Development of a field hockey stick with adaptive properties

Identification of influential stick properties and a design of an adaptive mechanism

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by

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Preface

Sports have always played an important role in my life; besides the enjoyment of exercising, I am interested in the improvement of sports performance. I have played field hockey for many years and therefore have experienced the feel and performance of different sticks; moreover, I believe that the benefits of exercising and playing in a team are very valuable. Therefore, when Arno discussed this project with me, I was enthusiastic and motivated to work on this project.

I want to thank Arno for his supervision of this project, his enthusiasm and guidance encouraged me during my thesis. I would also like to thank Wouter for his enthusiasm and involvement in this project, and Jonathan and Karien for their help during the measurements. And I would like to express my gratitude to Winfred Mugge and Freek Broeren for their commitment to take part in my graduation committee.

Lastly, I would like to thank my friends and family not only for their support during my thesis, but also for making my years as a student in Delft fun and special.

M.R. Overweel
Delft, February 2025

Abstract

Field hockey sticks are interesting hand-held sports equipment, due to their duality in preferred stick behaviour. The stick is used for striking, where a high power is desired; but also for stopping, where good control is required. This report aims *to identify the properties that influence stick performance and to design a field hockey stick with adaptable properties to improve stick performance*; where performance is defined as *the ability of the stick to develop a high velocity when hitting a ball, and to provide proper control when stopping the ball*.

The stiffness, damping and mass of the stick are properties influencing the stick behaviour; these properties are present locally, at the impact location, as well as over the full length as deflective properties due to the moment originating from the ball impact.

The deflective stiffness and damping properties are identified by applying a disturbance force on the stick tip and measuring the displacement; this shows a range of stick stiffness from 1.4 to $3.0 kN/m$ and a stick damping from 0.5 to $2.7 Ns/m$. Measurements are performed analysing the influence of stiffness, damping, mass and effective mass; this is done by a setup where a stick falls down towards a ball and the ball distance is measured. Additionally, a mathematical model is developed for the analysis of these stick properties. This consists of a collision model, including the coefficient of restitution reflecting the stiffness and damping properties. It can be concluded that the effective stick mass is most influential and the desired properties are opposite for striking and stopping.

A design of a mechanism that fits inside the stick is proposed, this mechanism reacts to the angular acceleration of the stick, and thereby changes its properties between striking and stopping a ball. It adapts the effective mass of the stick, by two weights moving towards the head of the stick when striking a ball. By this increase in effective stick mass, an increase of 7% (compared to the original effective mass) of the ball velocity after hitting the ball is expected.

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1 Introduction

The field hockey game shows an interesting controversy in the desired mechanical behaviour of the hockey stick, as it is used not only to strike the ball but also to stop and control the ball (Allen et al., 2012). Therefore, the player desires a stick with high power to obtain a high ball velocity when striking; and at the same time, the stick should allow the player to stop and control the ball when receiving it. Consequently, the desired characteristics of the stick change based on the type of action for which it is used.

However, it is not only a stick - ball interaction, the human is also involved. Experienced players developed skills to overcome this controversy in desired stick characteristics; by changing their human characteristics, the stick - ball contact behaves as desired. However, for beginners it can be difficult to develop these skills; even more so for adult beginners, as they play with experienced team members.

It is well known that sports improve people's well-being (Iulian-Doru and Maria, 2013), team sports are even more beneficial because they positively influence the continuation and participation in sports and improve the social and mental health of individuals (Andersen et al., 2019); therefore, it is important to enable and stimulate participation in team sports. However, it can be difficult for inexperienced adults to join a team sport with experienced players. Especially field hockey shows difficulties, as it requires technical abilities of the player.

Beginners in field hockey are advised to play with a stick with low stiffness, sometimes a wooden stick or a stick with a low carbon percentage. These sticks make it easier to stop and control the ball; however, they have low power when striking the ball (KNHB, 2023). This complicates the participation of inexperienced adults in a team of experienced players. It would be beneficial if the stick had both properties: high power when hitting and good control when stopping. However, this requires a stick with adjustable behaviour; and it is not yet clear which properties would be most important to change the stick characteristics. The variation in stick behaviour is visually displayed in the performance index by Wibbens (Wibbens, 2015) in Figure 1.1; where the trade-off between handling and hitting and the distinction between beginners and experienced players is shown. The interesting part of this graph is the top right part, where the stick would be optimal for both handling and hitting.

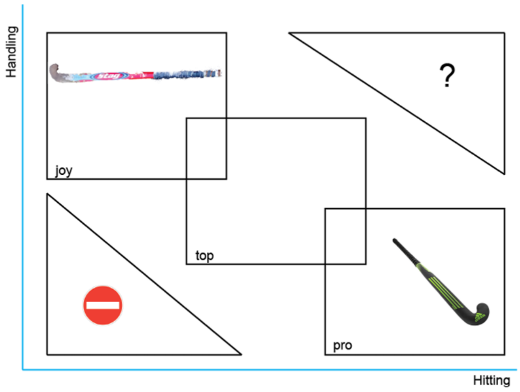


Figure 1.1: Trade-off in field hockey sticks and the distinction between beginners (joy) and experienced (top / pro) players (Wibbens, 2015)

Many research has been done on sports equipment like ice hockey sticks, tennis rackets, baseball bats and golf clubs; however, these sports only require high power and stopping the ball is not important. Research into field hockey sticks is scarce, and therefore research into the sports equipment mentioned above is needed to get a grip on the important properties of a field hockey stick.

