MASTER THESIS

Prediction of the nonlinear dynamic behaviour of a concrete slab subjected to blast load

Appendix IV – Analytical solution to the SDOF mass-spring system

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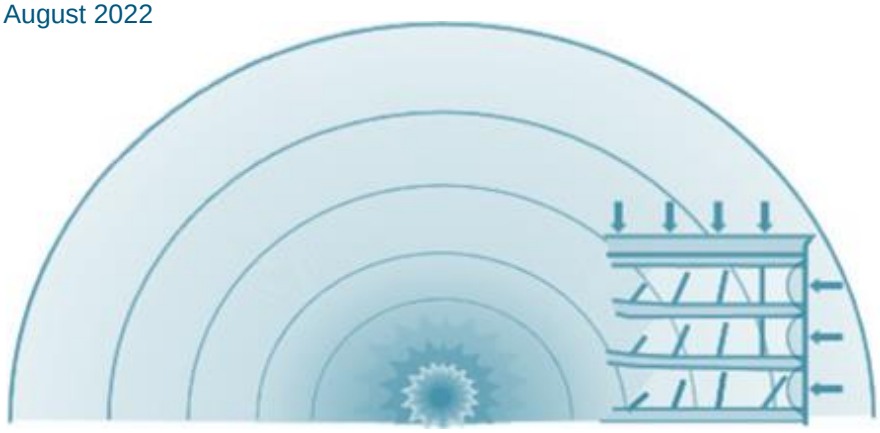
Status: Final

Date: 23 August 2022



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Document title: Prediction of the nonlinear dynamic behaviour of a concrete slab subjected to blast load

Subtitle: Appendix IV – Analytical solution to the SDOF mass-spring system

Status: 1.0/Final

Date: 23 August 2022

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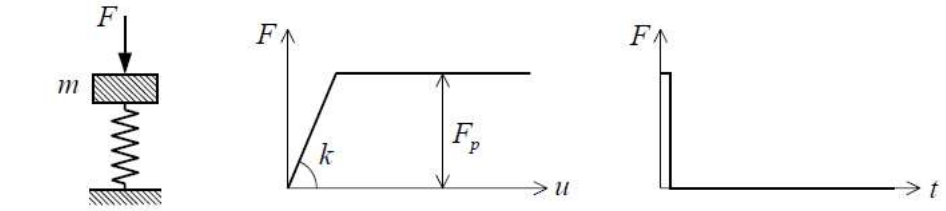
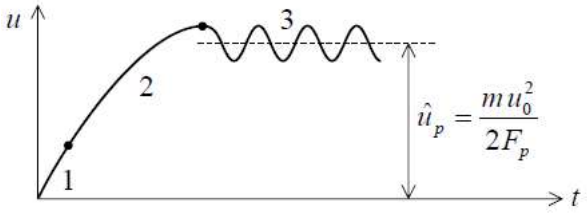
restart;
with(*plots*) :





