Start-up & Shut-down Trajectory Constraints in an Energy System Optimisation Model with Fully-Flexible Temporal Resolution

Introducing Trajectory Constraints in Tulipa

Yurian Lagrand[1]
Supervisor(s): Germán Morales-España[1]**, Maaike Elgersma**[1]
[1]EEMCS, Delft University of Technology, The Netherlands

A Thesis Submitted to EEMCS Faculty Delft University of Technology,
In Partial Fulfilment of the Requirements
For the Bachelor of Computer Science and Engineering
June 20, 2025

Name of the student: Yurian Lagrand
Final project course: CSE3000 Research Project
Thesis committee: Germán Morales-España, Maaike Elgersma, Jérémie Decouchant

An electronic version of this thesis is available at http://repository.tudelft.nl/.

Abstract

This research explores the integration of start-up and shut-down trajectory constraints into the Tulipa energy system optimisation model, which uses fully-flexible temporal resolution. These constraints aim to more realistically represent the behaviour of large thermal generators during operation. Case studies with varying time resolutions and generator configurations were evaluated to measure impacts on computation time and solution accuracy. Results show a significant increase in computation time with small gains in accuracy, compared to introduction of minimal down-time constraints.

1 Introduction

The transition towards renewable energy has been an increasingly important topic over the last decade [1]. To achieve their goals, many countries are taking steps to reduce their emissions [2]. The most popular choice to analyse energy systems currently are optimisation models [3]. For this problem specifically, Energy System Optimisation Models (ESOM)s are used. These are models which represent the energy generation capabilities, energy consumption and energy transformation technologies [4]. An important use of these models is determining good investment plans, taking into account renewability and cost of energy [4].

This can be achieved by determining operational details (productive periods, modulation) of existing generators (Unit Commitment problem), or investing in new generators (Generation Expansion Planning). Models made for Generation Expansion Planning rarely include the Unit Commitment problem to integrate operational details of the generators potentially invested in [5]. This is due to the high computation time caused by such high level of detail in a MIP model [3, 6]. However, excluding these details can lead to a significant reduction in accuracy of the model [7]. To attempt speeding up the computation without affecting the accuracy of the model significantly, there are numerous existing techniques, four of which will be introduced here.

The first approach to reduce the computation time is by uniformly reducing the temporal resolution [8, 6]. Usually this is done by averaging values of several timesteps in the higher resolution to compute one value in the lower resolution [9]. However, [9] also discusses different methods to down-sample the resolution, incurring a lower cost in accuracy. Still, these models are unable to solve sufficiently large problems in a high enough resolution [7].

The second, more effective technique frequently used is to reduce the amount of timesteps (and thus computation time) for which the solution is computed. This is done by using time-series aggregation, which splits the considered time frame for the model into representative periods [10, 11]. These are periods that are very similar to a set of other actual periods. Taking all the representative periods together should form a set of periods such that different situations for the the actual periods are covered. The model then computes the solution for the representative period, after which all actual periods represented by the representative period are assigned this solution [11].

The third potential solution for reducing computation time is omitting some constraints from the model, however, this significantly reduces the accuracy of the optimal solution [3, 7]. Some of the constraints that are important to include in ESOMs are minimum up- and down-time [12], start-up and shut-down costs [13], and ramping limits [14, 15], as shown in [7]. Having detailed Unit Commitment constraints is also important for Generation Expansion Planning, as omitting or simplifying them significantly affects the outcome of the models [16].

The fourth technique that speeds up the computation is clustered unit commitment [17]. In clustered unit commitment, the model is made using an integer variable representing all machines of a single type, rather than using a binary variable for each machine. Clustering introduces a very slight error in the final solution [18], while significantly reducing the computation time.

Finally, a new promising approach of reducing computation time is making the temporal resolution flexible, rather than uniform. One of the first models implementing some flexibility is presented in [19]. Here, the resolutions are multiples of each other, rather than all identical. Additionally, time blocks are allowed to be clustered together [20]. The next step in flexibility is a fully-flexible resolution. This means replacing the uniform discrete resolution with a (possibly) non-uniform, discrete resolution, where every resolution can be completely unique, and unrelated to the other resolutions [6].

An example of such a model is Tulipa, developed by TNO [21, 22]. Tulipa implements fully-flexible temporal resolution [21], and solves both the Unit Commitment problem and Generation Expansion Planning. However, Tulipa does not yet model any of the minimum up- and down-time [12], start-up and shut-down costs [13], or ramping [14, 15] constraints, which are critical for the models' accuracy [7]. Another type of constraint not yet in Tulipa are start-up and shut-down trajectory constraints, which describe complex start-up or shut-down trajectories for complicated or large generators [23], e.g., thermal or nuclear reactors. These can model realistic power output of generators in their start-up and shut-down phases, not being limited to linear constraints. This reduces the flexibility of the unit, which means a solution without these constraints might not be valid when they are included. The model with these constraints is more realistic, which thus makes the solution more accurate. The effects of excluding any of the aforementioned constraints in fully-flexible models, or the varying possibilities for time resolutions in them, on the solution and computation time has not been researched yet.

This paper aims to fill in part of this knowledge gap, by implementing trajectory constraints in Tulipa, and assessing how their addition affects the computation time and optimal solution. The optimal solution is analysed in terms of investments, operational schedules and objective function values.

The main contributions of this research are formulating trajectory constraints for fully-flexible ESOMs using MILP, and showing that their inclusion leads to a large increase in runtime, whereas the accuracy is only slightly higher.

This paper starts with section 2, explaining what trajectory constraints are, and formulating the mathematics for the model. Section 3 provides an overview of the experiments done, and their results. Section 4 provides a discussion of the results, along with the ethical aspects of the project and its reproducibility. Section 5 concludes the paper and presents suggestions for future work.

2 Mathematical formulation

This section introduces the mathematical formulation of the constraints which were added. It aims to illustrate the concepts and mathematics that support the final equations. For this, it first describes the background concepts in subsection 2.1. Then it introduces some expressions to simplify the resulting equations in subsection 2.2. Finally, it presents the complete equation in subsection 2.3, along with some limitations, requirements, and a supporting example.

Nomenclature

Sets	Meaning
$\mathcal{Y}$	Set of milestone years for which the model is considered
$\mathcal{P}_y$	Set of periods that make up a milestone year $y \in \mathcal{Y}$
$\mathcal{K}_y$	Set of representative periods s.t. $\forall p_y \in \mathcal{P}_y : (\exists k_y \in \mathcal{K}_y : represents(k_y, p_y))$ where $represents(x, y)$ means that x is representative for y.
$\mathcal{A}_y$	Set of assets in the model in year $y \in \mathcal{Y}$
$\mathcal{A}_y^{\text{uc}}$	Set of assets in the model that have UC constraints in year $y \in \mathcal{Y}$
$\mathcal{F}_{a,y}^{\text{out}}$	Set of possible flows out of asset $a \in \mathcal{A}_y$ in year $y \in \mathcal{Y}$
$\mathcal{T}$	Timeblock, a set containing consecutive discrete timestamps

Temporal Partitions	Meaning
$\mathcal{B}_{a,y,k_y}^{\text{uc}}$	Partitions (timeblocks) for the unit commitment of asset $a \in \mathcal{A}_y$ in representative period $k_y \in \mathcal{K}_y$
$\mathcal{B}_{f,y,k_y}^{\text{flow}}$	Partitions (timeblocks) for the flow $f \in \mathcal{F}_{a,y}^{\text{out}}$ in representative period $k_y \in \mathcal{K}_y$
$\mathcal{B}_{a,y,k_y}^{\text{highest}}$	Set $\mathcal{B} = \{\bigcap_{b_i \in b} b_i \mid b \in (\bigtimes_{f \in \mathcal{F}_{a,y}^{\text{out}}} \mathcal{B}_{f,y,k_y}^{\text{flow}}) \times \mathcal{B}_{a,y,k_y}^{\text{uc}}\}$
$\mathcal{B}_{a,y,k_y}^{\text{su}}$	Set $\mathcal{B} = \{b \in \mathcal{B}_{a,y,k_y}^{\text{highest}} \mid \exists b' \in \mathcal{B}_{a,y,k_y}^{\text{uc}} : start(b) = start(b')\}$, all blocks $b_{k_y} \in \mathcal{B}_{a,y,k_y}^{\text{highest}}$, that start
$\mathcal{B}_{a,y,k_y}^{\text{sd}}$	Same as $\mathcal{B}_{a,y,k_y}^{\text{su}}$

Functions	Meaning
$B_{a,y,k_y}^{\text{uc}}(b_{k_y})$	Returns the timeblock $b'_{k_y} \in \mathcal{B}_{a,y,k_y}^{\text{uc}}$ where $b_{k_y} \subseteq b'_{k_y}$
$B_{f,y,k_y}^{\text{flow}}(b_{k_y})$	Returns the timeblock $b'_{k_y} \in \mathcal{B}_{f,y,k_y}^{\text{flow}}$ where $b_{k_y} \subseteq b'_{k_y}$
$\text{start}(b_{k_y})$	Returns the lowest element of the set
$\text{end}(b_{k_y})$	Returns the highest element in the set
$\text{next}(b_{k_y})$	Returns the first following timeblock $b' \in \mathcal{B}_{a,y,k_y}^{\text{su}}$, that has not started yet
$\text{last}(b_{k_y})$	Returns the last started time block $b' \in \mathcal{B}_{a,y,k_y}^{\text{sd}}$

Parameters	Meaning
$p_{a,y,p_y}^{\text{availability profile}}$	Availability profile of asset $a \in \mathcal{A}_y$ in representative period $p_y \in \mathcal{P}_y$ of year $y \in \mathcal{Y}$
$p_{a,y}^{\text{min operating point}}$	Minimal operation point of asset $a \in \mathcal{A}_y$ in year $y \in \mathcal{Y}$
p_a^{capacity}	Capacity of asset $a \in \mathcal{A}_y$
b^{start}	First time block for which the model is computed, in $\mathcal{B}_{a,y,k_y}^{\text{su}}$

Variables	Meaning
$v_{a,y,k_y,b_{k_y}}^{\text{units on}}$	Amount of units of asset $a \in \mathcal{A}_y$ on in timeblock $b_{k_y} \in \mathcal{B}_{a,y,k_y}^{\text{uc}}$
$v_{a,y,k_y,b_{k_y}}^{\text{start up}}$	Units starting up in timeblock $b_{k_y} \in \mathcal{B}_{a,y,k_y}^{\text{su}}$
$v_{a,y,k_y,b_{k_y}}^{\text{shut down}}$	Units shutting down in timeblock $b_{k_y} \in \mathcal{B}_{a,y,k_y}^{\text{sd}}$
$v_{f,y,k_y,b_{k_y}}^{\text{flow}}$	Amount of flow in $f \in \mathcal{F}_{a,y}^{\text{out}}$ in timeblock $b_{k_y} \in \mathcal{B}_{f,y,k_y}^{\text{flow}}$

2.1 Concepts

The framework into which the trajectory constraints will be introduced has some novel concepts, which will be shortly explained in this subsection. The final constraint will build upon these concepts to define a lower and upper bound for the flows out of assets.

