

Strategic Stand Planning With Selection Of New Flights To Optimize Hub Connections

A Case Study

Master of Science Thesis

J. P. Ferreira



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by

J. P. Ferreira

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Student number:	4922425	
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Thesis committee:	Prof. Dr. Ir. M. Snellen,	TU Delft
	Ir. P. C. Roling,	TU Delft
	Dr. A. Bombelli,	TU Delft
	Ir. G. Földes,	BEONTRA GmbH
	Ir. M. Verhees,	BEONTRA GmbH

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List of Abbreviations

B&B	Branch and Bound
BLIF	Binary Linear Integer Formulation
BQF	Binary Quadratic Formulation
CPP	Clique Partitioning Problem
EU	European Union
GA	Genetic Algorithm
GAP	Gate Assignment Problem
IATA	International Air Transport Association
ICAO	International Civil Aviation Organization
ILP	Integer Linear Programming
MCNFP	Multi-Commodity Network Flow Problem
MCT	Minimum Connection Time
MILP	Mixed Integer Linear Programming
MLIF	Mixed Linear Integer Formulation
MQIF	Mixed Quadratic Integer Formulation
O&D	Origin and Destination
OF	Objective Function
OR	Operations Research
QAP	Quadratic Assignment Problem
SA	Simulated Annealing
SAP	Stand Assignment Problem
TA	Turnaround
TLSAP	Two-Level Stand Planning Model
TS	Tabu Search

Introduction

Nowadays, a large number of airports have to deal with transfer passengers. In particular, hub airports, which are home to the operations of one (or more) airlines, see a significant percentage of their passengers connecting from one arriving flight to a departing flight without leaving the facilities. In fact, at hub airports, statistics show that connecting passengers usually outnumber O&D passengers. For instance, in 2009, 64.2% of passengers at Atlanta HartsfieldJackson Airport were in fact making their connection between two flights [8]. Back in the 1970's, this was already a reality. In 1979, almost half of the passengers boarding a plane at Chicago OHare Airport were connecting passengers [9]. The importance of transfer passengers for airports is so significant that there has been an increasing investment of airports and airlines to optimize hub operations and make them more efficient. Furthermore, the consistent increase of air travellers over time has put hub airports under pressure, since they have to deal with an increasing demand while facing the challenge of a limited airport capacity. This makes it even more important to make an efficient use of the infrastructure. This challenge has motivated researchers to propose new methods and models to tackle it. In this field, the Stand Assignment Problem (SAP) is very commonly applied. The SAP is one of the most researched problems in the airline industry and it has been proven that it effectively contributes to achieve different objectives proposed by several stakeholders, namely airports, airlines and passengers.

Research Goal

This research focuses on developing a new strategic stand planning model that covers a gap on the current literature, thus continuing the work from previous authors on the different applications of the SAP. This research will contribute to the expansion of the current knowledge on the field of the SAP and to a more efficient future use of hub airports infrastructure. The research goals are summarized below:

- Develop and explore the applicability of a new strategic stand planning model capable of selecting profitable routes that will expand the network of a hub, integrating those routes in the schedule and efficiently capturing transfer passengers at that hub
- Confirm the correct implementation of the strategic stand planning model and assess its effectiveness
- Apply the strategic stand planning model on a case study airport, retrieve results from it and draw conclusions, including recommendations for the future

The scientific relevance of this research is found on the first and second goals shown above, since a new version of the SAP that aims to optimize different objectives will be developed and tested. The relevance of the research extends to the aviation industry, as the third goal points out. With the results obtained from the case study, specific industry applications of the strategic stand planning model will be recommended for the future.

To achieve these goals, this research was developed in partnership with BEONTRA GmbH, a leading airport scenario planning company. BEONTRA GmbH provided constant support and supervision along the entire duration of the research and it contributed with concepts and feedback from the perspective aviation industry. The company further provided the input data of the case study airport considered in this research, which not only included the flight schedule but also airport and terminal characteristics and other operational rules.

Finally, the most sensitive information included in this report (such as airport codes, countries and regions names) has been made anonymous to protect the privacy of the case study airport.

Research Scope

This research focuses on an optimization problem within the field of Operations Research (OR). Furthermore, the research covers the development of a stand assignment problem that includes transfer passengers, so the analysis and implementation of formulations and solving methods that efficiently model these passengers

in the SAP will be studied. Besides transfer passengers, the scope also includes other operational features of airports, namely certain stand restrictions and schedule robustness. Nonetheless, it is important to mention that the gate assignment is out of the scope.

The forecast of transfer passengers between origin-destination pairs is also briefly covered in this research, even though this is not the main focus.

Report Structure

This report is divided into 3 major parts. Part I includes the scientific paper that concisely describes this research, the model developed and the results obtained. In Part II, the Literature Study, which was previously graded under the course AE4020 as part of the Tu Delft - Air Transport Operations profile programme, is provided.

Finally, Part III is divided into several chapters, each portraying in detail different elements of the research. [chapter 1](#) provides a comprehensive explanation of the Two-Level Stand Planning Model (TLSAP). [chapter 2](#) shows the process of verification and validation of the same model. In [chapter 3](#), relevant background information regarding the case study airport is shown. In the same chapter, the values used for different model parameters and inputs are presented and several arguments are proposed to support those choices. In [chapter 4](#), a detailed analysis of all results is presented. With this analysis, each research question defined in the Literature Study is answered in the same chapter. [chapter 5](#) performs a sensitivity analysis of the model to the change in the most important parameters. Finally, [chapter 6](#) presents the major conclusions taken from the research, including the limitations found, scientific relevance, industry applicability and also future recommendations to improve and expand the strategic stand planning model.

I

Scientific Paper

Strategic Stand Planning With Selection Of New Flights To Optimize Hub Connections

J. P. Ferreira¹

under supervision of M. Snellen¹, P. Roling¹, A. Bombelli¹, G. Földes² and M. Verhees²

1. Faculty of Aerospace Engineering, Delft University of Technology, The Netherlands 2. BEONTRA GmbH, Germany

Abstract For a hub airport, capturing transfer passengers and offering better connections are key elements that largely influence its future growth and profitability. This research aims to develop a strategic stand planning model capable of selecting profitable turnarounds that will expand the network of a hub, integrating those turnarounds into the pre-existing schedule and efficiently capturing transfer passengers. The model includes 2 main goals, namely i) the maximization of the number of captured transfer passengers and the ii) minimization of connection times, which are integrated with 2 other objectives, iii) the minimization of unassigned turnarounds and iv) the minimization of tow moves. The 4 objectives are solved hierarchically in the order: iii)-i)-ii)-iv). In an attempt to tackle the inherent complexity of models that include transfer passengers, the problem is split into two less complex levels. One solves the multi-objective function and the other delivers a specific stand assignment. This approach significantly reduced the computational load for large input sets. The results show that an efficient stand planning can maximize the transfer passenger throughput by up to 38.66% and reduce the average connection by 17.86%. It was also possible to conclude that objectives i) and ii) conflict with each other. Further results confirmed that the time slot to which each new turnaround is allocated is the best slot for the airport, but it is not necessarily the most profitable for the airlines. This research could lead to the development of tools used by rapidly expanding airports to assess the connection potential of new routes. It can also be utilized by busy airports to optimize the current schedule and to make important time slot decisions.

1 Introduction

Over the past few decades, a large percentage of passengers flying through hub airports were connecting passengers. In fact, at hub airports, statistics show that connecting passengers may outnumber Origin & Destination (O&D) passengers. For instance, in 2009, 64.2% of passengers at Atlanta Hartsfield–Jackson Airport were making their connection between two flights (Kim et al., 2013a). Back in the 1970's, this was already a reality. In 1979, almost half of the passengers boarding a plane at Chicago's O'Hare Airport were connecting passengers (Kanafani and Ghobrial, 1985). The importance of transfer passengers for airports is so significant that there has been an increasing investment to optimize hub operations and make them more efficient. Furthermore, the consistent increase of air travellers over time has put hub airports under pressure, since they have to deal with an increasing demand while facing the challenge of a limited airport capacity. This makes it even more important to make an efficient use of the infrastructure.

This challenge has motivated researchers to propose new methods and models to tackle it. In this field, the Stand Assignment Problem (SAP) is very commonly applied. The SAP is one of the most researched problems in the airline indus-

try and it has been proven that it contributes to achieve different objectives proposed by airports. It provides a specific stand (or gate) assignment to the airport that follows certain rules and aims to certain goals. In particular, several authors have attempted to include transfer passengers in the SAP and aimed, for instance, to minimize the passenger walking times between arriving and departure gates for passengers (Xu and Bailey, 2001). However, previous research has not yet focused on quantitatively analyzing how much the number of captured transfer passengers by a hub can be optimized by manipulating the stand assignments. In particular, it is relevant to understand the impact of adding new flights to an airport's schedule and how those flights, together with the original schedule, can maximize the number of transfer passengers captured by the hub. The literature is quite extensive when it comes to the qualitative analysis of hub networks, the impact of future expansions and the relationships between airlines and how that affects the dynamics of an airport. However, no research was found that would attempt to select new, profitable turnarounds, integrate them in an airport's schedule, and quantify the new hub connections and the number of passengers.

In an attempt to cover the research gap mentioned above, the main objective of this research is to develop and explore

the applicability of a strategic stand planning model capable of selecting profitable routes that will expand the network of a hub, integrating those routes in the schedule and efficiently capturing transfer passengers at that hub. This model uses a list of proposed turnarounds, each with different time alternatives, and the turnarounds from the original schedule of an airport, which are fixed in time. The model then assigns as many operations as possible to the schedule, while being able to choose the best time alternative for each proposed turnaround. Finally, it delivers a stand assignment that optimizes 2 main goals, i.e., the maximization of the number of captured transfer passengers but also the minimization of connection times. These 2 objectives are integrated with other 2 into a multi-objective function, and solved hierarchically in the following order: i) minimization of ungated turnarounds, ii) maximization of captured transfer passengers, iii) minimization of connection times and iv) minimization of tow moves. Solving any SAP that considers transfer passengers is inherently a hard task due to the commonly large size of the problems. For this reason, this research proposes a new approach to solve the SAP, which henceforth will be called the Two-Level Stand Planning Model (TLSAP). In this approach, the problem is divided into two smaller levels. In the first level, where the computationally demanding multi-objective is solved, operations are assigned to zones (consisting of multiple stands) and on the second level an individual stand assignment is performed.

This research was developed in partnership with BEON-TRA GmbH, a leading airport scenario planning company, and the TLSAP is tested with a case study airport. Note that this model focuses mainly on the airport and not on other stakeholders (with the exception of passengers with the minimization of connection times). Several questions are expected to be answered. First, the growth of captured transfer passengers and the reduction of connection times will be analyzed for different scenarios, as well as the relationship between them. It is important to perform this analysis since these two objectives are the main goals of the research. It is also relevant to understand how the new turnarounds are distributed over the day of operations, how profitable for the airlines those turnarounds may be and how the network of the hub expanded.

2 Previous Literature

A. The Stand Assignment Problem: Objectives, Formulations and Solving Methods

It is important to understand how the Stand Assignment Problem (SAP) has been researched in the past, since the SAP is the means through which the final results will be obtained.

The objective function chosen for the SAP can assume different forms, depending on the intended results of the research. A very common objective, that is included in most

of the researches is the minimization of passenger walking distance (Mangoubi and Mathaisel, 1985; Ding et al., 2005, 2004), which can sometimes be written as the minimization of the passengers walking time (Xu and Bailey, 2001; Kim et al., 2013b). One should note that minimizing the connection times is relevant for this thesis, since it will be one of the main objectives.

Other authors extended the SAP with objectives that go beyond the consideration of passengers walking times. In the work developed in (Ding et al., 2004, 2005; Drexel and Nikulin, 2008), the authors minimize the number of ungated aircraft. In (Kim et al., 2013a), a model that takes into consideration the aircraft congestion on ramps is proposed. Fulfilling gate assignment preferences can also be one objective of the SAP, as it is the case in (Xu and Bailey, 2001; Neuman and Atkin, 2013).

Transfer passengers are usually a key part of the SAP, and the same applies to this research. However, including them in the SAP formulations can be challenging. Because of the greater complexity, some authors propose simplifications that give origin to a more manageable problem.

Authors very commonly use the mixed quadratic integer formulation (MQIF), as it is the case in (Xu and Bailey, 2001). However, the same authors later linearize their model to a mixed linear integer formulation (MILP). Other works use MILP formulation, such as (Bolat, 2001). Several other authors model the SAP as a quadratic integer formulation with only binary decision variables (Ding et al., 2004). Simpler problems use binary linear integer formulations (BLIF), which are more popular by virtue of their simplicity. (Mangoubi and Mathaisel, 1985) use one of these formulations, and the authors reach linearity by assuming *a priori* an average walking distance for transfer passengers arriving at each gate. It is also possible to find linear formulations with both binary and integer variables, of which the work of (Lim et al., 2005) is an example.

Less frequently, authors formulate the SAP as a Clique Partitioning Problem (CPP) (Dorndorf et al., 2008) or as a Multi-Commodity Network Flow Problem (MCNFP) (Yan and Chang, 1998).

With respect to the nature of the solution method, it is known that exact methods always reach the optimal solution in a finite (yet indefinite) amount of time and are the base of commercial solvers. However, they are not suitable to solve non-linear problems (such as the SAP when transfer passengers are considered). Usually, research that uses exact methods presents a linearization first (Xu and Bailey, 2001). In (Maharjan and Matis, 2012), the authors divide their problem into smaller sub-problems. The division is based on the layout of the airport and it leads to three levels of assignment: zone, sub-zone and the specific gate assignment. Each level uses the assignment of the previous level and since each sub-problem is considerably smaller than the original, a commercial solver *CPLEX* can be used.

Simple heuristic methods are easier to understand and,

usually, easier to design than meta-heuristic methods. However, they generally present more flaws, they are problem-dependent and do not reach global optimality as often (Mangoubi and Mathaisel, 1985; Haghani and Chen, 1998). The most researched meta-heuristic solving methods in the field of the GAP are Tabu Search (TS) (Ding et al., 2004, 2005), Simulated Annealing (SA), which is studied in detail in (Kirkpatrick et al., 1983) and Genetic Algorithms (GA), which have proven to be robust algorithms (Al-Tabtabai and Alex, 1999) and are used in different research (Wei and Liu, 2007; Hu and Paolo, 2007). It is also very common to incorporate neighbourhood searches with these methods because they promote the exploration of the solution space.

B. Hub Network Analysis

The analysis and optimization of airport networks has been performed by several authors for the past years. In (Redondi et al., 2011), the authors present a case study, with several European airports, which aims to evaluate the impact of creating future new connections to their network. The work in (Sun et al., 2014) analyses the temporal evolution of the European air transportation system between 2011 and 2013. On the other hand, (Burghouwt and Hakfoort, 2001) study the evolution of the European aviation network, with the objective of determining whether deregulation in the EU has led to changes in the route structures. In (Jimenez et al., 2012), an analysis is made to the evolution of the aviation network of the three main airports of mainland Portugal and it is concluded that it was influenced by the emergence of low-cost carriers.

However, proposing a possible change in the airport's schedule and network and specifically quantifying the new connections and the number of passengers flying through the hub has not been researched so far. However, this information is valuable for hub airports. In a study conducted in (Lin and Tsai, 2008) it was shown that expanding the airport's network is the third highest contributor to the maximization of its revenue.

To cover this gap in the current literature, the present work tries to integrate new objectives in the SAP that were never combined before to efficiently expand a hub's network. The results are analyzed and the gap in the literature is better explored.

3 Hub Connection Types

The main objective of this research is to develop a model that aims to capture as many transfer passengers as possible while connection times are minimized. For this reason, it is of extreme importance to define different types of connections between flights and how they are modelled in the context of this problem. Let the time between an arriving flight i (denoted by a_i) and any departing flight j be henceforth denoted as T . Then, Figure 1 shows that the type of

connection created between i and j is dependent on T .

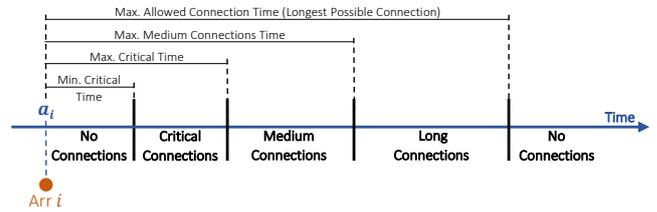


Figure 1: Time division used in the Strategic Stand Planning Model.

If T is smaller than a minimum threshold, denoted as **Minimum Critical Time**, it is assumed that the connection time is too small. Thus, no connection is possible between the two flights. On the other hand, if the value of T lies in between the Minimum Critical Time and the **Maximum Critical Time**, a connection between i and j may be possible. However, due to the limited time that passengers have to walk between the arrival of i and the departure of j , this type of connection is called a **Critical Connection**. Inside the Critical Connections period, passengers will have time to make the connection if they are assigned to stands that are close enough to each other, but will not be able to make the connection otherwise.

If T lies in between the Maximum Critical Time and the **Maximum Medium Connection Time**, a connection is always possible between i and j (provided they are both assigned to a stand). This period is called the **Medium Connections** period. When T increases even more, it falls somewhere between the Maximum Medium Connection Time and the **Maximum Allowed Connection Time**. In this period, called the **Long Connections** period, layovers will be significantly long. Since one of the objectives of this research is to minimize connection times, a penalty is added to any connection longer than the Maximum Critical Time. The value of this penalty will be equal to the connection time. This linear relationship is adequate since, the longer a certain connection is, the less desired it is by passengers, potentially making the airport less competitive.

Finally, when T is greater than the Maximum Allowed Connection Time, it is assumed, in the model, that no connection can occur between i and j . It is reasonable to include a maximum connection time value since passengers may disregard connections with very large layovers. Furthermore, long layovers may not be as profitable for airlines. In fact, it is shown in (Luttmann, 2019) that airlines tend to offer discounted fares for longer layover times.

Figure 2 shows the threshold values chosen for this research. The values for the critical connection period (40min to 1h30min) is set based on the definition of this period: if T is smaller than 40min, passengers do not have time to reach any stand, but if it is larger than 1h30, there will always be time to make the connection, and thus the connection is not critical anymore. On the other hand, a value of 7h was cho-

sen for the longest possible connection. This is a reasonable value taking into consideration that only a period of 24h (one day) will be tested. Furthermore, if the model works for this value, it is possible to extend the longest connection as much as desired.

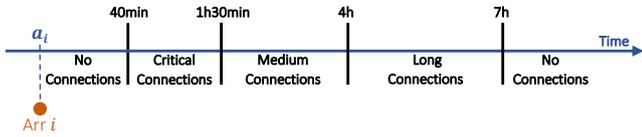


Figure 2: Time limits for each connection time period. The time limits are not to scale.

4 The Two-Level Stand Planning Model

In line with what was explained in section 2, modelling transfer passengers in the stand assignment problem often leads to problems with large solution spaces and with second order objective elements which, in turn, are inherently harder to solve when compared to linear problems. Several authors have proposed different approaches to reformulate problems in order to avoid second order terms in the model (Bouras et al., 2014). The definitive approach proposed for this research is based on several methods, but particularly on the work of (Maharjan and Matis, 2012). In the definitive approach, the operations are not directly assigned to stands. Instead, the problem is divided into two levels, which are explained below.

- **First Level:** operations are assigned to zones of the airport. These zones are defined based on several criteria. It is during the first level that the four objectives considered in this research are optimized.
- **Second Level:** the results obtained in the first level are used as input to the second level. In this level, operations are simply assigned to individual stands, inside the zone to which they were previously attributed.

In short, the first level of this approach is equivalent to the original approach because it is during this level that the multi-objective function is solved. The main difference is that, instead of assigning operations to stands, they are assigned to zones. While the first level is more demanding because all objectives are being solved there, the fact that zones instead of stands are being considered reduces considerably the number of variables needed. Consequently, the computational load is also significantly lower. With the main problem optimized, there are no demanding tasks that need to be solved during the second level, so in this level stands can already be considered. The second level completes the whole problem by assigning the operations to specific stands, based on the zone assignment obtained after the

first level. Note that, as a consequence of the reduced size of the two levels, they are both solved using the commercial solver *Gurobi*. Furthermore, both levels are formulated as MILP problems since they include binary, integer and continuous variables

A pre-processing algorithm was introduced before running the MILP model. In this algorithm, variables that, *a priori*, are known to be equal to zero, are disregarded and not included in the model. With this, the size of the state space can be reduced without affecting the results.

4.1 Sets and Parameters

The most relevant parameters are shown in Table 1, while Table 2 shows all the sets used in this research.

Param.	Description
f_i^u	Number of forecast pax wanting to connect from flight i to the destination-airline pair u
TE_k	Time from stand k to the entrance of its corresponding zone
S^i	Seat capacity of flight i
M	Parameter used in the big-M method. Refer to Equation 11 - Equation 14 for more information about how this method is applied
P	Penalty coefficient for unassigned turnarounds. P is 10 times larger for original turnarounds than it is for proposed turnarounds
Dummy Zone	Fictional zone where operations are assigned when not added to the schedule. In the model, the dummy zone is represented by index 0

Table 1: Parameters used to define the TLSAP.

It is important to mention that, in this problem, 3 types of stands are considered: *C*, *E* and *MARS*. The latter is in fact a group of stands: it contains 2 type *C* stands and one type *F* stand. However, in the first level of assignment, the group is considered to be one single type. Furthermore, a buffer time of 5 minutes was added at the beginning of each operation to avoid consecutive operations from being assigned to a single stand and, thus increasing the schedule robustness.

4.2 First Level of Assignment: Zone Assignment

On a first, more general level, aircraft are assigned to zones and, for that reason, it gets the name **zone assignment**.

First Level of Assignment: Decision Variables

One thing to notice is that the model developed for the first level is similar to the model developed in the original approach. The main difference lies in the fact that, now, decision variables refer to zones and not stands. Refer to Table 3

Set	Description	Set	Description
N	All operations	N_{crit}^i	Departures that have a critical connection with arrival i
N_{arr}	Arrival operations	N_{ncrit}^i	Departures that have a non-critical connection with arrival i
N_{dep}	Departure operations	N_{dep}^u	Departure operations with destination-airline pair u
N_{tow}	Operations that can be towed	U_{dep}	Destination-airline pairs for departure flights
N_{no_tow}	Operations that cannot be towed	R_{new}	New turnarounds
N_v	Operations assigned to zone v in the first level of assignment	Z	Zones, excluding the dummy zone
N_v^{towed}	Towed operations in the first level of assignment	Z_0	Zones, including the dummy zone
$N_v^{not_towed}$	Operations not towed in the first level of assignment	T_i^v	Types of stands of zone v that are compatible with flight i
N_{ov}^i	Operations overlapping in time with i	$T_i^v \setminus MARS$	Same set as above, but excluding MARS groups of zone v
$N_{ov}^{i,inc}$	Same set as above, but including operation i itself	K	All stands
N_{small}	Operations corresponding to small aircraft (type C or smaller)	K_v	Stands belonging to zone v
N_{large}	Operations corresponding to large aircraft (type E or larger)	K_t^v	Stands of type t belonging to zone v
$N_{ov,small}^{i,inc}$	$N_{ov}^{i,inc} \cap N_{small}$	K_{MARS}^v	MARS groups belonging to zone v . Contains large stand of each group
$N_{ov,large}^{i,inc}$	$N_{ov}^{i,inc} \cap N_{large}$	K_{blocks}^k	Small stands of the MARS group that has k as the large stand

Table 2: Sets used to define the TLSAP.

to see the decision variables used in the zone assignment. It is important to mention that z_{ijvw} variables are exclusively used in the Critical Connections period of a flight i , while x_{ij} are used in the Medium Connections and Long Connections period.

Variable	Variable Type	Variable Definition
y_{iv}	Binary	$y_{iv} = 1$ if operation i is assigned to zone v . Otherwise, $y_{iv} = 0$.
ϵ_{iot}	Binary	$\epsilon_{iot} = 1$ if operation i is assigned to a stand of type t inside zone v . Otherwise, $\epsilon_{iot} = 0$.
z_{ijvw}	Integer	Number of transfer passengers connecting from arrival operation i assigned to zone v to departure operation j assigned to zone w . Used for critical connections.
x_{ij}	Integer	Number of transfer passengers connecting from arrival operation i to departure operation j . Used for non-critical connections.
τ_i	Continuous	$\tau_i = 1$ if operation i is towed. Otherwise, $\tau_i = 0$.

Table 3: Definition of decision variables for the first-level of assignment.

First Level of Assignment: Complete Model

The complete model, including the different objective functions and constraints, is presented and explained below.

$$\min \sum_{i \in N} P y_{i0} \quad (1)$$

$$\max \sum_{i \in N_{arr}} \sum_{j \in N_{crit}^i} \sum_{v \in Z} \sum_{w \in Z} z_{ijvw} + \sum_{i \in N_{arr}} \sum_{j \in N_{ncrit}^i} x_{ij} \quad (2)$$

$$\min \sum_{i \in N_{arr}} \sum_{j \in N_{ncrit}^i} (d_j - a_i) x_{ij} \quad (3)$$

$$\min \sum_{i \in N_{tow}} \tau_i \quad (4)$$

Subject to:

$$\sum_{v \in Z_0} y_{iv} = 1, \quad \forall i \in N \quad (5)$$

$$\sum_{a \in R} \sum_{v \in Z} y_{iarr}^a v \leq 1, \quad \forall r \in R_{new} \quad (6)$$

$$\epsilon_{iot} - \epsilon_{i_{next}vt} = 0, \quad \forall i \in N_{no_tow}, \quad \forall v \in Z_0, \quad \forall t \in T_i^v \quad (7)$$

$$\epsilon_{iot} - \epsilon_{i_{next}vt} \leq \tau_i, \quad \forall i \in N_{tow}, \quad \forall v \in Z, \quad \forall t \in T_i^v \quad (8)$$

$$\epsilon_{iot} + \sum_{j \in N_{ov}^i} \epsilon_{jot} \leq |K_t^v|, \quad \forall i \in N, \quad \forall v \in Z, \quad \forall t \in T_i^v \setminus MARS \quad (9)$$

$$\sum_{j \in N_{ov,small}^{i,inc}} \epsilon_{jzt} + 2 \sum_{j \in N_{ov,large}^{i,inc}} \epsilon_{jzt} \leq 2 \times |K_{MARS}^v|, \quad \forall i \in N, \quad \forall v \in Z \quad (10)$$

$$z_{ijvw} \leq M y_{iv}, \quad \forall i \in N_{arr}, \quad \forall j \in N_{crit}^i, \quad \forall v, w \in Z \quad (11)$$

$$z_{ijvw} \leq M y_{jw}, \quad \forall i \in N_{arr}, \quad \forall j \in N_{crit}^i, \quad \forall v, w \in Z \quad (12)$$

$$x_{ij} \leq M(1 - y_{j0}), \quad \forall i \in N_{arr}, \quad \forall j \in N_{ncrit}^i \quad (13)$$

$$x_{ij} \leq M(1 - y_{j0}), \quad \forall i \in N_{arr}, \quad \forall j \in N_{ncrit}^i \quad (14)$$

$$\sum_{j \in N_{dep}^u} \sum_{v \in Z} \sum_{w \in Z} z_{ijvw} + \sum_{j \in N_{dep}^u} x_{ij} \leq f_i^u, \quad \forall i \in N_{arr}, \quad \forall u \in U_{dep} \quad (15)$$

$$\sum_{i \in N_{crit}^j} \sum_{v \in Z} \sum_{w \in Z} z_{ijvw} + \sum_{i \in N_{crit}^j} x_{ij} \leq S^j, \quad \forall i \in N_{dep} \quad (16)$$

$$\sum_{t \in T_i^v} \epsilon_{iot} = y_{iv} \quad \forall i \in N, \quad \forall v \in Z \quad (17)$$

$$y_{iv} \in \{0, 1\}, \quad \forall i \in N, \quad \forall v \in Z_0 \quad (18)$$

$$\epsilon_{i\omega t} \in \{0, 1\}, \quad \forall i \in N_{tow}, \quad \forall v \in Z, \quad \forall t \in T_i^v \quad (19)$$

$$z_{ijv\omega} \geq 0, \quad \forall i \in N_{arr}, \quad \forall j \in N_{critical}^i, \quad \forall v, \omega \in Z \quad (20)$$

$$x_{ij} \geq 0, \quad \forall i \in N_{arr}, \quad \forall j \in N_{ncrit}^i \quad (21)$$

$$\tau_i \geq 0, \quad \forall i \in N_{tow} \quad (22)$$

The four objectives of this model are solved in a hierarchical approach. The problem of this research only contains linear objectives and O. Grodzevich and O. Romanko explain in (Grodzevich and Romanko, 2006) that hierarchical approaches work well with this kind of objectives. It is significantly easy to find a sequence of priorities among the four objectives of this model. This sequence is shown below.

- **Obj.1:** Minimize Number of Non-Allocated Turnarounds (refer to Equation 1)
- **Obj.2:** Maximize Number of Captured Transfer Passengers (refer to Equation 2)
- **Obj.3:** Minimize Transfer Passenger Connection Times (refer to Equation 3)
- **Obj.4:** Minimize Number of Towed Operations (refer to Equation 4)

Equation 5 states that each operation has to be assigned to one and only one zone, including the dummy zone. Equation 6 allows, at most, one time alternative of each proposed turnaround to be selected to the schedule. Equation 7 and Equation 8 are the tow operations constraint. The first forbids certain operations from being towed, while the second allows tow operations to take place, at the expense of activating a τ_i variable. Equation 9 corresponds to the zone capacity constraints, and it states that, at any given time, the number of aircraft using a certain stand type (MARS stands not included) inside a zone cannot exceed the corresponding stand capacity. Similarly to the previous constraint, Equation 10 corresponds to the zones MARS capacity constraint. Equation 11 and Equation 12 relate the variables y_{iv} and $z_{ijv\omega}$ by stating that there can only be a critical connection between two flights i and j from zone v and w if both flights are respectively assigned to those zones. Equation 13 and Equation 14 are similar to the previous two constraints. They relate the variables y_{iv} and x_{ij} and they state that a non-critical connection between two flights i and j can only take place if neither of those two flights is assigned to the dummy zone. The control of the arrivals' transfer passenger forecast is enforced using Equation 15. With this constraint, the number of passengers connecting from a flight i to a certain airport-airline pair u cannot exceed the forecast

demand of people willing to make that connection. Equation 16, on the other hand, states that the number of passengers transferring to a departing flight cannot exceed the aircraft's capacity. Equation 17 relates the variables y_{iv} and $\epsilon_{i\omega t}$. This constraint is needed to keep the model's consistency. It forces that an operation is assigned to one and only stand type of a zone if that operation is assigned to that zone. On the other hand, it also forbids the operation from being assigned to any stand type of a zone if it is not assigned to that zone in the first place. Finally, Equation 18, Equation 19, Equation 20, Equation 21 and Equation 22 correspond to the definition of types and bounds of all 5 variables used in the model.

4.3 Second Level of Assignment: Individual Stand Assignment

After the first-level assignment, operations are already assigned to zones and the core problem is solved. However, while assigning operations to zones reduces considerably the size of the problem, it does not deliver a specific stand assignment that can be used, in practice, by the airport. This makes the second level of assignments just as relevant. The second level assignment is also called the **individual stand assignment**.

Second Level of Assignment: Decision Variable

The only variable type defined for this level is shown in Table 4. This variable has a similar meaning to the y_{iv} variable used in the first level, with the difference that the former refers to operation-to-stand assignments and the latter refers to operation-to-zone assignments.

Variable	Variable Type	Variable Definition
η_{ik}	Binary	$\eta_{ik} = 1$ if operation i is assigned to stand k . Otherwise, $\eta_{ik} = 0$.

Table 4: Definition of the decision variable for the second-level of assignment.

Second Level of Assignment: Complete Model

The second-level is divided into several sub-problems, one for each zone v (except the dummy zone). The model hereby presented is run once for each of those sub-problems with the specific characteristics of the zones (stand composition and assigned operations).

$$\forall v \in Z :$$

$$\min \sum_{i \in N_v} \sum_{k \in K_v} TE_k \eta_{ik} \quad (23)$$

Subject to:

$$\sum_{k \in K_z} \eta_{ik} = 1, \quad \forall i \in N_v \quad (24)$$

$$\eta_{ik} - \eta_{i_{next}k} = 0, \quad \forall i \in N_v^{not_towed}, \quad \forall k \in K_v, \quad \text{if } i_{next} \in N_v \quad (25)$$

$$\eta_{ik} - \eta_{i_{next}k} \leq 1, \quad \forall i \in N_v^{towed}, \quad \forall k \in K_v, \quad \text{if } i_{next} \in N_v \quad (26)$$

$$\eta_{ik} + \sum_{j \in N_{ov}^i \cap N_z} \eta_{jk} \leq 1, \quad \forall i \in N_v \quad (27)$$

$$\sum_{k' \in K_{blocks}^k} \sum_{j \in N_{ov,small}^{i,inc} \cap N_v} \eta_{jk'} + 2 \sum_{j \in N_{ov,large}^{i,inc} \cap N_v} \eta_{jk} \leq 2, \quad \forall i \in N_v, \quad \forall k \in K_{MARS}^v \quad (28)$$

$$\eta_{ik} \in \{0, 1\}, \quad \forall i \in N_v, \quad \forall k \in K_v \quad (29)$$

In the second level, and for each zone v , the model aims to minimize the walking time (measured in minutes) between the stand where each operation is assigned and the entrance of the corresponding zone. In other words, it places all operations as close as possible from the zone entrance. This objective is shown in Equation 23.

Equation 24 states that each operation has to be assigned to one and only one stand. Since the capacity constraint is verified during the first level, it is guaranteed that there is at least one stand available during the second level. Thus, a "dummy stand" is not needed. Equation 25 and Equation 26 translate the tow moves constraints. The number of tow movements to take place during the day of operations is obtained in the zone assignment, and that number cannot be changed in the second level. If an operation i is not towed, as determined by the zone assignment, then it can never be towed in the individual stand assignment, and constraint Equation 25 is used. If however, i is towed to a different stand type of the same zone, then Equation 26 is used. If i is towed from one zone to the other, then none of the above two constraints are applicable, since i and i_{next} do not co-exist in the same second level sub-problem. Equation 27 corresponds to the overlapping constraint, and it states that two operations cannot be assigned to the same stand if they overlap in time at some point. Equation 28 translates the overlapping constraint to MARS stands. It either allows for one large aircraft (design group D or larger) or two small aircraft (design group C or smaller) to be parked at a MARS group. Lastly, Equation 29 defines the type of the decision variable η_{ik} .

Algorithm 1 sums up all steps that integrate the TLSAP.

Algorithm 1 The Two-Level Stand Planning Model

- 1: **Input** Original turnarounds from airport schedule. Proposed turnarounds. Percentages of transfer passenger forecast (origin-destination pair)
// *First Level Pre-Processing*
 - 2: Elimination of variables know to be equal to zero *a priori*
// *First Level Assignment: Zone Assignment*
 - 3: Multi-objective optimization following hierarchy order:
 - 4: **I.** Minimization of Unassigned Turnarounds
 - 5: **II.** Maximization of Captured Transfer Passengers
 - 6: **III.** Minimization of Connection Times
 - 7: **IV.** Minimization of Tow Moves
 - 8: **First Level Output** Operation to zone assignment. Used as input for second level
// *Second Level Pre-Processing*
 - 9: Elimination of variables know to be equal to zero *a priori*
// *Second Level Assignment: Individual Stand Assignment*
 - 10: **for** v in Z **do**
 - 11: **I.** Minimization of Walking Times Between Operations and Zone v Entrance
 - 12: **end for**
 - 13: **Output** Individual stand assignment that maximizes transfer passenger throughput and minimizes connection times
-

5 Input Data

To obtain the results presented here, several operation days from December 2020 were selected. The original schedule for this day was provided by BEONTRA GmbH, Germany. The proposed turnarounds are obtained using *Beontra BRoute Development Tool*. All time alternatives of each proposed turnaround were chosen based on their profitability to the airline and on the forecast of transfer passengers, both indicated by the tool. The profitability is related to the operating margin of that specific alternative when compared to some similar markets. The list of proposed turnarounds and corresponding time alternatives is shown in Table 7. There are 2 types of proposed turnarounds: those that will **expand the frequency** of original routes and those that will open **new routes**.

With respect to the transfer passenger forecast, the total number of passengers willing to connect from each arrival was also provided by the case study airport. However, the specific percentages of passengers wanting to connect from an arrival to each specific destination-airline pair were computed using an algorithm that is based on two factors: i) the angle, centered at the hub, between the origin and the departure airports and ii) the relationship between the airline that operates the first flight and the second flight, i.e., if it is the same airline, if they belong to the same alliance (or share the same codes) or if they are not related at all. With respect to factor i), the higher the angle is between the origin and the destination, the more likely passengers will want

to make that connection, because it means that travel distances and flight times are generally shorter. For factor ii), it is more likely that a connection will take place between two flights operated by the same airline, but it is also possible (but less likely) that the two flights are operated by airlines belonging to the same alliance. Finally, it is assumed that no connection happens between two flights if they are operated by non-related airlines.

The times to reach different zones inside the airport (used in the first level) were also estimated based on the relative position of the different zones. Finally, the towing rules were provided by the case study airport.

6 Results and Discussion

All results in this section are obtained from the day 08/12/2020. Several research scenarios are proposed. These scenarios may differ on the input given to each of them or on the definition of certain parameters. Table 5 presents a summary of the research scenarios and the major elements that characterize them. Refer to section 4.2 to recall the 4 objective functions considered in this research. The MIP gap is a stopping criterion used by *Gurobi*. The lower its values is, the close the Branch and Bound gets to the optimal solution.

Research Scenario	Original Turnarounds	Proposed Turnarounds	MIP Gap	Objectives Solving Order
S0	Included	Not Included	0.01% (default)	Obj.1 → Obj.2 → Obj.3 → Obj.4
S1	Included	Included	0.01% (default)	Obj.1 → Obj.4
S2	Included	Included	0.01% (default)	Obj.1 → Obj.2 → Obj.4
S3	Included	Included	0.01% (default)	Obj.1 → Obj.2 → Obj.3 → Obj.4
S4	Included	Included	5%	Obj.1 → Obj.2 → Obj.3 → Obj.4
S5	Included	Included	0.01% (default)	Obj.1 → Obj.3 → Obj.2 → Obj.4

Table 5: Research scenarios and major elements that characterize each of them.

A. Balance Between Passengers and Connection Times

In general, the airport is more interested in maximizing captured transfer passengers while passengers prefer shorter connection times, for their convenience. However, the maximization of captured transfer passengers and minimization of connection times are two objectives that conflict with each other. For that reason, it is important to analyse the dynamic between them and how it affects the final results. It is paramount that the airport understands how this dynamic behaves and chooses the right balance between both objectives, in order to achieve its desired outcome. To better analyse the dynamic, Table 6 shows information regarding the captured passengers and connection times obtained in scenarios S1 to S5. Scenario S0 is not included in this Table since it does not include the proposed turnarounds.

First of all, note that all available turnarounds were added to the schedule in all scenarios, which means that the number of flights is constant throughout all scenarios.

Scenario	Captured Transfer Passengers				Avg. Conn. Times	Assigned TAs [%]
	Critical Conns.	Medium Conns.	Long Conns.	Total		
S1	642 (14.40%)	3051 (68.45%)	764 (17.14%)	4457	2h41min	100
S2	1177 (20.75%)	3354 (59.14%)	1140 (20.10%)	5671	2h48min	100
S3	1402 (24.72%)	3367 (59.37%)	902 (15.91%)	5671	2h34min	100
S4	1451 (26.10%)	3243 (58.34%)	865 (15.56%)	5559	2h32min	100
S5	1631 (32.32%)	2807 (55.62%)	609 (12.07%)	5047	2h18min	100

Table 6: Results regarding the number of captured transfer passengers and connection times (TA = Turnaround).

In scenario S1, where neither of the two main objectives are optimized, the number of passengers able to make a connection at the hub registers the lowest number of all scenarios in Table 6. The average connection is the second highest. This shows that assigning turnarounds without aiming to optimize both objectives, i.e., performing a more random assignment, is not efficient. Scenario S2 shows that it is possible to increase the number of captured passengers 27.24% to 5671. However, the average connection time in S2 is even higher than in S1. Note that, in both scenarios, connection times are disregarded, so the average connection time does not necessarily improve or get worse from S1 to S2.

Scenario S3 is the first to optimize both the number of transfer passengers and the connection times. The result was expected. The number of passenger throughput is the same as in S2, because both maximize that throughput, but in S3 connections are further re-arranged in such a way that reduces the average connection time from 2h48min to 2h34min (8.33%). Table 6 shows that in S3, around 225 more passengers are offered a critical connection. On the other hand, fewer passengers will have to wait for non-critical connections. The third period of time (long connections) sees a reduction of 238 transfer passengers. Ideally, the goal with the minimization of connection times was to push as many connections as possible to the critical zone. The comparison between scenarios S2 and S3 showed that this is indeed verified with the given input data.

It is important to remind that only the proposed turnarounds, which make up only 16% of all turnarounds, can be moved in time. Consequently, the flexibility given to the model is quite limited. This situation asks for the introduction of other KPIs to better analyse the results and investigate if there is in fact a substantial change in the connection times. To this end, Figure 3 shows the transfer passenger distribution as a function of connection times.

The horizontal axis of the plot shown in Figure 3 is divided in 3 periods that correspond to the connections periods defined before (critical, medium and the long connections). The plot also depicts an overlap of the results obtained for scenarios S2 and S3, and each bar represents a 5 minutes interval. The purple areas of each bar correspond to actual overlap of transfer passengers between the two scenarios, the light blue areas are related to an excess of passengers in S3, while light red areas show a similar excess, but now for scenario S2.

Figure 3 shows the effect of optimizing connection times. There is a clear tendency for passengers to be allocated to

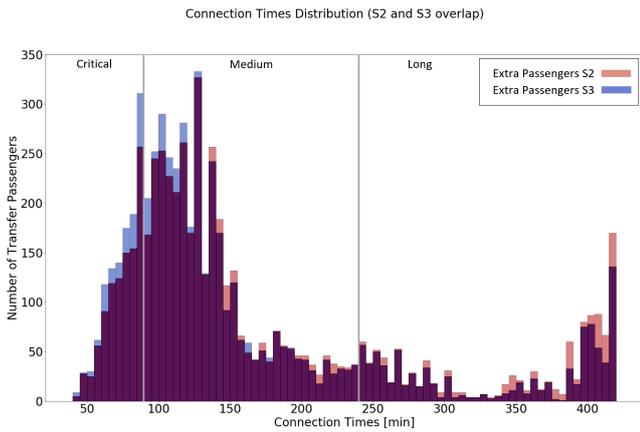


Figure 3: Change in the connection times distribution between scenarios S2 and S3.

short connections in scenario S3. Note how in this scenario most of the passenger connections that were moved in time are almost completely clustered in the critical connections or in the beginning of the medium connections, as suggested by the light blue portion of the bars. This transference of passengers from longer connections to the shortest connections is obtained by offering passengers arriving from several flights a connection to departures that are closer in time.

In S4, the MIP gap parameter is changed from the standard 0.01% to 5%. There is a reduction of the average waiting times when compared to both S2 and S3 and the critical connections period is the only period that saw both the number of passengers and respective percentage increase with respect to S3. It is confirmed that it is possible to obtain an even lower value for the waiting times by giving the objectives that are lower in the hierarchy more flexibility. This further reduction of the waiting times is, however, followed by a decrease of the number of captured transfer passengers, but that was expected. Let's suppose that it would be possible to obtain the average connection time verified in S4 (2 hours and 27 minutes) without reducing the number of passengers in scenarios S2 and S3 (5671). Then, in scenarios S2 and S3, it should have also been obtained an average waiting time of 2 hours and 27 minutes.

The last scenario to be analysed is S5. The first thing to notice is that S5 registers the highest percentage of passengers offered a critical connection. Almost a third of the passengers will have a connection shorter than 1 hour and 30 minutes, which corresponds to an increase of almost 12% from scenario S2, in which connection times were completely disregarded. The average connection time of scenario S5 is, by far, the lowest of all scenarios, with a reduction of 30 minutes (17.86%) when compared with S2. However, at the expense of reducing connection times, the total number of captured transfer passengers in S5 is also lower than any other scenario from S2 to S4. There is a reduction of 11.00% compared to S2 and S3, which can have an impact in the performance of the airport in terms of captured passengers.

B. Time Distribution of New Turnarounds

It is relevant to study how the proposed turnarounds are distributed over time and how that affects the hub performance. The process of choosing which routes should be expanded or opened, and deciding which time alternatives should be created for each proposed turnaround was strongly based on *Beontra's BRoute Development Tool*. During this process, an effort was put into selecting the most profitable alternatives to the airline or, at least, alternatives that would create some competition to other similar markets.

Table 7 indicates, highlighted in blue, the time alternative of each proposed turnaround that was selected in scenario S3. Only 2 turnarounds have both the arrival and the departure during an off-peak period. On the other hand, out of the 23 selected alternatives, 9 have both their arriving and departing operations happening during peak hours. There is a general tendency to select this kind of alternatives. It is also very common to find selected alternatives starting on a peak and ending off-peak, or vice-versa. This was verified for 12 of the 23 proposed turnarounds.

Airport [IATA]	Arr. Time [hh:mm]	Dep. Time [hh:mm]	Transfer Pax Forecast	On-Peak? (Arr/Dep)	Airport [IATA]	Arr. Time [hh:mm]	Dep. Time [hh:mm]	Transfer Pax Forecast	On-Peak? (Arr/Dep)
Airport1	07:35	14:10	159	YES/YES	Airport12	22:55	23:40	53	NO/NO
	18:50	20:50	89	NO/NO		07:30	14:30	63	YES/YES
Airport2	18:50	20:55	22	NO/NO	Airport13	00:25	01:30	125	YES/NO
	00:20	08:40	80	YES/NO		17:45	21:30	125	YES/YES
Airport3	13:00	20:55	80	YES/NO	Airport14	18:30	20:50	112	NO/NO
	00:30	01:55	125	YES/NO		00:05	01:35	89	YES/NO
Airport4	20:10	21:55	125	NO/YES	Airport15	00:05	13:50	89	YES/NO
	12:00	14:20	125	YES/YES		22:15	23:55	89	NO/NO
Airport5	00:30	01:30	85	YES/NO	Airport16	00:50	01:30	89	YES/NO
	17:00	20:55	76	YES/NO		01:00	01:45	35	YES/NO
Airport6	00:30	08:30	85	YES/NO	Airport17	13:05	14:10	34	NO/YES
	00:45	02:00	46	YES/YES		00:50	01:45	65	YES/NO
Airport7	07:30	14:10	63	YES/YES	Airport18	07:35	14:10	89	YES/YES
	06:00	04:50	89	YES/NO		00:30	02:00	89	YES/YES
Airport8	00:30	01:30	85	YES/NO	Airport19	21:40	22:30	89	NO/NO
	22:10	23:00	84	NO/NO		18:45	21:40	73	NO/YES
Airport9	09:20	13:00	164	NO/NO	Airport20	00:30	01:35	73	YES/NO
	11:00	13:00	104	NO/NO		00:30	08:15	72	YES/NO
Airport10	07:35	13:35	135	YES/NO	Airport21	00:15	01:30	80	YES/NO
	00:50	02:00	99	YES/YES		13:10	14:10	76	NO/YES
Airport11	10:50	14:10	56	NO/YES	Airport22	00:30	02:00	72	YES/YES
	00:30	01:30	80	YES/NO		00:15	01:35	115	YES/NO
Airport12	13:10	14:10	69	NO/YES	Airport23	21:55	23:30	118	NO/NO
	13:10	20:55	80	NO/NO		17:55	23:30	107	YES/NO
Airport13	07:30	08:30	146	YES/NO	Airport24	00:15	01:35	66	YES/NO
	00:15	08:30	155	YES/NO		13:10	20:55	48	NO/NO
Airport14	00:15	01:45	126	YES/NO	Airport25	07:35	14:10	117	YES/YES
	07:30	14:10	124	YES/YES		00:00	01:50	94	YES/NO
	22:25	23:05	149	NO/NO					

Table 7: Time alternative of each proposed turnaround that was added to the schedule (highlighted in blue). The turnarounds associated with new routes are highlighted in orange. Information retrieved from BEONTRA GmbH Route Development Tool.

This overview shows that, for the most part, *Gurobi* selected time alternatives with at least the arrival or the departure taking place during a peak hour. In particular, and when it comes to maximizing transfer passengers, it seems significantly beneficial to select alternatives that arrive during a peak hour.

To better visualize how exactly the new flights were distributed in S3 over the day, Figure 4 shows the number of arrivals per hour. Out of the 23 new arrivals, 18 take place during a peak hour. The total percentage of arrivals on-peak increases slightly from 46.53% when only the original flights are considered to 50.90% when all flights are included. This means that the model makes slightly more use of peak hours,

but it still approximately maintains the distribution pattern for the arrivals.

Distribution of Arrivals throughout the day for 08/12/2020 (scenario S3)

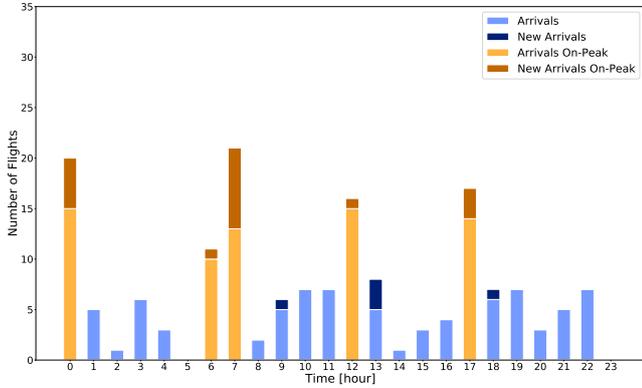


Figure 4: Arrivals distribution for scenario S3.

On the other hand, Figure 5 shows that new departures are more scattered during the day. As a consequence, in S3 some peak hours are not as outstanding when the new departures are added. In fact, the last peak hour of the day, at 23h, is not significantly busier than the periods between 20h and 22h. Despite this more evenly distribution of departures, 14h corresponds, by far, to the busiest period of the entire day.

Distribution of Departures throughout the day for 08/12/2020 (scenario S3)

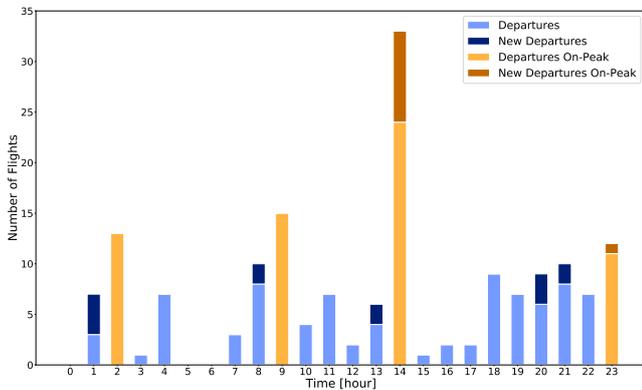


Figure 5: Departures distribution for scenario S3.

A comparison between S3 and S2 shows that the model selected the same time alternatives for all new proposed turnarounds in both scenarios, so they share the same complete flight schedule. For each arrival, there can be several connection opportunities to the same destination. In S2, one of those departures is chosen at random to establish the connection. In S3, the closest departure is picked. This is how S3 has a lower average connection time without the need of moving flights in time. A simple example of this situation is shown in Table 8.

In Table 8, the connections established from arrival XX

Scenario	XX 200 Arriving From Airport17							
	Airport17 - Airport24		Airport17 - Airport25		Airport17 - Airport12		Airport17 - Airport26	
	Conn. Time [min]	Pax.	Conn. Time [min]	Pax.	Conn. Time [min]	Pax.	Conn. Time [min]	Pax.
S2	340	1	340	1	205	3	145	2
S3	85	1	70	1	160	3	85	2

Table 8: Examples showing how connections are shortened in S3, when compared to S2.

200 are presented. For all connections, the waiting time is reduced from S2 to S3 without having to move the flight. This behaviour is generally verified with the other connections, even though there are exceptions.

The results comparison of S3 with S4 also reveal some new information. From S3 to S4, 4 turnarounds are moved in time and there is a clear tendency to move flights to the beginning of the day. This means that moving flights to that period of the day allows a large number of connections to still happen, while significantly reducing the connection times. Recall, from Table 6, that in S4 the passenger waiting times are lower.

The arrival peak at midnight is busier in S4, but overall it is not possible to find a clear connection between minimization of connection times and assignment of flights to peaks. In fact, from S3 to S4, only one off-peak flight was moved to a peak hour. Similarly, the departures are not preferably assigned to peak hours, but rather to the early hours of the day.

Similar conclusions are taken when S3 and S5 are compared. 13 proposed turnarounds were moved in time from S3 to S5, and 7 of them were pushed to the beginning of the day. It appears that, the more flexible the model is to minimize connection times, the more turnarounds are assigned in the early hours, which once again suggests that this configuration promotes shorter hub connections.

This outcome was most likely a consequence of the duration of those new turnarounds. It was verified that the time alternatives of the new turnarounds that take place in the beginning of the day are generally shorter than others that take place during the day. Having shorter turnarounds allows to cluster more arrivals and departures in time which leads to shorter connection times, and that is what happens in scenarios S4 and S5

C. Connectivity and Network Expansion

From Table 9, it is trivial that, for any scenario that includes the proposed turnarounds, there is always an increase in the airport connectivity in terms of number of captured passengers. Furthermore, in S2 and S2 the number of passengers registers the highest increase (38.66%).

Table 9 also shows the number of connections established in each scenario (it is assumed that, when there is at least one connection opportunity from airport AAA to airport BBB at the hub, a connection is established). According to what was expected, S0 creates the least amount of connections, followed by S1 and then by S2 and S3, both es-

establishing the highest number of connections. Scenario S5 establishes 1236, which is very close to the value observed in S4. Recall that S5 captures approximately 500 fewer transfers than S4. The difference in the number of passengers is not proportional to the difference in the number of connections.

Scenario	Total Captured Passengers	Growth w.r.t. S0 [%]	Number of Connections	Avg. Number of pax per Connection
S0	4090	0	941	4.35
S1	4457	8.97	1126	3.96
S2	5671	38.66	1267	4.48
S3	5671	38.66	1267	4.48
S4	5559	35.92	1239	4.49
S5	5047	23.40	1236	4.08

Table 9: Total number of captured transfer passengers, number of connections and connection density in the different scenarios.

The same Table also depicts the connection density in each scenario. The density value of 3.96 obtained in S1 is the lowest of all scenarios, reinforcing the inefficiency of randomly assigning operations (i.e. ignoring captured passengers and connection times). This value shows that the connectivity potential of the hub is wasted in S1. On the other hand, even though fewer passengers are captured in S0 because no new flights are added, the available connections are efficiently used, as shown by density value of 4.35.

Finally, scenarios S2 and S3 show a relatively high density, only surpassed by S4 (and the difference is almost negligible). This fact, combined with all results shown up until now, shows that S3 delivers the best results out of all scenarios.

Scenario S3 was chosen to make an analysis to the new routes added to the schedule and to the expanded routes. The 5 new turnarounds that established new routes created 224 completely new connections through the hub. Different parts of the world that were previously not connected with each other through the hub are now linked. For instance, with the addition of the route to Airport10, it is possible to connect from several places in RegionA (Airport1, Airport22, Airport7, Airport31) to Airport10, and vice-versa, intensifying the market between RegionA and CountryB.

When it comes to the alternatives that expanded the frequency of previous routes, there are instances in which the new turnaround was (partially) overlapping with one of the original turnarounds. In some cases, it even happens that the original and the new arrivals (or departures) take place at the same time or too close to each other. This is the case with the route expansion to Airport6, which has 2 simultaneous departures at 14:55 in S3.

For most of the expansions, however, this overlap does not happen because the original schedule only includes one daily turnaround. This is the case with the route to Airport23, with its only turnaround taking place in the evening. The new alternative starts exactly at midnight and lasts until 01:50. Operating these new flights will not only increase

the flexibility to O&D passengers (since now they have an opportunity to fly from/to Airport23 in the early morning and in the late afternoon) but it will also create several extra connections involving that airport during the first hours of the day. In fact, the new turnaround in the early hours of the day creates 21 new connections from/to Airport23 carrying 51 passengers.

Figure 6 shows in more detail the change in the number of transfer passengers arriving from different world regions/countries and for all scenarios. The same is shown in Figure 7, but for passengers departing to those regions.

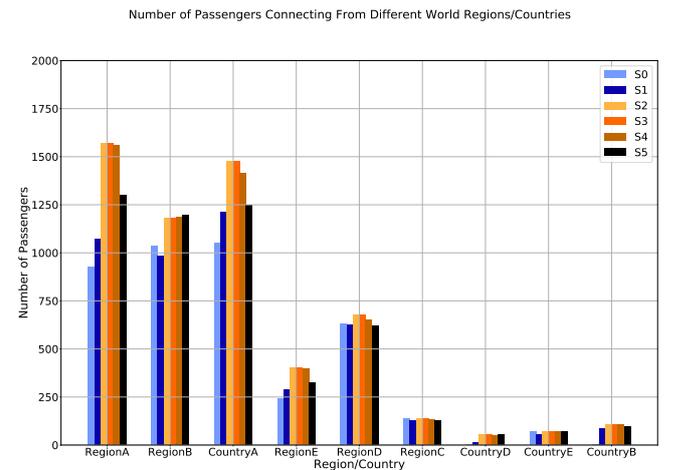


Figure 6: Number of passengers connecting from different world regions/countries.

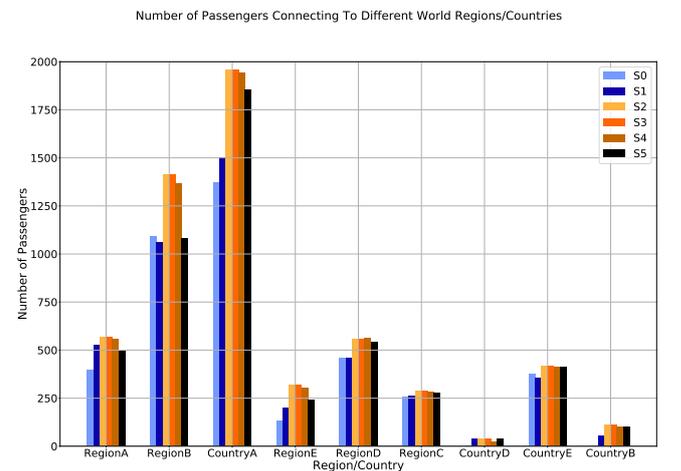


Figure 7: Number of passengers connecting to different world regions/countries.

Both pictures show that the network expands considerably to all regions when the proposed turnarounds are added to the schedule. In S0, the airport was not even connected to CountryB and the CountryD, so the network not only became stronger but it also expanded to new markets. More importantly, it was possible, with the same set of proposed

set of turnarounds, to optimize and strengthen the network to the different regions by simply including the maximization of captured transfer passengers in the model (compare S1 with S2 and S3).

It is also worth mentioning that when a scenario performs worse than others overall, it does not necessarily perform worse in every region's market. Note, for instance, that S5 captures fewer total passengers than S2, S3 and S4 but it establishes more connections from the RegionB. The same can be concluded from the analysis of other regions in Figure 6 and Figure 7.

D. Profitability of New Turnarounds

The final aspect to analyze is the profitability of the new turnarounds. The results show that the process of selecting proposed turnarounds follows some clear trends. First, turnaround alternatives with a higher transfer passenger forecast are usually prioritized over alternatives with lower forecasts. Second, the flights with the highest transfer forecasts are also usually the most profitable and the opposite is true for those with lower forecasts. As a consequence of the first and the second points, it is possible to conclude that there is a significant relationship between the number of passengers predicted in the forecast and the profitability of those turnarounds to the airline: when one increases, the other also tends to increase.

However, this is not always verified. Sometimes, the most profitable alternative for the airport (the one that captures more passengers) does not generate profit to the airline. For instance, in the expansion to Airport15, the time alternative that offers the highest amount of captured passengers captured by the airport was selected, but that alternative is also the least profitable of all to the airline. Something similar can be concluded with the route expansion to Airport19, in which almost all operating margins of the new flight are negative, but still a significant amount of passengers is expected to connect to/from it.

Finally, and with respect to the flights that open new routes, it is generally interesting in the airport's point of view to expand the hub connections to new parts of the world. However, it appears that this kind of expansion is not always profitable for the airline. For instance, the new expansion to Airport21 and Airport22 do not seem profitable investments for the near future, since most of their operating margins are negative.

