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CROSS-SHORE SEDIMENT TRANSPORT

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ABSTRACT

In the first phase of the detailed modelling of cross-shore sediment transport under random waves a model is constructed which adopts a vertically integrated transport description for sheetflow situations. The formulation of the transport as a function of the instantaneous velocity field is based on the approach of Bailard (1981). This approach assumes in essence simply that the instantaneous transport is proportional with some power of the instantaneous near-bottom velocity. Implementation of this transport description in a time-dependent model requires a formulation of the time-mean and some low order moments of the near-bottom velocity field. An ad-hoc formulation based on a monochromatic, second order Stokes wave representation is presented. A numerical research model, based on the above formulations, is described and limitedly checked on its performance on the basis of an available field data set. Some consequences for further study are indicated.

1. INTRODUCTION

The particular role of a nearly two-dimensional wave motion in the movement of sediment normal to the shore is poorly understood. It is generally assumed that a number of interaction mechanisms between this wave motion and the sediment motion contribute to the formation of the beach profile, also in the three-dimensional topographies that occur on a natural coast. Full account of all mechanisms can be taken when a description of both the horizontal velocity field, $u(x,z,t)$, and the sediment concentration field, $c(x,z,t)$, in space and time is available, so that the net cross-shore sediment transport, $\langle q(x)\rangle$, may be calculated from

$$\langle q(x)\rangle = \langle \int_d u(x,z,t).c(x,z,t)\, dz\rangle \qquad (1)$$

where the integration is performed over the instantaneous depth d and the brackets indicate time averaging. From the cross-shore variation of $\langle q(x)\rangle$ the bottom changes may be derived.

Visual and experimental observation of random waves on a two-dimensional beach indicates that one of the more important mechanisms under active surf conditions may be the transport of sediment by the time mean, seawards directed flow near the bottom induced by the breaking of waves. It was shown (Stive and Battjes, 1984) that this mechanism is so dominant that a vertically integrated model incorporating this mechanism alone describes the bottom variations in the surf zone to a satisfactory, first approximation. Extension of this model with other transport mechanisms is a logical step towards a more complete cross-shore sediment transport model. Here some first suggestions are made to extend the model with transport due to the asymmetry of the wave motion.

2. TRANSPORT FORMULATION

In principle the net cross-shore sediment transport may be calculated from Equation (1). There are, however, two reasons persuading us to rely on a simplified, vertically integrated form of Equation (1). Firstly, our knowledge of the velocity and concentration field in time and space is very limited. Secondly, a simpler - but qualitative correct - formulation of the sediment transport provides a better insight in the mechanisms. Since we are interested in a transport formulation which takes also the effects of wave asymmetry into account, it is essential to adopt a formulation describing the transport instantaneously. A simple approach would be to assume that the instantaneous sediment transport rate, q, is proportional to some power of the local relative velocity between the bed and the fluid outside the boundary layer. For example,

$$q(t) = A\, u(t)\,|u(t)|^n \qquad (2)$$

where $u(t) = u_b \cos\omega t$ with u_b the orbital velocity amplitude just outside the boundary layer and ω the angular frequency.

The latter approach has been elaborated consistently for surf zones on a plane sloping beach by Bailard (1981), who extended the work of Bailard and Inman (1981). Based on Bagnold's (1963) energetics concept these authors use as a starting point a description of the instantaneous sediment transport basically in the form of Equation (2), extended with the effect of a bottom slope. Bailard (1981) distinguishes between bedload transport in a granular-fluid shear layer of a thickness in the order of the wave boundary layer and suspended transport in a layer of greater thickness, typically in the order of several centimeters. For the bedload transport the power n as introduced by equation (2) is given by Bailard (1981) as 2, while for the suspended transport it is given as 3. Here his general two-dimensional horizontal formulation is reduced for application in the cross-shore direction which yields the instantaneous total load sediment transport equation (see also Bailard, 1982):

$$i(t) = i_B(t) + i_S(t) =$$

$$= \rho c_f \frac{\varepsilon_B}{\tan\phi}\left[|u(t)|^2\, u(t) - \frac{\tan\beta}{\tan\phi}|u(t)|^3\right] +$$

