

Uncertainty quantification in particle image velocimetry and advances in time-resolved image and data analysis

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Uncertainty quantification in particle image velocimetry and advances in time-resolved image and data analysis

PROEFSCHRIFT

ter verkrijging van de graad van doctor
aan de Technische Universiteit Delft,
op gezag van de Rector Magnificus prof. ir. K.C.A.M. Luyben,
voorzitter van het College voor Promoties,
in het openbaar te verdedigen op vrijdag 05 september 2014 om 10.00 uur

door

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Dit proefschrift is goedgekeurd door de promotor:
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This research has been conducted as part of the Adaptive PIV project funded by LaVision GmbH, Göttingen.

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ISBN: 978-94-6186-353-9

SUMMARY

Particle image velocimetry (PIV) is an experimental technique for flow field measurements over a two- or three-dimensional domain. After over 30 years from its first application, nowadays PIV is acknowledged as a standard diagnostic tool for fluid dynamics research and is widespread in universities, research institutes and industrial facilities. Despite its maturity, PIV features several limitations that leave room to further research; among those, the most relevant include the limited measurement volume, the need of optical access for illumination and imaging system, the reduced accuracy of time-resolved measurements and the lack of an acknowledged methodology for uncertainty quantification.

The present work initially discusses the state-of-the-art of PIV image and data analysis; numerous efforts are reviewed that have led to the development of advanced image processing algorithms for image reconstruction and enhancement, data processing approaches for increasing the measurement accuracy and post-processing methodologies for invalid vector detection and uncertainty quantification.

A novel methodology for the data-based uncertainty quantification in PIV is introduced in chapter 4. The method relies upon the concept of image-matching: the PIV recordings are matched based on the measured velocity field. The positional disparity between paired particle images is then computed to retrieve the measurement uncertainty. Both the numerical assessment via Monte Carlo simulations and the experimental assessment show that the image-matching approach allows estimating the measurement uncertainty in good agreement with the actual error value.

A collaborative framework for PIV uncertainty quantification has been setup to compare different uncertainty quantification methodologies and investigate strengths and weaknesses of those. A dedicated experimental data base has been produced where the reference (“exact”) velocity field is known with a good degree of accuracy. A major outcome of the collaborative framework is that the uncertainty evaluated with image-based methods such as image-matching and correlation statistics approaches exhibits high sensitivity to the measurement error. In contrast, methods based on numerical simulations as the uncertainty surface method are more accurate in presence of small particle images and low seeding density.

The present work also deals with advanced approaches for time-resolved image and data analysis. Chapter 6 introduces a novel technique to enhance the precision and robustness of time-resolved particle image velocimetry measurements. The innovative element of the technique is the linear combination of correlation functions computed at different separation time intervals. The resulting ensemble-averaged correlation function features a significantly higher signal-to-noise ratio and yields a more precise velocity estimation due to the evaluation of a larger particle image

displacement. The technique enables the accurate estimate of the flow acceleration even in cases of low image quality.

The issue of undesired laser light reflections in PIV images is addressed in chapter 7. A simple approach for the attenuation of those is proposed. The approach relies upon the decomposition of the pixel intensity in the frequency domain: the high frequency content of the signal is due to the transit of seeding particles, whereas undesired reflections will appear in the low frequency range. Applying a high pass filter on the light intensity time history retains only the contribution of the seeding particles and rejects the undesired light reflections. The method can be applied for both stationary interfaces as well as when the image of the interface is moving due to vibration of either the model or the imaging system. The application to real experiments shows that the approach can mostly eliminate the trace of the reflection, making it possible to measure the velocity vectors in proximity of the solid surface even when the cross-correlation window overlaps with the surface.

When laser lights reflections have not been correctly removed in the images, they may yield clusters of corrupted or missing data in the velocity field, namely gaps. Gaps can originate also from shadows produced by the model or limited optical access of laser or imaging system. The presence of gaps in velocity field data poses a major limitation for the computation of integral quantities, which are required for several applications including the determination of the noise source in a turbulent flow. Chapter 8 discusses a novel approach that relies upon the solution of the Navier-Stokes equations within small gaps to reconstruct velocity distributions of arbitrary wavelength.

The manuscript ends with a summary of the main results and conclusions from the preceding chapters, highlighting the possible directions for further research on the topic of PIV uncertainty quantification and advanced image and data analysis.

SAMENVATTING

Particle Image Velocimetry (PIV) is een gevestigde techniek voor de bepaling van de stroomsnelheid in twee- of driedimensionale domeinen. Meer dan 30 jaar na het eerste PIV experiment wordt PIV tegenwoordig erkend als een standaard methode voor onderzoek naar vloeistofdynamica en is de methode wijdverspreid op universiteiten, onderzoekscentra en in de industrie. Ondanks zijn volwassenheid kent PIV enkele beperkingen die ruimte laten voor verder onderzoek. Tot de meest relevante behoren het beperkte meetvolume, de noodzaak voor optische toegang voor de belichting en opname systemen, de verminderde nauwkeurigheid van tijd-opgeloste metingen en het ontbreken van een erkende methodologie voor kwantificatie van de meetonzekerheid.

Het huidige werk richt zich eerst op state-of-the-art PIV beeld en data analyse. Verschillende studies worden besproken die geleid hebben tot de ontwikkeling van geavanceerde beeld verwerkingsalgoritmen voor beeld reconstructie en verbetering van de beeldkwaliteit en data verwerkingsmethoden voor verhoging van de meetnauwkeurigheid en nabewerkingsmethoden voor detectie van ongeldige vectoren en kwantificatie van de meetonzekerheid.

Hoofdstuk 4 van het huidige werk introduceert een nieuwe methode voor het kwantificeren van onzekerheden in PIV data. De methode is gebaseerd op het concept van beeld matching: de PIV beelden worden gekoppeld op basis van het gemeten snelheidsveld. Het positionele verschil tussen de gekoppelde particle images wordt vervolgens berekend om de meetonzekerheid te bepalen. Zowel uit de numerieke beoordeling via Monte Carlo simulaties als uit de experimentele evaluatie blijkt dat de beeld matching aanpak meetonzekerheden bepaalt die in goede overeenstemming zijn met de werkelijke foutwaarde.

Een samenwerkingskader voor PIV onzekerheidskwantificatie is opgezet om de verschillende onzekerheidskwantificatie methoden te vergelijken en om de sterke en zwakke punten van deze te onderzoeken. Een specifieke experimentele database is geproduceerd waar het referentie (“exacte”) snelheidsveld bekend is tot op een hoge graad van nauwkeurigheid. Een belangrijke uitkomst van het samenwerkingskader is dat de onzekerheid bepaald met beeld-gebaseerde methoden, zoals image-matching en correlation statistics methoden, hoge gevoeligheid voor de meetfout toont. In tegenstelling tot deze methoden zijn methoden gebaseerd op numerieke simulatie, zoals de uncertainty surface methode, nauwkeuriger in aanwezigheid van kleine particle images en lage seeding density.

Verder zijn ook geavanceerde methoden voor tijd-opgeloste PIV beeld en data analyse onderzocht. Hoofdstuk 6 introduceert een nieuwe techniek voor de verbetering van precisie en robuustheid van tijd-opgeloste PIV metingen. Het innovatieve element van de techniek is dat een lineaire combinatie van de correlatie

functies op verschillende separatie tijd intervallen wordt berekend. De resulterende ensemble-gemiddelde correlatie functie heeft een significant hogere signaal-ruis verhouding en levert een meer nauwkeurige snelheidsbenadering door evaluatie van een grotere particle image verplaatsing. De techniek maakt zelfs in geval van lage beeldkwaliteit nauwkeurige schatting van de versnelling mogelijk.

De kwestie van ongewenste laserlicht reflecties in PIV beelden wordt behandeld in hoofdstuk 7 en een eenvoudige methode voor de demping van de reflecties wordt voorgesteld. Deze is gebaseerd op de decompositie van de pixel intensiteit in het frequentie domein: de hoge frequentie inhoud van het signaal wordt veroorzaakt door de doorstroom van seeding deeltjes, terwijl de ongewenste reflecties verschijnen in het lage frequentiegebied. Het toepassen van een high-pass filter op de lichtintensiteit tijd historie behoudt alleen de contributie van de seeding deeltjes en verwerpt de ongewenste licht reflecties. De methode kan worden toegepast voor zowel stationaire interfaces als wanneer het beeld van de interface beweegt door trillingen van hetzij het model of het opnamesysteem. Toepassing op echte experimenten laat zien dat de methode het spoor van de reflectie voor het grootste deel kan elimineren, wat het mogelijk maakt snelheidsvectoren in de nabijheid van een vast oppervlak te meten, zelfs wanneer het cross-correlatie venster overlapt met het oppervlak

Wanneer laser licht reflecties niet correct verwijderd zijn uit de beelden kunnen ze clusters van beschadigde of ontbrekende data in het snelheidsveld veroorzaken. Zulke gaten in het snelheidsveld kunnen ook afkomstig zijn van schaduwen veroorzaakt door het model of gelimiteerde optische toegang van het laser of opnamesysteem. De aanwezigheid van gaten in het snelheidsveld vormt een belangrijke beperking voor de berekening van geïntegreerde waarden, die essentieel zijn voor verschillende toepassingen waaronder het bepalen van de geluidsbron in een turbulente stroming. Hoofdstuk 8 bespreekt een nabewerkingsmethode gebaseerd op de oplossing van de Navier-Stokes vergelijkingen voor het schatten van het snelheidsveld in gebieden waar geen experimentele gegevens beschikbaar zijn.

Het manuscript eindigt met een samenvatting van de belangrijkste resultaten en conclusies uit de voorgaande hoofdstukken, waarbij gewezen wordt op mogelijke richtingen voor verder onderzoek op het gebied van PIV onzekerheidskwantificatie en geavanceerde beeld en data analyse.

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CHAPTER 1

INTRODUCTION

1.1 Background

Aircraft have been flying over our heads for more than a hundred years. Already in the early 1900s, the Wright brothers carried out wind tunnel experiments to investigate the flow over their Flyer (figure 1.1). Since then, aerodynamic investigation has advanced rapidly to improve the aircraft design and provide a better understanding of aerodynamic phenomena. Nowadays, several applications require a thorough understanding of the flow physics, including the design of efficient aerodynamic devices from subsonic to hypersonic flows, turbomachinery, combustion and aeroacoustics. The latter topic has received increasing interest in the last years due to the necessity of complying with strict international regulations on the noise emitted by aircraft, especially during take-off and landing. Furthermore, fundamental research on turbulence is conducted worldwide to investigate the physical processes governing turbulent flows. Three means of analysis are typically distinguished in aerodynamics investigation: theoretical, computational and experimental.

The theoretical approach relies upon the analytical solution of the fluid dynamics non-linear differential equations, namely the Navier-Stokes equations. Although this approach would lead to the exact solution of the flow field, in practice it is used only for extremely simple flows and geometries (e.g. Taylor-Green vortex and Poiseuille flow, see figure 1.2) due to the difficulty of solving those equations analytically*.

*In fact, the existence and smoothness of the Navier-Stokes equations is still to be demonstrated. In case the reader has the solution of this problem at hand, he/she is advised to apply for the Millennium prize established by the Clay Mathematics Institute (<http://www.claymath.org/millennium-problems/navier%E2%80%93stokes-equation/>) to win 1,000,000 dollars!

Introduction

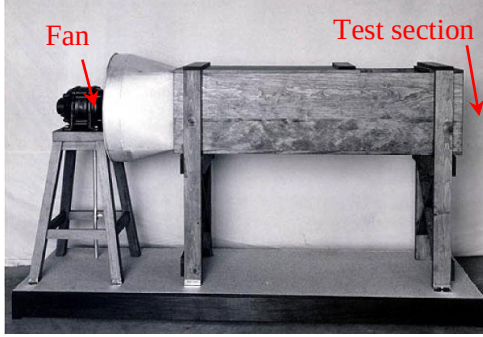


Figure 1.1. Wind tunnel designed, built and used by the Wright brothers in Dayton, Ohio, during 1901-1902. Figure from Anderson (2001).

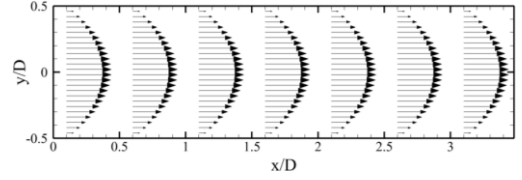


Figure 1.2. Two-dimensional Poiseuille flow (parallel flow between two horizontal plates separated by the distance D). The velocity field is given by $u = \frac{1}{2\mu} \left(\frac{dp}{dx} \right) \left(y^2 - \frac{D^2}{4} \right)$, being μ the dynamic viscosity of the fluid.

In computational fluid dynamics (CFD), instead, the continuous flow equations are discretized and then solved numerically with the aid of computers. In most cases, assumptions on the behaviour of the small turbulent scales are made to reduce the computational burden (as it is done in *large eddy simulation*, LES, and *Raynolds averaged Navier-Stokes* equations, RANS). Here, model validations via comparison with experimental data are required to assess the correctness of the assumptions (figure 1.3). In contrast, *direct numerical simulation* (DNS) solves the Navier-Stokes equations without introducing models for the turbulent properties; therefore, all motions in the turbulent flow are resolved. However, in order to ensure that all the eddies composing the turbulent flow have been captured, all flow scales from largest (dimension L) to smallest (Kolmogorov length scale η) need to be resolved. Hence, the computational domain must be composed by at least L/η grid points in each dimension. It can be proved that the ratio L/η increases with $Re^{3/4}$ (Tennekes and Lumley, 1972), being Re the flow Reynolds number; as a result, a dramatic increase in computational time occurs when the Reynolds number is increased. Consequently, with the current computational capabilities DNS is used to investigate flows only up to Reynolds number of few thousands (Pirozzoli *et al*, 2010, see figure 1.4).

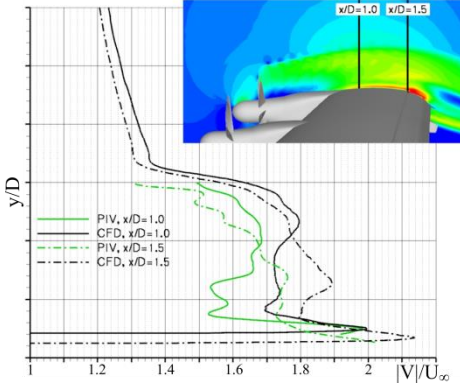


Figure 1.3. Validation of unsteady RANS calculations (CFD) with experimental data (PIV) in a propeller slipstream. Illustration from Roosenboom *et al*, 2009.

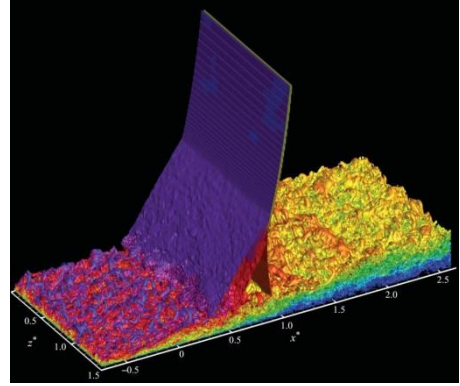


Figure 1.4. DNS of a shock-wave boundary layer interaction at $M_\infty = 2.0$ and $Re_\theta \approx 1200$. Iso-surfaces of pressure gradient modulus in the proximity of the interaction zone. Illustration from Pirozzoli *et al*, 2010.

The experimental approach consists of in-situ sampling of one or more flow properties. This is required for the study of high Reynolds number and complex flows for which the accuracy of numerical simulations relying upon turbulence models is questionable. Before the advent of the laser in the 1960s, velocity measurements were conducted with intrusive probe-based techniques such as Pitot-tubes (Pitot, 1732) and *hot-wire anemometry* (HWA, Fingerson and Freymuth, 1983). The development of *laser Doppler velocimetry* (LDV, Adrian, 1983) allowed the first non-intrusive velocity measurements. However, similarly to HWA and Pitot tube, LDV is a point-wise technique that allows measuring the velocity in one specific location at the time. Hence, the possibility of determining the components of the velocity gradient, crucial for the calculation of derived fluid dynamics variables such as vorticity, strain and velocity divergence, is precluded. Furthermore, the characterization of the velocity field in a plane or a volume is possible only from a statistical point of view and requires the laborious process of scanning the measurement domain point by point.

Flow visualization techniques have revealed that turbulence is not a random process, but consists of coherent flow structures (see figure 1.5 and the works by Brown and Roshko, 1974, and Van Dyke, 1982, among others). Most of those techniques rely upon the introduction of tracer particles (such as smoke or microspheres) or dye into the flow in order to make the flow features visible. With visualization, one obtains only a qualitative picture of those structures, which is immediately insightful but often not sufficient for a thorough and quantitative characterization of an aerodynamic phenomenon.

Introduction

An early technique to extract quantitative information on the flow velocity from the tracer images is *particle streak velocimetry*. Here the tracer particles are illuminated by a continuous light source so that their motion during the exposure time produces small streaks in the recorded images. The direction and magnitude of the fluid velocity are determined by measuring the orientation and length of the streaks. The technique was introduced by Prandtl (1905) to illustrate the motion of fluids and has been more recently applied by Flór and van Heijst (1996) to investigate the behaviour of vortices in stratified fluids (see figure 1.6). However, particle streak velocimetry does not reveal the sign of the fluid motion and only a few marker particles can be used, otherwise the streaks overlap and individual paths cannot be recognized.

Adequate selection of tracer particles, flow illumination and processing of the recorded images permits extracting accurate quantitative information on the flow velocity, with no directional ambiguity and higher spatial resolution with respect to particle streak velocimetry. This is the aim of a technique introduced in the early eighties and now become very popular among the fluid dynamics community, namely *particle image velocimetry* (PIV).

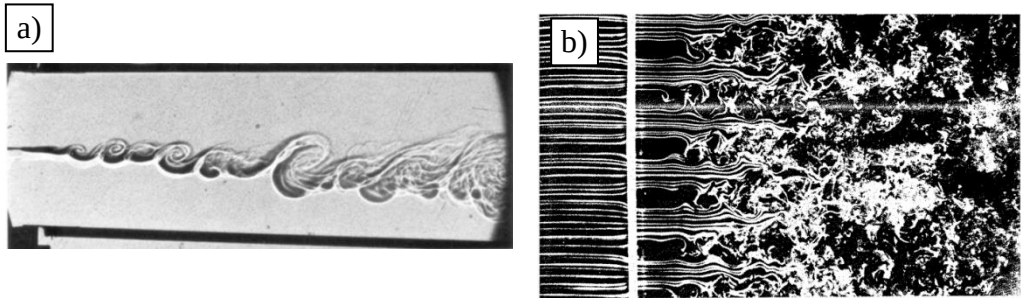


Figure 1.5. (a) Shadowgraph of a mixing layer; from Brown and Roshko (1974). (b) Smoke visualization of the generation of turbulence by a grid; from Van Dyke (1982).

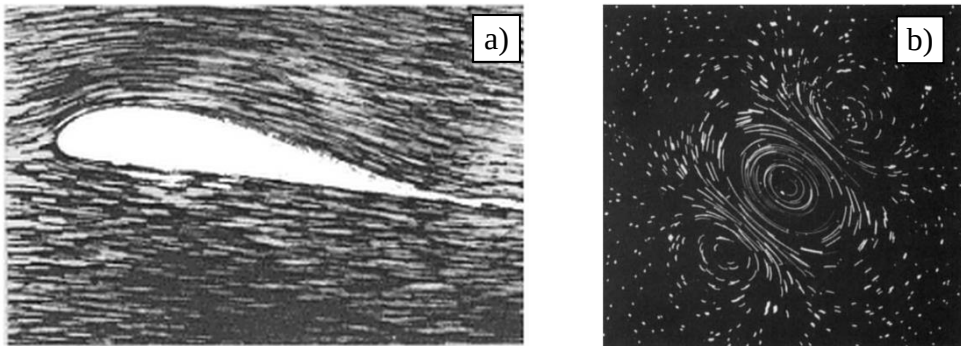


Figure 1.6. Particle streak velocimetry images. (a) Visualization of the flow around an airfoil; from Oertel *et al* (2010). (b) Streak photograph of a tripolar vortex; from Flór and van Heijst (1996).

1.2 General aspects of particle image velocimetry

1.2.1 Operational principle

The operational principle of PIV relies upon the measurement of the displacement, within a short time separation, of small tracer particles inserted into the flow and travelling with the fluid. In order to make them visible, the particles are illuminated by a light source, typically a pulsed laser. Pairs of images are recorded by a digital imaging system composed by a CCD or a CMOS camera. To evaluate the fluid velocity, the images are divided into interrogation windows and, in each window, the average particle displacement is estimated, typically via the spatial cross-correlation operator (figure 1.7). Details on the components of the experimental setup and the evaluation of the flow velocity are given in chapter 2.

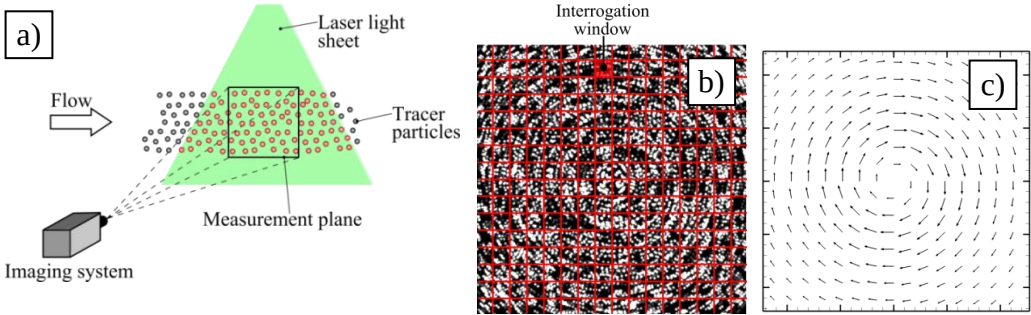


Figure 1.7. (a) Typical layout of a single-camera PIV setup (the tracer particles illuminated by the laser are displayed in red). (b) Superposition of two PIV images divided into interrogation windows (vortex flow field). (c) Vector field computed from the recordings of figure 1.7-b.

1.2.2 Technique development

The feasibility of employing PIV for the measurement of the velocity field in air and water flows was first demonstrated by Meynart in the early eighties (Meynart, 1983). At the time, images were formed on a photographic film and then transferred to a computer for the automatic analysis to retrieve the fluid motion. The technique received large attention within the fluid dynamics community due to its non-intrusive character and the capability to measure the fluid velocity vectors at a very large number of points simultaneously.

The development of PIV during the last thirty years has been strongly connected with the technological advancement of hardware components. Powerful lasers nowadays provide pulse energy up to 1 J to adequately illuminate the tracer particles over large regions (up to 1 m²). The repetition rate of high-speed lasers reaches several kilohertz. Digital cameras have been

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developed that record image pairs within less than 1 μs . Furthermore, high-speed cameras based on the CMOS sensor technology allow recording rates up to 10 kilohertz at resolution exceeding 1 megapixel.

An extensive survey of the technological developments and improvements of PIV during the last decades is reported in Raffel *et al* (2007), Adrian (2005) and Grant (1994), among others. In the reminder of this section, only a brief summary of the milestones of the PIV history is given, based on the measurement domain and the measured velocity components:

- **2D 2C:** two-component planar PIV, as illustrated in the setup of figure 1.7-a, where a single camera normal to the laser sheet (2D measurement volume) is used to record images and in turn measure two velocity components (2C).
- **2D 3C:** using two cameras at different observation angles, information on the third (out-of-plane) velocity component can be retrieved (figure 1.8-a). This technique is referred to as *stereoscopic PIV* and was introduced in the late nineties (Soloff *et al*, 1997; Willert, 1997; Prasad, 2000).
- **3D 3C:** the so-called *tomographic PIV* was proposed by Elsinga and co-workers (Elsinga *et al*, 2006) to measure the three-dimensional velocity field in a 3D measurement domain. A typical setup of a tomographic PIV experiment is illustrated in figure 1.8-b[†].
- **4D 3C:** when the tomographic PIV experiment is conducted with a high acquisition frequency cameras, time-resolved (TR) 3D velocity fields can be obtained (Schröder *et al*, 2008; Violato and Scarano, 2011, among others).

The present work focuses on advanced algorithms for planar PIV, where two velocity components are measured in a two-dimensional domain. The proposed methodologies could be extended to stereoscopic and tomographic PIV, although a thorough discussion on the topic goes beyond the aim of the work and is not reported here.

[†] Many other techniques have been developed for 3D measurements based on holography (Hinsch, 2002), scanning light sheet (Brüker, 1995), particle tracking (Maas *et al*, 1993) and digital defocusing (Pereira *et al*, 2000). Tomographic PIV is to date the most widespread method for 3D measurements among fluid dynamic laboratories.

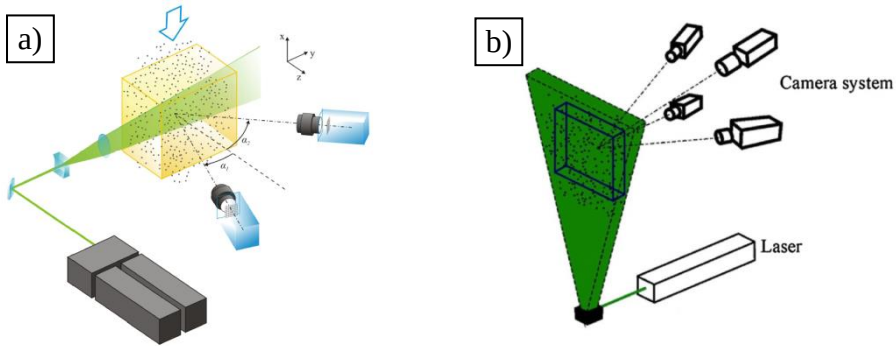


Figure 1.8. Example of camera arrangements for stereoscopic PIV (a) and tomographic PIV (b). Illustrations from Westerweel and Scarano, 2007, and Elsinga *et al*, 2006, respectively.

1.2.3 Applications

PIV is acknowledged as a standard diagnostic tool for fluid dynamics investigation.

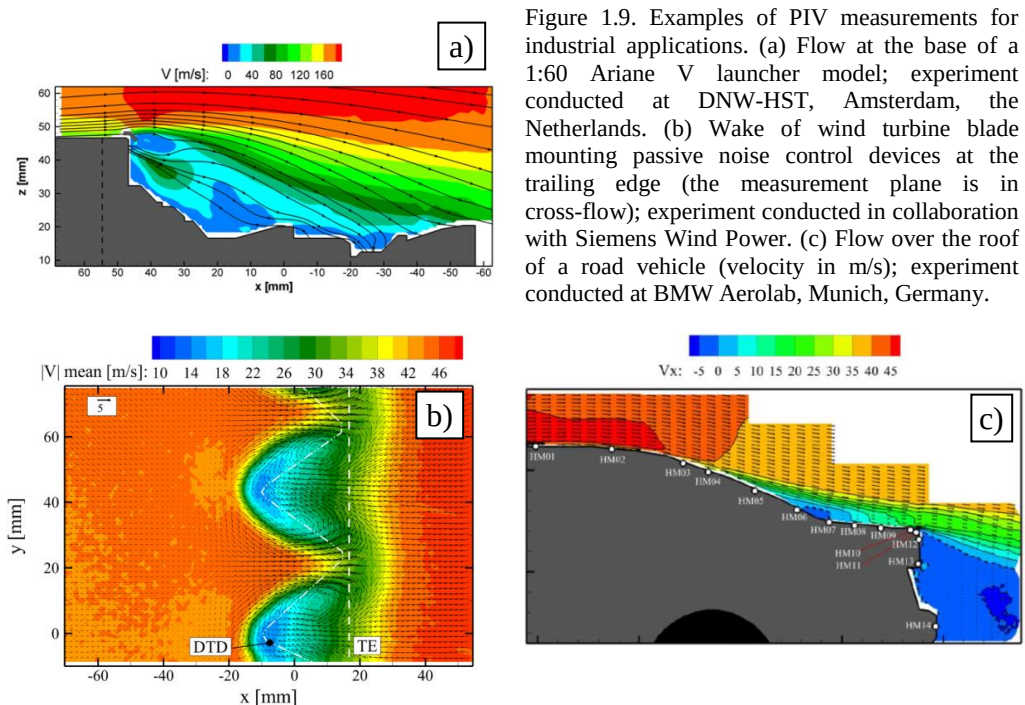


Figure 1.9. Examples of PIV measurements for industrial applications. (a) Flow at the base of a 1:60 Ariane V launcher model; experiment conducted at DNW-HST, Amsterdam, the Netherlands. (b) Wake of wind turbine blade mounting passive noise control devices at the trailing edge (the measurement plane is in cross-flow); experiment conducted in collaboration with Siemens Wind Power. (c) Flow over the roof of a road vehicle (velocity in m/s); experiment conducted at BMW Aerolab, Munich, Germany.

The technique is nowadays spread worldwide in universities, research institutes and industrial facilities; its range of application covers a wide gamut

Introduction

of fluid dynamics problems including fundamental turbulent research (Westerweel *et al*, 2013), hypersonic flows (Schrijer *et al*, 2006), flows in turbo machines (Wernet, 1997), aeroacoustics (Tuinstra *et al*, 2013), flows in micro-channels (Meinhart *et al*, 1999), combustion (Honoré *et al*, 2000) and biomedical flows (Jamison *et al*, 2012).

In the recent years, PIV has received increasing interest as a tool for industrial design that validates numerical simulations, especially in the aerospace, automotive and wind-energy sectors. Examples of results obtained in industrial measurement campaigns are illustrated in figure 1.9.

1.2.4 Current limitations

Although PIV is nowadays considered a mature technique, several limitations are recognized that leave room to further research. One of those is the limited measurement domain, that can reach 1 m^2 with low repetition rate lasers and CCD cameras, whereas it seldom exceeds $4 \cdot 10^{-2} \text{ m}^2$ for time-resolved measurements conducted with high-speed systems (Stanislas *et al*, 2008). The situation becomes even more critical in tomographic PIV, where measurement volumes not exceeding 200 cm^3 are typically achieved (Scarano, 2013). Larger domains are required for measurement campaigns in industrial facilities, where the three velocity components over large areas of the flow field (typically exceeding 1 m^2) need to be measured (Schröder and Willert, 2008). The relevance of the problem is confirmed by the recent collaboration among TU Delft, LaVision GmbH and the German Aerospace Center (DLR) for the study of the properties of tracer particles (namely helium-filled soap bubbles) with enhanced light scattering characteristics. These particles require lower light intensity to be visible and therefore allow a larger measurement domain.

As a second major limitation, PIV requires optical access for laser and imaging system from at least two directions. The investigation of the flow around a model of complex geometry (e.g. the wheel wells of a car, Schröder and Willert, 2008) is typically characterized by limited optical access: some regions of the flow field are either not illuminated by the laser or not imaged by the camera. Consequently, the velocity field exhibits regions where no velocity is measured; therefore, any further analysis relying upon the computation of integral quantities is precluded, as will be discussed in chapter 3.

The repetition rate of high-speed PIV systems seldom exceeds 10 kHz, which is well below the typical sampling rate of point-wise measurement

techniques such as hot-wire anemometry and laser Doppler velocimetry (Drain, 1980). Hence, PIV is characterized by lower temporal resolution than HWA and LDV, reason why the latter techniques are still considered a valuable tool for the investigation of turbulent flows where high frequency fluctuations (above 5 kHz) are present.

Furthermore, the analysis of PIV images requires a laborious processing. The computational cost depends on the experimental configuration (planar, stereoscopic or tomographic), the image resolution and the processing parameters. The processing time for a single velocity field is of few seconds for planar PIV, while it rises up to several minutes for tomographic PIV.

The measurement sensitivity is greatly related to the quality of the recorded images and the characteristics of the flow under investigation. Measurements with high-speed systems are reported to suffer of lower image quality due to the reduced sensitivity of CMOS cameras and the weaker illumination provided by high-repetition rate lasers (Stanistlas *et al*, 2008). When standard interrogation algorithms are employed (a detailed description of those is given in chapter 3), the uncertainty on the measured velocity is typically of the order of 1 to 5 percent; if the acceleration is extracted from the velocity, the uncertainty on that may be of the order of 50 to 100% with a signal to noise ratio seldom above 2 (this topic will be discussed in detail in chapter 3, where figure 3.14 shows an example of velocity and acceleration time histories obtained with a high-speed PIV system). Several works report that reliable temporal information can only be extracted after extensive data filtering in the space and time domain in order to reduce the amplitude of noisy fluctuations (Vétel *et al*, 2011; Oxlade *et al*; 2012, see figure 1.10), posing severe doubts on the reliability of the overall result.

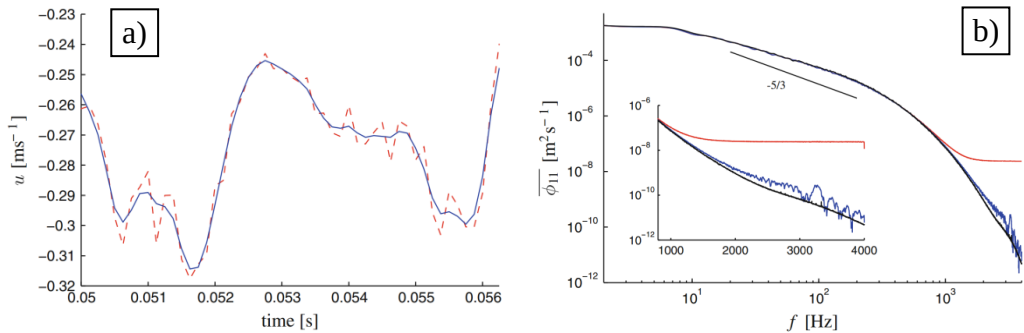


Figure 1.10. Effect of denoising on velocity time series (a) and temporal energy spectrum (b) in a grid turbulence flow. Red line: raw PIV data; blue line: denoised PIV data; black line (in the spectrum plot): hot-wire data. Figure from Oxlade *et al*, 2012.

The present work mainly focuses on the measurement errors in PIV. Errors are defined as the difference between the measured value and the true value of the quantity of interest. Since the latter is typically unknown, also the error is unknown: aim of uncertainty quantification is estimating a possible value of the error magnitude. These concepts will be discussed in detail in section 2.8. On the one hand, the experimenter would like to minimize the errors so to maximize the accuracy of the results. The careful design and choice of the experimental parameters is a key point for the successful experiment. Furthermore, the choice of the image interrogation algorithm and processing parameters is crucial for the accuracy of the final results. Here two important considerations should be made:

1. The optimization of the experimental and processing parameters largely relies upon the user's expertise. In many cases, it is not trivial which choice of those yields more accurate results. A typical example that will be further discussed in the reminder of this work is the selection of the interrogation window size reported in figure 1.11: a smaller window enhances the spatial resolution but yields higher error probability and a noisier velocity field measurement. Thus, the experimenter has to select the interrogation window based on his/her own perception of which result is more accurate: an objective indicator that guides the choice of the processing parameters is currently missing.
2. Due to the widespread use of low repetition rate lasers and CCD cameras, most of the interrogation algorithms have been developed and optimized for single-pair PIV; here, the velocity field is obtained from the analysis of two adjacent image recordings, which sample the tracer particles position only at two time instants. For this case, a thorough investigation has been conducted in the past (see for instance Willert and Gharib, 1991; Keane and Adrian, 1992; Westerweel *et al*, 1997; Scarano and Riethmuller, 2000; Scarano, 2002) and limited room for further enhancement of the measurement accuracy is envisaged. However, the technological developments of the recent years have brought to high-speed PIV systems (Hain *et al*, 2007) that allow time-resolved measurements. The temporal information provided by those can be exploited via advanced multi-frame

algorithms (Hain and Kähler, 2007) for further enhancing the accuracy of the results.[‡]

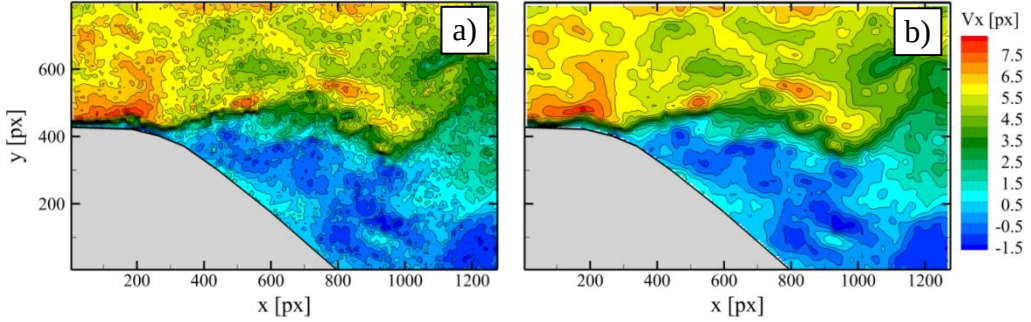


Figure 1.11. Flow around a hill (experiment conducted by Cierpka *et al*, 2013): (a) interrogation window size of 16×16 pixels; (b) interrogation window size of 32×32 pixels. The former result exhibits higher spatial resolution but also higher noise level.

On the other hand, measurement errors need to be estimated in order to assess the goodness of the results. Uncertainty quantification is required for evaluating the significance of the fluctuations in the measurement, i.e. to evaluate the level of noise embedded in the measured velocity. The topic has high relevance especially when PIV data are employed for validation of numerical simulations.

Not only uncertainty quantification provides information on the measurement accuracy, but also may serve as a tool for accuracy enhancement. In fact, the estimated uncertainty is an objective indicator for the optimization of the processing parameters. To date, despite the relevance of the subject, no consolidated approach for the uncertainty quantification of PIV data exists.

[‡] The possibility of using advanced algorithms in time-resolved PIV can be explained with the following analogy. In computer science, compression algorithms are used to reduce the storage requirements of images and videos by reducing the amount of redundant information. Videos are obtained from sequences of images displayed so fast (typically 20 to 30 per second) to yield the illusion of continuous motion. Because the images in a video are so closely linked, not everything in the image changes from frame to frame. Consequently, the video compression can store only the changes in the frame instead of the entire frame, thus yielding a reduction in the size of the video with respect to the ensemble of frames.

The main difference between the two cases is that in TR-PIV the advanced algorithms aim at enhancing the measurement accuracy, while in video compression they are used to reduce the file size.

1.3 Motivation and objectives of the present work

From the discussion above and from what will be shown in chapters 2 and 3, it emerges that two of the major limitations of PIV are the lack of a consolidated uncertainty quantification methodology and the limited accuracy of time-resolved data. Both topics are dealt with in the present work.

Uncertainty quantification of PIV data

In the present discussion, a distinction is drawn between uncertainty estimation methods based on *a-priori* analysis of the measurement chain properties and data-based uncertainty quantification, which is a procedure applied *a-posteriori* with respect to the experiment. The relevance of *a-posteriori* uncertainty quantification of PIV data is twofold: on the one hand, it provides a more thorough control of the measured quantities, allowing an accurate evaluation of the noise level embedded in the measured velocity; on the other hand, it can be used to optimize the processing parameters so to enhance the measurement accuracy. The importance of the subject is confirmed by the numerous recent works on the topic (Timmins *et al*, 2012; Wilson and Smith, 2013-a and -b; Charonko and Vlachos, 2013) and by an international framework that has been set up among different research group, which will be discussed in chapter 5.

To date, a consolidated approach for uncertainty quantification of PIV data is missing. Thus, a major goal of the present work is to develop an uncertainty quantification methodology and assess the reliability of such a procedure.

Advanced multi-frame algorithm for time-resolved PIV

High-speed PIV systems enable time-resolved measurements, from which time-correlated velocity fields are observed. From those, the flow acceleration can be evaluated, which is required by several applications including the determination of the unsteady pressure field and aerodynamic loads (Ghaemi *et al*, 2012; van Oudheusden, 2013).

However, it has been shown that the sensitivity of standard interrogation algorithms for single-pair recordings is not sufficient for accurate acceleration evaluation. In the last years, advanced multi-frame algorithms have been developed that make use of the temporal information to enhance the measurement accuracy. Those algorithms will be discussed in chapters 3 and 6. The relevance of TR-PIV data analysis is evidenced by the third (Stanistlas *et al*, 2008) and fourth (still in progress at the moment of writing this thesis)

international PIV challenges, which both present test cases where the accuracy of multi-frame algorithms for time-resolved PIV is assessed.

Thus, investigating an advanced multi-frame approach that enhances the measurement accuracy and precision and allows accurate acceleration evaluation is one of the objectives of the present work.

Generalized pre-processing technique for time-resolved PIV

Strong laser light reflections on solid interfaces are a major limitation to the accuracy of the measurement near the model surface (Honkanen and Nobach, 2005). When the reflections have stronger intensity than the particle images, they may preclude the correct evaluation of the displacement of the latter. Numerous existing strategies for reducing the detrimental effect of undesired reflections have been proposed in literature and are discussed in chapter 3. Most of the approaches rely upon statistical operators that estimate the image background, which is successively subtracted to the individual recordings.

However, in several measurements the interface image is unsteady, either because of a controlled motion of the model (e.g. pitching airfoil) or because of vibrations of the camera. In this case, standard approaches based on statistical operators fail in removing the image background.

Goal of the present work is investigating a generalized technique that makes use of the temporal information provided by time-resolved measurements to remove the laser light reflections even when they are unsteady.

Treatment of clusters of missing data

When laser lights reflections have not been correctly removed in the images, they may yield clusters of corrupted or missing data in the velocity field, namely gaps. Other sources of gaps are shadows produced by the model or limited optical access of laser or imaging system. Those gaps pose a major limitation for the computation of integral quantities, which are required e.g. for the evaluation of the loads acting on a body or the determination of the noise source in a turbulent flow. The state-of-the-art of the techniques for the reconstruction of the velocity field within the gaps is discussed in chapter 3. However, current techniques only allow reconstructing a velocity distribution with just one peak (a half wave) within the gap. The present work aims at introducing a novel approach that relies upon the solution of the Navier-Stokes equations within small gaps to reconstruct velocity distributions of arbitrary wavelength.

1.4 Outline of the thesis

This chapter has briefly discussed the operational principle of PIV and the evolution of the technique during the last decades. Furthermore, the current limitations of PIV have been outlined and the motivation and objectives of the present work have been defined.

Chapter 2 discusses the fundamental components of PIV experiments, introducing the characteristics and requirements of tracer particles, illumination systems and imaging systems. The mathematical background for the determination of the tracer particles motion via cross-correlation is given; the concepts of dynamic ranges and measurement error and uncertainty are introduced. The state-of-the-art of PIV image analysis is discussed in **chapter 3**, distinguishing the phases of data pre-processing, processing and post-processing.

Chapter 4 introduces a novel strategy for the uncertainty quantification of PIV data, namely the *image-matching approach*. It is demonstrated that the ensemble of particle images within the interrogation window contains sufficient information to retrieve the uncertainty of the measured velocity field. The method is assessed with both Monte Carlo simulations and experimental verification in presence of error sources representative of typical PIV experiments. **Chapter 5** presents the main results of an international collaboration for PIV uncertainty quantification, during which a comparative assessment of existing uncertainty quantification methodologies has been conducted.

In **chapter 6**, a multi-frame algorithm for increasing the dynamic velocity range in time-resolved measurements is proposed. The approach, named *pyramid correlation*, combines the accuracy of measurements conducted at large time separation with the robustness of correlation functions computed from adjacent recordings.

The issue of unsteady laser light reflections on the solid interfaces is dealt in **chapter 7**. It is shown that for several applications a simple filter in the frequency domain is effective in removing the undesired effect of the light reflections while retaining the light scattered by the tracer particles.

The treatment of gaps in PIV data and the reconstruction of the velocity distribution within those based on the solution of the unsteady Navier-Stokes equations is the topic of **chapter 8**.

Finally, a summary of the main results and conclusions from the preceding chapters is reported in **chapter 9**.

CHAPTER 2

PARTICLE IMAGE VELOCIMETRY

Abstract Here the fundamental aspects of particle image velocimetry are discussed, including the characteristics and requirements of tracer particles, illumination systems and imaging systems of PIV experiments. The chapter also addresses the determination of quantitative information on the tracers motion from the recordings. Two parameters related to the measurement quality are introduced: the dynamic spatial range and the dynamic velocity range. Finally the concepts of measurement error and uncertainty are discussed.

2.1 Working principle

Particle image velocimetry measures the fluid velocity from the displacement of tracer particles inserted into the flow and carried by the fluid. The particles are illuminated twice within a short time interval in a planar measurement domain by a light source, typically a laser, and the light scattered is recorded by a camera. A typical layout of a planar PIV measurement is sketched in figure 2.1.

Following the formulation of Westerweel (1997), the local fluid velocity $\mathbf{u}(\mathbf{x}, t)$ is measured indirectly as a function of the tracer displacement $\Delta\mathbf{x}(\mathbf{x}, t)$ occurring in the time interval Δt between two laser pulses (figure 2.2-a); using a forward formula for the determination of the tracer displacement, it is obtained:

$$\Delta\mathbf{x}(\mathbf{x}, t_0, \Delta t) = \int_{t_0}^{t_0 + \Delta t} \mathbf{u}_p(\mathbf{x}, t) dt \quad (2.1)$$

The quantity $\mathbf{u}_p(\mathbf{x}, t)$ represents the tracer particle velocity, which for ideal particle tracers coincides with the local fluid velocity $\mathbf{u}(\mathbf{x}, t)$. Assuming that the laser pulse separation is sufficiently small so that the effects of the flow acceleration during Δt can be neglected, the equation for the particle velocity reads:

$$\mathbf{u}_p(\mathbf{x}, t_0) = \lim_{\Delta t \rightarrow 0} \frac{\Delta\mathbf{x}(\mathbf{x}, t_0, \Delta t)}{\Delta t} \cong \frac{\Delta\mathbf{x}(\mathbf{x}, t_0, \Delta t)}{\Delta t} = \frac{\mathbf{x}(t_0 + \Delta t) - \mathbf{x}(t_0)}{\Delta t} \quad (2.2)$$

The equation (2.2) for $\mathbf{u}_p$ corresponds to the velocity obtained by truncating the Taylor series expansion of $\mathbf{x}(t_0 + \Delta t)$ to the first order term. From equation (2.2) it emerges that the particle velocity can be retrieved from the positions occupied by the particle at two distinct time instants.

So far, ideal tracers have been considered, which exactly follow the fluid motion, do not interact with each other and do not alter the flow or the fluid properties (see figure 2.2-a). Instead, in reality the velocity of tracer particles has always a small departure from that of the surrounding fluid (slip velocity), resulting in a relative motion (usually negligible) between tracers and fluid, as illustrated in figure 2.2-b.

The next sections provide guidelines for the selection of tracer particles that faithfully follow the fluid motion and scatter sufficient light to be distinguished from the background in the image recordings. Subsequently, the main features of typical PIV setups and the evaluation of the tracer particles motion are discussed.

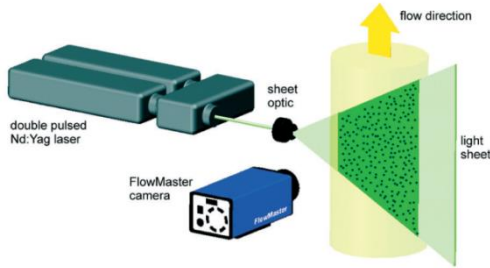


Figure 2.1. Typical layout of a planar PIV measurement system (www.lavision.de).

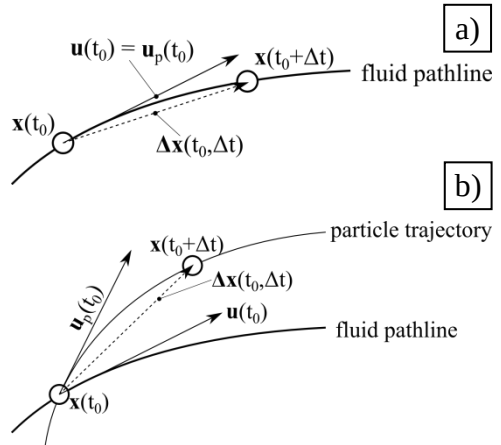


Figure 2.2. Displacement of a tracer particle and fluid pathline (following Westerweel, 1997). (a) Ideal tracer particle (null slip velocity); (b) real tracer particle (non-null slip velocity: the particle trajectory does not coincide with the fluid pathline).

2.2 Tracer particles

2.2.1 Flow tracking characteristics

When external forces (gravitational, centrifugal and electrostatic) are negligible, the motion of a small particle immersed in a fluid can be modelled

as that of a sphere of diameter d_p in a fluid in Stokes flow regime (Melling, 1997). The *slip velocity* $\mathbf{u}_s$, defined as the difference between the particle velocity and the fluid velocity, can be estimated as (Raffel *et al*, 2007):

$$\mathbf{u}_s = \mathbf{u}_p - \mathbf{u} = d_p^2 \frac{(\rho_p - \rho)}{18\mu} \frac{d\mathbf{u}_p}{dt} \quad (2.3)$$

wherein ρ_p is the particle density and ρ and μ are the density and dynamic viscosity of the fluid, respectively. Equation (2.3) shows that perfect tracking (i.e. null slip velocity) may only occur in a steady uniform flow (where the acceleration is null) or for neutrally buoyant particles: $(\rho_p - \rho)/\rho \ll 1$. The former case (null fluid acceleration) has low relevance in fluid dynamics and can be investigated using less expansive point-wise measurement techniques. Instead, the condition of buoyancy neutral particles is easily satisfied for liquid flows, whereas it is typically not achievable in gas flows, where $\rho_p/\rho = O(10^3)$.

To assess the capability of the tracer particles to follow the flow, the *relaxation time* τ_p is defined as response time of the particle to a sudden change in the fluid velocity; from equation (2.3), it is obtained:

$$\tau_p = d_p^2 \frac{(\rho_p - \rho)}{18\mu} \quad (2.4)$$

The relaxation time is function of the particle diameter and density. To obtain tracers that follow the flow faithfully, low values of the relaxation time are desired, which can be achieved either with low particle diameter or with tracer materials having density similar to that of the fluid. The fidelity of the flow tracers in turbulent flows is usually quantified by the particles' Stokes number S_k , defined as the ratio between relaxation time and characteristic flow time scale τ_{flow} :

$$S_k \equiv \frac{\tau_p}{\tau_{\text{flow}}} \quad (2.5)$$

For good tracing capabilities, the particles' Stokes number should not exceed 0.1 (Samimy and Lele, 1991).

When gas flows are considered, the density of the seeding material is often hundreds of times larger than that of the gas (Melling, 1997). Hence, to achieve good tracking capabilities, small particle diameters of the order of $1\text{ }\mu\text{m}$ are usually selected. Common materials employed for seeding gas flows are titanium dioxide (TiO_2), alumina (Al_2O_3), glass, olive oil and di-ethyl-hexyl-sebacate (DEHS) (Melling, 1997; Raffel *et al*, 2007). Typical particle response times are reported to range between less than $1\text{ }\mu\text{s}$ to more than $20\text{ }\mu\text{s}$ (see table 2.1). Solid particles made out of ceramic materials (e.g. Al_2O_3 and TiO_2) are suited for seeding flames, combustion and high temperature flows due to their inertness and high melting point (Willert *et al*, 2008). Non-compact particles (e.g. nanostructured fractal particles) have been recently proposed (Ghaemi *et al*, 2010) for high-speed flows to achieve small relaxation time (below $0.3\text{ }\mu\text{s}$) and scatter enough light to be detectable by the measurement system. For measurements of large scale flows (field of view of the order of 4 m^2), helium-filled soap bubbles have been employed (HFSB, Bosbach *et al*, 2009): those are neutrally buoyant particles of approximately 1 mm diameter where the helium filling compensates for the weight of the soap.

Table 2.1. Examples of seeding particles for gas flows.

Material	$\rho_p\text{ [kg/m}^3\text{]}$	$d_p\text{ [}\mu\text{m]}$	$\tau_p\text{ [}\mu\text{s]}$	Reference
TiO_2	4,230	$0.01\div0.5$	$0.4\text{--}3.7$	Ragni <i>et al</i> (2011a)
Al_2O_3	4,000	0.3	20-28	Urban and Mungal (2001)
Hollow glass	2,600	1.67	22.6	Melling (1997)
Olive oil	970	3	22.5	Melling (1997) [§]
DEHS	912	1	2	Ragni <i>et al</i> (2011a) Kähler <i>et al</i> (2002)

In liquid flows, the neutral buoyancy condition can be met with a wide variety of materials such as polyamide, latex and TiO_2 (Melling, 1997; Raffel *et al*, 2007). Consequently, larger particle diameters of the order of $50\text{ }\mu\text{m}$ can be selected to enhance the light scattering characteristics (Melling, 1997).

[§] Experiment in air at 220 K.

2.2.2 Light scattering properties

PIV images are composed by bright particles on a dark background. The contrast between particle images and background is directly proportional to the light intensity scattered by the tracer particles. The scattered light intensity is a function of the ratio of the refractive index of the particles to that of the fluid, of the particles size, shape and orientation and of polarization and observation angles (Raffel *et al*, 2007). For spherical particles of diameter d_p larger than the wavelength of incident light λ , Mie's scattering theory applies (Mie, 1908). Figure 2.3 illustrates the polar distribution of the scattered light intensity of a 1 μm oil particle in air with light wavelength of 532 nm according to Mie's theory. From the diagram, it emerges that the light is not blocked by the particle, but is scattered in all directions instead. In fact, the maximum light intensity is scattered in the forward direction, while the light scattered in backward and side directions is several orders of magnitude lower. Although it would be advantageous to record images in forward scatter, limitations on depth of field and optical access typically impose to position the imaging system in side scatter. Moreover, acquiring images from an angle between the side and the backward direction typically causes a reduction of the recorded light intensity, as shown in figure 2.4.

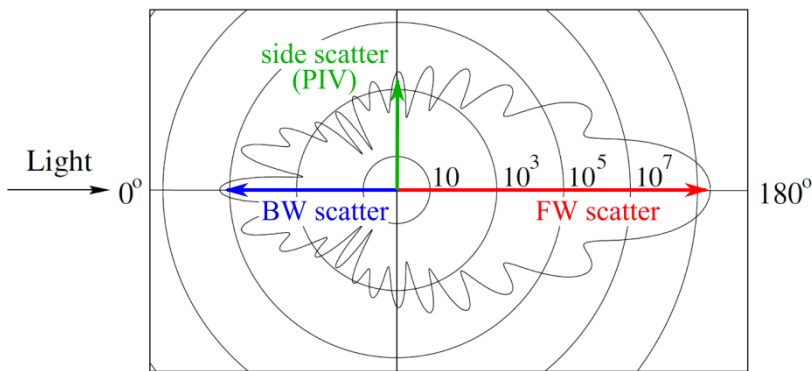


Figure 2.3. Light scattering by a 1 μm oil particle in air, adapted from Raffel *et al* (2007).

Following Mie's theory, the ratio between forward and backward scattering intensity and the overall scattered light intensity rapidly increase with the particle diameter, meaning that larger particles yield greater light scattering.

Furthermore, the scattering efficiency strongly depends on the ratio between particle and fluid refractive indexes. Consequently, ceramic materials as TiO_2 (refractive index 2.5) have better scattering properties in air than DEHS and

olive oil (refractive index 1.5). Moreover, since the refractive index of air is 30% lower than that of water, the scattering of particles in air is greater than that of particles of the same size and material in water (at least one order of magnitude according to Raffel *et al*, 2007). Hence, to achieve comparable scattering performance, larger tracer particles (diameter above $10\text{ }\mu\text{m}$) need to be used in water experiments, which can mostly be accepted because particle and water densities typically match yielding good flow tracking capability.

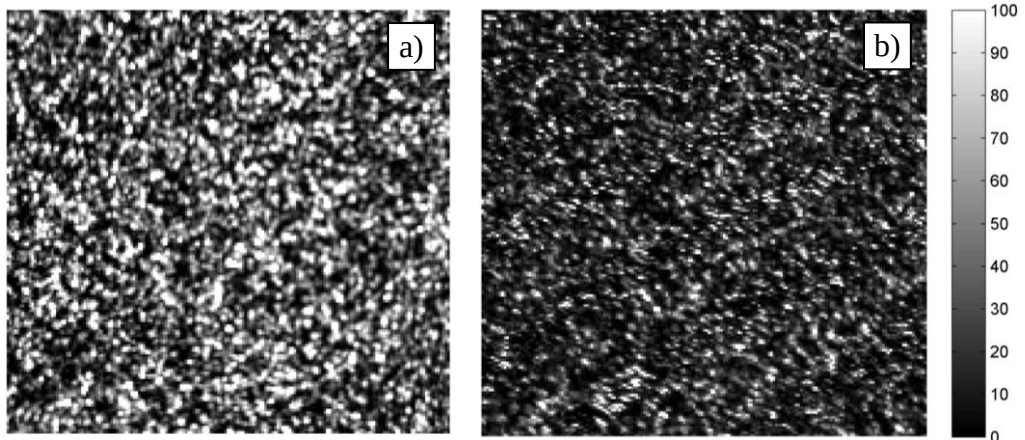


Figure 2.4. Example of PIV images acquired simultaneously in side scatter (a) and at an angle of 45 degrees between side and backward direction (b). The f-number of the lens of both cameras is set to the same value, namely 5.6. In image (a), the recorded light intensity is approximately twice as high as in (b). The images have been acquired within the collaborative framework for PIV uncertainty quantification, which will be discussed in chapter 5.

2.3 Illumination of the flow

The illumination system provides the light that allows the tracer particles to be visible in the recordings. The measurement camera projects the three-dimensional environment onto a two-dimensional image, averaging the spatial information along the camera's optical axis direction (Adrian, 1988), as illustrated in figure 2.5.

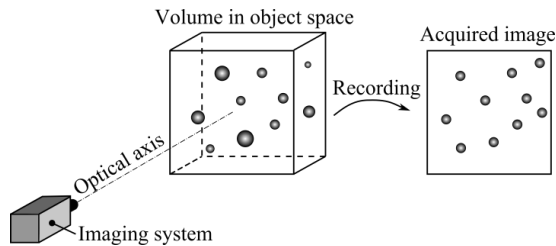


Figure 2.5. Effect of averaging of the information along the optical axis direction.

As a consequence, it is not possible to distinguish the tracer motion occurring at different locations along the optical axis without making use of additional information (e.g. provided by an additional camera in a stereoscopic setup, as discussed by Prasad, 2000). Hence, to obtain a non-ambiguous representation of the fluid motion from the recorded images, the measurement volume must be a slice. Optics composed by spherical and cylindrical lenses are employed to shape the illuminated volume into a sheet, as shown in figure 2.6. The thickness of the measurement volume typically ranges between 1 and 3 mm. For a detailed discussion on lens configurations for the production of a thin light sheet, the reader is referred to the book of Raffel *et al* (2007).

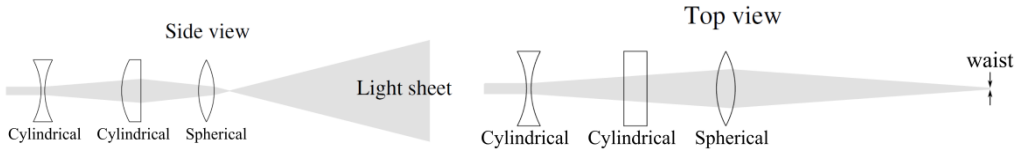


Figure 2.6. Example of optics arrangement composed by two cylindrical lenses and one spherical lens. Illustration adapted from Raffel *et al*, 2007.

Lasers are largely employed in PIV due to their ability to emit monochromatic light with high energy density, which can easily be shaped into thin light sheets by means of spherical and cylindrical lenses without chromatic aberrations. The most common lasers for PIV applications are pulsed Q-switched solid-state lasers, such as Nd:YAG and Nd:YLF. Nd:YAG lasers can provide up to 1 J of monochromatic light (532 nm) with pulse width below 10 ns and repetition rates up to 50 Hz. Instead, Nd:YLF lasers operate in the kilohertz range and therefore are usually employed in high-speed applications. They produce light of comparable wavelength to Nd:YAG lasers (527 nm) but with significantly lower pulse energy (up to 60 mJ at 1 kHz and below 10 mJ at 10 kHz).

Three parameters must be selected during the image acquisition phase: the *light pulse width* δt , the *pulse separation* Δt and the *time interval* between subsequent image pairs ΔT (see figure 2.7). The latter is usually indicated by its inverse, that is the measurement rate or acquisition frequency f_{acq} , and determines whether subsequent velocity fields are correlated or uncorrelated in time.

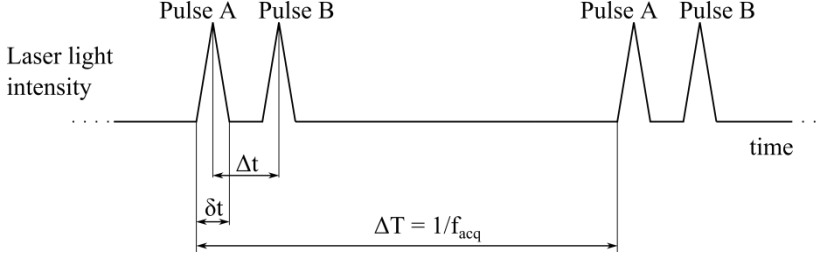


Figure 2.7. Illustration of laser pulse width and pulse separation.

The pulse width determines whether the tracer particles are imaged as dots or streaks. Ideally, the PIV images should record the tracer particles as if they were frozen at one time instant, therefore the particles should not appear as streaks. Hence, an upper bound for the pulse width can be estimated as the time interval during which the particle image displacement is equal to the particle image diameter:

$$\delta t \ll \frac{d_\tau}{M |\mathbf{u}_p|} \quad (2.6)$$

being d_τ the particle image diameter and M the optical magnification factor. Condition (2.6) is verified in most of the experiments conducted with pulsed lasers, which have pulse width far below 1 μ s; instead, it may become relevant for hypersonic flows, where the large velocity may yield particle streaks that increase the uncertainty in velocity and velocity gradient (Ganapathisubramani and Clemens, 2006).

The pulse separation is the time interval elapsing between two adjacent light pulses and must be selected based on considerations on (Keane and Adrian, 1990):

- a) Minimization of particle images loss-of-pairs due to out-of-plane motion (Lin and Perlin, 1998). In a three-dimensional flow, a larger pulse separation yields a larger displacement in the direction normal to the laser sheet: when a particle travels outside the illuminated region, it becomes not visible and leads to a so-called loss-of-pair.
- b) Truncation errors due to fluid acceleration and to the assumption of constant velocity between the laser pulses (Boillot and Prasad, 1996; Westerweel, 1997).
- c) Minimization of the relative error on the displacement estimate. Assuming a measurement error $\varepsilon_{\Delta x}$ approximately constant with the

displacement, a larger pulse separation results in a larger measured displacement and therefore yields lower relative error $|\varepsilon_{\Delta x}|/\Delta x$.

2.4 Imaging of tracer particles

A scattering particle within the light sheet behaves as a point light source. According to Fraunhofer diffraction theory, when the particle is imaged by a camera that mounts an aberration-free lens, the image of the particle does not appear as a point but forms a circular pattern that can be mathematically described by the Airy function (Hecht, 2002). The latter represents the *impulse response* (often defined also *point-spread function*) of the optical system in the ideal case of aberration-free lens. For convenience, it is usually approximated by a Gaussian function (figure 2.8).

The particle image diameter associated with the diffraction effect can be evaluated from the first zero of the Airy function and is equal to (Raffel *et al*, 2007):

$$d_{\text{diff}} = 2.44 f_{\#} (1 + M) \lambda \quad (2.7)$$

wherein $f_{\#}$ is the objective f-number and M represents the optical magnification. Those two quantities, and therefore the particle image diffraction diameter, depend on the characteristics of the optical system (figure 2.9):

$$M = \frac{z_o}{Z_o} \quad (2.8)$$

$$f_{\#} = \frac{f}{D} \quad (2.9)$$

being z_o the image distance (between lens and image plane), Z_o the object distance (between lens and object plane), f the lens focal length and D the aperture diameter. Under the assumption of thin lens (lens thickness negligible compared to the focal length), focal length and optical system distances are related via the thin lens equation (Hecht, 2002):

$$\frac{1}{f} = \frac{1}{z_o} + \frac{1}{Z_o} \quad (2.10)$$

If lens aberration can be neglected, the particle image diameter can be evaluated as (Adrian and Yao, 1984):

$$d_\tau = \sqrt{d_{\text{diff}}^2 + (M d_p)^2} \quad (2.11)$$

wherein $(M d_p)$ is the particle image geometric diameter, that is the projection of the particle diameter from the object plane onto the image plane. In presence of lens aberration, the particle image diameter may differ significantly from the value obtained with equation (2.11), especially for low f-numbers. A detailed discussion on the effect of lens aberration on the imaging of small particles is reported in Raffel *et al* (2007).

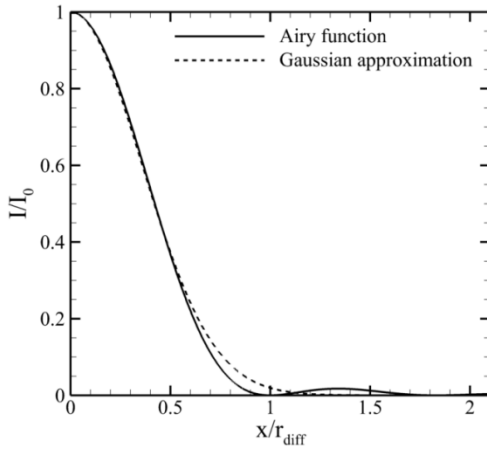


Figure 2.8. Normalized intensity distribution of the Airy function and its approximation via a Gaussian curve; r_{diff} represents the diffraction radius, equal to half of the diffraction diameter.

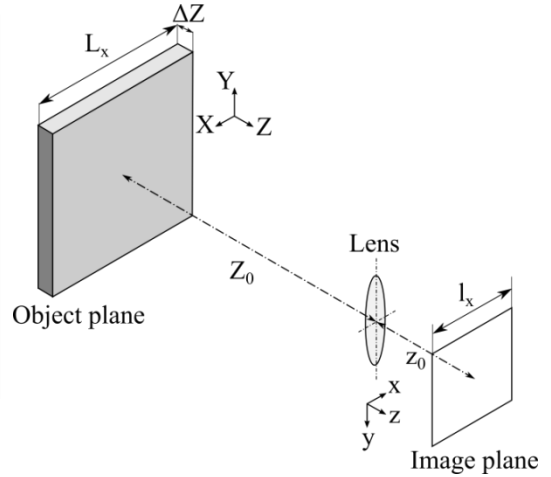


Figure 2.9. Optical arrangement of the PIV system.

Two important imaging parameters are controlled by means of the f-number: the particle image diameter and the depth-of-field. Small and sharp particle images are essential to achieve sufficient contrast between particle images and image background. However, particle images smaller than the sensor's pixel size are under-sampled, i.e. are imaged as individual pixels. As a consequence, the information on the particle position and on its light distribution is irreversibly lost. This effect is known as *peak-locking* (or *pixel-locking*, Westerweel, 1997) and yields systematic errors towards integer values (in pixel units) of the estimated particle displacement, as illustrated in figure 2.10.

To avoid peak-locking errors, the f-number is usually set in such a way that the particle image diameter ranges between two and three pixels.

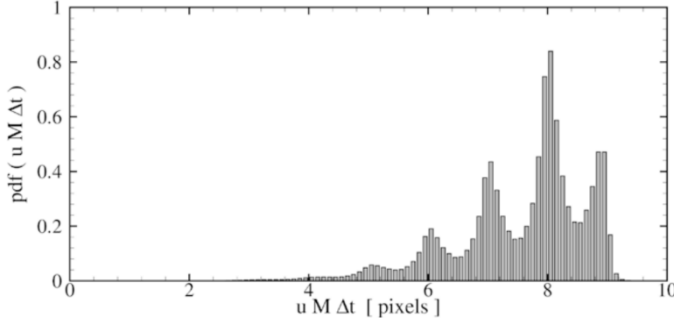


Figure 2.10. Histogram of PIV displacement obtained in a turbulent flow illustrating the peak locking effect associated with the small particle image size: integer displacements have higher frequency of occurrence than non-integer displacements. Figure from Christensen (2004).

The *depth-of-field* δz is defined as the thickness of the region containing in-focus particles in the object space. The equation for the depth-of-field reads (Raffel *et al*, 2007):

$$\delta z = 4.88 \left(\frac{1+M}{M} \right)^2 f_{\#}^2 \lambda \quad (2.12)$$

To minimize the background noise induced by out-of-focus particles, the depth-of-field should be at least equal to the laser sheet thickness.

2.5 Image recording

In early PIV measurements, photographic films were widely used as recording medium to store the optical information. The photographic film consists of silver halide grains in a gelatine support. When photons impinge on silver halide crystals, small patches of metallic silver are formed; those patches undergo development and turn the entire crystal into metallic silver. Areas of the film receiving larger amounts of light undergo the greatest development, resulting in the brightest regions in the image (Raffel *et al*, 2007).

To record the position of the tracer particles at two time instants, the shutter of the photographic camera was opened during two consecutive light pulses. As a result, each image recorded the particle position at two time instants, leading to ambiguity in the evaluation of the displacement direction (figure

2.11). Together with the directional ambiguity, a major limitation of the photographic recording was the lack of on-line control of the imaging parameters: in fact, the images were available only after a photochemical processing and not during the recording phase. Furthermore, the maximum number of PIV photographs taken by the investigators seldom (if ever) exceeded 1,000 (Adrian, 2005), which in most cases is not sufficient for statistical convergence in turbulent flows investigation.

