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Adaptive Fuzzy Finite-Time Tracking Control of Uncertain Non-Affine Multi-Agent Systems With Input Quantization

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ABSTRACT In this paper, the finite-time tracking control problem of a class of multi-agent systems with nonaffine functions and uncertain nonlinearity is investigated, which is different from the existing on high-order multi-agent systems with pure feedback forms. The multi-agent systems considered in this paper, moreover, the nonaffine functions and uncertain nonlinearities are completely unknown, and the input of each follower agent is quantized through a hysteresis quantizer. Based on the help of the fuzzy logic systems approximator, an adaptive fuzzy finite-time tracking control protocol with adaptive update laws is presented by the backstepping technique. On the basis of the finite-time stability strategy and Bhat and Bernstein theorem, the finite-time stability of designed control protocol is fully analyzed. Under the proposed control protocol, it is indicated that the tracking error of each follower agent can press on a small neighborhood in a finite time. Finally, the effectiveness of designed control protocol in this paper is analyzed by numerical examples.

INDEX TERMS Finite-time tracking control, uncertain nonaffine mutli-agent systems, fuzzy logic systems, input quantization.

I. INTRODUCTION

Over the past twenty years, the cooperative control problem of multi-agent systems has paid extensively attention in many fields, such as multiple unmanned aerial vehicles, flexible robot manipulator systems, wireless sensor networks and large-scale systems [1]–[6]. The basis issue of cooperative control problem is consensus, where the objective is to achieve the agreement or track a given trajectory under a designed control protocol. In the existing literature, there are many results on the first- and second-order multi-agent systems with undirected graph, directed graph and switching graph have been obtain, see [7]–[12] and references therein.

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For the most practical systems, however, they need to be modeled as the high-order system rather than simple first- or second-order system. Moreover, the relevant results on first- and second-order multi-agent systems cannot be used in high-order multi-agent systems. The control problem of high-order multi-agent systems has been considered in some literature. In [13], a reduced-order observer was presented to solve the distributed tracking control problem of high-order nonlinear multi-agent systems with heterogeneous leader. In [14], a distributed region regulator was presented to deal with the synchronization problem of a class of high-order multi-agent systems. For the high-order nonlinear strict-feedback multi-agent systems, the authors of [15] used the distributed extended state observer and backstepping technique to solve the practical time-varying output formation tracking problem, and in [16], [17], the consensus tracking problem of high-order multi-agent systems with high-dimensional leader was firstly considered, where a distributed robust adaptive consensus control protocol with local observer was designed. On the basis of the results of [13]–[17], the containment tracking problem of multi-agent systems with pure-feedback form was analyzed in [18], where the distributed adaptive fuzzy control protocol and local quanzied controller were considered. Based on the advantages of the fuzzy control method, some control problems are well solved by designing the control law based on with the fuzzy control method [19], [20]. To the best of author's knowledge, however, only a few results focus on the control problem of nonaffine multi-agent systems. For instance, in [21], the agreement control of a class of nonaffine multi-agent systems was addressed, where a distributed neural adaptive consensus control scheme with guaranteed performance was designed. In [22], [23], the tracking problem of uncertain nonlinear nonaffine multi-agent systems was studied, and the distributed output-feedback control protocol with prescribed tracking performance and the distributed containment controller were investigated, respectively.

Referring to the literature of above-mentioned, it is easy to found that the control problem of multi-agent systems is based on the case that the control time is close to infinite. In some cases, it is desirable that the control objective of a multi-agent system be achieved in a finite time. Recent years, the finite-time control problem of multi-agent systems has been greatly developed. The finite-time control scheme for the attitude tracking problem of the spacecraft systems was investigated in [24]. Considering the input saturation, a sliding mode observer was designed to solve the global finite-time tracking problem of second-order multi-agent systems in [25]. For the heterogeneous nonlinear high-order multi-agent systems with mismatched disturbances, the authors of [26] designed the families of consensus protocols and solved the finite-time output consensus problem. Furthermore, the containment control of a class of nonaffine pure-feedback multi-agent was addressed in [27], where a distributed neural adaptive control strategy was proposed. Moreover, the finite-time consensus tracking control of a class of nonlinear multi-agent systems with prescribed performance was studied in [28], where the error constrained control was applied to obtain satisfactory control performances. In addition, the finite-time control problems of Markov jump systems and nontriangular stochastic nonlinear systems are ananlyzed in [29]–[33]. However, it is worth noting that the results above-mentioned cannot work for the special case that input quantization. Therefore, it is necessary to design a suitable control protocol for uncertain nonaffine multi-agent systems with input quantization to achieve the desired control objective in a finite time.

This observation inspires our current study. This paper investigates the finite-time tracking control problem of a type of nonaffine multi-agent systems with input quantization and uncertain nonlinearity. The main contributions of this paper are outlined as follows:

1. A class of uncertain nonaffine multi-agent systems with input quantization is considered in this paper. Considering the input of each agent is quantized by an introduced quantizer, therefore, the design of control protocol is more difficult and complex.

2. With the help of the differential midvalue theorem, the nonaffine functions of multi-agent systems can be decomposed into a strict-feedback form. Therefore, the model considered in this paper is more versatile than [26]–[28]. Furthermore, the nonaffine functions and uncertain nonlinearities are completely unknown.

3. Based on the fuzzy logic systems approximator and backstepping technique, an adaptive fuzzy finite-time tracking control protocol with adaptive update laws is designed. By the simulation analysis, the control protocol proposed in this paper can guarantee the tracking error of each follower agent converges to a small interval within a finite time.

The rest of this paper is planned as follows. In Section II, the preliminaries and problem formulation are introduced. The detailed process of adaptive fuzzy finite-time tracking control protocol design is given in Section III, and Section IV provides the simulation example to verify the effectiveness of the theoretical analysis. Finally, the conclusions are drawn in Section V.

II. PRELIMINARIES

A. GRAPH THEORY

Let the directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ represent the communication relationship among agents, where $\mathcal{V} = \{v_1, \cdots, v_n\}$ is the set of follower agents and $\mathcal{E} = \{(i, j)|i, j \in \mathcal{V}, \text{ and } i \neq j\}$ is the set of edges. The adjacency matrix is denoted as $\mathcal{A} = \left[a_{ij}\right] \in R^{N\times N}$. If there exists an edge between follower agent i and j, then $a_{ij} = 1$, and $a_{ij} = 0$ otherwise. The neighbors of follower agent i is defined as $\mathcal{N}_i = \left\{v_j : (v_j, v_i) \in \mathcal{E}\right\}$ with the information of follower agent j can be received by follower agent i. The Laplacian matrix $\mathcal{L}$ is described as $\mathcal{L} = \mathcal{D} - \mathcal{A}$, where $\mathcal{D} = \text{diag}\{d_1, \cdots, d_N\}$ with $d_i = \sum_{j=1}^{\mathcal{N}_i} a_{ij}$.

Furthermore, let the augmented graph $\bar{\mathcal{G}}$ consist of the graph $\mathcal{G}$ and the leader agent. The connection matrix is given as a diagonal matrix $\mathcal{B} = \text{diag}\{b_1, \cdots, b_N\}$ with b_i denoting the connection between the leader agent and the follower agent i. If the follower agent i obtains the information of leader, one has $b_i = 1$ and $b_i = 0$ otherwise. It can be obtained that $\mathcal{H} = \mathcal{L} + \mathcal{B}$ is a matrix associated with $\bar{\mathcal{G}}$. The graph $\bar{\mathcal{G}}$ is connected if there exists a spanning tree in the graph $\bar{\mathcal{G}}$ and the leader agent is the root node.

B. PROBLEM FORMULATION

Consider a class of uncertain nonaffine multi-agent systems with $N(N > 1)$ follower agents and one leader agent. The number from 1 to N represents follower agent and 0 is leader agent. The dynamics of the ith follower agent is given as:

$$\begin{aligned}
\dot{\chi}_{i,m} &= \phi_{i,m}\left(\bar{\chi}_{i,m}, \chi_{i,m+1}\right) + \Delta_{i,m}\left(\bar{\chi}_{i,m}, t\right), m = 1, \cdots, n-1 \\
\dot{\chi}_{i,n} &= \phi_{i,n}\left(\bar{\chi}_{i,n}, Q(u_i)\right) + \Delta_{i,n}\left(\bar{\chi}_{i,n}, t\right) \\
y_i &= \chi_{i,1}
\end{aligned} \tag{1}$$

where $\chi_{i,m} \in R$ is the mth state of follower agent i, $\bar{\chi}_{i,m} = \left[\chi_{i,1}, \cdots, \chi_{i,m}\right]^{\mathrm{T}} \in R^m$ is the state vector with $m = 1, \cdots, n$, $y_i \in R$ is the output, $u_i \in R$ is the control input, $\phi_{i,m}\left(\bar{\chi}_{i,m}, \chi_{i,m+1}\right) : R^{m+1} \to R$ is an unknown smooth function, $\Delta_{i,m}\left(\bar{\chi}_{i,m}, t\right)$ represents the unknown uncertain nonlinearity, and n is the order of follower agent i. $Q(u_i)$ represents the quantized input, which is described as:

$$Q(u_i) = \begin{cases} \alpha_{i,j}\mathrm{sgn}(u_i) & \begin{cases} \dfrac{\alpha_{i,j}}{1+\beta} < |u_i| \leq \alpha_{i,j}, & \dot{u}_i < 0, \text{ or} \\ \alpha_{i,j} < |u_i| \leq \dfrac{\alpha_{i,j}}{1-\beta}, & \dot{u}_i > 0 \end{cases} \\ \alpha_{i,j}(1+\beta)\mathrm{sgn}(u_i) & \begin{cases} \alpha_{i,j} < |u_i| \leq \dfrac{\alpha_{i,j}}{1-\beta}, & \dot{u}_i < 0, \text{ or} \\ \dfrac{\alpha_{i,j}}{1-\beta} < |u_i| \leq \dfrac{\alpha_{i,j}(1+\beta)}{1-\beta}, & \dot{u}_i > 0 \end{cases} \\ 0 & \begin{cases} 0 \leq |u_i| < \dfrac{\alpha_{\min}}{1+\beta}, & \dot{u}_i < 0, \text{ or} \\ \dfrac{\alpha_{\min}}{1+\beta} \leq |u_i| < \alpha_{\min}, & \dot{u}_i > 0 \end{cases} \\ Q(u_i(t^-)) & \text{otherwise} \end{cases} \tag{2}$$

where $\alpha_{i,j} = \rho^{1-j}\alpha_{\min} (j = 1, 2, \cdots)$ and $\beta = \frac{1-\rho}{1+\rho}$ with parameters $\alpha_{\min} > 0$ and $0 < \rho < 1$ being the quantization density.

