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Using reset control for improvement of transient response

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Using reset control for improvement of transient response

by

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Abstract

High-tech industry has been developed to meet the more and more restrict demands, regarding precision, speed and robustness. The example is given as atomic force microscope and it is used to measure physical properties with the resolution requirements of nanometer level. In this case, the high disturbance rejection ability, high robustness as well as higher precision are all desired. Now days, Proportional Integral Derivative, as known as PID, has been widely used in industrial field due to its simple structure and easy to implement. However, PID is not sufficient to meet the increasing requirements because of the functional restrictions, like bode's phase gain relation and waterbed effect. Nonlinear controllers is then come up with to overcome the limitations by researchers. Since most of the nonlinear controllers are hard to implement and the structure of them are mostly complicated, reset control is chosen to be investigated because it is easier to implement and can be used with loop shaping method.

In order to improve the tracking performance and disturbance rejection, more integrators are considered to be added. Adding reset control into the system can solve the problem of limitations of the linear control based on current research. But the majority of current research has focused on the frequency domain analysis, while the time domain should be also focused. In time domain, the transient response is the term that can be analysed. There is a special kind of reset control proposed in

literature 'Constant in Gain, Lead in Phase'(CgLp). This kind of reset control provides broadband phase compensation without changing the gain behavior before the end of the broadband. Because there is still no systematical method to tune CgLp so this work will focus on by tuning different parameters in CgLp to investigate.

When exploring the transient response of designed reset control, the requirement of keeping phase margin constant at the bandwidth should be met. The methods that can achieve the goal of improving transient response as well as performance of system are presented in this thesis. During the process, the rule of tuning CgLp was obtained.

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Preface and Acknowledgements

This report is my master thesis of the MSc Mechanical Engineering in the High-Tech Engineering (HTE) track at TU Delft. During the two years of studying at TU Delft and living in Delft, I have gained lots of knowledge and really make sure what kind of person that I want to be. I am so glad I am here to be closer to the life that I want to live. And every time when I think about it, I just feel so lucky. This thesis can not be completed without the help of the following people, thus I want to thank:

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Chapter 1

Introduction

PID controllers, consisting of Proportional element, Integral element and Derivative element, are the most used linear controller in industries and our lives [2]. Because PID controller is easy to tune with its simple structure. The three different parts of PID controller are used for setting the desired bandwidth, creating high gain at low frequencies and achieving phase lead at crossover frequency respectively.

But there are some functional limitations of PID controller and with the higher and higher requirements in precision industries, these functional limitations should be overcome. Now days the high tech industries require nanometer accuracy and high robustness and precision which linear controllers can not meet the requirements. Performance of the whole system will be affected by external disturbance and also the input reference tracking. So it is important to have good disturbance rejection and reference tracking performance to meet the highly desired requirements.

I term in PID controller creates high gain at low frequencies which gives the benefits for reference tracking ability of the system. Therefore more integrators added into the system is supposed to have better reference tracking performance while because I term introduces phase lag in to system and that would destroy the system in transient response and frequency domain due to the limitations of linear control [3]. More integrators in the system means less phase margin at bandwidth and also end up with increased overshoot in transient response thus damage the stability of the system.

The functional limitations are bode's phase-gain relationship [4], which means there is a trade off between noise attenuation and tracking performance, and waterbed effect [5].Nonlinear controllers can overcome the limitations of linear controller but most of them are complicated. Reset control is paid a lot attention because of its flexibility and can be used with loop shaping method.

The first reset controller was created by Clegg in 1958 [6],which surpass the limitations of linear controllers. This reset controller is called Clegg integrator (CI), and it is an integrator that reset its state when the input signal - error of the system is zero. The Clegg integrator only introduces 38° phase loss and that is a big compensation compared to the 90° phase loss given by linear integrator. With this property of Clegg integrator, it overcomes the bode's phase-gain limitation in linear controller.

There are also many other reset controllers extended by Clegg integrator. For example, First Order Reset Element(FORE) [7], Second Order Reset Element [8], Constant in Gain Lead in Phase(CGLP) [9], etc. One of the reasons that researchers favor reset controller is that it can be approximately analyzed in frequency domain by describing functions. That is investigated in the literature. But besides that, time domain performance should be also focused. Therefore in this thesis, the transient response in time domain will be investigated. To do so, the goal of the research is set:

Improvement of transient response with multiple integrators.

To achieve the goal of improving the transient response, Chengwei has proved that there's a optimal sequence of controllers to get the less overshoot in the step response, which improves that transient response[10]. This thesis will base on this thought then design and tune the reset element to improve the transient response after adding more integrators into the system.

Chapter 2

Literature Review

This chapter is the literature review in scientific paper format. The limitation of linear controllers currently is introduced. Thus the reset control is introduced to overcome the limitation. Further more, the state of the art of reset strategies are also included therefore leads to the research goal of this report.

Using reset control for improvement of transient response

Tingting Wang

***Abstract*—PID, the Proportional-Integral-Derivative controller, is widely used in today's industries due to its simple structure and easy to tune. However, with the development of the high-tech industries, more and more rigorous demands are needed to achieve more precision and robustness. In this case, PID and other linear controllers get limited by their functional limitations and that is why researchers turn into nonlinear controller. Among the nonlinear controllers, reset controller can overcome the limitations and it is paid a lot attention because of its flexibility. This paper will study the different strategies by using reset control to achieve the rigorous requirements while improve transient response.**

I. INTRODUCTION

PID is being dominantly used in our lives and industries since it is easy to implement and the simple structure, like for example using it from our home air conditioning system to gas flow control in industry field. PID consists of Proportional element, Integral element and Derivative element, which are used for setting the desired bandwidth, creating high gain at low frequencies and achieving phase lead at crossover frequency respectively.

Tracking, bandwidth, and precision are the three important design objectives in precision mechanisms design. However, with high-tech technology developing, high-tech industries require more strict demands and do not just satisfy with current status. They want higher precision, bandwidth and robustness, which push PID into its limitations. Like for example, the atomic force microscope to measure physical properties and the resolution of the measurement is in a nanometer level so it needs more disturbance rejection as well as high robustness and precision. It is the functional limitations of linear controllers that restrict their work in high precision mechanisms.

One of the limitations is bode's phase-gain relationship [**1**]. It demonstrates that the phase at a certain frequency is always dependent on the slope of gain. To be illustrated, it is shown in Fig.1 where the blue line is a single mass system while the red line represents the system added a tamed derivative element. Although the additional tamed derivative element makes the system more stable because of the introduced phase lead, the slope of the system gets increased which makes gain decreased at low frequencies and increased at high frequencies, which just deteriorate the tracking performance as well as noise attenuation.

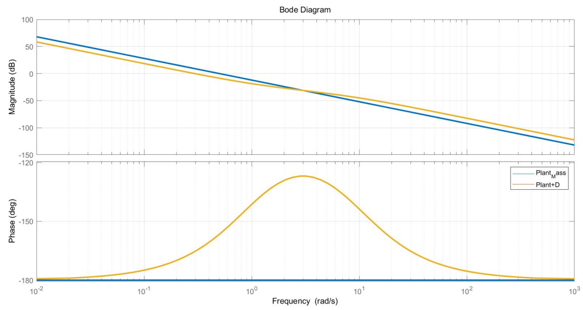

Figure 1: Trade-off between Stability and Performance

The other limitation is waterbed effect, which is represented in Eqa.1 [**2**]. The bode sensitivity integral is always 0, which means for one frequency the sensitivity function is lowered while for other frequencies the sensitivity function will be increased.

$$\int_0^\infty ln\left|S(\omega)\right|d\omega = 0 \tag{1}$$

Nonlinear controllers can overcome the limitations of linear controller but most of them are complicated. Reset control is paid a lot attention because of its flexibility and can be used with loop shaping method.

The first reset controller was created by Clegg in 1958 [**3**], which surpass the limitations of linear controllers. This reset controller is called Clegg integrator (CI), and it is an integrator that reset its state when the input signal - error of the system is zero. It is shown the comparison between Clegg integrator and linear integrator in time domain in Fig.2 that ever time when input gets zero, the output of Clegg integrator also goes zero. In frequency domain, Clegg integrator has little bit higher gain and less phase loss than linear integrator. It is shown in Fig.3 that CI reduces phase lag from 90 degrees to 38 degrees. This means Clegg integrator is more stable than linear integrator and less overshoot in time domain. The researchers realized the potential of reset controller so other reset controllers came out latter.

Horowitz et al. extended Clegg integrator into First Order Reset Element(FORE), which is that low pass filter gets applied with reset action [**4**]. And latter Hazeleger et al. did the similar but extended the second order low pass filter and got the Second Order Reset Element (SORE), which can be tuned by different reset values, damping ratios and frequencies [**5**].

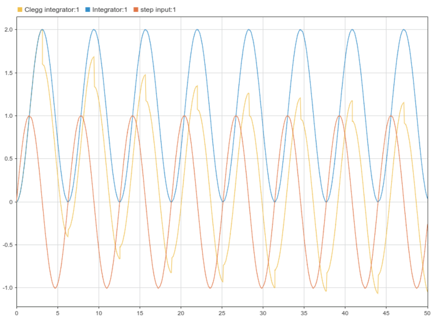

Figure 2: Time domain sinusoidal response of integrator and Clegg integrator

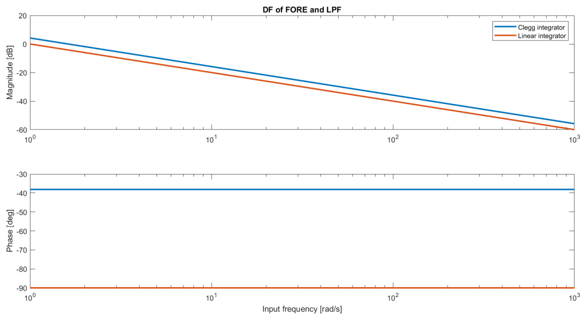

Figure 3: Bode plot of Linear integrator and Clegg integrator

It is roughly seen from Fig.3 that reset control compensates the phase loss so that increases the phase margin, gets higher bandwidth and less overshoot, which also means improving the transient response. So far the reset control technology has investigated in precision stage [**6**] [**7**], serve system [**8**], HDD system [**9**] [**10**] and etc. This review paper will focus on using reset control to improve transient response.

The structure of the paper is as follows. Section 2 provides basic knowledge of reset control and how to analyze it. Section 3 gives different kinds of reset control that have been researched. Section 4 describes the existing strategies for improvement of transient response. The last one, Section 5 will give the conclusion.

II. Preliminary

A. Definition of Reset Control

A reset controller can reset the parameters when a certain condition occurs, which is called reset law. The state space representation of general reset system is:

$$\begin{cases} \dot{x}(t) = A_r x(t) + B_r e(t) \\ x(t^+) = A_\rho x(t) \\ u(t) = C_r x(t) + D_r e(t) \end{cases} \tag{2}$$

where A_r, B_r, C_r and D_r are the state matrices of corresponding basic linear system, A_ρ is the reset matrices [**12**]. $x(t)$ is the state vector, while $e(t)$ is error signal put into controller and $u(t)$ is the output of controller. To be specific, the reset matrices A_ρ can imply how much the reset action can occur. When $A_\rho = 1$, the system behaves linear controller while $A_\rho = 0$, the system is a full reset controller. The diagonal form of the reset matrices can be shown:

$$A_\rho = \gamma I_{n\times n} \tag{3}$$

where the γ is reset value and it is only considered the range of $\gamma \in [0, 1]$ in this paper, n is the order of reset controller. For the reset law, it is normally set $e(t) = 0$.

B. Describing Function Analysis

Transfer function is used to analyze the system in frequency domain for linear control. While it does not exist in nonlinear system, so describing function (DF) is proposed to overcome the problem. But since describing function is a pseudo-linearization, it can only approximately describe nonlinear controller in frequency domain because it only considers the first order harmonic of the output. Among the several types of DF, the sinusoidal input describing function (SIDF) is used in this paper and the reset law is set when $e(t) = 0$. So based these conditions the describing function is given as [**7**]:

$$G(j\omega) = C_r(j\omega I - A_r)^{-1}(I + j\Theta_D(\omega))B_r + D_r \tag{4}$$

where,

$$\Theta_D(\omega) = -\frac{2\omega^2}{\pi}\Delta(\omega)[\Gamma_D(\omega) - \Lambda^{-1}(\omega)]$$

with

$$\begin{aligned} \Lambda(\omega) &= \omega^2 I + A_r^2 \\ \Delta(\omega) &= I + e^{\frac{\pi}{\omega}A_r} \\ \Delta_D(\omega) &= I + A_\rho e^{\frac{\pi}{\omega}A_r} \\ \Gamma_D &= \Delta_D^{-1} A_\rho \Delta(\omega)\Lambda^{-1}(\omega) \end{aligned}$$

C. HOSIDFs

The describing function only considers the first harmonic of the output of the nonlinear system, which is not sufficient. It is required to extend the DF to analyze higher order harmonics and make the frequency domain analysis more accurate. The High Order Sinusoidal Input Describing functions (HOSIDFs) was first proposed by Nuij [**13**]. Instead of calculating the Fourier series of the high order output, he found an analytical solution of regarding the nonlinear element as a virtual harmonic generator to create different order of harmonics and then deal with the describing function, which is shown in Fig.4. The set of different order of describing function forms HOSIDFs.

III. Reset control

As mentioned in Section I and II, reset action occurs into the reset controller when input is zero. Section III will show some types of reset controller and also details of them.

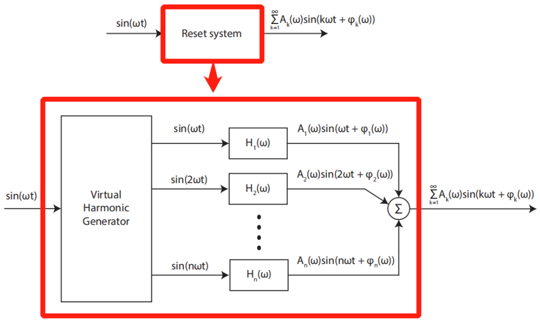

Figure 4: Graphical representation of HOSIDFs [12]

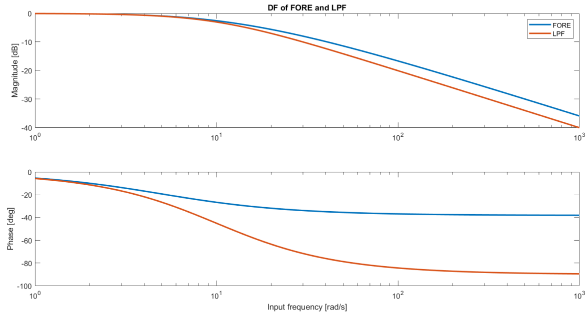

Figure 5: Bode plot of FORE and LPF

A. Clegg Integrator

Clegg integrator was first proposed by J. C. Clegg in 1958 [3]. The parameters in Eqa.(2) are

$$A_r = 0, B_r = 1, C_r = 1, D_r = 0, A_\rho = 0$$

When a linear integrator is applied by reset action, a Clegg integrator can be created. Clegg integrator has a similar gain behavior with linear integrator while the phase lag reduces from 90° to 38°, which means Clegg integrator will contribute more in stability of the system compared to normal linear integrator.

B. First Order Reset Element

The FORE is a reset integrator with reset action applied into first order low pass filter when input is 0 [4]. The parameters in FORE are given:

$$A_r = -\omega_r, B_r = \omega_r, C_r = 1, D_r = 0, A_\rho = 0$$

where ω_r is the corner frequency of the corresponding basic low pass filter.

FORE can be generalized into Generalized FORE, which means there is an additional freedom to tune the reset matrices A_ρ. This is achieved by reset value γ due to the relation of $A_\rho = \gamma$, and the value of range can be modified for $\gamma \in [0, 1]$ to consider the partial reset controller. The describing function for GFORE is given:

$$GFORE(s) = \frac{1}{\frac{s}{\omega_r} + 1} \nearrow \gamma \quad (5)$$

The bode plot of FORE and LPF is shown in Fig.5. The corner frequency in Fig.5 is $\omega_r = 10 rad/s$. It is shown that after the corner frequency ω_r, the FORE element has less phase lag than corresponding LPF.

C. Second Order Reset Element

Hazeleger applied reset action into a second order LPF so the second order reset element can be created with the reset condition when input is zero [5]. The parameters in the state space representation are:

$$A_r = \begin{bmatrix} 0 & 1 \\ -\omega_r^2 & -2\beta\omega_r \end{bmatrix}, B_r = \begin{bmatrix} 0 \\ \omega_r^2 \end{bmatrix}$$

$$C_r = \begin{bmatrix} 1 & 0 \end{bmatrix}, D_r = \begin{bmatrix} 0 \end{bmatrix}$$

$$A_\rho = \begin{bmatrix} \gamma_1 & 0 \\ 0 & \gamma_2 \end{bmatrix}$$

where ω_r is the corner frequency of the corresponding second order LPF and β is the damping ratio. With bode plot of different damping ratio of SORE for full reset in Fig.6, it is illustrated that β is an additional parameter that can be tuned compared to FORE. The controller will be a full reset controller only if γ_1 and γ_2 are all zero. The describing function of SORE is given:

$$GSORE = \frac{1}{(\frac{s}{\omega_r})^2 + \frac{2\beta s}{\omega_r} + 1} \nearrow \gamma \quad (6)$$

The way how damping ratio matters the SORE for full reset control is shown in Fig.6 and obviously the trade-off between delayed phase loss and resonance peak in linear second order LPF is reduced in SORE.