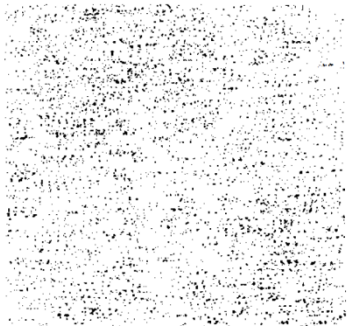


Figure 2.11. Single-frame/double-pulsed photograph of 5- μm Al_2O_3 spheres in a turbulent water channel. Mean flow is from left to right. The image grey-levels have been inverted. Figure from Adrian, 1991.

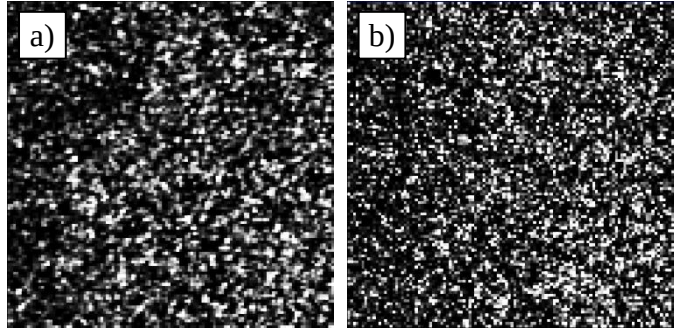


Figure 2.12. Example of images recorded with a CMOS camera (LaVision HighSpeedStar 6, pixel size: 20 μm). (a) Optimal imaging conditions (particle image diameter encompassing 2÷3 pixels); (b) sub-optimal imaging condition (particle image diameter of 1 pixel).

For these reasons and thanks to recent advances in electronic imaging, the use of digital image recording devices has nowadays superseded photographic recording. The most common electronic image sensors are charge couple devices, or CCD, and CMOS devices (Complementary Metal Oxide Semiconductor). Both sensor types consists of an array of sensitive elements, the pixels, that are capable of converting the incoming light (i.e. photons) into electric charge, which is further converted into a voltage and finally into a digital signal (Falchi and Romano, 2009). The sensor size is of the order of $1,000 \times 1,000$ pixels, and each pixel has area of the order of $10 \times 10 \mu\text{m}^2$. CCD sensors are characterized by a very limited number of output nodes, often one, that convert the charge into voltage. As a result, the repetition rate of those sensors is limited to about 10 Hz (Hain *et al*, 2007). On the other hand, each pixel of the CMOS sensors has its own charge-to-voltage conversion and is treated individually, yielding higher acquisition frequency (up to 10 kHz at 1 Mpx resolution); for this reason, CMOS cameras are the standard choice for

time-resolved PIV measurements. To overcome the lower light sensitivity of CMOS sensors associated with the lower fill factor (ratio between optically sensitive area and total area of the pixel, Raffel *et al*, 2007), larger pixel size is typically used (10 to 20 μm against 5 to 7 μm of CCD sensors). Examples of PIV images recorded with a CMOS camera in optimal and sub-optimal imaging conditions are shown in figure 2.12.

Recent developments in electronic imaging have led to a new generation of scientific CMOS (sCMOS) sensor that combines the advantages of up-to-date CCD and CMOS sensor technologies resulting in outstanding image quality and system performance. sCMOS sensors have resolution of about 5 mega-pixels and are characterized by low readout noise (factor 3 to 4 below that of a CCD sensor, www.lavision.de) and high frame rates, of the order of 100 frames per second. As a result, they yield a major enhancement of the image quality especially in measurements characterized by limited laser power and low contrast between particle tracers and background.

2.6 Image interrogation

2.6.1 Motion evaluation techniques

Once the images have been recorded, the fluid motion is evaluated by tracking the displacement of the tracer particles. For this purpose, two approaches are commonly employed: *particle tracking velocimetry* (PTV) and *particle image velocimetry* (PIV). In PTV, individual particle images are tracked throughout the entire recording sequence. Because the spatial resolution of the resulting velocity field is ultimately limited by the spacing between tracer particles, in theory PTV allows achieving the highest spatial resolution. In practice, the capability to correctly identify and link particles in consecutive frames strongly degrades for low ratios between the particles spacing and their displacement (Malik *et al*, 1993). Consequently, large particle spacing is required for unambiguous particle pairing, thus limiting the achievable spatial resolution (figure 2.13). Overall, accuracy and precision of PTV approaches are currently inferior to those of PIV algorithms owing to the uncertainty in locating the particle images centroids, as demonstrated by the results of three international PIV challenges (Stanislas *et al*, 2003, 2005 and 2008).

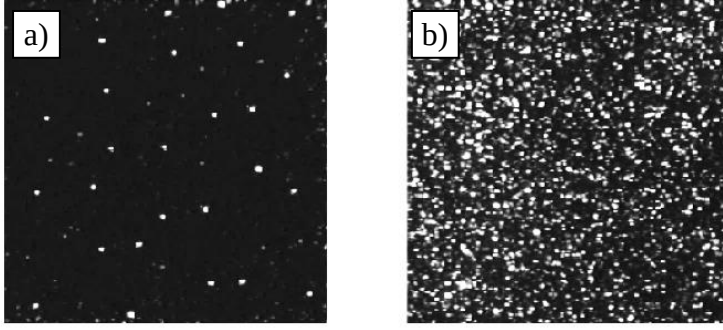


Figure 2.13. Two modes of particle image density: (a) low seeding density (PTV): the image is composed by isolated particles; (b) medium seeding density (PIV): individual particles can be detected but it is not possible to identify image pairs by visual inspection of the recordings. Figure from Raffel *et al* (2007).

On the other hand, particle image velocimetry operates by tracking the motion of ensembles of particle images contained within regularly spaced interrogation windows. In each interrogation window, the average particle image displacement $\Delta \mathbf{x} = (\Delta x, \Delta y)$ is evaluated as the shift that maximizes the degree of matching between the pixel grey levels at time instants t and $t + \Delta t$. In most of the current PIV interrogation techniques, the determination of the displacement relies upon the statistical operator of *cross-correlation*. Details on the mathematical background and implementation are discussed hereafter.

2.6.2 PIV mathematical background

The cross-correlation function C between two continuous intensity distributions $I_a(\mathbf{x}, t)$ and $I_b(\mathbf{x}, t + \Delta t)$ is a measure of the degree of matching of those:

$$C(\Delta \mathbf{x}) = \int I_a(\mathbf{x}) I_b(\mathbf{x} + \Delta \mathbf{x}) d\mathbf{x} \quad (2.13)$$

The evaluation of the particle image displacement is in fact an optimization process applied to C : the displacement $\Delta \mathbf{x}$ that maximizes the cross-correlation function corresponds to the shift that yields the highest matching between I_a and I_b , as shown in figure 2.14.

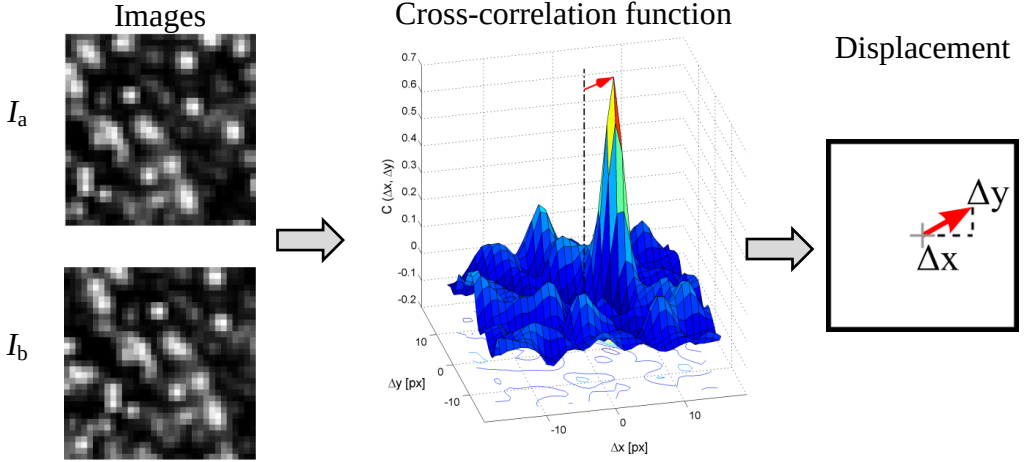


Figure 2.14. Determination of the particle image displacement via cross-correlation.

A detailed mathematical formulation of the properties of the cross-correlation operator applied to PIV images is reported in Keane and Adrian (1992) and Westerweel (1993), among others. In the present work, only the rationale behind the use of cross-correlation is discussed. Following the formulation of Theunissen (2010), assume that the intensity distribution I_a is composed by N particle images, the i -th particle image being located at $\mathbf{x}_i$, represented as an intensity impulse and having peak intensity I_i :

$$I_a(\mathbf{x}) = \sum_{i=1}^N I_i \cdot \delta(\mathbf{x} - \mathbf{x}_i) \quad (2.14)$$

wherein $\delta(\mathbf{x})$ is the Dirac delta function. Under the hypothesis that the tracer particles undergo a uniform in-plane translation $\mathbf{d}$ between the two exposures, the equation for the image intensity distribution at time $t + \Delta t$ reads:

$$I_b(\mathbf{x}) = \sum_{i=1}^N I_i \cdot \delta(\mathbf{x} - \mathbf{x}_i - \mathbf{d}) \quad (2.15)$$

Thus, the cross-correlation function of the two intensity distributions is given by:

$$C(\Delta \mathbf{x}) = \sum_{i=1}^N \sum_{j=1}^N \left[I_i I_j \int \delta(\mathbf{x} - \mathbf{x}_i) \delta(\mathbf{x} - \mathbf{x}_j - \mathbf{d} + \Delta \mathbf{x}) d\mathbf{x} \right] \quad (2.16)$$

Due to the definition of the Dirac delta function, the integral is non-null if and only if:

$$\mathbf{x}_i = \mathbf{x}_j + \mathbf{d} - \Delta\mathbf{x} \quad (2.17)$$

Two cases occur that yield non-null values of the cross-correlation function:

- a) $\mathbf{x}_i = \mathbf{x}_j$. This case represents the condition when the particle images of I_a and I_b are correctly paired, which based on equation (2.17) occurs when $\Delta\mathbf{x} = \mathbf{d}$, i.e. when the estimated shift is equal to the particle image translation. In this case, the cross-correlation function reaches the global maximum $C_{\max} = C(\mathbf{d}) = \sum_{i=1}^N (I_i)^2$.
- b) $\mathbf{x}_i \neq \mathbf{x}_j$. Following equation (2.17), this condition corresponds to $\Delta\mathbf{x} \neq \mathbf{d}$. The cross-correlation function is non-null due to contributions ascribed to particles of image I_a that correlate with different particles (and not with themselves) in image I_b . This condition is referred to as *spurious pairing* of the particle images.

Hence, the determination of the displacement $\Delta\mathbf{x}$ that leads to the global maximum of the correlation function allows evaluating the average particle displacement within the interrogation spot.

2.6.3 Discrete cross-correlation

Due to the use of digital image recording systems, the images I_a and I_b are spatially discretized at integer pixel locations (i, j) . The grey level in each pixel corresponds to the integral of the light gathered by the pixel. For two discrete image distributions, the equation for the (discrete) cross-correlation function reads:

$$C(m, n) = \sum_{i=1}^{Ws} \sum_{j=1}^{Ws} I_a(i, j) I_b(i + m, j + n) \quad (2.18)$$

wherein Ws is the linear size of the interrogation window (assumed to be squared). As for the continuous case, an optimization process is conducted to determine the displacement that maximizes C .

Huang *et al* (1997) report that the cross-correlation function resulting from equation (2.18) is sensitive to changes in intensities between I_a and I_b (e.g. associated with a linear transformation of the intensity). To overcome this issue, the normalized cross-correlation function is defined, which assumes values only in the range $[-1, 1]$:

$$C(m, n) = \frac{\sum_{i=1}^{Ws} \sum_{j=1}^{Ws} [I_a(i, j) - \bar{I}_a] \cdot [I_b(i + m, j + n) - \bar{I}_b]}{\sqrt{\sum_{i=1}^{Ws} \sum_{j=1}^{Ws} [I_a(i, j) - \bar{I}_a]^2 \cdot \sum_{i=1}^{Ws} \sum_{j=1}^{Ws} [I_b(i, j) - \bar{I}_b]^2}} \quad (2.19)$$

being $\bar{I}_a$ and $\bar{I}_b$ the spatial average of I_a and I_b , respectively, within the interrogation window:

$$\bar{I} = \frac{1}{Ws^2} \cdot \sum_{i=1}^{Ws} \sum_{j=1}^{Ws} I(i, j) \quad (2.20)$$

The computation of the cross-correlation via the normalized function (2.19) has the advantage of making C independent of linear transformations. Furthermore, the spatial mean subtraction removes the DC component of the signal that would otherwise yield a significant systematic error (Huang *et al*, 1997).

The calculation of C through equation (2.19) is referred to as *direct cross-correlation* (DCC). An efficient way to obtain the same result with lower computational cost relies upon the convolution theorem (De Groot, 1989) and the use of the fast Fourier transform algorithm (Raffel *et al*, 2007).

2.6.4 Estimation of the fractional displacement

The cross-correlation function computed with equations (2.19) is a discrete function. As a result, if the particle image displacement is retrieved only from the maximum of C , an integer displacement in pixel units would be estimated. In this case, the displacement resolution would be half a pixel, due to the uncertainty in locating the cross-correlation function peak. This means that, for a typical displacement of 10 pixels, a relative uncertainty of 5% would arise just from the discrete nature of the correlation function. It is evident that

a measurement of this kind would not be sufficiently accurate for most applications.

However, when the correlation peak covers more than one pixel, the displacement can be determined with sub-pixel accuracy by peak interpolation. The rationale behind the peak interpolation relies upon the use of information not only from the correlation maximum, but also from neighbouring samples to achieve a more accurate estimate of the correlation peak location (figure 2.15). Several peak fitting algorithms have been proposed in literature, such as the centre of mass method, the parabolic fit and the Gaussian fit (Raffel *et al*, 2007). The equation for the Gaussian peak fit for the horizontal displacement reads (Willert and Gharib, 1991):

$$\Delta x = i + \frac{1}{2} \frac{\ln C(i-1, j) - \ln C(i+1, j)}{[\ln C(i-1, j) + \ln C(i+1, j) - 2 \ln C(i, j)]} \quad (2.21)$$

where (i, j) is the discrete location of the cross-correlation maximum.

The 3-point Gaussian peak fit algorithm is the most frequently implemented due to its accuracy in the estimation of the actual peak shape. In fact, as discussed in section 2.4, for diffraction limited optics the particle image is mathematically described by the Airy function, which can be accurately approximated by a Gaussian function. Consequently, the correlation function peak has a Gaussian shape because it results from the correlation of Gaussian functions.

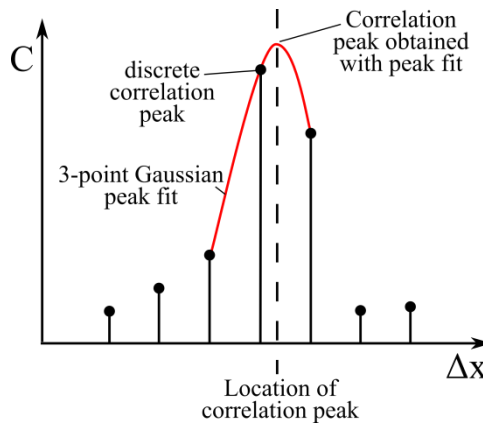


Figure 2.15. Sub-pixel interpolation of the correlation peak.

2.7 PIV dynamic ranges

The performance of a PIV measurement is typically expressed in terms of *dynamic spatial range DSR* and *dynamic velocity range DVR* (Adrian, 1997), which are related to the measurement spatial resolution and the velocity resolution and accuracy, respectively. The dynamic spatial range is defined as the ratio of the largest to the smallest resolvable spatial variation. Since the largest measurable wavelength is equal to the field-of-view in the object space, and the smallest resolvable variation is represented by the interrogation window size, the equation for the *DSR* reads:

$$DSR = \frac{M L_x}{W_s} = \frac{l_x}{W_s} \quad (2.22)$$

where L_x and l_x are the field-of-view linear dimensions in the object space and in the image space, respectively.

In practice, the dynamic spatial range coincides with the number of independent vector measurements (i.e. resulting from adjacent interrogation windows) that can be made across the linear dimension of the field-of-view. The dynamic spatial range is indicative of the ability of the PIV system to measure small-scale variations embedded in larger scale motions. Considering a sensor of 1 Mpx resolution and assuming that the PIV interrogation is conducted using interrogation windows of 32×32 px² size, a typical figure of approximately 30 for the *DSR* is obtained. Higher values exceeding 100 are achieved either by using larger sensors or by decreasing the interrogation window size (Westerweel *et al*, 2013).

The dynamic velocity range is defined as the ratio between the largest measurable displacement (viz. velocity) $\Delta x_{\max}$ and the smallest resolvable displacement fluctuation $\sigma_{\Delta x}$ (which in fact corresponds to the measurement uncertainty):

$$DVR = \frac{\Delta x_{\max}}{\sigma_{\Delta x}} \quad (2.23)$$

Conventional PIV systems with state-of-the-art interrogation algorithms achieve values of *DVR* up to 200-300 (Hain and Kähler, 2007). However, lower *DVR* (typically in the range 50-100) are obtained when using high-speed PIV systems due to the lower image quality of CMOS cameras and the weaker illumination of Nd:YLF lasers (Stanislas *et al*, 2008).

As suggested by Adrian (1997), the product $DSR \cdot DVR$ is often used to express the capability of a PIV system to have both a large dynamic spatial range and a large velocity range:

$$DSR \cdot DVR = \frac{l_x \cdot \Delta x_{\max}}{Ws \cdot \sigma_{\Delta x}} \quad (2.24)$$

From equation (2.24) it emerges that the quality of a PIV measurement depends upon four parameters:

1. The sensor size l_x
2. The maximum measured displacement $\Delta x_{\max}$, which can be tuned with the laser pulse separation Δt
3. The interrogation window size Ws
4. The smallest resolvable displacement fluctuation $\sigma_{\Delta x}$, which corresponds to the measurement uncertainty.

While the determination of the former three parameters is straightforward, the evaluation of the measurement uncertainty is far from trivial and requires a dedicated investigation, as it will be discussed in the remainder of this work.

2.8 PIV errors and uncertainty

Errors are defined as the difference between the measured velocity and the true (unknown) value. Since the velocity is computed from the particle image displacement during the time interval Δt , the equation for the measurement error reads:

$$\varepsilon_u = u_{\text{meas}} - u = u_{\text{meas}} \cdot \sqrt{\left(\frac{\varepsilon_{\Delta x}}{\Delta x}\right)^2 + \left(\frac{\varepsilon_{\Delta t}}{\Delta t}\right)^2} \quad (2.25)$$

being u and u_{meas} the true and the measured velocities, respectively, and $\varepsilon_{\Delta x}$ and $\varepsilon_{\Delta t}$ the errors on measured displacement and pulse separation. The typical error on the laser pulse separation is below one nanosecond, while Δt typically exceeds one microsecond; consequently, the relative error on the time separation is usually below 0.1%. In contrast, the typical measured displacement is about 10 pixels, while the uncertainty on the displacement is of the order of 0.1 pixels. As a result, the relative error on the displacement is of the order of 1% and dominates the error on the measured velocity (Raffel *et*

al, 2007). Although this error may be considered acceptable for the instantaneous velocity, it may preclude the accurate evaluation of statistical quantities such as the Reynolds stresses and other derived quantities, as acceleration, velocity gradient, divergence and vorticity.

It is common practice to distinguish the errors into *systematic* (or bias) and *random* (Coleman and Steele, 2009). The former are defined as errors that are a fixed function of their sources, such as calibration errors and truncation errors (errors occurring when a fluid parcel has a non-null acceleration due to the assumption of linear trajectory and constant velocity). A well-known example of systematic error in digital PIV is the *peak-locking* (Westerweel, 1997), which arises when the particle image diameter is of the order of one pixel or below: in this case, the peak-fitting algorithm fails in evaluating the particle image displacement with sub-pixel accuracy, yielding a systematic error towards the closest integer displacement, as it has been shown in figure 2.10. The random errors, instead, are different for each measurement and are associated with noise in the recordings, out-of-plane motion, low tracer density, displacement gradients and poor image quantization, among other error sources.

Those errors typically range between 0.03 and 0.1 pixels, which makes their identification challenging. In contrast, *outliers* are erroneous displacement estimates occurring when the interrogation windows contain insufficient particle image pairs and high noise level, resulting in a correlation function where the random peaks dominate the true displacement peak. The error associated with the outliers is of the order of several pixels and therefore yields a major bias on the flow statistics (time average and in particular Reynolds stresses); however, the large error magnitude makes the outliers easy to detect. Common strategies for outliers detection are discussed in section 3.4. Figure 2.16 illustrates three velocity time series measured by PIV affected by different error sources, namely random noise, outliers and peak-locking.

The *uncertainty* U is an estimate of the interval ($\pm U$) that likely contains the magnitude of the error affecting the measurement, as illustrated in figure 2.17. Following the definition of Coleman and Steele (2009), the *standard uncertainty* U_s is an estimate of the standard deviation of the actual error distribution. The definition of standard uncertainty does not require any hypothesis on the error distribution, therefore it is not possible to determine the probability that the error magnitude is contained within the uncertainty bounds.

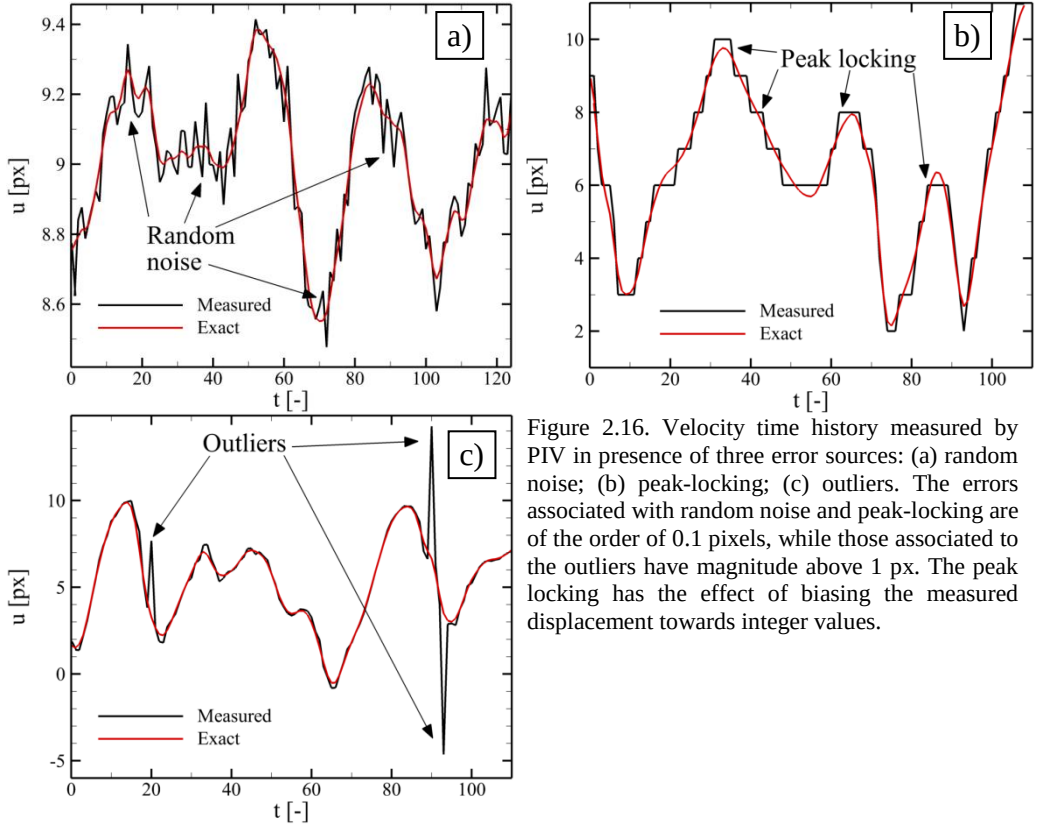


Figure 2.16. Velocity time history measured by PIV in presence of three error sources: (a) random noise; (b) peak-locking; (c) outliers. The errors associated with random noise and peak-locking are of the order of 0.1 pixels, while those associated to the outliers have magnitude above 1 px. The peak locking has the effect of biasing the measured displacement towards integer values.

Hence, the *expanded uncertainty estimate* U is introduced so that we can say that the error magnitude falls within $\pm U$ with probability P , which is referred to as *confidence level*. The expanded uncertainty is calculated from the standard uncertainty as:

$$U = k \cdot U_s \quad (2.26)$$

where the *coverage factor* k depends upon the error distribution and the confidence level. For example, assuming that the error has a Gaussian distribution, $k = 1$ and $k = 1.96$ yield a confidence level of 68.3% and 95%, respectively.

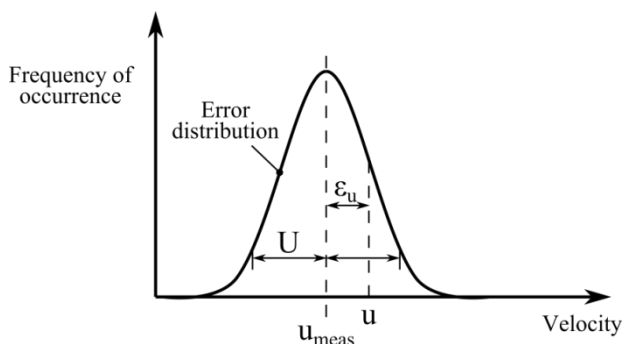


Figure 2.17. Schematic representation of measured velocity, true velocity, measurement error and uncertainty.

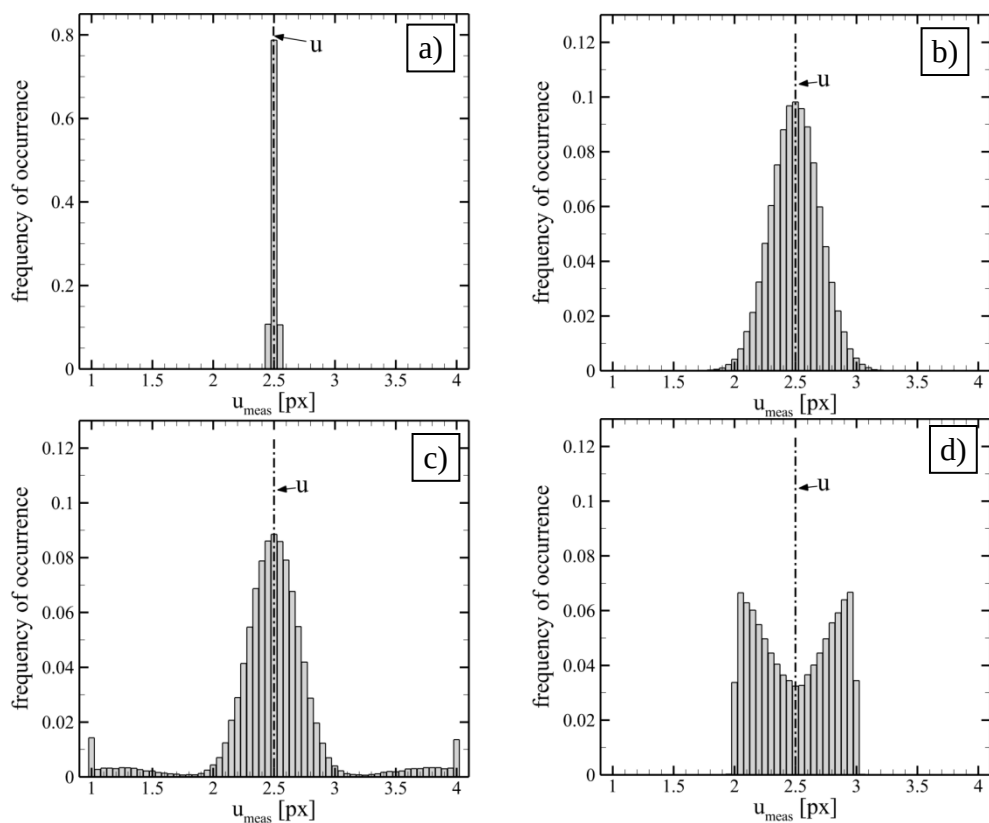


Figure 2.18. Measured velocity distribution in presence of different error sources. (a) Minor noise level; (b) large random noise in the measurement; (c) presence of outliers; (d) peak-locking.

Figure 2.18 illustrates how different error sources affect the distribution of the measured velocity and in turn of the measurement error; in the example,

the flow is assumed to be steady, hence the actual velocity has a constant value u . When the measurement noise level is minor, the measured velocity exhibits a narrow distribution centred around the actual velocity value, which in the example is $u = 2.5$ px (figure 2.18-a); in contrast, large noise magnitude has the effect of broadening the velocity distribution (figure 2.18-b). When outliers are present, the distribution assumes larger values at the tails, owing to the great discrepancy (often several pixels) between exact velocity and measured velocity (figure 2.18-c). The two peaks at the edges of figure 2.18-c ($u_{\text{meas}} = 1$ px and 4 px) indicate that for a significant number of samples the measured velocity is below 1 pixel or exceeding 4 pixels. Finally, when the measurement is characterized by peak-locking errors, the velocity distribution exhibits two distinct peaks at integer velocity values, in the example $u_{\text{meas}} = 2$ and 3 px (figure 2.18-d).

2.9 Concluding remarks

The background of particle image velocimetry has been discussed, introducing the fundamental components of the setup, the principle of image interrogation and the concepts of dynamic ranges, error and uncertainty.

It has been shown that quantitative information on the fluid motion can be obtained by sampling the position of tracers particles in the fluid at two time instants and measuring the displacement of those within the small time interval. Several forms of error contribute to the overall uncertainty of the measurement, such as random errors, systematic errors and outliers.

Numerous studies reported in literature have focused on the evaluation and minimization of those errors, aiming at increasing the dynamic velocity range of the measurement. The approaches proposed in literature include enhancement of the quality of the PIV recordings, advanced interrogation algorithms and data validation methodologies for the detection and removal of spurious measurements. The state-of-the-art of those approaches will be the topic of chapter 3.

CHAPTER 3

STATE-OF-THE-ART OF PIV IMAGE ANALYSIS

Abstract The present chapter covers the state-of-the-art of the approaches and algorithms used to extract quantitative information on the fluid motion from the PIV recordings. PIV image analysis consists of three fundamental steps:

- *Digital image processing*, often named *data pre-processing*, is used to enhance the image quality by strengthening the tracer particles signal with respect to background noise and unwanted light reflections.
- *Data processing* is the actual interrogation of the PIV images that allows obtaining quantitative information on the fluid motion.
- *Data post-processing* is the series of operations conducted for vector validation, outlier removal, data refill and estimation of the measurement accuracy.

3.1 Data pre-processing: digital image processing

Ideal PIV recordings are composed by bright particle images superimposed on a perfectly dark background; consequently, the particle images are easily detectable due to the high image contrast. However, in most practical applications the image background is not perfectly dark, e.g. due to thermal noise in digital imaging cameras. Furthermore, the background intensity may be non-uniform as a result of spatial variations of the illumination light intensity or due to light reflections from nearby solid objects. The aim of digital image processing is twofold: a) remove undesired effects that deteriorate the image quality; b) amplify any favourable aspect of the image that carries information on the fluid motion. These two processes are referred to as *image restoration* and *image enhancement*, respectively, and are discussed in the reminder of this section.

3.1.1 Image restoration

Image restoration refers to the process of removing the unwanted background from the images. In certain cases, the background removal is simply achieved by recording an image without tracer particles and subtracting it from all the PIV recordings. When a background image has not been acquired, an estimate of that can be computed as the minimum or the average of the intensity time-series at each pixel location, as illustrated in figure 3.1-a

and -b. In stationary problems (no moving interfaces or pulse-to-pulse laser intensity variations) the time history minimum or average intensity is a good estimator for the background, which can be removed by subtraction from the instantaneous recordings (figure 3.1-c). However, the time-average intensity subtraction may lead to removing particle images from the recordings, thus reducing the information on the fluid motion contained in the images (Adrian and Westerweel, 2011).

In practice, effective background removal without particles loss is achieved by normalizing the pixel intensity with respect to the time-averaged value, which allows homogenizing the signal variance in the image (Adrian and Westerweel, 2011), as shown in figure 3.1-d.

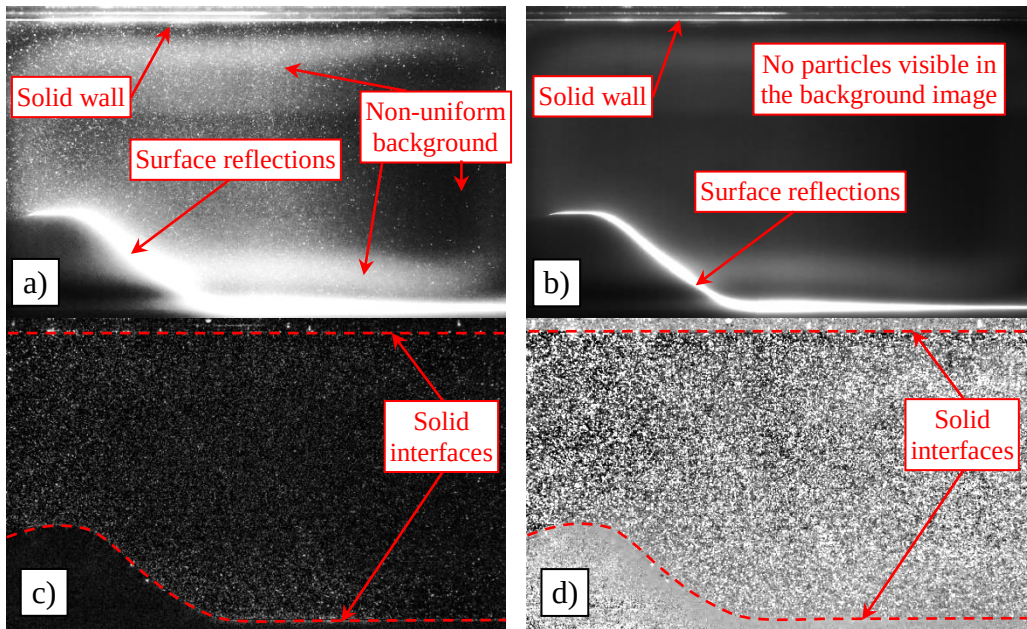


Figure 3.1. PIV recording of a water flow around a hill. (a) Raw image; (b) background obtained as time-averaged intensity; (c) image after subtraction of the time-averaged intensity; (d) image after intensity normalization with respect to the time-averaged value. Experiment conducted by Cierpka *et al* (2013).

Nevertheless, those approaches are typically not effective when the illumination intensity fluctuates between subsequent exposures or in presence of moving solid interfaces. In these circumstances, statistical approaches applied to the ensemble of the recordings fail to produce an accurate estimate of the background. On the other hand, procedures that estimate the background from instantaneous images can be employed. Most approaches rely on the consideration that the particle images have shorter length scales

(typically up to 3 to 5 pixels) than the background, which may cover the entire image. Therefore, the contribution of the particle images can be isolated by applying a spatial high-pass filter to the image, where the filter kernel must be larger than the particle image size. Several filters have been proposed in literature and implemented on commercial software, such as the top-hat filter (see figure 3.2), the Gaussian filter and the median filter, with the latter exhibiting the highest performance even in presence of sharp interfaces (Adrian and Westerweel, 2011).

In contrast, noisy fluctuations associated with thermal effects in digital recording are mainly uncorrelated in space and can be attenuated by low-pass filtering of the image. Guezennec and Kiritsis (1990) proposed the use of a 3×3 uniform smoothing filter as a good compromise between noise reduction and inherent blurring resulting from the use of the filter.

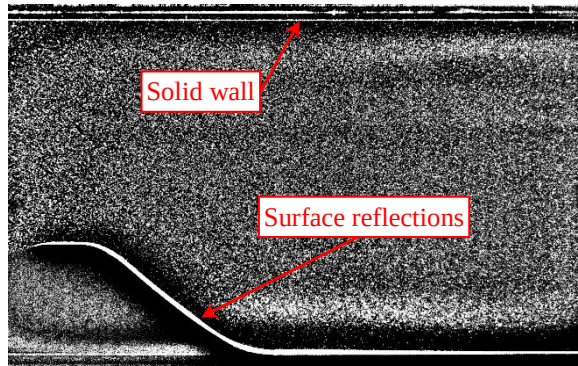


Figure 3.2. Raw image of figure 3.1-a after subtraction of the spatial sliding average intensity in a kernel of 11×11 pixels.

3.1.2 Image enhancement

Image enhancement methods aim at increasing the detectability of the particle images by improving the contrast of PIV recordings. Two techniques are discussed here: the *histogram equalization* and the *min-max filtering*. The histogram equalization is a standard technique used in digital image processing for maximizing the image contrast (Jain, 1989). The histogram of an image is the distribution of the intensity level values within the image. Usually, this distribution is non-uniform, meaning that certain pixel grey values occur more frequently than others. The histogram equalization allows redistributing the grey values over the total range, so that the intensity distribution becomes approximately uniform, allowing areas of lower local contrast to gain higher contrast (see figure 3.3). Although this technique is

useful for high-seeding density recordings, it is not suitable for low-seeding density images because it reassigns non-background pixels with the same grey-level value, thus causing the loss of a significant amount of information on the particle images intensity variations (Adrian and Westerweel, 2011).

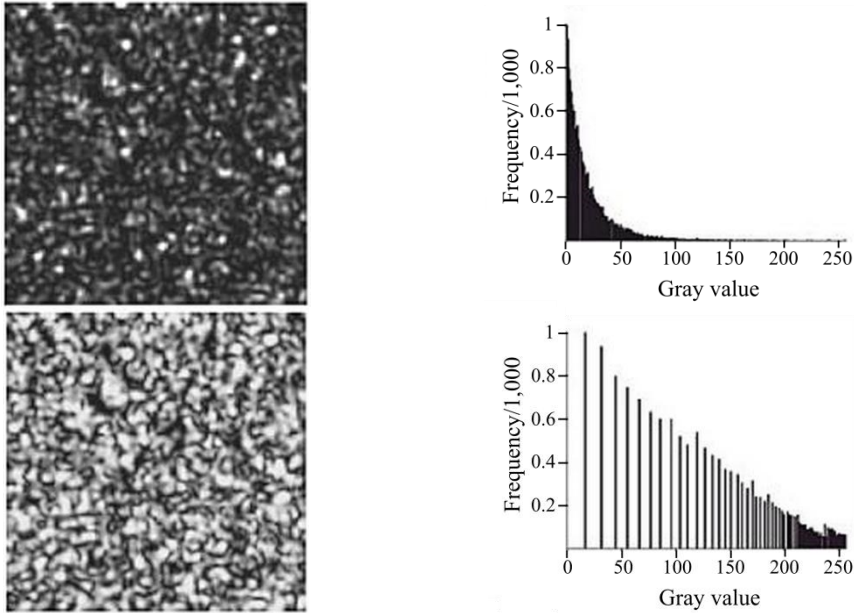


Figure 3.3. Histogram equalization of a speckle image. Top row: raw image (left) and its intensity histogram (right). Bottom row: image after histogram equalization. From Adrian and Westerweel (2011).

An alternative approach for the normalization of the image contrast is the min-max filter proposed by Westerweel (1993). This is a nonlinear filter that determines the upper and lower envelopes of the local image intensity, subtracts the lower envelope, and then normalizes the local image intensity with respect to the difference between upper and lower envelopes. The rationale behind the min-max filter relies upon the removal of the local background (lower envelope subtraction) and the normalization with respect to the local image contrast (division by the difference between upper and lower envelopes). Details on the implementation are reported in Adrian and Westerweel (2011). As illustrated in figure 3.4, the min-max filter allows enhancing the image contrast and equalizing the intensity distribution even in presence of non-uniform illumination. The main limitation of the approach is the poor performance in the vicinity of sharp interfaces, where particle images tend to be removed due to the large estimated local image contrast.

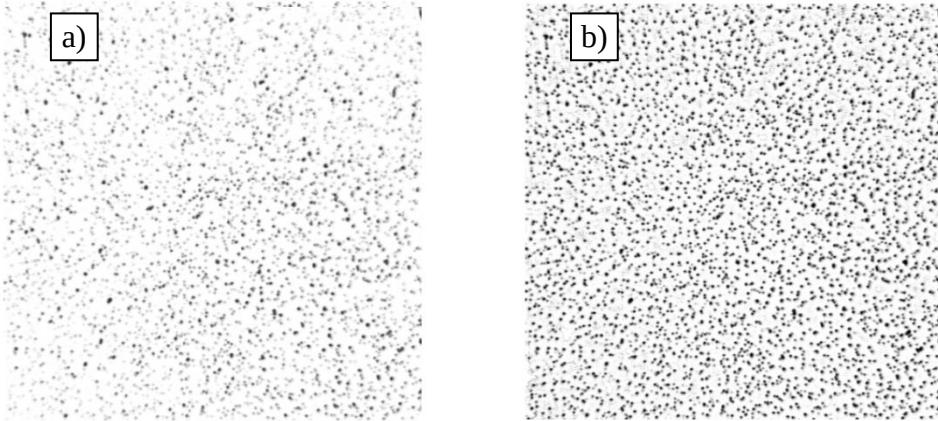


Figure 3.4. PIV picture with non-uniform illumination (the grey values have been inverted); (a) raw image; (b) image after min-max filter. Illustration from Westerweel (1993).

3.2 Data processing: evaluation of tracer motion

The basic principle of PIV image interrogation via cross-correlation has been presented in chapter 2. This approach is referred to as *single-step cross-correlation analysis* and can be divided into the following steps:

- a) Division of two consecutive PIV recordings into interrogation windows
- b) Cross-correlation between corresponding interrogation windows
- c) Determination of the discrete position of the peak of the cross-correlation function, which corresponds to the average particle image displacement within the interrogation window
- d) Evaluation of the fractional (viz. sub-pixel) displacement via cross-correlation peak fitting
- e) Determination of the velocity from the displacement, knowing the time separation between the two exposures and the optical magnification.

The single-step cross-correlation analysis is suitable for cases where the variation of the particle image displacement within the interrogation window is negligible. However, many flows investigated by PIV involve the presence of velocity gradients which are responsible of non-uniform particle image displacements. In this case, the cross-correlation analysis may fail in determining the average displacement: in fact, the non-uniform particle motion yields multiple peaks of the cross-correlation function which are associated with the different displacements occurring within the interrogation window (figure 3.5). For preserving the displacement peak detectability, Keane and

Adrian (1990) suggested an upper limit of 5% for the displacement differences within the interrogation window.

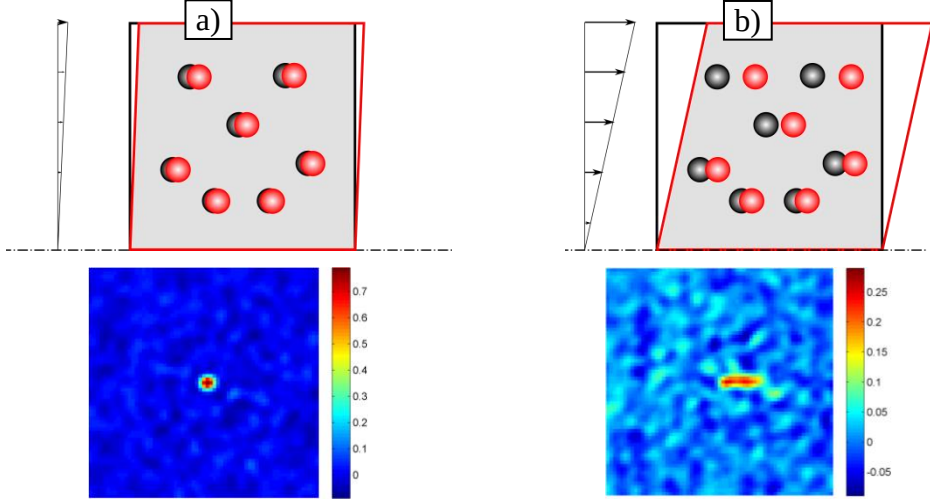


Figure 3.5. Weak (a, 0.03 pixels/pixel) and strong (b, 0.2 pixels/pixel) velocity gradient with typical correlation functions. The particle images of the first image are displayed in grey, those of the second image in red. The high velocity gradient decreases the number of paired particle images (the common area between the two interrogation window is depicted in grey), yielding a cross-correlation peak that broadens in the shear direction.

Furthermore, the performance of the PIV interrogation is strongly related to the amount of particle images correctly paired. The *image density* N_1 is defined as the average number of particle images contained within an interrogation window (Adrian and Yao, 1984):

$$N_1 = \frac{c \cdot \Delta z_0}{M^2} W_s^2 \quad (3.1)$$

where c is the particles concentration, Δz_0 the laser sheet thickness, M the optical magnification and W_s the interrogation window linear size.

Because of the fluid motion, not all the particles appear within the same interrogation window in both PIV exposures: some of the particles travel either outside of the interrogation window or outside of the laser sheet, as illustrated in figure 3.6.

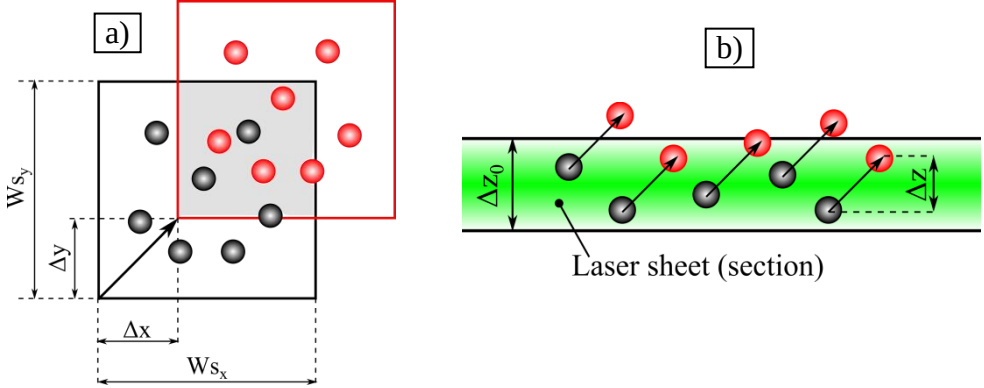


Figure 3.6. Illustration of the in-plane (a) and out-of-plane (b) loss of pairs. The particle images of the first image are displayed in grey, those of the second image in red.

Hence, the *mean effective particle image density* is defined as $N_I F_I F_O$, being F_I and F_O two parameters that account for the fraction of particle pairs lost due to in-plane and out-of-plane motion, respectively (Keane and Adrian, 1992):

$$F_I = \left(1 - \frac{\Delta x}{W_{s_x}}\right) \left(1 - \frac{\Delta y}{W_{s_y}}\right), \quad F_O = \left(1 - \frac{\Delta z}{\Delta z_0}\right) \quad (3.2)$$

where W_{s_x} and W_{s_y} are the interrogation window dimensions in x - and y -direction, respectively.

For values of $N_I F_I F_O$ above 10, the probability of valid peak detection exceeds 90%, meaning that in more than 90% of the cases the cross-correlation peak corresponds to the average particle displacement (Keane and Adrian, 1992).

The interrogation window size, the laser pulse separation and, to some extent, the laser sheet thickness should be optimized in order to minimize the amount of particle images lost due to in-plane and out-of-plane motion, thus enhancing the probability of valid peak detection. A general recommendation widely accepted among the PIV community is the so-called *one-quarter rule*: the in-plane displacement should be less than one quarter of the interrogation window size and the out-of-plane displacement should be less than one quarter of the laser sheet thickness (Westerweel, 1997).

Note that the use of a fixed interrogation window size is a major limitation of the single-pass cross-correlation analysis. First of all, the presence of velocity gradients biases the estimated displacement towards smaller values,

because the particles moving at low velocity tend to remain within the interrogation window, contrarily to those moving at large velocity. Moreover, a large interrogation window, which is required for fulfilling the one-quarter rule and minimize the in-plane loss-of-pairs, increases the errors associated with *spatial modulation*. In fact, it has been shown that the displacement measured by image cross-correlation is approximately equal to that evaluated by applying a moving average filter to the exact displacement field (Scarano and Riethmuller, 2000; Nogueira *et al*, 2005; Schrijer and Scarano, 2008). Hence, the frequency response of the cross-correlation operator is equivalent to that of a moving average filter having a kernel size equal to the interrogation window linear size W_s . This frequency response is given by the sinc function, which is illustrated in figure 3.7. As a consequence, amplitudes of signals having wavelength λ equal to or smaller than twice the interrogation window size are modulated by approximately 40% or above.

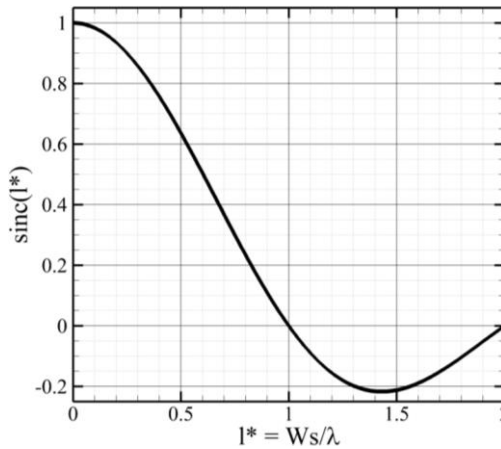


Figure 3.7. Amplitude response of the cross-correlation operator modelled by sinc function. W_s is the interrogation window linear size, λ is the signal wavelength, $l^* = W_s/\lambda$ is the normalized window size.

3.2.1 Multi-grid iterative approaches for single-pair recordings

Iterative approaches have been conceived with the purpose of compensating for the loss-of-pairs due to in-plane motion. The basic idea is to provide the interrogation windows with two (in-plane translation) or more (in-plane translation and deformation) degrees of freedom to maximize the number of particle images that pair correctly. The local displacement or deformation of the interrogation window is determined by the flow field estimated in a previous interrogation (*predictor*), which implies that an iterative procedure is needed. The image interrogation yields a *corrector* which is added up to the

predictor to update the measured velocity field; the block diagram of a general iterative approach for PIV recordings is reported in figure 3.8. Note that at the start of the process no information on the flow field is considered available and the first predictor is set uniformly to zero.

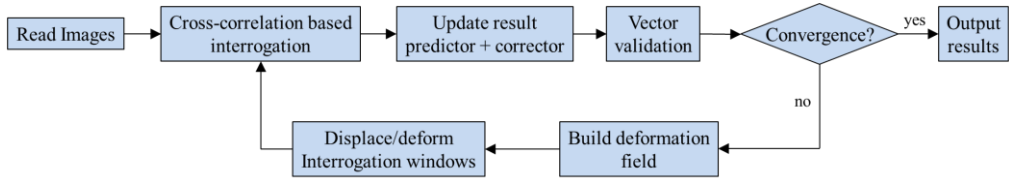


Figure 3.8. Block diagram of a general iterative approach for PIV recordings. Adapted from Schrijer and Scarano (2008).

Because after the window shift/deformation the residual displacement is typically reduced to sub-pixel values (or at most a few pixels), the one-quarter rule is usually satisfied for the iterations after the first. Hence, the successive iterations can be performed with smaller interrogation windows to enhance the measurement spatial resolution and decouple it from the dynamic velocity range (Scarano and Riethmuller, 1999). The refinement of the interrogation window size during the iterations is referred to as *multi-grid* iterative approach.

Iterative analysis with window offset

An effective way to reduce the loss-of-pairs due to in-plane motion consists of providing the interrogation windows with an in-plane displacement according to the fluid motion. In this case, the interrogation window undergoes a rigid translation in x - and y -direction. This can be done by moving the interrogation window by an integer number of pixels, which is equal to the integer part of the particle image displacement estimated in the previous iteration (Scarano and Riethmuller, 1999). A further improvement to this approach is achieved by imposing a sub-pixel offset (Scarano and Riethmuller, 2000), which allows practically removing the systematic errors and reducing the random errors by one order of magnitude (figure 3.9).

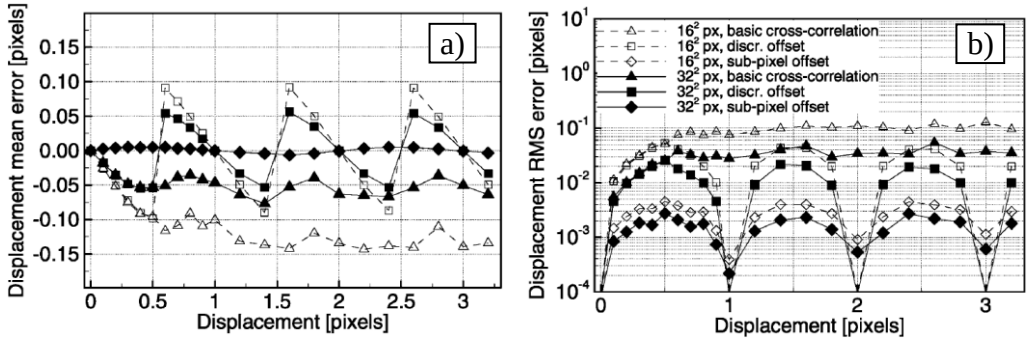


Figure 3.9. Mean displacement error (a) and displacement rms error (b) as a function of the displacement for different window sizes and interrogation methods (from Scarano and Riethmuller, 2000).

Iterative analysis with window deformation

Although the window offset approach partly compensates for the in-plane loss-of-pairs, it makes use of a zero-order displacement model (simple translation) that does not overcome the issues related to non-uniform displacement within the interrogation window. Thus, to resolve regions of strong velocity gradient, a small interrogation window should be adopted to minimize the variation of particles displacement; as a result, less particle images are contained within the interrogation window, yielding reduced strength of the correlation signal.

The use of an iterative approach allows predicting not only the displacement field but also the spatial distribution of the displacement gradient. The knowledge of these two quantities can be employed to replace the rigid translation of the interrogation window with a first-order approximation of the displacement distribution that includes both translation and deformation, as illustrated in figure 3.10. This approach, defined *iterative interrogation with window deformation*, allows compensating also for in-plane loss-of-pairs due to velocity gradients up to 0.5 pixels per pixel (Huang *et al*, 1993; Jambunatan *et al*, 1995; Scarano and Riethmuller, 2000), thus enhancing the correlation strength and the probability of valid peak detection in regions of shear (figure 3.11).

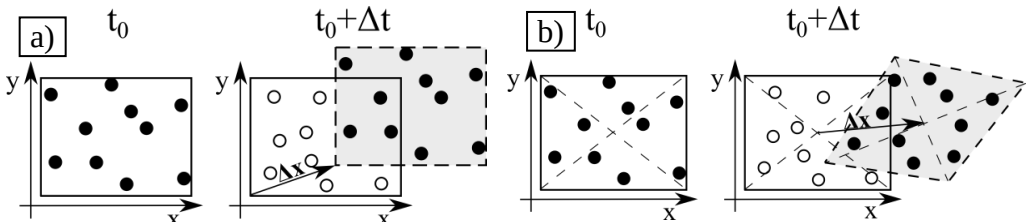


Figure 3.10. Schematic illustration of window offset approach (a) and window deformation approach (b).

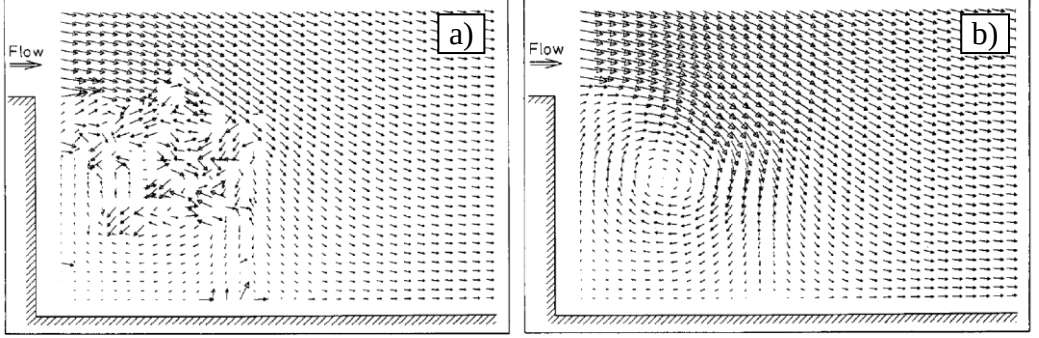


Figure 3.11. Comparison between the vector fields as obtained with single-pass correlation analysis (a) and iterative analysis with window deformation (right) for the measurement of the water flow downstream of a backward-facing step (illustration from Huang *et al*, 1993).

The iterative analysis with window deformation makes use of a continuous displacement field $(\Delta x, \Delta y)$ to deform to the intermediate time instant $t_0 + \Delta t/2$ the PIV recordings defined at discrete pixel locations (i, j) and at times t_0 and $t_0 + \Delta t$:

$$\begin{aligned}\tilde{I}_a\left(i, j; t_0 + \frac{\Delta t}{2}\right) &= I_a\left(i - \frac{\Delta x}{2}, j - \frac{\Delta y}{2}; t_0\right) \\ \tilde{I}_b\left(i, j; t_0 + \frac{\Delta t}{2}\right) &= I_b\left(i + \frac{\Delta x}{2}, j + \frac{\Delta y}{2}; t_0 + \Delta t\right)\end{aligned}\quad (3.3)$$

It should be noticed that in expression (3.3) the indices (i, j) of the image array correspond to spatial coordinates and can be summed or subtracted with the pixel displacements $(\Delta x, \Delta y)$.

Hence, an interpolation scheme is required to evaluate the intensity of images I_a and I_b between discrete pixel locations. The choice of the interpolation scheme is crucial for the accuracy of the deformed images. Several schemes have been proposed in literature, including the Whittaker interpolation (Scarano and Riethmuller, 2000) and the B-spline interpolation (Astarita and Cardone, 2005). For a thorough discussion on the accuracy, precision and computational cost of different interpolation schemes, the interested reader is referred to Astarita and Cardone (2005), Astarita (2006) and Kim and Sung (2006). Based on their investigation, Astarita and Cardone concluded that highly accurate image reconstructions are achieved with B-spline, Whittaker and Fourier interpolation using a kernel of at least 8×8 samples. When speed is the major concern, simplex or bicubic interpolation

should be employed which allow a reduction in the processing time by up to one order of magnitude at the cost of lower reconstruction accuracy.

Stability of the iterative approaches

The results of three international PIV challenges (Stanislas *et al* 2003, 2005 and 2008) have demonstrated the superior precision and accuracy of iterative approaches based on image deformation with respect to those making use of window offset. However, Nogueira *et al* (1999) reported that iterative methods with image deformation are subject to numerical instabilities as a consequence of the negative lobes of the frequency response of the cross-correlation operator (e.g. for values of the normalized window size l^* between 1 and 2, as illustrated in figure 3.7). To understand the reason of the instability, consider an ideal sinusoidal displacement of wavelength λ as that depicted in figure 3.12. In the example, the interrogation window has size $W_s = 1.5\lambda$. The true displacement at the centre of the interrogation window is equal to $-d$; in contrast, the displacement d_{meas} measured by the cross-correlation algorithm is the mean displacement within the interrogation window, and therefore is opposite to the true displacement. Thus, when conducting successive iterations, the images are deformed in opposite direction with respect to the true particles displacement, decreasing the probability to detect a valid vector.

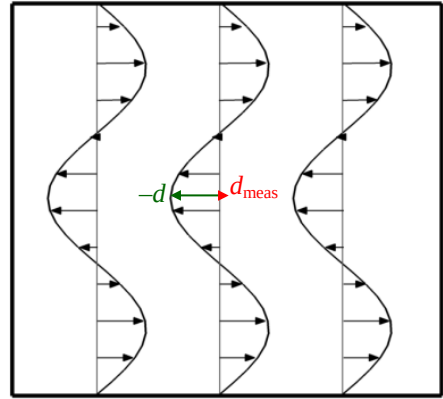


Figure 3.12. Example of instability occurring for iterative image analysis with window deformation: interrogation window containing a sinusoidal displacement field of wavelength equal to $2/3$ of the interrogation window size. The displacement d_{meas} measured by means of cross-correlation analysis is opposite to the exact displacement $-d$ at the centre of the interrogation window.

Schrijer and Scarano (2008) investigated the effect of spatial filtering of the velocity distribution to prevent the amplification of fluctuations at specific wavelengths during the iterative interrogation. Two strategies have been proposed, namely the *predictor filtering* and the *corrector filtering* approach. In the former approach, the velocity predictor is filtered before being applied for image deformation and then summed up to the corrector (figure 3.13), while in the corrector filtering the filter is applied to the result of the

cross-correlation (velocity corrector). The authors found that the corrector filtering strategy yields higher spatial resolution than the predictor filtering, but causes lower convergence rate and higher random errors in the results. Based on Schrijer and Scarano's findings, the current work makes use of a second order polynomial regression predictor filter on a kernel of size 1.5 W_s .

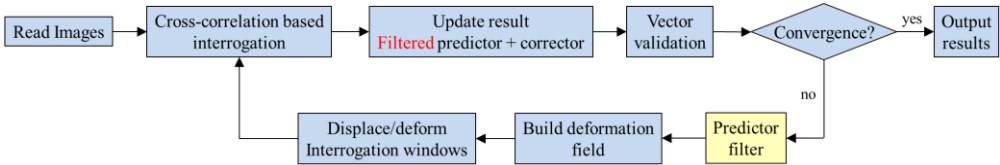


Figure 3.13. Block diagram of the iterative interrogation algorithm for PIV recordings with the predictor filter approach. Adapted from Schrijer and Scarano (2008).

3.2.2 Multi-frame processing techniques for time-resolved PIV

When PIV images are acquired using high-quality CCD cameras together with high-energy double pulse Nd:YAG lasers, the tracer motion can be evaluated with accuracy within 0.03 to 0.1 pixels by means of iterative interrogation algorithms with window deformation (Stanislas *et al*, 2003, 2005, 2008). While CCD cameras and Nd:YAG lasers are widely employed in experiments aiming at extracting information on the flow statistics (typically mean and Reynolds stresses), they are not suited for time-resolved measurements due to the low sampling rate (typically few Hertz). The latter are required for the investigation of unsteady flows where time and frequency domain analyses are necessary. Consequently, time resolved PIV measurements are typically conducted with CMOS cameras and diode-pumped Nd:YLF lasers to achieve a sampling rate up to 10 kHz. The combination of lower image quality of CMOS cameras and lower pulse energy of Nd:YLF lasers yields higher measurements uncertainty, which often exceeds the tenth of a pixel, leading to values of *DVR* typically below 100 (Stanislas *et al*, 2008). The situation becomes even more critical when temporal derivatives need to be evaluated, because the measured acceleration might be of the same order of the measurement uncertainty. Figure 3.14 shows an example of velocity and acceleration time-histories at the centreline of a laminar water jet. The velocity time-series exhibits low-frequency physical fluctuations of amplitude exceeding 3 pixels (figure 3.14-a); on top of those, high-frequency spurious fluctuations of 0.1 px magnitude occur due to noise in the measurement. In this case, the magnitude of the spurious fluctuations is much smaller than that of the physical fluctuations, therefore the former do not compromise the reliability of the latter. The acceleration is computed from the

velocity with a 3-point central difference scheme (figure 3.14-b). Here, the physical fluctuations are of the same order of the spurious fluctuations (approximately 0.4 px); consequently, the reliability of the measured acceleration is questionable.

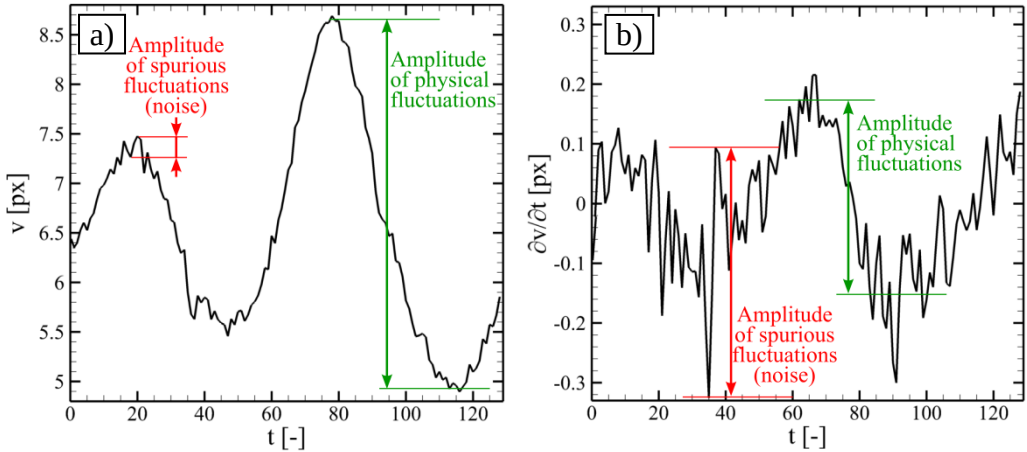


Figure 3.14. Axial velocity time-history (a) and axial acceleration time-history (b) at the centreline of a laminar jet in water ($y/D = 3.5$, being D the nozzle diameter and y the axial direction). Details of the experiment are reported in Violato and Scarano (2011).

In order to overcome these fundamental limitations of high-speed PIV systems, multi-frame approaches have been proposed in the last years. The basic idea of these is to make use of temporal information provided by the time-resolved measurement to achieve higher dynamic velocity range. In particular, the velocity field at time $t_0 + \Delta t/2$ is obtained using the information provided not only by two consecutive images at times t_0 and $t_0 + \Delta t$, but also by additional images at $t_0 \pm k\Delta t$.

Multi-frame PIV

The multi-frame PIV has been developed in the last decade by Pereira *et al* (2004), Hain and Kähler (2007) and Persoons *et al* (2011) with the purpose of enhancing the dynamic velocity range of time-resolved PIV measurements. The operational principle of the approach consists in locally optimizing the temporal separation between the PIV recordings in order to reduce the relative error on the measured displacement. Consider a fluid parcel that moves at constant velocity u . If the PIV interrogation is conducted between two consecutive recordings (time separation Δt), the distance travelled by the

parcel during Δt is $\Delta x = u \Delta t$ and the relative error on the measured displacement is equal to:

$$\varepsilon_{r,\Delta t} = \frac{\varepsilon_{\Delta x}}{\Delta x} = \frac{\varepsilon_{\Delta x}}{u \Delta t} \quad (3.4)$$

where $\varepsilon_{\Delta x}$ is the absolute error on the measured displacement, which in first approximation is independent of the measured displacement (Westerweel, 1997). In contrast, when a larger time separation $k\Delta t$ (with $k>1$) is considered, the parcel displacement is equal to $k\Delta x = k u \Delta t$, resulting in a reduction of the relative error by factor k :

$$\varepsilon_{r,k\Delta t} = \frac{\varepsilon_{\Delta x}}{k \cdot \Delta x} = \frac{\varepsilon_{\Delta x}}{k \cdot u \Delta t} = \frac{\varepsilon_{r,\Delta t}}{k} \quad (3.5)$$

Equation (3.5) shows that the larger the time separation, the lower the relative error on the displacement. However, the time separation cannot be increased indefinitely for two main reasons. First, in presence of out-of-plane motion, large values of $k\Delta t$ yield higher probability of out-of-plane loss-of-pairs, thus reducing the correlation signal strength and in turn the accuracy of the measurement. Furthermore, the method relies upon the assumptions of linear trajectory and constant velocity within the time interval $k\Delta t$: when the acceleration is not negligible, the fluid parcel trajectory may be non-linear and its velocity varies in time; consequently, truncation errors arise owing to the assumptions made (Boillot and Prasad, 1996).

From the above discussion, it emerges that the selection of a proper temporal separation is crucial for the performance of the multi-frame approach. Several strategies have been proposed for the determination of the optimal value of $k\Delta t$. Pereira *et al* (2004) suggested to use an inter-frame time such that the local displacement exceeds a minimum value, provided that the correlation signal-to-noise ratio (*SNR*, defined as the ratio between the correlation highest peak and the second highest peak) is above an arbitrary threshold. Persoons *et al* (2011) defined the *weighted peak ratio SNR'* as a measure of the local vector quality, that combines correlation strength and precision:

$$SNR' = SNR \cdot \left(1 - \frac{\varepsilon_{\Delta x}}{|\Delta \mathbf{x}|} \right) \quad (3.6)$$

being $|\Delta \mathbf{x}|$ the magnitude of the measured displacement and $\varepsilon_{\Delta x}$ the minimum resolvable displacement, arbitrarily considered equal to 0.1 px. Thus the time separation is considered optimal when the weighted peak ratio is the maximum. Finally, Hain and Kähler (2007) introduced an approach that determines the optimal $k\Delta t$ based on criteria that account for the local in-plane and out-of-plane loss-of-pairs, the velocity gradients and the flow acceleration.

The main limitation of the multi-frame PIV is that the increase of the separation time, which would be beneficial for reducing the relative error on the displacement, is often limited by the reduction of correlation signal strength due to out-of-plane loss-of-correlation. In other terms, the technique requires limited out-of-plane displacement and good image quality to yield enhanced measurement accuracy with respect to the single-pair correlation. Furthermore, the approach relies upon the hypothesis of constant velocity within the observation time; as a result, it is subject to truncation errors when the flow acceleration is non-null.