curve 1 : $m\ddot{u} + k u = 0$

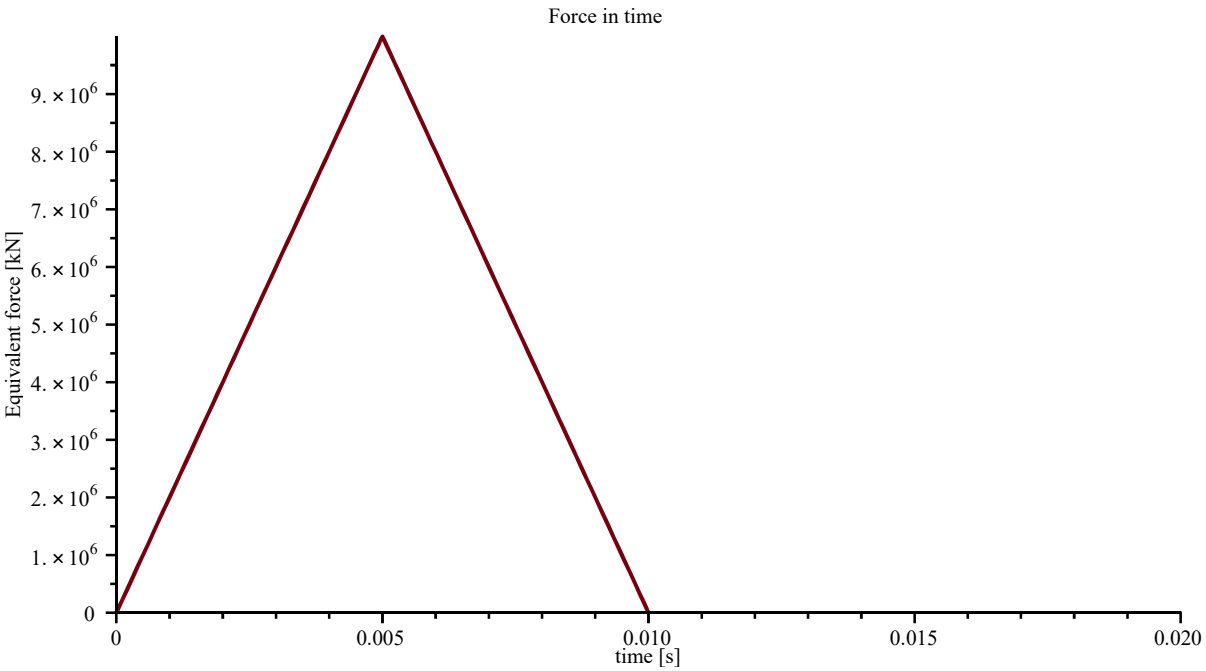
curve 2 : $m\ddot{u} + F_p = 0$

curve 3 : $m\ddot{u} + k\left(u - \hat{u}_p\right) = 0$

Blast force

$F_0 := 1e7 : t_0 := 0 : t_1 := 0.005 : t_2 := 0.010 : t_{max} := 1 :$

$$Force := \begin{cases} 0 & t \le 0 \\ \frac{F_0\, t}{t_1} & 0 < t \le t_1 \\ 2\,F_0 - \frac{2\,F_0\, t}{t_2} & t_1 < t \le t_2 \\ 0 & t2 < t \end{cases} : plot\left(Force, t = 0 .. 2\cdot t_2, labels = [\text{"time [s]"}, \text{"Equivalent force [kN]"}], labeldirections = [\text{"horizontal"}, \text{"vertical"}], title = \text{"Force in time"}\right)$$



Curve 1 equations

$$u_1(t) := \frac{1}{m\,\omega_D} \int_{t_0}^{t} \left(\frac{F_0 \cdot \tau}{t_1}\right) \cdot e^{-\xi\,\omega_n \cdot (t-\tau)} \cdot \sin\left(\omega_D \cdot (t-\tau)\right) d\tau :$$

$$u_2(t) := \frac{1}{m\,\omega_D}\int_{t_0}^{t_1}\left(\frac{F_0\cdot\tau}{t_1}\right)\cdot e^{-\xi\,\omega_n\cdot(t-\tau)}\cdot\sin\left(\omega_D\cdot(t-\tau)\right)\,d\tau + \frac{1}{m\,\omega_D}\int_{t_1}^{t}\left(2\,F_0-\frac{2\,F_0\,\tau}{t_2}\right)\cdot e^{-\xi\,\omega_n\cdot(t-\tau)}\cdot\sin\left(\omega_D\cdot(t-\tau)\right)\,d\tau :$$

$$u3(t) := \frac{1}{m\,\omega_D}\int_{t_0}^{t_1}\left(\frac{F_0\cdot\tau}{t_1}\right)\cdot e^{-\xi\,\omega_n\cdot(t-\tau)}\cdot\sin\left(\omega_D\cdot(t-\tau)\right)\,d\tau + \frac{1}{m\,\omega_D}\int_{t_1}^{t_2}\left(2\,F_0-\frac{2\,F_0\,\tau}{t_2}\right)\cdot e^{-\xi\,\omega_n\cdot(t-\tau)}\cdot\sin\left(\omega_D\cdot(t-\tau)\right)\,d\tau :$$

$$v_1(t) := \frac{d}{d\,t}u_1(t) : v_2(t) := \frac{d}{d\,t}u_2(t) : v_3(t) := \frac{d}{d\,t}u3(t) : a_1(t) := \frac{d}{d\,t}v_1(t) : a_2(t) := \frac{d}{d\,t}v_2(t) : a_3(t) := \frac{d}{d\,t}v_3(t) :$$

Curve 2 equations

$$EoM := m\cdot\frac{d^2}{d\,t^2}u4(t) + F_{pl} = 0 : dsol := dsolve(EoM); assign(dsol) :$$

$$dsol := u4(t) = -\frac{F_{pl}\,t^2}{2\,m} + \_C1\,t + \_C2 \qquad (1)$$

$$v_4(t) := \frac{d}{d\,t}u4(t) : a_4(t) := \frac{d}{d\,t}v_4(t) :$$

$$BC1 := eval\left(u4(t), t = t_{pl}\right) = eval\left(u3(t), t = t_{pl}\right) : BC2 := eval\left(diff\left(u4(t), t\right), t = t_{pl}\right) = eval\left(diff\left(u3(t), t\right), t = t_{pl}\right) : sol := solve\left(\{BC1, BC2\}, \{\_C1, \_C2\}\right) : assign(sol) :$$