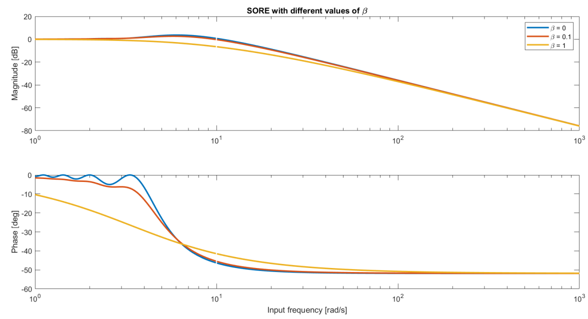

Figure 6: Bode plot of SORE with different damping ratios

D. CgLp

Reset control has the advantage of phase loss reduction, which FORE and SORE can achieve by tuning their parameters in reset matrices. The low pass filter is in control of reducing gain at high frequency to improve noise attenuation, which can be replaced by reset low pass filter in GSORE or GFORE. And also to explore the reset for phase lead applied in region of bandwidth, a new reset element CgLp, constant in gain lead in phase, is introduced to provide broadband phase compensation in required frequencies [**15**]. The structure of CgLp is a reset lag filter R in series with a corresponding order linear lead filter L.

$$R(s) = \frac{1}{(\frac{s}{\omega_{\gamma\alpha}})^2 + \frac{2s\beta_\gamma}{\omega_{\gamma\alpha}} + 1}^{\gamma} \; or \; R(s) = \frac{1}{\frac{s}{\omega_{\gamma\alpha}} + 1}^{\gamma} \tag{7}$$

and

$$L(s) = \frac{(s/\omega_r)^2 + (2s\beta_r/\omega_r) + 1}{(s/\omega_f)^2 + (2s/\omega_f) + 1} \; or \; L(s) = \frac{s/\omega_r + 1}{s/\omega_f + 1} \tag{8}$$

where $\omega_{\gamma\alpha} = \frac{\omega_\gamma}{\alpha}$ representing the corner frequency of the shift, and α is a fraction of shift in corner frequency of reset element. Its corresponding basic linear element and the specific value can be taken from [**15**].

The Fig.7 shows the broadband phase lead achieved by CgLp using GFORE with the frequency range of $[\omega_\gamma, \omega_f] = [10, 100]$ with the value of $\gamma = 0$ and $\beta = 1$.

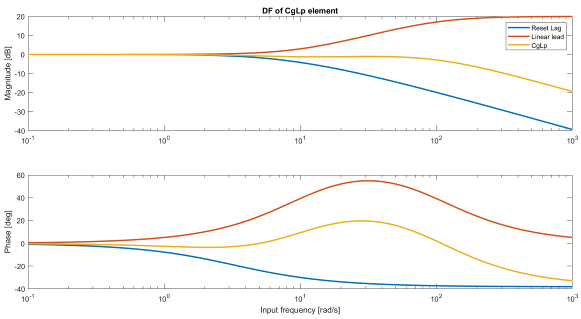

Figure 7: Bode plot of CgLp

Generally in linear case, the gain and phase difference between linear lead filter and linear lag filter should be zero respectively. While after the applied reset action into lag filter, the gain difference still stays zero while the phase difference over zero.

IV. Existing strategies for improvement of transient response

In this section, a few strategies are discussed to enhance the transient response in time domain. The improved transient response could be obtained less overshoot, faster settling time or rise time, etc and less overshoot will be focused.

A. PI + CI

The structure of PI+CI configuration is given in Fig.8, which combines a linear PI controller and Clegg integrator in parallel.

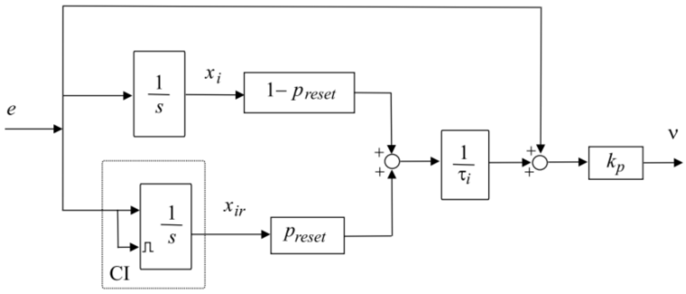

Figure 8: Block diagram of PI+CI [**16**]

Based on the block diagram of PI+CI structure, it can be represented as:

$$PI + CI = kp(1 + \frac{1}{\tau_i}(\frac{1 - P_{reset}}{s} + \frac{P_{reset}}{s})) \tag{9}$$

The configuration can be tuned by parameter P_{reset}, which is the weight of Clegg integrator compared to linear PI controller. To explore how the configuration will perform, it is assumed the plant $\frac{1}{s+0.5}$ controlled by PI + CI with $k_p = 1$, $\tau_i = 1$ and $P_{reset} = 0.2$. The comparison of the system with linear PI controller, PI reset controller and PI+CI controller of step response is shown in Fig.9.

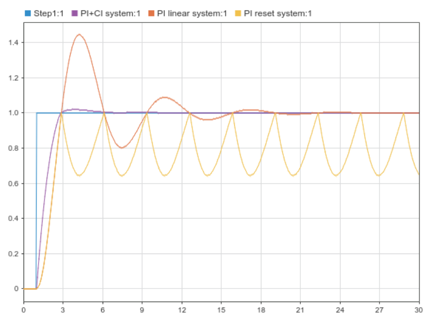

Figure 9: Step response of PI, PI+CI and reset PI

It is illustrated from Fig.9 that PI+CI configuration lessen the overshoot compared to linear PI controller and also reduce the oscillations that caused by nonlinear controller. Less overshoot means better transient response. The tuning rule of PI+CI configuration is summarized in [**19**].

B. Phase Loss Compensation

In time domain, the peak of resonance is related to overshoot and both of them are related to phase margin in open loop. So improving the phase margin is equivalent to improve the overshoot.

In Erdi's thesis, CgLp is introduced into the system with additional integrator in controller [**15**]. Considering the transfer function of the mass-damper-spring system is represented as:

$$P(s) = \frac{1}{ms^s + cs + k} \tag{10}$$

where the parameters are chosen as the mass $m = 45kg$, damping constant $c = 1114N.m$ and spring constant $k = 1e5N/m$ respectively. The bandwidth ω_c is chosen at $100Hz$, and other parameters in PID can be designed according to the rule of thumb [**17**].

As discussed before, additional integrator increases gain at low frequency so that the system would have better tracking performance and higher stiffness so based on PID, PI^2D is created. However, the open loop bode plot of PID and PI^2D in Fig.10 shows that phase margin gets loss because of the additional integrator.

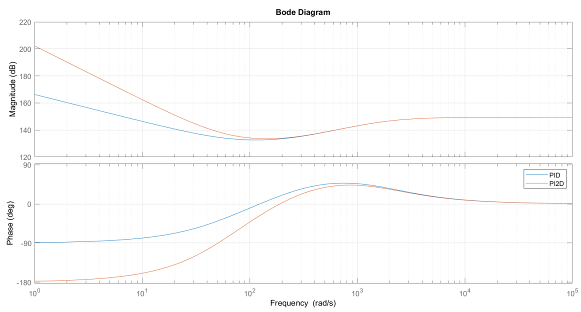

Figure 10: Bode plot of PID, PI^2D in open loop

To overcome the reduced phase margin, there are two methods mentioned in Erdi's work [**21**], to replace one of integrator into clegg integrator or introduce cglp into the system. The CgLp is combined with reset lag filter and linear lead filter in series, which can be designed as: reset lag filter(FORE),

$$R(s) = \frac{1}{\frac{s}{\omega_{\gamma\alpha}} + 1} \tag{11}$$

and linear lead filter,

$$L(s) = \frac{\frac{s}{\omega_r} + 1}{\frac{s}{\omega_f} + 1} \tag{12}$$

The frequency response between PID, PI^2D, PI(CI)D and PI^2D_CgLp can be observed in Fig.11. It is shown that replaced reset integrator and added CgLp compensate the phase loss and increases more phase lead, so the phase margin even higher than system with PID controller. More phase margin in open loop of the system means better improved overshoot and enhanced transient response.

The step response of PID, PI^2D, PI(CI)D and PI^2D_CgLp in Fig.12 confirms that with more phase margin in open loop, the less overshoot in the time domain.

C. Sequence of Linear Lead Element and Reset Element

As introduced in section III, CgLp is a reset controller with reset lag element and linear lead element. In Chengwei's thesis, he listed 4 different sequences of controllers which can be shown in tableI and analyse them from sensitivity function, output, transient response and etc to find the optimal sequence among the configurations by using HOSIDFs tool [**18**]. It is

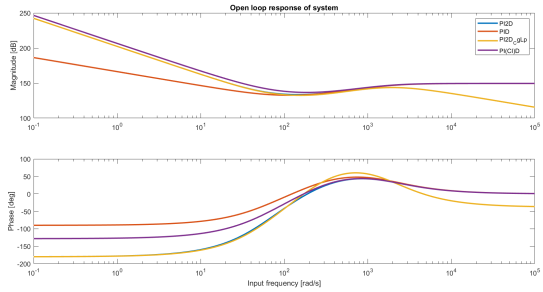

Figure 11: Frequency response of PID, PI^2D, PI(CI)D and PI^2D_CgLp

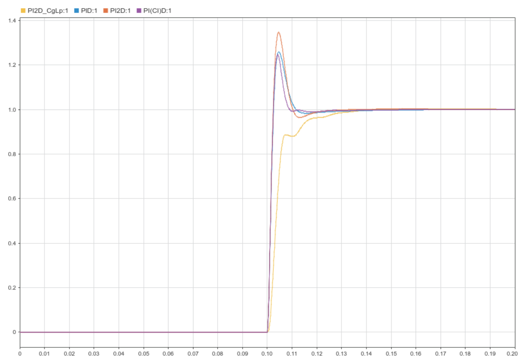

Figure 12: Step response of PID, PI^2D, PI(CI)D and PI^2D_CgLp

because of the deterioration of the higher harmonics [**15**] [**20**], and chengwei has found the optimal configurations in his paper through analysing the 4 configurations to show the least effect by higher harmonics. These combinations can be seen as a linear PI controller and a reset element CgLp, including FORE controller as well as a linear lead controller. It is found that no matter which configurations, they all have the same describing functions as shown in Fig.13.

The configurations are used to control second order mass-damper-spring system, which is represented as:

$$P(s) = \frac{1}{1.077 \times 10^{-4}s^2 + 0.0049s + 4.2218} \tag{13}$$

The Lead-Reset-Lag configuration has no overshoot and no state error when analyze the step response of the different configurations in Fig.14. And the overshoot occurs when reset element is set before lead element, while state error occurs when reset element is located after lag element. The reason why Lead-Reset-Lag configuration does not have steady state error is because lag filter after the reset element would help

No.	Sequence
1	Lead-Reset-Lag
2	Lag-Reset-Lead
3	Reset-Lead-Lag
4	Lead-Lag-Reset

Table I: Configurations of different sequences [**18**]

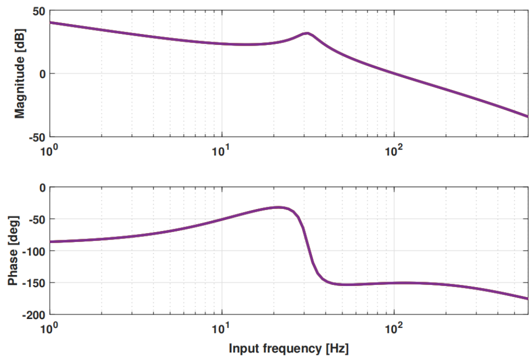

Figure 13: Describing function of the system with different configurations [**18**]

to reduce the influence of jumps, which would involve high frequencies.

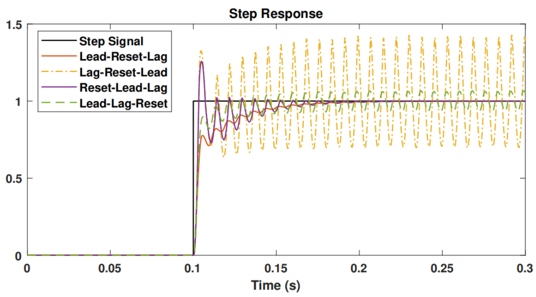

Figure 14: Step response of configurations [**18**]

The different sequence of the elements of reset lag filter and linear lead filter in CgLp can have different step response that can observed in Fig.14. And the configuration of Lead-reset has less overshoot and improve the transient response compared to Reset-lead.

V. Conclusions

High-tech industries and researchers put lots of efforts on reset controllers due to the simple structure, easy to implement and the ability to overcome functional limitations of linear controllers. Reset controllers are reviewed from basic definition, the method that can analyze in frequency domain and different types of the reset controllers.

Reset controllers are investigated and used to improve the transient response. The first strategy is configuration PI+CI. PI+CI strategy can not only solve the problem of limit cycles of pure reset controller Clegg integrator but also reduce the overshoot compared to linear PI controller. However, there is still a trade-off between linear and reset controller and it saves the potential of reset controller. The second strategy is to compensate the phase margin in the open loop. It is because the peak of resonance is related to overshoot and both them are related to phase margin, so dealing with phase margin is equivalent to deal with the overshoot. It is observed that the controller PI^2D_CgLp and PI(CI)D has more phase margin than either PI^2D or PID. The third strategy is to consider the different sequence of the controllers. The linear lead element and the reset can be seen as a CgLp element. When the sequence of reset element and linear lead element changes, the step response of the designed system shows that Lead-reset configuration does not have overshoot while Reset-lead occurs. This phenomenon shows the proper sequence of controllers in reset controller can also improve the transient response.

There are lots of literature focus on phase compensation but leave the blank of transient response. The configuration Lead-reset provides the possibility that this sequence can improve transient response than the other configuration. Based on that, the wildly used version of reset controller is asked to improve the transient response.

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Chapter 3

Objective

3-1 Problem Definition

Integrator in PID plays an important role to have high gains at low frequencies. High tech companies require higher precision so more integrators are trying to be added. In literature review part, it shows that the state of art of reset control. Therefore the system with more integrators can also use controller with reset element to avoid the functional limitation. It is a big advantage of analysing reset control in frequency domain by describing function compared to other nonlinear controllers but high order harmonics are introduced and needed to be considered, which has been investigated by lots of researchers [11][12]. Reset control can overcome the limitation of linear controller. Besides that, time domain analysis should also consider. And in [10], it has been already proved that changing the sequences of the controllers in system can improve transient response. Based on above, the objective of this thesis is set:

Improve the transient response of system with multiple integrators and check if their performances also be improved.

3-2 Approach

The research is conducted by following approach.

- Based on optimal sequence of controllers in system design the reset control suits multiple integrator system
- Using different methods to tune the reset control and keep frequency domain behavior while improve transient response.
- Check the close loop performance of systems with the designed controller in Matlab simulation.

3-3 Thesis Outline

The structure of the thesis is as follows.

Chapter 1 gives the brief introduction of how high tech industry field needs nonlinear controller to reach higher precision requirements. In chapter 2, there is the state of the art about reset control strategy. This chapter gives the idea about the definition of the problem, and objective of this thesis based on the background of the state of art and industrial requirements. Chapter 4 is the core of this thesis report, including the controller design with multiple integrators to improve the transient response of the system. The conclusion and further research direction are included in chapter 5. And more details about thesis work can be found in appendices.

Chapter 4

Improvement of transient response with multiple integrators

As discussed in previous chapters, PID controller is widely used in industries. While with the increasing demands of higher precision, PID controller can not meet the higher requirements because of the functional limitations. The integral element in PID provides the gain at low frequencies, which is related to the tracking performance. Besides that, more integrators in the system can also achieve good disturbance rejection [3]. But adding more integuators into the system also introduce instability because the phase loss is added so that the transient response will be ruined. In this case, this chapter will focus on improving the transient response by using reset controllers to reduce the overshoot in time domain.