According to the differential midvalue theorem [34], the dynamics of follower agents (1) can be re-described as:

$$\begin{aligned} \dot{\chi}_{i,m} &= \phi_{i,m}(\bar{\chi}_{i,m}, 0) + \varphi_{i,m}(\bar{\chi}_{i,m}, \eta_{i,m})\chi_{i,m+1} + \Delta_{i,m}(\bar{\chi}_{i,m}, t) \\ \dot{\chi}_{i,n} &= \phi_{i,n}(\bar{\chi}_{i,n}, 0) + \varphi_{i,n}(\bar{\chi}_{i,n}, \eta_{i,n})Q(u_i) + \Delta_{i,n}(\bar{\chi}_{i,n}, t) \\ y_i &= \chi_{i,1}, \end{aligned} \tag{3}$$

where $\varphi_{i,m}(\bar{\chi}_{i,m}, \eta_{i,m}) = \partial\phi_{i,m}(\bar{\chi}_{i,m}, \chi_{i,m+1})\big/\partial\chi_{i,m+1} |_{\chi_{i,m+1}=\eta_{i,m}}$ with $\eta_{i,m}$ being some points between 0 and $\chi_{i,m+1}$, $\varphi_{i,n}(\bar{\chi}_{i,n}, \eta_{i,n}) = \partial\phi_{i,n}(\bar{\chi}_{i,n}, Q(u_i))\big/\partial Q(u_i)|_{Q(u_i)=\eta_{i,n}}$ with $\eta_{i,n}$ being some points between 0 and $Q(u_i)$.

Assumption 1: The signs of $\varphi_{i,m}(\bar{\chi}_{i,m}, \eta_{i,m})$ are known, and $\varphi_{i,m}(\bar{\chi}_{i,m}, \eta_{i,m})$ are bounded such that $0 < \underline{\varphi}_{i,m} \leq \left|\varphi_{i,m}(\bar{\chi}_{i,m}, \eta_{i,m})\right| \leq \bar{\varphi}_{i,m} < \infty (m = 1, \cdots, n)$. Without loss of generality, it is assumed that $\varphi_{i,m}(\bar{\chi}_{i,m}, \eta_{i,m}) > 0$.

Assumption 2: The uncertain nonlinearity $\Delta_{i,m}(\bar{\chi}_{i,m}, t)$ $(m = 1, \cdots, n)$ are bounded such that $\left|\Delta_{i,m}(\bar{\chi}_{i,m}, t)\right| \leq \bar{\Delta}_{i,m}$ with $\bar{\Delta}_{i,m} > 0$ being unknown constants.

Lemma 1 [35]: The quantized input $Q(u_i)$ can be decomposed into as follows.

$$Q(u_i) = H(u_i)u_i + G(t) \tag{4}$$

where $H(u_i)$ and $G(t)$ satisfy:

$$1 - \beta \leq H(u_i) \leq 1 + \beta, |G(t)| \leq \alpha_{\min} \tag{5}$$

In the subsequent analysis, some necessary lemmas will be considered.

Lemma 2 [36]: Consider differential equation $\dot{\hat{\nu}}(t) = -a\hat{\nu}(t) + b\psi(t)$, where $a > 0$ and $b > 0$ represent design parameters, if there exist $\hat{\nu}(t_0) \geq 0$ and $\psi(t) \geq 0$, then one has $\nu(t) \geq 0$ for $t > t_0$, which also implies $\hat{\nu}(t) > 0$ for $t > t_0$.

Lemma 3 [37]: Consider the system $\dot{\varsigma} = f(\varsigma, s)$ for smooth positive definite function $V(\varsigma) \in C^1$, if there exist constants $c > 0$, $d > 0$ and $\mu \in (0, 1)$, satisfying that:

$$\dot{V}(\varsigma) \leq -cV^{\mu}(\varsigma) + d, \forall t \geq 0 \tag{6}$$

Then the system $\dot{\varsigma} = f(\varsigma, s)$ is semi-global practical finite-time stable (SGPFS).

Lemma 4 [38]: For $\iota_k \in R (k = 1, \cdots, n)$ and $0 < \mu \leq 1$, one has:

$$\left(\sum_{k=1}^{n} \iota_k\right)^{\mu} \leq \sum_{k=1}^{n} |\iota_k|^{\mu} \leq n^{1-\mu}\left(\sum_{k=1}^{n} |\iota_k|\right)^{\mu} \tag{7}$$

Lemma 5 [39]: For any real variables o and ω, the following inequality is held:

$$|o|^{\tau} |\omega|^{\gamma} \leq \frac{\tau}{\tau+\gamma}\varrho\, |o|^{\tau+\gamma} + \frac{\gamma}{\tau+\gamma}\varrho^{-\frac{\tau}{\gamma}} |\omega|^{\tau+\gamma} \tag{8}$$

where $\tau > 0$, $\gamma > 0$ and $\varrho > 0$ denote design parameters.

Lemma 6 [40]: For any $h \in R$ and $\kappa > 0$, the hyperbolic tangent function $\tanh(\cdot)$ has the following property:

$$0 \leq |h| - h\tanh\left(\frac{h}{\kappa}\right) \leq 0.2785\kappa \tag{9}$$

Lemma 7 [41]*:* Let $\mathcal{F}(x) : \Omega \to R$ be a continuous function given on a compact set $\Omega \in R^n$. Then, for any constant $\varepsilon > 0$, there exist a fuzzy logic systems $W^{*\mathrm{T}}\xi(x)$ such that $\sup_{x\in\Omega}\left|\mathcal{F}(x) - W^{*T}\xi(x)\right| \leq \varepsilon$, where $W^* = \left[W_1^*, \cdots, W_L^*\right]^{\mathrm{T}}$ is the ideal weight vector, and $\xi(x) = [\xi_1(x), \cdots, \xi_L(x)]^{\mathrm{T}}$ is fuzzy basis function composed of Gaussian functions. L represents the number of fuzzy rules.

The control objective of this paper is to design the adaptive fuzzy control law for each follower agent such that the output of all follower agents can track the output trajectory of leader agent.

III. ADAPTIVE FUZZY TRACKING CONTROL DESIGN

According to the multi-agent systems (3), the tracking error of follower agent i can be designed as $e_i = y_i - y_0 (i = 1, \cdots, N)$ and y_0 is the output of leader agent, then one has $e = [e_1, \cdots, e_N]^{\mathrm{T}}$.

Let $z_{i,1}$ be the local consensus error of follower agent i, hence, we have:

$$z_{i,1} = \sum_{j\in\mathcal{N}_i} a_{ij}(y_i - y_j) + b_i(y_i - y_0) \tag{10}$$

Furthermore, the entire consensus error $z_{e,1}$ is given as:

$$z_{e,1} = (\mathcal{D} - \mathcal{A})y + \mathcal{B}(y - \mathbf{1}y_0) = (\mathcal{L} + \mathcal{B})e = \mathcal{H}e \tag{11}$$

where $\mathbf{1} = [1, \cdots, 1]^{\mathrm{T}}$.

For (10), taking the time derivative of $z_{i,1}$ gets:

$$\dot{z}_{i,1} = \sum_{j\in\mathcal{N}_i} a_{ij}(\dot{y}_i - \dot{y}_j) + b_i(\dot{y}_i - \dot{y}_0)$$

$$= (d_i + b_i)\left[f_{i,1}(Z_{i,1}) - \frac{1}{d_i + b_i}\sum_{j\in\mathcal{N}_i} a_{ij}\Delta_{j,1}(\chi_{j,1}, t) + \varphi_{i,1}(\chi_{i,1}, \eta_{i,1})\chi_{i,2} + \Delta_{i,1}(\chi_{i,1}, t)\right] \tag{12}$$

where $f_{i,1}(Z_{i,1}) = -\frac{1}{d_i+b_i}\sum_{j\in\mathcal{N}_i} a_{ij}(\phi_{j,1}(\chi_{j,1}, 0) + \varphi(\chi_{j,1}, \eta_{j,1})\chi_{j,2}) + \phi_{i,1}(\chi_{i,1}, 0) - \frac{1}{d_i+b_i} b_i\dot{y}_0$ and $Z_{i,1} = [\chi_{i,1}, \chi_{j,1}, \chi_{j,2}, b_i\dot{y}_0]^{\mathrm{T}}$ with $j \in \mathcal{N}_i$.

The adaptive fuzzy tracking control law for follower agent i is designed by applying the backstepping technique with n steps.

Step 1: According to the Lemma 7, over a compact set Ω, the function $f_{i,1}(Z_{i,1})$ can be approximated by:

$$f_{i,1}(Z_{i,1}) = W_{i,1}^{*T}\xi(Z_{i,1}) + \varepsilon(Z_{i,1}) \tag{13}$$

where $W_{i,1}^*$ is the ideal weight, $\varepsilon(Z_{i,1})$ is the approximation error and satisfying $|\varepsilon(Z_{i,1})| \le \bar{\varepsilon}_{i,1}$ with $\bar{\varepsilon}_{i,1} > 0$ being a constant. Then, we have:

$$z_{i,1}f_{i,1}(Z_{i,1}) \le |z_{i,1}|\left(\frac{1}{2}\left\|W_{i,1}^*\right\|^2\xi^{\mathrm{T}}(Z_{i,1})\xi(Z_{i,1}) + \frac{1}{2} + \bar{\varepsilon}_{i,1}\right) \tag{14}$$

Define $\upsilon_{i,1} = \frac{1}{2\bar{\varphi}_{i,1}}\left\|W_{i,1}^*\right\|^2$ and $\delta_{i,1} = \frac{1}{\bar{\varphi}_{i,1}}\left(\frac{1}{d_i+b_i}\sum_{j\in\mathcal{N}_i} a_{ij}\bar{\Delta}_{j,1} + \frac{1}{2} + \bar{\varepsilon}_{i,1} + \bar{\Delta}_{i,1}\right)$. Due to $\upsilon_{i,1}$ and $\delta_{i,1}$ are unknown, let $\hat{\upsilon}_{i,1}$ and $\hat{\delta}_{i,1}$ be their estimates, respectively, and their estimation error are defined as $\tilde{\upsilon}_{i,1} = \upsilon_{i,1} - \hat{\upsilon}_{i,1}$ and $\tilde{\delta}_{i,1} = \delta_{i,1} - \hat{\delta}_{i,1}$. Thereafter, consider the following Lyapunov function candidate

$$V_{i,1} = \frac{1}{2\bar{\varphi}_{i,1}}z_{i,1}^2 + \frac{1}{2\theta_{i,1}}\tilde{\upsilon}_{i,1}^2 + \frac{1}{2\pi_{i,1}}\tilde{\delta}_{i,1}^2 \tag{15}$$

where $\theta_{i,1} > 0$ and $\pi_{i,1} > 0$ are constants.