E. Run Times

Table 10 shows detailed information about the model's characteristics.

Note that S0 registers the lowest run time because it does not include proposed turnarounds. However, S0 is a complete scenario with all objectives active simultaneously, which means that solving the full problem with the original schedule takes only 9.54s using the two-level approach. As

Scenario	#TAs	#Operations	Active Objectives	Run Time [s]	#Variables (1st Level)	#Constraints (1st Level)	#Variables (2st Level)	#Constraints (2st Level)
S0	144	312	4	9.54	10750	20739	15079	3628
S1	203	449	2	10.41	27899	59681	27838	4750
S2	203	449	3	522.64	27899	59681	26951	4557
S3	203	449	4	1705.52	27899	59681	28339	4790
S4	203	449	4	616.85	27899	59681	31397	4657
S5	203	449	4	3609.06	27899	59681	23121	4542

Table 10: Information regarding the size, complexity and run time of each research scenarios (TA = Turnaround).

the complexity of the scenarios increases, the run time also increases, but the times never exceed 30 minutes, except for S5. This scenario registers the highest value, with approximately 1 hour of run time. In reality, *Gurobi* was still solving objective 4 (minimization of tow moves) after 1 hour. It was interrupted since the Branch and Bound was stuck at a value of 1 tow move with a minimum bound of 0 (thus, the optimal solution was either 0 or 1 tow moves).

Table 10 also portrays the size of the model in each scenario. Again, S1 has the lowest number of variables and also the lowest number of constraints in both the first and the second level. For scenarios S1 to S5, the number of variables and constraints in the first level is constant, which is logical since the input to the first level is the same in all these scenarios. However, the number of variables and constraints changes in each scenario, because the specific assignments to each zone also vary.

F. Tow Moves

The minimization of tow moves is the last objective to be optimized in the hierarchy, so its results are highly dependent on the results obtained in all previous objectives. For scenario S0, no tow moves are needed, which is logical since in this scenario the amount of capacity available is the highest and this gives more flexibility to the model. In S1 no tow moves are added either, because the two main objectives are not considered. Scenarios S2 and S4 also register 0 tow moves in their output. However, 2 tow moves are planned in S5 and finally, S3 registers 3 towing operations. Note that S3 and S5 are the most complex scenarios (they include all turnarounds, all objectives and a small MIP gap), so it was expected that the minimization of tow moves objective would be more limited.

7 Sensitivity Analysis

MIP Gap

With the results, it became apparent that the changing in the MIP gap may influence the final results. While it could lower the connection times, the number of captured transfer passengers also decreases. Figure 8 shows the variation in the number of captured transfer passengers and average connection times as a function of the MIP gap (for the day 08/12/2020). The range of values chosen for the gap vary from 0.01% (standard) to 40%.

The results show that, for MIP gap ranges from 0.01% to 18%, there is a clear tendency for the average connection time to decrease when the MIP gap increases, which matches the results obtained for scenarios S3 and S4. However, it is not advantageous for the airport to increase the gap even more, since that action will not reduce the average connection time but the capacity of the hub to capture transfer passengers continues to decrease. With very large MIP gap values, the model becomes more unstable because the Branch and Bound stops too soon. With very large MIP gap values, the model becomes more unstable because the Branch and Bound stops too soon.

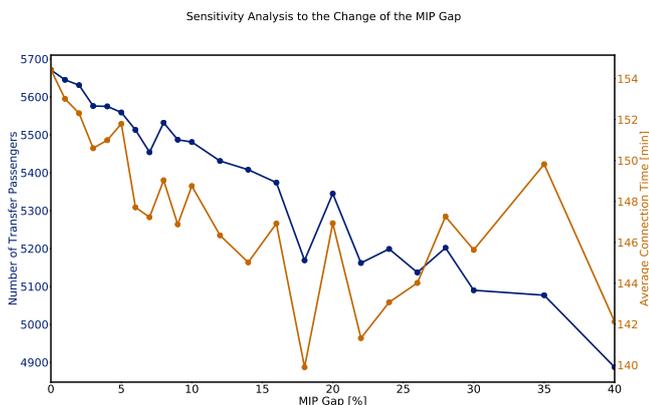


Figure 8: Sensitivity analysis to the change of the MIP gap.

The same MIP gap test was run for other days of operation, namely 03/12/2020, 04/12/2020 and 12/12/2020 and the same tendencies were verified. Thus, this behaviour is consistent for different input sets.

Different Days of Operation

It is also relevant to analyze the behaviour of the model when the input changes, particularly the original schedule. Four different days are considered in this sensitivity test: 03/12/2020 (Thursday), 04/12/2020 (Friday), 08/12/2020 (Tuesday, base day) and 12/12/2020 (Saturday). Different days of the week were purposely chosen because passenger patterns change over the week. When it comes to the proposed turnarounds, the same set as the one originally used for 08/12/2020 is re-used for all the other days. The demand and profit of a flight usually changes over time, but not significantly within the same month. An effort was put into choosing days sufficiently far from Christmas, to avoid having the effect of the holiday season. *Beontra BRoute Development* tool also assumes an approximately constant demand and operating margins for each month, further validating the use of the same set of proposed turnarounds.

Figure 9 and Figure 10 show, respectively, the change in the number of transfers and the number of average connection times over all scenarios, for each day of operations.

Figure 9 points out the stability of the captured transfer passengers throughout all days. S1 is always the sce-

nario with the least amount of passengers captured. In S2 and S3, the number of captured reaches the highest value and it slightly decreases in S4. Finally, S5 registers a relative decrease of transfer passengers from S4, but this value is always higher than in S1. When it comes to the average connection times, Figure 10 also shows some stability of the model. The average connection time seems to increase from S1 to S2 and it obviously decrease from S2 to S3, because in S3 connection times are minimized. There is also a general tendency for average connection times to slightly decrease from S3 to S4 and even more from S4 to S5. The only exception is on day 04/12/2020, where the average connection times slightly increase from scenario S3 to S4, thus showing that it is not always efficient to increase the MIP gap and each case needs to be analyzed in detail.

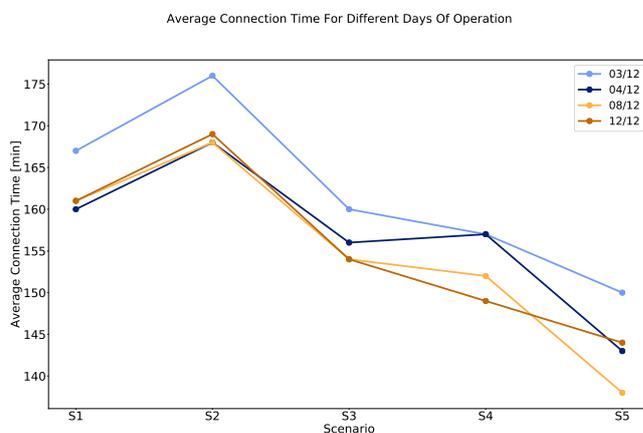


Figure 9: Change in the number of transfer passengers throughout all scenarios, for all days of operation.

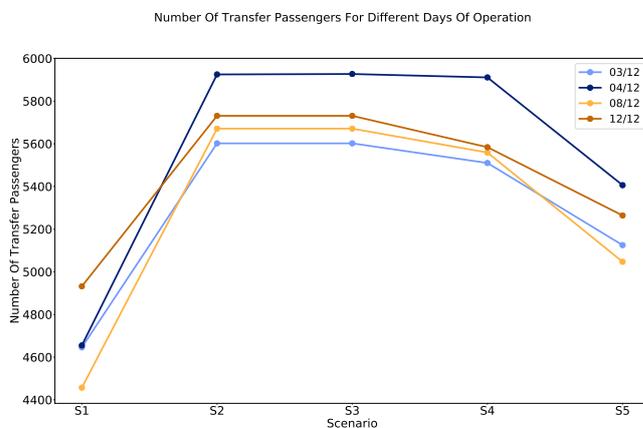


Figure 10: Change in the average connection time throughout all scenarios, for all days of operation.

8 Conclusions

The results presented in this work refer to a specific case study, but they allow to formulate important conclusions. It is clear that there is a conflicting relationship between the maximization of transfer passengers and minimization of connection times. The comparison between scenarios showed that it is possible to reduce the average connection time without affecting the number of passengers, but only to a certain extent. If a further reduction is desired, the hub risks to lose some connecting passengers. Capturing more passengers is advantageous for the airport but having shorter connection times increases the service quality offered to passengers. Thus, hubs capacity management teams may have to find a balance between what is favourable for the airport but also what is convenient for passengers. A maximum reduction of 17.86% in the average connection time was obtained in this research.

The results also showed that, when new flights were added to the schedule, it was easy to expand the number of captured transfer passengers, to a maximum of 38.66%. The extent of that expansion, however, depends on the optimization process, i.e., on which objectives are considered and how flexible the model is (compare S1, S2, S3, S4 and S5).

The profitability analysis of the selected time alternative of each proposed turnaround reinforces the idea that this research is mainly focused on the optimization of the airport and its infrastructure and does not look directly in the perspective of other stakeholders such as the airline. The selected time alternatives were in fact the most adequate for the airport (otherwise the model would not select them) but they were not always the most profitable from the list. It is also possible to infer that creating brand new routes opens up a vast number of new connections that were never possible before, potentially connecting the hub (directly or indirectly) to other regions of the globe and making it competitive with other hubs in the vicinity.

The results also showed that the selection of the time alternative for each proposed turnarounds is highly dependent on the relative importance given to the maximization of transfer passengers and minimization of connection times. When more importance is given to the second objective, the model tends to group the new turnarounds together in the beginning of the day, because those turnarounds are shorter. With shorter turnarounds, it is possible to cluster more arrivals and departures in time which leads to shorter connection times.

It is also important to point out that each scenario establishes connections that may not exist in any of the other scenarios. Note that the model maximizes number of passengers but not explicitly number of connections. If a hub is looking for a specific connection, this should be taken into account.

This research finds its scientific relevance not only by covering the research gap mentioned in the Introduction but

also by proposing a novel approach that efficiently tackles large sized stand assignment problems that include transfer passengers, which was confirmed by the low run times. At the same time, the current research may be the root of the development of tools used by airports to assess the connection potential of new routes or to make an informed decision on which flights of which airlines should be selected. A tool that provides information about transfer passenger growth, network expansion and connection times is valuable by allowing a hub airport to grow in a sustainable way while consistently making an optimized use of its infrastructure. This is particularly important for hubs that are expanding at rapid rates, with evolving schedules and a large percentage of free capacity.

The results of this research could also be applied to very busy hub airports by allowing, for instance, to switch an original turnaround with a new turnaround (positively affecting the connectivity) or to make time slot decisions in a very limited capacity.

9 Recommendations

The run time of this model is relatively slow, but there is still some room for improvement. For most instances, the Branch and Bound would spend most of the run time solving the last objective of the hierarchy, the minimization of tow moves. Two recommendations are proposed. Either reduce the towing possibilities (for instance, only tows from/to remote zones) or simply remove this objective from the hierarchy since this is not a main goal of the research. Furthermore, in the first phases of market studies and stand planning, towing operations are not as important.

Related to the zone division made in the first level, it seems that for relatively small airports that task is easy but it can become too complex or even impossible for large airports that use very specific stand rules (for instance, complex reserved areas for certain airlines). In the future, the robustness of the zone definition can be improved by the introduction of other techniques that facilitate this process.

Some extra recommendations can be made when it comes to the objectives optimized by the TLSAP. Note that in the current research the profitability of flights to airlines (the operating margins) are not included in the objectives, but this can be considered if airlines' interests are taken into account. Furthermore, this research optimizes the number of transfer passengers but not the number of hub connections. But if a hub is more concerned with expanding the network connections, it is always possible, in the future, to formulate a new objective function that explicitly maximizes the number of O&D connections established. Still related with the O&D connections, if a specific connection is particularly important for a hub, it would be convenient for the capacity teams if the model could guarantee one (or several) specific connections to be established. This could be done by adding a "reward objective" or a hard constraint that forces

that connection to take place.

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II

Literature Study
previously graded under AE4020

1

Introduction to The Literature Study

According to what was explained in the Introduction of this report, transfer passengers are a key stakeholder for hub airports. Furthermore, passenger hubs have been facing a consistent increase of air travellers over time, putting those airports and their capacity under pressure. The capacity and marketing teams of hub airports are required, more than ever, to make an efficient use of the infrastructure. This need to achieve higher efficiencies when it comes to the use of an airport's infrastructure has motivated researchers to propose new methods and models. In this field, the Stand Assignment Problem (SAP) is very commonly applied. The SAP is one of the most researched problems in the airline industry and it has been proven that it effectively contributes to achieve different objectives proposed by several stakeholders, namely airports, airlines and passengers.

The SAP provides a specific stand (or gate) assignment to the airport that follows certain rules and aims to certain goals. However, previous research has not yet focused on quantitatively analyzing how much the number of captured transfer passengers by a hub can be optimized by manipulating the stand assignments. In particular, it is relevant to understand the impact of adding new flights to an airport's schedule and how those flights, together with the original schedule, can maximize the number of transfer passengers captured by the hub. The literature is quite extensive when it comes to the qualitative analysis of hub networks, the impact of future expansions and the relationships between airlines and how that affects the dynamics of an airport. However, no research was found that would attempt to select new, profitable turnarounds, integrate them in an airport's schedule, and quantify the new hub connections and the number of passengers.

In an attempt to cover the research gap mentioned above, the main objective of this research is to develop and explore the applicability of a strategic stand planning model capable of selecting profitable routes that will expand the network of a hub, integrating those routes in the schedule and efficiently capturing transfer passengers at that hub. This model should be able to deliver a stand assignment that optimizes 2 main goals, namely the maximization of the number of captured transfer passengers but also the minimization of connection times. Solving any SAP that considers transfer passengers is inherently a hard task due to the commonly large size of the problems and its quadratic nature. For this reason, it is necessary to understand what is the state-of-the-art when it comes to the methods and approaches to solve the SAP with transfer passengers.

The previous paragraphs highlighted the need to perform a literature review of all the research related to the SAP that has been developed over the years. It is essential to understand exactly what kind of objectives are optimized in the SAP, how it can be formulated and more importantly how it can be efficiently solved when transfer passengers are modelled. Furthermore, it is relevant to understand how detailed the study of hub network expansion is, since this part is also a key element of the current research.

Hence, this literature study will cover two areas. On the one hand, the SAP formulations, solving methods, common objective functions and constraints need to be reviewed. On the other hand, works related to airport networks and route expansion will also be part of the reviewing process.

It is important to note that the SAP and the Gate Assignment Problem (GAP) are strongly related to each other and they are essentially the same problem. The objective of both is to assign operations to the airport's infrastructure. The major difference is that the SAP focuses on assigning operations to stands while the GAP focuses on the gate assignment. This research is focused on the SAP, but it is very common to find literature on the more generic GAP. For this reason, along this literature review, the term GAP will be more often used

than the term SAP. Nonetheless, everything that will be reviewed and explained for the GAP is applicable and valid for the SAP.

1.1. Research Questions

The model that will be developed in this research will be applied to a case study airport. With the results taken from this practical application, several questions are expected to be answered. These questions are shown below below:

- How can the Stand Assignment Problem (SAP) be formulated in order to be integrated in this research?
- How does the individual importance given to the maximization of transfer passenger through-put and the minimization of connection times influence the final solution and how does it affect the satisfaction of the airport authorities and passengers?
- Which Heuristic or Meta-Heuristic method is the most fitting to solve the SAP?
- What is the nature of the relationship between the way new flights are distributed over time (in peak or off-peak hours) and the types of solutions obtained? In particular, does allocating flights between peaks increase the number of possible connections and/or reduce the connection times?
- As a consequence of the growth in the number of captured transfer passengers, how much do hub connections and the network expand and how profitable could the proposed turnarounds potentially be?

1.2. Report Structure

The literature review presented in the next sections, which is part of the course AE4020: Literature Stud, materializes the first step of this research, which is to study previous work developed by several authors, clearly outline the gap of the current research and define the questions that should be answered to reach the final objective. The literature review is divided into different areas. In [chapter 2](#), the foundations of the SAP are reviewed. Then, [chapter 3](#) and [chapter 4](#) present, respectively, different formulations of the SAP when it considers transfer passengers and several solving methods. At the end of each of these chapters, it is possible to find a table that summarizes all the relevant information. The state of the research on airport performance analysis and network connectivity is examined in [chapter 5](#). Finally, in [chapter 6](#), the previous chapters are linked and the literature gap is explained. The same chapter includes a note on the possible applications of this research and finally, the research questions are formulated.

2

Foundations of the Gate Assignment Problem (GAP)

The gate assignment problem (GAP) is already extensively explored and reviewed. The fundamental idea of the problem is to assign a certain set of flights to a set of available airport gates, while some criteria and constraints have to be met [10]. Every gate assignment problem incorporates two fundamental parts:

- **Objective Function:** refers to the objectives one wants to achieve with the gate assignment.
- **Constraints:** specific restrictions that the problem is subject to, making it applicable in real life scenarios.

The objective function and the constraints that a problem is subject to can be very different in each specific case. This makes the GAP a very flexible problem that can be applied in different scenarios.

In this chapter, examples of objective functions will be presented. Most of the times, however, the GAP includes a multi-objective function that makes the model more applicable to real-life scenarios. This concept will also be discussed. Then, relevant constraints that some authors used in the past are also presented. Finally, at the end of the chapter, some information regarding the insertion of transfer passengers in the gate assignment problem is also provided.

2.1. Objective Function

The objective function chosen for the GAP can assume different forms, depending on the intended results of the research. A very common objective, that is included in most of the researches is the minimization of passenger walking distance [2] [11] [7] [12]. There are some authors that mention the minimization of the passengers' connection time [6], which in a sense is equivalent to the first objective. Indeed, S. H. Kim et al. ([13]) show how the transit time of passengers depends on the distance between two points in the airport. Obviously, that dependence is translated into the passengers' average walking speed inside the terminals. The two points between which one needs to consider the distance depends on the type of passenger, that is, if the passenger is arriving, departing or transferring between flights. If the passenger is either arriving or departing, the distance between the airport's exit/entrance is needed. If the passenger is connecting at the airport, the inter-gate distances should be considered. B. Maharjan and T. I. Matis ([14]) aim to minimize the passenger discomfort for critical connections. This objective is translated to a connection penalty that is a function of the connection time and the distance between gates.

Other authors extended the GAP with objectives that go beyond the consideration of passengers walking times. In [8], for instance, S. H. Kim et al. propose a model that takes into consideration the aircraft congestion on ramps. The authors try to minimize weighted taxi time and weighted taxi delay.

Another objective that is fairly common is the assignment of flights so that the resulting schedule is robust with respect to delays. With this robustness incorporated in the model, small perturbations on the departure or arrival of flights do not invalidate the obtained gate assignment. In the example shown in [10], the authors try to reach this objective by maximizing the intervals of time between which two consecutive flights use the

same gate. Because these intervals are dynamic in that example, i.e., their values are not constant for every situation, their model becomes more robust when compared to models that assume a fixed buffer time.

In busy airports, it happens very often that the number of aircraft exceed the number of available gates, specially during peak hours, which leads to delays and unassigned aircraft. Several authors have proposed models to solve this problem by minimizing the flights that are assigned to the apron [11] [7] [15], by explicitly minimizing the costs of delays [16] or by minimizing the aircraft waiting time on the apron [12] [4]. Another way to deal with delays in a busy airport is by minimizing the dispersion of gates idle time periods [3]. Achieving this objective is important because, if the idle times are uniformly distributed among all gates, the probability that a delayed departing flight will still leave earlier than the next arrival is minimum.

Fulfilling gate assignment preferences can also be one objective of the GAP. The authors of [10], [15] and [17] take this into consideration. These preferences can represent aircraft that have to be assigned to specific gates because of their size, or represent the fact that some airlines have reserved usage of a group of gates. Another objective of the GAP that hasn't been so commonly discussed is the baggage transport distance [18] [19], even though the algorithms used to minimize the passengers walking distance can be adapted to the scenario that also considers baggage transport distance [19].

All these variations show how flexible the GAP can be and how sometimes it can be easily adapted by just performing some changes the objective function. Nonetheless, for the current research it is of paramount importance to take into consideration terminal activities, namely the usual passenger transit times. Table 2.1 contains an overview of all types of objective functions mentioned in this subsection, and the research in which they are mentioned.

Type of Objective	Ref.	Observations
Minimization of Passenger Walking Distances	[2] [7] [11] [12]	Most common objective function in the GAP. Usually used together with other objectives.
Minimization of Passenger Connection Times	[6]	Equivalent to the minimization of passenger walking distances.
Minimization of Passenger Discomfort for Critical Connections	[14]	Adds a connection penalty that is a function of the connection time and the distance between gates
Minimization of Weighted Nominal Taxi Time and Weighted Taxi Delay	[8]	Not very common. Main objective is to consider aircraft congestion on ramps.
Maximization of Intervals of Time Between Which two Consecutive Flights use the Same Gate	[10]	Increases robustness with respect to small perturbations on the departure or arrival of flights.
Minimization of Flights Assigned to the Apron	[11] [7] [15]	Looks for a solution at peak hours in busy airports, when the number of aircraft exceeds gate availability.
Minimization of Delay Costs	[16]	
Minimization of Aircraft Waiting Time on the Apron	[4] [12]	
Minimization of Dispersion of Gates Idle Time Periods	[3]	Minimizes probability that a delayed departing flight will still leave earlier than the next arrival.
Maximization Gate Assignment Preferences	[17] [15] [10]	Gate preferences include aircraft that have to be assigned to specific gates because of their size, or airline preference
Minimization of Baggage Transport Distance	[18] [19]	Not very common. Algorithms that minimize passengers walking distance can be adapted to baggage transport problems.

Table 2.1: Overview of different types of objective functions used in the Gate Assignment Problem.

2.2. Multi-Objective Functions

Previously, some examples of typical objective functions (OFs) used in the GAP were shown. However, it is hard to find models that consider a single objective. Most of the times, those functions are a linear combination of several OFs (see Equation 2.1, where w_i are the weights of each individual OF z_i). By implementing this strategy, authors are able to create more realistic models with extra reliability.

$$OF = \sum_i w_i z_i \quad (2.1)$$

Nonetheless, choosing the weights w_i of the individual terms is not straightforward and is heavily influenced by the relative influence intended for each of them.

Some considerations have to be taken into account when working with multi-objective functions. For instance, when an objective function is composed by several terms, it may be possible to eliminate those that don't conflict with each other as these are redundant terms. This leads to a simpler, lower dimension

problem [20]. Other times, extra parameters need to be multiplied to some terms so that all of them can be comparable. For instance, the multi-objective function of [4] includes (i) the minimization of total passenger transfer distance, (ii) the minimization of baggage transferring distance and (iii) the minimization of total passenger waiting time. To be able to sum all of them as in Equation 2.1, an extra system parameter is added to (iii) to convert time to distance.

U. Dorndorf et al. ([21]), as an example, consider several airport operations. The authors aim at finding an assignment that is robust with respect to flight delays while it maximizes an aircraft's preference score to specific gates and minimizes the number of tows. H. Ding et al. ([11]) try to minimize passenger walking distances between gates while aiming to minimize the number of unassigned flights. S. Yan and C. M. Huo [22] formulate their version of the GAP as a dual-objective problem, and aim to minimize the total passenger walking distance while minimizing the passenger waiting times.

2.3. Constraints

Constraints translate real-life requirements to mathematical formulas that can be included in the gate assignment model. For the GAP, two essential constraints are always included, as they are needed to create physically achievable outputs. These two constraints are shown below.

- Each flight must be assigned to one and only one gate.
- No two aircraft may be assigned to the same gate concurrently.

As it was already mentioned in 2.1, there may not be enough available gates for all aircraft at busy airports. In this cases, it seems impossible to fulfill the first constraint shown above and consequently, the GAP doesn't seem applicable anymore. However, as H. Ding et al. show in [11], it is possible to mathematically adapt the GAP so that it can be still applicable. The authors want to minimize the number of aircraft assigned to the apron in an over-constrained airport (more flights than gates) and for that they add a "dummy" gate that represents the apron. Modelling the apron as a gate allow them to apply the first constraint without any restrictions. For the second constraint, they simply don't consider it for the "apron gate", since several aircraft may be assigned concurrently to the apron.

Some authors decide to formulate these two constraints in a different way. For instance, D. Wei and C. Liu ([23]) define two constraints, one that imposes each flight to have at most one preceding flight, and another which specifies that every flight has at most one succeeding flight. The effect of these two constraints combined is that no two aircraft will be assigned simultaneously to the same gate, even though it is not explicitly presented this way.

But besides the two main constraints, every model of the GAP also include non-negativity requirements, that state that variables can only either assume positive values or be equal to 0. If the GAP is formulated in a specific form, for instance only using binary variables, then an extra constraint stating that the variables can only assume the values 0 or 1 is also needed [6].

Even though these two constraints are fundamental, most of the times other requirements are present, such as individual airport constraints. Three important constraints belong to this category, namely (i) the creation of subdivisions of gates for different airlines, (ii) not allowing some aircraft to park at specific gates and (iii) forcing flights to be assigned to nearby gates [2]. The first one (i) is needed as it is very common for specific airlines (or groups of airlines) to have exclusive use of a set of gates. The second constraint (ii) prevents wide-bodied aircraft to be assigned to small gates. Finally, the third constraint (iii) can be convenient when two flights carry the same transfer passengers, so it is more advantageous for them to be as close as possible to each other. This last constraint is relevant for the current research, as it can help reducing the minimum connection time between several flights. It should be noted, however, that this constraint is non-linear and thus not as straightforward to solve. The non-linearity of the GAP is discussed further ahead in chapter 3 and chapter 4.

D. Wei and C. Liu ([23]) add an extra constraint stating that a flight can only be assigned to a gate after the preceding flight has left that gate for a certain buffer time. This restriction allows for a more robust flight schedule, because flights can still be delayed for a limited amount of time without disturbing the gate planning.

Another important constraint is the so called "shadow restriction". The authors of [17] and [21] include it in their models. It doesn't allow two big aircraft to be parked at adjacent gates as their wing tips can get dangerously close to each other or even collide.

Sometimes, auxiliary constraints are required to define the decision variables [12], or even to establish relationships between variables, so that they assume consistent values [12] [6].

Table 2.2 collects all the examples given in this section regarding the use of constraints.

Type of Constraint	Ref.	Observations
Assignment of one flight to one and only one gate	All	Essential constraint of the GAP, needed in every formulation to obtain physically achievable solutions.
Prevention of the concurrent assignment of two aircraft to the same gate	GAP	
Every flight can only have, at most, one preceding flight and one succeeding flight	[23]	Has the same effect as the constraint " <i>Prevention of the concurrent assignment of two aircraft to the same gate</i> "
Creation of subdivisions of gates for different airlines	[2]	Used when specific airlines have exclusive use of a set of gates.
Prevention of the assignment of aircraft to specific gates	[2]	Prevents wide-bodied aircraft to be assigned to small gates.
Forcing flights to be assigned to nearby gates	[2]	Used when two flights carry the same transfer passengers, so it is advantageous for them to be close to each other.
Assignment of a flight to a gate after the preceding flight has left for a certain buffer time	[23]	Creates a more robust flight schedule, capable of coping with small flight delays.
Shadow restriction	[17] [21]	Blocks two wide bodied aircraft to be assigned to adjacent gates, preventing their wing tips to get close to each other.
Auxiliary Constraint	[12]	Used to mathematically define decision variables.
	[6]	Used to establish relationships between decision variables.
	[12]	

Table 2.2: Overview of different types of constraints used in the Gate Assignment Problem.

2.4. Integration of Transfer Passengers in the GAP

The current research focuses on the maximization of the number of transfer passengers captured by the airport. For this reason, it becomes paramount to find ways of portraying this kind of passengers in the GAP. Transfer passengers differ from arriving and departing passengers because the former travel between gates, while the latter travel between a gate and the exit/entrance of the airport. Mathematically speaking, there is also a significant difference, since it is required to add non-linear terms to model transfer passengers in the GAP. Generally, when in the GAP some costs depend on the assignment of pairs of flights to pairs of gates, the objective function will contain non-linear terms [24], more specifically, quadratic terms. To better understand why this is the case for transfer passengers, a simple example is presented. Let's suppose that a set of flights has to be assigned to a set of gates. Then, i and j are two arbitrary flights and k and l are two arbitrary gates. Let's also define the distance between two arbitrary gates k and l as d_{kl} and the decision variable x_{ik} as being 1 if flight i is assigned to gate k and 0 otherwise. In this case, the hypothetical distance that transfer passengers have to walk from gate k to l if their arriving and departing flights are respectively assigned to those two gates is mathematically given by $d_{kl} \times x_{ik} \times x_{jl}$, which represents a non-linear term.

Transfer passengers are recurrently included in the GAP, mainly for two reasons. First, it is important to model this kind of passengers for its significance in big airports, namely hub airports. Second, the additional difficulty that is inherent to the representation of transfer passengers motivates authors to find and develop more efficient approaches to the problem. When it comes to the type of objectives, it is fairly common to see research that tries to minimize the total transfer passenger distance for a given arrival-departure cycle [2] [7] [25]. On the other hand, research that involves minimizing passenger connection times between flights is quite relevant for this research. For instance, J. Xu, and G. Bailey ([6]) were the first to directly apply the GAP to the minimization of customer connection time.

Because of the relevance of transfer passengers in the current research, the next two chapters will be dedicated to this topic. In chapter 3, different ways of mathematically formulating the GAP with transfer passengers are shown and briefly explained, and in chapter 4, the most used methods to solve the problem are explored in some detail.

3

Integration of Transfer Passengers in the GAP: Formulations

Transfer passengers are usually a key part of the GAP, and the same applies to this research. However, including them in the GAP formulations can be challenging. Because of the greater complexity, some authors propose simplifications that give origin to a more manageable problem without compromising the validity of the model or invalidating the obtained results. This chapter will reveal which types of formulations are more common in the GAP when it includes transfer passengers, and how it can be adapted. As it was explained in the previous chapter, transfer passengers naturally lead to the emergence of quadratic terms in the GAP formulations. For this reason, this chapter starts with examples of quadratic formulations, first purely binary and then mixed. After that, linear adaptations of the problem are also shown. Then, more complex and less common formulations are briefly explained. The criteria used to define and split different formulations is based on the work of A. Bouras et al. ([26]). At the end of the chapter, a table with an overview of the most relevant examples explored here is presented.

3.1. Binary Quadratic Integer Formulation

The binary quadratic formulation (BQF) is a simpler version of the mixed binary quadratic formulation, which is shown on the next section. The difference is that the former only contains binary decision variables. H. Ding et al. ([7]) formulate the GAP as a purely binary quadratic problem, with the objective to minimize the number of ungated flights and the total walking distances (connection times). The only variable y_{ik} used in this formulation is defined in [Equation 3.1](#).

$$y_{ik} = \begin{cases} 1, & \text{if flight } i \text{ is assigned to gate } k \\ 0, & \text{otherwise} \end{cases} \quad (3.1)$$

It is worth understanding the technique used by the authors to represent the ungated flights. Let m be the number of gates. Then, an extra gate $m + 1$ is added that represents the apron or tarmac, which is where aircraft have to stay when no gates are available. This means that minimizing the number of ungated flights i is simply minimizing the use of the variables $y_{i,m+1}$. There is another dummy gate (Gate 0) which is defined as the entrance or exit or the airport. If $f_{i,j}$ is the number of passengers transferring from flight i to flight j , then $f_{i,0}$ represents the number of arriving passengers from flight i and $f_{0,i}$ is the number of departing passengers to flight i . It is now possible to construct the objective function. Refer to [Equation 3.2](#), where the first set of term depicts the minimization of ungated flights and the second, third and fourth represent, respectively, the minimization of transfer, departing and arriving passengers walking distance. Note that $w_{k,l}$ is the walking distance from gate k to gate l .

$$\min \left(\sum_{i=1}^n y_{i,m+1} + \sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^{m+1} \sum_{l=1}^{m+1} f_{i,j} w_{k,l} y_{i,k} y_{j,l} + \sum_{i=1}^n f_{0,i} w_{0,i} + \sum_{i=1}^n f_{i,0} w_{i,0} \right) \quad (3.2)$$

The same authors, H. Ding et al., have developed more work related to the GAP. In particular, in [11] they aim to the same objectives, that is, they try to minimize the number of ungated flights and the total

passenger walking distances. For that, the GAP is again formulated as a binary quadratic problem, but the solving methods are different. These will be further explored in [chapter 4](#).

3.2. Mixed Quadratic Integer Formulation

J. Xu and G. Bailey ([6]) describe a problem whose objective is to minimize the passenger connection time, and use a mixed quadratic integer formulation (MQIF). It includes three types of decision variables, and it is said to be a mixed formulation because two of them are binary variables and the other is a non-binary variable, as it can be seen below.

$$y_{ik} = \begin{cases} 1, & \text{if flight } i \text{ is assigned to gate } k \\ 0, & \text{otherwise} \end{cases} \quad z_{ijk} = \begin{cases} 1, & \text{if flights } i, j \text{ are assigned to gate} \\ & k \text{ and } i \text{ immediately precedes } j \\ 0, & \text{otherwise} \end{cases} \quad (3.3)$$

$$t_i: \text{ time the gate opens for boarding for flight } i \quad (3.4)$$

[Equation 3.3](#) defines the two binary decision variables, while [Equation 3.4](#) defines the non-binary variable. The latter is simply an integer (discrete) time variable, that is used in the constraints that assure gates are opened for boarding after the flight's arrival and before its departure. It is also used in the constraint that assures that each gate is only used by one aircraft at each point in time. For this last constraint, the decision variables z_{ijk} are also used. Due to the extension of the formulation, no more constraints will be mentioned here, also because they do not feature any particularity worth mentioning. However, it is relevant to analyse the objective function, presented in [Equation 3.5](#).

$$\text{Min} \sum_{i,j \in N} \sum_{k,l \in K} f_{ij} c_{kl} y_{ik} y_{jl} \quad (3.5)$$

where N is the set of flights, K is the set of gates, f_{ij} is the number of transfer passengers from flight i to flight j and c_{kl} is the connection time from gate k to gate l . The quadratic nature of this formulation is visible in [Equation 3.5](#), in the product of two decision variables.

Having presented this formulation, the authors of [6] use an approach to reformulate it as a mixed binary integer problem with linear objective function and constraints. For that, a new binary variable is defined (see [Equation 3.6](#)). This variable will replace the quadratic term $y_{ik}y_{jl}$ in the original objective function ([Equation 3.5](#)), making it linear in x .

$$x_{ijkl} = \begin{cases} 1, & \text{if flight } i \text{ is assigned to gate } k \text{ and} \\ & \text{flight } j \text{ is assigned to gate } l \\ 0, & \text{otherwise} \end{cases} \quad (3.6)$$

The original constraints do not change, and only a few extra need to be added to guarantee the link between the original and the new variables. The two constraints in [Equation 3.7](#) state that a variable x_{ijkl} can be equal to 1 if $y_{ik} = 1$ and $y_{jl} = 1$, while the constraint in [Equation 3.8](#) reinforces that x_{ijkl} has to be equal to 1 if $y_{ik} = 1$ and $y_{jl} = 1$. Together, the three constraints create the necessary and sufficient condition: $x_{ijkl} = 1 \Leftrightarrow y_{ik} = 1 \wedge y_{jl} = 1$.

$$x_{ijkl} \leq y_{ik}, \forall i, j \in N, \forall k, l \in K \quad x_{ijkl} \leq y_{jl}, \forall i, j \in N, \forall k, l \in K \quad (3.7)$$

$$y_{ik} + y_{jl} - 1 \leq x_{ijkl}, \forall i, j \in N, \forall k, l \in K \quad (3.8)$$

3.3. Binary Linear Integer Formulation

The binary (0, 1) linear integer formulations (BLIF) are quite popular when it comes to solving the GAP by virtue of its simplicity. R. S. Mangoubi and D. F. X. Mathaisel ([2]) use one of these formulations to minimize arriving (p_i^a), departing (p_i^d) and transfer (p_i^t) passenger walking distances. The objective function is given in [Equation 3.9](#).

$$\text{Min} \sum_{i=1}^N \sum_{j=1}^N (p_i^a d_j^a + p_i^d d_j^d + p_i^t d_j^t) x_{ij} \quad (3.9)$$

where N is the total number of gates. In this case, the formulation is linear because all terms in the objective function are linear. It is also said to be binary integer because each decision variable x_{ij} can only take the values 0 and 1, and it is defined the following way:

$$x_{ij} = \begin{cases} 1, & \text{if flight } i \text{ is assigned to gate } j \\ 0, & \text{otherwise} \end{cases} \quad (3.10)$$

Another relevant side of the formulation in [2] is how the authors reduce the quadratic nature of the GAP to achieve a simpler linear problem, just like the objective function (Equation 3.9) shows. For that, it is assumed that a transfer passenger arriving at a gate j is equally likely to board any gate. A uniform probability distribution of all integrate walking distances is then used to determine the average passenger transfer walking distance. If a passenger arrives at a gate j and w_{jk} is the distance between that gate and a generic gate k , then the average walking distance for that passenger is:

$$d_j^t = \frac{1}{N} \sum_{k=1}^N w_{jk} \quad (3.11)$$

Even though this is a reasonable assumption that simplifies the problem, it might be biased, specially when transfer patterns are already known. For instance, just like the authors point out, if there is a group of attractive gates all close to each other, the real walking distances between them could be shorter than the ones obtained in the uniform distribution. As a result, the model will be adding extra costs to pairs of gates that, in reality, are very suitable for transfer passengers.

3.4. (Mixed) Linear Integer Formulation

Usually, it is assumed that flights are immediately assigned to a gate when they arrive at an airport, so the schedules are fixed. But A. Lim et al. ([12]) propose a gate scheduling model in which a flight can be assigned to a gate anytime inside an established time window, so not necessarily right after it lands. The amount of time an aircraft needs to stay assigned to a gate is fixed, and it can be positioned anywhere between the limits of the time window. Figure 3.1 illustrates how this concept works. On the left-hand side, the aircraft is assigned to a gate right after it arrives, at 15 : 00, and stays there for 3 hours, until 18 : 00. On the right-hand side, however, it is only assigned one hour later, at 16 : 00, and it must stay there for the same 3 hours, so it only leaves at 19 : 00, right when its dedicated time window is over.

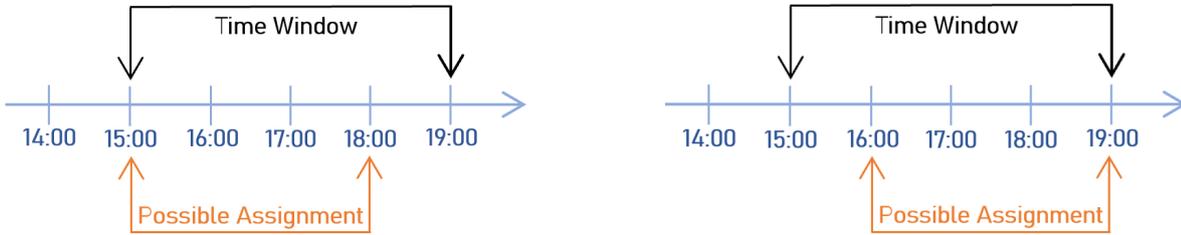


Figure 3.1: Graphical representation of two possible assignments of a flight inside its dedicated time window.

This problem is modelled with a linear integer formulation (LIF), with both binary and integer variables. The definition of the decision variables is shown in Equation 3.12 and Equation 3.13 and it is crucial to understand the model proposed by the authors. It is important to mention that, for the sake of simplicity, c_i only assumes non-negative integer values. All the remaining decision variables are binary. Thus, the final formulation is linear, integer and composed of mixed types of variables.

$$x_{ik} = \begin{cases} 1, & \text{if flight } i \text{ is assigned to gate } k \\ 0, & \text{otherwise} \end{cases} \quad z_{ijkl} = \begin{cases} 1, & \text{if flight } i \text{ is assigned to gate } k \text{ and} \\ & \text{flight } j \text{ is assigned to gate } l \\ 0, & \text{otherwise} \end{cases} \quad (3.12)$$

$$y_{ij} = \begin{cases} 1, & \text{if flight } i \text{ departs no later than} \\ & \text{flight } j \\ 0, & \text{otherwise} \end{cases} \quad c_i: \text{ time when flight } i \text{ starts to occupy a gate} \quad (3.13)$$

The complete objective function is shown in Equation 3.14. Note that f_{ij} represents the number of passengers between flights i and j and w_{kl} is the walking distance between gates k and l . The first set of terms of the objective function represents the minimization of the transfer passenger walking distance. The second set of terms has a different purpose. Even though the model becomes more flexible with the introduction of time windows, it is assumed that it is more convenient if flights are assigned to a gate as soon as they land, so a penalty is added in the objective function for the cases in which the assignment is delayed. The second set of terms mathematically describes this penalty. The unit delay penalty of flight i is given by p_i and a_i marks the beginning of flight i 's time window.

$$\sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^m \sum_{l=1}^m f_{ij} w_{kl} z_{ijkl} + \sum_{i=1}^n p_i (c_i - a_i) \quad (3.14)$$

Two fundamental constraints, shown in Equation 3.15, are needed to define the variable y_{ij} . The one on the left states that $y_{ij} = 1$ if flight i departs before or at the time a gate opens for gate j , while the one on the right states that $y_{ij} = 0$ in the opposite case.

$$(c_i + d_i) - c_j + y_{ij}M > 0, \quad 1 \leq i, j \leq n \quad (c_i + d_i) - c_j - (1 - y_{ij})M \leq 0, \quad 1 \leq i, j \leq n \quad (3.15)$$

The relationship between the variables x_{ik} and z_{ijkl} needs to be established as well, and this is done the same way J. Xu and G. Bailey did in [6], using the constraints shown in Equation 3.7 and Equation 3.8.

This model proposed by A. Lim et al. is able to solve the over-constrained gate assignment problem, in which there are more flights than available gates, more efficiently than other proposals, from which [11] is an example. This is because in the former model flights can be assigned a bit later to a gate, instead of being automatically considered ungated if there is no space for them at the time of arrival.

Several authors have attempted to minimize the variance or the dispersion of gate idle time periods. In [3], A. Bolat approaches this objective with a framework consisting of five different models. Two are formulated as mixed integer **linear** programming (MILP) and three as mixed integer **non-linear** programming. Some decision variables are redefined in order to transform the non-linear models into linear models.

3.5. Other Formulations

Formulation as a Quadratic Assignment Problem

The Quadratic Assignment Problem (QAP) is a problem where the costs of assigning a pair of facilities to a pair of locations depend on the flow between the facilities and the distance between the locations [27]. In this sense, if one considers the facilities to be flights and the locations to be gates, the GAP can be seen as a QAP when it includes transfer passengers, as they basically represent flow between gates. In fact, it is generally accepted that the GAP is a QAP, but it is not always explicitly formulated as one. The examples presented previously are proof of that.

There are, however, authors that still formulate the GAP as a QAP, as it is the case with A. Haghani and M. C. Chen ([27]). To be able to solve the problem, they linearize it into an integer programming formulation. A Drexl and Y. Nikulin ([15]) formulate the multi-criteria GAP as a Quadratic Assignment Problem, but instead propose a Pareto simulated annealing to solve it.

Formulation as a Clique Partitioning Problem

The Clique Partitioning Problem (CPP) is part of graph theory. In short, a *clique* of an undirected graph is a subset W of the vertices of that graph such that for every pair of vertices in W , there exists an edge connecting them. In other words, every pair of vertices in W are adjacent. The *Clique Partitioning Problem* consists in finding the minimum number of cliques in the graph so that each vertex is represented in one and only one clique [28].

U. Dorndorf et al. ([21]) tried something never considered before in the literature by presenting a regular optimization GAP and then transforming it into a CPP. The authors support this choice with two arguments. First, there are efficient heuristics to solve the CPP, and second, this method simplifies the incorporation of schedule robustness (that is, keeping a buffer time before a flight's arrival and after its departure) in the objective function. Besides this objective, the authors also propose to maximize the total assignment preference score and minimize both the number of unassigned flights during overload periods and the number of tows. As it is possible to tell, none of these objectives contemplate transfer passengers. This method is still included here because of its rareness.

Formulation as a Multi-Commodity Network Flow

It is possible to model the GAP as a multi-commodity network flow problem (MCNFP). In the work developed in [1], S. Yan and C. M. Chang suggested the division of flights into three classes: (1) arriving flights, (2) departing flights and (3) a pair of flights served by the same aircraft. In the last case, the aircraft arrives from a flight, stays in the airport for some time, and finally departs for a new flight. In between that time, it may be towed from one gate to the other. The authors created a time-space network for each flight or pair of flights, in order to model the corresponding gate assignment. The generic time-space network contains two dummy nodes: an initial node and a final node. The remaining nodes correspond to a gate assignment at a certain time point. There are three types of arcs. Entering arcs are added from the initial node to every available gate at the starting time (here, gate availability also accounts for cases in which an aircraft type or airline are not allowed to park at a gate). Holding arcs connect two time-adjacent nodes at the same gate. Leaving arcs connect available gates at the ending time to the final dummy node. Flights are treated as the commodity that has to flow through the networks. Figure 3.2 illustrates a time-space network, where the horizontal axis is time and the vertical axis is space.

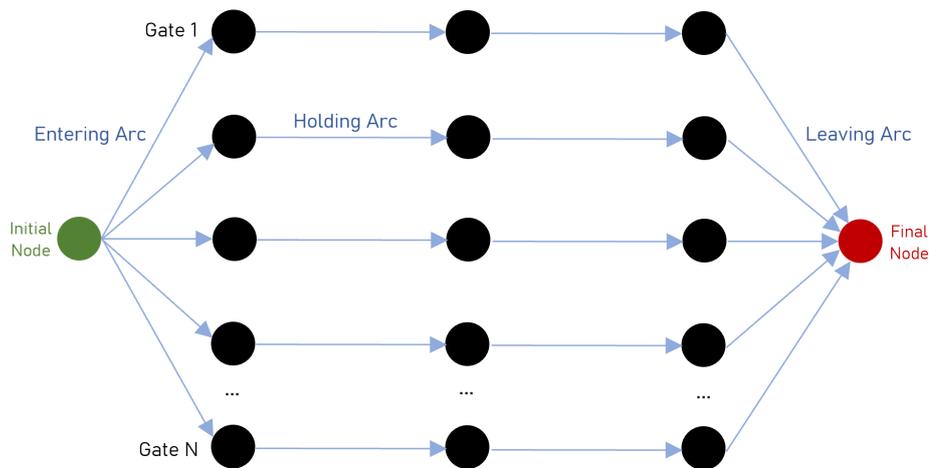


Figure 3.2: Generic time-space network developed in [1] to model the assignment of a flight or a pair of flights to a gate.

The supply and demand of each time-space network is one (one aircraft), which means that the objective is to flow each aircraft through the corresponding network by solving the shortest path problem. Since this is a MCNFP, the constraint that ensures the flow conservation at each node of each network has to be added.

B. Maharjan and T. I. Matis ([14]) also use a MCNFP of the GAP, but the approach is substantially different. In this case, gates are defined as the commodity that has to flow from a source to a terminal node, while going through nodes that represent flight demand. The authors show that this approach is computationally efficient. The objective of this model is to minimize the fuel burn cost of aircraft taxi while minimizing the passenger discomfort for critical connections. The second part of this objective has some relevance for the current research. Each existing connection has an associated multidimensional cost that is a function of the distance between the connecting gates and the time left until the departure of the connecting flight. The cost is formulated in such a way that assigning a connecting flight far from the arriving flight when the connection time is small leads to a higher cost. On the other hand, the longer the connection time is, the more irrelevant it becomes where the connecting flight is assigned, since passengers have time to reach more gates.

Even though the last example is a multi-commodity network flow formulation, the objective function still contains a quadratic term, that is simplified in a similar way to the linearization done in Equation 3.7 and Equation 3.8.

3.6. Overview of GAP Formulations that include transfer passengers

Table 3.1 contains a summary of the current chapter. The different formulations are shown together with the objective function(s) and specific observations of the examples explored.

Authors (Reference)	Formulation	Objective Function	Observations
H. Ding et al. (7)	Binary Quadratic Integer Formulation	1) Minimization of ungated flights 2) Minimization of transfer, departing and arriving passengers walking distance	Only one purely binary variable is used. The apron and the exit/entrance of the airport are represented as a dummy gate, which facilitates the development of a uniform mathematical model.
H. Ding et al. (11)			Very similar to the previous formulation. The difference between the two works is the type of solving method used. This will be further explored in chapter 4 .
J. Xu and G. Bailey (6)	Mixed Quadratic Integer Formulation Transformed to a Mixed Linear Integer Formulation	1) Minimization of transfer passengers connection times	Three types of decision variables are used: two binary and one non-negative integer. The latter is a discrete time variable. A new decision variable is created to transform the quadratic problem into a linear problem. Some extra constraints need to be added to link the original variables with the new variable.
R. S. Mangoubi and D. F. X. Mathaisel (2)	Binary Linear Integer Formulation	1) Minimization of transfer, departing and arriving passengers walking distance	Only one binary variable is used. The quadratic nature of this problem is simplified to a linear formulation by using the following assumption: a transfer passenger arriving at a gate is equally likely to board any gate for their connecting flight. The average walking distance of that passenger is then computed based on this assumption and used as the "transfer cost" in the objective function for that gate. It is a valid assumption, but it may over-estimate the "transfer" cost of attractive gates.
A. Lim et al. (12)	Linear Integer Formulation	1) Minimization of transfer passengers walking distance 2) Minimization of delayed flight assignments to gates	Does not assume fixed flight schedules, but rather allows aircraft to be assigned to a gate after waiting on the ground for a limited amount of time (within a time window). Because it is more flexible, it leads to more efficient solutions of the over constrained GAP. This formulation uses 4 types of decision variables: 3 binary and 1 integer (discrete) time variable, which indicates when a flight is effectively assigned to a gate.
A. Bolat (3)	Mixed (Non-)Linear Integer Formulation	1) Minimization of the variance or the dispersion of gate idle time periods	This framework consists of five different models. Two are formulated as mixed integer linear programming (MILP) and three as mixed integer non-linear programming. Some decision variables are redefined in order to transform the non-linear models into linear models.
A. Haghani and M. C. Chen (27)	Quadratic Assignment Problem (QAP) Formulation	1) Minimization of transfer, departing and arriving passengers walking distance	When the GAP includes transfer passengers, it can be considered a QAP, due to its inherent quadratic nature. Even though it does not have to be explicitly formulated as one, some authors still do that.
A. Drexel and Y. Nikulin (15)		1) Minimization of ungated flights 2) Minimization of transfer, departing and arriving passengers walking distance 3) Maximization of the total gate assignment preferences	
U. Dorndorf et al. (21)	Clique Partitioning Problem (CPP) Formulation	1) Maximize Schedule Robustness 2) Maximization of the total gate assignment preference score 3) Minimization of ungated flights 4) Minimization of number of tows	These authors were the first to adapt the CPP to the GAP. This choice was supported with two arguments. First, there are efficient heuristics to solve the CPP. Second, the incorporation of objective 1) in the model is simplified when CPP is used.
S. Yan and C. M. Chang (1)	Multi-Commodity Network Flow Formulation	1) Minimization of transfer, departing and arriving passengers walking distance	In this formulation, flights are divided into three classes: arriving, departing and pairs of flights served by the same aircraft. The authors created a time-space network for each flight or pair of flights, in order to model the corresponding gate assignment. The generic time-space network can be seen in Figure 3.2 . Flights are treated as the commodity that has to flow through the networks. The solution is obtained by solving the shortest path problem for each network.
B. Maharjan and T. I. Matis (14)		1) Minimization of the fuel burn cost of aircraft taxi 2) Minimization of passenger discomfort for critical connections	In this case, gates (not flights) are defined as the commodity that has to flow from a source to a terminal node, while going through nodes that represent flight demand. Passenger discomfort is a function of the distance between the connecting gates and the time left until the connecting flight departs.

Table 3.1: Overview of different approaches used to formulate transfer passengers in the Gate Assignment Problem.

4

Integration of Transfer Passengers in the GAP: Solving Methods

As it was already mentioned in [chapter 2](#), the GAP may naturally contain quadratic terms when transfer passengers are considered. More specifically, in [chapter 3](#) it is shown that, under this condition, the GAP is generally considered to be a Quadratic Assignment Problem. The generic quadratic gate assignment problem was already shown to be NP-hard [29] [30], meaning that it is an extremely difficult problem to solve - there are no known algorithms capable of solving it in a polynomial-bounded amount of time. In order to obtain feasible solutions in a reasonable amount of time, several authors have tried different approaches.

It is worth exploring which particular approaches are used to solve the GAP when transfer passengers are included, since this version is an inevitable part of the current research. In this chapter, the application of exact methods to solve the GAP is presented, followed by the more in-depth explanation of several heuristic and meta-heuristic methods specially designed to solve the GAP. Finally, at the end of the chapter, the reader can find a summary of the methods and remarks taken from their analysis.

4.1. Exact Methods

Different methods are used to solve the GAP. Some problems may be solved with exact methods. These yield the optimal solution of a model. Due to the predominant binary nature of the GAP, a method that is very commonly used to solve this operations research problem is the Branch and Bound (B&B), which also serves as the base to some optimization software such as CPLEX. Most of the times, however, it becomes impractical to make direct use of these exact methods. In particular, when the GAP contemplates non-linear terms, which is the case when connecting passengers are considered, it is not even possible to directly use those methods. Besides, the GAP can easily become a problem with large dimensions, that includes thousands or millions of decision variables. As a consequence, it requires large amounts of memory and a large computational time to be solved [14].

Authors that are still willing to apply exact methods ought to find alternatives. This is the case with L. Wang ([31]), who solves the GAP with a method based on the Branch and Bound technique. In [2], R. S. Mangoubi and D. F. X. Mathaisel do not use B&B either, but rather suggest a linear programming relaxation of an integer program formulation. The model proposed by these two authors is linear, so it is automatically easier and faster to solve. J. Xu and G. Bailey ([6]) follow a similar path. They originally propose a quadratic formulation, which is then simplified to an equivalent linear model, so that it can be solved using the Branch and Bound technique (CPLEX). However, a heuristic method proves to be faster and just as efficient.

B. Maharjan and T. I. Matis ([14]) develop a rather efficient technique to be able to solve their version of the GAP with less effort. The original problem is substantially large, so it is split into a series of sub-problems of smaller dimension. The division is based on the layout of the airport's infrastructure and it leads to three levels of assignment, which are solved in hierarchical order. Each level makes use of the assignment performed on the previous level. See below the division made for the study case airport used in [14]:

- **Level 1 (Zone Assignment):** assigns flights to 1 of the 3 terminals of the airport

- **Level 2 (Sub-zone Assignment):** assigns flights already assigned to one of the terminals on Level 1 to one respective sub-zone. A sub-zone represents a group of gates in the same corridor.
- **Level 3 (Gate Assignment):** assigns flights already assigned to one sub-zone on Level 2 to a gate inside that sub-zone.

As a consequence of dividing the original problem into sub-problems, some changes have to be applied to the parameters, objective function and constraints so that the grouping of gates is effectively contemplated in the model. However, the basic model remains unchanged. Each smaller sub-problem is then solved in a reasonable amount of time, using CPLEX. The method of zone-based decomposition of large problems proves to be extremely useful to solve large gate assignment problems with exact methods. It has to be pointed out, however, that even though an exact method can be applied in this situation, the hierarchical division approach is not strictly an exact method, but rather heuristic. This is because the specific division chosen does not guarantee that the global optimum will be achieved.

4.2. Introduction to Heuristic and Meta-Heuristic Methods

Heuristic and Meta-Heuristic methods have proven to be essential to solve most of the non-linear versions of the GAP. Unlike exact solving methods, neither of these two kinds of methods guarantee that the global optimum will be reached. Nonetheless, they always arrive at feasible solutions for any kind of GAP problem, they are usually faster than exact methods and generally don't require linearizations of the original model to be applied.

However, meta-heuristic methods have advantages when compared to heuristic methods. The latter are problem-dependent techniques, specifically adapted to the characteristics of each problem, but that can easily get stuck in local optima because they are too greedy. Meta-heuristics, on the other side, seem to be more flexible because they are not as dependent on each particular problem (they do not require knowledge on the problem) [32]. On top of that, they tend to explore the solution space with more depth, so they will more often lead to the global optimum of the problem.

4.3. Simple Heuristic Methods

R. S. Mangoubi and D. F. X. Mathaisel ([2]) try to minimize departing, arriving and transfer passengers walking distances and for that, two approaches are presented. The first one uses a rather simple heuristic method that gives quite satisfactory results. The main idea is to list the flights in descending order of number of passengers, and then assign one at a time along that list to the gate with the corresponding shortest walking distances (see Figure 4.1). However, due to the simplicity of the method, it won't probably give the optimal solution. Indeed, assigning aircraft with higher number of passengers first to the optimal gate does not guarantee that the average walking distance per passenger will be minimal.

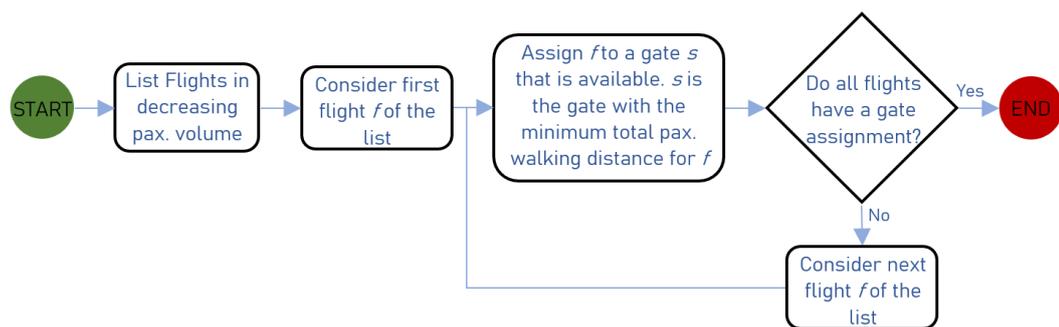


Figure 4.1: Flow chart of the heuristic method used in [2].

The second approach is simply a linear programming relaxation to an integer programming formulation. This approach delivered the optimal solution to the problem (and thus, better results when compared to the heuristic method). Nonetheless, the heuristic approach was near optimal and it was significantly faster than the linear programming relaxation, making it more convenient. Another interesting conclusion taken from [2] is that combining different approaches may be beneficial. If the solution of the heuristic method is used as an initial solution to the LP relaxation, the latter's computational time is substantially reduced.

A. Haghani and M. C. Chen ([27]) reinforce the idea that giving priority to aircraft that carry more passengers is not an optimum strategy to tackle the problem, specially when it considers long time periods. Instead, the real problem is dynamic and features an inherent combinatorial nature. They propose a more complex, yet quite logical heuristic approach. The algorithm tries to assign successive flights to the same gate - the gate that leads to the minimum walking distance - as long as those flights' schedules do not overlap. Let's imagine the unrealistic case in which no two flight schedules overlap with each other. Then, it becomes clear why this algorithm is logical, since in that case all flights would be assigned to the same gate (the one closest to the terminal's entrance/exit). Nonetheless, in reality several flights overlap with each other. When this happens, the algorithm successively assigns overlapping flights to available gates in increasing order of their objective function coefficients.

4.4. Tabu Search with Neighbourhood Search Moves

Tabu Search (TS) is a meta-heuristic very commonly used as an auxiliary procedure to other methods, by preventing them from getting stuck in local optima. It uses memory of different time spans that stores previously obtained solutions and uses that information to increase the explored space state, by freeing or blocking certain moves. Usually, it temporarily blocks the algorithm from going back to the most recent solutions or from performing opposite moves one after the other. How long they will be blocked depends on the definition of certain parameters [6] [33] [12].

J. Xu and G. Bailey ([6]) focus solely on the minimization of passenger total connection times. They initially formulate the GAP as a mixed 0-1 integer programming problem with a quadratic objective function, which is then re-formulated as a mixed 0-1 integer problem, with a fully linear objective function. The resulting linear problem is extremely large, so the authors designed a tailor-made Tabu Search algorithm with Neighbourhood Search Moves to obtain faster results (see Table 4.1). As explained before, the authors take advantage of the short-term memory of the Tabu Search and block recently made movements. The blockage of a certain movement remains valid for a fixed number of iterations. Besides, the Tabu procedure is applied independently to each of the three move types.