4-1 System overview

Most mechanical system can be represented to be modeled as mass-spring-damper system. For mechanical system, stiffness is associated with the spring to give the property of resistance against deformation, which is very important [13]. And body in the system can not be ignored and follows Newton's second law. For damper of the mechanical system can be very small. The transfer function of a single mass-spring-damper system is

$$P(s) = \frac{\frac{\omega_0^2}{k}}{s^2 + 2\zeta\omega_0 s + \omega_0^2} \tag{4-1}$$

where $\omega_0 = \sqrt{\frac{k}{m}}$ is the undamped natural resonance frequency, $\zeta = \frac{c}{2m\omega_0}$ is the damping ratio [13]. And based on the Eq.4-1, the plant used to make the simulation is given as below [14].

$$P(s) = \frac{2.262e5}{s^2 + 133.7s + 1.185e5} \tag{4-2}$$

4-2 Controller design

PID controller is widely used in modern industries and there is rule of thumb for designing general industry standard PID. And CgLp is an abbreviation for Constant in gain Lead in phase, which is combined with a linear lead element and a reset lag element with the corresponding order. CgLp provides phase lead at a broad band frequency and without changing the gain behavior [15] and that's why it can be made use of in this paper.

4-2-1 PID design

PID used in industry standard can be defined as below,

$$PID(s) = kp\left(\frac{s+\omega_i}{s}\right)\left(\frac{1+\frac{s}{\omega_d}}{1+\frac{s}{\omega_t}}\right)\left(\frac{1}{1+\frac{s}{\omega_f}}\right) \tag{4-3}$$

where kp is the proportional gain, ω_i is the frequency where integrator action in PID stops, ω_d is where differentiator action starts and ends at ω_t and ω_f is the corner frequency where starts the tamed differtiating action. The proportional action works on setting desired bandwidth, integrator is used for creating high gain at low frequencies for good tracking performance and disturbance rejection and the derivative element can achieve phase lead at crossover frequency.

4-2-2 CgLp design

CgLp, Constant in gain Lead in phase, consists of linear lead filter and reset lag filter. Generally there are two orders getting extended into CgLp, GFORE and GSORE. In this thesis, the reset element in CgLp is only considered as GFORE with the corresponding order of linear lead filter. The CgLp thus is defined as

$$CgLp = R(s) * L(s) = \left(\frac{1}{\cancel{\frac{s}{\omega_{\gamma\alpha}} + 1}}\right)\left(\frac{\frac{s}{\omega_r} + 1}{\frac{s}{\omega_f} + 1}\right) \tag{4-4}$$

while the arrow indicates the nature of reset. And $\omega_{\gamma\alpha}$ is accounting the shift in coroner frequency so $\omega_{\gamma\alpha} = \frac{\omega_\gamma}{\alpha}$. α is the function of γ and the value of α is to make sure the corner frequency is located at desired place. The magnitude of gain keeps constant at $0dB$ and the phase lead is added within the broad band.

4-3 Performances with multiple integrators

The performances with multiple integrators are discussed as disturbance rejection, reference tracking and transient response. While disturbance rejection and reference tracking are improved because more integrators means higher gain at low frequency according to the loop shaping. But the transient response would be destroyed because of the additional integrators.

4-3-1 PI^{n}D Disturbance Rejection

In Fig.4-1, it shows that the general feedback system, where $P(s)$ is the plant in the system, $C(s)$ is the controller, r is the reference signal applied into the system, d is the process disturbance and n is the noise.

In order to observe the ability of disturbance rejection of the system, the normalized sinusoidal signal should be applied after controller but before plant then measure the error from the system. The less error signal shows the better disturbance rejection is.

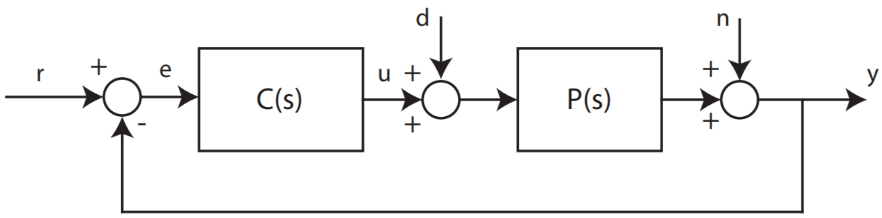


Figure 4-1: Feedback system [1]

For example, the sinusoidal signal of 30 Hz is applied into the designed system [16]. The error signal with non-reference signal is given in Fig.4-2. In the figure, it is obviously seen that more integrators applied into the system, the better disturbance rejection of the system at certain frequency.

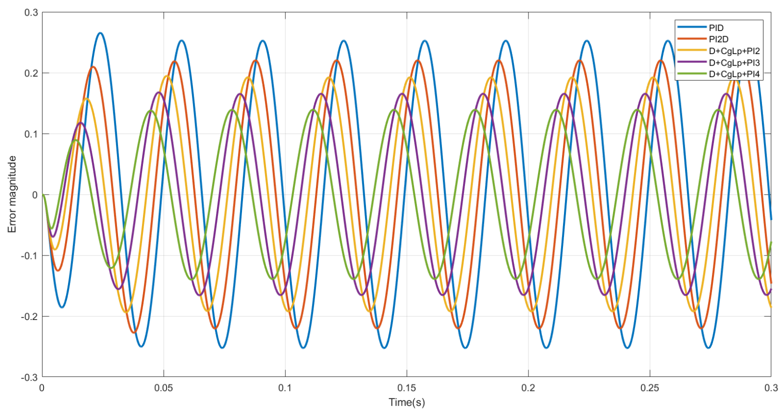


Figure 4-2: Disturbance rejection of PI^{n}D

4-3-2 PI^{n}D Reference tracking

Based on the Fig.4-1, in order to check the reference tracking of the system, the signal input for reference is needed. Sinusoidal signal of 10 Hz is applied into the system and by tracking the error signal, the reference tracking performance should be known [16]. The result of reference tracking performance for PI^{n}D is given in Fig.4-3. More integrators of controller in the system outperforms less integrators of controller.

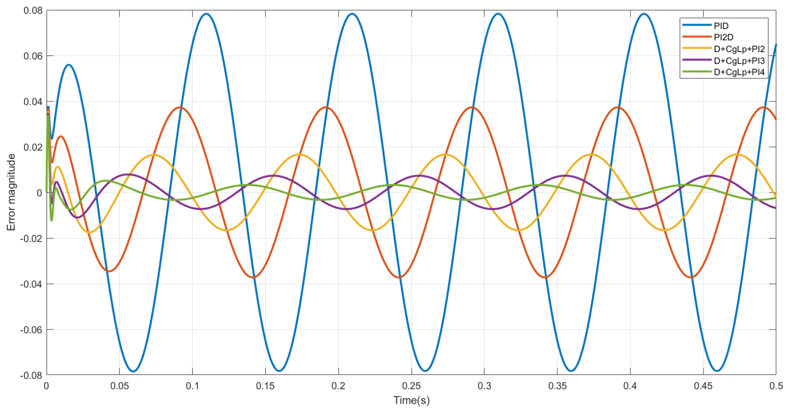


Figure 4-3: Reference tracking

4-3-3 Transient Response

However, additional integrators will destroy the transient response. While overshoot is related to phase margin in open loop. And integrators introduce more phase loss into the system thus cause the instability. The transient response of different systems PI^{n}D controller is shown in Fig.4-4. With the more integrators added into the system, the overshoot gets larger and larger.

To overcome the increased overshoot, reset control is considered to be introduced into the system because of its phase advantage.

4-4 Improve with reset control

More integrators in the system can improve the performance of both tracking and disturbance rejection. While integrators bring phase loss into the system and reset control has the phase advantage so it can compensate the phase loss causing by the multiple integrators. On the other hand, reset control is easy to implement through loop shaping techniques compared to other types of nonlinear control. Because loop shaping techniques are very popular used in industries. In this section,

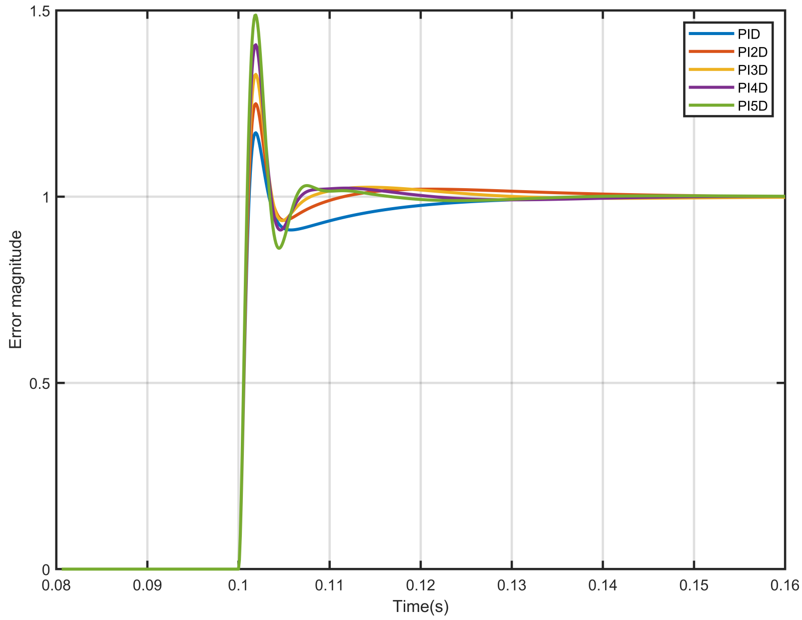


Figure 4-4: Transient response of PI^{n}D

there are three methods to overcome the issue will be shown. Changing the sequence of the controller, by putting differentiator (Lead) in front of reset control can reduce overshoot. For the reset controller, tuning the band of CgLp and γ value can also change the performance of reset controller.

4-4-1 Different sequence of controller

In [10], it has been proved that by changing the sequence of controller can improve transient response with less overshoot and less settling time. When the sequence of controller is linear lead-reset lag-linear lag, there will be no overshoot in transient response [10].

Inspired by [10], the controller with different sequences driving the same plant as designed in last section can be shown here. And in [3], it also shows that double integrators added into the system can improve the robustness and precision. So the controller consisting of PI^{2}D and CgLp based on [10] and [3] is considered as CgLp-PI^{2}D and D-CgLp-PI^2.

The describing function of the two different controllers is shown in Fig.4-

5. From Fig.4-5, when there is one more integrator added into PID controller and turn into PI^{2}D, the phase margin at bandwidth frequency gets less. But when add reset element CgLp into the system, no matter the sequence is, their describing function behaves the same and the gain behavior also doesn't change before the end of cglp band.

In this case, the band of the CgLp is chosen as $[0.9\omega_c, 3.7\omega_c]$ so that after the phase compensation by CgLp the phase margin at bandwidth is 50°. CgLp compensates the phase loss introducing by the additional integrator.

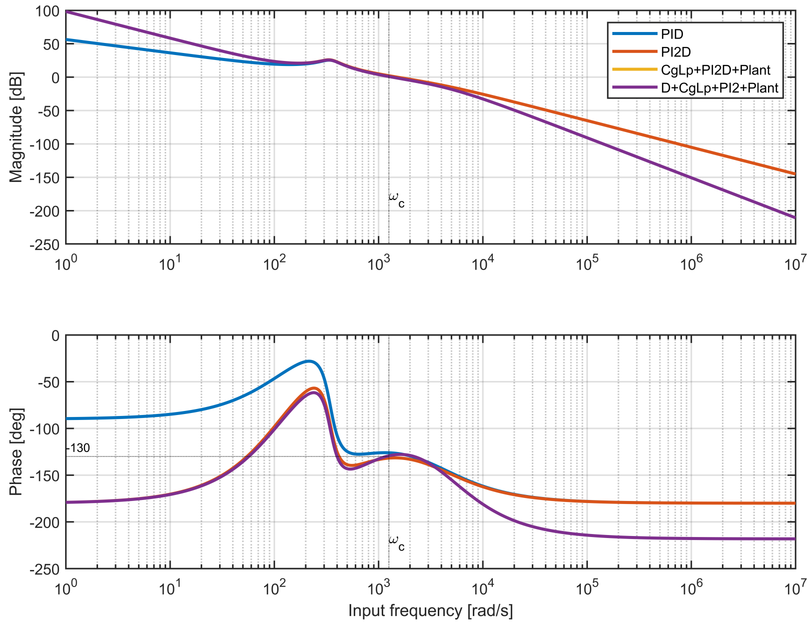


Figure 4-5: Describing function of systems

While in time domain, the step responses of the two different sequence of controllers are not the same. In Fig.4-6, the overshoot of the controller with D-CgLp-PI^2 is reduced compared to CgLp-PI^{2}D. So based on the result of this simulation, it shows that system with controller of D-CgLp-PI^2 has less overshoot thus better transient response.

4-4-2 Varying band

The band of CgLp determines the frequency ω_r for where the broad band starts and ends at ω_f. Within the band, it provides broadband phase

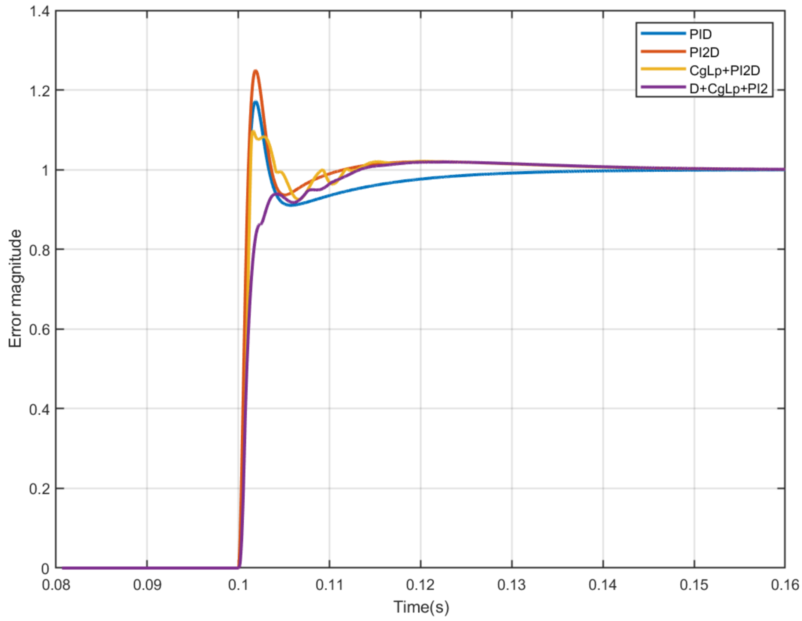


Figure 4-6: Step response of systems

compensation. So different band of the CgLp can provide different phase compensation at bandwidth. Transient performance is focused in this report so firstly phase margin should be fixed in frequency domain and then vary band of CgLp to get same phase margin at bandwidth.

For different number of integrators in the system, the band of $[\omega_r, \omega_f]$ is varied to give the same phase margin at bandwidth as the system only with PID controller. It shows in Fig.4-7 that with different bands of CgLp can make different systems to have the same phase margin. In Fig.4-7, it shows the describing function when the phase margin is 50°. More fixed phase margin describing function configurations are shown in Appendix A.

The Table4-1 gives the bands of D+CgLp+PI^n to get different desired phase margin at bandwidth. With the increasing number of integrators, the bands are needed wider and wider. Because the broadband starts near the bandwidth so enlarging the band by modifying ω_f to be larger can compensate more phase loss. And when the same phase margin is required, with more integrators added, the band is also enlarged by giving larger ω_f.