2.1.1 Trajectory Constraints

Trajectory constraints bound the minimum and maximum flow out of an asset during a start-up or shut-down procedure. This ensures that technically complex and large generators, like large thermal generators or nuclear reactors, cannot instantly start up/shut down in the model, which makes it more realistic. Instead, they must follow a pre-defined trajectory the same way real-world generators do. Figure 1 shows an example of such trajectories, as indicated. In this example, the start-up and shut-down trajectories are symmetric, but they can be asymmetric if necessary.

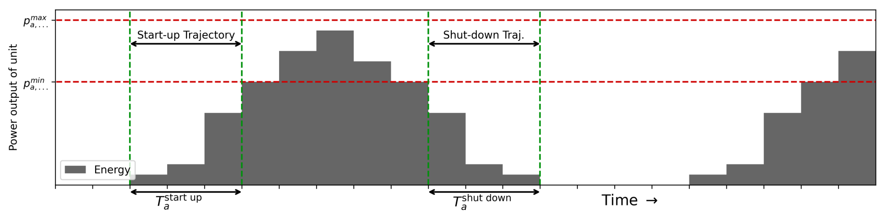


Figure 1: Example of start-up & shut-down trajectories.

2.1.2 Fully-Flexible Time Resolution

The model considers milestone years $y \in \mathcal{Y}$, consisting of periods $p_y \in \mathcal{P}_y$. Instead of considering every period, the model uses representative periods $k_y \in \mathcal{K}_y$. These are periods that somewhat accurately describe other similar periods. For example, a milestone year can be modelled as 10 days, each representing some other days in the year. Each of these representative periods consists of smaller timer periods $t \in k_y$.

To decrease size and computation time of the model, the temporal resolution can be decreased. In a fully-flexible model, each asset and each flow between two assets is allowed to have a unique resolution. An example of some possible resolutions is shown in figure 2.

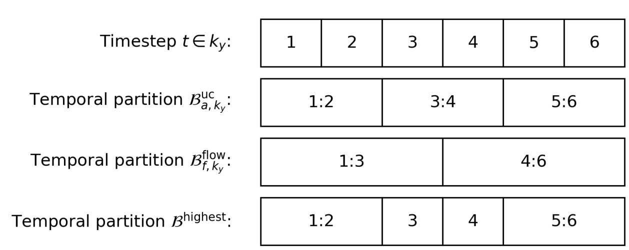
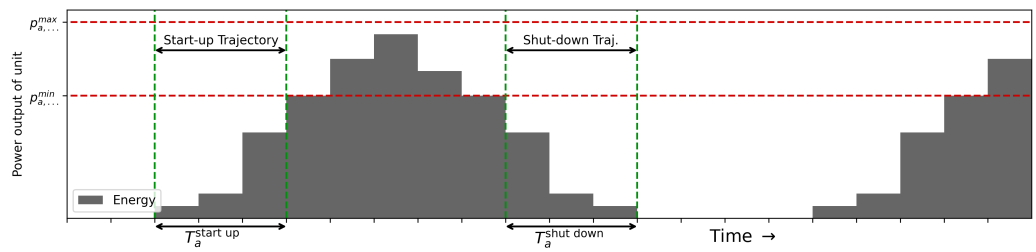


Figure 2: Example of possible resolutions, and the highest resolution computed from them.

2.1.3 Start-up & Shut-down Variables

Start-up & shut-down variables represent the number of units starting up and shutting down in some timeblock $b_{k_y} \in \mathcal{B}^{\text{uc}}_{a,y,k_y}$. They are both defined in terms of $\mathcal{B}^{\text{highest}}_{a,y,k_y}$, but only for the timeblocks where there also exists a timeblock in $\mathcal{B}^{\text{uc}}_{a,y,k_y}$ with the same start time. The values of the variables are fully determined by the values of $v^{\text{units on}}_{a,y,k_y,b_{k_y}}$. Figure 3 shows an example, with the same graph as from figure 1, but now including a table with the variable assignments for each timeblock. Their exact definitions are shown in (3a)-(3d) in section 2.2. An example of trajectories and start-up & shut-down variables from resulting data from an experiment is given in appendix D.



$v^{\text{units on}}_{...}$	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	1	1
$v^{\text{start up}}_{...}$	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
$v^{\text{shut down}}_{...}$	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0

Figure 3: Example of start-up & shut-down trajectories, with additional variables for unit commitment.

2.2 Expressions

To implement trajectory constraints into this framework, it is useful to first define some expressions, which will simplify the formulation of the final constraints. This subsection will introduce expressions for total outgoing flow of an asset, minimal and maximum production of an asset, and equations for the start-up & shut-down variables, explained in section 2.1.3

For the total outgoing flow of an asset in some timeblock, $v^{\text{flow total}}_{a,y,k_y,b_{k_y}}$, we define it as described by (1). It is the sum of all flows going out of the asset during the timeblock.

$$v^{\text{flow total}}_{a,y,k_y,b_{k_y}} = \sum_{\substack{f \in \mathcal{F}^{\text{out}}_{a,y} \\ b' = B^{\text{flow}}_{f,y,k_y}(b_{k_y})}} v^{\text{flow}}_{f,y,k_y,b'} \qquad \forall y \in \mathcal{Y}, a \in \mathcal{A}^{uc}_y, k_y \in \mathcal{K}_y, b_{k_y} \in \mathcal{B}^{\text{highest}}_{a,y,k_y} \tag{1}$$

For expressions for minimum and maximum output of an asset, $p^{\text{min}}_{a,y,k_y,b_{k_y}}$ and $p^{\text{max}}_{a,y,k_y,b_{k_y}}$ respectively, we define them in terms of input parameters provided to the model. Equations (2a) & (2b) describe these formulations.

$$p^{\text{max}}_{a,y,k_y,b_{k_y}} = p^{\text{availability profile}}_{a,y,p_y} \cdot p^{\text{capacity}}_{a} \qquad \forall y \in \mathcal{Y}, a \in \mathcal{A}^{uc}_y, k_y \in \mathcal{K}_y, b_{k_y} \in \mathcal{B}^{\text{highest}}_{a,y,k_y} \tag{2a}$$

$$p^{\text{min}}_{a,y,k_y,b_{k_y}} = p^{\text{max}}_{a,y,k_y,b_{k_y}} \cdot p^{\text{min operating point}}_{a,y} \qquad \forall y \in \mathcal{Y}, a \in \mathcal{A}^{uc}_y, k_y \in \mathcal{K}_y, b_{k_y} \in \mathcal{B}^{\text{highest}}_{a,y,k_y} \tag{2b}$$

Finally, the equations for the start-up and shut-down variables, for which their meaning is explained in subsection 2.1 are shown in (3a)-(3d). These include the restriction for minimal down-time, as described in section 2.3. In these equations, $T^{\text{min down}}_a$ represents the amount of consecutive hours a unit must be offline after shutting down.

$$v^{\text{units on}}_{a,y,k_y,B^{\text{uc}}_{a,y,k_y}(b_{k_y})} - v^{\text{units on}}_{a,y,k_y,B^{\text{uc}}_{a,y,k_y}(b_{k_y}-1)} = v^{\text{start up}}_{a,y,k_y,b_{k_y}} - v^{\text{shut down}}_{a,y,k_y,b_{k_y}} \qquad \forall y \in \mathcal{Y}, a \in \mathcal{A}^{uc}_y, k_y \in \mathcal{K}_y, b_{k_y} \in \mathcal{B}^{\text{su}}_{a,y,k_y} \setminus \{b^{\text{start}}\} \tag{3a}$$

$$v^{\text{start up}}_{a,y,k_y,b_{k_y}} \leq v^{\text{units on}}_{a,y,k_y,B^{\text{uc}}_{a,y,k_y}(b_{k_y})} \qquad \forall y \in \mathcal{Y}, a \in \mathcal{A}^{uc}_y, k_y \in \mathcal{K}_y, b_{k_y} \in \mathcal{B}^{\text{su}}_{a,y,k_y} \setminus \{b^{\text{start}}\} \tag{3b}$$

$$\sum_{\substack{i \in \mathcal{B}^{\text{sd}}_{a,y,k_y} \\ \text{start}(b_{k_y}) - T^{\text{min down}}_a + 1 \leq \text{start}(i) \leq \text{start}(b_{k_y})}} v^{\text{shut down}}_{a,y,k_y,i} \leq v^{\text{available units}}_{a,y} - v^{\text{units on}}_{a,y,k_y,B^{\text{uc}}_{a,y,k_y}(b_{k_y})} \qquad \forall y \in \mathcal{Y}, a \in \mathcal{A}^{uc}_y, k_y \in \mathcal{K}_y, b_{k_y} \in \mathcal{B}^{\text{sd}}_{a,y,k_y} \setminus \{b^{\text{start}}\} \tag{3c}$$

$$v^{\text{start up}}_{a,y,k_y,b_{k_y}}, v^{\text{shut down}}_{a,y,k_y,b_{k_y}} \in \mathbb{Z}_{\geq 0} \qquad \forall y \in \mathcal{Y}, a \in \mathcal{A}^{uc}_y, k_y \in \mathcal{K}_y, b_{k_y} \in \mathcal{B}^{\text{su}}_{a,y,k_y} \setminus \{b^{\text{start}}\} \tag{3d}$$

2.3 Constraint formulation

This subsection describes how the final constraint formulations came to be from the original formulations, and some limitations and requirements for it to be valid. It ends by showing a small example.

