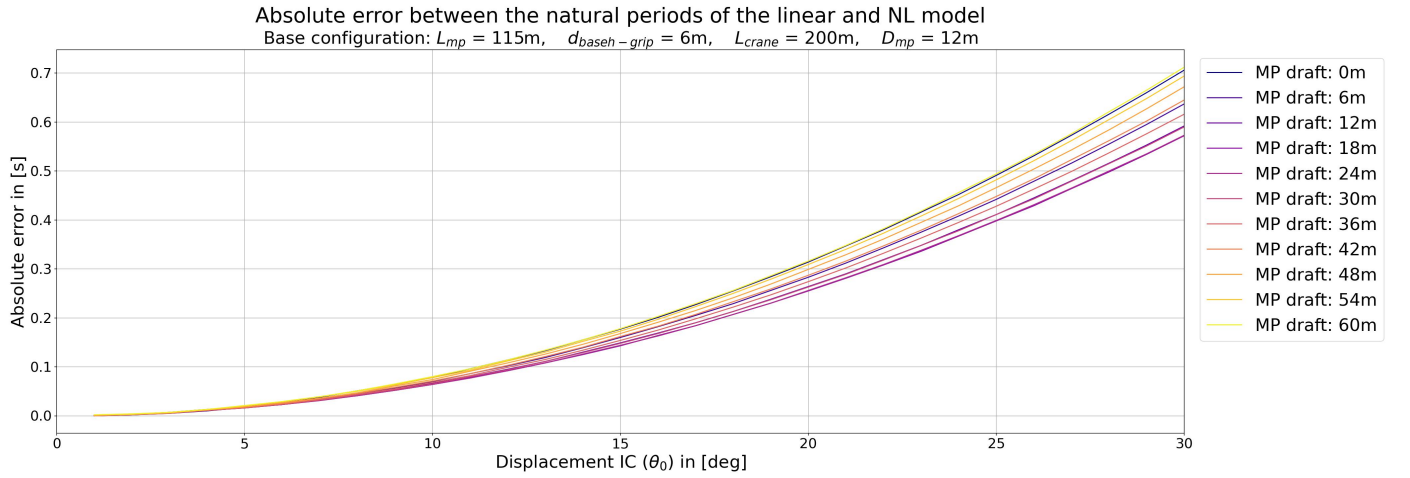


Figure 70: Absolute error of the natural period between the linear and non-linear EOM

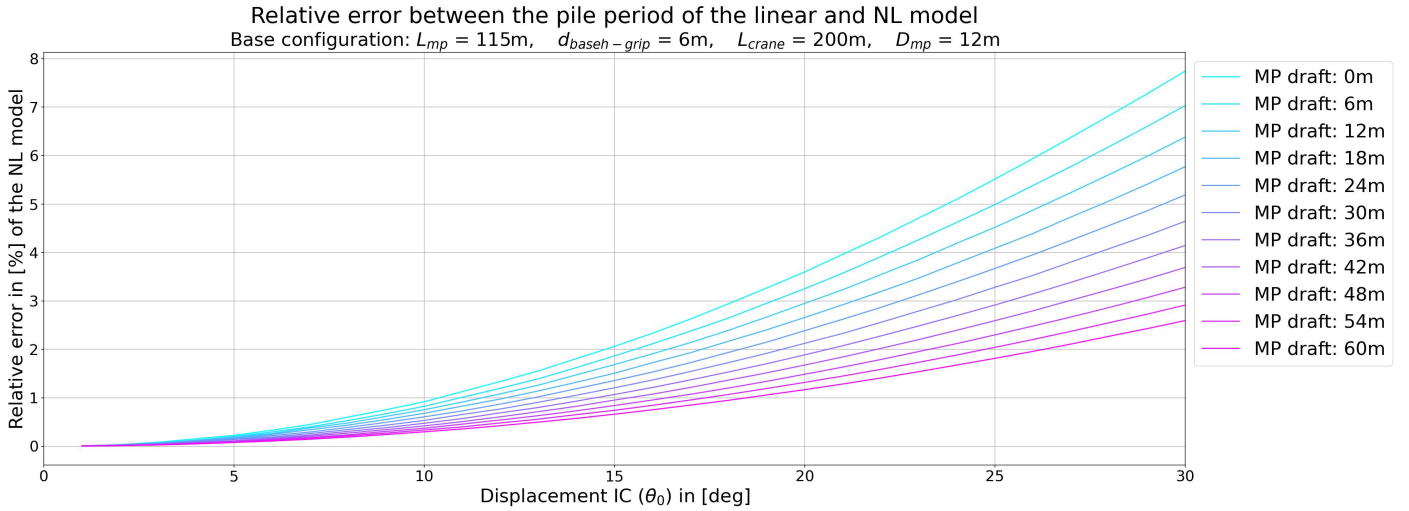


Figure 71: Relative error of the natural period between the linear and non-linear EOM

### B.2.2 Uniform mass distribution

It is assumed within this MSc thesis that the mass distribution of the MP is uniform. This implies that the mass moment of inertia of the MP can be evaluated as if the MP were a cylinder with a constant diameter and thickness, see Figure 60. In reality, the MP is tapered at the top and therefore has a non-uniform mass distribution w.r.t. its COG, see Figure 72. In this subsection, the natural periods of the pile-cable subsystem will be evaluated assuming that the mass is non-uniformly distributed due to the tapered part at the top of the MP.

The contribution to the mass moment of inertia due to the added mass of the entrapped water and the water outside of the MP is evaluated in the same way as was done in subsection 5.2, whilst assuming a pile period of motion of 8s. For the evaluation of the mass moment of inertia of the tapered MP, the MP is divided into 3 sections, see Figure 72. Because of confidentiality reasons, the dimensions depicted in this figure will not be shared. The mass moment of inertia of each of the 3 individual sections is evaluated w.r.t. the C.O.G. of that section, with the equations of the mass moment of inertia used for a cylinder, see Figure 60, and Equation 133. For the tapered part the cylinder is assumed to be straight, and the average diameter between  $D_1$  and  $D_2$  is considered. Hereafter, the mass of all the 3 sections is added up to check that this mass is equal to the uniformly mass-distributed MP. As this was not exactly the case, the density of the steel was scaled in such a way that the mass of the tapered MP was equal to the mass of the uniformly mass-distributed MP. Hereafter, the mass matrix for every individual section w.r.t. the COG of that section was established. All the individual mass matrices were then moved towards the COG, through the utilization of a transformation matrix as was also done and explained in subsubsection 3.2.3. This mass matrix was hereafter added to the mass matrices of the added mass due to the entrapped water and the water outside of the MP w.r.t. the gripper node, after which solely the mass moment of inertia in the pitch-DOF was extracted.

Because the mass of the MP is not assumed to be uniformly distributed anymore, the moment due to weight of the steel of the MP,  $M_w$ , also changes, see Figure 56, and Equation 112. For the uniform distributed mass assumption there is one term in the equation for  $M_w$  that models the mass of the MP above the gripper node. For the tapered part of the MP, there are now 3 different terms that contribute to the mass above the gripper node; one term for the MP part with diameter  $D_2$ , one term for the tapered part, and one term for the MP part with diameter  $D_1$ . Modifying the  $M_w$  in the stiffness term of the 1-DOF pile-cable model and solving for the natural periods of the system, in the same procedure as was done in subsection 5.2, yields Figure 73.

From Figure 73 it can be seen that the natural periods of the 1-DOF pile-cable model, assuming a non-uniform mass distribution, is in general lower as opposed to the natural period assuming a uniform mass distribution. This is concerning, considering the assumed periods of excitation, as discussed in subsubsection 3.2.1, since the natural period of the system is now closer to the assumed periods of excitation. There is however one other thing to consider which is not modeled in this MSc thesis, this is the lifting tool. The lifting tool, which lifts the MP into the gripper and lowers it through the splash zone has two effects: it decreases the stiffness of the pile-cable model because of the interface between the MP and the lifting tool, and it increases the mass moment of inertia due to the mass of the lifting. Both of these effects increase the natural periods of the 1-DOF pile-cable model, and it is assumed that these effects will cancel out the effects of the non-uniform mass distribution of the MP. Because of this, the uniform mass distribution assumption is reasonably valid and will be used throughout this MSc thesis. However, the above effect should be kept in mind.

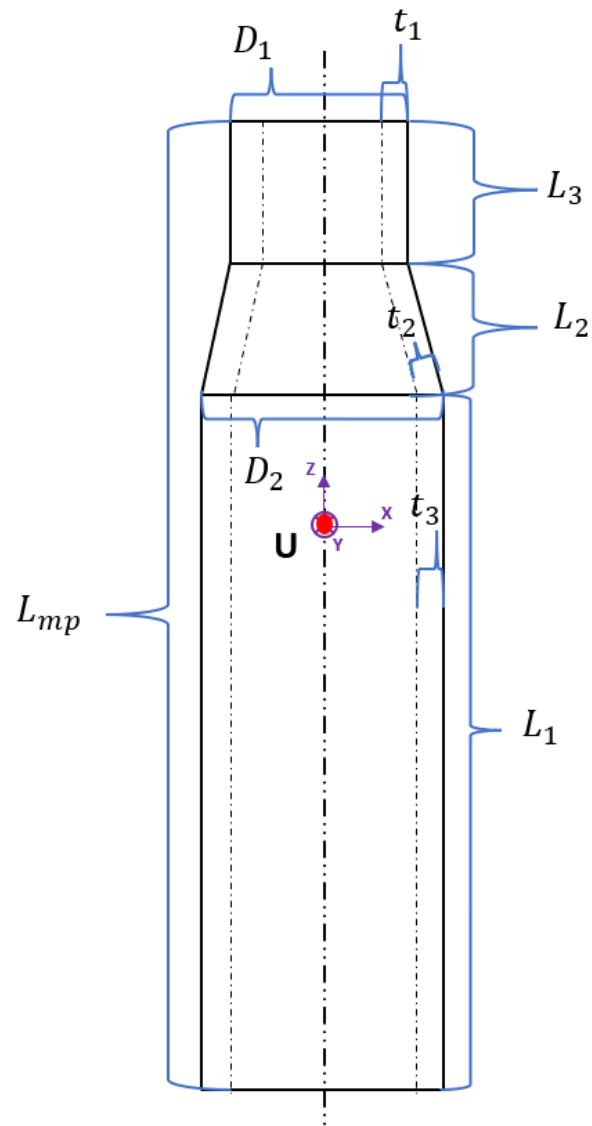


Figure 72: Sketch of the tapered MP

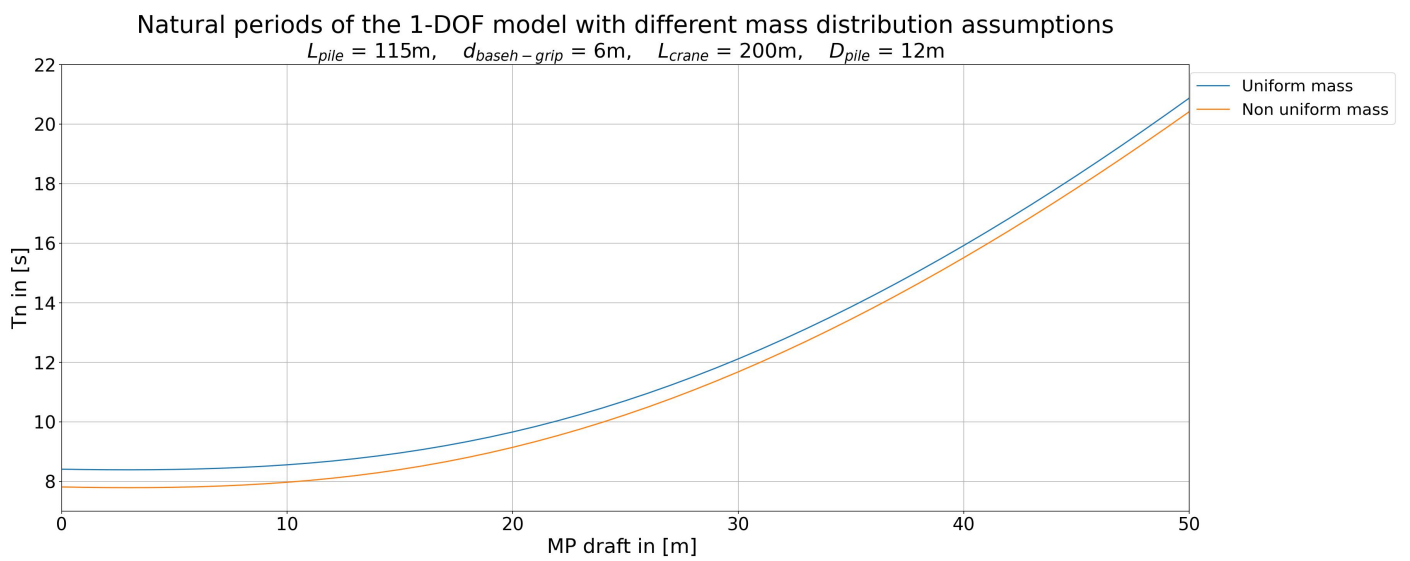


Figure 73: Natural period of the 1-DOF model with a uniform and non-uniform mass distribution

### B.3 Effects of different jack-up layouts on the natural periods

In section 4 it was discussed that the red parameters in Figure 4 would be fixed at the values defined in Table 3. It would be convenient, to know when designing for example a jack-up, what the impact would be of some of these fixed parameters on the natural periods of the pile-cable model. For that reason, the effect of the MP diameter, MP length, crane height, and gripper height on the natural periods of the 1-DOF pile cable will be shown in this subsection. For the mentioned variables, the minimum, mid, and maximum values will be plotted while maintaining the other parameters at their base values, see Table 21. These values are established in consultation with GustoMSC, such that the most convenient variation of the mentioned variables was established. The added mass on the outside of the MP, which contributes to the mass moment of inertia of the pile-cable model is evaluated with diffraction, assuming a pile-period of motion of 8s.