Sliding-average correlation

The sliding-average correlation approach (Scarano *et al*, 2010) aims at reducing the random component of the measurement error by averaging the correlation functions of a small temporal kernel (figure 3.15).

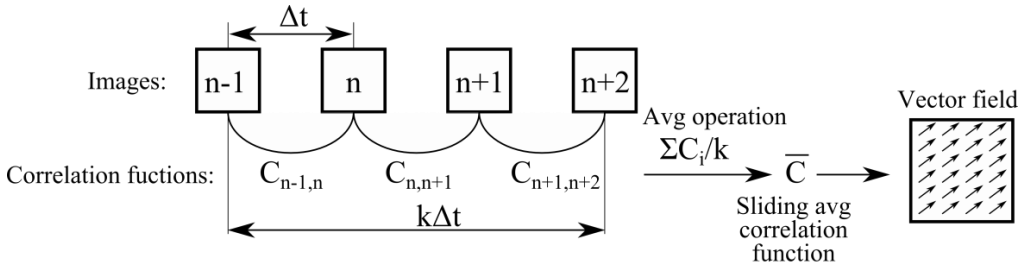


Figure 3.15. Scheme of the sliding average correlation.

The idea was first introduced by Meinhart *et al* (2000) (*ensemble average correlation*) for estimating time-averaged velocity fields in situations where the signal strength is not sufficient for standard single-pair correlation. The technique can be applied to steady flows or unsteady flows where the fluctuations displacement does not exceed the particle image diameter. As a result of the enhanced signal strength, the ensemble average correlation allows a significant reduction of the interrogation window size, virtually down to a

single pixel (Westerweel *et al*, 2004; Kähler *et al*, 2006), without lack of robustness. When only a small temporal kernel composed by k correlation functions is chosen (small in the sense that $k\Delta t \ll \tau$, being τ the characteristic time scale of the flow fluctuations) the result can be regarded as instantaneous. Assuming that the random component of the error is uncorrelated in time, the correlation averaging operation yields a reduction of the error magnitude by $\sqrt{k}$. However, possible systematic errors are not attenuated with this approach.

3.3 Data post-processing

After the velocity field has been computed, post-processing operations are conducted to validate the results and remove the outliers, fill in the regions (if any) where no information on the flow motion is attained and quantify the measurement accuracy or uncertainty. This section discusses the common strategies for validation and uncertainty quantification of PIV data.

3.3.1 Data validation approaches

Data validation is needed to remove spurious vectors from the measured velocity field. This step is crucial to achieve unbiased statistical results, because an erroneous measurement may alter dramatically the estimated values of flow mean and Reynolds stresses. The issue is even more relevant in iterative multi-grid approaches, where an erroneous evaluation of the displacement at early stage can compromise the final result in a relatively large region. A common strategy to detect spurious vectors is based on the correlation signal-to-noise ratio SNR , defined as the ratio between the highest correlation peak, associated with the average particle displacement, and the second highest peak, ascribed to the combined effects of all error sources stemming from the image quality and flow field. Since high values of SNR indicate large confidence in the measured displacement, a threshold criterion can be employed to individuate possible outliers; typical threshold values are set in the range 1.3-1.5 (figure 3.16).

Although often used due to its simplicity, the outliers detection via correlation signal-to-noise ratio may fail in presence of strong laser light reflections on solid surfaces. In fact, these reflections yield a strong auto-correlation peak that may exceed any other peak (including that associated with the particles displacement) even of one order of magnitude. As a result, even if the detected signal-to-noise ratio is above the threshold, the measured vector may still be an outlier.

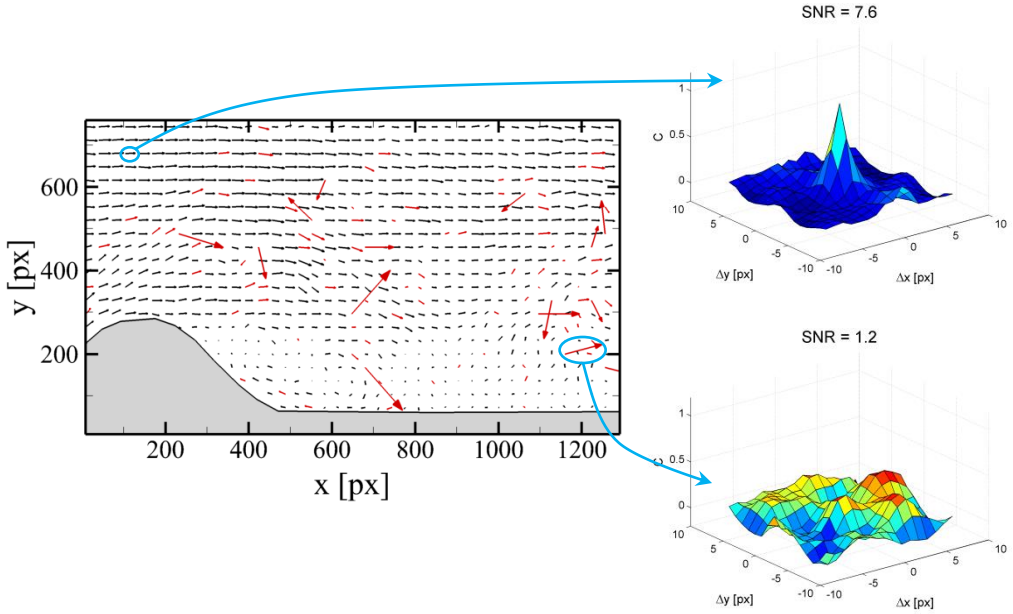


Figure 3.16. Example of velocity field with outliers. The valid vectors are displayed in black, the outliers in red. The correlation functions on the right show that valid vectors typically stem from high correlation SNR (top), while the outliers typically come from correlation functions with low SNR (bottom).

Westerweel (1994) proposed an approach (*local median test*) for outliers detection based on the deviation of a vector with respect to neighbouring vectors. The technique has been further improved and generalized by Westerweel and Scarano (2005) through the *normalized median test*, which has been shown to be effective for a wide gamut of flow conditions ranging from micro-channel flow to supersonic wake. According to the normalized median test, a vector of velocity V_0 should be retained when:

$$\frac{|V_0 - V_m|}{r_m + \delta} \leq r_c \quad (3.7)$$

where V_m is the median velocity of the neighbouring vectors, r_m the median of the residuals $r_i = |V_i - V_m|$ (being V_i the velocity of the i -th neighbouring vector) and δ the acceptable fluctuation level, suggested equal to 0.1 px. An appropriate choice for the threshold r_c is indicated as $r_c = 2$ by the authors. In figure 3.17 an example of application of the universal outliers detection method to the velocity field of figure 3.16 is shown; the wrong vectors have been removed and substituted with a bilinear interpolation of neighbouring vectors.

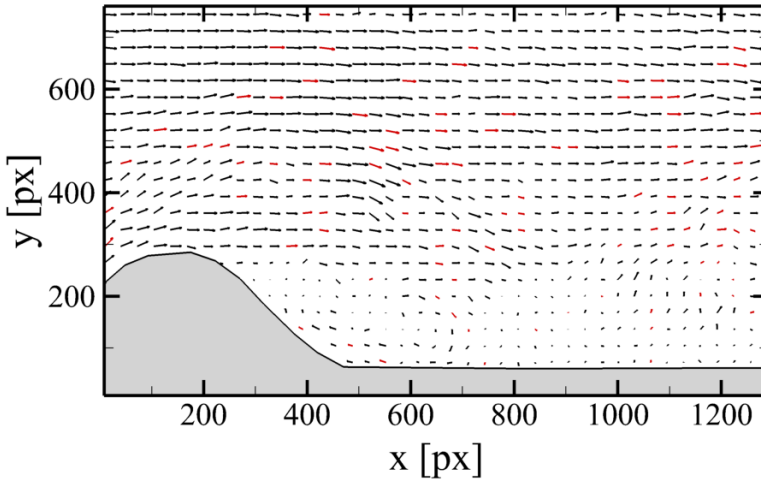


Figure 3.17. Velocity field of figure 3.16 after outlier detection via the universal outlier detection approach and data refill. The vectors obtained by means of interpolation of neighbouring vectors are displayed in red.

3.3.2 Uncertainty and accuracy

Several works have focused on the estimation of measurement errors of digital PIV. Two approach types are possible, namely *a-priori* and *a-posteriori* uncertainty quantification.

3.3.2.1 A-priori uncertainty quantification

A-priori uncertainty quantification aims at providing an estimate of the magnitude of the systematic and random errors of an interrogation algorithm.

As mentioned in chapter 2, errors in PIV are typically distinguished into systematic (or bias) and random (Fincham and Spedding, 1997). Despite the attention devoted to the properties of bias errors in PIV (see Westerweel, 1997; Scarano and Riethmuller, 2000; Nogueira *et al*, 2005; Schrijer and Scarano, 2008, among others), the random component most often dominates the measurement error. A first estimate of the measurement error is based on evaluating the width of the particle images (Adrian, 1991). It is theoretically demonstrated that the autocorrelation peak width is proportional to the particle image diameter. A simple predictive model is that the error is directly proportional to the correlation peak width. However, many more parameters play a role for random-type errors, such as particles out-of-plane motion (Nobach and Bodenschatz, 2009), fluctuating background intensity and camera noise. Several studies have indicated typical values for the random

errors. Westerweel (1993) reported a typical figure of 0.05 pixels from Monte Carlo simulations. Very similar values were obtained by Raffel *et al* (2007). The random error is also reported to be highly sensitive to the interrogation procedure; for instance, Scarano and Riethmuller (2000) measured an RMS error three times smaller when comparing iterative window deformation to the discrete window shift technique (Westerweel *et al*, 1997). The use of window weighting functions and advanced interpolators is also shown to affect the amplitude of the random error (Astarita, 2007).

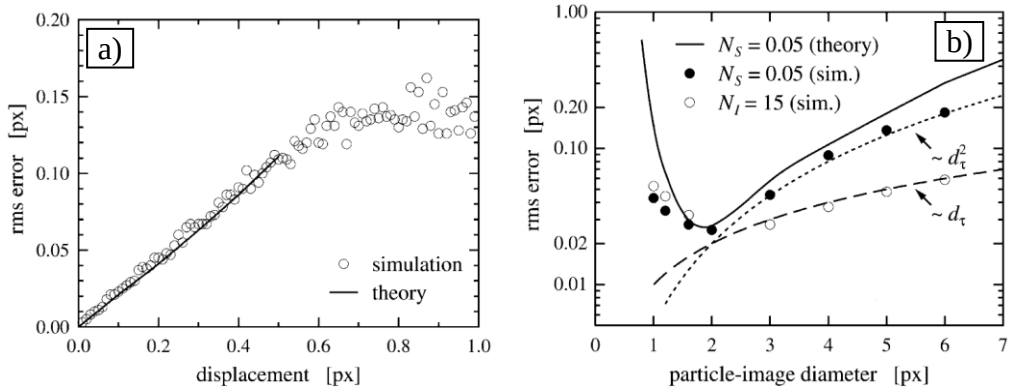


Figure 3.18. Random error as a function of the displacement (a) and particle image diameter (b) as obtained with theoretical modelling and Monte-Carlo simulations (N_I : image density; N_S : source density). Illustrations from Westerweel (2000).

Efforts to estimate the error of PIV measurements have mostly followed a-priori approaches based on modelling the measurement chain, from the tracers motion and imaging to the digital image analysis (Westerweel, 1997). In most cases, the results are obtained with Monte Carlo simulations. Theoretical models have been formulated to evaluate the effects of several measurement conditions (velocity gradient, particle-image diameter and image intensity, among others) on the measurement precision (Westerweel, 2000; Westerweel, 2008; see figure 3.18). However, most results are valid only for simple interrogation methods such as discrete window shift (Westerweel *et al*, 1997). In contrast, state-of-the-art methods such as iterative window deformation are less prone to some of the errors (Fincham and Delerce, 2000; Scarano and Riethmuller, 2000), but their theoretical modelling is more complex. Finally, the Monte Carlo approach reaches its limits when more realistic measurement conditions are to be numerically modelled and simulated. It is generally accepted that computer-based simulations lead to a significant underestimation of the measurement error due to the adoption of

too idealized conditions (Megerle *et al*, 2002; Stanislas *et al*, 2008). Nevertheless, to date, very realistic simulations of turbulent flow measurements have been achieved in the study of Lecordier *et al* (2001).

Despite the above efforts, the a-priori estimate of the measurement error cannot account for the many parameters that affect the measurement precision. As a result, the investigator is too often left with the *empirical universal constant* of 0.1 pixels as a typical figure for measurement error.

3.3.2.2 A-posteriori uncertainty quantification

A-posteriori uncertainty quantification is required to assess the goodness of the computed velocity. Contrarily to the a-priori approach that provides only general information on the limitations of an interrogation algorithm, the a-posteriori approach aims at estimating the uncertainty of a specific velocity field in order to evaluate the significance of the scatter in the data. In fact, the measured velocity distribution typically exhibits fluctuations in the spatial domain (and also in the temporal domain for time-resolved measurements): aim of a-posteriori uncertainty quantification is to determine to which extent those fluctuations are ascribed to measurement errors instead of physical phenomena. The topic is crucial when PIV measurements are used for validating computational fluid dynamics (CFD) results. Several strategies have been proposed in the past, although none of them has been shown to have a general applicability or is acknowledged as a standard uncertainty quantification approach. A brief survey of techniques available at the moment of the start of this PhD project are reported hereafter; more recent techniques are discussed in the introduction of chapters 4 and 5.

Spatial/temporal coherence

Consider an error-free velocity field. In this case, if the distance between adjacent grid points is much smaller than the characteristic dimension of the resolved flow structures, the velocity distribution varies smoothly between contiguous grid points, i.e. it is coherent in space. Instead, in presence of spatially uncorrelated measurement errors, the velocity distribution exhibits high-frequency spurious fluctuations ascribed to noise (figure 3.19-a). Hence, an estimate of the error can be achieved by filtering the velocity field via a low-pass spatial filter to get a noise-free velocity distribution (figure 3.19-b) and computing the difference between measured and filtered velocity (figure 3.19-c).

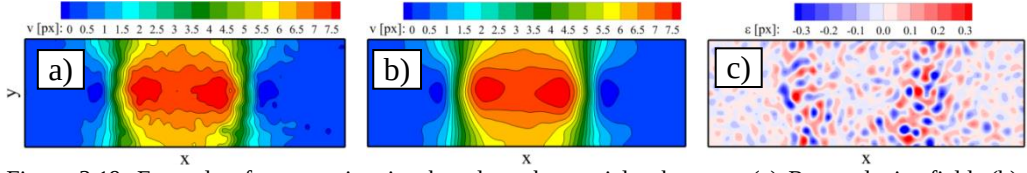


Figure 3.19. Example of error estimation based on the spatial coherence. (a) Raw velocity field; (b) velocity field filtered in space; (c) measured error computed as the difference between measured and filtered velocities.

For time-resolved measurements, the same approach can be employed taking into account the temporal evolution of the velocity field; the velocity time-history is filtered in time and the error is estimated as the difference between the measured and the filtered velocity (figure 3.20).

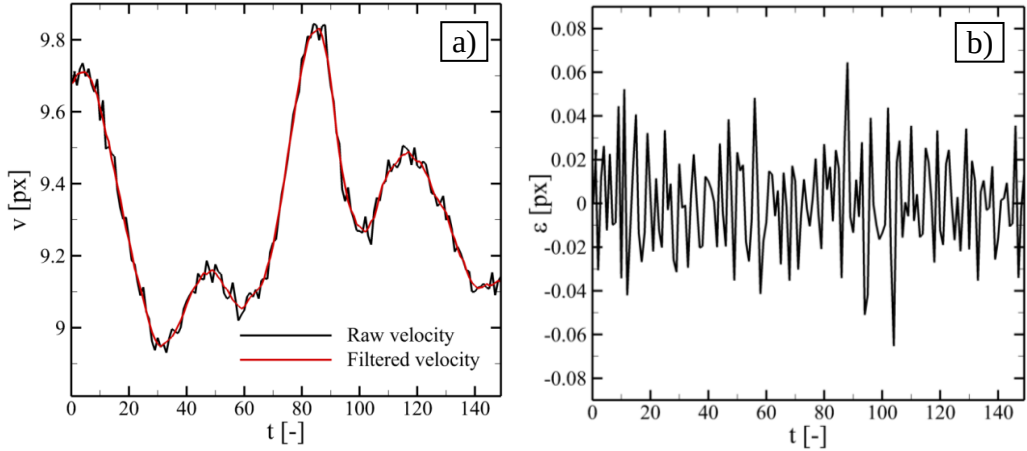


Figure 3.20. Example of raw and filtered velocity time histories (a). (b) Error computed as the difference between the measured and the filtered velocity.

The main advantage of the uncertainty quantification based on the spatial/temporal coherence is its simplicity and ease to implement. However, the approach suffers of two major drawbacks. First, the choice of the filter type and its kernel size is arbitrary: different choices can lead to largely different uncertainty estimates. Second, this strategy accounts only for random error sources, while possible systematic errors remain unnoticed.

Signal-to-noise ratio

As discussed in section 3.3.1, the signal-to-noise ratio is strongly related to the quality of the measurement and can be used as an indicator of the confidence of the measured displacement. However, the determination of the uncertainty interval from the SNR value is far from trivial and has been

investigated only recently by Charonko and Vlachos (2013). The topic will be further discussed in chapters 4 and 5.

Compliance with physical laws

A conclusive way to assess the uncertainty of a velocity measurement is to verify its compliance with physical laws. Since the utmost majority of flows is three-dimensional, this approach yields unbiased results only when instantaneous three-dimensional velocity fields are available, as provided by tomographic PIV (Elsinga *et al*, 2006). For incompressible flows, the compliance with the continuity equation

$$\nabla \cdot \mathbf{V} = 0 \quad (3.8)$$

has been proposed to infer the uncertainty on the measured velocity (Zhang *et al*, 1997; Scarano and Poelma, 2009; Violato and Scarano, 2011; see figure 3.21).

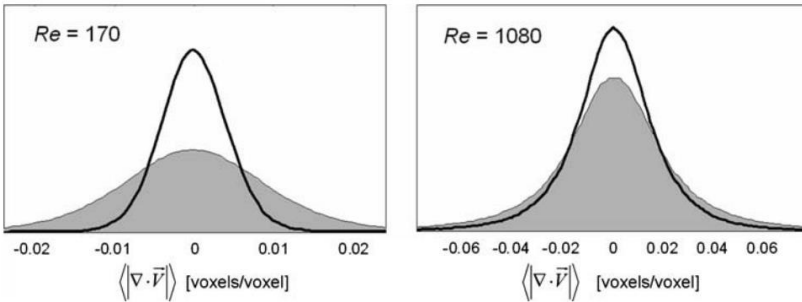


Figure 3.21. Probability density of measurement error estimated by velocity divergence. Raw data (grey area) and filtered (solid line). Illustration from Scarano and Poelma (2009).

For time-resolved measurements of incompressible flows, the vorticity transport equation can be used in lieu of the continuity equation to account also for the temporal evolution of the velocity field (Novara, 2013):

$$\frac{D\boldsymbol{\omega}}{Dt} = (\boldsymbol{\omega} \cdot \nabla) \mathbf{V} + \nu \nabla^2 \boldsymbol{\omega} \quad (3.9)$$

where $\boldsymbol{\omega}$ is the vorticity vector, ν the kinematic viscosity of the fluid and D/Dt indicates the material derivative operator.

Spectral analysis

A further uncertainty quantification methodology for time-resolved measurements has been proposed by Ghaemi *et al* (2012). This approach relies upon the analysis of the signal in the frequency domain. In turbulent flows, the energy content of the physical fluctuations is known to decrease with the frequency and to be null for frequencies above that corresponding to the Kolmogorov time microscale (Tennekes and Lumley, 1972). However, random measurement errors add approximately white noise with constant power spectral density to the signal, preventing the power spectrum to drop to zero. Therefore, the minimum value of the measured power spectrum can be used to estimate the measurement noise level, as illustrated in figure 3.22.

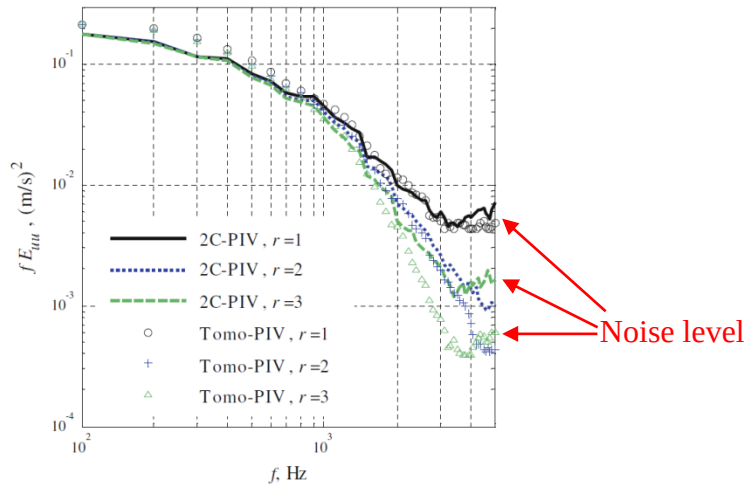


Figure 3.22. Premultiplied power spectral density of the streamwise velocity fluctuations in a turbulent boundary layer. Figure from Ghaemi *et al* (2012).

3.3.3 Advanced data refill

A further issue to be tackled in the post-processing phase is the treatment of missing data (*gaps*) in the measured velocity distribution. PIV vector fields often exhibit clusters of missing data, typically caused by limited optical access for the imaging system or shadows generated by the presence of objects in the light path. Several applications require the computation of integral quantities, for which the presence of gaps in the data is an issue that needs to be solved: examples of those are the integration of the force equation over a control volume (Kurtulus *et al*, 2007) and the determination of the production of acoustic pressure fluctuations in turbulent flow (Violato and Scarano, 2011). Little attention has been paid to the treatment of gaps in PIV. Typically,

data refill procedures based on bilinear or bicubic interpolation of neighbouring vectors have been employed (Ragni *et al*, 2011c; see figure 3.23). A more advanced approach has been proposed by Gunes and Rist (2008) based on the application of the kriging method for stereoscopic data reconstruction. Venturi and Karniadakis (2004) investigated the possibility of using proper orthogonal decomposition (POD) for reconstructing flow fields in gappy data. However, the aforementioned methods can only reconstruct a velocity field that is monotonic or at most with a single peak inside the gap (that is a half wave), meaning that the reconstructed velocity becomes highly inaccurate when the velocity distribution to reconstruct has wavelength equal to or smaller than the gap size.

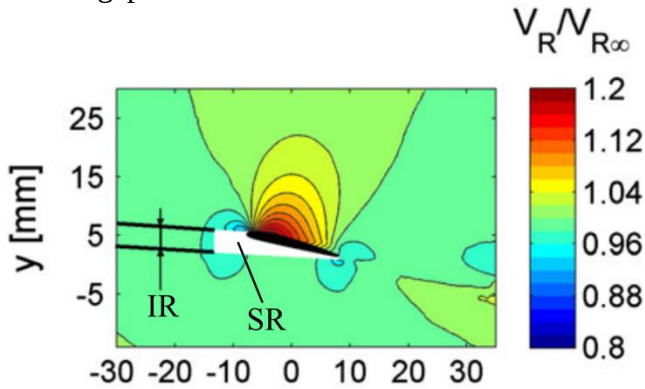


Figure 3.23. Planar PIV visualization of the relative velocity around a propeller blade. SR: shadow region produced by the blade; IR: interpolated region, where the missing data has been reconstructed via bicubic interpolation of neighbouring values. Illustration from Ragni *et al* (2011c).

3.4 Summary of research statements

From the above discussion, it emerges that several approaches have been proposed to enhance the reliability, accuracy and dynamic range of PIV measurements, so that nowadays the technique is recognized as a mature tool for fluid dynamics investigation. Nevertheless, various limitations are encountered which leave room to further research and development:

- *Pre-processing*: a standard technique for image restoration and enhancement even in presence of moving interfaces (owing to a controlled motion of the model or to vibrations of the model or the imaging systems) is still missing.
- *Processing*: an advanced multi-frame approach for time-resolved PIV that combines high dynamic velocity range with robustness is needed for accurate evaluation of the flow temporal evolution.

State-of-the-art of PIV image analysis

- *Post-processing:*
 - A major limitation of the state-of-the-art PIV image evaluation is the lack of a generalized strategy for a-posteriori uncertainty quantification, which is required to assess the reliability of the measurement.
 - A data refill technique that allows reconstructing flow structures of arbitrary wavelength within regions of missing experimental data is not yet available.

The methodologies proposed to overcome the current limitations of PIV image analysis and uncertainty quantification are discussed in chapters 4 to 8.

CHAPTER 4

PIV UNCERTAINTY QUANTIFICATION BY IMAGE MATCHING^{*}

Abstract This chapter proposes a novel method to quantify the uncertainty of PIV data. The approach is a-posteriori, meaning that the measurement error is estimated using the original images and the velocity field as input. The principle of the method relies on the concept of super-resolution: the image pair is matched according to the cross-correlation analysis and the residual distance between matched particle image pairs (particle disparity vector) due to incomplete match between the two exposures is measured. The ensemble of disparity vectors within the interrogation window is analysed statistically. The dispersion of the disparity returns the estimate of the random error, whereas its mean value indicates the occurrence of a systematic error.

The validity of the working principle is first demonstrated via Monte Carlo simulations. Two different interrogation algorithms are considered, namely the cross-correlation with discrete window offset and the multi-pass with window deformation. In the simulated recordings, the effects of particle image displacement, its gradient, out-of-plane motion, seeding density and particle image diameter are considered. In all cases a good agreement is retrieved, indicating that the uncertainty estimator is able to consistently follow the trend of the actual error with satisfactory precision.

Experiments where time-resolved PIV data is available are used to prove the concept in realistic measurement conditions. In this case the “exact” velocity field is unknown; however, a highly-accurate estimate is obtained with an independent, more accurate measurement system or with an advanced interrogation algorithm that exploits the redundant information of temporally oversampled data (*pyramid correlation*; the algorithm is the topic of chapter 6 and is discussed in Sciacchitano *et al*, 2012). Dedicated experiments are conducted that make use of two concurrent PIV measurement systems, one of which has significantly higher digital resolution than the other, thus providing an increased velocity dynamic range. The measurement error is therefore approximated by the difference between the results obtained with the two systems. The acquired data base covers a gamut of flow conditions and error sources representative of typical PIV experiments, including low-speed flow around a prism, fully turbulent wake and supersonic boundary layer. Finally, a criterion for evaluating the accuracy of the estimated uncertainty is proposed and applied to the current image matching estimator, showing that the uncertainty scales correctly with the measurement error. Most results yield an uncertainty estimate within 10% of the actual error value independently of the measurement conditions and flow regime.

^{*} Part of this work has been published in Sciacchitano *et al* (2013) *Measurement Science and Technology* **24**: 045302.

4.1 Introduction

Chapters 2 and 3 have introduced the concepts of error and uncertainty and the distinction between a-priori and a-posteriori uncertainty quantification approaches. The objective of the present chapter is to establish the founding principles of the a-posteriori uncertainty quantification technique, which aims at quantitative and objective evaluation of the measurement error and its statistical properties[†].

In the last years, the topic of PIV uncertainty quantification has received increasing attention, especially when PIV is used to assess the validity of results obtained with computational fluid dynamics. The “PIV uncertainty workshop” held in Las Vegas in 2011 is only one of the events that demonstrates such attention. Only in a recent work, Timmins *et al* (2012) introduced a method for the automatic uncertainty estimation of PIV measurements. The approach consists in identifying the main error sources and determining their contribution to the measurement error via Monte Carlo simulation. The method can be categorized as a-posteriori because it makes use of information taken from the measurement conditions (particle image diameter, particle density, particle displacement and velocity gradient). The measurement uncertainty is retrieved from numerical simulations that reproduce the magnitude of the error sources encountered in the experiment. As already stated, the amplitude of the errors returned from the numerical simulations is often lower than the actual experimental error. Furthermore, the method relies upon the measured magnitude of the error sources, itself affected by uncertainty. Finally, only a limited number of error sources is taken into account, thus only yielding a lower bound for the total uncertainty. For instance, errors associated with out-of-plane particles motion are not accounted for, unless the out-of-plane displacement is measured with a stereoscopic PIV system.

Charonko and Vlachos (2013) empirically determined the relationship between the correlation peak ratio and the measurement uncertainty. The method has been shown to be effective for the specific case of the robust phase

[†] Despite the incredible development of hardware and image analysis techniques that have rendered PIV a reliable, accurate and versatile measurement technique as we know it today, little advancement can be claimed on the side of uncertainty quantification. An average-level PIV user, when asked about the uncertainty of the just completed PIV measurements, would first put his finger in the mouth, then raise it as of sampling the wind direction and then answer candidly: “well, about one-tenth of a pixel”.

correlation (Eckstein and Vlachos, 2009), whereas it yields a less reliable uncertainty prediction for the standard cross-correlation technique.

Other methods for a-posteriori error estimation are based on the application of governing laws to the measured velocity field. In 3D data from incompressible flows, the fluctuating velocity divergence (Liu and Katz 2006, Scarano and Poelma, 2009) indicates the error level of the velocity and its derivatives, as discussed in chapter 3. Unfortunately, in planar experiments of turbulent flows, mass conservation cannot be imposed due to the three-dimensional flow motion.

The present work describes a method for quantifying a-posteriori the uncertainty of PIV measurements. The work focuses on the errors arising from the vector computation and puts most attention on the random error. The methodology overcomes the limitation of Timmins and co-workers' approach, where the uncertainty associated with a limited number of error sources of measured magnitude is estimated. The proposed approach relies upon the concept of super-resolution (Keane *et al*, 1995; Cowen and Monismith, 1997), in the sense that the contribution of individual particle images to the correlation peak is analysed to infer the measurement standard uncertainty. The latter is defined as the estimate U_s of the magnitude of the unknown actual error ε . As discussed in chapter 3, the expanded uncertainty estimate $U = k \cdot U_s$ is finally defined as the interval about the measurement which contains the true value at a confidence level that depends on the chosen coverage factor k .

4.1.1 Validation of the uncertainty quantification method

The validation of uncertainty quantification methods requires the measurement error to be known or at least estimated to a good degree. When Monte Carlo simulations are used, the validation is straightforward because the exact velocity field is known and the measurement error can be computed directly (Timmins *et al*, 2012; Charonko and Vlachos, 2013). In contrast, knowing the exact value of the velocity within an experiment requires that an additional independent measurement is conducted, usually of higher precision than the first one. For this purpose, Timmins *et al* (2012) used a hot wire anemometer to provide more accurate values of mean velocity and root-mean-square fluctuations. The use of an additional independent measurement system allowed Timmins and co-workers to achieve an accurate estimate of the measurement error on time-averaged velocity and Reynolds stresses. However, since PIV and hot wire measurements were not conducted

simultaneously, the approach did not provide information on the instantaneous measurement error. Charonko and Vlachos (2013) measured the steady laminar flow around the stagnation region of a rectangular body and compared it with the exact velocity field obtained from the fluid dynamics equations. In their case, the instantaneous measurement error could be retrieved; however, the analytical solution can be computed only for a very limited number of flows with simple geometry.

To assess the methodology against experimental data, the approach of concurrent measurements is followed. Two PIV systems are used to measure simultaneously the same velocity field. The first is referred to as the *measurement system*, of which the uncertainty needs to be determined. The second system has higher digital imaging resolution and is referred to as the *high dynamic range* (HDR) system. The latter provides a more accurate evaluation of the velocity field that is used to determine the instantaneous local measurement error. This approach forms the basis of the experimental validation of the image matching approach for uncertainty quantification. Different flow regimes are considered, in the attempt to obtain results that encompass a wide range of PIV measurement conditions in wind tunnel experiments. The experiments include time-resolved measurements around a bluff body, measurements in the turbulent wake and in a supersonic turbulent boundary layer. Finally, the results are synthesized proposing a criterion to evaluate the statistical measurement uncertainty of all experiments on a common ground.

4.2 Image-matching uncertainty quantification

4.2.1 The method in brief

It is assumed here that the PIV interrogation process is performed by means of spatial cross-correlation of a pair of images. The analysis of the correlation map yields the position of the peak with sub-pixel precision obtained by correlation peak fit. The result is the instantaneous velocity field $\mathbf{V} = (u, v)$ defined at the centre of each interrogation window.

In the present discussion we consider three possible interrogation methods with increasing level of complexity:

- 1) single pass cross-correlation (following e.g. Keane and Adrian, 1992);

- 2) double-step analysis with discrete window offset (Westerweel *et al*, 1997);
- 3) image deformation (Huang *et al*, 1993) with iterative multigrid analysis (Scarano and Riethmuller, 2000).

For simplicity, the working principle is explained considering the second method. However, the proposed methodology is generally applicable to these three methods and a few other, based on cross-correlation analysis.

Let us consider two PIV exposures. Cross-correlation analysis yields the mean particle image displacement for each interrogation window, which is denoted as $\Delta \mathbf{x}$ here; this pixel displacement can be decomposed into an integer part $\Delta \mathbf{x}_{\text{int}}$ and a fractional (sub-pixel) part $\Delta \mathbf{x}_{\text{frac}}$:

$$\Delta \mathbf{x} = \Delta \mathbf{x}_{\text{int}} + \Delta \mathbf{x}_{\text{frac}} \quad (4.1)$$

Now, if the window from the first exposure is shifted towards the second exposure with the closest integer approximation $\Delta \mathbf{x}_{\text{int}}$ (see Westerweel *et al*, 1997, for details), most particle images will come to partly superimpose. However, a number of particle pairs will not correspond exactly (figure 4.1). This can be caused by several factors: first, the particles images displacement is generally different from an integer number of pixels, i.e. $\Delta \mathbf{x}_{\text{frac}} \neq 0$; second, the particles displacement may be not uniform in presence of a velocity gradient; particle images may partly or totally disappear due to the out-of-plane particle motion (e.g. particle image at the centre of the window in figure 4.1-b). Furthermore, a randomly distributed disparity vector with fractional pixel amplitude will also occur due to the presence of noise in the recordings.

Thus, a residual distance $\mathbf{d}$ between the pair of particle images after the matching arises, which is called here *matched particle image disparity*. If the interrogation analysis is conducted with the discrete window offset technique, the residual distance includes also the fractional displacement $\Delta \mathbf{x}_{\text{frac}}$ that can be easily accounted for. The basis of the present method is the statistical analysis of such disparity, which returns the estimate for the velocity vector measurement error.

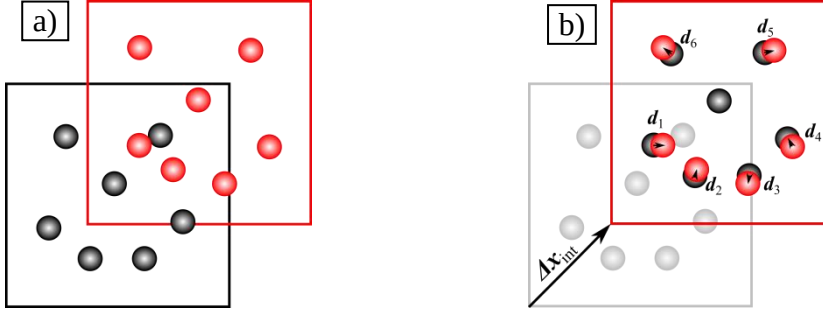


Figure 4.1. (a) Particle images at time instants t_1 (black) and t_2 (red). (b) The interrogation window of time t_1 is shifted based on the integer part $\Delta \mathbf{x}_{int}$ of the displacement measured at the previous iteration. A positional disparity (indicated with $d_1, d_2 \dots d_6$) occurs between the particle images of the two images. In the example, the particle at the centre of the window is not correctly paired due to out-of-plane loss-of-pairs.

4.2.2 Detailed implementation

The main elements composing the principle of the proposed approach are four:

- a) *Image matching*: the particle pairs are matched at the best of the velocity estimator.
- b) *Particle image pairs detection*: particle images occurring in both exposures and falling close to each other are detected as a pair.
- c) *Disparity vector computation*: the distance between the particle image pair is evaluated as the distance between the particles centroids.
- d) *Statistical analysis of disparity vector ensemble*: the mean value and the statistical dispersion of the disparity vectors of the particle pairs belonging to the interrogation window are used to estimate the uncertainty of the velocity vector.

Since the measurement error is estimated via a statistical analysis of the disparity vector ensemble, the accuracy of the method depends on the number of paired particle images.

a) Image matching

Let us consider a pair of images I_1 and I_2 separated by the time interval Δt . The procedure of image matching is based on the principle that one image is taken as reference pattern and the second one is subject to a transformation that minimizes the difference (typically L^2 -norm) or maximizes the cross-correlation coefficient between the two images. The match between the

two images is based on an estimate of the velocity field, commonly done by multi-step interrogation, either with window shift or by deformation. The latter estimate is denoted as displacement predictor, following the discussion in Scarano (2002).

The degree of matching will depend upon many parameters: the size of the interrogation window, the flow length-scales, the out-of-plane motion and the matching algorithm used. The method based on discrete offset will match the particle motion in each window within uniform (planar) velocity approximation; the image deformation method will match the particle motion up to a piecewise linear or non-linear approximation (Scarano, 2002; Astarita and Cardone, 2005; Astarita, 2008). The image interrogation procedure is performed by symmetrical deformation of both images (Wereley and Meinhart, 2001); the deformed images are indicated with $\tilde{I}_1$ and $\tilde{I}_2$ respectively.

It is important to remark that, after matching, the particle image pairs should superimpose to a large degree. To explain the working principle of the technique, the distinction between the ideal case (perfect particle images with only planar motion and no noise) and real case is remarked hereafter. In the ideal case, the particle images of a pair superimpose perfectly in the matched images $\tilde{I}_1$ and $\tilde{I}_2$; therefore, the cross-correlation function exhibits a displacement peak centred at the origin (null relative displacement) or at the fractional displacement (equal for all the particle images within the window) with a peak width proportional to the particle image diameter d_p , as illustrated in the first row of figure 4.2. As discussed by Adrian (1991), the uncertainty of the displacement measurement is also proportional to the particle image diameter d_p . Later studies have also discussed the dependence upon the peak-fit scheme that locates the particle centroid with sub-pixel precision (Lourengo and Krothapalli, 1995).

In actual experiments, the velocity predictor only yields an approximation of the overall particle motion and individual particle images will not match perfectly. As a consequence, the correlation between different particle image pairs yields contributions to the peak at positions scattered around the origin of the correlation space. Each relative displacement obtained from a particle image pair can be regarded as the elemental contribution to the actual cross-correlation. Assuming that only the particle image pairs contribute to the correlation peak (and not e.g. the background noise in the recordings), the correlation peak for the entire window equals the sum of all individual peaks from particle pairs. Clearly, the dispersion of the data causes the window

cross-correlation peak to broaden as shown in the second row of figure 4.2. Moreover, the correlation peak may not be centred at the origin of the correlation space, yielding a non-null displacement. Furthermore, the peak magnitude drops as a result of the signal broadening. The position of the maximum in the broadened signal is more sensitive to the noisy contributions resulting in a higher measurement uncertainty. In this case, the uncertainty of the displacement measurement is proportional to the positional disparity of the particle pairs in the matched images.

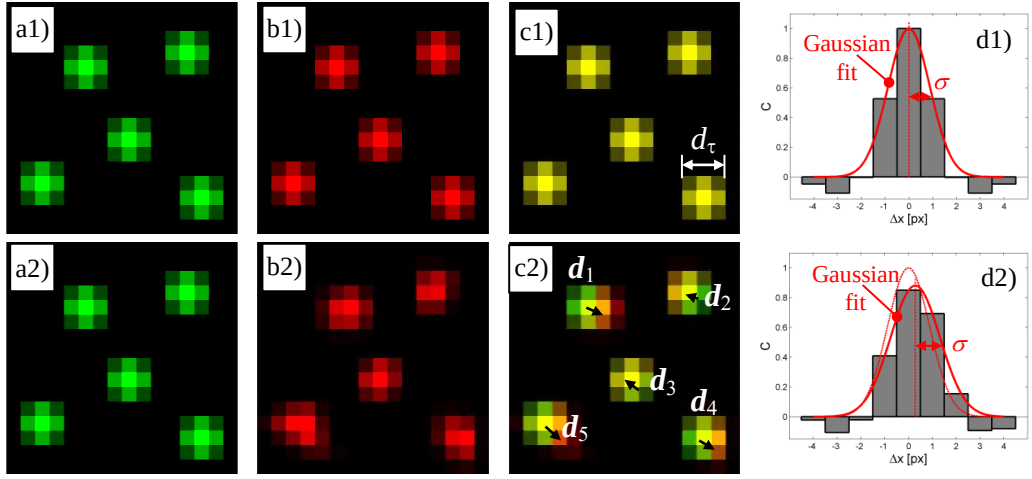


Figure 4.2. First row: image matching in the ideal case. (a1) Particle images of $\tilde{I}_1$; (b1) Particle images of $\tilde{I}_2$; (c1) Superposition of $\tilde{I}_1$ and $\tilde{I}_2$: the green particles of $\tilde{I}_1$ and the red particles of $\tilde{I}_2$ superimpose perfectly, yielding the particles displayed in yellow; (d1) Correlation function (profile) between $\tilde{I}_1$ and $\tilde{I}_2$: the width of the cross-correlation peak is proportional to the particle image diameter. Second row: image matching in the real case. (a2) Particle images of $\tilde{I}_1$; (b2) Particle images of $\tilde{I}_2$; (c2) Superposition of $\tilde{I}_1$ and $\tilde{I}_2$: the particle images do not superimpose perfectly (yellow: particles correctly superimposed; green: portion of $\tilde{I}_1$ not paired in $\tilde{I}_2$; red: portion of $\tilde{I}_2$ not paired in $\tilde{I}_1$); (d2) Correlation function (profile) between $\tilde{I}_1$ and $\tilde{I}_2$: the positional disparity of the particle image pairs yields a larger cross-correlation peak width with respect to the ideal case.

b) Particle image pairs detection

The concept applied here is taken from the particle tracking technique. Because the particle pairs detection is done on the matched images, the principle can be assimilated to that of super resolution (Keane *et al*, 1995). The latter applies particle image pairing after that the images have been matched by means of cross-correlation. In the present case, the image pairs are detected considering the *intensity product image* $\Pi = \tilde{I}_1 \tilde{I}_2$. The peaks inside Π correspond to particle images that candidate to form a pair (figure 4.3) and as

such they contribute to the build-up of the correlation peak. Let us define the *peaks image* φ the binary image composed by the peaks of Π :

$$\varphi(i, j) = \begin{cases} 1 & \text{if } \Pi(i, j) \text{ is a relative maximum} \\ 0 & \text{otherwise} \end{cases} \quad (4.2)$$

Each point (i, j) where φ is non-null indicates a particle image pair; the peak of the corresponding particle images is detected in $\tilde{I}_1$ and $\tilde{I}_2$ in a neighbourhood of search radius r (typically 1 or 2 pixels), centred in (i, j) .

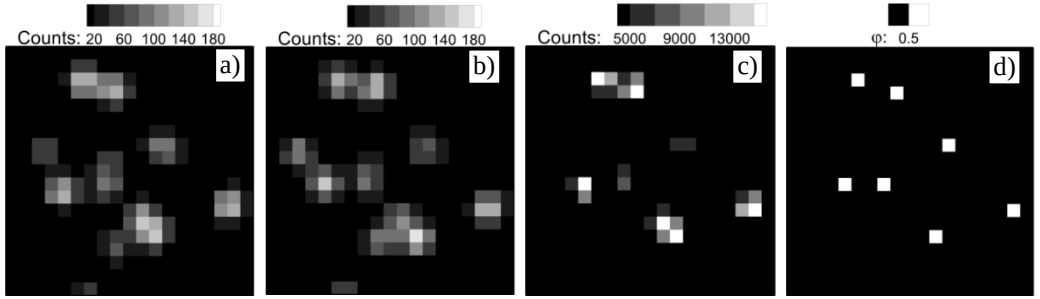


Figure 4.3 (a) Matched image $\tilde{I}_1$; (b) Matched image $\tilde{I}_2$; (c) Image intensity product Π ; (d) Peaks image φ .

c) Disparity vector computation

Sub-pixel precision is required to determine the disparity of the particles position in the matched images. The sub-pixel peak position estimator adopted here is the standard 3-point Gaussian fit as introduced by Willert and Gharib (1991). The fit returns sets of particle positions $\mathbf{X}^1$ and $\mathbf{X}^2$ at times t_1 and t_2 respectively:

$$\begin{aligned} \mathbf{X}^1 &= \{\mathbf{x}_1^1, \mathbf{x}_2^1, \dots, \mathbf{x}_N^1\} \\ \mathbf{X}^2 &= \{\mathbf{x}_1^2, \mathbf{x}_2^2, \dots, \mathbf{x}_N^2\} \end{aligned} \quad (4.3)$$

where x_i^j is the position occupied by the i -th particle in the matched image j and N is the number of particle pairs in the interrogation window. Figure 4.4 illustrates two matched windows, where the position of the particle images at time t_1 (hollow squares) and t_2 (filled circles) is not exactly corresponding.

The small red arrows are the disparity vectors $\mathbf{d}_i$, which form the disparity set $\mathbf{D}$:

$$\mathbf{D} = \{\mathbf{d}_1, \mathbf{d}_2, \dots, \mathbf{d}_N\} = \mathbf{X}^2 - \mathbf{X}^1 \quad (4.4)$$

The histogram of the disparity vector component is shown in figure 4.5 (for sake of clarity, a region containing more particles than those shown in figure 4.4 is considered in the histogram; the horizontal component d_x of the disparity is displayed). For a sufficiently large number of particles within the window (typically above 6), the analysis of the disparity vector dispersion becomes statistically significant.

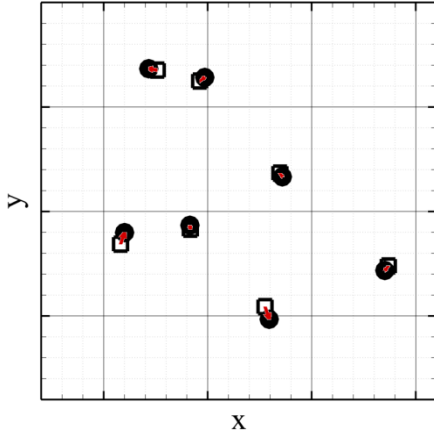


Figure 4.4. Particle images of $\tilde{I}_1$ (hollow squares) and $\tilde{I}_2$ (full circles) and disparity vectors (in red).

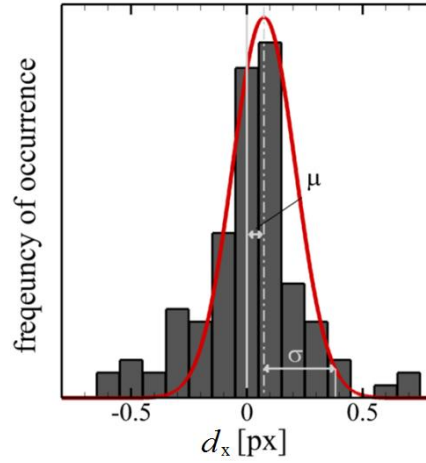


Figure 4.5. Distribution of the disparity vector.

d) Statistical analysis of disparity vector ensemble

Let us assume for simplicity that the components of the disparity set are statistically independent and that they are randomly distributed around the zero. The error committed on each particle pair can be propagated to that of the ensemble considered by the cross-correlation operator. In particular, the error scales with the standard deviation $\sigma = \{\sigma_x, \sigma_y\}$ of $\mathbf{D}$, and is inversely proportional to the square root of the number of particle pairs N . Therefore, the random error can be estimated as $\sigma/\sqrt{N}$ (Coleman and Steele, 2009).

In the more general case, the mean of the disparity set $\mu = \{\mu_x, \mu_y\}$ may be nonzero, when detectable bias errors are present (figure 4.5).

When the values of μ and σ are computed as the arithmetic mean and the standard deviation of the disparity set, all particles are assumed to make the

same contribution to the correlation peak. However, brighter particle images make a larger contribution than dimmer ones. This is taken into account by computing μ and σ as weighted mean and standard deviation of the disparity set; the weight is chosen as the square-root of the particle image intensity product Π :

$$\mu = \frac{1}{N} \sum_{i=1}^N c_i d_i, \quad \sigma = \sqrt{\frac{\sum_{i=1}^N c_i (d_i - \mu)^2}{\sum_{i=1}^N c_i}}, \quad \text{with } c_i = \sqrt{\Pi(\mathbf{x}_i)}, \text{ for } i = 1, 2 \dots N \quad (4.5)$$

In conclusion, the expression of the instantaneous standard uncertainty U_s reads:

$$U_s = \{U_{sx}, U_{sy}\} = \sqrt{\mu^2 + \left(\frac{\sigma}{\sqrt{N}}\right)^2} \quad (4.6)$$

U_s represents an estimation of the magnitude of the actual error ε committed in the measurement of the velocity vector. Expression (4.6) may be interpreted as follows: when the systematic error μ is negligible, the error is dominated by the dispersion of the disparity vectors and decreases with the square root of the number of particle pairs. In contrast, in case of negligible dispersion of the disparity set, the error will be mostly due to the systematic component μ . This is for instance the case when very large interrogation windows are used and N is very large. It should be remarked here that the term μ allows detecting systematic errors stemming from spatial modulation effects due to the finite interrogation window size (Nogueira *et al*, 2005). However, other systematic error sources such as those due to peak-locking, temporal modulation (truncation errors) and other aspects of the measurement chain (e.g. timing errors, perspective errors, magnification errors, calibration errors...) are not accounted for. In particular, when the particle image diameter is 1 pixel or below, the position of individual particle images is “locked” at integer pixel positions, thus precluding an accurate estimate of the particle positional disparity.

Note also that errors stemming from outliers are not given attention in the present framework.

The expanded uncertainty U can be retrieved from the standard uncertainty U_s to determine an interval about the measurement which contains the true value at a given confidence level:

$$U = k \cdot U_s \quad (4.7)$$

Assuming a Gaussian distribution of the measurement error, $k = 1.96$ yields a confidence of 95% (Coleman and Steel, 2009), meaning that the error magnitude is contained within the uncertainty bounds in 95% of the measurements. In the reminder of this work $k = 1$ is used (confidence level of 68%).

Note that the standard deviation of the estimated uncertainty scales with $1/\sqrt{2(N-1)}$ (Ahn and Fessler, 2003), with N number of particle image pairs. Hence, a minimum number of about $6 \div 7$ particle images per interrogation window is required for error estimates accurate at 70% or above.

The computational cost of the uncertainty quantification depends on the number of particle images in the recordings and is typically about 10% of that of the vector computation.

In summary, the overall procedure can be schematically visualized by the following flow-chart (figure 4.6).

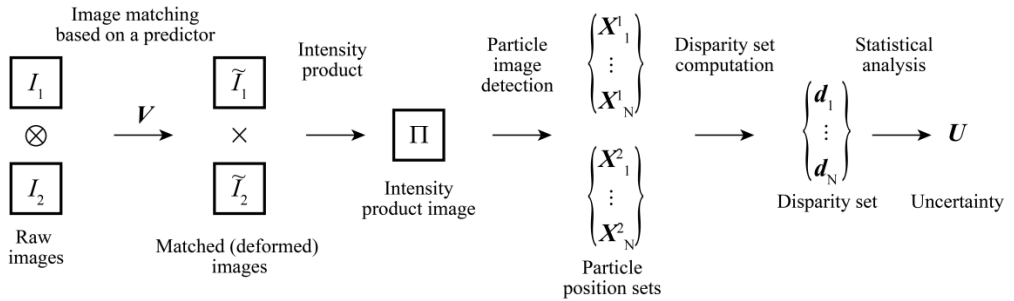


Figure 4.6. Scheme of the procedure for uncertainty quantification by image matching.

4.3 Numerical assessment

The image-matching approach for uncertainty quantification of PIV data is assessed via Monte Carlo simulation. Synthetic images of 400×400 pixels are generated with a random distribution of 16,000 particles (seeding density of 0.1 particles per pixel). The image intensity is rounded off with 8 bits quantization (0-255) and the particle images are assumed to have a Gaussian shape with 2 px mean diameter and 0.2 px standard deviation. A

Gaussian-shaped laser sheet is simulated having width $\Delta z = 30$ px. The camera read-out error is modelled as white noise of average intensity equal to 5 counts; the pixels fill factor equals 1. A uniform displacement is considered with values ranging from 0 to 2 pixels.

The recordings are processed with the three interrogation algorithms introduced in section 4.2.1:

- Single-pass
- Double-step with discrete window offset
- Multi-step with window deformation (WIDIM)

The window size is chosen of 17×17 and 33×33 px; 50% overlap factor is selected. In WIDIM, three iterations are performed.

A statistical analysis is conducted considering the root-mean-square (RMS) of both actual error and estimated uncertainty:

$$\overline{\varepsilon}_{\text{RMS}} = \sqrt{\overline{\varepsilon^2}}, \quad U_{\text{RMS}} = \sqrt{\overline{U^2}} \quad (4.8)$$

where the over-bar indicates the time average. The effect of several error sources typical in PIV measurements is analysed hereafter.

Uniform in-plane displacement

The RMS error for single-pass correlation increases linearly in the range of displacement between 0 and 0.5 pixels and stays nearly constant for larger displacements (figure 4.7-a). This agrees with well-known results from Westerweel (1993) and Raffel *et al* (2007). The estimated uncertainty follows the behaviour of the actual error with good accuracy. Oscillations of small amplitude (0.005 pixels) of U_{RMS} about the actual value have a wavelength of one pixel and are ascribed to numerical errors associated to the particle image peak fit algorithm. From the analysis of the results at different interrogation window size, this simple test already shows that the $1/\sqrt{N}$ scaling correctly takes into account the statistical properties of the error dispersion.

The discrete window offset method introduces a periodic behaviour of the RMS error in agreement with the early simulations of Scarano and Riethmuller (1999). The error increases for a displacement increasing from 0 and 0.5 pixels and decreases in the range 0.5-1 pixels. Also in this case the proposed method yields an accurate error estimate, with discrepancy between U_{RMS} and ε_{RMS} of less than a hundredth of a pixel.

The image deformation technique further lessens the random errors, which agrees well with the abundant literature on the subject (Lecordier *et al*, 2001; Scarano, 2002; Astarita and Cardone, 2005; Fincham and Delerce, 2000, among others). In the present test, the actual RMS error does not exceed 0.02 px for the smallest window. In this case, the uncertainty is clearly overestimated by 30% to 50% due to the limited precision of the individual particle image peak fit (Astarita and Cardone, 2005). A minimum error level is thus introduced which may be regarded as a “fog level” for the present estimator and is considered to be between 0.005 and 0.01 pixels for window sizes of 33×33 and 17×17 px, respectively.

Again, the scaling with the interrogation window size (viz. $1/\sqrt{N}$) is reproduced correctly and agrees fairly well with known results (Raffel *et al*, 2007, among others).

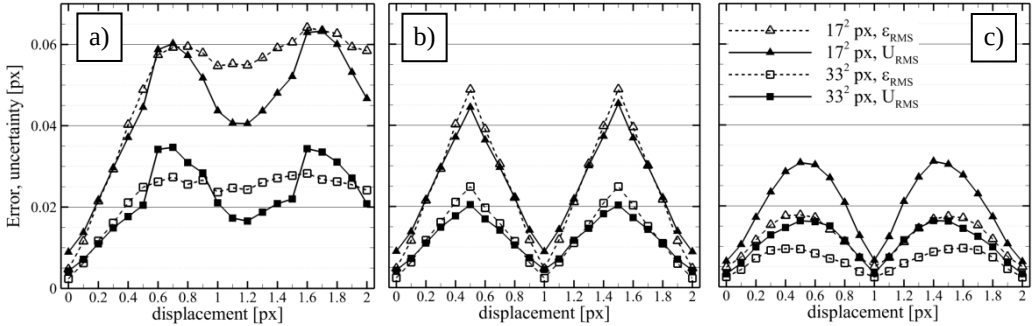


Figure 4.7. Error and uncertainty RMS as a function of the in-plane displacement for different interrogation algorithms. (a) single-pass correlation; (b) double-step cross-correlation with discrete window offset; (c) multi-pass cross-correlation with window deformation. The symbol key applies to the three plots.

The analysis in the remainder focuses on double-step interrogation with discrete window offset and multi-step correlation with window deformation (WIDIM); the single-pass cross-correlation method is not considered. The interrogation window size is kept equal to 33×33 pixels.

Displacement gradient

A displacement gradient ranging from 0 to 0.2 pixels per pixel is considered here. The displacement RMS error varies over about two orders of magnitude in the analysed gradient range (figure 4.8) when the double-step correlation with discrete window offset is employed. This variation is ascribed to the correlation peak broadening that increases with the displacement gradient (Westerweel, 2008).

The WIDIM algorithm applies in-plane deformation and compensates the gradient effect. As a result, the measurement error is reduced by one order of magnitude. This result agrees well with previous findings (Scarano and Riethmuller, 2000).

For the latter interrogation algorithm, a good agreement between actual error and estimated uncertainty is obtained, as the error levels lie clearly above the fog-level. For the technique based on the discrete window offset, the image-matching approach provides a good error estimate for displacement gradients below 0.1 px/px. For even larger values of the displacement gradient, the error is underestimated. This is ascribed to the particles pair detection technique, which finds a pair as long as their disparity does not exceed a particle image diameter. For a value of the gradient (shear rate in this case) of 0.2 pixels/pixel, the particle disparity at the edge of the window is already 3 px, which cannot be retrieved with the current technique. Further improvements would be needed if the method is to be generalized to flows where the value of the gradient exceeds the measurable range of particle disparity.

Out-of-plane motion

Particle motion through the measurement plane is recognized as one of the main sources of measurement uncertainty (Adrian, 1991; Nobach and Bodenschatz, 2009). The current simulation considers a mildly varying in-plane displacement distributed over 0-2 pixels (displacement gradient of 0.005 pixels/pixel) and uniform out-of-plane displacement w ranging from 0 to 0.4 Δz (different sets of images are generated for different values of w).

The out-of-plane motion causes a variation of the particle images intensity and contributes to the measurement errors in several ways:

- At high seeding density, overlapping particle images that vary their relative intensity lead to a biased displacement estimate; the magnitude of this error depends on particle image intensity, diameter and overlap, as thoroughly investigated by Nobach and Bodenschatz (2009). This effect alone was reported to cause random errors of the order of 0.1 pixels.
- At low particle image density, the out-of-plane displacement may cause a significant reduction of the correlation signal strength: as a result, in real experiments, the correlation peak shape may become strongly affected by camera read-out noise (Raffel *et al*, 2007).

Nobach and Bodenschatz (2009) report an exponential increase of the error with the out-of-plane displacement. The latter is qualitatively retrieved in the present simulation (figure 4.9). The image-matching uncertainty estimation appears to take into account the out-of-plane motion consistently: the reduction of particle image pairs yields higher measurement error according to the $1/\sqrt{N}$ scaling. For both interrogation methods, the uncertainty follows the actual error increase within 0.005 pixels accuracy.

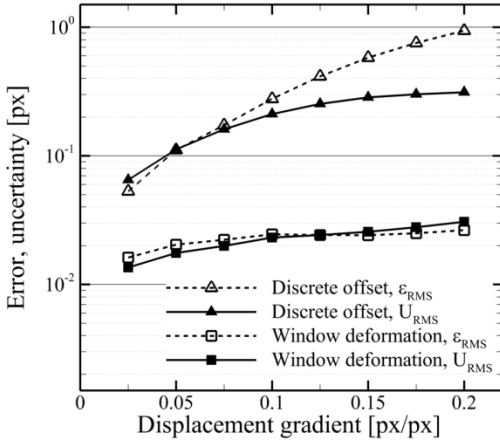


Figure 4.8. Error and uncertainty RMS as a function of the displacement gradient.

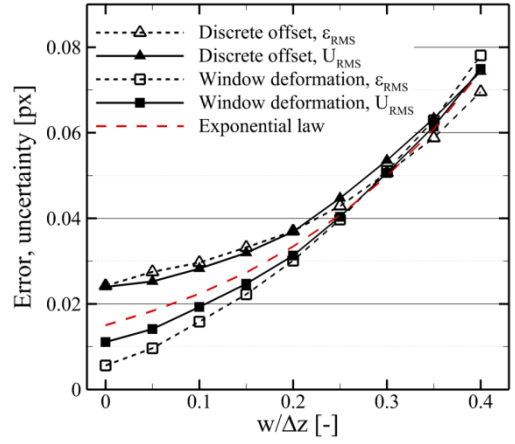


Figure 4.9. Error and uncertainty RMS as a function of the out-of-plane displacement. The red dashed line represents the exponential behaviour reported by Nobach and Bodenschatz (2009).

Seeding density

The same planar displacement of the previous case (with no out-of-plane motion) is considered here to evaluate the correctness of the estimator with respect to the particles image density. The latter is varied between 0.005 and 0.15 particles per pixel (*ppp*).

The RMS error decreases for increasing seeding density, which is known from previous studies (Raffel *et al*, 2007, among others). When more particle

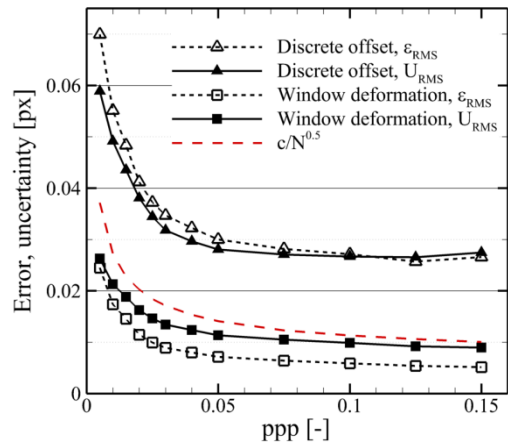


Figure 4.10. Error and uncertainty RMS as a function of the seeding density.

image pairs are present in the interrogation spot, a stronger correlation signal-to-noise is achieved. The plots of figure 4.10 show that the scaling rule implied in the model ($1/\sqrt{N}$) is consistent with the behaviour of the actual error. In particular, one can see that for a fourfold increase of the seeding density the RMS error approximately halves. Also in this case, the agreement between U_{RMS} and ε_{RMS} is within 0.005 pixels.

A large seeding density enhances the amount of information on the fluid motion contained in the PIV recordings. However, when more than one particle is located along a line of sight of the imaging system, the particle images overlap in the image plane. As a result of the superposition between particle images, the determination of their position via intensity peak fit may be affected by significant errors of the order of 0.1 px, as shown in figure 4.11.

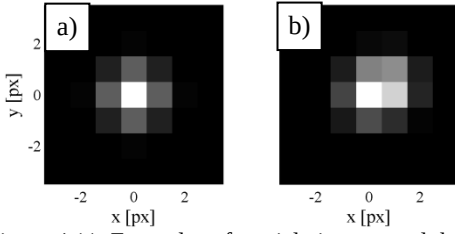


Figure 4.11. Examples of particle images and their estimated position. (a) Single particle image: the estimated position coincides with the actual position (0, 0). (b) Superposition of two particle images at (0, 0) and (0.8, 0.4) px: the image-matching algorithm detects a single particle image located at (0.37, 0.14) px.

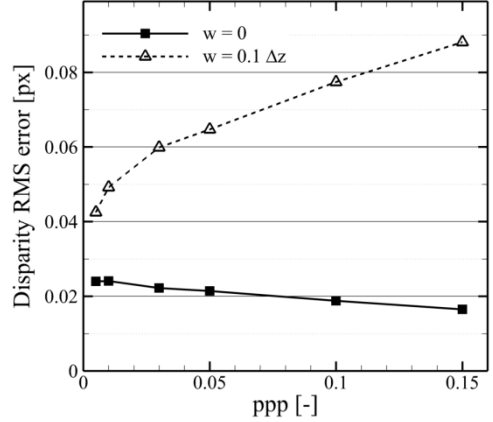


Figure 4.12. RMS error on the estimated disparity as a function of the seeding density for the cases of null out-of-plane motion ($w = 0$) and out-of-plane displacement equal to 10% of the laser sheet thickness ($w = 0.1 \Delta z$).

When the particle images superposition is not accompanied by image noise or out-of-plane motion, it has minor consequences on the accuracy of the image-matching approach. In fact, in this case the same systematic error affects the determination of the particle image position in two PIV recordings and it cancels out when the disparity is evaluated from the difference between the particle positions. Instead, in presence of out-of-plane particle displacement or background noise, the relative variation of the intensity of superimposed particle images yields a fictitious in-plane displacement, as

discussed by Nobach and Bodenschatz (2009). Hence, severe errors on the estimated disparity may arise, as illustrated in figure 4.12 for the case of out-of-plane displacement equal to 10% of the laser sheet thickness. A possible way to reduce those errors is indicated: the particle image disparity may be determined directly from the cross-correlation of the images within a small interrogation area, without computing the position of the individual particle images.

Particle image diameter

The effect of the particle image diameter d_τ onto the measurement precision is well known and documented. In the present simulation, particles with an image diameter varying in the range 0.5-5 pixels are considered; the displacement field is the same as that used in the previous case. The RMS error for discrete window offset sharply decreases for any increase of the particle image diameter in the range between a fraction of a pixel and 2 pixels (figure 4.13). For any further increase of d_τ , the error increases approximately linearly. This agrees with known results from Westerweel (1997), who reports that *peak locking* errors dominate for $d_\tau \leq 1$ px and random errors prevail for $d_\tau \gg 1$ px. For particle image diameter in the sub-pixel regime, the random errors obtained with

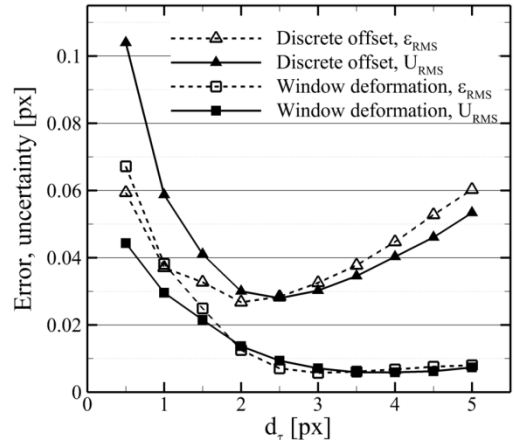


Figure 4.13. Error and uncertainty RMS as a function of the mean particle image diameter.

with window deformation are equivalent to those obtained with the discrete shift technique. When d_τ is larger than a pixel, a marked difference is observed: the actual error continues decreasing in the observed range, in contrast with the discrete offset technique. This result is also in contrast with the model recently stated by Westerweel *et al* (2013) whereby the RMS error always increases with the d_τ , irrespective of the interrogation method used for the analysis.

In this case, the image-matching error estimator shows a marked difference from the actual error when d_τ is less than 1 pixel. This result does not come unexpected: in the peak-locking regime also the estimator of the individual particles peak position is biased to the closest integer. As a result, the estimator is not expected to detect the presence of peak-locking. Nevertheless,

U_{RMS} follows the same trend as ε_{RMS} . For measurements with particle image diameter of one pixel or larger, the error estimator becomes again very accurate.

Background noise

The effect of the background noise on the measurement uncertainty is evaluated in this section. The main noise sources for conventional CCD and CMOS cameras are classified as follows (<http://www.emva.org>):

- *read noise* σ_r , which appears on each pixel readout and reflects the electronic noise;
- *dark noise* σ_d , function of chip temperature, pixel size and exposure time;
- *shot noise* σ_s , due to the fluctuations in the number of photoelectrons within a pixel. According to quantum mechanics, the probability of those fluctuations is Poisson distributed, therefore the variance σ_s^2 of the fluctuations equals the mean number N_e of accumulated electrons.

Following the EMVA (European Machine Vision Association) standard (<http://www.emva.org>), a linear model for the camera noise is employed here, therefore the variance of the total noise (expressed in electrons) is calculated as the sum of the variances of each sources:

$$\sigma_n^2 = \sigma_r^2 + \sigma_d^2 + \sigma_s^2 \quad (4.9)$$

The noise level NL , expressed in counts, is obtained dividing σ_n by the conversion factor, i.e. the number of electrons per count. Values of NL ranging between 0 and 25% of the maximum particle image intensity I_{max} are selected, which are considered the standard for state-of-the-art CCD and CMOS cameras used in PIV (Raffel *et al*, 2007).

For both WIDIM and discrete window offset, the error is below 0.02 pixels for noise levels below 5%. Larger background noise levels have a detrimental effect on the correlation peak shape and cause the error to increase. The uncertainty quantification consistently reproduces the error increase with background noise (figure 4.14). Only for high noise level ($NL > 20\%$), the position of the particle image itself is strongly affected by the noise, therefore the accuracy of the uncertainty estimate is reduced.