$$\rho c_f \frac{\varepsilon_S}{w}\left[|u(t)|^3\, u(t) - \frac{\varepsilon_S}{w}\tan\beta\, |u(t)|^5\right] \qquad (3)$$

where i is the total cross-shore immersed weight sediment transport rate (composed of the bedload transport rate, i_B, and the suspended load transport rate, i_S), ρ is the water density, c_f is the drag coefficient for the bed, $\tan\beta$ is the slope of the bed, ϕ is the internal angle of friction of the sediment, w is the sediment's fall velocity and ε_B and ε_S are bedload and suspended load efficiencies, respectively. The efficiency factors ε_B and ε_S denote those (constant) fractions of the total power produced by the fluid motion which are expended in transporting. The immersed weight sediment transport rate is linked to the volumetric transport rate by

$$q = \frac{i}{(\rho_s - \rho) g N} \qquad (4)$$

where ρ_s is the sediment density, g the gravitational acceleration and N the local volume concentration of solids.

The above sediment transport formulation uses vertically integrated equations. As a consequence, the sediment transports are assumed to respond to the near bottom water velocity in an instantaneous, quasi-steady manner. This assumption is probably valid for bedload transport on a flat bed (except for a phase lag which is neglected for simplicity) because the bedload layer has a small thickness and it can respond quickly to the instantaneous shear stress. The suspended sediment transport, however, is distributed over a layer thickness of several centimeters. The characteristic time constant for this layer is the ratio of its thickness and the sediment fall velocity which is typically in the order of 1-2 seconds. For most natural beaches with prevailing plane bed conditions and incident wave periods of 5-10 seconds, it appears that the quasi-steady assumption is reasonable.

Another uncertainty in the transport formulation concerns the use of bedload and suspended load efficiency factors. Although constant values have been found adequate for certain types of flow (see Table 1), their variations with the type of flow considered leaves at least some quantitative uncertainty.

3. THE CROSS-SHORE VELOCITY FIELD

Given the variation of the cross-shore velocity field the mean cross-shore sediment transport rate may in principle be calculated from the time averaged Equation (3):

$$\langle i\rangle = \rho\, c_f \frac{\varepsilon_B}{\tan\phi}\left[\langle|u|^2 u\rangle - \frac{\tan\beta}{\tan\phi}\langle|u|^3\rangle\right] + \rho\, c_f \frac{\varepsilon_S}{w}\left[\langle|u|^3 u\rangle - \frac{\varepsilon_S}{w}\tan\beta\langle|u|^5\rangle\right] \quad (5)$$

where the total velocity u is composed of a mean (overbar) and an oscillatory (tilde) flow component.:

$$u = \bar{u} + \tilde{u} \quad (6)$$

Thus, the problem to be evaluated here is how to predict the cross-shore variation of the velocity moments appearing in Equation (5).

Conceptual simplifications follow by assuming that the oscillatory velocity is due to a single plane wave of frequency ω and some small nonlinear harmonics:

$$\tilde{u} = u_m \cos\omega t + u_{2m}\cos 2\omega t + \ldots \quad (7)$$

in which $u_m > u_{2m} > \ldots$

Using Equations (6) and (7) in Equation (5) yields:

$$\langle i\rangle = \rho c_f u_m^3 \frac{\varepsilon_B}{\tan\phi}\left[\psi_1 + \frac{3}{2}\delta_u - \frac{\tan\beta}{\tan\phi}(u3)^*\right] + \rho c_f u_m^4 \frac{\varepsilon_S}{w}\left[\psi_2 + \delta_u (u3)^* - \frac{u_m}{w}\varepsilon_S \tan\beta (u5)^*\right] \quad (8)$$

in which the relative current strength, δ_u, is

$$\delta_u = \bar{u}/u_m \quad (9)$$

and the odd velocity moments, ψ_1 and ψ_2, are:

$$\psi_1 = \langle\tilde{u}^3\rangle/u_m^3 \quad (10a)$$

$$\psi_2 = \langle|u|^3\,\tilde{u}\rangle/u_m^4 \quad (10b)$$

The even velocity moments $(u3)^*$ and $(u5)^*$ are defined as:

$$(u3)^* = \langle|u|^3\rangle/u_m^3 \quad (11a)$$

$$(u5)^* = \langle|u|^5\rangle/u_m^5 \quad (11b)$$

Retaining first order in the relative current strength and odd moments only three velocity moments may be simplified further, i.e.