Taking the time derivative of $V_{i,1}$ and considering (12), (14), $\upsilon_{i,1}$ and $\delta_{i,1}$, we have:

$$\begin{aligned}
\dot{V}_{i,1} &= \frac{z_{i,1}\dot{z}_{i,1}}{\bar{\varphi}_{i,1}} - \frac{\tilde{\upsilon}_{i,1}\dot{\hat{\upsilon}}_{i,1}}{\theta_{i,1}} - \frac{\tilde{\delta}_{i,1}\dot{\hat{\delta}}_{i,1}}{\pi_{i,1}}\\
&= \frac{1}{\bar{\varphi}_{i,1}}(d_i+b_i)z_{i,1}\left[f_{i,1}(Z_{i,1}) - \frac{1}{d_i+b_i}\sum_{j\in\mathcal{N}_i} a_{ij}\Delta_{j,1}(\chi_{j,1}, t) + \Delta_{i,1}(\chi_{i,1}, t)\right] + \frac{1}{\bar{\varphi}_{i,1}}(d_i + b_i)z_{i,1}\varphi_{i,1}(\chi_{i,1}, \eta_{i,1})\chi_{i,2}\\
&\quad - \frac{\tilde{\upsilon}_{i,1}\dot{\hat{\upsilon}}_{i,1}}{\theta_{i,1}} - \frac{\tilde{\delta}_{i,1}\dot{\hat{\delta}}_{i,1}}{\pi_{i,1}}\\
&\le \frac{1}{\bar{\varphi}_{i,1}}(d_i + b_i)|z_{i,1}|\left[\frac{1}{2}\left\|W_{i,1}^*\right\|^2\xi^{\mathrm{T}}(Z_{i,1})\xi(Z_{i,1}) + \frac{1}{2} + \bar{\varepsilon}_{i,1} + \bar{\Delta}_{i,1} + \frac{1}{d_i + b_i}\sum_{j\in\mathcal{N}_i} a_{ij}\bar{\Delta}_{j,1}\right] - \frac{\tilde{\upsilon}_{i,1}\dot{\hat{\upsilon}}_{i,1}}{\theta_{i,1}} - \frac{\tilde{\delta}_{i,1}\dot{\hat{\delta}}_{i,1}}{\pi_{i,1}}\\
&\quad + \frac{1}{\bar{\varphi}_{i,1}}(d_i + b_i)z_{i,1}\varphi_{i,1}(\chi_{i,1}, \eta_{i,1})\chi_{i,2}\\
&= (d_i + b_i)|z_{i,1}|\upsilon_{i,1}\xi^{\mathrm{T}}(Z_{i,1})\xi(Z_{i,1}) + (d_i + b_i)|z_{i,1}|\delta_{i,1}\\
&\quad + \frac{1}{\bar{\varphi}_{i,1}}(d_i + b_i)z_{i,1}\varphi_{i,1}(\chi_{i,1}, \eta_{i,1})\chi_{i,2} - \frac{\tilde{\upsilon}_{i,1}\dot{\hat{\upsilon}}_{i,1}}{\theta_{i,1}} - \frac{\tilde{\delta}_{i,1}\dot{\hat{\delta}}_{i,1}}{\pi_{i,1}}
\end{aligned} \tag{16}$$

Let the error variable $z_{i,2} = \chi_{i,2} - \sigma_{i,2}$, then we get:

$$\begin{aligned}
\dot{V}_{i,1} &\le (d_i + b_i)|z_{i,1}|\upsilon_{i,1}\xi^{\mathrm{T}}(Z_{i,1})\xi(Z_{i,1}) + (d_i + b_i)|z_{i,1}|\delta_{i,1}\\
&\quad + \frac{1}{\bar{\varphi}_{i,1}}(d_i + b_i)z_{i,1}\varphi_{i,1}(\chi_{i,1}, \eta_{i,1})\sigma_{i,2} - \frac{\tilde{\upsilon}_{i,1}\dot{\hat{\upsilon}}_{i,1}}{\theta_{i,1}}\\
&\quad + \frac{1}{\bar{\varphi}_{i,1}}(d_i + b_i)z_{i,1}\varphi_{i,1}(\chi_{i,1}, \eta_{i,1})z_{i,2} - \frac{\tilde{\delta}_{i,1}\dot{\hat{\delta}}_{i,1}}{\pi_{i,1}}.
\end{aligned} \tag{17}$$

Design the virtual control law $\sigma_{i,2}$ as:

$$\begin{aligned}
\sigma_{i,2} &= -\xi^{\mathrm{T}}(Z_{i,1})\xi(Z_{i,1})\hat{\upsilon}_{i,1} \times \tanh\left(\frac{(d_i + b_i)z_{i,1}\xi^{\mathrm{T}}(Z_{i,1})\xi(Z_{i,1})\hat{\upsilon}_{i,1}}{\kappa}\right)\\
&\quad - \hat{\delta}_{i,1}\tanh\left(\frac{(d_i + b_i)z_{i,1}\hat{\delta}_{i,1}}{\kappa}\right) - \frac{1}{d_i + b_i}p_{i,1}z_{i,1}^{2\mu-1}
\end{aligned} \tag{18}$$

where $p_{i,1} > 0$ and $\mu = \frac{2m-1}{2m+1}(m \ge 2, m \in n)$ are design parameters, respectively.

The adaptive update laws of $\hat{\upsilon}_{i,1}$ and $\hat{\delta}_{i,1}$ are designed as:

$$\begin{aligned}
\dot{\hat{\upsilon}}_{i,1} &= \theta_{i,1}(d_i + b_i)|z_{i,1}|\xi^{\mathrm{T}}(Z_{i,1})\xi(Z_{i,1}) - q_{i,1}\hat{\upsilon}_{i,1},\ \hat{\upsilon}_{i,1}(0) \ge 0\\
\dot{\hat{\delta}}_{i,1} &= \pi_{i,1}(d_i + b_i)|z_{i,1}| - r_{i,1}\hat{\delta}_{i,1},\ \hat{\delta}_{i,1}(0) \ge 0
\end{aligned} \tag{19}$$

where $q_{i,1} > 0$ and $r_{i,1} > 0$ represent design constants.

According to the Lemma 2, we easily obtain $\hat{\upsilon}_{i,1} \ge 0$ and $\hat{\delta}_{i,1} \ge 0$. Substituting (18) and (19) into (17) yields:

$$\begin{aligned}
\dot{V}_{i,1} &\le (d_i + b_i)|z_{i,1}|\xi^{\mathrm{T}}(Z_{i,1})\xi(Z_{i,1})\hat{\upsilon}_{i,1} + (d_i + b_i)|z_{i,1}|\hat{\delta}_{i,1} - \frac{\tilde{\upsilon}_{i,1}\dot{\hat{\upsilon}}_{i,1}}{\theta_{i,1}}\\
&\quad + (d_i+b_i)|z_{i,1}|\xi^{\mathrm{T}}(Z_{i,1})\xi(Z_{i,1})\tilde{\upsilon}_{i,1} + (d_i+b_i)|z_{i,1}|\tilde{\delta}_{i,1}\\
&\quad + \frac{1}{\bar{\varphi}_{i,1}}(d_i + b_i)z_{i,1}\varphi_{i,1}(\chi_{i,1}, \eta_{i,1})\sigma_{i,2} - \frac{\tilde{\delta}_{i,1}\dot{\hat{\delta}}_{i,1}}{\pi_{i,1}}\\
&\quad + \frac{1}{\bar{\varphi}_{i,1}}(d_i + b_i)z_{i,1}\varphi_{i,1}(\chi_{i,1}, \eta_{i,1})z_{i,2}\\
&\le -p_{i,1}z_{i,1}^{2\mu} + (d_i + b_i)|z_{i,1}|\hat{\delta}_{i,1} + (d_i + b_i)\\
&\quad \times\left[|z_{i,1}|\xi^{\mathrm{T}}(Z_{i,1})\xi(Z_{i,1})\hat{\upsilon}_{i,1} - z_{i,1}\xi^{\mathrm{T}}(Z_{i,1})\xi(Z_{i,1})\hat{\upsilon}_{i,1} \times \tanh\left(\frac{(d_i + b_i)z_{i,1}\xi^{\mathrm{T}}(Z_{i,1})\xi(Z_{i,1})\hat{\upsilon}_{i,1}}{\kappa}\right)\right]\\
&\quad - (d_i + b_i)z_{i,1}\hat{\delta}_{i,1} \times \tanh\left(\frac{(d_i + b_i)z_{i,1}\hat{\delta}_{i,1}}{\kappa}\right) - \frac{\tilde{\upsilon}_{i,1}\dot{\hat{\upsilon}}_{i,1}}{\theta_{i,1}} - \frac{\tilde{\delta}_{i,1}\dot{\hat{\delta}}_{i,1}}{\pi_{i,1}}\\
&\quad + (d_i + b_i)|z_{i,1}|\xi^{\mathrm{T}}(Z_{i,1})\xi(Z_{i,1})\tilde{\upsilon}_{i,1} + (d_i+b_i)|z_{i,1}|\tilde{\delta}_{i,1}
\end{aligned}$$

$$
\begin{aligned}
&+\frac{1}{\bar{\varphi}_{i,1}}(d_i+b_i)z_{i,1}\varphi_{i,1}(\chi_{i,1},\eta_{i,1})z_{i,2}\\
&\le -p_{i,1}z_{i,1}^{2\mu}+0.557\kappa+\frac{q_{i,1}\tilde{\upsilon}_{i,1}\hat{\upsilon}_{i,1}}{\theta_{i,1}}+\frac{r_{i,1}\tilde{\delta}_{i,1}\hat{\delta}_{i,1}}{\pi_{i,1}}\\
&+\frac{1}{\bar{\varphi}_{i,1}}(d_i+b_i)z_{i,1}\varphi_{i,1}(\chi_{i,1},\eta_{i,1})z_{i,2} \qquad (20)
\end{aligned}
$$