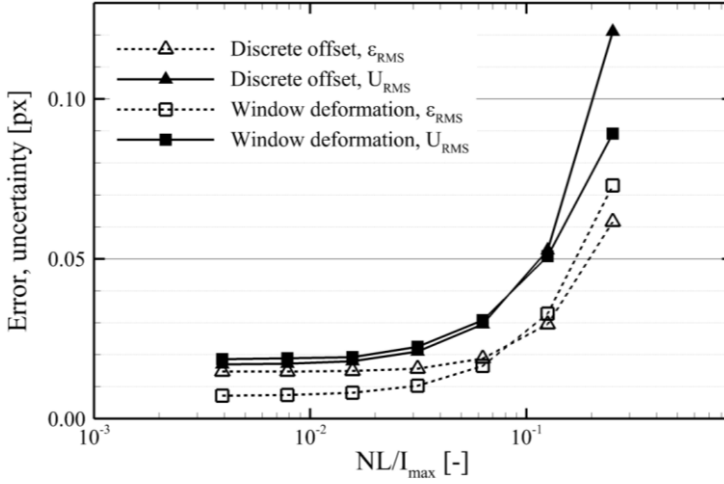


Figure 4.14. Error and uncertainty RMS as a function of the background noise level.

4.4 Experimental assessment

4.4.1 Methodology

The experimental validation of uncertainty quantification methods is recognized as a challenge because the exact velocity field and therefore the true measurement error are unknown. Here several strategies are proposed to estimate the latter to a good degree of accuracy.

Concurrent experiments

A PIV system named the *measurement system* is used to acquire images in conditions representative of a typical PIV experiment. The interrogation of these images yields the so-called *measured velocity field*, whose uncertainty is quantified via the image-matching approach. An additional PIV system is employed, defined the *high dynamic range (HDR) system*, that records images at digital resolution typically 3 to 4 times higher. In the HDR system a particle displacement in the physical space is discretized with a larger number of pixels than for the measurement system. As a result, errors due to spatial discretization affect the HDR output to a lesser extent. Considering the absolute displacement error $\epsilon_{\Delta x}$ as approximately invariant with the measured displacement Δx (Raffel *et al*, 2007), the HDR system yields lower relative errors ϵ_r and in turn a larger dynamic velocity range (DVR, Adrian, 1997), as illustrated in figure 4.15. Furthermore, the HDR measurement is conducted

with optimized imaging conditions, i.e. with particle images encompassing approximately two pixels. By this arrangement, the *DVR* gain with respect to the measurement system typically achieves a factor 5. Under these conditions, the velocity field retrieved with the HDR system may well be considered as the reference velocity measurement. Its difference to the velocity measured by the regular system is taken as the measurement error of the latter.

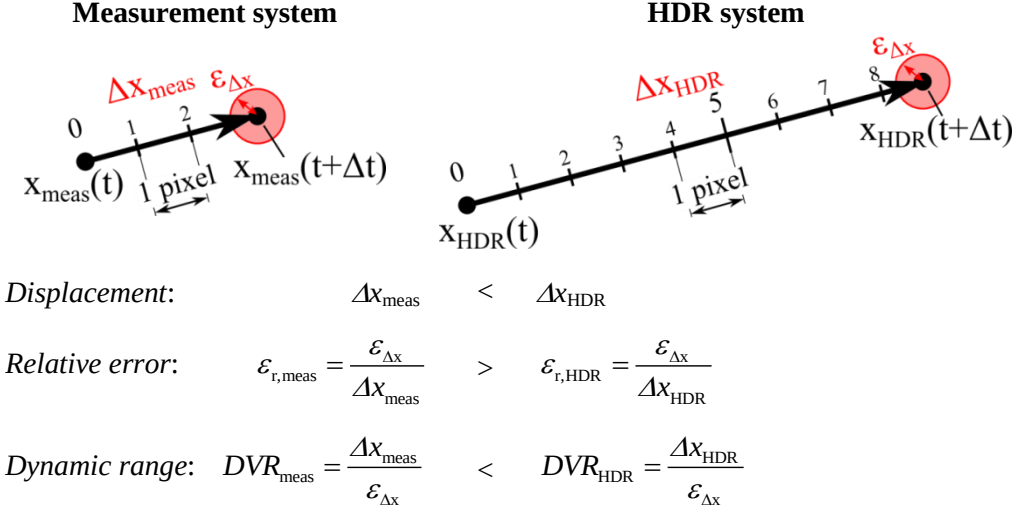


Figure 4.15. Relative error and dynamic velocity range of measurement and HDR systems. The displacements are shown in the image space.

However, most PIV experiments reported in the scientific literature as well as within industrial reports are performed with a single measurement system. Even when such an auxiliary, more accurate system is not present, quantification of uncertainty for time-resolved measurements can be done by the strategies reported hereafter.

Advanced multi-frame interrogation

In time-resolved measurements, multi-frame interrogation (Hain and Kähler, 2007) and advanced variants such as sliding-average correlation (Scarano *et al*, 2010) and pyramid correlation (Sciacchitano *et al*, 2012) allow obtaining velocity fields with typically 2 to 5 times lower uncertainty than those computed with single-pair correlation. The former velocity fields can be regarded as a reference and used to calculate the measurement error of the single-pair correlation analysis.

Spectral analysis

Physical considerations based on the velocity power spectrum can be used to retrieve the measurement uncertainty. In flows exhibiting fluctuations with broadband spectrum such as in developed turbulence, the energy content of the physical fluctuations decreases monotonically with the frequency. However, measurement errors are assumed to be uncorrelated in time and contribute to the spectrum in form of white noise (constant power spectral density). Consequently, the power spectrum of a signal with embedded white noise does not drop to zero but to a positive level also referred to as the noise floor. As proposed by Ghaemi *et al* (2012), the minimum value of the measured power spectrum may be taken as representative to determine the measurement noise level. This approach is further discussed and used in section 4.4.3.2.

Temporal coherence

When the flow temporal evolution is oversampled (i.e. the flow characteristic frequency is significantly smaller than the acquisition frequency, like for instance in slowly varying laminar flows), the velocity time history is expected to return a smooth signal, where the spatio-temporally resolved physical fluctuations are represented by sufficient number of samples. Also in this case, the measurement noise adds spurious high-frequency fluctuations that can be assimilated to white noise. Given the oversampling regime, the latter can be largely diminished by low-pass filtering the velocity time history, e.g. using a polynomial least-square time regression. The denoised velocity time history thus obtained can be considered as representative of the exact velocity, from which the measurement noise magnitude can be estimated.

4.4.2 Experimental apparatus and setup

A number of experiments have been conducted that encompass very different flow regimes, from a low speed jet in water to a fully developed turbulent boundary layer in supersonic flow. The associated flow features and error sources cover several situations encountered in typical PIV experiments. The main characteristics of the experiments are reported in table 4.1. In three experiments, the use of an ancillary HDR system is made, with 3 to 4 times higher image digital resolution than the measurement system. The reported gain of *DVR* is taken conservatively as it does not account also for the improved imaging conditions. The setup of the experiments is discussed in detail in the following sections.

Table 4.1. Description of the experiments.

Experiment	Re or Mach [-]	Wind tunnel	Type of assessment	Image digital resolution [px/mm]		DVR [-]	
				Measurement	HDR	Measurement	HDR
Separated shear layer	$Re = 12,000$	V-tunnel	Concurrent experiments	12.8	36.6	94	270
Turbulent wake behind a prism	$Re = 100,000$	V-tunnel	Spectral analysis and temporal coherence	13.8	/	43	/
Uniform transverse flow	$V_{\infty} = 2.1$ m/s	V-tunnel	Concurrent experiments	12.6	48.3	27	93
Transitional jet	$Re = 5,000$	JTF	Advanced multi-frame interrogation	22.0	22.0	80	250 [‡]
Supersonic boundary layer	$M = 2.0$	ST-15	Concurrent experiments	20.9	80.0	63	249

4.4.2.1 Low-speed flow measurements

The low-speed experiments in air (separated shear layer, turbulent wake and uniform transverse flow) are conducted in the vertical wind tunnel (V-tunnel) of the Aerodynamics Laboratories of Delft University of Technology. A schematics of the V-tunnel is shown in figure 4.16. The wind tunnel has an open test section with circular exit of 60 cm diameter. The contraction ratio of 150:1 yields a turbulent intensity of about 0.02% at free stream velocity of 10 m/s.

A dual cavity diode pumped Nd:YLF laser (Litron Lasers, LDY303HE) provides light at wavelength $\lambda = 527$ nm. Each cavity delivers a pulse energy of 22.5 mJ/pulse at 1 kHz. The

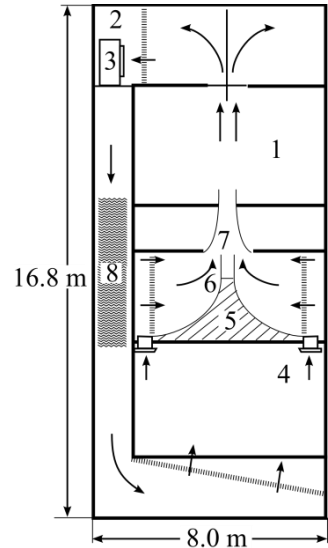


Figure 4.16. Overview of the V-tunnel: 1) measuring room; 2) fan room; 3) fan; 4) settling chamber; 5), 6) and 7) suction devices; 8) sound damper.

[‡] DVR of the velocity field computed with the pyramid correlation algorithm.

PIV uncertainty quantification by image matching

laser beam has an output diameter of 3 mm and is shaped into a sheet approximately 2 mm thick using spherical and cylindrical lenses. Seeding particles of mean diameter $d_p = 1 \mu\text{m}$ are generated by a SAFEX smoke generator and dispersed in the settling chamber. Images of the seeding particles are recorded by two Photron *FastCAM SA1* CMOS cameras, (12-bit, 1024×1024 pixels, $20 \mu\text{m}$ pixel pitch). The cameras are equipped with Nikon objectives of focal length 105 mm and 200 mm respectively.

Separated shear layer

The flow around a prism of rectangular cross section $40 \times 60 \text{ mm}^2$ ($W \times H$) and spanwise length $L = 700 \text{ mm}$ is considered. The free-stream velocity is set to $V_\infty = 3.6 \text{ m/s}$, yielding a Reynolds number $Re_H = 12,000$ based on the prism height. Images are recorded in continuous mode at $f_{\text{acq}} = 5,000 \text{ Hz}$. The fields of view of the measurement and HDR systems (FOV_{Meas} and FOV_{HDR} , respectively) are $2.0 W \times 2.0 W$ and $0.7 W \times 0.7 W$, respectively (figure 4.17); the magnification factors are $M_{\text{Meas}} = 0.25$ and $M_{\text{HDR}} = 0.74$, respectively, yielding a magnification factor ratio $M_{\text{HDR}}/M_{\text{Meas}} = 3$. The origin of the coordinate system $(x_0, y_0) = (0, 0)$ is chosen at the bottom left corner of the model base. The recordings are processed with LaVision Davis 8.1, which makes use of a multi-grid iterative interrogation algorithm with window deformation. The interrogation window size is set to 16×16 pixels for the measurement system and to 48×48 pixels for the HDR system, so that the velocity is estimated at the same spatial resolution by the two systems. In both cases, the interrogation windows are Gaussian weighted and the overlap factor equals 75%.

Turbulent wake behind a prism

The analysis in the turbulent wake of the same rectangular model is conducted with one PIV system (i.e. the *measurement system*). The field of view (FOV_{TW}) has dimensions of $18.5 \times 18.5 \text{ mm}^2$ ($0.46 W \times 0.46 W$) and is centred along the base centreline 180 mm ($4.5 W$) downstream of the prism (figure 4.17). In order to achieve an acquisition frequency of 10,000 Hz and to increase the number of stored images, the active region of the sensor is reduced to 256×256 pixels. The optical magnification equals $M = 0.28$. The free-stream velocity is $V_\infty = 24 \text{ m/s}$ corresponding to a Reynolds number of about 100,000 based on the model height. The image interrogation is conducted with LaVision Davis 8.1. Different sizes of the interrogation

windows are investigated, ranging from 64×64 to 16×16 pixels; the overlap factor is set to 75%.

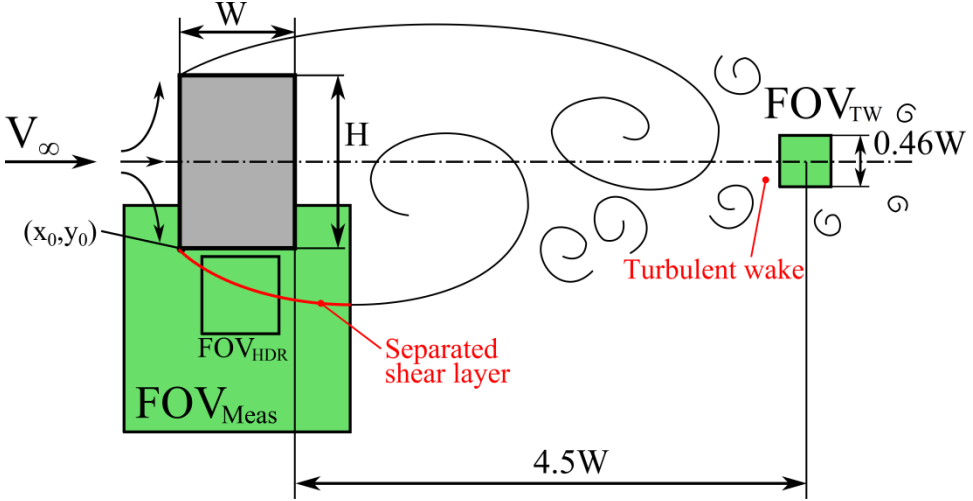


Figure 4.17. Fields of view of separated shear layer and turbulent wake tests.

Uniform transverse flow

The effect of the out-of-plane motion is investigated by tilting the light sheet with respect to the wind tunnel free stream direction. The wind tunnel is run at free-stream velocity $V_\infty = 2.1$ m/s without any model mounted in the test section. The laser sheet has 2 mm thickness (Δz) and is tilted by $\alpha = 9$ deg with respect to the free-stream to realize a velocity component $V_{\infty\perp}$ orthogonal to the laser sheet that causes an out-of-plane particle displacement (figure 4.18). The latter can be controlled through the pulse separation time. The sensor size of the two cameras is cropped to 512×512 pixels to achieve an acquisition frequency of $f_{acq} = 10,000$ Hz in continuous mode, yielding a minimum out-of-plane displacement $w_0/\Delta z = 0.016$. Larger out-of-plane displacements (multiples of $w_0/\Delta z = 0.016$) are obtained by skipping recordings from the sequence; for instance, when the first image is correlated with the fourth one, the out-of-plane displacement is equal to $3 \cdot w_0 = 0.048 \Delta z$.

The fields of view of HDR and measurement system are 10.6×10.6 mm² and 40.5×40.5 mm² respectively, yielding magnification factors $M_{HDR} = 0.97$ and $M_{Meas} = 0.25$ ($M_{Meas}/M_{HDR} = 3.9$).

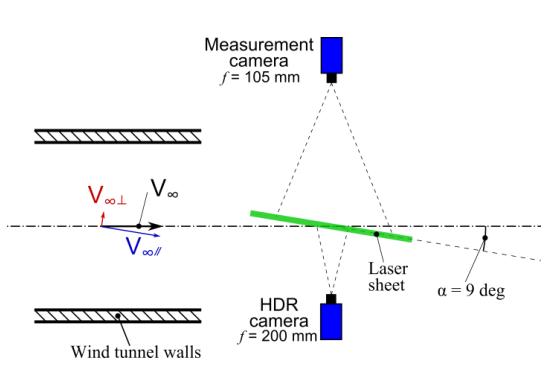


Figure 4.18. Experimental setup for the uniform transverse flow experiment.

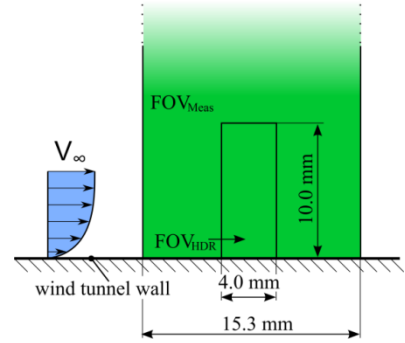


Figure 4.19. Measured and HDR fields of view (FOV_{Meas} and FOV_{HDR} , respectively) for the supersonic boundary layer experiment.

Transitional jet

Experiments in water are carried out in the Jet Tomographic Facility (JTF) of TU Delft (Violato and Scarano, 2011). A laminar submerged water jet is issued from a circular nozzle of 10 mm diameter D at 0.45 m/s, yielding a diameter based Reynolds number of 5,000. The jet is enclosed in an octagonal water tank of 600 mm diameter and 800 mm height built in Plexiglas.

The flow is homogeneously seeded with neutrally buoyant polyamide particles of 56 μm diameter; the seeding density is equal to 0.65 particles/ mm^3 . Images are acquired with two Photron *FastCAM SA1* CMOS cameras in single frame mode at frame rate of 1.2 kHz. The optical magnification is equal to 0.44. The illumination is provided by a Quantronix *Darwin-Duo* solid-state diode-pumped Nd:YLF laser. For a thorough description of jet facility and experimental setup, the reader is referred to Violato and Scarano (2011).

The recordings are processed with WIDIM with interrogation windows of 33×33 pixels and 50% overlap factor.

4.4.2.2 Supersonic boundary layer

A high-speed experiment is performed in the ST-15 wind tunnel of the Aerodynamics Laboratories of Delft University of Technology. The wind tunnel has a test section of $150 \times 150 \text{ mm}^2$ and is operated at Mach 2.0 and total pressure $p_0 = 3.1 \text{ bar}$.

The measurements are conducted to investigate the boundary layer generated at the wind tunnel's wall. The flow is seeded with micron size di-ethyl-hexyl-

sebacate (DEHS) particles, with a nominal median diameter $d_p = 1 \mu\text{m}$. Experiments conducted by Ragni *et al* (2011a) showed a typical relaxation time of $2 \mu\text{s}$ of such particles in a Mach 2.0 flow. The seeding particles are injected into the flow in the settling chamber.

The particle tracers are illuminated by a Quantel *CFR PIV-200* laser (double-pulsed Nd:YAG laser, with 200 mJ pulse energy and 9 ns pulse duration at 532 nm wavelength). Laser optics shape the beam into a plane of 1 mm thickness. The images are recorded by two PCO *Sensicam QE* CCD cameras (1376×1040 pixels, pitch of $6.45 \mu\text{m}$, 12-bit quantization). The sensor is cropped to 320×800 pixels in the streamwise and vertical directions respectively to achieve an acquisition rate of 10 Hz in frame-straddling mode. The pulse separation time is set to $0.6 \mu\text{s}$, the minimum inter-frame time for the camera.

The measurement camera is equipped with a Nikon objective of focal length 60 mm and images a region of 15.3×38.2 mm (magnification factor $M_{\text{Meas}} = 0.14$); a 105 mm focal length Nikon objective is mounted on the HDR camera, which images a region of 4.0×10.0 mm (magnification factor $M_{\text{HDR}} = 0.52$, figure 4.19). The f-number is set to $f_{\#} = 11$ and $f_{\#} = 16$ for the two cameras, respectively.

The recordings are processed with LaVision DaVis 8.1, using interrogation windows of 64×64 pixels for the HDR camera and 16×16 pixels for the measurement camera, both with Gaussian weighting and 75% overlap factor.

4.4.3 Results

4.4.3.1 Separated shear layer

The shear layer emanating from the sharp separation at the prism corner ($x = 0, y = 0$) divides the outer flow from the recirculating region adjacent to the prism (figure 4.20-a). Coherent vortices are formed in the separated shear layer under the effect of the Kelvin-Helmholtz instability. The frequency of shedding of these vortices can be determined already from visual inspection and corresponds to Strouhal number $St_H = 3.5$, which is in good agreement with the value reported by de Kat *et al* (2008). In the outer region, the potential flow exhibits small amplitude velocity fluctuations (below 0.5 pixels) as shown in figure 4.20-b. The fluctuations magnitude becomes higher in the separated region (about 1 pixel) where the flow exhibits a more chaotic behaviour. The largest velocity fluctuations (exceeding 3 pixels) occur within the shear layer and are associated with the vortex formation and the flapping motion of the shear layer. The vortex breakdown determines the transition

from the laminar to the turbulent regime, which takes place downstream the location $x = 300$ px ($x/W = 0.6$).

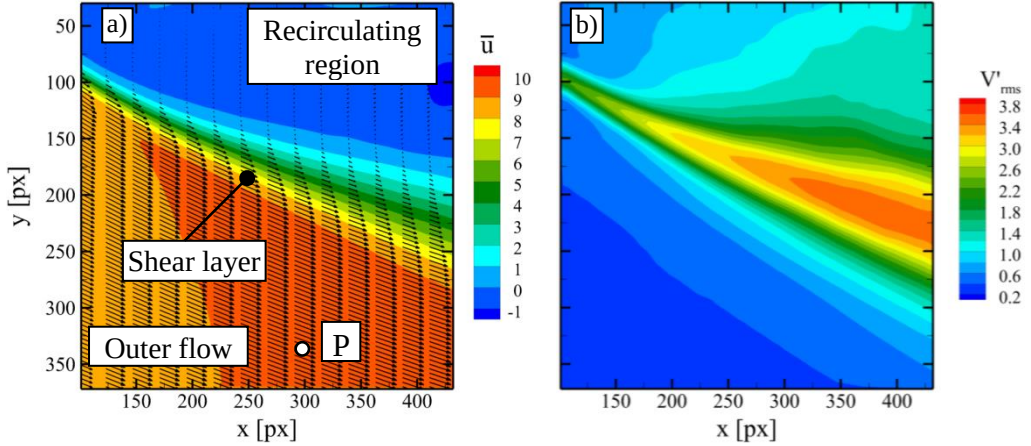


Figure 4.20. (a) Mean horizontal velocity with velocity vectors (one every 6 vectors is displayed in the x -direction, for clarity). (b) Root mean square of velocity fluctuations. Both quantities are measured with the HDR system and reduced to pixel units of the measurement system.

The velocity time history yielded by the measurement and the HDR system is extracted from a point P in the outer region (figure 4.20) and displayed in figure 4.21. For completeness, the particle image displacement in pixels is given on the left axis for the measurement system and on the right for the HDR system. The plot legend also shows a first estimate of the uncertainty bars, corresponding to 0.1 pixels of each system. The higher magnification of the PIV-HDR yields narrower uncertainty bars: the lower uncertainty of the latter system is confirmed by the velocity time history, which exhibits only low-frequency fluctuations associated with the physical process of vortex shedding. In contrast, the data points of the measurement system exhibit higher scatter due to random noise that causes high-frequency spurious fluctuations.

The power spectral density (PSD) of the streamwise velocity component confirms that the HDR system yields lower noise level than the measurement system (figure 4.22). The minimum PSD computed with the HDR system is about 12.5 times lower than that obtained with the measurement system, meaning that the HDR system yields a reduction by factor $\sqrt{12.5} = 3.5$ of the noise level (for a more thorough discussion on how the noise level is evaluated from the PSD, the reader is referred to section 4.4.3.2).

As a result, the PIV HDR velocity can be considered as a “reference” because it has significantly lower uncertainty and can be employed to determine the actual measurement error of the PIV measurement system.

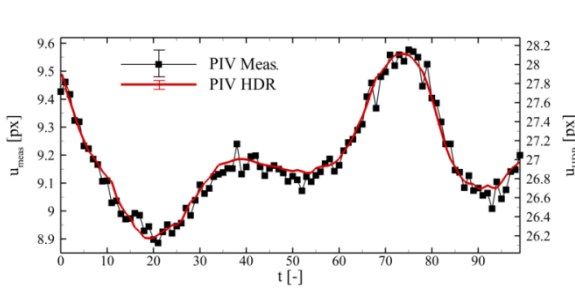


Figure 4.21. Time history of horizontal velocity component. Black full squares: measured velocity; continuous red line: velocity measured by the PIV HDR system.

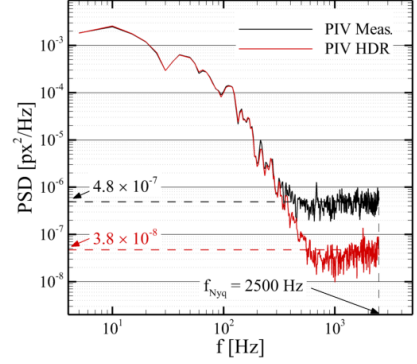


Figure 4.22. Power spectral density (PSD) of the streamwise velocity component in P.

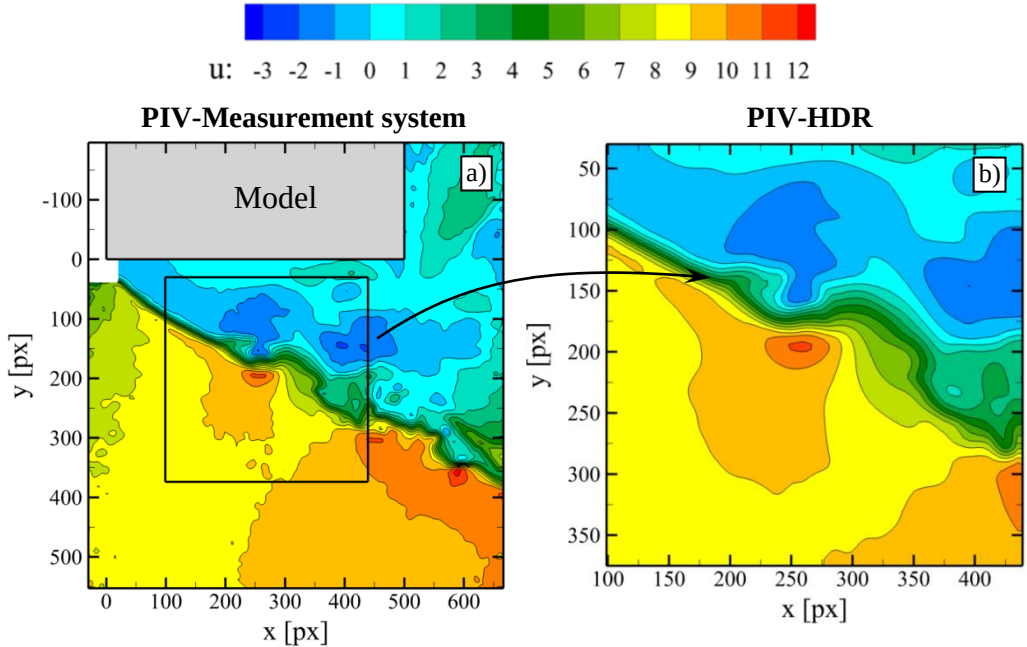


Figure 4.23. Instantaneous horizontal velocity component, expressed in pixels of the measurement system. (a) Measurement PIV system; (b) PIV HDR system.

The instantaneous velocity field obtained simultaneously by the two systems is compared in figure 4.23. The large-scale velocity distribution shows a good correspondence, with the maximum horizontal velocity (equal to 11.2 px) in both cases associated with a vortex generated along the shear layer. Nevertheless, the iso-contours produced by the measurement system exhibit a jittery pattern, which is ascribed to the larger noise magnitude.

Error assessment and uncertainty quantification

Three error sources dominate the present case:

- The velocity gradient across the shear layer, which reaches 0.4 pixels per pixel
- The streamlines curvature due to the vortex shedding
- The out-of-plane motion downstream of the transition point

The spatial distributions of the root-mean-square of actual error and estimated uncertainty are compared in figure 4.24. Three regions can be distinguished based on the error magnitude:

- the shear layer, where the measurement error peaks, exceeding 0.15 px
- the outer flow region, characterized by errors below 0.06 px
- the separated region, where the error assumes intermediate values

The evaluated uncertainty allows distinguishing the three regions and estimating the error magnitude within 0.03 pixels.

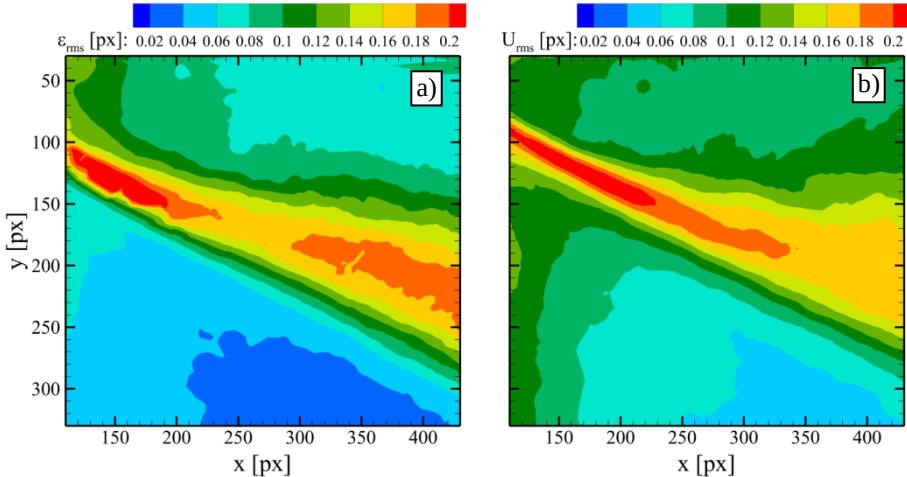


Figure 4.24. Actual error (a) and estimated uncertainty (b) root mean square.

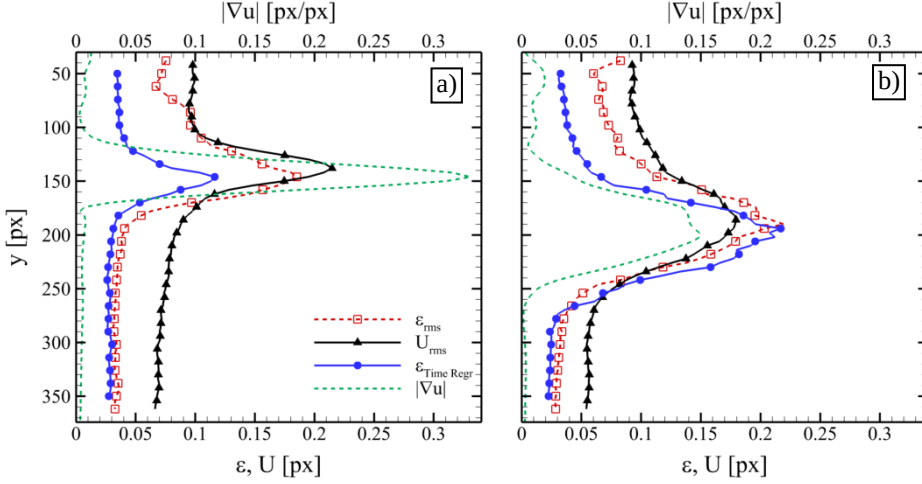


Figure 4.25. Root-mean-square error and uncertainty profiles at $x = 200$ pixels ($x/W = 0.4$, a) and $x = 340$ pixels ($x/W = 0.68$, b). For clarity, one every three samples is displayed. Error and uncertainty are expressed in pixels (bottom horizontal axis), the velocity gradient is expressed in pixels per pixel (top horizontal axis). The symbols key applies to both plots.

Two profiles are extracted from the contours above, respectively before and beyond transition ($x = 200$ px and $x = 340$ px, respectively, see figure 4.25). For comparison, the error is also estimated via the temporal coherence analysis (see section 4.4.1), using a second order polynomial regression on a kernel of nine samples.

In the laminar region ($x = 200$ px), the error peak occurs at the location where the mean horizontal-velocity gradient $|\nabla u|$ is the maximum ($y = 150$ px), which suggests that the error is primarily due to the in-plane velocity gradient. The estimated peak uncertainty is in good agreement with the actual error (0.2 pixels). In the separated region and the outer flow, the velocity gradient is significantly lower and the measurement error drops down to about 0.03 pixels. Here, the uncertainty is overestimated to about 0.06 pixels, which can be considered as the “fog level” for the image-matching estimator in this experiment. The error estimated with the time-regression method, here indicated as $\epsilon_{Time Regr}$, reproduces the peak at the mean shear layer location, but underestimates by 50% the actual value of the error.

Beyond transition, the shear layer flapping motion increases, resulting in higher velocity fluctuations spread over a larger region. Also at this location, the measurement error peaks at the point of maximum velocity gradient. In this case, the measurement error is primarily due to the streamlines curvature

and to three-dimensional fluid motion. As a consequence of the larger amplitude of the shear layer flapping motion, the error peak also broadens (figure 4.25-b). The image-matching approach consistently reproduces the broadening of the error peak; however, the peak value is underestimated by 20%. In the separated region and the outer flow, the same considerations as before apply: the error is overestimated because it lies below the fog level. Also the time-regression method estimates accurately the error peak location and its value.

4.4.3.2 Turbulent wake

The flow behind the prism is characterized by the transition to the turbulent regime, where small-scale structures and three-dimensional motion occur. The instantaneous contour of Q (Jeong and Hussain, 1995) depicted in figure 4.26 illustrates the presence of vortical structures having wavelength below 50 pixels (in the measurement system units). Figure 4.27 shows a fragment of the streamwise velocity time history extracted along the centreline at $x/H = 3$ (point P of figure 4.26), where velocity fluctuations of 1 pixel magnitude are clearly visible. The plot exhibits also high-frequency spurious fluctuations of the order of 0.1 pixels ascribed to measurement random noise.

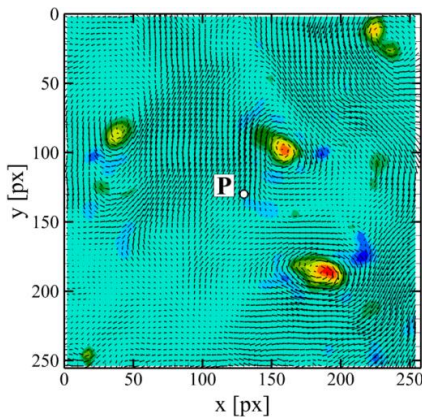


Figure 4.26. Instantaneous value of Q and velocity vectors relative to the convective velocity.

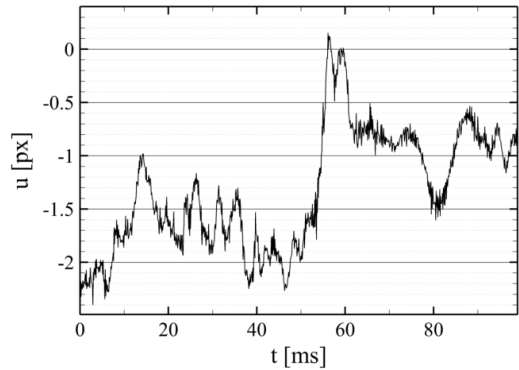


Figure 4.27. Fragment of the streamwise velocity time history in P expressed in pixel units of the measurement system.

Error assessment via spectral analysis

In this case, the measurement noise is inferred by the analysis of the velocity power spectral density (PSD). From turbulence dynamics, the PSD of the velocity fluctuations decreases monotonically for sufficiently high frequency

(inertial subrange, Tennekes and Lumley, 1972). However, the measurement noise, which in its simpler form can be modelled as white noise having constant power spectral density PSD_{noise} , prevents the PSD of the signal from decreasing to zero. Hence, the power associated to the measurement noise can be calculated as:

$$P_N = \int_0^{f_{\text{Nyq}}} PSD_{\text{noise}} df = PSD_{\text{noise}} \cdot f_{\text{Nyq}} \quad (4.10)$$

The Nyquist frequency f_{Nyq} is equal to half of the sampling rate. Finally, the magnitude associated to the measurement noise is:

$$NL = \sqrt{P_N} = \sqrt{PSD_{\text{noise}} \cdot f_{\text{Nyq}}} \quad (4.11)$$

The velocity time history is measured at location P with interrogation window sizes ranging from 16×16 pixels to 64×64 pixels. The PSD of the streamwise velocity component is computed and displayed in figure 4.28 for interrogation windows of 16×16 and 64×64 pixels, respectively. The former spectrum (window size 16×16 pixels, figure 4.28-a) exhibits a power distribution varying over five decades up to a frequency of approximately 1,000 Hz, reaching a minimum of approximately $1.69 \times 10^{-6} \text{ px}^2/\text{Hz}$. The noise level estimated with equation (4.11) is equal to $NL = 0.092 \text{ px}$. When the recordings are processed with a larger interrogation window, a reduction in random errors is expected due to the increased correlation signal strength. This is confirmed by the PSD of figure 4.28-b (interrogation window of 64×64 pixels), where the power distribution varies over six decades and reaches a minimum of $PSD_{\text{noise}} = 1.18 \times 10^{-7} \text{ px}^2/\text{Hz}$, corresponding to a noise magnitude of 0.024 px.

Figure 4.29 depicts the noise level evaluated with the spectral analysis and the estimated uncertainty for different interrogation window sizes. The results evidence that the error scales with the linear size Ws of the interrogation window. This finding does not come unexpected, because it has been shown in section 4.3 that the error scales with $\sqrt{N}$ and, for homogeneous seeding density, N is proportional to the interrogation area. The measurement error is also estimated through the temporal coherence analysis, using a second-order polynomial least-square regression. In the present experiment, the integral time scale computed from the velocity autocorrelation function is equal to $T = 33 \text{ ms}$, therefore a time kernel $t_{\text{ker}} = 2.1 \text{ ms} \ll T$ is selected for the time

regression. Such approach shows a correct scaling with the interrogation window size and provides a good estimate of the random errors magnitude, although underestimated by about 30% in the present test.

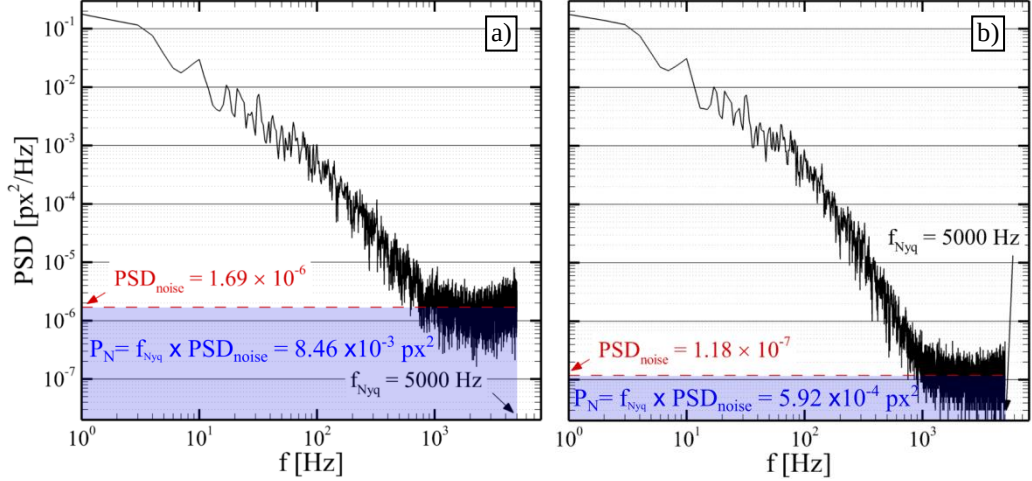


Figure 4.28. Power spectral density of the streamwise velocity component in P. (a) Interrogation window size 16×16 px. (b) Interrogation window size 64×64 px. The power associated to measurement noise P_N is displayed as a shaded region.

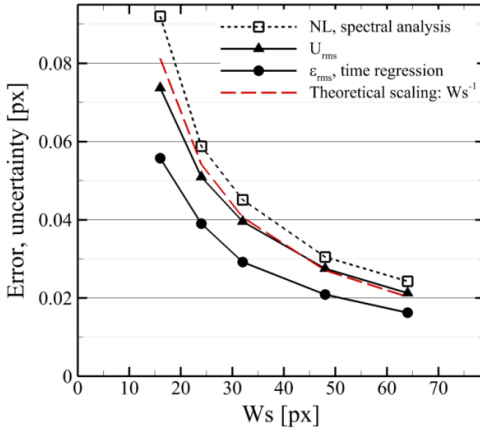


Figure 4.29. Root mean square of the measurement error and uncertainty as a function of the interrogation window size for the turbulent wake test case.

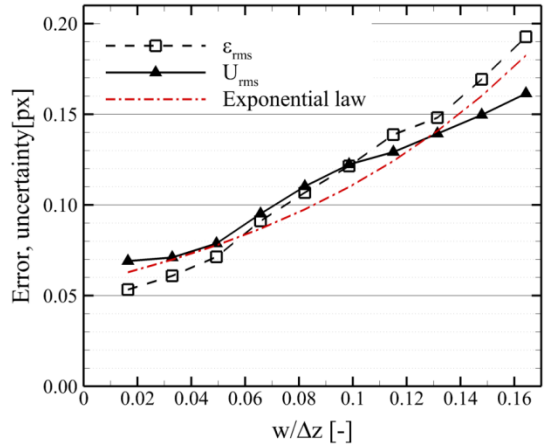


Figure 4.30. Root mean square of the measurement error and uncertainty as a function of the out-of-plane displacement.

4.4.3.3 Uniform transverse flow

The uniform transverse flow experiment has been conducted to isolate the error associated with out-of-plane displacement from other error sources. The contribution of the out-of-plane motion to the measurement error is well documented in literature (Keane and Adrian, 1995; Raffel *et al*, 2007; Nobach and Bodenschatz, 2009, among others) and has been discussed already in the numerical assessment (section 4.3). An exponential increase of the error with the out-of-plane displacement has been reported.

The exponential trend is retrieved also in the present experiment (figure 4.30), where the error ranges from $1/20^{\text{th}}$ to $1/5^{\text{th}}$ of a pixel. The estimated uncertainty reproduces the exponential trend and matches the actual error value within 20%.

Note that the image-matching approach is able to quantify the uncertainty associated with the out-of-plane motion without requiring the measurement of the out-of-plane displacement, contrarily to other approaches proposed in literature (e.g. the uncertainty surface method by Timmins *et al*, 2012).

4.4.3.4 Transitional jet

In the transitional jet test case, the reference velocity is obtained with an advanced multi-frame technique, namely the *pyramid correlation* (Sciacchitano *et al*, 2012), which will be discussed in detail in chapter 6 (figure 4.31-a). The accuracy of the technique has been shown to exceed 0.01 pixels. An example of instantaneous actual error, determined as the difference between measured and reference velocity, is depicted in figure 4.31-b. Two regions of high error are identified. The first one is the highly sheared region at the jet exit, where the displacement gradient reaches 0.25 px/px. The second region is beyond transition ($y/D \geq 5$), where the turbulent motions cause significant out-of-plane particle displacement.

The estimated uncertainty returns a qualitatively similar picture to the actual error magnitude (figure 4.31-c), with peaks near the jet exit along the shear layer and distributed randomly in the turbulent region. Two statistical profiles of error and uncertainty are extracted before and beyond transition ($y/D = 2.5$ and $y/D = 6.5$ respectively) and illustrated in figure 4.32-a and -b. These profiles evidence that in both flow regimes the agreement between uncertainty and error is within 0.03 pixels.

An additional outcome of the a-posteriori uncertainty quantification is that the estimated uncertainty can be used to drive an optimization process to

further enhance the measurement accuracy. An example of optimization procedure for the interrogation window size is presented in figure 4.33. Typically, this is done by gradually reducing the window size: at each reduction step, the spatial resolution increases at the cost of higher noise (figure 4.33, top row). When the measurement uncertainty is not quantified with an a-posteriori estimator, the interrogation window is selected as the smallest window before the noise level grows to unacceptable level. Clearly, such procedure is largely dependent on human perception of the measurable signal and the embedded noise and does not guarantee an optimal setting of the interrogation window size that minimizes measurement error.

Uncertainty estimation with the image-matching approach enables an objective choice of the interrogation window that minimizes the error magnitude. This is shown in the bottom row of figure 4.33 and in figure 4.34. It is observed that the large interrogation window (51×51 pixels) yields error due to spatial modulation, whereas the small one (11×11 pixels) produces a result dominated by random errors. The optimal interrogation window is in the range between 31×31 and 41×41 pixels.

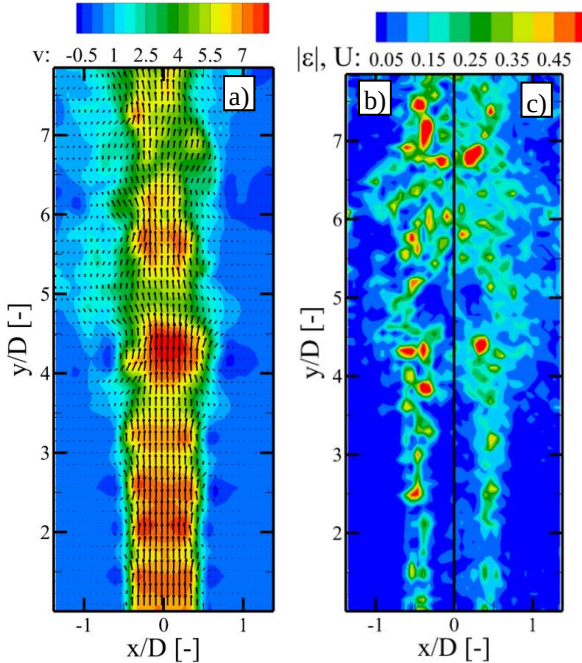


Figure 4.31. (a) Instantaneous reference axial velocity; (b) instantaneous error magnitude; (c) instantaneous estimated uncertainty. All the quantities are expressed in pixels

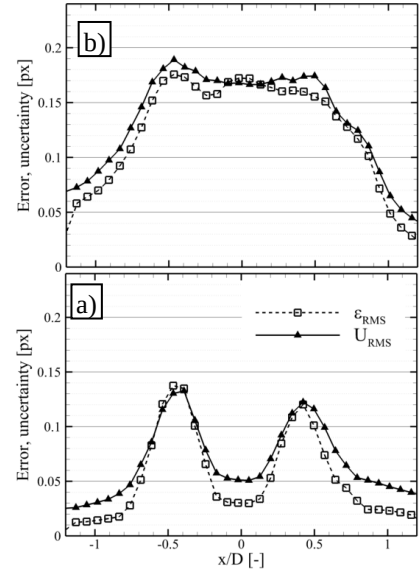


Figure 4.32. Statistical error and uncertainty profiles at $y/D = 2.5$ (a) and $y/D = 6.5$ (b).

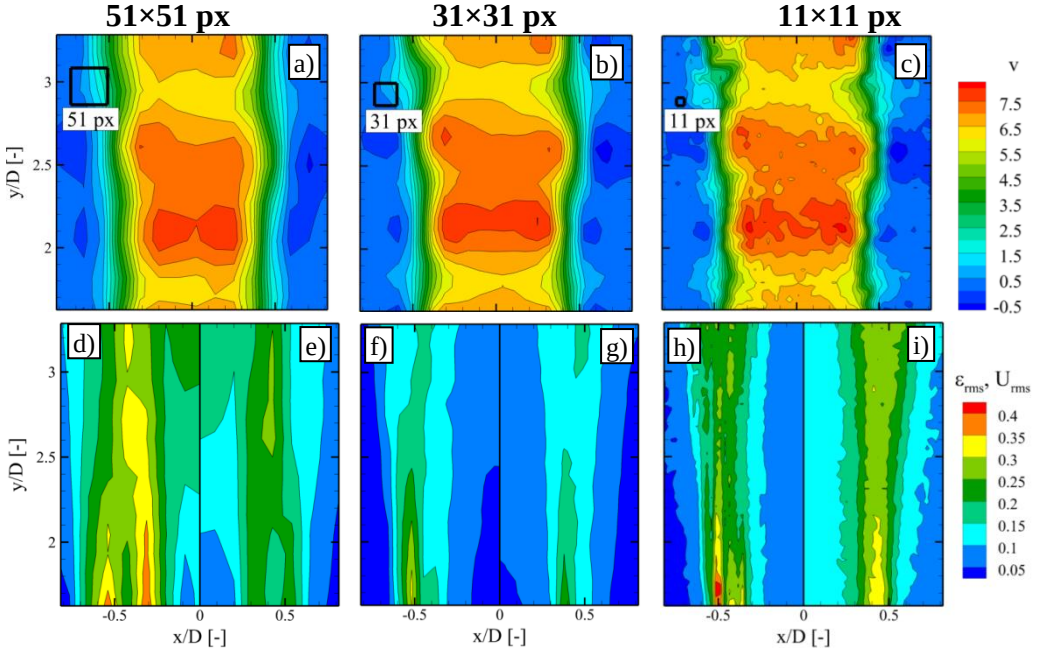


Figure 4.33. Top row: instantaneous axial velocity fields at different interrogation window sizes. Bottom row: root-mean-square of actual error magnitude (d, f, h) and uncertainty (e, g, i) at different interrogation window sizes. Quantities expressed in pixels.

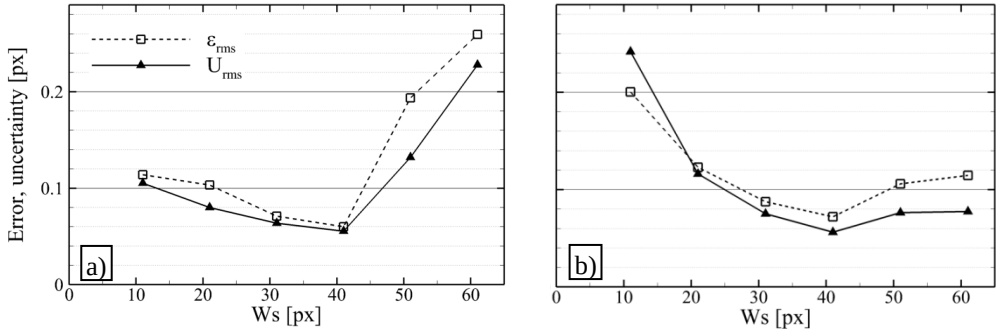


Figure 4.34. Root-mean-square of actual error and uncertainty as a function of the interrogation window size in two points: (a) point $(x/D, y/D) = (0.3, 2.4)$ along the shear layer; (a) point $(x/D, y/D) = (0.0, 5.7)$ in the turbulent region.

4.4.3.5 Supersonic boundary layer

A turbulent boundary layer develops for a length of approximately 1 m on a smooth surface under nearly adiabatic conditions, reaching a thickness of $\delta_{99} = 5.5$ mm. The characteristics of the boundary layer are evaluated from the recordings of the HDR system using the sum-of-correlation algorithm (Meinhart *et al*, 2000) with interrogation windows of 8x8 pixels and 75%

overlap. These characteristics are reported in table 4.2 and show good agreement with values measured in previous experiments conducted by Sun *et al* (2012).

Table 4.2. Supersonic boundary layer parameters. δ^* : displacement thickness; θ : momentum thickness; H_{inc} : incompressible shape factor; Re_θ : Reynolds number based on the incompressible momentum thickness; C_f : skin friction coefficient (computed with the Van Driest II formula in combination with the Crocco-Buseman relation, according to White, 2006; a recovery factor of 0.89 is assumed); u_τ : friction velocity.

Quantity	Value	Quantity	Value
M_∞ [-]	2.0	θ [mm]	0.52
u_∞ [m/s]	501	H_{inc} [-]	1.29
p_0 [Pa]	3.1×10^5	Re_θ [-]	21,000
T_0 [K]	285	C_f [-]	1.8×10^{-3}
δ_{99} [mm]	5.5	u_τ [m/s]	19.7
δ^* [mm]	0.67		

Figure 4.35 shows the boundary layer profile expressed in inner units $u^+ = u/u_\tau$ and $y^+ = y u_\tau / \nu$, being ν the fluid kinematic viscosity at the wall. The HDR and the measured profiles are plotted together for comparison. The HDR profile follows the log law ($u^+ = \frac{1}{0.41} \ln(y^+) + 5.0$, White, 2006) in the range $60 < y^+ < 2,000$, which corresponds to $0.01 < y/\delta_{99} < 0.32$. The linear sublayer (where $u^+ = y^+$) is not visible in the present experiment due to the limited spatial resolution of PIV. For $y^+ > 2000$, the reference profile departs from the log law due to a mild adverse pressure gradient in the outer region (wake component, White, 2006). The measured boundary layer profile falls on top of the reference one for $y^+ \geq 200$ ($y/\delta_{99} \geq 0.03$), indicating that the present experiment is not affected by any major systematic error source. The measurement point closest to the wall is in $y^+ = 90$, where the data departs already from reference profile and log law due to the limited spatial resolution of the measurement system.

The streamwise velocity fluctuations (figure 4.36) exhibit the typical trend of a turbulent boundary layer with zero pressure gradient (Klebanoff, 1954), with reference fluctuations up to $u'_{rms}/u_\tau = 2.2$ ($u'_{rms}/u_\infty = 0.09$) in proximity of the wall. Low reference velocity fluctuations are found in the free-stream region ($y/\delta_{99} > 1$), where the flow is nearly uniform. The measured velocity fluctuations show the same trend as the reference ones. However, the

fluctuations root-mean-square is slightly overestimated in the free-stream region due to random noise in the measured velocity fields. The overestimation is larger in the near wall region ($y/\delta_{99} < 0.2$, $u'_{\text{rms}}/u_{\infty}$ overestimated by 0.04) due e.g. to laser light reflections not completely removed in the pre-processing phase.

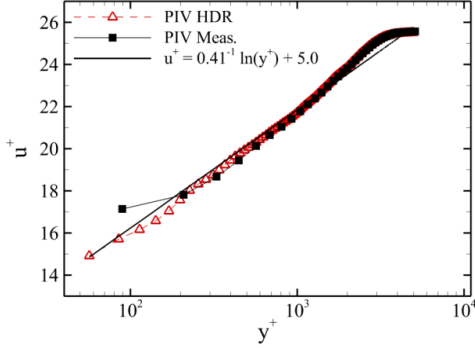


Figure 4.35. Mean boundary layer profile in inner units; HDR data (hollow triangles) and measurement data (full squares).

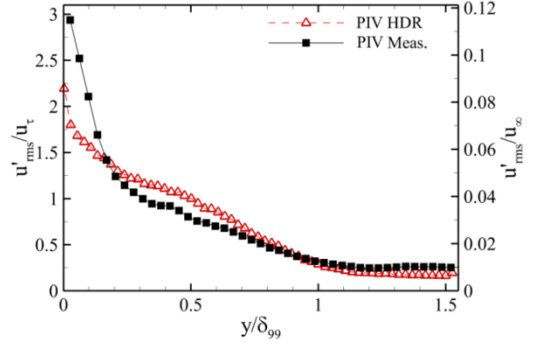


Figure 4.36. Velocity fluctuations in the boundary layer; HDR data (hollow triangles) and measurement data (full squares). For clarity, every three reference data point is displayed.

Error assessment and uncertainty quantification

The error is expected to be the minimum in the free-stream region, and to grow towards the wall due to high shear and velocity fluctuations. This is confirmed by the instantaneous velocity profile of figure 4.37, where the actual error and the uncertainty are the largest in the wall proximity.

The root-mean-square of error and uncertainty follows a trend similar to the velocity fluctuations, being the minimum in the free-stream region ($\varepsilon_{\text{rms}}/u_{\infty} = 0.01$) and the maximum in the vicinity of the wall ($\varepsilon_{\text{rms}}/u_{\infty} = 0.035$), as shown in figure 4.38. It is noticed that in the logarithmic-law region ($y^+ < 1500$, $y/\delta_{99} < 0.25$), the measurement uncertainty is underestimated by about 20%. This is attributed to an inadequate particle image detection and pairing in presence of disparities exceeding one pixel. In this case, the particle image disparity is “clipped” and the estimated uncertainty is biased towards lower values.

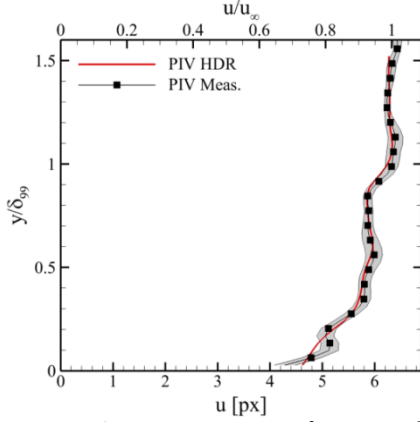


Figure 4.37. Instantaneous reference and measured velocity profile in the boundary layer with error bars. For clarity, every other data point is displayed.

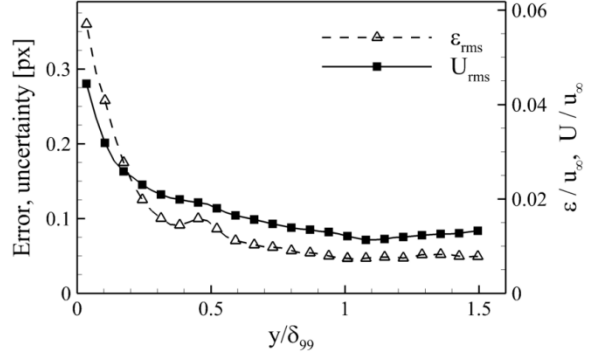


Figure 4.38. Profiles of the root-mean-square of actual error and uncertainty in the boundary layer. For clarity, every other data point is displayed.

4.5 Synthesis of uncertainty assessment

A methodology is presented here to assess the accuracy of the uncertainty quantification by image-matching in all the flow regimes discussed in the previous section.

The cumulative histograms of the actual error magnitude $|\varepsilon|$ (figure 4.39) show how the measurement error depends on the experimental conditions. The results put in evidence that the value 0.1 pixels often considered as rule-of-thumb for the typical uncertainty of PIV measurements is far from being generally valid: for some experiments (e.g. uniform transverse flow with $w/\Delta z = 0.02$) it is a too conservative estimate, while for other cases (e.g. uniform transverse flow with $w/\Delta z = 0.17$) it is instead too optimistic.

Based on the definitions of error and expanded uncertainty, when the uncertainty is correctly quantified at 68% confidence level, it must satisfy:

$$\begin{cases} |\varepsilon| \leq U & \text{for 68\% of the measurements} \\ |\varepsilon| > U & \text{for the remaining 32\% of the measurements} \end{cases} \quad (4.12)$$

Relationship (4.12) has been used in the past in a global way, i.e. considering the ensemble of all measurements that have different errors and uncertainty, to assess the accuracy of the uncertainty quantification (*uncertainty effectiveness* by Timmins *et al*, 2012). However, the global

approach can yield misleading results, because relationship (4.12) could be satisfied globally even if the uncertainty is for instance overestimated for low errors and underestimated for large errors.

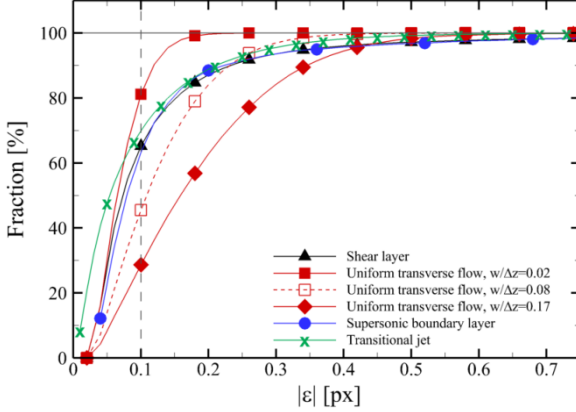


Figure 4.39. Cumulative histograms of the actual error magnitude. On the vertical axis, the fraction of vectors having error magnitude below a certain $|\varepsilon|$ is represented.

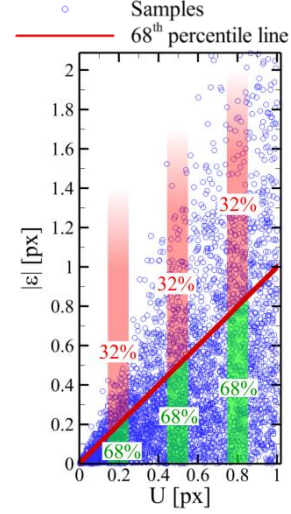


Figure 4.40. Ideal relationship between error and uncertainty. The plot has been generated synthetically to illustrate the relationship between error and uncertainty.

In the present work, a local approach is proposed that consists in dividing the estimated uncertainty into bins and verifying whether relationship (4.12) holds in each bin. In the ideal case, in each uncertainty bin the estimated uncertainty equals the 68th percentile of the error, which by definition is the value $|\varepsilon|_{68}$ such that $|\varepsilon| \leq |\varepsilon|_{68}$ for 68% of the measurements. Hence, the theoretical 68th percentile line $|\varepsilon|_{68} = U$ with unity slope is defined (figure 4.40).

In the uncertainty quantification of real experimental data, the 68th percentile departs from the theoretical value: $|\varepsilon|_{68} \neq U$. Where the estimated 68th percentile line (estimated in the sense that it stems from the estimated uncertainty) is above the theoretical line, the uncertainty is underestimated because less than 68% of the measurements have error magnitude below the theoretical value $|\varepsilon|_{68} = U$. Vice versa, where the estimated line is below the theoretical line, the uncertainty is overestimated. The discrepancy between

theoretical and estimated 68th percentile line is used to evaluate the goodness of the uncertainty estimation provided by the image-matching approach.

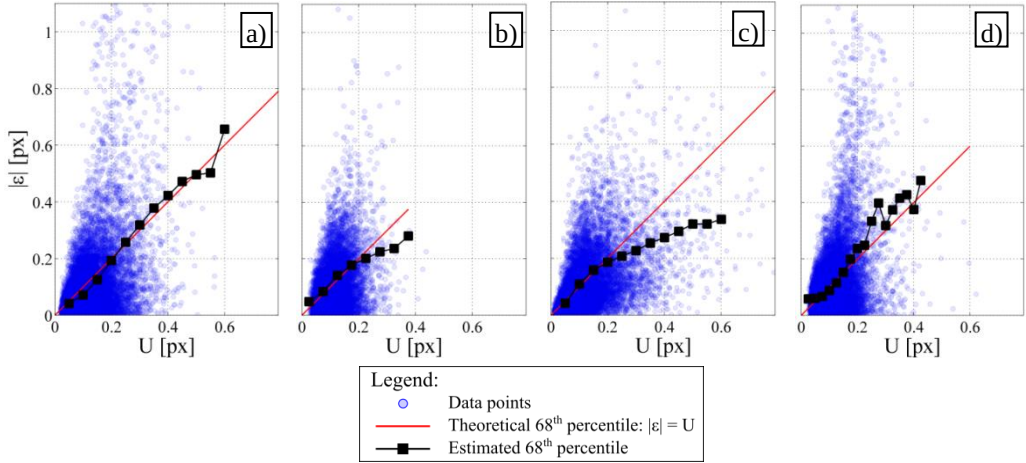


Figure 4.41. Actual error magnitude against estimated uncertainty; (a) shear layer; (b) uniform transverse flow; (c) transitional jet; (d) supersonic boundary layer.

The uncertainty assessment is conducted on the shear flow, uniform transverse flow, transitional jet and supersonic boundary layer test cases. It cannot be applied to the turbulent wake test case due to the lack of knowledge of the instantaneous local error magnitude in this case.

The results of figure 4.41 indicate that the estimated 68th percentile line has positive slope for all the test cases, meaning that at higher uncertainty corresponds higher error magnitude; in other terms, the uncertainty scales consistently with the error. In all the four cases, a good agreement is retrieved between the theoretical and estimated 68th percentile line. Uncertainty values below 0.3 pixels are estimated within 20% of their actual value.

4.6 Conclusions

A novel principle for uncertainty estimation based on image-matching has been introduced to quantify the local uncertainty in planar PIV data. The working principle is general and can be applied to any cross-correlation based interrogation algorithms. The velocity field measured from two images is used as a predictor to match the images with a degree of approximation depending upon the interrogation algorithm. Due to bias and random errors, the images do not match perfectly and the particle images exhibit a residual positional disparity. The disparity distribution is considered to infer the instantaneous uncertainty of the measured displacement. The approach accounts for random

errors stemming to e.g. background noise, out-of-plane motion and seeding density.

The performance of the estimator is assessed via Monte Carlo simulation. The algorithms employed for the analysis range from simple ones (*single-pass correlation*, *double-step with discrete window offset*) to what is considered the state-of-the-art of PIV interrogation (*multi-step with window deformation*). The uncertainty associated with several measurement conditions (namely in-plane and out-of-plane displacement, in-plane displacement gradient, particle image diameter, seeding density and image noise) has been investigated. The estimated uncertainty follows the behaviour of the actual error with accuracy in most cases within a hundredth of a pixel; also the validity of the model based on the $1/\sqrt{N}$ scaling has been proven. The present approach shows its limitations for particle images having diameter of 1 pixel or below; in this case, the determination of the particle image position is affected by peak-locking errors and leads to lower accuracy in the estimated uncertainty.

The experimental assessment has been conducted using a dedicated experimental database that reproduces flow regimes and error sources representative of typical PIV experiments. The analysed flow fields include laminar shear layer and transition to turbulent regime, turbulent wake behind a bluff body, uniform transverse flow, transitional jet in water and supersonic boundary layer.

For the experimental assessment of the uncertainty quantification method, the knowledge of the exact measurement error is required, which is obtained either via concurrent measurements conducted with a high dynamic range system (HDR), or with an advanced multi-frame interrogation algorithm, or by means of physical considerations. In the test cases that have been investigated, the image-matching approach estimates the error root-mean-square within 30% of the actual value.

A novel methodology is introduced to assess the goodness of the instantaneous and local uncertainty estimation. In such methodology, the error magnitude $|\varepsilon|$ is plotted against the uncertainty U and the 68th percentile line of the error magnitude is compared with the theoretical line $|\varepsilon| = U$. It is shown that the uncertainty scales consistently with the error and uncertainty values below 0.3 pixels are estimated within 20% of their actual value independently of the flow regime.

A final remark should be made to comment on cases where large-magnitude errors (exceeding 1 pixel) are present, namely outliers. In these cases, the

image-matching approach does not provide an accurate estimate of the error magnitude; in fact, here the error is not associated with the spread of the particle pairs positional disparity, but with an erroneous evaluation of the displacement peak in the cross-correlation function. However, the presence of spurious vectors causes only a very limited number of particle images (in the worst case zero) to pair correctly. Consequently, the occurrence of an outlier can be detected by monitoring the ratio between number of particle image pairs and total number of particle images within an interrogation window: when the ratio approaches low values (typically below $0.3\div 0.4$), most particle images are not paired correctly and the estimated vector is likely an outlier.

CHAPTER 5

THE COLLABORATIVE FRAMEWORK FOR PIV UNCERTAINTY QUANTIFICATION*

Abstract This chapter presents the main results of the *collaborative framework for PIV uncertainty quantification*, which involves four research groups (namely three universities and a company) from Europe and United States of America. Within the framework, a reference data base has been produced to conduct a comparative assessment of different uncertainty quantification methodologies, so to achieve a more thorough understanding on the performance of those.

5.1 Background of the collaborative framework

The collaborative framework for PIV uncertainty quantification has been set up with the objective to foster the collaboration among research groups currently active on the topic of PIV uncertainty quantification. It is recognized that a step beyond the current level of interaction, typically based on journal article publication and reviews, requires that the different groups adopt a common basis (nomenclature, definitions, procedures, methodology) and even better share a common data base to test and verify the validity of several approaches for the purpose of estimating PIV uncertainty. The framework involves four research groups, namely three universities (TU Delft, Utah State University and Purdue University) and an imaging systems company (LaVision GmbH).

The activity focuses on performing a number of well controlled experiments with the purpose of delivering several datasets, each with unique features, which are regarded as a reference to validate different algorithms for PIV uncertainty quantification. The datasets cover a wide range of experimental and flow conditions considered representative of typical PIV measurements. The data base is made available to all participants in this initiative and is

* The present work has been conducted in collaboration with Utah State University, Purdue University, LaVision GmbH and LaVision Inc. The work has been presented at the 17th International Symposium on Applications of Laser Techniques to Fluid Mechanics in Lisbon, Portugal, in July 2014.

The valuable contribution of dr. Doug Neal, prof. Bart Smith, Scott Warner, Bernd Wieneke and prof. Pavlos Vlachos, co-authors of the conference papers, is kindly acknowledged.

expected to be opened to external contributors after the experiments have been documented in literature via publications on peer reviewed journals.

Goal of the framework is to achieve a thorough understanding of the performance of the uncertainty quantification approaches in presence of different error sources, highlighting strengths and weaknesses of those. It is expected that the results of the collaboration will constitute the basis for developing a generalized and consolidated methodology that combines the strengths of existing methods.

5.1.1 Description of the uncertainty quantification methods

Four approaches for the *a-posteriori* quantification of the instantaneous local uncertainty are assessed. Those approaches make use of information on the PIV recordings or the cross-correlation function to determine the uncertainty of the PIV data and are described hereafter.

Image matching method

The image matching method (*IM*, Sciacchitano *et al*, 2013) has been discussed in chapter 4: the reader is referred to that chapter for a thorough description of the method. In brief, the measured velocity field is used as a predictor to match the particle images of the recordings at the best of the processing algorithm (e.g. by image deformation or window shift). In each interrogation window, particle image pairs are sought for: in the ideal case (exact measurement), the particle images of the two recordings match perfectly; instead, in a real measurement the paired particle images do not match exactly and a positional disparity is present. The latter can be estimated with sub-pixel accuracy by particle image detection and a peak fitting algorithm. The measurement uncertainty is finally retrieved from the statistical dispersion and the mean value of the disparity vector within the interrogation window.

Uncertainty surface method

The uncertainty surface method (*US*) developed by Timmins *et al* (2012) uses the known response of a PIV algorithm to a number of error sources along with the magnitudes of the error sources to determine the uncertainty of each vector. Since the uncertainty of a PIV calculation depends on the algorithm used (i.e. the software package along with all settings), the response of the algorithm to varying magnitude of each error source must be

systematically tested to generate an *uncertainty surface* for that algorithm. Synthetic images are used for this purpose. Once a surface has been generated, the uncertainty of each vector may be determined by measuring the value of each error source (particle image diameter, particle image density, particle displacement and shear, in work to date), and then querying the uncertainty surface based on those results.