The goal of this project is *to identify the properties that influence stick performance and to design a field hockey stick with adaptable properties to improve stick performance.* Stick performance is defined as *the ability of the stick to develop a high ball velocity when hitting a ball, and to provide proper control when stopping the ball*. Hence, the adaptable stick will minimise the trade-off between hitting power and stopping control of a hockey stick. Obviously, field hockey includes more actions than only hitting and stopping, but the biggest difference in desired stick behaviour is shown for these actions; therefore, these are used to develop the adaptable design.

To obtain this goal, the following questions are answered:

- What properties influence stick performance?
- What properties have the greatest influence on stick performance?
- How can these properties be adapted in a regular field hockey stick during the game?

The next chapter will describe the background needed to design an adjustable field hockey stick. It includes a summary of the game, an overview of interesting stick properties (taken from research into field hockey sticks as well as other sports equipment) and an analysis of the evolution of the stick over the years. The background is used to get an overview of the properties that would be of most interest in the design of an adjustable hockey stick. The subsequent chapter shows an experimental analysis of several hockey sticks and the influence of their properties, used to determine which property is most efficient to become adjustable during the game. This analysis is followed by a similar theoretical analysis, which uses a mathematical model instead of experiments to determine the most influential properties. The last chapter describes the design of the adjustable hockey stick. The report ends with a discussion and conclusion.

2 Background

2.1. Field hockey

Since its debut at the Olympics more than 100 years ago (Allen et al., 2012), field hockey has developed into a fast sport in which the stick plays an important role. The stick is used to pass, score, receive and conquer the ball during the game; different techniques are used to perform these actions. Passing can be done by pushing (the stick and the ball are in contact during the push, and they both are in contact with the ground); sweeping (a swinging movement of the stick while staying in contact with the ground); striking (a sweeping upward movement, no contact with the ground); or scooping (lifting the ball of the ground). The same techniques can be used for scoring; in addition, the drag flick is used (the ball is raised by a dragging movement, while the stick and ball are in contact) (KNHB, 2023). For receiving and conquering the ball, several techniques can be used, for which control of the ball and the stick plays an important role.

The stick head has a flat and a rounded side, where it is only allowed to play with the flat side. Therefore, the actions mentioned above can be played with the forehand and most of them can also be played with the backhand.

The actions played during the game require a stick with high power and proper stick and ball control, meaning control of ball placement while hitting the ball, as well as ball control when receiving the ball. The variation in the game creates a situation in which the desired stick behaviour and thereby its properties, differ for different actions while playing (Allen et al., 2012).

This control and power improve as the player gets more experienced and depend on the stick properties. The combination of the player and the stick should be optimal for the best performance; and it depends on several variables, like the player's position, length, weight, age and experience. The available sticks vary in shape, weight, weight distribution, stiffness, length, curvature and material (KNHB, 2023).

The properties of the stick are limited by the rules set by the International Hockey Federation (FIH). The following is a summary of the most important rules for this project:

- The stick can be made of any material, except metal.
- The shape of the stick is specified and has a maximum length of $1050mm$.
- The maximum diameter of the stick is $51mm$.
- The maximum weight of the stick is $737g$.
- The entire stick should be smooth (with smooth meaning that it cannot have rough or sharp parts).
- The maximum ball speed after striking is set at 98% of the initial stick head speed.

The FIH rules do not specify the adaptability of the stick during the game (FIH, 2023); interestingly, the Tennis Federation has stated that the use of adjustable rackets is not allowed (KNLTB, 2024). However, the FIH rules do specify the prohibition of metal in the stick; therefore, an adaptable stick has to work fully mechanically, without any electrical components.

2.2. Field hockey stick properties

2.2.1. Local and bending properties

The literature on the properties of field hockey sticks is scarce; therefore, this research takes into account comparable sports equipment, namely golf clubs, tennis rackets, baseball bats and ice hockey sticks. However, it is important to mention that these sports and consequently the literature focus on striking a ball, so only the power property mentioned before is taken into account. The interesting feature of field hockey sticks is the duality in

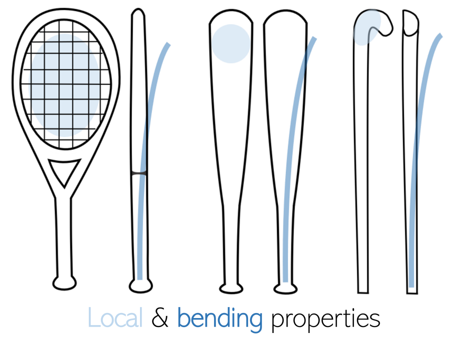


Figure 2.1: Definition of local and bending properties of hand held equipment - front and side view of a tennis racket, baseball bat and field hockey stick

	Influence on power	Influence on control
	Bending	
Stiffness	Positive	Unknown
Damping	Unknown	Unknown
Total mass	Positive - limited	Negative
	Local	
Stiffness	Negative	Unknown
Damping	Unknown	Unknown
Effective mass	Positive - limited	Negative
	Shape and dimensions	
Width	Positive - limited	Unknown
Shaft curvature	Unknown	Positive - limited

Table 2.1: Stick properties and their influence on striking power and stopping control - a summary of literature on hand-held equipment (where the influence describes the effect of increasing the parameter)

power and handling control, which is not reflected in most of the literature. Therefore, an adjustment in the properties of other hand-held sports equipment may have a positive effect on the power performance of a hockey stick, but the (possibly negative) effect on the handling control is not taken into account.