For limiting the flow in the model without trajectory constraints, the equations for bounding the total flow are the minimal/maximal production multiplied by the amounts of units on, as described in (4a) & (4b).

$$v^{\text{flow total}}_{a,y,k_y,b_{k_y}} \leq p^{\text{max}}_{a,y,k_y,b_{k_y}} \cdot v^{\text{units on}}_{a,y,k_y,B^{\text{uc}}_{a,y,k_y}(b_{k_y})} \qquad \forall y \in \mathcal{Y}, a \in \mathcal{A}^{uc}_y, k_y \in \mathcal{K}_y, b_{k_y} \in \mathcal{B}^{\text{highest}}_{a,y,k_y}, b' = B^{\text{uc}}_{a,y,k_y}(b_{k_y}) \tag{4a}$$

$$v^{\text{flow total}}_{a,y,k_y,b_{k_y}} \geq p^{\text{min}}_{a,y,k_y,b_{k_y}} \cdot v^{\text{units on}}_{a,y,k_y,B^{\text{uc}}_{a,y,k_y}(b_{k_y})} \qquad \forall y \in \mathcal{Y}, a \in \mathcal{A}^{uc}_y, k_y \in \mathcal{K}_y, b_{k_y} \in \mathcal{B}^{\text{highest}}_{a,y,k_y}, b' = B^{\text{uc}}_{a,y,k_y}(b_{k_y}) \tag{4b}$$

To include the start-up trajectory constraints, the contribution of machines that are starting up is added as a term. To this end, some new model parameters are introduced. These are $p^{\text{start up trajectory}}_{a,i}$ for all timesteps i in the start-up trajectory. This is used to denote the power output in hour i of the trajectory. Additionally, we also add $T^{\text{start up}}_a$ and $T^{\text{shut down}}_a$ which are the start-up trajectory length, and the shut-down trajectory length respectively. The new term added to the constraint is defined as a sum that scans ahead to determine if any units are starting up in timeblocks after the current one and if so, sums up their contributions. These contributions are averaged over the length of the timeblock, to make sure that they are not summed together in larger timeblocks, as this would lead to a increased flow limit. For shut-down trajectories, $p_{a,i}^{\text{shut down trajectory}}$ is introduced, which denotes the power output in hour i of the shut-down trajectory. For the sum for this term, the same concept is used, except the it checks if any machines shut down recently, rather than if any are starting up soon. To simplify the formulations, we define a variable representing the trajectories, as shown in (5). Equations (6a) & (6b) show the constraints with the new term.

$$
\begin{aligned}
p_{a,y,k_y,b_{k_y}}^{\text{trajectories}} = v_{a,y,k_y,b'}^{\text{start up}} \cdot & \sum_{\substack{1 \le i \le T_a^{\text{start up}}: \\ \text{start}(b_{k_y})+i \le \text{start}(b') \le \text{end}(b_{k_y})+i}} \frac{p_{a,T_a^{\text{start up}}-i+1}^{\text{start up trajectory}}}{\text{end}(b_{k_y}) - \text{start}(b_{k_y}) + 1} \\
+ v_{a,y,k_y,b''}^{\text{shut down}} \cdot & \sum_{\substack{0 \le i \le T_a^{\text{shut down}}-1: \\ \text{start}(b_{k_y})-i \le \text{start}(b'') \le \text{end}(b_{k_y})-i}} \frac{p_{a,i+1}^{\text{shut down trajectory}}}{\text{end}(b_{k_y}) - \text{start}(b_{k_y}) + 1} \\
& \forall y \in \mathcal{Y}, a \in \mathcal{A}_y^{uc}, k_y \in \mathcal{K}_y, b_{k_y} \in \mathcal{B}_{a,y,k_y}^{\text{highest}}, b' = \text{next}(b_{k_y}), b'' = \text{last}(b_{k_y})
\end{aligned} \tag{5}
$$

$$
\begin{aligned}
v_{a,y,k_y,b_{k_y}}^{\text{flow total}} \le p_{a,y,k_y,b_{k_y}}^{\max} \cdot v_{a,y,k_y,B_{a,y,k_y}^{\text{uc}}(b_{k_y})}^{\text{units on}} + p_{a,y,k_y,b_{k_y}}^{\text{trajectories}} \\
\forall y \in \mathcal{Y}, a \in \mathcal{A}_y^{uc}, k_y \in \mathcal{K}_y, b_{k_y} \in \mathcal{B}_{a,y,k_y}^{\text{highest}}
\end{aligned} \tag{6a}
$$

$$
\begin{aligned}
v_{a,y,k_y,b_{k_y}}^{\text{flow total}} \ge p_{a,y,k_y,b_{k_y}}^{\min} \cdot v_{a,y,k_y,B_{a,y,k_y}^{\text{uc}}(b_{k_y})}^{\text{units on}} + p_{a,y,k_y,b_{k_y}}^{\text{trajectories}} \\
\forall y \in \mathcal{Y}, a \in \mathcal{A}_y^{uc}, k_y \in \mathcal{K}_y, b_{k_y} \in \mathcal{B}_{a,y,k_y}^{\text{highest}}
\end{aligned} \tag{6b}
$$

It should be noted that there are three requirements for these constraints to be valid. The first, each timeblock in $\mathcal{B}_{a,y,k_y}^{\text{uc}}$ has to contain an integer amount of timeblocks from $\mathcal{B}_{f,y,k_y}^{\text{flow}}$. If any timeblock in $\mathcal{B}_{f,y,k_y}^{\text{flow}}$ spans more than 1 timeblock from $\mathcal{B}_{a,y,k_y}^{\text{uc}}$, the trajectories will not be limited correctly.

Second, to ensure the trajectories are not allowed to overlap, a minimum down-time constraint should be used. The minimum downtime of an asset should be at least as long as the length of the start-up trajectory plus the length of the shut-down trajectory.

Finally, it is also required that the timeblocks from $\mathcal{B}_{a,y,k_y}^{\text{uc}}$ are at least as long as the longest trajectory relating to the asset. If this is not the case, the trajectory gets cut off, and not fully accounted for.

To show correctness of the added terms, the remainder of this section will describe a small working example, showing that the terms add up the correct values for the trajectories, and that other ones are ignored. Assume the following scenario: we have an asset $a \in \mathcal{A}_y$ during some year $y \in \mathcal{Y}$ and some representative period $k_y \in \mathcal{K}_y$ with the following parameters:

- $p_{a,1}^{\text{start up trajectory}} = 1$; $p_{a,2}^{\text{start up trajectory}} = 3$; $p_{a,3}^{\text{start up trajectory}} = 7$
- $p_{a,1}^{\text{shut down trajectory}} = 8$; $p_{a,2}^{\text{shut down trajectory}} = 4$; $p_{a,3}^{\text{shut down trajectory}} = 2$

This means that $T_a^{\text{start up}} = T_a^{\text{shut down}} = 3$. Consider an 8 hour window with 4-hourly uniform UC resolution, and 2-hourly uniform resolution for each of its outgoing flows, as shown in figure 4.

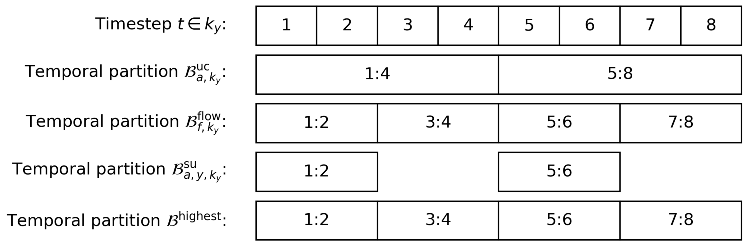


Figure 4: Example resolutions for a unit with trajectory constraints.

Assume the asset starts up in timeblock $[5:8]$[1], thus $v^{\text{start up}}_{a,y,k_y,[5:6]} = 1$, whereas $v^{\text{start up}}_{a,y,k_y,[1:2]} = 0$. Additionally, assume $v^{\text{start up}}_{a,y,k_y,[9:X]} = 0$, to ensure nothing is starting up after the considered window. This means the desired flow bounds for each timeblock are increased by:

- $v^{\text{flow total}}_{a,y,k_y,[1:2]}$ by $\frac{p^{\text{start up trajectory}}_{a,1}}{2-1+1} = \frac{1}{2}$, as $b_{k_y} = [1:2], b' = [5:6]$, and for the i's in the sum:
  - $i = 1 \rightarrow 0$, as $5 = \text{start}(b') \not\leq \text{end}(b_{k_y}) + i = 3$
  - $i = 2 \rightarrow 0$, as $5 = \text{start}(b') \not\leq \text{end}(b_{k_y}) + i = 4$
  - $i = 3 \rightarrow \frac{p^{\text{start up trajectory}}_{a,1}}{2-1+1} = \frac{1}{2}$, as $\text{start}(b_{k_y}) + i = 4 \leq 5 = \text{start}(b') \leq \text{end}(b_{k_y}) + i = 5$
- $v^{\text{flow total}}_{a,y,k_y,[3:4]}$ by $\frac{p^{\text{start up trajectory}}_{a,2} + p^{\text{start up trajectory}}_{a,3}}{4-3+1} = 5$, as $b_{k_y} = [3:4], b' = [5:6]$, and for the i's in the sum:
  - $i = 1 \rightarrow \frac{p^{\text{start up trajectory}}_{a,3}}{4-3+1} = \frac{7}{2}$, as $\text{start}(b_{k_y}) + i = 4 \leq 5 = \text{start}(b') \leq \text{end}(b_{k_y}) + i = 5$
  - $i = 2 \rightarrow \frac{p^{\text{start up trajectory}}_{a,2}}{4-3+1} = \frac{3}{2}$, as $\text{start}(b_{k_y}) + i = 5 \leq 5 = \text{start}(b') \leq \text{end}(b_{k_y}) + i = 6$
  - $i = 3 \rightarrow 0$, as $6 = \text{start}(b_{k_y}) + i \not\leq \text{start}(b') = 5$

For $v^{\text{flow total}}_{a,y,k_y,[5:6]}$ & $v^{\text{flow total}}_{a,y,k_y,[7:8]}$, a value of 0 will be assigned regardless of the sum, as the $v^{\text{start up}}_{a,y,k_y,[9:X]} = 0$. In this case, for every flow timeblock, the correct start-up trajectory parts are included, and the rest is excluded.

Similarly, for the shut-down trajectories, assume the asset shuts down in timeblock $[5:8]$ instead. Thus $v^{\text{shut down}}_{a,y,k_y,[5:6]} = 1$, whereas $v^{\text{shut down}}_{a,y,k_y,[1:2]} = 0$. This means the desired flow bounds for each timeblock are increased by:

- $v^{\text{flow total}}_{a,y,k_y,[5:6]}$ by $\frac{p^{\text{shut down trajectory}}_{a,1} + p^{\text{shut down trajectory}}_{a,2}}{6-5+1} = 6$, as $b_{k_y} = [5:6], b'' = [5:6]$, and for the i's in the sum:
  - $i = 0 \rightarrow \frac{p^{\text{shut down trajectory}}_{a,1}}{6-5+1}$, as $\text{start}(b_{k_y}) - i = 5 \leq 5 = \text{start}(b'') \leq \text{end}(b_{k_y}) - i = 6$

[1] Here, $[x:y]$ denotes an ordered set, ranging from x to y, both inclusive.

- $i = 1 \rightarrow \frac{p^{\text{shut down trajectory}}_{a,2}}{6-5+1}$, as $\text{start}(b_{k_y}) - i = 4 \leq 5 = \text{start}(b'') \leq \text{end}(b_{k_y}) - i = 5$
- $i = 2 \rightarrow 0$, as $\text{start}(b'') = 5 \not\leq \text{end}(b_{k_y}) - i = 4$

- $v^{\text{flow total}}_{a,y,k_y,[7:8]}$ by $\frac{p^{\text{start up trajectory}}_{a,3}}{4-3+1} = 5$, as $b_{k_y} = [7:8], b'' = [5:6]$, and for the i's in the sum:
  - $i = 0 \rightarrow 0$, as $7 = \text{start}(b_{k_y}) - i \not\leq \text{start}(b'') = 5$
  - $i = 1 \rightarrow 0$, as $6 = \text{start}(b_{k_y}) - i \not\leq \text{start}(b'') = 5$
  - $i = 2 \rightarrow \frac{p^{\text{shut down trajectory}}_{a,3}}{8-7+1}$, as $\text{start}(b_{k_y}) - i = 5 \leq 5 = \text{start}(b'') \leq \text{end}(b_{k_y}) - i = 6$

For $v^{\text{flow total}}_{a,y,k_y,[1:2]}$ & $v^{\text{flow total}}_{a,y,k_y,[3:4]}$, a value of 0 will be assigned regardless of the sum, as the $v^{\text{shut down}}_{a,y,k_y,[1:2]} = 0$. Here, for every flow timeblock, the correct shut-down trajectory parts are included, and the others are excluded.