In Figure 74 the natural periods of the 1-DOF pile cable model are depicted for different MP diameters. It can be seen that for lower MP drafts the natural periods remain unchanged. For increasing MP drafts the smaller MP diameters generally result in a lower natural period. This effect is however not that important since for larger MP drafts the natural period of the system moves away from the excitation periods. This means that a variation in natural periods of the system at these drafts will not result in a significant change in dynamic motion response. Hence it can be concluded that the diameter of the MP does not have a big influence on the response of the system.

In Figure 75 the natural periods of the 1-DOF pile cable model are depicted for different MP lengths. It can be seen that the 3 different plots start at the same natural period, but that the natural period for the largest MP length will increase the slowest. This means that, in general, resonance has more time to build up for larger MP lengths since the associated natural periods will remain for a longer time close to the expected wave-excitation periods. In addition to this, the natural period for MP drafts bigger than 10m will also be lower for a larger MP length.

In Figure 76 the natural periods of the 1-DOF pile cable model are depicted for different crane heights. It can be seen from this figure that the natural periods for larger crane heights are in general lower as opposed to smaller crane heights. The slope of the 3 curves are roughly the same.

In Figure 77 the natural periods of the 1-DOF pile cable model are depicted for different gripper heights. This is a plot with a large variety in natural periods. It can be seen that higher gripper positions start at the lowest natural period, but also increase the fastest. This means that for lower gripper heights there is less time for resonance to build up, as the natural periods move away quicker from the excitation periods. For smaller gripper heights the natural periods of the system start at the lowest value, however the slope of the graph is also the lowest. This means that resonance has a lot of time to build up at the gripper heights since the natural periods remain for a longer time closer to the excitation periods. Because of this, the dynamic motion response for a jack-up vessel with a gripper positioned at a low height is expected to be higher as opposed to a configuration with a high gripper height.

| Parameter           | Min value | mid value | Max value | Base value | Unit | Description    |
|---------------------|-----------|-----------|-----------|------------|------|----------------|
| $D_{pile}$          | 8.5       | 10        | 12        | 12         | m    | MP diameter    |
| $L_{pile}$          | 95        | 100       | 115       | 115        | m    | MP length      |
| $L_{crane}$         | 180       | 200       | 220       | 200        | m    | Crane height   |
| $d_{baseh-gripper}$ | 0         | 6         | 18        | 18         | m    | Gripper height |

Table 21: Jack-up configuration: min/mid/max/base values

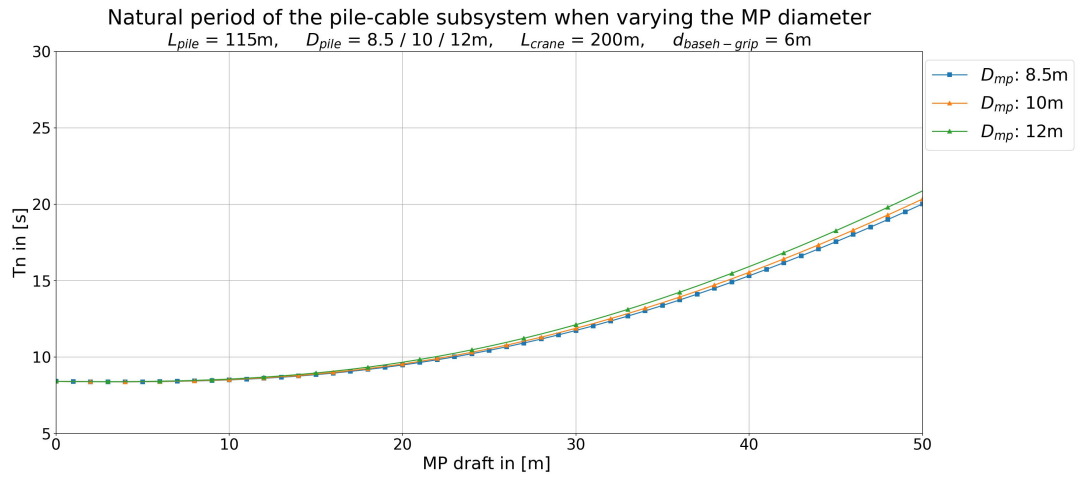


Figure 74: Natural periods when varying the MP diameter

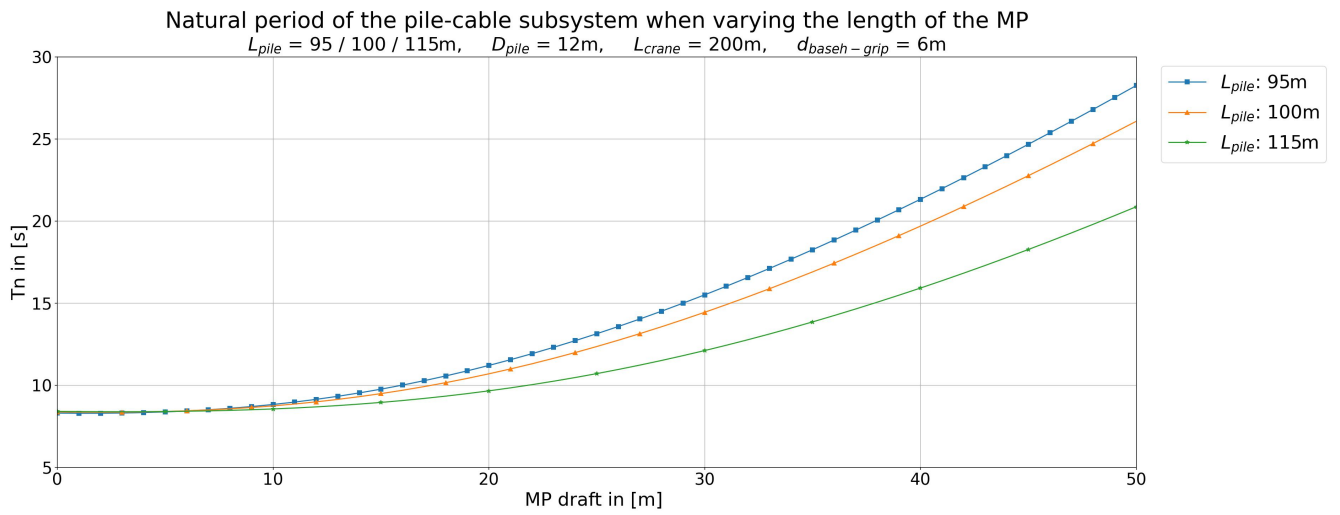


Figure 75: Natural periods when varying the MP length

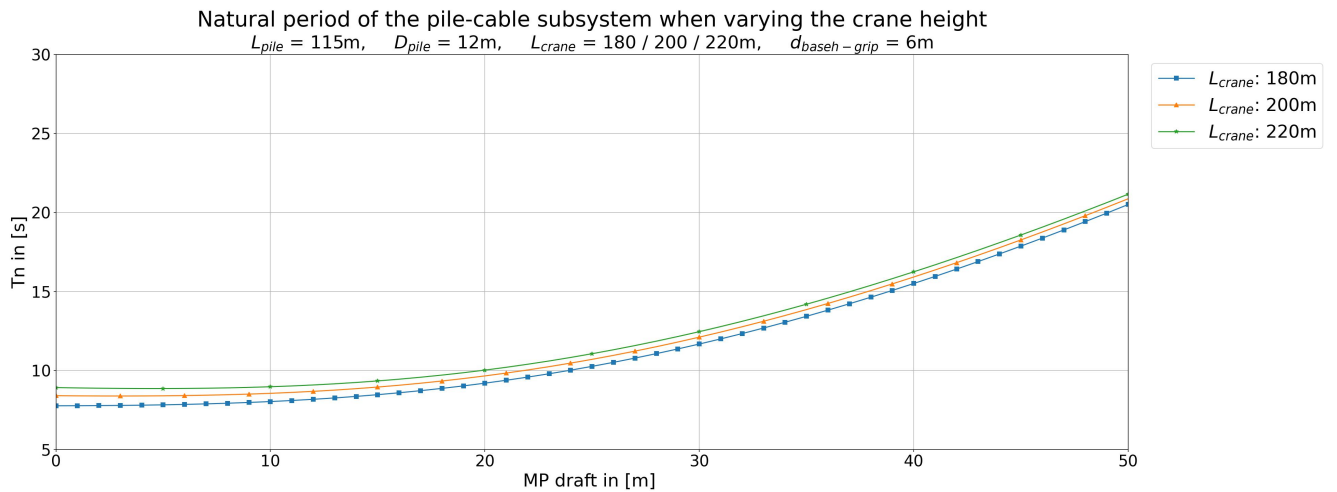


Figure 76: Natural periods when varying the crane height

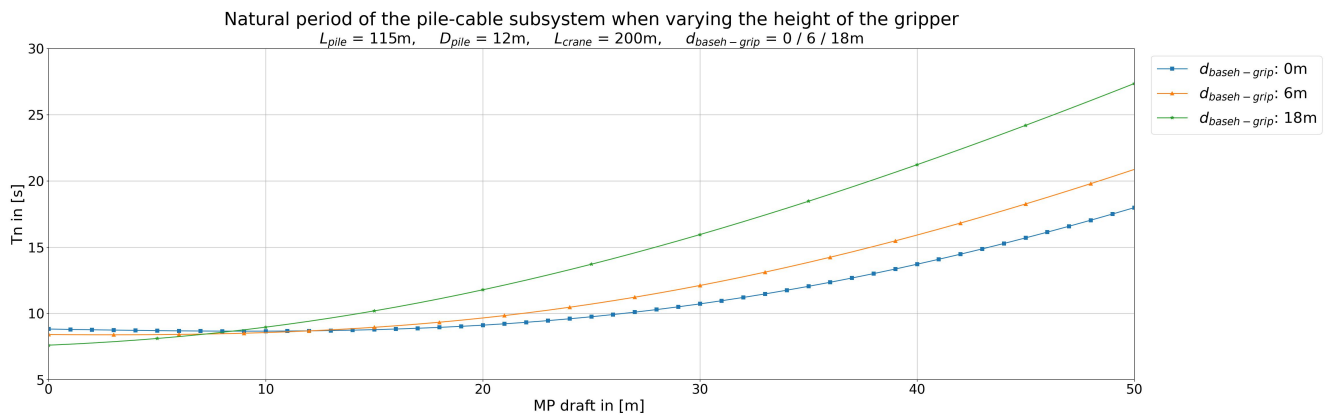


Figure 77: Natural periods when varying the gripper height

## B.4 Validation of the hydromechanical loads on the MP

### B.4.1 Wave forces due to a regular wave

The force on the MP in the pitch DOF of the 1-DOF model for a regular wave with of  $2.5\text{m}$  high and a period of 9 seconds at a MP draft of  $20\text{m}$ , in the time domain is depicted in Figure 79. Comparing this with the force in the pitch DOF on the MP of the multi-DOF model under similar conditions, see Figure 78, shows to be near perfect coincident, note that the maximum forces of both plots in the legend and the subtitle are almost equal. The minor observed differences between the two models have to do with a different interpolation technique used within both Python models. Note that the wave-excitation forces on the MP of both models are slowly build up in order to avoid the transient effect. This build up of the load can be observed in Figure 78 since the full range of the time-simulation is shown. This slow build up of the load is also done in the 1-DOF model, however this is not observable since only a small range after this build up of the load is depicted in Figure 79.