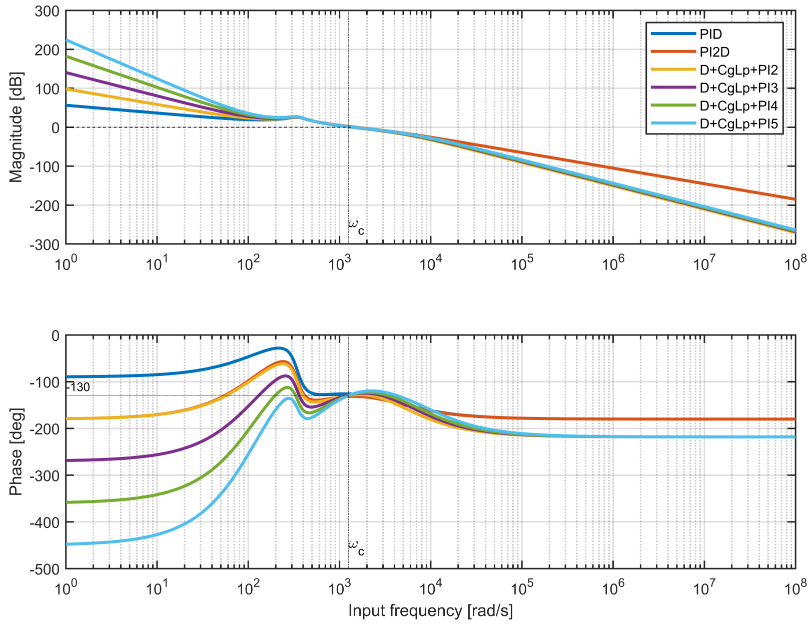


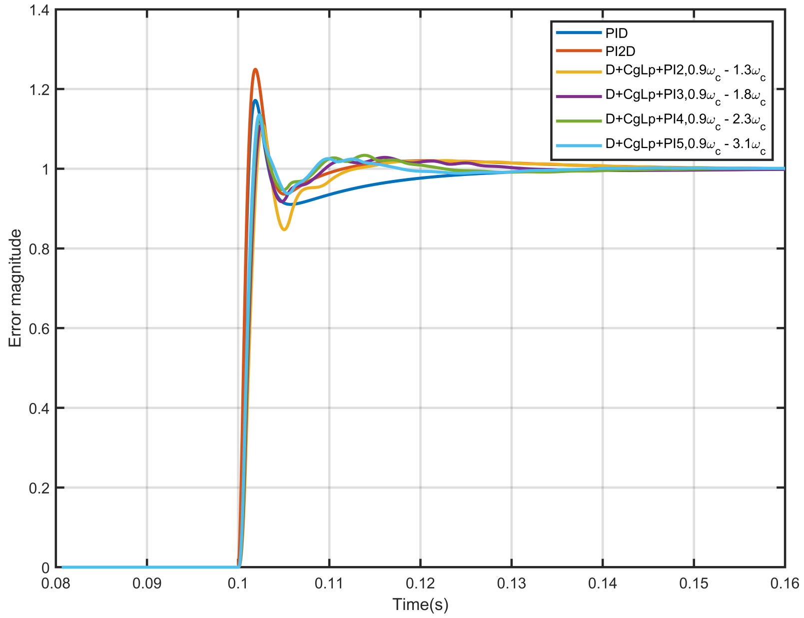
Figure 4-7: Describing function of system with different controllers when vary band

Controllers	PM = 50°		PM = 40°		PM = 30°	
	ω_r	ω_f	ω_r	ω_f	ω_r	ω_f
D+CgLp+PI^2	$0.9\omega_c$	$3.7\omega_c$	$0.9\omega_c$	$1.7\omega_c$	$0.9\omega_c$	$1.3\omega_c$
D+CgLp+PI^3	$0.8\omega_c$	$5.0\omega_c$	$0.9\omega_c$	$2.9\omega_c$	$0.9\omega_c$	$1.8\omega_c$
D+CgLp+PI^4	$0.7\omega_c$	$6.1\omega_c$	$0.9\omega_c$	$4.1\omega_c$	$0.9\omega_c$	$2.3\omega_c$
D+CgLp+PI^5	$0.6\omega_c$	$8.3\omega_c$	$0.9\omega_c$	$7.1\omega_c$	$0.9\omega_c$	$3.1\omega_c$

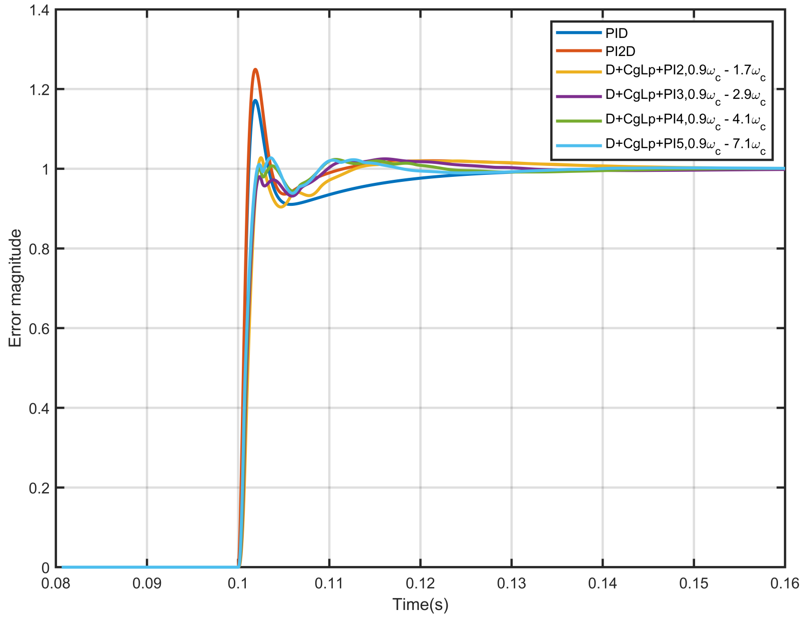
Table 4-1: Bands of controllers with different phase margin

Base on the band regions given in Table4-1, the transient performance of systems with different numbers of integrators is in Fig.4-6. After adding the reset element CgLp into PI^{n}D, the overshoot of the system is decreased even if there are more integrators added, the overshoot is still less than system with PID or PI^{2}D. When varying the band of CgLp, γ value is constant zero.

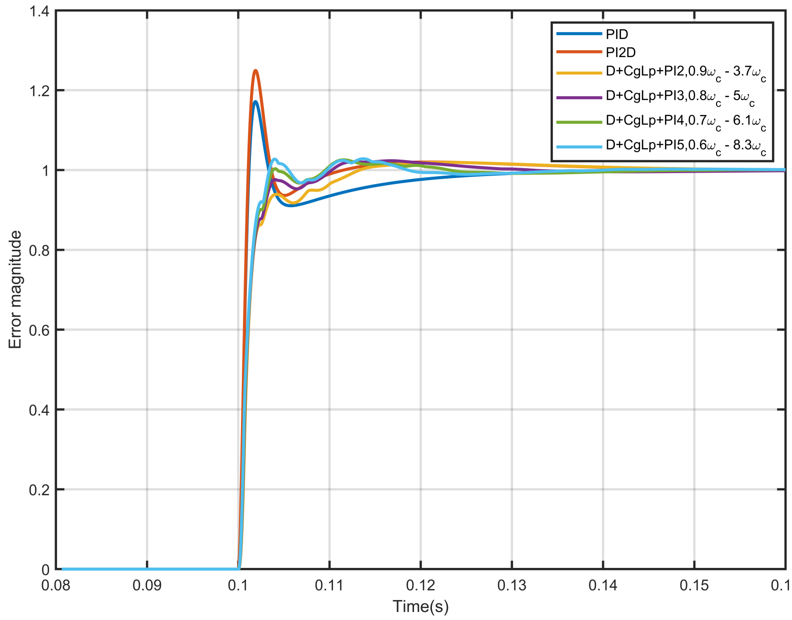
From the step response of the system with multiple integrators in different constant PM in Fig.4-6, they showed that the less phase margin required, the more overshoot will be in transient response. But system with reset controller CgLp still has less overshoot than system with PID and PI2D, which means by varying the band of CgLp can improve the transient response.



(a) Step response when PM = 30



(b) Step response when PM = 40



(c) Step response when PM = 50

Figure 4-8: Step response of systems with multiple integrators when vary band

Since it is hard to observe how the settling time behaves through the picture, the settling time of different systems is shown in Fig.4-9. Settling time is decreased when there is more integrators added, especially after adding reset control into PI^2. Thus the transient response of D+CgLp+PI^n system improves than system with either PID or PI^2.

4-4-3 Varying γ value

Besides the band of CgLp, the γ value can also determine the performance of reset element. From Fig.4-10, it shows that the reduction in phase lag with different reset values and with the increasing of γ value, the phase lag also increase while different types of reset controller also perform differently.

To vary the γ value in CgLp, the point is still to keep the same phase margin in frequency domain and compare their behaviors in time domain. The frequency domain of system with D+CgLp+PI^n is shown in Fig.4-11 and the γ value is chosen to have same phase margin for

PM=50Hz at bandwidth. More required phase margin describing function configurations are given in Appendix B.

The Table 4-2 gives γ value in below. The γ value is chosen based on D+CgLp+PI^2 and when more integrators are added into the system, in order to keep phase margin constant as required, the change of γ value is given in Table 4-2.

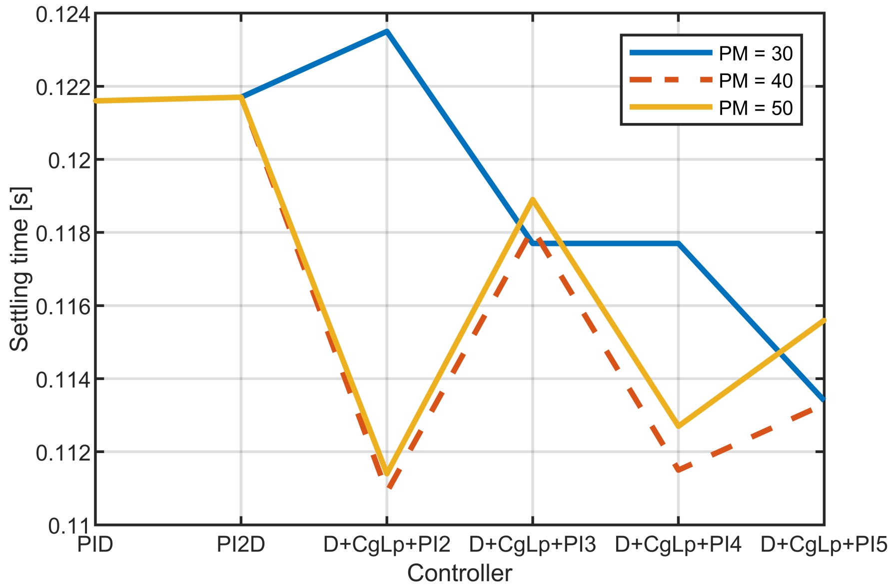


Figure 4-9: Settling time after step response by varying the band

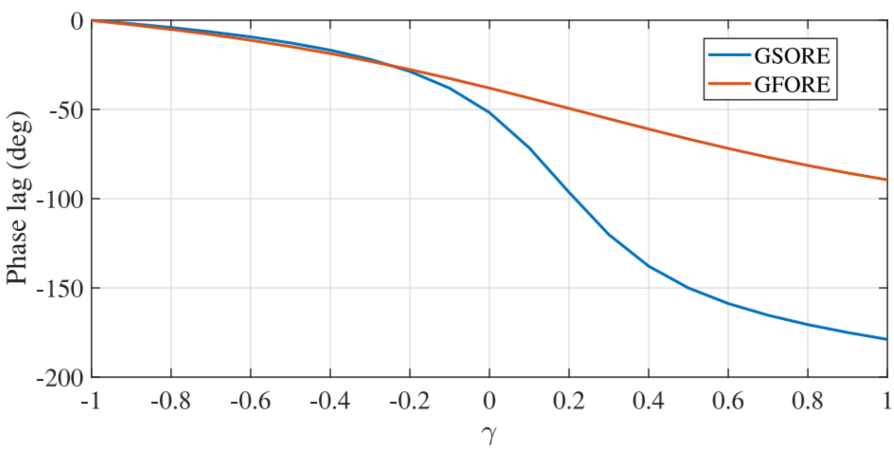


Figure 4-10: Reduction in phase lag with reset controller

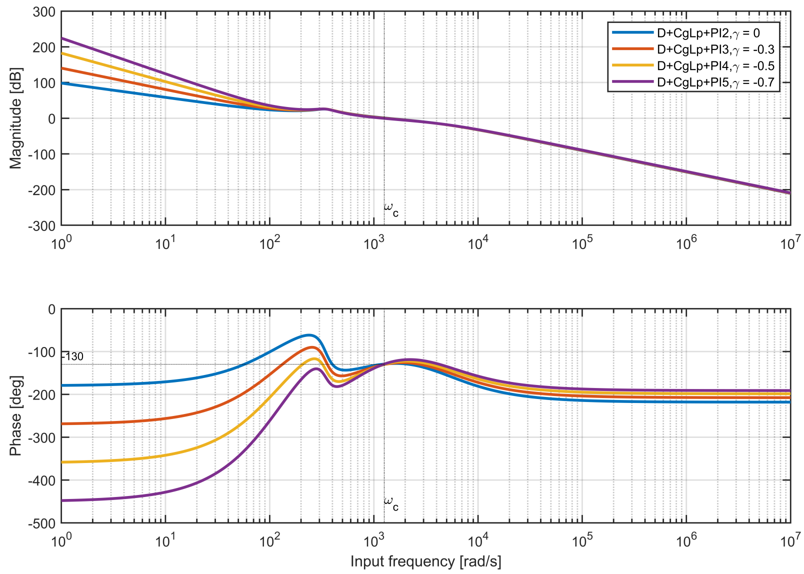


Figure 4-11: Describing function of system with different controller when vary γ

Controllers	PM = 50° $0.9\omega_c - 3.7\omega_c$	PM = 40° $0.9\omega_c - 1.7\omega_c$	PM = 30° $0.9\omega_c - 1.3\omega_c$
D+CgLp+PI^2	0	0	0
D+CgLp+PI^3	-0.2	-0.5	-0.3
D+CgLp+PI^4	-0.4	-0.7	-0.5
D+CgLp+PI^5	-0.6	-0.8	-0.7

Table 4-2: γ value with different constant PM in bands

The simulation of system with D+CgLp+PI^n in time domain is shown in Fig.4-13. It is shown that transient response gets better than system with only PID or PI^{2}D due to their reduced overshoot and settling time. With more integrators added, the transient response is not damaged and still keeps the advantages compared to PID or PI^{2}D.

Compared with varying band of CgLp, varying γ value to keep required phase margin performs better at reducing the overshoot while needs more settling time. On the other hand, the settling time is less when higher phase margin is required from Fig.4-12

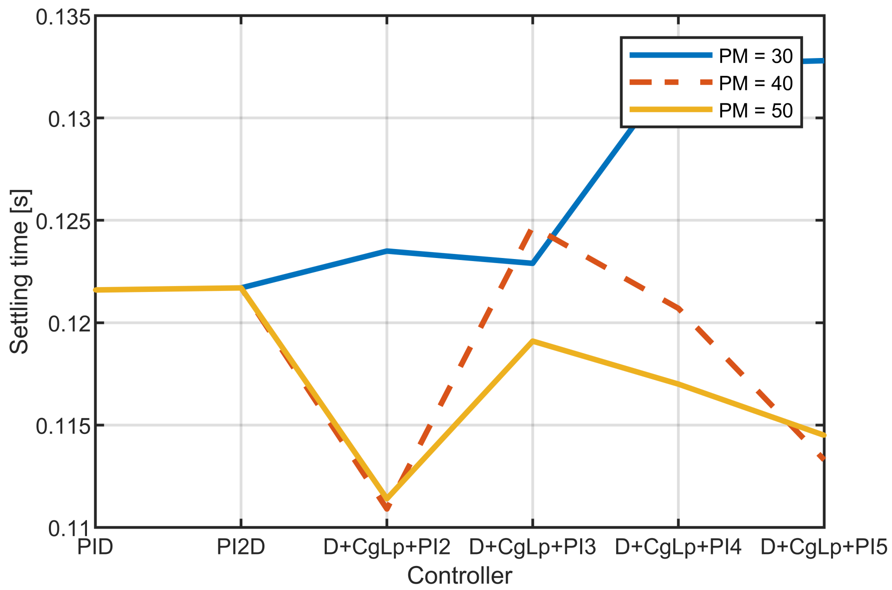


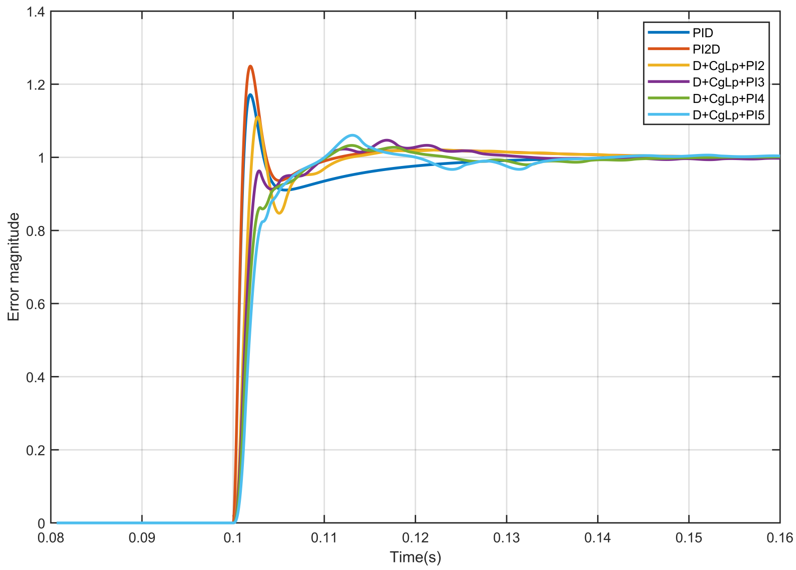
Figure 4-12: Settling time by varying γ

4-5 Checking performance

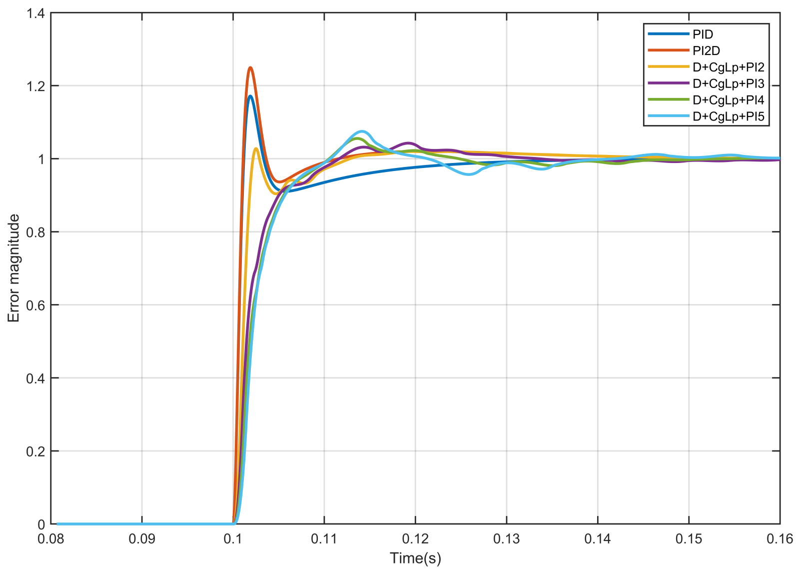
As the results showed before, adding integrators will improve the disturbance rejection and also the reference tracking. It has to be proved that after adding reset element into the system that the advantages still stay.