Step 2: Due to $z_{i,2}=\chi_{i,2}-\sigma_{i,2}$, then the time derivative of $z_{i,2}$ is given as:

$$
\dot{z}_{i,2}=\phi(\bar{\chi}_{i,2},0)+\varphi_{i,2}(\bar{\chi}_{i,2},\eta_{i,2})\chi_{i,3}+\Delta_{i,2}(\bar{\chi}_{i,2},t)-\dot{\sigma}_{i,2} \qquad (21)
$$

Considering (18), the derivative of $\sigma_{i,2}$ can be described as:

$$
\begin{aligned}
\dot{\sigma}_{i,2}=&\frac{\partial\sigma_{i,2}}{\partial\chi_{i,1}}\left(\phi(\chi_{i,1},0)+\varphi_{i,1}(\chi_{i,1},\eta_{i,1})\chi_{i,2}\right)+\vartheta_{i,2}+\zeta_{i,2}\\
&+\sum_{j\in\mathcal{N}_i}\frac{\partial\sigma_{i,2}}{\partial\chi_{j,1}}\left(\phi(\chi_{j,1},0)+\varphi_{j,1}(\chi_{j,1},\eta_{j,1})\chi_{j,2}\right) \qquad (22)
\end{aligned}
$$

where $\vartheta_{i,2}=\frac{\partial\sigma_{i,2}}{\partial y_0}\dot{y}_0+\frac{\partial\sigma_{i,2}}{\partial\dot{y}_0}\ddot{y}_0+\frac{\partial\sigma_{i,2}}{\partial\hat{\upsilon}_{i,1}}\dot{\hat{\upsilon}}_{i,1}+\frac{\partial\sigma_{i,2}}{\partial\hat{\delta}_{i,1}}\dot{\hat{\delta}}_{i,1}$, $\zeta_{i,2}=\frac{\partial\sigma_{i,2}}{\partial\chi_{i,1}}\Delta_{i,1}(\chi_{i,1},t)+\sum_{j\in\mathcal{N}_i}\frac{\partial\sigma_{i,2}}{\partial\chi_{j,1}}\Delta_{j,1}(\chi_{j,1},t)$.

Applying the Lemma 7, we get:

$$
\begin{aligned}
&\phi_{i,2}(\bar{\chi}_{i,2},0)-\dot{\sigma}_{i,2}+\frac{1}{\bar{\varphi}_{i,1}}\bar{\varphi}_{i,2}(d_i+b_i)z_{i,1}\varphi_{i,1}(\chi_{i,1},\eta_{i,1})\\
&=W_{i,2}^{*\mathrm{T}}\xi(Z_{i,2})+\varepsilon(Z_{i,2}) \qquad (23)
\end{aligned}
$$

where $W_{i,2}^*$ is the ideal weight, and $\varepsilon(Z_{i,2})$ is the approximation error satisfying $|\varepsilon(Z_{i,2})|\le\bar{\varepsilon}_{i,2}$ with $\bar{\varepsilon}_{i,2}$ being a positive constant, $Z_{i,2}=\left[\bar{\chi}_{i,2},\bar{\chi}_{j,2},z_{i,1},\vartheta_{i,2},\zeta_{i,2},(\partial\sigma_{i,2}/\partial\chi_{i,1}),(\partial\sigma_{i,2}/\partial\chi_{j,1})\right]^{\mathrm{T}}$ with $j\in\mathcal{N}_i$. Hence, we have:

$$
\begin{aligned}
&z_{i,2}\left(\phi_{i,2}(\bar{\chi}_{i,2},0)-\dot{\sigma}_{i,2}+\frac{\bar{\varphi}_{i,2}}{\bar{\varphi}_{i,1}}(d_i+b_i)z_{i,1}\varphi_{i,1}(\chi_{i,1},\eta_{i,1})\right)\\
&\le|z_{i,2}|\left(\frac{1}{2}\left\|W_{i,2}^*\right\|^2\xi^{\mathrm{T}}(Z_{i,2})\xi(Z_{i,2})+\frac{1}{2}+\bar{\varepsilon}_{i,2}\right) \qquad (24)
\end{aligned}
$$

Define $\upsilon_{i,2}=\frac{1}{2\bar{\varphi}_{i,2}}\left\|W_{i,2}^*\right\|^2$ and $\delta_{i,2}=\frac{1}{\bar{\varphi}_{i,2}}\left(\frac{1}{2}+\bar{\varepsilon}_{i,2}+\bar{\Delta}_{i,2}\right)$, consider the following Lyapunov function candidate:

$$
V_{i,2}=V_{i,1}+\frac{1}{2\bar{\varphi}_{i,2}}z_{i,2}^2+\frac{1}{2\theta_{i,2}}\tilde{\upsilon}_{i,2}^2+\frac{1}{2\pi_{i,2}}\tilde{\delta}_{i,2}^2 \qquad (25)
$$

The time derivative of $V_{i,2}$ can be written as:

$$
\begin{aligned}
\dot{V}_{i,2}=&\dot{V}_{i,1}+\frac{z_{i,2}\dot{z}_{i,2}}{\bar{\varphi}_{i,2}}-\frac{\tilde{\upsilon}_{i,2}\dot{\hat{\upsilon}}_{i,2}}{\theta_{i,2}}-\frac{\tilde{\delta}_{i,2}\dot{\hat{\delta}}_{i,2}}{\pi_{i,2}}\\
\le&-\frac{\tilde{\upsilon}_{i,2}\dot{\hat{\upsilon}}_{i,2}}{\theta_{i,2}}-\frac{\tilde{\delta}_{i,2}\dot{\hat{\delta}}_{i,2}}{\pi_{i,2}}-\frac{1}{\bar{\varphi}_{i,1}}(d_i+b_i)z_{i,1}\varphi_{i,1}(\chi_{i,1},\eta_{i,1})z_{i,2}\\
&+\dot{V}_{i,1}+\frac{1}{\bar{\varphi}_{i,2}}|z_{i,2}|\left(\frac{1}{2}\left\|W_{i,2}^*\right\|^2\xi^{\mathrm{T}}(Z_{i,2})\xi(Z_{i,2})+\frac{1}{2}\right.\\
&\left.+\bar{\varepsilon}_{i,2}+\bar{\Delta}_{i,2}\right)+\frac{1}{\bar{\varphi}_{i,2}}z_{i,2}\varphi_{i,2}(\bar{\chi}_{i,2},\eta_{i,2})\chi_{i,3} \qquad (26)
\end{aligned}
$$

Consider the error variable $z_{i,3}=\chi_{i,3}-\sigma_{i,3}$, $\upsilon_{i,2}$ and $\delta_{i,2}$, then we have:

$$
\begin{aligned}
\dot{V}_{i,2}\le&|z_{i,2}|\xi^{\mathrm{T}}(Z_{i,2})\xi(Z_{i,2})\upsilon_{i,2}+\frac{1}{\bar{\varphi}_{i,2}}z_{i,2}\varphi_{i,2}(\bar{\chi}_{i,2},\eta_{i,2})\sigma_{i,3}\\
&+\frac{1}{\bar{\varphi}_{i,2}}z_{i,2}\varphi_{i,2}(\bar{\chi}_{i,2},\eta_{i,2})z_{i,3}-\frac{\tilde{\upsilon}_{i,2}\dot{\hat{\upsilon}}_{i,2}}{\theta_{i,2}}-\frac{\tilde{\delta}_{i,2}\dot{\hat{\delta}}_{i,2}}{\pi_{i,2}}+\dot{V}_{i,1}\\
&-\frac{1}{\bar{\varphi}_{i,1}}(d_i+b_i)z_{i,1}\varphi_{i,1}(\chi_{i,1},\eta_{i,1})z_{i,2}+|z_{i,2}|\delta_{i,2} \qquad (27)
\end{aligned}
$$

where $p_{i,2}>0$ is a constant.

Design the virtual control law $\sigma_{i,3}$ as:

$$
\begin{aligned}
\sigma_{i,3}=&-\xi^{\mathrm{T}}(Z_{i,2})\xi(Z_{i,2})\hat{\upsilon}_{i,2}\tanh\left(\frac{z_{i,2}\xi^{\mathrm{T}}(Z_{i,2})\xi(Z_{i,2})\hat{\upsilon}_{i,2}}{\kappa}\right)\\
&-\hat{\delta}_{i,2}\tanh\left(\frac{z_{i,2}\hat{\delta}_{i,2}}{\kappa}\right)-p_{i,2}z_{i,2}^{2\mu-1} \qquad (28)
\end{aligned}
$$

The adaptive update laws $\hat{\upsilon}_{i,2}$ and $\hat{\delta}_{i,2}$ are designed as:

$$
\begin{aligned}
\dot{\hat{\upsilon}}_{i,2}&=\theta_{i,2}|z_{i,2}|\xi^{\mathrm{T}}(Z_{i,2})\xi(Z_{i,2})-q_{i,2}\hat{\upsilon}_{i,2},\ \hat{\upsilon}_{i,2}(0)\ge 0\\
\dot{\hat{\delta}}_{i,2}&=\pi_{i,2}|z_{i,2}|-r_{i,2}\hat{\delta}_{i,2},\ \hat{\delta}_{i,2}(0)\ge 0 \qquad (29)
\end{aligned}
$$

where $q_{i,2}>0$ and $r_{i,2}>0$ are design constants.