Type of Move	Description
Insertion Move	Moves one flight to a different gate from the one currently assigned. $(i, k) \leftrightarrow (i, l)$
Exchange I Move	Exchanges two flights and their gate assignment. Can be seen as the composition of two Insertion Moves. $(i, k) \leftrightarrow (i, l)$ and $(j, l) \leftrightarrow (j, k)$
Exchange II Move	Exchanges two flight pairs in the current assignment. Can be seen as the composition of two Exchange I Moves.

Table 4.1: Types of Neighbourhood Search Moves used in [6]. Note that i and j represent flights, and k and l represent gates

This approach is quite effective and is used to solve the GAP in other literature. However, it presents some flaws. First, it generates the initial solution by performing a random assignment, meaning that an initial feasible solution is not guaranteed. Secondly, *Exchange I Moves* and *Exchange II Moves* are not very flexible, so it can be hard to find good solutions, specially when the considered flight schedule is dense. H. Ding et al. (in [7] and [11]) try to solve these two flaws in their work. The first one is solved by adding an initial **Greedy Algorithm** that sorts flights by departure time and assigns them only to available gates. This Greedy Algorithm leads to an initial feasible solution that is then used in the Tabu Search Algorithm. The second flaw is solved by replacing the Exchange Moves by a new one, called *Interval Exchange*. It is a generalization of the moves it replaces, as more than two flight pairs can be exchanged. Because one of the objectives of [7] is to minimize the number of ungated flights, another move, the *Apron Exchange Move* is created. It allows to exchange one flight assigned to the apron with a flight assigned to a gate. The new Neighbourhood Moves proposed by [7] are listed in Table 4.2.

In 2005, A. Lim et al. ([12]) also created a solving algorithm based on the *Insertion Move* and the *Interval Exchange Move*. However, since they don't assume fixed flight schedules but rather allow for delayed gate assignments inside a time window, new time shift subroutines have to be defined, which are then incorporated to the two neighbourhood search moves. The neighbourhood search is, in turn, linked to a Tabu Search heuristic, just like in the previous examples.

Type of Move	Description
Insertion Move	Moves one flight to a different gate from the one currently assigned. $(i, k) \leftrightarrow (i, l)$
Interval Exchange Move	Generalization of Exchange I and II Moves. Several flights arriving and departing at gate k in the time interval $[a, b]$ can be exchanged with several flights arriving and departing in gate l in the time interval $[c, d]$. $(a, b, k) \leftrightarrow (c, d, l)$
Apron Exchange Move	Exchanges one flight assigned to the apron with a flight assigned to a gate.

Table 4.2: Types of Neighbourhood Search Moves used in [7]. Note that i and j represent flights, and k and l represent gates

4.5. Simulated Annealing with Neighbourhood Search Moves

The application of Simulated Annealing (SA) to the field of optimization was studied in [34]. It was concluded that it is possible to obtain good results with SA, and that the increase in the problem size is followed by a slow increase in the computational load. The authors also point out that SA has promising applications in the optimization field due to the generality of the method.

In a different approach to solve the over-constrained GAP, in which some flights cannot be assigned to gates, H. Ding et al. develop a new solving algorithm. In the previous section, their proposed TS algorithm combined with neighbourhood moves that starts with a greedy algorithm [11] was presented. In the same work, ([11]), they also developed a Simulated Annealing framework. The SA approach is then again combined with the same Neighbourhood Search Moves shown in Table 4.2.

For each iteration, one of the three moves is chosen based on a uniform distribution. Then, a random neighbourhood based on that move is generated and the change in the objective function Δ in case that move is performed is computed. Finally, it is decided whether that move is indeed performed or not, based on the annealing process and using the formula shown in Equation 4.1.

$$p_{acceptance} = a * e^{-\Delta/(k \times T)} \quad (4.1)$$

In the formula, T is the annealing temperature and k and a determine the acceptance rate. At the end of each iteration the temperature is decreased by a factor of d : $T = T \times d$.

This SA approach can decrease the value of the objective function value faster than the TS, but it cannot improve much more after that. Thus, the same authors proposed a final hybrid framework ([11]) that combines Simulated Annealing and Tabu Search, to try to get lower objective function values faster (with the SA) without getting stuck after a while (with the TS). The base structure of this new framework is the same as in the SA algorithm, but now some TS iterations are performed if the result is not improved or if the neighbourhood moves are not accepted for more than a specified maximum number of iterations. As expected, the results found for the hybrid framework are better than those obtained for the TS and SA alone.

4.6. Genetic Algorithms

Genetic algorithms (GA) have been commonly used to solve optimization problems. They have proven to be robust algorithms. Even though it is not guaranteed they will reach the global optimum, they can reach significantly good solutions in a reasonable amount of time, while avoiding convergence on local optima [35]. In the field of the GAP, some authors have already tried to approach the problem using genetic algorithms.

4.6.1. Different Representations of Chromosomes in a Genetic Algorithm

Chromosomes represent a solution of the real problem and they are a key element of genetic algorithms. A GA does not work directly with solutions, but rather with the associated chromosomes, so their structure is quite important in the problem description and in the efficiency of the evolutionary algorithm [36]. The structure of a chromosome In the GAP, different chromosome representations have been designed in the past. A simple representation, used in [3] and [37], is shown in Figure 4.2. Here, a chromosome is an integer string with size N (the number of flights considered). The value of each gene $C(i) = g$ in the i th cell indicates that flight i is assigned to gate g .

Figure 4.2 also highlights how easy it is to find similarities between two chromosomes. The genes with a blue background contain the same value in both chromosomes, which means that the assignment of those



Figure 4.2: **Representation 1** - Examples of two chromosomes in [3]. The genes with a blue background correspond to flights that were assigned to the same gate in both solutions.

flights does not change from one solution to the other. However, this design doesn't allow to identify common relative positions between aircraft in queues for a gate. X. B. Hu and E. Di Paolo ([4]) overcome this obstacle by designing new chromosomes that contain not only the gate assignment as in Figure 4.2, but also the relative positions of aircraft. A visual representation is shown in Figure 4.3. Each chromosome has a dimension of $(N + 1) \times N$, where N is again the number of flights. The first $N \times N$ genes contain the relative positions of aircraft and the last row $(N + 1)$ contains the gate assignments. If $C(i, i) = 1$ and $C(N + 1, i) = g$ then flight i is the first one to be assigned to gate g . If $C(i, j) = 1$ and $C(N + 1, j) = g$, then flight j is assigned after flight i to gate g .

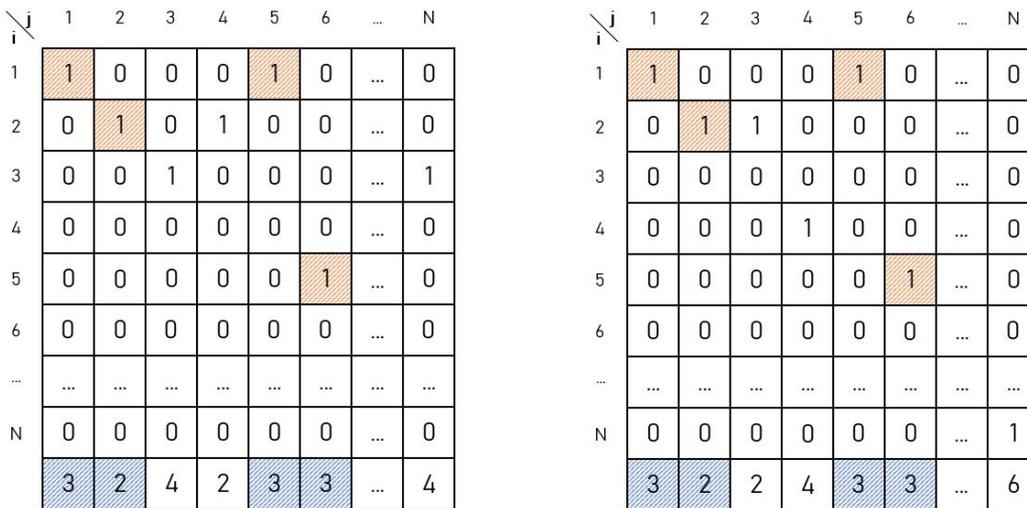


Figure 4.3: **Representation 2** - Examples of two chromosomes in [4]. The genes with a blue background correspond to flights that were assigned to the same gate in both solutions. The genes with an orange background correspond to pair of flights that have the same relative positions in both solutions.

Just like the previous representation, it is also possible to visualize identical gate assignments in different solutions. The genes with a blue background in Figure 4.3 correspond to similar assignments in both chromosomes. Besides, it is also possible to identify common relative positions of aircraft. The genes with orange background highlight this feature. By analyzing these genes, one can now tell that, in both solutions, flight 5 is assigned after flight 1 to the same gate, and flight 6 is assigned after flight 5. This new design is used in [4] to create a new, more efficient formulation of the GA that will be explored in the next subsection, together with other formulations.

4.6.2. Different Solving Methods Using Genetic Algorithms

In this subsection, different formulations of the Genetic Algorithm are shown with some detail. D. Wei and C. Liu ([23]) approach the gate assignment problem using a genetic algorithm in combination with a tabu search method heuristic. The objective is not only to minimize passenger walking distance, but also to minimize the dispersion of gates idle time periods. The genetic algorithm is the main part of the implementation and it follows the standard operators of this kind of algorithms. First, there is a selection of chromosomes (the name given to each individual solution). Then, the crossover operator will combine two chromosomes to obtain new solutions, which may contain some infeasible assignments. If this is the case, the algorithm will reassign the flights that are creating infeasibility to new gates. Finally, a random exchange method called multi-exchange mutation (that simulates the mutation of chromosomes) is applied from time to time in order to swap flights between gates, leading to a bigger exploration of the solution space. The tabu search method is used to transfer the second part of the objective function (the minimization of the dispersion of gates idle

time periods) to a dynamic constraint in the genetic algorithm operators.

On the previous formulation, it was mentioned that the chromosomes obtained from the crossover operator can be infeasible. This is quite common in evolutionary algorithms, due to the stochastic nature of both the crossover and mutation operators [4]. To overcome this obstacle, different authors focus on designing efficient GA operators that lead to feasible solutions only. Uniform crossover (UC), as an example, is one of the most used crossover types, because it is efficient in identifying, inheriting and protecting common genes, but also in recombining non-common genes [4] [38]. Nonetheless, some chromosome designs are not compatible with UC. X. B. Hu and E. Di Paolo ([4]) propose a new Genetic Algorithm with an effective uniform crossover operator that uses the chromosome design shown in Figure 4.3. The results from that research proved that the incorporation of the new chromosome design in the uniform crossover operator is advantageous.

4.7. Overview of the GAP Solving Methods

The analysis developed throughout this chapter allows for some remarks to be made with respect to the GAP solving methods. These remarks are shown below. Table 4.3 summarizes all instances of solving methods included in this chapter.

- Exact methods are not directly applicable to solve non-linear problems. If one wants to use exact methods to solve this kind of problems, from which the GAP with transfer passengers is an example, at least one of two things should be done: either linearize the problem ([6]) or divide the problem into smaller sub-problems. This division should not be random, but rather follow a certain logic/pattern [14]
- Exact methods always reach the optimal solution, but generally they are significantly slower than heuristic or meta-heuristics (refer to [2] and [6]). For large problems, it may not even be possible to reach the optimal solution in a reasonable amount of time
- Even though exact methods are slower, commercial solvers that apply this kind of methods (CPLEX, for instance) already exist, which means it is less time consuming and more convenient to directly use them. There is no need to build a new heuristic or meta-heuristic method
- As a consequence of the previous bullet points, one can conclude that it is possible to solve the GAP with exact methods, even when it includes connecting passengers (if some adaptations are made) and despite its quadratic nature
- Simple heuristic methods are easier to understand and, usually, easier to build than meta-heuristic methods. However, they generally present more flaws and do not reach global optimality as often (refer to [2] and [27])
- TS, SA and GA are the most researched meta-heuristic solving methods in the field of the GAP that considers transfer passengers [32] (refer to section 4.4, section 4.5 and section 4.6). It is also very common to incorporate Neighbourhood Searches with these methods because they promote the exploration of the solution space. A hybrid solving method based on SA with TS iterations has been already successfully implemented [11]
- Some authors have tried to solve the same GAP with different methods and then compared them, even though this may not be clear in this report (because different methods were split into different sections). This was the case with R. S. Mangoubi and D. F. X. Mathaisel ([2]), J. Xu and G. Bailey ([6]) and H. Ding et al. ([11]). Since exact methods always return the optimal solution, they can be used to obtain a benchmark with respect to which the efficiency of heuristic and meta-heuristic methods can be compared [2].

Authors (Reference)	Type Of Solving Method	Solving Method	Observations
R. S. Mangoubi and D. F. X. Mathaisel ([2])	Exact Method	Linear programming relaxation	The integer programming formulation is relaxed, making it easier to reach optimality. The fact that the model presented by the authors is linear facilitates the solving process. The computational time is higher than that of a heuristic method.
J. Xu and G. Bailey ([6])	Exact Method	Branch and bound (B&B)	The original quadratic formulation is linearized, and the solver CPLEX is used to obtain the global optimum. The computational time is still high when compared to the use of a heuristic method.
B. Maharjan and T. I. Matis ([14])	Heuristic, but based on an exact algorithm	Hierarchical division of the infrastructure and Branch and Bound	The large original problem is split into smaller sub-problems. The division is based on the layout of the airports infrastructure and it leads to three levels of assignment, which are solved in hierarchical order: zone, subzone and finally gate assignment. Each level makes use of the assignment performed on the previous level. Each sub-problem is solved using Branch and Bound (CPLEX).
R. S. Mangoubi and D. F. X. Mathaisel ([2])	Heuristic	Successive assignment of flights based on the number of passengers	Simple heuristic in which the flights start to be listed in descending order of number of passengers. Then, they are successively assigned along that list to the available gate with the corresponding shortest walking distances (see Figure 4.1). It is likely that this method does not reach the optimal solution, because assigning aircraft with higher number of passengers first is not necessarily the best strategy.
A. Haghani and M. C. Chen ([27])	Heuristic	Successive assignment of flights	The algorithm tries to assign successive flights to the same gate - the gate that leads to the minimum walking distance - as long as those flightsschedules do not overlap. When this happens, the algorithm successively assigns overlapping flights to available gates in increasing order of their objective function coefficients.
J. Xu and G. Bailey ([6])	Meta-Heuristic	Tabu Search (TS) with Neighbourhood Search Moves	The originally quadratic problem is linearized and solved with a TS to take advantage of its short-term memory (blocks recently made moves). Three Neighbourhood Search moves are designed to obtain faster results (see Table 4.1). The moves present two shortcomings: 1) the initial solution is obtained by performing a random assignment, so it may be infeasible and 2) the Exchange moves are not very flexible.
H. Ding et al. ([7] [11])	Meta-Heuristic	Tabu Search (TS) with Neighbourhood Search Moves and Greedy algorithm	In these two works, the authors try to solve the two flaws presented in the previous solving algorithm. Flaw 1) is solved by adding an initial Greedy Algorithm which only assigns flights to available gates, and thus the initial solution is always feasible. Flaw 2) is solved by replacing the Exchange moves by a new, more generic Interval Exchange move. A new type of move that allows to exchange a flight assigned to the apron with one assigned to a gate is also created. Table 4.2 shows the new moves.
A. Lim et al. ([12])	Meta-Heuristic	Tabu Search (TS) with Neighbourhood Search Moves and Greedy algorithm	Based on the Insertion Move and Interval Exchange Move (Table 4.2). Because these authors don't assume fixed flight schedules but rather allow for delayed gate assignments inside a time window, new time shift subroutines have to be defined, which are then incorporated to the two neighbourhood search moves.
H. Ding et al. ([11])	Meta-Heuristic	Simulated Annealing (SA) with Neighbourhood Search Moves	This SA approach is combined with the moves shown in Table 4.2. For each iteration, one of the three moves is chosen based on a uniform distribution. Then, a random neighbourhood based on that move is generated. Finally, it is decided whether that move is indeed performed or not, based on the annealing process (Equation 4.1).
H. Ding et al. ([11])	Meta-Heuristic	Simulated Annealing (SA) with Tabu Search (TS) iterations	This hybrid algorithm uses the SA framework presented in the previous method, but now some TS iterations are performed if the result is not improved or if the neighbourhood moves are not accepted for more than a specified maximum number of iterations. It combines the faster nature of SA with the TS ability of not getting stuck.
D. Wei and C. Liu ([23])	Meta-Heuristic	Genetic Algorithm (GA) with Tabu Search (TS)	The GA is the base of this solving method, and it follows the standard operators: selection, crossover and mutation. The tabu search method is used to transfer the second part of the objective function (minimization of the dispersion of gates idle time periods) to a dynamic constraint in the genetic algorithm operators.
X. B. Hu and E. Di Paolo ([4])	Meta-Heuristic	Genetic Algorithm with Uniform Crossover (UC)	The authors propose A new GA with an effective UC operator that uses the innovative chromosome design shown in Figure 4.3. The results from that research proved that the incorporation of the new chromosome design in the uniform crossover operator is advantageous.

Table 4.3: Overview of different methods used to solve the Gate Assignment Problem, when transfer passengers are considered.

5

Airport Performance and Hub Network Connectivity

Up until now, a review of the GAP, its formulations and solving methods was presented. However, since one of the major objectives of this research is to increase the number of hub connections and thus, improve an airport's network efficiency, it is important to be familiar with the current state of the art when it comes to airport performance and network connectivity analysis. Finding out how, in the past, authors have tried to optimize the network of airports is also paramount.

To this end, this chapter starts with a general introduction of the research on airport performance and quality analysis. Then, hub airport network and connectivity are presented as a way of measuring airport performance and reviewed in more detail due to their relevance to the current research. In particular, the concepts of accessibility and centrality are explained, and different methods to analyse and optimize networks are presented. Finally, a brief reference to hub airports competition based on connectivity is made.

5.1. Airport Performance and Quality Analysis

In a world where the aviation industry keeps growing and the passenger demand increases, the analysis of an airport's efficiency when it comes to passenger processing and capacity management has become more relevant. Several authors have made different studies on airport efficiency and service quality, how airport demand will evolve with time and how that demand will affect the future performance of an airport. Conducting these studies can be extremely relevant as they help airport authorities decide when actions need to be taken [39]. Even though most of the times the quality of an airport is determined from the passenger point of view, it is also relevant to approach the problem from the airlines' perspective. In fact, airports' service quality and efficiency influence the airlines' choice of hubs [40]. The next paragraphs show some examples of research that analyses airport demand and performance.

C. Y. Hsiao, and M. Hansen ([41]) propose a passenger demand model for the US domestic hub-and-spoke network which is able to predict passenger demand between specific pairs of airports, at the route level. The model is based on random utility theory, and it represents an advance when compared to previous models, because it incorporates the classic demand allocation (which explains the distribution of traffic among alternatives airports, routes and others) with demand generation, by giving passengers the choice to travel (or not) by air.

In the work developed by E. Fernandes and R. R. Pacheco ([39]), the authors analyse 35 Brazilian airports using a support instrument called Data Envelopment Analysis (DEA), which allows to determine which airports use their resources efficiently and which offered surplus in their facilities. They also predict, for each airport, when future capacity expansions may be needed, based on passenger demand forecasts. A similar study is made by W. H. K. Tsui et al. ([42]). A forecast of future passenger throughput at Hong Kong's airport is made, and the authors believe the projection obtained can be used by policy makers, airport authorities and airline management as a tool to find which future challengers need to be faced.

M. Turcotte et al. ([43]), analysed the impact of redesigning the connection banks of Air Canada at its hub, in Toronto, on the gate assignment performance. In particular, the authors compared the effects of having directional waves (as opposed to the already implemented non-directional waves) on the percentage

of unassigned demand, the preference satisfaction level and the first preference satisfaction percentage. The results show there is a slight improvement when directional waves are considered.

5.1.1. Hub Congestion Analysis

Hub congestion (in the form of terminal passenger congestion and airside congestion) is a phenomenon that is repeatedly present in hub airports. Airside congestion is usually higher during peak hours, in which several flights arrive or depart from the airport. It is believed that a primary contributor to congestion is the fact that airlines operate a considerable amount of flights with small aircraft, which leads to a higher frequency [44]. Better hub connections, with a small layover times, and larger airport networks seems to increase congestion as well [44].

Some authors propose measures to alleviate hub congestion and subsequent delays. One suggestion could be the investment on new runways or the improvement of air traffic control [45]. A commonly suggested strategy is congestion pricing, which encourages airlines to increase aircraft size and decrease flight frequency. The current pricing system is weight based, and it produces the opposite effect [44]. J. K. Brueckner ([45]), on the other hand, suggests that the application of a congestion pricing system may be more limited than what other researches suggest. K. Kemppainen et al. ([46]) claim that an airline with its operations based at a hub airport can considerably reduce their peak congestion costs if some flights' schedules are adjusted. The results of their work are also relevant for airport service providers to estimate the economic effects of service peaks.

5.2. Hub Airport Network and Connectivity

A useful way to measure if a hub airport is using its infrastructure efficiently is to analyse its network and connectivity. Connectivity has been more commonly used as a measure of airports' performance [5]. In fact, in a study conducted by C. W. Lin and C. Tsai ([47]) regarding the revenue maximization of airport cities, it was shown that expanding the airport's network is the third highest contributor for that maximization. The biggest contributor is the efficient operation and turnaround times of aircraft. The authors in [47] also point out that expanding networks includes connecting traffic opportunities, and thus, special attention has to be given to transfer times. However, any expansion should be based on a meticulous plan, otherwise it may not necessarily lead to the increase of the airport's efficiency and revenue. This idea of planning an efficient expansion of an airport's network is one of the focuses of the current research.

The importance of network connectivity led several authors to explore the connectivity of airports and study specific features of their networks. In this section, the two main concepts of airport connectivity (accessibility and centrality) will be defined, as well as some ways of measuring them. Then, a few concrete examples that apply these concepts to analyse airport networks are presented.

5.2.1. Accessibility and Centrality: Definition and Measurements

When it comes to airport connectivity, two different perspectives are defined: accessibility and centrality [48].

- **Accessibility:** measures how easy it is to travel between that airport and any other airport of the network. It takes into account the number and quality of direct or indirect connections that are possible to take.
- **Centrality:** it is defined as the number of transfer opportunities available through the airport.

In terms of accessibility, an airport can have direct connections, when a passenger is able to reach another airport with one flight, and indirect connections, when at least two flights are needed to fly from the airport to a different one. Figure 5.3 shows two different situations that highlight the difference between the concepts of accessibility and centrality. Figure 5.1 shows the case of an airport (in red) with a relatively high accessibility, since it is possible to reach several airports from there. However, its centrality is quite low, because there are not significant connections from a generic airport to another that pass through it (low number of transfer opportunities). On the other hand, the case in Figure 5.2 shows an airport with both high accessibility and centrality [48].

For hub airports, it is of paramount importance to have high centrality. This concept is so profoundly related to hub airport that is very often called *hub connectivity*. Centrality can be used by those airports to compare themselves with other competing hubs and assess how relevant they are in a certain O&D market [48]. Since this research focuses in creating as many transfer connections as possible inside a hub airport,

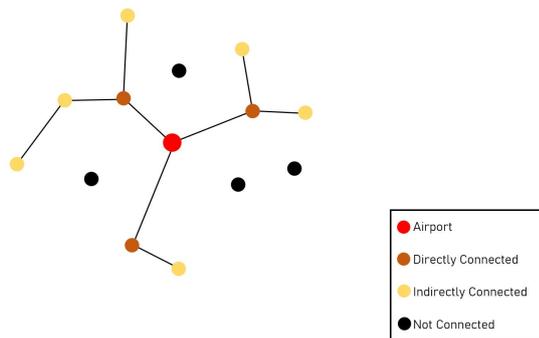


Figure 5.1: Representation of an airport with a relatively high accessibility but poor centrality.

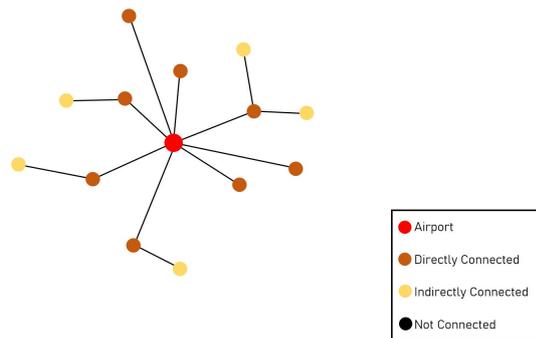


Figure 5.2: Representation of an airport with high accessibility and centrality (strong hub connectivity).

Figure 5.3: Visual representation of the difference between the concepts of accessibility and centrality.

the concept of centrality is also the most relevant here. Nonetheless, an airport's accessibility will naturally increase by increasing the number of connections made available by it.

G. Burghouwt and R. Redondi ([48]) show that the concepts of accessibility and centrality can be quantified using several different connectivity measurements. These measurements are important as they allow airport authorities to assess the efficiency and performance of the airport network and to evaluate the impact of future actions. The authors gather an extensive collection of previously developed connectivity measurement models and use them to assess the accessibility and centrality of several European airports.

5.2.2. Analysis and Optimization of Airport Networks

In this subsection, some research regarding the analysis and optimization of airport networks is presented. A very interesting approach to airport connectivity is proposed by R. Redondi et al. ([5]). The authors present a case study, with several European airports, which aims to evaluate the impact of creating future new connections to their network. Their approach is based on the division of the European network into different modules, and simulated annealing is used to find the best division possible. The objective is to create modules as compact as possible. A module is said to be compact if there are several connections between airports inside that module. Ultimately, the authors defend that their research should help airports decide whether opening up a new route really contributes to an increase in their connectivity.

In the end, 13 modules were obtained for the considered case study. Three of these modules correspond to massive international modules, highlighted in Figure 5.4 with three different colours. If the European network was redesigned, the airport of each of those modules with the highest number of available seats inside that module could be considered its "key airport", in an analogy to the hub airports, as it would work as a central connection between the airports belonging to that module. In Figure 5.5, on the other hand, shows the remaining 10 national modules. Each of these modules represents a domestic network in which domestic airports are connected to the most important ones of the country.

X. Sun et al. ([49]) analyse the temporal evolution of the European air transportation system between 2011 and 2013. The analysis was split in two layers, namely the air navigation route network and the airport network. Besides, the temporal analysis of each layer was made per season and per week. The authors concluded that the air navigation route network is characterized by summer/winter seasonal variations, and that the airport network shows not only summer/winter variations but also weekly peak/off-peak changes.

G. Burghouwt, and J. Hakfoort ([50]) also study the evolution of the European aviation network, but their objective is to determine whether deregulation in the EU has led to changes in the route structures and to the adoption of hub-and-spoke systems. They concluded that, at the airport level, there is no evident trend of concentration of intra-European traffic at the primary hubs, but that is not the case for intercontinental flights. At the route level, it was found that a hub-and-spoke structure is indeed present.

E. Jimenez et al. ([51]) analyse the evolution of the aviation network of the three main airports of mainland Portugal and conclude that it was influenced by the emergence of low-cost carriers which, in turn, was a consequence of the deregulation. They also point out that the three airports are becoming more dependent

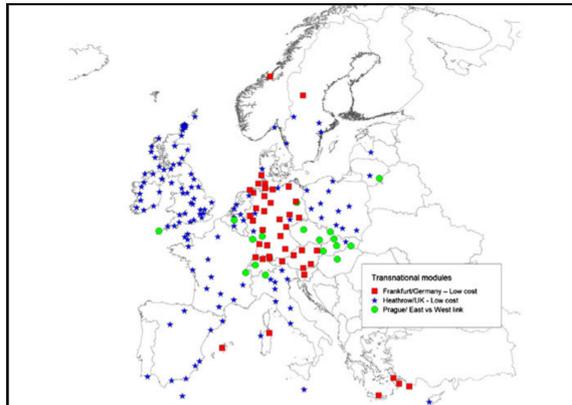


Figure 5.4: Map of the three international modules, represented in blue, green and red [5].

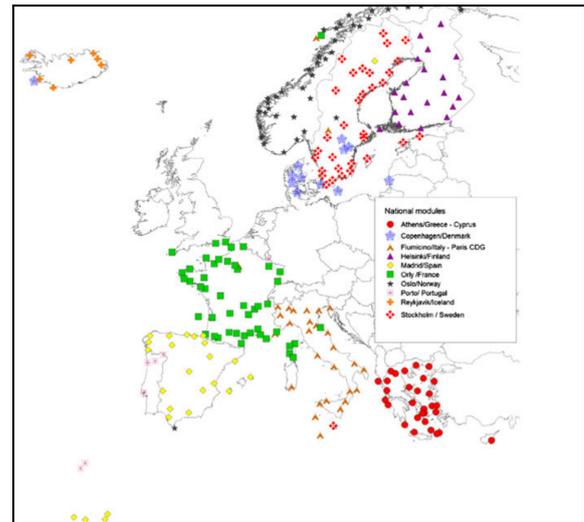


Figure 5.5: Map of the ten national modules, each represented with a different colour [5].

Figure 5.6: General results obtained in [5], showing both the international and the national modules obtained.

on fewer airlines, most of them foreign low-cost. These are becoming dominant carriers and may threaten the position of the national carrier TAP.

Even though it is true that airport networks and corresponding assessment and optimization have been extensively studied, it is also important to point out that real structures of networks rarely match their theoretical representations and simulations on the research papers. This is not only a consequence of possible political barriers but also a result of the increasing congestion costs that come with the inclusion of new routes in busy hub airports [5]. It is also relevant to mention that the requirements of different types of airlines (for instance, carrier airlines and low cost airlines) have a significant influence in the expansion of an airport's network, and for that reason it has become a key responsibility of airport managers to assess and satisfy those requirements [51].

5.2.3. Hub Airports Competition

The study of an airport's network and corresponding performance can be extended and used as a means to evaluate competition between airports. In particular, hub competition started to be more relevant after the liberalization of the air transport market. One important aspect of this kind of competition is that flying through hub airports should only lead to a small increase in travel time and distance. [52]. One commonly adopted way to compensate for longer trips is to increase service frequency in airports [52], but the strategic choice for a hub location on a global network can also have a heavy impact on an airport's competitiveness [53]. By comparing an airport with others, it is possible to position it in the market and eventually develop future action plans.

The importance of airport competition led to the development of several studies in the area, which show that it can be quantified in distinct ways. For instance, A. R. Feighan and P. McLay ([54]) compare different airports' accessibility based on the importance of destinations served and routes capacity. G. Burghouwt and J. Veldhuis ([55]), on the other hand, argued that measuring airport connectivity and competitiveness should consider both direct and indirect connections, and developed a model accordingly, which was applied to the network between Northwest Europe and the US. R. Redondi et al. ([52]) developed a measure that determines which hub airports directly compete for a certain O&D market, which means that, similarly to the previous authors, they also consider direct and indirect connections. However, they were the first to be able to apply the measure on a global scale, to a network of more than 200 airports.

6

Current Literature Gap and Relevance of this Research

6.1. Understanding the Current Literature Gap

Throughout this literature study, the Gate Assignment Problem was extensively reviewed and analysed. It became clear how flexible the problem is from all the different formulations (refer to [chapter 3](#)) and solving methods (refer to [chapter 4](#)) that were developed in the past. The most common objective functions are shown in [Table 2.1](#). Amongst them, minimizing passenger walking distance (or time) and minimization of aircraft assigned to the apron are very frequently used. In particular, for hub airports, some authors focus on minimizing the walking connection times for transfer passengers, which leads to a reduction of the overall time (and thus, distance) that passengers need to connect from their arrival flight to their departure flight.

On the other hand, several authors have analysed airport performance (refer to [chapter 5](#)). There are several study cases that assess whether airports offer a reasonable service quality or if they use their infrastructure efficiently. Research on the analysis of airport demand evolution over time is also fairly common. Sometimes, demand forecasts are used to predict if an airport needs future action or if its infrastructure is able to cope with extra passengers. This type of analysis is extremely valuable to airports as it raises awareness of future challenges that the authorities will have to face.

As part of performance analysis, one can find specific research on airport network and connectivity. The network of hub airports, and how closely connected it is to other airports, tells the authorities how easy it is to fly through the airport, which is important to determine how efficient the airport is to capture transfer passengers. Some authors have already analyzed patterns and changes on network systems. Others aimed to evaluate the impact of creating new, better routes in a network of airports.

Let's now suppose that a hub airport's authority wants to assess the impact of adding new flights (based, for instance, on future passenger demand) on the efficiency of the airport's network and the number of captured transfer passengers. One way to do this is to strategically assign the old and potential new flights to gates so that the walking times between pairs of flights is minimized as much as possible. This should reduce the overall Minimum Connection Time (MCT) of the airport and thus open up new connections inside the hub. The Gate Assignment Problem can be used here as the tool to find the optimal strategic flight assignment. As it was already said before, minimizing passenger connection times using the GAP was already researched in the past, but integrating it with the specific objective of maximizing transfer passenger throughput has never been explored before. On the other hand, it was shown that airport performance and network connectivity analysis has been researched as well. However, proposing a possible change in the airport's network and specifically quantifying the new connections and the number of passengers flying through that network hasn't been researched so far either. The current research gap can thus be seen as being the link between the improvement of a hub network's performance and the gate assignment problem. This link is graphically described in [Figure 6.1](#), and the research gap is stated as follows: **the literature has not yet researched the impact of a strategic gate planning that identifies (future) profitable flights, on the hub airports transfer passenger throughput and its network performance.**

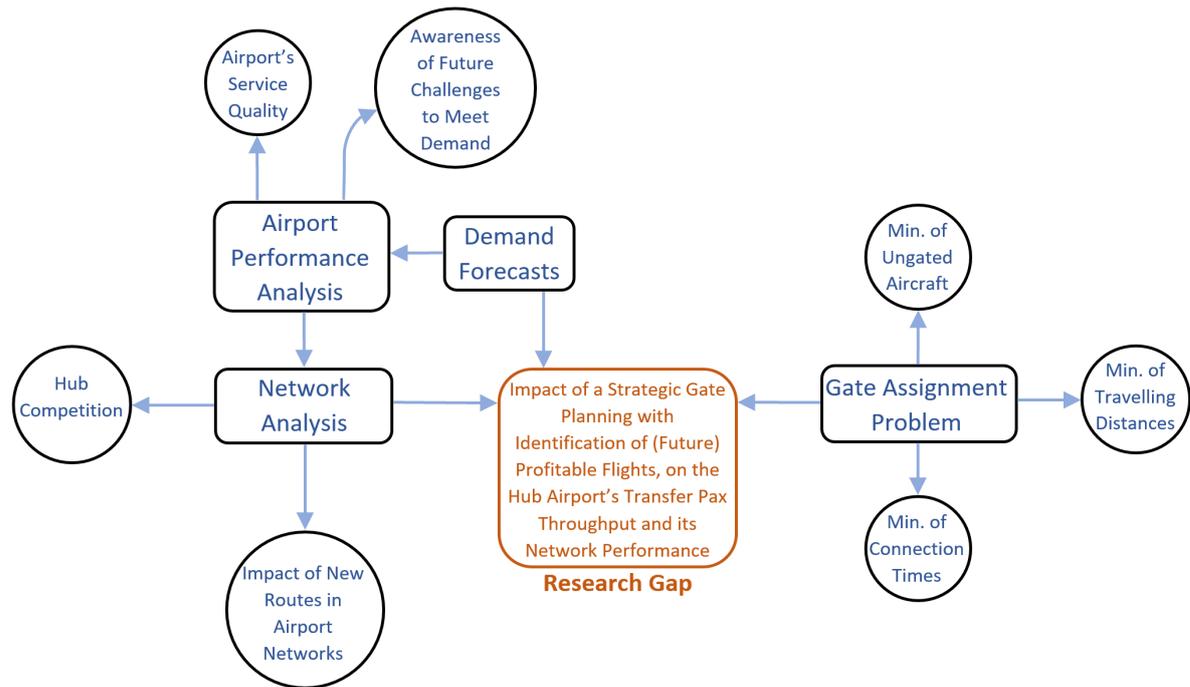


Figure 6.1: Graphical representation of the current literature gap, presented as the link between the assessment of a hub network's performance and the gate assignment problem.

6.2. Practical Applications of this Research

The literature gap was already found, but it is just as important to understand why this research is relevant. For the aviation industry, the results obtained from this research can be used as the base to develop airport software tools with various applications. With the passenger demand growing and changing, there may be the need to readjust gate allocations to create a more efficient use of the current infrastructure (capacity). This readjustment should create more connections in the hub, making it possible for more passengers to travel through it, without physically expanding the infrastructure. This is specially important for busy hub airports, as these cannot allocate all the potential new flights due to the lack of capacity.

Even though it is the airlines' decision to ultimately choose which flights to operate from and to an airport and which routes to open [5], the results of this research can be used as a tool for airports to assess which flights are the most profitable and how those flights can be placed in space (gates) and possibly time (flight schedule) in order to maximize the captured transfer passengers, the hub connections and thus the airport's network efficiency. This should aid airport authorities to make a more informed decision on which flights to prioritize, and also to recommend profitable new routes to airlines [5]. In fact, airlines can also indirectly benefit from the effects of increasing the hub's connections. A study conducted by W. Wei and M. Hansen in [56] concluded that airlines can attract more transfer passengers if the connection opportunities in the network are increased.

6.3. Research Questions

The main research question is significantly broad, but looking for an answer to it is the motivation that will drive this research and ultimately cover the gap of the current researches. It is directly derived from the research gap statement shown before, and it is formulated below:

- **What is the impact of a strategic gate planning that includes the identification of profitable (future) flights and their incorporation in the schedule, on a hub airport's network and its potential to capture more transfer passengers and offer more hub connections?**

In order to steer the research in the right direction, and reach the final answer of the main research, several other questions were formulated. These questions are more specific, so it should be less demanding to answer them individually. They are presented below:

1. How can the Stand Assignment Problem (SAP) be formulated in order to be integrated in this research?

It was already mentioned that the SAP can be used as the tool to obtain a strategic gate planning that maximizes the number of captured transfer passengers and minimizes connection times. But since this was never done before, it is important to determine how it can be specifically formulated and how the model can be mathematically written. This includes the development of the objective function (potentially with several individual terms) and specific constraints. To help answering this question, [chapter 2](#) and [chapter 3](#) can be consulted.

1.1. How does the individual importance given to the maximization of transfer passenger throughput and the minimization of connection times influence the final solution and how does it affect the satisfaction of the airport authorities and passengers?

This question emerges in alignment with **Question 1**. Since the main goals of this research are to maximize transfer passenger throughput and minimize connection times, and because the two stakeholders considered (airport authorities and passengers) have different interests, an equilibrium between the importance given to each of the goals has to be found. Indeed, paying more attention to one goal than the other may lead to different solutions which, in turn, may be more attractive to one stakeholder and less attractive to the other.

2. Which Heuristic or Meta-Heuristic method is the most fitting to solve the SAP?

Choosing the most adequate solving method of the SAP and corresponding algorithm is the key to efficiently obtain a solution that is as close to optimality as possible, without the need of a great computational load. Thus, this question is not intended to study in detail how the different solving methods work, nor to develop a completely new algorithm, but rather to find out which methods can be used and combined in the most adequate for this research. Several methods were already presented in [chapter 4](#), which will be used as the base to answer this question.

3. What is the nature of the relationship between the way new flights are distributed over time (in peak or off-peak hours) and the types of solutions obtained? In particular, does allocating flights between peaks increase the number of possible connections and/or reduce the connection times?

At hub airports, a considerable percentage of the flights are scheduled to arrive or depart during peak hours (or flight waves). Several flights arrive at the airport during a peak, which is followed by an off peak period. After a certain period of time, a new peak with several departing flights takes place. The time in between peaks should be enough for passengers to connect between flights. Some considerations regarding flight waves were already made by some authors in the past. It was shown that re-designing flight waves may be more advantageous [43] and that adjusting some flights during rush hours could have positive economic effects on airlines, authorities and airport services providers [46].

With this question, the distribution of the proposed turnarounds (and thus the new flights) over time will be analyzed. Furthermore, changing some parameters of the model, namely the relative importance of the maximization of captured transfer passengers and the minimization of connection times, may cause the model to change the time distribution of the new turnarounds. It is relevant to understand whether this happens, and if it does, what is the nature of that change. This research question asks explicitly if the model will distribute the new flights more evenly along the day (in between peaks) to increase the number of captured passengers or reduce connection times, but other questions are implicit. For instance, if the relative importance given to the maximization of captured transfer passengers and the minimization of connection times changes, will the new flights be clustered in certain periods of the day that do not correspond to peaks?

4. As a consequence of the growth in the number of captured transfer passengers, how much do hub connections and the network expand and how profitable could the proposed turnarounds potentially be?

To better understand whether this research is leading to meaningful results, it is important to measure the change in the airport's connectivity. In fact, P. Malighetti et al. ([57]) show that there is a positive

relation between efficiency and airport's connectivity. The competitive pressure is then forcing airports to promote their connectivity [5].

Thus, this last question tries to find answers on the specific network expansion of the hub. In particular, how much did the original routes expand and what are the new connection opportunities brought with the new routes? Also in this question, the selected time alternative of each proposed turnaround will be analyzed in terms of how large its transfer passenger forecast is, but also how profitable it might potentially be for the airline.

6.4. Research Plan

It is expected that the research which follows this literature study will take a period of approximately 6 months (24 weeks) to be completed. The partitioning of that period into different tasks, and the time that should be reserved for each individual task is shown in the Gantt Chart of Figure 6.2. A similar timeline is shown in Figure 6.3.

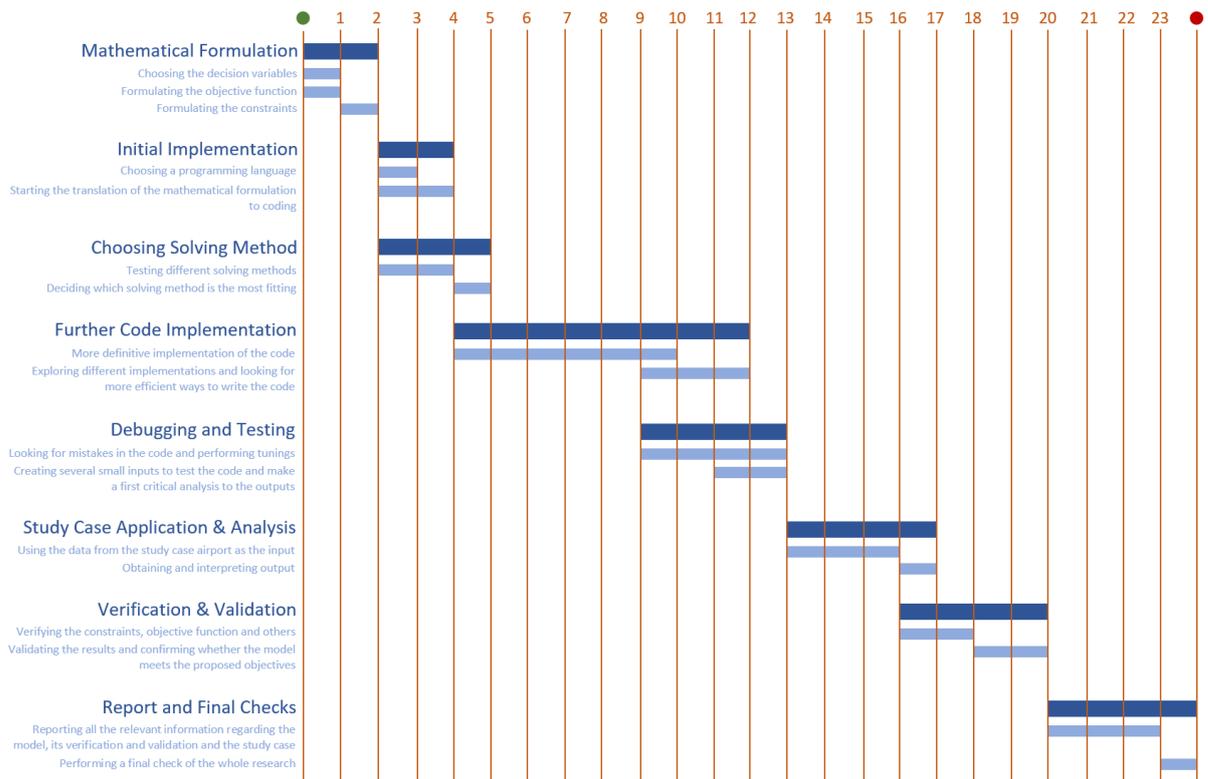


Figure 6.2: Gantt Chart with the proposed timeline for this research.

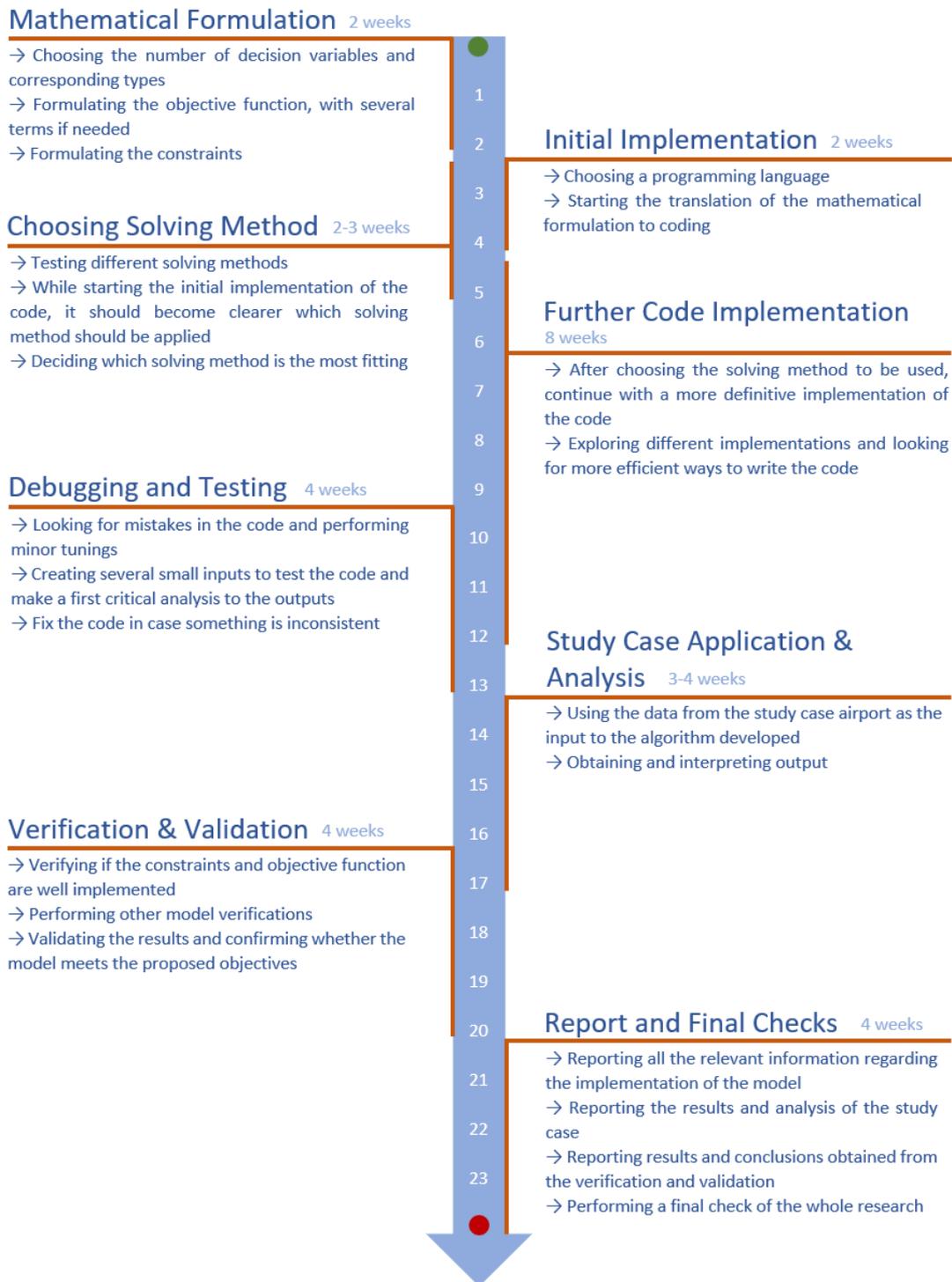


Figure 6.3: Timeline of the planned tasks and corresponding execution times, to be developed during this research.

III

Supporting work

1

Detailed Definition of The Two-Level Stand Planning Model

In this chapter, the complete strategic stand planning model is presented. It starts with an explanation of different connection types and how they will be defined during this research in [section 1.1](#). In [section 1.2](#), a brief description of the first model created to approach this problem is shown. With this description, the reader should understand why that this kind of problem is usually complex to solve and why it portrays a large solution space, thus requiring alternative solving methods. Also in the same section, it is shown how some unnecessary decision variables were eliminated through a pre-processing algorithm, before running the full model. The Two-Level Stand Assignment Model (TLSAP) - the definitive approach used in this research - is introduced in [section 1.3](#). The model developed for the first level of assignment is explained in great detail in [section 1.4](#), which includes its decision variables, its multi-objective function and corresponding solving method and its constraints. On the other hand, the model developed for the second level, the defined decision variables, objective function and constraints are presented in [section 1.5](#). Finally, some final remarks regarding the whole framework are shown in [section 1.6](#).

1.1. Relevant Information Regarding the SAP

Relationship Between Turnarounds and Operations

Along this Appendix, the terms turnaround and operation will be very frequently used, and it is important to clearly define each of them and explain how they are related. A turnaround refers to the period of time during which an aircraft is on the ground of an airport. All actions and events taking place during that time - from its arrival and disembarking of passengers, going through the cleaning of the aircraft, the eventual movement of the aircraft between airport stands, to the embarking of new passengers, refueling and finally ending with the departure - are part of the turnaround.

However, each turnarounds can be split into different operations for modelling purposes. For aircraft that stay on the ground for short periods of time, the turnaround is split into two operations, an arrival during the first half of the turnaround and a departure during the second half. However, when an aircraft stays on the ground longer than a certain threshold, it remains inactive at the airport for a certain period of time, after the arrival operation is over and before the departure operation starts. This period of inactivity is considered a third operation, called the idle operation.

It now becomes clear that the turnaround of an aircraft consists on the entire stay of the aircraft at the airport and it includes either 2 or 3 operations, depending on the ground time.

Relationship Between Gates and Stands

Every airport has a certain number of gates and stands. A gate is the area inside the terminal where departing passengers have to walk prior to embarking to a flight and it includes facilities for checking boarding passes. It is also the place where arriving passengers enter the terminal after landing. The stand is the area outside the terminal where aircraft can park. It is important to note that gates are always located inside the buildings, while stands can be either built right next to the terminal - the contact stands - or somewhere far from any building - the remote stands. Furthermore, it is not common that gates and stands are perfectly paired. There

are cases in which two gates share the same stand. On other situations, several stands may be served by the same gate. This is the case, for instance with remote stands.

When gate and stand planning are being developed, this complex relationship between both infrastructures has to be taken into account. It happens frequently in the literature that authors choose to disregard one of the sides. They either consider the stand allocation, the gate allocation or sometimes it is assumed that gates and stands are perfectly paired. In the current research, and since the main focus is based on the assignment of aircraft, the model will only focus on the stand assignment. It is assumed that gates will always have capacity for any stand assignment that is performed. The areas inside the terminal where each remote stand is connected to (i.e., the areas in the terminal where passengers embark/disembark to remote stands) are based on the rules used by the study case airport and are explained in detail in [section 3.1](#).

Percentages of Transfer Demand Between Origin and Destination Pairs

In this research, it is assumed that the transfer demand between an origin-destination pair depends on 2 factors, $factor_{A,B}$ and $factor_{XX,YY}$.

Angle Between Origin and Destination: $factor_{A,B}$

The first factor is related to the angle $\theta_{A,B}$, centered at the hub, between the origin A and the destination B airports. This angle $\theta_{A,B}$ is represented in [Figure 1.1](#) and it is defined in the interval $\theta_{A,B} \in [0, 180]$.

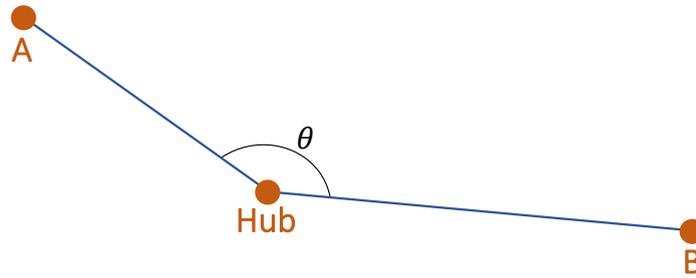


Figure 1.1: Angle $\theta_{A,B}$ between origin and destination airport.

To obtain $\theta_{A,B}$, the bearings between the hub and A and between the hub and B are first computed. The general formula for the bearing between any two points p_1 and p_2 on the surface of a sphere is given in [Equation 1.1](#).

$$bearing(p_1, p_2) = \arctan2(\sin(lon_2 - lon_1) \times \cos(lat_2), \cos(lat_1) \times \sin(lat_2) - \sin(lat_1) \times \cos(lat_2) \times \cos(lon_2 - lon_1)) \quad (1.1)$$

In the Equation, lat_1 and lat_2 correspond respectively to the latitudes of points p_1 and p_2 and lon_1 and lon_2 correspond to the longitudes of points p_1 and p_2 . The angle θ is then given by:

$$\theta_{A,B} = |bearing(Hub, A) - bearing(Hub, B)| \quad (1.2)$$

To obtain a normalized value for $factor_{A,B}$, the angle $\theta_{A,B}$ is divided by 180:

$$factor_{A,B} = \frac{\theta_{A,B}}{180} \quad (1.3)$$

This factor is related to the angle $\theta_{A,B}$ because this partially represents the behaviour of passengers when connecting at hub airports. Let's suppose that two airports lie approximately to the east of the hub. Then, the bearing of each one with respect to the hub is somewhere close to 90. Thus, using [Equation 1.2](#), one can conclude that $\theta_{A,B}$ is very close to 0. Consequently, [Equation 1.3](#) states that $factor_{A,B}$ is very close to 0. If, on the other hand, one airport lies to the east of the hub but the other lies to the west, $\theta_{A,B}$ will be very close to 180 and $factor_{A,B}$ will be approximately 1. This factor represents how likely it is for a passenger to make a connection between two airports, based on their relative positions with respect to the hub. If the factor is close to 0, it means that a passenger would have to fly to the hub from the origin airport and then almost fly back to the destination airport. It is not probable that a passenger will want this connection. The same reasoning can be done for any other value of $factor_{A,B}$.

Relationship Between Arriving and Departing Flights: $factor_{XX,YY}$

The second factor takes into account the relationship between the airline operating the first leg, arriving to the hub, (airline XX) and the airline operating the second leg, departing from the hub (airline YY). Normally, it is much more likely that passengers will choose a connection between two flights from the same airline. Less likely, but still quite often, a passengers may choose a connection between two flights from airlines that belong to the same alliance or that share the same codes. However, it is significantly rare the two flights of the connections are not related in any way. In fact, most of the times, connections like this are not even offered by airlines or by websites. Based on the information above, $factor_{XX,YY}$ can assume different values:

- **XX and YY are same airline:** $factor_2 = 0.67$
- **XX and YY are airlines belonging to the same alliance or codesharing:** $factor_2 = 0.33$
- **XX and YY are not related:** $factor_2 = 0$

Note that the sum of $factor_{XX,YY}$ for all 3 situations presented above equals 1. Thus, the probability that a passenger will choose a connection between flights from the same airline is 67%. Note that this value is twice as large as the value chosen from connections between airlines that belong to the same alliance (or that code share). Finally, the research assumes that it is not possible to have a connection between flights operated by non-related airlines at this hub. Even though these values are not based on any specific example, they represent the general reality of hub connections. It is significantly likely that passengers are offered connections between flights operated by the same airline, but it is also possible to establish connections between airlines belonging to the same alliance. Finally, it is reasonable to assume that no connections are established between non-related airlines (but it should be taken into account that it can happen in reality, especially at large hubs).

The final factor for each arrival-departure pair $(A, XX) - (B, YY)$ is given by the product of both $factor_{A,B}$ and $factor_{XX,YY}$:

$$factor_{XX,YY}^{A,B} = factor_{A,B} \times factor_{XX,YY} \quad (1.4)$$

Final Demand Percentage for Each Arrival-Departure Pair

Now that the factor for each arrival-departure pair $(A, XX) - (B, YY)$ has been obtained, it is necessary to normalize that factor to obtain a percentage value. This value will correspond to the percentage of passengers arriving from an origin B via airline XX willing to connect to a destination B via airline YY . The formula is shown below:

$$percentage_{XX,AA}^{A,B} = \frac{factor_{XX,YY}^{A,B}}{\sum_i \sum_j factor_{XX,j}^{A,i}} \quad (1.5)$$

In the Equation above, i corresponds to a generic airport and j to a generic airline. Thus, the denominator represents the sum of all factors between the arrival combination (A, XX) and all departure combinations (i, j) , while the numerator corresponds to only one of those factors. In conclusion, Equation 1.5 represents a ratio.

Combining the effects of airports' relative positions and airlines' relationships does not constitute a flawless method to obtain the percentages of transfer passenger demands between origin-destination pairs, but it is a reasonable approximation. The two factors are indeed a great influence on the passengers' choices and on the airport's decisions. More factors could be taken into consideration to make the values even more realistic. One of these factors could be the distance between the origin and destination airports.

Adding New Turnarounds To The Schedule

The base input of this model is the pre-existing flight schedule of the airport, henceforth called the **original schedule**. However, one of the main goals of this research is to study the effect of adding new turnarounds to the schedule of the airport to and find the best time slot to insert those turnarounds. To better understand how that influences the hub's route network and the number of passengers able to connect at the airport, a list of turnarounds is proposed for this research. These are called the **proposed turnarounds**. For each proposed turnaround, and every time that it is possible, several **time slot alternatives** will be proposed. This gives flexibility to the model to adapt and find the combination of slots that best adapts to the original schedule

and most expands the connections at the hub. Different time alternatives of the same proposed turnaround may have different duration and, more importantly, different transfer passenger demands, which makes the problem more dynamic.

In this research, two types of proposed turnarounds are considered. First, the turnarounds that will **expand the frequency** of routes already established at the hub. Second, the turnarounds that will create completely **new routes** that connect the hub to new airports. The selection of routes to be expanded is based on the flights that carry large amounts of passengers and, specifically, transfer passengers. For the new routes, an effort was put into finding parts of the world originally not connected to the hub, or finding airports that can indirectly expand the hub's network to other areas.

BEONTRA GmbH *BRoute Development Tool* was used to obtain all the proposed turnarounds. Generally speaking, the time alternatives of each turnaround were chosen based on profitability to the airline and on the forecast of transfer passengers. The profitability is related to the operating margin of that specific alternative when compared to some like markets, that is, to routes that are similar to the one proposed. These routes do not necessarily directly connect the hub and the other airport considered. For instance, if a route expansion is being considered from the hub to airport B , one like market could be the route from airport A to B , in which A is geographically close to the hub and can consequently capture transfer traffic from it.

It is also important to mention that the information regarding the transfer passenger demand of each time alternative is based on the assumption that the daily frequency of that turnaround is 1. Consequently, only one time slot alternative can be selected. It is not valid to include several alternatives of the same proposal.

Definition and Visualization of Connection Types

The main objective of this research is to develop a model that aims to capture as many transfer passengers as possible while connection times are minimized. For this reason, it is of extreme importance to define different types of connections between flights and how they are modelled in the context of this problem. Let the time between an arriving flight i (denoted by a_i) and any departing flight j be henceforth denoted as T . Then, Figure 1.2 shows that the type of connection created between i and j is dependent on T .

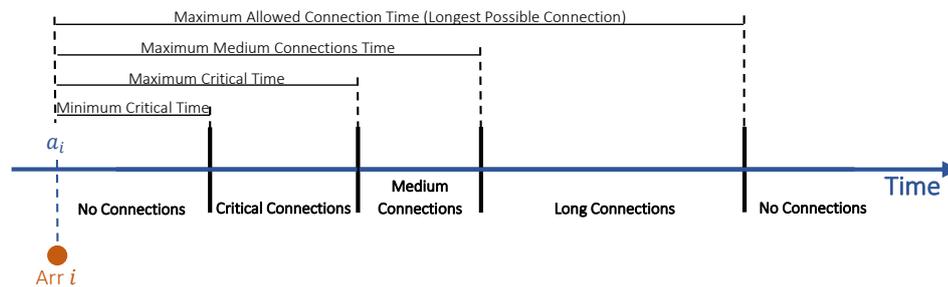


Figure 1.2: Time division used in the Strategic Stand Planning Model.