3 Experimental Setup and Results

This section describes the exact experiments ran and their outcomes. Subsection 3.1 explains which cases were used, which data was used to create these cases, and why the data is realistic. Subsection 3.2 discusses what metrics were measured and how the measurements were collected. Finally, 3.3 shows the results of the measurements.

3.1 Case Studies

The following case studies were each run with hourly, 2-hourly, 4-hourly, and 6-hourly resolution, always with 0 initial generators:

- 7 countries (Netherlands, Belgium, Luxembourg, France, Germany, Switzerland, United Kingdom), where each country has the access to onshore and offshore wind farms, solar farms, battery output connectors, battery storage, CCGT, OCGT, coal and nuclear power plants.
- same as previous, but the coal and nuclear generators are switched to minimum downtime constraints.
- same as previous, but the coal and nuclear generators are switched to trajectory constraints.

Each case contains 10 representative periods, the exact data for each case can be found here: [24]. The assets with trajectory constraints were always set to a resolution of at least the length of their trajectory, but their outgoing flows matched the general resolution. This is because, as explained in section 2.3, the constraint is only valid in the case where the UC timeblocks are at least as long as the trajectory. For comparing accuracy across resolutions, the hourly resolution was taken as 100% accurate.

For the data for generators, demands of countries, and other important modelling details, the following sources were used:

- Countries, connections between them, and their peak demands: [6].
- Availability profiles, demands profiles: personal communcation with TNO.

- Maximum capacities for assets: averaged from [25].
- Investment costs for assets: [26].
- Minimum operating point of assets: [27].
- Cost per MWh for assets: [28].

3.2 Experimental Setup

The metrics collected from the experiments are the runtime, investments made by the model, operational schedules of assets with unit commitment, and objective function values. These have been collected by extending the existing Tulipa ESOM [21, 22] with minimal down-time constraints and trajectory constraints. The implementation for minimal down-time constraints was taken from [29]. Tulipa creates a MILP model using JuMP [30, 31], which is then solved using Gurobi [32]. To collect the data, 100 samples were run for each test case.

All experiments were ran on a laptop with a 12th Gen Intel(R) Core(TM) i7-12700H @ 2.30 GHz, with 16.0GB of installed RAM. The used device runs Windows 11.

3.3 Results

This subsection discusses the changes found in the metrics after adding trajectory constraints, compared to the original model, and the original model extended with just minimal down-time constraints. It first presents the difference in computation time. Second, the changes in investment plan are compared. Third, the operational schedules are examined. Extended visualisations of the results can be found in appendix B.

3.3.1 Computation Time

Figure 5 shows that the computation time of the model went up after adding the trajectory constraints for all resolutions, compared to adding only minimal down-time.

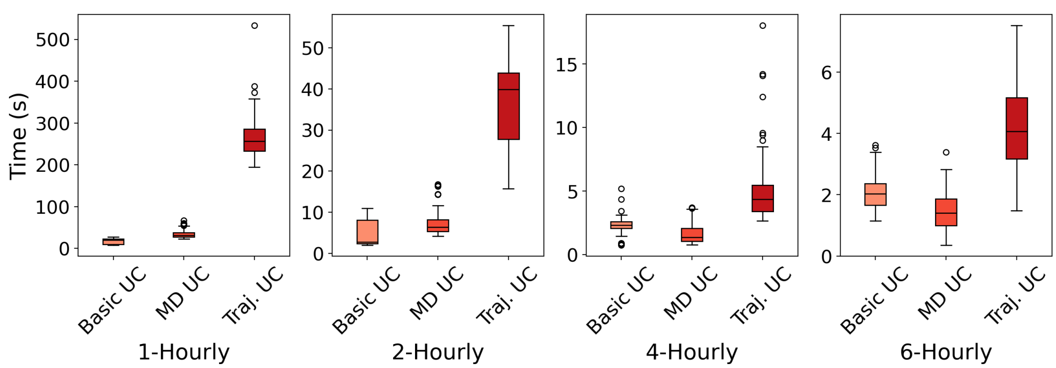


Figure 5: Model solving times for all cases. Comparison of the existing model (Basic UC), an extended version with just minimal down-time (MD UC), and an extended version with both minimal down-time and trajectories (Traj. UC).

3.3.2 Investment Plan

Figure 6 shows that the inclusion of trajectories has an effect on the investment choices made by the solver. For most assets, small deviations are visible compared to including only the minimal down-time constraints. However, for storage, a larger amount of both storage capacity and output capacity is bought. Neither of these observations were seen in lower resolutions.

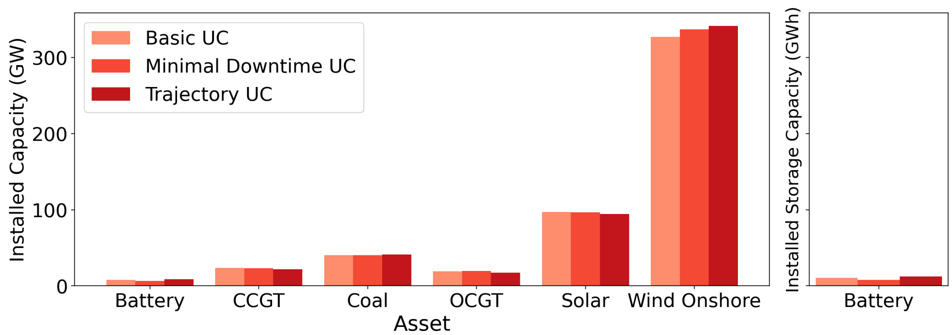


Figure 6: Model investment plans for hourly resolution. Comparison of the existing model, an extended version with just minimal down-time, and an extended version with both minimal down-time and trajectories.

3.3.3 Operational Schedules

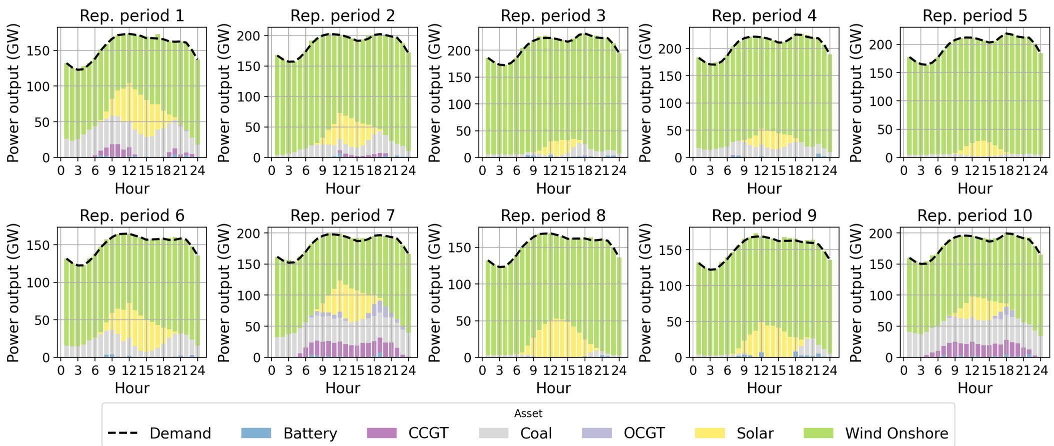
The operational schedules for hourly resolution, shown in figure 7, have only insignificant differences between minimum down-time UC and trajectory UC. However, introducing trajectories reduced the number of start-ups and shut-downs of generators. These results hold for all tested resolutions, where the reduction increases as resolution decreases.

3.3.4 Objective Function

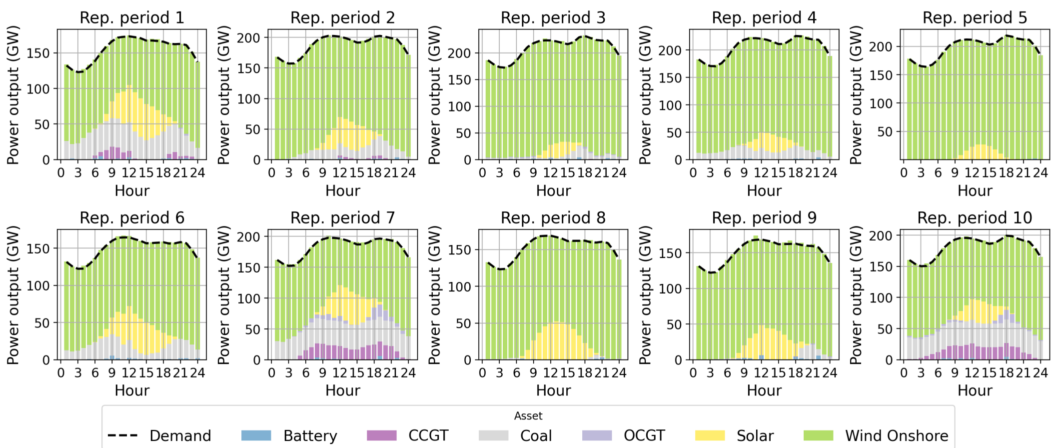
The difference in objective functions between cases with minimal down-time and trajectories is larger than the MIP gap, the accuracy which the solver for the model can achieve. This means trajectories have a noticeable impact on the objective function value. This pattern was visible for each resolution, with the difference decreasing for lower resolutions. Table 1 presents a brief overview of the exact numbers, an exhaustive overview is included in appendix C.

UC Variant	Total Cost	Investment Cost	Operational Cost
Basic	29,496,058	22,422,192	7,073,866
Min. down-time	29,712,401	22,835,519	6,876,882
Trajectories	29,815,366	23,048,621	6,766,745

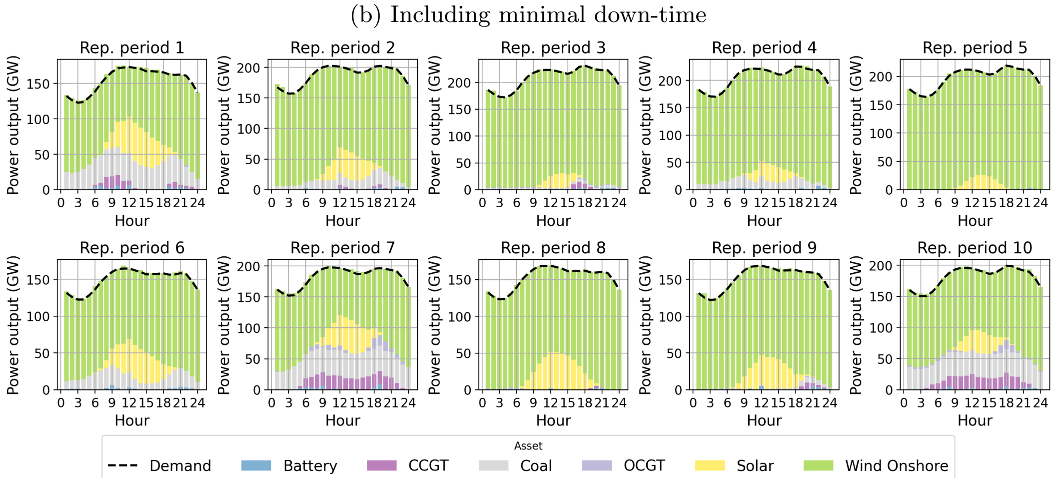
Table 1: Model costs (total cost = objective function value) in kEUR for hourly resolution. The total cost is accurate to 0.01%.