efficiency factor	steady stream flow (Bagnold, 1966)	longshore current flow (Bailard, 1981)	cross-shore current flow (Bailard, 1982)
ε_B	0.13	0.21	0.10
ε_S	0.01	0.025	0.020

Table 1 Estimates of the efficiency factors

$$u_m^4 \psi_2 \simeq 2\bar{u}\langle|\tilde{u}|^3\rangle + \langle|\tilde{u}|^3 \tilde{u}\rangle \quad (12)$$

$$\text{and } u_m^3(u3)^* \simeq |\bar{u}|\langle\tilde{u}^2\rangle + \langle|\tilde{u}|^3\rangle \quad (13a)$$

$$u_m^5(u5)^* \simeq |\bar{u}|\langle\tilde{u}^4\rangle + \langle|\tilde{u}|^5\rangle \quad (13b)$$

Inspection of the above expressions indicates that the following low order velocity moments are of importance:

- the four lowest even moments $\langle\tilde{u}^2\rangle$, $\langle|\tilde{u}|^3$, $\langle\tilde{u}^4\rangle$, $\langle|\tilde{u}|^5\rangle$, which are non zero for symmetric velocities,
- the two lowest odd moments $\langle\tilde{u}^3\rangle$, $\langle|\tilde{u}|^3\tilde{u}\rangle$, which are zero for symmetric velocities.

The latter moments are the most difficult to estimate: they are nonzero only for nonlinear waves that actually occur nearshore. The shoreward velocities are typically stronger and of shorter duration than the offshore flows, leading to nonzero values for the odd moments. Calculation of the odd moments requires a nonlinear wave shoaling and decay model.

A theoretical evaluation of the even moments for both a monochromatic, linear sea (sinusoidal model) and a random, linear sea (Gaussian model) is given by Guza and Thornton (1985). The theoretical moments are compared to field observations from the NSTS study. A summary of observations and theory for the several cross-shore moments is given in Table 2 below. The moments are normalized by the local variance.
The above results indicate that even moments do not critically depend on cross-shore velocity asymmetry. This is due to the fact that also for symmetric velocities these terms are nonzero. At the present stage we will therefore rely on the Gaussian estimates for the even moments. The odd moments are zero for a symmetric velocity field, but can be nonzero for asymmetric (nonlinear) motions. Here we suggest the following ad-hoc formulation.

As indicated above calculation of the odd velocity moments requires a shoaling and decay model which predicts certain nonlinear properties of the presently considered random, breaking waves. A relevant nonlinear property is the asymmetry of the wave surface about the horizontal axis. For non-breaking waves this asymmetry may to a first aprroximation well be predicted on the basis of a horizontal bottom, nonlinear wave theory, assuming that due to gradual bottom variations the waves locally behave as on a horizontal bottom (see Flick, et al., 1981). However, in the horizontal bottom, nonlinear wave theories the phases of the harmonics are locked to zero and there is no vertical wave profile asymmetry possible. This asymmetry about the vertical plane is an essential property of the sawtooth shaped breaking waves in the surf zone.

moment	observations Nov. 17	Nov. 20	theory Gaussian	sinusoid
$\langle \lvert\tilde{u}\rvert^3 \rangle / \langle \tilde{u}^2 \rangle^{3/2}$	1.60	1.69	1.60	1.20
$\langle \tilde{u}^4 \rangle / \langle \tilde{u}^2 \rangle^{2}$	2.86	3.50	3.00	1.50
$\langle \lvert\tilde{u}\rvert^5 \rangle / \langle \tilde{u}^2 \rangle^{5/2}$	7.77	8.58	6.38	1.92
$\langle \tilde{u}^3 \rangle / \langle \tilde{u}^2 \rangle^{3/2}$	0.55	0.50	0	0
$\langle \lvert\tilde{u}\rvert^3 \tilde{u} \rangle / \langle \tilde{u}^2 \rangle^{2}$	~1.20	~1.20	0	0
$\langle \tilde{u}^5 \rangle / \langle \tilde{u}^2 \rangle^{5/2}$	4.95	5.39	0	0