If T is smaller than a minimum threshold, denoted as **Minimum Critical Time**, it is assumed that the connection time is too small. Thus, no connection is possible between the two flights.

On the other hand, if the value of T lies in between the Minimum Critical Time and the **Maximum Critical Time**, a connection between i and j may be possible. However, due to the limited time that passengers possess to walk between the arrival of i and the departure of j , this type of connection is called a **Critical Connection**. Inside the Critical Connections period, passengers may or may not have time to make the connection, depending on where both flights are assigned - if they are assigned to stands that are close enough to each other, then a transfer passenger will have time to walk from i to j and make the connection. Otherwise, that is not possible. As a consequence of how critical connections are defined, the Minimum and Maximum Critical Time values correspond, respectively, to the minimum and maximum walking times between any two stands. In other words, if T is smaller than the time between the closest pair of stands, a connection between i and j is never possible. If T is greater than the time between the pair of stands farthest away from each other, a connection is always possible.

If T lies in between the Maximum Critical Time and the **Maximum Medium Connection Time**, a connection is always possible between i and j (provided they are both assigned to any stand). This period is called the **Medium Connections** period. When T increases even more, it falls somewhere between the Maximum Medium Connection Time and the **Maximum Allowed Connection Time**. In this period, called the **Long**

Connections period, layovers will be significantly long. Note that in the Medium Connections and the Long Connections periods, passengers have time to walk from the arriving stand to the departing stand. For this reason, these connections are non-critical.

Since one of the objectives of this research is to minimize connection times, a waiting time penalty is added to any connection longer than the Maximum Critical Time, i.e., to any non-critical connection. The value of this penalty will be equal to the connection time. This linear relationship is adequate since, the longer a certain connection is, the less desired it is by passengers. Penalizing long connections is important because this type of connection may turn an airport less competitive.

Finally, when T is greater than the Maximum Allowed Connection Time, it is assumed, in the model, that no connection can occur between i and j . This assumption of the existence of an upper time limit for connections is based on two arguments. The first one comes in line with what was explained in the previous paragraph. When layovers increase, passengers may start looking for alternatives in which they do not need to wait as long for a connecting flight. If the value for the Maximum Allowed Connection Time is set to be high enough, then layovers start getting excessively large and the probability that a passenger will find a different alternative becomes significantly large. As a consequence, it becomes valid to ignore possible connections beyond the Maximum Allowed Connection Time threshold because, in reality, they will most likely not be chosen by any passenger. Furthermore, long layovers may not be as profitable for airlines. In fact, A. Luttmann shows in [58] that airlines tend to offer discounted fares for longer layover times. The second argument is related to the problem size. Extending the Maximum Waiting Time Penalty will increase the number of variables needed, but it is not worth to sacrifice the size of the problem with connections that are not relevant in real life.

It should be noted that all the threshold values shown in Figure 1.2 can be defined by the airport planners and changed accordingly to different scenarios. The values chosen for this research will be shown later.

In Figure 1.3, a simple graphical example is presented, showing connection possibilities between an arrival flight i and several departing flights.

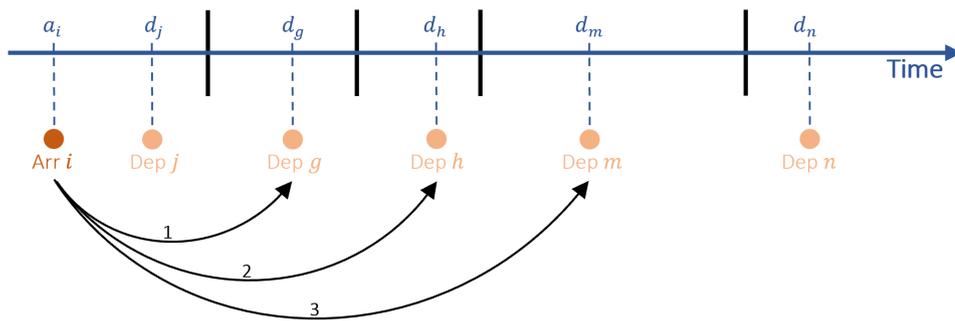


Figure 1.3: Example showing which types of connections are possible from an arrival flight **Arr i**.

First of all, no connection will certainly occur between flights i and j since the period of time between both operations is too short. The same will happen between flights i and n , but in this case, the cause is the long waiting time between the arrival and the departure. Case 1 shows a critical connection between flights i and g . The connection will only effectively be possible if those two flights are assigned to stands close enough to each other. In case 2, a non-critical connection is guaranteed to happen between flights i and h and this is also called a Medium Connection, according to the definition. Finally, case 3 also shows a non-critical connection between flights i and m that, although guaranteed, will be considerably long and thus will carry a large waiting time penalty with it.

1.2. Initial Approach to the Problem

Transfer passengers have already been previously modelled in the stand (or gate) assignment problem. A characteristic of this version of the SAP/GAP is the need to relate pairs of flights and pairs of stands. This relationship often leads to quadratic assignment problems (QAP) with large solution spaces and with second order objective functions and/or constraints which, in turn, are inherently harder to solve when compared to linear problems. Several authors have proposed different approaches to reformulate problems in order to avoid second order terms in the model [26]. Some of this proposals have been extensively analysed in the Literature Study. Nonetheless, even after reformulation, the problems tend to be excessively large, with

too many constraints and a very broad solution space. In fact, in some instances, reformulating the model implies adding new variables and constraints.

On the first approach to solve the current problem, henceforth called the original approach, a Mixed Integer Linear Programming (MILP) model with a multi-objective function was formulated. In the original approach, solved using the commercial solver *Gurobi*, operations were directly assigned to stands, and the different individual objectives were solved using a hierarchical method, that gives preference to the optimization of certain objectives (more detail on the multi-objective function and the hierarchical method will be shown in subsection 1.4.3). The fact that the original approach was formulated as a MILP means that no second order terms were present in it. However, and in line with what was explained before, linear SAP problems can still be significantly large when transfer passengers are included.

To better understand the dimension of the problem under these circumstances, an example will be shown in which it will become clear how many variables are needed. For the convenience of the reader, the types of variables used in the original approach are shown in Table 1.1. These variables are significantly similar to the ones used in the definitive approach (refer to section 1.3). Thus, the following explanation is also valid for that approach.

Decision Variable	Type of Variable	Variable Definition
y_{ik}	Binary	$y_{ik} = 1$ if operation i is assigned to stand k . Otherwise, $y_{ik} = 0$.
z_{ijkl}	Integer	Number of transfer passengers connecting from arrival operation i assigned to stand k to departure operation j assigned to stand l . Used for critical connections.
x_{ij}	Integer	Number of transfer passengers connecting from arrival operation i to departure operation j . Used for non-critical (Medium and Long) connections.
τ_i	Continuous	$\tau_i = 1$ if operation i is towed. Otherwise, $\tau_i = 0$.

Table 1.1: Decision variables used in the initial approach to solve the problem.

First of all, it should be noted that this example only involves z_{ijkl} variables, used for critical connections. These variables represent the relationship between pairs of flights and pairs of stands, so they will also be the most numerous type of variables. It is also important to explain that the choice of numbers in this example is supported by the data retrieved from the real case study of the current research. Let us suppose that an airport has 68 stands and in a regular day the airport operates around 150 arrivals and 150 departures. Let's assume that all arrivals and departures can be assigned to any stand and that there may be a critical connection between every arrival-departure pair. Then, an upper bound for the number of z_{ijkl} variables is given by $150 \times 150 \times 68 \times 68 = 104\,040\,000$, which is infeasible in terms of computer memory in a regular laptop. However, in reality there is not a critical connection between every arrival-departure pair and not every stand is compatible with every aircraft. Supposing that, on average, each arriving flight may have a critical connection with 40 departing flights and that each flight (arrival or departure) is compatible with 45 stands, then the model will create $150 \times 40 \times 45 \times 45 = 12\,150\,000$ z_{ijkl} variables, which still makes it impractical to run the model.

1.2.1. Elimination of Variables

As a first attempt to reduce the problem size, a pre-processing algorithm was introduced before running the original MILP approach. In this algorithm, variables that, *a priori*, are known to be equal to zero, are disregarded and not included in the model. This elimination process was applied to every type of decision variable defined in Table 1.1 in the following way:

- **y_{ik} variables:** for each operation i , the variables y_{ik} corresponding to stands k that are not compatible with i are not added to the model.
- **z_{ijkl} variables:** three reduction types are considered in the pre-processing of these variables. First, the same elimination described above is applied here. Not only does that reduce the problem size, but it also contributes to the consistency with the previous variables. If a certain assignment y_{ik} is not included in the model, it does not make sense to add a variable z_{ijkl} that premises the existence of that

same assignment. The second reduction type is in line with what was explained in [section 1.1](#) regarding critical connections. For each pair of flights $i-j$, the algorithm disregards the variables z_{ijkl} when there is not enough time to connect from a stand k to another stand l . The third and final reduction takes into account the forecast of transfer passengers between i and j . If there are no potential passengers willing to make the connection, the corresponding variable is disregarded.

- **x_{ij} variables:** for these variables, the algorithm checks if the number of forecast transfer passengers between i and j is zero. If that is the case, the corresponding x_{ij} variable is not considered.
- **τ_i variables:** the algorithm only selects τ_i variables for operations i that can be towed, i.e., operations that belong to long turnarounds..

Because of the similarity in the types of variables used in the original approach and in the definitive approach, the pre-processing is applied to both without the need of great changes. The pre-processing algorithm proved to be quite efficient when it comes to the reduction of the number of variables. After running it, the new number of z_{ijkl} variables (the most constraining type, used to represent connections) decreases from 12 150 000 to around 800 000. However, the computational load to solve a problem this size is still quite high. In fact, after performing some test runs using this pre-processing algorithm and solving the multi-objective function of the original approach with the hierarchical method, it was possible to conclude that *Gurobi* was taking more than 24 hours to reach a result. At this point, it becomes clear that a new approach is needed to achieve a final problem that can be successfully solved. This is explored in great detail on the next section.

1.3. TLSAP: Introduction to the Definitive Approach

In the previous section, it was shown that the size of this research's problem poses a barrier to the process of obtaining a final result. It was previously mentioned that several techniques exist to linearize problems that include transfer passengers, in order to reduce the complexity of those problems. However, the previous section showed that linear problems can still be extremely hard to solve, due to their large solution space. The two main alternatives to overcome this difficulty are to either find ways of reducing the solution space (one of which - the pre-processing algorithm - was already shown in the previous section) or to choose a different way to solve the problem, instead of simply performing a direct operation-to-stand assignment using a commercial solver.

It is clear that, for this problem, reducing the solution space through a pre-processing algorithm is not enough. Thus, a new solving method is needed. During the literature review, several methods used in the past to solve large problems were analysed. In particular, several authors have solved the stand assignment problem using heuristic or meta-heuristic algorithms. Genetic Algorithms, Simulated Annealing and algorithms based on a Tabu Search are common amongst the literature [26].

A Simulated Annealing algorithm was initially being developed for this research. However, because meta-heuristic algorithms can be hard to implement and adapt to each specific problem, other alternatives were being analysed simultaneously. After some research and try-outs, the definitive approach proposed for this research was fully developed. It is based on several methods, but particularly on the work of B. Maharjan and T. I. Matis in [14].

In the definitive approach, the operations are not directly assigned to stands at first. Instead, the problem is divided into two levels.

- **First Level:** in this level, operations are assigned to zones of the airport. These zones are defined based on several criteria, which will be further explained in [chapter 3](#). It is during the first level that the four objectives considered in this research are optimized.
- **Second Level:** the results obtained in the first level are used as input to the second level. In this level, operations are simply assigned to individual stands, inside the zone to which they were previously attributed.

In short, the first level of this approach is equivalent to the original approach because it is during this level that the multi-objective function is solved. The main difference is that, instead of assigning operations to stands, they are assigned to zones. While the first level is more demanding because all objectives are being solved there, the fact that zones instead of stands are being considered reduces considerably the number of variables needed, as it will be showed during this chapter. Consequently, the computational load is also

significantly lower. With the main problem optimized, there are no demanding tasks that need to be solved during the second level, so in this level stands can already be considered. The second level simply completes the whole problem by assigning the operations to specific stands, based on the zone assignment obtained after the first level. Note that, as a consequence of the reduced size of the two levels, they are both solved using the commercial solver *Gurobi*. Besides that, both levels are formulated as MILP problems including binary, integer and continuous variables.

1.3.1. Sets and Parameters Used In The TLSAP

Defining sets, parameters and nomenclature is essential to create a rigorous and unambiguous model that can be read by everyone. The most relevant parameters and nomenclature are shown in [Table 1.2](#), while [Table 1.3](#) shows all the sets used in this research.

Param.	Description	Param.	Description
i, j	Refer to a single operation	r	Refers to a single turnaround
i_{next}	Refers to the operation that succeeds i in its turnaround. If i is the last operation of the turnaround, i_{next} does not exist	a	Refers to a single schedule alternative of a new turnaround
k	Refers to a single stand	TE_k	Time from stand k to the entrance of its corresponding zone
t	Refers to a single type of stand. In this problem, 3 types of stands are considered: C , E and $MARS$. The latter is in fact a group of stands: it contains 2 type C stands and one type F stand. However, in the first level of assignment, the group is considered to be one single type	Dummy Zone	Refers to the fictional zone where operations are assigned when they are not added to the schedule. It allows the model to be always feasible. In the model, the dummy zone is represented by the index 0
u	Refers to a single destination-airline pair	M	Parameter used in the big-M method. Check Equation 1.42 to Equation 1.45 for more information on this method
v, w	Refer to a single zone	S^i	Seat Capacity of flight i
f_i^u	Refers to the number of forecast passengers wanting to connect from flight i to the destination-airline pair u	p	Penalty coefficient for unassigned turnarounds. Refer to Equation 1.32

Table 1.2: Parameters used to define the TLSAP.

Set	Description	Set	Description
N	All operations	N_{crit}^i	Departures that have a critical connection with arrival i
N_{arr}	Arrival operations	N_{ncrit}^i	Departures that have a non-critical connection with arrival i
N_{dep}	Departure operations	N_{dep}^u	Departure operations with destination-airline pair u
N_{tow}	Operations that can be towed	U_{dep}	Destination-airline pairs for departure flights
N_{no_tow}	Operations that cannot be towed	R_{new}	New turnarounds
N_v	Operations assigned to zone v in the first level of assignment	Z	Zones, excluding the dummy zone
N_v^{towed}	Towed operations in the first level of assignment	Z_0	Zones, including the dummy zone
$N_v^{not\,towed}$	Operations not towed in the first level of assignment	T_i^v	Types of stands of zone v that are compatible with flight i
N_{ov}^i	Operations overlapping in time with i	$T_i^v \setminus MARS$	Same set as above, but excluding MARS groups of zone v
$N_{ov}^{i,inc}$	Same set as above, but including operation i itself	K	All stands
N_{small}	Operations corresponding to small aircraft (type C or smaller)	K_v	Stands belonging to zone v
N_{large}	Operations corresponding to large aircraft (type E or larger)	K_t^v	Stands of type t belonging to zone v
$N_{ov,small}^{i,inc}$	$N_{ov}^{i,inc} \cap N_{small}$	K_{MARS}^v	MARS groups belonging to zone v . Contains large stand of each group
$N_{ov,large}^{i,inc}$	$N_{ov}^{i,inc} \cap N_{large}$	K_{blocks}^k	Small stands of the MARS group that has k as the large stand

Table 1.3: Sets used to define the TLSAP.

1.4. First Level of Assignment: Zone Assignment

On a first, more general level aircraft are assigned to zones and, for that reason, it gets the name of **zone assignment**. It is during this level that the core problem with the multi-objective function is solved. Unlike stands, which are already well characterized by an airport's documentation, the first level of assignment requires each zone considered by the model to be consistently defined. The definition of zones is based on the terminal's layout, on the proximity between stands and on the rules used by the airport for gate-stand dependencies. The specific zoning layout defined for this research's case study can be consulted in [section 3.1](#).

1.4.1. First Level of Assignment: Decision Variables

One thing to notice is that the model developed for the first level is similar to the model developed in the original approach. The main difference lies on the fact that, now, decision variables refer to zones and not stands (compare with [Table 1.1](#)). It is also necessary to introduce a new type of decision variable. This newly introduced variable type ϵ_{ivt} is particularly relevant to write the zones capacity constraints, which will be explained in [subsection 1.4.4](#). Please, refer to [Table 1.4](#) to see the decision variables used in the zone assignment.

To better understand in which cases z_{ijvw} and x_{ij} are used, [Figure 1.4](#) presents a similar schematic to the one shown in [section 1.1](#). It is now clear that z_{ijvw} variables are exclusively used in the Critical Connections period of a flight i , while x_{ij} is used in the non-critical connections periods, that is, the Medium Connections and the Long Connections. For the last two periods, the relative positions of flights i and j are not relevant because passengers always have time to make the connections, and that is why x_{ij} variables dropped the zones indices (v and w).

Decision Variable	Type of Variable	Variable Definition
y_{iv}	Binary	$y_{iv} = 1$ if operation i is assigned to zone v . Otherwise, $y_{iv} = 0$.
ϵ_{ivt}	Binary	$\epsilon_{ivt} = 1$ if operation i is assigned to a stand of type t inside zone v . Otherwise, $\epsilon_{ivt} = 0$.
z_{ijvw}	Integer	Number of transfer passengers connecting from arrival operation i assigned to zone v to departure operation j assigned to zone w . Used for critical connections.
x_{ij}	Integer	Number of transfer passengers connecting from arrival operation i to departure operation j . Used for non-critical connections.
τ_i	Continuous	$\tau_i = 1$ if operation i is towed. Otherwise, $\tau_i = 0$.

Table 1.4: Definition of decision variables for the first-level of assignment.

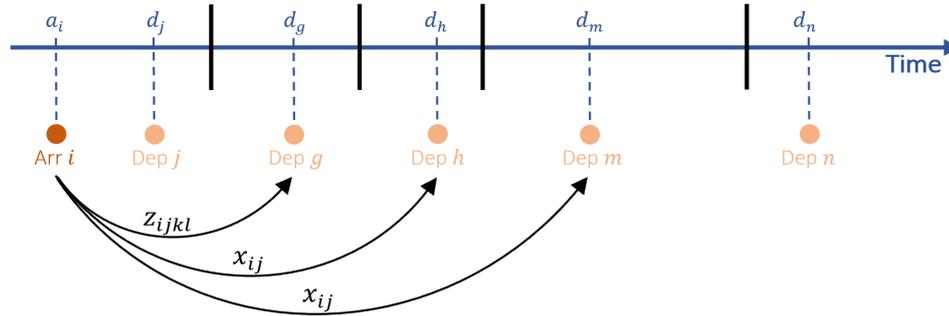


Figure 1.4: Relationship between types of decision variables and types of connections.

Elimination of Variables

In line with what was mentioned in subsection 1.2.1, variables that will certainly be equal to zero are unnecessary. The pre-processing algorithm described before disregards these variables and reduces the size of the problem without affecting its output. The type of elimination performed on each variable is very similar to the one described in subsection 1.2.1, but it is shown again below with more detail and applied to the first level of assignment.

- y_{iv} : for this type of variable, if an operation i is incompatible with a zone v , the corresponding variable is not added to the model.

To find out if a zone v is incompatible with an operation i , an analysis is initially made to the stands of v . If all those stands are incompatible with i , it follows that the entire zone is incompatible with i . If at least one stand is compatible with i , then v is also compatible with i .

There can be several reasons why an operation is not compatible with a stand. In this research, the incompatibility arises from two distinct situations. The first and most common one happens when an aircraft belongs to a larger ICAO Airplane Design Group than the largest allowed for a certain stand. This means that the aircraft does not fit on that stand, so it cannot be assigned there. The second situation is specific to the case study airport of this research and it was considered in the problem to increase its level of detail. At this airport, domestic arrivals are only allowed to park in a very specific area of the airport (Group 6C). In the model, and for simplicity (taken into consideration the zone division), it was assumed that domestic flights can only be assigned to Groups 6C and 5C, since both belong to the same Pier. All operations are assumed to be compatible with the dummy zone.

It should be noted that by eliminating variables, the incompatibility constraint, which is a primal element of every SAP problem, does not need to be added to the model, further contributing to the problem's size reduction.

- ϵ_{ivt} : in this case, and for each operation i , the pre-processing algorithm disregards variables when i is incompatible with a zone v or with a specific stand type t .

- z_{ijvw} : three reduction types are considered in the pre-processing of these variables. First, if operation i is not compatible with zone v or if operation j is not compatible with zone w , then the corresponding z_{ijvw} variable is not added. The second reduction is related to critical connection times. For each arrival-departure pair $i - j$, the algorithm disregards the variables z_{ijvw} when there is not enough time to connect from a zone v to another zone w . The third reduction takes into account the forecast of transfer passengers between i and j . If there are no passengers that want to make the connection, the corresponding variable is disregarded.
- x_{ij} : the algorithm checks the number of transfer passengers willing to make the non-critical connection between i and j . If that number is zero, then the corresponding x_{ij} variable is disregarded.
- τ_i : for these variables, the pre-processing algorithm only takes into consideration the operations i that can be towed, i.e., only operations belonging to the set N_{tow} .

1.4.2. First Level of Assignment: Complete Model

In this subsection, the complete model, including the different objective functions and constraints, is presented. The multi-objective function is approached in detail in [subsection 1.4.3](#) while the constraints are analysed in [subsection 1.4.4](#).

$$\min \sum_{i \in N} P y_{i0} \quad (1.6)$$

$$\max \sum_{i \in N_{arr}} \sum_{j \in N_{critical}^i} \sum_{v \in Z} \sum_{w \in Z} z_{ijvw} + \sum_{i \in N_{arr}} \sum_{j \in N_{ncrit}^i} x_{ij} \quad (1.7)$$

$$\min \sum_{i \in N_{arr}} \sum_{j \in N_{ncrit}^i} (d_j - a_i) x_{ij} \quad (1.8)$$

$$\min \sum_{i \in N_{tow}} \tau_i \quad (1.9)$$

Subject to:

$$\sum_{v \in Z_0} y_{iv} = 1, \quad \forall i \in N \quad (1.10)$$

$$\sum_{a \in R} \sum_{v \in Z} y_{arr,v}^a \leq 1, \quad \forall r \in R_{new} \quad (1.11)$$

$$\epsilon_{ivt} - \epsilon_{i_{next}vt} = 0, \quad \forall i \in N_{no_tow}, \quad \forall v \in Z_0, \quad \forall t \in T_i^v \quad (1.12)$$

$$\epsilon_{ivt} - \epsilon_{i_{next}vt} \leq \tau_i, \quad \forall i \in N_{tow}, \quad \forall v \in Z, \quad \forall t \in T_i^v \quad (1.13)$$

$$\epsilon_{ivt} + \sum_{j \in N_{ov}^i} \epsilon_{jvt} \leq |K_i^v|, \quad \forall i \in N, \quad \forall v \in Z, \quad \forall t \in T_i^v \setminus MARS \quad (1.14)$$

$$\sum_{j \in N_{ov,small}^{i,inc}} \epsilon_{jzt} + 2 \sum_{j \in N_{ov,large}^{i,inc}} \epsilon_{jzt} \leq 2 \times |K_{MARS}^v|, \quad \forall i \in N, \quad \forall v \in Z \quad (1.15)$$

$$z_{ijvw} \leq M y_{iv}, \quad \forall i \in N_{arr}, \quad \forall j \in N_{critical}^i, \quad \forall v, w \in Z \quad (1.16)$$

$$z_{ijvw} \leq M y_{jw}, \quad \forall i \in N_{arr}, \quad \forall j \in N_{critical}^i, \quad \forall v, w \in Z \quad (1.17)$$

$$x_{ij} \leq M(1 - y_{i0}), \quad \forall i \in N_{arr}, \quad \forall j \in N_{ncrit}^i \quad (1.18)$$

$$x_{ij} \leq M(1 - y_{j0}), \quad \forall i \in N_{arr}, \quad \forall j \in N_{ncrit}^i \quad (1.19)$$

$$\sum_{j \in N_{dep}^u} \sum_{v \in Z} \sum_{w \in Z} z_{ijvw} + \sum_{j \in N_{dep}^u} x_{ij} \leq f_i^u, \quad \forall i \in N_{arr}, \forall u \in U_{dep} \quad (1.20)$$

$$\sum_{i \in N_{crit}^j} \sum_{v \in Z} \sum_{w \in Z} z_{ijvw} + \sum_{i \in N_{ncrit}^j} x_{ij} \leq S^j, \quad \forall i \in N_{dep} \quad (1.21)$$

$$\sum_{t \in T_i^z} \epsilon_{ivt} = y_{iv} \quad \forall i \in N, \forall v \in Z \quad (1.22)$$

$$y_{iv} \in \{0, 1\}, \quad \forall i \in N, \forall v \in Z_0 \quad (1.23)$$

$$\epsilon_{ivt} \in \{0, 1\}, \quad \forall i \in N_{tow}, \forall v \in Z, \forall t \in T_i^v \quad (1.24)$$

$$z_{ijvw} \geq 0, \quad \forall i \in N_{arr}, \forall j \in N_{critical}^i, \forall v, w \in Z \quad (1.25)$$

$$x_{ij} \geq 0, \quad \forall i \in N_{arr}, \forall j \in N_{ncrit}^i \quad (1.26)$$

$$\tau_i \geq 0, \quad \forall i \in N_{tow} \quad (1.27)$$

1.4.3. First Level of Assignment: Multi-Objective Function

It was mentioned in subsection 1.4.2 that the complete objective function of this problem consists of several individual objectives. Besides that, some objectives need to be maximized while other should be minimized. Solving multi-objective functions is a recurring problem in the field of Operations Research. Due to the increased difficulty of this type of problem when compared to single-objective functions, several authors have tried to find ways to approach it using efficient methods that deliver the best results possible.

Unlike single-objective functions, in multi-objective functions there might not be a solution that optimizes all objectives at the same time. In other words, there is no global optimum. In fact, most of the time there is an infinite number of optimal solutions. As a consequence, the definition of optimal solution in a single-objective optimization is not applied anymore. For a multi-objective optimization, a solution is said to be optimal if there are no other feasible solutions that improve the value of at least one objective function, without worsening any other objective. This type of optimality is called Pareto optimality [59] [60]. Thus, a multi-objective optimization is solved, not by finding a single solution, but rather a set of solutions named Pareto front, which have the Pareto optimality property [61]. Any solution that is part of this set is optimal for the corresponding multi-objective problem.

On the other hand, it is often the case that different individual objectives cannot directly be related to each other. For instance, if a problem aims to minimize number of tows and minimize passenger walking times, it becomes clear that one cannot simply add both individual terms to create the multi-objective function. In addition, the decision maker may determine that one individual objective has priority over others, or that it should weight more on the final objective function. In order to ensure the consistency of optimal solutions that respect the decision maker's preferences, the normalization of objective functions becomes important [59]. Several approaches to normalize the terms of multi-objective functions were developed in the past. The two approaches that are considered in this research are the weighted sum approach and the hierarchical method. Since the latter was the one chosen to solve the multi-objective function of this research, its concept will be explained in more detail. It will also be shown how the method was adapted to the problem.

Weighted Sum Approach

The weighted sum approach allows to write all objectives of the objective function as a single mathematical function, and each objective is multiplied by a coefficient, called weight [59]. The objective function is formulated as follows:

$$OF = \sum_{i=1}^N w_i z_i \quad (1.28)$$

where N is the number of individual objectives, z_i is an individual objective and w_i is its corresponding weight. The weights w_i are, in reality, obtained from the product of two terms. The first term comes from the actual weights that the decision maker wants to assign to the individual objectives, giving different priorities to each. The second term comes from the normalization of the individual objectives. According to what was mentioned before, different individual objectives may not be compatible. Usually, the fact that individual objectives have different magnitudes requires a subsequent normalization, which is obtained by multiplying a coefficient with each term [59].

The normalization can be achieved through different methods. One of them normalizes each objective using the magnitude of that objective at the initial point. A second method uses the minimum value of each objective for the normalization. While relatively simple, these two methods are not efficient. A third method uses objective values at two specific points: the Nadir and Utopia points. In short, the Utopia point is the ideal objective vector because it is defined as the vector that optimizes each term individually. For the same reason, the Utopia point is very often infeasible. The Nadir point, on the other hand, corresponds to the vector containing the worst possible value of each term. The normalization coefficient of each individual objective is then given by the difference between the corresponding Nadir point Utopia point values. At the end of this normalization process, the objectives become dimensionless and, consequently, comparable with each other, which means that the different weights added by the decision maker will effectively make some objectives more important than others [59] [60].

Hierarchical Method

In the hierarchical method, the individual objectives are not combined together in a single function. Instead of solving all objectives at the same time, the objectives are ranked with respect to their relative importance and are solved, one at a time, from the most to the least important. Thus, when this method is used, the decision maker defines the order in which the objectives are solved, which is the equivalent of choosing the weights of the individual objectives in the weighted sum approach.

When a certain objective is being optimized, some extra constraints are added in order to prevent the previously solved objective from deviating more than a predefined threshold from its optimal solution. This guarantees that the optimal solution from a certain objective is achieved without severely deteriorating the previous solution. This threshold can be also defined by the decision maker, and its value (usually given as a percentage) represents how much one is willing to sacrifice the previously solved objectives to improve the current objective. In *Gurobi*, the threshold is called MIP gap. Let f_1, f_2, \dots, f_k be the order chosen to solve a certain multi-objective function problem. The general representation of the hierarchical approach to solve that problem is presented by O. Grodzevich and O. Romanko in [59]. First, the following sub-problem should be solved:

$$\min f_1 \tag{1.29}$$

$$\text{s.t. } x \in \Omega \tag{1.30}$$

where Ω corresponds to the feasible region. The optimal solution $x^{[1]}$ is obtained. Then, a similar sub-problem is solved for each remaining individual objective j , to find the corresponding optimal solution $x^{[j]}$. However, extra constraints of the form:

$$f_l(x) \leq (1 + \delta_l) f_l(x^{[l]}), \quad \forall l = 1, \dots, j - 1 \tag{1.31}$$

are added to guarantee that the previous objectives don't deviate more than a percentage of δ_l from the corresponding optimal values.

The hierarchical method may become hard to solve when the original problem contains one (or more) quadratic individual objectives, as solving one of these will force the introduction of quadratic constraints on subsequent levels of the hierarchy. However, the problem of this research does not contain any objective with a quadratic nature. On the other hand, the method works well with linear objectives, as O. Grodzevich and O. Romanko explain in [59]. Besides, the same authors also mention in their work that the weighted sum approach may fail when all objectives are linear and a simplex method is used to solve them, which is the case in this research.

For the reasons above, it seems that solving this problem using the hierarchical method is the most appropriate approach, and the most intuitive at the same time. Besides that, it is significantly easy to find a sequence of priorities among the four objectives of this model. This sequence is shown below, as well as the argument that supports the choice.

- **1st Objective:** Minimize Number of Unassigned Turnarounds (refer to Equation 1.32)

$$\min \sum_{i \in N} P y_{i0} \quad (1.32)$$

- **2nd Objective:** Maximize Number of Captured Transfer Passengers (refer to Equation 1.33)

$$\max \sum_{i \in N_{arr}} \sum_{j \in N_{critical}^i} \sum_{v \in Z} \sum_{w \in Z} z_{ijvw} + \sum_{i \in N_{arr}} \sum_{j \in N_{ncrit}^i} x_{ij} \quad (1.33)$$

- **3rd Objective:** Minimize Connection Times for Medium and Long Connections (refer to Equation 1.34)

$$\min \sum_{i \in N_{arr}} \sum_{j \in N_{ncrit}^i} (d_j - a_i) x_{ij} \quad (1.34)$$

- **4th Objective:** Minimize Number of Towed Operations (refer to Equation 1.35)

$$\min \sum_{i \in N_{tow}} \tau_i \quad (1.35)$$

Maximizing the number of captured transfer passengers is the main objective of this research. However, it is important to first allocate as many aircraft as possible to the schedule, and that is why the objective with the highest priority is the minimization of the number of non-allocated turnarounds. As a consequence of the hierarchical method's nature, the solver can, at most, swap an allocated aircraft with a non-allocated aircraft in subsequent levels of optimization (if that leads to a better solution in the corresponding individual objective), thus guaranteeing that the maximum number of aircraft is always allocated. Note that a penalty P is added for each unassigned turnaround. This penalty is defined as 10 times larger for original turnarounds than for the proposed turnarounds. While this model allows to remove original turnarounds from the schedule, it is more desired and logistically easier to give priority to these turnarounds over the proposed ones. That is why the latter have a lower associated penalty coefficient P .

After this first step, which can be interpreted as a "setup" step, the number of captured transfer passengers should be maximized. Again, the reason why this objective is placed in second is because it is the main goal of the research. Equation 1.33 shows that this objective is composed of two terms. In fact, they both represent transfer passengers but, as it was explained before, the first term is related to critical connections and the second represents non-critical connections. Both need to be accounted for the total number of connections created. They can, nonetheless, be simply added to each other.

The third objective to be optimized is the minimization of passenger connection times. In line with what was explained in section 1.1 and shown in Figure 1.2, a penalty is added when the passenger layover is higher than a certain threshold, and the minimization of this penalty comes in third place in the priority order. Even though minimizing this penalty is not as important as capturing a large number of transfer passengers, it is still possible to change the scheduled arrival and departure times of some turnarounds in order to reduce connections times, while not interfering with the number of connections created. It should be noted, however, that this objective is only relevant when the connection is medium or long (refer again to Figure 1.2 to recall the types of connection). Critical connections are significantly short by definition and the Critical Period is small when compared to the other periods. This means that it is not relevant to minimize short connections. Figure 1.5 shows how the connection times can be reduced when one of the turnarounds is new. The simple scenario shown in the figure portrays two turnarounds: i is an original turnaround (fixed in time) and j is a new turnaround with two time alternatives (j_1 and j_2). In case **I**, the waiting time for passengers arriving in $Arri$ and departing in Dep_{j_1} is significantly long. However, if alternative j_2 is chosen, the connection is not affected but the waiting time is reduced, leading to a better solution in case **II**.

Finally, the solver aims to minimize the number of towed operations. This objective was added last in the list because it is not one of the main goals of the research and it will not be analysed in great detail. Instead, it was added to increase airport operations efficiency. It is not convenient and sometimes not even profitable for an airport to constantly tow aircraft between stands. More infrastructure and people may be needed, the ground operations become more complex and the taxiways will get more crowded. By leaving the optimization of this objective to the end, one can guarantee that unnecessary tow moves which were

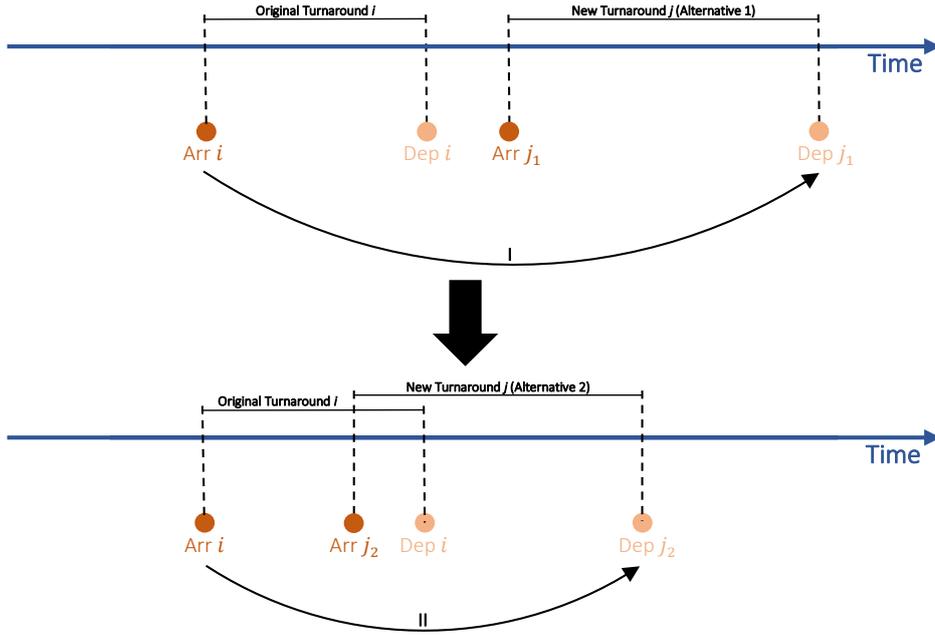


Figure 1.5: Simple example showing how the passenger waiting time can be reduced by moving new turnarounds in time.

randomly generated during the previous optimizations are eliminated, without affecting the results of the main goals.

1.4.4. First Level of Assignment: Constraints

In this section, the different constraints shown in subsection 1.4.2 are individually explained in detail. Besides the core constraints that are part of any stand assignment problem, this research requires the introduction of specific constraints. In particular, the problem includes restrictions and relationships between variables that correctly model connections inside an airport.

Correct Stand Assignment Constraints

A constraint that is part of every stand assignment problem and that is necessary to assure the correct allocation of operations to stands is shown in Equation 1.36. It states that every operation must be assigned to exactly one stand. This includes the dummy stand, to which the operations not added to the schedule are assigned.

$$\sum_{v \in Z_0} y_{iv} = 1, \quad \forall i \in N \quad (1.36)$$

Another constraint that assures a correct use of the stands is shown in Equation 1.37. This constraint is specific to this research and it is only applicable to the new turnarounds. It guarantees that, at most, only one schedule alternative of a new turnaround can be added to the schedule. Let r be a new turnaround, A be the set containing the different schedule alternatives for r and a a certain alternative contained in A . It should be noted that, in equation Equation 1.37, only the arrival operations i_{arr} of the different alternatives are being explicitly considered. However, making sure that, at most, only one arrival operation i_{arr}^a can be added to the schedule is enough to guarantee that the complete corresponding turnaround a is fully assigned to the schedule and that the other alternatives from A are not. This statement is made under the assumption that different operations of the same alternative a cannot be split between the dummy zone and real zones. This assumption is, in turn, always true, as a consequence of how the towing constraints are defined.

$$\sum_{a \in r} \sum_{v \in Z} y_{i_{arr}^a v} \leq 1, \quad \forall r \in R_{new} \quad (1.37)$$

Towing Constraints

In a regular turnaround, an aircraft arrives at the airport, arriving passengers disembark and cargo is unloaded, then departing passengers embark and new cargo is loaded, the aircraft is refueled, a quick maintenance check may be done and it departs again. A turnaround like this is called a short turnaround. However, there are cases in which an aircraft's ground time is longer than a certain threshold. In those cases, it may be more efficient to tow that aircraft to a different stand after its arrival, potentially freeing a valuable stand for another aircraft. Then, the aircraft can again be towed to a better stand some time prior to its departure, so that passengers have time to embark and cargo can be loaded. In a case like this, the turnaround is said to be long.

The threshold that splits short turnarounds from long turnarounds is not necessarily a fixed value and it is usually defined by an airport. In the case of this research's case study airport, there are specific rules for towing activities. These rules are explained in [section 3.1](#). In essence, short turnarounds can never be towed, while long turnarounds may be moved between stands if that is more efficient.

Since some turnarounds may be towed but others may not, it is mandatory to define two towing constraints. [Equation 1.38](#) states that operations belonging to the set N_{no_tow} cannot be towed. This set includes not only operations from short turnarounds, but also all operations that are assigned to the dummy zone. This forces all operations of a single turnaround to either be all assigned to the dummy zone or all assigned to a real zone, thus guaranteeing consistency with the concept of "dummy zone": a turnaround cannot be simultaneously assigned and unassigned to the schedule. This is also what allows [Equation 1.37](#) to consider only the arrival operations of the alternatives of a new turnaround.

On the other hand, [Equation 1.39](#) allows certain operations to be towed. This is only possible for operations belonging to long-stay turnarounds that are not assigned to the dummy zone. Note how the variable τ_i becomes active when operation i and i_{next} are not assigned to the same zone and consequently penalizes the objective function in [Equation 1.35](#).

$$\epsilon_{i_vt} - \epsilon_{i_{next}vt} = 0, \quad \forall i \in N_{no_tow}, \forall v \in Z_0, \forall t \in T_i^v \quad (1.38)$$

$$\epsilon_{i_vt} - \epsilon_{i_{next}vt} \leq \tau_i, \quad \forall i \in N_{tow}, \forall v \in Z, \forall t \in T_i^v \quad (1.39)$$

Note that ϵ_{i_zt} variables are used to write the towing constraints. This means that aircraft are allowed to be towed between different zones and, inside the same zone, they can be towed between different stand types. However, they are not allowed to be towed between two stands of the same type that are also located in the same zone.

Zone Capacity Constraint

The zone capacity constraint is shown in [Equation 1.40](#). Traditionally, this constraint is called the overlapping constraint and it prevents operations taking place simultaneously at some point in time from being assigned to the same stand. In this problem, the constraint takes into account the whole capacity of a zone and not just a single stand. Let's first define how an operation is considered to be overlapping with another operation. Consider i to be a certain operation and N_{ov}^i the set of operations overlapping with i . The algorithm checks which operations are taking place at the moment i starts. Those operations are considered to be overlapping with i and are added to N_{ov}^i . [Figure 1.6](#) shows visually how this process works. For all operations i , the model checks which operations intersect the vertical red line (the start of operation i) and those operations are overlapping with i . It is important to note that, under this definition, j overlaps with i but i does not overlap with j . This differentiation is vital to correctly write the Zone Capacity Constraint.

Mathematically, an operation j is added to the set N_{ov}^i if $t_{begin}^j \leq t_{begin}^i < t_{end}^j$, where t_{begin}^i and t_{begin}^j are the initial times of operation i and j , respectively, and t_{end}^j is the end time of operation j .

Because each zone v may contain more than one type of stand t (either C, E or MARS (2C + 1F)), the overlapping constraint of v has to be split into the different types. This makes the variable ϵ_{i_vt} extremely useful to define the constraint. [Equation 1.40](#) shows that, inside each zone, and for each stand type t , the assignment of operation i is summed together with its overlapping operations. That sum cannot exceed the capacity of stand type t inside v , $|K_t^v|$.

$$\epsilon_{i_vt} + \sum_{j \in N_{ov}^i} \epsilon_{j_vt} \leq |K_t^v|, \quad \forall i \in N, \forall v \in Z, \forall t \in T_i^v \setminus MARS \quad (1.40)$$

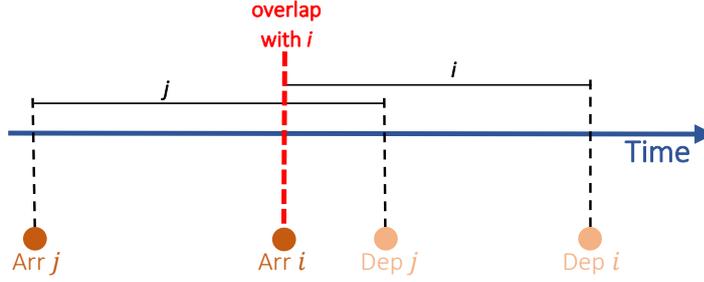


Figure 1.6: Schematic explanation of how an operation j can overlap with operation i .

This constraint is applicable to every zone, except the dummy zone, which is assumed to have an infinite capacity. Note how this constraint is not applicable to MARS groups.

The stand assignment problem very often takes into account that there might be small delays when an aircraft is departing or arriving at the stand. To avoid a domino effect on the delay of several aircraft when one flight is not at its designated stand on time, it is common to consider a small buffer time t_{buffer} before each operation starts. This buffer time improves the robustness of a schedule since it increases its tolerance to slight delays [62] [63]. The schedule robustness is added to the model by following a very similar process as the one described before. Let the buffer period of operation i be denoted by $buffer_i$ (the interval where $buffer_i$ is active is given by $[t_{begin}^i - t_{buffer}, t_{begin}^i]$). Then, it is assumed that each operation i starts at $t_{begin}^i - t_{buffer}$, and the capacity constraint for i is verified at that point instead of its real starting time. Figure 1.7 shows that, even though operations i and j do not physically overlap with each other, j is still taking place when the buffer period of i starts, so for the model that overlap exists, and operation j will still be added to the set N_{ov}^i .

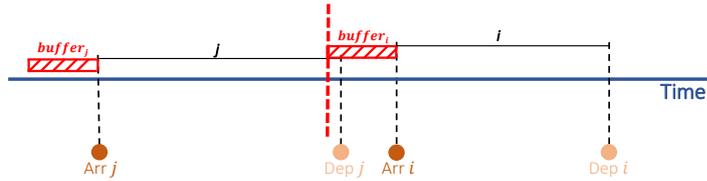


Figure 1.7: Schematic explanation of how an operation j may overlap with the buffer period $buffer_i$ of an operation i .

Zone MARS Capacity Constraint

In order to consider the MARS stand capacity of each zone, a new constraint is developed. The constraint is shown in Equation 1.41. To better understand this constraint, it will be split into its different terms. The first term represents the assignment of small aircraft operations j overlapping with operation i , and are multiplied by a factor of 1. The second term represents the assignment of large aircraft operations j that overlap with operation i , and are multiplied by a factor of 2. The third term, on the right hand side of the equation, takes into account the number of MARS groups inside zone v , $|K_{MARS}^v|$. This value is multiplied by 2 because this makes the equation mathematically correct. Figure 1.8 shows how the MARS constraint equation works, and it helps visualizing why some of its terms need to be multiplied by 2. Note that each MARS stands contains one large block that is suitable for large aircraft (E or bigger) and two small blocks compatible with small aircraft (C or smaller).

$$\sum_{j \in N_{ov,small}^{i,inc}} \epsilon_{jzt} + 2 \sum_{j \in N_{ov,large}^{i,inc}} \epsilon_{jzt} \leq 2 \times |K_{MARS}^v|, \forall i \in N, \forall v \in Z \quad (1.41)$$

Different factors are needed to correctly model the layout of a MARS group: parking a small aircraft takes up one small block, while parking a big aircraft takes up one block to big aircraft, eliminating two blocks to small aircraft. Without loss of generalization, let's consider that a certain zone has one MARS stand. In **Case 1**, the stand is empty, which means it is available for two small aircraft or one large aircraft. The overlapping constraint will have the form $0 \leq 2$. In **Case 2**, one of the small blocks (401R) is being used by a small aircraft, and the overlapping constraint will change to $1 \leq 2$. The constraint allows another small aircraft to be parked

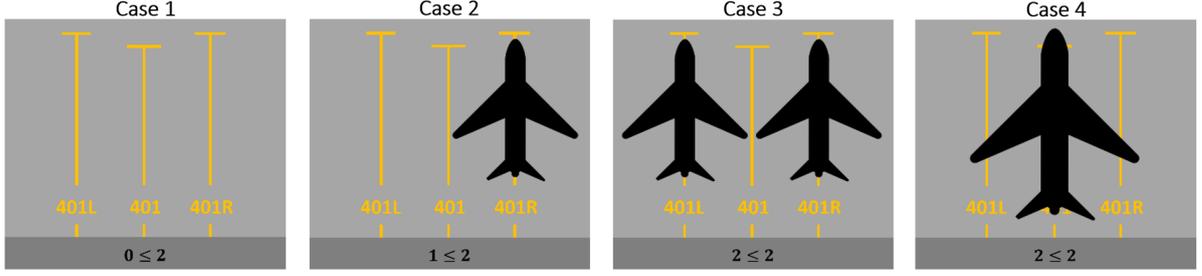


Figure 1.8: Simple example portraying the dynamics of a MARS group.

at the stand, because its factor is equal to 1. However, the stand is not suited anymore for a large aircraft, since its factor equals to 2, and thus the constraint would take the form $3 \leq 2$. In **Case 3**, the two small aircraft are being used by two small aircraft, leaving no space for any other aircraft, which is also visible in the overlapping constraint $2 \leq 2$. Finally, **Case 4** portrays the scenario in which a large aircraft parks in the main stand (401). The constraint changes to $2 \leq 2$, leaving no space for any other aircraft. This logic can be extended from one to $|K_{MARS}^v|$ MARS stands, if the right hand side of the constraint is multiplied by $|K_{MARS}^v|$.

It is also important to note that the set of operations overlapping with i is united with i itself to form a new set: $N_{ov}^i \cup \{i\} = N_{ov}^{i,inc}$. This set is in turn split into a subset with only small aircraft ($N_{ov,small}^{i,inc}$) and another with only large aircraft ($N_{ov,large}^{i,inc}$). This adds operation i to the correct subset, which means it will be added to the correct term of the equation: either the first if it is a small aircraft operation, or the second if it is a large aircraft operation.

The Zone MARS Capacity Constraint is also extended to include the buffer period $buffer_i$ for every operation i and following the exact same logic and steps as explained in the Zone Capacity Constraint.

Hub Connections Constraints

Because of the nature of this research, some specific constraints to model hub connections and relate different decision variables were added. Equation 1.42 and Equation 1.43 link the variables z_{ijkl} with the variables y_{ik} . They state that a connection between i (parked at stand k) and j (parked at stand l) is only possible if both operations are individually assigned to those stands, i.e., if i is effectively assigned to k and j is effectively assigned to l .

$$z_{ijvw} \leq My_{iv}, \quad \forall i \in N_{arr}, \quad \forall j \in N_{critical}^i, \quad \forall v, w \in Z \quad (1.42)$$

$$z_{ijvw} \leq My_{jw}, \quad \forall i \in N_{arr}, \quad \forall j \in N_{critical}^i, \quad \forall v, w \in Z \quad (1.43)$$

A similar pair of constraints is needed to relate variables x_{ij} and y_{ik} . As it was already explained before, x_{ij} variables refer to non-critical connections. Thus, the constraints shown in Equation 1.44 and Equation 1.45 imply that a connection between operations i and j can happen (regardless of where they are assigned), provided neither of them is assigned to the dummy zone.

$$x_{ij} \leq M(1 - y_{i0}), \quad \forall i \in N_{arr}, \quad \forall j \in N_{ncrit}^i \quad (1.44)$$

$$x_{ij} \leq M(1 - y_{j0}), \quad \forall i \in N_{arr}, \quad \forall j \in N_{ncrit}^i \quad (1.45)$$

It is important to remark that the four equations above make use of the big-M method, since both z_{ijvw} and x_{ij} variables are integer. Notice how in the equations the right-hand side of the equations is multiplied by the constant M . This constant is defined to be larger than the highest value allowed for both types of variables, which in this case is the highest number of transfer passengers connecting from a flight i to a certain destination-airline pair u .

Arrivals Transfer Passenger Forecast Constraint

In this research, the total number of forecast transfer passengers of each arrival flight i , $Pax_{transfer}^i$, is already given as an input. However, the number of passengers wanting to transfer between i and each departure j still needs to be computed.

The percentage of passengers wanting to connect from an origin-airline pair to a destination-airline pair is also given to the model as an input matrix. For instance, consider that airline XX flies from an origin airport AAA to the hub and airline YY flies from the hub to a destination airport BBB . The input data will contain an entry of the form $(AAA, XX; BBB, YY)$, and a percentage value x will be associated with it. This means that, for every flight i arriving from AAA via airline YY , x percent of its transfer passengers will want to transfer to a flight departing to destination BBB via airline YY . The percentage is then multiplied by $Pax_{transfer}^i$ to obtain the effective number of passengers f_i^u willing to make that connection, where u represents the pair (BBB, YY) .

Note that the passengers f_i^u wanting to make the connection may be assigned to different departing flights, provided they all fly to BBB via YY . The constraint is shown in Equation 1.46 and it is schematically explained for a simple example in Figure 1.9.

$$\sum_{j \in N_{dep}^u} \sum_{v \in Z} \sum_{w \in Z} z_{ijvw} + \sum_{j \in N_{dep}^u} x_{ij} \leq f_i^u, \quad \forall i \in N_{arr}, \quad \forall u \in U_{dep} \quad (1.46)$$

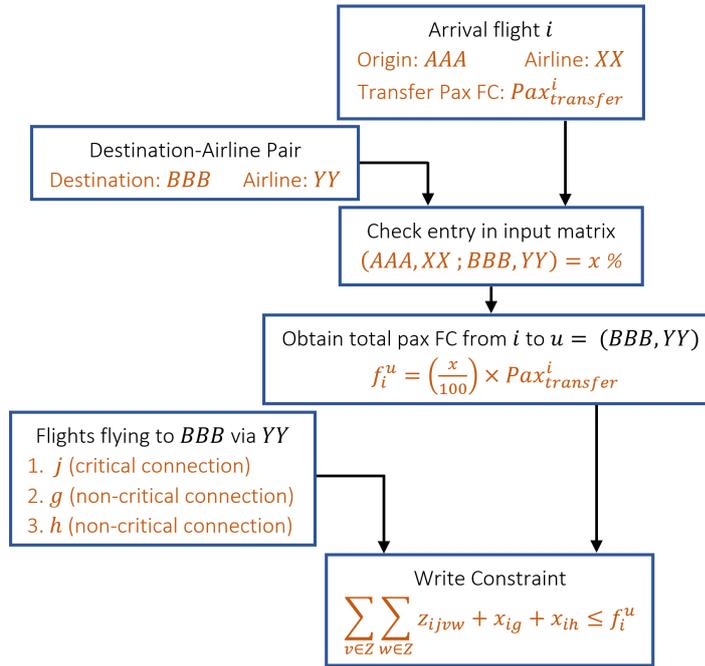


Figure 1.9: Example showing the dynamic behind Equation 1.46. Note that, for each flight i , the process is repeated for all destination-airline pairs contained in U_{dep} .

Departures Seat Capacity Constraint

The number of passengers transferring from each arrival was already verified with the previous constraint. But each departing aircraft has a limited seat capacity and it cannot accommodate an infinite number of transfer passengers. Equation 1.47 shows the constraint that establishes an upper bound to the sum of all passengers flowing to departure j , and that upper bound is that flight's seat capacity, S^j .

$$\sum_{i \in N_{crit}^j} \sum_{v \in Z} \sum_{w \in Z} z_{ijvw} + \sum_{i \in N_{ncrit}^j} x_{ij} \leq S^j, \quad \forall i \in N_{dep} \quad (1.47)$$

The first term of the equation adds up all passengers connecting to j via critical connections, while the second term adds up passengers incoming from non-critical connection to j .

Zone/Stand Type Relation Constraint

The first-level assignment introduces the variable type ϵ_{iut} , which is particularly useful to write the zone stand capacity constraints. Variable type y_{iu} relates to zone assignment and thus to a more generic assign-

ment, while ϵ_{ivt} relates to stand type assignment inside each zone, so it represents a more specific assignment. There is a clear relationship between these variables and y_{iv} variables, that needs to be translated to a constraint. The relationship can be intuitively explained as follows: if an operation is assigned to a zone, it has to be assigned to exactly one stand type of that zone. Otherwise, it cannot be assigned to any stand type of the zone. The constraint in Equation 1.48 translates the relationship mentioned in the previous paragraph to a mathematical relation.

$$\sum_{t \in T_i^z} \epsilon_{ivt} = y_{iv} \quad \forall i \in N, \forall v \in Z \quad (1.48)$$

Figure 1.10 is included below to clarify the relationship between ϵ_{ivt} variables and y_{iv} . It also shows the different scenarios that can occur.

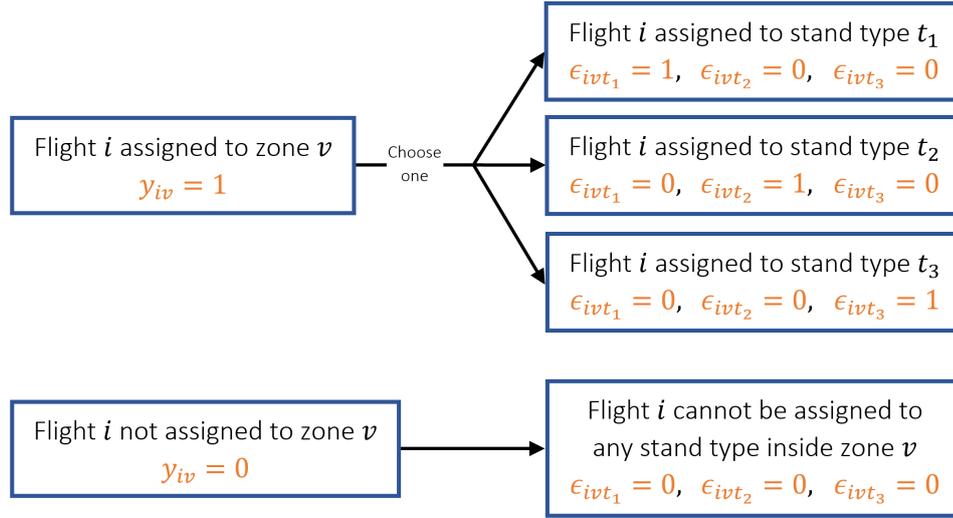


Figure 1.10: Relationship between ϵ_{ivt} variables and y_{iv} variables, and possible scenarios.

Variable Bounds Constraints

Finally, it is important to define the lower and upper bounds of each variable type. Variables y_{iv} and ϵ_{ivt} are binary, which means they can only either assume the value 1 or 0 - see Equation 1.49 and Equation 1.50.

$$y_{iv} \in \{0, 1\}, \quad \forall i \in N, \forall v \in Z_0 \quad (1.49)$$

$$\epsilon_{ivt} \in \{0, 1\}, \quad \forall i \in N_{tow}, \forall v \in Z, \forall t \in T_i^v \quad (1.50)$$

Variables z_{ijvw} and x_{ij} are integer but, since they represent a number of transfer passengers, they cannot be negative, which leads to the constraints in Equation 1.51 and Equation 1.52, respectively.

$$z_{ijvw} \geq 0, \quad \forall i \in N_{arr}, \forall j \in N_{dep}, \forall v, w \in Z \quad (1.51)$$

$$x_{ij} \geq 0, \quad \forall i \in N_{arr}, \forall j \in N_{dep} \quad (1.52)$$

Finally, variables τ_i are allowed to be continuous as a consequence of how the variables y_{ik} are defined and how towing constraints are written. However, τ_i variables still need to be constrained to non-negative values, which this is depicted in Equation 1.53.

$$\tau_i \geq 0, \quad \forall i \in N_{tow} \quad (1.53)$$

1.5. Second Level of Assignment: Individual Stand Assignment

After the first-level assignment, operations are already assigned to zones in a way that maximizes the number of captured transfer passengers, while minimizing transfer passenger connection times and the number of unassigned aircraft and towing operations. It can be concluded that the core of the problem is solved. However, while assigning operations to zones reduces considerably the size of the problem, it does not deliver a specific stand assignment that can be used, in practice, by the airport. This makes the second level of assignments just as relevant. During this level, operations previously attributed to a zone v will be assigned to its available stands. This process is repeated until all zones have been considered. The second level assignment is also called the **individual stand assignment**.

This section follows a similar structure to the previous section. The full problem, including decision variables, is portrayed. Then, the objective function and constraints will be explained in more detail.

1.5.1. Second Level of Assignment: Decision Variable

The only variable type defined for this level is shown in Table 1.5. This variable has a similar meaning to the y_{iv} variable used in the first level, with the difference that the former refers to operation-to-stand assignments and the latter refers to operation-to-zone assignments.

Decision Variable	Type of Variable	Variable Definition
η_{ik}	Binary	$\eta_{ik} = 1$ if operation i is assigned to stand k . Otherwise, $\eta_{ik} = 0$.

Table 1.5: Definition of the decision variable for the second-level of assignment.

Elimination of Variables

Even though each sub-problem of the second-level is significantly small and easy to solve, the pre-processing algorithm is still applied to them in order to reduce the number of variables as much as possible. For each operation i , the algorithm disregards η_{ik} variables when a certain stand k is incompatible with i .

1.5.2. Second Level of Assignment: Complete Model

The second-level is divided into several sub-problems, one for each zone v (except the dummy zone). The model hereby presented is run once for each of those sub-problems with the specific characteristics of the zones (stand composition and assigned operations). The objective function is discussed in more detail in subsection 1.5.3 while the constraints are analysed in subsection 1.5.4.