Peak ratio method

The peak ratio method (PR, Charonko and Vlachos, 2013) relies upon the assumption that, for any correlation plane, the error on the measured displacement is related to the cross-correlation peak ratio. The latter is defined as the ratio between the largest detectable peak (representing the particles displacement) and the second highest peak, which is linked to the combined effects of all error sources stemming from the image quality and flow field. The uncertainty (U) of the measured displacement magnitude is retrieved from the peak ratio (PPR) by means of the empirical equation (5.1):

$$U = 0.402 \cdot PPR^{-0.84} \quad (5.1)$$

Based on the error propagation formula, the uncertainty of individual displacement components is retrieved from the output of equation (5.1) via division by the square-root of 2.

Correlation statistics method

Similarly to the image-matching method, the correlation statistics method (CS) proposed by Wieneke and Prevost (2014) quantifies the differences between the two interrogation windows mapped onto each other by the computed displacement field. However, instead of identifying the contribution of individual particles, this method analyses the overall contribution from all pixels to the shape of the correlation peak. The approach relies on the assumption that the PIV interrogation algorithm with predictor-correction scheme should always yield a symmetrical correlation peak when brought to convergence, i.e. with a final zero corrector displacement. However, the symmetrical correlation peak arises from the contributions not only of the particle images correctly matched, but also of the noise in the recordings. The correlation statistics method estimates the contribution of all pixels in the interrogation window to the asymmetry of the correlation peak, which on average leads to a symmetrical correlation peak. The standard deviation of the

contributions provides an estimate of the expected asymmetry due to the image noise, which is related to the uncertainty of the displacement vector. In principle, this method takes all factors into account like remaining particle disparities, background image noise or out-of-plane particle motion affecting the correlation function.

From the description of the four methods, it emerges that image-matching and correlation statistics approaches rely upon the use of the PIV image contributions to the cross-correlation peak shape to determine the measurement uncertainty; such quantity directly affects the measured displacement. The main difference between the two approaches is that the image-matching method accounts for the contribution of the detected particle image pairs, whereas the correlation statistics method includes the effect of all pixels in the interrogation window. Due to the intrinsic uncertainty in determining the position of individual particle images (Sciacchitano *et al*, 2013), the image-matching method is expected to provide less accurate uncertainty estimates compared with the correlation statistics approach.

The peak ratio method quantifies the uncertainty from the ratio between highest and second highest peak of the correlation function. Such ratio is more traditionally regarded as an indicator of the reliability of the result, but it is indirectly related to the measured displacement, which is determined only by the position of the highest peak. The method is based on the empirical relation defined in equation (5.1) and requires input of calibration coefficients that are obtained with synthetic data.

Finally, the surface method is the only of the four approaches where the uncertainty quantification does not rely upon the analysis of the correlation function or of the image contributions to that. Monte Carlo simulations are conducted varying the magnitude of the chosen error sources. The uncertainty surface for the processing algorithm is thus built, which univocally associate a measurement uncertainty with a combination of error sources. The approach accounts for a limited number of error sources.

5.1.2 Uncertainty assessment by experiments

To evaluate the validity and accuracy of any uncertainty estimation method, the true measurement error should be known (Moffat, 1988). While the true error is readily available when conducting Monte Carlo simulations, it is typically unknown in real experiments and is usually difficult to access. For this reason, the experimental assessment of uncertainty quantification methods is recognized as a challenge.

In the present framework, a dedicated series of experiments is conducted with two PIV systems in order to produce a very accurate estimate of the exact velocity field. The methodology is the same as that defined *concurrent experiments* in chapter 4. The *measurement system* records images in conditions representative of typical PIV experiments. At the same time, the second system, named *high dynamic range (HDR) system*, records images at higher image digital resolution (typically by factor 3 to 4) and with optimized imaging conditions (particle image size of two pixels). As a result, the HDR system delivers measurements at significantly higher dynamic velocity range (Adrian, 1997).

Furthermore, a concurrent, independent hot-wire measurement is compared to the HDR results to further verify that, for the purpose of the present work, the HDR measurements may be treated as “reference” due to the lower relative uncertainty with respect to the measurement system. Thus, the HDR system output is used to estimate the actual error, which is employed to validate the different PIV uncertainty quantification methods. Note that, although it is common practice to evaluate the uncertainty at 95% confidence level (Coleman and Steele, 2009), in the present work 68% confidence level is used for a more accurate assessment of the uncertainty quantification methods via the uncertainty coverage analysis, which will be discussed in section 5.3.

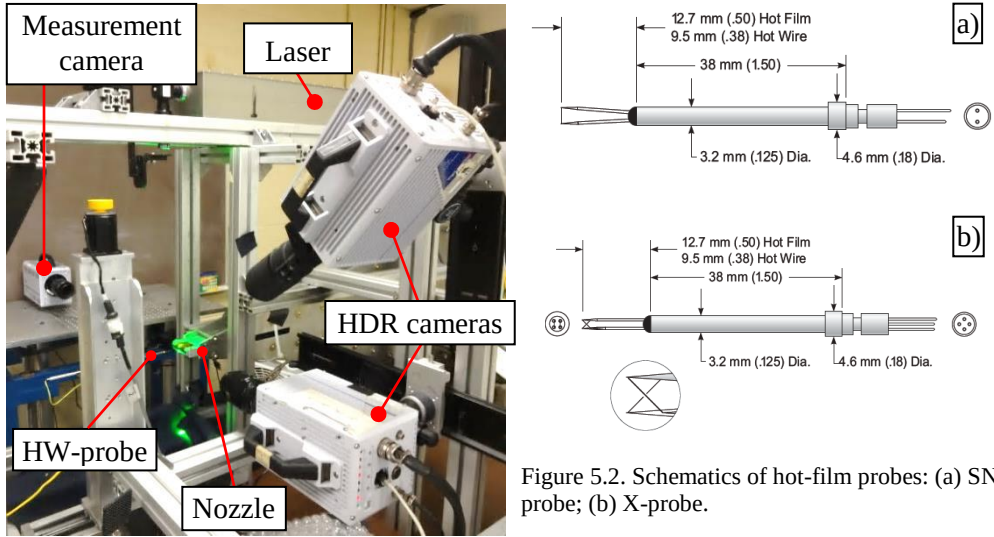
5.2 Setup of the experiments

The experiments were conducted at the Experimental Fluid Dynamics Laboratory of Utah State University using a jet facility with rectangular nozzle of aspect ratio 7.2 (width $w = 72.8$ mm, height $h = 10.2$ mm). The facility is the same as that used in the experiments reported by Wilson and Smith (2013b). The jet exit velocity was set to 5 m/s, yielding a Reynolds number based on the nozzle height of $Re_h = 3,000$. The flow was seeded with a Rocket Portable Smoke System (model #PS23 - 1.1 kW) that used a glycerine-water solution to produce particles of 1 μm median diameter. The illumination was provided by a Photonics Industries laser (model DM40-527, pulse energy of 40 mJ at 1 kHz).

Recordings were acquired in continuous mode at sampling rate of 4,000 to 10,000 Hz. One camera normal to the measurement plane was used for the measurement system (LaVision HighSpeedStar 5, CMOS, 12 bit, $1,024 \times 1,024$ pixels, 17 μm pixel pitch, 3,000 frames per second at full resolution). The PIV HDR system was composed by two cameras in stereoscopic configuration (LaVision HighSpeedStar 6, CMOS, 12 bit, $1,024 \times 1,024$ pixels, 20 μm pixel

pitch, 5,400 frames per second at full resolution). The setup of the experiment is illustrated in figure 5.1.

The recordings were processed with the LaVision DaVis 8.1.6 software. For the measurement system, the final interrogation window size was 16×16 pixels; for the HDR system, final windows of 32×32 pixels size were employed. In both cases, the overlap factor was set to 75%.



For the hot-wire system, four probes have been tested, namely:

- 5 μm single normal hot-wire probe (SN-probe), built by the Turbulent Shear Flows Laboratory (TSFL) of Michigan State University (MSU)
- 5 μm cross-wire probe (X-probe), also built by the TSFL of MSU
- 50 μm hot-film SN-probe, TSI model 1210-20 (figure 5.2-a)
- 50 μm hot-film X-probe, TSI model 1241-20 (figure 5.2-b)

Since the hot-wire measurements were carried out simultaneously with PIV, tracer particles impacting on the hot-wire sensor could have affected the response of the latter. It was found that the 5 μm hot-wire output exhibited high frequency spurious fluctuations when the measurements were conducted in presence of seeding particles. In contrast, the larger sensor of the hot-film probes was approximately insensitive to those. For this reason, the hot-film probes were used for almost the totality of the measurements.

Furthermore, both hot-wire and hot-film are sensitive to changes in temperature: if the sensor is heated by the laser light, the sensor output would be biased towards lower velocity. To avoid this systematic error, it was decided to clip sharply the edge of the light sheet using a knife-edge filter. The hot-wire probe was placed approximately 1.0-1.5 mm downstream of the light sheet edge, as shown in figure 5.3.

Measurements were conducted in several regions that exhibited characteristic flow features encountered in typical PIV experiments. These regions included the potential core ($x/h \leq 1$, where x indexes the streamwise direction), the unsteady inviscid jet core ($x/h \approx 3-4$) and developed turbulent region ($x/h \approx 19$). Moreover, different imaging conditions were taken into account in terms of particle image diameter and seeding density. Also the effect of out-of-plane motion was investigated by tilting the laser sheet with respect to the streamwise direction by 16 degrees.

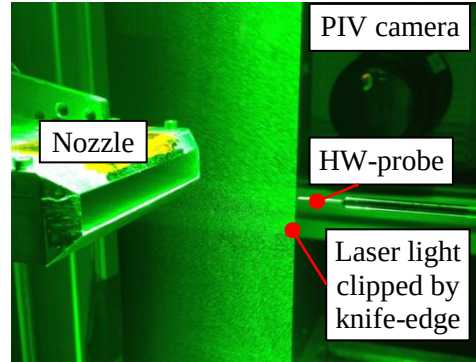


Figure 5.3. Picture showing the proximity of the HW probe to the PIV measurement region. Flow is from left to right.

5.3 Results

5.3.1 Validation of the HDR system

In order to establish the HDR-PIV as the “reference measurement”, a comparison is made between the HWA and PIV data. Since HWA is a point measurement, a single location in the PIV domain (directly upstream of the HW probe) is selected. Based on the assumption that the flow structures are convected at velocity V_{conv} , the velocity V_{HW} measured by the hot-wire at $(x_{\text{HW}}; t)$ is compared with that obtained with the PIV system at $(x_{\text{PIV}}; t - k\Delta t)$, where $x_{\text{PIV}} = x_{\text{HW}} - V_{\text{conv}} k\Delta t$ and k is the minimum integer number such that x_{PIV} falls within the PIV domain. The convective velocity is determined from the correlation of the velocity time-series at two points of the PIV-HDR field, following the approach proposed by Willis (1964); the distance between the two points is set equal to that between laser sheet edge and hot-wire probe. A

sample of the velocity time-history at the location $x/h=4$, $y/h=0$ is shown in figure 5.4-a. At this location, the flow is laminar and low-frequency velocity fluctuations are expected due to acceleration induced by the formation of Kelvin-Helmholtz vortices. The measured fluctuation frequency corresponds to a Strouhal number of 0.39, which is consistent with the value reported by Krothapalli *et al* (1981) for their jet flow of aspect ratio 16.7 and Reynolds number 12,000. The results of figure 5.4-a show the excellent agreement between PIV-HDR and HWA data, with differences typically below 0.05 m/s. In contrast, the PIV-measurement velocity exhibit spurious high-frequency fluctuations up to 0.2 m/s; due to the laminar regime of the flow at the considered location, those fluctuations are ascribed to measurement errors.

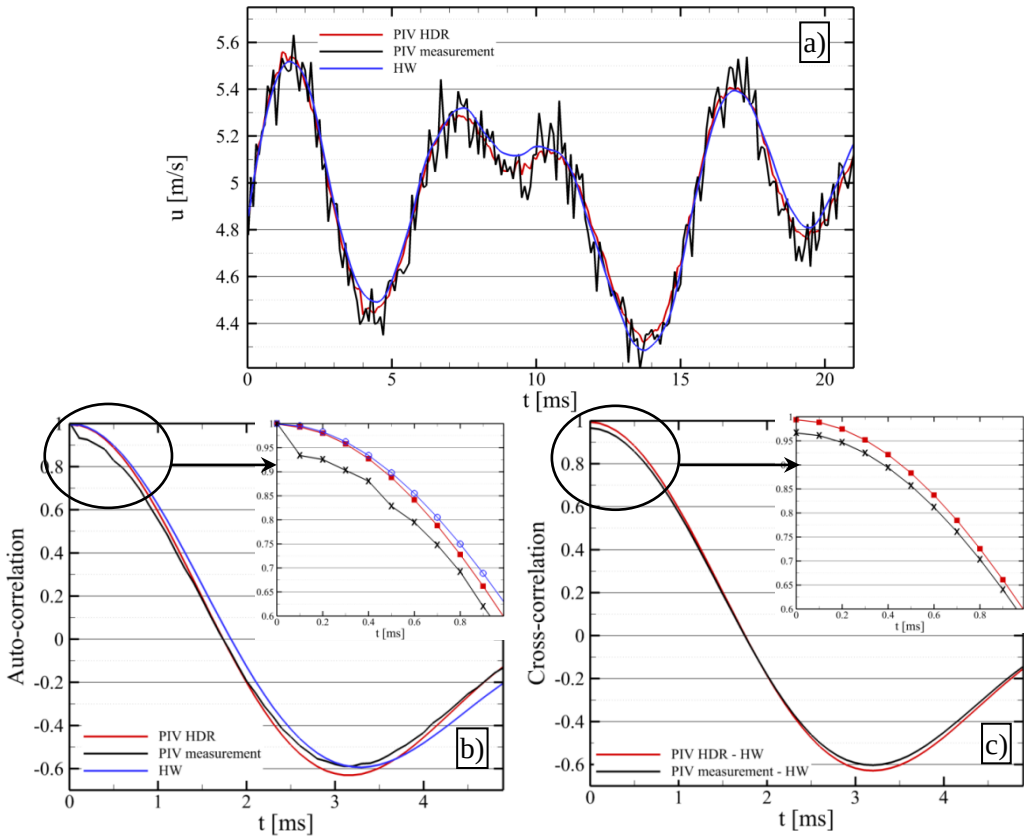


Figure 5.4. (a) Velocity time history at $x/h = 4$, $y/h = 0$; (b) auto-correlation of the velocity time history; (c) cross-correlation between PIV and HW velocity time histories.

The auto-correlation function of the time histories is displayed in figure 5.4-b. As it will be discussed in detail in chapter 6, the drop of correlation after the first sample is indicative of the noise level embedded in the measurement.

While for HW and PIV-HDR the drop of correlation is below 1% (0.4% and 0.7%, respectively), it approaches 7% for the PIV-measurement system, meaning that the latter exhibits significantly larger measurement noise. This is also confirmed by the cross-correlation function between PIV and HW velocity time histories of figure 5.4-c: the cross-correlation between hot-wire and PIV-HDR velocities returns a maximum value of 99.4%; in contrast, the maximum of the cross-correlation between HW and PIV-measurement drops to 96.7% due to the noise embedded in the latter.

From the comparison with the HW measurement, the PIV-HDR uncertainty is estimated to be about fourfold lower than that of the PIV-measurement. Hence, the PIV-HDR velocity can be considered as a *reference* to estimate the actual error of the PIV-measurement system.

5.3.2 Comparative assessment of uncertainty quantification methods

5.3.2.1 Unsteady inviscid jet core

In this section, the comparative assessment of uncertainty quantification methods is conducted in the unsteady jet core region where vortices are formed periodically due to the growth of Kelvin Helmholtz instabilities ($x/h \approx 3-4$).

The cross-stream profiles of the time-averaged velocity magnitude and normal stress are displayed in figure 5.5. The time-averaged profile is flat in the jet core and features a shear layer resulting in a shear rate of the images of approximately 0.15 px/px at the locations $y/h = \pm 0.5$. The normal stress profile shows the presence of large fluctuations within the shear layer, ascribed to the formation and growth of Kelvin-Helmholtz vortices causing the shear layer to oscillate. Fluctuations of the order of 10% of the centreline time-averaged axial velocity are found within the jet core, which are associated with the accelerations induced by the Kelvin-Helmholtz vortices. The best estimate of the actual error, computed as the difference between measured velocity and HDR velocity, exhibits a normal distribution centred about zero (figure 5.6). The contribution of random errors is more than one order of magnitude larger than that of systematic errors (mean bias error: $\mu = -0.005$ px; standard deviation of the actual error: $\sigma = 0.056$ px).

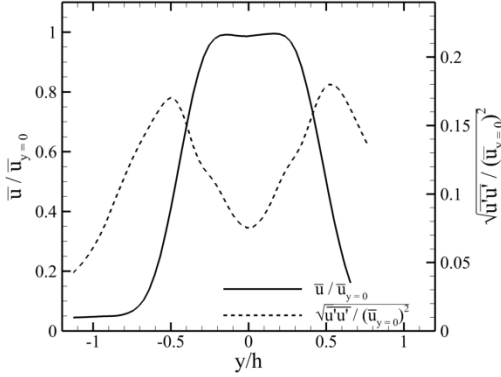


Figure 5.5. Cross-stream profile of the time-average velocity magnitude (continuous line) and of the normal stress (dashed line).

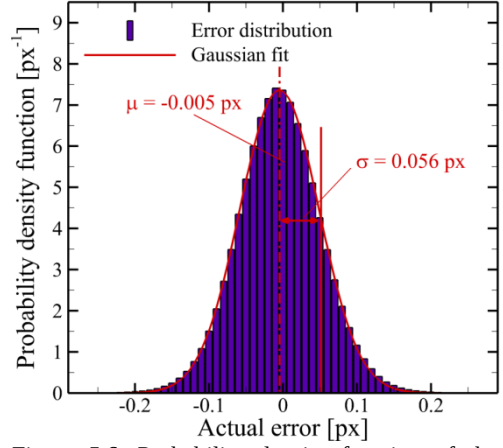


Figure 5.6. Probability density function of the actual error.

Figure 5.7 shows an instantaneous cross-stream profile of the stream-wise velocity as computed with the measurement and the HDR-PIV system. In the former profile, the uncertainty bands at 68% confidence level are evaluated with the four uncertainty quantification methods. The measured profile is consistent with the HDR profile, with differences up to 0.1 pixels in the jet core ($y/h \approx 0$) and in the shear layer ($y/h \approx \pm 0.5$). Note that the HDR profile is defined only in $y/h = [-1.1, 0.8]$ due to the smaller size of the HDR measurement domain. The results show that for all the methods, the magnitude of the uncertainty bands is comparable with the actual error magnitude. However, the peak ratio method appears to overestimate the uncertainty; in fact, the HDR profile falls within the uncertainty bands for most of the measurements, while this was expected to occur only for 68% of the measurements. The root-mean-square profiles of figure 5.8 confirm that, in the present test case, the peak ratio overestimates the uncertainty by about factor 2. The image-matching method also overestimates the measurement uncertainty, but to a lesser extent (approximately 30%). Instead, the uncertainty estimated with the correlation statistics and uncertainty surface approaches follows more closely the actual error value.

A more representative way to evaluate the reliability of the estimated uncertainty consists in computing the *uncertainty coverage*, which is defined as the percentage of measurements for which the exact value is contained within the uncertainty bands (Timmins *et al*, 2012). By definition, the uncertainty coverage should equal the confidence level (Coleman and Steele, 2009), that in this case is 68%; therefore, the closer the uncertainty coverage to 68%, the more accurate the uncertainty estimation. The results reported in

table 5.1 confirm that in the present test case the peak ratio method overestimates the uncertainty; in fact, for more than 90% of the measurements the true value is found within the uncertainty bounds. Also the coverage obtained with the image-matching method slightly exceeds the expected value. Instead, uncertainty surface and correlation statistics methods slightly underestimate the uncertainty, yielding a coverage between 50% and 60%.

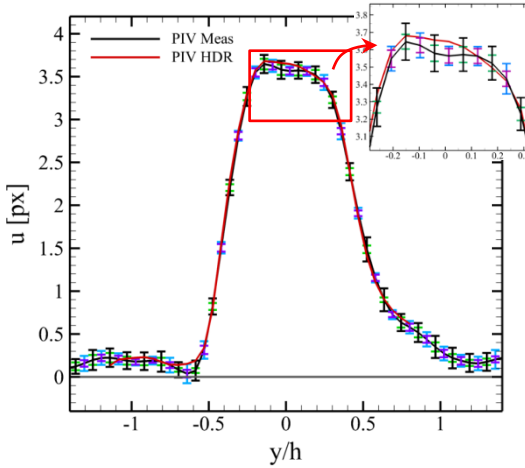


Figure 5.7. Instantaneous cross-stream profile obtained with the measurement system (black line) and with the HDR system (red line) with uncertainty bands at 68% confidence level: green: surface method; light blue: image-matching method; black: peak ratio method; purple: correlation statistics method.

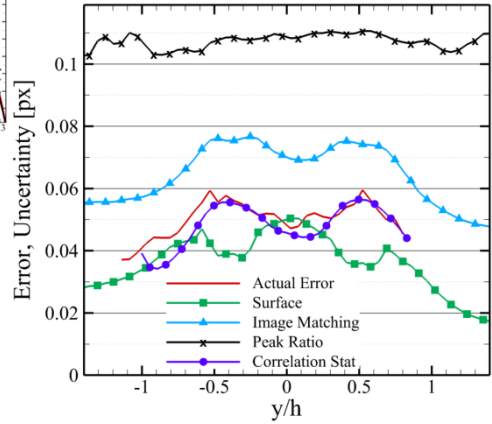


Figure 5.8. Comparison between actual error standard deviation and estimated uncertainty root-mean-square at $x/h = 2.9$.

Table 5.1. Minimum and maximum estimated uncertainties and uncertainty coverage for the four methods: uncertainty surface (US), image matching (IM), peak ratio (PR) and correlation statistics (CS). The root-mean-square of the actual error is 0.057 pixels. The uncertainty has been estimated at 68% confidence level, thus the uncertainty coverage should ideally be 68%.

	US	IM	PR	CS
Min estimated uncertainty [px]	0.012	0.012	0.046	0.000
Max estimated uncertainty [px]	0.152	0.442	0.280	0.500
RMS estimated uncertainty [px]	0.041	0.073	0.107	0.049
Uncertainty coverage	52%	76%	94%	58%

The velocity time series is extracted from a point of the jet core to investigate the instantaneous measurement error (figure 5.9-a). Whereas the

actual velocity is characterized only by low frequency fluctuations (below 100 Hz) ascribed to the acceleration induced by the Kelvin Helmholtz vortices, the measurement system returns a velocity time history that exhibits also high frequency fluctuations due to random noise. In figure 5.9-b, the actual error time series is computed as the difference between HDR and measurement velocity: the error is dominated by the random component, has peaks exceeding 0.1 px and follows a Gaussian distribution (figure 5.9-c; skewness: -0.080; excess kurtosis: 0.028). Ideally, the standard uncertainty, which for Gaussian error distribution corresponds to the expanded uncertainty at 68% confidence level, should yield a constant value equal to the standard deviation of the error distribution (Coleman and Steele, 2009). The time series of figure 5.10 show that the uncertainty evaluated with the four methods is approximately constant in time, but exhibits random fluctuations due to the finite information employed in the uncertainty quantification. In fact, according to Ahn and Fessler (2003), the expected spread of uncertainty is proportional to $1/\sqrt{2(N-1)}$, being N the number of particle image pairs contained in the interrogation window, which in the present test case is about 11. Hence, the uncertainty is expected to exhibit random fluctuations of the order of 20% of the mean estimated value.

The results also show that the uncertainty root-mean-square (rms) agrees with the actual error rms within 0.005 px when using the surface or correlation statistics methods; in contrast, peak ratio and image-matching methods yield an uncertainty rms that overestimates the actual error by above 0.02 px.

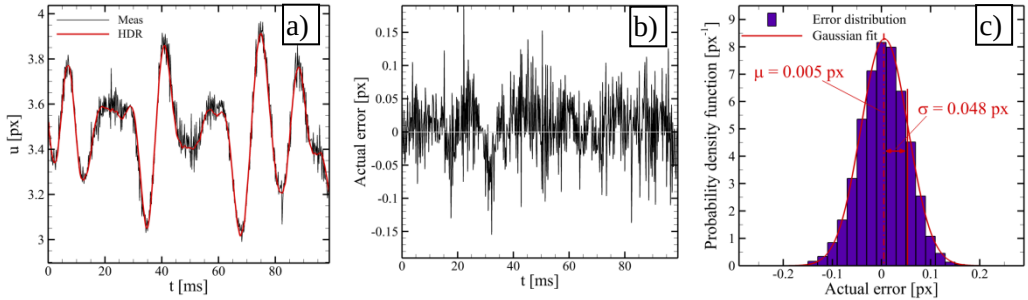


Figure 5.9. (a) Velocity time series at $x/h = 2.9$ and $y/h = 0$ obtained with the measurement and the HDR system. (b) Actual error time series. (c) Actual error distribution.

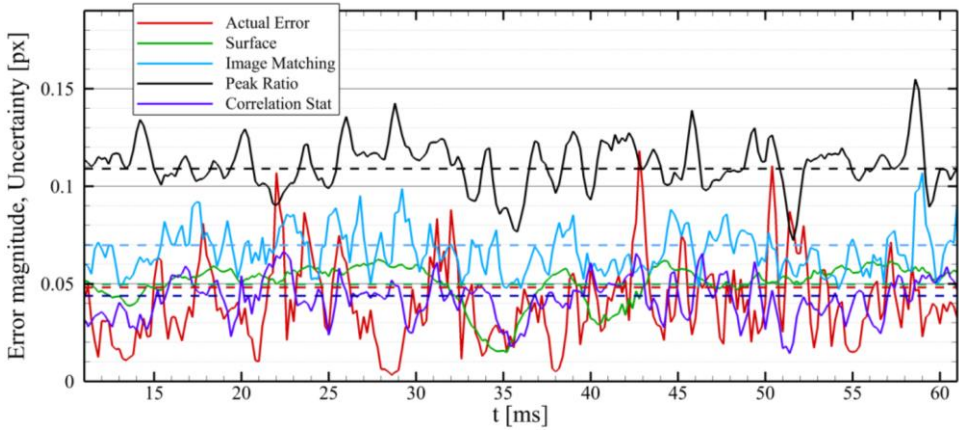


Figure 5.10. Comparison among the time series of actual error magnitude and uncertainty evaluated from the four methods. The root-mean-square of each series is displayed as a dashed curve. The time series have been extracted from $x/h = 2.9$ and $y/h = 0$.

5.3.2.2 Effect of out-of-plane motion

The effect of the out-of-plane motion is investigated in the potential core region. The laser sheet is tilted by 16 degrees with respect to the jet exit direction so that a non-null component in the direction normal to the laser sheet is realized (indicated with z in figure 5.11). In the present test case, the laser sheet thickness is $\Delta z = 1.7$ mm (about 12 px of the measurement system).

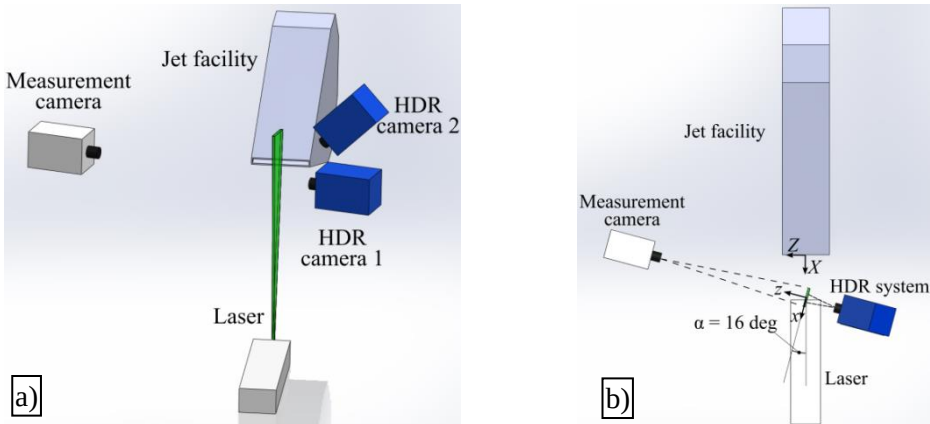


Figure 5.11. Schematic representation of the experimental setup. (a) Three-dimensional view; (b) top view. The laser sheet is tilted by 16 degrees with respect to the jet axis direction. The jet axes system is indicated with (X, Y, Z) , the measurement axes system with (x, y, z) . In the actual experiment, the flow is illuminated from above the jet facility.

Figure 5.12 reports the time-averaged x - and the z -velocity components (being z the out-of-plane direction) measured by the HDR system and

expressed in pixel units of the measurement system. As expected, the largest out-of-plane displacement occurs in the jet core, where also the axial velocity is the highest; the maximum out-of-plane displacement between subsequent recordings is approximately 18% of the laser sheet thickness. Outside the jet, the fluid is approximately at rest.

The measurement error is strongly affected by the out-of-plane displacement. The error distribution of figure 5.13 shows how the out-of-plane motion yields a major increase of the random error component, with the actual error standard deviation increasing by factor 5 (from 0.04 px to 0.23 px). It is also noticed that the two distributions are centred about zero, but exhibit opposite mean bias errors (-0.07 px in the jet core, 0.06 px in the stagnant region). Although perspective errors may be the cause of this behaviour (the optical axis of the measurement camera might be not exactly normal to the laser sheet), a conclusive explanation has not been found yet.

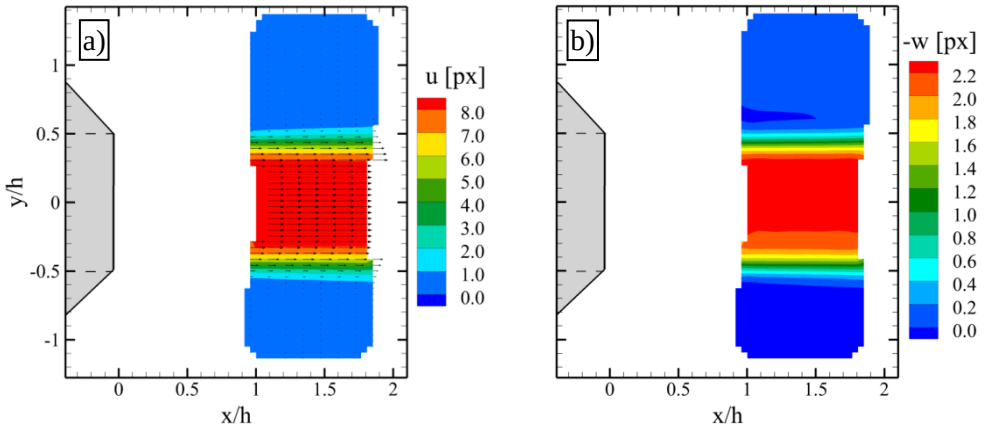


Figure 5.12. Time-averaged x-velocity component (a) and z-velocity component (b) measured by the HDR system. Both components are expressed in pixel units of the measurement system.

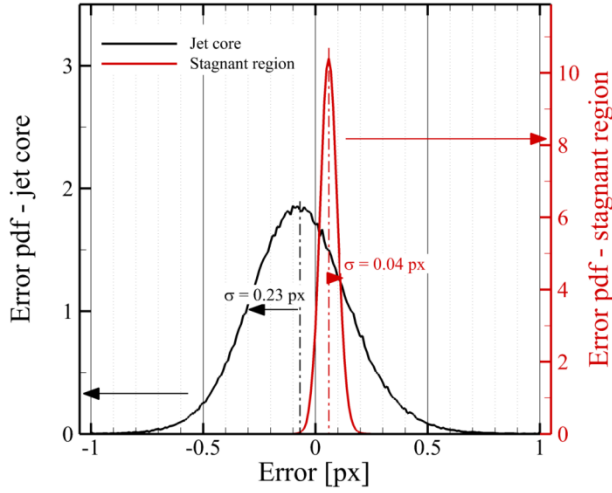


Figure 5.13. Actual error distribution in the jet core and in the stagnant region.

The comparison between estimated uncertainty and actual error is reported in figure 5.14. The actual error distribution of figure 5.14-e clearly shows that the largest error occur in the jet core, where the out-of-plane displacement is the highest, while errors below 0.05 px take place in the outer region. The uncertainty estimation obtained from image-matching and correlation statistics methods is more consistent with the actual error distribution (figure 5.14-b and -d): the estimated uncertainty exceeds 0.2 pixels in the jet core and drops below 0.1 px in the stagnant region, where the out-of-plane displacement (as well as the in-plane displacement) is negligible. Here, the image-matching method yields overestimated uncertainty values due to the intrinsic inaccuracy in the determination of the particle image disparity (see figure 5.14-f). The uncertainty evaluated with the peak ratio method exhibits lower sensitivity to the actual error value (figure 5.14-c): as can be seen also from the uncertainty profile extracted of figure 5.14-f, the estimated uncertainty is approximately uniform along the y -direction and only minor differences of the order of 0.06 px are retrieved between jet core and outer region. Finally, the surface method strongly underestimates the uncertainty, especially in the jet core (figure 5.14-a); the latter result was anticipated, because the approach does not account for errors stemming from out-of-plane motion.

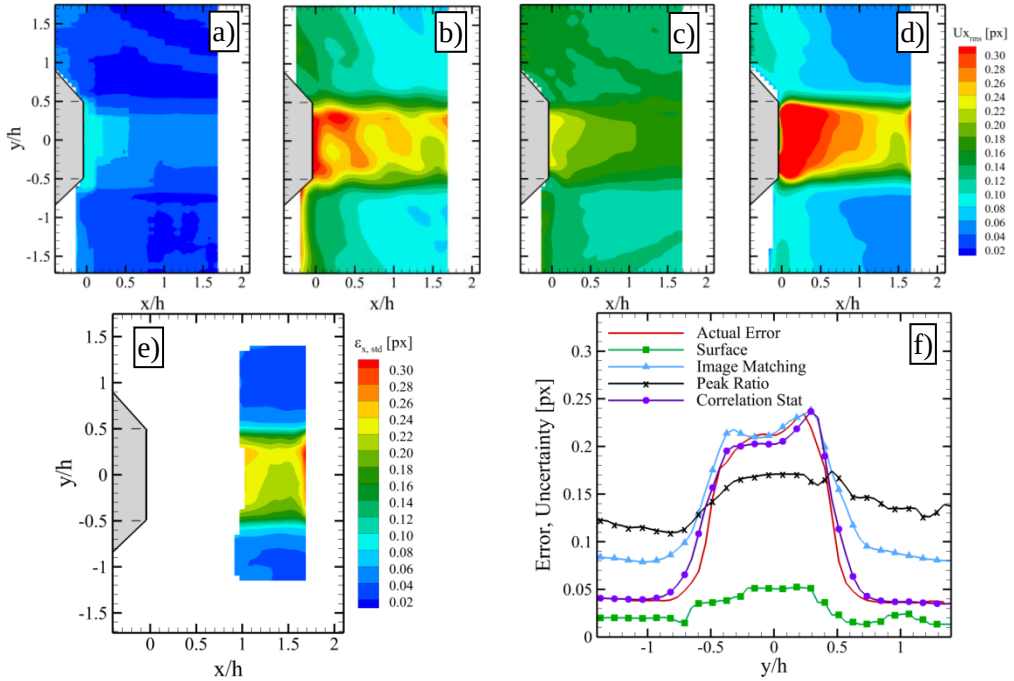


Figure 5.14. Top row: root-mean-square of the estimated uncertainty. (a) Surface method; (b) image matching method; (c) peak ratio method; (d) correlation statistics method. Bottom row: (e) actual error standard deviation; (f) comparison between actual error standard deviation and estimated uncertainty root-mean-square along a profile at $x/h = 1.5$.

The uncertainty statistics and coverage in the jet core are reported in table 5.2. The results confirm that both image-matching and correlation statistics methods consistently estimate the uncertainty, yielding a coverage that approaches the theoretical value (68%). It is also confirmed that the peak ratio method slightly underestimates the uncertainty in this region, while for the surface method the underestimation is major.

Table 5.2. Uncertainty statistics and coverage in the jet core. Uncertainty surface (US), image-matching (IM), peak ratio (PR) and correlation statistics (CS) methods. The root-mean-square of the actual error in this region is 0.237 pixels. The uncertainty has been estimated at 68% confidence level, thus the uncertainty coverage should ideally be equal to 68%.

	US	IM	PR	CS
Min estimated uncertainty [px]	0.012	0.054	0.053	0.030
Max estimated uncertainty [px]	0.160	0.783	0.284	0.707
RMS estimated uncertainty [px]	0.052	0.229	0.172	0.233
Uncertainty coverage	19%	64%	53%	65%

5.3.2.3 Effect of small particle images

A test case with poorly sampled particle images (diameter of 1 pixel) is selected to examine how the different uncertainty quantification algorithms cope with peak-locking errors. The seeding density is approximately 0.05 particles per pixel (ppp) for the measurement system. The potential core region is considered in this case. Due to the small particle image diameter in the measurement system recordings, peak locking errors occur that bias the estimated displacement towards integer values, as illustrated in figure 5.15.

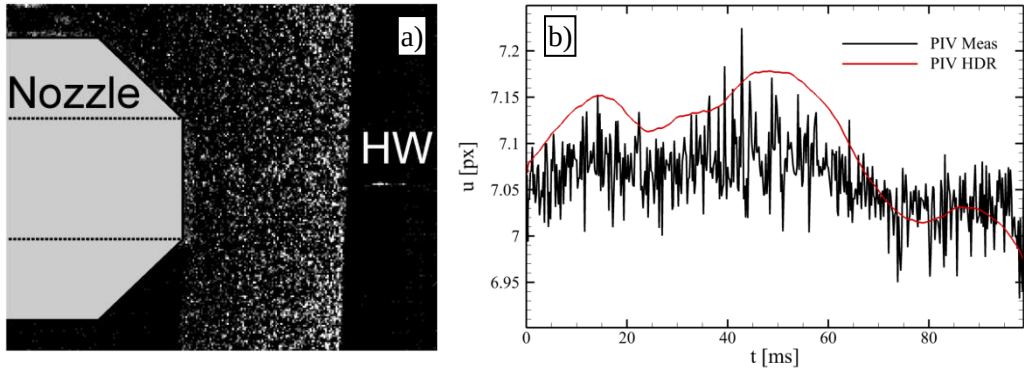


Figure 5.15. (a) Raw image. The nozzle and the hot-wire probe (HW) on the right of the image are indicated. Note that the particle images have diameter of approximately 1 pixel. (b) Fragment of the velocity time history in the potential core: in the measurement system, actual displacements of $7.1\div 7.2$ pixels are typically biased towards 7 pixels due to peak locking errors.

The actual error is approximately 0.08 px in the jet core, whereas it rises by up to 0.15 px in the shear layer (see figures 5.16-e and -f). The uncertainty contours of figures 5.16-a to -d show that the four methods consistently evaluate higher uncertainty in the shear layer. Nevertheless, the surface method provides values underestimated by above factor 2 both in the jet core and in the shear layer (figure 5.16-a). It is noted that the particle image diameter found with the method of Warner and Smith (2014), which is used in the US method code, returned a particle image diameter of 1.7 pixels. The difference in this method and the method of Adrian and Westerweel (2011) (which results in a particle image diameter of 1.4) is only pre-processing of the image to remove background noise. Modifying the particle image size to 1.4 results in a doubling of the uncertainty from the US method, which shows that the uncertainty is extremely sensitive to particle image size near 1.4 pixels.

Correlation statistics and image-matching approaches yield an uncertainty distribution that is consistent with the actual error (see figures 5.16-b and -d). Instead, the uncertainty estimated with the peak ratio method exhibits lower variations (about 0.06 px) among the different regions (core, shear layer and stagnant flow) of the flow field (figure 5.16-c).

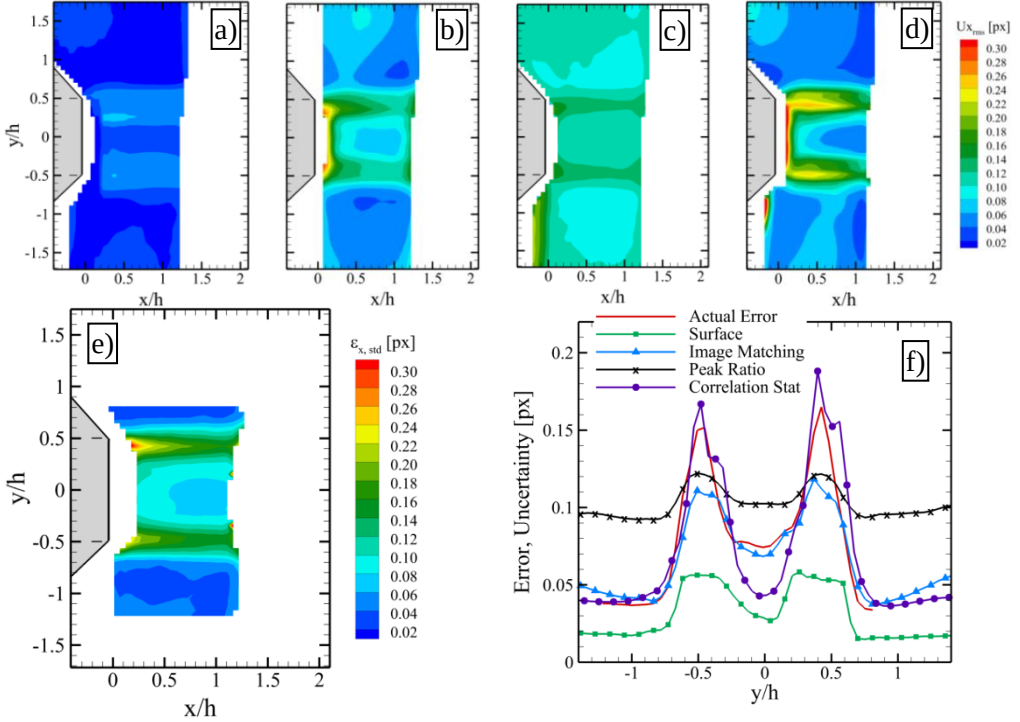


Figure 5.16. Top row: root-mean-square of the estimated uncertainty. (a) Surface method; (b) image matching method; (c) peak ratio method; (d) correlation statistics method. Bottom row: (e) actual error standard deviation; (f) comparison between actual error standard deviation and estimated uncertainty root-mean-square along a profile at $x/h = 1.0$.

The error distribution at a point of the potential core ($x/h = 1$, $y/h = 0$) is shown in figure 5.17. Such distribution is not Gaussian: the highest peak at $\epsilon = 0$ px does not correspond to the mean error ($\mu = -0.033$ px). Both skewness and excess kurtosis, which are much larger with respect to those evaluated in section 5.3.2.1 (-0.191 and 0.153 , respectively), confirm that the error distribution is not normal. The latter finding suggests that the errors at that point stem from multiple parent populations. This can be explained by the presence of peak locking: when the actual fractional displacement is null, peak locking errors are negligible and the error parent population is expected to be narrow and centred around zero; instead, for fractional displacements of

0.1-0.2 px, larger peak locking errors are expected (Raffel et al, 2007), resulting in a wider error parent population not centred around zero.

The time series of the error magnitude is extracted from the point $x/h = 1$, $y/h = 0$ and compared with the estimated uncertainty (see figure 5.18). Contrary to the error time series shown in figure 5.10, where a constant error in time with only random fluctuations is found, the present result yields a systematic error component, which is not constant in time, but varies between less than 0.01 px (see for instance $550 \text{ ms} \leq t \leq 600 \text{ ms}$ and $t \approx 950 \text{ ms}$) and 0.1 px (e.g. $400 \text{ ms} \leq t \leq 450 \text{ ms}$ and $800 \text{ ms} \leq t \leq 900 \text{ ms}$). The highest systematic errors occur when the actual displacement is about 7.3 pixels (figure 5.19). This, along with the small particle image diameter, suggests that the systematic error is mainly caused by peak locking (Westerweel, 1997). The uncertainty estimated with the four methods is also displayed in figure 5.18 for comparison. Even though different uncertainty values are obtained (e.g. the peak ratio method returns an uncertainty time-series approximately constant about 0.1 px, whereas the surface approach yields a minimum uncertainty estimate below 0.02 px), all the time series exhibit a correlation with the actual error magnitude. Higher uncertainty is estimated at the locations where the error magnitude is the highest (e.g. at time instants about $t = 400 \text{ ms}$ and 800 ms), whereas low error magnitudes are associated with uncertainty values typically below 0.05 px, except for the peak ratio method.

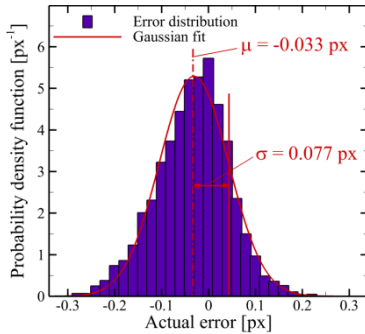


Figure 5.17. Actual error distribution at $x/h = 1$, $y/h = 0$.

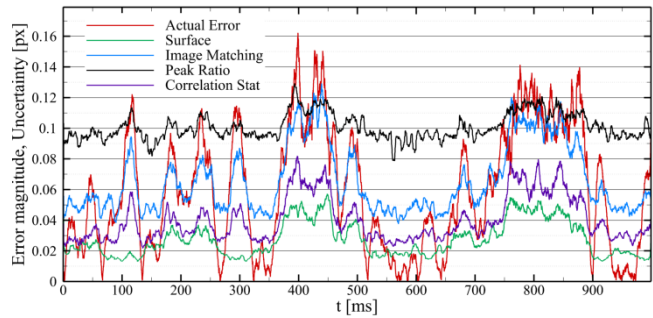


Figure 5.18. Actual error magnitude and uncertainty time histories at $x/h = 1$, $y/h = 0$.

The normalized cross-correlation function between actual error magnitude and estimated uncertainty has been computed to assess the dependence between the two quantities. The normalised cross-correlation attains a unit value when the two variables are identical or linearly dependent, while it drops to 0 when they are linearly independent (De Groot, 1989). The results of figure 5.20 show that all the four methods yield maximum cross-correlation

values exceeding 0.70, meaning that the estimated uncertainty is strongly correlated with the actual error magnitude. The highest correlation is achieved with the particle disparity method, which returns a value exceeding 0.90.

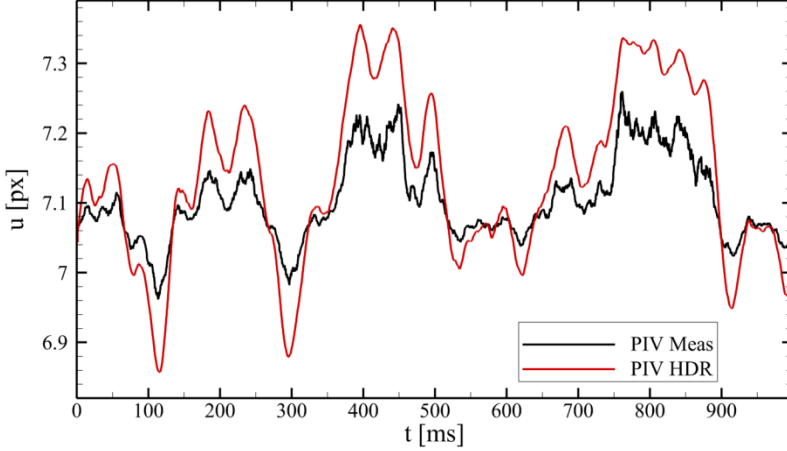


Figure 5.19. Velocity time history at $x/h = 1$, $y/h = 0$ measured by the HDR and the measurement systems. The time series have been filtered with a moving average top-hat filter on a kernel of 6 ms to attenuate random fluctuations.

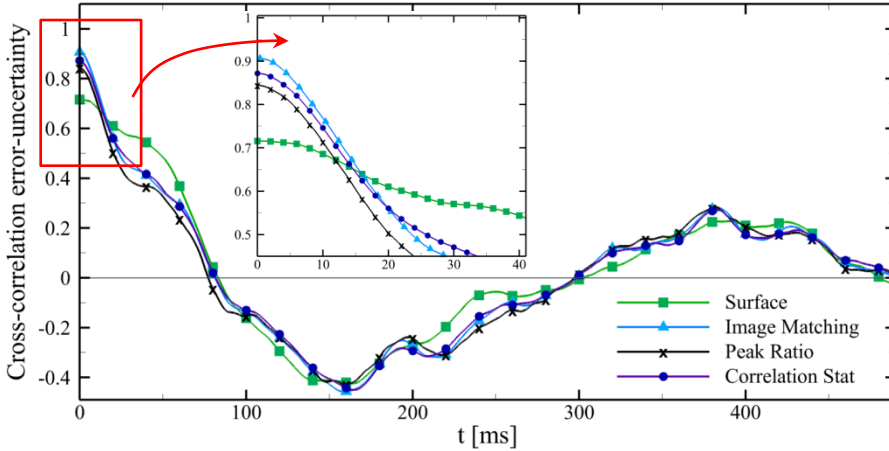


Figure 5.20. Normalized cross-correlation function between actual error magnitude and uncertainty time series.

The systematic and random error components are extracted from the error time series of figure 5.18 to assess how the four methods are able to estimate the uncertainty stemming from those. Figure 5.21 puts in evidence that severe peak locking errors occur: the mean bias error is the minimum for zero fractional displacement and raises up to above 0.1 px for fractional

displacements exceeding 0.2 px. Also the random error component is the minimum at zero fractional displacement, which is consistent with the numerical simulations of Scarano and Riethmuller (2000).

Both random and systematic errors increase with the fractional displacement. As a result, regions of fractional displacement close to zero are characterized by both low mean bias error (below 0.01 px) and low random error (below 0.03 px); here, also the uncertainty is expected to be low, because by definition it is an estimate of the standard deviation of the parent population from which the error stems. Instead, for fractional displacements exceeding 0.15 px both systematic and random errors are significantly larger; hence, also the uncertainty is expected to rise. This explains the correlation between uncertainty and error magnitude shown in figures 5.18 and 5.20.

It is important to remark here that in more conventional PIV experiments where the error is dominated by the random component, the uncertainty is typically uncorrelated from the error magnitude, as it has been shown in section 5.3.2.1. Instead, the presence of systematic errors comparable to or larger than random errors may lead to correlation between error magnitude and uncertainty.

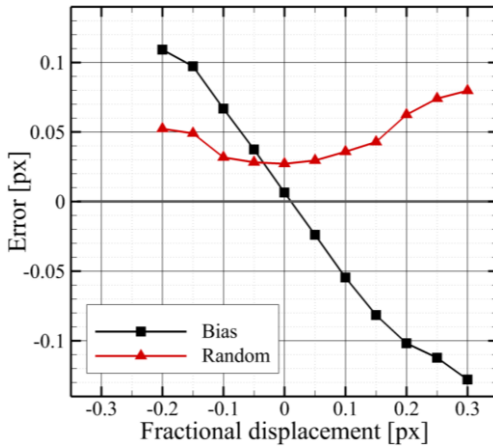


Figure 5.21. Bias and random error components as a function of the fractional displacement at $x/h = 1$, $y/h = 0$.

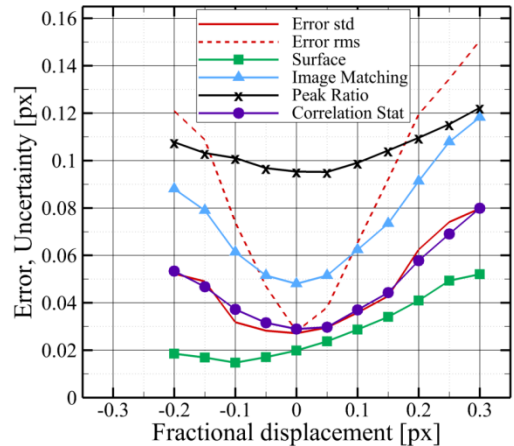


Figure 5.22. Error and uncertainty as a function of the fractional displacement at $x/h = 1$, $y/h = 0$.

The comparison between actual error and estimated uncertainty is shown in figure 5.22. The results show that US and CS methods estimate only the random component of the error; in particular, the CS-uncertainty exhibits excellent agreement with the actual error standard deviation. The IM approach

partly detects also the bias error component and estimates larger uncertainty (up to 0.12 px) for fractional displacement of 0.3 pixels. Finally, the PR method shows a higher “floor” for fractional displacement close to zero, meaning that for sub-pixel displacements between -0.1 and 0.1 px the uncertainty due to both random and bias error components is largely overestimated.

5.3.2.4 Effect of low seeding density

The same test case of section 5.3.2.3 is replicated with lower seeding density (approximately 0.02 ppp, see figure 5.23) to investigate the response of the four methods to such error source. In this case, each interrogation window of size 16×16 pixels contains on average approximately 5 particle images. Consequently, the accuracy of those methods that quantify the uncertainty from a statistical analysis of the particles contribution to the correlation peak is expected to decrease due to the reduced information contained in each window.

Figure 5.24 shows the comparison between actual error and estimated uncertainty along a profile at $x/h = 1$. With respect to the case presented in figure 5.16, higher error up to 0.25 pixels is obtained in the shear layer. The peak ratio method yields an estimated uncertainty profile that resembles that for the larger seeding density case: the uncertainty is approximately uniform about 0.1 px and slightly larger values (0.13 px) are achieved in the shear layer. Also the image-matching approach provides an uncertainty estimate that does not differ significantly from that of the previous test case. The method underestimates the uncertainty in the shear layer; as anticipated, this is attributed to the reduced number of particle images contained in the interrogation window, which precludes the convergence of the statistical analysis from which the uncertainty is evaluated. For the correlation statistics method, the underestimation is lower and uncertainty peaks up to 0.2 pixels are estimated at the shear layer locations. Finally, the surface method yields uncertainty values that better approximate the error peaks.

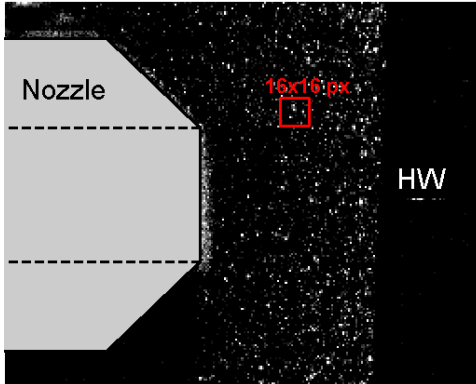


Figure 5.23. Raw image for the test case of seeding density of approximately 0.02 ppp.

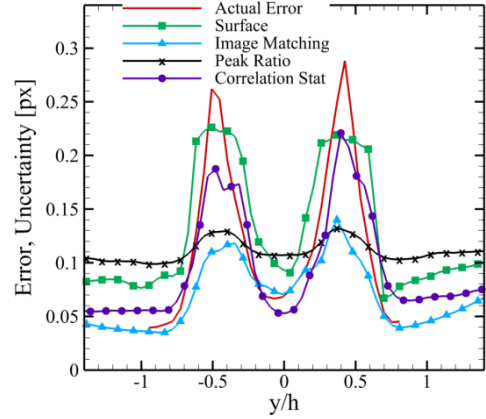


Figure 5.24. Comparison between actual error standard deviation and estimated uncertainty root-mean-square along a profile at $x/h = 1.0$.

The uncertainty statistics and coverage reported in table 5.3 confirm that for the present test case the surface method provides a more reliable uncertainty estimate than those methods that rely upon the analysis of the image contributions to the correlation. This is ascribed to the fact that the surface method makes use of numerical simulations that reproduce the experimental conditions; therefore, its performance does not degrade for low seeding density.

Table 5.3. Uncertainty statistics and coverage. Uncertainty surface (US), image matching (IM), peak ratio (PR) and correlation statistics (CS) methods. The root-mean-square of the actual error in this region is 0.258 pixels. The uncertainty has been estimated at 68% confidence level, thus the uncertainty coverage should ideally be equal to 68%.

	US	IM	PR	CS
Min estimated uncertainty [px]	0.012	0.008	0.006	0.006
Max estimated uncertainty [px]	0.416	1.156	0.284	1.286
RMS estimated uncertainty [px]	0.189	0.106	0.117	0.155
Uncertainty coverage	60%	38%	51%	47%

5.4 Concluding remarks

The collaborative framework for PIV uncertainty quantification has been set up to foster the collaboration among research groups working on the topic. A dedicated experimental data base has been generated to enable the

comparative assessment of four uncertainty quantification methods; those are the image-matching method (Sciacchitano *et al*, 2013), the uncertainty surface approach (Timmins *et al*, 2012), the peak ratio method (Charonko and Vlachos, 2013) and the correlation statistics approach (Wieneke and Prevost, 2014). The peculiarity of the experimental data base is that the reference instantaneous velocity field is known via a more accurate measurement conducted by an auxiliary PIV system, defined the HDR (high dynamic range) system. The measurement error is computed as the difference between measured and HDR velocity and therefore is estimated with high accuracy. This quantity is used to evaluate the goodness of the uncertainty estimate.

The data base has been used to investigate the capability of the four uncertainty quantification methods to estimate the instantaneous local uncertainty in presence of error sources typically encountered in PIV experiments. From this collaboration, strengths and weaknesses of the existing uncertainty quantification approaches have been put in evidence.

It has been shown that the approaches that quantify the uncertainty from the image contributions to the shape of the correlation peak (namely image-matching and correlation statistics methods) exhibit satisfactory sensitivity to the actual measurement error: higher uncertainty is typically estimated in regions of larger error. In the presented test cases, uncertainty peaks up to 3-4 times the minimum estimated value have been retrieved consistently with the actual error trend. The sensitivity of those methods is ascribed to the fact that the uncertainty is estimated from the shape of the correlation peak, which affects directly the measured displacement. This work has also revealed that the image-matching method typically overestimates the uncertainty associated with error values below 0.04 pixels: such behaviour has been anticipated and is due to the intrinsic uncertainty of the approach in determining the position of individual particle images.

The peak ratio method exhibits lower sensitivity to variations of the actual error: in the presented cases, the peak uncertainty typically varies of about 50% between regions of highest and lowest error, even if the error variation exceeds factor 5. The lower sensitivity with respect to e.g. correlation statistics and image-matching approaches can be explained by the fact that the method quantifies the uncertainty from a quantity, namely the cross-correlation peak ratio, which is not directly related to the measured displacement.

The surface method does not make use of information stemming from the cross-correlation function or the image contributions to that. The main advantage is that the performance of the approach does not degrade for e.g.

low seeding density, when little information is contained in the PIV recordings and image-based methods (image-matching and correlation statistics) yield results where the statistical convergence is not reached. The main limitation is that not all the error sources are accounted for: in the current implementation, the method is insensitive to errors arising from out-of-plane motion, which constitute a relevant component in turbulent flow investigation.

The lessons learned in the present investigation are expected to promote further advances in the direction of the development of a consolidated methodology for the a-posteriori uncertainty quantification of PIV data.

CHAPTER 6

MULTI-FRAME PYRAMID CORRELATION FOR TIME-RESOLVED PIV*

Abstract The chapter introduces a novel technique to increase the precision and robustness of time-resolved particle image velocimetry (TR-PIV) measurements. The innovative element of the technique is the linear combination of the correlation signal computed at different separation time intervals. The domain of the correlation signal resulting from different temporal separations is matched via homothetic transformation prior to the averaging of the correlation maps. The resulting ensemble-averaged correlation function features a significantly higher signal-to-noise ratio and yields a more precise velocity estimation due to the evaluation of a larger particle image displacement. The method relies on a local optimization of the observation time between snapshots taking into account the local out-of-plane motion, continuum deformation due to in-plane velocity gradient and acceleration errors. The performance of the *pyramid correlation* algorithm is assessed on a synthetically generated image sequence reproducing a three-dimensional Batchelor vortex. Experiments conducted in air and water flows are used to assess the performance on time-resolved PIV image sequences.

The numerical assessment demonstrates the effectiveness of the pyramid correlation technique in reducing both random and systematic errors by factor 3 and one order of magnitude, respectively. The experimental assessment shows a significant increase of signal strength indicating enhanced measurement robustness. Moreover, the amplitude of noisy fluctuations is considerably attenuated resulting in higher precision of the measured velocity and acceleration.

6.1 Introduction

In order to follow the temporal evolution of the flow field, high-speed digital cameras with CMOS sensors and high repetition rate lasers are usually employed. As explained in chapter 2, modern CMOS cameras can acquire images at comparable frame size to CCD cameras (typically $2,000 \times 2,000$ pixels per image) and much higher frames per second (above 1 kHz). Though, the larger pixel size, higher noise and lower sensitivity yield inferior image quality (Hain *et al*, 2007). Furthermore, while current flashlamp-pumped Nd:YAG lasers usually deliver pulses up to 1 J at typical repetition rate of

* This work has been published in Sciacchitano *et al* (2012) Experiments in Fluids 53: 1087–1105, DOI 10.1007/s00348-012-1345-x. The pyramid correlation algorithm has been implemented in the commercial software Davis 8 of LaVision GmbH.

10 Hz, the pulse energy of diode-pumped Nd:YLF lasers does not exceed 20 mJ at 1 kHz and drops well below 10 mJ at 10 kHz. Thus, the combination of lower image quality of CMOS cameras and weaker illumination of high repetition rate lasers are the principal causes of the reduced accuracy and precision of high-speed PIV systems, as also documented in the results of the third PIV challenge (Stanislas *et al*, 2008).

The current interest in time-resolved PIV for the determination of the unsteady pressure field requires an accurate measurement of the velocity temporal derivatives in order to evaluate the Lagrangian acceleration of fluid particles and in turn integrate the spatial field of the pressure gradient (Liu and Katz, 2006; Haigermoser, 2009; Charonko *et al*, 2010; Violato *et al*, 2010). The requirements for accurate measurements become even stricter for applications in aeroacoustics, where the use of PIV data in combination with aeroacoustic analogies often requires a double temporal derivative (Violato and Scarano, 2011). The reader is addressed to the recent review due to Morris (2011) for a detailed discussion of current trends.

In many works it is reported that reliable temporal information can only be extracted after data filtering in the space and time domain in order to reduce the amplitude of noisy fluctuations (Vétel *et al*, 2011; Oxlade *et al*, 2012). The need to increase the measurement robustness and accuracy has led to the development of several interrogation techniques that make use of more image pairs to extract an instantaneous velocity field, as discussed in chapter 3. The *ensemble average correlation* proposed by Meinhart *et al* (2000) for computing time-averaged velocity fields has received the most attention. The concept has been extended to unsteady flows by Scarano *et al* (2010): in this case, the average correlation function $\bar{C}$ is evaluated from a small kernel of N image pairs at fixed temporal separation (*sliding-average correlation*):

$$\bar{C} = \frac{1}{N} \sum_{i=-(N-1)/2}^{(N+1)/2} C_{i,i+1} \quad (6.1)$$

being $C_{i,i+1}$ the correlation function obtained from the interrogation of windows I_i and I_{i+1} . The evaluation of the correlation peak position and in turn of the fluid parcels displacement is based on the average correlation function $\bar{C}$. The most important advantage of such approach to the single-pair cross-correlation analysis is that the random error component is lowered by factor $\sqrt{N}$ (assuming that it is uncorrelated in time). However, the choice of an optimal duration of the image sequence used for the averaging is far from

trivial. In this respect, the work of Boillot and Prasad (1996) that indicates a way to optimize the temporal separation between two exposures can be used as a guide.

Nogueira *et al* (2009, 2010, 2011) introduced a multiple Δt strategy for the quantitative estimate of the magnitude of peak-locking and CCD read-out bias errors. However, the method can only be applied to lower the error of the time-averaged velocity field and no correction can be obtained for the instantaneous velocity measurements.

The use of multiple time separations has been investigated also in the works of Pereira *et al* (2004), Hain and Kähler (2007) and Persoons *et al* (2011). Here, the concept of multi-frame PIV relies upon the local optimization of the separation time between the image pair to reduce the relative error and enhance the dynamic velocity range. Though, the increase of the inter-frame time interval is limited by the occurrence of loss-of-pairs ascribed to out-of-plane motion (Keane and Adrian, 1992).

The present chapter investigates an approach, namely the *pyramid correlation*, which aims at solving the main limitations of the aforementioned methods. The technique is based on the combination of correlation maps from different image pairs, which are obtained also at different temporal separation[†].

The case of image sequences acquired at constant time separation is treated for sake of simplicity, although the working principle may be easily generalized to the case of sequences with unequally separated images.

6.2 The pyramid correlation

The *pyramid correlation* aims at reducing the magnitude of both systematic and random errors in time-resolved PIV measurements. The method can be placed across the adaptive multi-frame technique (Hain and Kähler, 2007) and the ensemble average correlation approach (Meinhart *et al*, 2000): correlation functions computed at different inter-frame time intervals are averaged to build an ensemble map with higher signal-to-noise ratio, which yields a more robust estimate of the particle image displacement. An innovative element is introduced: the correlation space matching by homothety, which is necessary

[†] Note that the expression *pyramidal correlation* has been used by Bonmassar and Schwartz (1998) to indicate a multi-resolution cross-correlation performed on Laplacian pyramid image architectures (Burt and Adelson, 1983). The pyramid correlation introduced in the present work has no connection with the technique used by Bonmassar and Schwartz: the expression is used to indicate the way correlation functions are computed at different temporal separation.

to the linear combination of the correlation signal obtained at different temporal separation.

The method takes as input a short sequence of recordings separated by a constant time interval; the optimum observation time, i.e. the time interval elapsing between first and last image of the set, is locally evaluated based on error-minimization criteria discussed in the remainder.

The underlying principle of the method is that the correlation averaging obtained from snapshots at short time separation contributes to the robustness of correlation, while the information built from larger temporal separation is exploited to achieve higher measurement precision.

6.2.1 Algorithm description

The pyramid correlation algorithm is based on the WIDIM interrogation algorithm (Scarano and Riethmuller, 2000). For each iteration and in each interrogation window, a set of n_{opt} successive image pairs is employed to compute the correlation functions following the cross-combinatorial approach illustrated in the scheme of figure 6.1. The temporal interval between two subsequent recordings is indicated with Δt_0 , whereas the optimum temporal separation is $\Delta t_{\text{opt}} = n_{\text{opt}} \cdot \Delta t_0$. The generic temporal separation between two recordings is indicated as $\Delta t = n \cdot \Delta t_0$, with $1 \leq n \leq n_{\text{opt}}$.

The instantaneous correlation functions form a pyramid whose base is composed by n_{opt} functions, as shown in figure 6.1. The height of the pyramid is constituted of n_h correlation functions, with n_h ranging from 1 to n_{opt} . If $n_h = n_{\text{opt}}$, the full pyramid is constructed, whereas if $n_h < n_{\text{opt}}$ the pyramid is truncated and obtained from the sole time separations from 1 to n_h . The approach for the selection of n_{opt} and n_h is explained in section 6.2.2.

The maps obtained from snapshots at the same separation time are averaged, yielding n_h averaged correlation maps. When n_{opt} is an odd number, the expression for the averaged cross-correlation function reads as:

$$\bar{C}_n(\Delta \mathbf{x}) = \frac{1}{n_{\text{opt}} - n + 1} \sum_{i=-(n_{\text{opt}}-1)/2}^{(n_{\text{opt}}+1)/2-n} C_{i,i+n}(\Delta \mathbf{x}), \quad \text{for } n = 1, 2 \dots n_h \quad (6.2)$$

wherein $\Delta \mathbf{x} = (\Delta x, \Delta y) \in \mathcal{D}$ are the coordinates of the correlation function in the two-dimensional domain of discrete pixels displacements $\mathcal{D}$. $\bar{C}_n$ is the ensemble averaged correlation function obtained from image pairs at the same separation time $\Delta t = n \cdot \Delta t_0$. In case that n_{opt} is an even number, one frame is

added to obtain a velocity measurement centred in between two frames. The correlation map from the largest time separation (i.e. $n_{\text{opt}}+1$) in this case is not utilized to avoid the drop of signal-to-noise ratio.

A possible variant of the method makes use of a weighted average (e.g. triangular or Gaussian) in equation (6.2) to avoid discontinuities associated to the variation of the data at the edges of the kernel. The process is analogous to the use of weighting windows for cross-correlation as discussed by Astarita (2006). From a preliminary analysis, the authors observed that a Gaussian weighting yields a visible improvement in the measurement accuracy.

At each separation time Δt , the ensemble of image pairs yields an average correlation $\bar{C}_n$ with reduced noise level with respect to $C_{i,i+n}$. Moving from small separations to larger ones (increasing n), a more accurate estimate of the particle image displacement is expected because of the larger absolute displacement. However, the correlation peak height is likely to weaken due to the combined effect of loss-of-pairs related to the out-of-plane motion and correlation peak broadening due to local variations of the velocity field not accounted for by the window deformation method.

Combination of correlation functions at different Δt – homothetic transformation

The information obtained at different time separations cannot be directly combined, because it refers to displacements occurring at different time intervals. For instance, if the correlation map obtained for $n = 1$ shows a peak at location (2, 0), the map corresponding to $n = 2$ is expected to exhibit a peak around (4, 0), as illustrated in figure 6.2 left. Despite the two correlation functions yield different displacements, assuming that the fluid acceleration is negligible, they measure the same velocity.