Curve 3 equations

$$u_5(t) := u_{pl} + e^{-\xi\,\omega_n\cdot\left(t - t_{pl,\,max}\right)}\cdot\left(u_{5,\,0}\cdot\cos\left(\left(t - t_{pl,\,max}\right)\cdot\omega_D\right)\right) :$$

$$v_5(t) := \frac{d}{d\,t}u_5(t) : a_5(t) := \frac{d}{d\,t}v_5(t) :$$

Example

$$\xi := 0.00 :$$

$$\omega_D := \omega_n\cdot\sqrt{1-\xi^2} :$$

$$\omega_n := 10.58287382 : m := 8\cdot0.5\cdot\frac{625}{2}\cdot0.25\cdot10\cdot10 : u_{pl} := 0.035 : F_{pl} := 123920 : t_{pl} := solve\left(u3(t) = u_{pl}\right); u_{5,\,0} := eval\left(subs\left(t = t_{pl}, u3(t)\right)\right); t_{pl,\,max} := eval\left(solve\left(v_4(t) = 0, t\right)\right); u_{pl} := subs\left(t = t_{pl,\,max}, eval(u4(t))\right) - eval\left(subs\left(t = t_{pl}, u3(t)\right)\right);$$

$$t_{pl} := 0.02708050451$$

$$u_{5,\,0} := 0.035000003$$

$$t_{pl,\,max} := 0.4195091653$$

$$u_{pl} := 0.3053393810 \qquad (2)$$

$simplify(evalf(u_1(t)))$; $simplify(evalf(u_2(t)))$; $simplify(evalf(u3(t)))$; $simplify(eval(u4(t)))$; $simplify(evalf(u_5(t)))$;

$$-53.99693635\sin(10.58287382\,t) + 571.4427641\,t$$

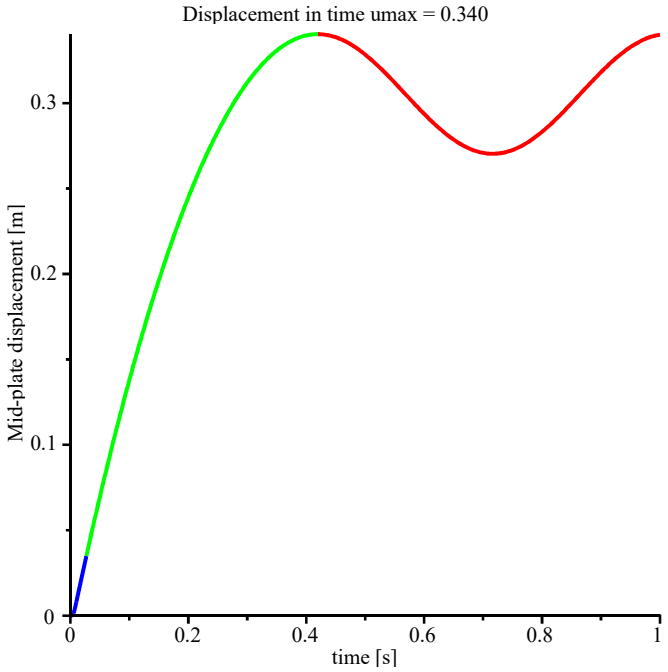
$$53.84578395\sin(10.58287382\,t) - 5.711761347\cos(10.58287382\,t) + 5.714427641 - 571.4427641\,t$$

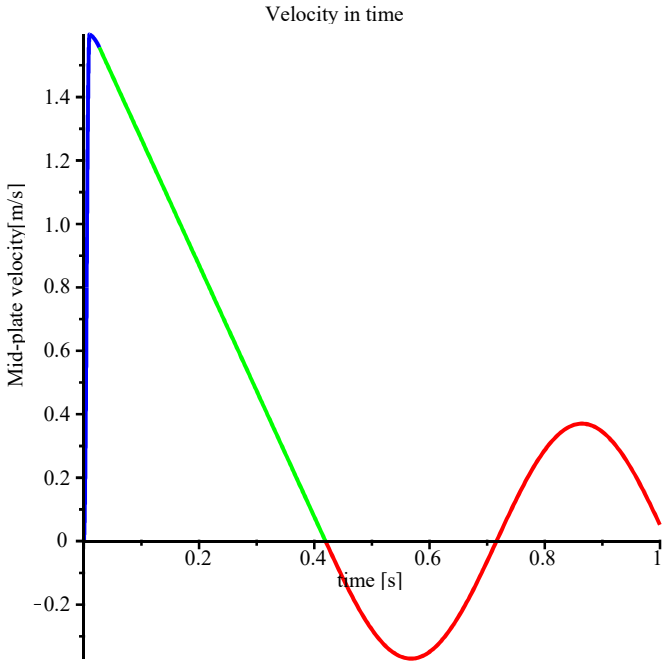
$$0.1509408\sin(10.58287382\,t) - 0.007994403\cos(10.58287382\,t)$$