$\forall v \in Z :$

$$\min \sum_{i \in N_v} \sum_{k \in K_v} TE_k \eta_{ik} \quad (1.54)$$

Subject to:

$$\sum_{k \in K_z} \eta_{ik} = 1, \quad \forall i \in N_v \quad (1.55)$$

$$\eta_{ik} - \eta_{i_{next}k} = 0, \quad \forall i \in N_v^{not_towed}, \quad \forall k \in K_v, \quad \text{if } i_{next} \in N_v \quad (1.56)$$

$$\eta_{ik} - \eta_{i_{next}k} \leq 1, \quad \forall i \in N_v^{towed}, \quad \forall k \in K_v, \quad \text{if } i_{next} \in N_v \quad (1.57)$$

$$\eta_{ik} + \eta_{jk} \leq 1, \quad \forall i \in N_v, \quad \forall j \in N_{ov}^i \cap N_z \quad (1.58)$$

$$\sum_{k' \in K_{blocks}^k} \sum_{j \in N_{ov,small}^{i,inc} \cap N_v} \eta_{jk'} + 2 \sum_{j \in N_{ov,large}^{i,inc} \cap N_v} \eta_{jk} \leq 2, \quad \forall i \in N_v, \quad \forall k \in K_{MARS}^v \quad (1.59)$$

$$\eta_{ik} \in \{0, 1\}, \quad \forall i \in N_v, \quad \forall k \in K_v \quad (1.60)$$

1.5.3. Second Level of Assignment: Single-Objective Function

The individual stand assignment uses a simple single-objective function. Since all the important objectives of the research are already optimized, it is only important to assign operations to stands. Equation 1.61 shows the objective function defined for the second level. It aims to minimize the walking time (measured in minutes) between the stand where each operation is assigned and the entrance of the corresponding zone. In other words, it places all operations as close as possible from the zone entrance. It is important to keep operations as close as possible to the entrance of zones for passenger convenience. This is particularly important when passengers need to transfer between zones inside the terminal of the airport. It is also convenient for O&D passengers, since, for these passengers, reaching the stand or the airport's exit will take less time.

$$\min \sum_{i \in N_v} \sum_{k \in K_v} TE_k \eta_{ik} \quad (1.61)$$

1.5.4. Second Level of Assignment: Constraints

Fewer constraints are needed to formulate the second-level assignment, and all of them are somehow similar to some constraints from the first level. They will be individually analysed in this subsection.

Correct Stand Assignment Constraint

Once again, it is paramount to guarantee a correct use of stands which avoids, for instance, that one operation is assigned to several stands. This constraint is shown in Equation 1.62. The major difference between this constraint and the one defined for the first-level is that this one does not include a dummy stand in the summation. In the second level, all operations of each zone must be assigned to a real stand. This assumes that there is always space for every operation, which is possible to guarantee since zone capacity was taken into account during the zone assignment. If there would be no space inside a zone for a certain operation, it would either be assigned to a different zone or to the dummy zone. Thus, there is no risk of infeasible problems.

$$\sum_{k \in K_z} \eta_{ik} = 1, \quad \forall i \in N_v \quad (1.62)$$

Towing Constraints

In the individual stand assignment, the towing constraints have several restrictions and they need special attention. The number of towing movements to take place during the day of operations were obtained in the zone assignment, and that number cannot be changed in the second level. If an operation is not towed, as determined by the zone assignment, then it can never be towed in the individual stand assignment. The set $N_v^{not_towed}$ includes all operations that verify the condition above and, in this case, Equation 1.63 is used.

$$\eta_{ik} - \eta_{i_{next}k} = 0, \quad \forall i \in N_v^{not_towed}, \quad \forall k \in Z_v, \quad \text{if } i_{next} \in N_v \quad (1.63)$$

If in the first level it is determined that an operation is towed to a different stand type of the same zone, then it is allowed to effectively move inside the zone. In this case, Equation 1.64 is used. The set N_v^{towed} contains the operations that verify this condition.

$$\eta_{ik} - \eta_{i_{next}k} \leq 1, \quad \forall i \in N_v^{towed}, \quad \forall k \in Z_v, \quad \text{if } i_{next} \in N_v \quad (1.64)$$

When an operation i and its succeeding operation i_{next} are assigned to different zones, it necessarily means that i was towed. However, from the second level's model point of view this tow movement does not exist, since in this level each zone is analysed individually and independently from the others. None of the equations shown above can be applied in this scenario (notice that both of them are conditioned by the statement $\text{if } i_{next} \in N_v$) because the variables η_{ik} and $\eta_{i_{next}k}$ do not coexist in the same sub-problem.

Overlapping Operations Constraint

This constraint is the equivalent to the zone stand capacity constraint of the first-level (Equation 1.40), except now only one stand is considered at a time. Thus, the "capacity" is reduced to 1. The assignment variables of each pair of operations i and j that overlap are added and the sum cannot exceed one (here, the concept of overlapping operation follows the same definition as in the first-level, refer to subsection 1.4.4 for more details).

The overlapping constraint is depicted in Equation 1.65. Note how the set of i 's overlapping operations, N_{ov}^i , is intersected with the set of operations assigned to z , N_z . This is done for two reasons. First, the operations j that are not assigned to z will never physically overlap with i . Secondly, those operations do not have the corresponding η_{jk} variables defined, so they cannot be included in the constraint.

$$\eta_{ik} + \eta_{jk} \leq 1, \quad \forall i \in N_v, \forall j \in N_{ov}^i \cap N_z \quad (1.65)$$

Similarly to the first level of assignment, the Overlapping Operations Constraint is also extended to include the buffer period $buffer_i$ of each operation i . Recall the explanation in subsection 1.4.4 (Zone Capacity Constraint) on how the buffer time is considered in the overlapping operations.

MARS Constraint

The MARS constraint is equivalent to the zone MARS capacity constraint defined on the first-level (refer to subsection 1.4.4). The constraint is written in Equation 1.66. The first term considers the small aircraft operations that can be assigned to one of the two small stands k' contained in the MARS group, and it is multiplied by a factor of 1. The second term considers large aircraft that might be assigned to the large stand of the MARS group, and it is multiplied by a factor of 2. Each term is multiplied by a different factor to take into account the dynamics of a MARS group. Please, refer again to subsection 1.4.4 for a detailed explanation on the usage of factors.

The set of i 's overlapping operations is again intersected with the set of operations assigned to v , N_v , and for the same reason as explained in the previous constraint. Also, and similarly to what was explained in the Zone MARS Capacity Constraint (subsection 1.4.4), the subsets $N_{ov,small}^{i,inc}$ and $N_{ov,large}^{i,inc}$ were created. This adds operation i to the correct subset, which means it will be added to the correct term of Equation 1.66: either the first if it is a small aircraft operation, or the second if it is a large aircraft operation.

$$\sum_{k' \in K_{blocks}^k} \sum_{j \in N_{ov,small}^{i,inc} \cap N_v} \eta_{jk'} + 2 \sum_{j \in N_{ov,large}^{i,inc} \cap N_v} \eta_{jk} \leq 2, \quad \forall i \in N_v, \forall k \in K_{MARS}^v \quad (1.66)$$

The MARS Constraint also considers the buffer period $buffer_i$ added before each operation i by following the same logic and procedures explained previously in the Overlapping Operations Constraint and also in subsection 1.4.4 (Zone Capacity Constraint).

Variable Bounds Constraint

The only variable defined for each second-level sub-problem represents an operation assignment to a stand and it is binary, thus it can only assume either the value 0 or 1 (Equation 1.67).

$$\eta_{ik} \in \{0, 1\}, \quad \forall i \in N_v, \forall k \in K_v \quad (1.67)$$

1.6. Remarks and General Framework of the Strategic Stand Assignment Problem

In the final section of the chapter, some important remarks with respect to the model are presented. Finally, the framework of the Strategic Stand Assignment Problem depicted in Figure 1.11 summarizes the whole chapter, and it contains an indication about the sections the reader should refer to in order to read about a particular aspect.

- It was explained in subsection 1.5.4 the reason why ϵ_{ivt} variables are useful as a complement to y_{iv} variables, especially to write some constraints. A closer analysis to these types of variables reveals that it is possible to eliminate all y_{iv} variables and replace them with ϵ_{ivt} variables. Some constraints need to be re-written but it is still feasible. Thus, one might argue this replacement is the best option since it reduces the overall number of variables. However, the number of z_{ijvw} would in fact increase because there would be more possible combinations. The connection combinations would not be of the type zone-zone anymore, but rather stand_type-stand_type. This goes against the idea of zone assignment, which was implemented in the first place precisely to reduce the number of z_{ijvw} variables. In the end, making the replacement would in fact make the problem much slower. Keeping ϵ_{ivt} allows more detailed capacity constraints to be written, but without interfering with the problem size since connections can be established between zones with the use of y_{iv} variables.

- One important consequence of performing the pre-processing described in [subsection 1.2.1](#) is that some constraints are already being checked, so they do not need to be explicitly included in the model anymore. This helps to make the problem smaller and faster. Notice, for instance, that when an operation i is not compatible with a zone v its corresponding variable y_{iv} is not included in the model. The same is true for hub connections, since in the pre-processing a certain variable is not added if a connection cannot happen due to lack of time.
- Even though this model was applied to a single case study airport, the general framework, shown in [Figure 1.11](#), can be adapted to any airport. In that case, it is of extreme importance to first carefully define the zones of the airport. Then, extra measures or specific rules of that airport can also be added as constraints or penalties in the objective function

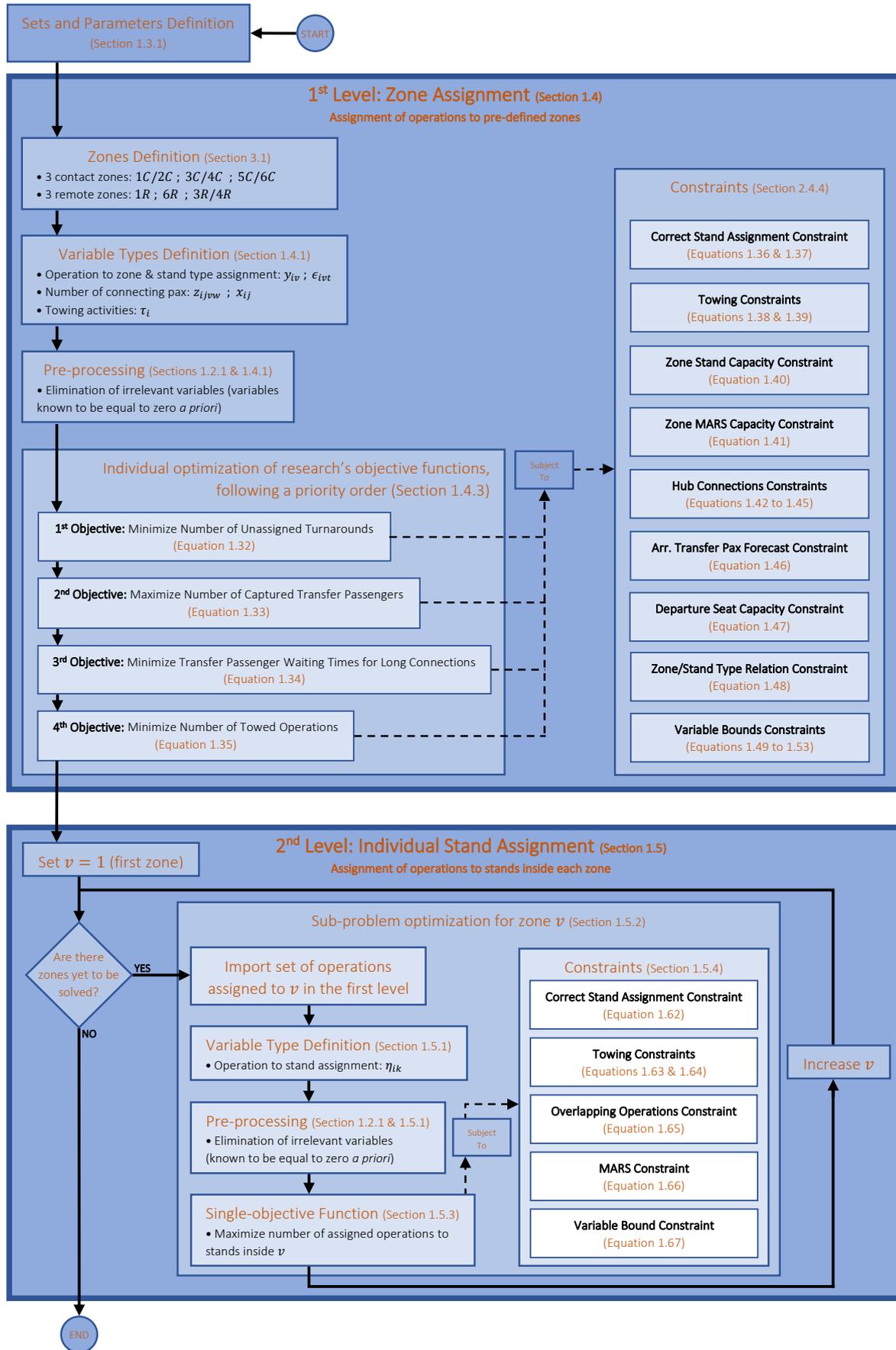


Figure 1.11: Complete flow diagram showing the framework of the TLSAP.

2

Verification and Validation of The Two-Level Stand Planning Model

This chapter cover the verification of the TLSAP in [section 2.1](#). It is important to understand whether all elements of the model are correctly implemented (objective functions and constraints). To achieve this, several unit tests are run for all elements separately in [subsection 2.1.1](#) and in [subsection 2.1.2](#) an integrated test including all OFs and constraints is run in order to verify the system as a whole. In [subsection 2.1.3](#), the transfer passenger forecast algorithm is verified.

At the end of this chapter, in [section 2.2](#), the values chosen for the parameters used in the model and the overall results obtained are validated, in order to confirm that this model delivers correct and realistic results.

2.1. Verification of the TLSAP

The verification of the TLSAP is divided into two parts. In the first one, each element of the model will be analysed individually, while in the second part a system test integrating all elements is performed in order to verify the global correctness of the model.

2.1.1. Objective Functions and Constraints Verification: Individual Tests

Along this section, each element of the TLSAP (OFs and constraints) is verified using unit tests. A unit test is an experiment based on a simple, controlled input and from which a certain output is expected. The input is carefully chosen so that it is possible to analyse the behaviour of the element being tested. If the output matches the expected result, that particular element is considered verified.

When verifying a certain element, it is important to remove other elements from the model, as far as possible. This allows to perform a more specific test to that particular element without the potential interference of others and, consequently, the result is more reliable. However, some elements of the model need others to be properly verified. For instance, it is not possible to test the zone capacity constraint without first introducing the correct zone assignment constraint, since without the latter the model is able to simply not assign some operations to any zone, or assign other operations multiple times. For this reason, important elements will be tested first, so that they can be verified and safely used in the unit tests of others.

It is also important to mention that, for some elements, more than one unit test is performed in order to check different input scenarios and ultimately give a more robust verification.

Some constraints are present in the first level and in the second level of assignment. For these cases, a unit test is suggested for the first level, and the corresponding outcome is used to verify the constraint also on the first level. However, the same outcome is directly used as the input for the second level (similarly to what happens with the complete model) and the final output is used to verify the constraint on the second level.

Finally, for each turnaround Tn , An represents its arrival, Dn represents its departure and In relates to its idle period, if it exists.

Test 1.1.: Correct Zone Assignment Constraint: Single Zone Assignment

The first constraint to be verified is the correct stand assignment constraint. It states that each operations must be assigned to one and only one zone, including the "dummy zone". This constraint is the base for any

SAP problem, and for that reason it is the first to be introduced. The input data for this unit test is shown in Table 2.1 and it consists of three turnarounds that do not overlap in time and two zones (A and B) with a certain stand capacity. However, zone capacity is not relevant for this particular test, since this will be verified later. The walking times between the two zones is also showed but not used for now.

Turnaround	Arrival Flight	Arrival Time	Departure Flight	Departure Time	ICAO design group	Transfer Pax Arriving	Zone	C-stand Capacity	E-stand Capacity	MARS-stand Capacity	Time to Zone_A [min]	Time to Zone_B [min]
T1	XX 110	07:30	XX 111	08:30	C (738)	75	Zone_A	1	1	0	40	60
T2	XX 112	08:40	XX 113	11:50	C (738)	90	Zone_B	1	0	0	60	40
T3	YY 114	12:00	YY 115	15:10	E (789)	100						
T4	YY 116	15:20	YY 117	19:30	E (789)	100						

Table 2.1: Turnarounds and Zones information, used in Unit Test 1.1.

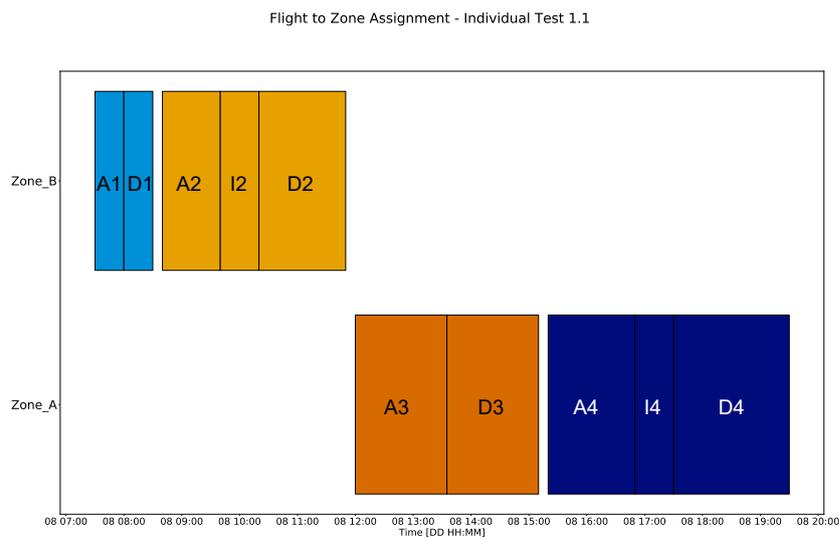


Figure 2.1: Assignment of operations to zones for Unit Test 1.1.

The resulting allocation for this unit test is shown in Figure 2.1. The picture clearly shows that each of operation from each turnaround is assigned to one and only one zone. The Correct Zone Assignment Constraint is thus verified.

From this verification, other conclusions can be drawn. The first is that the pre-processing algorithm that eliminates assignment variables when an operation is incompatible with a zone is working. Note how turnarounds T3 and T4 had to be assigned to Zone A since only this zone contains an E-type stand. If neither of the zones had an E-type stand, the problem would be infeasible because T3 and T4 would not have any assignment variables.

Figure 2.1 also allows to verify that the division between short and long connections is working, according to Table 3.3. Recall that small aircraft (type C or smaller) can be towed if their ground time is larger or equal than 180 minutes (3 hours), while large aircraft (type E or larger) can be towed when the ground time is larger or equal than 240 minutes (4 hours). When a turnaround can be towed, it is divided into three operations. Turnaround T1 (type C) stays on the ground for 60, so it is short and it is only divided into 2 operations. T2 (type C) stays on ground for 190 minutes, which means it is long and it is divided into 3 operations. T3 (type E) also stays on ground 190 minutes, but since the aircraft is large, the rules do not allow it to be towed, so it is only divided into 2 operations. Finally, T4 is on the ground for 250 minutes, allowing it to be towed and divided into 3 operations.

Test 2.1.: Correct Stand Assignment Constraint: Single Stand Assignment

According to what was explained in the beginning of this section, some constraints are present in the first and second levels of operation. This is the case with the Correct Assignment Constraint. Since it was already verified for the first level in Test 1.1., the output from that test is used now as input to test the constraint in the second level. The flight schedule is again shown in Table 2.1. The stands belonging to each zone are shown in

Stand	Zone	Stand Size	Time to Zone Entrance [min]
A.01	A	C	2
A.02	A	E	5
B.01	B	C	2

Table 2.2: Stands information, used in Unit Test 2.1.

Table 2.2. In the same Table, the distance of each stand to the corresponding zone entrance is also presented, but this information is not needed for this unit test.

The output of this test is shown in Figure 2.2. Each operation was assigned to one and only one stand, which verifies the Correct Stand Assignment Constraint. Similarly to the first level, the effects of the pre-processing algorithm are visible here. The two E-type turnarounds, T3 and T4, were assigned to the only E-type stand, A.02, since only this assignment variable was created for these two turnarounds.

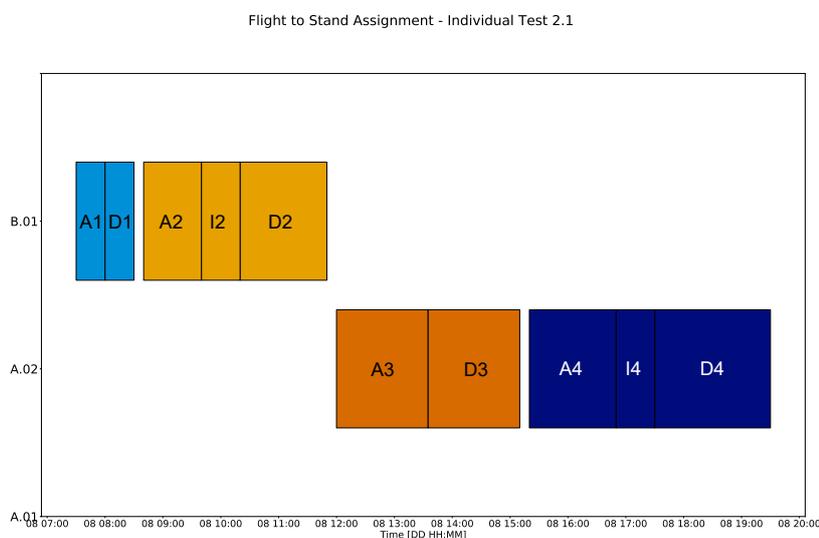


Figure 2.2: Assignment of operations to stands for Unit Test 2.1.

Test 1.2.: Zone/Stand Type Relation Constraint

The Zone/Stand Type Relation Constraint relates the y_{iv} and ϵ_{ivt} variables and it states that, when an operation is assigned to a zone, it has to be assigned to one and only one stand type inside that zone. If an operation is not assigned to a zone, it cannot be assigned to any stand type of that zone. Both variable types are important to the model definition, and their relationship is just as important to make the model consistent. This relationship is what allows to properly write the zone capacity constraints.

The input used for this unit test is the same as for Test 1.1., so it is shown in Table 2.1. The y_{iv} and ϵ_{ivt} variable values for all operations before the constraint is introduced is shown in Table 2.3.

Operation i	A1	D1	A2	I2	D2	A3	D3	A4	I4	D4
$y_{i,Zone_A}$	0	0	0	0	0	1	1	1	1	1
$y_{i,Zone_B}$	1	1	1	1	1	0	0	0	0	0
$\epsilon_{i,Zone_{A,C}}$	0	0	0	0	0	0	0	0	0	0
$\epsilon_{i,Zone_{A,E}}$	0	0	0	0	0	0	0	0	0	0
$\epsilon_{i,Zone_{B,C}}$	0	0	0	0	0	0	0	0	0	0

Table 2.3: Variable values for all operations before the Zone/Stand Type Relation Constraint is activated.

The result shown in Table 2.3 is illogical in the context of this problem. For every operation used in this unit test, *Gurobi* sets all ϵ_{ivt} variables to zero. For the model, this means that the operations are assigned to

one zone but they are not assigned to any stand type inside that zone, which is impossible. For instance, for arrival A1, the variables show that this operation is assigned to zone B (which is compatible with the result from Test 1.1), but, inside zone B, it is not assigned to any stand type. In this case, the variable $\epsilon_{A1,Zone_B,C}$ should be equal to 1. When the constraint is introduced, the results change to what is presented in Table 2.4.

Operation i	A1	D1	A2	I2	D2	A3	D3	A4	I4	D4
$y_{i,Zone_A}$	0	0	0	0	0	1	1	1	1	1
$y_{i,Zone_B}$	1	1	1	1	1	0	0	0	0	0
$\epsilon_{i,Zone_A,C}$	0	0	0	0	0	0	0	0	0	0
$\epsilon_{i,Zone_A,E}$	0	0	0	0	0	1	1	1	1	1
$\epsilon_{i,Zone_B,C}$	1	1	1	1	1	0	0	0	0	0

Table 2.4: Variable values for all operations after the Zone/Stand Type Relation Constraint is activated.

When the constraint is introduced, each operation is assigned to a zone and one and only one stand type inside that zone. For instance, arrival A1 is assigned to zone B and to a stand of type C belonging to zone B. This was the expected output and it verifies that the Zone/Stand Type Relation Constraint is properly implemented. The pre-processing algorithm is also verified again here, but now for the ϵ_{ivt} variables. Note that no constraints that block the E-type turnarounds from being assigned to a type-C stand were introduced. Nonetheless, both T3 and T4 are assigned to a stand of type E inside zone A. This is not a coincidence. but rather the result of eliminating beforehand all other epsilon variables for these two turnarounds.

Test 1.3.: Zone Capacity Constraint

The Zone Capacity Constraint is fundamental for every SAP problem, since it takes into consideration the number of available stands at any time at the airport and prevents several aircraft simultaneously present at the airport from being assigned to the same stand which, in reality, is impossible to happen. The correct implementation of the buffer time will also be tested together with the Zone Capacity Constraint.

The flight schedule used for this unit test is shown in Table 2.5. T1 and T2 overlap in time, so they should not be assigned to the same stand. T3 does not overlap in time with T1 nor T2, but since a buffer time of 5 minutes is added before each operation, T3 should not be assigned to the same stand (or even the same zone in some cases) as T1 and T2. Finally, T4 does not overlap with T1 and T2 but it does with T3. Zone_A has a capacity of 1 C stand and 1 E stand, while Zone_B has a capacity of a single C stand.

Turnaround	Arrival Flight	Arrival Time	Departure Flight	Departure Time	ICAO design group	Transfer Pax Arriving	Zone	C-stand Capacity	E-stand Capacity	MARS-stand Capacity	Time to Zone_A [min]	Time to Zone_B [min]
T1	YY 114	08:30	YY 115	11:00	C (738)	50	Zone_A	1	1	0	40	60
T2	XX 110	09:00	XX 111	11:00	C (738)	70	Zone_B	1	0	0	60	40
T3	XX 112	11:00	XX 113	12:00	C (738)	70						
T4	XX 114	11:30	XX 115	12:40	E (789)	100						

Table 2.5: Turnarounds and Zones information, used in Unit Test 1.3.

The output of this unit test is shown in Figure 2.3. For the convenience of the reader, this Figure does not show the operations to zone assignment as before, but rather the distribution of operations over time. Inside each operation, the assignment of that operation is portrayed in the form: Zone/Stand Type.

Note that the operations belonging to T1, T2 and T3 can be assigned to any of the 3 available stands in this unit test. Operations A1 and D1 were assigned to a stand of type E inside Zone_A. On the other hand, A2 was assigned to a stand of type C in Zone_A, but D2 was moved to Zone_B. In theory, arrival A3 could have been assigned to any of the three stands, but because a buffer time of 5 minutes is being considered, it could not be assigned to either the E stand from Zone_A nor the C stand from Zone_B, since those were being used by D1 and D2. Instead, A3 was forced to be assigned to the only remaining stand (the C stand in Zone_A). D3 remained in a stand C of Zone_A. Finally, the whole turnaround T4 had to be assigned the stand E from Zone_A, due to the aircraft size. This stand is available at the starting time of A4, so this assignment is possible.

This simple set of operations and the explanation above show that the zone capacity is correctly verified by the constraint. Besides, adding the buffer time produces the predicted effect. Overall, unit Test 1.3 confirmed the correctness of the Zone Capacity Constraint.

Finally, operation A2 is towed to different zones or stand types. With the full model working properly, this could not happen since turnaround T2 is short and thus the airport's rules do not allow the corresponding

Flight Schedule - Individual Test 1.3

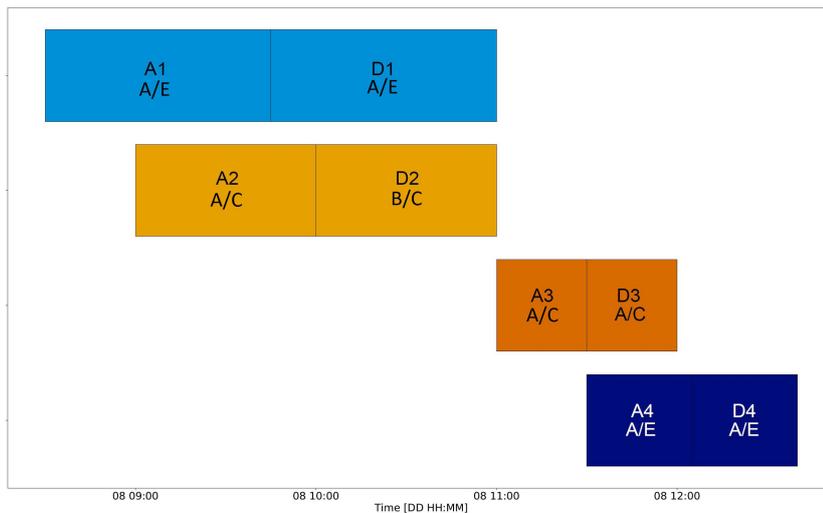


Figure 2.3: Flight schedule for Unit Test 1.3. The assignment results of each operation are showed inside the corresponding bar in the following way: Zone/Stand Type.

aircraft to be towed. However, since the towing constraints were not introduced yet, this is not an unexpected behaviour and does not interfere with the verification of the Zone Capacity Constraint.

Test 2.2.: Overlapping Constraint

The Overlapping Constraint is the second level equivalent to the Zone Capacity Constraint. While in the first level the capacity of each zone could vary, in the second level, the "capacity" of each stand is simply 1. This fact allows to reduce the capacity constraint to a regular overlapping constraint present in most SAP problems. Since this constraint is related to the Overlapping Constraint, the output from Test 1.3 will be used as the input to this test. Thus, the flight schedule can be recalled in Table 2.5, while the stands information can be seen in Table 2.6. Note that the walking times to the zones entrances are not relevant for this particular unit test.

In this unit test, it expected that turnarounds T1 and T2 are placed in different stands, since their operations overlap (A1 with A2, and D1 with A and D2). Arrival A3 should be placed in a different stand than departures D1 and D2. Furthermore, turnaround T4 should be assigned to the only E-type stand, A.02, as a consequence of that aircraft's size, and it should not be assigned to the same stand as T3 since these two turnarounds overlap in time.

Stand	Zone	Stand Size	Time to Zone Entrance [min]
A.01	A	C	2
A.02	A	E	5
B.01	B	C	2

Table 2.6: Stands information, used in Unit Test 2.2.

Figure 2.4 contains the output of Test 2.2. First, it should be noted that the overlapping operations A1 and A2 are placed in different stands, and the same is true for A2 and D1 and D1 and D2. Recall that operation A3 does not overlap with T1 and T2, but because a buffer time of 5 minutes is added before each operations starts, A3 cannot be assigned to the same stand as D1 and D2. For that reason, it is placed in the only available stand, A.01. D3 is also assigned to stand A.01. The last turnaround, T4, is assigned to A.02, the only E-type stand (once again showing that the pre-processing algorithm is working correctly, since the only available stand for T4 the entire day is A.02).

Some operations were towed, even though that cannot happen in reality. Similarly to test 1.3, this is the case for now since the towing constraints are not active yet.

Flight to Stand Assignment - Individual Test 2.2

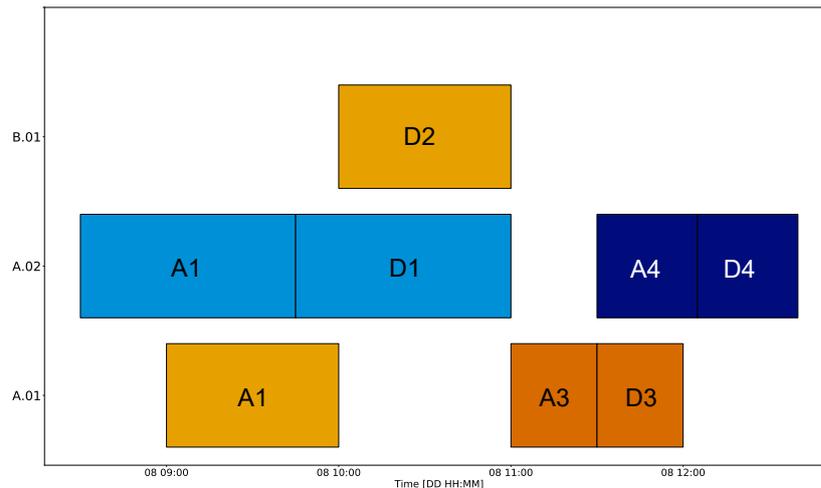


Figure 2.4: Assignment of operations to stands for Unit Test 2.2.

It is relevant to note that, even though in this case the stand type assignment made in the first level matches exactly with the specific stand assignment in the second level, this is not forced by the model. The specific stand type assignment of the first level is not transmitted to the second level, but only the zone assignment. The stand type assignment is only needed during the first level to verify whether or not there is enough space inside a zone. This means that, if a solution is found in the first level, at least that solution exists in the second level. This does not exclude scenarios in which there are multiple solutions, which is the case in this example. Consequently, the specific stand type assignment is indeed irrelevant for the second level. In fact, this gives more flexibility to the second level during the minimization of walking times from aircraft to the zones entrance (this objective is verified later in this section).

The constraints tested up until now are the base of the TLSAP. For this reason, they will be active during all unit tests performed until the end of this section.

Test 1.4.: Maximization of Captured Transfer Passengers Objective

The first objective to be verified is the Maximization of Captured Transfer Passengers. For this unit test, the flight schedule is shown in Table 2.7. There is one flight arriving from and then departing to LKO, and four other flights arriving from and then departing to FRA. Since T2, T3, T4 and T5 all fly the same route, there will not be any transfer passenger demand between them. With this flight schedule, the only demand possible happens from the arrival A1 to departures D2, D3, D4 and D5. For this test, it was defined that the minimum connection equals 40min, the maximum critical connection is 3h long, while the longest possible connection lasts 7h. Thus, it is expected that a critical connection is established between A1 and all the flights departing within 40min and 3h from A1, and if passengers have enough time to reach those departures. At the same time, a non-critical connection is expected between A1 and all flights departing within 3h and 7h from A1.

Two zones are used for this unit test. Zone_A has a capacity of 2 C-type stands, while zone_B has the capacity of 1 E-type stand. It is also important to mention that the walking times within the same zone is defined as 40min (minimum connection time) and the inter-zone walking times is 180min or 3h (maximum critical connection time). These values are not realistic but are used here in order to obtain better results from the test. Finally, and to keep the consistency of the model, all constraints tested until this point will remain active.

Turnaround	Arrival Flight	Arrival Time	Arrival Airport IATA	Departure Flight	Departure Time	Departure Airport IATA	ICAO design group	Transfer Pax Arriving	Zone	C-stand Capacity	E-stand Capacity	MARS-stand Capacity	Time to Zone_A [min]	Time to Zone_B [min]
T1	XX 110	8:30	LKO	XX 111	09:30	LKO	C	80	Zone_A	2	0	0	40	180
T2	XX 112	10:00	FRA	XX 113	11:00	FRA	C	70	Zone_B	0	1	0	180	40
T3	XX 114	10:10	FRA	XX 115	11:20	FRA	C	70						
T4	YY 110	10:20	FRA	YY 111	11:00	FRA	E	120						
T5	YY 112	11:10	FRA	YY 113	12:20	FRA	E	110						

Table 2.7: Turnarounds and Zones information, used in Unit Test 1.4.

After running Test 1.4, the output was analysed. The output can be used to verify this objective, but without the introduction of other constraints (showed in the next tests), it has no meaning nor applicability. First of all, since there is still no relationship between z_{ijvw} and y_{iv} variables, the model is not able to recognize that there can only be a critical connection between flight i , assigned to zone v , and flight j , assigned to zone W if both flights are individually assigned to those zones. Consequently, and because the model is maximizing transfer passengers, *Gurobi* establishes all possible connections, as Table 2.8 shows. For instance, the output recognizes two critical connections from A1 to D2: i) when both are assigned to Zone_A and ii) when both are assigned to Zone_B. In reality, this is not possible since each operation is only assigned to one and only one zone. According to the output from this unit test, only case i) should exist since both A1 and D2 are assigned to Zone_A.

Connection Variable	Value
$Z_{A1,D2,Zone_A,Zone_A}$	∞
$Z_{A1,D2,Zone_B,Zone_B}$	∞
$Z_{A1,D3,Zone_A,Zone_A}$	∞
$Z_{A1,D3,Zone_B,Zone_B}$	∞
$Z_{A1,D4,Zone_B,Zone_B}$	∞
$X_{A1,D5}$	∞

Table 2.8: Table containing all connection variables created in Test 1.4, and the final value attributed to each.

Besides, since the model is maximizing the number of captured transfer passengers and there are no restrictions to aircraft capacities yet, *Gurobi* assigns an infinite value to all connection variables (z_{ijvw} and x_{ij}).

It is important to mention that variables of the type $Z(i, j, Zone_A, Zone_B)$, which represent a connection from any arrival i to any departure j , from Zone_A to Zone_B were not created. This proves that the pre-processing algorithm is also being effective here. Since there cannot be critical connections from Zone_A to Zone_B due to lack of time, the corresponding variables are not added to the model, which simplifies the model.

Even though this unit test is not entirely satisfactory, it is still enough to confirm that the Maximization of Captured Transfer Passengers Objective is indeed maximizing the number of captured transfer passengers. This objective will be further verified as some extra constraints, namely the relationship between z_{ijvw} and y_{iv} , the aircraft capacity and transfer passenger demand, will be introduced in the next sections.

Test 1.5.: Hub Connections Constraints: Relationship between z_{ijvw} and y_{iv} variables

In Test 1.4, the Maximization of Captured Transfer Passengers Objective was verified. However, the output from that test does not have any applicability nor real meaning, since some important constraints are needed. In Test 1.5, the constraint that establishes the relationship between z_{ijvw} and y_{iv} variables is verified and incorporated with the previous objective function. Since Test 1.4 and Test 1.5 are so closely related, the same input, shown in Table 2.7 is used for both.

It is expected that the output of this unit test will recognize that there can only be a critical connection between flight i , assigned to zone v , and flight j , assigned to zone W if both flights are individually assigned to those zones. In other words, a critical connection from i cannot happen from different zones, and a critical connection to j cannot happen to different zones, since every i and j flights are always assigned to one and only one zone.

The output of Test 1.5 is shown in Figure 2.5. The Figure portrays the operations distribution over the day, the connections established (with red arrows) and the assignment of operations to zones and stand types, in the form: Zone/Stand Type.

The results from Figure 2.5 show that a critical connection is established between A1 and D2, which is correct since the time between arrival A1 and departure D2 is 2h30min and the walking time between stands within Zone_A is only 40 minutes. Another critical connection is established between A1 and D3. This is also expected, since the time between arrival A1 and departure D3 is 2h50min. On the other hand, it is not possible to establish a critical connection between A1 and D4. First of all, the turnaround T4 has to be assigned to Zone_B due to the size of the aircraft. This means that passengers need at least 180min to walk from Zone_A

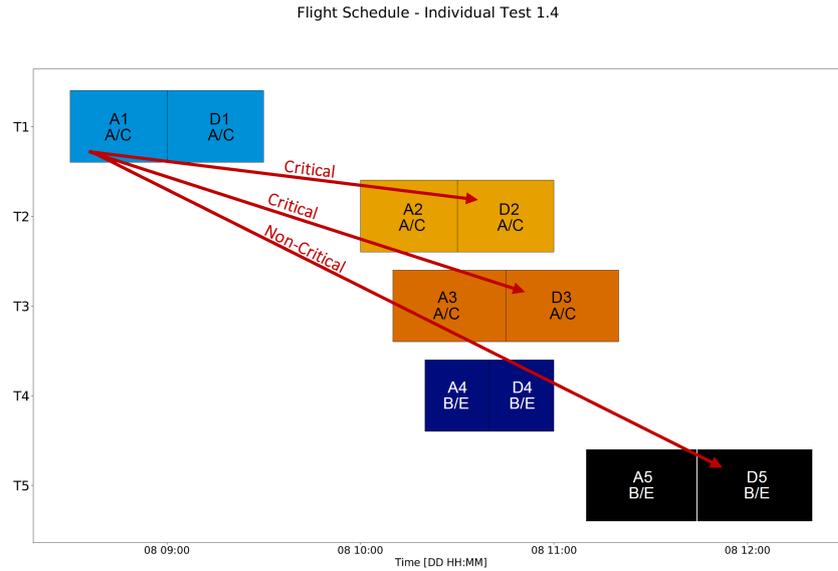


Figure 2.5: Flight Schedule for Individual Test 1.4.

to Zone_B, but in fact they only have 2h30min, so that connection is not feasible. Finally, turnaround T5 has to be assigned to Zone_B, also as a consequence of the aircraft size. Nonetheless, the departure D5 takes place 3h50min after A1 arrives at the airport. This period of time is greater than the longest time passengers have to walk inside the airport to reach any zone. As a consequence, the connection from A1 to D5 is non-critical and it is activated by the model.

The connection variables values are shown in Table 2.9.

Connection Variable	Value
$Z_{A1,D2,Zone_A,Zone_A}$	500
$Z_{A1,D2,Zone_B,Zone_B}$	0
$Z_{A1,D3,Zone_A,Zone_A}$	500
$Z_{A1,D3,Zone_B,Zone_B}$	0
$Z_{A1,D4,Zone_B,Zone_B}$	0
$X_{A1,D5}$	500

Table 2.9: Table containing all connection variables created in Test 1.5, and the final value attributed to each.

Unlike Test 1.4, some connection variables are now equal to 0. An analysis to the Table shows that those values are consistent with the zones where each operation is assigned. For instance, the first variable of the Table is not 0 since both A1 and D2 are assigned to Zone_A. As a consequence, the second variable of the Table has to be 0 because neither A1 nor D2 are assigned to Zone_B. Following the same logic, the fourth variable can be greater than 0 and the fifth variable of Table 2.9 has to be equal to 0 since A1 was not assigned to Zone_B. Finally, the last connection variable of the Table can be greater than 0 since it represents a non-critical connection.

This analysis shows that the model is still maximizing transfer passengers, but now the connections are consistent with the assignment of each operation. This not only shows that the Hub Connections Constraint that establishes the relationship between z_{ijvw} and y_{iv} is correctly implemented but also how important this constraint is for the robustness of the model.

One should note that the value 500 attributed to some connection variables in Table 2.9 is a consequence of how the big-M method is used in this constraint. The value for M was set to 500, and since there are no aircraft capacity constraints yet, **Gurobi** attributes the maximum possible value to those connection variables in order to maximize the number of captured transfer passengers.

Similarly to the Hub Connections Constraint that establishes the relationship between z_{ijvw} and y_{iv} , a

constraint that establishes the relationship between x_{ij} and y_{iv} variables is used in this model. However, it will be verified at a later stage of this verification process, since it is not important to have that constraint active for now.

Test 1.6.: Arrivals Transfer Passenger Capacity Constraint

Up until now, the transfer passenger demand arriving from a flight was not taken into consideration. In Test 1.6, the Arrivals Transfer Passenger Capacity Constraint is tested and verified. Since this unit test is still closely related to Test 1.4, the same flight schedule, shown in Table 2.7, is reused here. According to that Table, arrival A1 has 80 transfer passengers wanting to connect to other flights. Since only one destination airport, FRA, is considered here, all 80 passengers will connect to flights departing to that airport. In other words, it is expected that those passengers are assigned to connections with either D2, D3, D4 or D5 or any combination of these flights. The results are shown in Table 2.10.

Connection Variable	Value
$Z_{A1,D2,Zone_A,Zone_A}$	0
$Z_{A1,D2,Zone_B,Zone_B}$	0
$Z_{A1,D3,Zone_A,Zone_A}$	0
$Z_{A1,D3,Zone_B,Zone_B}$	0
$Z_{A1,D4,Zone_B,Zone_B}$	0
$X_{A1,D5}$	80

Table 2.10: Table containing all connection variables created in Test 1.6, and the final value attributed to each.

The results from Table 2.10 show the expected output. The sum of all passengers departing from A1 to the other arrivals equals 80, which corresponds to the transfer passenger demand given as the input to the model. Coincidentally, all those passengers were allocated to the same departure flight D5. This shows that the Arrivals Transfer Passenger Capacity Constraint is correctly implemented.

Test 1.7.: Departures Seat Capacity Constraint

Besides taking into account the transfer passenger forecast for the arrivals, it is also important to guarantee that each departure is not assigned more passengers than the maximum seat capacity of the corresponding aircraft. This is why the Departures Seat Capacity Constraint is important. This constraint is verified through Test 1.7. The Departures Seat Capacity Constraint is closely related to the elements verified in Test 1.4, 1.5 and 1.6, and these elements are activated for Test 1.7. Nonetheless, the flight schedule and the zone/stand configuration used now is different from previous tests. This is presented in Table 2.11. There are now 3 turnarounds: T1 arriving from and departing to LKO and T2 and T3 arriving from and departing to FRA. Since T2 and T3 fly the same route, there will not be any transfer passenger demand between them. Consequently, there can only be a connection from A1 to D2 and D3. With respect to the terminal configuration, this test makes use of a single zone, Zone_A, with 4 C-type stands. The 200 passengers wanting to transfer from A1 are expected to be assigned to both D and D3, since neither of those aircraft has enough seat capacity to fit all those passengers.

Turnaround	Arrival Flight	Arrival Time	Arrival Airport IATA	Departure Flight	Departure Time	Departure Airport IATA	ICAO design group	Transfer Pax Arriving	Seat Capacity	Zone	C-stand Capacity	E-stand Capacity	MARS-stand Capacity
T1	XX 110	8:30	LKO	XX 111	09:30	LKO	E (789)	200	281	Zone_A	1	1	0
T2	XX 112	10:00	FRA	XX 113	11:00	FRA	C (738)	60	162				
T3	XX 114	10:10	FRA	XX 115	11:20	FRA	C (738)	70	162				

Table 2.11: Turnarounds and Zones information, used in Unit Test 1.7.

The output is shown in Figure 2.6. The zone assignment is not as relevant in this test since only one zone is considered, so the Figure only portrays the individual stand assignment. The connections established are marked with red arrows and the number of passengers making each connection is also shown close to the corresponding arrow.

The results show once again that the Arrivals Transfer Passenger Capacity Constraint is correctly implemented, since a maximum of 200 passengers transferred from A1. It also shows that only 162 out of the 200

Flight to Stand Assignment - Individual Test 1.7

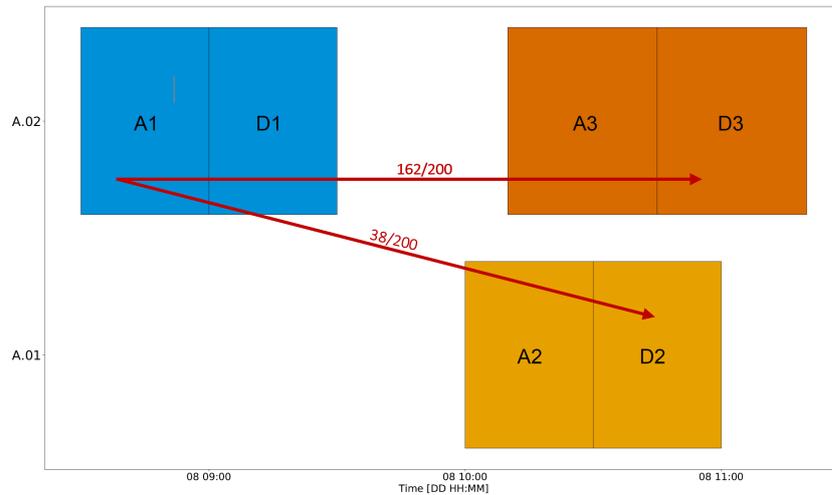


Figure 2.6: Assignment of operations to stands for Unit Test 1.7.

passengers connected from A1 to D3. The remaining transfer passengers were allocated to D2. This happened because the seat capacity of D3 is exactly 162. Consequently, the Departures Seat Capacity Constraint is verified with Test 1.7.

Test 2.3.: Minimization of Walking Times to Zone Entrance Objective

The Minimization of Walking Times to Zone Entrance Objective is considered in the second level of assignment. For passenger convenience, it is important to keep operations as close as possible to the entrance of zones. This is particularly important when passengers need to transfer between zones inside the terminal of the airport. It is also convenient for O&D passengers, since, for these passengers, reaching the stand or the airport's exit will take less time. For this unit test, only the constraints verified in Tests 1.1, 1.2, 1.3, 2.1 and 2.2 (the fundamental constraints of the Two Level Stand Planning Model) are used. The flight schedule and the zone configuration are shown in Table 2.12. Note that only one zone is defined in this case, but it has a capacity of 4 C-type stands. Furthermore, all aircraft belong to the ICAO design group C.

Turnaround	Arrival Flight	Arrival Time	Departure Flight	Departure Time	ICAO design group	Transfer Pax Arriving	Zone	C-stand Capacity	E-stand Capacity	MARS-stand Capacity
T1	XX 110	9:50	XX 111	10:40	C (738)	50	Zone_A	4	0	0
T2	XX 112	10:50	XX 113	12:40	C (738)	50				
T3	XX 114	11:30	XX 115	12:30	C (738)	70				
T4	YY 110	12:20	YY 111	14:00	C (738)	70				
T5	YY 112	13:00	YY 113	14:30	C (738)	90				

Table 2.12: Turnarounds and Zones information, used in Unit Test 2.3.

The information regarding each of the 4 stands inside Zone_A are presented in Table 2.13. The time it takes to walk from each stand to the entrance of the zone is shown on the last column of the Table and it will influence the outcome of this unit test. Since stand A.01 is the closest stand, it is expected that a large number of operations are assigned there. Then, a decreasing number of operations are expected to be assigned to A.02, A.03 and A.04.

The outcome obtained with this unit test is shown in Figure 2.7. This Figure shows the assignment of operations to the stands of Zone_A.

The results in Figure 2.7 perfectly match the predictions made. 6 operations are assigned to the closest stand from the entrance, A.01. No more operations are assigned to this stand because of overlapping conflicts. The second most used stand, A.02, is also the second closest to the zone entrance. Then, stand A.03 has 1 operations assigned to it and finally, no operations are assigned to the farthest stand from the entrance, A.04. This verifies the correct implementation of Minimization of Walking Times to Zone Entrance Objective.

Stand	Zone	Stand Size	Time to Zone entrance [min]
A.01	A	C	5
A.02	A	C	10
A.03	A	C	15
A.04	A	C	20

Table 2.13: Stands information, used in Unit Test 2.3.

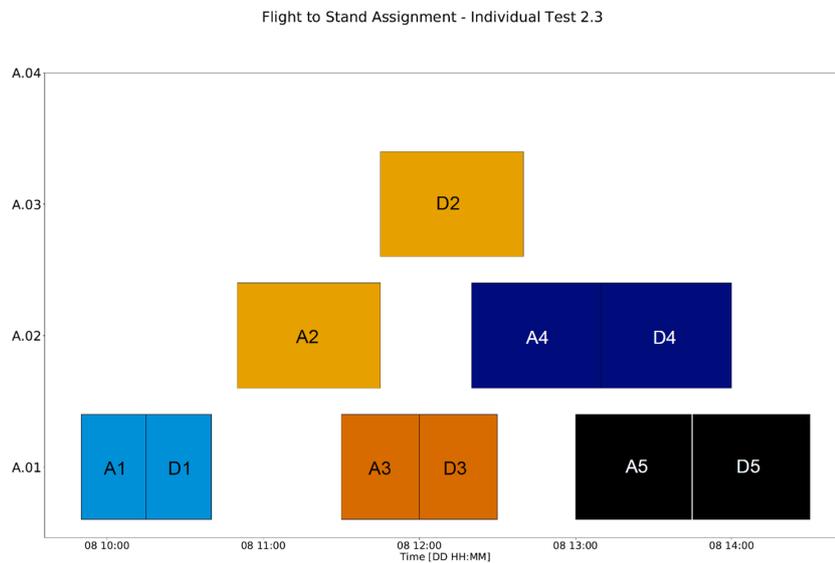


Figure 2.7: Assignment of operations to zones for Unit Test 2.3.

It is worth mentioning that, in the complete model, the aircraft performing turnaround T2 cannot be towed from A.02 to A.03, since this is a short turnaround (only divided into arrival and departure). As a consequence, the real problem will most likely assign A2 also to stand A.03, since this is the closest stand from the entrance available to accommodate both A2 and D2. For now, this tow move is considered valid because the towing constraints were not verified yet.

Test 1.8.: Tow Operations Constraints (First Level)

The Tow Operations Constraints used in the first level of assignment are tested in this subsection. For this unit test, the Maximization of Captured Transfer Passengers Objective is used, together with the constraints linked to that objective (hub connections relationship, arrivals transfer forecast and departures seat capacity). Besides, and as always, the fundamental constraints of the Two Level Stand Planning Model are also active. The flight schedule and zone information used in this unit test is presented in Table 2.14. There are two turnarounds, T1 and T2, which partially overlap with each other. This characteristic will be important to test the Tow Operations Constraint. This example also counts with two zones: Zone_A with 1 C-type stand, and Zone_B with 2 C-type stands.

Turnaround	Arrival Flight	Arrival Time	Arrival Airport IATA	Departure Flight	Departure Time	Departure Airport IATA	ICAO design group	Transfer Pax Arriving	Zone	C-stand Capacity	E-stand Capacity	MARS-stand Capacity	Time to Zone_A [min]	Time to Zone_B [min]
T1	XX 110	9:00	LKO	XX 111	12:30	LKO	C	70	Zone_A	1	0	0	40	240
T2	YY 112	10:50	FRA	YY 113	12:40	FRA	C	50	Zone_B	2	0	0	240	240

Table 2.14: Turnarounds and Zones information, used in Unit Test 1.8.

The assignment of operations to zones is shown in Figure 2.8. It is important to mention that it is only possible to have a connection from A1 to D2. Besides, the walking time between Zone_A and Zone_B is 240min or 4h, but the period of time between arrival A1 and departure D2 is only 3h40min. Hence, the model will try to assign A1 and D2 to the same zone since this is the only assignment that allows the connection fro

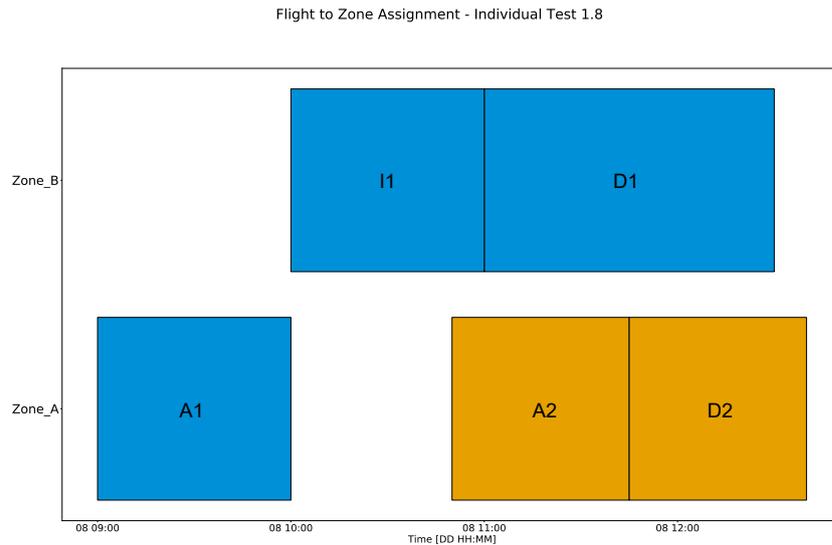


Figure 2.8: Assignment of operations to zones for Unit Test 1.8.

A1 to D2 and thus maximizes the number of captured transfer passengers. Figure 2.8 confirms that this is the case. However, two of the operations from T1, namely I1 and D2, overlap with the two operations from turnaround T2. This implies that it is only possible to have A1 and D2 in the same zone if one operation is towed. Again, the Figure confirms this. After its arrival, the aircraft from turnaround T1 is towed from Zone_A to Zone_B, leaving space for T2.

An alternative solution would be to assign operations A1 and I1 to Zone_A and operation D1 to Zone_B. Then, operation A2 would be assigned to Zone_B and D2 to Zone_A. This way, the connection would still be possible and the solution would be feasible. However, the rules state that only turnaround T2 can be towed, so that solution would not be valid. In conclusion, the assignment shown in Figure 2.8 follows the towing rules and this verifies the correct implementation of the Tow Operations Constraints used in the first level.

Test 2.4.1: Tow Operations Constraints (Second Level)

Two unit tests are proposed to verify the Tow Operations Constraints used in the second level. The first one, Test 2.4.1, uses the same input as Test 1.8 and, once again, the output of the first level of assignment is used as input to this level. Refer to Table 2.14 to recall the turnarounds and the zones information. Furthermore, Table 2.15 contains the information relative to each specific stand of Zone_A and Zone_B. The objective used in the second level (Minimization of Walking Times to Zone Entrance Objective) is not active for this test.

In line with what was explained in chapter 1, the number of tow operations and the operations subject to tows are defined in the first level and this cannot be changed in the second level. For this reason, and according to the result from Test 1.8, it is expected that only operation A1 is towed from Zone_A to Zone_B. Inside Zone_B, operations I1 and D1 are not allowed to be towed.

Stand	Zone	Stand Size	Distance to Zone entrance [min]
A.01	A	C	5
B.01	B	C	5
B.02	B	C	10

Table 2.15: Stands information, used in Unit Test 2.4.1.

The results in Figure 2.9 correspond to the predicted outcome. Turnaround T2 is not towed. On the other hand, operation A1 is towed from Zone_A to Zone_B and operations I1 and D1 are not allowed to tow inside Zone_B, so they have to be assigned to the same stand. This assignment complies with the zone assignment output obtained on the first level. Thus, the Tow Operations Constraints used in the second level of assignment are partially verified. However, to obtain a more solid conclusion about the verification of these constraint, Test 2.4.2 is proposed next.

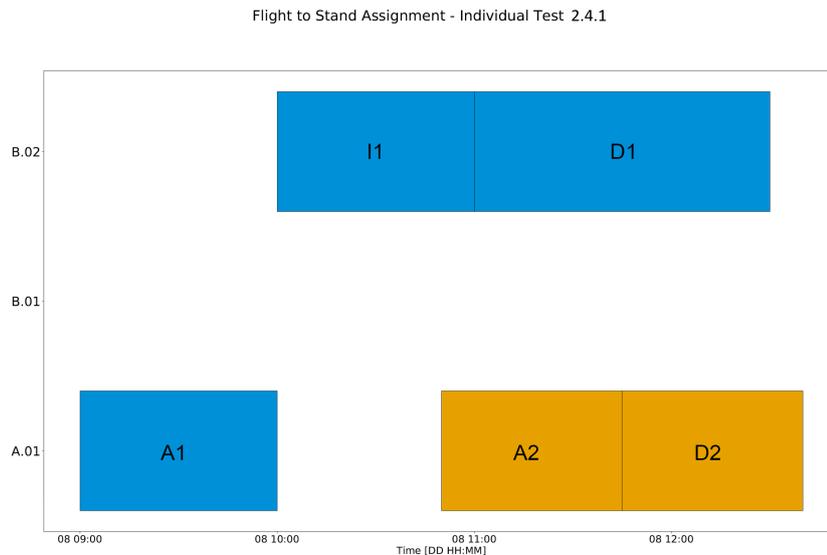


Figure 2.9: Assignment of operations to stands for Unit Test 2.4.1.

Test 2.4.2: Tow Operations Constraints (Second Level)

This unit test is also dedicated to verify the Tow Operations Constraints used in the second level. Unlike the previous test, this one does not use the same flight schedule and zone information as Test 1.8. The turnarounds used now are presented in Table 2.16. Note that T1 and T2 partially overlap with each other. The same Table shows the zone information. In this case, only 1 zone with a capacity of 1 C-type stand and 1 E-type stand is used. The information regarding each stand is shown in Table 2.17. Note that stand A.01 is closer to the entrance than A.02, as this will be relevant during this test.

No objectives from the first level are used in this test, but the Minimization of Walking Times to Zone Entrance Objective is now active. Furthermore, the towing constraints from the first level are active, together with the fundamental constraints from the Two Level Stand Planning Model.

It is also important to mention that, since the Minimization of Tow Operations Objective is not active yet, turnaround T1 can be towed from one stand to the other without any restrictions. This gives flexibility to test the tow moves in the second level of assignment. Nonetheless, T2 cannot be towed because it is a short turnaround.

In the end, it is expected that the model tries to fit as many operations as possible to stand A.01 (the closest to the zone entrance), but since some operations overlap, at least one tow move will be performed.

Turnaround	Arrival Flight	Arrival Time	Departure Flight	Departure Time	ICAO design group	Transfer Pax Arriving	Zone	C-stand Capacity	E-stand Capacity	MARS-stand Capacity
T1	XX 110	9:00	XX 111	12:30	C (738)	70	Zone_A	1	1	0
T2	YY 112	10:50	YY 113	12:40	E (789)	50				

Table 2.16: Turnarounds and Zones information, used in Unit Test 2.4.2.