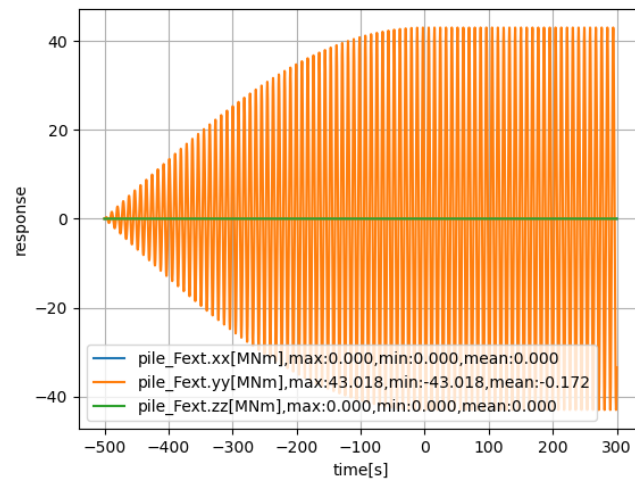


Figure 78: Force response in the pitch DOF of the multi-DOF model, steady-state simulation,  $H=2.5\text{m}$ ,  $T=9\text{s}$ , MP draft= $20\text{m}$

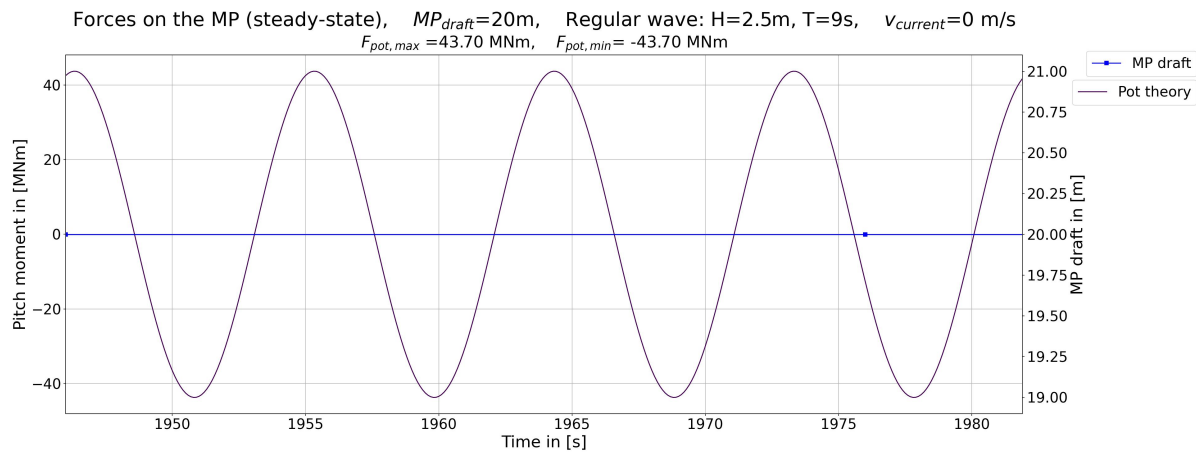


Figure 79: Force response in the pitch DOF of the 1-DOF model, steady-state simulation,  $H=2.5\text{m}$ ,  $T=9\text{s}$ , MP draft= $20\text{m}$

### B.4.2 Viscous dragforces

The drag force in the pitch DOF of the 1-DOF model, without wave-excitation forces, and hence solely due to a current of  $1.5 \frac{m}{s}$  at a MP draft of 15m is depicted in Figure 80. A hand calculation is made to check if this value is properly evaluated in the Python model of the 1-DOF pile-cable model. In Figure 81 the current profile along the MP is depicted, where  $L_{subm} = 15m$ . In the hand calculation the dragforce per meter length with the 2nd term of the separate flow form of Morison is utilized, see Equation 61. In this equation, the velocity along the MP draft is obtained with Equation 52. The resulting dragforces per meter length at various MP drafts are then multiplied by their distance w.r.t. the gripper node, and hereafter integrated over the submerged length of the MP such that the pitch moment w.r.t. the gripper node due to the current is obtained. The result of the hand calculation is a total dragforce w.r.t. the gripper node U of  $3.07 MNm$ , which shows that the Python model is properly evaluating the dragforce due to the current.

Dragforce due to current,  $MP_{draft} = 15m$ , Regular wave:  $H=2.5m$ ,  $T=9s$ ,  $v_{current} = 1.5 \frac{m}{s}$   
 $F5_{drag-current,max} = 2.98MNm$

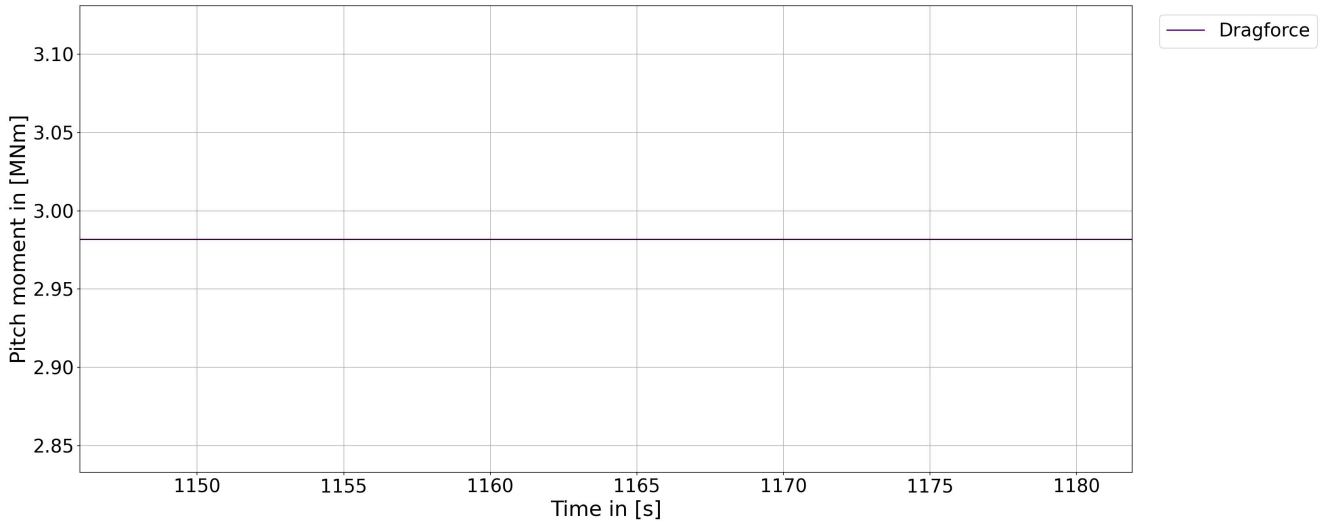


Figure 80: Dragforce on the MP in the pitch DOF of the 1-DOF model, solely due to current

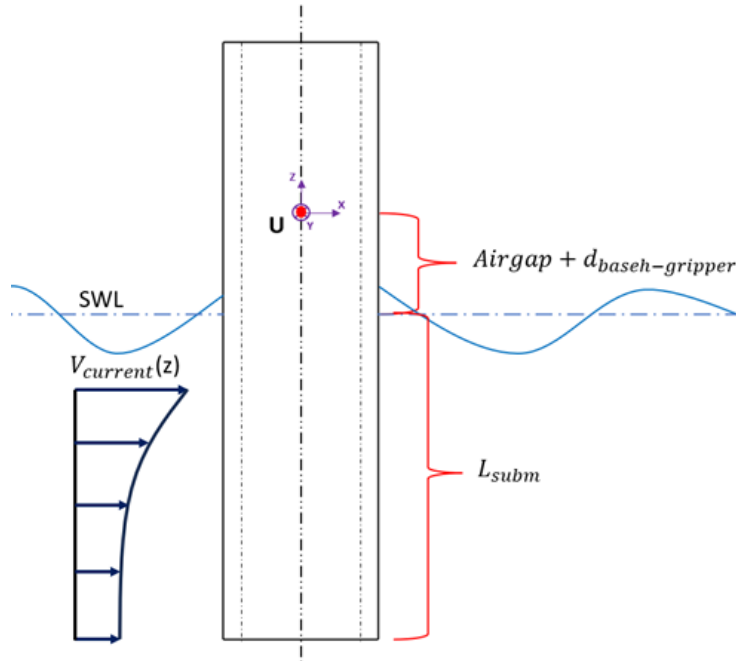


Figure 81: Dragforce on the MP due to current sketch

Earlier it was mentioned that the viscous drag due to the MP motion through the water is evaluated with the



2th term in the separate flow form of the Morison equation, see Equation 61. It is stated in (Moe & Verley, 1980) that this drag force would be a more conservative estimate as opposed to the 2th term in the relative flow form of the Morison equation, see Equation 60. To demonstrate the difference between the 2 equations, the motion response with a regular wave of 2.5m high, a wave period of 9 seconds, and a current of  $1.5 \frac{m}{s}$  at a MP draft of 15m is evaluated with one model that considers the relative flow form of the Morison equation, and one model that considers the separate flow form of the Morison equation for the viscous drag force, see Equations 60, and 61. Keep in mind that at a MP draft of 15m, during a regular wave of 9 seconds, the system is excited very close to the natural period, see Figure 29. Under these circumstances, the response is expected to be relatively high, which implies that the viscous drag due to the motion of the MP will also be high. Therefore the difference between the drag force with the separate and relative flow form can more easily be distinguished.

See Figure 82 for the motion response of the MP in the pitch DOF w.r.t. the gripper node with the mentioned models. It can be seen that the motion response with potential theory and the viscous drag utilizing the separate flow form is higher as opposed to modeling the viscous drag with the relative flow form. It can therefore be concluded that this separate flow form of Morison is indeed a more conservative option as the relative flow form when evaluating the viscous drag due to the MP motion. Hence, the viscous drag due to the MP motion will be estimated with the 2nd term from the separate flow form of the Morison equation.

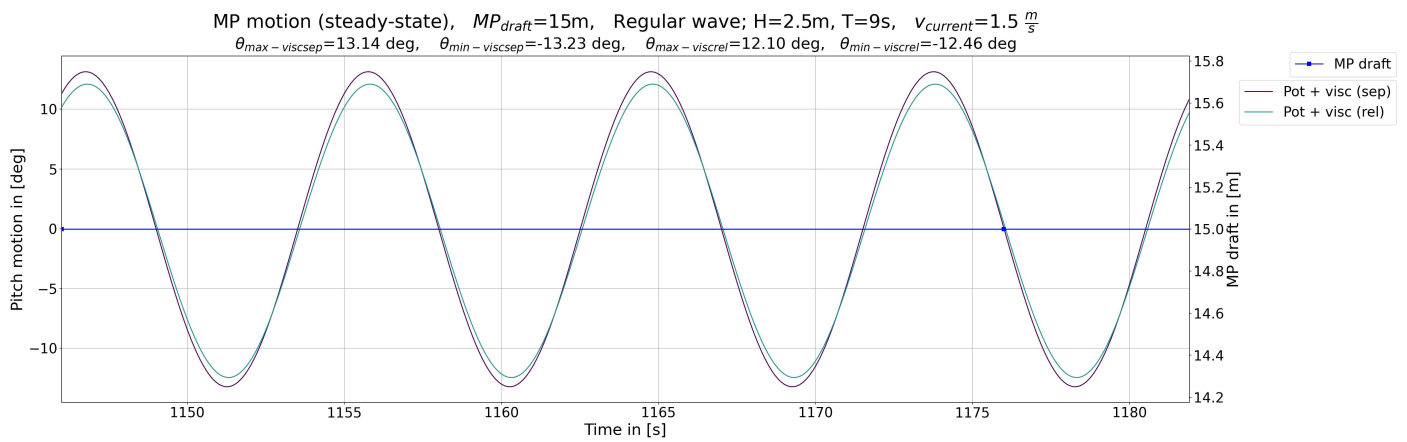


Figure 82: Motion response steady-state simulation, 1-DOF model:  $H=2.5m$ ,  $T=9s$ , MP draft=20m,  $V_c = 1.5 \frac{m}{s}$ , damping due to the viscous drag with the separate flow form and the relative flow form of the Morison eq

In Figure 83, the drag force in the pitch DOF whilst utilizing Equation 61, considering a current of  $1.5 \frac{m}{s}$  and the MP motion through the water, due to the wave-excitation forces obtained from WAMIT for a regular wave of 2.5m high and a period of 9 seconds at a MP draft of 15m, is plotted for a steady-state simulation. The associated velocity of the MP during the same time simulation is depicted in Figure 84. To check if the dragforces of Figure 83 are evaluated correctly in the Python model, a hand calculation is performed. At different drafts along the MP, the tangential velocity at that draft due to the mentioned velocity components are evaluated. See Figure 81 for the current profile along the MP draft, and see Figure 85 for the velocity profile along the MP draft due to the pitch velocity of the MP around the gripper node U. Both of these velocity components are then used to evaluate the dragforce per meter length with the 2nd term in the separate flow form of the Morison equation along the MP draft, see Equation 61. These dragforces per meter length are hereafter multiplied by the distance w.r.t. the gripper node, after which the obtained values are integrated over the MP draft such that the total pitch moment due to the viscous dragforces are found. The result for a regular wave of 2.5m high, and a period of 9 seconds, with a current of  $1.5 \frac{m}{s}$ , at a MP draft of 15m is a total maximum dragforce of  $-25.03 MNm$ . Because this is coincident with the Python model results, see Figure 83, it is assumed that the dragforce due to the MP velocity and the current is properly evaluated.