Hence, in order to superimpose correlation functions at different Δt , those need to be expressed in the velocity domain instead of the displacement domain. The correlation planes matching is conducted via the homothetic transformation:

$$\bar{C}_n^u(\mathbf{u}) = \bar{C}_n \left(\frac{\Delta \mathbf{x}}{n \Delta t_0} \right) \quad (6.3)$$

wherein the superscript u indicates that the correlation function is expressed in the velocity domain. Equation (6.3) can be applied under the hypothesis that

the particle image displacement in the observation time interval $n_h \Delta t_0$ can be linearized, i.e. the fluid acceleration is negligible.

Each scaled map $\bar{C}_n^u$ is computed from the values of $\bar{C}_n$ in a subset of $\mathcal{D}$, displayed as the colour region of the maps on the left in figure 6.2. The match at sub-pixel precision requires a spatial interpolation of $\bar{C}_n$, which is performed by a 2D cubic spline; this causes a broadening of the maps at low n . According to Astarita and Cardone (2005), this choice prevents the total mean errors due to interpolation to exceed 0.009 pixels. The scaled functions are subsequently averaged yielding the ensemble correlation map. In the current work, the averaging process is performed through an arithmetic mean, but weighted averages that take into account the signal-to-noise ratio of the mean maps and the measured displacement may also be suitable.

From the analysis of the results of figure 6.2, it emerges that the correlation functions at low separation exhibit a clear displacement peak, which is broadened by the homothety; this broad peak can be considered as locally flat, therefore it has minor contribution to the sub-pixel accuracy of the measurement, but it enhances the detectability of the (usually weak) peak of the maps at larger Δt . The ensemble correlation function C_{ens}^u is computed averaging the contributions of all the maps at different separation time: the broad peak of the maps at low separation can be regarded as the base of a pyramid and contributes to the robustness, while the maps at large separation have a sharp peak (top of the pyramid) which enhances the sub-pixel accuracy of the measurement.

The whole procedure of determining the ensemble correlation function is repeated at each iteration of the WIDIM interrogation algorithm. The velocity predictor used for the image deformation is the same for all the images and is equal to the velocity field retrieved from the ensemble correlation function at the preceding iteration. When performing the cross-correlation on the deformed images, the displacement peak progressively tends to the centre of the correlation plane; in this case, the ensemble correlation function is computed in the same way as it has been previously explained.

A special case: sliding-average correlation

A simplified version of the pyramid algorithm is obtained when only one time separation (typically the shortest) is selected. In this case only the base of the pyramid is used ($n_h = 1$) and the correlation maps are averaged; no homothetic transformation is performed, since only one time separation is selected. The method practically corresponds to that of Meinhart *et al* (2000),

with the only difference that the ensemble average is not made on a long sequence, but rather on a short one. This method is referred to as *sliding-average correlation* and has been already employed in a number of applications (Scarano *et al*, 2010; Scarano and Moore, 2011; Violato and Scarano, 2011, among others).

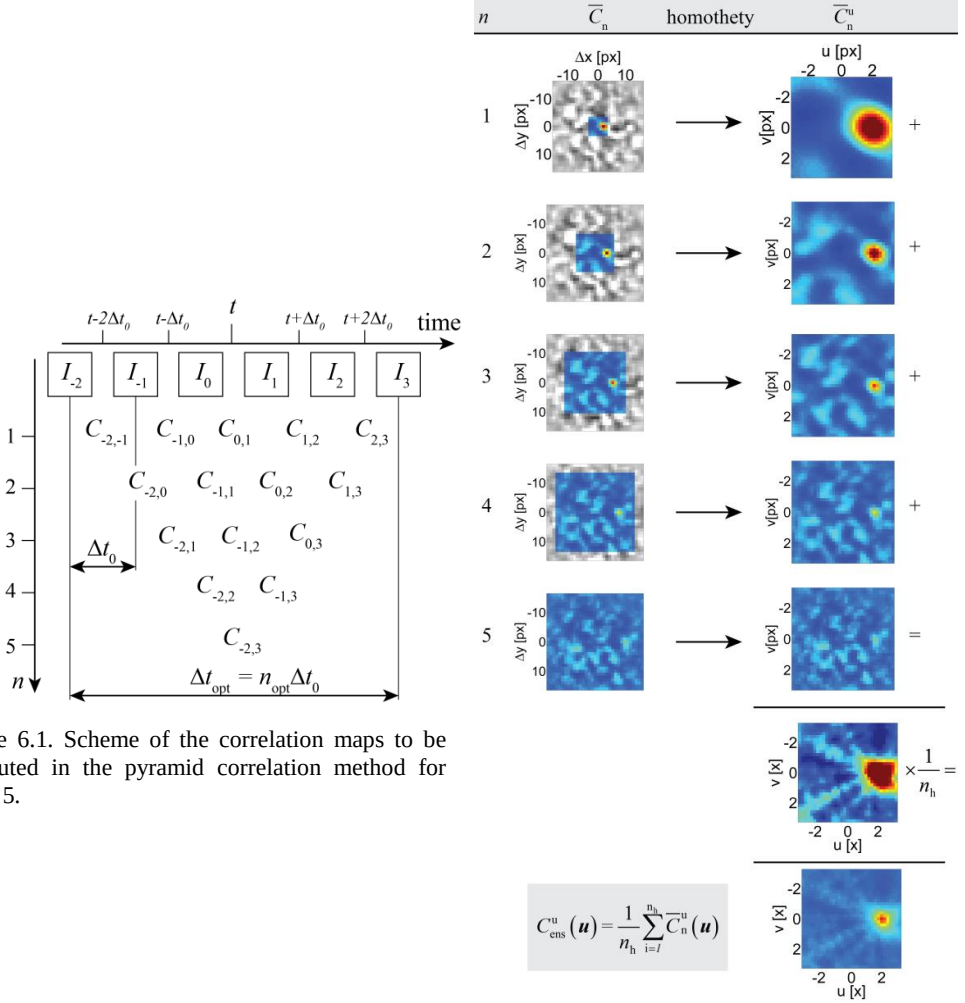


Figure 6.2. Mean correlation maps before (left) and after (right) the homothetic transformation ($n_h = n_{\text{opt}} = 5$ has been assumed); the black-and-white region of the maps on the left does not undergo the transformation.

6.2.2 Optimum sequence length and pyramid height

In order to build the pyramid of correlation functions, two parameters need to be determined: the optimal sequence length n_{opt} , which defines the optimum measurement time $\Delta t_{\text{opt}} = n_{\text{opt}} \cdot \Delta t_0$, and the correlation pyramid height n_h , which governs the maximum separation between the images of a pair. For this purpose, four criteria are considered, which take into account the random error on velocity and acceleration, the particle image pattern deformation due to in-plane displacement gradient and the loss-of-pairs related to the out-of-plane motion.

The first condition is based on that proposed by Boillot and Prasad (1996) for computing the optimum separation time between snapshots; the optimum Δt minimizes the combined effect of relative random and acceleration error (i.e. the truncation errors due to the curvature of the trajectory and the velocity temporal variation):

$$\Delta t_{\text{opt}} = \sqrt{\frac{2cd_\tau}{|\mathbf{Du} / Dt|}} \quad (6.4)$$

wherein cd_τ is an estimate of the absolute random error, assumed to be proportional to the particle image diameter d_τ . The latter is estimated from the auto-correlation function peak width, as suggested by Adrian and Westerweel (2011). The constant c has often been proposed in the order of 0.05-0.1 (Adrian 1991). The term $|\mathbf{Du}/Dt|$ is the norm of the material derivative of the velocity, which represents the contribution of the acceleration error. Assuming that the velocity at the previous time instant $t - \Delta t_0$ is known, the material derivative at t is estimated with a backward scheme, with the approximations that the local trajectory is a straight line and that locally the flow is convected at the local instantaneous velocity $\mathbf{u}(\mathbf{x}, t)$:

$$\frac{D\mathbf{u}}{Dt}(\mathbf{x}, t) \approx \frac{\mathbf{u}(\mathbf{x}, t) - \mathbf{u}[\mathbf{x} - \mathbf{u}(\mathbf{x}, t)\Delta t_0, t - \Delta t_0]}{\Delta t_0} \quad (6.5)$$

Since the velocity field provided by the predictor and employed in equation (6.5) is affected by random noise, the contribution of the acceleration error in equation (6.4) is often overestimated with respect to the contribution of the random error, biasing the estimate of Δt_{opt} towards low values. This effect may be mitigated by filtering the velocity field and choosing a conservative

estimate for the constant c . In the current work $c = 0.25$, is proposed. Therefore, the first estimation of n_{opt} used in the pyramid correlation, indicated with n_{BP} , is equal to:

$$1. \quad n_{\text{BP}} = \frac{1}{\Delta t_0} \sqrt{\frac{2cd_\tau}{|D\mathbf{u} / Dt|}} \quad (\text{BP})$$

The obtained value is rounded to the closest integer number. This criterion is applied locally; as a result different values of n_{BP} are chosen in the measurement domain, depending on the local flow conditions. Note that when the material derivative tends to zero, e.g. in case of uniform flow, n_{BP} tends to infinity because the contribution of the acceleration error vanishes: the only other error source considered in criterion 1 is the relative random error, which according to the model by Boillot and Prasad (1996) becomes negligible for very large displacements.

The first condition takes into account only the random and acceleration errors, not the broadening of the correlation peak and its magnitude reduction that occur in presence of a severe in-plane displacement gradient $\nabla \Delta \mathbf{x}$. These effects need to be accounted for when the image deformation technique is used, which is reported to cope with deformations not exceeding 0.5 pixels/pixel (Scarano and Riethmuller, 2000). Hence a limiting condition on n_{opt} must be introduced to ensure that for the optimum separation time the in-plane gradient remains within 0.5 pixels/pixel:

$$2. \quad n_{\text{def}} = \frac{0.5 \text{ pixels / pixel}}{|\nabla \Delta \mathbf{x}^{n=1}|} \quad (\text{deformation})$$

In regions where the velocity is close to zero, both the aforementioned criteria may be fulfilled even for unrealistically large values of n_{opt} (order of hundred). However, this causes a dramatic increase of the processing time, because hundreds of image pairs would need to be analysed, despite the fact that still fluid or uniform flow regions are of minor interest and such high precision is seldom required. Therefore, an upper-bound n_{max} must be selected to n_{opt} in the entire measurement domain. Typical values of n_{max} suggested in the present work range between 5 and 11:

$$3. \quad n_{\text{opt}} \leq n_{\text{max}} \quad (n_{\text{max}})$$

A possible choice for $n_{\max}$ is based on the requirement that the maximum particle image displacement should not exceed twice the final interrogation window size (Ws) to avoid modulation of physical fluctuations: $n_{\max} \leq (2 \cdot Ws) / |\Delta \mathbf{x}|$.

Finally, an effect not accounted for by the previous criteria is the particle image loss-of-pairs due to out-of-plane motion, which affects the reliability of the correlation peak (Keane and Adrian, 1992). In absence of information on the out-of-plane displacement (for instance obtained with a stereoscopic measurement system), the detrimental effect of the out-of-plane loss-of-pairs is indirectly monitored with the correlation signal-to-noise ratio, which at largest separation time must not fall below a prescribed threshold, typically set to 1.5:

$$4. \quad SNR \geq 1.5 \quad (SNR)$$

The local determination of n_{opt} and n_h is performed in two steps, as illustrated in figure 6.3; first, the optimal sequence length is calculated based on the most restrictive among the former three conditions:

$$n_{\text{opt}} = \min\{n_{\text{BP}}, n_{\text{def}}, n_{\text{max}}\}.$$

Which condition is locally the most restrictive depends upon the local flow field and image quality and will be discussed in the reminder of the chapter. The value of n_{opt} locally determines the width of the base of the pyramid of correlations.

Successively, the pyramid is built starting from the base (separation time $n = 1$) up to separation $n = n_{\text{opt}}$. At each separation time, the average correlation function $\bar{C}_n$ is computed and its signal-to-noise ratio is queried: if it does fulfil criterion 4 ($SNR \geq 1.5$), the height of the pyramid is temporarily set to $n_h = n$ and the successive separation time is considered; if instead $SNR < 1.5$, n_h is chosen equal to $n-1$ (or to 1 when $n = 1$). Therefore, criterion 4 limits the height n_h of the pyramid.

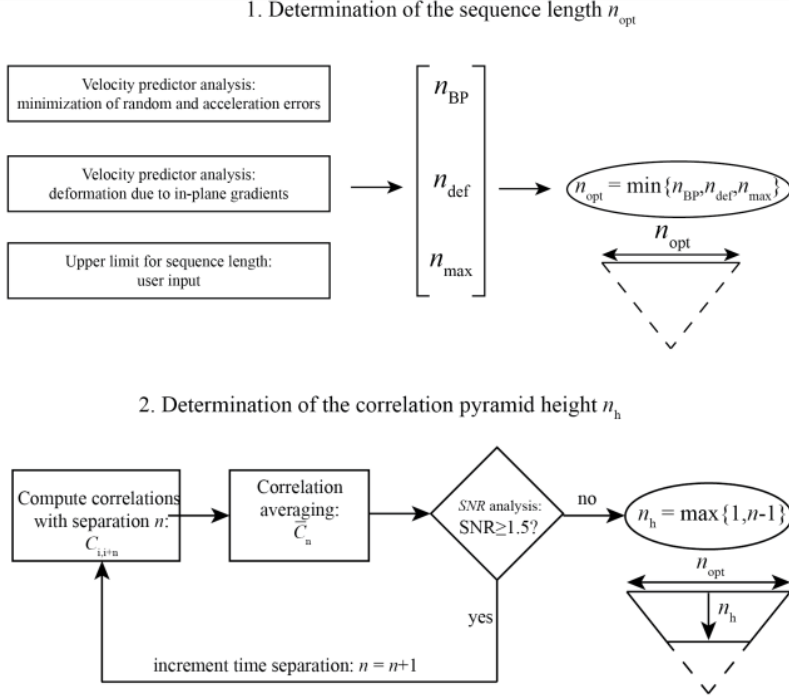


Figure 6.3. Flow chart describing the algorithm for optimal choice of the measurement temporal sequence length n_{opt} and the correlation pyramid height n_h .

6.3 Numerical assessment

6.3.1 3D vortex flow field

The pyramid correlation is assessed via Monte Carlo simulation. A sequence of 500 synthetic images of 400×400 pixels resolution is generated simulating a Gaussian light sheet intensity profile of width $\Delta z = 30$ pixels. The seeding concentration is 0.1 particles per pixel. The particle images have normal distribution, with mean diameter of 0.5 pixels: this represents the condition that may be encountered with CMOS sensors (pixel pitch $> 10 \mu\text{m}$) and relatively large aperture ($f_{\#} \leq 4$), which causes important peak-locking effects (Overmars *et al*, 2010). The maximum particle image intensity is set to 255 counts.

The motion of the particles follows the Batchelor vortex law (Batchelor, 1964):

Radial velocity: $V_r = 0$

Tangential velocity: $V_\theta = V_{\theta, \max} \left(1 + \frac{0.5}{\alpha} \right) \frac{R_0}{r} \left[1 - \exp \left(-\alpha \frac{r^2}{R_0^2} \right) \right],$

Axial velocity: $V_z = V_{z, \max} \cdot \exp \left(-\frac{r^2}{R_0^2} \right)$

wherein $\alpha = 1.26$, $R_0 = 50$ pixels is the vortex core radius, where the maximum tangential velocity $V_{\theta, \max}$ is encountered, and r is the radial position. The maximum tangential displacement within a single time separation Δt_0 is set to $0.06 R_0$, while the maximum out-of-plane displacement is $0.4 \Delta z$ (see figure 6.4).

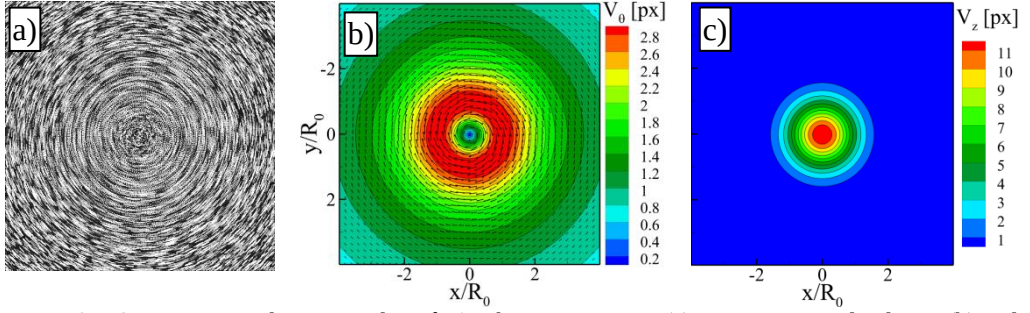


Figure 6.4. 3D vortex simulation: overlay of 10 subsequent images (a), exact tangential velocity (b) and axial velocity (c).

Four methods are compared, namely:

1. Single-pair correlation with constant inter-frame time interval ($SP-c\Delta t$);
2. Ensemble correlation with constant inter-frame time interval ($EC-c\Delta t$);
3. Single-pair correlation with adaptive inter-frame time interval ($SP-a\Delta t$);
4. Ensemble correlation with adaptive inter-frame time interval (i.e. pyramid correlation).

All methods use multi-grid analysis and window deformation (*WIDIM*, Scarano and Riethmuller, 2000). The methods differ only for the way of determining the correlation function employed to measure the displacement vector:

- the $SP-c\Delta t$ method consists in the *WIDIM* algorithm;

- for the *EC-cΔt* method, the average of correlation functions (Meinhart *et al*, 2000) over subsequent image pairs at the same separation time is considered: it corresponds to the *sliding-average correlation* discussed in chapter 3 and section 6.2.1.
- in the *SP-aΔt* method, the correlation function is computed from two recordings at a temporal separation which is locally optimized, as in the works of Pereira *et al* (2004), Hain and Kähler (2007) and Persoons *et al* (2011);
- the pyramid correlation is based on the cross-combinatorial superposition of the cross-correlations after homothetic transformation, as explained in section 6.2.1.

The schemes of the four techniques are graphically illustrated in figure 6.5. The computation of the total observation time $n_{\text{opt}}\Delta t$ for the *EC-cΔt*, *SP-aΔt* and pyramid correlation is based on the criteria discussed in section 6.2.2, setting the value of n_{max} to 9. The interrogation window size is set to 17×17 pixels with 75% overlap.

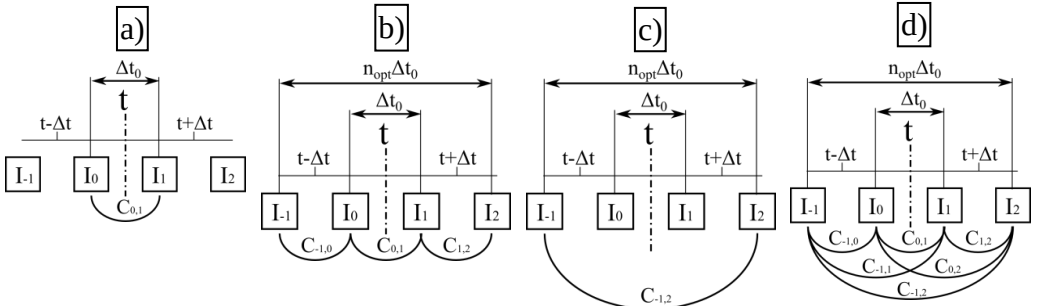


Figure 6.5. Schemes of the interrogation algorithms compared in the numerical assessment (the case of $n_{\text{opt}} = 3$ is considered here): *SP-cΔt* (a), *EC-cΔt* (b), *SP-aΔt* (c), *pyramid correlation* (d).

Figure 6.6 shows the mean bias and random errors on the horizontal displacement obtained with the four interrogation algorithms (each quadrant corresponds to one algorithm). Both the error components are significantly reduced when using a technique that employs an adaptive temporal separation (*SP-aΔt* and *pyramid correlation*): the error reduction is major outside the vortex core (factor 10 for the mean bias error, factor 3 for the random error), while it is less pronounced in the core, where the strong out-of-plane displacement precludes the use of a large temporal separation. Analogous results are obtained for the vertical displacement component.

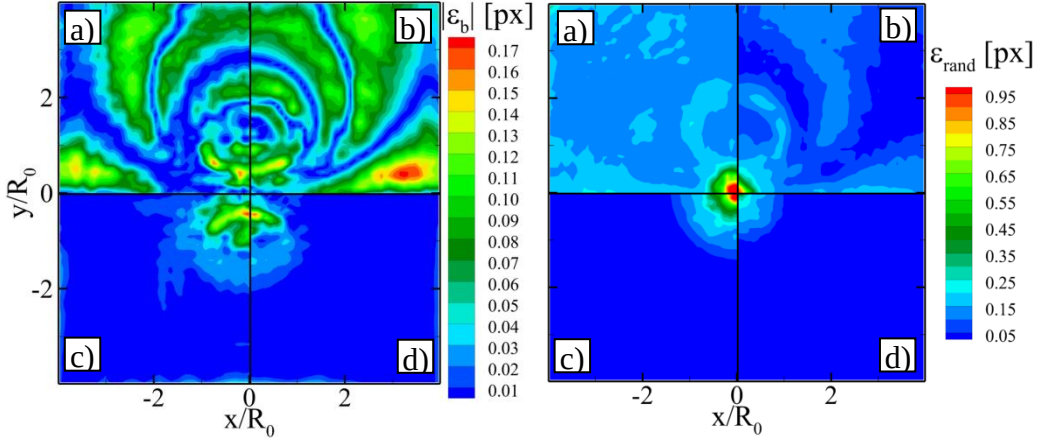


Figure 6.6. Mean bias error (left) and random error (right) on the horizontal displacement; (a) *SP-cΔt*; (b) *EC-cΔt*; (c) *SP-aΔt*; (d) *pyramid correlation*.

For a thorough investigation of the measurement errors of the four algorithms, the error along a profile *a* at $x/R_0 = 0$ is shown in figure 6.7. Figure 6.7-a shows the mean horizontal and out-of-plane actual displacement along the profile: the maximum out-of-plane displacement occurs at the vortex core and equals 40% of the laser sheet thickness, while the horizontal displacement is the maximum at $y/R_0 = \pm 1$. The statistical error along the profile obtained with the four processing algorithms is displayed in figure 6.7-b to -e: the symbols indicate the mean bias component, whereas the error bars express the random component. The error of the algorithms adopting a constant time separation is dominated by the random component, especially in the vortex core where the out-of-plane displacement is the largest. Furthermore, the mean bias error exhibits peaks exceeding 0.1 pixels ascribed to peak locking. The *SP-aΔt* strongly reduces bias and random errors outside the jet core due to the measurement of a larger displacement. In contrast, no major improvement is noticed in the jet core where the loss-of-pairs due to out-of-plane motion precludes the use of a large Δt . Instead, the pyramid correlation yields a reduction of both error components both in jet core and outer region.

The average *SNR* contours are depicted in figure 6.8 to assess the robustness of the algorithms (each quadrant corresponds to a different processing algorithm). The *SNR* is always computed from the correlation function employed for determining the measured velocity, which in the pyramid correlation is the ensemble average correlation function C_{ens}^u . The *SP-cΔt* returns a wide region of low signal strength in the vortex core: more than 60% of the computed vectors have average signal-to-noise below 4; this number rises to 85% when increasing the temporal separation through the *SP-aΔt*

algorithm. The signal robustness is significantly enhanced through the ensemble correlation, especially when the separation is chosen adaptively: less than 7% of the vectors have signal-to-noise ratio drops below 4.

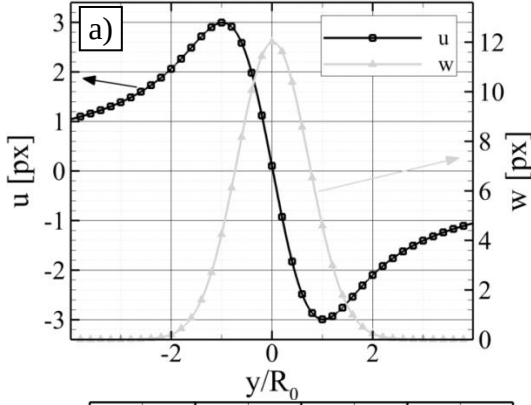
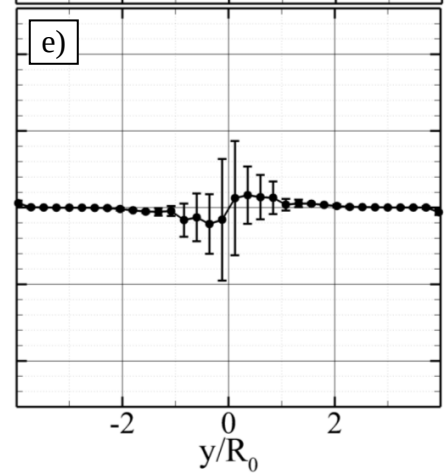
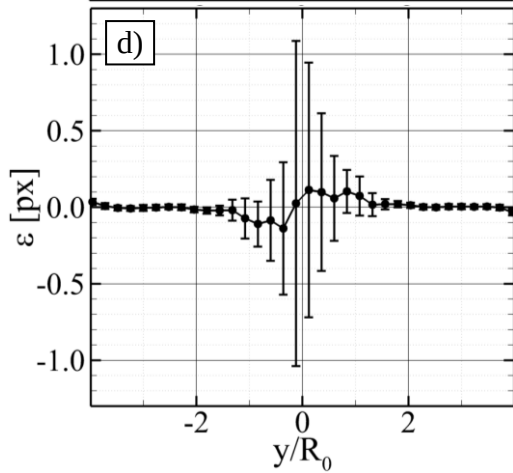
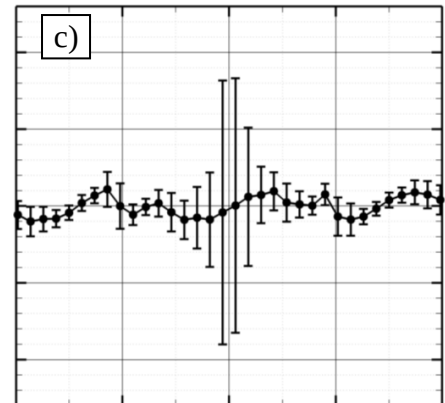
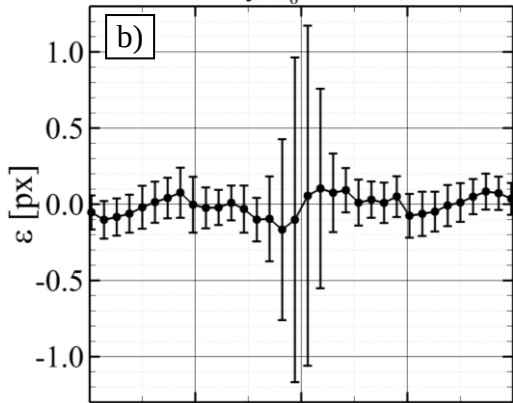


Figure 6.7. (a) Mean horizontal and out-of-plane actual displacement profile at $x/R_0=0$. (b) to (e): error along the profile $x/R_0=0$; (b) $SP-c\Delta t$; (c) $EC-c\Delta t$; (d) $SP-a\Delta t$; (e) pyramid correlation.



In order to clarify how the pyramid of correlations is built and which constraints are the most effective, the contours of figure 6.9 illustrate the mean values of the pyramid base and height and the criteria limiting the value of n_{opt} . Outside the vortex core ($r/R_0 > 1.5$), material acceleration and in-plane displacement gradients are small enough to fulfil criteria 1 and 2, therefore n_{opt} rises up to the upper bound $n_{\text{max}} = 9$. For $r/R_0 < 1.5$, truncation errors arise from the curvature of the trajectories, thus n_{opt} is limited by criterion 1 (BP). Inside the vortex core ($r/R_0 < 0.5$), in addition to the strong material acceleration, significant in-plane gradients occur; as a consequence, n_{opt} is limited by the criterion on the continuum deformation (criterion 2). Furthermore, here the low SNR due to the large out-of-plane displacement limits the height n_h of the pyramid to values smaller than n_{opt} (figure 6.9-c). In contrast, outside the core the out-of-plane motion is negligible and n_h rises up to n_{opt} .

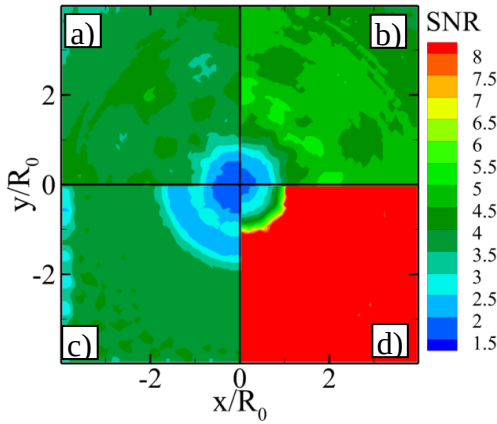
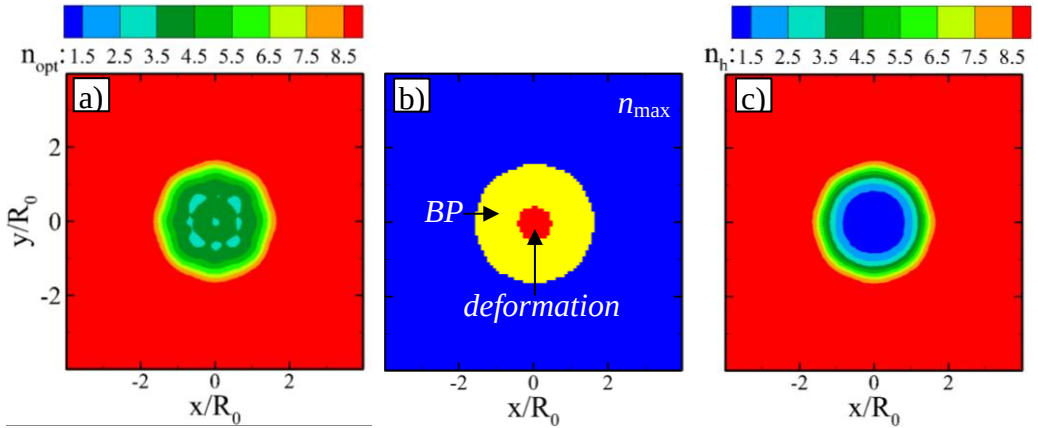


Figure 6.8 (Left). Average signal-to-noise ratio: (a) *SP-cAt*; (b) *EC-cAt*; (c) *SP-aAt*; (d) *pyramid correlation*.

Figure 6.9 (Bottom). (a) Mean optimum temporal separation n_{opt} ; (b) color-coded most frequently occurring criterion determining n_{opt} : deformation (red), BP (yellow), n_{max} (blue); (c) mean height n_h of the pyramid of correlations.



6.3.2 Temporal response

The temporal response of the pyramid correlation algorithm is compared to that of single-pair correlation and sliding-average correlation. Synthetic images of resolution 400×400 pixels are generated with seeding density of 0.1 particles per pixel. The particle images have Gaussian shape with mean diameter of 2 pixels and move in the horizontal direction according to the sinusoidal displacement law:

$$u(t) = u_0 + u_A \cos\left(\frac{2\pi t}{T}\right) \quad (6.6)$$

The mean displacement is chosen as $u_0 = 0.3$ pixels, while the displacement amplitude is $u_A = 0.2$ pixels. A period $T = 20$ is selected, meaning that a complete wave is sampled by 20 successive PIV recordings.

The results of the temporal sine-wave test are illustrated in figure 6.10. Let us consider first the single-pair correlation algorithm. When the temporal separation $n_{\text{opt}}\Delta t_0$ between the images of the pair is increased, the measured amplitude u_m drops. This occurs because only the information at the beginning and the end of the time interval is retained, and the particle images are assumed to move at constant velocity along a linear trajectory between initial and final position. The results of figure 6.10 show that the time response of the single-pair correlation is equivalent to that of a top-hat temporal filter of kernel size $n_{\text{opt}}\Delta t_0$, which is a sinc function. This means that the single-pair algorithm returns the average velocity in the time interval $n_{\text{opt}}\Delta t_0$.

In the sliding-average correlation (indicted as $EC-c\Delta t$ in figure 6.10), n_{opt} correlation maps from consecutive images (at time separation Δt_0) are averaged for a total observation time $n_{\text{opt}}\Delta t_0$. The temporal response is the same as for the single-pair correlation. In fact, even if information at smaller time separation Δt_0 is used, this is averaged over the time interval $n_{\text{opt}}\Delta t_0$: as a result, the measured velocity is equivalent to that obtained by filtering the actual velocity with a top-hat filter of kernel size $n_{\text{opt}}\Delta t_0$. Higher temporal response is expected when using a weighted average of the correlation maps; however, the analysis of this case goes beyond the scope of the present work.

The pyramid correlation returns a slightly higher temporal response. This result does not come unexpected and is explained in figure 6.11 considering the case $n_{\text{opt}} = 3$. When the pyramid of correlation is built, four correlation maps provide information on the displacement between images 0 and 1, while

only three contribute to the measurement of the displacement between images -1 and 0 and images 1 and 2 . Consequently, the pyramid does not behave as a top-hat filter in time, but more as the superposition of a top-hat and a triangular filter, which is indicated as pyramid filter in figure 6.10.

Although the pyramid correlation yields only a minor increase in the temporal response, this is not considered a limitation of the technique. In fact, the algorithm aims at enhancing the dynamic range and robustness of the measurement and not at achieving higher temporal response. The latter can be obtained with recent multi-frame algorithms relying upon a Lagrangian flow description, which are introduced in section 6.6.

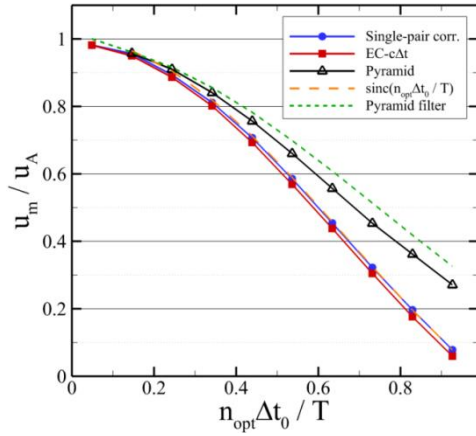


Figure 6.10. Temporal response of single pair correlation, sliding-average correlation and pyramid correlation.

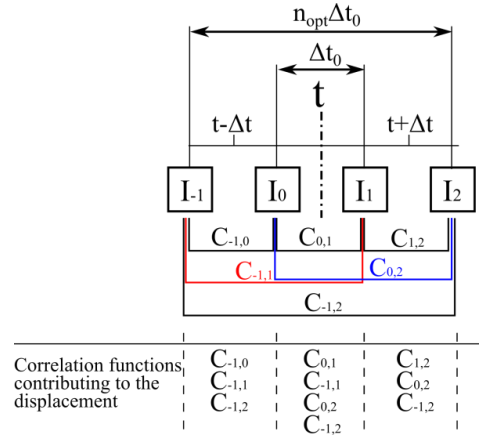


Figure 6.11. Explanation of the temporal response of the pyramid correlation algorithm.

6.4 Experimental assessment

Two experiments are conducted in the Aerodynamics Laboratory of TU Delft: the first one is performed in the near wake of a NACA airfoil (section 6.4.1) and the second one is a transitional jet in water (section 6.4.2).

6.4.1 Near wake of a NACA airfoil

The near wake of a 40 cm chord length NACA-0012 airfoil is analysed through the pyramid correlation to assess the effectiveness of the technique in a turbulent shear flow. The experiment is conducted in a low-speed wind tunnel at free-stream velocity of 14 m/s (chord based Reynolds number of 370,000); the boundary layer transition is forced at 30% of the chord. The airfoil is placed in the test section at zero angle of incidence.

A Quantronix Darwin-Duo laser (Nd:YLF diode pumped, 2×25 mJ at 1,000 Hz, $\lambda = 527$ nm) is employed to illuminate the field of view. Images are recorded by a Photron FASTCAM-SA1 camera (CMOS, 1024×1024 pixels at 5,400 Hz, 10 bits, pixel pitch $20 \mu\text{m}$). The camera is equipped with a Nikon objective of 105 mm focal length set at $f_{\#} = 2.8$; the magnification factor is equal to 0.39. The active sensor size is reduced to 736×256 pixels to achieve an acquisition rate of 10,000 image pairs per second. The inter-frame time interval is set to $50 \mu\text{s}$, so that the effective acquisition rate of 20 kHz. Additional details of the experimental apparatus are reported by Ghaemi *et al* (2011); table 6.1 summarizes the measurement conditions.

A region of 300×256 pixels downstream of the airfoil's trailing edge is selected for the analysis as illustrated in figure 6.12.

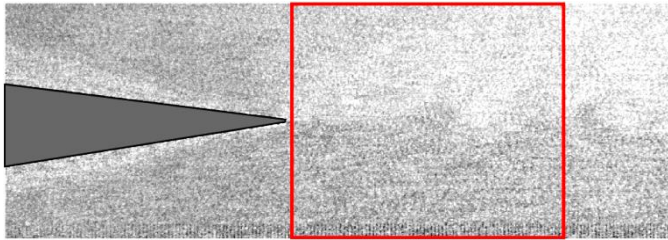


Figure 6.12. Region selected for the analysis (superposition of two successive recordings).

A set of 2,000 images are interrogated with windows of 17×17 pixels at 75% overlap, which yields a vector pitch of 0.20 mm. The maximum observation time is set to $n_{\text{max}} = 9$.

In figure 6.13-a, the average velocity field measured with pyramid correlation is shown to illustrate how the four conditions listed in section 6.2.2 limit the observation time and the pyramid height.

Multi-frame Pyramid Correlation for time-resolved PIV

Table 6.1. System parameters of the near wake experiment.

Seeding	Smoke droplets, 1 μm diameter
Illumination	Quantronic <i>Darwind-Duo</i> Nd:YLF laser (2×25mJ @ 1 kHz)
Recording device	Photron <i>Fast CAM SA1</i> camera (1024×1024 pixels @ 5.4 kHz) 20 μm pitch
Imaging	<i>Nikon</i> objectives $f = 105 \text{ mm}$, $f_{\#} = 2.8$
Field of view	32.0×12.8 mm ² , 736×256 px ²
Acquisition frequency	20,000 Hz
Magnification factor	0.39
Number of images	2,000

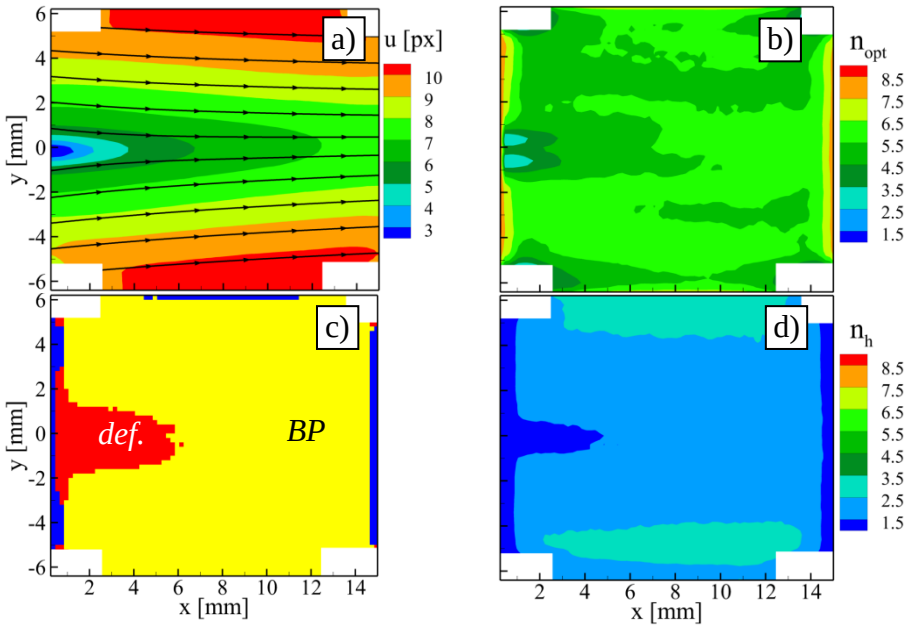


Figure 6.13. Near wake experiment. (a) Mean velocity field; (b) mean optimum temporal separation n_{opt} ; (c) color-coded most frequently occurring criterion determining n_{opt} : *deformation* (red), *BP* (yellow), n_{max} (blue); (d) mean height n_h of the pyramid of correlations.

In most of the domain, the optimum temporal separation is limited to 5 or 6 (figure 6.13-b) by the strong acceleration errors present in the turbulent flow (Boillot and Prasad condition). Only in the immediate vicinity of the trailing

edge ($x < 6$ mm, $y \in [-2, 2]$ mm) the criterion on the continuum deformation is active (figure 6.13-c). On average, the height of the pyramid is smaller than the base due to the significant out-of-plane motion responsible of low signal-to-noise ratio values (figure 6.13-d).

The robustness of the measurement is analysed by the SNR of the correlation functions. In figure 6.14 the four methods are compared in terms of probability distribution and cumulative distribution functions.

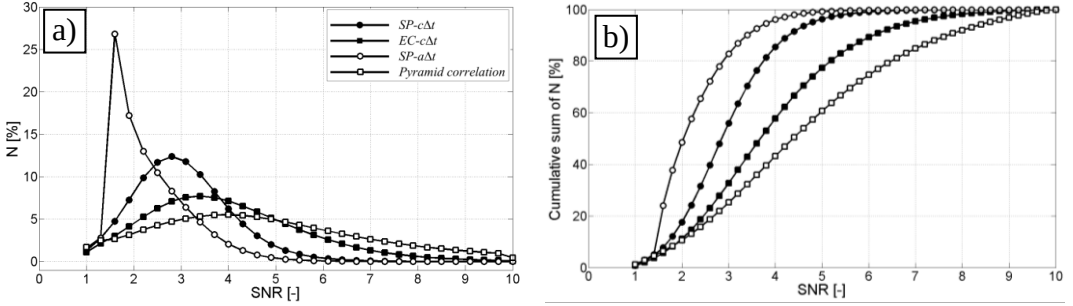


Figure 6.14. Probability distribution (a) and cumulative distribution (b) of SNR. N : number of correlation functions with a signal-to-noise ratio within a bin of width 0.2.

The results of figure 6.14-a show a Gaussian-shaped SNR distribution centred around 3 for the $SP-c\Delta t$. The use of the adaptive separation time ($SP-a\Delta t$) causes a dramatic drop of the signal strength, mainly due to the particle image out-of-plane motion. The large skewness of the SNR distribution for the $SP-a\Delta t$ method is due to the truncation imposed by the threshold on SNR set at 1.5: the time separation is increased until the signal-to-noise ratio drops below 1.5. When the ensemble correlation with constant time separation ($EC-c\Delta t$) is employed, the signal strength increases and the SNR distribution broadens with an average value of 4. The pyramid correlation returns a distribution similar to the latter but allows higher reduction of the relative error ascribed to the larger measured displacement.

From the cumulative distribution plot of figure 6.14-b, it emerges that the pyramid correlation technique allows a significant improvement in terms of measurement robustness, with less than 10% of the measurements having SNR below 2, as opposed to 50 % for $SP-a\Delta t$.

Figure 6.15 shows the instantaneous velocity field computed with WIDIM (a) and the pyramid correlation (b): most non-physical fluctuations ascribed to random errors are lessened with the advanced multi-frame analysis.

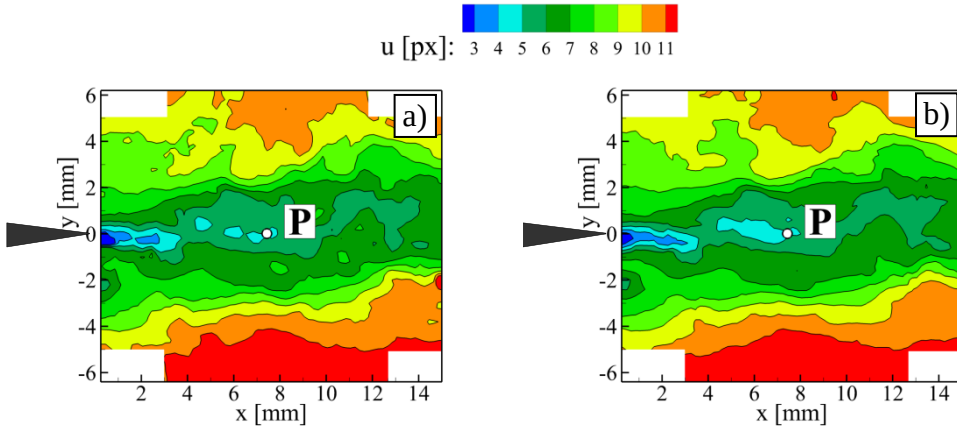


Figure 6.15. Instantaneous velocity field in the near wake of the NACA-0012 airfoil: *SP-cAt* (a) and *pyramid correlation* (b).

The velocity time-history is extracted from a point of the wake to evaluate the temporal coherence of the measured velocity field. In this case, the comparison is also made with a data post-processing technique based on a 5-point regression in time (second order polynomial) of the velocity fields obtained with *SP-cAt*.

The comparison among the techniques is shown in figure 6.16 for the vertical velocity component; similar results are obtained for the horizontal component and are not reported for brevity. The velocity time series computed with single-pair correlation (*SP-cAt* and *SP-aAt*) exhibit the larger amount of spurious fluctuations with amplitude of approximately 0.2 pixels. However, no major discrepancies between single-pair and ensemble correlation algorithms can be found from the velocity probability density functions (figure 6.16-c). This can be justified by the fact that, for an error level of approximately 0.2 pixels, the contribution of noise to the pdf is negligible with respect to the amplitude of the physical fluctuations (approximately 2 pixels). When considering the Eulerian acceleration (computed with a three-point central difference scheme applied to the velocity time series), the differences among the techniques are more pronounced (figure 6.16-b and -d). The width of the probability density function is considerably smaller (approximately one pixel); in this case, the contribution of the random errors (roughly 0.2 pixels) becomes significant and introduces a broadening of the pdf. The pyramid correlation algorithm exhibits a higher and narrower peak of the acceleration pdf curves, suggesting that spurious fluctuations due to random noise have been eliminated.

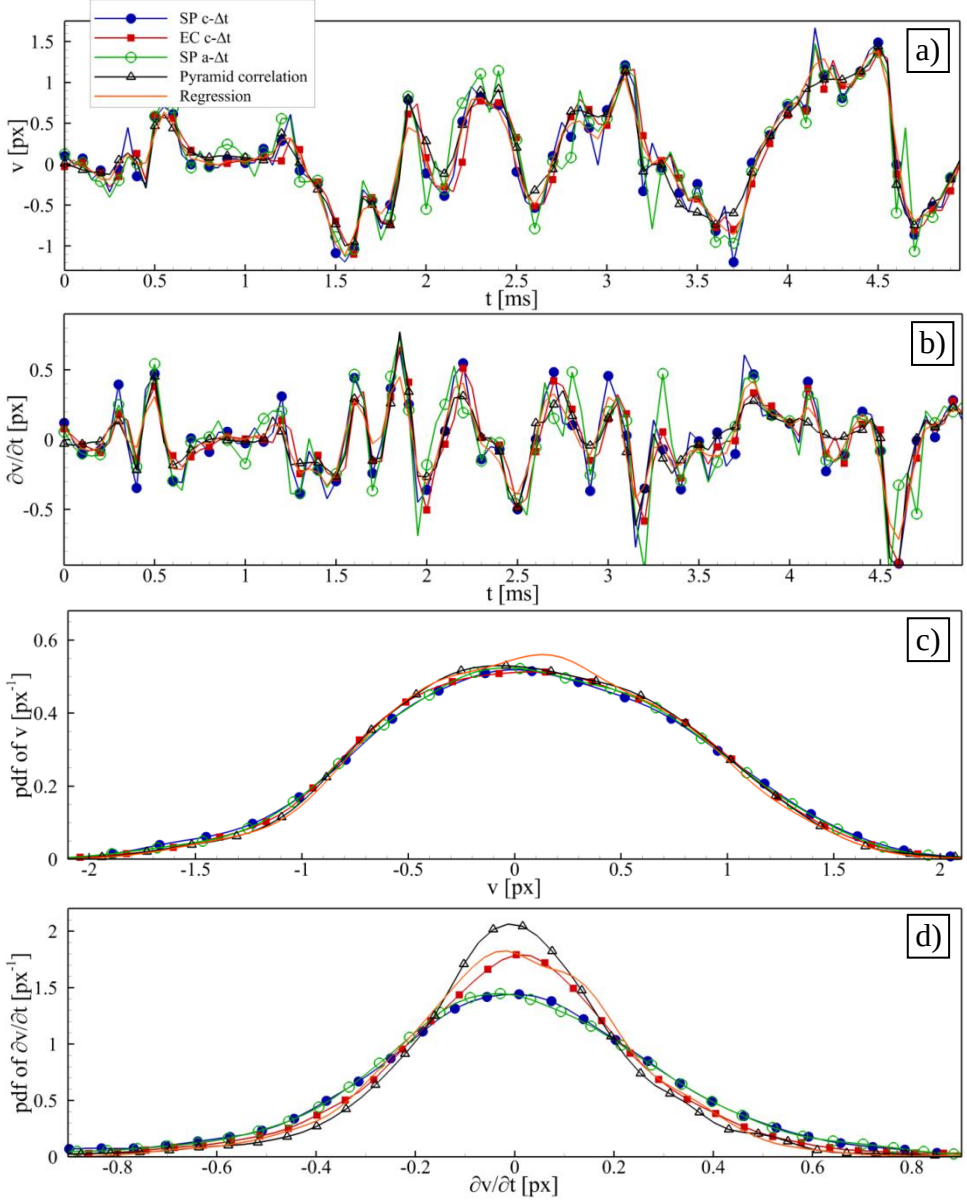


Figure 6.16. (a) Vertical velocity time history in P; (b) vertical Eulerian acceleration time history in P; (c) probability density function (pdf) of v ; (d) pdf of $\partial v / \partial t$.

6.4.2 Transitional jet in water

For the transitional jet test case, the experiment described in section 4.4.2.1 and in Violato and Scarano (2011) is considered. The system parameters are summarized in table 6.2.

Two regions of edge $2D$ both centred along the jet axis are selected as shown in figure 6.17. The first one extends from D to $3D$ from the nozzle exit and is characterized by a laminar flow, whereas in the second one, from $5D$ to $7D$, a turbulent flow takes place. The two regions having a 440×440 pixels resolution are interrogated with 17×17 pixels windows with 75% overlap, yielding a vector pitch of 0.25 vectors/pixel. The maximum observation time is set to $n_{\max} = 9$.

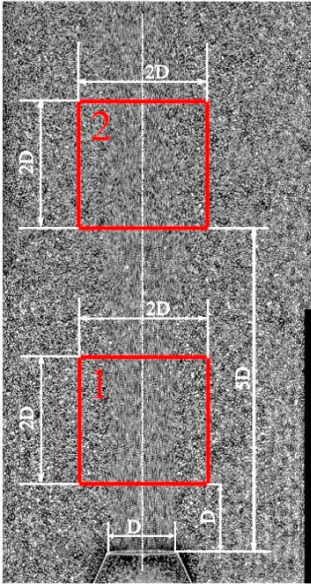


Figure 6.17. Measurement regions within the water jet (overlay of two images).

Table 6.2. System parameters of the water jet experiment.

Seeding	Polyamide particles, 10 μm diameter
Illumination	Quantronic <i>Darwin-Duo</i> Nd:YLF laser ($2 \times 25\text{mJ}$ @ 1 kHz)
Recording device	Photron <i>Fast CAM SA1</i> (1024×1024 pixels @5.4 kHz), 20 μm pitch
Imaging	<i>Nikon</i> objectives $f = 105\text{ mm}$, $f_{\#} = 2.8$
Field of view	$4.7D \times 9.2D$, $1,024 \times 1,932\text{ px}^2$
Acquisition frequency	1,200 Hz
Magnification factor	0.44
Number of images	1,000

Laminar region

The laminar region is considered first. As illustrated in figure 6.18, in the jet core and in the stagnant region the value of n_{opt} is limited by $n_{\max}$, because acceleration errors and in-plane gradients are negligible. The constraint on the continuum deformation is active in the shear layer, where the in-plane displacement gradient is severe (figure 6.18-c). The high quality of the recordings and the minor out-of-plane displacement make the *SNR* criterion ineffective, except for two thin regions at the top and bottom of the jet core affected by edge effects: in the rest of the domain, the height n_h of the pyramid

equals the base width n_{opt} , as it can be seen by the comparison of figure 6.18-b and -d.

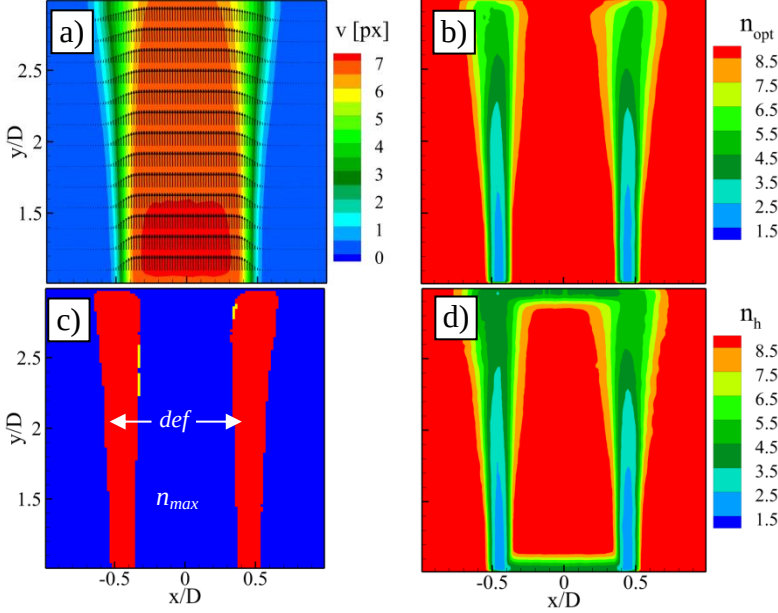


Figure 6.18. Water jet, laminar region. (a) Mean velocity field; (b) mean optimum temporal separation n_{opt} ; (c) colour-coded most frequently occurring criterion determining n_{opt} : deformation (red), BP (yellow), n_{max} (blue); (d) mean height n_h of the pyramid of correlations.

The contours of figure 6.19 represent the instantaneous axial velocity field in the laminar region computed with the four interrogation algorithms. The conventional single-pair correlation ($SP-c\Delta t$) is affected by the highest noise level that results in low spatial coherence of the flow structures both inside and outside the jet core. The random noise is significantly lessened both employing the sliding-average correlation ($EC-c\Delta t$) and increasing the temporal separation based on an adaptive criterion ($SP-a\Delta t$).

From the velocity fields of figure 6.19, the difference between $SP-a\Delta t$ and *pyramid correlation* is barely appreciable. A time series is extracted from a point P along the jet axis to further evaluate the effectiveness of the techniques in reducing the measurement errors. The axial velocity in the point exhibits physical oscillations of 0.5 pixels amplitude (figure 6.20-a); in addition to those, the $SP-c\Delta t$ technique measures spurious fluctuations of 0.1 pixels which can be ascribed to measurement errors. The spurious fluctuations are reduced to 0.05 pixels when using the sliding-average correlation ($EC-c\Delta t$) and to

below 0.01 pixels when a technique based on adaptive temporal separation ($SP-a\Delta t$ or pyramid correlation) is employed.

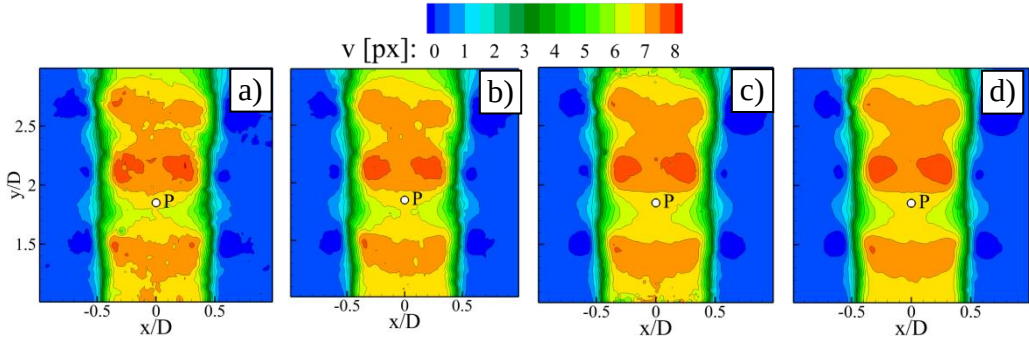


Figure 6.19. Instantaneous axial velocity field in region 1: (a) $SP-c\Delta t$; (b) $EC-c\Delta t$; (c) $SP-a\Delta t$; (d) pyramid correlation.

The auto-correlation function of the velocity (figure 6.20-b) is computed to assess the temporal coherence of the velocity time series. It is known that the autocorrelation of a sine wave is a sinusoidal function, whereas a random signal yields a Dirac-type autocorrelation, which is 1 at the origin and 0 otherwise (De Groot, 1989). By definition, the autocorrelation coefficient of a measured signal is always unit at the origin; one sample away from the origin, it drops below 1. This drop of correlation can be employed to quantify the noise level embedded in the signal: the larger the drop of correlation, the higher the noise level. The results of figure 6.20-b evidence that the adaptive techniques ($SP-a\Delta t$ and pyramid correlation) and the sliding-average correlation ($EC-c\Delta t$) yield a minor drop of correlation (within 2%), whereas the $SP-c\Delta t$ algorithm exhibits a reduction of 20% ascribed to random noise.

The Eulerian acceleration time history in P is shown in figure 6.20-c. The results evidence that the acceleration obtained with the $SP-c\Delta t$ is dominated by noise: physical and spurious fluctuations are of the same order of magnitude. The corresponding auto-correlation function (figure 6.20-d) is approximately a discrete Dirac function, which is the auto-correlation function of white noise. The drop of correlation still remains major (about 80%) when the velocity data are computed with $EC-c\Delta t$. The use of adaptive temporal separation has a beneficial effect in reducing the spurious fluctuations: the drop of correlation

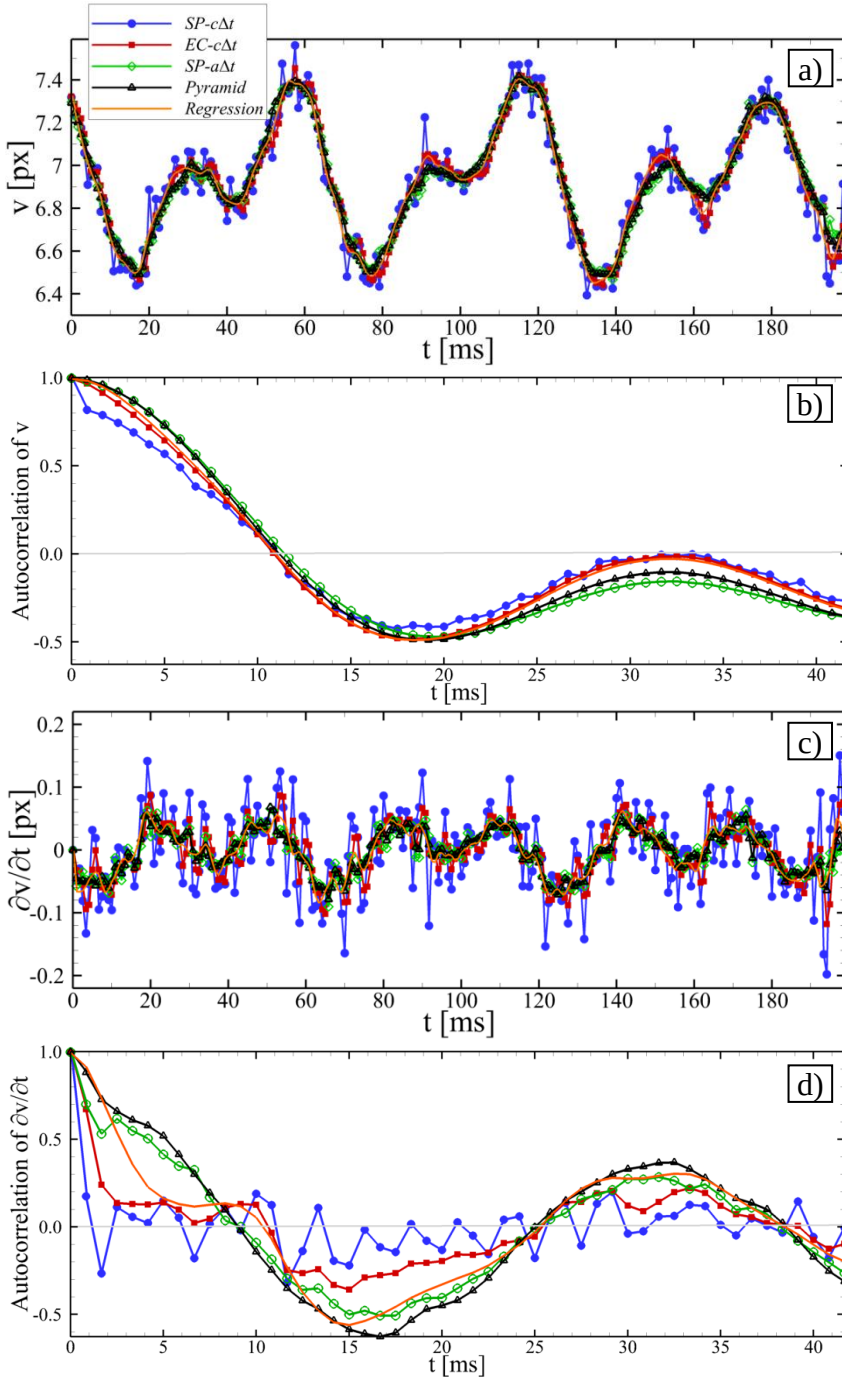


Figure 6.20. Time series in P. (a) Axial velocity; (b) auto-correlation function of the axial velocity; (c) axial acceleration; (d) auto-correlation function of the axial acceleration.

is reduced to 40% and 35% when using $SP-a\Delta t$ and pyramid correlation, respectively.

One of the chief outcomes of the pyramid correlation technique is the possibility of measuring velocity temporal derivatives with enhanced accuracy. The contours of figure 6.21 depict the standard deviation of the axial acceleration computed with $SP-c\Delta t$ and pyramid correlation. The single-pair correlation yields high fluctuations not only in the shear layer, but also in the jet core and in the stagnant region (standard deviation of $\partial v/\partial t$ of the order of 0.05 pixels), where they are supposed to be negligible due to the uniform or stagnant flow. These values are ascribed to random noise that causes spurious unphysical fluctuations. In contrast, the pyramid correlation strongly damps the spurious fluctuations, returning values below 0.02 pixels in those regions, and enhances the spatial coherence of the acceleration standard deviation.

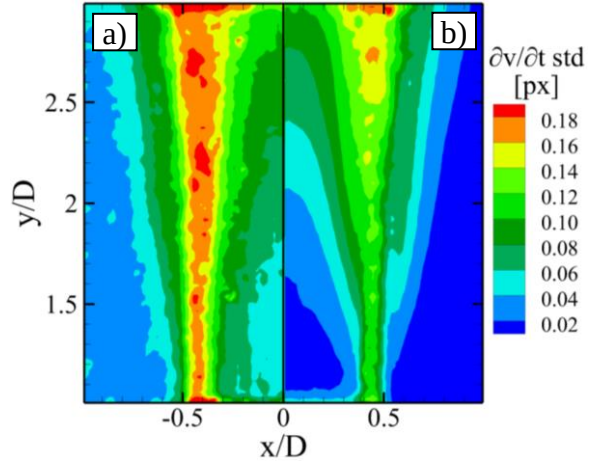


Figure 6.21. Standard deviation of the axial acceleration; (a) $SP-c\Delta t$; (b) pyramid correlation.

Turbulent region

In the upper region, the jet flow is in transition to the turbulent regime; the shear layer about $x/D = \pm 0.5$ is still visible, but it is wider than that shown in the laminar region. As in the previous case, n_{opt} rises up to n_{max} in the jet core and in the stagnant region, while it is limited by the deformation criterion along the shear layer (figures 6.22-b and -c). A difference with respect to the laminar case is that, due to the wider shear layer, the rate-of-change of the in-plane displacement is lower, therefore n_{opt} assumes higher values. Because of the more isotropic fluctuations occurring in the turbulent regime, the out-of-plane motion becomes severe and causes localized reduction of the signal strength. As a consequence, in most of the domain the pyramid height is limited by the constraint on the signal-to-noise ratio and n_h is smaller than n_{opt} , as depicted in figure 6.22-d.

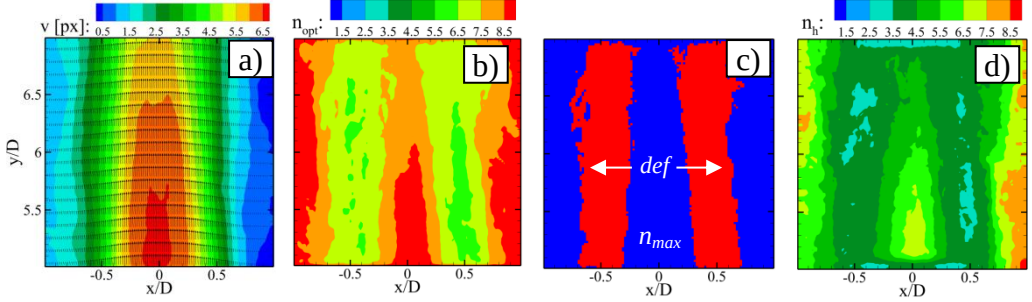


Figure 6.22. Water jet in the turbulent region. (a) Mean axial velocity (one vector every 6 is displayed in the axial direction); (b) mean optimum temporal separation n_{opt} ; (c) colour-coded most frequently occurring criterion determining n_{opt} : deformation (red), BP (yellow), n_{max} (blue); (d) mean height n_h of the pyramid of correlations.

It should be noticed that the average pyramid height n_h in the jet core is larger than in the shear layer. This is mainly due to higher out-of-plane displacement occurring in the shear layer, which is related to growing instabilities that contribute to radial and azimuthal vorticity, as discussed by Violato and Scarano (2011). Because of the higher out-of-plane displacement, in the shear layer the SNR criterion becomes effective at lower separation time, limiting the value of n_h .

The effect of the pyramid correlation algorithm on the temporal coherence of the measured signal is analysed via the standard deviation of the axial acceleration depicted in figure 6.23. As in the laminar region, the result obtained with $SP-c\Delta t$ (figure 6.23-a) exhibits large accelerations (of the order of 0.1 pixels or above) not only in the shear layer but also in the jet core and in the outer region: these are ascribed to noise in the measurement and are largely suppressed by the pyramid correlation (figure 6.23-b).

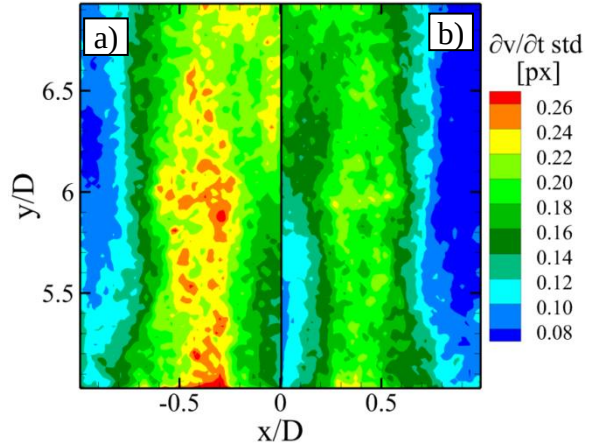


Figure 6.23. Standard deviation of the axial acceleration; (a) $SP-c\Delta t$; (b) pyramid correlation.

6.5 Discussion and conclusions

An adaptive multi-frame technique for time-resolved PIV has been proposed, based on the average of correlation functions computed at variable temporal separation.

The technique locally optimizes the observation time taking into account random and acceleration errors and continuum deformation. All the correlation functions obtained from a small sequence of recordings are employed for building a more robust ensemble function, from which the displacement vector is computed. Due to the assumption of constant velocity within the observation time, highly-resolved flows are required.

The technique is suited to enhance precision and robustness for time-resolved PIV experiments performed using high-speed systems (CMOS cameras and high repetition rate lasers), in particular when the imaging conditions are sub-optimal. It has been proved to increase the reliability of the measured vectors and to reduce the measurement errors with respect to the state-of-the-art of PIV processing techniques. Higher temporal coherence of the vector field is achieved without encountering significant modulation issues. Enhanced derivability of the signal in time is gained. Reduced peak-locking errors are obtained for recordings with small particle images (diameter equal to or below 1 pixel).

In extreme conditions of major out-of-plane motion (out-of-plane displacement exceeding 40% of the laser sheet thickness), strong shear (above 0.5 pixels/pixel) or large curvature of the trajectories, the pyramid correlation technique is not expected to yield significant benefits; in fact, in those conditions the technique would correspond to the *sliding-average correlation* approach ($n_h = 1$) or to the conventional single-pair correlation ($n_{opt} = 1$).

Table 6.3. Number of image deformations per iteration and number of correlations per iteration and grid point with the four PIV processing techniques (in the pyramid correlation, $n_h = n_{opt}$ is considered).