Stand	Zone	Stand Size	Distance to Zone entrance [min]
A.01	A	E	5
A.02	A	C	10

Table 2.17: Stands information, used in Unit Test 2.4.2.

Figure 2.10 shows the output of Test 2.4.2. Since there is only one zone, it is not relevant to show the operation to zone assignment in this scenario.

Recall that, for now, the first level is not restricting the tow moves in the second level, since the objective that minimizes tow moves was still not introduced. At the same time, in the second level the model tries to

Flight to Stand Assignment - Individual Test 2.4.2

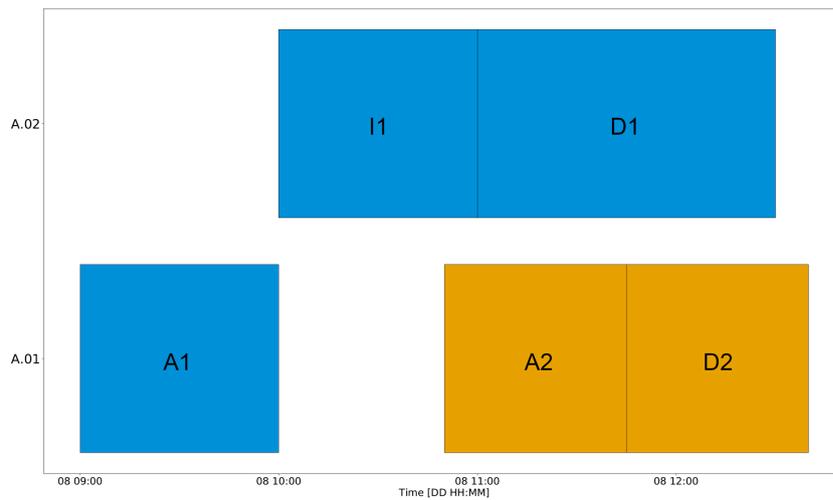


Figure 2.10: Assignment of operations to stands for Unit Test 2.4.2.

minimize times from operations to the zone entrance. Consequently, Figure 2.10 shows that operations are preferably assigned to stand A.01. Since I1 and D1 overlap with A2 and D2, the first two operations are towed to A.02. This shows that, when a tow move within the same zone is allowed in the first level, it can take place in the second level (the opposite scenario was verified in Test 2.4.1). Note that turnaround T2 had to be assigned to stand A.01 due to its aircraft's size. This was purposely defined this way to force the tow move of operation A1.

With Tests 2.4.1 and 2.4.2, it is possible to conclude that the Tow Operations Constraints used in the first level are correctly implemented.

Test 1.9.: Minimization of Tow Operations Objective

After introducing all Tow Operations Constraints, it is now possible to verify the Minimization of Tow Operations Objective. In this unit test, the Tow Operations Constraints are active, together with the fundamental constraints of the Two Level Stand Planning Model. The flight schedule and zone layout used in this scenario is shown in Table 2.18. Turnarounds T1 and T2 are long so they can be towed, while T3 is short. This test also includes 2 zones: Zone_A with 2 type C stand and Zone_B with 1 type C stand. Table 2.19 contains the stand information. Since there are 3 available stands and 3 turnarounds, no tow moves are needed, even if they all overlap with each other.

Turnaround	Arrival Flight	Arrival Time	Departure Flight	Departure Time	ICAO design group	Transfer Pax Arriving	Zone	C-stand Capacity	E-stand Capacity	MARS-stand Capacity	Time to Zone_A [min]	Time to Zone_B [min]
T1	YY 114	06:00	YY 115	11:00	C (738)	50	Zone_A	2	0	0	40	60
T2	XX 110	07:20	XX 111	11:00	C (738)	70	Zone_B	1	0	0	60	40
T3	XX 114	11:30	XX 115	12:40	C (738)	100						

Table 2.18: Turnarounds and Zones information, used in Unit Test 1.9.

Stand	Zone	Stand Size	Time to Zone Entrance [min]
A.01	A	C	2
A.02	A	C	5
B.01	B	C	2

Table 2.19: Stands information, used in Unit Test 2.2.

For this unit test, two assignments will be used for comparison. On the left, Figure 2.11 shows the operation to stand assignment before the Minimization of Tow Operations Objective is activated. On the right,

Figure 2.12 shows the operation to stand assignment after that same objective is added to the model.

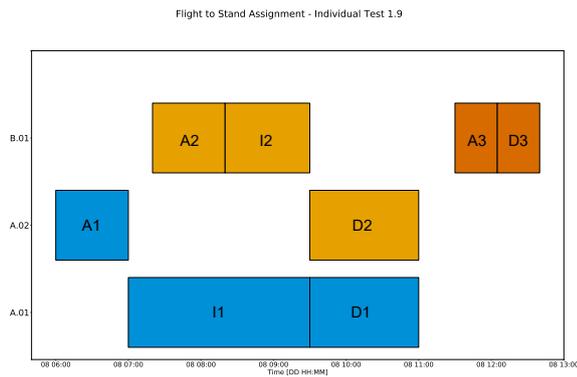


Figure 2.11: Assignment of operations to stands for Unit Test 1.9 before activating the Minimization of Tow Operations Objective.

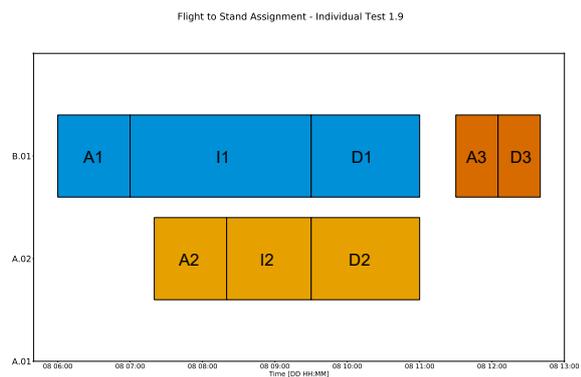


Figure 2.12: Assignment of operations to stands for Unit Test 1.9 after activating the Minimization of Tow Operations Objective.

The comparison between both Figures clearly shows the effect of trying to minimize the number of tow moves. In Figure 2.11, the long turnarounds T1 and T2 are towed once. However, these moves are unnecessary. In line with what was predicted in the beginning of this test, Figure 2.12 shows that it is not needed to move any of the turnarounds between stands, as there is space for all of them. Figure 2.12 further confirms that the Minimization of Tow Operations Objective is working properly by eliminating redundant tow movements.

Test 1.10.: Minimization of Unassigned Turnarounds Objective

The Minimization of Unassigned Turnarounds Objective is verified with Test 1.10. It is now important to introduce the Dummy Zone, to which unassigned turnarounds are attributed. The Dummy Zone has an infinite capacity. This way, the model can never become infeasible for lack of airport stands capacity.

The flight schedule and the zone information are presented in Table 2.20. For this unit test, 3 zones are considered. Two of them are real zones (Zone_A and Zone_B) and the other one is the Dummy Zone. Zone_A and Zone_B only have a capacity of one C stand each. A more detailed analysis to the flight schedule reveals that, during several points of the day, 3 operations coexist in the airport. Since the airport stand capacity is only 2, it is expected that some turnarounds need to be assigned to the Dummy Zone (i.e., they cannot be assigned to the real schedule). Since the model is trying to minimize the number of turnarounds assigned to the Dummy Zone, it is also anticipated that Gurobi will, at least, try to assign most of the turnarounds to either Zone_A or Zone_B.

Finally, only the fundamental constraints of the Two Level Stand Planning Model and the Towing Constraints are activated during this test.

Turnaround	Arrival Flight	Arrival Time	Departure Flight	Departure Time	ICAO design group	Transfer Pax Arriving	Zone	C-stand Capacity	E-stand Capacity	MARS-stand Capacity	Time to Zone_A [min]	Time to Zone_B [min]
T1	XX 110	10:50	XX 111	12:00	C (738)	50	Zone_A	1	0	0	40	60
T2	XX 112	11:00	XX 113	14:10	C (738)	50	Zone_B	1	0	0	60	40
T3	XX 114	11:10	XX 115	11:50	C (738)	50	Dummy	∞	∞	∞	N.D.	N.D.
T4	XX 116	11:55	XX 117	12:30	C (738)	50						
T5	XX 118	12:10	XX 119	14:20	C (738)	50						

Table 2.20: Turnarounds and Zones information, used in Unit Test 1.10.

The result presented in Figure 2.13 shows that 2 turnarounds were assigned to Zone_A, 2 others were assigned to Zone_B, and the largest turnaround, T2, was assigned to the dummy zone. Note that the model could have chosen to assign turnarounds T1 and T5 to the Dummy Zone, leaving space for T2 in Zone_A, or even assign T3 and T4 to the Dummy Zone, now leaving space for T2 in Zone_B. However, if that would have been the case, 2 turnarounds instead of 1 were being assigned to the Dummy Zone, which would correspond to a sub-optimal solution. The model has indeed assigned as few turnarounds as possible to the Dummy Zone, leading to the conclusion that the Minimization of Unassigned Turnarounds Objective is verified.

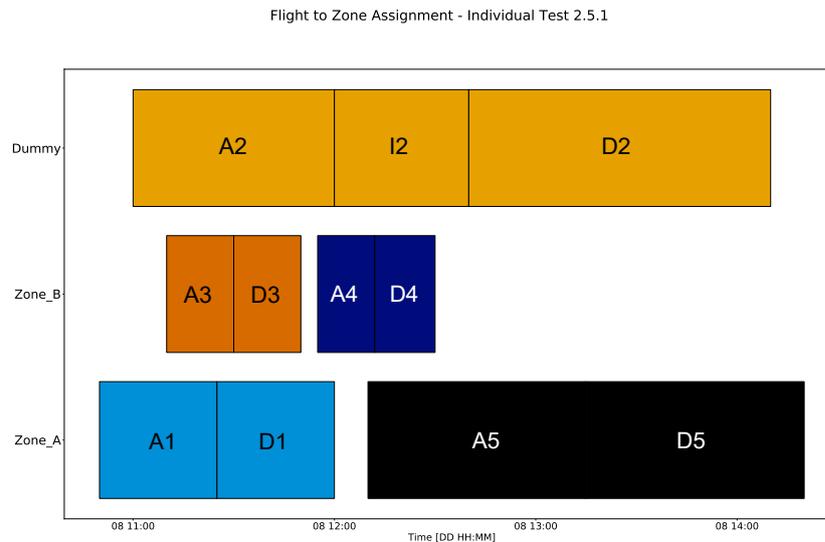


Figure 2.13: Assignment of operations to zones for Unit Test 1.10.

Test 1.11.: Correct Stand Assignment Constraint: Single Time Alternative for Proposed Turnarounds

One important constraint of the Two Level Stand Planning Model is the constraint that allows the selection of, at most, one time alternative of the proposed turnarounds. It is paramount that the same turnaround is not repeated several times during the same day. To verify this constraint, the flight schedule shown in Table 2.21 is proposed. Two new turnarounds are considered: T1 has 3 time alternatives (T1.1, T1.2 and T1.3) and the same is defined for T2 (T2.1, T2.2 and T2.3). For the sake of focusing this test on the Single Time Alternative for Proposed Turnarounds Constraint, none of these proposed time alternatives overlap with one another. Table 2.21 also contains information about the zones. For this test, and besides the Dummy Zone, only one other zone with a capacity of 1 C stand is considered.

Test 1.11 also uses the Minimization of Unassigned Turnarounds Objective, together with the Tow Moves Constraints and the base constraints of the Two Level Stand Planning Model. Taken into account all the information presented before, it is expected that one and only one time alternative of each of the turnarounds T1 and T2 is assigned to the stand in Zone_A. All the other alternatives should be assigned to the Dummy Zone.

Turnaround	Arrival Flight	Arrival Time	Departure Flight	Departure Time	ICAO design group	Transfer Pax Arriving	Zone	C-stand Capacity	E-stand Capacity	MARS-stand Capacity
T1.1	XX 110/1.1	8:40	XX 111/1.1	09:30	0	65	Zone_A	1	0	0
T1.2	XX 110/1.2	9:40	XX 111/1.2	10:30	0	80	Dummy	∞	∞	∞
T1.3	XX 110/1.3	10:40	XX 111/1.3	11:30	0	80	Dummy	∞	∞	∞
T2.1	YY 114/2.1	12:00	YY 115/2.1	12:50	0	70	Dummy	∞	∞	∞
T2.2	YY 114/2.2	13:00	YY 115/2.2	13:50	0	70	Dummy	∞	∞	∞
T2.3	YY 114/2.3	14:00	YY 115/2.3	14:50	0	95	Dummy	∞	∞	∞

Table 2.21: Turnarounds and Zones information, used in Unit Test 1.11.

Figure 2.14 shows the results of unit test 1.11. The expected outcome was verified in the solution. For T1, only the time alternative T1.1 was selected while for T2 only the alternative T2.1 was chosen. In this unit test, it is irrelevant which time alternative of each turnaround is assigned to Zone_A, so the model was able to randomly choose one. In conclusion, Test 1.11 verified the correct implementation of the Single Time Alternative for Proposed Turnarounds Constraint.

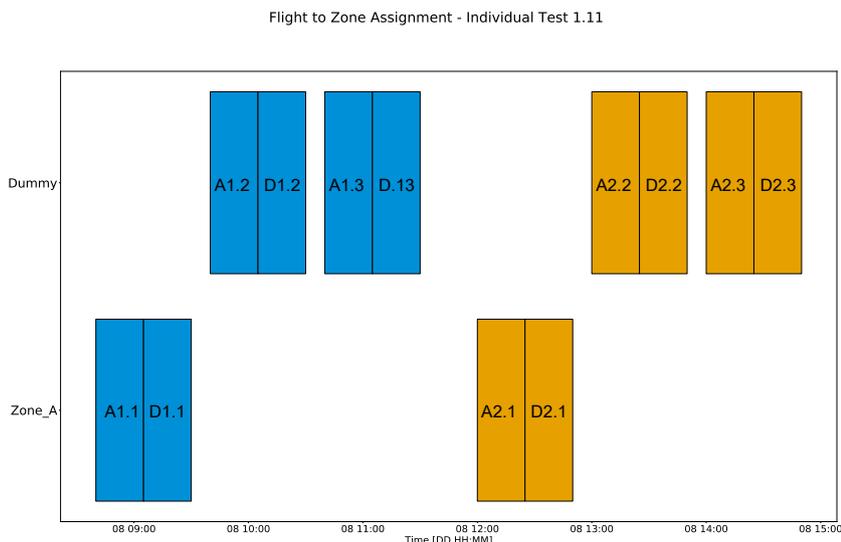


Figure 2.14: Assignment of operations to zones for Unit Test 1.11.

Test 1.12.: Hub Connections Constraints: Relationship between x_{ij} and y_{iv} variables

Before in the verification process, the correct implementation of the Hub Connections Constraints that relate z_{ijvw} and y_{iv} variables was already tested. Now that the Dummy Zone has been introduced, it is possible to check the Hub Connections Constraints that relate x_{ij} and y_{iv} variables. These constraints are important to block the model from creating a non-critical connection between operations that are assigned to the Dummy Zone. The flight schedule is visible in Table 2.22. Turnaround T1 does not overlap with any other turnarounds, but T2 and T3 overlap with each other, which means that only one of them can be selected. A connection can be established from A1 to D2 or from A1 to D3. This connection is non-critical because the span of time from arrival A1 to both departures D2 and D3 is 3h, and the minimum time for non-critical connections was established at 2h30min for this test. Table 2.22 also contains information about the only zone of the airport, Zone_A, which contains 1 E stand.

Besides the base constraints of the Two Level Stand Planning Model, this test uses the Maximization of Captured Transfer Passengers Objective, the Arrivals Transfer Passenger Capacity Constraint and the Departures Seat Capacity Constraint. It is expected that only one non-critical will be established (either A1-D2 or A1-D3), since T2 and T3 overlap with each other and there is only one stand available.

Turnaround	Arrival Flight	Arrival Time	Arrival Airport IATA	Departure Flight	Departure Time	Departure Airport IATA	ICAO design group	Transfer Pax Arriving	Seat Capacity	Zone	C-stand Capacity	E-stand Capacity	MARS-stand Capacity
T1	XX 110	10:00	LKO	XX 111	11:00	LKO	E (789)	180	281	Zone_A	0	1	0
T2	YY 112	12:00	FRA	YY 113	13:00	FRA	C (738)	50	162	Dummy	∞	∞	∞
T3	YY 114	12:00	FRA	YY 115	13:00	FRA	C (738)	50	162				

Table 2.22: Turnarounds and Zones information, used in Unit Test 1.12.

The result in Figure 2.15 matches the predictions made before. First of all, turnaround T2 is assigned to the Dummy Zone. Besides, there are 180 passengers wanting to connect from arrival A1. 162 of those passengers connect to departure D3, but no passengers connect to departure D2 (in the model, the values of the corresponding connection variables are given by $X(A1, D3) = 162$ and $X(A1, D2) = 0$). It is important to note that the model is trying to maximize the number of captured transfer passengers, but no passengers were allowed to connect from A1 to D2, even though there were still 18 travellers without a connection. In conclusion, the model is not selecting a more profitable solution because the Hub Connections Constraints that relate x_{ij} and y_{iv} variables are preventing it from doing so. Thus, Test 1.12 successfully verified those constraints.

Flight to Zone Assignment - Individual Test 1.12

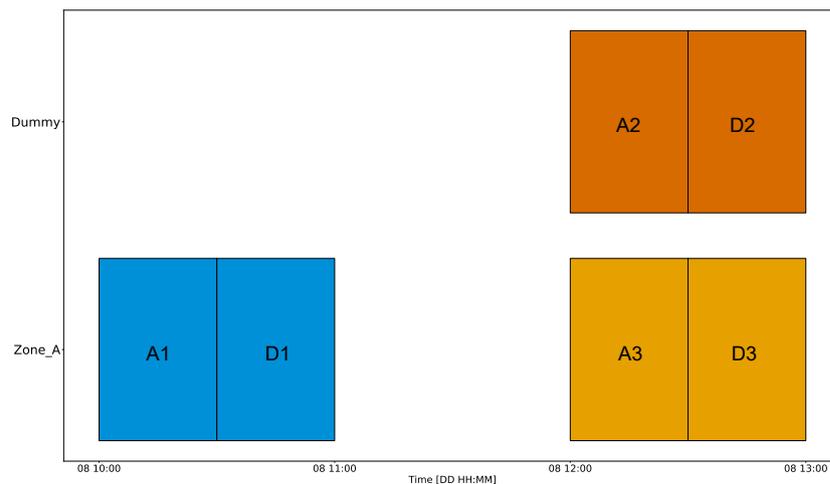


Figure 2.15: Assignment of operations to zones for Unit Test 1.12.

Test 1.13.1: Zone MARS Capacity Constraint (First Level)

The MARS Capacity Constraint used in the first level is complementary to the Zone Capacity Constraint, which was already verified. Together, they guarantee that the maximum capacity of the airport (single stand and MARS stands) is not surpassed. Due to the bigger complexity of the MARS Constraints, two tests were developed for the first level (1.13.1 and 1.13.2) and their outputs were used for another two tests in the second level (Tests 2.5.1 and 2.5.2). The flight schedule for test 1.13.1 is shown in Table 2.23. All 3 turnarounds overlap with each other. T1 and T2 are operated by large aircraft and T3 is operated by a small aircraft. The zone capacity information is also presented in Table 2.23. The Dummy Zone is once again present, and Zone_A contains 2 MARS stands. Recall that each MARS stand can either accommodate two small aircraft (C or smaller) or one large aircraft (E or larger).

The only elements active for this test besides the base constraints are the Minimization of Unassigned Turnarounds Objective and the Tow Moves Constraints. With all the previous information, one can expect that one operation has to be left at the Dummy Zone. There are 2 MARS stands. The large block of each can be taken by the two large aircraft, but in this case the small aircraft has no space left. Alternatively, one large aircraft can take the large block of one of the MARS stands, and the small aircraft can take a small block from the other MARS stand. With this configuration, the remaining large aircraft has to be left out.

Turnaround	Arrival Flight	Arrival Time	Departure Flight	Departure Time	ICAO design group	Transfer Pax Arriving	Zone	C-stand Capacity	E-stand Capacity	MARS-stand Capacity
T1	YY 110	12:00	YY 111	13:40	E (789)	50	Zone_A	0	0	2
T2	YY 112	12:10	YY 113	13:30	E (738)	60	Dummy	∞	∞	∞
T3	YY 114	12:15	YY 115	13:50	C (738)	70				

Table 2.23: Turnarounds and Zones information, used in Unit Test 1.13.1.

The zone assignment obtained in this test was the following: $Y(A1, Zone_A) = 1$, $Y(A2, Zone_A) = 1$, $Y(A2, Dummy) = 1$, $Y(D2, Dummy) = 1$, $Y(A3, Zone_A) = 1$ and $Y(D3, Zone_A) = 1$. In practice, turnarounds T1 and T3 were assigned to Zone_A, while T2 was assigned to the Dummy Zone due to lack of space. It is guaranteed that there was no space for T2 because the Minimization of Unassigned Turnarounds Objective is being used. Thus, the Zone MARS Capacity Constraint used in the first level is partially verified, but a second test will be run later.

Test 2.5.1: Zone MARS Capacity Constraint (Second Level)

This test follows Test 1.13.1 and it uses its results as input. The information about the flight schedule and zones can be recalled in Table 2.23. The results from 1.13.1 determine that turnarounds T1 and T3 are assigned to Zone_A and T2 is assigned to the Dummy Zone. The specific stand information is shown in Table 2.24. Note that the small stand blocks A.01L and A.01R are connected to the large block A.01 and blocks

A.02L and A.02R are connected to A.02.

Since T1 is a large aircraft, it has to be assigned to a large MARS block (either A.01 or A.02), and the smaller T3 has to be assigned to a small block (either A.01L, A.01R, A.02L or A.02R).

Stand	Zone	Stand Size	MARS Group	Block Size	Time to Zone Entrance [min]
A.01	A	F	A.01	Large	5
A.01L	A	C	A.01	Small	5
A.01R	A	C	A.01	Small	5
A.02	A	F	A.02	Large	10
A.02L	A	C	A.02	Small	10
A.02R	A	C	A.02	Small	10

Table 2.24: Stands information, used in Unit Test 2.5.1.

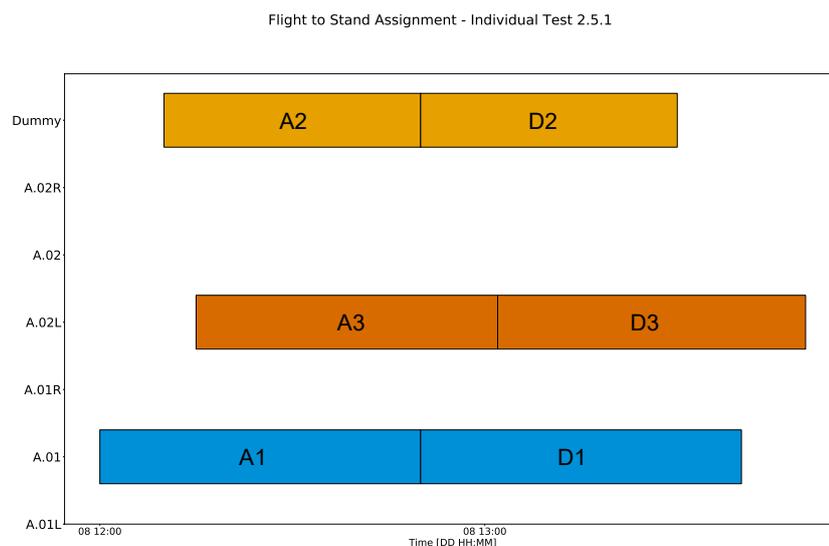


Figure 2.16: Assignment of operations to stands for Unit Test 2.5.1.

The results shown in Figure 2.16 confirm the expected outcome. Turnaround T1 is assigned to stand A.01 and T3 is assigned to stand A.02L. This confirmation partially verifies the Zone MARS Capacity Constraint used in the second level. However, a second test will be run to further analyse this constraint.

Test 1.13.2: Zone MARS Capacity Constraint (First Level)

In line with what was mentioned in 1.13.1, a second test to verify the Zone MARS Capacity Constraint used in the first level is performed. The flight schedule for this test is shown in Table 2.25. There are 4 turnarounds, all overlapping with each other. Three of the turnarounds, T2, T3 and T4 are operated by a small aircraft. T1 is operated by a large aircraft. Furthermore, the only zone defined besides the Dummy Zone is Zone_A, with a capacity of 2 MARS stands.

The only elements active for this test besides the base constraints are the Minimization of Unassigned Turnarounds Objective and the Tow Moves Constraints.

Turnaround	Arrival Flight	Arrival Time	Departure Flight	Departure Time	ICAO design group	Transfer Pax Arriving	Zone	C-stand Capacity	E-stand Capacity	MARS-stand Capacity
T1	YY 110	12:00	YY 111	13:40	E (789)	50	Zone_A	0	0	2
T2	YY 112	12:10	YY 113	13:40	C (738)	50	Dummy	∞	∞	∞
T3	YY 114	12:20	YY 115	14:00	C (738)	60				
T4	YY 116	12:25	YY 117	13:35	C (738)	80				

Table 2.25: Turnarounds and Zones information, used in Unit Test 1.13.2.

It is important to highlight that two MARS stand have capacity for (the following alternatives are all mutually exclusive):

- i) 4 small aircraft and 0 large aircraft, or
- ii) 3 small aircraft and 0 large aircraft, or
- iii) 2 small aircraft and 1 large aircraft, or
- iv) 1 small aircraft and 1 large aircraft, or
- v) 0 small aircraft and 2 large aircraft

Based on the previous information, it is expected that at least one turnaround is assigned to the Dummy Zone. With 3 small aircraft and 1 large aircraft, the best the model can do is to choose either alternative ii) or iii), which allows 3 turnarounds to be assigned to Zone_A. The remaining turnaround has to be attributed to the Dummy Zone.

The zone assignment obtained in this test was the following: $Y(A1, Zone_A) = 1$, $Y(A2, Zone_A) = 1$, $Y(A3, Zone_A) = 1$, $Y(D2, Zone_A) = 1$, $Y(D3, Zone_A) = 1$, $Y(A4, Dummy) = 1$ and $Y(D4, Dummy) = 1$. In practice, this means that turnarounds T1, T2 and T3 were assigned to Zone_A, while T4 had to be left in the Dummy Zone. This outcome matches the prediction. Indeed, the model picked alternative iii) by selecting one large aircraft and two small aircraft which, according to what was explained in the previous paragraph, is the best result it can achieve.

This test, together with Test 1.13.1, enable to confidently validate the Zone MARS Capacity Constraints, used in the first level.

Test 2.5.2: Zone MARS Capacity Constraint (Second Level)

The second unit test used to verify the second level's Zone MARS Capacity Constraint follows Test 1.13.2 and it uses its outcome as input. The flight schedule used here is the same as in Table 2.23 and the specific stand information can be recalled in Table 2.24.

According to the results obtained in Test 1.13.2 for the first level of assignment, turnarounds T1, T2 and T3 are assigned to Zone_A. Since T1 is operated by a large aircraft, it has to occupy the large block of one of the MARS stands (either block A.01 or A.02). Consequently, T2 and T3, which are operated by small aircraft, have necessarily to be assigned to the small blocks of the other MARS stand. This is the only configuration that works, but the specific assignments can vary.

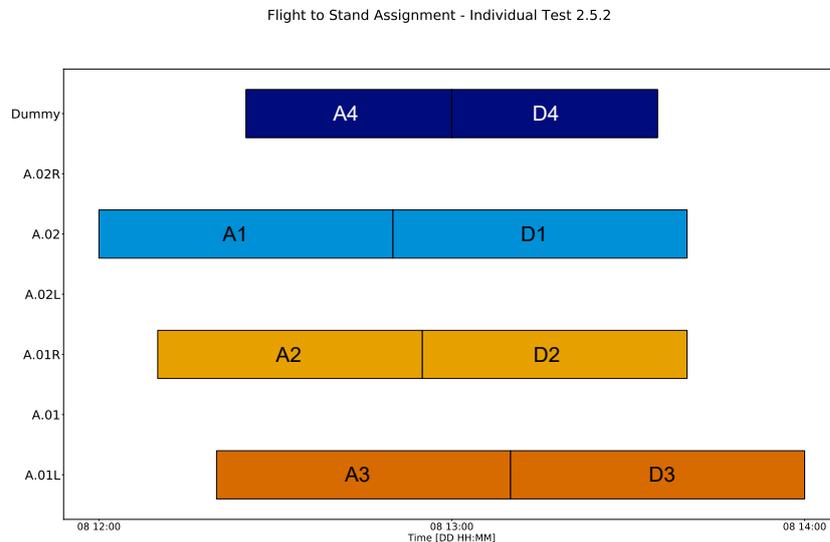


Figure 2.17: Assignment of operations to stands for Unit Test 2.5.2.

The results of Test 2.5.2 are presented in Figure 2.17. The model assigned the small turnarounds T2 and T3 to the small blocks A.01L and A.01R and the large turnaround T1 to the large block A.02. In line with the explanation made before, this is the only configuration possible in this scenario.

The MARS stand rules were complied in this second level assignment. For this reason, Test 2.5.2 allows to verify the correct implementation of the second level's Zone MARS Capacity Constraint. This verification is further supported by the result obtained from Test 2.5.1, which was already performed.

Test 1.14: Minimization of Connection Times Objective

The last element of the Two Level Stand Planning Model to be verified is the Minimization of Connection Times Objective. The flight schedule used in this unit test is shown in Table 2.26. Turnaround T1 arrives from and departs to HKG. Turnaround T2 has 3 time alternatives (T2.1, T2.2 and T2.3) and it arrives from and departs to LKO. Finally, T3 has 3 time alternatives (T3.1, T3.2 and T3.3) and it flies the route to FRA. Each time alternative of T2 overlaps with the corresponding alternative of T3. Table 2.26 also shows that this test uses the Dummy Zone and one zone with a capacity of 1 E stand.

It is important to mention that only non-critical connections are subject to the current objective. For this unit test, it was established that non-critical connections last from 2h to 7h. For this reason, a non-critical connection can happen between A1 and any time alternative of T2 and also between A1 and any alternative of T3. Some non-critical connection might also be established between T2 and T3, but these will not be relevant in this unit test.

Table 2.26 shows that a total of 210 passengers want to transfer from A1. Out of these 210, 5 passengers want to connect to LKO (and thus, to T2) and 105 passengers want to connect to FRA (and thus, to T3). Since more people are making the connection T1-T3, this connection has a higher impact on the total connection time (#pax × single connection time). Thus, T3 should be placed as close as possible from A1, i.e., the model should prioritize the connection T1-T3.

All constraints, except for the Zone MARS Constraints, are active during Test 1.14. The Maximization of Captured transfer Passengers is also active.

Turnaround	Arrival Flight	Arrival Time	Arrival Airport IATA	Departure Flight	Departure Time	Departure Airport IATA	ICAO design group	Transfer Pax Arriving	Seat Capacity	Zone	C-stand Capacity	E-stand Capacity	MARS-stand Capacity
T1	XX 110	8:00	HKG	XX 111	09:00	HKG	E (789)	210	281	Zone_A	0	1	0
T2.1	XX 112/1.1	10:00	LKO	XX 113/1.1	10:50	LKO	C (738)	80	162	Dummy	∞	∞	∞
T2.2	XX 112/1.2	11:00	LKO	XX 113/1.2	11:50	LKO	C (738)	80	162				
T2.3	XX 112/1.3	12:00	LKO	XX 113/1.3	12:50	LKO	C (738)	80	162				
T3.1	XX 114/2.1	10:10	FRA	XX 115/2.1	10:50	FRA	C (738)	80	162				
T3.2	XX 114/2.2	11:10	FRA	XX 115/2.2	11:50	FRA	C (738)	80	162				
T3.3	XX 114/2.3	12:10	FRA	XX 115/2.3	12:50	FRA	C (738)	80	162				

Table 2.26: Turnarounds and Zones information, used in Unit Test 1.14.

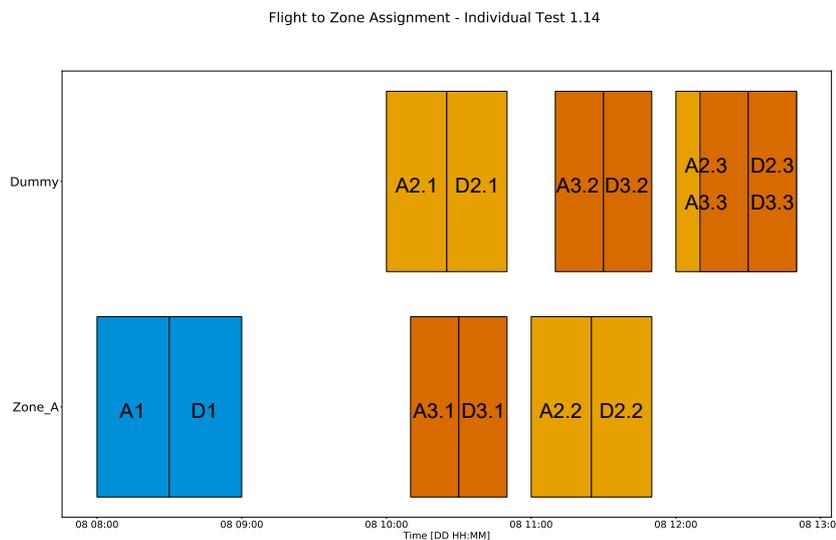


Figure 2.18: Assignment of operations to zones for Unit Test 1.14.

The output of the test presented in Figure 2.18 shows that alternatives T3.1 and T2.2 were assigned to Zone_A, and T3.1 is closer to A1 than alternative T2.2. According to what was predicted, the model prioritized T3 over T2. Alternative T2.1 would be the closest from A1, but since it cannot be assigned to Zone_A due to lack of space (T3.1 is using that slot), the second best option in terms of minimization of connection times is T2.2. This is why this alternative is picked by the model. Clearly, if T3.1 and T2.2 switched time slots, the total connection time would increase considerably.

All 5 passengers wanting to connect from T1 to T2 were able to make the connection. However, the seat capacity of the aircraft operating T3.1 is 162 people, which is lower than the transfer demand of 205 to that

flight. For this reason, in the output of the model, only 162 passengers were in fact offered the connection T1-T3. Nonetheless, this does not affect the explanation made on the previous paragraph nor the validity of Test 1.14. In the end, this test verified the correct implementation of the Minimization of Connection Times Objective.

2.1.2. Objective Functions and Constraints Verification: Integrated Test

In the previous section, each element of the TLSAP was individually tested. For some unit tests, it was necessary to activate previously verified elements to obtain proper results. It is now time to design and run a system test that will integrate all elements. This way, the TLSAP will be verified as a complete model. Table 2.27 contains the information for all turnarounds used in this test. Two zones are considered for this test. Zone_A has a capacity of only 1 C-stand while Zone_B contains one MARS stand, which means it can accommodate either 2 small aircraft or 1 large aircraft.

Turnaround	Arrival Flight	Arrival Time	Arrival Airport IATA	Departure Flight	Departure Time	Departure Airport IATA	ICAO design group	Transfer Pax Arriving	Seat Capacity	Zone	C-stand Capacity	E-stand Capacity	MARS-stand Capacity	Time to Zone_A [min]	Time to Zone_B [min]
T1.1	XX 112/1.1	8:20	LKO	XX 113/1.1	9:30	LKO	C (738)	80	162	Zone_A	0	1	0	40	300
T1.2	XX 112/1.2	8:00	LKO	XX 113/1.2	10:50	LKO	C (738)	90	162	Zone_B	0	0	1	300	40
T1.3	XX 112/1.3	8:30	LKO	XX 113/1.3	12:30	LKO	C (738)	50	162	Dummy	∞	∞	∞	-	-
T2	XX 114	8:00	LKO	XX 115	08:50	LKO	C (738)	80	162						
T3	XX 116	10:40	FRA	XX 117	15:00	FRA	E (789)	120	281						
T4	XX 118	9:00	DEL	XX 119	13:10	DEL	E (789)	60	281						
T5	XX 120	9:00	LHR	XX 121	9:40	LHR	C (738)	70	162						
T6	XX 122	7:50	LHR	XX 123	11:50	LHR	C (738)	80	162						

Table 2.27: Turnarounds and Zones Information, used in the System Test.

The information regarding each individual stand is shown in Table 2.28, including MARS stand relationships when that is relevant and also the time that it takes to walk from each stand to the corresponding zone's entrance.

Stand	Zone	Stand Size	MARS Group	Block Size	Time to Zone Entrance [min]
A.01	A	E	-	-	5
B.01	B	F	B.01	Large	10
B.01L	B	C	B.01	Small	5
B.01R	B	C	B.01	Small	15

Table 2.28: Stands information, used in the System Test.

Flight to Stand Assignment - System Test

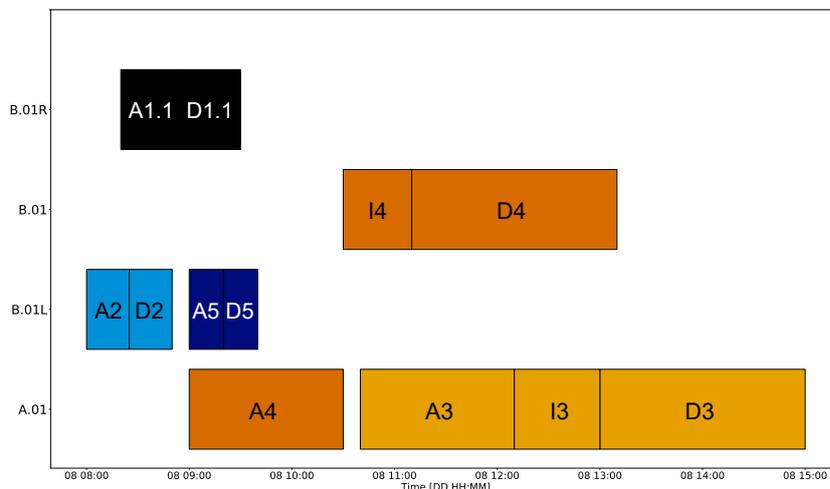


Figure 2.19: Assignment of operations to stands for the System Test.

The stand assignment of the system test is shown in Figure 2.19. Note that the unassigned turnarounds (T1.2, T1.3 and T6) are not shown in the Figure, but they were all assigned to the Dummy Zone. The first

aspect to notice is that only 1 time alternative of turnaround T1 was selected. Furthermore, turnaround T3 and T4 were assigned to one of the two only possible stands, since those aircraft are large (type E). Note that the only unassigned turnaround, T6, could theoretically be assigned to either stands B.01R, B.01L or A.01. However, due to its length, this theoretical assignment would result in at least one less turnaround being assigned to the schedule, so the number of unassigned turnarounds would not be minimized. Furthermore, the number of captured transfer passengers would not be maximized either.

In terms of number of captured transfer passengers, they are distributed in the following way:

- 40 passengers offered a critical connection from A1.1 to D5
- 40 passengers offered a critical connection from A2 to D5
- 70 passengers offered a critical connection from A5 to D4
- 40 passengers offered a non-critical connection from A1.1 to D3
- 40 passengers offered a non-critical connection from A2 to D3
- 30 passengers offered a non-critical connection from A4 to D3

A more detailed analysis to these values shows that the number of captured transfer passengers was indeed maximized. At the same time, the transfer passenger forecast from each arrival was never exceeded (recall Table 2.28 to confirm the number of transfer passengers arriving for each turnaround). As an example, note that the number of passengers transferring from arrival A2 (to D3 and D5) adds up to 80, which corresponds exactly to the number of passengers willing to connect from that arrival. Departures D3 and D5 still have seat capacity available, but no more passengers transfer from A2 because the forecast is 80.

Furthermore, the seat capacity of each departure was also verified for all cases. Note that the number of passengers transferring to any departure never exceeds the corresponding seat capacity.

The minimization of walking times inside each zone (performed in the second level of assignment) is also visible in Zone_B. Note that turnarounds T2 and T5 could have been swapped with T1.1, but this would increase the total walking times to the entrance of Zone_B (stand B0.1L is closer to the entrance).

It is important to mention that turnaround T4 is towed from Zone_A to Zone_B after its arrival. This happens because T3 takes its place at stand A.01, so to minimize the number of unassigned operations, *Gurobi* moves T4 to a different stand. This is a necessary tow move and that is why it is included. No unnecessary towing operations are added in this test, which means that the Minimization of Tow Moves Objective is working, as well as the Towing Constraints.

Finally, note that arrival A4 could have been assigned to B.01, eliminating the need for the tow move. However, this would violate the dynamic of the MARS stands (there would be 2 C aircraft and 1 E aircraft parked simultaneously at that stand). Consequently, this proves that MARS Constraints are behaving correctly.

2.1.3. Transfer Passenger Forecast Algorithm Verification

In chapter 1, the algorithm used in this research to compute the percentage of transfer demand between each arrival-departure pair was presented. This algorithm is based on i) the angle, centered at the hub, between the origin and the departure airports and ii) the relationship between the airline that operates the first flight and the second flight, i.e., if it is the same airline, if they belong to the same alliance (or share the same codes) or if they are not related at all. Each case i) and ii) have a factor associated to them that is then converted into a percentage of demand. In this section, the algorithm is verified. For this verification, 3 different tests are defined:

- **Test 1:** Only the factor related to the angle is analyzed
- **Test 2:** Only the factor related to the relationship between the airlines is analyzed
- **Test 3:** Both factors are tested simultaneously

The ICAO code of the hub used in this example is *LFPG*. This airport is located in Paris, France. The ICAO of the other airports used are *BIRK* (Reykjavík, Iceland), *ENGM* (Oslo, Norway), *UUDD* (Moscow, Russia) and *LGAV* (Athens, Greece). The relative position of these 4 airports is presented in Figure 2.20.

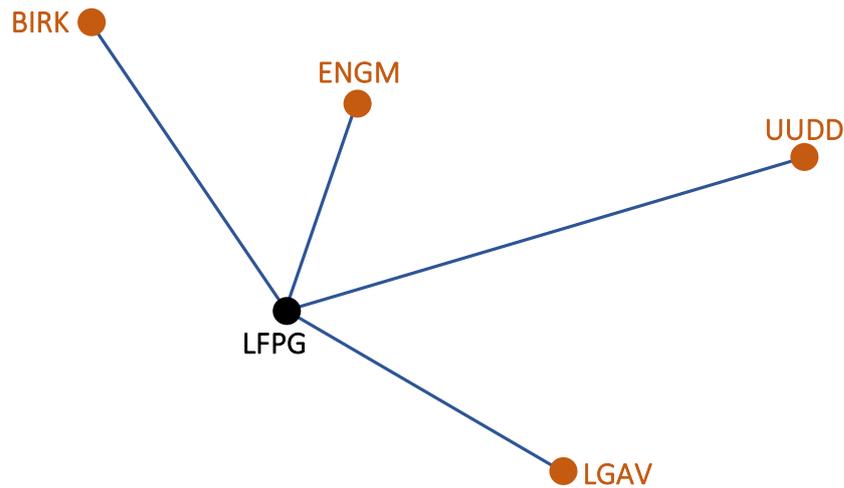


Figure 2.20: Relative positions of the 5 airports used in this example.

Test 1: Angle Between the Origin and the Departure Airports

For this test, it is assumed that the hub is connected to 4 other airports, *BIRK*, *ENGM*, *UDD* and *LGAV*. It is further assumed that only one airline, *XX*, flies to and from the hub. This way, case ii) does not have an influence on the results of this test. The results for each origin-destination pair for Test 1 is shown in Table 2.29. Note that, for each pair *AAAA – BBBB* in the Table, the cell corresponds to the percentage of passengers wanting to connect from a flight arriving from *AAAA* to a departure flying to *BBBB* (both in airline *XX* since that is the only airline considered here).

From \ To	BIRK	ENGM	UDD	LGAV
BIRK	0	17.826	31.249	50.924
ENGM	27.703	0	20.861	51.437
UDD	48.563	20.861	0	30.576
LGAV	49.109	31.918	18.973	0

Table 2.29: Percentage of passengers willing to connect from the airports in each row to the airports in each column, for Test 1.

The results show that the transfer passengers between flights arriving from and departing to the same airport is 0, which is logical. An analysis to the geographical position of all airports (visualized in Figure 2.20) confirms that the values in Table 2.29 are correct. Note that the *ENGM* is the closest to *BIRK* in terms of angular position, so the percentage of passengers wanting to connect from *ENGM* to *BIRK* is also the smallest (17.826%). Then, *UDD* follows with a percentage of 31.249%. Finally, the angle between *BIRK* and *LGAV* is almost 180 (they are located opposite of each other), Thus, the percentage forecast to the connection *BIRK – LGAV* is also the highest (50.924%). The same observations are made for the other 3 airports.

Note, however, that the demand matrix is not symmetric. For instance, the percentage for the connection *BIRK – ENGM* is 17.826%, while the corresponding value for the opposite connection is 27.703%. This is expected and it also reveals that the algorithm is working properly. Indeed, the percentage values for each origin airport should only be related to how that airport is connected to all the others.

It is important to mention that the sum of percentages for each line equals 100%, which is a consequence of the normalization performed in the algorithm.

Test 2: Relationship Between the Two Airlines

Unlike the previous test, this one considers that 4 airlines, *XX*, *YY*, *ZZ* and *TT*, fly to and from the hub, but only 2 airport are connected to it, namely *BIRK* and *LGAV*. This eliminates the interference from case i) of having multiple airport choices. It is assumed that *XX*, *YY* and *TT* belong to the same alliance, but neither of the 3 have any relationship with *ZZ*. Recall, from section 1.1, the connection probabilities between an arrival and a departure based on the airlines' relationship:

- Arrival and departure are operated by **the same airline**: 67%

- Arrival and departure are operated by **airlines belonging to the same alliance or codesharing**: 33%
- Arrival and departure are operated by **airlines not related to each other**: 0%

With this in mind, it is expected that connections between XX , YY and TT will take place, but not between these 3 airlines and ZZ , since the latter does not belong to any airline alliance. The results of Test 2 are presented in Table 2.30.

From \ To	XX-BIRK	YY-BIRK	ZZ-BIRK	TT-BIRK	XX-LGAV	YY-LGAV	ZZ-LGAV	TT-LGAV
XX-BIRK	0	0	0	0	50.376	24.812	0	24.812
YY-BIRK	0	0	0	0	24.812	50.376	0	24.812
ZZ-BIRK	0	0	0	0	0	0	100	0
TT-BIRK	0	0	0	0	24.812	24.812	0	50.376
XX-LGAV	50.376	24.812	0	24.812	0	0	0	0
YY-LGAV	24.812	50.376	0	24.812	0	0	0	0
ZZ-LGAV	0	0	100	0	0	0	0	0
TT-LGAV	24.812	24.812	0	50.376	0	0	0	0

Table 2.30: Percentage of passengers willing to connect from the airports in each row to the airports in each column, for Test 2.

The first results taken from Table 2.30 is that the transfer passengers between flights arriving from and departing to the same airport is 0. This was expected and it follows from Test 1. Furthermore, the arrivals operated by airline ZZ can only connect to departures operated by the same airline. Once again, and according to what was mentioned in section 1.1, it is assumed that there can only be connections between flights operated by the same airline or airlines belonging to the same alliance. ZZ does not have any arrangements with the other airlines, so this result is correct. On the other hand, and for the same reason, airlines XX , YY and TT do not establish any connection with ZZ .

For the arrival $XX - BIRK$, that is, the arrival from $BIRK$ operated by airline XX , the highest percentage of transfer passenger demand is verified to the departure operated by the same airline ($XX - LGAV$). Note that this percentage is twice as large as the other two percentages, namely the connections $(XX - BIRK) \rightarrow (YY - LGAV)$ and $(XX - BIRK) \rightarrow (TT - LGAV)$. This outcome was predicted based on the connection probabilities shown before: even though there is the possibility to connect between any of the airlines XX , YY and TT , the probability to connect to the same airline is twice as large as the probability to connect to other airlines from the same alliance.

Note that the values 50.376% and 24.812% visible on the first line are repeated in a pattern through all the other airlines. This confirms the consistency of the algorithm. The value 50.376% is always associated with a connection between two flights operated by the same airline, while the value 24.812% is related to the connections between two flights operated by different airlines of the same alliance.

Similarly to Test 1, the sum of percentages for each line equals 100%.

Test 3: Combined Analysis

The final test is a combination of Tests 1 and 2. It is assumed that the hub is connected to 3 other airports, $BIRK$, $ENGM$ and $LGAV$ via 4 different airlines, XX , YY , ZZ and TT . Similarly to Test 2, airlines XX , YY and TT belong to the same alliance but they are not related to ZZ . The results of Test 3 are shown in Table 2.31.

From \ To	XX-BIRK	YY-BIRK	ZZ-BIRK	TT-BIRK	XX-ENGM	YY-ENGM	ZZ-ENGM	TT-ENGM	XX-LGAV	YY-LGAV	ZZ-LGAV	TT-LGAV
XX-BIRK	0	0	0	0	13.062	6.433	0	6.433	37.314	18.379	0	18.379
YY-BIRK	0	0	0	0	6.433	13.062	0	6.433	18.379	37.314	0	18.379
ZZ-BIRK	0	0	0	0	0	0	25.929	0	0	0	74.071	0
TT-BIRK	0	0	0	0	6.433	6.433	0	13.062	18.379	18.379	0	37.314
XX-ENGM	17.634	8.685	0	8.685	0	0	0	0	32.742	16.127	0	16.127
YY-ENGM	8.685	17.634	0	8.685	0	0	0	0	16.127	32.742	0	16.127
ZZ-ENGM	0	0	35.005	0	0	0	0	0	0	0	64.995	0
TT-ENGM	8.685	8.685	0	17.634	0	0	0	0	16.127	16.127	0	32.742
XX-LGAV	30.532	15.038	0	15.038	19.844	9.774	0	9.774	0	0	0	0
YY-LGAV	15.038	30.532	0	15.038	9.774	19.844	0	9.774	0	0	0	0
ZZ-LGAV	0	0	60.608	0	0	0	39.392	0	0	0	0	0
TT-LGAV	15.038	15.038	0	30.532	9.774	9.774	0	19.844	0	0	0	0

Table 2.31: Percentage of passengers willing to connect from the airports in each row to the airports in each column, for Test 3.

The analysis developed in this paragraph concerns the first line of [Table 2.31](#), which corresponds to an arrival from *BIRK* via airline *XX*. Once again, 0 connections are forecast to departures flying back to *BIRK* and to departures operated by airline *ZZ*. For each destination, the percentages are highest for departures operated by the same airline (*XX*). This percentage is twice as large as the forecast percentages for departures operated by *YY* and *TT*. One last remark taken from [Table 2.31](#) is that the percentages are generally higher for the connections *BIRK* → *LGAV* than for the connections *BIRK* → *ENGM*. Once again, this is explained by the relative position of the airports, based on the angle. The analysis done to the first line of the Table can be extended to all other lines (unless for lines in which the arrival is operated by airline *ZZ*). Besides that, the

The arrivals operated by airline *ZZ* only establish connections with departures also operated by that airline.

Once again, and following the results of Tests 1 and 2, the sum of percentages for each line equals 100%.

Overall, the analysis made to the results of Test 3, together with the partial results obtained from Tests 1 and 2, show that both the angle between origin and destination airports and the relationship between the airlines are correctly incorporated into the algorithm. They further show that the algorithm itself is properly implemented and that the outcome is translated to plausible, real life values for transfer passenger forecasts.

2.2. Validation of the TLSAP

The validation of the TLSAP is just as important as the verification of the correct implementation of the model. In this research, it will not be possible to directly compare the output obtained in the different proposed scenarios with real life data. The validation will be done in two fronts. First, the consistency and the level of applicability of the model is validated with the help from 2 product managers from BEONTRA GmbH. The two experts provided valuable feedback on the values of the parameters chosen for this research, on the usage of BEONTRA GmbH *BRoute Development Tool* and also on the results obtained and discussed in [chapter 4](#). Secondly, the sensitivity analysis developed in [chapter 5](#) will be used to confirm the robustness of the model and eventually conclude that it is delivering the correct results.

It is important to understand whether BEONTRA GmbH *BRoute Development Tool* was correctly used for this research. In line with previous paragraphs, the tool was used to obtain the different time alternatives for the proposed turnarounds. The decision to accept a certain time slot was based on the number of passenger forecast for that slot but also on the profitability for the airline, when compared to similar markets. Two types of turnarounds were searched: i) turnarounds that will expand the frequency of already existing routes and ii) turnarounds that will create completely new routes to other airports.

It is important to mention that this tool was mainly designed and it is more accurate in the forecast of new routes, that is, flights that were previously not operated by a certain airline. Consequently, the search for turnarounds that increase the frequency of old routes will result in less accurate transfer forecasts. Nonetheless, one of the experts from BEONTRA GmbH stressed that the values are still valid if the original frequency of the flights is not too high (e.g. if there is just a single flight operating everyday). This is the case with most of the routes that were expanded in this research. In some cases, the original schedule already featured several daily flights, but it is important to mention that those were operated by different airlines. Thus, the real frequency for each airline was significantly low.

Comparing each time slot of a route with some of its similar markets, and using the corresponding operating margins to decide whether that time slot is added to the input or not is also a valid assumption.

It is also relevant to mention that the tool already tries to optimize captured passengers, by delivering the best time alternatives for a certain route. Thus, there is a common line of reasoning between the tool and the model, in the sense that they are both aiming to the same optimization. This reinforces the idea that the tool is appropriate for this research.

Overall, BEONTRA GmbH *BRoute Development Tool* was adequate for this research and it was correctly consulted. The transfer passenger forecasts are not optimal and are prone to some errors as a consequence of what was explained in the previous paragraphs, but it is still acceptable for this model.

The definition of values for all the parameters and the arguments why those values were selected are presented in [section 3.1](#). All these arguments and assumptions were presented to one of the experts and it was confirmed that they are all valid and adequate to the research. In particular, it is adequate to establish a limit of 7h for a connection time. Only one day of operations is being analyzed. so using large connection times (for instance, 15h) would be disproportional to the time span of the input. Furthermore, since the model gives satisfactory results for connections within 40min and 7h, it will also deliver adequate results for larger intervals. Finally, if the time threshold would be larger, the problem size would increase, making it

potentially slower to solve.

With respect to the buffer time, it was assumed that a value of 5min would be reasonable. One of the experts confirmed that any value within 5min to 10min are acceptable for an airport.

It is now time to validate the results obtained for the different scenarios. It was possible to conclude that all the results for captured transfer passengers and connection times were within reasonable values. However, since for this research it is not possible to compare the absolute values obtained in the results, it becomes even more important to analyze the changes and tendencies between scenarios. It is paramount to understand whether the relative values of the scenarios and the comparison between them are valid. For instance, the increasing importance given to the minimization of connection times from scenario S2, to S3, S4 and S5 was translated in an effectively lower average connection time and general establishment of shorter connections, which confirms the validity of these results.

A detailed description of the results between scenarios and how those results are related was already performed in [chapter 4](#). This chapter can be used to confirm the consistency of results obtained throughout the research. There are exceptions that do not follow the general tendency or that do not have the expected behaviour, but these cases are not relevant enough to invalidate the results.

A part of the conclusions taken from the sensitivity analysis can also be used to validate the model. [Table 5.1](#) shows a comparison of the results for all scenarios S1 to S5 and for 4 days of operations. These days were not randomly chosen, but rather based on different days of the week - Tuesday, Thursday, Friday and Saturday. The Table shows a very stable behaviour along scenarios S1 to S5 for all days of operation. Scenario S1 is always the one with the lowest number of captured passengers, while S2 and S3 always have the highest value. On the other hand, the capacity of the hub to capture more transfer passengers decreases from S2/S3 to S4 and to S5.

The average connection times also have a quite stable behaviour. They tend to increase from S1 to S2, and they always decrease from S2 to S3 and from S4 to S5. Nonetheless, the change between S3 and S4 is not as stable. It seems that, for one of the operation days, the average connection time increases from S3 to S4, which shows that increasing the MIP gap in an attempt to obtain lower connection times is not always an efficient action.

Lastly, the run times of all scenarios were considered to be quite reasonable, especially when considering the large dimension of problems that model transfer passengers. Scenario S5 was clearly slower, but since this is a strategic stand planning more focused on the future expansion and not on short-term operations, that run time is still acceptable.

The evolution of values along all scenarios for the captured transfer passengers and connection times are logical, taking into account how each scenario was defined. This fact shows that the model is robust and delivers expected and consistent results. It is plausible to conclude that, by feeding real data into the model, it is possible to obtain realistic outputs and, more importantly, one can expect to obtain improvements when it comes to the maximization of transfer passenger throughput at the hub and to the minimization of connection times.

3

Case Study: Airport Information and Research Scenarios

This chapter is fundamentally divided into two parts. In the first part, in [section 3.1](#), important background information regarding the case study airport is provided. Furthermore, the values of some parameters used in the TLSAP and the arguments supporting the choice of those values are also presented in this section. The second part of this chapter, presented in [section 3.2](#), contains information about the research scenarios that will be run for the case study airport (using the parameters values and the information defined in [section 3.1](#)). The research scenarios will be needed to obtain results and ultimately answer the research questions. Finally, [section 3.3](#) explains in more detail how each research question will be approached and which research scenarios will be used to answer each of them.

3.1. Background Information on The Case Study Airport

Zones Definition for The TLSAP

Defining airport zones is a key aspect of the TLSAP. The division of the airport into zones is extremely versatile and can be easily adapted to different characteristics of the airport or requirements of authorities and stand planners. For instance, if a certain airline cannot park their aircraft in small group of stands (or even a single stand), it is possible to define that group as an independent zone and then state that any operation belonging to that airline is incompatible with that zone. For this research, however, such complex considerations were not accounted for. The definition of zones in this case study is based on the terminal's layout, on the proximity between stands and on the rules used by the airport for gate-stand dependencies. The zones can be visualized in [Figure 3.1](#). It is important to note, however, that one rule of this airport is that domestic arrivals can only be assigned to stands of Group 6C (on the bottom-left area of the terminal. For simplicity, it was assumed that domestic arrivals can be assigned to any stand of Zone 5C/6C, which includes Group 6C but also Group 5C.

The zones in orange correspond to "contact zones" and accommodate contact stands only. The terminal consists of three piers, and the stands connected to the same pier were grouped in the same zone. This leads to three contact zones, namely 1C/2C, 3C/4C and 5C/6C. On the other hand, zones in dark blue correspond to "remote zones", as they only contain remote stands. There are three zones of this type. Zone 1R is a small isolated group located Northeast of the terminal, and for that reason it was considered an independent zone. It is mainly served by the bus gates from pier A (Zone 1C/2C). Similarly, zone 6R is isolated Southeast of the terminal, and it is mainly served by the bus gates from pier B (Zone 5C/6C). Finally, zone 3R/4R is located East of the terminal and consists on large remote zone that is mostly served by the bus gates of pier C (zone 3C/4C). [Table 3.1](#) summarizes the most relevant information about all the zones. It should be noted that the term "MARS stand" refers to a set of 2 type C stands and 1 type F stand. No type F stands are included in [Table 3.1](#) because the case study airport does not have independent stands of this type. In the same Table, "Gate-Stand Dependency" refers to the combined use of stands and gates of the airport.

Walking Times Between Zones

The walking times between zones are a fundamental part of the current research. These times will determine whether critical connections can be established or not. [Table 3.2](#) shows the time, in minutes, needed to travel

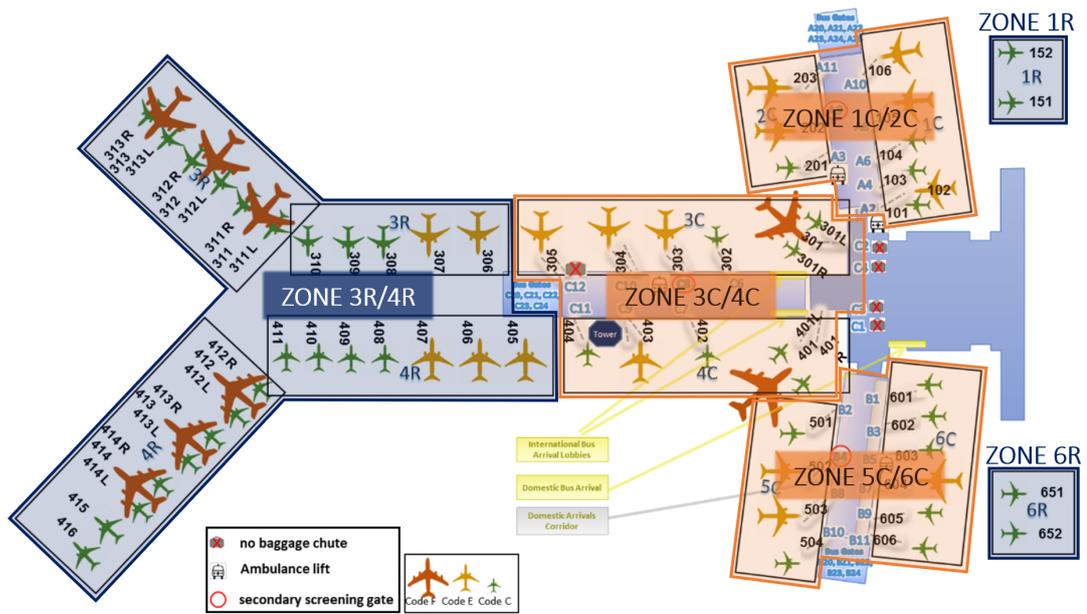


Figure 3.1: Zoning layout used in the TLSAP. Information provided BEONTRA GmbH, Germany.