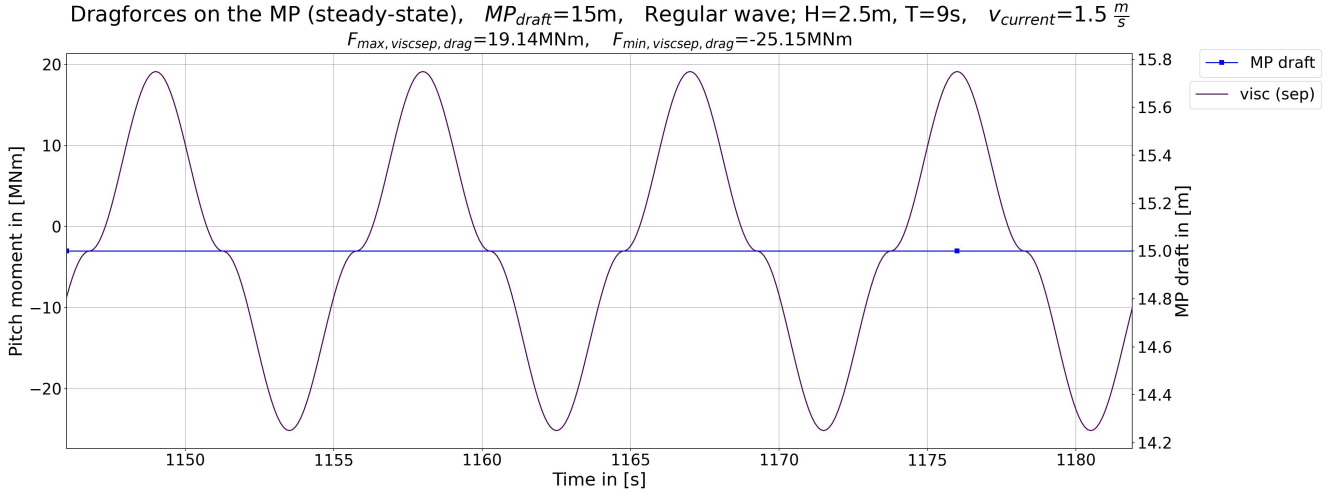


Figure 83: Dragforce in the pitch DOF of the 1-DOF model during a steady-state simulation,  $H=2.5m$ ,  $T=9s$ ,  $MP_{draft}=20m$ ,  $V_c = 1.5 m/s$

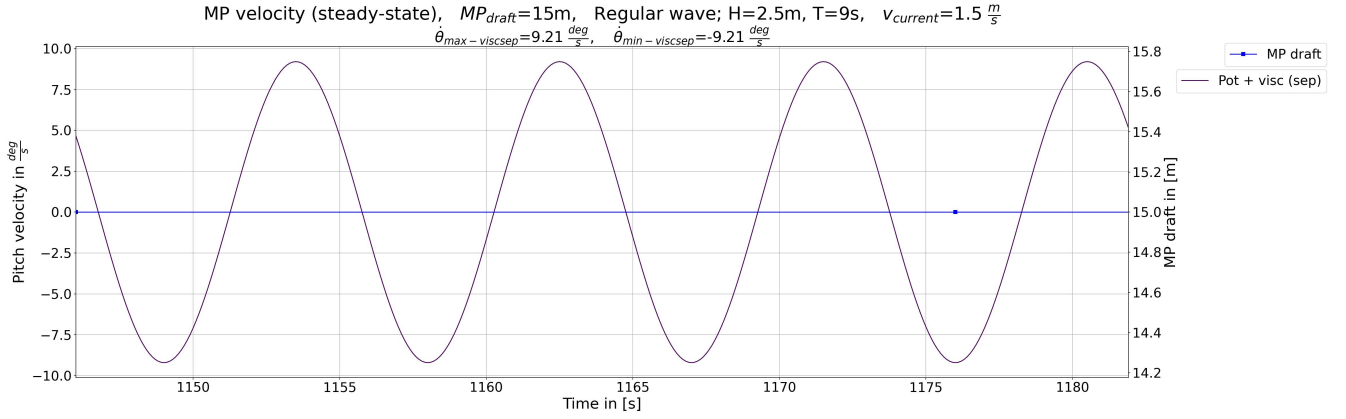


Figure 84: Velocity response in the pitch DOF of the 1-DOF model during a steady-state simulation,  $H=2.5m$ ,  $T=9s$ ,  $MP_{draft}=20m$ ,  $V_c = 1.5 m/s$

### B.4.3 Wave-excitation forces due to irregular waves

In this subsection the wave-excitation forces due to irregular waves on the MP in the 1-DOF model are evaluated and compared with the same forces in the multi-DOF model, to make sure that the wave forces due to irregular waves are evaluated correctly. The wave excitation forces at a fixed MP draft during a steady-state simulation is evaluated where the current is assumed to be  $0 \frac{m}{s}$ , the significant waveheight is set at 2.5m, the peak period at 7 seconds, and the MP draft is fixed at 20m. In Figure 86 the wave excitation forces in the pitch DOF on the MP due to irregular unidirectional waves are evaluated with the 1-DOF model. The same is done with the multi-DOF model, see Figure 87. It can be seen from both figures that the standard deviation of the wave-excitation forces is almost identical,  $\sigma_{F5,pot} = 22.78MNm$  in the 1-DOF model, and  $\sigma_{F5,pot} = 22.431MNm$  in the multi-DOF model. This indicates that the irregular unidirectional wave-forces in the 1-DOF model for a steady-state simulation are most likely evaluated correctly. Note that the multi-DOF model does not consider viscous drag forces, hence the standard deviation of the 1-DOF model without the damping due to viscous drag is considered, i.e. the 'Pot' model, see the legend of Figure 86.

The exact same under equal conditions is done for irregular waves with spreading, where the amount of spreading angles is set at 11 for both the 1-DOF model and the multi-DOF model. See Figure 88, and Figure 89 for the results. From the figures it can be observed that the standard deviation of the wave-excitation forces in the pitch DOF is almost equal,  $\sigma_{F5,pot} = 19.69MNm$  in the 1-DOF model, and  $\sigma_{F5,pot} = 19.744MNm$  in the multi-DOF model.

The slight discrepancies from both results likely occur due to the stochastic nature of the waves due to which small differences in the standard deviation occur depending on the wave-seed.

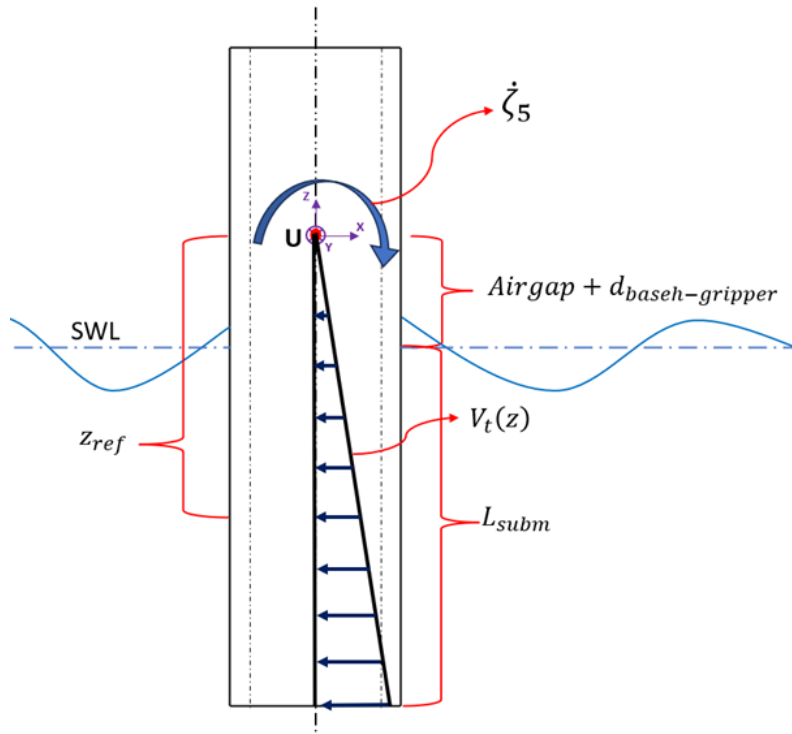


Figure 85: Dragforce due to MP velocity through the water

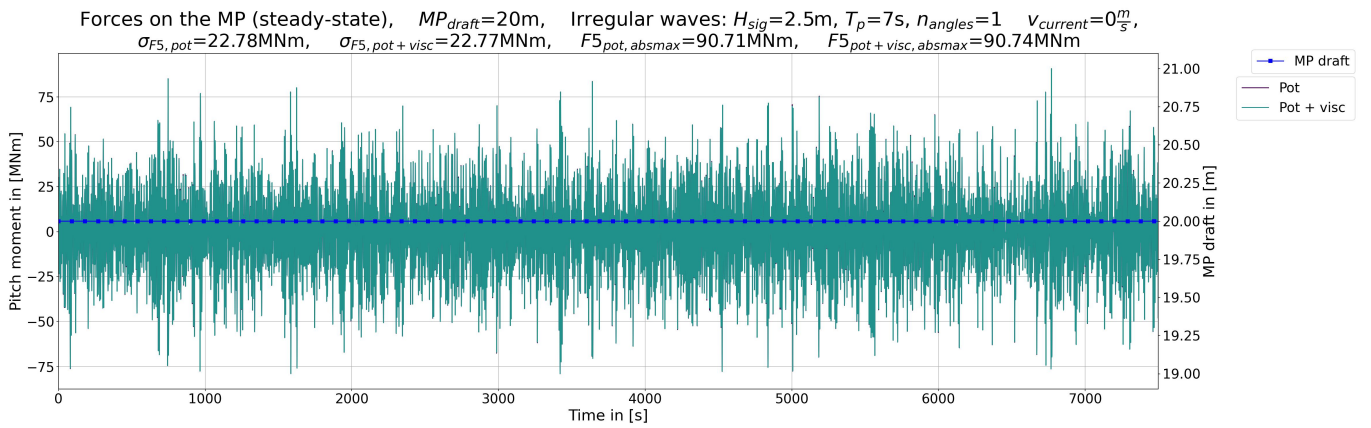


Figure 86: Forces on the MP in the pitch DOF of the 1-DOF model, during a steady-state simulation at a MP draft of 20m, with irregular waves (no spreading):  $H_{sig}=2.5m$ ,  $T_p=7s$ ,  $V_c = 0$  m/s

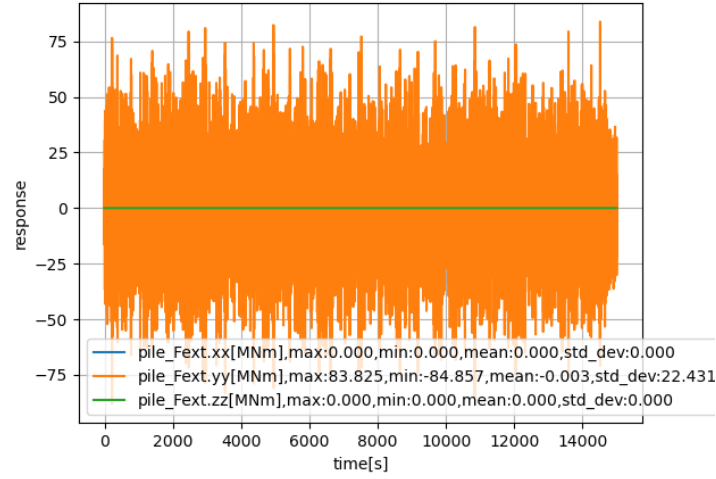


Figure 87: Forces on the MP in the pitch DOF of the multi-DOF model, during a steady-state simulation at a MP draft of 20m, with irregular waves (no spreading):  $H_{sig}=2.5\text{m}$ ,  $T_p=7\text{s}$ ,  $V_c = 0 \text{ m/s}$

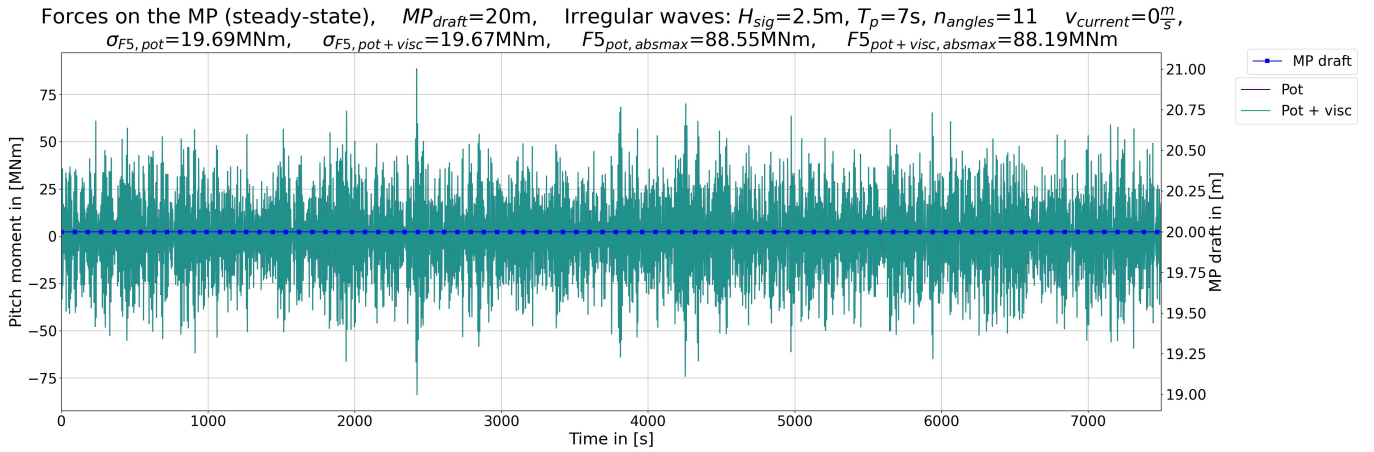


Figure 88: Forces on the MP in the pitch DOF of the 1-DOF model, during a steady-state simulation at a MP draft of 20m, with irregular waves (with spreading):  $H_{sig}=2.5\text{m}$ ,  $T_p=9\text{s}$ ,  $V_c = 0 \text{ m/s}$

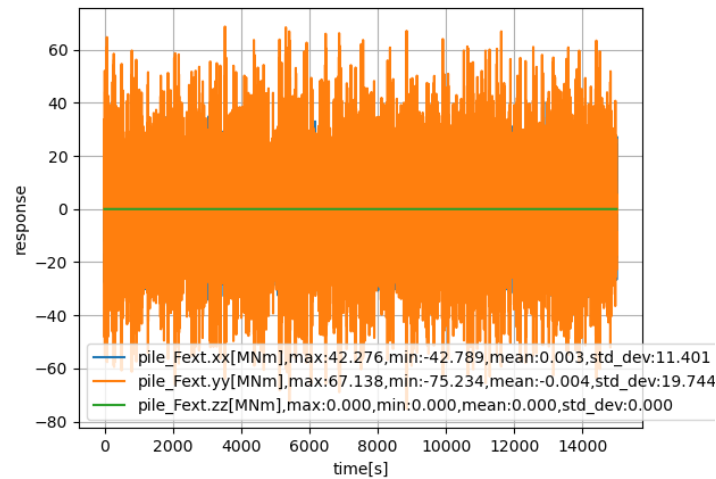


Figure 89: Forces on the MP in the pitch DOF of the multi-DOF model, during a steady-state simulation at a MP draft of 20m, with irregular waves (with spreading):  $H_{sig}=2.5\text{m}$ ,  $T_p=9\text{s}$ ,  $V_c = 0 \text{ m/s}$

## C Supplementary results and plots

The most important results are discussed and summarized in section 6, in this appendix the plots are shown from which these summarized results are obtained.