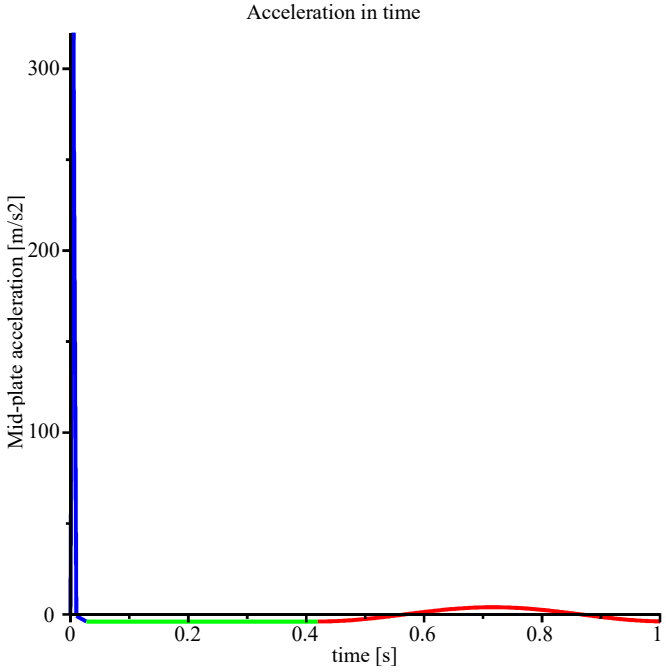
$$-1.982720000\,t^2 + 1.663538425\,t - 0.008595424120$$

$$0.3053393810 + 0.035000003\cos(10.58287382\,t - 4.439612563) \quad \textbf{(3)}$$

$plotu1 := plot(u_1(t), t = 0..t_1, color = 'blue', title = ["Displacement in time umax = 0.340"], labels = ["time [s]", "Mid-plate displacement [m]"], labeldirections = ["horizontal", "vertical"])$: $plotu2 := plot(u_2(t), t = t_1..t_2, color = 'blue')$: $plotu3 := plot(u3(t), t = t_2..t_{pl}, color = 'blue')$: $plotu4 := plot(u4(t), t = t_{pl}..t_{pl,\,max}, color = 'green')$: $plotu5 := plot(u_5(t), t = t_{pl,\,max}..t_{max}, color = 'red')$: $display(plotu1, plotu2, plotu3, plotu4, plotu5)$; $plotv1 := plot(v_1(t), t = 0..t_1, color = 'blue', title = ["Velocity in time"], labels = ["time [s]", "Mid-plate velocity[m/s]"], labeldirections = ["horizontal", "vertical"])$: $plotv2 := plot(v_2(t), t = t_1..t_2, color = 'blue')$: $plotv3 := plot(v_3(t), t = t_2..t_{pl}, color = 'blue')$: $plotv4 := plot(v_4(t), t = t_{pl}..t_{pl,\,max}, color = 'green')$: $plotv5 := plot(v_5(t), t = t_{pl,\,max}..t_{max}, color = 'red')$: $display(plotv1, plotv2, plotv3, plotv4, plotv5)$; $plota1 := plot(a_1(t), t = 0..t_1, color = 'blue', title = ["Acceleration in time"], labels = ["time [s]", "Mid-plate acceleration [m/s2]"], labeldirections = ["horizontal", "vertical"])$: $plota2 := plot(a_2(t), t = t_1..t_2, color = 'blue')$: $plota3 := plot(a_3(t), t = t_2..t_{pl}, color = 'blue')$: $plota4 := plot(a_4(t), t = t_{pl}..t_{pl,\,max}, color = 'green')$: $plota5 := plot(a_5(t), t = t_{pl,\,max}..t_{max}, color = 'red')$: $display(plota1, plota2, plota3, plota4, plota5)$;




Velocity in time
Mid-plate velocity[m/s]
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
-0.2
0.2
0.4
0.6
0.8
1
time [s]


Acceleration in time
Mid-plate acceleration [m/s2]
300
200
100
0
0
0.2
0.4
0.6
0.8
1
time [s]