Zone	Type	C Stands	E Stands	MARS Stands	Location	Gate-Stand Dependency
1C/2C	Contact	4	5	0	Pier A	Each stand is directly connected to a single gate (assumption)
3C/4C	Contact	3	4	2	Pier C	Each stand is directly connected to a single gate (assumption)
5C/6C	Contact	7	3	0	Pier B	Each stand is directly connected to a single gate (assumption)
1R	Remote	2	0	0	Northeast of Terminal	Bus gates from pier A (zone 1C/2C)
6R	Remote	2	0	0	Southeast of Terminal	Bus gates from pier B (zone 5C/6C)
3R/4R	Remote	9	5	6	West of Terminal	Bus gates from pier C (zone 3C/4C)

Table 3.1: Type, stand composition, location and gate-stand dependency information of the different airport zones defined in this problem. Information provided BEONTRA GmbH, Germany.

between zones. Note that the walking time matrix is symmetric. The time required to go from Zone_A to Zone_B is the same as the time needed to walk the opposite direction.

From \ To	1C/2C	1R	5C/6C	6R	3C/4C	3R/4R
1C/2C	40	70	50	80	50	80
1R	70	60	80	90	80	90
5C/6C	50	80	40	70	50	80
6R	80	90	70	60	80	90
3C/4C	50	80	50	80	40	70
3R/4R	80	90	80	90	70	60

Table 3.2: Walking times between zones of the airport, in minutes.

Within the same contact zone, it was established that the minimum time required to move from one stand to the other is 40 minutes. This time corresponds to the movement of passengers between stands within the

same zone of the airport. Even though in this airport a person can take less than 40 minutes to reach stands in the same zone, it is assumed that this value includes the whole process from leaving one aircraft, arrive in the terminal, walk to the other stand, potentially wait in a line and enter the other aircraft.

Within the same remote zone, it is assumed that 60 minutes are needed to make the connection. This is because passengers may need to enter the terminal and then head back to the zone, or be transported by bus, which logistically requires more time than simply walking inside the terminal.

It was assumed that the movement of passengers between contact zones takes 50min. It is slightly higher than the previous 40min since this movement implies a greater walking distance for passengers. However, note that moving between contact zones does not imply moving out of the terminal. Besides, the terminal dimensions are relatively small which makes the extra 10min enough to reach other contact zones.

The time between one contact zone and one remote zone depends on the relative position of both. If the contact zone contains the bus gates that serve the remote zone (this is the case with the pairs 1C/2C-1R, 5C/6C-6R and 3C/4C-3R/4R), it is assumed that those two areas are relatively close. However, passengers still need some time to walk to the bus gate, wait for a bus and be transported to the aircraft. Taken this into account, it was defined that the travel time between a contact zone and a remote zone that are close to each other is approximately 70min. If, however, the bus gates for the remote zone are not located in the contact zone's area, passengers are required to walk further. In this case, the travel time is set to 80min, so slightly higher than the previous case.

The maximum walking time was set to 1h30min (90min) and this corresponds to the movement of passengers from one remote zone to a different remote zone. Again, it is likely that this journey will take less than 1h30 if everything goes well. However, going from one remote zone to another may require passengers to board a bus, be transported to one of the terminal's bus gates, walk inside the terminal to reach the other bus gate, board a second bus and finally arrive at the other aircraft. The bussing process may be quite slow, especially when the flights are crowded and this is taken into account.

In general, it is assumed that it is easier for passengers to walk inside the terminal, and thus all travel times between contact zones are lower than the travel times that involve at least one remote zone. The more obstacles passengers have to face to reach a zone, the higher the travel times are set. It is important to recall that these are just assumptions based on the layout and size of the airport, so they can be easily modified. It is possible, for instance, to have more zones with smaller sizes each. This may increase the level of accuracy of time travels. Nonetheless, in the context of this research, assuming constant walking times for all stands inside a zone is an accurate approximation, since those stands are very close to each other and the walking time differences can be neglected.

Connection Times Thresholds

One of the central aspects of this research is the creation of connections between arrival-departure pairs. For this reason, it was mentioned in [section 1.1](#) that connection times can be divided into 3 periods: critical connections, medium connections and long connections. The connection time limits that split these 3 periods vary per airport and can be defined by the airport's authorities according to their objectives and needs. The threshold values of each zone can be consulted in [Figure 3.2](#) and an explanation of why each value is chosen is portrayed below:

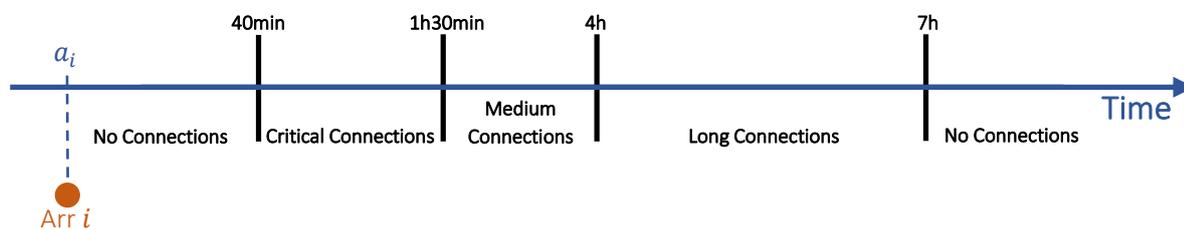


Figure 3.2: Time limits for each connection time period defined in the Two Level Stand Assignment Problem for this case study. The time limits are not to scale.

- **Critical Connections Period:** the time limits of this period are highly dependent on the airport's terminal layout. First, the lower limit of the Critical Connections Period corresponds to the shortest walking time shown in [Table 3.2](#): 40min. If the elapsed time between an arrival and a departure is shorter than 40min, passengers do not have enough time to reach any other stand, according to [Figure 3.2](#). On the

other hand, if the elapsed time between an arrival and a departure is larger than the longest walking time in the airport (1h30min according to Table 3.2), then passengers will always have time to make that connection. Consequently, and following the definition given in section 1.1, that connection is non-critical. Hence, the upper limit of the Critical Connections Period has to be established at 1h30min to guarantee consistency with the period definition.

- **Medium Connections Period:** this period extends from the end of the Critical Connections Period (1h30min) until the 4h mark. It is important to note that, for the model, the Medium Connections Period and the Long Connections Period are treated the same way. This division is made to obtain more segmented results, which makes the analysis easier and more robust. The value of 4h as the upper limit for the Medium Connections Period was chosen because a layover with a length ranging from 1h30min to 4h is still fairly acceptable for passengers. Furthermore, 4h roughly marks the midpoint of the considered connection lengths (40min to 7h).
- **Long Connections Period:** this period extends from the end of the Medium Connections Period (4h) to the largest possible connection time, 7h. This value was chosen for two reasons. Firstly, waiting for a connection that takes more than 7h can make passengers slightly unhappy and they may look for alternatives at other airports. Thus, these connections may not be profitable for the hub. Secondly, the results of this research show that it is academically relevant to focus more on connections that are relatively short. If the model proves efficient for the range [40min,7h], it is also efficient for wider ranges. It is also important to mention that setting the maximum connection to 7h still allows for overnight layovers to take place (for instance, from 2a.m to 8a.m.). In reality, this value can be extended as much as desired. For instance, it is possible to consider very long connections (15h-20h) which sometimes happen at large hub airports.

Towing Rules

Each airport has its own rules that define when an aircraft can be towed and who is responsible for that move. Usually, airports define a ground time threshold beyond which aircraft are allowed to be towed. This time threshold splits turnarounds into two types:

- **Short-Stay Aircraft:** the aircraft's ground time is smaller than the threshold, and it cannot be towed. The turnaround is divided into two operations - {**Arrival, Departure**}.
- **Long-Stay Aircraft:** the aircraft's ground time is larger or equal than the threshold, and it may be towed. The turnaround is divided into three operations - {**Arrival, Idle, Departure**}.

In the case study of the current research, the ground time threshold that indicates when an aircraft can be towed depends on the ICAO design group to which that aircraft belongs to. Furthermore, the dedicated time for disembarking during the arrival operation and for embarking during the departure can be dependent on the ICAO design group as well. Table 3.3 summarizes how those rules were applied in this research, and it also shows how a turnaround is divided in time into its corresponding operations.

Aircraft Type	Turnaround Type	Ground Time (hours)	Operations Duration (minutes)		
			Arrival	Idle	Departure
C	Short	< 3	$(ETD - ETA)/2$	-	$(ETD - ETA)/2$
	Long	≥ 3	60	$(ETD - ETA - 60 - 90)$	90
D,E,F	Short	< 4	$(ETD - ETA)/2$	-	$(ETD - ETA)/2$
	Long	≥ 4	90	$(ETD - ETA - 90 - 120)$	120

Table 3.3: Rotation types and duration of the corresponding operations, based on ground times. Information provided BEONTRA GmbH, Germany.

Peak Hours Definition

The day of operation chosen to obtain the results showed in [chapter 4](#) was the 08/12/2020. The definition of the peak hours for arrivals and departures was based on how busy each hour is by itself, but also on how much busier each hour is relatively to the neighbouring hours. If, for a certain hour, the number of arrivals or departures is higher than 10 and the surrounding hours are, for the most part, not was busy, then that period will be considered a peak. [Figure 3.3](#) shows the arrival peak hours (in yellow) while [Figure 3.4](#) shows the departure peak hours (also in yellow). It is important to note that 6h and 7h were considered two consecutive arrival peak hours for being particularly busy with arrivals and much busier than periods like 4h, 5h and 8h.

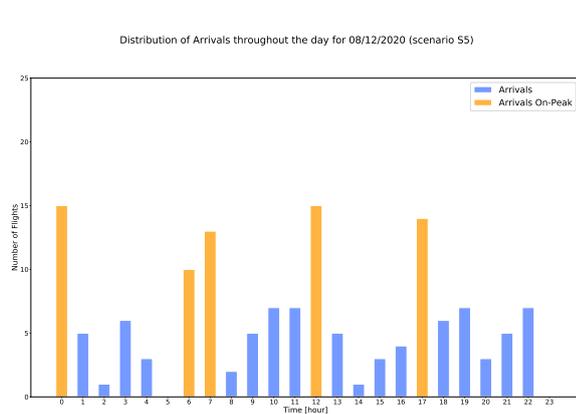


Figure 3.3: Identification of arrival peaks for the day 08/12/2020.

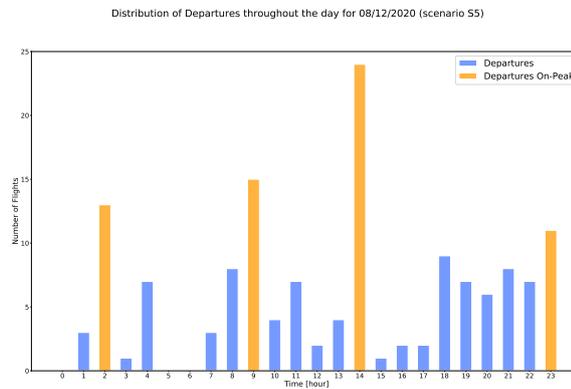


Figure 3.4: Identification of departure peaks for the day 08/12/2020.

Schedule Robustness

In line with what was explained in [subsection 1.4.4](#), a buffer time is added before each operation to increase the robustness of the model. As a consequence of this buffer time, two non-overlapping operations cannot be assigned to the same stand if the time gap between the end of one and the beginning of the other is smaller than the buffer time.

The value of the buffer time usually varies for each different airport and there are no strict rules that define that value. The airport in this research's case study is not particularly busy during a regular day of operations. This fact leads to three conclusions: i) it is less likely that one operation will be delayed, ii) if one operation is delayed it will not necessarily create a domino effect because all operations are generally more spaced in time and iii) in case an operation indeed causes a domino effect of delays, it is possible to reassign one operation to a different stand and that can easily solve the delay issue. With these three notes in mind, and using the assumption that most of the delays during a regular day of operations are not significantly high, it was concluded that the buffer time chosen for this research does not need to be significantly high.

In previous research, a buffer time of 10 minutes was chosen for a busy airport [64]. Taking this into account, it was decided that a buffer time of 5 minutes was adequate for this research. It gives a sufficient and realistic schedule robustness for the airport considered in this research without significantly disturbing potential hub connections.

Big-M Method

In [chapter 1](#), the Big-M method was introduced in some constraints (refer to [Equation 1.42-Equation 1.45](#)). The value of M has to be sufficiently large to avoid the risk of capping the z_{ijvw} and x_{ij} variables. Simultaneously, very high values for M could make the model unstable. Both these variables types correspond to transfer passengers connection between two flights. Thus, it is enough to define a value for M that is higher than the seat capacity of any aircraft travelling through the airport, since z_{ijvw} and x_{ij} variables can, at most, be equal to the seat of capacity of the largest aircraft. For this research, and after an analysis of all flights, it was defined that $M = 500$. For safety reasons, there is a margin between the largest seat capacity and this value of M . Thus, the Big-M method can now be applied without concerns.

3.2. Research Scenarios

The day of operations selected to obtain results is the 08/12/2020. The original schedule for this day was provided by BEONTRA GmbH, Germany. The proposed turnarounds are obtained using BEONTRA GmbH *BRoute Development Tool*. All time alternatives of each turnaround were chosen based on their profitability to the airline and on the forecast of transfer passengers, both indicated by the tool. The profitability is related to the operating margin of that specific alternative when compared to some similar markets, that is, to routes that are similar to the one proposed. The list of proposed turnarounds and corresponding time alternatives is shown in Table 3.4. Recall that there are 2 types of proposed turnarounds: those that will expand the frequency of original routes and those that will open new routes. In Table 3.4, the turnarounds associated to the new routes are highlighted in orange.

Airport [IATA]	Arr. Time [hh:mm]	Dep. Time [hh:mm]	Ground Time [min]	Transfer Pax Forecast	On-Peak? (Arr/Dep)	Airport [IATA]	Arr. Time [hh:mm]	Dep. Time [hh:mm]	Ground Time [min]	Transfer Pax Forecast	On-Peak? (Arr/Dep)
Airport1	00:10	02:00	110	134	YES/YES	Airport12	18:55	20:45	110	44	NO/NO
	07:35	14:10	395	155	YES/YES		22:55	23:40	45	53	NO/NO
	18:50	20:50	120	89	NO/NO		07:30	14:30	420	63	YES/YES
Airport2	18:50	20:55	125	22	NO/NO	Airport13	00:25	01:30	65	125	YES/NO
	00:20	08:40	500	80	YES/NO		17:45	21:30	225	125	YES/YES
	13:00	20:55	475	80	YES/NO		18:30	20:50	140	112	NO/NO
Airport3	00:30	01:55	85	125	YES/NO	Airport14	00:05	01:35	90	89	YES/NO
	20:10	21:55	105	125	NO/YES		00:05	13:50	825	89	YES/NO
	12:40	14:20	100	125	YES/YES		22:15	23:55	100	89	NO/NO
Airport4	00:30	01:30	60	86	YES/NO	Airport15	00:30	01:30	60	80	YES/NO
	17:00	20:55	235	76	YES/NO		01:00	01:45	45	35	YES/NO
	00:30	08:30	480	85	YES/NO		13:05	14:10	65	34	NO/YES
Airport5	00:45	02:00	75	46	YES/YES	Airport16	00:50	01:45	55	65	YES/NO
	07:30	14:10	400	63	YES/YES		07:35	14:10	395	89	YES/YES
	00:50	04:50	240	89	YES/NO		00:30	02:00	90	89	YES/YES
Airport6	06:00	14:55	535	89	YES/YES	Airport17	21:40	22:20	40	89	NO/NO
	22:10	23:00	50	84	NO/NO		18:45	21:40	175	75	NO/YES
	09:20	13:00	220	164	NO/NO		00:30	01:35	65	73	YES/NO
Airport7	11:00	13:00	120	104	NO/NO	Airport18	00:30	08:15	465	72	YES/NO
	07:35	13:35	360	135	YES/NO		00:15	01:30	75	80	YES/NO
	00:50	02:00	70	99	YES/YES		13:10	14:10	60	76	NO/YES
Airport8	10:50	14:10	200	56	NO/YES	Airport19	18:50	20:55	125	72	NO/NO
	00:30	01:30	60	80	YES/NO		00:15	01:35	80	115	YES/NO
	13:10	14:10	60	69	NO/YES		21:55	23:30	95	118	NO/NO
Airport9	13:10	20:55	465	80	NO/NO	Airport20	17:55	23:30	335	107	YES/NO
	07:30	08:30	60	146	YES/NO		00:15	01:35	80	66	YES/NO
	00:15	08:30	495	155	YES/NO		13:10	20:55	465	48	NO/NO
Airport10	00:15	01:45	90	126	YES/NO	Airport21	07:35	14:10	395	117	YES/YES
	07:30	14:10	400	124	YES/YES		00:00	01:50	110	94	YES/NO
	22:25	23:05	40	149	NO/NO						

Table 3.4: List of proposed turnarounds and corresponding time alternatives. The turnarounds associated with new routes are highlighted in orange. Information retrieved from BEONTRA GmbH *BRoute Development Tool*.

Several research scenarios are proposed. These scenarios may differ on the input given to each of them or on the definition of certain parameters. The scenarios are especially designed to be used as a guide to answer the research questions, formulated in the literature study.

To better understand the table, please refer again to subsection 1.4.2 to review the hierarchical order in which the multi-objective function terms are solved.

Research Scenario	Day of Operation	Original Turnarounds	Proposed Turnarounds	MIP Gap	Objectives Solving Order
S0	08/12/2020	Included	Not Included	0.01% (default)	Obj.1 → Obj.2 → Obj.3 → Obj.4
S1	08/12/2020	Included	Included	0.01% (default)	Obj.1 → Obj.4
S2	08/12/2020	Included	Included	0.01% (default)	Obj.1 → Obj.2 → Obj.4
S3	08/12/2020	Included	Included	0.01% (default)	Obj.1 → Obj.2 → Obj.3 → Obj.4
S4	08/12/2020	Included	Included	5%	Obj.1 → Obj.2 → Obj.3 → Obj.4
S5	08/12/2020	Included	Included	0.01% (default)	Obj.1 → Obj.3 → Obj.2 → Obj.4

Table 3.5: Research scenarios and major elements that characterize each of them.

The first research scenario, **S0**, is the simplest scenario. Only the original schedule is considered here but the two main goals of this research - maximizing transfer passenger throughput (objective 2) and minimize connection times (objective 3) - are included. S0 can be seen as a control scenario to which other scenarios will be compared, portraying how the aircraft would be hypothetically assigned to stands before any attempt to include new turnarounds.

Unlike scenario S0, **S1** includes both the original and the proposed set of turnarounds. However, the algorithm disregards both main goals of the research. This scenario simulates the case in which new flights are added to the schedule but they are not efficiently assigned to improve the hub connectivity.

Scenario **S2** includes both the original set and the proposed set of turnarounds. However, passenger connection times are not minimized (objective 3). Scenario **S3** covers this gap by attempting to maximize the number of captured transfer passengers and also to minimize connection times.

The MIP gap, as defined by *Gurobi*, is a stopping criterion. When the value obtained by the solver is within the defined gap from the best bound, that value is accepted as the optimal solution and the branch and bound stops. In a hierarchical approach, the MIP gap has two different applications. Let's consider, without loss of generalization, that the hierarchy consists of two objectives. The first objective is optimized until a value that is within the defined gap from the best bound is found. Then, the second objective is optimized following the same logic. However, at the same time, the solver is also able to deteriorate the first objective if the value obtained after that deterioration is still within the first objective's gap, and if it improves the second objective. This dynamic is represented in Figure 3.5. In this simple example (using once again two objectives), it is shown how the optimization of a certain objective can affect the optimal value of the previous objective. The schematic also portrays the relationship between the gaps of both objectives' solutions and the decision of accepting/rejecting a solution. Note how the solution gaps of both objectives need to be verified for a solution to be accepted.

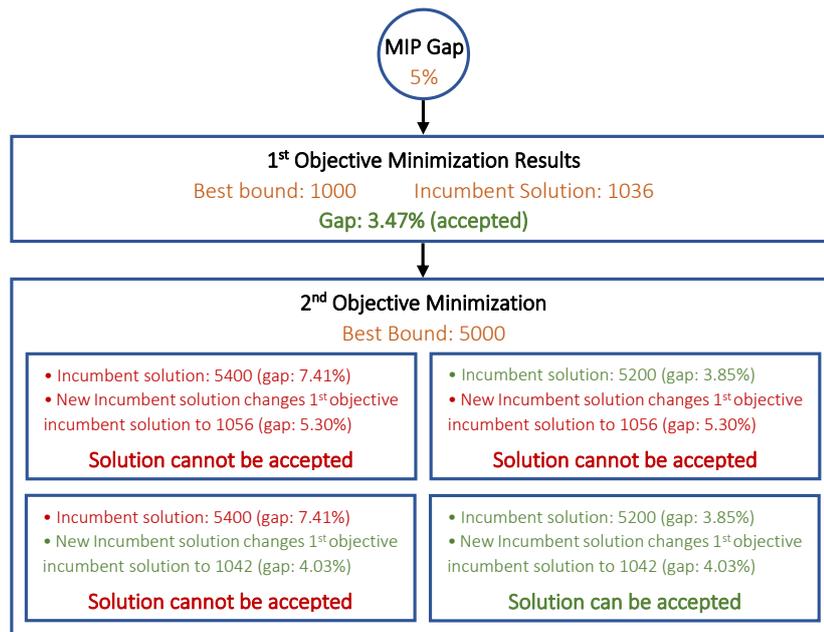


Figure 3.5: Schematics showing the dynamics of the MIP gap, defined in *Gurobi*. Solutions that exceed the gap are shown in red. Solutions that fall within the gap are shown in green.

If the MIP gap is large enough, this gives flexibility to obtain slightly better results on lower steps of the hierarchy while still giving priority to higher steps.

Up until now, all scenarios are run in *Gurobi* with a MIP gap of 0.01%. This corresponds to the default value defined by the solver. This value is so low, that in practice it makes the hierarchy virtually inflexible. For most of the cases, the gap value is not large enough to allow the deterioration of previous objectives. For instance, the number of captured transfer passengers is in the order of 10^3 . If this value is multiplied by a gap of 0.01%, one obtains 0.1. This translates to a loss of one tenth of a passenger, which is not allowed to happen due to the integer nature of the variables. Thus, all previous scenarios follow a strict priority of objectives.

Scenario **S4** breaks this pattern and introduces different MIP gaps in order to test the effects of increasing the flexibility of the algorithm. Following the model definition in [subsection 1.4.2](#), maximizing the number of transfer passengers has a higher priority than the minimization of connection times. Thus, increasing the MIP gap will, in theory, compromise some airport connections to reduce connection times. Apart from the MIP gap, **S4** is very similar to **S3**, because both the original and the proposed flight sets are considered.

In line with what was mentioned in the previous paragraph, the maximization of captured transfer passengers is higher in the hierarchy with respect to the minimization of connection times. However, it may be helpful to understand what can happen if this order is changed, that is, if connection times are minimized before captured transfer passengers are maximized. This change is materialized in scenario **S5**. With this scenario, it may also be possible to infer how the TLSAP will react to this change and to which extent it will be applicable to a real case.

3.3. Plan to Answer Research Questions

The ultimate objective of this research is to be able to answer the questions formulated during the literature study conducted prior to this work and draw conclusions from them. To help answering those questions the previous section will be used, as well as the research scenarios defined in [section 3.2](#). These scenarios were carefully developed to be applied in the resolution of the research questions. During the remainder of the current section, the plan to answer each question is explained, as well as the expected results and conclusions that can be taken from them.

- **Question 1: How can the Stand Assignment Problem (SAP) be formulated in order to be integrated in this research?**

This introductory, more theoretical question, is based on [chapter 1](#) and on the conclusions taken from the literature study.

- **Question 1.1: How does the individual importance given to the maximization of transfer passenger throughput and the minimization of connection times influence the final solution and how does it affect the satisfaction of the airport authorities and passengers?**

This is a core question in the sense that it analyses the dynamics of the two main goals of the research. It is thus important to create a broad number of scenarios that allow to reach a consistent and complete answer. To this end, results taken from scenarios **S1**, **S2**, **S3**, **S4** and **S5** will be used to answer Question 1.1.

S1 should be the least efficient scenario since neither the number of captured passengers nor the connection times are minimized. **S2** does not minimize connection times, which means that this objective is completely disregarded. The main optimization is focused on the maximization of the number of captured transfer passengers. **S3**, on the other hand, minimizes connection times after maximizing captured transfer passengers. For this reason, an improvement in connection times is expected from **S2** to **S3**. An improvement in the number of connection times from **S1** to both **S2** and **S3** is also predicted. With respect to the connection times comparison between **S1** and **S2**, it is not possible to have any concrete hypothesis. Since neither consider the minimization of connection times, no specific tendency will necessarily be found. Nonetheless, this will still be analyzed in [chapter 4](#).

Scenario **S4** gives even more importance to the optimization of connection times, by increasing the MIP gap value and, thus, the flexibility of the algorithm. As a consequence of the trade-off, it is very likely that several passenger connection opportunities will be lost with this flexibility.

Finally, **S5** is characterized by an inversion of the priority order of objectives 2 and 3. Connection times will be minimized before the number of transfer passengers are maximized. This priority inversion translates to a significant change in the original model that could lead to unexpected outputs. Consequently, the results might not be entirely realistic nor directly applicable in real life. This situation will be further analysed in [chapter 4](#). Nonetheless, **S5** should achieve a relatively low value in the average connection times, probably the lowest compared to scenarios **S1** to **S4**. Consequently, the number of passengers should also decrease.

It is expected that, overall, **S3** will deliver the best results, but this shall be confirmed in the results.

- **Question 2: Which Heuristic or Meta-Heuristic method is the most fitting to solve the GAP?**

In line with what was said before, choosing an adequate solving method is the key to reach good results within an acceptable amount of time. It was shown in [chapter 1](#) that a two-level assignment approach is a plausible approach in the context of this research, but this will be further discussed in [chapter 4](#).

For this research question, the run time results from scenarios **S0** to **S5** will be used specifically to check the speed of the algorithm, that is, how much time, on average, it takes to obtain an output. Potentially, these results could reinforce the choice of the two-level assignment model. It is likely that the run times will differ according to the complexity of each scenarios. Scenarios that make use of more objectives should take longer to run.

- **Question 3: What is the nature of the relationship between the way new flights are distributed over time (in peak or off-peak hours) and the types of solutions obtained? In particular, does allocating flights between peaks increase the number of possible connections and/or reduce the connection times?**

The flight peaks were identified in [section 3.1](#). It is important to analyze how the new turnarounds (and thus the new flights) are distributed over time. Furthermore, changing some parameters of the model, namely the relative importance of the maximization of captured transfer passengers and the minimization of connection times, may cause the model to change the selected time alternative for each new turnaround. It is relevant to understand whether this happens, and if it does, what is the nature of that change. This research question asks explicitly if the model will distribute the new flights more evenly along the day (in between peaks) to increase the number of captured passengers or reduce connection times, but other questions can be asked. For instance, if the relative importance given to the maximization of captured transfer passengers and the minimization of connection times changes, will the new flights be clustered in certain periods of the day that do not correspond to peaks?

For this question, the distribution of the new turnarounds (namely their arrivals and departures) of scenarios S2, S3, S4 and S5 will be analyzed and compared with the other scenarios. Furthermore, some specific connections will also be showed to make some important comparisons.

- **Question 4: As a consequence of the growth in the number of captured transfer passengers, how much do hub connections expand and how profitable could the routes potentially be?** The last question tries to find answers on the specific network expansion of the hub. In particular, how much did the original routes expand and what are the new connection opportunities brought with the new routes? Will the airport be connected to different parts of the world?

In this question, the selected time alternative of each proposed turnaround will be analyzed in terms of how large its transfer passenger forecast is, but also how profitable it might potentially be for the airline. This analysis could reveal if the selected alternatives might be operated by the airlines in the future.

Some information will be taken from all scenarios to answer this question. However, most of the results will correspond to scenario S3, since this is the most standard scenario (it considers all objective functions and uses the standard MIP gap).

4

Research Scenarios Results

In this Chapter, the results obtained from the research scenarios are extensively analyzed. Each research question is answered individually by resorting to the results of some (or all) research scenarios. In [section 4.1](#), Question 1 answered and Question 1.1 is answered in [section 4.2](#). Then, Question 2 is analyzed in [section 4.3](#), followed by Question 3 in [section 4.4](#) and finally Question 4 in [section 4.5](#).

4.1. Question 1: Analysing the Formulation of the SAP Used in this Research

The Formulation of the Stand Allocation Problem used in this research is explained in great detail in [chapter 1](#). Many formulations were analysed during the Literature Study, including linear and quadratic formulations. Since the beginning of this research, an effort was put to develop a Mixed Integer Linear Programming (MILP) formulation since this formulation is extensively studied in previous works and it is very flexible because it allows to combine different variable types. In the model developed for this research, variables of 3 types are used, namely binary, integer and continuous. At the same time, the formulation is linear, which usually means that solving the problem is substantially easier and a commercial solver based on the branch and bound method can be used. It is important to mention that commercial solvers can also integrate quadratic constraints or objective functions, but the time to solve those grows significantly.

In the definitive approach to the problem, a two-level model was proposed. A different set of objective functions and constraints were defined for the first and second levels, but both formulations are mixed integer and linear. It was possible to write all elements using only linear terms, so the MILP proved to be a suitable formulation of the problem. In [section 4.3](#), where Research Question 2 is answered, it is possible to conclude that this linear mixed formulation also contributed (together with the solving method developed) to lower the computational and memory demand needed to solve the whole problem.

4.2. Question 1.1: Studying the Relative Importance Given to the Main Goals

The dynamic of the relative importance given to the maximization of captured transfer passengers and minimization of connection times is rather complex. Changing the importance given to each of the 2 main goals can significantly change the final output. In general terms, the airport is more interested in maximizing of captured transfer passengers while passengers prefer shorter connection times, for their convenience. In this section, it will be shown how these two objectives conflict with each other. Thus, the airport authorities have to find a balance between the two objectives that sufficiently satisfies the two stakeholders. [Table 4.1](#) shows information regarding the captured passengers and connection times obtained as a result of the 4 scenarios used in this question. Scenario S0 is not included in this Table since it does not include the proposed turnarounds.

First of all, note that all available turnarounds were added to the schedule in all scenarios, which means that the number of flights is constant throughout all scenarios. Consequently, the airport has stand capacity for even more turnarounds, which can be added in the future.

Scenario	Captured Transfer Passengers				Avg. Conn. Times	Assigned TAs [%]
	Critical Conns.	Medium Conns.	Long Conns.	Total		
S1	642 (14.40%)	3051 (68.45%)	764 (17.14%)	4457	2h41min	100
S2	1177 (20.75%)	3354 (59.14%)	1140 (20.10%)	5671	2h48min	100
S3	1402 (24.72%)	3367 (59.37%)	902 (15.91%)	5671	2h34min	100
S4	1451 (26.10%)	3243 (58.34%)	865 (15.56%)	5559	2h32min	100
S5	1631 (32.32%)	2807 (55.62%)	609 (12.07%)	5047	2h18min	100

Table 4.1: Results regarding the number of captured transfer passengers and connection times (TA = Turnaround).

Analysis to Scenario S1

In the first scenario to be considered, S1, the proposed flights are added to the input of the model, but neither the maximization of transfer passengers nor the minimization of connection times are regarded by the model. The results for this scenario show that a total of 4457 transfer passengers are able to connect at the airport. From the results of scenario S1, it is also possible to conclude that 605 of transfer passengers have a connection time between 40 minutes and 1 hour and 30 minutes (interval corresponding to critical connections), which corresponds to only 14.40% of the total amount of transfers. On the other hand, most passengers have connection that last from 1 hour and 30 minutes to 4 hours. This interval corresponds to the non-critical period. Finally, there is also a considerable part of passengers that was offered a long connection, with waiting times between 4 hours and 7 hours. The average connection time is 2 hours and 41 minutes.

Analysis to Scenario S2

In scenario S2, the model maximizes the number of captured transfer passengers, but the newly proposed flights are not allocated in a way that minimizes connection times. The results from this scenario show a significant improvement when compared to scenario S1. The number of transfer passengers now captured increases 27.24% to 5671. As it was expected, the number of passengers increased in all three periods considered in this research. It is worth mentioning that, because connection times are not minimized in scenario S2, the average connection time increased by 7 minutes to 2 hours and 48 minutes, again compared to S1. By maximizing the number of transfer passengers, the model had to select certain connections that, overall, raised the average connection times. This outcome is not surprising because in S2 connections are established randomly and not based on connection times.

Analysis to Scenario S3

In scenario S3, connection times are minimized after transfer passenger throughput is maximized. The results obtained from this scenario were expected. The first thing to notice is that the number of captured transfer passengers does not change from S2 to S3, which is logical taking into account that both maximize the passenger throughput and use the same parameters (same hierarchy order and MIP gap). However, several connections were either shortened or replaced by shorter connections from S2 to S3. In S3, around 225 more passengers are offered a short, critical connection. On the other hand, fewer passengers will have to wait for non-critical connections. The number of passengers with connection times laying on the second period of time (from 1 hour and 30 minutes to 4 hours) slightly increases by 13, while the third period of time (from 4 hours to 7 hours) sees a reduction of 238 transfer passengers. Ideally, the goal with the minimization of connection times was to push as many connections as possible to the critical zone, as this assigns more passengers to shorter connections. The comparison between scenarios S2 and S3 showed that this is indeed verified with the given input data.

Previously, the change in the passenger distribution was analysed, but it is also important to study how connection times were in fact affected. The results from the table show that the average waiting times reduced 14 minutes from scenario S2 to scenario S3, which corresponds to a change of 8.33%. Without any further analysis, one can already conclude that this is a significant change. However, it is worth mentioning that the high number of total transfer passengers, 5671, implies that a significantly large number of passengers needs to be moved to shorter connections in order to obtain big reductions of the average connection time. It is also important to remind that only the proposed turnarounds, which make up only 16% of all turnarounds, can be moved in time. Consequently, the flexibility given to the model is quite limited. This situation asks for the introduction of other KPIs to better analyse the results and investigate if there is in fact a substantial change in the connection times. To this end, [Figure 4.1](#) shows the transfer passenger distribution as a function of connection times.

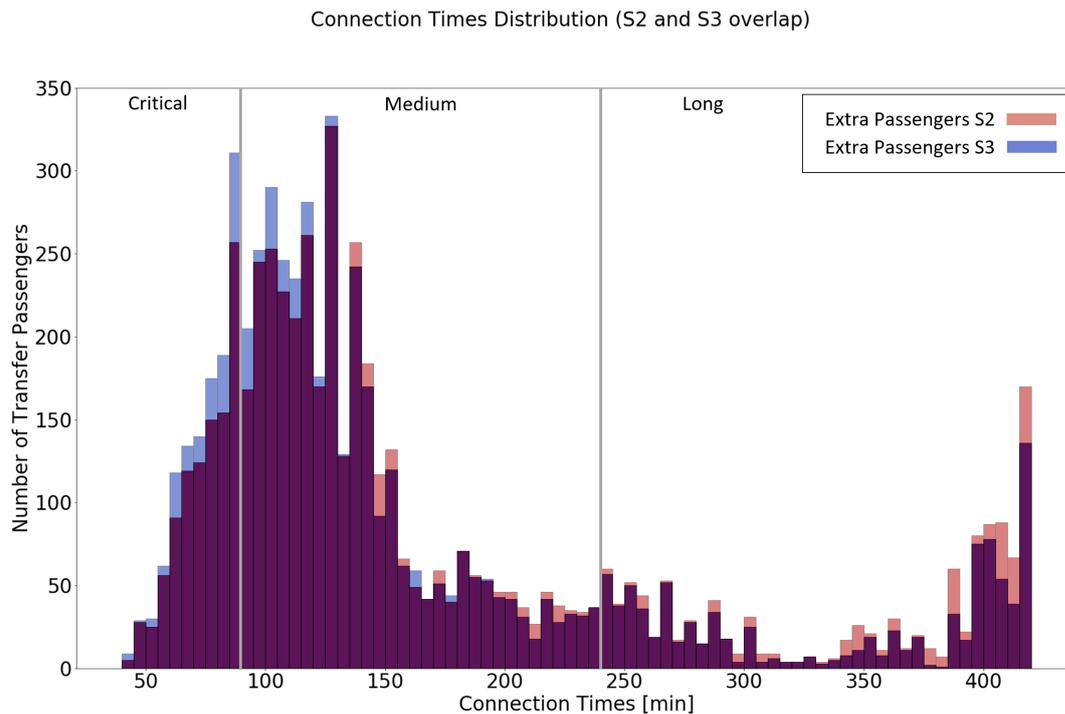


Figure 4.1: Change in the connection times distribution between scenarios S2 and S3. Each bar corresponds to a 5 minute interval. The light blue portion of each bar is related to extra passengers in S3 and the light red portion corresponds to extra passengers in S2.

The horizontal axis of the plot shown in Figure 4.1 is divided in 3 periods that correspond to the connections periods defined before: the critical connections period, the medium connections period, and the long connections period. The plot also depicts an overlap of the results obtained for scenarios S2 and S3, and each bar represents a 5 minutes interval. The darker, purple areas of the bars correspond to actual overlap of transfer passengers between the two scenarios. On the other hand, the light blue areas are related to an excess of passengers in that specific 5 minutes period for scenario S3, while light red areas show a similar excess, but now for scenario S2.

Figure 4.1 shows the effect of optimizing connection times. There is a clear tendency for passengers to be allocated to short connections in scenario S3. Note how in this scenario most of the passenger connections that were moved in time are almost completely clustered in the critical connections period or in the beginning of the medium connections period, as suggested by the light blue portion of the bars. At the same time, in scenario S3, considerably fewer passengers have long connection (higher than 350 minutes). This transference of passengers from the longest connections to the shortest connections was desired when the minimization of connection times was included in the hierarchy. It is important to stress that almost all 5-minute interval in the long connections period registered a decrease in the number of transfer passengers.

It is also possible to see an increase in the number of passengers making short non-critical connections (between 90 minutes and 100 minutes). These are still reasonably short connections, close to the critical period. The highest peak in the number of connections (S2) happens in the 125-130 minutes interval, but it becomes less significant in S3, since the number of passengers with connections in the interval 85-90 minutes increases quite significantly in S3. This contributes to the overall reduction of connection times.

Analysis to Scenario S4

It is now important to introduce scenario S4 and compare it with the previous scenarios. In S4, the maximization of captured transfer passengers is still prioritized over the minimization of connection times. However, the MIP gap parameter is changed from the standard 0.01% to 5%. In theory, this should give the objectives that are lower in the hierarchy some room to be further optimized, when compared to previous scenarios. There is, in fact, a reduction of the average waiting times when compared to both S2 and S3. The change with respect to S3 is of 2 minutes and of 16 minutes when compared with S2. The critical connections period is the only period that saw both the number of passengers and respective percentage increase, with a change of 49 passengers or 3.50%, which is quite significant for a MIP gap of only 5%. This outcome was desired and

predicted to be obtained when the change in the MIP gap value was applied. It is confirmed that it is, in fact, possible to obtain an even lower value for the waiting times.

This further reduction of the waiting times is, however, followed by a decrease of the number of captured transfer passengers. It is logical that this value is lower than that of scenarios S2 and S3 and it could not be differently. Let's suppose that it would be possible to obtain the average connection time verified in S4 (2 hours and 32 minutes) without reducing the number of passengers in scenarios S2 and S3 (5671). Then, in scenarios S2 and S3, it should have also been obtained an average waiting time of 2 hours and 32 minutes. This is true because *Gurobi* always tries to optimize all objectives, even in a hierarchical approach. The solver only deteriorates previous objectives if it leads to better results on the objective being currently solved.

Analysis to Scenario S5

The last scenario to be analysed is S5. In this scenario, the model presented in subsection 1.4.2 is slightly adapted. The hierarchical priority of the minimization of connection times and maximization of captured transfer passengers is swapped. The results for this scenario are also included in Table 4.1. The first thing to notice is that S5 registers the highest percentage of passengers offered a critical connection. Almost a third of the passengers will have a connection shorter than 1 hour and 30 minutes, which corresponds to an increase of almost 12% from scenario S2, in which connection times were completely disregarded. Simultaneously, both the medium connection and the long connection periods have considerably fewer passengers and lower percentages than any other scenario shown so far. As a result of this change, the average connection is considerably reduced. There is a reduction of 14 minutes (9.21%) with respect to S4 and of 30 minutes from the scenario in which connection times are not minimized, S2. This corresponds to a much more significant change than the one found between scenarios S2 and S3, 8.33%. In fact, the average connection time of scenario S5 is, by far, the lowest of all scenarios used in this question.

However, at the expense of reducing connection times, the total number of captured transfer passengers in S5 is also lower than any other scenario from S2 to S4. There is a reduction of 9.21% compared to S4 and a reduction of 11.00% compared to S2 and S3 (the scenarios with the highest number of captured passengers). While the change from S2/S3 to S4 was not really significant, the change from S2/S3 to S5 can already have an impact in the performance of the airport in terms of captured passengers.

Analysis to Transfer Passengers Connecting To/From New Flights

Since several flights are being added to the schedule, it is relevant to analyse how long transfer passengers connecting via those flights need to wait at the airport. To this end, a more detailed result analysis of scenarios S2 to S5 is shown in Table 4.2. This Table only shows the transfer passengers connecting between two flights in which at least one is a newly proposed flight.

Scenario	Captured Transfer Passengers				Avg. Conn. Times
	Critical Conns.	Medium Conns.	Long Conns.	Total	
S2	664 (31.51%)	894 (42.43%)	549 (26.06%)	2107	2h58min
S3	822 (36.60%)	976 (43.46%)	448 (19.95%)	2246	2h39min
S4	900 (42.43%)	833 (39.27%)	388 (18.29%)	2121	2h31min
S5	1044 (56.04%)	619 (33.23%)	200 (10.74%)	1863	2h00min

Table 4.2: Results regarding the number of captured transfer passengers and connection times, obtained from scenarios S2 to S5, but only for connections between two flights in which at least one is a newly proposed flight.

Table 4.2 shows that, for connections involving at least one new flight, there is an increase of over 5% in the number of critical connections, from scenario S2 to scenario S3. However, the long connections period registers a reduction of 6%. It is also worth mentioning that the average waiting time decreases significantly from S2 to S3. There is a change of 19 minutes, which corresponds to a decrease of 10.67%. This decrease is more significant than that of the global output, which was previously shown to be equal to 8.33%. This result is logical if one recalls that only the new flights can be moved in time, which means that connections involving at least one of these flights are more flexible.

With Table 4.2, the effect of changing the MIP gap to 5% is even more noticeable. The percentage of passengers being offered a critical connection gets closer to half in S4. Furthermore, it increases around 12% when compared to scenario S2, in which no connection times optimization is performed. The average time also decreases significantly in scenario S4. It is reduced by 27 minutes with respect to scenario S2 (15.17%).

The change in S5 is the most significant out of all scenarios. The number of passengers with short connections (in the critical period) increase from 42.43% in scenario S4 to 56.04% in scenario S5. For the first time, more than half of the connections making use of new flights take between 40min and 1h30min. It is worth to note that there is a decrease of almost 1 hour in the average connection time from S2, where waiting times are not minimized, to S5, the scenario that attributes the highest importance to the optimization of connection times. The reduction of captured passengers in S5 is also visible in Table 4.2.

Overall, the results in Table 4.2 show that, similarly to the global output, the tendency to push connections to the critical period along scenarios S2, S3, S4 and S5 is also verified for connections involving at least one flight from a new turnaround. This similarity strengthens the validity and the consistency of the results presented.

4.3. Question 2: Analysing Selected Solving Method

In chapter 1, it was explained why a simple operation-to-stand assignment using a commercial solver is not the best approach to a problem that involves the modelling of transfer passengers, and that is the case in this research.

Initially, a simple operation-to-stand problem was proposed, but the problem size and computational load was significantly high and was not showing desirable results. The definitive approach to the problem solves it in two levels or steps. In the first level, operations are assigned to zones and not stands, and the multi-objective function is solved. In the second level, operations that were assigned to a zone in the first level are assigned to specific stands inside that zone.

First, it is important to note that the solving method, i.e., the TLSAP was verified and validated in chapter 2. Hence, it is possible to claim that the solving method is correctly implemented, with all objectives and constraints properly defined and with all decision variables properly related with each other. Still in chapter 2, the link between the first and second level of assignment was tested and it was possible to conclude that this link is also correctly implemented. Both levels follow the same line of reasoning and there is a consistent flow of information from one to the other. For instance, each zone in the second level only considers the operations assigned to it during the first level. Another example is the dynamic of the tow operations: all tow movements are defined in the first level and communicated to the second level, but no new movements can be generated in the latter.

The correct implementation of the solving method is thus confirmed. However, it is also possible to analyze its speed performance. Table 4.3 compares the run times and sizes of the different scenarios.

Scenario	Number of Turnarounds	Number of Operations	Active Objectives	Run Time [s]	#Variables (1st Level)	#Constraints (1st Level)	#Variables (2st Level)	#Constraints (2st Level)
S0	144	312	4	9.54	10750	20739	15079	3628
S1	203	449	2	10.41	27899	59681	27838	4750
S2	203	449	3	522.64	27899	59681	26951	4557
S3	203	449	4	1705.52	27899	59681	28339	4790
S4	203	449	4	616.85	27899	59681	31397	4657
S5	203	449	4	3609.06	27899	59681	23121	4542

Table 4.3: Information regarding the size, complexity and run time of each the research scenarios.

According to what was previously said, Stand Allocation Problems that include transfer passengers are typically quite large and may take long periods of time if they are not approached in a more efficient way. With the original approach proposed in this research (a simple MILP formulation with a operation-to-stand assignment) it was not possible to reach a final result due to the long run time in *Gurobi*. The solver was looking for a solution for over 13 hours and it was not yet close to reach an optimum solution. It is, however, always possible to define a higher MIP gap or set a time limit, but usually authors want to avoid this and look for new ways of solving the SAP when transfer passengers and hub connections.

With the definitive approach, the TLSAP, the run times are significantly low, especially when compared with the previous situation. According to Table 4.3, S5 was the scenario that took the longest to finish, with a run time of 3609.06s or approximately 1 hour. It is important to recall that, in S5, the hierarchical order of objectives 2 and 3 (maximization of captured transfer passengers and minimization of connection times, respectively) are swapped. Apparently, this change significantly impacts the time needed to solve the problem. In fact, the algorithm was forced to stop after 1 hour. The problem was almost solved but the solver was stuck in the branch and bound of the minimization of tow moves objective. The incumbent solution at that point equaled 2 tow moves, so it seemed reasonable to stop the algorithm at that point.

The second scenario to take the longest to finish is S3 and it is considerably faster than S5, with a run time

of 1705.52s or 28min26s. S3 is the scenario that considers all 4 objectives solved in the standard hierarchical order (as defined in [chapter 1](#)) and with the MIP gap defined to a very small value (0.01%). Thus, the value 28min26s can be seen as the average time needed to solve the standard version of the TLSAP.

Scenario S4 takes around 10min17s to run, which indicates a reduction of computational load when the MIP gap is increased to 5%. Even less time is needed in S2 because only 3 objectives are considered here, and the same is valid for S1, where 2 objectives are active. Clearly, and as it was expected, a larger number of objectives added to the hierarchy increases the complexity of the problem.

Finally, the only scenario that only includes the original schedule, S0, registers the smallest run time value, taking only 9.54s to finish. Including the proposed turnarounds in the problem expands the solution space of the problem considerably. Note that most of the proposed turnarounds offer a list of 2 or 3 time slot alternatives which are modelled as extra turnarounds and operations. Consequently, even though only 23 turnarounds were proposed in the other scenarios, the corresponding number increased from 144 to 203, and the number of operations increased from 312 to 449. This explains why the run time grows so notoriously from S0 to the other scenarios.

The last 4 columns of [Table 4.3](#) show that scenario S0 has the lowest number of variables and also the lowest number of constraints in both the first and the second level. This was expected, because S0 includes fewer turnarounds than all the other scenarios.

For scenarios S1 to S5, the number of variables and constraints in the first level is constant, which is logical since the input to the first level is the same in all these scenarios. However, the number of variables and constraints changes in each scenario, because the specific assignments to each zone also vary.

4.4. Question 3: Analysing Time Distribution of Proposed Turnarounds and Corresponding Effects

Answering Question 3 should help to conclude what is the preferred time distribution of flights for the model. The process of choosing which routes should be expanded or opened, and deciding which time alternatives should be created for each proposed turnaround and added to the list was strongly based on BEONTRA GmbH *BRoute Development Tool*. During this process, an effort was put into selecting the most profitable alternatives for airlines or, at least, alternatives that would create some competition to other similar markets. It is also relevant to analyze if, for some scenarios, the model selects time alternatives with a lower number of transfer passengers in an attempt to reduce connection times.

It is important to refer that, for the sake of testing the model and obtaining significant results, some time alternatives that were not profitable were still added to the list. If this would have not been done, the diversity of time alternatives would be more limited.

Results Drawn From Scenario S3

The first results to be analysed are related to scenario S3. Once again, it is important to mention that all proposed turnarounds were allocated to the schedule. [Table 4.4](#) shows, highlighted in blue, the time alternative of each proposed turnaround that was selected in S3.

[Table 4.4](#) shows that several proposed turnarounds have time alternatives with both the departure and arrival on-peak, others have one on-peak and the other off-peak and some others have both the arrival and the departure off-peak. Each case will be analysed in the following paragraphs.

First of all, only two turnarounds, namely the route to Airport9 and the route to Airport21, have both the arrival and the departure during an off-peak period. It is also worth mentioning that neither of the two had an alternative with both the arrival and the departure taking place on-peak.

On the other hand, out of the 23 selected alternatives (1 for each proposed turnaround), 9 have their arriving and departing operations happening during peak hours. There is a general tendency to select this kind of alternatives. It is also very common to find selected alternatives starting on a peak and ending off-peak, or vice-versa. This was verified for 12 of the 23 proposed turnarounds.

This overview shows that, for the most part, *Gurobi* selected time alternatives with at least the arrival or the departure taking place during a peak hour, so this seems to be beneficial when it comes to maximizing transfer passengers.

To better visualize what was explained in the previous paragraphs, and how exactly the new flights were distributed over the day of operations, the figures below portray an overlap of original and new flights from S3. [Figure 4.2](#) shows the original and new arrivals. [Figure 4.3](#) can be interpreted similarly, but for the departures.

Airport [IATA]	Arr. Time [hh:mm]	Dep. Time [hh:mm]	Ground Time [min]	Transfer Pax Forecast	On-Peak? (Arr/Dep)	Airport [IATA]	Arr. Time [hh:mm]	Dep. Time [hh:mm]	Ground Time [min]	Transfer Pax Forecast	On-Peak? (Arr/Dep)	
Airport1	00:10	02:00	110	134	YES/YES	Airport12	18:55	20:45	110	44	NO/NO	
	07:35	14:10	395	155	YES/YES		22:55	23:40	45	53	NO/NO	
	18:50	20:50	120	89	NO/NO		07:30	14:30	420	63	YES/YES	
Airport2	18:50	20:55	125	22	NO/NO	Airport13	00:25	01:30	65	125	YES/NO	
	00:20	08:40	500	80	YES/NO		17:45	21:30	225	125	YES/YES	
	13:00	20:55	475	80	YES/NO		18:30	20:50	140	112	NO/NO	
Airport3	00:30	01:55	85	125	YES/NO	Airport14	00:05	01:35	90	89	YES/NO	
	20:10	21:55	105	125	NO/YES		00:05	13:50	825	89	YES/NO	
	12:40	14:20	100	125	YES/YES		22:15	23:55	100	89	NO/NO	
Airport4	00:30	01:30	60	86	YES/NO	Airport15	00:30	01:30	60	80	YES/NO	
	17:00	20:55	235	76	YES/NO		01:00	01:45	45	35	YES/NO	
	00:30	08:30	480	85	YES/NO		13:05	14:10	65	34	NO/YES	
Airport5	00:45	02:00	75	46	YES/YES	Airport16	00:50	01:45	55	65	YES/NO	
	07:30	14:10	400	63	YES/YES		07:35	14:10	395	89	YES/YES	
	00:50	04:50	240	89	YES/NO		00:30	02:00	90	89	YES/YES	
Airport6	06:00	14:55	535	89	YES/YES	Airport17	21:40	22:20	40	89	NO/NO	
	22:10	23:00	50	84	NO/NO		18:45	21:40	175	75	NO/YES	
	09:20	13:00	220	164	NO/NO		00:30	01:35	65	73	YES/NO	
Airport7	11:00	13:00	120	104	NO/NO	Airport18	00:30	08:15	465	72	YES/NO	
	07:35	13:35	360	135	YES/NO		00:15	01:30	75	80	YES/NO	
	00:50	02:00	70	99	YES/YES		13:10	14:10	60	76	NO/YES	
Airport8	10:50	14:10	200	56	NO/YES	Airport19	18:50	20:55	125	72	NO/NO	
	00:30	01:30	60	80	YES/NO		00:15	01:35	80	115	YES/NO	
	13:10	14:10	60	69	NO/YES		21:55	23:30	95	118	NO/NO	
Airport9	13:10	20:55	465	80	NO/NO	Airport20	17:55	23:30	335	107	YES/NO	
	07:30	08:30	60	146	YES/NO		00:15	01:35	80	66	YES/NO	
	00:15	08:30	495	155	YES/NO		13:10	20:55	465	48	NO/NO	
Airport10	00:15	01:45	90	126	YES/NO	Airport22	07:35	14:10	395	117	YES/YES	
	07:30	14:10	400	124	YES/YES		Airport23	00:00	01:50	110	94	YES/NO
	22:25	23:05	40	149	NO/NO							

Table 4.4: Time alternatives of the proposed turnarounds that were selected and added to the schedule highlighted in blue. The turnarounds associated with new routes are highlighted in orange.

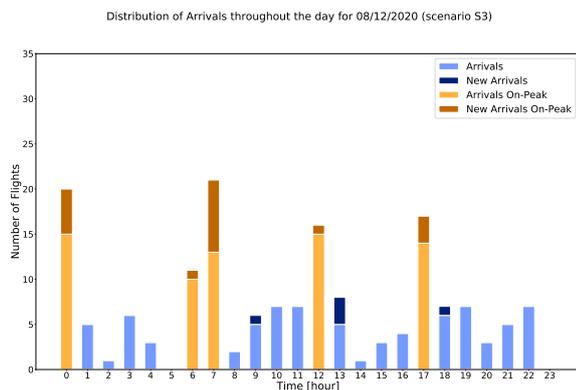


Figure 4.2: Arrivals distribution for scenario S3.

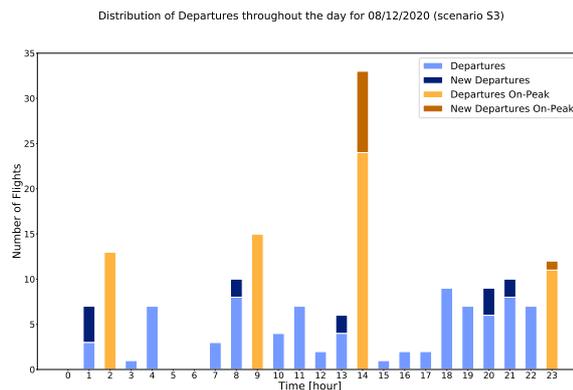


Figure 4.3: Departures distribution for scenario S3.

The exact distribution of arrivals is now more clear in Figure 4.2. In scenario S3, there is a clear preference to assign new arrivals to the peak hours. In fact, out of the 23 new arrivals, 18 take place during a peak hour. Table 4.5 shows that the proportion of arrivals on-peak hours increases slightly when the new turnarounds are added. There is an increase from 46.53% when only the original flights are considered to 50.90% when all flights are included.

On the other hand, the preference of assigning departures to the peak hours is not as clear. These are more scattered during the day, including less busy hours, as it is the case at 0h and 13h. As a consequence, in S3 some departure waves are not as outstanding. Despite this more evenly distribution of departures, 14h corresponds, by far, to the busiest period of the entire day, with 24 original departures and an extra 9 new departures happening during that time. Table 4.5 shows that there are no significant changes in the proportion of departures in each peak and off-peak hours.

There is one last important note to take from scenario S3. Some new turnarounds have their arrival and/or departure taking place very close in time to the original turnarounds. For instance, the new departure to Airport6 takes place exactly at the same time as one of the original departures. This is because the model does not take into account the proximity between flights. The list of alternatives is based on the flights that will allow for more passengers to transfer at the hub. In other words, the research is mainly based on the perspective of the airport and on what is most profitable for the stand planners. Sometimes, the most profitable

Flights (S3)	Arrivals							Departures					
	On-Peak Hours [%]						Off-Peak Hours [%]	On-Peak Hours [%]					Off-Peak Hours [%]
	00:00	06:00	07:00	12:00	17:00	Total		02:00	09:00	14:00	23:00	Total	
Only Original	10.42	6.94	9.03	10.42	9.72	46.53	53.47	9.03	10.42	16.67	7.64	43.75	56.25
Original + New	11.98	6.59	12.57	9.58	10.18	50.90	49.10	7.78	8.98	19.76	7.19	43.71	56.29

Table 4.5: Proportion of arrivals and departures taking place at the different peak hours and in off-peak hours.

alternative for the airport overlaps with existent flights.

Comparison Between Scenarios S2 and S3

It is important to recall that scenario S2 does not minimize connection times, while S3 does. The most relevant aspect taken from the comparison between these two scenarios is that the model selected the same time alternatives for all new proposed turnarounds, so the complete flight schedule is the same in both cases. It is likely that no changes were made to the schedule since the maximization of transfer passengers is still a priority in S3 and the MIP gap is considerably small in this scenario.

The lower average connection time in S3 is obtained by offering transfer passengers a different departure or arrival flight. A simple example of this situation is shown in Table 4.6.

Scenario	XX 100 Arriving From Airport27						XX 200 Arriving From Airport17					
	Airport27 - Airport24		Airport27 - Airport25		Airport27 - Airport26		Airport17 - Airport24		Airport17 - Airport25		Airport17 - Airport12	
	Conn. Time [min]	Pax.	Conn. Time [min]	Pax.	Conn. Time [min]	Pax.	Conn. Time [min]	Pax.	Conn. Time [min]	Pax.	Conn. Time [min]	Pax.
S2	345	1	345	1	150	1	340	1	340	1	205	3
S3	90	1	75	1	90	1	85	1	70	1	160	3

Table 4.6: Examples showing how connections are shortened in S3, when compared to S2.

In Table 4.6, some of the connections established from two arrivals (XX 100 and XX 200) are presented. For all connections, the waiting time is reduced from S2 to S3 because passengers were offered a connection to a departure that is closer in time to that arrival. This behaviour is generally verified with the other connections, leading to the lower average connection time in S3.

It is relevant to mention that certain connections are established in S2 and not in S3, and vice-versa. This cannot be directly controlled since the model decides by itself how to allocate flights and to establish connections, but the overall results are not affected by this dynamic.

Comparison Between Scenarios S3 and S4

During Question 1.1., it became clear that it is possible to get better values of the connection times by sacrificing some transfer passengers. It is now time to analyse how the selected proposed flights are moved in time from one scenario to the other in order to achieve the lower connection times. Table 4.7 shows the turnarounds that were moved in time from S3 to S4, and their arrival and departure times in each scenario.