Table 2 Observed and theoretical velocity moments (after Guza and Thornton, 1985)

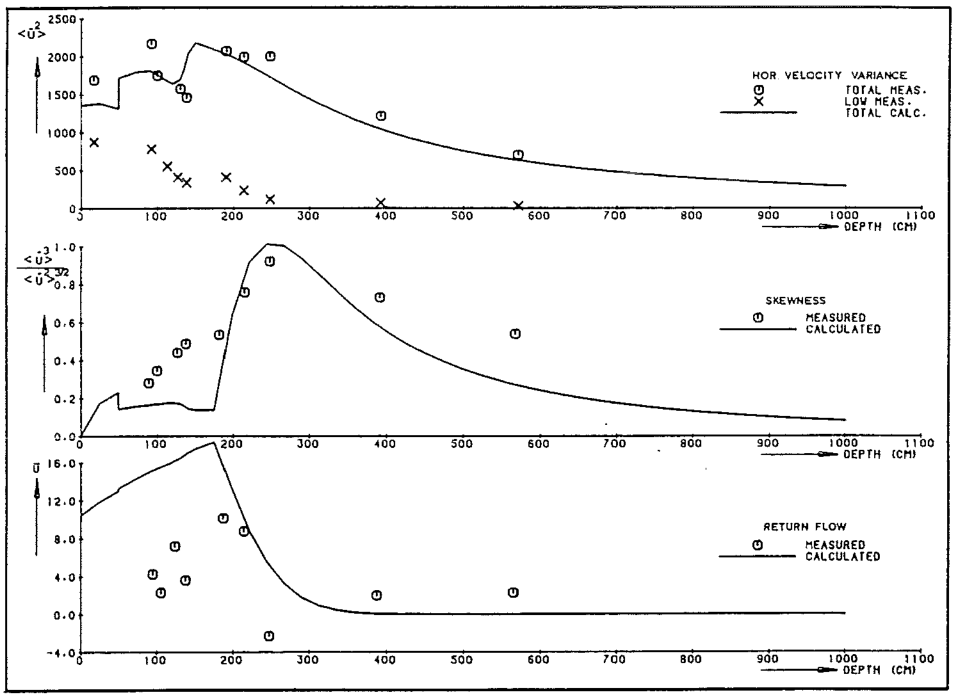

Fig. 1 Cross-shore velocity characteristics NSTS Torrey Pines measurements November 20 (after Guza and Thornton, 1985) compared to present theory

These theories are deficient in this respect and thus particularly unsuitable for calculations of odd velocity moments which depend critically on phase. To illustrate this we calculate the two lowest order odd moments

assuming that the velocity fluctuation is described by a second order approximation with a locked but nonzero phase between the two components:

$$\tilde{u} = u_m \cos \omega t + u_{2m} \cos (2\omega t + \phi_2) \qquad (14)$$

in which $u_m > u_{2m}$. After some algebraic manipulation it may be show that to lowest order the two odd velocity moments are given by:

$$\langle \tilde{u}^3 \rangle = \frac{3}{4} u_m^2 u_{2m} \cos \phi_2 \qquad (15a)$$

$$\langle |\tilde{u}|^3 \tilde{u} \rangle = \frac{12}{5\pi} u_m^3 u_{2m} \cos \phi_2 \qquad (15b)$$

An interesting perspective now arises when we combine these results with the following observations. In the inner surf zone where the breaking waves are quasi-steady the relative phase of the second harmonic increases smoothly toward the asymptotic value (see Flick et al., 1981):

$$\phi_2 \rightarrow \pi/2 \qquad (16)$$

Thus, according to Eq. 15a, 15b, the odd velocity moents for breaking waves vanish ultimately.