Substituting (28) and (29) into (27) has:

$$
\begin{aligned}
\dot{V}_{i,2}\le&\dot{V}_{i,1}-z_{i,2}\xi^{\mathrm{T}}(Z_{i,2})\xi(Z_{i,2})\hat{\upsilon}_{i,2}\\
&\times\tanh\left(\frac{z_{i,2}\xi^{\mathrm{T}}(Z_{i,2})\xi(Z_{i,2})\hat{\upsilon}_{i,2}}{\kappa}\right)\\
&-z_{i,2}\hat{\delta}_{i,2}\tanh\left(\frac{z_{i,2}\hat{\delta}_{i,2}}{\kappa}\right)-\frac{\tilde{\upsilon}_{i,2}\dot{\hat{\upsilon}}_{i,2}}{\theta_{i,2}}-\frac{\tilde{\delta}_{i,2}\dot{\hat{\delta}}_{i,2}}{\pi_{i,2}}+|z_{i,2}|\hat{\delta}_{i,2}\\
&+\frac{1}{\bar{\varphi}_{i,2}}z_{i,2}\varphi_{i,2}(\bar{\chi}_{i,2},\eta_{i,2})\sigma_{i,3}+\frac{1}{\bar{\varphi}_{i,2}}z_{i,2}\varphi_{i,2}(\bar{\chi}_{i,2},\eta_{i,2})z_{i,3}\\
&-\frac{1}{\bar{\varphi}_{i,1}}(d_i+b_i)z_{i,1}\varphi_{i,1}(\chi_{i,1},\eta_{i,1})z_{i,2}+\tilde{\delta}_{i,2}|z_{i,2}|-p_{i,2}z_{i,2}^{2\mu}\\
&+|z_{i,2}|\xi^{\mathrm{T}}(Z_{i,2})\xi(Z_{i,2})\hat{\upsilon}_{i,2}+\tilde{\upsilon}_{i,2}|z_{i,2}|\xi^{\mathrm{T}}(Z_{i,2})\xi(Z_{i,2})\\
\le&\dot{V}_{i,1}+\frac{1}{\bar{\varphi}_{i,2}}z_{i,2}\varphi_{i,2}(\bar{\chi}_{i,2},\eta_{i,2})z_{i,3}+\frac{q_{i,2}\tilde{\upsilon}_{i,2}\hat{\upsilon}_{i,2}}{\theta_{i,2}}\\
&+\frac{r_{i,2}\tilde{\delta}_{i,2}\hat{\delta}_{i,2}}{\pi_{i,2}}\\
&-\frac{1}{\bar{\varphi}_{i,1}}(d_i+b_i)z_{i,1}\varphi_{i,1}(\chi_{i,1},\eta_{i,1})z_{i,2}+0.557\kappa-p_{i,2}z_{i,2}^{2\mu} \qquad (30)
\end{aligned}
$$

Considering (20), then:

$$
\begin{aligned}
\dot{V}_{i,2}\le&-\sum_{k=1}^{2}p_{i,k}z_{i,k}^{2\mu}+\sum_{k=1}^{2}\frac{q_{i,k}\tilde{\upsilon}_{i,k}\hat{\upsilon}_{i,k}}{\theta_{i,k}}+\sum_{k=1}^{2}\frac{r_{i,k}\tilde{\delta}_{i,k}\hat{\delta}_{i,k}}{\pi_{i,k}}\\
&+\frac{1}{\bar{\varphi}_{i,2}}z_{i,2}\varphi_{i,2}(\bar{\chi}_{i,2},\eta_{i,2})z_{i,3}+2\times 0.557\kappa \qquad (31)
\end{aligned}
$$

Step m $(m\in\{3,\cdots,n-1\})$: The time derivative of $z_{i,m}=\chi_{i,m}-\sigma_{i,m}$ is given by:

$$
\begin{aligned}
\dot{z}_{i,m}=&\phi_{i,k}(\bar{\chi}_{i,m},0)+\varphi_{i,m}(\bar{\chi}_{i,m},\eta_{i,m})\chi_{i,m+1}\\
&+\Delta_{i,m}(\bar{\chi}_{i,m},t)-\dot{\sigma}_{i,m} \qquad (32)
\end{aligned}
$$

The derivative of $\sigma_{i,m}$ can be written as:

$$
\dot{\sigma}_{i,m}=\sum_{k=1}^{m-1}\frac{\partial\sigma_{i,k}}{\partial\chi_{i,k}}\left(\phi_{i,k}(\bar{\chi}_{i,k},0)+\varphi_{i,k}(\bar{\chi}_{i,k},\eta_{i,k})\chi_{i,k+1}\right)+\vartheta_{i,m}
$$

$$+\sum_{j\in\mathcal{N}_i}\frac{\partial\sigma_{i,m}}{\partial\chi_{j,1}}\left(\phi_{j,1}(\chi_{j,1},0)+\varphi_{j,1}(\chi_{j,1},\eta_{j,1})\chi_{j,2}\right)+\zeta_{i,m} \tag{33}$$

where $\vartheta_{i,m}=\frac{\partial\sigma_{i,m}}{\partial y_0}\dot{y}_0+\frac{\partial\sigma_{i,m}}{\partial\dot{y}_0}\ddot{y}_0+\sum_{k=1}^{m-1}\frac{\partial\sigma_{i,k}}{\partial\hat{\upsilon}_{i,k}}\dot{\hat{\upsilon}}_{i,k}+\sum_{k=1}^{m-1}\frac{\partial\sigma_{i,k}}{\partial\hat{\delta}_{i,k}}\dot{\hat{\delta}}_{i,k}$, $\zeta_{i,m}=\sum_{k=1}^{m-1}\frac{\partial\sigma_{i,k}}{\partial\chi_{i,k}}\Delta_{i,k}(\bar{\chi}_{i,k},t)+\sum_{j\in\mathcal{N}_i}\frac{\partial\sigma_{i,m}}{\partial\chi_{j,1}}\Delta_{j,1}(\chi_{j,1},t)$.

Applying the Lemma 7, we obtain:

$$\begin{aligned}&\phi_{i,m}(\bar{\chi}_{i,m},0)\\&\quad-\dot{\sigma}_{i,m}+\frac{1}{\bar{\varphi}_{i,m-1}}\bar{\varphi}_{i,m}z_{i,m-1}\varphi_{i,m-1}(\bar{\chi}_{i,m-1},\eta_{i,m-1})\\&\quad=W_{i,m}^{*\mathrm{T}}\xi(Z_{i,m})+\varepsilon(Z_{i,m})\end{aligned} \tag{34}$$

where $W_{i,m}^*$ is the ideal weight, and $\varepsilon(Z_{i,m})$ is the approximation error satisfying $|\varepsilon(Z_{i,m})|\le\bar{\varepsilon}_{i,m}$ with $\bar{\varepsilon}_{i,m}>0$ being a constant. $Z_{i,m}=\left[\bar{\chi}_{i,m},\bar{\chi}_{j,2},z_{i,m-1},(\partial\sigma_{i,m}/\partial\chi_{i,1}),\cdots,(\partial\sigma_{i,m}/\partial\chi_{i,m}),(\partial\sigma_{i,m}/\partial\chi_{j,1}),\zeta_{i,m},\vartheta_{i,m}\right]^{\mathrm{T}}$ with $j\in\mathcal{N}_i$. Thus, we get:

$$\begin{aligned}&z_{i,m}\Big(\phi_{i,m}(\bar{\chi}_{i,m},0)\\&\qquad+\frac{1}{\bar{\varphi}_{i,m-1}}\bar{\varphi}_{i,m}z_{i,m-1}\varphi_{i,m-1}(\bar{\chi}_{i,m-1},\eta_{i,m-1})-\dot{\sigma}_{i,m}\Big)\\&\quad\le|z_{i,m}|\left(\frac{1}{2}\left\|W_{i,m}^*\right\|^2\xi^{\mathrm{T}}(Z_{i,m})\xi(Z_{i,m})+\frac{1}{2}+\bar{\varepsilon}_{i,m}\right)\end{aligned} \tag{35}$$

Let $\upsilon_{i,m}=\frac{1}{2\bar{\varphi}_{i,m}}\left\|W_{i,m}^*\right\|^2$ and $\delta_{i,m}=\frac{1}{\bar{\varphi}_{i,m}}\left(\frac{1}{2}+\bar{\varepsilon}_{i,m}+\bar{\Delta}_{i,m}\right)$, consider the following Lyapunov function candidate:

$$V_{i,m}=V_{i,m-1}+\frac{1}{2\bar{\varphi}_{i,m}}z_{i,m}^2+\frac{1}{2\theta_{i,m}}\tilde{\upsilon}_{i,m}^2+\frac{1}{2\pi_{i,m}}\tilde{\delta}_{i,m}^2 \tag{36}$$

The derivative of $V_{i,m}$ can be expressed as:

$$\begin{aligned}\dot{V}_{i,m}&=\dot{V}_{i,m-1}+\frac{z_{i,m}\dot{z}_{i,m}}{\bar{\varphi}_{i,m}}-\frac{\tilde{\upsilon}_{i,m}\dot{\hat{\upsilon}}_{i,m}}{\theta_{i,m}}-\frac{\tilde{\delta}_{i,m}\dot{\hat{\delta}}_{i,m}}{\pi_{i,m}}\\&\le\dot{V}_{i,m-1}+\frac{1}{\bar{\varphi}_{i,m}}|z_{i,m}|\Big[\frac{1}{2}\left\|W_{i,m}^*\right\|^2\xi^{\mathrm{T}}(Z_{i,m})\xi(Z_{i,m})+\frac{1}{2}\\&\quad+\bar{\varepsilon}_{i,m}+\bar{\Delta}_{i,m}\Big]+\frac{1}{\bar{\varphi}_{i,m}}z_{i,m}\varphi_{i,m}(\bar{\chi}_{i,m},\eta_{i,m})\chi_{i,m+1}\\&\quad-\frac{1}{\bar{\varphi}_{i,m-1}}z_{i,m-1}\varphi_{i,m-1}(\bar{\chi}_{i,m-1},\eta_{i,m-1})z_{i,m}\\&\quad-\frac{\tilde{\upsilon}_{i,m}\dot{\hat{\upsilon}}_{i,m}}{\theta_{i,m}}-\frac{\tilde{\delta}_{i,m}\dot{\hat{\delta}}_{i,m}}{\pi_{i,m}}\end{aligned} \tag{37}$$

Define the virtual control law $z_{i,m+1}=\chi_{i,m+1}-\sigma_{i,m+1}$, and considering $\upsilon_{i,m}$ and $\delta_{i,m}$, we have:

$$\begin{aligned}\dot{V}_{i,m}&\le\frac{1}{\bar{\varphi}_{i,m}}z_{i,m}\varphi_{i,m}(\bar{\chi}_{i,m},\eta_{i,m})\sigma_{i,m+1}+\dot{V}_{i,m-1}+|z_{i,m}|\,\delta_{i,m}\\&\quad-\frac{1}{\bar{\varphi}_{i,m-1}}z_{i,m-1}\varphi_{i,m-1}(\bar{\chi}_{i,m-1},\eta_{i,m-1})z_{i,m}\\&\quad+\frac{1}{\bar{\varphi}_{i,m}}z_{i,m}\varphi_{i,m}(\bar{\chi}_{i,m},\eta_{i,m})z_{i,m+1}-\frac{\tilde{\upsilon}_{i,m}\dot{\hat{\upsilon}}_{i,m}}{\theta_{i,m}}\\&\quad+|z_{i,m}|\,\upsilon_{i,m}\xi^{\mathrm{T}}(Z_{i,m})\xi(Z_{i,m})-\frac{\tilde{\delta}_{i,m}\dot{\hat{\delta}}_{i,m}}{\pi_{i,m}}\end{aligned} \tag{38}$$

Design the virtual control law $\sigma_{i,m+1}$ as:

$$\begin{aligned}&\sigma_{i,m+1}\\&\quad=-\xi^{\mathrm{T}}(Z_{i,m})\xi(Z_{i,m})\hat{\upsilon}_{i,m}\tanh\left(\frac{z_{i,m}\xi^{\mathrm{T}}(Z_{i,m})\xi(Z_{i,m})\hat{\upsilon}_{i,m}}{\kappa}\right)\\&\qquad-\hat{\delta}_{i,m}\tanh\left(\frac{z_{i,m}\hat{\delta}_{i,m}}{\kappa}\right)-p_{i,m}z_{i,m}^{2\mu-1}\end{aligned} \tag{39}$$

where $p_{i,m}>0$ is a constant.