(a) Existing model



(b) Including minimal down-time



(c) Including minimal down-time and trajectories

Figure 7: Comparison of the model operational schedules for hourly resolution for each variant.

4 Discussion

4.1 Discussion of results

The results show a significant increase in computation time when adding trajectory constraints, compared to the other scenarios. This increase comes with a small increase in accuracy, as shown by figures 5, 6, 7 and table 1. As the resolution gets lower, including only the minimal down-time constraint adds a similar amount of accuracy as including the trajectory constraints, for a much smaller increase in computational time. The trajectory constraints only having a small effect is likely because the tests were run on large models. The high number of large generators in the used model allows already running generators to compensate for ramps caused by generators starting up or shutting down, reducing their effect.

However, trajectory constraints could be more useful in other types of fully-flexible models that involve UC. For example, [33] showed its importance when considering a single, self-scheduling generator. The experiments ran for this paper also showed larger increases in accuracy on smaller models, but analysing this is beyond the scope of this research.

There are four limitations to these conclusions. First, the trajectories used are short, with the longest one being 4 hours. Their impact could be larger for longer trajectories, as this would restrict the generators more.

Second, the dataset used is relatively small for a large-scale model, and contains a small number of generator types. This is necessary because the computation times for trajectory constraints are very large, due to the limited computation resources available for the research.

Third, all experiments were conducted as greenfield experiments. This means that the model started with no initial units, and could freely choose what to invest in. If the model is given real-world data about what generators already exist, results may be different.

Fourth, the constraint only works for resolutions where each timeblock in $\mathcal{B}^{\text{uc}}_{a,y,k_y}$ contains an integer amount of timeblocks from $\mathcal{B}^{\text{flow}}_{f,y,k_y}$, as explained in section 2.3. An alternative formulation could remove this limitation, but also potentially cause even longer computation times.

4.2 Responsible Research

This section will shortly discuss the ethical aspects of the research and explain why it is reproducible.

There are two important considerations to the research when it comes to ethics. Firstly, the current objective of the model is to purely minimise the cost of the solution. Emissions of thermal generators are not taken into account in this cost. A cost for this could be modelled as some amount of cost per emitted unit. Alternatively, a limit on emission could be set, which forces the model to stay under it.

Secondly, the accuracy of the model is important, because its goal is to be used to provide governments with advice about their energy networks. If there are inaccuracies or big oversights in the model creation, this could lead to real-world consequences for the energy network of a country, if they follow the investment plan provided by the model.

The research is easily reproducible, all code is available open source[2], and so is all the data used for creating and running the models[3]. Additionally, the results from the runs

[2] `https://github.com/Cerberus22/TulipaEnergyModel.jl/`

[3] `https://github.com/Cerberus22/RP_Data`

are also available with the input data. All technologies used for running the code are also available for free, except Gurobi, which can be replaced for free by the HiGHS optimiser.

5 Conclusions and Future Work

This research discusses the trade-off between accuracy and computation time for addition of trajectory constraints to a fully-flexible ESOM. This was done by extending the Tulipa Energy Model with trajectory constraints, to reflect more realistic operational behaviour of large thermal generators. Several case studies were run, and the resulting computation times, investment plans, objective function values and operational schedules were recorded. An important requirement for trajectory constraints to be valid, is the inclusion of minimal down-time constraints. The vast majority of changes between the original model, and the model including trajectory constraint was introduced by the minimal down-time constraints. This suggests that the increased computational cost outweighs the accuracy improvements.

Nonetheless, some further research should be done in two areas. The first area is reformulating the trajectory constraints to more flexible versions, as this could provide more accurate results, since the unit commitment resolution could be defined in higher resolution.

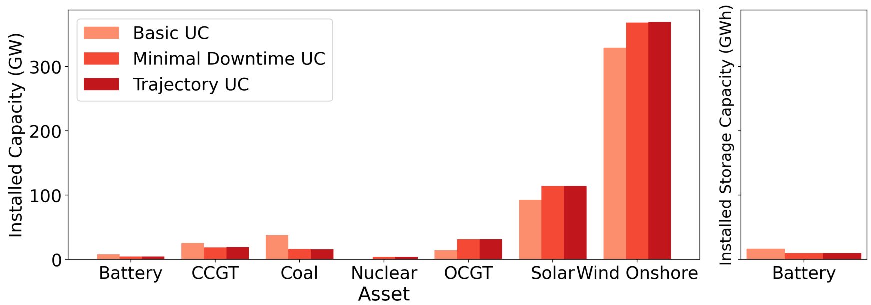
The second area is formally proving the correctness of the constraints. Although it is a challenging mathematical task, it is possible to prove the constraints are correct, and this might be an important proof to have in cases where accuracy of the model is critical.

A Acknowledgements

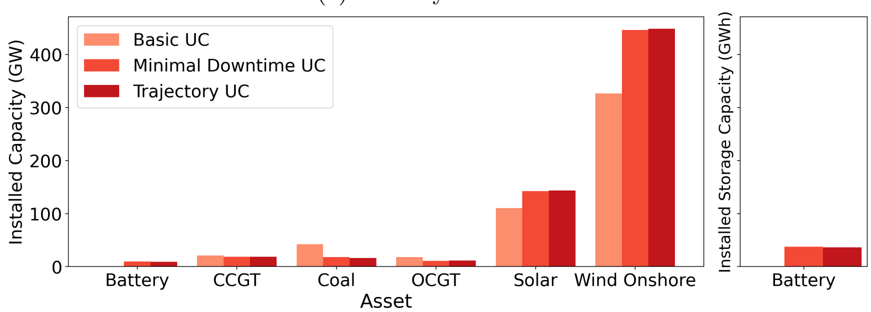
I would like to express my gratitude to my supervisor, Maaike Elgersma, and my responsible professor, Germán Morales-España, for their help and feedback during the project. Additionally, I would like to thank Diego Tejada Arango and Ni Wang from TNO for assisting with the programming and data collection.

B Extended Result Visualisations

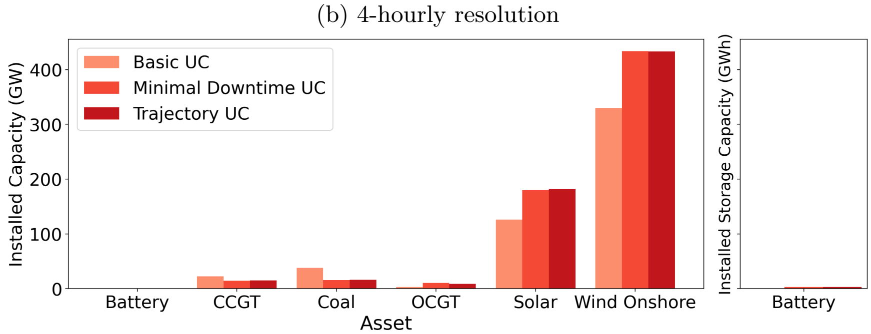
B.1 Investment plans



(a) 2-hourly resolution



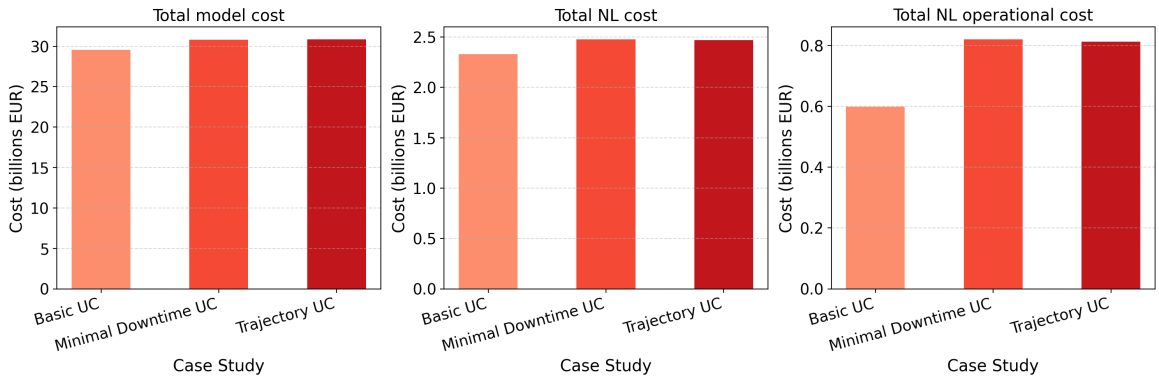
(b) 4-hourly resolution



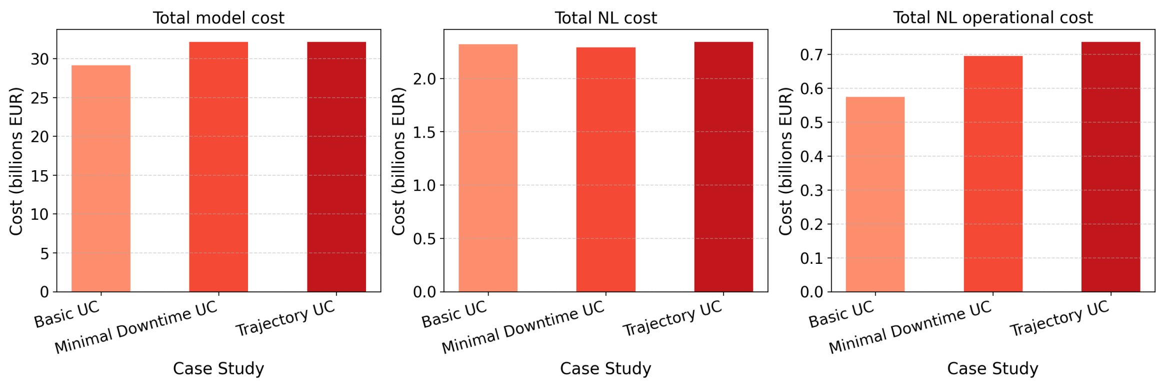
(c) 6-hourly resolution

Figure 8: Model investment plans for 2-hourly resolution (a), 4-hourly resolution (b), and 6-hourly resolution (c). Comparison of the existing model, an extended version with just minimal down-time, and an extended version with both minimal down-time and trajectories.