At this point we may formulate and ad-hoc wave decay model which predicts linear and nonlinear properties necessary to derive the velocity moments. As a starting point Battjes and Janssen's (1978) wave decay model is adopted to predict the variance of the wave elevation in cross-shore direction. The propagation properties of this model are linear; the dissipation process due to breaking is based on a Gaussian wave description. Given the wave variance variation linear theory may be applied to provide the variation of the near-bottom velocity variance and thus the even velocity moments. In the random wave model there is a gradual transition in the breaking fraction of the wave field on a beach of monotoneously decreasing depth. Without the risk of discontinuities we may therefore safely estimate the odd velocity moments from the nonbreaking fraction of waves only and assume that the contribution of the breaking waves is negligible in view of the above conclusions. To provide results from this model we use the second order Stokes expansion with

$$\tilde{u} = u_m \cos \omega_p t$$

$$+ \frac{3}{4} \frac{u_m^2}{c} \sinh^{-2}(k_p h) \cos 2\omega_p t \qquad (17)$$

and choose $u_m = u_{rms}$ from the consideration that the monochromatic representation of the random wave field should have to same variance.

Here we conclude with a comparison between observations of the undertow, the velocity variance and the skewness (i.e. the first odd velocity moment normalized by the variance, $\langle \tilde{u}^3 \rangle / \langle \tilde{u}^2 \rangle^{3/2}$) and calculations with the present model (see Figure 1). The observations are by Guza and Thornton (1985) and concern rather long wave conditions.
The comparison shows that qualitatively the predictions are reasonable; quantitavely there are discrepancies indicating that improvements should be made.

4. THE COMPUTATION OF TRANSPORT AND BOTTOM CHANGES

In the present model the local mean, volumetric cross-shore sediment transport rate, $\langle q \rangle$, is calculated according to the following expressions, where use has been made of expressions (4) and (8)...(12):

$$\langle q \rangle = B_{as} \langle q_{as} \rangle + B_{un} \langle q_{un} \rangle - B_{sl} \langle q_{sl} \rangle \quad (18a)$$

$$\langle q_{as} \rangle = F_B \psi_1 + F_S \psi_2 \quad (18b)$$

$$\langle q_{un} \rangle = F_B \frac{3}{2} \delta_u + F_S 3\delta_u (u3)^* \quad (18c)$$

$$\langle q_{sl} \rangle = F_B \frac{\tan\beta}{\tan\phi} (u3)^* + F_S \frac{u_m}{w} \varepsilon_S \tan\beta (u5)^* \quad (18d)$$

$$F_B = \frac{c_f u_{rms}^3 \varepsilon_B}{\Delta g N \tan\phi} \quad (18e)$$

$$F_S = \frac{c_f u_{rms}^4 \varepsilon_S}{\Delta g N w} \quad (18f)$$

Here c_f is the drag coefficient equal to $\frac{1}{2} f_w$ with f_w the friction factor as defined in Stive and Battjes (1984) and B_{as}, B_{un} and B_{sl} are proportionality constants which should be O(1) if the description is right. The free parameters in the above expressions are ε_B and ε_S which for cross-shore transport are given by Bailard (1982) on the basis of field observations as 0.10 and 0.02 respectively. These values are in principle adopted here.

The cross-shore variation of the local, mean sediment transport may now be calculated with the above expressions (18a...f) given the results of the wave height decay and kinematics model. Through application of the mass balance for the sediment (of which the properties are assumed constant) the bottom changes may be calculated. This procedure may be repeated for the new beach profile.

In the numerical evaluation of the above procedure a second order Runge-Kutta algorithm is used in the wave decay model and a modified Lax scheme in the bottom change calculations.
As a boundary condition on the waterline the present formulation yields $\langle q \rangle = 0$. To simulate the smoothing effect of swash motion on the sediment transport near the waterline $\langle q(x) \rangle$ was damped starting from a depth of approximately half the initial wave height in proportion to the mean water depth.