	<i>SP-cΔt</i>	<i>SP-aΔt</i>	<i>EC-cΔt</i>	<i>Pyramid correlation</i>
Number of image deformations	2	2	$2 \cdot n_{opt}$	$n_{opt}^2 + n_{opt}$
Number of correlations	1	1	n_{opt}	$(n_{opt}^2 + n_{opt})/2$

The pyramid correlation algorithm has higher computational cost than the other PIV processing techniques, mainly because of the larger amount of image deformations and correlations to be computed. Considering an optimum separation n_{opt} , the increase of computational cost is proportional to n_{opt}^2 , as

reported in table 6.3. In contrast, the increase in computational cost for determining the optimum temporal separation and the pyramid height applying the criteria of section 6.2.2 is negligible.

6.6 Outlook: Lagrangian techniques

As the multi-frame and the sliding-average correlation algorithms, the pyramid correlation relies upon an Eulerian flow description: each displacement vector is retrieved from information collected in the same location at different time instants. The fluid parcels are assumed to move along linear trajectories with null acceleration during the time interval containing a short sequence of PIV recordings. When the time interval is increased to enhance the measurement dynamic velocity range, velocity variations may become important and truncation errors arise, which can compromise the accuracy of the measurement.

To overcome this limitation and allow a further increase in the dynamic velocity range via a larger observation time, multi-frame interrogation algorithms that rely upon a Lagrangian flow description have been recently developed. In the *fluid trajectory correlation* (FTC, Lynch and Scarano, 2013), a dense collection of fluid elements is tracked along curvilinear trajectories through an image sequence. The technique abandons the assumption of linear, nonaccelerating particle motion between frames, instead opting to explicitly capture nonlinear trajectories. This potentially allows for reductions in truncation errors and random errors. Furthermore, the FTC algorithm permits a direct computation of the fluid parcel acceleration, which is retrieved from the temporal derivative of the velocity along the trajectory.

The technique has been further improved by Jeon *et al* (2014), who proposed a multi-frame approach that combines the main features of FTC and pyramid correlation, namely the *fluid trajectory evaluation based on ensemble averaged cross-correlation* (FTEE). In this technique, the most appropriate fluid trajectory is determined by means of ensemble average correlation in order to enhance the measurement robustness and accuracy, especially in presence of severe out-of-plane motion or sub-optimal imaging conditions.

CHAPTER 7

PIV LIGHT REFLECTIONS ELIMINATION VIA TEMPORAL HIGH-PASS FILTER^{*}

Abstract In this chapter, a novel approach for PIV image pre-processing is proposed to deal with the undesired effect of laser light reflections from solid walls in wind tunnel experiments. The method can be applied for both stationary interfaces as well as when the image of the interface is moving due to vibration of either the model or the imaging system. The working hypothesis is that the motion of the interface is resolved temporally, which is typically the case when employing high-speed PIV systems. The method is based on the decomposition of the pixel intensity in the frequency domain. The high frequency content of the signal is due to the transit of seeding particles, whereas undesired reflections will appear in the low frequency range. Applying a high pass filter on the light intensity time history retains only the contribution of the seeding particles and rejects the undesired light reflections.

Two experiments show the application of the method. In the low-speed flow regime around a pitching airfoil, the trace of the laser impinging on the moving surface can be mostly eliminated, enabling cross correlation analysis of the flow closer to the wall. In the transonic regime, experiments are performed in an industrial wind tunnel around the base region of the ARIANE V launcher model. Here the high-pass filter eliminates all secondary reflections and enhances the particles peak intensity relative to the reflections and the background light.

7.1 Introduction

Good image quality is crucial for accurate PIV measurements. Ideally, PIV recordings should be composed by bright particle images having much larger intensity than the background or the camera noise. However, when conducting measurements close to a solid body, it is not always possible to avoid that the laser light impinges on the surface and strong reflections arise with higher intensity than the surrounding scattering particles. As a result, when the image analysis is performed by cross-correlation, such features dominate the correlation map introducing a self-correlation stripe-like region of high intensity that typically precludes the detection of the displacement peak (Theunissen *et al*, 2009a). Because the light reflection dominates the pixel intensity with respect to particles peak, even when the correlation window minimally includes the reflection region, the cross-correlation signal degrades and an erroneous vector estimate (outlier) is produced.

^{*} This work has been published in Sciacchitano and Scarano (2014), *Measurements Science and Technology* 25 084009. The algorithm has been implemented in the commercial software Davis 8 of LaVision GmbH.

Despite the problem relevance, only few studies can be found that are specifically dedicated to this effect or that attempt at least minimizing light reflection from interfaces. Lindken and Merzkirch (2002) made use of fluorescent particles and shadowgraphy in an attempt to filter out these unwanted reflections. Although the approach is the most followed for multi-phase flows, it cannot be employed in wind tunnel experiments due to the strict health and safety regulations applying for fluorescent particles. Kähler (2009) investigated the influence of the model material and surface treatment on the reflections and found out that aluminum models with highly polished surface have very low diffuse reflectivity with respect to the other tested materials (steel, carbon fiber reinforced plastic, glass, PMMA).

Depardon *et al* (2005) on the other hand reduced the effect of optical disturbances by painting the complete test section and object with fluorescent paint. By placing the camera under the Brewster angle with the interface, Lin and Perlin (1998) were able to minimize the mirror-like behavior. Kähler *et al* (2006) used tangential model illumination to achieve perfect suppression of undesired wall reflections for the study of wall-shear-stress and near-wall turbulence by means of long-distance micro-PIV.

However, setup modifications such as surface treatments or changes in the laser or camera orientation are not always possible or become unpractical when dealing with curved interfaces. Consequently it is interesting to investigate the possibility to minimize the effects of undesired light reflections at an image pre-processing stage.

Chapter 3 has discussed the common approaches reported in literature to eliminate reflections and background from PIV recordings, for both cases of steady and moving interfaces. It has been shown that the situation is more critical for moving interfaces, where the background reflections are unsteady and cannot be estimated simply by stationary statistics such as the minimum of the time series or the time-averaged light intensity. Honkanen and Nobach (2005) faced the problem of moving interfaces with measurements in bubbly flows and performed a thorough analysis of several background elimination techniques for moving interfaces. They also proposed a simple but effective approach in their case, based on the subtraction of first and second exposure, which appears to be the most appropriate in case of moving interfaces. However, as also reported by the authors, issues of particles image cancellation occur both at high image source density and in regions of low velocity, where the distance travelled by the particle images is not exceeding the particle image diameter.

Furthermore, Theunissen *et al* (2009a, 2009b) thoroughly investigated possible strategies to improve the image interrogation in proximity of interfaces also considering adaptive resolution close to the wall, which demonstrated to be an effective approach. Nevertheless, it was found that a small relative uncertainty in the intensity of the reflection has profound effect on the final result. Moreover, it is common in PIV experiments that the laser pulse energy fluctuates from shot to shot, making the determination of the appropriate factor for the background attenuation not straightforward.

Clearly, the treatment of the undesired laser light reflections requires a robust approach that takes into account the spatio-temporal fluctuations of the light intensity. The objective is to enhance the ratio between particle images peak intensity and the background light reflections without incurring into cancellation of particle images.

The method proposed here is mostly applicable when using high-speed PIV systems, whereby the interface motion is temporally resolved (along with the particles motion). The frequency range of mechanical systems vibrations encountered in aerodynamic laboratories typically ranges between 10 and 100 Hz. Considering that high-speed systems operate in the kilohertz range, these vibrations are usually well captured in time and can therefore be filtered out, provided that their frequency range is well separated from that of the particles motion.

The present chapter illustrates the working principle of the method and the application to two real experiments of increasing difficulty in terms of ratio between peak intensity of particle images and light reflections.

7.2 Working principle

When measuring near solid interfaces, the image properties strongly vary along the wall-normal direction, with reflections that are orders of magnitude more intense than the individual particle images and often saturate the range of the electronic imaging system. The typical signature of the interface is a stripe of saturated pixels (e.g. figure 7.1-a). Under these circumstances, the image interrogation by cross-correlation often fails to return a valid peak that corresponds to the particles motion because of the presence of a self-correlation stripe in the correlation function (figure 7.1-b).

Time-resolved PIV measurements provide the intensity time-history at each pixel location, which can be used to attenuate the undesired light reflections; such quantity can be decomposed in the frequency domain to separate the contribution due to particles transit from the spurious reflections.

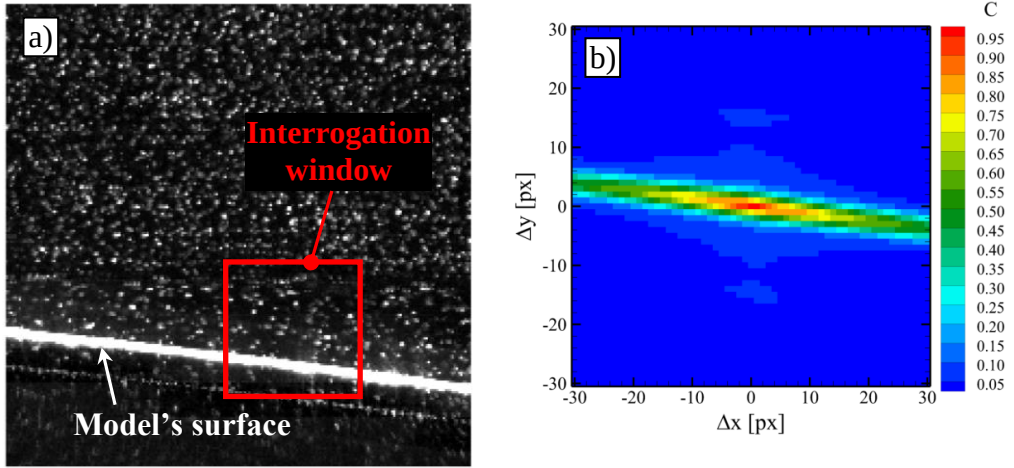


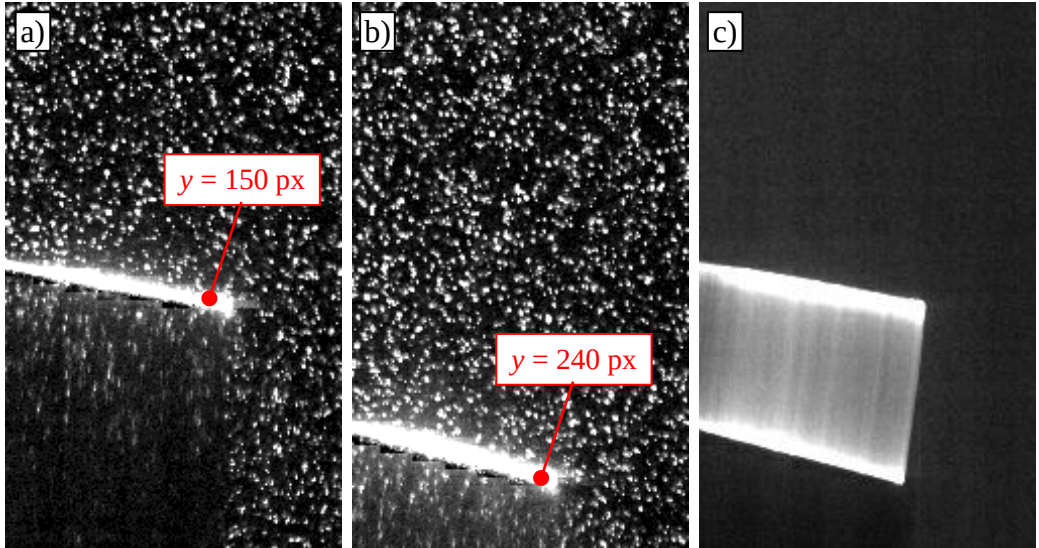
Figure 7.1. Raw image pair (a) and cross-correlation function in an interrogation window (b).

The working principle of the approach is presented by an example. Consider the case of a pitching airfoil, where strong laser light reflections take place on the model surface and the interface image motion spans approximately 100 pixels (figure 7.2-a and -b). The measurement parameters are summarized in table 7.1; the test case will be analysed more in detail in section 7.4.1. Due to the interface motion, no stationary background can be built from the image sequence, as illustrated in the time-averaged image of figure 7.2-c. The application of existing pre-processing techniques such as minimum subtraction and normalization with respect to the time-average intensity (figure 7.2-d and -e) or the subtraction of the neighbourhood minimum from each instantaneous image with a careful choice of the kernel size (slightly larger than the particle image size) only yields a marginal reduction of the surface reflection. Also the difference between the two exposures (Honkanen and Nobach, 2005) would lead to high percentage of particle images cancellation in regions of high source density or small particle displacement. Moreover, the application of a simple intensity threshold filter that sets to zero all the intensity values above a chosen threshold (in the example 500 counts) would allow removing only the strong reflection on the surface, but not the reflections at the surface neighbourhood. Furthermore, the intensity threshold filter may reduce to zero the peak intensity of several particles, as illustrated in figure 7.2-f.

PIV light reflections elimination via temporal high-pass filter

Table 7.1. Measurement parameters of the pitching airfoil test case.

Acquisition frequency f_{acq}	2,700 Hz
Time interval between image pairs: $\Delta T = 1/f_{\text{acq}}$	370 μs
Pulse separation Δt	100 μs
Magnification factor M	0.30
Digital imaging resolution	15.2 px/mm
Particle image diameter d_τ	2 px, 40 μm
Airfoil chord c	80 mm
Airfoil center of rotation	$c/4$
Airfoil pitching frequency f_{airfoil}	4.5 Hz
Airfoil pitching range $\Delta\alpha$	4 degrees
Free-stream velocity V_∞	5.7 m/s
Free-stream particle image displacement between image pairs: V_{part}	37 px
Characteristic particle time scale: $\tau_{\text{part}} = d_\tau / M V_\infty$	0.02 ms
Mean trailing-edge velocity V_{TE}	38 mm/s
Mean trailing-edge image displacement between image pairs: V_{refl}	0.2 px
Reflection image thickness d_{refl}	5 px, 100 μm
Characteristic reflection time scale: $\tau_{\text{refl}} = d_{\text{refl}} / M V_{\text{TE}}$	8.8 ms



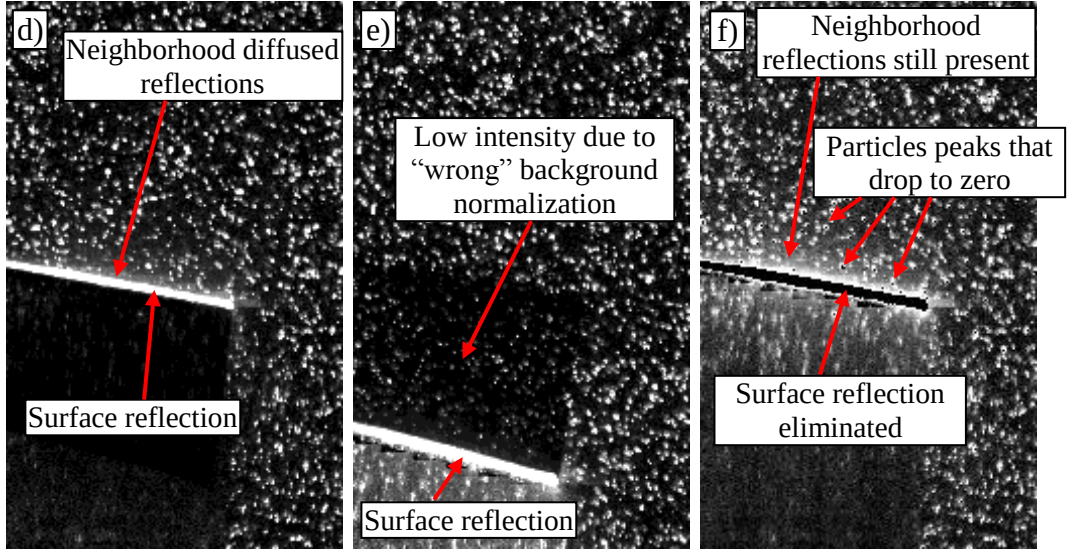


Figure 7.2. (a) Raw image at time $t_1 = 0$: the trailing edge is at $y = 150$ px; (b) raw image at time $t_2 = 137$ ms: the trailing edge is at $y = 240$ px; (c) time-average image of the entire sequence; (d) and (e) images at time instants $t_1 = 0$ and $t_2 = 137$ ms, respectively, after pre-processing by minimum subtraction and normalization with respect to the time average; (f) raw image at time $t_1 = 0$ after the application of an intensity threshold filter.

In the present example, the seeding particles move at a speed that scales with the free-stream velocity $V_\infty = 5.7$ m/s. The time scale τ_{part} of the light intensity fluctuations due to the transit of a particle across an individual pixel can be evaluated as:

$$\tau_{\text{part}} = \frac{d_\tau}{MV_\infty} = 0.02 \text{ ms} \quad (7.1)$$

being d_τ the mean particle image diameter and M the optical magnification factor. Instead, the airfoil's trailing edge moves at much lower speed (average velocity $V_{\text{TE}} = 0.038$ m/s) and therefore the time scale τ_{refl} of the light intensity fluctuations owing to the transit of the reflection across a pixel is several orders of magnitude larger:

$$\tau_{\text{refl}} = \frac{d_{\text{refl}}}{MV_{\text{TE}}} = 8.8 \text{ ms} \quad (7.2)$$

wherein d_{refl} is the thickness of the reflection image.

When considering the time-history of the recorded intensity at an individual pixel (figure 7.3), the contributions of particle images and reflections can be easily distinguished. The time-history exhibits short low-intensity peaks corresponding to the presence of a particle image and longer high-intensity peaks caused by the reflection region crossing the pixel. Since the particle time scale τ_{part} is much smaller than the time interval ΔT between image pairs, the particle images do not occupy the same pixel during two subsequent recordings and only appear as a pulse in the time history. Instead, the light reflection region moves at a relative slow velocity and has a characteristic time scale τ_{refl} much larger than ΔT . As a result, typically 10 to 12 samples are observed between rise and fall of intensity due to the reflection.

In common PIV experiments, the two main causes of interface image motion are vibration of the cameras (typically occurring at frequency in the order of 10 to 100 Hz) and controlled (e.g. oscillating airfoil) or uncontrolled (e.g. flexible wings, aero-elastic interactions) model motion. In both cases, the transit time τ_{refl} for the reflection across a pixel is estimated in the order of several milliseconds.

Furthermore, the maximum light intensity reflected by the interface is usually more than one order of magnitude higher than that scattered by the particles in air flows and often exceeds the saturation value (4095 counts for 12 bits cameras) of the CMOS imager. As a result, the values of the high intensity region may also be clipped to the maximum, introducing a top-hat like profile in the time series of some pixels.

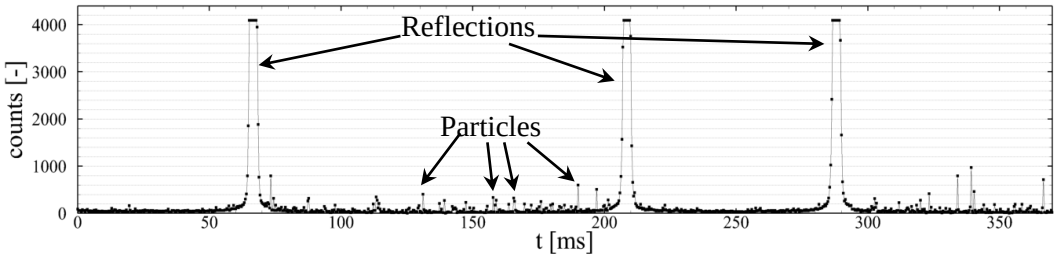


Figure 7.3. Time-history of the intensity recorded in a pixel location.

From the above discussion it emerges that, for a broad range of problems, the time-history of the pixel intensity recorded by a high-speed PIV system can be regarded as composed by two separate contributions having different time scales: a first contribution associated with the transit of seeding particles (characteristic time scale typically smaller than the image-pairs separation), and a second contribution ascribed to undesired light reflections (characteristic

time scale larger than the image-pairs separation). The two contributions can be separated when the recorded intensity signal is analysed in the frequency domain: the particle images' transit produces a high-frequency contribution usually not sampled within the Nyquist criterion, whereas the reflection's transit yields a low-frequency contribution (up to a few hundreds Hertz) generally captured by the high-speed imaging system. The working principle of the proposed pre-processing method is based on the decomposition of the signal in the frequency domain and the removal of the low-frequency content representative of the unwanted reflections.

7.3 High-pass filter characteristics

The elimination of the low-frequency component of the signal is performed by a *high-pass filter* (HPF) conducted in the frequency domain. The use of a third order Butterworth filter is proposed here due to its maximally flat frequency response in the pass-band and the computational speed (Butterworth, 1930). The Butterworth filter is fully specified by two parameters, namely the filter order N and the cut-off frequency f_c . A higher filter order allows faster roll-off between pass-band and stop-band (i.e. shorter transition band), but causes wider impulse response that yields negative and positive artefacts on adjacent images and can lead to the filter instability (Smith, 1997); for this reason, values of N below 6 are suggested. The cut-off frequency is selected to eliminate the intensity component due to steady or slow-moving reflections and retain the light scattered by the seeding particles. Therefore, f_c is chosen as the minimum frequency above the frequency content ascribed to the light reflection crossing the pixels.

The Butterworth filter is a recursive filter, hence it yields a phase distortion of the signal due to its non-linear phase response. However, the phase distortion can be eliminated by applying the filter in both forward and reverse directions, which produces a zero-phase recursive filter with the only penalty of a factor two in execution time (Smith, 1997).

The HPF is numerically implemented in MATLAB (a synthetic pseudo-code is given in the appendix) with a 3rd order Butterworth algorithm and a cut-off frequency set at 0.3 of the Nyquist frequency $f_{Nyq} = f_{acq}/2$. The latter choice typically suffices the scope of reflection elimination while avoiding images peak intensity attenuation. The use of the MATLAB function *filtfilt* allows performing the zero-phase digital filtering so that no phase distortion occurs. The frequency response of such filter is illustrated in figure 7.4: fluctuations occurring at frequency exceeding 0.5 f_{Nyq} are attenuated by less than 2%, while those occurring at frequency below 0.15 f_{Nyq} are reduced by at least two

orders of magnitude. The impulse response of figure 7.5 shows that the high-pass filter causes a drop by 30% of the signal peak and that negative intensity artefacts exceeding 5% of the particle peak value are produced only on the two preceding and two successive recordings. As expected, no phase distortion is encountered.

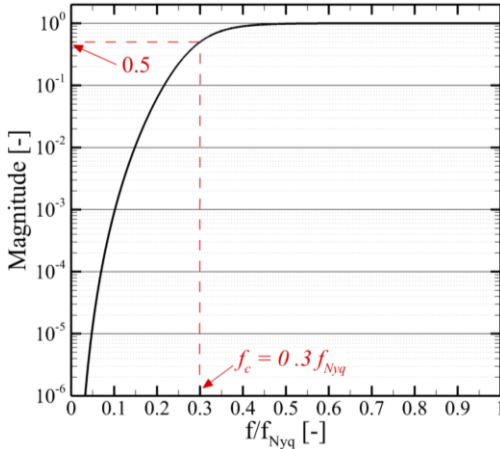


Figure 7.4. Frequency response of the 3rd order high-pass Butterworth filter with cut-off frequency $f_c = 0.3 f_{Nyq}$.

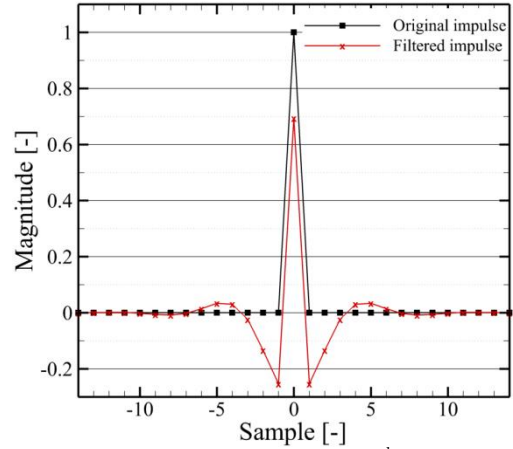


Figure 7.5. Impulse response of the 3rd order high-pass Butterworth filter with cut-off frequency $f_c = 0.3 f_{Nyq}$.

When the HPF is applied to the raw intensity time history, a major reduction of the reflections intensity is achieved while retaining the intensity peaks due to the seeding particles (figure 7.6). Two cases can be distinguished. When the pixel intensity due to reflections slowly grows from 0 to the maximum value (typically in more than 5 time steps), the reflection is completely eliminated by the filter (figure 7.6-a); as a result, no bias error associated with the reflection is expected when performing measurements close to the interface. Instead, when the pixel intensity grows quickly from 0 to the maximum reflection value (typically in less than 5 time steps), the reflection is strongly attenuated by the filter but not removed completely (figure 7.6-b). Hence, bias errors in the measured velocity field may occur when the intensity of the particle images is low with respect to the attenuated reflections.

The signal decomposition in the frequency domain is analysed by Fourier transformation of the time-history. Figure 7.7 illustrates that the raw signal contains significant energy in the very low frequency ($0-0.1 f_{Nyq}$) corresponding to the varying speed of the interface motion. When the airfoil has maximum pitching rate, the time scale of the interface travelling across a

pixel is shorter and produces a small peak about $0.25 f_{Nyq}$. At higher frequencies up to the Nyquist frequency, the signal appears as white noise (constant power spectral density) due to the random occurrence of particles onto the selected pixel. The high-pass filter at the selected cut-off frequency of $0.3 f_{Nyq}$ essentially eliminates the contribution at low frequency dominated by the reflection while retaining that at high frequency.

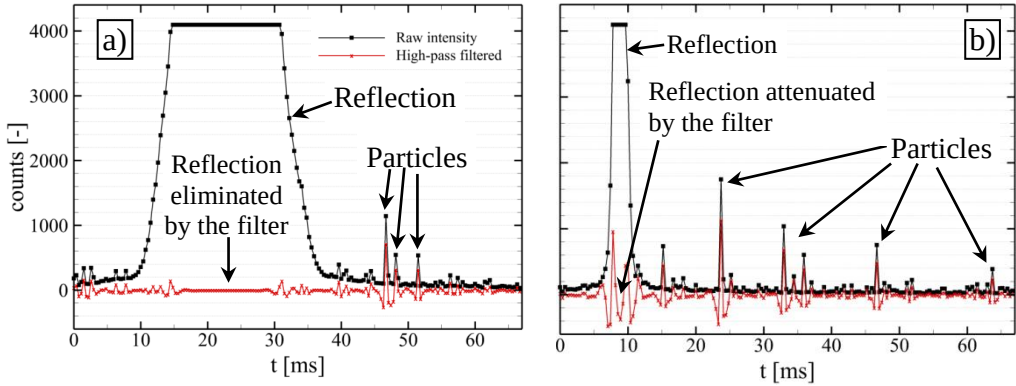


Figure 7.6. Time-history of the raw and filtered intensity recorded in a pixel location. (a) Slow motion of the interface across the pixel; (b) fast motion of the interface across the pixel.

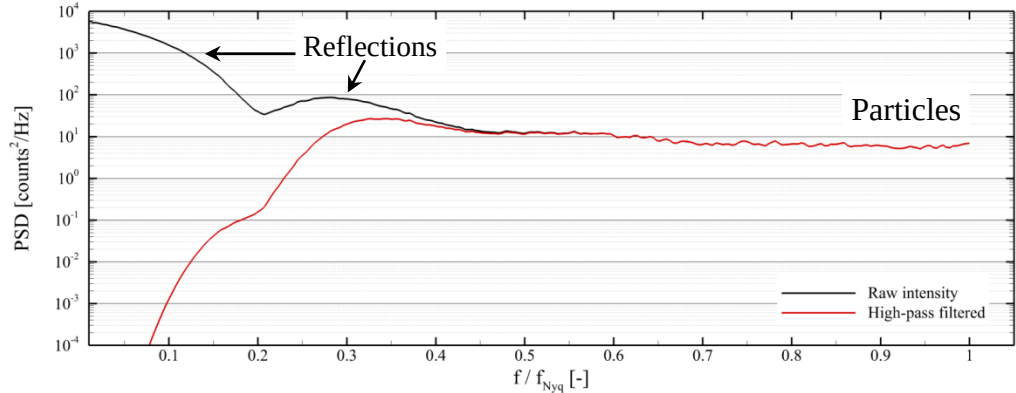


Figure 7.7. Power spectral density (PSD) of the raw and filtered intensity time histories for the case of fast motion of the interface across the pixel.

7.4 Applications

Two experiments are presented that demonstrate the performance of the method. The first is a pitching airfoil for the study of an unsteady laminar separation bubble (Nati, 2011). High-quality imaging conditions are obtained in this case, with bright particle images and no camera vibration. The laser

light reflection from the surface is also very bright and confined to a small region of the image.

The second example represents more critical experimental conditions, with the after-body of a rocket launcher (ARIANE V) model operating in the transonic regime for the study of the buffeting phenomenon (Schrijer *et al*, 2011). A number of degrading effects are present: direct light reflection from the model surface, motion of the image due to camera vibrations, out-of-focus reflections in the background. Moreover, the peak intensity of particle images is below 100 counts. As a result the camera noise is significant, with the extreme case that a few pixels appear to be barely active.

Table 7.2 reports the measurement parameters and the characteristic frequency and time scales of the two experiments. Note that for both the experiments the characteristic particle frequency is at least two orders of magnitude larger than the reflection frequency, therefore the high-pass filter approach is suited to separate the two contributions.

Table 7.2. Measurement parameters and characteristic time and frequency scales of the two experiments.

	Oscillating airfoil experiment	ARIANE V experiment
Free-stream velocity V_∞ [m/s]	5.7	180
Acquisition frequency f_{acq} [Hz]	2,700	2,700
Time interval between image pairs ΔT [μ s]	370	370
Pulse separation Δt [μ s]	100	5
Magnification factor M [-]	0.30	0.19
Digital imaging resolution [px/mm]	15.2	9.3
Particle image diameter d_τ [px, μ m]	2.0, 40	1.6, 32
Characteristic particle time scale τ_{part} [ms]	0.02	0.94×10^{-3}
Characteristic reflection time scale τ_{refl} [ms]	8.8	33
Characteristic particle frequency scale: $f_{\text{part}} = 1/\tau_{\text{part}}$ [Hz]	50,000	1.1×10^6
Characteristic reflection frequency scale: $f_{\text{refl}} = 1/\tau_{\text{refl}}$ [Hz]	114	30

7.4.1 Oscillating airfoil

The flow over the suction side of a SD7003 airfoil with a chord length of 80 mm is measured by time-resolved PIV at a rate of 2,700 Hz. The free-stream velocity is 5.7 m/s and the airfoil oscillates with a frequency of 4.5 Hz around an axis placed at $\frac{1}{4}$ chord between an angle of 4 and 8 degrees. The airfoil is placed in the test section of the M-tunnel of the Delft

Aerodynamics Laboratory, which has a $40 \times 40 \text{ cm}^2$ cross section. The Reynolds number based on the airfoil chord is 30,000 and the flow regime exhibits laminar separation and reattachment at the airfoil trailing edge.

The flow is seeded with $1 \text{ }\mu\text{m}$ diameter water-glycol droplets produced by a SAFEX smoke generator. The measurement domain is a sheet 100 mm wide and 2 mm thick illuminated with a Quantronix *Darwin-Duo* Nd:YLF diode-pumped laser, that delivers pulses of 12 mJ at a repetition rate of 2,700 Hz. The laser pulse separation is 100 μs , whereas the interval between pairs is 370 μs . The field of view of $108 \times 54 \text{ mm}^2$ is covered using two LaVision *HighSpeedStar* 6 CMOS cameras (1024×1024 pixels, 20 μm pixel pitch, acquisition frequency of 5,400 frames/s at full resolution) equipped with Nikon-Nikkor objectives of 105 mm focal length set at $f_{\#} = 2.8$. The illumination and imaging configuration is shown in figure 7.8. The resulting combined image has 1640×1007 pixels and a digital imaging resolution of 15.2 pixels/mm. The motion of the airfoil surface at the trailing edge has a maximum velocity of 42 mm/s. In the image space, the trace of the surface moves of approximately 0.3 pixels between subsequent image pairs, which indicates that the interface motion is well captured by the recording system. The particle image displacement in the free-stream is approximately 10 pixels between exposures and 37 pixels between subsequent pairs. Even for particles moving of only 1 pixel between exposures, the distance travelled between subsequent pairs is approximately 4 pixels, that is one order of magnitude larger than the interface motion. As a result, the time scales of the interface and the particles image motion are separated by approximately two orders of magnitude (table 7.2).

The raw image pair shown in figure 7.9-top exhibits strong laser light reflections on the airfoil surface that cannot be eliminated via existing pre-processing techniques. The result of the application of the HPF to the entire image is shown in figure 7.9-bottom, where the reflection is eliminated.

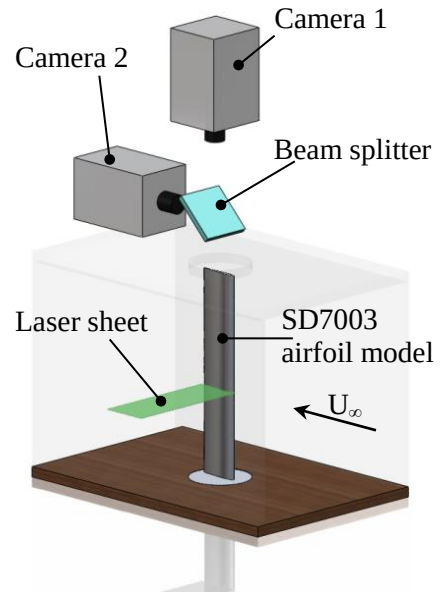


Figure 7.8. Illumination and imaging configuration of the pitching airfoil experiment.

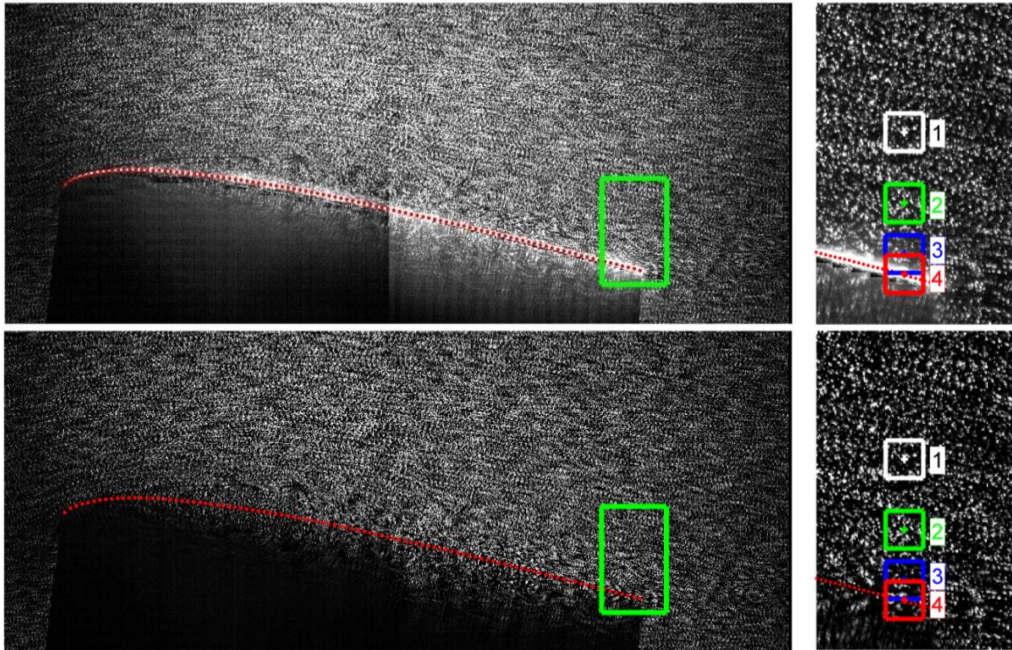


Figure 7.9. Raw image pair (top) and after pre-processing by HPF (bottom). Details of surface reflection on the right. The airfoil surface is highlighted by a line of red dots.

The impact of the pre-processing to the spatial cross-correlation is illustrated in figure 7.10 by selecting several interrogation windows along a vertical profile (the selected windows are shown in the right of figure 7.9). The correlation signal is not affected by the reflection of the interface outside of the boundary layer (regions 1 and 2). As a consequence, the correlation map shows a distinct peak, with a higher signal-to-noise ratio (*SNR*) in the outer stream than inside the turbulent region (region 2). When the interrogation window contains even few pixels of the reflection (case 3) or it is entirely crossed by it (case 4) the map essentially yields the interface autocorrelation featuring a stripe with the same orientation. As a result, the peak corresponding to the average particle displacement cannot be recovered. The elimination of the interface intensity distribution by HPF enables a particles displacement peak to be detected even when the interrogation window is centred on the interface, as illustrated in table 7.3 and in the right of figure 7.10. It should be retained in mind that in this case other sources of error also affect the measurement as discussed by Theunissen *et al* (2009a) and Kähler *et al* (2012).

PIV light reflections elimination via temporal high-pass filter

Table 7.3. Correlation signal-to-noise ratio (SNR) and measured displacement in the four interrogation windows.

Region	Raw image			High-pass filtered image		
	SNR	Δx [px]	Δy [px]	SNR	Δx [px]	Δy [px]
1	2.7	10.0	1.2	2.7	10.0	2.0
2	2.2	7.6	4.0	2.4	7.7	4.0
3	< 1	0.0	0.0	1.4	1.3	0.1
4	< 1	0.0	0.0	2.0	1.3	0.1

An instantaneous velocity field is computed with both the raw and the high-pass filtered image-pair and displayed in figure 7.11 to show the effect of HPF pre-processing. When the raw image-pair is employed (figure 7.11-a), the measurement near the wall is locked to null displacement due to laser light reflections, precluding the accurate evaluation of separation point location (region A), recirculation region (region B) and attached flow close to the trailing edge (region C). The reflections on the solid walls are essentially eliminated in the high-pass filtered image-pair, allowing a more accurate measurement close to the model's surface (figure 7.11-b).

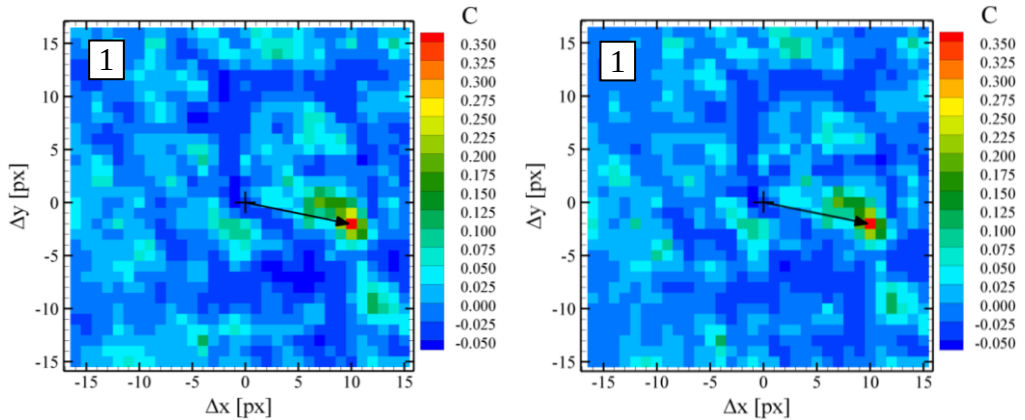
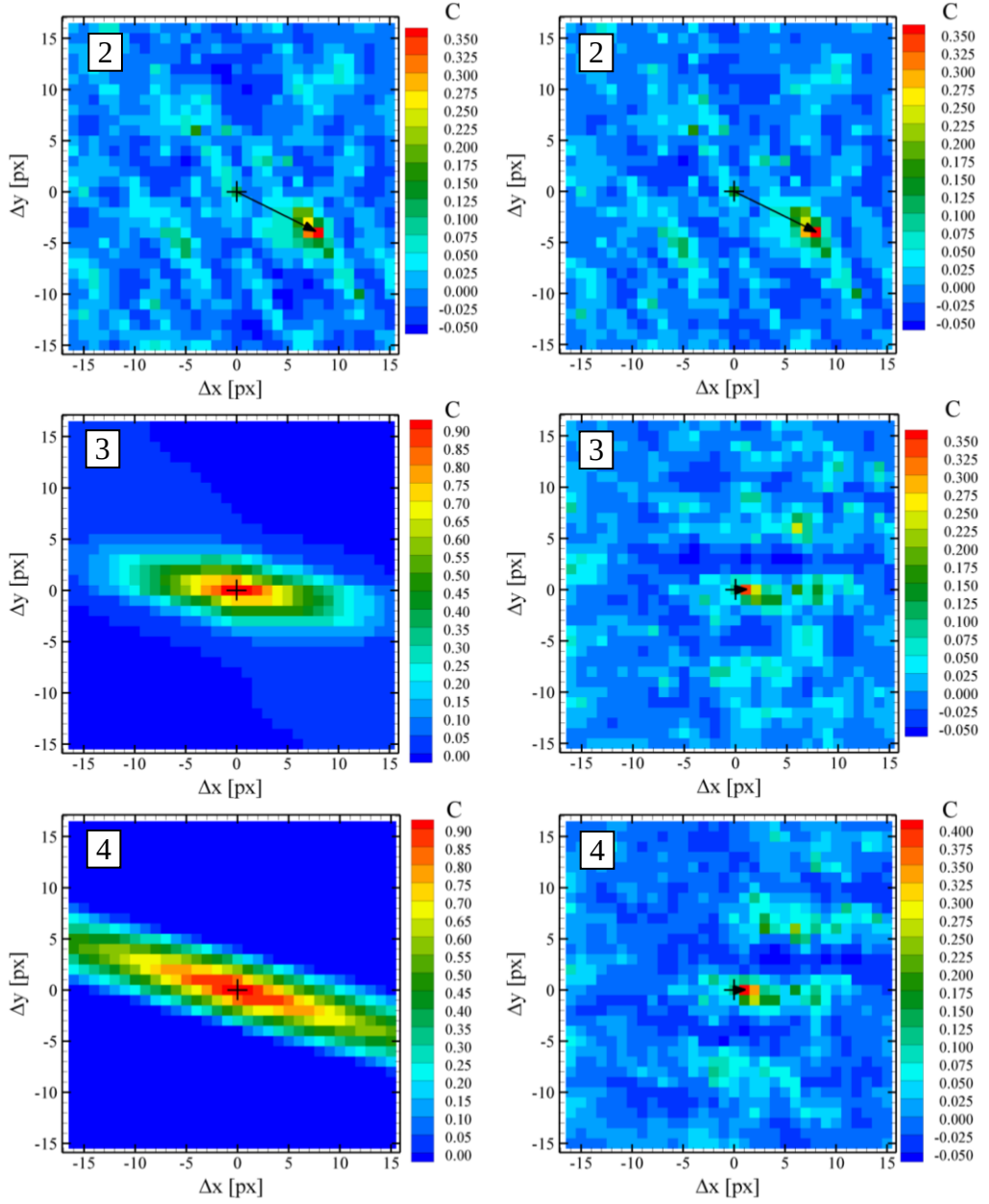


Figure 7.10. Maps of cross-correlation coefficient from windows evaluated at different distance from the airfoil boundary: (1) outside the boundary layer; (2) inside the boundary layer; (3) overlapping with reflection region by 4 pixels; (4) centred on the reflection. Results from raw image pair (left) and pre-processed by HPF (right).

PIV light reflections elimination via temporal high-pass filter



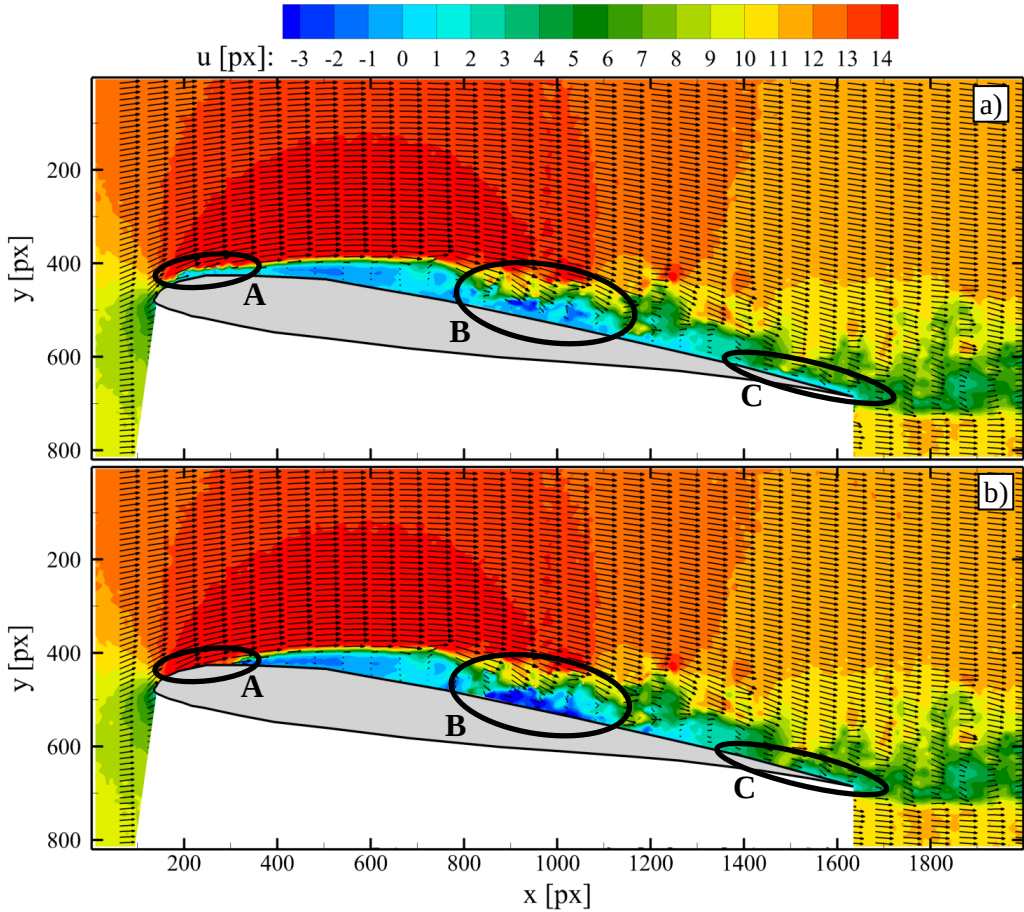


Figure 7.11. Instantaneous velocity field measured from raw images (a) and from high-pass filtered images (b). For sake of clarity, the vectors are displayed one every 15 in x -direction and one every 3 in y -direction.

7.4.2 Transonic flow over the ARIANE V after-body

The experiments are conducted in the industrial transonic wind tunnel DNW-HST on a 1:60 scaled model of the ARIANE V launcher to study the phenomenon of buffeting encountered in the transonic regime (Schrijer *et al*, 2011). The model is installed in the wind tunnel test section at zero incidence and is supported by a z -sting to reduce acoustic interference. Illumination is provided by a Litron LDY303HE Nd:YLF laser delivering approximately 10 mJ/pulse at 2.7 kHz. The laser is installed above of the test section and the light sheet is reflected towards the model symmetry plane. The free-stream

Mach number is set to 0.5. The experimental setup is illustrated in figures 7.12 and 7.13.

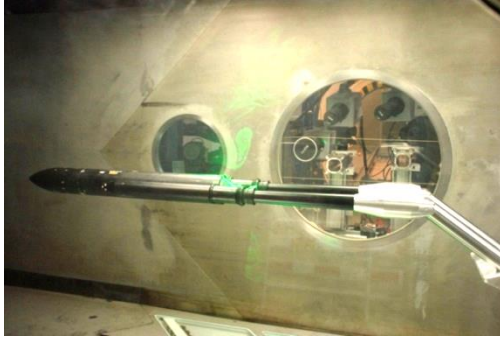


Figure 7.12. ARIANE V model installed in DNW-HST with laser illumination and imaging systems.

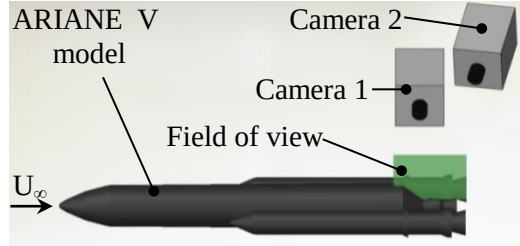


Figure 7.13. Schematic views of illumination and imaging systems layout during experiments.

The laser sheet is approximately 150 mm wide and 2 mm thick. Liquid DEHS tracer particles of approximately $1\text{ }\mu\text{m}$ median diameter are dispersed in the settling chamber. The light scattered by the particle images is recorded by two LaVision HighSpeedStar 6 CMOS cameras (1024×1024 pixels, acquisition frequency of 5,400 frames/s at full resolution). Camera 1 is placed exactly at 90 degrees with respect to the measurement plane and is equipped with a Nikon-Nikkor objective of 200 mm focal length set at $f_{\#} = 2$; the field of view of this camera is 92×92 mm. The second camera is equipped with a Nikon-Nikkor objective of 180 mm focal length set at $f_{\#} = 2.8$ and images a field of view of 110×110 mm. Camera 2 is placed at a slight angle due to the large size of the cameras imaging adjacent regions. This position causes stronger reflections by direct imaging of the laser light against the model and a secondary (out-of-focus) reflection due to the oblique path through the wind tunnel window. The discussion in the present section is based on the recordings acquired by camera 2.

Due to the large imaging distance and the limited laser power, the light scattered by the particle tracers has low intensity (typically 50 to 100 counts). The direct reflections from the model surface exceed the saturation level in the sensor, whereas the indirect reflections do not saturate the range of the sensor, but have higher intensity than the particle images. An additional difficulty is introduced by the slight vibration of the imaging system that causes a motion of the field of view in the order of 0.5 mm (5 pixels). The vibration frequency

is approximately 30 Hz from the measurement of the reflection motion in the PIV recordings.

The raw images are shown in figure 7.14-left, together with the result after a pre-processing by means of subtraction of the time-minimum intensity and the HPF pre-processing. The direct reflection of the model surface dominates the light intensity distribution. The indirect reflections also exhibit pixel intensity larger than the particle peaks, although saturation is not reached. This is also due to the fact that indirect reflections are out of focus because their optical path is made longer by internal reflections in the glass window of the wind tunnel. As a result of these conditions, the particle images can be clearly distinguished only in the region far from the model. The application of the cross-correlation analysis to the raw measurements fails almost in the entire domain due to the background-dominated signal (figure 7.14-a right).

A significant improvement is already obtained when the background intensity at each pixel is reduced by subtraction of the temporal-minimum value (figure 7.14-b). However, the background is not fully compensated due to the motion of the reflection and the cross-correlation signal is still dominated by the reflection auto-correlation leading to spurious vectors in the regions where secondary reflections are present. This effect is particularly pronounced on the right edge of the field of view, where the seeding particles are imaged in out-of-focus condition, with consequently reduced strength of cross-correlation signal.

The application of the image pre-processing based on HPF mostly eliminates the direct reflections and only minor traces of the interface are left. The secondary reflections appear to be entirely removed. An additional effect of this filtering technique is that the particle intensity distribution is normalized over the entire measurement domain, which may be beneficial in case of sharp variations of illumination. As a final result, the cross-correlation analysis of the images pre-processed by HPF appears unaffected by reflections even in the regions where the interrogation windows partly overlap with the model's surface (figure 7.14-c). From this experiment it can be concluded that the features of HPF can be crucial to the successful cross-correlation analysis in presence of model or imaging system vibrations.

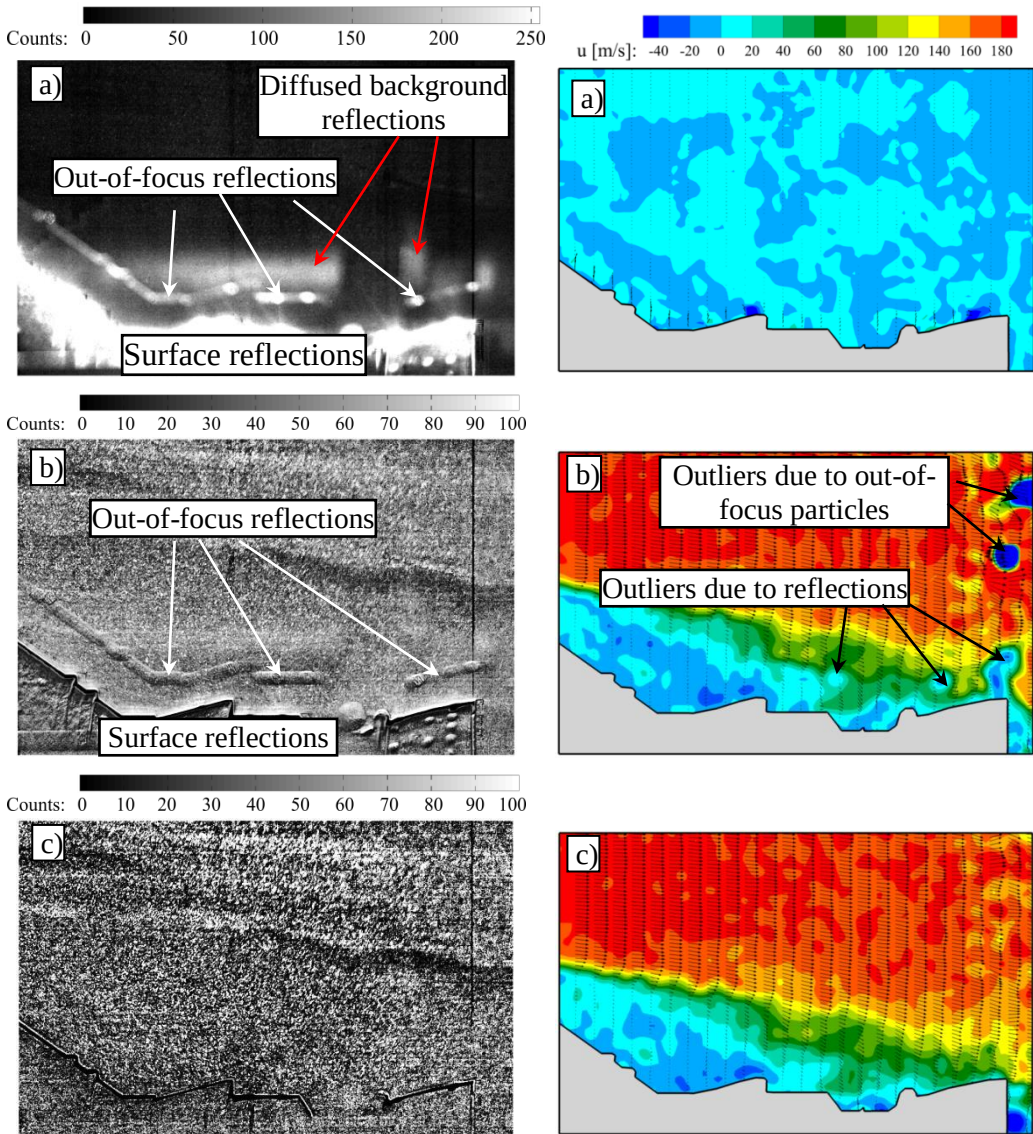


Figure 7.14. Left: Single exposure PIV recording of the ARIANE V after-body. (a) Raw image; (b) minimum intensity subtraction; (c) HPF with cut-off at 30% of Nyquist frequency. Right: instantaneous velocity vector field and contours of horizontal velocity component. (a) Raw image; (b) minimum intensity subtraction; (c) HPF with cut-off at 30% of Nyquist frequency. For clarity, one vector every five is displayed in the horizontal direction.

7.5 Conclusions

A novel image pre-processing approach is introduced for time-resolved PIV that deals with the undesired effect of light reflections. The working principle relies upon an efficient decomposition in the frequency domain of the intensity time-history recorded at individual pixel locations. For both the cases of steady and unsteady reflections, the application of a high-pass filter to the intensity time-history allows eliminating the undesired reflections while retaining the contribution of the seeding particles. The working hypothesis is that the time scale of the reflection crossing an individual pixel is well separated from the transit time of a particle on a pixel. It is indicated as a requirement that these two characteristic times should be separated by at least a factor three.

The application of the HPF to real experiments shows that, under well-controlled measurement conditions, the approach can mostly eliminate the trace of the reflection, making it possible to measure the velocity vectors in proximity of the solid surface even when the cross-correlation window overlaps with the surface.

Under more difficult experiments, for instance conducted in industrial facilities where the particle signal is low with respect to reflections and background, the HPF approach yields a significant reduction of all intensity spurious components (steady or slowly moving) with a consequent relative enhancement of the particles signal.

CHAPTER 8

NAVIER-STOKES SIMULATIONS IN GAPPY PIV DATA^{*}

Abstract Velocity measurements conducted with PIV often exhibit regions where the flow motion cannot be evaluated. The principal reasons for this are the absence of seeding particles or limited optical access for illumination or imaging. Additional causes can be laser light reflections and unwanted out-of-focus effects. As a consequence, the velocity field measured with PIV contains regions where no velocity information is available, namely gaps.

This chapter investigates the suitability of using the unsteady incompressible Navier-Stokes equations to obtain accurate estimates of the local transient velocity field in small gaps. The method relies upon a finite volume discretization with partitioned time stepping to solve the governing equations in the incompressible regime. The measured velocity distribution at the gap boundary is taken as time varying boundary condition (BC), and an approximate initial condition (IC) inside the gap is obtained via low-order spatial interpolation of the velocity at the boundaries. The influence of this IC is seen to diminish over time, as information is convected through the gap. Due to the form of the equations, no initial or boundary conditions on the pressure are required. The approach is evaluated by a time-resolved experiment where the true solution is known a-priori. The results are compared with a boundary interpolation approach. Finally, an application of the technique to an experiment with a gap of complex shape is presented.

8.1 Introduction

PIV measurements are often affected by gaps, i.e. regions where no information regarding the velocity field is obtained. The gaps occur in areas where the particle image displacement cannot be evaluated. There is a wide variety of reasons for this, including:

- 1) absence of seeding particles due to inhomogeneous tracer dispersion or centrifugal forces acting in vortices and wakes of high-speed flows (Schrijer and Walpot, 2010; Bitter *et al*, 2010). In this case the gaps occur irregularly in space and time;
- 2) laser light reflections from the surface of objects leading to corrupted tracer particle images;
- 3) inaccessible regions for the imaging system;
- 4) shadows due to the presence of objects in the light path.

^{*} This work has been published in Sciacchitano *et al* (2012) Experiments in Fluids, 53:1421-1435 DOI 10.1007/s00348-012-1366-5.

Of this last problem, a very frequently occurring case is the presence of regions with reduced or no particle tracer signal when the flow around a solid model is investigated: the object blocks the laser light, producing a shadow where the seeding particles are not illuminated and therefore not visible. To eliminate the shadow region, a more complicated setup must be employed, where the laser light illuminates the model from at least two different angles, as proposed by Ragni *et al* (2011b) and illustrated in figure 8.1. Even when the model is manufactured from a transparent material (Kurtulus *et al*, 2007), shadow regions due to total internal reflection or refraction can still be present. Moreover, when the out-of-plane dimension of the model is much larger (typically factor 2) than the in-plane dimensions, perspective effects produce an optical blockage unless the imaging system uses a telecentric lens (figure 8.2).

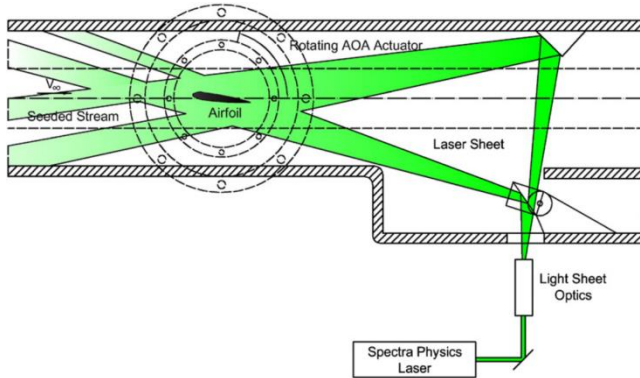


Figure 8.1. Experimental setup used by Ragni *et al* (2011b) with illumination from two different angles to avoid shadows around the model.

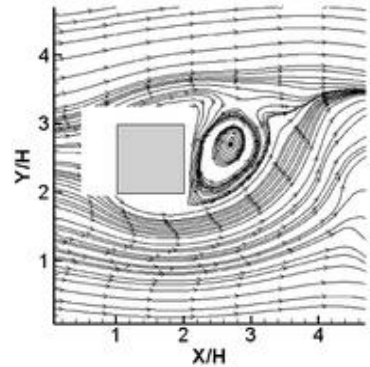


Figure 8.2. Instantaneous velocity streamlines measured by Kurtulus *et al* (2007) around a square cylinder. No velocity is measured in a region close to the model ($x/H \approx 1$, $2 \leq y/H \leq 3$) due to the optical blockage caused by perspective effects.

As shown in chapter 7, measurements performed close to the surface of a solid object usually involve strong laser light reflections which may lead to erroneous velocity evaluations. Other elements contributing to poor particle image signal are camera noise or out-of-focus imaging: in these regions the velocity can still be evaluated, but its reliability is compromised by the low

signal-to-noise ratio. Consequently, the measured velocity field may exhibit regions of highly uncertain vectors or even where no vector is measured.

The presence of gaps in PIV data is an important issue to be solved for several applications that require the computation of integral quantities. Two examples are the measurement of unsteady aerodynamic loads and aeroacoustic investigations. In the former, the force equations are integrated over a control volume enclosing the object (Kurtulus *et al*, 2007). Recent studies (Violato and Scarano, 2011) propose the use of PIV to evaluate the production of acoustic pressure fluctuations in turbulent flows, which can be predicted by a volume integration of a kinematic property of the flow over the entire source region (e.g. the Lamb-vector for Powell's analogy, Powell, 1964). In both circumstances, gappy data are not suited to integration and prevent these techniques from being applied.

As discussed in chapter 3, the treatment of gaps in PIV data has received little attention and has been generally considered as a by-product of the vector validation problem, (detection and replacement of false vectors, see Westerweel, 1994, for instance). Existing data refill procedures essentially rely upon the spatial interpolation of neighbouring vectors and can only reconstruct a monotonic velocity field or at most a field with a single peak inside the gap (a half wave).

The main idea behind the present study is to overcome the limitations of the existing methods by using the full governing equations of the flow to predict the velocity distribution inside the gap. The method proposed herein relies upon the numerical solution of the Navier-Stokes equations. The technique makes use of a staggered-grid finite volume method to solve the two-dimensional unsteady incompressible Navier-Stokes equations inside the gap, with time-varying Dirichlet boundary conditions imposed using the velocity provided by the PIV data. The partial differential equation problem is well-posed with these boundary conditions; in particular, conditions on the pressure are not required. Finally, initial conditions on the velocity in the gap are unknown, but have a diminishing effect on the solution over time provided that information is advected out of the gap. Therefore it is sufficient to specify any initial condition consistent with the boundary conditions at time zero, and integrate sufficiently far forward in time.

The combined use of experimental measurements and numerical simulations in aerodynamics belongs to the domain of data assimilation. Application of

data assimilation to PIV data is not completely new but very recent, and until now attention has been focused on using PIV data as a conditioner to enhance the accuracy of numerical solutions (see the works of Ma *et al*, 2002, and Suzuki *et al*, 2009a and 2009b, among others). In contrast, the current study does not propose a superposition of experimental and computational data to improve the latter, but rather aims at using the solution of the Navier-Stokes equations to resolve regions where the experimental data is missing or corrupted.

Standard approaches to problems of time-dependent gap-filling exist in meteorology and oceanography; two common approaches are deterministic variational methods such as 4DVAR (Vidard *et al*, 2000), and stochastic filtering methods such as the ensemble Kalman smoother (Evensen, 2003). These methods are able to deduce initial conditions, handle large amounts of noise in data, and account for unknown or uncertain boundary conditions and modelling parameters. They are also very computationally expensive, requiring a large number of forward simulations (Navier-Stokes in this case) to obtain a solution. The direct method we propose here costs only a single forward simulation. This is made possible by the observations that (a) all the data necessary for the boundary conditions are measured, (b) the data is relatively free from noise, and (c) reconstruction of initial conditions is not required.

8.2 Working principle

The case of two-dimensional incompressible flow will be considered. The present technique employs a two-dimensional approximation of the flow field, assuming that the velocity component w normal to the measurement plane is negligible with respect to the in-plane components (u, v) . To verify the validity of this hypothesis, the magnitude of w , if not measured with a stereoscopic system, can be estimated through the correlation peak height or the signal-to-noise ratio. When the out-of-plane velocity is of the same order of the in-plane components, the hypothesis of two-dimensional flow does not hold and the application of the two-dimensional assumption (c.f. (8.1) and (8.2)) would lead to inaccurate results.

In figure 8.3, a concrete example of experiment where the PIV data is not available in the entire measurement domain is shown: the velocity is measured in the domain $\mathcal{D}$ at sampling frequency f_{acq} , whereas the domain Ω is

composed of the gaps Ω_1 and Ω_2 ($\Omega = \Omega_1 \cup \Omega_2$) where no velocity information is present.

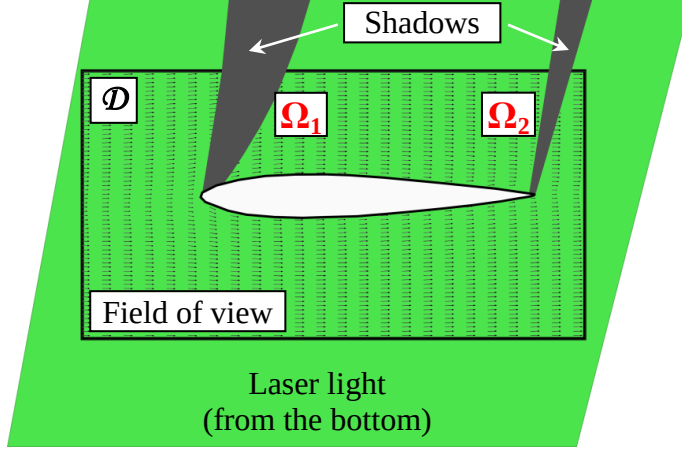


Figure 8.3. Schematic representation of measurement domain ($\mathcal{D}$) with two gaps (Ω_1 and Ω_2).

The velocity field in Ω is computed by solving the unsteady Navier-Stokes equations, which in non-dimensional form read:

$$\text{Conservation of mass:} \quad \nabla \cdot \mathbf{V} = 0 \quad (8.1)$$

$$\text{Conservation of momentum:} \quad \frac{\partial \mathbf{V}}{\partial t} + \nabla p = \mathbf{R}, \quad (8.2)$$

The following units are chosen for writing equations (8.1) and (8.2) in non-dimensional form:

- velocity: V_{ref}
- length: L
- density: ρ_{ref}
- pressure: $\rho_{\text{ref}} V_{\text{ref}}^2$

The vector $\mathbf{R} = [Ru \ Rv]^T$ contains the contributions of advection and diffusion and is given by:

$$\mathbf{R} = \nabla \cdot \left[-\mathbf{V} \otimes \mathbf{V} + \frac{1}{Re} (\nabla \mathbf{V} + \nabla \mathbf{V}^T) \right] \quad (8.3)$$

with $Re = \frac{\rho_{\text{ref}} V_{\text{ref}} L}{\mu}$ the nominal Reynolds number, and μ the coefficient of dynamic viscosity.

To solve the Navier-Stokes equations in Ω , the finite volume method is employed using a Cartesian staggered grid (Welch *et al*, 1966) as illustrated in figure 8.4.

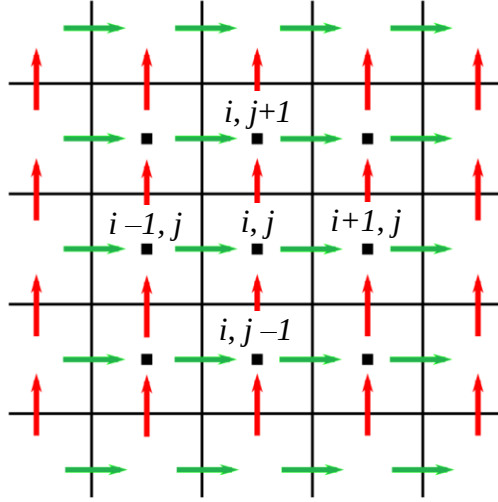


Figure 8.4. Staggered grid: the horizontal velocity component is defined at the midpoints of the vertical sides of the cells (green arrows); the vertical velocity component is defined at the midpoints of the horizontal sides of the cells (red arrows); the pressure is defined at the centre of the cells (black squares).