The adaptive update laws $\hat{\upsilon}_{i,m}$and $\hat{\delta}_{i,m}$ are designed as:

$$\begin{aligned}\dot{\hat{\upsilon}}_{i,m}&=\theta_{i,m}|z_{i,m}|\,\xi^{\mathrm{T}}(Z_{i,m})\xi(Z_{i,m})-q_{i,m}\hat{\upsilon}_{i,m},\ \hat{\upsilon}_{i,m}(0)\ge 0\\\dot{\hat{\delta}}_{i,m}&=\pi_{i,m}|z_{i,m}|-r_{i,m}\hat{\delta}_{i,m},\ \hat{\delta}_{i,m}(0)\ge 0\end{aligned} \tag{40}$$

where $q_{i,m}>0$ and $r_{i,m}>0$ are design parameters.

Substituting (39) and (40) into (38) gets:

$$\begin{aligned}\dot{V}_{i,m}&\le\dot{V}_{i,m-1}-p_{i,m}z_{i,m}^{2\mu}+\tilde{\delta}_{i,m}|z_{i,m}|\\&\quad+\Big[|z_{i,m}|\,\xi^{\mathrm{T}}(Z_{i,m})\xi(Z_{i,m})\hat{\upsilon}_{i,m}-z_{i,m}\xi^{\mathrm{T}}(Z_{i,m})\xi\\&\quad\times(Z_{i,m})\hat{\upsilon}_{i,m}\tanh\left(\frac{z_{i,m}\xi^{\mathrm{T}}(Z_{i,m})\xi(Z_{i,m})\hat{\upsilon}_{i,m}}{\kappa}\right)\Big]\\&\quad+\left[|z_{i,m}|\,\hat{\delta}_{i,m}-z_{i,m}\hat{\delta}_{i,m}\tanh\left(\frac{z_{i,m}\hat{\delta}_{i,m}}{\kappa}\right)\right]\\&\quad-\frac{\tilde{\upsilon}_{i,m}\dot{\hat{\upsilon}}_{i,m}}{\chi_{i,m}}-\frac{\tilde{\delta}_{i,m}\dot{\hat{\delta}}_{i,m}}{\pi_{i,m}}\\&\quad+\frac{1}{\bar{\varphi}_{i,m}}z_{i,m}\varphi_{i,m}(\bar{\chi}_{i,m},\eta_{i,m})z_{i,m+1}\\&\quad+\tilde{\upsilon}_{i,m}|z_{i,m}|\,\xi^{\mathrm{T}}(Z_{i,m})\xi(Z_{i,m})\\&\quad-\frac{1}{\bar{\varphi}_{i,m-1}}z_{i,m-1}\varphi_{i,m-1}(\bar{\chi}_{i,m-1},\eta_{i,m-1})z_{i,m}\\&\le-\sum_{k=1}^{m}p_{i,k}z_{i,k}^{2\mu}+0.557m\kappa+\sum_{k=1}^{m}\frac{q_{i,k}\tilde{\upsilon}_{i,k}\hat{\upsilon}_{i,k}}{\theta_{i,k}}\\&\quad+\sum_{k=1}^{m}\frac{r_{i,k}\tilde{\delta}_{i,k}\hat{\delta}_{i,k}}{\pi_{i,k}}\\&\quad+\frac{1}{\bar{\varphi}_{i,m}}z_{i,m}\varphi_{i,m}(\bar{\chi}_{i,m},\eta_{i,m})z_{i,m+1}\end{aligned} \tag{41}$$

Step n. This is the final step of follower agent i, similar to step i, m, the dynamics of $z_{i,n}$ can be directly obtained by replacing m with n in (32). Then we have:

$$\begin{aligned}&z_{i,n}\Big[\phi_{i,n}(\bar{\chi}_{i,n},0)-\dot{\sigma}_{i,n}\\&\qquad+\frac{1}{\bar{\varphi}_{i,n-1}}\bar{\varphi}_{i,n}z_{i,n-1}\varphi_{i,n-1}(\bar{\chi}_{i,n-1},\eta_{i,n-1})\Big]\\&\quad\le|z_{i,n}|\left(\frac{1}{2}\left\|W_{i,n}^*\right\|^2\xi^{\mathrm{T}}(Z_{i,n})\xi(Z_{i,n})+\frac{1}{2}+\bar{\varepsilon}_{i,n}\right)\end{aligned} \tag{42}$$

where $W_{i,n}^*$ is the ideal weight, and $\varepsilon(Z_{i,n})$ is the approximation error satisfying $|\varepsilon(Z_{i,n})|\le\bar{\varepsilon}_{i,n}$ with $\bar{\varepsilon}_{i,n}>0$ being a constant, $Z_{i,n} = \left[\bar{\chi}_{i,n}, \bar{\chi}_{j,2}, z_{i,n-1}, (\partial\sigma_{i,n}/\partial\chi_{i,1}), \cdots, (\partial\sigma_{i,n}/\partial\chi_{i,n}), (\partial\sigma_{i,n}/\partial\chi_{j,1}), \zeta_{i,n}, \vartheta_{i,n}\right]^{\mathrm{T}}$ with $j \in \mathcal{N}_i$.

Let $\upsilon_{i,n} = \frac{1}{2\bar{\varphi}_{i,n}}\left\|W_{i,n}^*\right\|^2$ and $\delta_{i,n} = \frac{1}{\bar{\varphi}_{i,n}}\left(\frac{1}{2} + \bar{\varepsilon}_{i,n} + \bar{\Delta}_{i,n}\right)$, and consider the Lemma 1, hence, the time derivative of $V_{i,n}$ can be written as:

$$\begin{aligned}\dot{V}_{i,n} \le\ & \dot{V}_{i,n-1} + \frac{1}{\bar{\varphi}_{i,n}} z_{i,n}\varphi_{i,n}(\bar{\chi}_{i,n}, \eta_{i,n})H(u_i)u_i + \left|z_{i,n}\right|\delta_{i,n} \\ & + \frac{1}{\bar{\varphi}_{i,n}} z_{i,n}\varphi_{i,n}(\bar{\chi}_{i,n}, \eta_{i,n})G(t) - \frac{\tilde{\upsilon}_{i,n}\dot{\hat{\upsilon}}_{i,n}}{\theta_{i,n}} \\ & + \left|z_{i,n}\right|\upsilon_{i,n}\xi^{\mathrm{T}}(Z_{i,n})\xi(Z_{i,n}) - \frac{\tilde{\delta}_{i,n}\dot{\hat{\delta}}_{i,n}}{\pi_{i,n}} \\ & - \frac{1}{\bar{\varphi}_{i,n-1}} z_{i,n-1}\varphi_{i,n-1}(\bar{\chi}_{i,n-1}, \eta_{i,n-1})z_{i,n} \end{aligned} \tag{43}$$

Design the control law u_i as:

$$\begin{aligned} u_i = & -\frac{1}{1-\beta}\left[\frac{z_{i,n}}{2} + p_{i,n}z_{i,n}^{2\mu-1} + \hat{\delta}_{i,n}\tanh\left(\frac{z_{i,n}\hat{\delta}_{i,n}}{\kappa}\right)\right. \\ & \left. + \xi^{\mathrm{T}}(Z_{i,n})\xi(Z_{i,n})\hat{\upsilon}_{i,n}\tanh\left(\frac{z_{i,n}\xi^{\mathrm{T}}(Z_{i,n})\xi(Z_{i,n})\hat{\upsilon}_{i,n}}{\kappa}\right)\right] \end{aligned} \tag{44}$$

where $p_{i,n} > 0$ is a constant.

The adaptive update laws $\hat{\upsilon}_{i,n}$ and $\hat{\delta}_{i,n}$ are designed as:

$$\begin{aligned} \dot{\hat{\upsilon}}_{i,n} &= \theta_{i,n}\left|z_{i,n}\right|\xi^{\mathrm{T}}(Z_{i,n})\xi(Z_{i,n}) - q_{i,n}\hat{\upsilon}_{i,n},\ \hat{\upsilon}_{i,n}(0) \ge 0, \\ \dot{\hat{\delta}}_{i,n} &= \pi_{i,n}\left|z_{i,n}\right| - r_{i,n}\hat{\delta}_{i,n},\ \hat{\delta}_{i,n}(0) \ge 0 \end{aligned} \tag{45}$$

Substituting (44) and (45) into (43) yields:

$$\begin{aligned}\dot{V}_{i,n} \le\ & \dot{V}_{i,n-1} - p_{i,n}z_{i,n}^{2\mu} - \frac{z_{i,n}^2}{2} + \frac{p_{i,n}\tilde{\upsilon}_{i,n}\hat{\upsilon}_{i,n}}{\theta_{i,n}} + \frac{r_{i,n}\tilde{\delta}_{i,n}\hat{\delta}_{i,n}}{\pi_{i,n}} \\ & + \left|z_{i,n}\right|G(t) - \frac{1}{\bar{g}_{i,n-1}} z_{i,n-1}\varphi_{i,n-1}(\bar{\chi}_{i,n-1}, \eta_{i,n-1})z_{i,n} \\ & + 2\times 0.557\kappa. \end{aligned} \tag{46}$$

Due to $\left|z_{i,n}\right|G(t) \le \frac{z_{i,n}^2}{2} + \frac{\alpha_{\min}^2}{2}$, and considering $\dot{V}_{i,n-1}$, then we have:

$$\begin{aligned}\dot{V}_{i,n} \le\ & -\sum_{k=1}^{n} p_{i,k}z_{i,k}^{2\mu} + \sum_{k=1}^{n}\frac{q_{i,k}\tilde{\upsilon}_{i,k}\tilde{\upsilon}_{i,k}}{\theta_{i,k}} + \sum_{k=1}^{n}\frac{r_{i,k}\tilde{\delta}_{i,k}\hat{\delta}_{i,k}}{\pi_{i,k}} \\ & + \frac{\alpha_{\min}^2}{2} + 0.557n\kappa. \end{aligned} \tag{47}$$

In what follows, the main work is shown in Theorem 1.