5. MODEL VERIFICATION

A laboratory measurement programme aimed at the verification of the present model has not yet been conducted. Instead we present a preliminary comparison of model calculations with observed bar formation and deformation in an estuary region in the South of the Netherlands, the socalled Voordelta, which
occurred after closure of one of the Southern Dutch estuaries. The profile deformation in cross-shore direction is appreciable (see Figure 2). The comparison between the hindcast results and the measurements is satisfactory, despite the fact that the wave climate and hydraulic conditions were schematized to one value for the incident wave characteristics and a fixed waterlevel. The proportionality constants B_{as}, B_{un} and B_{sl} were set at 1.0.
Some characteristic parameters of this case are collected in Table 4 below.

case	profile	grain diameter (µm)	$H_{rms,incident}$ (m)	f_p (Hz)
field	barred	225	1.50	0.17

Table 4 Characteristic parameters hindcast case

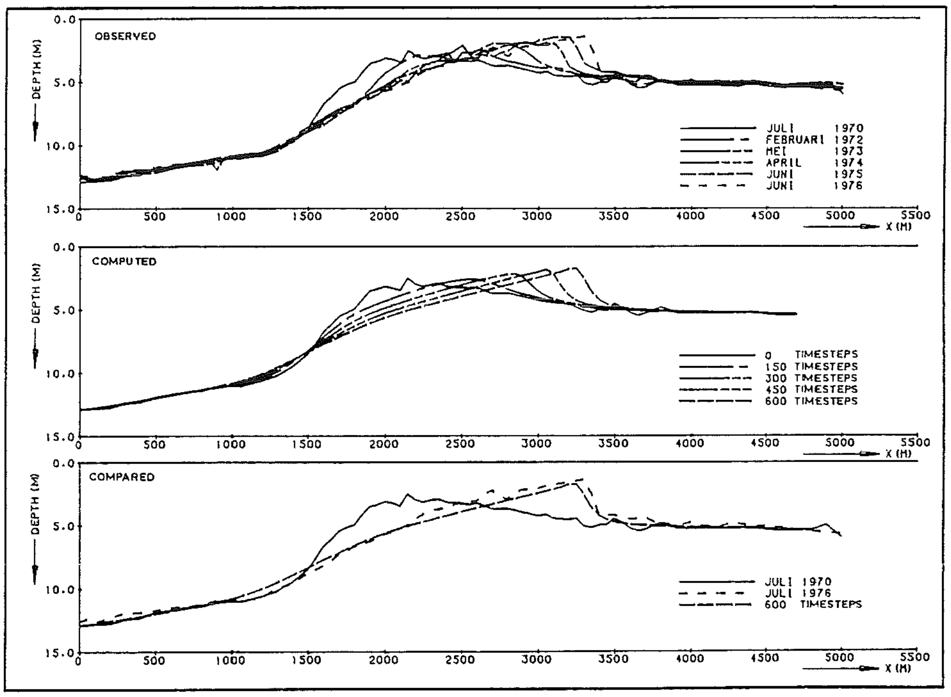

Fig. 2 Comparison between profile development at the Voordelta and present theoretical prediction

6. DISCUSSION AND CONCLUSION

In this paper a first suggestion is made to extend the earlier formulated model for offshore sediment transport due to undertow (Stive and Battjes, 1984) with the effects due to horizontal asymmetry in the wave motion.

To arrive at these results it was necessary to model some low order odd moments of the near-bottom velocity field. An ad-hoc formulation based on a monochromatic, second order Stokes wave representation is shown to give a reasonable, first approximation to the odd velocity moments, but obviously the formulation needs improvement.

The odd velocity moments were readily used in the transport formulation after Bailard (1981). This concerns a vertically integrated description of the sediment transport in sheetflow conditions, which assumes that the instantaneous transport is proportional with some power of the instantaneous near-bottom velocity. The validity of this approach for natural surf zones needs further investigation. This requires study of the temporal and spatial variations of sediment load and/or sediment concentrations due to spatially varying waves in general and random waves breaking on a beach in particular.

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