	Airport4	Airport20	Airport21	Airport9
S3	17:00-20:55	17:55-23:30	13:10-20:55	13:10-20:55
S4	00:30-08:30	00:15-01:35	00:15-01:35	13:10-14:10

Table 4.7: Differences in the arrival and departure times of the new turnarounds between scenarios S3 and S4.

According to Table 4.7, there is a clear tendency to move flights to the beginning of the day in S4. This means that moving flights to that period of the day allows a large number of connections to still happen, while significantly reducing the connection times. On the other hand, the percentage of on-peak new arrivals changes from 50.90% in S3 to 51.50% in S4, which is not significant. Thus, it is not possible to find a correlation between the minimization of connection times and the distribution of flights on or off peaks.

Similarly to the arrivals, the tendency is to cluster more departures in the beginning of the day, but there is no clear preference to move departures to peak or off-peak hours. The percentage of on-peak new departures has the same value in S3 and S4.

To better understand how the model is able to reduce connections times by moving turnarounds in time, Figure 4.4 portrays a close analysis to the new turnaround to Airport20. It shows all the connections from/to Airport20 established simultaneously in S3 and S4 and how the total waiting times of each connection (i.e.,

connection time × number of passengers) changes from one scenario to the other when the turnaround is moved in time. Note that the airports' codes are abbreviated so that they fit in the plot (for instance, A20 = Airport20)

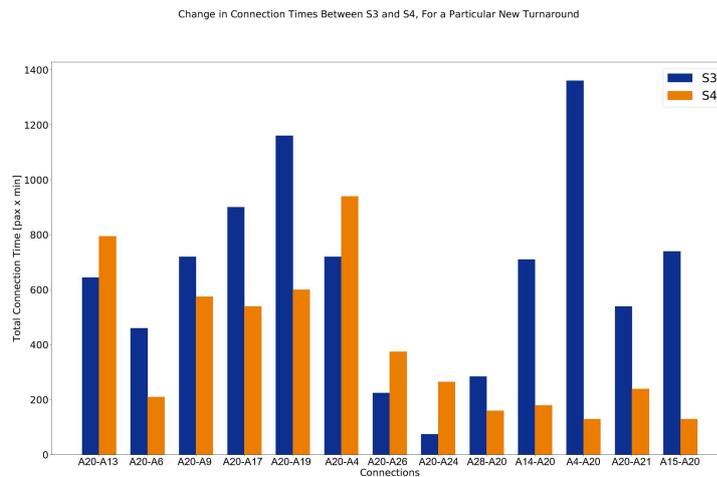


Figure 4.4: Change in the total connection times for connections from/to Airport20 established simultaneously in scenarios S3 and S4. Airports' codes are abbreviated so that they fit in the plot (A20 = Airport20).

From Figure 4.4, it is possible to infer that, overall, the waiting times for the connections are considerably shortened in S4. There are a few exceptions, namely the connections Airport20-Airport26, Airport20-Airport24, Airport20-Airport13 and Airport20-Airport4, but they are completely cancelled out by the lower waiting times of the remaining connections.

Without any further analysis, one may argue that the lower total connection times in S4 may happen because fewer passengers are making each connection. However, this is only the case for the connection Airport4-Airport20. For all the other cases shown in Figure 4.4, the number of passengers making the connection in S4 is equal or even higher than in S3. From this analysis, it follows that it is possible to shorten connections without sacrificing a significant number of passengers.

Another aspect that was observed with this analysis is that not all connection pairs offered in scenario S3 from/to Airport20 are offered in S4, and vice-versa. For instance, it is possible to fly from Airport20 to Airport29 through the hub in S3, but not in S4. On the other hand, it is possible to fly from Airport10 to Airport20 in S4, but not in S3. Thus, the decision of the airport authorities for one scenario or the other can also be influenced by which hub connections should be opened.

Comparison Between Scenarios S3 and S5

Table 4.8 contains the new turnarounds that were placed in different time slots in S3 and S5.

	Airport13	Airport3	Airport4	Airport20	Airport5	Airport11	Airport6	Airport16	Airport21	Airport17	Airport19	Airport9	Airport7
S3	17:45-21:30	12:40-14:20	17:00-20:55	17:55-23:30	07:30-14:10	07:30-14:10	06:00-14:55	07:35-14:10	13:10-20:55	18:45-21:40	13:10-14:10	13:10-20:55	09:20-13:00
S5	00:25-01:30	00:30-01:55	00:30-08:30	00:15-01:35	00:45-02:00	00:15-01:45	00:50-04:50	00:50-01:45	00:15-01:35	00:30-02:00	00:15-01:30	00:30-01:30	11:00-13:00

Table 4.8: Differences in the arrival and departure times of the new turnarounds between scenarios S3 and S5.

Several turnarounds were moved to the beginning of the day from S3 to S4 and the same tendency is verified between S3 and S5. However, it is clear that more turnarounds were moved in time from S3 to S5 than from S3 to S4 (compare Table 4.7 and Table 4.8). It is understandable that this happens since in S4 the flexibility to minimize connection times is not as significant as it is in S5.

Prioritizing the minimization of connection times over the maximization of captured transfer passengers significantly encourages new turnarounds to be allocated to the first hour of the day. In fact, Table 4.8 shows that, in S5, each of those turnarounds starts at midnight and that the corresponding aircraft leaves during the night or in the early morning. The only exception to this is the turnaround to Airport7. Scenarios S4 and S5 clearly show that allocating arrivals to the beginning of the day reduced connection times, but that also reduces the number of captured transfer passengers.

It is also relevant to analyse if there is any relationship between the minimization of connection times and the allocation of flights to peak or off-peak hours. Figure 4.5 and Figure 4.6 portray, respectively, the arrivals

and departures distribution in S5.

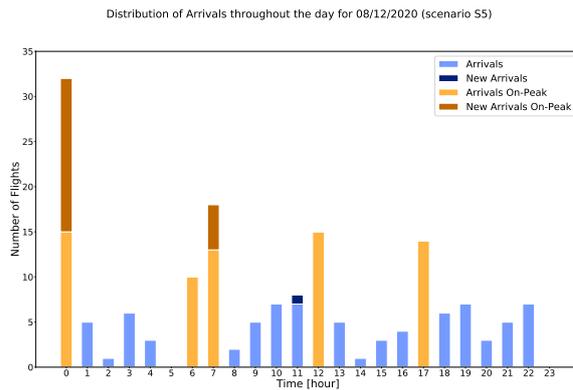


Figure 4.5: Arrivals distribution for scenario S5.

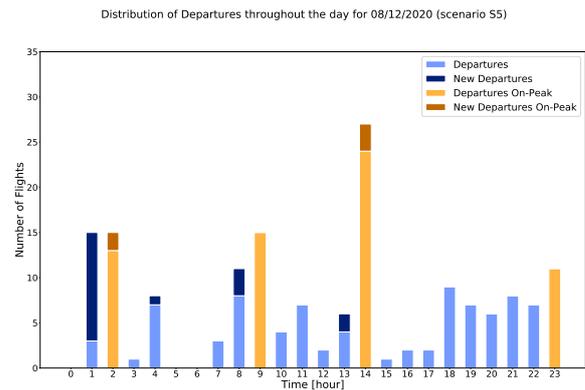


Figure 4.6: Departures distribution for scenario S5.

In S5, 17 of the 23 new arrivals were assigned to 12a.m. and 5 to 7a.m. Both these hours correspond to peak periods. The remaining arrival was assigned to 11h, which corresponds to an off-peak hour. Overall, S5 is the scenario with the most arrivals assigned to peak hours with 53.29% followed by S4 with 51.50% and finally S3 with 50.90%. Thus, it now seems that assigning arrivals to peaks may have a small influence in the reduction of connection times.

On the other hand, 12 out of the 23 departures are assigned to 1 a.m. This growth is so significant that this period can now be considered a peak hour, since it is as busy as or even busier than other peak periods. For the remaining departures, 6 were assigned to off-peak hours and only 5 were assigned to peak hours: 2a.m. and 2p.m. It is thus possible to infer that the percentage of departures on-peak reaches its lowest value in S5 with 40.72% and it increases to 43.71% in both S3 and S4. Unlike the arrivals, the new departures tend to be more spaced along the day even when more importance is given to the minimization of connection times.

4.5. Question 4: Analysing Airport Connectivity and Network Expansion

From Table 4.1, it is trivial that, for any scenario that includes the proposed turnarounds, there is always an increase in the airport connectivity in terms of number of captured passengers. The biggest increase happens for scenarios S2 and S3, with more 44.30% transfer passengers going through the airport compared to S0 (in which no new turnarounds are considered). If neither of the main objective functions are optimized (S1), the growth is smaller and equal to 18.14%. In scenario S4, the growth in the number of captured transfer passengers equals 40.48% and finally, S5 registers an increase of 32.67%.

Scenario	Total Captured Passengers	Growth w.r.t. S0 [%]	Number of Connections	Avg. Number of pax per Connection
S0	4090	0	941	4.35
S1	4457	8.97	1126	3.96
S2	5671	38.66	1267	4.48
S3	5671	38.66	1267	4.48
S4	5559	35.92	1239	4.49
S5	5047	23.40	1236	4.08

Table 4.9: Total number of captured transfer passengers, number of connections and connection density in the different scenarios.

Table 4.9 also shows the number of connections established in each scenario (it is assumed that, when there is at least one connection opportunity from airport AAA to airport BBB at the hub, a connection is established).

Unexpectedly, S0 is the scenario with the least number of connections. Even though the number of captured passengers is not maximized in S1, this scenario still has more connections. It is important to refer that when two scenarios capture the same number of transfer passengers, it does not necessarily imply that the number of connections in both has to be the same. It is possible to have more connections in one scenario, but if each of those connection captures fewer passengers, the total sum may still be the same in both. How-

ever, that is not the case between S2 and S3. They both capture the same amount of transfer passengers and, coincidentally, they also register the highest number of connections, 1267 each.

In total, S3 establishes 326 more connections than S0. Note that there are only 5 proposed turnarounds representing new routes, but those 5 turnarounds alone establish 224 (or 68.71%) of the extra 326 connections in S3. This makes sense since every connection established with a new route is completely new for the airport, while the connections established between original routes are, for the most part, already possible.

In, scenario S4 fewer connections are established when compared with S2 and S3, which is understandable because that scenario also captures fewer transfer passengers. Scenario S5 establishes 1236, which is very close to the value observed in S4. Recall that S5 captures approximately 500 fewer transfers than S4. The difference in the number of passengers is not proportional to the difference in the number of connections. A closer analysis shows that in S5 a large number of short connections are established, which effectively contributes to its lower average connection time. Those short connections are so numerous, that S5 ends up creating almost as many connections as S4. However, some connections in S5 capture fewer passenger than those in S4. That difference is so significant that it is translated in a lower number of captured passengers in S5.

The previous paragraphs show that there is not a direct relationship between number of passengers and number of connections. Common sense would say that a higher number of passengers captured would always have to be followed by more connections established, but that may not be the case. This would only be true if the number of passengers in each connection was always the same. In reality, it is more complex than that, and it highly depends on each scenario and each input set.

Table 4.9 also shows the average number of passengers assigned to each connection, or the connection density. The density value of 3.96 obtained in S1 is the lowest of all scenarios, reinforcing the inefficiency of randomly assigning operations (i.e. ignoring captured passengers and connection times). This value shows that the connectivity potential of the hub is wasted in S1. On the other hand, even though fewer passengers are captured in S0 because no new flights are added, the available connections are efficiently used, as shown by density value of 4.35. In short, more passengers make use of each connection in S0 than in S1.

Furthermore, scenarios S2 and S3 show a relatively high density, only surpassed by S4 (and the difference is almost negligible). This fact, combined with all results shown up until now, shows that S3 delivers the best results out of all scenarios.

Finally, the average number of passengers assigned to each connection in S5 drops significantly when compared to S2, S3 and S4. This is a direct consequence of swapping the optimization order of the two main objectives. Moving the maximization of the number of captured passengers to third place in the hierarchy severely affects the number of passengers that are offered each connection.

Detailed Network Expansion For Different World Regions

To obtain a more detailed understanding of how the case study hub's network expanded throughout the different scenarios, Figure 4.7 and Figure 4.8 are presented. The first shows, for each scenario, the number of passengers connecting **from** the different regions/countries of the hub's network. The second portrays, also for each scenario, how many passengers connect **to** the same regions/countries.

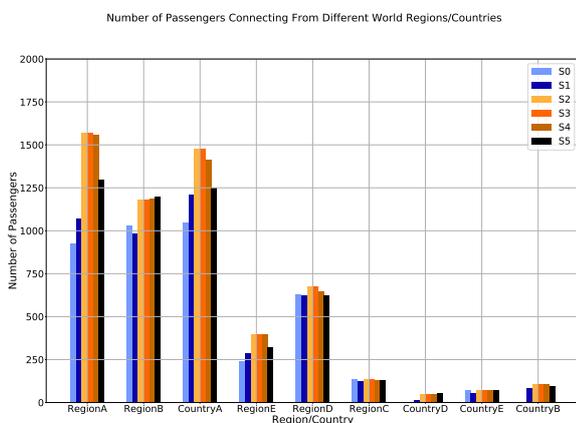


Figure 4.7: Number of passengers connecting from different world regions/countries.

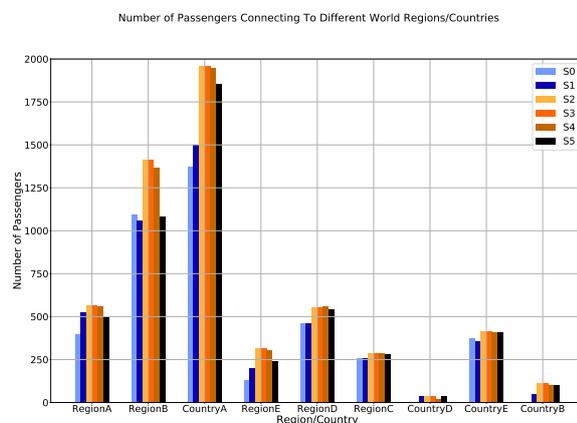


Figure 4.8: Number of passengers connecting to different world regions/countries.

Both pictures show that the network expands considerably to all regions when the proposed turnarounds are added to the schedule. In S0, the airport was not even connected to CountryB and the CountryD, so the network not only became stronger but it also expanded to new markets. More importantly, it was possible, with the same set of proposed set of turnarounds, to optimize and strengthen the network to the different regions by simply including the maximization of captured transfer passengers in the model. To confirm this, note how both scenarios S2 and S3 capture more passengers from/to all regions when compared to S1.

It is also worth mentioning that when a scenario performs worse than others overall, it does not necessarily perform worse in every region's market. For instance, S5 sees an overall decrease of transfer passengers compared to S2, S3 and S4. Nonetheless, S5 registers the highest number of transfer passengers arriving from the RegionB, out of all scenarios. In another example, more passengers connect from/to the RegionB in S0 than in S1, even though S0 performs worse overall, when compared to S1. More examples like these can be found from the analysis of [Figure 4.7](#) and [Figure 4.8](#).

The results from the previous paragraph confirm what was mentioned in previous sections. A scenario that captures fewer transfer passengers does not necessarily perform worse in every route. Before, it was possible to conclude that some O&D connections are exclusive to certain scenarios (independently of how well those scenarios would perform) and now it became apparent that scenarios that perform worse overall can in fact capture more passengers in specific markets.

It is also important to note that different regions of the world that were not connected before are now connected through the new routes. In S3, for instance, with the addition of the route to Airport10, it is possible to connect from several places in RegionA (Airport1, Airport22, Airport7, Airport31) to Airport10, and vice-versa, opening the market between RegionA and CountryB. This new route also led to the establishment of some connections between RegionB and CountryB, even though this is not as expressive. Creating a new route to Airport21 also promoted connections from CountryC to several airports in RegionA, such as Airport22, Airport1, Airport31, Airport8 and Airport32. In this case, however, the connection is only available in one way.

Profitability of New Turnarounds: Scenarios S0 and S3

According to what was previously explained, the alternatives of the new turnarounds were chosen based on profitability and transfer passenger forecast. In some cases, however, less profitable or even unprofitable alternatives were added to the input list in order to create schedule diversity among the proposed turnarounds and to allow for a more robust analysis of the academic value of the current research.

It is important to mention that, usually, the higher the forecast of transfer passengers of a certain time alternative is, the more profitable that route is for the airline. However, this is not always the case. Furthermore, this research focuses on the airport's point of view, so it is expected that sometimes the selected time alternative is not particularly profitable for the airline in question, but it is suitable for the airport's schedule. These topics and some others are discussed in [Table 4.11](#) (for proposed turnarounds that expanded the frequency of existing routes) and [Table 4.10](#) (for proposed turnarounds that opened new routes). These Tables provide a brief profit analysis of the time alternative selected by the model for each new turnaround added to the schedule.

Note, for instance, the route expansion to Airport19. The selected alternative is most likely not a viable option for the airline, since all operating margins seem to be negative. However, still quite a large amount of passengers is expected to connect from and to the added aircraft, so the expansion can be considered by the airport. Other route expansions appear to be a solid investment for the future to both the airline and the hub. This is true for expansions with relatively high operating margins for the airline and a high forecast of transfer passengers. Examples of this are the routes to Airport2, Airport9, Airport1 and especially Airport13 and Airport3. The last 2 have remarkably high operating margins. Note that all these 5 routes still spill a considerable amount of passengers, so it may be feasible to increase capacity even more.

It is important to mention that there is a very specific market that seems to be a promising expansion for the future. In the beginning of the research, it was possible to note that the original schedule of the airport features several flights from/to a large set of airports in CountryA, which leads to assume that this market is quite popular. For this reason, when looking for new route expansions, many attempts were made with airports in that country. This search proved successful. In fact, the airports of the first 10 routes presented in [Table 4.11](#) are all located in CountryA. Simultaneously, it was observed that these route expansions were consistently profitable and generally more profitable than routes to other parts of the world. They also registered very high numbers of passengers forecasts, often with a significant passenger spillage. In conclusion, it seems that the market to CountryA is indeed quite profitable and its expansion is most likely the most beneficial for the airport at the moment. Strengthening this market may give the case study airport the opportunity to become a gateway hub to CountryA by offering transfer passengers from different parts of the world a large number of connections to that country.

Route [IATA]	Type of Route	Profitability Analysis of the Selected Time Alternative
Airport21	New Route	The time alternatives of this route are generally not profitable to the airline. In fact, most of the operating margins are negative. Still, the model selected the most profitable alternative for the airline, which is also the alternative with the largest transfer forecast. Overall, this new route does not seem a solid choice, at least for the near future.
Airport10	New Route	The selected alternative is not the most profitable for the airline, but it is still significantly viable. The operating margins are generally high. The number of transfer forecast is also high. Since this is a new route, it creates several completely new connection at the hub. It seems to be worth investing on this route.
Airport20	New Route	The operating margins of times all alternatives of this new route are notably high. The chosen alternative is in fact the least profitable, but the differences are minimal. The transfer forecast is also the lowest, but again the differences are not relevant. Similarly to the previous case, it seems to be worth investing on this route, also due to the new connections.
Airport22	New Route	Only one suitable time alternative was found for this new route, which already indicates that it is not particularly profitable for the airline. Some operating costs are, in fact, negative. It is, however, still viable to open this route from the airport's point of view, due to the potential network expansion it may carry.
Airport11	New Route	All time alternatives are similarly profitable, but the one selected is not the best out of all. One of the operating margins is negative. The transfer passenger forecast is the lowest, but the differences are not significant. This new route also seems to be a solid addition to the network.

Table 4.10: Profitability analysis of the selected time alternative of the proposed turnaround that opened new routes.

Route [IATA]	Type of Route	Profitability Analysis of the Selected Time Alternative
Airport2	Frequency Increase	For this route, the second most profitable time alternative was picked. The operating margin when compared to some like markets is in fact negative, but overall it is profitable for the airline. Together with another time alternative, it registers the highest number of forecast transfer passengers. There is a high percentage of passengers spilled.
Airport13	Frequency Increase	The second most profitable alternative was chosen in this case. The operating margin of all like market comparisons is positive, which indicates a solid route for the airline. This alternative registers the highest number of the transfer passenger forecast, which is also positive for the airport. Even more passengers are spilled than in the previous case. Maybe consider increasing even more the frequency of this route.
Airport3	Frequency Increase	All time alternatives of this proposed turnaround are considerably profitable for the airline. Besides, the passenger forecast is high in all of them and a large number of passengers is spilled. Again, consider a further increase in the frequency of this route.
Airport14	Frequency Increase	The most profitable alternative for the airline was selected here. The operating margin w.r.t. all like markets is positive, but one value lands very close to 0% of margin. All time alternatives have a similar transfer forecast, so they are all regarded as equal for the maximization of connecting passengers.
Airport4	Frequency Increase	The least profitable alternative was selected. One operation margin value is significantly negative, while this does not happen with the other alternatives. The transfer forecast is not the highest either.
Airport15	Frequency Increase	All time alternatives of this route are similarly profitable, but the one selected is slightly less than the others. However, the transfer forecast of the selected alternative is the highest of all, since the aircraft considered in this alternative is also larger.
Airport17	Frequency Increase	The selected alternative is the least profitable for the airline. One of the operating margins is significantly negative and the others are very close to 0%. The selected alternative also has the lowest forecast.
Airport19	Frequency Increase	In general, this route is not profitable for the airline, since almost all operating margins are negative. It is, however, possible to capture potential transfer passengers. The selected time alternative is not the most profitable, but its transfer forecast considerably high.
Airport9	Frequency Increase	The alternative chosen is, in fact, the only one profitable for the airline, even though one of the operating margins is exactly 0%. It also has the highest transfer forecast and it registers a significantly high number of spilled passengers.
Airport18	Frequency Increase	The chosen alternative in this case is the most profitable and probably the only viable choice for the airline. The airport also benefits from it since it registers the highest transfer forecast.
Airport1	Frequency Increase	The most profitable alternative for the airline was picked here. All operating margins are positive and the transfer forecast is the highest, so it is also a solid choice for the airport.
Airport12	Frequency Increase	The selected time alternative selected is the most profitable, but one of the operating margins is too low, which may make this expansion infeasible for the airline. For the airport, however, a significant amount of transfer passengers are forecast in this route.
Airport8	Frequency Increase	The chosen alternative is, by far, the most profitable of all to the airline. However, one of the operating margins is negative. There is a large difference in the transfer forecast between this alternative and the others, making it also the most suitable for the airport's schedule. A high number of passengers are spilled when this alternative is chosen.
Airport23	Frequency Increase	Only one suitable time slot was found for this route, which suggests that this is an expansion without priority. The airline profit that would come with the expansion is not particularly high either.
Airport5	Frequency Increase	The selected alternative is, by far, the most profitable for the airline. However, some operating costs are remarkably low. The forecast is higher than the other alternative, but it is in fact quite low. This is not one of the preferred routes to expand.
Airport7	Frequency Increase	For this route, the most profitable time alternative was chosen. It is substantially more profitable than the other alternative and all its operating margins are positive. It is worth noting the high number of spilled passengers in this case.
Airport6	Frequency Increase	The selected alternative is, by far, the most profitable for the airline and its operating margins are all positive. However, the difference in terms of forecast is not as significant.
Airport16	Frequency Increase	The selected alternative is the most profitable and it is the only one with all positive operating margins. It also registers the highest forecast of passengers, so it is the best option for the airport too.

Table 4.11: Profitability analysis of the selected time alternative of the proposed turnarounds that expanded the frequency of existing routes.

4.6. Tow Moves

The fourth and last objective in the hierarchy of all scenarios is the minimization of tow movements at the airport, which means its results are highly dependent on all the previous objectives and on how flexible the model was set to be. There are no research questions directly related to the dynamic of tow moves in the different scenarios, since this is a secondary objective. However, it is still relevant to briefly analyse the corresponding results. For this effect, [Table 4.12](#) shows the number of tows assigned by *Gurobi* to each scenario.

Scenario	S0	S1	S2	S3	S4	S5
#Tow Moves	0	0	0	3	0	2

Table 4.12: Number of tow moves registered in each scenario.

For scenario S0, no tow moves are needed. Note that, in this scenario fewer turnarounds are considered, so the amount of capacity available is the highest and this gives more flexibility to the model. In S1, the proposed turnarounds are already considered, but the two main objectives (maximization of transfer passengers and minimization of connection times) are not considered. Scenarios S2 and S4 also register 0 tow moves in their output. S2 is still characterized by a large level of flexibility, since only 3 of the objectives are considered. In the case of S4, all objectives are considered, but the larger MIP gap allows the number of tow moves to be reduced to 0. Naturally, this comes at the expense of slightly deteriorating the previous objectives.

All scenarios mentioned so far registered 0 tow moves. However, 2 tow moves are planned in S5 and finally, S3 registers 3 towing operations. Note that S3 and S5 are the most complex scenarios. They include all turnarounds, all objectives and a small MIP gap. It was thus expected that the minimization of tow moves objective would be more limited in these two scenarios.

5

Sensitivity Analysis

In this chapter, a sensitivity analysis is performed to test the behaviour of the model when a parameter or an input changes. In [section 5.1](#), the reaction of the model to the change in the MIP gap used by *Gurobi* is tested and analyzed. In [section 5.2](#), the model is tested using different inputs. In this case, multiple days of operations are fed to the model one by one and the number of captured transfer passengers and connection times are analyzed and compared between all those days.

5.1. Changes in the MIP Gap

By default, *Gurobi* uses a gap of 0.01% but a gap of 5% was set for scenario S4. The minimization of connection times is placed in third place in the hierarchy, after the maximization of captured transfer passengers. In line with what was explained before, increasing the MIP gap should give more flexibility to the minimization of connection times objective, by allowing *Gurobi* to improve this objective while slightly degrading the maximization of captured transfer passengers.

In [chapter 4](#), it was shown that increasing the gap from 0.01% to 5% can indeed lead to a better average connection times. At the same time, it was verified that there is a slight decrease on the number of captured passengers.

In this section, the sensitivity of the model when the MIP gap varies is analyzed. Several values for the gap were chosen to perform this test. The first values are separated by 1%, then the separation increases to 2% and the last values are separated by 5%. The results are shown in [Figure 5.1](#). The lowest value to be used is 0.01% and the highest value is 40%. Note that the day of operation used in this test is 08/12/2020.

The results show that, for MIP gap ranges from 0.01% to 18%, there is a clear tendency for the average connection time to decrease when the MIP gap increases, which matches the results obtained for scenarios S3 and S4 in [chapter 4](#). However, for large MIP gap values, there is not a clear tendency of evolution. The average connection times fluctuate between higher and lower values.

On the other hand, the capacity of the hub to capture transfer passengers registers a consistent decrease from the first value, 0.01%, to the last value, 40%.

It seems that raising the MIP gap until a certain value, around 18%, can effectively reduce the average connection time while not losing too many transfer passengers. However, after that value, it may not be worth to increase the gap even more, since the average connection time will not reach more satisfactory values but the airport still loses transfer passengers.

The same MIP gap test was run for other days of operation, namely 03/12/2020, 04/12/2020 and 12/12/2020 and the same tendencies were verified. Thus, this behaviour is consistent for different input sets.

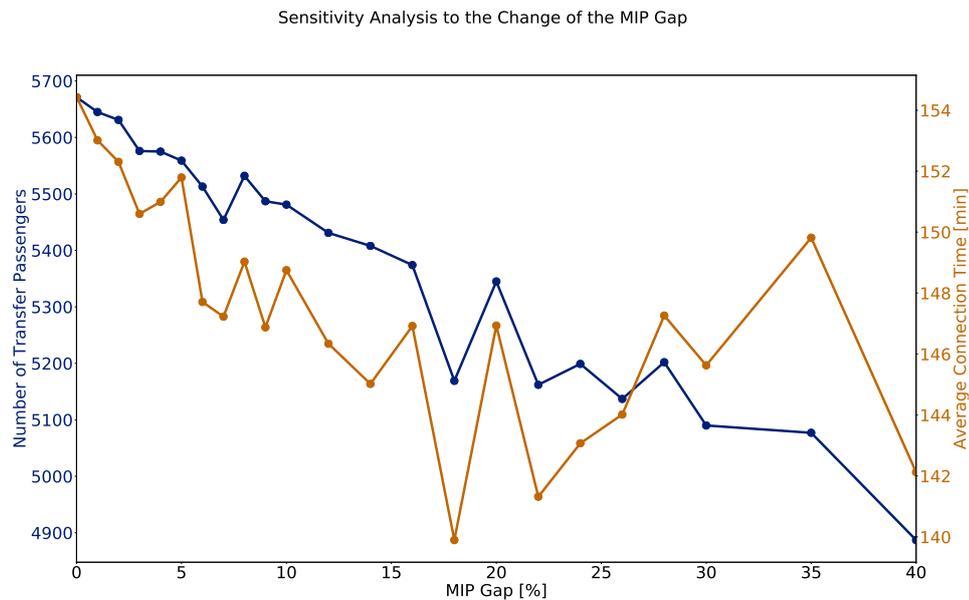


Figure 5.1: Sensitivity analysis to the change of the MIP gap.

5.2. Results Obtained in Different Days of Operations

In chapter 4, the results were analyzed in detailed for a single day operation, namely 08/12/2020. However, it is important to understand how the model reacts when the input schedule changes. All scenarios from S1 to S5 were run for different days in the beginning of December 2020. Since the travel patterns of passengers change over the week, each of the days corresponds to a different weekday. Two of the days take place in the middle of the week (Tuesday and Thursday), while the other two should carry the effect of the weekend (Friday and Saturday).

When it comes to the proposed turnarounds, the same set as the one originally used for 08/12/2020 is re-used for all the other days. The demand and profit of a flight usually changes over time. This change is usually more relevant when looking at different months. That is why all other days considered for this test belong to December 2020 and take place in the first half of the month. The closer they are, the more realistic it is to assume that the proposed turnarounds, their corresponding profitability and transfer forecast are constant. An effort was put into choosing days sufficiently far from Christmas, to avoid having the effect of the holiday season. BEONTRA GmbH *BRoute Development Tool* tool also assumes an approximately constant demand and operating margins for each month, further validating the use of the same set of proposed turnarounds. The days used in this section are shown below:

- **03/12/2020:** simulates operations on a Thursday
- **04/12/2020:** simulates operations on a Friday
- **08/12/2020:** simulates operations on a Tuesday
- **12/12/2020:** simulates operations on a Saturday

Finally, it is relevant to know that all 4 days of operations are similarly busy when it comes to the number of original turnarounds. 03/12/2020 registers 146 turnarounds, 04/12/2020 and 08/12/2020 144 and 12/12/2020 registers 140. Note that Thursday is the busiest day, while Tuesday and Friday are equally busy. Saturday, on the other hand, is the least busy day.

Table 5.1 shows the results of the captured transfer passengers, average connection times and tow moves for all scenarios and for each of the days presented above.

Table 5.1 shows that similar results to the ones taken for the day 08/12/2020 in chapter 4 can be extended to the other days. For all days of operation, S1 is the scenario that captures the least amount of passengers. On the other hand, S2 and S3 register the highest transfer throughput in all days and the average connection time in S3 is always lower than in S2. The tendency to have lower average connection times and fewer captured

Scenario	Day Of Operation	Captured Transfer Passengers				Avg. Conn. Times	Tow Moves
		Critical	Non-Critical	Non-Critical (Long Connections)	Total		
S1	03/12/2020	549 (11.82%)	3231 (69.54%)	866 (18.64%)	4646	2h47min	0
	04/12/2020	597 (12.82%)	3322 (71.36%)	736 (15.81%)	4655	2h40min	0
	08/12/2020	642 (14.40%)	3051 (68.45%)	764 (17.14%)	4457	2h41min	0
	12/12/2020	785 (15.92%)	3232 (65.53%)	915 (18.55%)	4932	2h41min	0
S2	03/12/2020	994 (17.74%)	3343 (59.68%)	1265 (22.58%)	5602	2h56min	3
	04/12/2020	1023 (17.27%)	3783 (63.85%)	1119 (18.89%)	5925	2h48min	0
	08/12/2020	1177 (20.75%)	3354 (59.14%)	1140 (20.10%)	5671	2h48min	0
	12/12/2020	1181 (20.61%)	3357 (58.58%)	1193 (20.82%)	5731	2h49min	6
S3	03/12/2020	1244 (22.21%)	3375 (60.25%)	983 (17.55%)	5602	2h40min	5
	04/12/2020	1242 (20.95%)	3794 (64.01%)	891 (15.03%)	5927	2h36min	5
	08/12/2020	1402 (24.72%)	3367 (59.37%)	902 (15.91%)	5671	2h34min	3
	12/12/2020	1413 (24.66%)	3374 (58.87%)	944 (16.47%)	5731	2h34min	6
S4	03/12/2020	1300 (23.59%)	3282 (59.56%)	928 (16.84%)	5510	2h37min	0
	04/12/2020	1203 (20.35%)	3807 (64.41%)	901 (15.24%)	5911	2h37min	0
	08/12/2020	1451 (26.10%)	3243 (58.34%)	865 (15.56%)	5559	2h32min	0
	12/12/2020	1441 (25.81%)	3272 (58.60%)	871 (15.60%)	5584	2h29min	0
S5	03/12/2020	1415 (27.61%)	2919 (56.96%)	791 (15.43%)	5125	2h30min	5
	04/12/2020	1430 (26.45%)	3332 (61.64%)	644 (11.91%)	5406	2h23min	6
	08/12/2020	1631 (32.32%)	2807 (55.62%)	609 (12.07%)	5047	2h18min	2
	12/12/2020	1584 (30.09%)	2896 (55.02%)	784 (14.89%)	5264	2h24min	6

Table 5.1: Results obtained with each scenario for different days.

passengers from S3 to S4 and from S4 to S5 is also verified for all days. In fact, in all 5 days, S5 registers the lowest value for the average connection time and the second lowest value for the captured passengers.

There is, however, something that is not verified in every day. In [chapter 4](#), it was mentioned that the minimum connection time dropped from S3 to S4 because the MIP gap in S4 is increases to 5% and this value gives more flexibility to the minimization of connection times objective. What the results in [Table 5.1](#) shows is that this reduction happens in most of the days of operation. However, there can be exceptions. On the 04/12/2020, the average connection time increases from S3 to S4, while the number of captured transfer passengers still decreases, so this output is undesirable. This leads to conclude that, while it is generally efficient to use a larger MIP gap when one wants lower connection times, this cannot be generalized, since in some of the days this may not happen. Each case must be carefully analyzed.

It is also possible to find small differences on the percentages of passengers taking short, medium and long connections, but these are not significant. In fact, for each scenario and each connection type, the differences in percentages never deviate more than 6% amongst the different days. This further confirms the consistency of results obtained with the TLSAP.

[Figure 5.2](#) confirms what was explained in the previous paragraphs. The general change of transfer passengers throughout all scenarios follows the same tendency in all days of operation considered here.

A similar plot for the average connection time is depicted in [Figure 5.3](#). Again, the Figure confirms what was presented before. The change in the average connection times throughout scenarios S1 to S5 follows approximately the same trend in all days. The only exception is now more clear. The day 04/12/2020 is the only one that registers a growth in the average connection time from scenario S3 to S4. [Figure 5.3](#) further highlights that the drop of connection times from S4 to S5 is much more significant than the drop from S3 to S4, which reinforces the idea that swapping the order of the maximization of captured transfer passengers and the minimization of connection times is more efficient (to minimize connection times) than simply increasing the MIP gap to 5%.

It is quite common to have full days of operations without any tow movements. This is especially true for scenarios S0 and S4. S2 registers some tows for two of the days, while the results from S3 and S5 dictate a reasonably high number of tow moves for all days of operation.

Generally speaking, the model brings improvements to the airport in every day considered in this test. Passenger throughput increases, and with it the hub connections and network. Simultaneously, it is always possible to obtain lower connection times for passenger convenience.

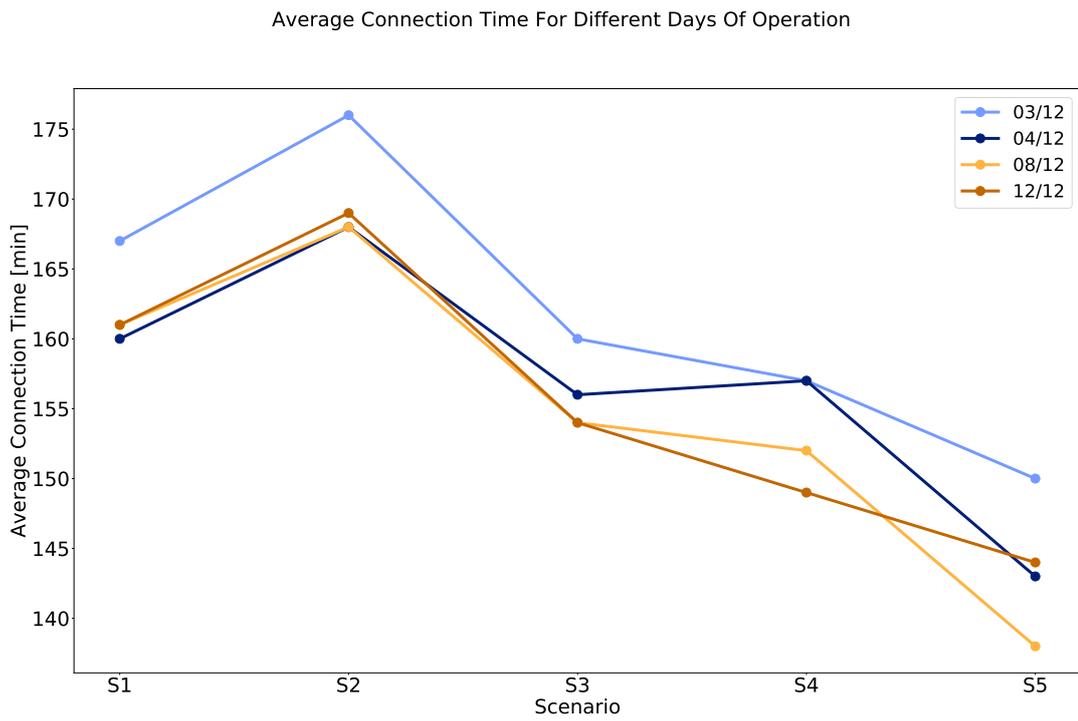


Figure 5.2: Change in the number of transfer passengers throughout all scenarios, for all days of operation.

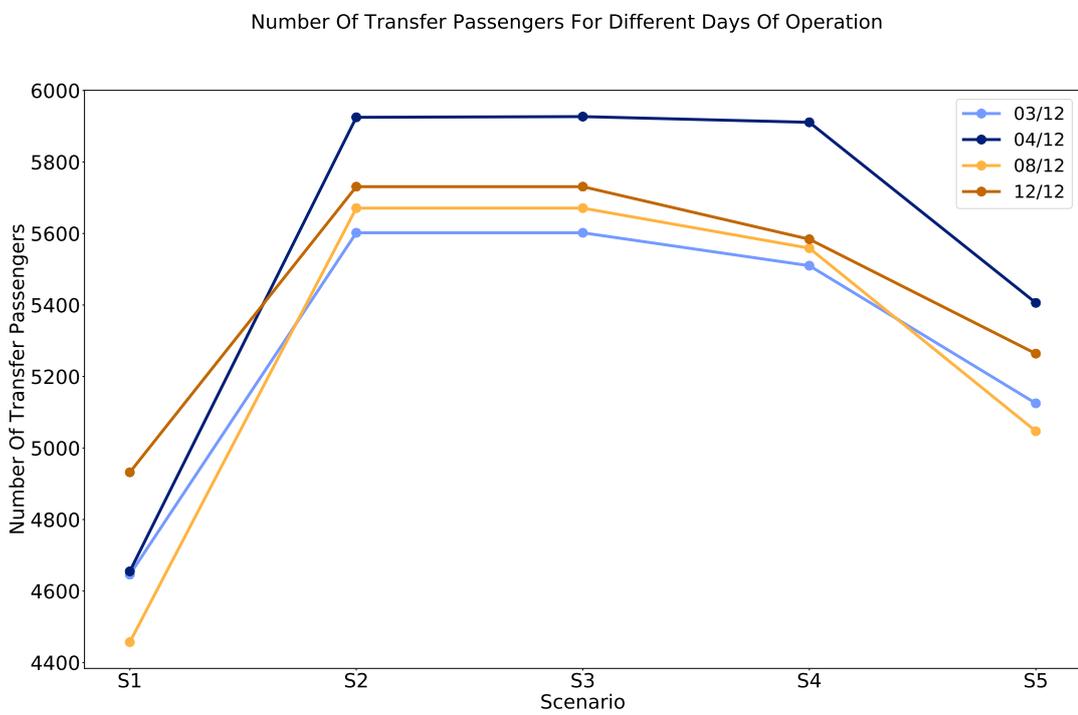


Figure 5.3: Change in the average connection time throughout all scenarios, for all days of operation.

6

Conclusions and Recommendations

The results taken from all scenarios were already thoroughly analyzed in [chapter 4](#). In the same chapter, some light conclusions and deductions were already made. The behaviour of the model was further analyzed in [chapter 5](#). Nonetheless, there are still some remarks to be made regarding the results, which will be presented in [section 6.1](#). Finally, it is also important to point out how the current model can be improved and propose recommendations for the future, which shall be explained in [section 6.3](#).

6.1. Global Conclusions Regarding the Research Results

The first remark to be made is related to the relationship between the main two objectives of this research, which are also the main KPIs, namely the maximization of captured transfer passengers and the minimization of connection times. Answering Question 1.1 helped to understand that this relationship is quite complex. Usually, both objectives conflict with each other, which means that it is usually not possible to get significantly lower connection times without losing captured passengers. The comparison between scenarios S3, S4 and S5 showed this. In both S4 and S5 it was possible to offer shorter connection times to passengers, but at the cost of losing some connections. In fact, the higher the significance given to the minimization of connection times, the higher the drop is in the hub's capacity of capturing more passengers. In S5, where the minimization of connection times is prioritized over the other objective, this is particularly significant. The behaviour of the model for different MIP gaps, explained in [section 5.1](#), also shows that increasing the gap in order to give more importance to the minimization of connection times leads to a consistent reduction of captured transfer passengers.

This leads to conclude that, in most real life cases, it may not be possible to reach the desired values for both KPIs. Note that the maximization of captured transfer passengers is more valuable for the airport, while the minimization of connection times is more important for passengers convenience. Thus, it is likely that the airport's capacity management teams will have to find a balance between what is favourable for the airport but also what is convenient for passengers. This balance really depends on how much the airport wants to expand and what kind of service quality the hub wants to offer to travellers.

When it comes to the distribution of new flights over time, it is also possible to withdraw some important conclusions taken from Question 3. Before running the model, it was expected that having more flights on peak hours could increase the number of captured passengers. This statement was confirmed with the results. However, there does not seem to be a very clear relationship between the distribution of flights in on-peak and off-peak hours and the minimization of connection times. The comparison of scenario S3 with scenarios S4 and S5 showed that there is a tendency to assign more of the new turnarounds to the early hours of the day when the model puts more effort into minimizing connection times. Indeed, several short connections were established in the beginning of the day for S4 and S5. This outcome was most likely a consequence of the duration of those new turnarounds. It was verified that the time alternatives of the new turnarounds that take place in the beginning of the day are generally shorter than others that take place during the day. Having shorter turnarounds allows to cluster more arrivals and departures in time which leads to shorter connection times, and that is what happens in scenarios S4 and S5. It is therefore possible to conclude that the selection of the time alternative for each proposed turnaround are, first of all, highly dependent on what is the desired outcome for the airport (more passengers or lower connection times). Furthermore, it is also

dependent on the duration of the time alternatives of each new proposed turnaround and how they fit in the schedule.

From Questions 3 and 4, it is reported that not all connections established in one scenario will be established in the others. It is expected that, if a scenario captures more passengers than another, the first will naturally open several connections not present in the second. But the statement is also true the other way around. Recall that scenarios S4 and S5, for instance, capture fewer passengers and establish fewer connections than S3 (refer to Table 4.9). Nonetheless, the results have shown that some connections in S4 and S5 are not possible in S3. The model aims to minimize the transfer passenger throughput but it does not look specifically into which connections are being created. Consequently, it is completely valid that scenarios that register a lower number of transfer passengers open very specific connections.

In line with the previous paragraph, it is important to stress that the model does not explicitly maximize the number of connections. It is logical that if more passengers are captured, more connections will most likely be established. But again, the results have shown that, between scenarios, the change in the number of passengers is not proportional to the change in the connections. For instance, there is an increase of approximately 600 passengers from S5 to S3, but the increase in the number of connections does not surpass 30. Obviously, these 30 extra connections do not carry all those 600 passengers. Instead, some of the connections in S3 were replaced with others that carry fewer passengers. Taking into account this conclusion, the decision of the airport between one scenario or the other may also depend on the type of connections the capacity management teams wish to open.

From Question 4, it was possible to analyze how the network of the airport was affected and how profitable the new flights can possibly be for the airlines. It became clear that what is more advantageous for the hub may not be for the airlines. The selected time alternatives of some turnarounds were in fact more adequate for the airport (otherwise the model would not select them) but they were not the most profitable from the list. Sometimes, its corresponding operating margin compared to similar markets was negative. In short, what is better for the airport does not always overlap with what is better for the airline. This result reinforces the idea that this research is mainly focused on the optimization of the airport and its infrastructure and does not look directly in the perspective of the airlines. However, it is important to note that when a hub airport is considering adding flights from its hub airline, the profitability criterion is more relevant, since their relationship is much stronger and both stakeholders are significantly dependent on one another.

Finally, the results from Question 2 evaluated the speed of the model developed. On average, the run time for each scenario is significantly low, especially when taking into account that transfer passengers are being modelled in this research. The decomposition of the main problem into smaller sub-problems by using a two-level approach proved efficient when it comes to the speed of the whole model.

6.2. Research and Model Limitations

Along the development of this research, it was possible to identify several limitations to the model and the assumptions that were made for it.

With respect to the proposed turnarounds, there are two drawbacks to point out. For the first one, it is important to remember that the current research's target is the optimization of the hub infrastructure. For this reason, the duration of each time alternative for each proposed turnaround was not taken into account. Consequently, there are several new turnarounds with considerably large ground times during the day. Usually, however, long turnarounds are more common during the night. In this case, the flight arrives at the airport and stays parked until the morning, when it departs again. This is called an overnight flight. Thus, accepting such long turnarounds during the day is not completely realistic and most likely will not be accepted by the airlines, unless it is a very profitable route. Nonetheless, this drawback is not particularly limiting. A very long turnaround can still be the most advantageous alternative for the hub. If the airlines in question are not satisfied with it, the capacity teams can always adapt the input to include smaller turnarounds and choose the best option out of those.

The second drawback related to the proposed turnarounds is that some arrivals and departures added to the schedule take place very close in time to original arrivals and departures from/to the same airports. In one case, there is even an overlap of two departures. If the two airlines in question are different, this is not as relevant, but if the airline is the same, this represents an unrealistic scenario. In most of the cases, an effort was made to avoid these overlaps. However, in some cases these overlaps were overlooked for the sake of creating more time slot diversity, which allows to obtain more robust results and take more complete results.

Related directly to the TLSAP, there are also a few limitations to point out. The division of an airport into

zones for the first level of assignment may be an easy task for relatively small airports, but that is not the case for large hubs. At very busy airports, for instance, there are several restrictions for different airlines. Some stand areas are reserved for certain airlines and cannot be occupied by other aircraft. With the two-level approach it is possible, to a certain extent, to make divisions based on airline or flight type restrictions. If a certain group of stands cannot be used a certain airline (or group of airlines) then that group can be defined as a zone and those airlines are not allowed into that zone. It is even possible to define zones with just one stand, if that is necessary. However, when the complexity of restrictions starts to increase, and some restrictions overlap with each, the task of defining zones becomes much harder. In these cases, even if it is possible to obtain a certain zone layout, the number of zones may be significantly high and the whole purpose of using zones instead of stands to reduce the problem size is lost.

Another thing related to the model is that estimated operating margins of the new turnarounds is not considered in any element of the model. The model purely looks for maximization of captured transfer passengers and minimization of connection times. Thus, an assumption is made that profitability is not relevant. However, and according to what was previously said, for the airport it is indeed more important, at least on a first level, to find flights that will bring more transfer passengers to the hub.

Still related to the model, the runway capacity is completely disregarded in this research. The model may be able to allocate a very large number of arrivals and departures to a certain hour, but in reality there may not be enough capacity to actually handle those operations. For instance, in scenario S3, it was shown that at 14h the number of departures is significantly high and this could potentially affect the free flow of aircraft taxiing, landing and taking off.

Finally, it is important to mention that, even though the run times of the model are significantly low, there is still some room for improvement. For all times that the model was ran in *Gurobi*, it was clear that the Branch and Bound would spend most of the time (in some cases, around 90% of the total time) trying to solve the last objective of the hierarchy, the minimization of tow moves. This happens because there are probably many different solutions that lead to the same optimal value and there are also several possible combinations. Consequently, the Branch and Bound needs more time to reach the MIP gap.

6.3. Future Recommendations

After presenting the major limitations of this research, it is now time to look for ways of overcoming those limitations while proposing future developments and ideas to expand the research and its model in the future.

The first recommendation follows the points mentioned in the previous section. It is recommended to add airline profitability as an extra objective, especially if this is important for the decision making of which new turnarounds to choose for the schedule. It also helps to obtain a more realistic and secure stand assignment for the airport.

The second recommendation is still in line with the previous section. It was mentioned that the run time of this model may be compromised by the minimization of tow moves. One of two measures can be taken to overcome this problem. One is to find a different way to define tow operations. For instance, it is possible to allow tow moves only from contact zones to remote zones, and then from remote zones back to the contact zones. This assumption would reduce the number of optimal solutions available and it is valid at the same time. Most instances of towing operations at an airport move an aircraft from a contact stand to remote stands, where they can stay idle for long periods of time without disrupting the availability of valuable contact stand capacity. In particular, during the night, several aircraft are moved inside the airport either to remote zones where they are temporarily parked or to a different contact stand to prepare that aircraft for the first flight in the following morning. Either way, restricting tow moves only for these cases could effectively make the problem simpler. The second measure has a bigger impact on the model itself and it was also suggested by one of BEONTRA's experts during the validation of the model. The focus of this research is to find a strategic stand planning for a future expansion at the hub. Usually, in the early stages of that planning, the airport's market analysis teams may not be as concerned with the reduction of tow movements. Consequently, this objective can even be completely removed from the hierarchy. Note, however, that this decision is highly dependent on the planning stage and also on the effort that the airport wants to put into having efficient tow operations. It is not a decision that should be taken just to reduce run times, since it can be extremely important to include it.

Similarly to the previous 2 recommendations, this one is also related to the previous section. It was explained why the zone division may be too complex or even impossible to achieve sometimes, while it can also lead to significantly large problems. The division considered in this research is very adequate for relatively

small airports, but it is not guaranteed that it works in every case study. In the future, the robustness of the zone definition can be improved by the introduction of other techniques to facilitate this process and make it flexible enough to be applied in any airport.

It was mentioned in [section 6.1](#) that in each scenario it is possible to obtain different O&D connections that do not exist in other scenarios. This is true even for those scenarios capturing fewer transfer passengers. It was further pointed out that the airport's decision for one or the other scenario may be influenced on which kinds of connections are opened in each. It is possible that a certain O&D connection is so important for the hub that the capacity teams may end up choosing that scenario and risking losing several transfer passengers. It would be much more convenient for the airport if the capacity teams were able to choose the scenario that captures the highest number of passengers and still have that one important connection. To solve this, it is recommended to add an element to the model that promotes the creation of specific connections. This can be done by adding a constraint that forces a certain connection to happen. However, with constraints like this there could be some risk of infeasibility, if that connection cannot happen at all. Instead, having an extra objective function that rewards every desired connection if it is established might work. Ideally, this extra objective should be placed in second in the hierarchy, right before the maximization of the number of captured passengers. This way, one can guarantee that the desired O&D connections are established, and then all other connections are established in function of the first. However, there is an even simpler way of considering connections preferences, by simply adding a reward coefficient directly on the maximization of captured transfer passengers objective. For all desired connections, a certain value R would be multiplied by the number of passengers making a connection (so either multiplied by the z_{ijvw} or x_{ij} variables). The larger R is, the more likely it is to create the desired connections.

It is already known that this model explicitly maximizes the number of captured transfer passengers, but as the results show, an increase in the number transfer passengers is not usually followed by the same increase in the number of connections. But if a hub is more concerned with expanding the network connections, and not necessarily the number of actual passengers transferring through the hub, it is always possible, in the future, to formulate a new objective function that explicitly maximizes the number of O&D connections established at the airport. Certainly, the results would be different than the ones obtained in this research.

Finally, many other objectives besides the ones already mentioned can be added to the model according to the specific requests of each airport. The Literature Study has shown that the SAP/GAP has included a vast number of objectives for all different purposes [26], that can always be added to the TLSAP and placed in different positions in the hierarchy.

6.4. Scientific Relevance and Industry Applications

It is fundamental to analyze how the current research can be a valuable contribution to the scientific and academic community and also to the aviation industry.

Modelling transfer passengers in the stand assignment problem is usually a complex task that leads to very large problems with big solution spaces. Furthermore, there might be quadratic objective functions or constraints added to the model, making it even harder to solve. The work of B. Maharjan and T. I. Matis in [14] claims and proves that by dividing the main problem into smaller sub-problems based on the airport's layout can reduce the run time of the model. This idea was the starting point for the TLSAP developed in this research. The model itself, however, is significantly different from the one proposed by those authors. In fact, the characteristics of this model make it different from models of previous researches. Completely new objectives and constraints had to be formulated to create a model that measures the number of transfer passengers and considers the creation of connections between pairs of flights. Some other new constraints were also added to correctly model hub connections and relate different decision variables. The introduction of two levels makes it really important to correctly and consistently connect those two levels, so that the final results are not distorted or affected by this division.

A two-level, hierarchical approach was never extensively researched in the past, in the field of the stand assignment problem. As far as the literature study conducted during this research extends, no work was found that attempts to develop such a model. Hence, this research finds its scientific relevance by proposing a novel approach to tackle large sized stand assignment problems that include transfer passengers, and without resorting to meta-heuristic approaches, which are fast algorithms but also harder to design, implement and verify.

It is important to recall that this research was conducted in order to cover a research gap in the field of the stand assignment problem. There were several studies on the future creation of certain routes for

a hub airport and also on the network analysis. However, no research has proposed to develop a specific stand assignment that can be used to obtain quantitative information regarding the optimization of a hub's connections. There was not yet a study focused on analysing how it is possible to capture more transfer passengers at a hub airport by manipulating the stand assignment, and how new flights can be inserted in the schedule in the most efficient way for the airport. In this context, more efficient means that the hub can i) capture more transfer passengers while ii) the connection times are shortened as much as possible. Previous works have focused in other objectives, such as passenger walking distance (or time) minimization [2, 6, 7], minimization of delay costs [16] or maximization of gate assignment preferences [15, 17] but not on objectives i) and ii) mentioned above.

Throughout this research, several assumptions were made to simplify the model and make it possible to run it, get the results and evaluate them to withdraw conclusions. However, in the future, it is possible to add more complexity and realism to the model. With this future effort, the current research may potentially be the root of the development of tools used by airports to assess the connection potential of new routes. In particular, let's suppose that an airport wants to expand a certain route or open a new route to a certain area. Several time slots could be proposed by the airport, and the model would reveal what would be the best combination. Once again, the best combination is not necessarily the most profitable for the airline but again, the focus of this research is on the optimization of the hub's schedule and of the captured transfer passengers. Other stakeholders' interests are not accounted for (with exception of the minimization of connection times for passengers convenience. The airport could thus propose the best option in its point of view, but the final decision of opening or not a route is always up to the airlines, so the airport might need to make some adaptations and find a good balance.

Furthermore, it is possible that one (or several) airlines show interest to open or expand new routes at an airport. In these cases, and with a tool designed based on the work developed on this research may give airports the opportunity to make an informed decision on which flights of which airlines the capacity and market teams should select. A tool that provides information about transfer passenger growth, network expansion and connection times is valuable by allowing a hub airport to grow in a sustainable way while consistently making an optimized use of its infrastructure. This sustainable growth is particularly important for smaller hubs that are in the process of expansion and still have a large percentage of its capacity available.

It is also possible to find an important application of this research for large hub airports. It is true that for a busy hub the capacity is almost fully used, so it may not be possible to simply add new turnarounds and flights to the schedule. But the capacity management team and the market team of large hubs may be interested in finding out whether switching an original turnaround with a new turnaround may positively affect the connectivity of the airport. Furthermore, recall that the model is able to consider several time alternatives to place a turnaround. With this in mind, and taking into account that busy hubs have a very limited available time slot capacity, a future tool derived from this research could prove to be valuable to make timing decisions.

Nonetheless, the applicability of this research in the aviation industry is not limited to the expansion of a hub's schedule. The results have shown that, even without adding any new flights, it is possible to improve the hub's capacity of capturing transfer passengers (comparison between scenarios S1, S2 and S3). Thus, an airport can also make use of this model to optimize the current schedule by re-arranging the stand assignments in a way that more connections are possible inside the terminal.

7

Operations To Stand Assignment for Scenario S3 (08/12/2020)

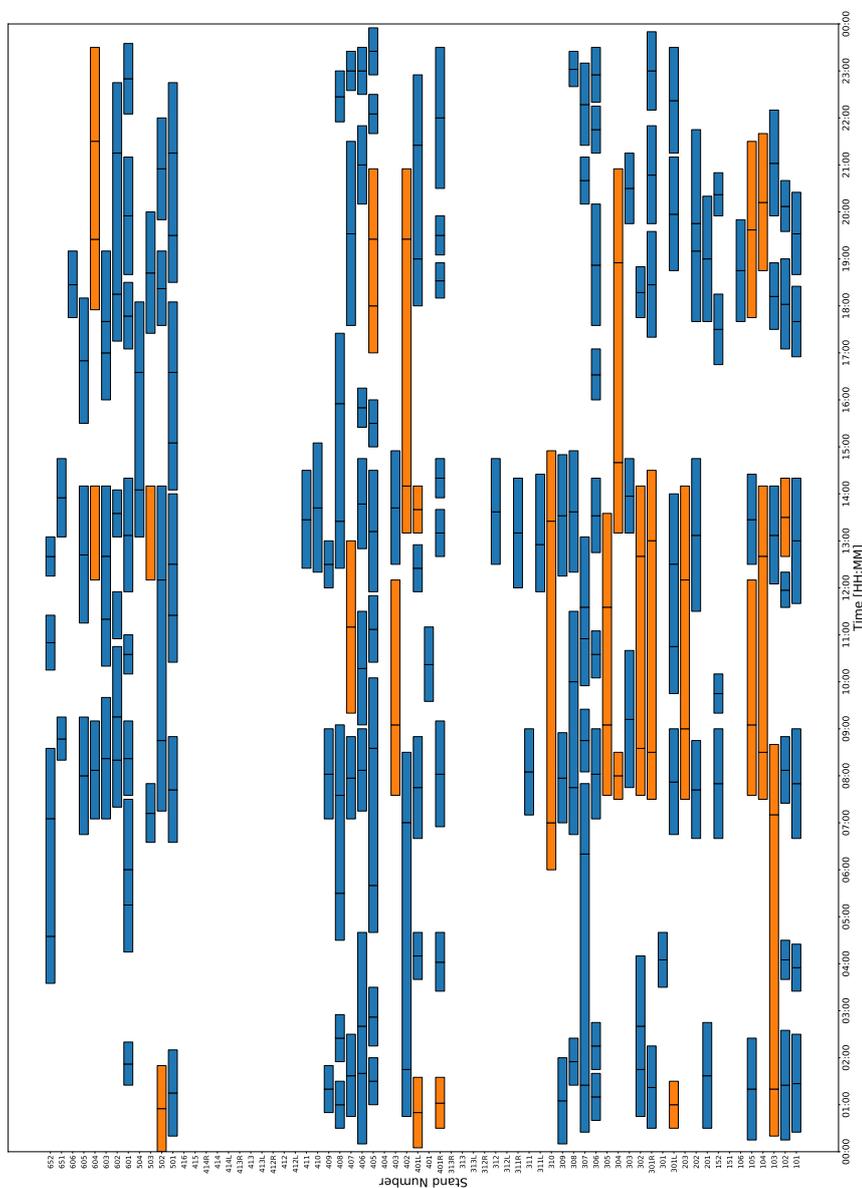


Figure 7.1: Operations To Stand Assignment for Scenario S3 (08/12/2020). Blue bars correspond to original turnarounds while orange bars correspond to new turnarounds added to the schedule.

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