Theorem 1. Consider a network of uncertain nonaffine multi-agent systems with input quantization (1), let the assumptions 1 and 2 hold, and if the control law is designed as (44) with the adaptive update laws (19), (29), (40) and (45), all the follower agents can track the output of leader agent after finite time, and the tracking error e_i going to stay a compact set, which is defined as:

$$\begin{aligned}\Omega_{e_i} & \\ = & \left\{(z_{i,n}, \upsilon_{i,n}, \delta_{i,n})\left| V(t) \le \left(\frac{d}{(1-\varpi)c}\right)^{\frac{1}{\mu}}, \frac{1}{2} < \varpi < 1\right.\right\} \end{aligned} \tag{48}$$

where $c = \min\left\{(2\bar{\varphi}_{i,k})^{\mu}p_{i,k}, q_{i,k}, r_{i,k}, k = 1, \cdots, n\right\}$, $d = \frac{\alpha_{\min}^2}{2} + 0.557n\kappa + (1-\mu)\mu^{\frac{\mu}{1-\mu}}\sum_{k=1}^{n}(q_{i,k} + r_{i,k}) + \sum_{k=1}^{n}\left(\frac{q_{i,k}\upsilon_{i,k}^2}{2\theta_{i,k}} + \frac{r_{i,k}\delta_{i,k}^2}{2\pi_{i,k}}\right)$.

Proof. Due to $\frac{q_{i,k}\tilde{\upsilon}_{i,k}\hat{\upsilon}_{i,k}}{\theta_{i,k}} \le -\frac{q_{i,k}\tilde{\upsilon}_{i,k}^2}{2\theta_{i,k}} + \frac{q_{i,k}\upsilon_{i,k}^2}{2\theta_{i,k}}$ and $\frac{r_{i,k}\tilde{\delta}_{i,k}\hat{\delta}_{i,k}}{\pi_{i,k}} \le -\frac{r_{i,k}\tilde{\delta}_{i,k}^2}{2\pi_{i,k}} + \frac{r_{i,k}\delta_{i,k}^2}{2\pi_{i,k}}$, and considering Lemma 6, let $\tau = \mu\gamma = 1-\mu$, $\varrho = 1/\mu$, $o = 1$ and $\omega = \frac{\tilde{\upsilon}_{i,k}^2}{2\theta_{i,k}}$ or $\omega = \frac{\tilde{\delta}_{i,k}^2}{2\pi_{i,k}}$, we get $q_{i,k}\left(\frac{\tilde{\upsilon}_{i,k}^2}{2\chi_{i,k}}\right)^{\mu} \le \frac{q_{i,k}\tilde{\upsilon}_{i,k}^2}{2\chi_{i,k}} + q_{i,k}(1-\mu)\mu^{\frac{\mu}{1-\mu}}$, $r_{i,k}\left(\frac{\tilde{\delta}_{i,k}^2}{2\pi_{i,k}}\right)^{\mu} \le \frac{r_{i,k}\tilde{\delta}_{i,k}^2}{2\pi_{i,k}} + r_{i,k}(1-\mu)\mu^{\frac{\mu}{1-\mu}}$. Thus, (47) can be re-written as:

$$\dot{V}_{i,n} \le -\sum_{k=1}^{n}\left(p_{i,k}z_{i,k}^{2\mu} + q_{i,k}\left(\frac{\tilde{\upsilon}_{i,k}^2}{2\theta_{i,k}}\right)^{\mu} + r_{i,k}\left(\frac{\tilde{\delta}_{i,k}^2}{2\pi_{i,k}}\right)^{\mu}\right) + d \tag{49}$$

Furthermore, considering the Lemma 4, we from (48) have:

$$\begin{aligned}\dot{V}_{i,n} &\le -c\sum_{k=1}^{n}\left(\frac{z_{i,k}^2}{2\bar{\varphi}_{i,k}} + \frac{\tilde{\upsilon}_{i,k}^2}{2\theta_{i,k}} + \frac{\tilde{\delta}_{i,k}^2}{2\pi_{i,k}}\right)^{\mu} + d \\ &= -cV_{i,n}^{\mu} + d \end{aligned} \tag{50}$$

According to [42], $\forall(z_{i,n}, \upsilon_{i,n}, \delta_{i,n})\bar{\in}\Omega_{e_i}$, for $\forall t \in [0, t_f]$, we have $V_{i,n} \ge \left(\frac{d}{[(1-\varpi)c]}\right)^{\frac{1}{\mu}}$, namely, $d \le \varpi cV_{i,n}^{\mu}$ for $\forall t \in [0, t_f]$. Hence, combining (50), for $\forall t \in [0, t_f]$, the following relationship is obtained:

$$\dot{V}_{i,n} \le -c(1-\varpi)V_{i,n}^{\mu} \tag{51}$$

Considering $V_{i,n} \ge \left(\frac{d}{\varpi c}\right)^{\frac{1}{\mu}}$ and (44), then we get:

$$t_f \le \frac{\left(V_{i,n}(t_0)\right)^{1-\mu}}{c(1-\varpi)(1-\mu)} \tag{52}$$

Let $t^* = \frac{\left(V_{i,n}(t_0)\right)^{1-\mu}}{c(1-\varpi)(1-\mu)}$, thus, it follows from Lemma 3 that for $t \ge t^*$, one has:

$$V_{i,n}(t) \le \left[\frac{d}{(1-\varpi)c}\right]^{\frac{1}{\mu}} \tag{53}$$

which means that all signal in the closed-loop systems are SGPFS.

Furthermore, according to the definition of $V_{i,n}$, for $\forall t \ge t^*$, one gets:

$$\begin{aligned}\left|z_{i,1}\right|^2 &\le 2\bar{\varphi}_{i,1}\left[\frac{d}{(1-\varpi)c}\right]^{\frac{1}{\mu}}, \left\|z_{e,1}\right\|_2^2 \\ &\le 2N\varphi_{\max}\left[\frac{d}{(1-\varpi)c}\right]^{\frac{1}{\mu}} \end{aligned} \tag{54}$$

where $\left\|z_{e,1}\right\|_2^2 = \sum_{i=1}^{n}\left|z_{i,1}\right|^2$ and $\varphi_{\max} = \max\left\{\bar{\varphi}_{i,1}, \cdots, \bar{\varphi}_{N,1}\right\}$.

In view of $\|e\|_2 \le \frac{1}{\lambda_{\min}(\mathcal{H})}\left\|z_{e,1}\right\|_2$, where $\lambda_{\min}(\mathcal{H})$ represents the minimum eigenvalue of $\mathcal{H}$, then we obtain:

$$\|e\|_2 \le \sqrt{2N\varphi_{\max}/\lambda_{\min}(\mathcal{H})}\left(\frac{d}{(1-\varpi)c}\right)^{\frac{1}{2\mu}} \tag{55}$$

and one has:

$$|y_i - y_0| \le \sqrt{2\varphi_{\max}/\lambda_{\min}(\mathcal{H})}\left(\frac{d}{(1-\varpi)c}\right)^{\frac{1}{2\mu}}, \forall t \ge t^* \tag{56}$$

It implies that the tracking error of each follower agent converges to a small neighborhood of the origin and remains there after the finite time t^*. The proof is completed.

Remark 1: Compared with the traditional methods which the convergence time is usually $t \to \infty$, we can obtain convergence in a finite time, which implies that the control method designed in this paper has a better convergence rate.

*Remark 2:*Throughout the analysis process and simulation results (they are given below), we have obtained the finite convergence time, namely $t_f \le t^*$ and where $t^* = \left(V_{i,n}(t_0)\right)^{1-\mu}\Big/c(1-\varpi)(1-\mu)$, which shows that the control laws designed in this paper are effective, and that the fuzzy logic systems and adaptive parameters are also convergent.

IV. SIMULATION RESULTS

In this part, a practical example with input quantization is given to verify the effectiveness of theproposed control law. Consider a class of uncertain nonaffine multi-agent systems with four follower agents and one leader agent, where each follower agent represents an inverted pendulum system and the leader is used as a signal generator to generate tracking signal. The communication graph among agents is shown as Figure 1.

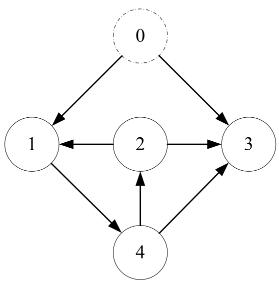

FIGURE 1. Communication graph among agents.

Example 1. The dynamic of the ith follower agent is given as:

$$\begin{aligned}
\dot{\chi}_{i,1} &= \chi_{i,2} + \frac{1-e^{-1-\chi_{i,1}|\cos(0.5)|}}{1+e^{-1-\chi_{i,1}|\cos(0.5)|}}, i = 1,2,3,4\\
\dot{\chi}_{i,2} &= (\chi_{i,1}^2 + \chi_{i,2}^2)e^{-|\cos(0.5)|^2} + 9.8\sin\chi_{i,1}\\
&\quad +0.1\sin(t) + Q(u_i(t))\\
y_{i,1} &= \chi_{i,1}
\end{aligned} \tag{57}$$

The dynamics of the leader agent is described as $\dot{\chi}_{0,1} = \chi_{0,2}\sin(\chi_{0,1}) + \sin(t)$ and $\dot{\chi}_{0,2} = \sin(0.4\pi t)$, and the output of leader agent is $y_0 = \chi_{0,1}$.