4-5-1 Disturbance rejection

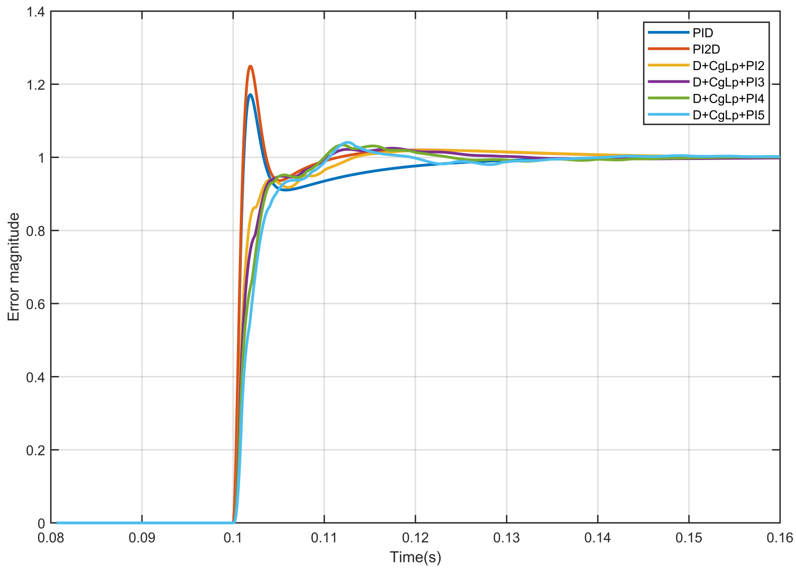
According to the theoretical controller analysis, reset control surpass linear control due to the phase loss advantage which is good for disturbance rejection. To check the disturbance rejection after adding reset element, the sinusoidal signal is applied between controllers and plant in the system. The frequency of the sinusoidal disturbance is given as 30 Hz. The error signal is measured and compared in Table 4-3 and it uses the method of varying the band of CgLp to keep phase margin fixed as 30°, 40° and 50° while γ value stays 0. It shows that after adding reset element into the multiple integrators system, the disturbance rejection has been also improved.



(a) Step response when PM = 30



(b) Step response when PM = 40



(c) Step response when PM = 50

Figure 4-13: Step response of systems with multiple integrators when vary γ

Controllers	PM = 30°		PM = 40°		PM = 50°	
	max(e(t))	Band	max(e(t))	Band	max(e(t))	Band
PID	0.2655	-	0.2655	-	0.2655	-
PI^2D	0.2204	-	0.2204	-	0.2204	-
$D+CgLp+PI^2$	0.2441	$0.9\omega_c - 1.3\omega_c$	0.2368	$0.9\omega_c - 1.7\omega_c$	0.2258	$0.9\omega_c - 3.7\omega_c$
$D+CgLp+PI^3$	0.2153	$0.9\omega_c - 1.8\omega_c$	0.2033	$0.9\omega_c - 2.9\omega_c$	0.2006	$0.8\omega_c - 5\omega_c$
$D+CgLp+PI^4$	0.1844	$0.9\omega_c - 2.3\omega_c$	0.1743	$0.9\omega_c - 4.1\omega_c$	0.1746	$0.7\omega_c - 6.1\omega_c$
$D+CgLp+PI^5$	0.1519	$0.9\omega_c - 3.1\omega_c$	0.1448	$0.9\omega_c - 7.1\omega_c$	0.1483	$0.6\omega_c - 8.3\omega_c$

Table 4-3: Maximum steady error for Disturbance rejection when vary band

In Fig.4-14, it shows the maximum steady error of different systems with numbers of integrators with reset control. From Fig.4-14, the steady error slight raises when adding CgLp into PI^2D but still lower than PID system and with the increasing number of integrators, the steady error keeps decreasing.

While in Table.4-4 below, it shows the disturbance rejection by only varying the γ value with the fixed band. In this table, it shows that when tuning the γ value to determine CgLp, the disturbance rejection does not always perform better when adding more integrators into the system.

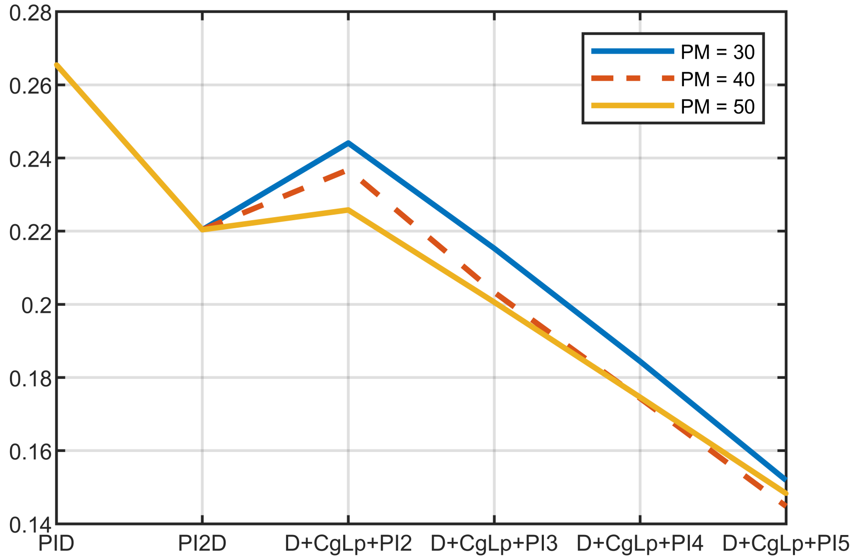


Figure 4-14: Maximum steady error of systems when vary the band

Controllers	PM = 30°			PM = 40°			PM = 50°		
	max(e(t))	γ	Band	max(e(t))	γ	Band	max(e(t))	γ	Band
PID	0.2655	-	-	0.2655	-	-	0.2655	-	-
PI^2D	0.2204	-	-	0.2204	-	-	0.2204	-	-
D+CgLp+PI^2	0.2433	0	$0.9\omega_c - 1.3\omega_c$	0.2368	0	$0.9\omega_c - 1.7\omega_c$	0.2258	0	$0.9\omega_c - 3.7\omega_c$
D+CgLp+PI^3	0.2176	-0.3	$0.9\omega_c - 1.3\omega_c$	0.4191	-0.5	$0.9\omega_c - 1.7\omega_c$	0.2098	-0.2	$0.9\omega_c - 3.7\omega_c$
D+CgLp+PI^4	0.4183	-0.5	$0.9\omega_c - 1.3\omega_c$	0.3813	-0.7	$0.9\omega_c - 1.7\omega_c$	0.2074	-0.4	$0.9\omega_c - 3.7\omega_c$
D+CgLp+PI^3	0.3918	-0.7	$0.9\omega_c - 1.3\omega_c$	0.4641	-0.8	$0.9\omega_c - 1.7\omega_c$	0.3082	-0.6	$0.9\omega_c - 3.7\omega_c$

Table 4-4: Maximum steady error for Disturbance rejection when vary γ

In Fig.4-15, the maximum steady error of the systems with different number of integrators is shown. It shows that it is not with more integrators the less steady error will be. So for disturbance rejection, varying band has the advantage than varying γ.

4-5-2 Reference tracking

Based on the theoretical analysis, the higher gain at low frequencies means better tracking performance. The normalized sinusoidal signal is applied as reference signal of different frequencies for f1 = 2Hz and f2 = 10Hz. The tracking error for the different controllers of PID, PI^2D and D+CgLp+PI^n is given in Table.4-5. From Table.4-5, it shows that the systems with more integrators have better reference tracking and with the larger frequencies input into the system the error signal also strengthens. But when different constant phase margins are required in the system, that doesn't effect the disturbance rejection since there is only slight change of error value in between.

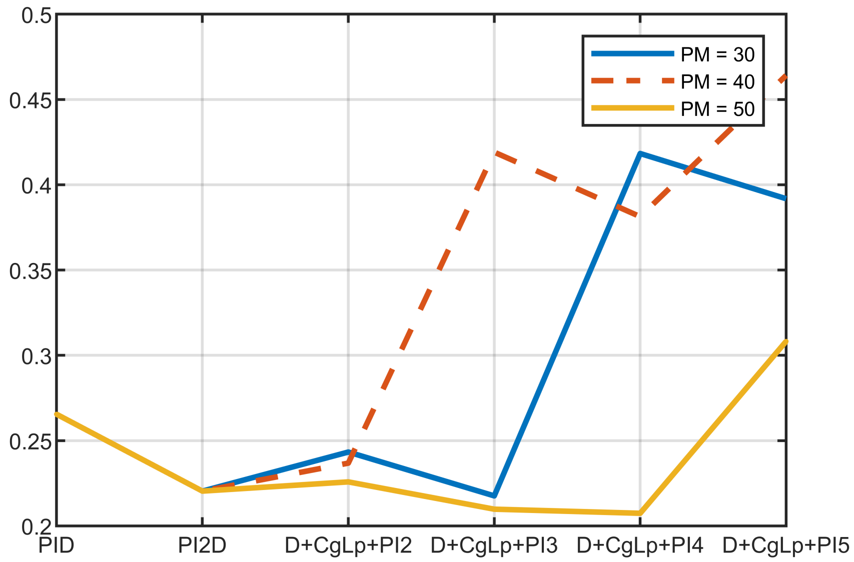


Figure 4-15: Maximum steady error of systems when vary γ

Controllers	PM = 30°			PM = 40°			PM = 50°		
	max(e(t)), f1	max(e(t)), f2	Band	max(e(t)), f1	max(e(t)), f2	Band	max(e(t)), f1	max(e(t)), f2	Band
PID	0.0189	0.0783	-	0.0189	0.0783	-	0.0189	0.0783	-
PI^2D	0.0019	0.0373	-	0.0019	0.0373	-	0.0019	0.0373	-
D+CgLp+PI^2	1.9e-3	3.8e-2	$0.9\omega_c - 1.3\omega_c$	1.9e-3	3.75e-2	$0.9\omega_c - 1.7\omega_c$	1.9e-3	3.74e-2	$0.9\omega_c - 3.7\omega_c$
D+CgLp+PI^3	1.87e-4	1.68e-2	$0.9\omega_c - 1.8\omega_c$	1.87e-4	1.66e-2	$0.9\omega_c - 2.9\omega_c$	2.00e-4	1.66e-2	$0.8\omega_c - 5\omega_c$
D+CgLp+PI^4	2.95e-4	8.1e-3	$0.9\omega_c - 2.3\omega_c$	2.83e-4	7.9e-3	$0.9\omega_c - 4.1\omega_c$	2.82e-4	7.9e-3	$0.7\omega_c - 6.1\omega_c$
D+CgLp+PI^5	2.49e-4	3.6e-3	$0.9\omega_c - 3.1\omega_c$	2.35e-4	3.6e-3	$0.9\omega_c - 7.1\omega_c$	2.27e-4	3.5e-3	$0.6\omega_c - 8.3\omega_c$

Table 4-5: Maximum steady error for tracking performance when vary band

In Fig.4-16, it shows when the frequency of input sinusoidal signal is given as 10Hz, the maximum steady error for tracking performance of systems with multiple integrators and reset control. The error decreases when there is more integrators in the system, which means that more integrators added helps to improve the reference tracking performance with the method of tuning CgLp by varying the band of it.

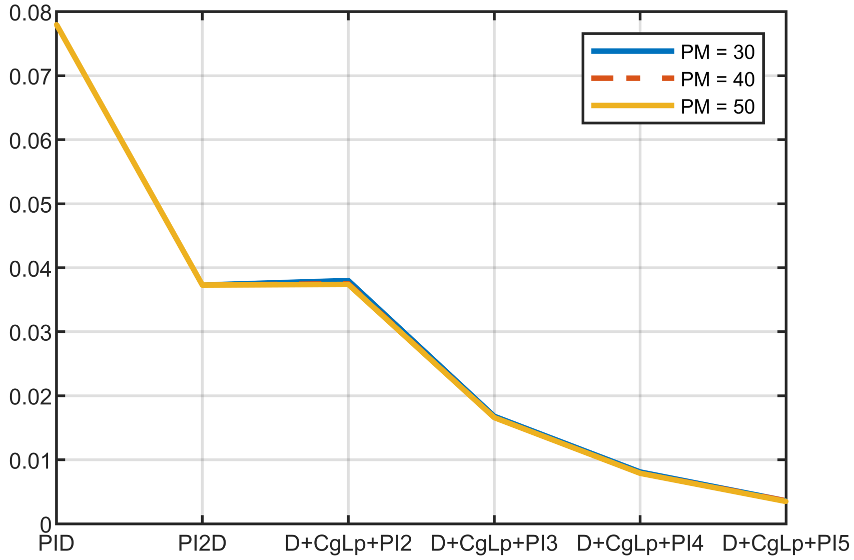


Figure 4-16: Maximum steady error for tracking performance when vary band

When the controller is designed by varying the γ value, the maximum steady errors for reference tracking are shown in Table 4-6. It also shows that system with more integrators can perform better than that of less integrators. While the error signal shown in Table 4-6 is less smooth than in Table.4-5 and plots of error signal can be seen in appendix D. The band keeps constant as $[0.9\omega_c, 1.3\omega_c]$ when phase margin equals 30°, $[0.9\omega_c, 1.7\omega_c]$ when PM is 40° and $[0.9\omega_c, 3.7\omega_c]$ when PM is 50°.

Controllers	PM = 30°			PM = 40°			PM = 50°		
	max(e(t)), f1	max(e(t)), f2	γ	max(e(t)), f1	max(e(t)), f2	γ	max(e(t)), f1	max(e(t)), f2	γ
PID	1.89e-2	7.8e-2	-	1.89e-2	7.8e-2	-	1.89e-2	7.8e-2	-
PI^2D	1.9e-3	3.73e-2	-	1.9e-3	3.73e-2	-	1.9e-3	3.73e-2	-
$D+CgLp+PI^2$	2.60e-3	3.8e-2	0	2.2e-3	3.75e-2	0	1.9e-3	3.74e-2	0
$D+CgLp+PI^3$	5e-4	3.19e-2	-0.3	7e-4	3.49e-2	-0.5	4.00e-4	1.67e-2	-0.2
$D+CgLp+PI^4$	3e-4	1.74e-2	-0.5	2e-4	1.91e-2	-0.7	1e-4	8.9e-3	-0.4
$D+CgLp+PI^5$	5e-4	1.68e-2	-0.7	9e-4	1.69e-2	-0.8	1e-4	1.61e-2	-0.6

Table 4-6: Maximum steady error for tracking performance when vary γ

The trend of the maximum steady error for tracking performance when design CgLp by tuning the γ value is shown in Fig.4-17. The trend is basically decreasing with more integrators added. What's more, the more phase margin is required at bandwidth, the less maximum error will be in the same system.

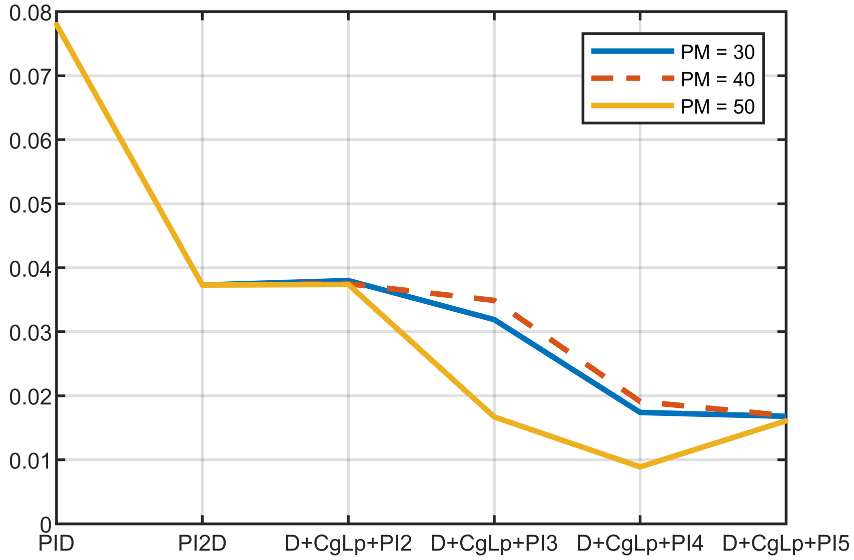


Figure 4-17: Maximum steady error for tracking performance when vary γ

In this case, the conclusion can be given that no matter the method of designing the controller, by varying the band of CgLp or varying γ value in reset element, they both improve the transient response by reducing the overshoot while still improve tracking performance. But for disturbance rejection, varying band to tune CgLp performs better than varying γ.