Following the approach by Veldman (1990), the continuity and momentum equations are combined to reformulate the problem as a Poisson equation for pressure, which in the discrete form reads:

$$D_h^\Omega G_h p^{n+1} = D_h^\Omega \left(\frac{1}{\delta t} \mathbf{V}_h^n + \mathbf{R}_h^n \right) + \frac{1}{\delta t} D_h^{\partial\Omega} \mathbf{V}_h^{n+1} \quad (8.4)$$

where h represents the mesh spacing, D_h^Ω and $D_h^{\partial\Omega}$ are the discrete divergence operators at the points of the volume mesh and boundary mesh respectively, G_h is the discrete gradient operator, δt is the temporal step and n indexes the time level. Finally $\mathbf{V}_h$, p and $\mathbf{R}_h$ are discrete quantities corresponding to $\mathbf{V}$, p (continuous pressure) and $\mathbf{R}$. Employing a central discretization for both convective and diffusive terms, equation (8.4) written out in full for the horizontal component reads:

$$\begin{aligned}
 \frac{p_{i+1,j}^{n+1} - 2p_{i,j}^{n+1} + p_{i-1,j}^{n+1}}{h_x^2} + \frac{p_{i,j+1}^{n+1} - 2p_{i,j}^{n+1} + p_{i,j-1}^{n+1}}{h_y^2} &= \frac{1}{\delta t} \left(\frac{u_{i+1/2,j}^n - u_{i-1/2,j}^n}{h_x} + \frac{v_{i,j+1/2}^n - v_{i,j-1/2}^n}{h_y} \right) + \\
 + \frac{Ru_{i+1/2,j}^n - Ru_{i-1/2,j}^n}{h_x} + \frac{Rv_{i,j+1/2}^n - Rv_{i,j-1/2}^n}{h_y}
 \end{aligned} \tag{8.5}$$

with i and j representing the position of a generic cell inside the domain and u and v the horizontal and vertical velocity components respectively.

Equation (8.5) is an elliptic system of $N_x \times N_y$ linear equations (being N_x and N_y the number of cells in x - and y -direction, respectively) for an equal number of unknowns p , therefore its solution is straightforward. Once the discrete pressure is computed, the velocity in each grid point is calculated through a forward Euler integration in time of equation (8.2):

$$\mathbf{V}_h^{n+1} = \mathbf{V}_h^n + \delta t \mathbf{R}_h^n - \delta t G_h p^{n+1} \tag{8.6}$$

which is explicitly written as:

$$\begin{aligned}
 u_{i+1/2,j}^{n+1} &= u_{i+1/2,j}^n + \delta t Ru_{i+1/2,j}^n - \delta t \frac{p_{i+1,j}^{n+1} - p_{i,j}^{n+1}}{h_x} \\
 v_{i,j+1/2}^{n+1} &= v_{i,j+1/2}^n + \delta t Rv_{i,j+1/2}^n - \delta t \frac{p_{i,j+1}^{n+1} - p_{i,j}^{n+1}}{h_y}
 \end{aligned} \tag{8.7}$$

The pressure values computed from equation (8.5) guarantee that the velocity components obtained by solving (8.6) satisfy the incompressibility constraint (8.1).

8.2.1 Treatment of boundary and initial conditions

Equations (8.4) and (8.6) can be solved imposing boundary conditions only on the velocity and not on the pressure (Ferziger and Peric, 2002). The boundary conditions on $\mathbf{V}$ are provided at discrete time instants by the PIV measurements and are imposed as Dirichlet conditions. In order for the numerical scheme to be stable, the spacing h of the computational mesh and

the time step δt are selected according to the conditions proposed by Ferziger and Peric (2002) for the linear convection-diffusion equation. To fulfil those conditions, δt often needs to be smaller than the time interval Δt between PIV recordings. Therefore, the velocity boundary conditions at intermediate time instants are computed through interpolation in time of the PIV velocity data.

The time interpolation is based on the advection model proposed by Scarano and Moore (2011), summarized as follows: assume that the velocity in a boundary point $\mathbf{X}_2$ has to be computed at time t_2 from known PIV velocity fields defined at t_1 and $t_3 = t_1 + \Delta t$ (with $t_1 < t_2 < t_3$). Under the hypothesis of *frozen turbulence*, the positions at times t_1 and t_3 of the fluid parcel located in $\mathbf{X}_2$ at t_2 are estimated through a Taylor expansion truncated at the first order:

$$\begin{aligned}\mathbf{X}_1 &= \mathbf{X}_2 - \mathbf{V}_{\text{conv}}(\mathbf{X}_2, t_1) \cdot \frac{t_2 - t_1}{\Delta t} + O(\Delta t^2) \\ \mathbf{X}_3 &= \mathbf{X}_2 + \mathbf{V}_{\text{conv}}(\mathbf{X}_2, t_3) \cdot \frac{t_3 - t_2}{\Delta t} + O(\Delta t^2)\end{aligned}\tag{8.8}$$

where $\mathbf{V}_{\text{conv}}$ indexes the convective velocity. If $\mathbf{X}_2$ is an inflow boundary point (as in figure 8.5), $\mathbf{X}_1$ is located in the measurement domain, while $\mathbf{X}_3$ falls inside the gap, where no experimental velocity is measured. Vice versa, if $\mathbf{X}_2$ is an outflow boundary point, $\mathbf{X}_1$ is inside the gap and $\mathbf{X}_3$ outside. Therefore, if the numerical simulation is conducted only in the gap (*direct numerical boundary case*: the numerical domain coincides with the gap region) at each boundary point the velocity at time t_2 is computed exploiting only the information either at time t_1 or at time t_3 , since only the experimental information available outside the gap can be used:

$$\begin{aligned}\text{inflow points:} \quad & \mathbf{V}(\mathbf{X}_2, t_2) = \mathbf{V}(\mathbf{X}_1, t_1) \\ \text{outflow points:} \quad & \mathbf{V}(\mathbf{X}_2, t_2) = \mathbf{V}(\mathbf{X}_3, t_3)\end{aligned}\tag{8.9}$$

Although this approach provides a simple estimate of the boundary velocity at intermediate time instants, it leads to boundary conditions that are discontinuous in time. In fact, considering an inflow boundary point, $\mathbf{V}(\mathbf{X}_2, t_2)$ is equal to $\mathbf{V}(\mathbf{X}_1, t_1)$ for time instants preceding t_3 , while at time $t_2 = t_3$ it is equal to $\mathbf{V}(\mathbf{X}_2, t_3)$, which is measured by PIV.

To avoid this discontinuity, it is proposed to take the numerical domain slightly larger than the actual gap Ω , by introducing the buffer region $\mathcal{B}$

(buffered numerical boundary case), as illustrated in figure 8.6. This way, when both $\mathbf{X}_1$ and $\mathbf{X}_3$ fall outside the gap where experimental data is present, the velocity at the boundary can be computed making use of the information at both times t_1 and t_3 :

$$\mathbf{V}(\mathbf{X}_2, t_2) = \frac{t_3 - t_2}{\Delta t} \mathbf{V}(\mathbf{X}_1, t_1) + \frac{t_2 - t_1}{\Delta t} \mathbf{V}(\mathbf{X}_3, t_3) \quad (8.10)$$

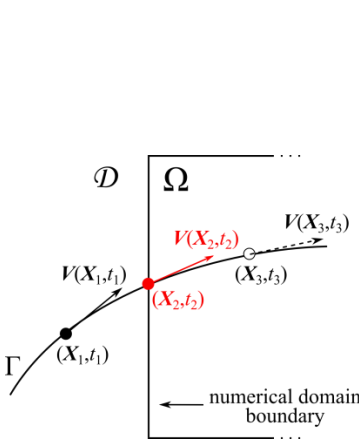


Figure 8.5. Motion of a fluid parcel along a trajectory Γ through an inflow boundary; direct numerical boundary case. $\mathcal{D}$: measurement domain; Ω : gap region.

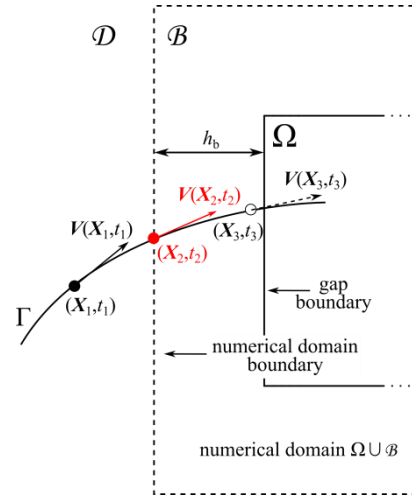


Figure 8.6. Motion of a fluid parcel along a trajectory Γ through an inflow boundary; buffered numerical boundary case. $\mathcal{D}$: measurement domain; Ω : gap region; $\mathcal{B}$: buffer region.

To guarantee that both $\mathbf{X}_1$ and $\mathbf{X}_3$ are outside Ω , the thickness of the buffer region has to be $h_b \geq |\mathbf{V} \cdot \hat{n}| \Delta t$, being $\mathbf{V} \cdot \hat{n}$ the projection of the velocity at the boundary along the normal to the boundary.

It is common that the boundary conditions based on the PIV measurements (or their temporal interpolation) do not satisfy the two-dimensional conservation of mass (8.1). The two main reasons are the presence of random noise in the measurement and the non-negligible derivative $\partial w / \partial z$ of the out-of-plane velocity component. Employing the present algorithm with such velocity boundary conditions leads to the divergence of the pressure. To avoid this, the velocity distribution along the boundary needs to be adjusted in such a

way that a zero mass flux is obtained through the numerical domain. This is accomplished by computing the surplus mass flux at the boundary, and subtracting it from the imposed mass flux. Typical values of the surplus mass flux range from 0.1% to 10% of the total mass flux; at each boundary point, the normal velocity V_n is corrected of a coefficient depending on surplus mass flux and local mass flux; the correction coefficient is typically of the order of the measurement error (0.1 px).

In general, the velocity boundary conditions are inexact because they contain random noise; as a consequence, an unphysical boundary layer or wiggles may generate at the outflow boundaries. However, for Reynolds numbers $Re \gg 1$ the erroneous information at the outflow boundaries has only minor influence on the numerical solution, because it propagates upstream only over a distance $O(Re^{-1})$ (Wesseling, 2001).

In contrast to the boundary conditions, the initial condition in the simulated domain $\mathbf{V}_h^0$ is typically completely unknown. It will be shown numerically that after a time interval τ that scales with the ratio between the gap's characteristic dimension and the advection velocity, the solution in Ω is no longer sensitive to the initial condition and is determined predominantly by the boundary conditions. We therefore choose as initial condition a bicubic spatial interpolation of the boundary conditions at the initial time level, $n = 0$. It is assumed that the measurement is performed over a time longer than τ such to reach independence from the initial condition.

8.3 Algorithmic assessment

To assess the technique, an artificial gap is constructed in existing full-field PIV data. Time-resolved measurements of the transitional jet in water discussed in chapters 4 and 6 and in Violato and Scarano (2011) are considered. The measurement parameters are the same as those reported in section 6.4.2 (the recordings have been processed with the pyramid correlation algorithm to reduce to noise level in the experimental data). The vector pitch is $h = 4$ pixels ($h = h_x = h_y = 0.25$ mm).

A data gap is simulated in the velocity field across the shear layer. The simulation is performed at the same Reynolds number as the flow ($Re_D = 5,000$) in the domain $\Omega \cup \mathcal{B}$ (see figure 8.7) of 40×40 PIV grid nodes; this region corresponds to $1.3\lambda \times 1.3\lambda$, with $\lambda = 0.55 D$ the wavelength of the large-scale vortices in the shear layer. In the following, the ratio between the

numerical domain dimensions and λ is indicated with L_x^* and L_y^* . The numerical mesh is composed of 400×400 cells. The reference velocity is selected as the jet exit velocity: $V_{\text{ref}} = 0.45$ m/s.

The region Ω simulates a gap where no data is available: here the numerical solution is computed without exploiting any information on the local experimental velocity. In contrast, $\mathcal{B}$ represents a buffer between PIV data and simulated data employed to compute the boundary conditions at each time step according to section 8.2.1. The width of such a region is presently set to 3 PIV grid nodes, i.e. 12 pixels, which is larger than the maximum particle image displacement occurring between two snapshots (8 pixels). Furthermore, in this region the velocity is obtained as a distance weighted linear combination of experimental and numerical data. After performing the numerical simulation, the computed velocity components and pressure are projected onto the original Cartesian grid of the PIV data.

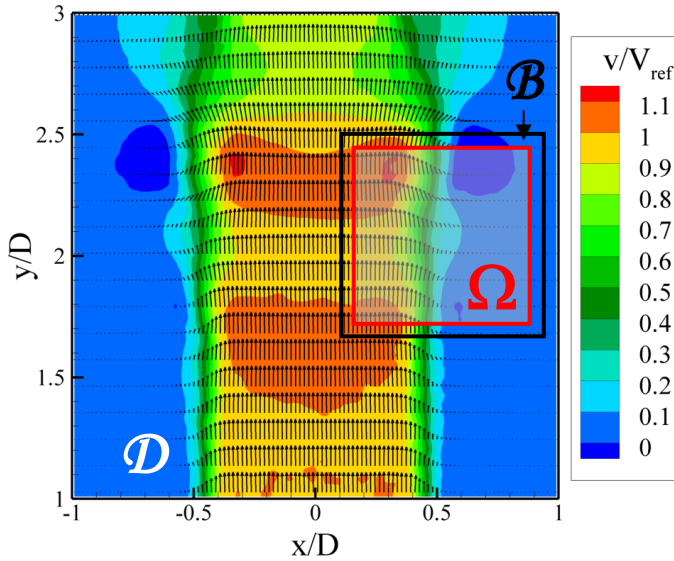


Figure 8.7. Laminar flow issued from a circular jet; axial velocity v . $\mathcal{D}$: measurement domain; Ω : simulated gap; $\mathcal{B}$: buffer region.

To run the numerical simulation, the initial condition on the velocity $\mathbf{V}_h^0$ must be prescribed. However, since in Ω no velocity data is acquired, the initial condition is not provided by PIV measurements and has to be estimated, for instance as a uniform velocity or as an interpolation of data at the gap

boundary. The prescribed initial condition affects only the solution $\mathbf{V}_h^n$ defined in the successive time interval τ ; after τ , $\mathbf{V}_h^n$ becomes predominantly independent of $\mathbf{V}_h^0$. Therefore, the choice of the initial condition is largely irrelevant for the solution after τ . For a flow where a convective component is present in the domain, such time scale can be estimated as $\tau \approx \frac{L}{\mathbf{V}_{\text{conv}} \cdot \hat{\mathbf{L}}}$, where

$\mathbf{V}_{\text{conv}}$ is the convection velocity and L is the dimension of the domain. The inner-product on the denominator indicates the projection of the convection velocity along the direction where the domain size is L . For flows with no dominant direction (e.g. recirculating regions within the gap), the estimation of τ is not as straightforward: the information on the flow field within the gap is transported from the boundaries mainly via diffusion, therefore the value of τ is expected to be substantially larger (above one order of magnitude) and can be estimated as $\tau \approx L^2 / \nu$, with ν the kinematic viscosity of the flow (Bird *et al*, 1976).

In this section, the time scale τ is evaluated experimentally by employing two different initial conditions, namely a uniform field of zero velocity and a bicubic interpolation of the velocity at the gap boundary. This is done to evaluate first whether the advanced time solution has any dependence upon the initial conditions and also to explore the possibility of accelerating the initial condition independence of the solution. In the present assessment, the temporal evolution of the velocity within the simulated gap is measured by PIV and will be considered as reference for validating the simulation.

The spatio-temporal evolution of the velocity field computed from the uniform and interpolated initial conditions is shown in figure 8.8; the time is expressed in the non-dimensional form $t = t_{\text{dim}} \times V_{\text{ref}} / L$, being t_{dim} the dimensional time and L the characteristic dimension of the numerical domain ($L = 7.3$ mm). The flow is characterized by a sequence of vortices entering the simulated domain from the centre of the lower boundary, convecting upwards with approximately constant speed, and exiting at the top boundary.

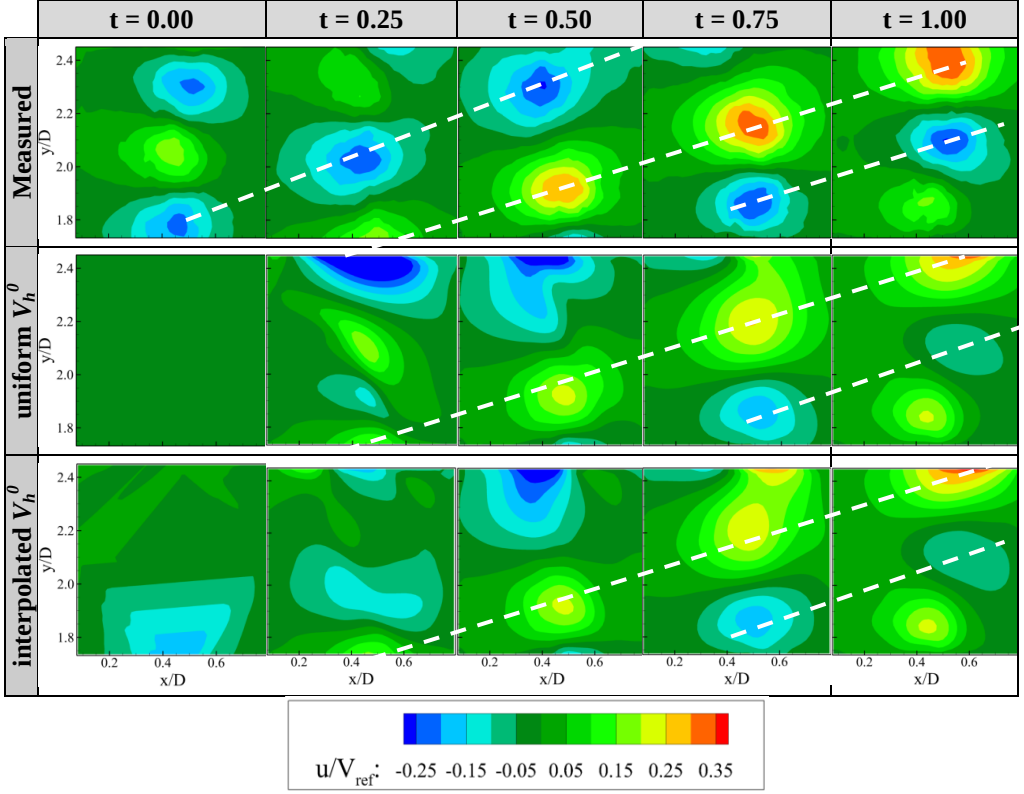


Figure 8.8. Contours of the computed radial velocity u starting from different initial conditions. First row: velocity field measured by PIV. Second row: numerical simulation starting from uniform zero velocity initial condition. Third row: numerical simulation starting from an initial condition obtained via bicubic interpolation of the boundary conditions. The white dashed lines indicate the temporal evolution of the flow structures.

Although the bicubic interpolation of the velocity is a closer approximation of the solution at $t = 0$ than the uniform stagnant flow, it cannot be considered as an acceptable representation of the flow. At $t = 0.25$, the numerical result does not show a visible similarity with the experimental flow field. At $t = 0.5$, about half the domain length has been swept by the flow information convected by the bottom boundary. As a consequence, the flow simulation resembles more the reference data in the lower half of the domain. In the upper half, the solution still depends on the initial conditions, therefore the numerical simulation is not able to reproduce the measured flow structure. The dependency on the initial conditions becomes less pronounced at $t \geq 0.75$, where the velocity contours exhibit only minor differences in shape. In

particular, at $t = 1$ the numerical simulations starting from different initial conditions produce nearly the same result. This confirms that, for convective flows, the time τ after which the solution becomes mostly independent of the initial condition scales with $L / (\mathbf{V}_{\text{ref}} \cdot \mathbf{L})$. In the cases where the flow is not dominated by convection, a larger value of τ is expected because the information from the boundary layer is transported within the domain by means of diffusion.

The results obtained with the Navier-Stokes solver are presented in comparison with a bicubic interpolation of the gap boundary, which is more accurate than the linear interpolation technique commonly used in PIV to refill missing data (Westerweel, 1994; Nogueira *et al*, 1997; Raffel *et al*, 2007). In the numerical simulation, the initial condition is computed as a bicubic interpolation of the boundary conditions at $t = 0$. In figure 8.9, the reconstructed instantaneous velocity fields are reported together with the experimental measurements.

The velocity field selected for the experimental assessment presents a vortex of $0.5 D$ wavelength (see figure 8.9); the radial velocity measured by PIV shows a positive and a negative peak about $y/D = 1.95$ and $y/D = 2.2$, respectively. The bicubic interpolation is not able to reproduce the vortex within the numerical domain. On the contrary, the numerical simulation based on Navier-Stokes equations allows reconstructing the flow structure with the two velocity peaks. The positive peak shows only minor differences with respect to the experimental one (peak velocity reproduced within 5%), while significant damping effects (up to 40% of the measured value) are noticed for the negative peak velocity. This discrepancy can be attributed to the fact that the negative peak velocity is further away from the inflow boundary, and numerical dissipation has reduced its strength.

The axial velocity contour computed by means of bicubic interpolation exhibits very low accuracy: the shear layer is substantially widened (the shear layer thickness increases by a factor 3 with respect to the experimental value) and again no velocity extrema within the domain are reproduced. In contrast, the numerical reconstruction shows good agreement with the PIV data: in particular, the shear layer has comparable thickness and the negative velocity region at $y/D = 2.3$ is well captured. However, discrepancies of the order of 3% of V_{ref} are noticed, for instance at the top-left corner of the numerical domain.

The effect of the techniques on a higher order quantity such as the vorticity ω is displayed in the third row of figure 8.9. The experimental vorticity exhibits two peaks, one close to the inflow boundary ($y/D = 1.85$) and one close to the outflow boundary ($y/D = 2.3$). The bicubic interpolation fails in reproducing the vorticity within the domain. In contrast, the numerical simulation allows reproducing the inflow peak within 7% accuracy, while the outflow peak is significantly damped by dissipation (reduction of 35% with respect to the experimental value).

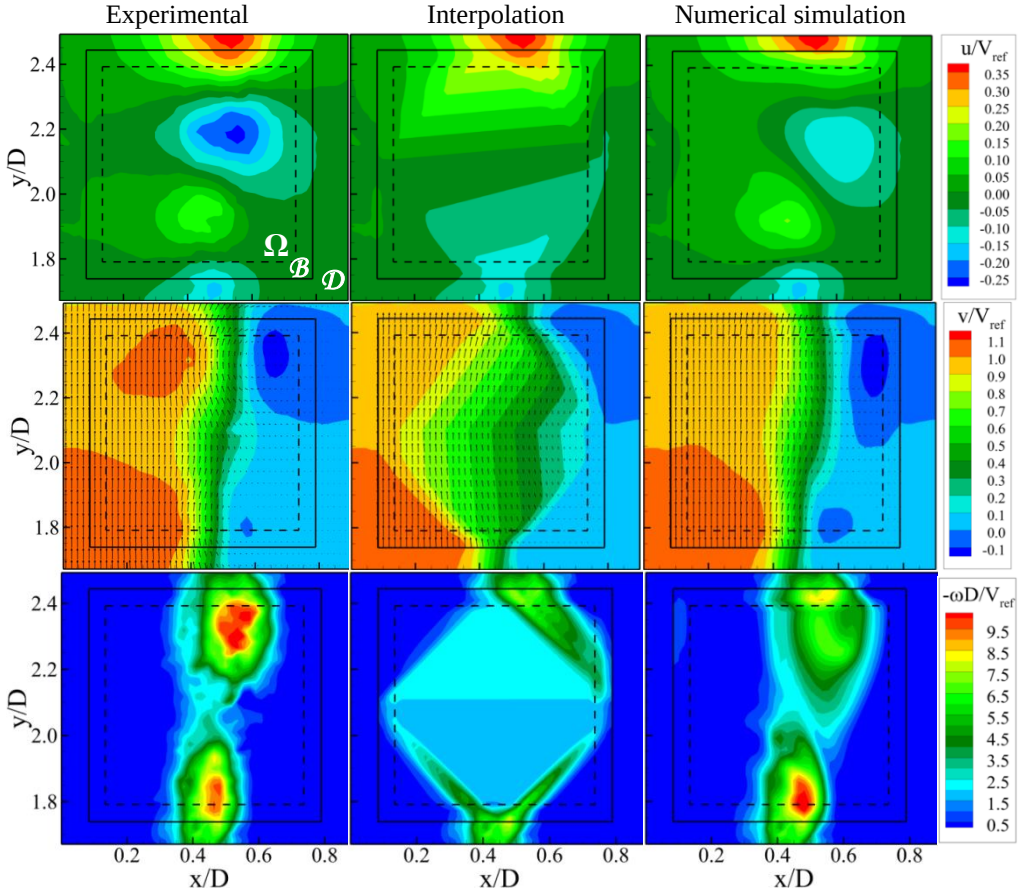


Figure 8.9. Instantaneous velocity field at $t = 3.4$; first row: radial velocity; second row: axial velocity; third row: vorticity. From left to right: experimental result, bicubic interpolation of the boundary conditions, numerical simulation.

To quantify the error due to bicubic interpolation and numerical reconstruction, the root-mean-square (rms) error (Fincham and Delerce, 2000)

with respect to the PIV velocity measurements in Ω is plotted as a function of the non-dimensional time t (see figure 8.10).

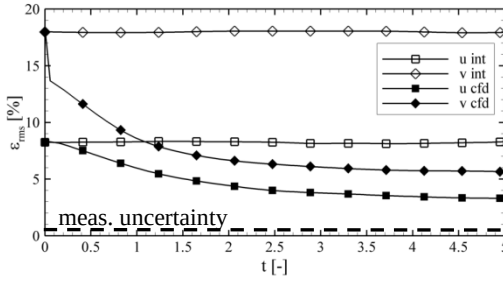


Figure 8.10. Root-mean-square error (expressed in percent of the reference velocity) as a function of t . Dashed line: measurement uncertainty; int: bicubic interpolation; cfd: results obtained through the Navier-Stokes solver.

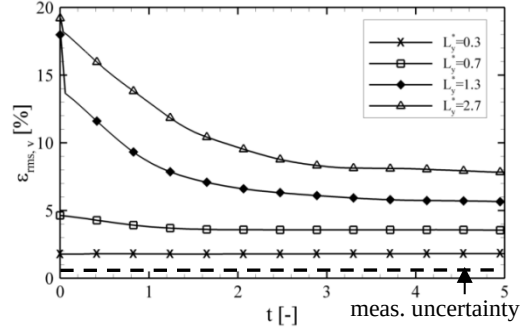


Figure 8.11. Root-mean-square error (expressed in percent of the reference velocity) on the axial velocity for different sizes of the numerical domain.

When the velocity field in Ω is computed through a bicubic interpolation of the boundary conditions, the root-mean-square error is approximately constant in time and equals 8% and 18% of V_{ref} for the radial and axial velocity components, respectively. In contrast, the result of the Navier-Stokes simulation shows a diminishing error in time. At the beginning of the simulation ($t = 0$), the numerical solution exhibits the same rms error as the interpolation, due to the choice of the initial conditions. In the successive time instants, the root-mean-square error decreases due to the information conveyed from the boundary conditions; a plateau is reached after $t = 3$, where ϵ_{rms} for the radial and axial velocity components equals 3% and 6% of V_{ref} respectively.

The effect of the numerical domain size on the accuracy of the results is investigated in figure 8.11, where the rms error of the axial velocity component is reported (analogous results are obtained for the radial velocity component). The height of the domain is set to 10, 20, 40 and 80 PIV grid points respectively, which correspond to $L_y^* = 0.3, 0.7, 1.3$ and 2.7 , respectively. The width of the domain is kept constant at 40 PIV grid points ($L_x^* = 1.3$). The choice of varying the sole vertical dimension is due to the fact that the flow evolves mainly along the jet axis, whereas the advection along the x -direction is minor. The plot of figure 8.11 shows that the accuracy of the simulation is strongly related to the size of the numerical domain. When the

domain height is $L_y^* = 0.3$, the discrepancy between numerical and experimental data is below 2%, while it increases up to 8% for $L_y^* = 2.7$.

Since the velocity at the boundaries of the numerical domain is computed through temporal interpolation of the PIV velocity values, the PIV acquisition frequency has major influence on the accuracy of the numerical simulation; the coarser the acquisition frequency, the higher the uncertainty of the interpolated values. In figure 8.12, the root-mean-square error of the numerical simulation is plotted as a function of the PIV data acquisition frequency, which is normalized with respect to V_{ref}/D . The latter represents the inverse of the time interval spent by a fluid parcel to cover a distance equal to the reference length D ; therefore, a non-dimensional acquisition frequency f implies that f successive PIV recordings are taken before a fluid parcel moving at velocity V_{ref} undergoes a displacement of D . The results reported in figure 8.12 are computed for $L_y^* = 1.3$; the same trend is obtained for other heights of the numerical domain.

As expected, the error in the numerical simulation is large at low acquisition frequency and drops with increasing f . A plateau is reached at $f = 5$: above this value, additional velocity measurements do not improve the simulation accuracy, and the total error is dominated by other effects, such as incorrect assumption of two-dimensional flow.

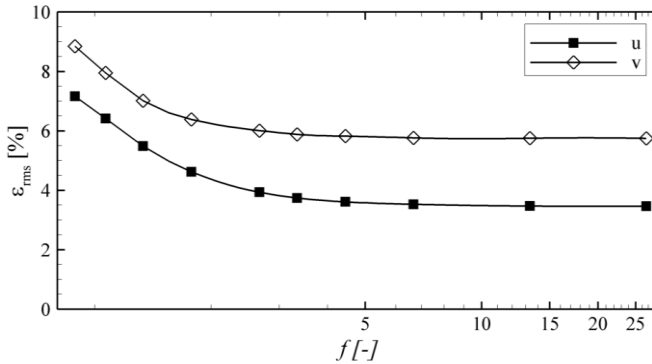


Figure 8.12. Root-mean-square error (expressed in percent of the reference velocity) as a function of the acquisition frequency of PIV data.

8.4 Application: treatment of shadow regions

The technique is applied to an experiment in air of a rod-airfoil configuration. A cylindrical rod of 0.6 cm diameter is mounted 10.2 cm upstream of a NACA 0012 airfoil, having a chord of 10 cm and placed at zero angle of attack. The airfoil is made out of Plexiglas in order to be transparent

to the laser light. The nominal free-stream velocity is 15 m/s, yielding Reynolds numbers of 6,000 and 100,000 with respect to the rod diameter and the airfoil chord respectively. The temporal separation between laser pulses is 50 μ s, while the acquisition frequency is 2,700 Hz ($f = 1.1$ with respect to the rod diameter, which is the characteristic length scale of the flow structures approaching the airfoil). The field of view size is 164×83 mm², imaged by 1939×1024 pixels. A thorough description of the experiment is reported in Lorenzoni *et al* (2009).

The illumination is directed from the bottom of the field of view upwards, tilted clockwise by 11 degrees. According to Snell's law (Born and Wolf, 1999), when a light ray passes from air (refractive index $n_1 = 1.000$) to Plexiglas (refractive index $n_2 = 1.488$), it undergoes a deflection that depends on the angle between the incident ray and the normal to the interface. The deflection is negligible in the central part of the airfoil, where the incident and refracted rays are roughly normal to the interface. In contrast, where the airfoil curvature is larger, i.e. at the leading and trailing edges, the refraction becomes so strong that non-illuminated regions are generated above the airfoil, as illustrated in figure 8.13.

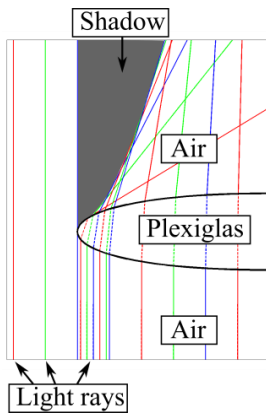


Figure 8.13. Shadow region generated above the airfoil leading edge due to refraction.

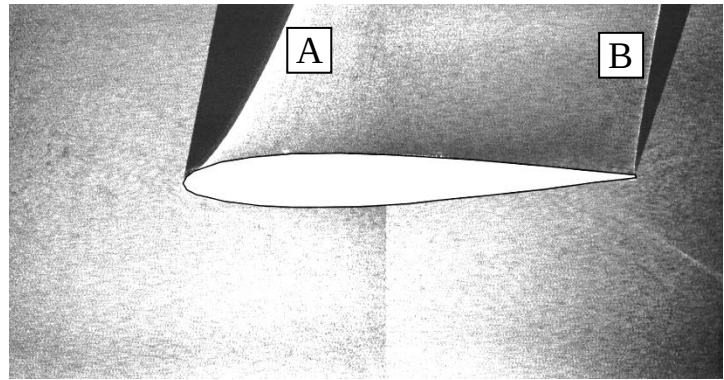


Figure 8.14. Double-frame recording of particle images on a transparent NACA 0012 airfoil (laser light inserted from the bottom) in the wake of a rod (outside the field of view, on the left).

In these regions, indicated with A and B respectively in figure 8.14, no particle images are visible and no displacement vector can be extracted through a correlation-based PIV algorithm.

The images are analysed with 32×32 pixels interrogation windows with 75% overlap, yielding a vector pitch of 8 pixels.

To reproduce the same physical behaviour of the flow, the simulation should be run at the same Reynolds number as in the experiment, that is 100,000 based on the chord. When this is done, the stability conditions by Ferziger and Peric (2002) yield a numerical grid spacing equal to $1/300^{\text{th}}$ of the experimental grid spacing and a time step of $0.15 \mu\text{s}$ ($1/2500^{\text{th}}$ of the time interval between image pairs). As a consequence, the computational time becomes so large that filling PIV gaps even in a small sequence of velocity fields is made impossible. Furthermore, the spatial resolution becomes hundreds of times higher than that of the PIV measurements: such a high spatial resolution is not required, because the simulation aims at filling in gaps of PIV data, not at resolving turbulent scales smaller than the PIV interrogation window. When a lower spatial resolution is employed (numerical grid only 1 to 10 times finer than the PIV ones), according to the aforementioned stability conditions, the Reynolds number needs to be lowered: this has the effect of adding numerical dissipation to stabilize the simulation. To investigate the effects of the Reynolds number on the accuracy of the results, the simulation is first conducted in a region where PIV measurements are available, placed upstream of the leading edge, where Kármán vortices are periodically shed by the rod.

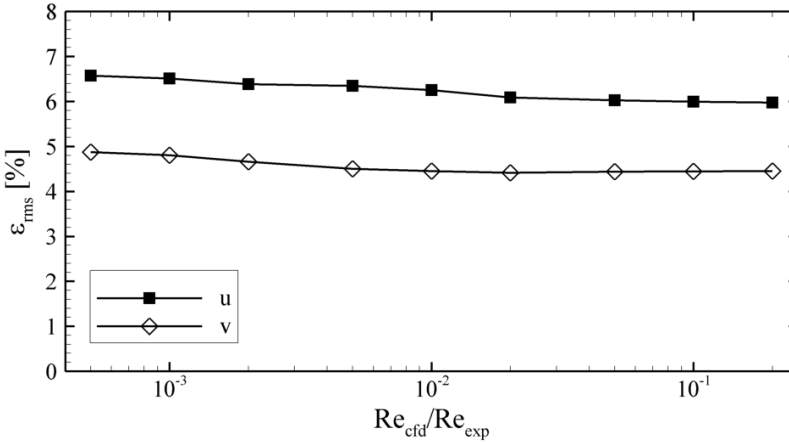


Figure 8.15. Rms error as a function of the Reynolds number in a region upstream of the leading edge. Re_{cfd} : Reynolds number of the numerical simulation; Re_{exp} : true Reynolds number in the experiment.

The results of figure 8.15 show that, in the considered range, the accuracy of the results is rather independent of the Reynolds number Re_{cfd} of the

simulation, with root-mean-square errors about 6% for the horizontal velocity component and 5% for the vertical velocity component. Only a slight reduction of the error is noticed for increasing Re_{cfd} , with a plateau reached at $1/20^{\text{th}}$ of the experimental Reynolds number Re_{exp} . This result is ascribed to the fact that, due to the small size of the computational domain (of the order of few centimetres) and the convection velocity of about 15 m/s, the transit time of a fluid parcel within the numerical domain is of the order of 1 ms, that is much shorter than the characteristic time of diffusion $O(10 \text{ s})$; hence, diffusive effects play a negligible role here.

Therefore, it has been decided to run the simulation at $Re_{\text{cfd}} = 1,000$, which equals $1/100^{\text{th}}$ of Re_{exp} . The gained reduction of computational time is remarkable, since the numerical grid can be just twice as fine as the PIV grid and the time step $1/40^{\text{th}}$ of the temporal separation between image pairs.

For both the shadow regions A and B, the numerical simulation is conducted in rectangular domains, indicated by Ω_A and Ω_B respectively in figure 8.16. Ω_A is composed by 112×88 cells, corresponding to 56×44 PIV grid nodes, while Ω_B is composed by 62×100 cells, which correspond to 31×50 PIV grid nodes. The region containing measurement data inside the numerical domains serves as the buffer region $\mathcal{B}$ as explained in section 8.2.1.

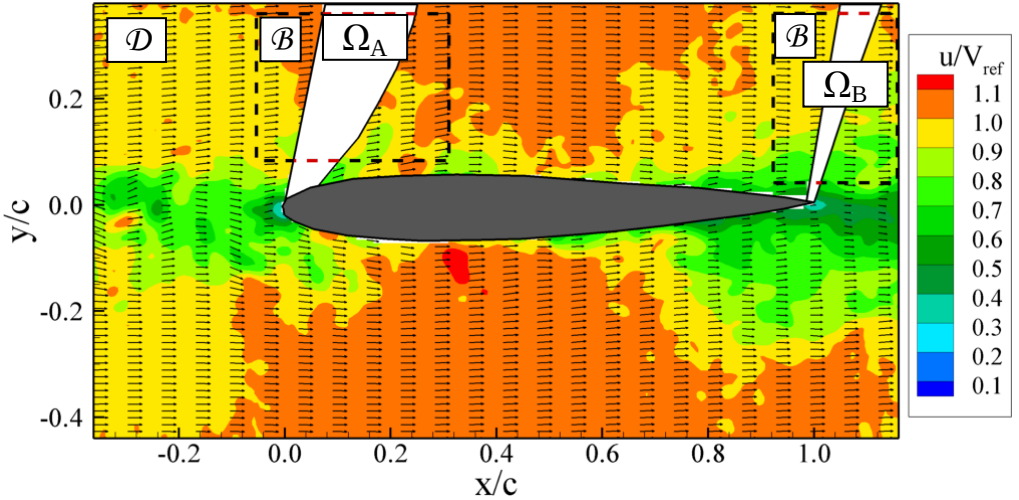


Figure 8.16. Numerical domain Ω and the regions A and B. The PIV velocity vectors are plotted every 10 sample in the horizontal direction and every 2 samples in the vertical direction.

Both numerical domains Ω_A and Ω_B exhibit a central region where no PIV data is present and two lateral regions where particle image velocimetry vectors have been computed; the numerical solution is calculated in the

entirety of the rectangular domains, but it is retained only in the central regions. At the unknown boundaries (the red dashed lines of figure 8.16), the boundary conditions are computed as a linear interpolation of the measured PIV velocity data.

Figure 8.17 shows an example of reconstructed instantaneous velocity fields. In both the shadow regions, the velocity components computed with the Navier-Stokes solver exhibit good agreement with the surrounding experimental data.

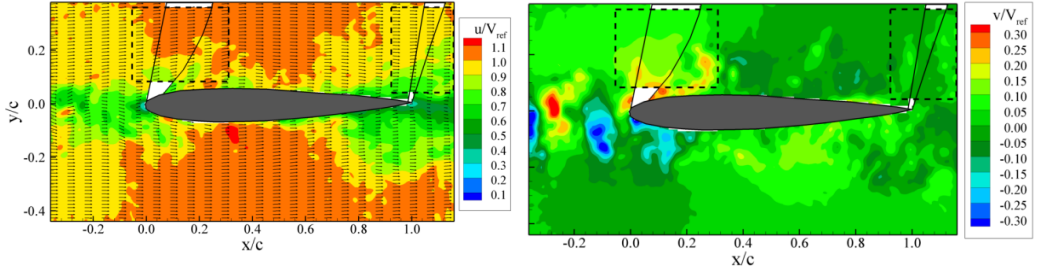


Figure 8.17. Instantaneous reconstructed velocity fields. Left: horizontal velocity component; right: vertical velocity component.

The statistical results (namely time average and fluctuations root-mean-square) obtained from the reconstructed velocity fields are shown in figure 8.18. The mean horizontal and vertical components (figure 8.18-a and -b respectively) are mainly continuous between the experimental region and the shadow region; moreover, the velocity components calculated with the Navier-Stokes solver fulfil the symmetry of the flow field with respect to the airfoil. The uncertainty in the mean components is estimated to be below 0.6% of V_{ref} (i.e. 0.05 pixels)[†]. In contrast, the fluctuations root-mean-square contours exhibit discontinuities between numerical and experimental values; the uncertainty on those components is of the order of 1% of V_{ref} (i.e. 0.09 pixels).

[†] The uncertainty of the numerical simulation is estimated from the direct comparison of the velocity in the buffer region, where both the numerical and the experimental velocities are available.

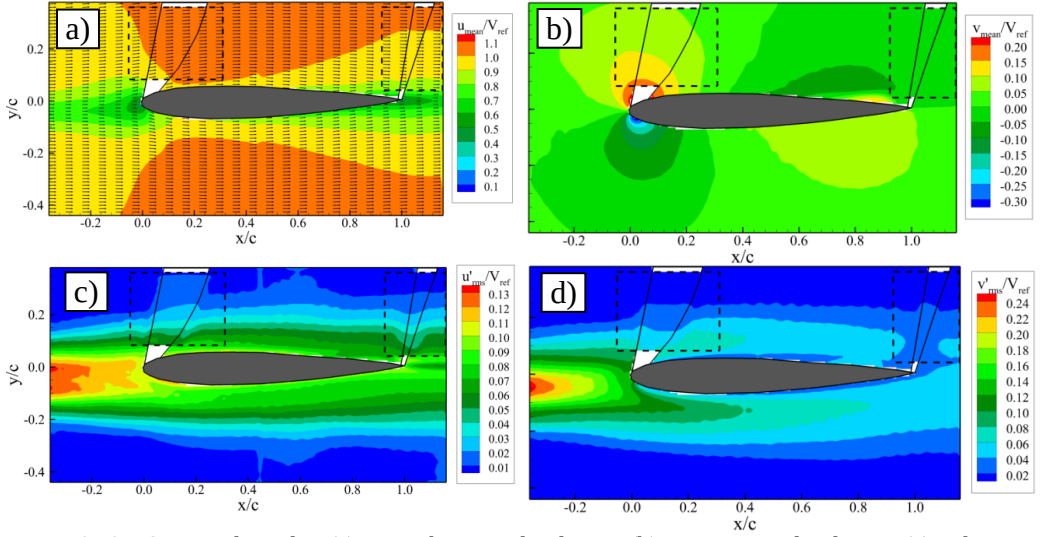


Figure 8.18. Statistical results: (a) Mean horizontal velocity; (b) mean vertical velocity; (c) velocity fluctuations in the horizontal direction; (d) velocity fluctuations in the vertical direction.

The added value of the present technique with respect to interpolation approaches is evident when small flow structures need to be reconstructed in the shadow region. In figures 8.19 and 8.20, the motion of two vortices *a* and *b* is illustrated. At time $t = 0$, the two vortices are in the measurement domain: vortex *a* has a single core, whereas vortex *b* has two distinct cores $0.037c$ apart ($d_{b1b2} = 0.037c$). The distance between vortices *a* and *b*, measured as the distance between the points of maximum vorticity, equals $d_{ab1} = 0.091c$. The two vortices are convected downstream by the flow and pass through the shadow region, where the velocity is calculated via the numerical simulation.

The proposed filling approach allows tracking the vortices within the shadow region and estimating the distance between them within 5% accuracy (figure 8.19, third row). However, modulation effects are noticed which have two main consequences: reducing the peak vorticity up to 50% of the original value and merging the two cores of vortex *b* in a single-core vortex (see figure 8.19). Downstream of the shadow region, the vorticity is measured from PIV data: the peak vorticity is recovered and the two cores of vortex *b* become distinct again. Minor modulation effects also occur in the immediate vicinity of the shadow region edges, where the velocity is computed as a distance-weighted combination of experimental and numerical values.

In contrast, when the velocity in the shadow region is computed via bicubic interpolation of the boundary values, the motion of the two vortices along the trajectory cannot be tracked, as illustrated in figure 8.20.

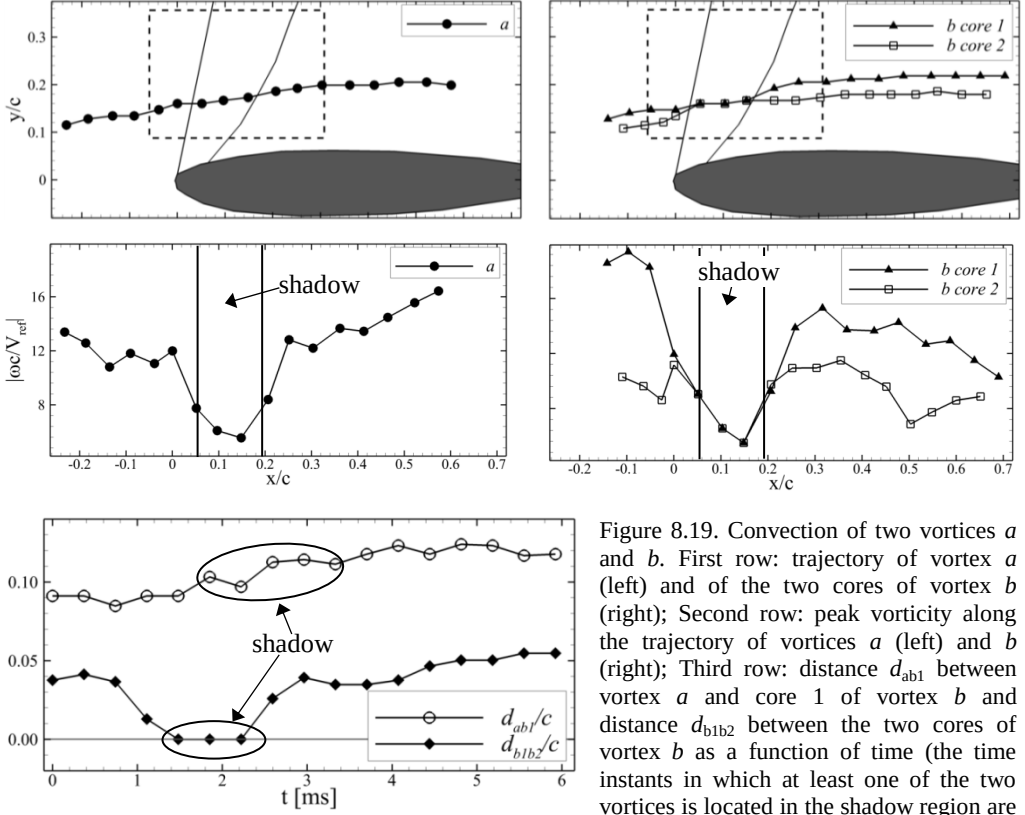


Figure 8.19. Convection of two vortices a and b . First row: trajectory of vortex a (left) and of the two cores of vortex b (right); Second row: peak vorticity along the trajectory of vortices a (left) and b (right); Third row: distance d_{ab1} between vortex a and core 1 of vortex b and distance d_{b1b2} between the two cores of vortex b as a function of time (the time instants in which at least one of the two vortices is located in the shadow region are indicated in the figure).

8.5 Conclusions

A novel technique for filling in gaps in PIV data has been proposed. The technique takes as input the measured velocity at the gap boundary and solves the unsteady incompressible Navier-Stokes equations within the gap. The finite volume approach is employed with central discretization of the convective and diffusive terms and explicit forward discretization in time. Because stability considerations require a numerical time step usually smaller than the time interval between the PIV velocity fields, the boundary conditions are interpolated in time employing an advection-based model.

The technique has been first assessed on an artificial gap constructed in existing full-field PIV data. The assessment has shown that the accuracy of the results depends on the numerical domain size and acquisition frequency of PIV data. In typical conditions of applicability (PIV data sufficiently resolved in time and numerical domain size of the order of the wavelength of the large-

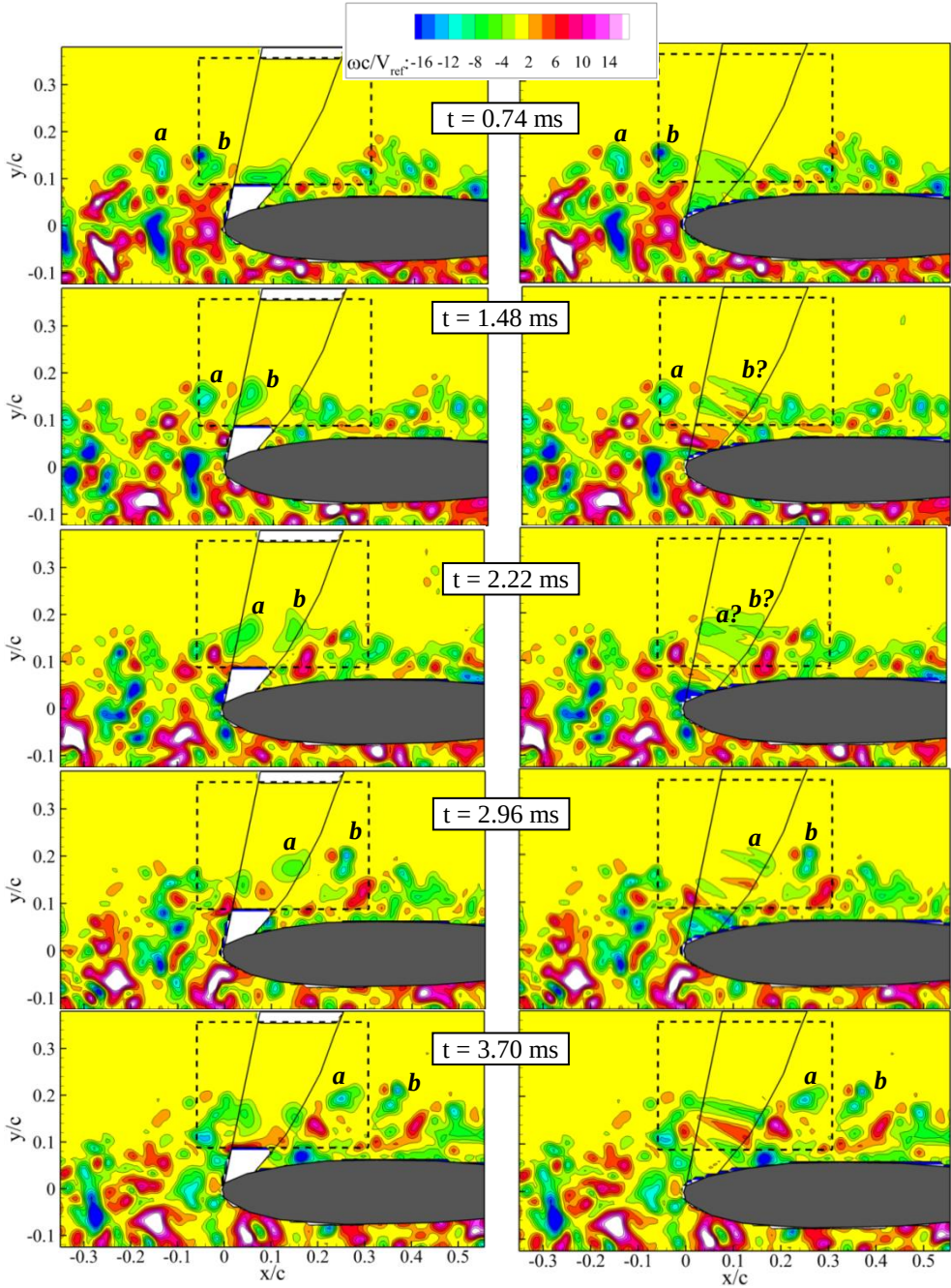


Figure 8.20. Vorticity contours showing the convection of two vortices *a* and *b* through the shadow region. Left: reconstruction through the Navier-Stokes solver; right: reconstruction through a bicubic interpolation of the velocity boundary values.

scale structures in the flow), the accuracy of the method is within 5% of the reference velocity. By using PIV data with real gaps, the capability of the method of reconstructing the velocity field where no velocity information is available has been shown.

CHAPTER 9

CONCLUSIONS

Abstract This chapter summarizes the main results and findings of the present dissertation on PIV uncertainty quantification and advanced image and data analysis. The proposed approaches are discussed with focus on operational principle, applicability, main improvements with respect to existing techniques and limitations. The perspectives of development of these approaches are also indicated.

9.1 A-posteriori uncertainty quantification of PIV data

A methodology for the uncertainty quantification of PIV data has been introduced, namely the *image-matching* uncertainty quantification. The approach is a-posteriori, meaning that the uncertainty of specific vector fields measured by PIV is quantified. The estimated uncertainty is instantaneous and local: each measured vector is associated with its own uncertainty interval.

The working principle is general and can be applied to any interrogation algorithm relying on cross-correlation, independently of whether it makes use of window deformation, window offset or single-pass interrogation. The methodology is an image-based technique, meaning that the measurement uncertainty is evaluated from the velocity field along with the PIV images used for the computation of the velocity. The velocity is used as a predictor to match the images with a degree of approximation depending upon the interrogation algorithm. The positional disparity between the particle images of a pair is computed and its statistical distribution is considered to infer the instantaneous uncertainty of the measured displacement.

The image matching estimator allows quantifying the uncertainty associated with random errors stemming e.g. from noise in the recordings, out-of-plane motion and low seeding density. Also systematic errors due to spatial modulation (i.e. caused by the finite interrogation window size) can be detected from the average particle image disparity. In contrast, systematic errors owing to truncation in time (i.e. lack of information on the actual particle trajectory between the two images) cannot be accounted for. Furthermore, the technique exhibits lower accuracy for particle images of diameter of 1 pixel or below: in this case, the estimation of the particle image position is affected by peak-locking errors and therefore may yield erroneous error estimates. Finally, the estimated uncertainty has its own spread due to the finite number of particle image pairs within the interrogation window. Due to

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the limited number of particle image pairs and the inaccuracy in determining their position, uncertainty values below a “fog level” of $0.03 \div 0.05$ pixels are typically overestimated.

Both Monte Carlo simulations and the experimental assessment have shown that the uncertainty scales consistently with the actual error for different error sources. It has been shown that both error and uncertainty scale with the inverse of the square root of the number of particle image pairs. On average, the image-matching uncertainty estimates the actual error magnitude within 30% of its value.

9.1.1 *Perspectives of the image-matching approach for PIV uncertainty quantification*

As discussed in chapter 4, the image-matching approach does not provide an accurate estimate of systematic errors stemming from peak-locking. Those are typically dominant in imaging conditions where the particles are sub-sampled (diameter of 1 pixel or below), which often occur in measurement conducted with CMOS cameras due to the large pixel size. Thus, the performance of the image-matching approach in presence of small particle images should be enhanced for a more accurate estimate of peak-locking errors.

Moreover, uncertainties below a fog level (usually $0.03 \div 0.05$ pixels) are typically overestimated mainly due to the inaccuracy in the determination of the particle image position. Higher accuracy in the evaluation of the pair positional disparity could be achieved by computing the latter via spatial cross-correlation in a small kernel (e.g. 3×3 or 5×5 pixels) instead of from the difference of individual particle images position. This approach is expected to reduce the fog level and in turn the minimum uncertainty that can be estimated.

Furthermore, the uncertainty associated with large-magnitude errors (exceeding 1 pixel), namely outliers, is typically underestimated. Approaches for the detection of those and a correct estimate of their uncertainty based on the number of paired particle images are currently under investigation.

9.1.2 *Outlook on PIV uncertainty quantification*

Chapter 5 has introduced the *collaborative framework for PIV uncertainty quantification*, which has been set up in collaboration with LaVision GmbH, Utah State University and Purdue University. Within the framework, a comparative assessment of different PIV uncertainty quantification techniques

has been conducted to achieve a more thorough understanding on how those perform in presence of different error sources. The results have been presented at the 17th International Symposium on Applications of Laser Techniques to Fluid Mechanics in Lisbon, Portugal, in July 2014; one or more publications on peer reviewed journals are expected to be produced.

From the comparative assessment, strengths and weaknesses of the different uncertainty quantification approaches have emerged. It has been shown that methods relying upon the analysis of the image contributions to the cross-correlation peak shape yield satisfactory sensitivity to the measurement error. Reduced sensitivity is achieved when the uncertainty is evaluated from the correlation peak ratio, which does not affect directly the measured displacement. The use of numerical simulations for uncertainty quantification has been proved to be valuable especially for low seeding density images, where image-based methodologies yield results that may not reach statistical convergence.

The analysis of the different approaches is considered a crucial step in the direction of developing a consolidated and generalized a-posteriori uncertainty quantification approach that combines the strengths of existing algorithms.

A related topic that should be addressed within a collaborative framework among research groups is the definition of standard procedures and methodologies for the estimation of the uncertainty associated with all the relevant error sources stemming from the entire measurement chain (e.g. calibration errors, timing errors, perspective errors, magnification errors...). This is recognized as relevant for a thorough evaluation of the overall measurement uncertainty that goes beyond the errors originating from the image interrogation.

9.2 Multi-frame pyramid correlation for time-resolved PIV

An adaptive multi-frame technique for time-resolved PIV has been proposed to enhance the accuracy and dynamic velocity range of TR-PIV data. The *pyramid correlation* relies upon the average of cross-correlation functions also at different time separation of a small sequence of images. Aim of the technique is to increase the measurement robustness and reduce the relative error on the displacement, thus allowing a more accurate estimate of the velocity field and in turn of the flow acceleration. The approach locally optimizes the observation time taking into account precision and acceleration errors and continuum deformation.

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The working hypothesis of the pyramid correlation is that the velocity is approximately constant in the time interval containing a short sequence of images (i.e. the fluid acceleration is negligible). This is typically achieved by using high-speed acquisition systems (CMOS cameras and high repetition rate lasers) to obtain data highly sampled in time. Due to its robustness, the pyramid correlation yields the major improvements with respect to state-of-the-art interrogation algorithms in all the cases where the signal strength is low due to either sub-optimal imaging conditions (e.g. noise in the recordings, particle images of diameter below one pixel, low seeding density) or out-of-plane motion. Higher temporal coherence of the velocity fields and a reduction of the measurement errors by factor 3 or above are typically achieved.

9.2.1 Outlook on advanced multi-frame algorithms for time resolved PIV

The main limitation of the pyramid correlation algorithm is that it does not account for velocity variations within the small sequence of PIV recordings. Consequently, when the flow acceleration is not negligible, truncation errors arise from the assumption of constant velocity, which reduce the overall measurement accuracy.

To overcome this limitation, multi-frame approaches that rely upon a Lagrangian flow description have been recently proposed by Lynch and Scarano (2013) and Jeon *et al* (2014). Those approaches track collections of fluid elements along their curvilinear trajectory, thus allowing a further enhancement of the *DVR* achieved via the larger measured displacement. The greatest improvements with respect to state-of-the-art interrogation algorithms are achieved in three-dimensional measurements, as those obtained with tomographic PIV; here fluid parcels can be tracked for several time instants without incurring particle image loss-of-pairs due to out-of-plane motion, contrarily to what occurs in planar or stereoscopic measurements.

Moreover, the fluid parcel acceleration can be directly computed by derivation in time of the parcel velocity along its trajectory.

9.3 Treatment of laser light reflections

An image pre-processing approach for time-resolved PIV has been introduced to deal with the undesired effect of laser light reflections. The approach is suited to remove the reflections in both cases when the reflection

image is steady or moving due to vibrations of the model or the cameras. The method relies upon an efficient decomposition in the frequency domain of the intensity time history recorded at individual pixel locations. The working hypothesis is that the characteristic time of the reflection crossing a pixel is lower than the characteristic transit time of a particle image on a pixel. It is indicated as a requirement that these two characteristic times should be separated by at least factor three.

It has been shown that the application of a high-pass filter to the intensity time history recorded by a pixel allows removing the undesired effects of light reflections while retaining the contribution of the particle images. In well-controlled experimental conditions, the laser light reflections have been mostly eliminated making it possible to measure the particle image displacement in proximity of a solid interface. In more difficult conditions, direct reflections on solid interfaces as well as secondary out-of-focus reflections have been eliminated thus enhancing the particles signal with respect to the background.

9.3.1 *Perspectives on the treatment of light reflections*

In the current implementation, the characteristics of the high-pass filter (in particular the cut-off frequency) are kept constant in the entire image. However, in many cases the imaging conditions are inhomogeneous: in regions with steady or slowly-moving reflections, lower cut-off frequency would be beneficial to reduce the drop of the particle image intensity; contrarily, regions affected by strong intensity reflections moving at high-frequency require higher cut-off frequency to achieve a complete removal of those. Thus, an adaptive optimization of the filter characteristics based on the local imaging conditions may yield a potential enhancement of the pre-processing technique.

9.4 **Treatment of gaps in PIV data**

Laser light reflections not correctly removed may be the cause of clusters of missing or corrupted data in the PIV velocity field, namely gaps. Shadows in the recorded images, lack of optical access for laser or camera, inhomogeneous seeding density are other possible causes.

The possibility of reconstructing the velocity field in small gaps based on the solution of the unsteady incompressible Navier-Stokes equations has been investigated. To solve the Navier-Stokes equations, boundary and initial

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conditions on the velocity are required. The boundary conditions are retrieved from the measured velocity at the gap boundary; in contrast, the initial condition is typically unknown. However, it has been shown that, in advective flows, the effect of the initial condition decays in time; after a characteristic time that scales with the ratio between gap dimension and advection velocity, the solution is mainly independent of the initial condition.

In the current implementation, the method is applicable to unsteady incompressible two-dimensional flows. The experimental assessment has showed the capability of reproducing more than a single velocity peak in the gap, contrarily to the state-of-the-art of refilling techniques which rely upon the interpolation of the velocity values at the boundary. The accuracy of the reconstructed velocity is within 5% of the reference velocity. Furthermore, the capability of the technique to track the motion of small vortical structures within the gap has been demonstrated.

9.4.1 Outlook on the treatment of gaps in PIV data

Several recent works have focused on the treatment of *temporal gaps* of PIV data, where information on the velocity spatial distribution is used to determine the velocity field at intermediate time instants, aiming at enhancing the temporal resolution of PIV measurements (*pouring space to time*, Scarano and Moore, 2011; Schneiders *et al*, 2014). The treatment of spatial gaps by using temporal information (*pouring time to space*) has received less attention so far, although it is crucial for determining the velocity distribution in regions where the measurement fails. Due to its relevance, the topic is expected to receive increasing consideration in the next years.

Three main directions are foreseen for improving the proposed methodology for the treatment of gaps in PIV data. The first is the generalization of the technique by extending its applicability to compressible and three-dimensional flows. This would allow treating gaps in a wide gamut of flow regimes (subsonic and supersonic flows) and measurement configurations (from planar to tomographic PIV) typical of PIV campaigns. In particular, gaps in three-dimensional measurement domains would require the solution of the 3D Navier-Stokes equations, which causes a major increase in the computational time.

Furthermore, the approach could be used to compute the velocity distribution within the boundary layer and especially in proximity of a solid surface, which is crucial for the calculation of quantities such as the wall shear

stress and the skin friction coefficient. Measurements close to solid walls are recognized as a challenge in PIV, owing to perspective effects and laser light reflections that often preclude the accurate velocity measurement.

Finally, the application of the approach to time-averaged velocity and Reynolds stresses is considered relevant for many applications where flow statistics is investigated; this is often the case in measurement campaigns in industrial facilities or those aimed at CFD data validation. In this case, approaches based on the solution of the Reynolds Averaged Navier-Stokes (RANS) equations may be taken into consideration which would significantly reduce the computational cost.

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APPENDIX

HPF PRE-PROCESSING APPROACH IN PSEUDO CODE AND IMPLEMENTED AS A MATLAB FUNCTION

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% %
%
% MATLAB routine for light reflections elimination
% via high-pass filter
%
% Authors: A. Sciacchitano, F. Scarano
% Delft University of Technology
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% %

% The routine inputs are:
% 1) the 3D matrix "Raw_Image_Seq" containing the sequence of images
%     to be pre-processed
% 2) the cut-off frequency "fc" of the Butterworth filter (normalized
%     with respect to the Nyquist frequency; "fc" ranges between 0
%     and 1)
% 3) the filter order "N_filt" (e.g. 3)

% "Raw_Image_Seq" has size JxIxN, where J is the number of rows of
% each image, I the number of columns and N the length of the image
% sequence.

% The routine output is the 3D matrix "Filt_Image_Seq" containing the
% sequence of pre-processed images. "Filt_Image_Seq" has size JxIxN.
% Note: the first image of the output sequence has significantly lower
% quality

function Filt_Image_Seq = HPF_pre_processing (Raw_Image_Seq,fc,N_filt)

% size of Raw_Image_Seq
[J, I, N] = size( Raw_Image_Seq );

% initialize the output matrix
Filt_Image_Seq = zeros( J, I, N );

% recursion coefficients of the high-pass Butterworth filter
% (MATLAB function)
[b, a] = butter( N_filt, fc, 'high' );

% loop on the rows
for j = 1:J

    % loop on the columns

```

Appendix

```
for i = 1:I

    % Raw time-history in the pixel location (j,i)
    Raw_time_history = squeeze( Raw_Image_Seq(j,i,:) );

    % Conversion to double precision
    Raw_time_history = double( Raw_time_history );

    % Zero-phase high-pass filtered time-history in the pixel
    % location(j,i)
    Filt_time_history = filtfilt( b, a, Raw_time_history );

    % Store the filtered time-history in the output matrix
    Filt_Image_Seq(j,i,:) = Filt_time_history;

end
end
```

LIST OF PUBLICATIONS

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- [1] Sciacchitano A and Scarano F (2014), PIV light reflections elimination via a temporal high pass filter, *Meas. Sci. Technol.* **25** 084009 (13pp)
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- [5] Sciacchitano A, Scarano F and Wieneke B, Performance of the image-matching approach for PIV uncertainty quantification on wind tunnel experiments, in preparation
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CONFERENCE PROCEEDINGS

- [1] Sciacchitano A, Neal DR, Smith BL, Warner SO, Vlachos PP, Wieneke B and Scarano F, Collaborative framework for PIV uncertainty

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- quantification: comparative assessment of methods, *17th International Symposium on Application of Laser Techniques to Fluid Mechanics*, Lisbon, Portugal, 07-10 July 2014
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ACKNOWLEDGEMENTS

This manuscript summarizes the main results of four years of hard work as a PhD student at TU Delft. Here I would like to acknowledge all the people that supported and motivated me during the entire PhD, making this thesis possible. A special thanks to my supervisor and *promotor* prof. Fulvio Scarano, who guided me through these years with his enthusiasm, competence and dedication. I would like to thank you, Fulvio, not only for being a careful supervisor who motivated me in achieving ambitious goals, but also for the many opportunities to grow you offered me via collaborations and projects.

I would like to thank Mr. Bernd Wieneke from LaVision GmbH for his support and useful discussions throughout the entire PhD research. Bernd, I have found very inspiring working with you, a rigorous word-class scientist and managing director of a successful company.

I am grateful to Colette Russo for her efficiency in helping me with any administrative issue. Thanks also to the members of the technical staff for their support: Eric de Keizer, Frits Donker Duyvis, Henk-Jan Siemer, Peter Duyndam, Nico van Beek, Stefan Bernardy and Leo Molenwijk. I express my gratitude to dr. Richard Dwight for his help on the work of numerical simulations in gappy PIV data.

I would like to thank all my colleagues PhD-students at the Aerodynamics section of TU Delft for the company during the long days at the office. Many thanks to Daniele Violato and Sina Ghaemi for providing the experimental database that has been used in chapters 4 and 6. I express my gratitude also to Matteo Novara, Daniele Ragni and Kyle Lynch for the many discussions on PIV. I am grateful to Wouter van der Velden and Jan Schneiders for the valuable help with the Dutch translations of propositions and summary. A special thanks to Mustafa Percin, Vahid Kazemi Kamyab and Ilya Popov who kept me company in the office during the last months of my PhD with the many serious and semi-serious conversations.

Chapter 5 of this thesis would not have been possible without the support of the people who actively worked on the conduction of the experiments and analysis of the data. I express my gratitude to prof. Bart Smith, who hosted me for three weeks in his group at Utah State University, allowing me to collect

the data required for this part of the research. I am thankful to dr. Doug Neal for having shared with me the hard work during the experiments, from dawn till night. Doug's expertise on hot-wire measurements and capability of solving technical problems of any sort have been crucial for the success of the measurement campaign. I would like to thank Scott Warner for the preparation of the experiments and processing of the data with the uncertainty surface method. I am grateful to prof. Bart Smith, Bernd Wieneke, Doug Neal, Scott Warner and prof. Pavlos Vlachos for the useful discussions on the topic of PIV uncertainty quantification.

Many thanks to all my friends for all the pleasant moments spent together during the last years. I owe special words of gratitude to my parents Anna Maria and Salvatore and my brother Luca, who have always supported me in the difficulties and shared with me my successes. Finally I want to thank you, Vir, for being always there, celebrating with me any achievement and making me smile even when nothing seemed to go right. Thank you for your unconditional support and for always taking care of me. And most important, thank you for making me love everything of our life together. To you, Vir, I dedicate this thesis.

Delft, July 15, 2014

CURRICULUM VITAE

Andrea Sciacchitano was born on May 9, 1986, in Bergamo, Italy. He graduated with honours in aerospace engineering in 2010 at Sapienza University of Rome, Italy, with a M.Sc. thesis on the topic “Experimental investigation of flow control devices for the reduction of transonic buffeting on rocket afterbodies”. This work was conducted at the Aerospace Engineering department of Delft University of Technology under the supervision of dr. F.F.J. Schrijer and prof. F. Scarano.

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