The initial conditions for four follower agents are set as $[\chi_{1,1}(0), \chi_{2,1}(0), \chi_{3,1}(0), \chi_{4,1}(0)]^{\mathrm{T}} = [0.3, 1.0, 1.5, 3.0]^{\mathrm{T}}$ and $[\chi_{1,2}(0), \chi_{2,2}(0), \chi_{3,2}(0), \chi_{4,2}(0)]^{\mathrm{T}} = [0.1, 0.5, 0.3, 0.4]^{\mathrm{T}}$. The initial conditions for the leader agent is set as $[\chi_{0,1}(0), \chi_{0,2}(0)]^{\mathrm{T}} = [0.7, 0.2]^{\mathrm{T}}$. The other control parameters are set as: $\rho = 0.1$, $\alpha_{\min} = 0.2$, $m = 2$, $\kappa = 2$, $\mu = 3/5$, $\theta_{1,j} = \theta_{2,j} = \theta_{3,j} = \theta_{4,j} = 2$, $\pi_{1,j} = \pi_{2,j} = \pi_{3,j} = \pi_{4,j} = 1.5$, $q_{1,j} = q_{2,j} = q_{3,j} = q_{4,j} = 0.5$, $r_{1,j} = r_{2,j} = r_{3,j} = r_{4,j} = 1$, $p_{1,j} = p_{2,j} = p_{3,j} = p_{4,j} = 0.5$, $\hat{\upsilon}_{1,j}(0) = \hat{\upsilon}_{2,j}(0) = \hat{\upsilon}_{3,j}(0) = \hat{\upsilon}_{4,j}(0) = \hat{\delta}_{1,j}(0) = \hat{\delta}_{2,j}(0) = \hat{\delta}_{3,j}(0) = \hat{\delta}_{4,j}(0) = 0$, where $j = 1,2,3,4$. In addition, let the fuzzy logic systems $W_{i,j}^{*\mathrm{T}}\xi(Z_{i,j})$ $(i,j \in \{1,2,3,4\})$ contain 11 nodes with centers evenly spaced in the range $[-5, 5]$ and widths be set as 1.5.

Considering the control law (44) with the adaptive update laws (19), (29), (40) and (45), the simulation results are given as Figures 2-6.

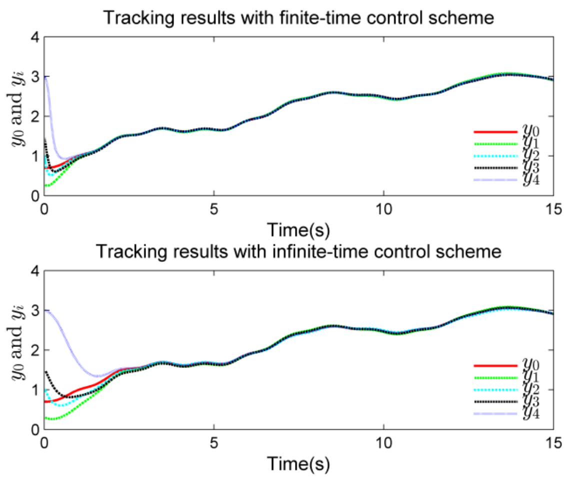

FIGURE 2. Position tracking results of four agents.

The output response curves with four follower agents and one leader agent under the general infinite-time control scheme and the finite-time control scheme are shown as Figure 2. It can be found that four follower agents can tracking the leader agent, and the better tracking effect can be obtained by applying the control scheme designed in this paper. The response curves for tracking error are given as Figure 3, which can be seen that the tracking problem is realized in finite time. The result also implies the validity of the theoretical analysis from another aspect. In addition, it is not difficult to find that the fuzzy logic systems and other adaptive parameters converge in a finite time by analyzing the simulation results. The reason is that the following four agents can track the leading agent in a finite time, which shows that the fuzzy logic system and adaptive parameters also converge in a finite time.

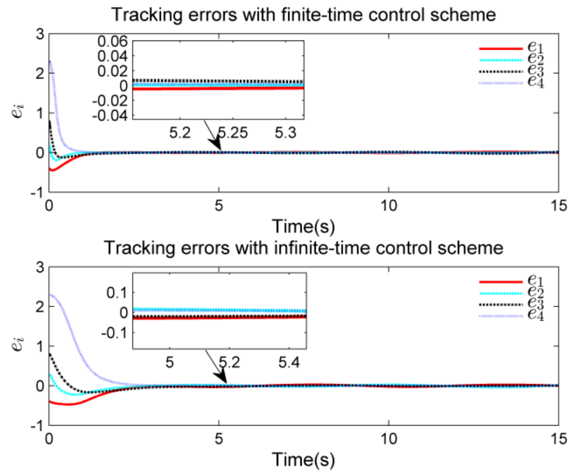

FIGURE 3. Tracking errors of four agents.

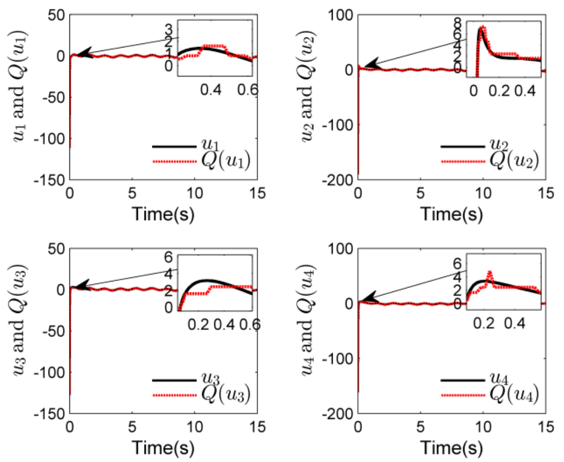

FIGURE 4. Control input and quantized control input.

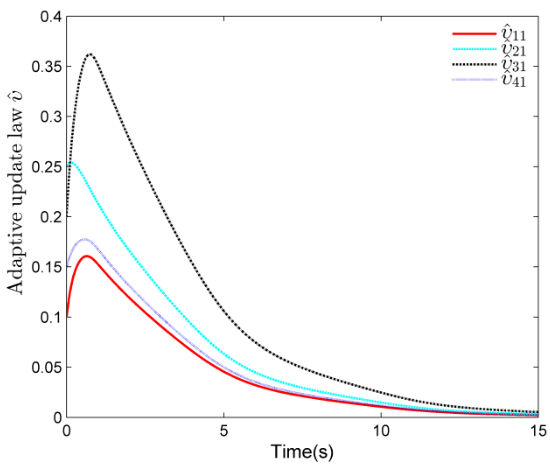

FIGURE 5. Adaptive update law $\hat{\upsilon}$.

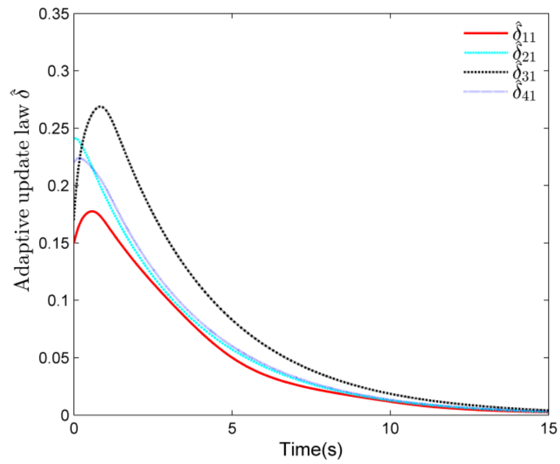

FIGURE 6. Adaptive update law $\hat{\delta}$.

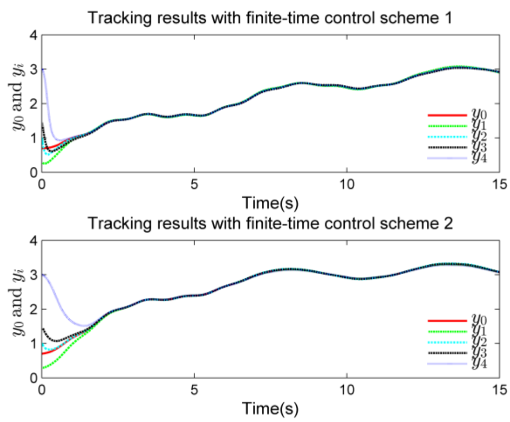

FIGURE 7. The comparative of tracking with other scheme.

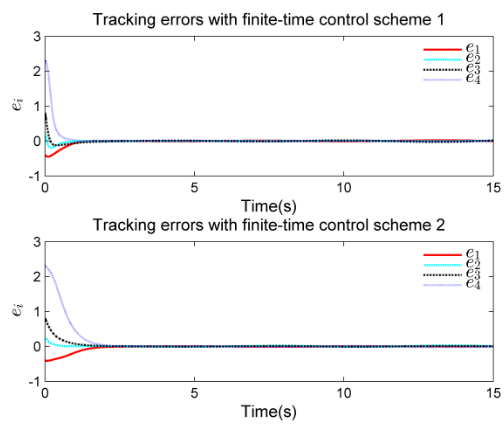

FIGURE 8. The comparative of tracking errors with other scheme.

Under the hysteresis quantizer, the control input and quantized control input are shown as Figure 4. Although the control input is quantized, the tracking control problem with the finite-time control law designed in this paper can be achieved. In addition, the response curves for the adaptive update laws $\hat{\upsilon}$ and $\hat{\delta}$ are displayed as Figures 5-6.

Example 2. In order to verify the effectiveness of the designed control law (44) (scheme 1), a comparative study with the finite-time control law (scheme 2) designed in literature [28] is given in this paper. The parameters setting are the same as that of Example 1. The simulation results are shown in Figures 7-8.

According to the Figure 7, the control scheme 1 designed in this paper and the control scheme 2 designed in [28] can realize the tracking control problem of given multi-agent system in finite time. Compared with the scheme 2, however, the control law designed in this paper can achieve the control result in a shorter time, and the control is better than scheme 2. The comparative results of tracking errors are given in Figure 8.

V. CONCLUSION

This paper has investigated the finite time tracking control problem of a type of uncertain nonaffine multi-agent systems. Compared with the existing multi-agent systems, a more general model with unknown nonaffine functions and uncertain nonlinearity has been considered. The control input of multi-agent systems has been quantized by the quantizer. With the help of a fuzzy logic systems and the backstepping technique, furthermore, the fuzzy finite-time tracking control protocol has been proposed. Based on the presented control protocol, it has been proved that the tracking error of all follower agents can be held on a small enough neighborhood in a finite time. Finally, the effectiveness of the proposed control law is shown by using simulation examples in simulation part. It should be pointed out that the differential of virtual control laws needs to be taken in this paper, which is going to cause ''explosion of complexity''. To avoid this phenomenon, the dynamic surface control method will be considered in our follow-up work.

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