Chapter 5

Conclusion

In this thesis, it proves that the transient response can be improved by reset control with multiple integrators. So that the objective of this thesis was achieved.

The system overview was given at first. Based on the plant showed, the designed controller was going to drive it. Adding integrators into the system with only PID controller can indeed improve the tracking performance while ruins the transient response. Then the controller consisting of both linear element and nonlinear element is added into the system.

In [10], it has been proved that changing the sequences of the controller can reduce the overshoot thus improve the transient response. In this case, the controller in the system CgLp-PI^{2}D was compared with controller with different sequence of D-CgLp-PI^2. After changing the sequence of the controller, there are less overshoot and less settling time in the time domain.

So far there are only two integrators in the system. While the more integrators are willing to be added in the system. This means there will be higher gain at low frequencies so better tracking performance but will ruin the transient response more than before. Therefore in order to further add integrators, the reset element should be considered to tune in some other way. The first way considered in the report is to vary the band of the CgLp. It is because the band of the CgLp defines the properties of the reset element, including both the region of the constant gain area and also phase lead area. Different bands of CgLp will compensate different phase at bandwidth when there is different number of integrators in the system. By doing so, both the overshoot and the settling time are reduced.

The second way was to vary the γ value of the reset element. γ value means the amount of reset thus influences the nonlinearity in the system. Considering this method, the band of CgLp is fixed and just vary γ value of reset element to keep phase margin constant. The result still shows that the overshoot and settling time are both improved.

For the performance of disturbance rejection and tracking performance, reference tracking stays the advantage no matter tuning the band of CgLp or γ of the reset element. While for the disturbance rejection, it goes well when vary the band of CgLp while when tuning the γ value the advantage is not shown as before.

For recommendations for the future research, noises can be added into the system. Besides the tracking performance and disturbance rejection, noise attenuation performance is another important performance that should be focused on. Although this thesis shows the guideline to tune CgLp for multiple integrators system, the exact relation between the design parameters in CgLp and improvement of transient response should be verified mathematically. Last but not least, the simulation results should be tested in practical.

Appendix A

Different bands of CgLp

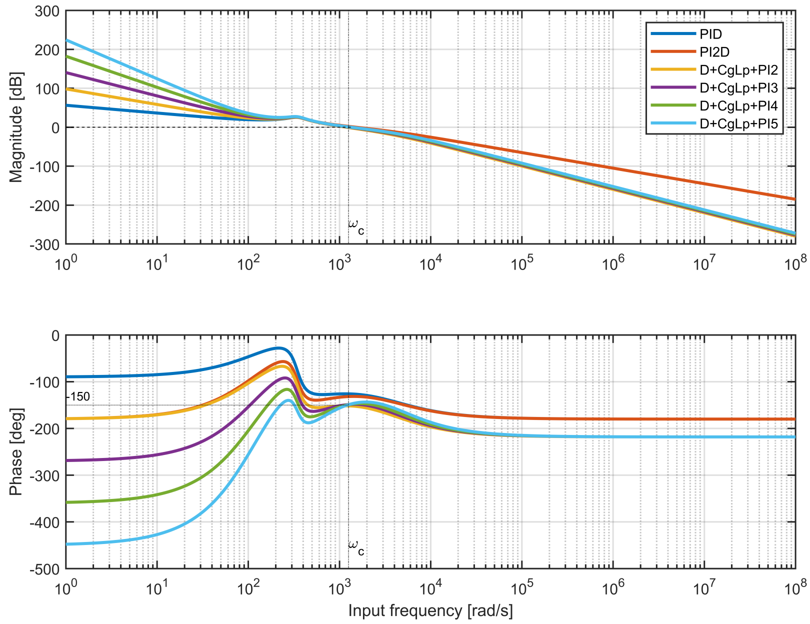
This appendix gives the additional information of different bands in CgLp. In Chapter 4, it shows that there are so many different bands can be given into the reset controller and keep the phase margin as required constant at bandwidth. Besides the mentioned bands of CgLp, there are more other options.

Phase margin	D+CgLp+PI^2	D+CgLp+PI^3	D+CgLp+PI^4	D+CgLp+PI^5
30°	$0.7\omega_c-$	$0.7\omega_c - 1.5\omega_c$	$0.7\omega_c - 1.8\omega_c$	$0.7\omega_c - 2.3\omega_c$
	$0.8\omega_c-$	$0.8\omega_c - 1.6\omega_c$	$0.8\omega_c - 2\omega_c$	$0.8\omega_c - 2.7\omega_c$
	$0.9\omega_c - 1.3\omega_c$	$0.9\omega_c - 1.8\omega_c$	$0.9\omega_c - 2.3\omega_c$	$0.9\omega_c - 3.1\omega_c$
40°	$0.7\omega_c - 1.6\omega_c$	$0.7\omega_c - 2.2\omega_c$	$0.7\omega_c - 2.9\omega_c$	$0.7\omega_c - 4.1\omega_c$
	$0.8\omega_c - 1.6\omega_c$	$0.8\omega_c - 2.5\omega_c$	$0.8\omega_c - 3.4\omega_c$	$0.8\omega_c - 5.3\omega_c$
	$0.9\omega_c - 1.7\omega_c$	$0.9\omega_c - 2.9\omega_c$	$0.9\omega_c - 4.1\omega_c$	$0.9\omega_c - 7.1\omega_c$
50°	$0.7\omega_c - 2.7\omega_c$	$0.7\omega_c - 3.5\omega_c$	$0.7\omega_c - 6.1\omega_c$	$0.7\omega_c - 16\omega_c$
	$0.8\omega_c - 3.1\omega_c$	$0.8\omega_c - 5\omega_c$	$0.8\omega_c - 9\omega_c$	$0.8\omega_c - 30\omega_c+$
	$0.9\omega_c - 3.7\omega_c$	$0.9\omega_c - 6\omega_c$	$0.9\omega_c - 15.3\omega_c$	$0.9\omega_c - 30\omega_c+$

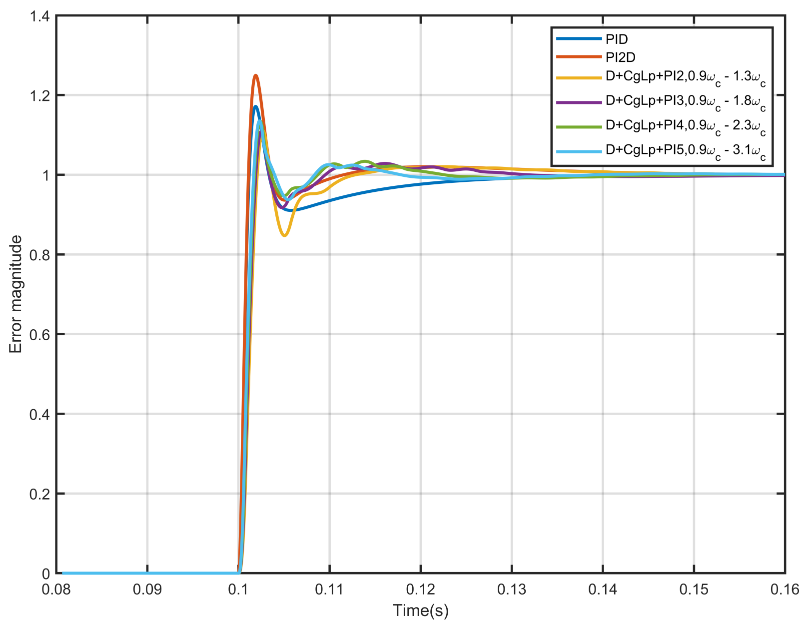
Table A-1: Bands of CgLp

The bands with colors are picked to analyze both in frequency domain and time domain. In frequency domain, it is the describing function that is used. In Fig.A-1, the describing function of controllers with different required phase margin with the corresponding of step response are both shown.

Based on the Table A-1, the conclusion can be made that with the same

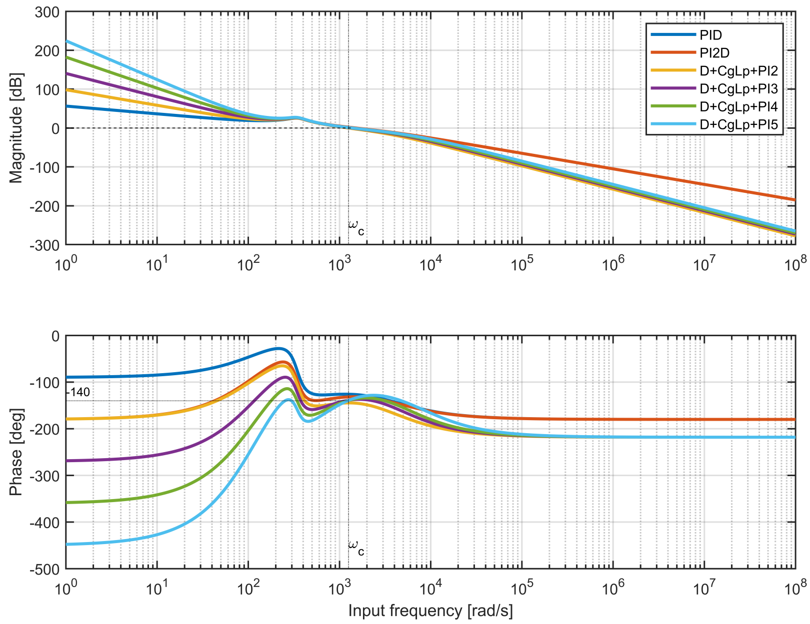


(a) DF when PM = 30

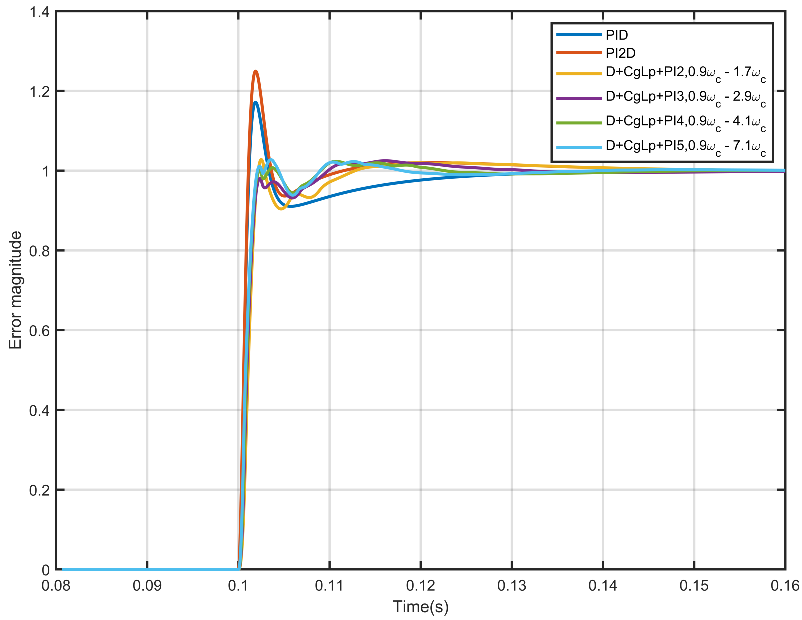


(b) Step response when PM = 30

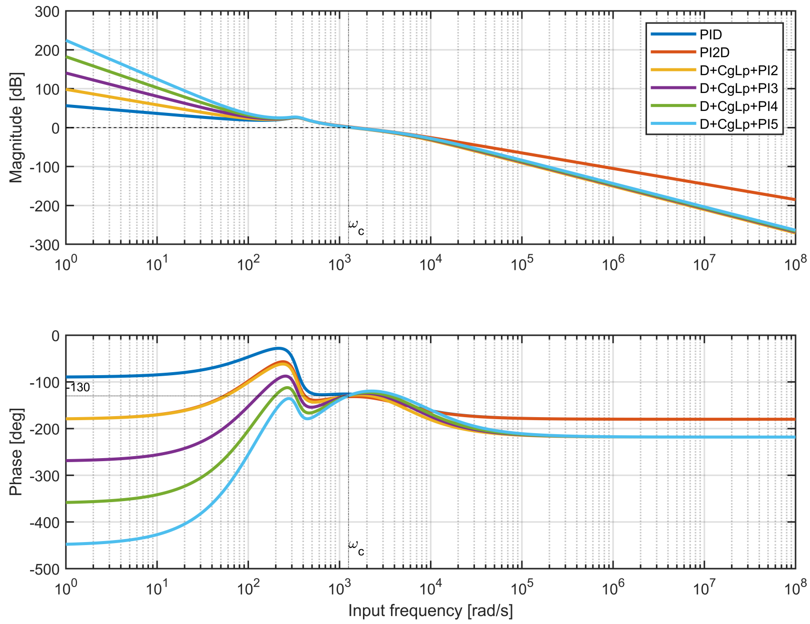
required phase margin, the more integrators added into the system, the wider band of CgLp is required. With the same controller in the system, the more phase margin required, the wider band is also required. In



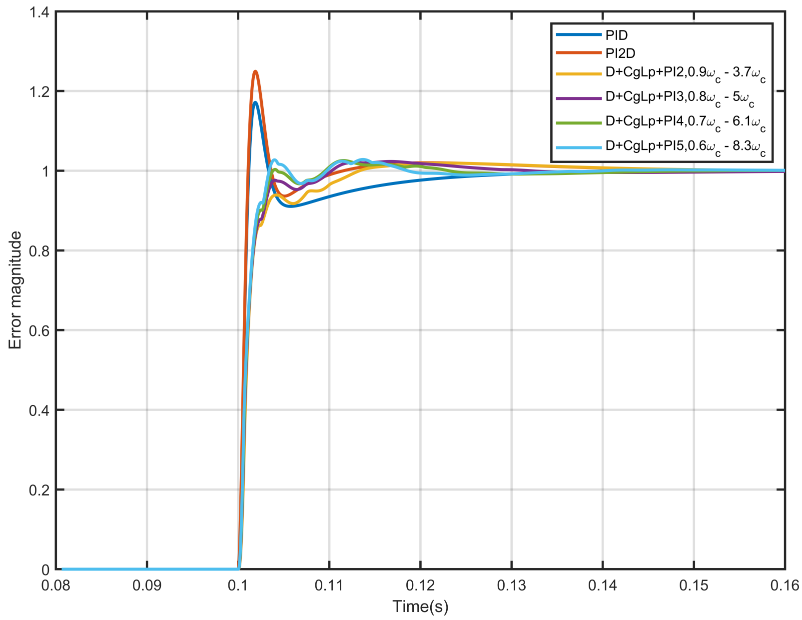
(c) DF when PM = 40



(d) Step response when PM = 40



(e) DF when PM = 50



(f) Step response when PM = 50

Figure A-1: Describing function and step response of systems with multiple integrators

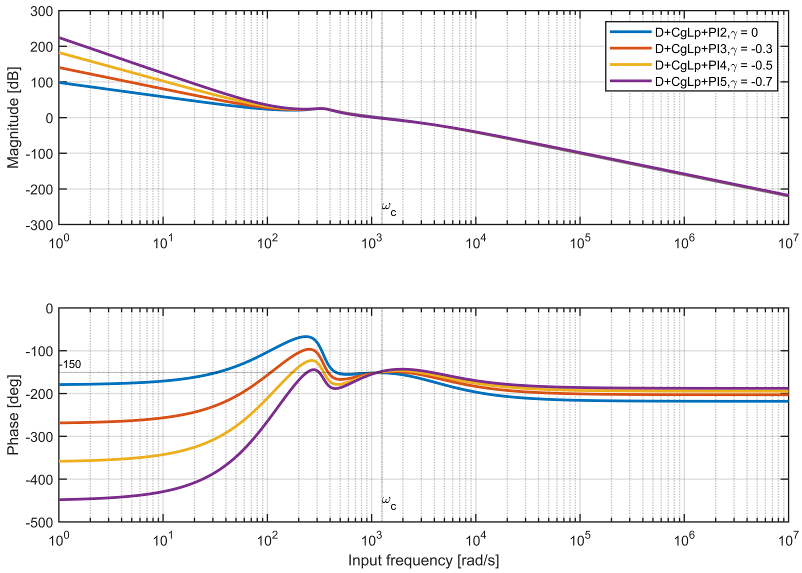
Fig.A-1, from transient response, the less phase margin required can have more overshoot but still performs better than linear controller. While with less required phase margin, the settling time is slightly improved.