### C.1 steady-state simulations at critical MP drafts

In this subsection the results of the motion in the steady-state simulations at the critical drafts defined in subsection 6.3 are shown, see Figures 90, 91, 92. Note that the standard deviation, the maximum pitch motion for the considered wave-seed, the zero up crossing period, and the MPM are depicted in the subtitle of the figures.

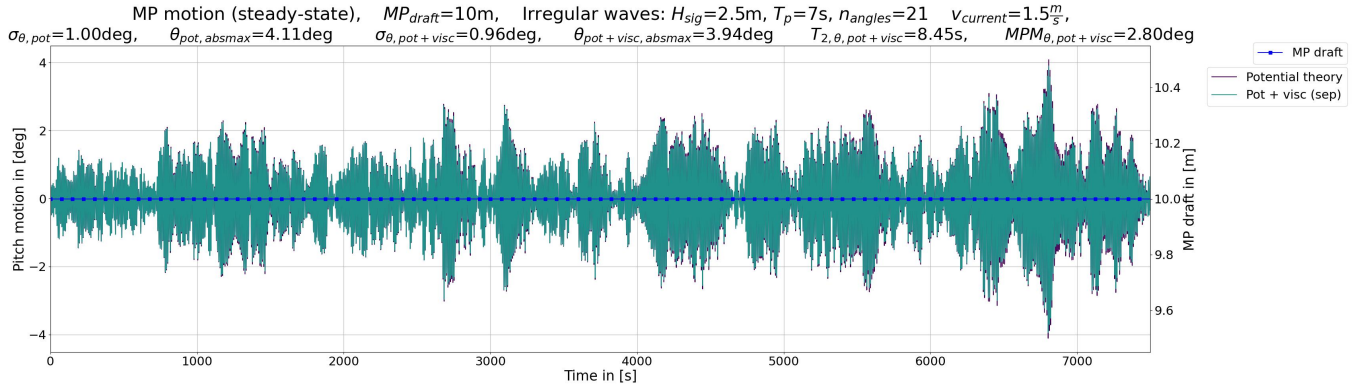


Figure 90: MP motion response during a steady-state simulation with irregular spread waves,  $MP_{draft} = 10m$ ,  $H_{sig}=2.5m$ ,  $T_p=7s$ ,  $V_c = 1.5\frac{m}{s}$

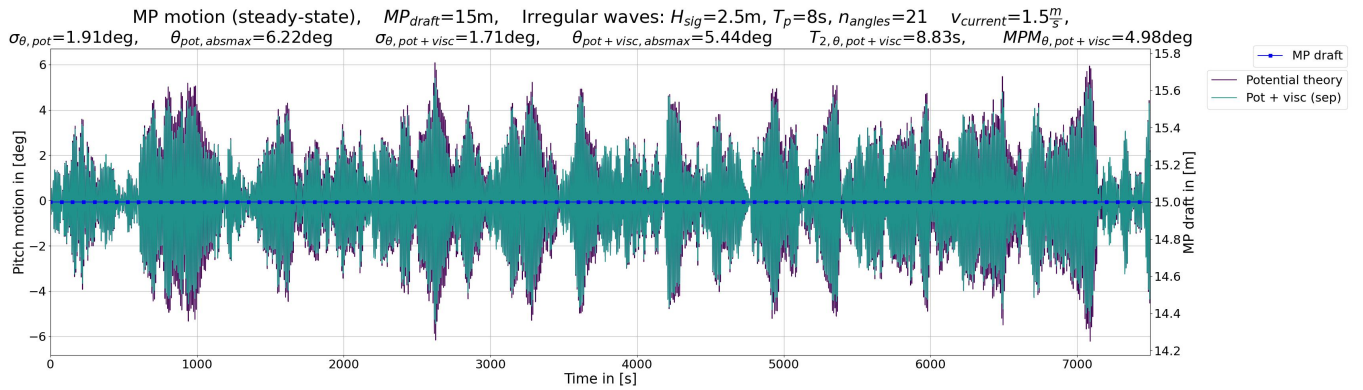


Figure 91: MP motion response during a steady-state simulation with irregular spread waves,  $MP_{draft} = 15m$ ,  $H_{sig}=2.5m$ ,  $T_p=8s$ ,  $V_c = 1.5\frac{m}{s}$

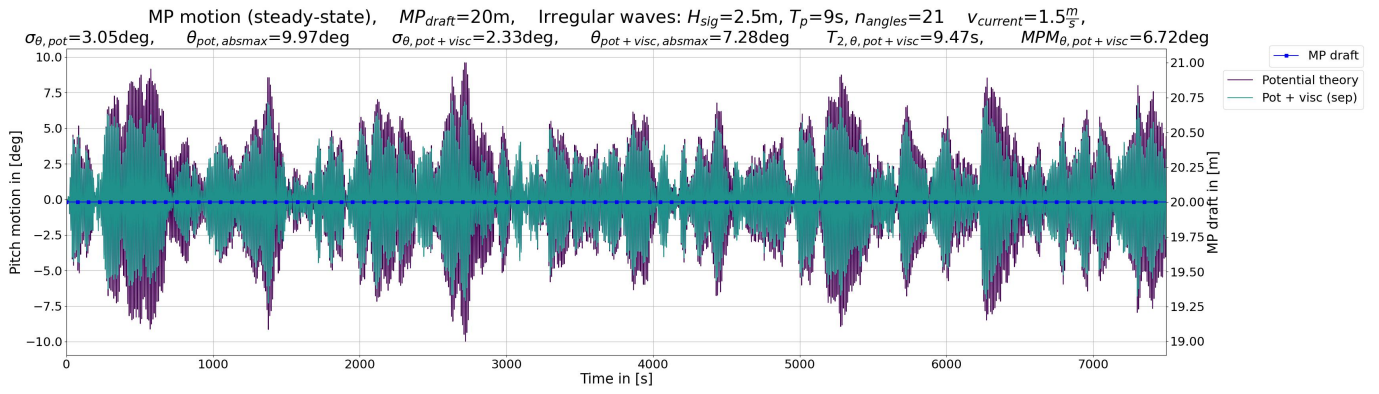


Figure 92: MP motion response during a steady-state simulation with irregular spread waves,  $MP_{draft} = 20m$ ,  $H_{sig}=2.5m$ ,  $T_p=9s$ ,  $V_c = 1.5\frac{m}{s}$

## C.2 MP motion response, dynamic lowering simulation

In Figures 93, 94, 95, the MP motion response for the dynamic lowering simulation under irregular spread waves for respectively peak periods of 7, 8 and 9 seconds all during a significant wave height of 2.5m are shown. In all 3 figures the left y-axis indicates the pitch motion of the MP w.r.t. the gripper in [deg], the right side of the y-axis indicates the MP draft in [m], and the x-axis depicts the time in [s]. The purple graph is the MP motion evaluated with the model that does not consider the viscous drag, and the cyan graph is the MP motion evaluated with the model which does consider the viscous drag. The maximum absolute motion response of each graph considering both models is shown in the subtitle of each plot. Note that because the viscous drag does not dampen the motion significantly enough in Figures 93, and 94, it looks like as if only the model with viscous drag is plotted. The motion response of both models is plotted in these figures but since they are almost exactly similar the model that does not include damping due to viscous drag is not observable. The difference between the two models can be observed in Figure 95, because the damping due to viscous drag is more dominant in this plot.

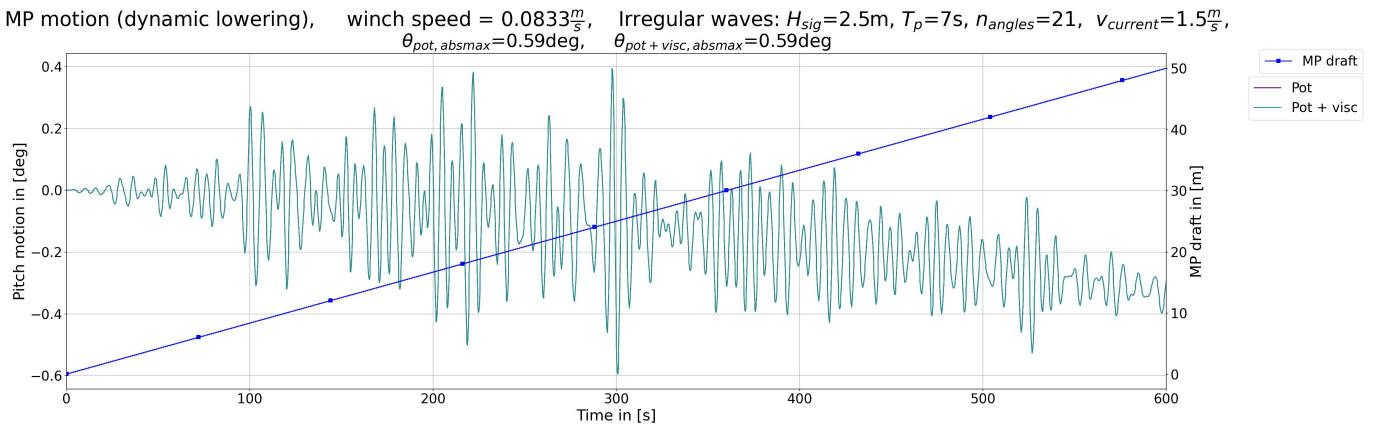


Figure 93: MP motion of the 1-DOF model, during the dynamic lowering simulation, irregular waves (with spreading):  $H_{sig}=2.5m$ ,  $T_p=7s$ ,  $V_c = 1.5 m/s$



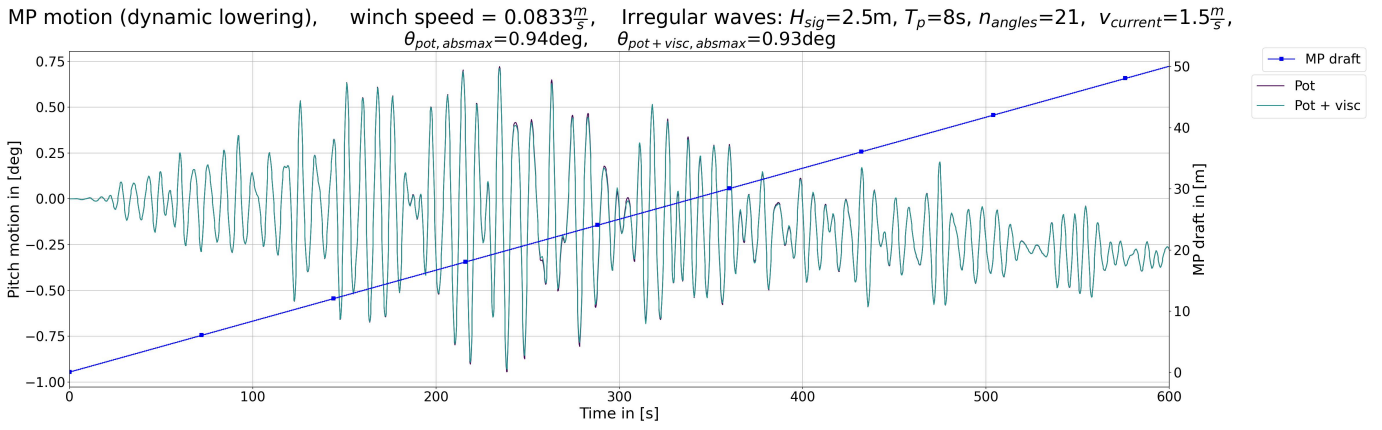


Figure 94: MP motion of the 1-DOF model, during the dynamic lowering simulation, irregular waves (with spreading):  $H_{sig}=2.5m$ ,  $T_p=8s$ ,  $V_c = 1.5 m/s$

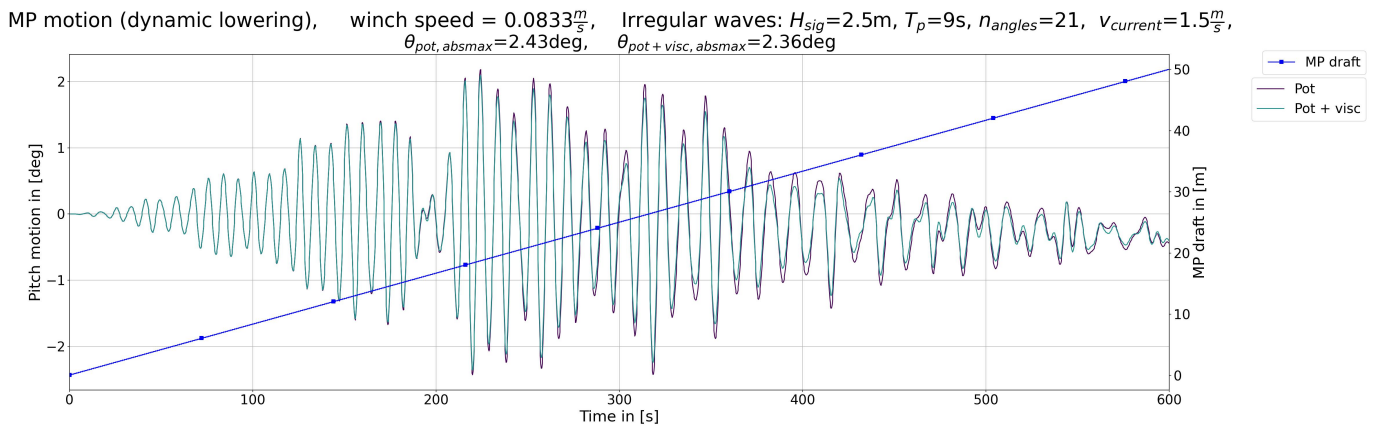


Figure 95: MP motion of the 1-DOF model, during the dynamic lowering simulation, irregular waves (with spreading):  $H_{sig}=2.5m$ ,  $T_p=9s$ ,  $V_c = 1.5 m/s$

### C.3 JONSWAP with ambient current

In Figures 96, 97, and 98 the modified JONSWAP spectrum for ambient current are depicted. In can be seen that the situation with current traveling in the opposite direction of the waves clearly do not yield a proper JONSWAP spectrum. As discussed in subsection 6.5, this is due to the non-linear waves that arise in this situation, hence the weird shape of these spectra. Because of this the modified JONSWAP spectrum for opposing current cannot be used to evaluate wave-excitation forces.