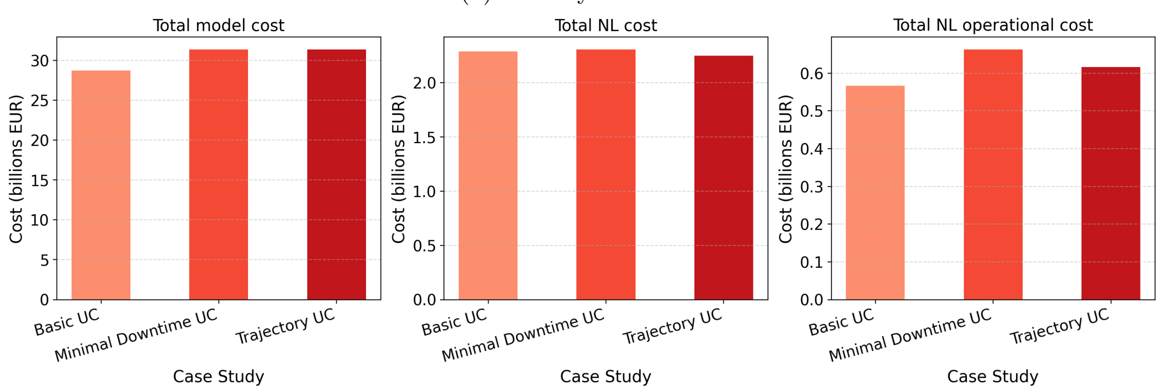
B.2 Objective Functions



(a) 2-hourly resolution



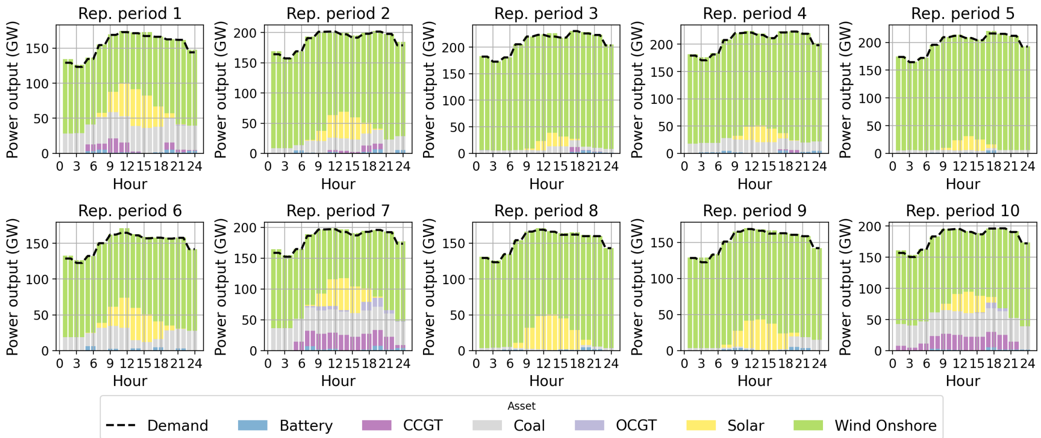
(b) 4-hourly resolution



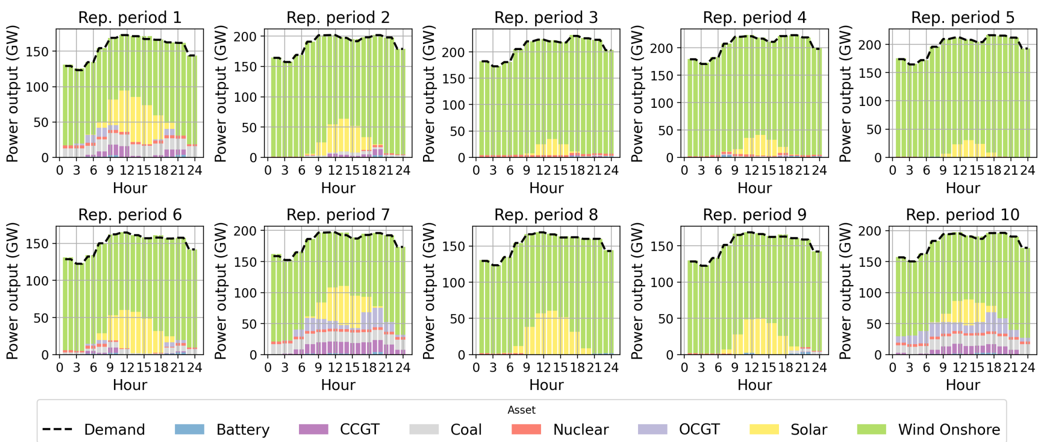
(c) 6-hourly resolution

Figure 9: Total and regional objective function costs for 2-hourly (a), 4-hourly (b), and 6-hourly (c) resolutions. Total cost (Left), total cost NL (middle), and just operating costs NL (right). Comparison of the existing model, an extended version with just minimal down-time, and an extended version with both minimal down-time and trajectories.

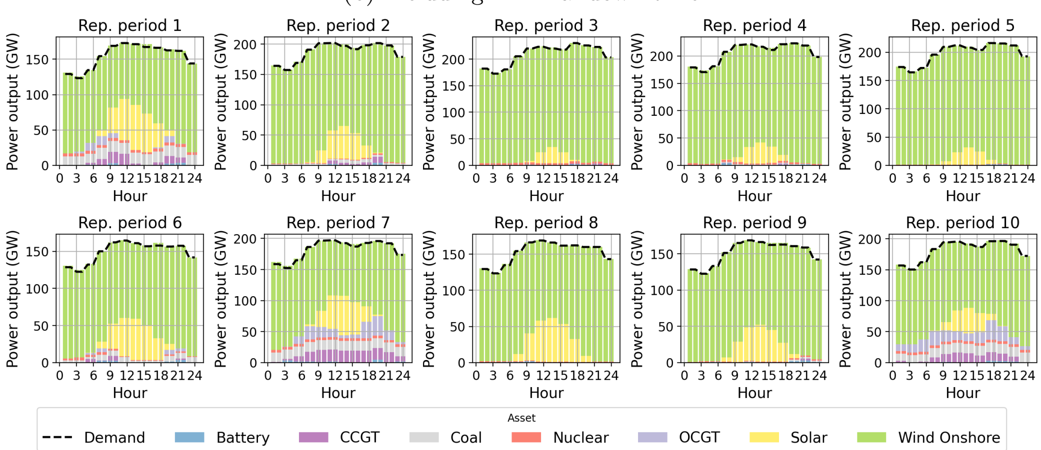
B.3 Operational Schedules



(a) Existing model

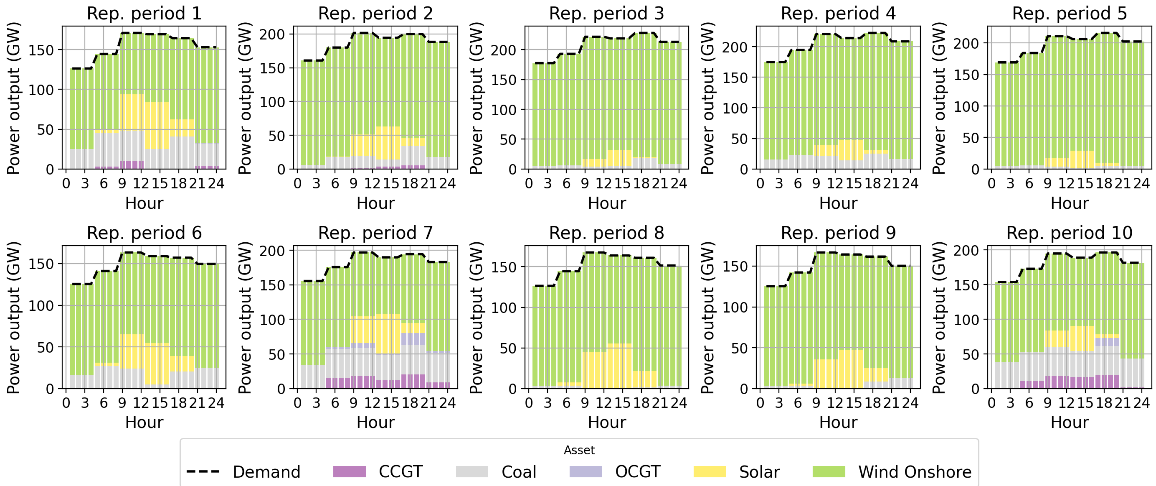


(b) Including minimal down-time

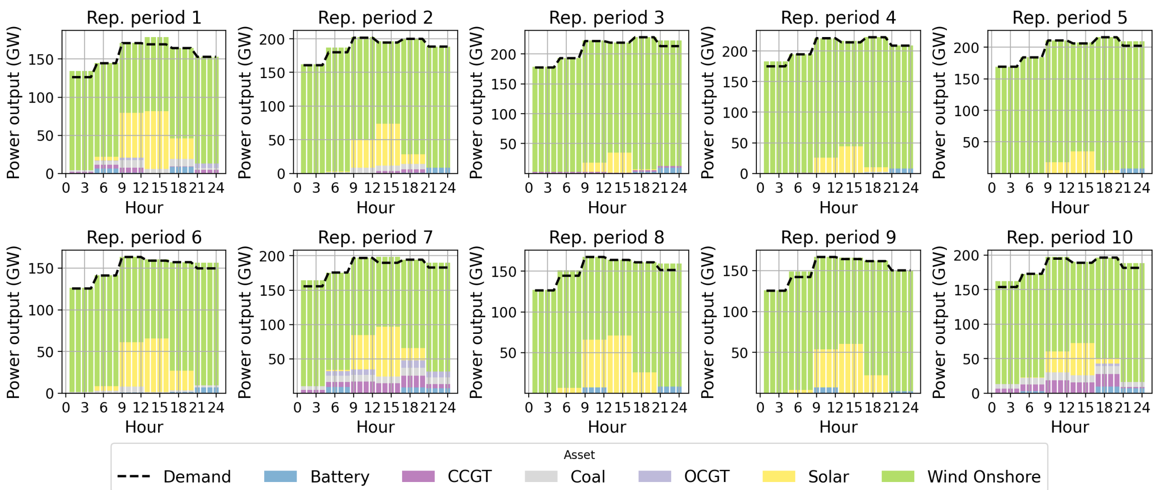


(c) Including trajectories

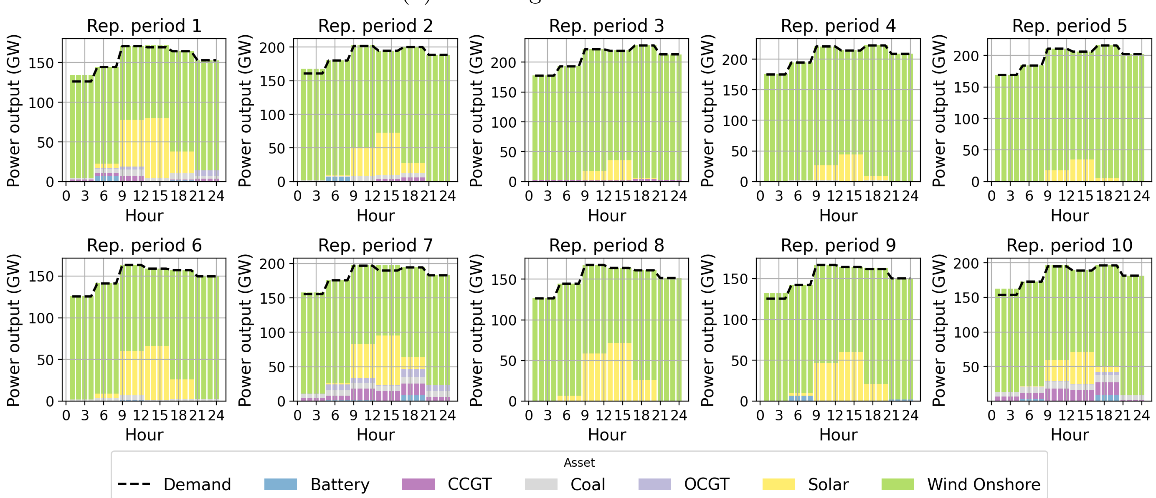
Figure 10: Comparison of the model operational schedules for 2-hourly resolution for each variant.