Appendix B

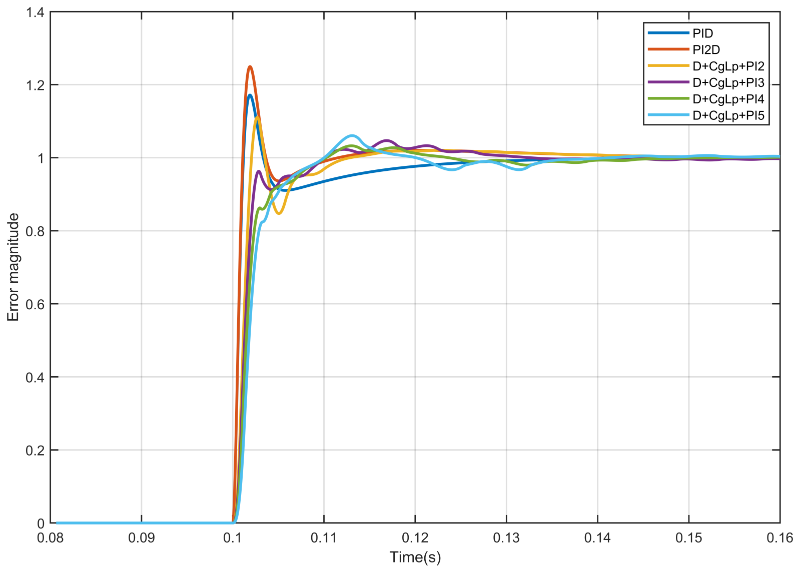
Different γ value of CgLp

This appendix gives more information about giving different γ value to the reset controller. The band of the CgLp is fixed when trying to tune the γ value. The γ value of picked band is shown in Chapter 4. Here the describing function of the system with varying γ and the corresponding step response are shown in Fig.B-1.

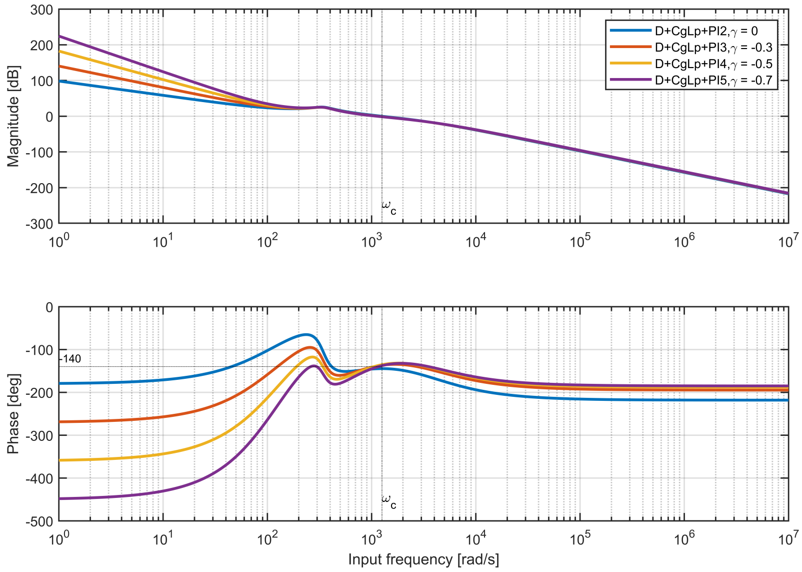
Based on Fig.B-1, the overshoot is less than using the bands to tune the reset controller. In contrast with varying the band, when more required phase margin is needed in this case, the settling time performs better.



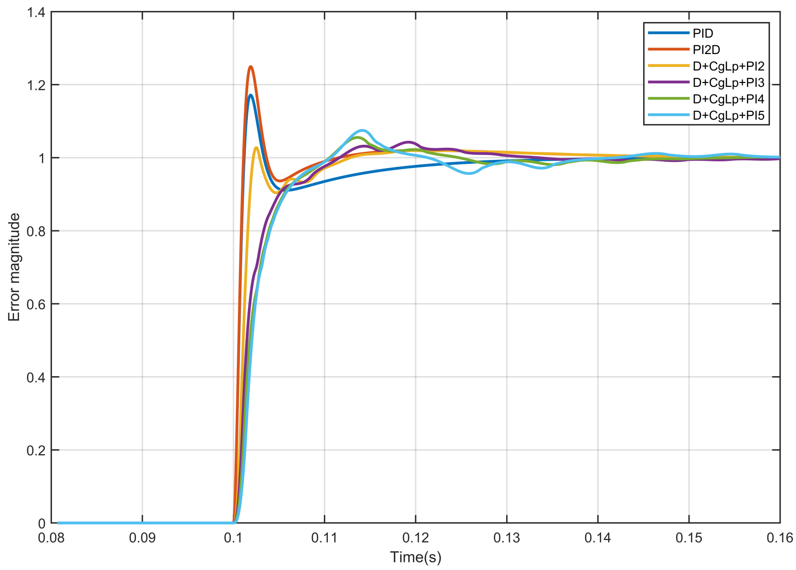
(a) DF when PM = 30



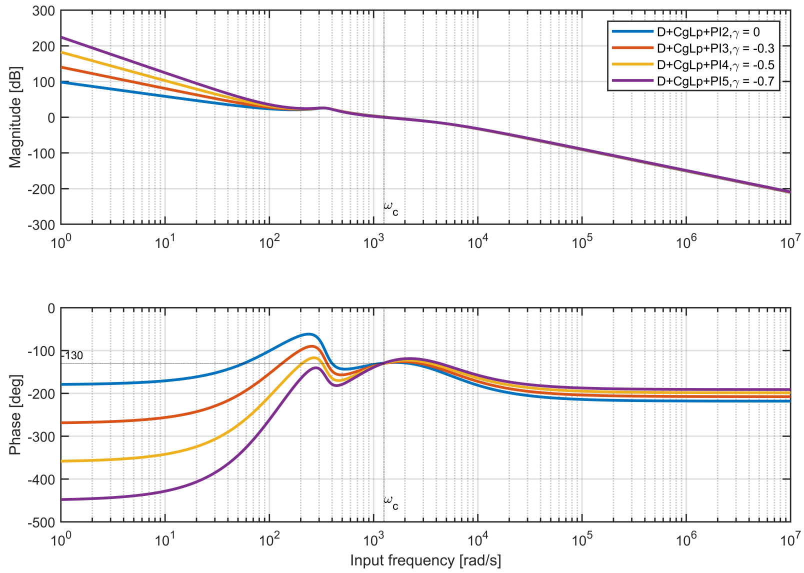
(b) Step response when PM = 30



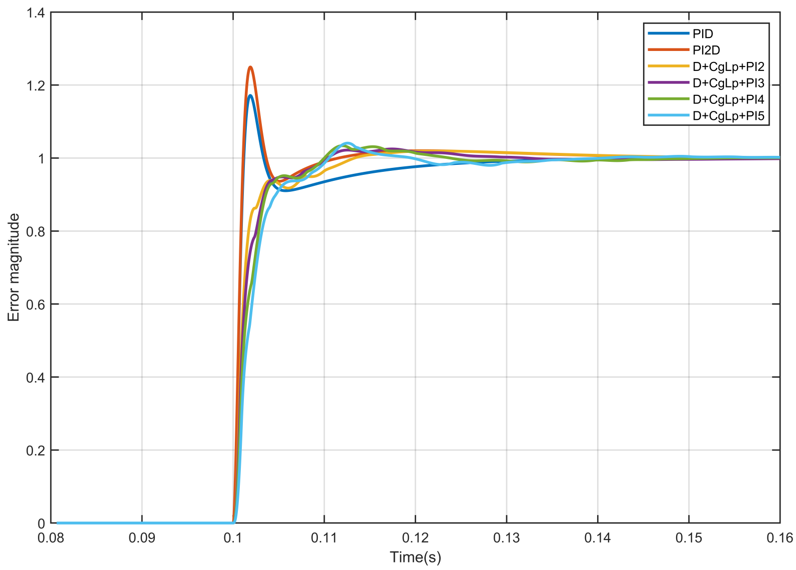
(c) DF when PM = 40



(d) Step response when PM = 40



(e) DF when PM = 50



(f) Step response when PM = 50

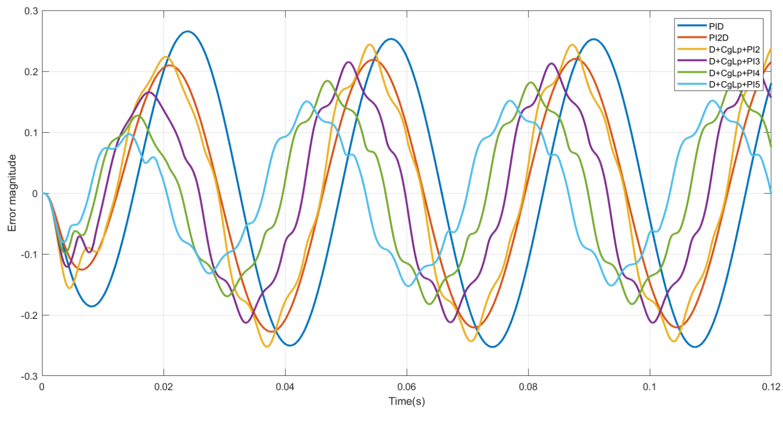
Figure B-1: Describing function and step response of systems with multiple integrators

Appendix C

Disturbance rejection performance

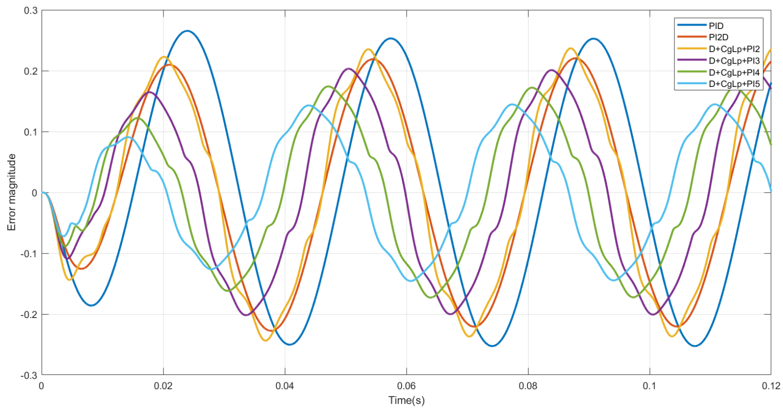
In Chapter 4, it shows the maximum steady error with different bands of CgLp and γ value. This appendix shows the picture of their time domain behavior.

In Fig.C-1, the pictures are when there are different phase margin the error value varying with time. The controllers in the system are tuned by varying the band. With the increasing number of integrators in the system, the maximum steady error is reduced.

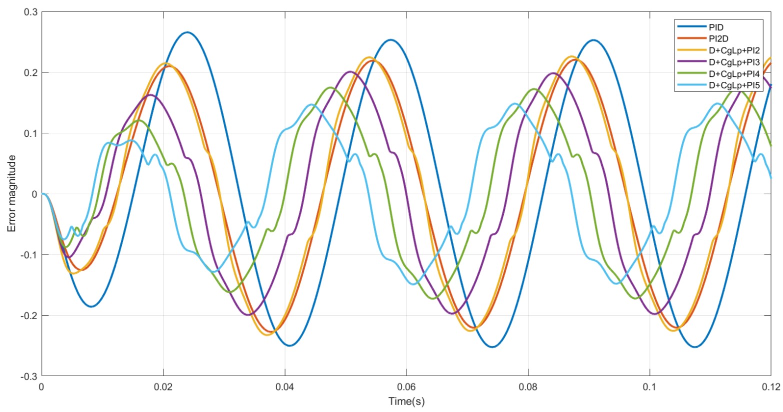


(a) Error when PM = 30

While in the Fig.C-2, the pictures are the maximum steady error by using the different γ value under different phase margin. The error keeps decreasing when the number of integrators in the system is below 3 only

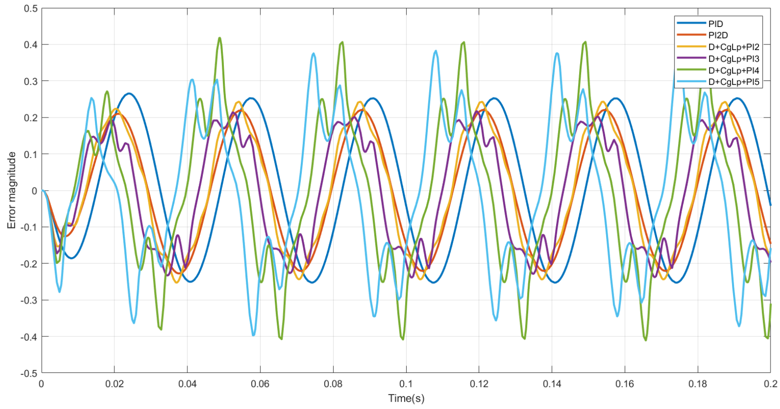


(b) Error when PM = 40

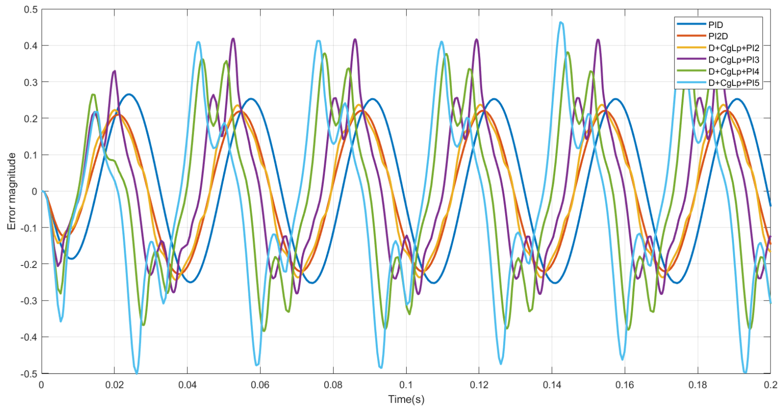


(c) Error when PM = 50

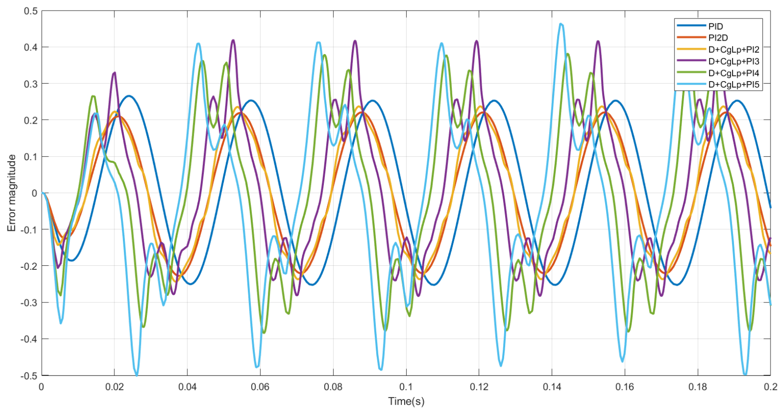
Figure C-1: Error signal after driven by multiple integrator systems by using the method of varying the band of CgLp when PM = 30. The rule still keeps if we only see D+CgLp+PI^4 and D+CgLp+PI^5 but the error is larger than linear controller system. So in this case, the method of varying band of CgLp is better than varying γ.



(a) Error when PM = 30



(b) Error when PM = 40



(c) Error when PM = 50

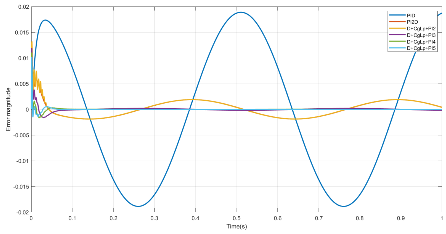
Figure C-2: Error signal after driven by multiple integrator systems by using the method of varying the γ of CgLp

Appendix D

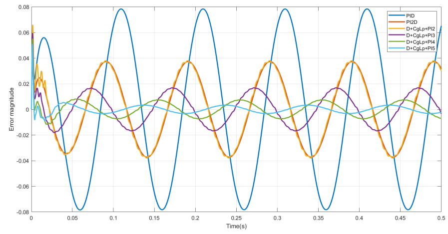
Tracking performance

To get known the tracking performance of the system, error signal is also measured. The maximum steady error by applying into different input sinusoidal signals is shown in Chapter 4 in Table 4-5 and Table 4-6.

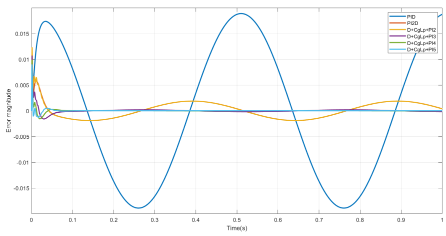
This appendix will show the error signal with various band and γ. In Fig.D-1, it is the method of using different bands of CgLp with applying f = 2Hz and f = 10Hz signals into the system. Fig.D-2 also shows the error signal with different applied input signal. Comparing with two different methods, it is clear that the error signal with method of using different γ value is more rough especially when the magnitude of the input signal is relatively high.