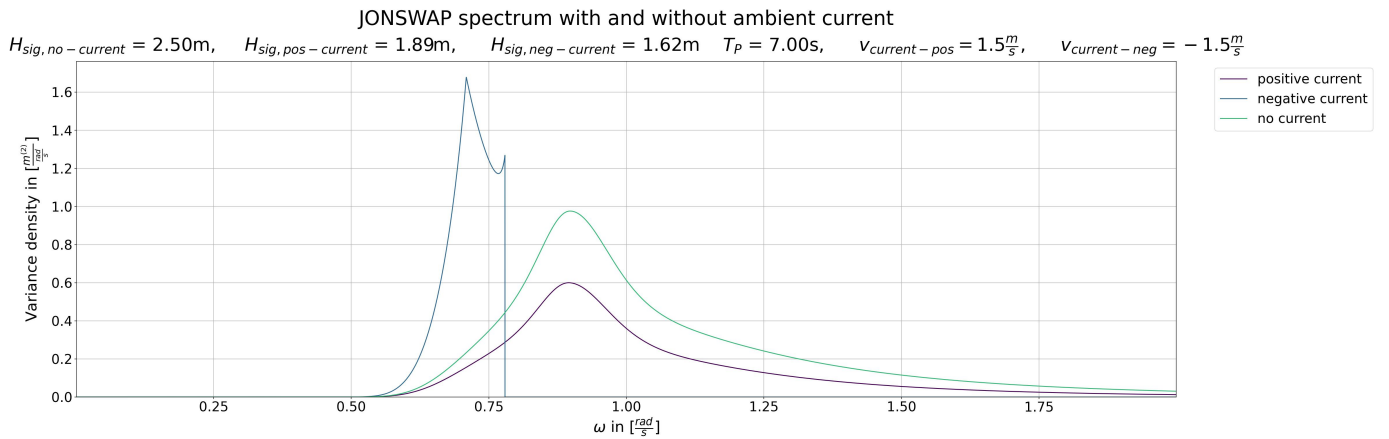


Figure 96: JONSWAP with ambient current,  $H_{sig} = 2.5m$ ,  $T_p = 7s$ , and  $v_{current} = 1.5 \frac{m}{s}$

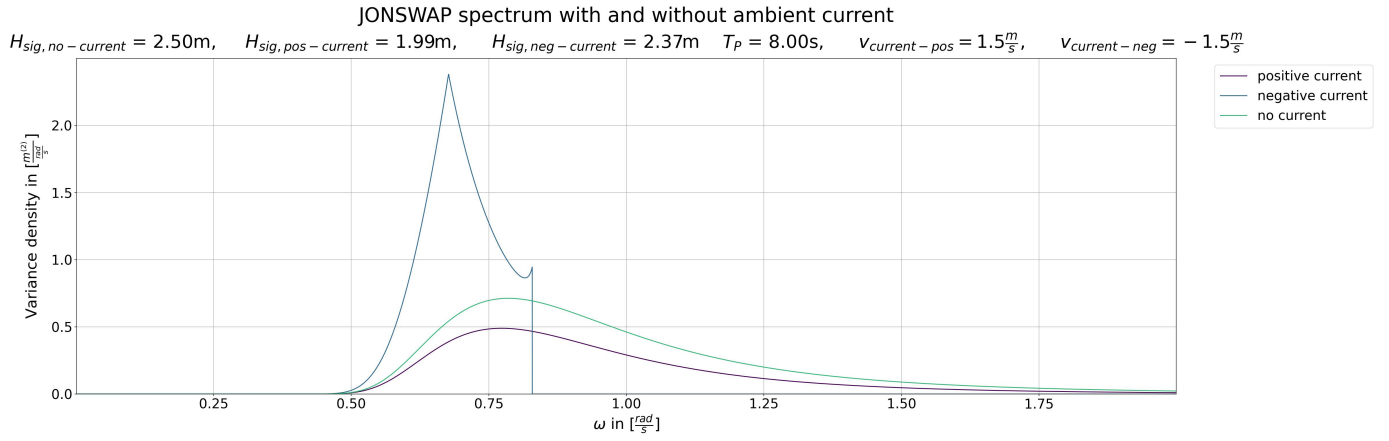


Figure 97: JONSWAP with ambient current,  $H_{sig} = 2.5m$ ,  $T_p = 8s$ , and  $v_{current} = 1.5\frac{m}{s}$

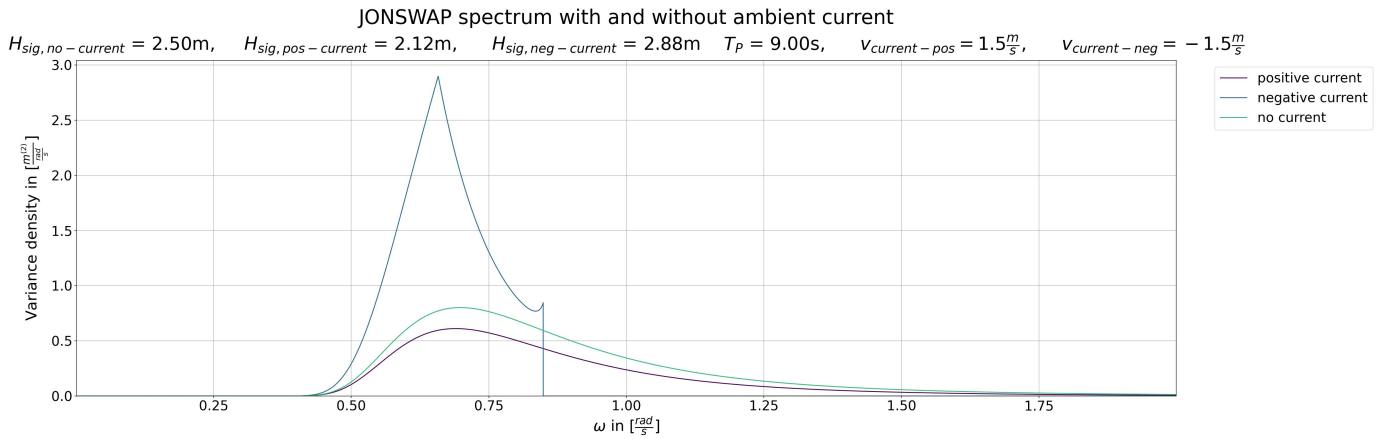


Figure 98: JONSWAP with ambient current,  $H_{sig} = 2.5m$ ,  $T_p = 9s$ , and  $v_{current} = 1.5\frac{m}{s}$

#### C.4 Dynamic lowering simulation, ambient current effect

In Figures 99, and 100 the dynamic lowering simulation is depicted for respectively the case without the ambient current effect and with the ambient current effect with a peak period of 7 seconds, a significant wave height of 2.5m, and a current of  $1.5\frac{m}{s}$ . Both models consider the damping due to viscous drag, irregular unidirectional waves, and the current which flows in the same direction as the propagation of the waves. The exact same simulations are shown for equal environmental conditions, except for different peak periods. See Figure 101, and Figure 102 for the simulations with a peak period of 8 seconds, and see Figure 103, and Figure 104 for the simulations with a peak period of 9 seconds.

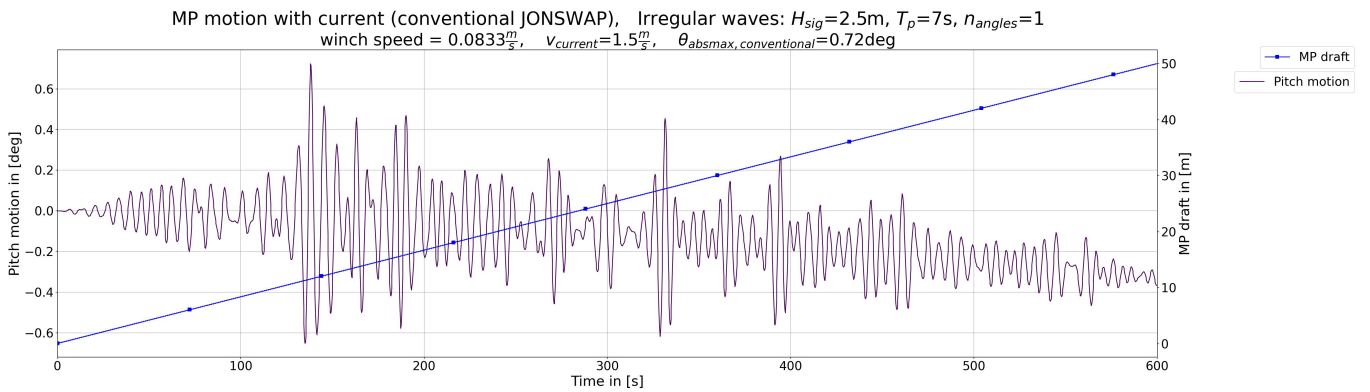


Figure 99: MP motion with the conventional JONSWAP spectrum, viscous drag, and without the spreading of waves, with:  $H_{sig} = 2.5m$ ,  $T_p = 7s$



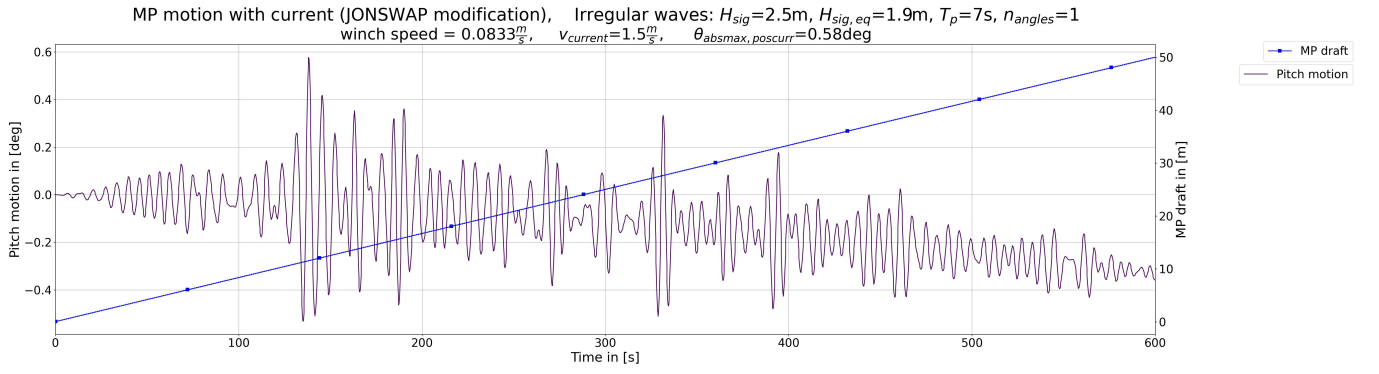


Figure 100: MP motion with the modified JONSWAP spectrum, viscous drag, and without the spreading of waves, with:  $H_{sig} = 2.5m$ ,  $T_p = 7s$