(a) Existing model

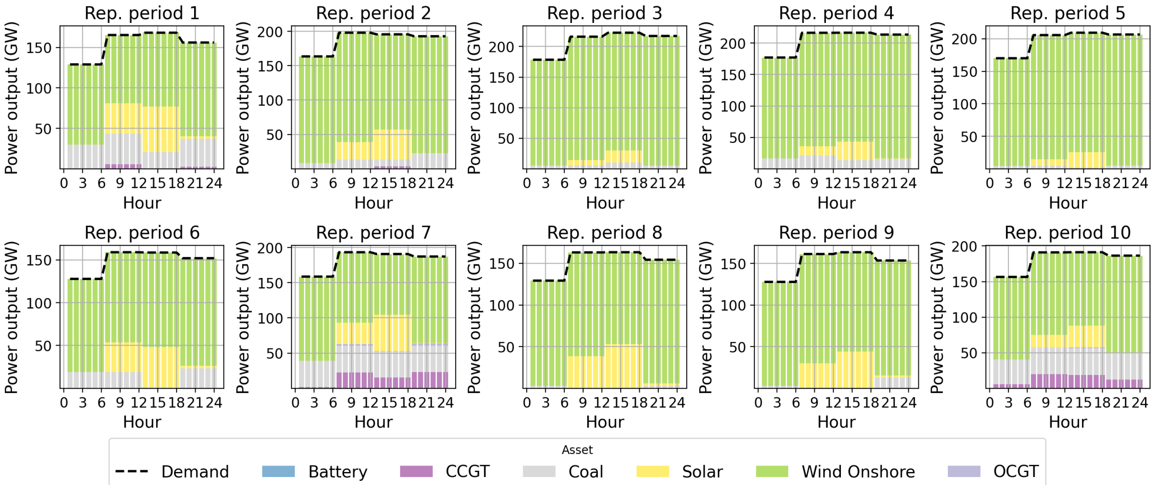


(b) Including minimal down-time

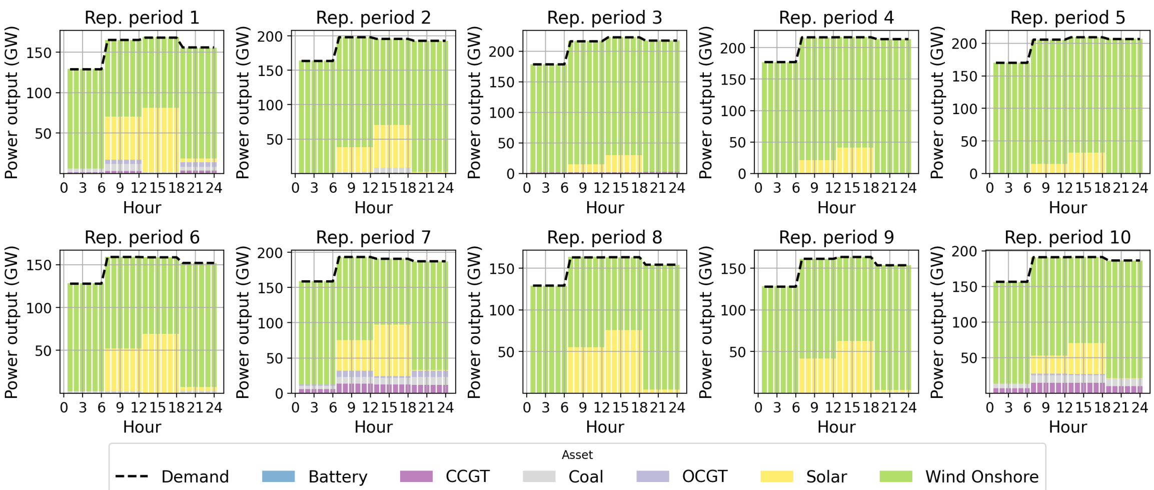


(c) Including trajectories

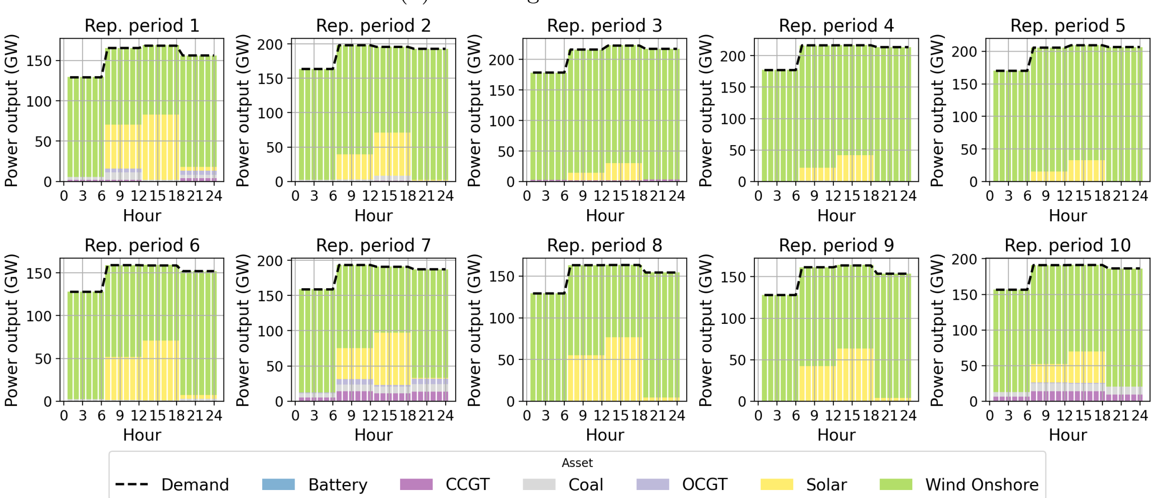
Figure 11: Comparison of the model operational schedules for 4-hourly resolution for each variant.



(a) Existing model



(b) Including minimal down-time



(c) Including trajectories

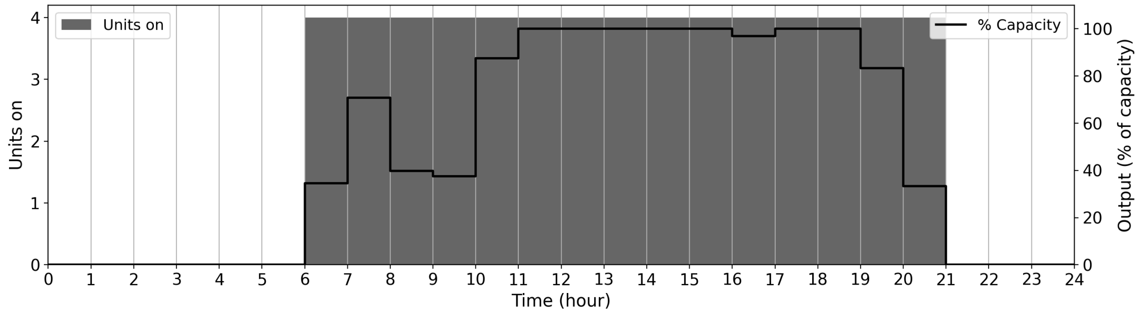
Figure 12: Comparison of the model operational schedules for 6-hourly resolution for each variant.

C Numerical results

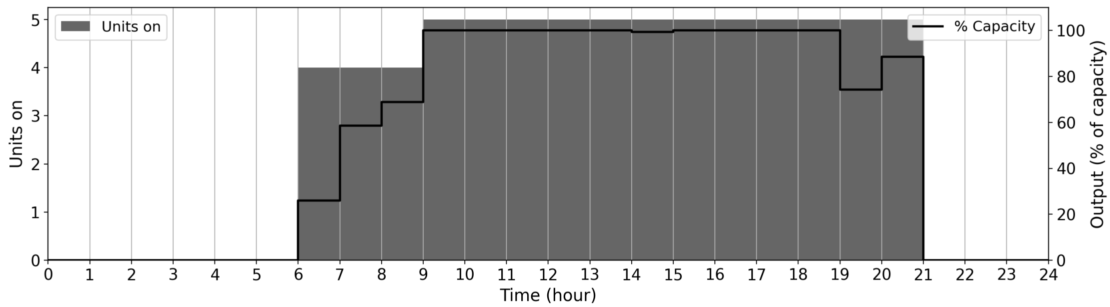
Resolution	Mean	Std. Dev.	Obj. Func. Value	Δ% from Basic	Δ% from Min. Down-time
1-Hourly Basic	17.00	6.79	29,496,059	0	-
1-Hourly Min. Down-time	34.27	9.77	29,712,402	0.73	0
1-Hourly Trajectory	264.32	47.82	29,815,367	1.08	0.35
2-Hourly Basic	4.90	2.96	29,540,197	0	-
2-Hourly Min. Down-time	7.17	2.68	30,811,312	4.30	0
2-Hourly Trajectory	37.00	9.86	30,838,601	4.40	0.09
4-Hourly Basic	2.29	0.64	29,188,493	0	-
4-Hourly Min. Down-time	1.65	0.73	32,159,543	1.018	0
4-Hourly Trajectory	5.00	2.53	32,172,492	1.022	0.04
6-Hourly Basic	2.07	0.50	28,753,448	0	-
6-Hourly Min. Down-time	1.41	0.62	31,357,199	9.06	0
6-Hourly Trajectory	4.24	1.43	31,364,996	9.08	0.02

Table 2: Model solving times means (s) and standard deviations, objective values (kEUR), and percentage differences between cases.