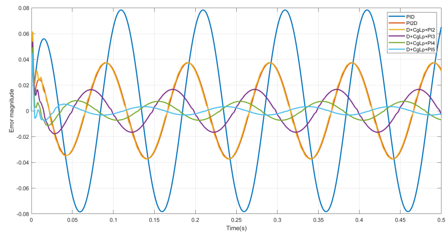
(a) Error when PM = 30,f = 2Hz



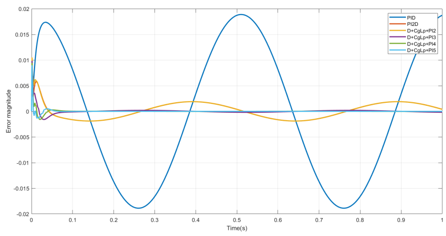
(b) Error when PM = 30,f = 10Hz



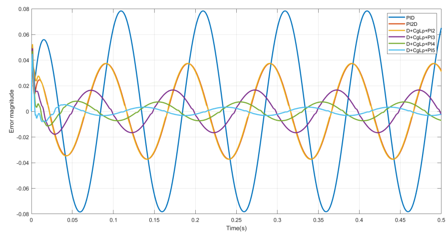
(c) Error when PM = 40,f = 2Hz



(d) Error when PM = 40,f = 10Hz

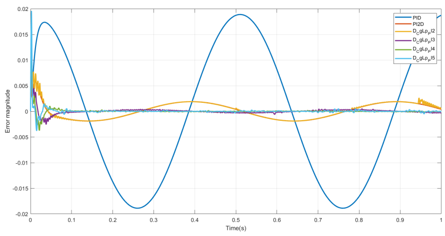


(e) Error when PM = 50,f = 2Hz

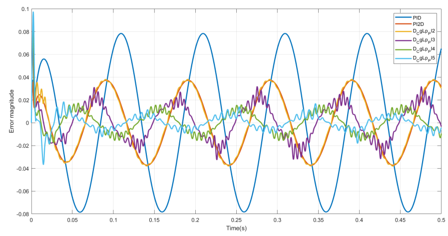


(f) Error when PM = 50,f = 10Hz

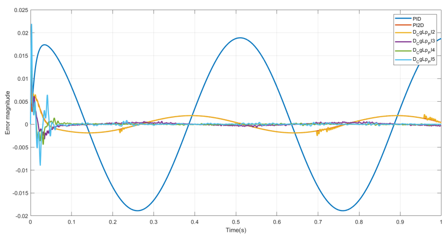
Figure D-1: Error signal after driven by multiple integrator systems by using the method of varying the band of CgLp



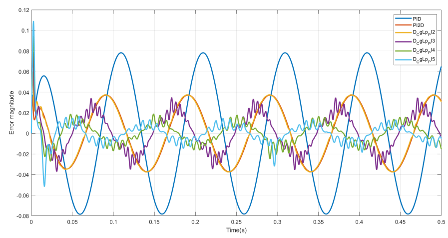
(a) Error when PM = 30,f = 2Hz



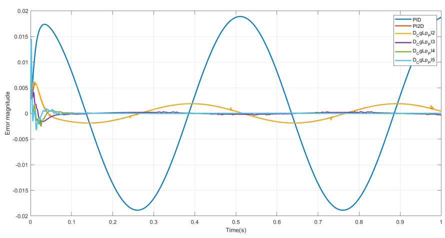
(b) Error when PM = 30,f = 10Hz



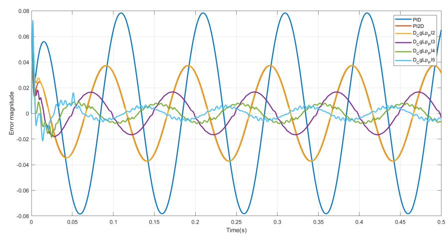
(c) Error when PM = 40,f = 2Hz



(d) Error when PM = 40,f = 10Hz



(e) Error when PM = 50,f = 2Hz



(f) Error when PM = 50,f = 10Hz

Figure D-2: Error signal after driven by multiple integrator systems by using the method of varying the γ of CgLp

Appendix E

Matlab code and Simulink model

E-1 hodf

This code is adapted from Kar's thesis[1]. It is basically used to calculate the higher order sinusoidal describing functions of the reset elements. In this thesis, only the first order harmonic is considered to be as a reference to keep the phase margin constant at bandwidth.

```
function [G] = hodf(sys,Ar,n,freqs)
%calculate FORE, basically n = 1, which means that the first order
    harmonic
%is considered.
%sys:state space matrix of linear base dynamics
%Ar:gamma
%n:the order of describing function(only odd number)\
%freqs:the frequencies that the describing funtion are calculated for
%A,B,C,D: state space matrix
%This code is adapted from Kar's thesis

if(mod(n,2)==0)
    G=0;
    return;
end

A = sys.a;
B = sys.b;
C = sys.c;
D = sys.d;
G = zeros(1,numel(freqs));
for i = 1:numel(freqs)
    w = freqs(i);
    lambda = w*w*eye(size(A))+A^2;
```

```
    Delta = eye(size(A)) + expm(A*pi/w);
    Delta_d = eye(size(A))+ Ar*expm(A*pi/w);
    gamma_r = inv(Delta_d)*Ar*Delta*inv(lambda);
    theta_D = (-2*w*w/pi)*Delta*(gamma_r-inv(lambda));

    if (n==1)
        G(i) = C*inv(j*w*eye(size(A))-A)*(eye(size(A))+j*theta_D)*B+D;
    else
        G(i) = C*inv(j*w*n*eye(size(A))-A)*j*theta_D*B;
    end
end
if (n==1)
    G = G+D;
end
```

E-2 Determine the band of CgLp

This code tries to find the band of CgLp with fixed phase margin and fixed start up frequencies. The band can be found by running this code.

```
clear
clc

s = tf('s');
m = ; c = ; k = ;% Plant in the system that needed to be driven
t = ;
Plant = tf(t,[m c k]);

wc = ; % designed bandwidth

%PID, designed by rule of thrumb
G = t/(m*wc^2+c*wc+k);
kp = 1/(3*G);
wi = wc/10;
wd = wc/3;
wt = 3*wc;

PI2D = kp*(1+wi/s)^2*((s/wd +1 )/(s/wt +1));
CP_PI2D = PI2D*Plant;
PInD = PI2D;

PM = ;%Desried Phase margin at bandwidth, for 30, 40 or 50


a = ; % coefficient in front start up frequency
wr = a*wc;
wf = [wc:0.5*wc:10*wc];% vary the end frequency in CgLp
Ar = 0; % gamma value keeps zero
n = 1;
```

```
% reset part of cglp
alpha = 1.44;
wra = wr/alpha;

freqs = logspace(0,7,1000);

for i = 1:1:numel(wf)
  sym = ss(-wra,wra,1,0);% FORE in cglp

  del = @(w)abs((-180+PM)-rad2deg(angle(hodf(sym,Ar,1,w).*((w*j/(wr)+1)/(w*j/(wf(i))+1))))-CP_PInD(w,PInD)); %looking for the frequency that really near BW

  lb = [0];
  ub = [ ];
  [w(i),fval] = fmincon(del,wr,[],[],[],[],lb,ub);
  if (wr<w(i))&&(w(i)<wf(i))
      k = 1;
      wc_c(k) = w(i);
      wf_f(k) = wf(i);% so that the corresponding end frequency for the CgLp band will be found.
      k = k+1;
  end
end
```

E-3 Determine the γ of CgLp

This code is when there is fixed required phase margin and with the same band of CgLp then only tune the γ value. The γ value can be found by running this code.

```
clear
clc

s = tf('s');
m = ; c = ; k = ; % plant that needs to be driven
t = ;
Plant = tf(t,[m c k]);

wc = ; % bandwidth of PID
%PID, rule of thrumb
G = t/(m*wc^2+c*wc+k);
kp = 1/(3*G);
wi = wc/10;
wd = wc/3;
wt = 3*wc;

PI2D = kp*(1+wi/s)^2*((s/wd +1 )/(s/wt +1));
```

```
PInD = PI2D;

PM = ; % required phase margin


a = ; % coefficient
wr = a*wc; % where band of cglp starts
wf = ; % where band of cglp ends
Ar = [-0.9:0.1:0.9]; % vary the gamma value of cglp
n = 1;

% reset part of cglp
alpha = [ ];% insert alpha values


freqs = logspace(0,7,1000);

for i = 1:1:numel(Ar)
   Ar(i);
   wra = wr/alpha(i);
  sym = ss(-wra,wra,1,0);% FORE in cglp

  del = @(w)abs((-180+PM)-rad2deg(angle(hodf(sym,Ar(i),1,w).*((w*j/(wr)
      +1)/(w*j/(wf)+1))))-CP_PInD(w,PInD));

  lb = [0];
  ub = [ ];
  [w(i),fval] = fmincon(del,wr,[],[],[],[],lb,ub);
end
```

E-4 D_CgLp_PIn

This code helps to plot the describing function of different systems.

```
function D_CgLp_PIn(sys,sys1,PInD,Plant,Ar,n,freqs,wc,PM)
% sys, reset system that need to know the phase at corner frequency
% sys1, linear lead part of the controller
% PInD, multiple integrators with PD controller. n is the number of
% integrators
% Ar, gamma value in the reset system
% n,order of describing function
% freqs: frequency
% wc, bandwidth
% PM, required phase margin at bandwidth.

    for i=1:numel(n) % numel: the number of the orders to display
        order = n(i);

        % linear lead filter
```

```
        [mag1, angle1] = bode(sys1, freqs*order);% Is freqs*order a vector of frequencies?
        abs1 = mag2db(squeeze(mag1)'); % squeeze: remove the dimension of 1.? but why
        angle1 = squeeze(angle1)';

        L = hodf(sys, Ar, order, freqs); % function hodf --Calculated the higher order (n) describing function for a reset system

        Gabs(i,:) = mag2db(abs(L)); % expresses in decibels (dB) the magnitude measurements specified in y
        Gphase(i,:) = unwrap(rad2deg(phase(L))); % rad2deg: Convert angle units from radians to degrees
                                               % unwrap: shift phase
        abs2 = Gabs(i,:);
        angle2 = Gphase(i,:);

        %PInD
        [mag, ph] = bode(PInD, freqs*order);% calculate the magnitude and phase at certain frequency
        abs3 = mag2db(squeeze(mag)');
        angle3 = squeeze(ph)';

        %Plant
        [mag2, ph2] = bode(Plant, freqs*order);
        abs4 = mag2db(squeeze(mag2)');
        angle4 = squeeze(ph2)';

        Gabs_C(i,:) = abs2  + abs3 + abs1 + abs4; % magnitude with unit of db of controller and plant
        Gphase_C(i,:) = angle2 + angle3 + angle1 +angle4; % phase with unit of degree of controller and plant

    end
    % plot
    ax1 = subplot(2,1,1);
    semilogx(freqs, Gabs_C, 'lineWidth', 2);hold on; grid on; % D_CgLp_PInD+plant gain behavior
    xline(wc,'-.')
    text(wc,-180,texlabel('omega_c'));

    legend('D+CgLp+PIn+Plant'); %
    ylabel('Magnitude [dB]');

    ax2 = subplot(2,1,2);
    semilogx(freqs,Gphase_C,'lineWidth', 2);hold on; grid on; % D_CgLp_PInD+plant phase behavior
    xline(wc,'-.')
    text(wc,-220,texlabel('omega_c'));
    line([1 wc],[-180+PM -180+PM],'linestyle','-.', 'Color','k', 'LineWidth', 0.25);
    text(1,-180+PM+10,texlabel('-140'),'FontSize',8);

```

```
    ylabel('Phase [deg]');
    xlabel('Input frequency [rad/s]');
    linkaxes([ax1,ax2], 'x')
end
```

E-5 Simulink model

This model is used to simulate the closed-loop performance of the system.

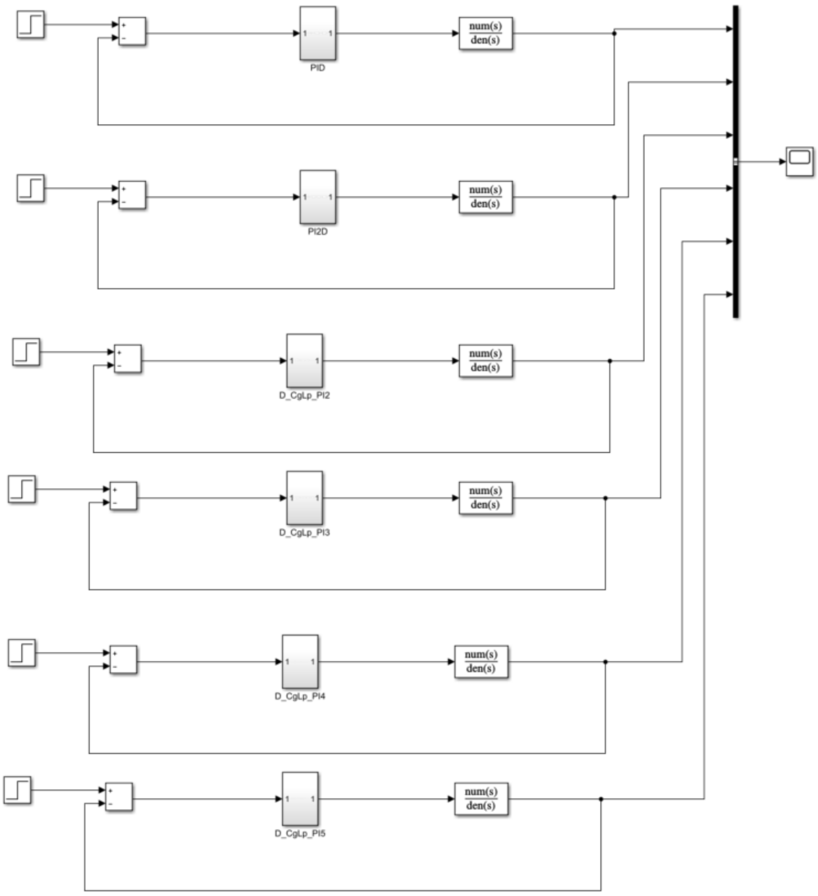


Figure E-1: Overview of the system comparison

One of the controllers of the system is shown in Fig.E-2.

The structure of CgLp is shown in Fig.E-3.

The reset element in CgLp is in Fig.E-4..

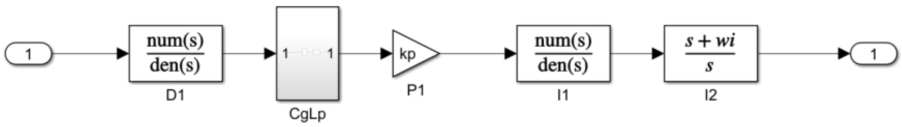


Figure E-2: Controller of the system

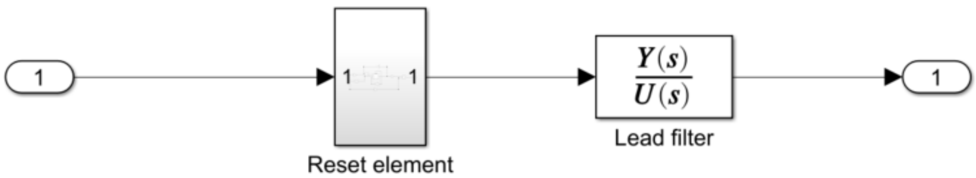


Figure E-3: Controller of the system

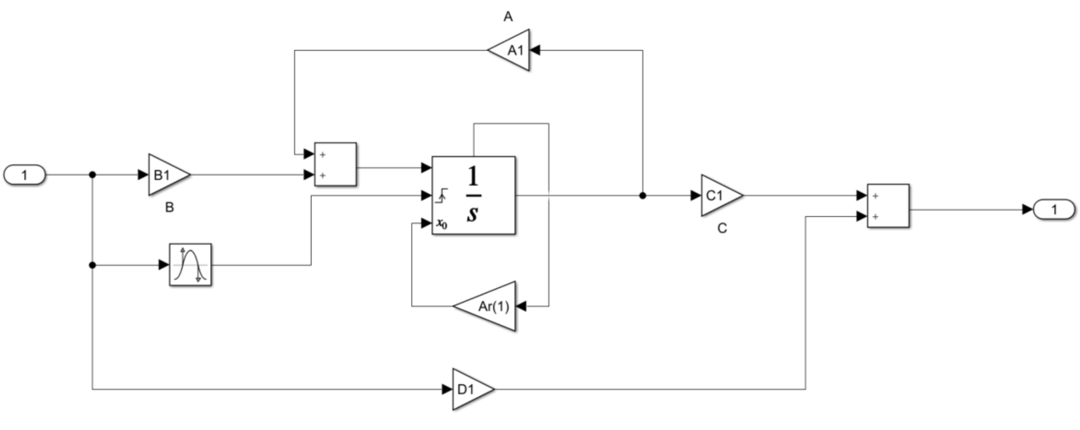


Figure E-4: Reset element in CgLp

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