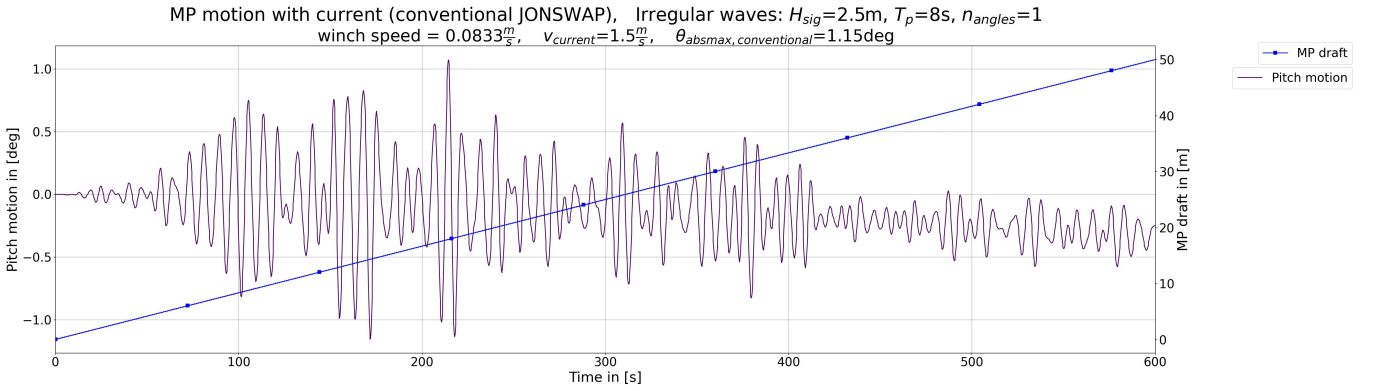


Figure 101: MP motion with the conventional JONSWAP spectrum, viscous drag, and without the spreading of waves, with:  $H_{sig} = 2.5m$ ,  $T_p = 8s$

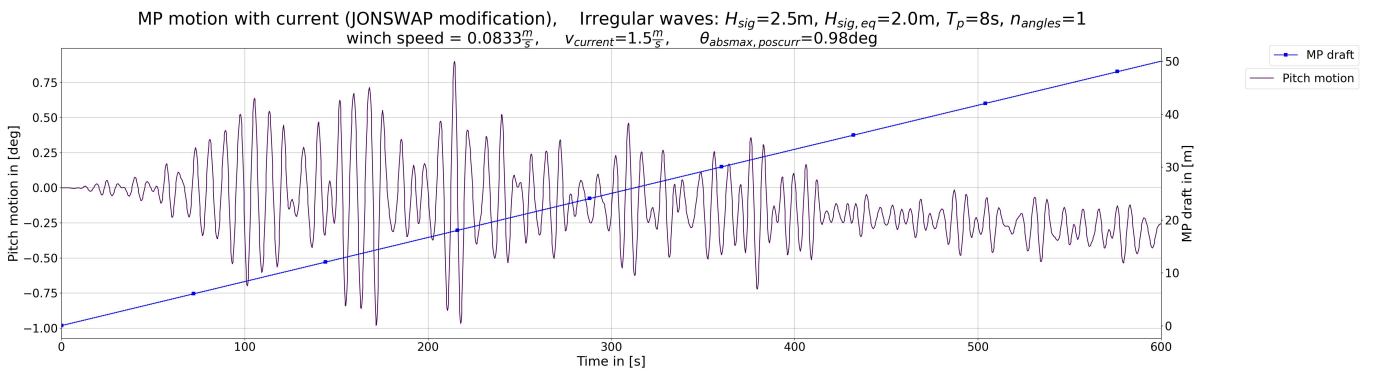


Figure 102: MP motion with the modified JONSWAP spectrum, viscous drag, and without the spreading of waves, with:  $H_{sig} = 2.5m$ ,  $T_p = 8s$

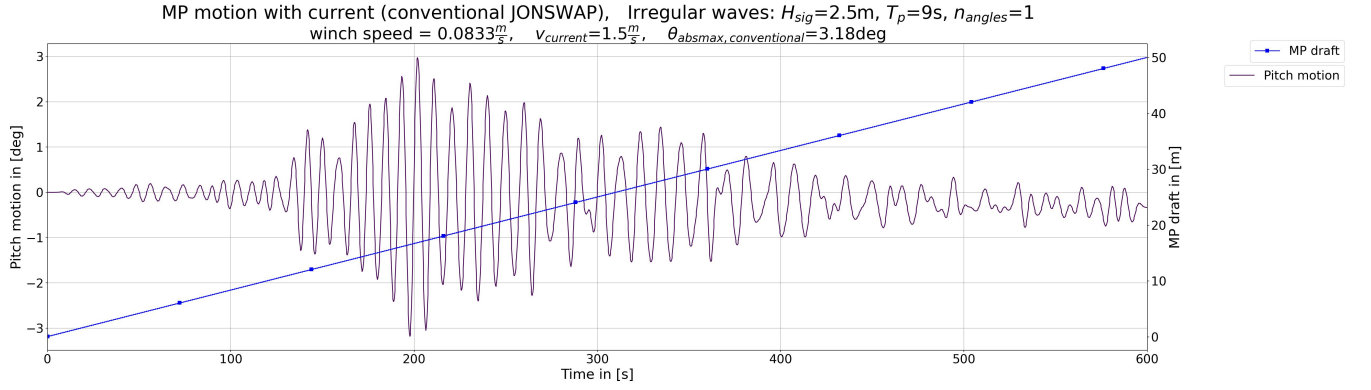


Figure 103: MP motion with the conventional JONSWAP spectrum, viscous drag, and without the spreading of waves, with:  $H_{sig} = 2.5m$ ,  $T_p = 9s$

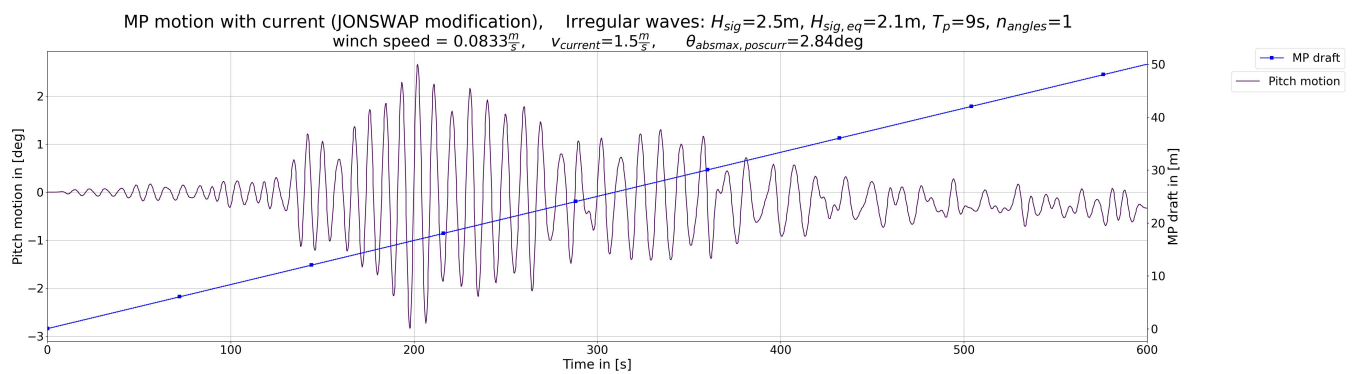


Figure 104: MP motion with the modified JONSWAP spectrum, viscous drag, and without the spreading of waves, with:  $H_{sig} = 2.5m$ ,  $T_p = 9s$

## C.5 2nd order difference frequency wave-drift forces

In Figures 105, 106, and 107 the 2nd order difference frequency wave-drift force spectra for respectively peak periods of 7, 8 and 9 seconds with a significant waveheight of 2.5m are shown. In subsubsection 6.5.3 the procedure of obtaining these spectra is explained.

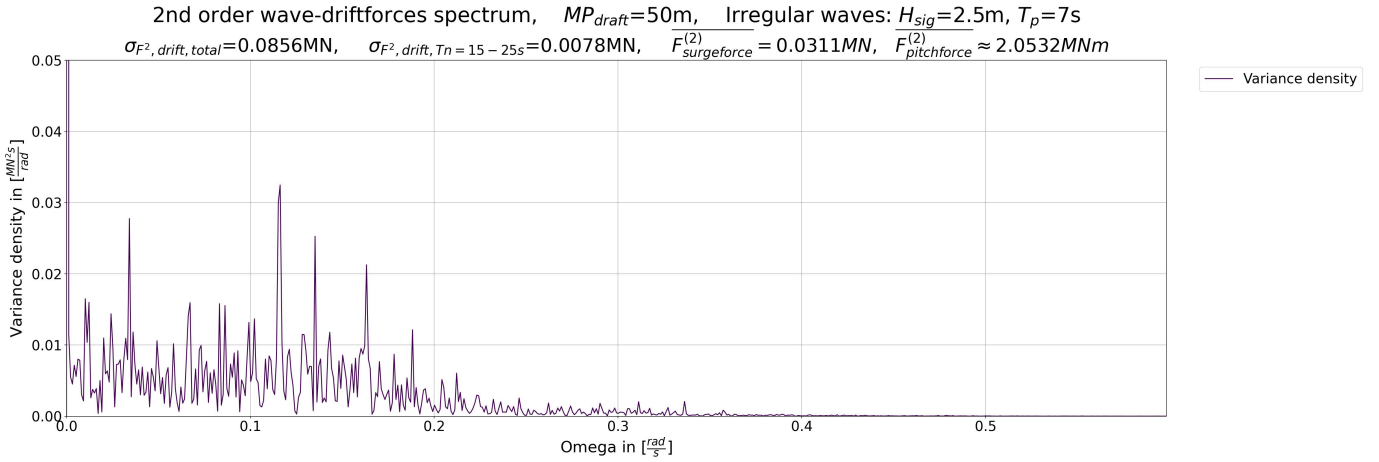


Figure 105: 2nd order difference frequency wave drift-force spectrum  $H_{sig} = 2.5m$ ,  $T_p = 7s$

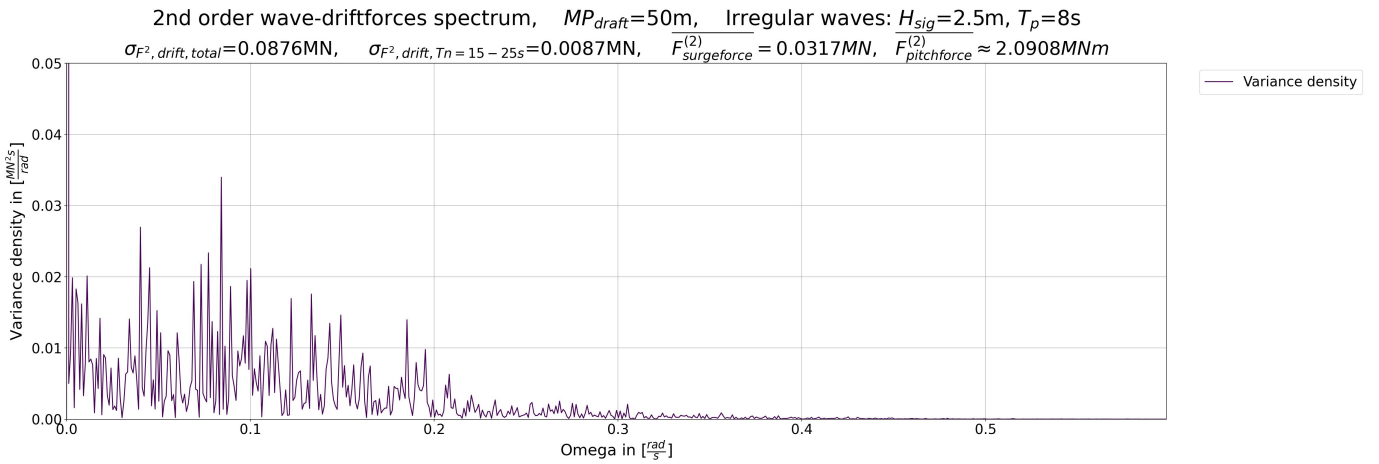


Figure 106: 2nd order difference frequency wave drift-force spectrum  $H_{sig} = 2.5m$ ,  $T_p = 8s$

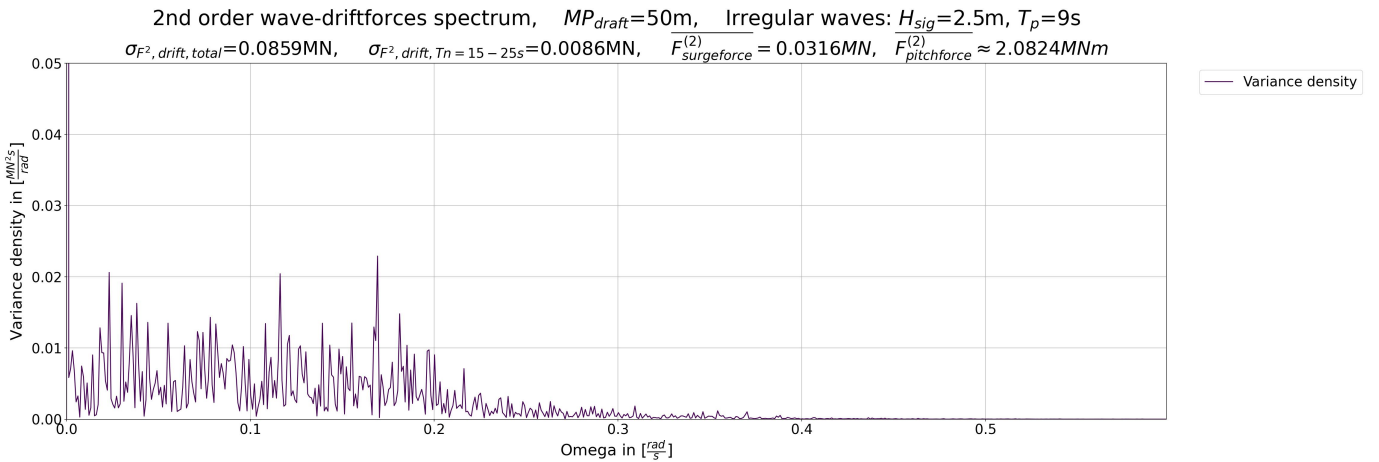


Figure 107: 2nd order difference frequency wave drift-force spectrum,  $H_{sig} = 2.5m$ ,  $T_p = 9s$

## C.6 Effect of added damping on the MP motion

In Figures 108, 109, and 110 the effect of varying the added damping through different assumed pile periods of motion is shown. In the figures the dynamic lowering simulations are depicted for irregular unidirectional waves with respectively peak periods of 7,8, and 9 seconds with a significant wave height of 2.5m, and a current of  $1.5 \frac{m}{s}$ . The winch speed in this dynamic lowering simulation is set at  $0.0833 \frac{m}{s}$ .