D Example Trajectory In Experiments

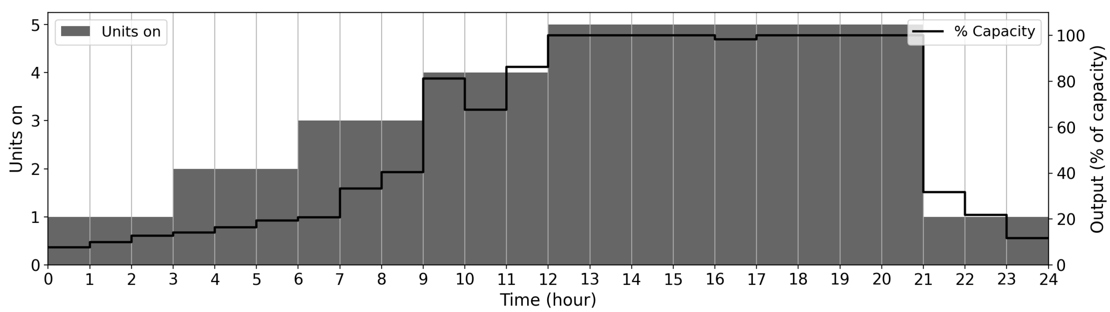


(a) Existing model



$v^{\text{units on}}_{\dots}$	0	0	4	5	5	5	5	0
$v^{\text{start up}}_{\dots}$	0	0	0	0	0	0	0	0
$v^{\text{shut down}}_{\dots}$	0	0	0	0	0	0	0	0

(b) Including minimal down-time



$v^{\text{units on}}_{\dots}$	1	2	3	4	5	5	5	1
$v^{\text{start up}}_{\dots}$	0	1	1	1	1	1	1	0
$v^{\text{shut down}}_{\dots}$	0	0	0	0	0	1	1	4

(c) Including trajectories

Figure 13: Output of a coal generator over 1 representative period with hourly resolution, for all variants of the model.

References

[1] Fadoua Chiba. "ENERGY TRANSITION: DIFFICULTIES, IMPLICATIONS AND SOLUTIONS". fr. PhD thesis. Universite de Bordeaux, Nov. 2018. URL: https://theses.hal.science/tel-02069926.

[2] "Net Zero by 2050 - A Roadmap for the Global Energy Sector". en. In: ().

[3] Jan Priesmann. "Are complex energy system models more accurate? An intra-model comparison of power system optimization models". en. In: (2019).

[4] Priyanka Laha and Basab Chakraborty. "Energy model – A tool for preventing energy dysfunction". en. In: *Renewable and Sustainable Energy Reviews* 73 (June 2017), pp. 95–114. ISSN: 13640321. DOI: 10.1016/j.rser.2017.01.106. URL: https://linkinghub.elsevier.com/retrieve/pii/S1364032117301193 (visited on 04/23/2025).

[5] Nikolaos E. Koltsaklis and Athanasios S. Dagoumas. "State-of-the-art generation expansion planning: A review". en. In: *Applied Energy* 230 (Nov. 2018), pp. 563–589. ISSN: 03062619. DOI: 10.1016/j.apenergy.2018.08.087. URL: https://linkinghub.elsevier.com/retrieve/pii/S0306261918312583 (visited on 04/25/2025).

[6] Zhi Gao et al. *Fully Flexible Temporal Resolution for Energy System Optimization*. en. Nov. 2025. DOI: 10.2139/ssrn.5214263.

[7] Rogier Hans Wuijts, Marjan Van Den Akker, and Machteld Van Den Broek. "Effect of modelling choices in the unit commitment problem". en. In: *Energy Systems* 15.1 (Feb. 2024), pp. 1–63. ISSN: 1868-3967, 1868-3975. DOI: 10.1007/s12667-023-00564-5. URL: https://link.springer.com/10.1007/s12667-023-00564-5 (visited on 04/23/2025).

[8] Behrang Shirizadeh. "Do multi-sector energy system optimization models need hourly temporal resolution? A case study with an investment and dispatch model applied to France". en. In: *Applied Energy* (2022).

[9] Laura C Brafine. "Exploring the Potential of Down-Sampling to Reduce Temporal Resolution in Energy System Optimisation Models". en. PhD thesis. Utrecht University, Sept. 2024.

[10] Holger Teichgraeber. "Time-series aggregation for the optimization of energy systems: Goals, challenges, approaches, and opportunities". en. In: *Renewable and Sustainable Energy Reviews* (2022).

[11] Diego A Tejada-Arango et al. "Enhanced Representative Days and System States Modeling for Energy Storage Investment Analysis". en. In: *IEEE Transactions on Power Systems* 33 (Nov. 2018), pp. 6534–6544.

[12] Pascale Bendotti, Pierre Fouilhoux, and Cecile Rottner. "The Min-up/Min-down Unit Commitment polytope". en. In: *Journal of Combinatorial Optimization* 36 (Nov. 2016), pp. 1024–1058.

[13] C Gentile, G Morales-España, and A Ramos. "A Tight MIP Formulation of the Unit Commitment Problem with Start-up and Shut-down Constraints". en. In: *EURO Journal on Computational Optimization* 5.1-2 (Aug. 2014), pp. 177–201. DOI: 10.1007/s13675-016-0066-y.

[14] Pelin Damcı-Kurt et al. “A polyhedral study of production ramping”. en. In: *Mathematical Programming* 158.1-2 (July 2016), pp. 175–205. ISSN: 0025-5610, 1436-4646. DOI: 10.1007/s10107-015-0919-9. URL: http://link.springer.com/10.1007/s10107-015-0919-9 (visited on 04/23/2025).

[15] German Morales-Espana, Jesus M. Latorre, and Andres Ramos. “Tight and Compact MILP Formulation of Start-Up and Shut-Down Ramping in Unit Commitment”. en. In: *IEEE Transactions on Power Systems* 28.2 (May 2013), pp. 1288–1296. ISSN: 0885-8950, 1558-0679. DOI: 10.1109/TPWRS.2012.2222938. URL: http://ieeexplore.ieee.org/document/6365287/ (visited on 04/23/2025).

[16] B. Palmintier and M. Webster. “Impact of unit commitment constraints on generation expansion planning with renewables”. en. In: *2011 IEEE Power and Energy Society General Meeting*. Journal Abbreviation: 2011 IEEE Power and Energy Society General Meeting. July 2011, pp. 1–7. ISBN: 1944-9925. DOI: 10.1109/PES.2011.6038963.

[17] Bryan S. Palmintier and Mort D. Webster. “Heterogeneous Unit Clustering for Efficient Operational Flexibility Modeling”. en. In: *IEEE Transactions on Power Systems* 29.3 (May 2014), pp. 1089–1098. ISSN: 0885-8950, 1558-0679. DOI: 10.1109/TPWRS.2013.2293127. URL: http://ieeexplore.ieee.org/document/6684593/ (visited on 04/25/2025).

[18] Jelle Meus, Kris Poncelet, and Erik Delarue. “Applicability of a Clustered Unit Commitment Model in Power System Modeling”. en. In: *IEEE Transactions on Power Systems* 33.2 (Mar. 2018), pp. 2195–2204. ISSN: 0885-8950, 1558-0679. DOI: 10.1109/TPWRS.2017.2736441. URL: https://ieeexplore.ieee.org/document/8003295/ (visited on 04/24/2025).

[19] Maren Ihlemann. “SpineOpt: A flexible open-source energy system modelling framework”. en. In: *Energy Strategy Reviews* (2022).

[20] Salvador Pineda and Juan M. Morales. “Chronological Time-Period Clustering for Optimal Capacity Expansion Planning With Storage”. en. In: *IEEE Transactions on Power Systems* 33.6 (Nov. 2018), pp. 7162–7170. ISSN: 0885-8950, 1558-0679. DOI: 10.1109/TPWRS.2018.2842093. URL: https://ieeexplore.ieee.org/document/8369128/ (visited on 04/29/2025).

[21] Diego A. Tejada-Arango et al. *Tulipa Energy Model: Mathematical Formulation*. en. arXiv:2309.07711 [math]. Sept. 2023. DOI: 10.48550/arXiv.2309.07711. URL: http://arxiv.org/abs/2309.07711 (visited on 04/30/2025).

[22] Abel Soares Siqueira et al. *Tulipa Energy Model*. Apr. 2025. DOI: 10.5281/ZENODO.8363262. URL: https://zenodo.org/doi/10.5281/zenodo.8363262 (visited on 04/23/2025).

[23] Tiziano Bacci et al. “New MINLP Formulations for the Unit Commitment Problems with Ramping Constraints”. en. In: (). DOI: 10.1287/opre.2023.2435.

[24] Yurian Lagrand. *RP Data*. Oct. 2025. URL: https://github.com/Cerberus22/RP_Data (visited on 06/10/2025).

[25] *Global Power Plant Database - Datasets*. en. URL: https://datasets.wri.org//datasets/global-power-plant-database (visited on 05/31/2025).

[26] *technology-data/inputs/EWG_ costs.csv at master · PyPSA/technology-data*. en. URL: https://github.com/PyPSA/technology-data/blob/master/inputs/EWG_costs.csv (visited on 05/31/2025).

[27] *pypsa-eur/data/unit_ commitment.csv at master · PyPSA/pypsa-eur*. en. URL: https://github.com/PyPSA/pypsa-eur/blob/master/data/unit_commitment.csv (visited on 06/10/2025).

[28] *IEA-ETSAP | Energy Supply Technologies Data*. URL: https://iea-etsap.org/index.php/energy-technology-data/energy-supply-technologies-data (visited on 05/31/2025).

[29] Gabriel Tertelici. *Effect of Minimum Up and Down Time Constraints with Fully Flexible Temporal Resolutions*.

[30] Iain Dunning, Joey Huchette, and Miles Lubin. "JuMP: A Modeling Language for Mathematical Optimization". en. In: *SIAM Review* 59.2 (Jan. 2017), pp. 295–320. ISSN: 0036-1445, 1095-7200. DOI: 10.1137/15M1020575. URL: https://epubs.siam.org/doi/10.1137/15M1020575 (visited on 06/02/2025).

[31] Miles Lubin et al. "JuMP 1.0: recent improvements to a modeling language for mathematical optimization". en. In: *Mathematical Programming Computation* 15.3 (Sept. 2023), pp. 581–589. ISSN: 1867-2949, 1867-2957. DOI: 10.1007/s12532-023-00239-3. URL: https://link.springer.com/10.1007/s12532-023-00239-3 (visited on 06/02/2025).

[32] LLC Gurobi Optimization. *Gurobi Optimizer Reference Manual*. 2024. URL: https://www.gurobi.com.

[33] J.M. Arroyo and A.J. Conejo. "Modeling of Start-Up and Shut-Down Power Trajectories of Thermal Units". en. In: *IEEE Transactions on Power Systems* 19.3 (Aug. 2004), pp. 1562–1568. ISSN: 0885-8950. DOI: 10.1109/TPWRS.2004.831654. URL: http://ieeexplore.ieee.org/document/1318694/ (visited on 04/23/2025).