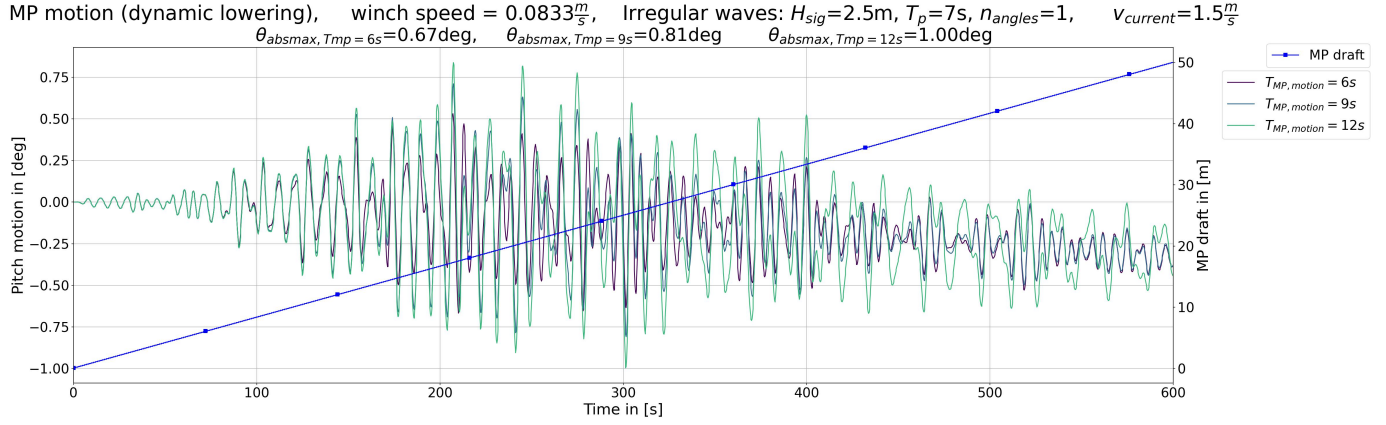


Figure 108: MP motion response for different assumed pile-periods of motion,  $H_{sig} = 2.5m$ ,  $T_p = 7s$

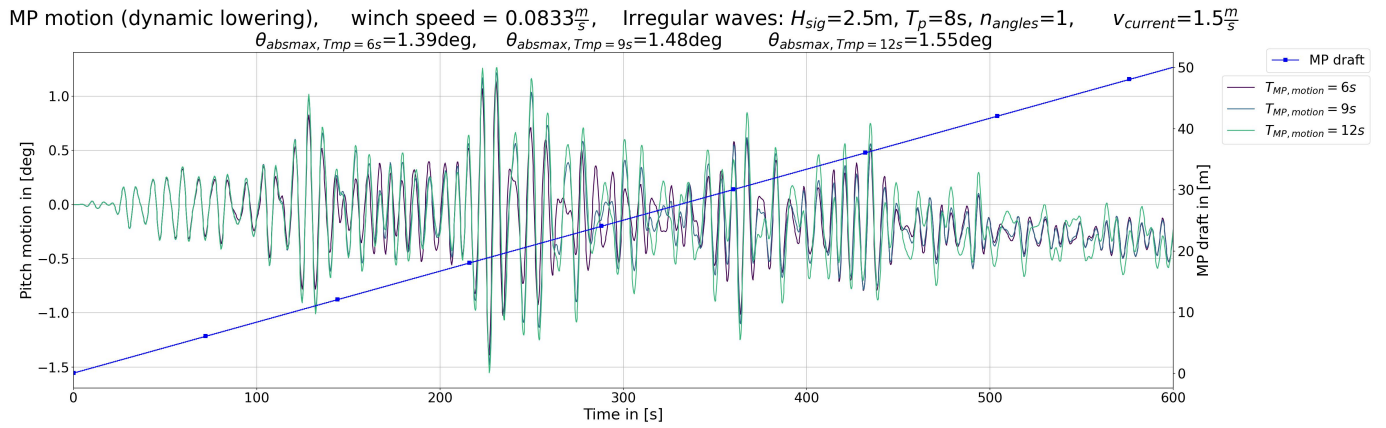


Figure 109: MP motion response for different assumed pile-periods of motion,  $H_{sig} = 2.5m$ ,  $T_p = 8s$

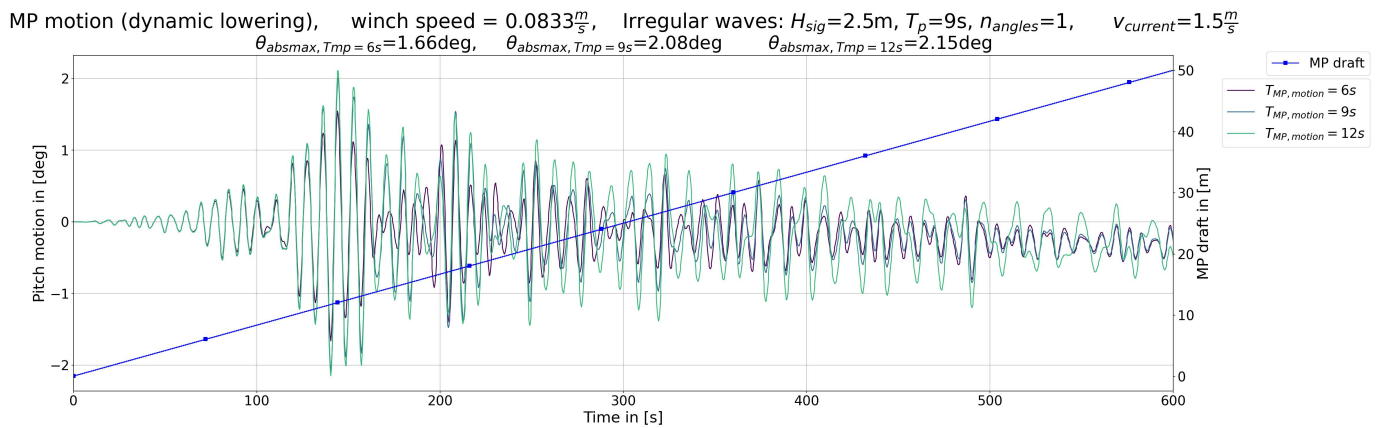


Figure 110: MP motion response for different assumed pile-periods of motion,  $H_{sig} = 2.5m$ ,  $T_p = 9s$

## C.7 Jack-up motions

In Figures 111, 112, and 113 the rotational MP motion response around the x and y-axis w.r.t. the gripper of the multi-DOF model are shown. The motion response is obtained at critical MP drafts, as defined in subsection 6.3, for irregular spread waves with respectively peak periods of 7, 8, and 9 seconds with a significant wave height of 2.5m, and a current of  $1.5 \frac{m}{s}$ . In Figures 114, 115, and 116 the translation motion of the gripper along the x and y-axis of the multi-DOF model are shown under the same conditions. See Figure 4 for the definition of the axis system in the gripper.

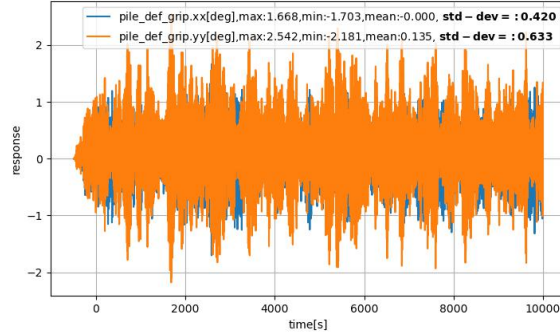


Figure 111: MP rotational motion response around the x and y axis in the gripper of the multi-DOF model,  $H_s = 2.5m$ ,  $T_p = 7s$ ,  $v_c = 1.5 \frac{m}{s}$ ,  $MP_{draft} = 10m$

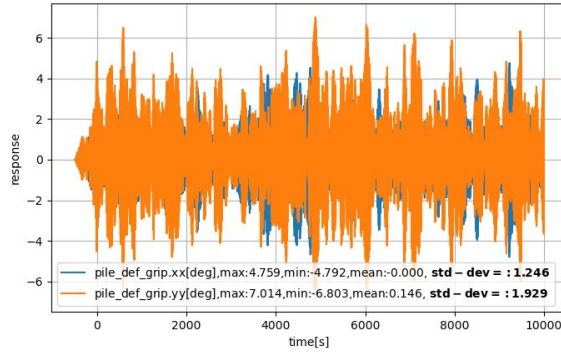


Figure 112: MP rotational motion response around the x and y axis in the gripper of the multi-DOF model,  $H_s = 2.5m$ ,  $T_p = 8s$ ,  $v_c = 1.5 \frac{m}{s}$ ,  $MP_{draft} = 15m$

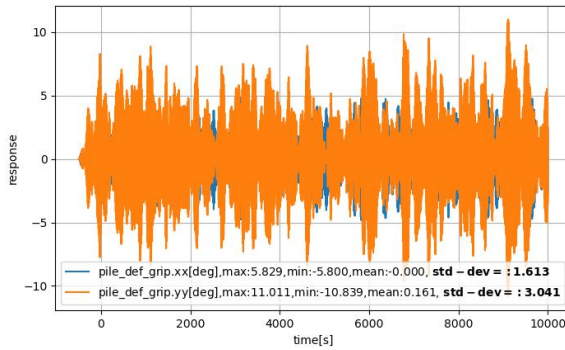


Figure 113: MP rotational motion response around the x and y axis in the gripper of the multi-DOF model,  $H_s = 2.5m$ ,  $T_p = 9s$ ,  $v_c = 1.5 \frac{m}{s}$ ,  $MP_{draft} = 20m$

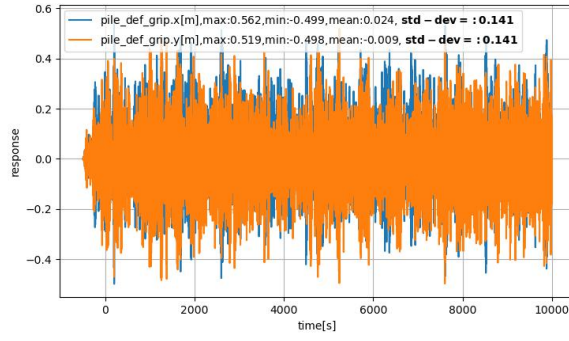


Figure 114: MP translation motion response along the x and y axis in the gripper of the multi-DOF model,  $H_s = 2.5m$ ,  $T_p = 7s$ ,  $v_c = 1.5 \frac{m}{s}$ ,  $MP_{draft} = 10m$

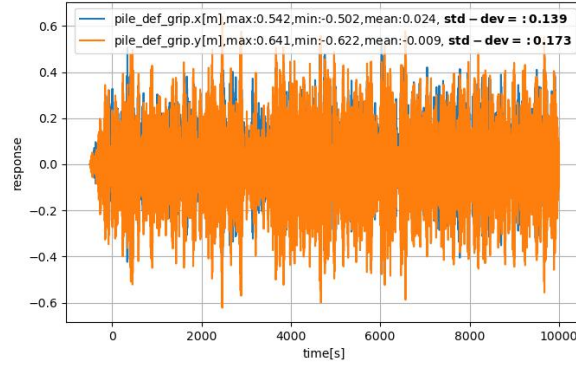


Figure 115: MP translation motion response along the x and y axis in the gripper of the multi-DOF model,  $H_s = 2.5m$ ,  $T_p = 8s$ ,  $v_c = 1.5 \frac{m}{s}$ ,  $MP_{draft} = 15m$

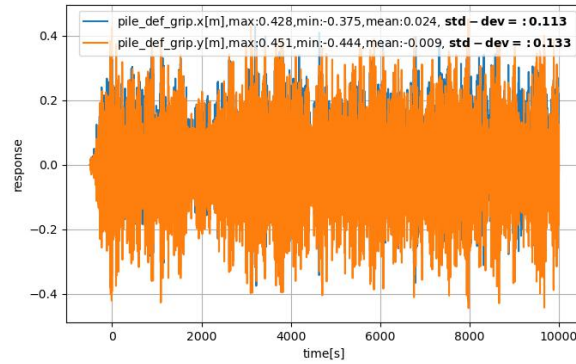


Figure 116: MP translation motion response along the x and y axis in the gripper of the multi-DOF model,  $H_s = 2.5m$ ,  $T_p = 9s$ ,  $v_c = 1.5 \frac{m}{s}$ ,  $MP_{draft} = 15m$



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