In the description of the influential properties, there is a distinction between the local and deflection properties. The local properties describe the properties of the location of impact between the ball and the stick / racket / club / bat, without taking into account the properties over the full length of the equipment. The deflection properties take into account the behaviour of the stick due to the moment applied on the stick during the impact (see Figure 2.1).

In the following sections, the properties and their influence on hitting power and stopping control are described, an overview is shown in Table 2.1.

2.2.2. Stiffness and damping

Literature concludes that the deflective stiffness of hand-held equipment has a small positive influence on the velocity of the outgoing ball after a hit (Betzler et al., 2012, Carlisle, 2011, Kawazoe, 2002, Covill et al., 2009 & Wibbens, 2015). This is mainly due to the decrease in energy loss when using stiffer equipment, as less energy is put into deformation (Miller, 2006) and unwanted vibrations (Allen et al., 2015) of the hand-held implement. However, this influence is small and almost zero for impacts located at the node (Nicholls et al., 2004 & Allen et al., 2015) (a perfect hit is when the impact location is at the sweet spot, where the output velocity is highest); and it has a greater influence for off-centre hits.

Most of the literature does not take into account the effect of the deflective stiffness of the stick on the handling control. However, it has become clear that a higher stiffness decreases energy loss and develops a higher ball velocity after an impact between the ball and the stick. For stopping, this is unwanted, as the energy has to be taken out of the ball and the ball and the stick should clamp to each other instead of bouncing off each other. Therefore, bending stiffness has a negative effect on stick handling control (Carlisle, 2011).

To compensate for this decrease in handling control, a higher deflective damping may be used. However, most literature does not include damping properties and no experimental research is done on this property. Nevertheless, in his research, Wibbens (Wibbens, 2015) suggests that higher deflective damping would be beneficial for handling control.

In addition to bending stiffness and damping properties, local stiffness and damping properties are also of interest. The local stiffness in a tennis racket, which is the stiffness of the string bed, has a negative impact on the ball velocity (Allen et al., 2015 & Miller, 2006). With a lower stiffness, more energy is transferred into the string bed, and the strings are more energy efficient and therefore return more of their absorbed energy into kinetic energy of the ball (Allen et al., 2015). This is also reflected in the core of a baseball bat, since a hollow implement shows more elasticity which gives a higher bounce to the outgoing ball (Cross, 2013). Although Miller (Miller, 2006) describes that a higher local stiffness improves hitting control, the impact on handling control is unclear.

Local damping is assumed to be beneficial for handling control; however, no literature is found on the influence of this property.

2.2.3. Mass and mass distribution

A higher mass creates a higher momentum, and thereby can increase the outgoing ball speed (Allen et al., 2015 & Miller, 2006). However, a heavier stick is harder to accelerate and therefore the maximum stick speed is lower, which decreases the total momentum. To conclude, an increase in the total mass of a stick has a positive impact on the power of the stick; however, this only applies to a certain limit, after which the addition of mass will have a negative influence due to the decrease in maximum stick velocity (Nicholls et al., 2004). And when relating mass to handling control, literature states that accuracy decreases with increasing mass when hitting a ball (Allen et al., 2015).

The mass distribution also plays an important role, as a higher stick velocity can be achieved when the centre of mass moves more toward the handle (Miller, 2006). Moreover, mass distribution also determines the effective mass at the location of impact. The relation between the effective mass and outgoing ball speed is similar to that described for the total mass. Because with a constant total mass, a higher effective mass increases the momentum of the stick, but it decreases the velocity of the stick in a hit which decreases the momentum; therefore, the effective mass has a limited positive effect on power (Allen et al., 2015 & Nicholls et al., 2005). A higher tip mass also makes it more difficult to control the stick when handling it (Miller, 2006). The effective mass is based on the total mass of the implement and decreases as the distance between the centre of mass and the location of impact increases. It can be defined by the following formula:

$$M_{\text{e}} = \frac{M}{1 + M(\frac{d^2}{I_{\text{COM}}} + \frac{R^2}{I_{\text{y}}})} \tag{2.1}$$

With M being the total mass of the implement, d is the longitudinal distance between the centre of mass and the location of impact, I_{COM} is the moment of inertia at the centre of mass, R is the radial distance between the centre of mass and the location of impact and I_{y} is the polar moment of inertia.

2.2.4. Shape and dimensions

The curvature of the stick shaft, defined by the bow of the stick, can influence the control of the stick (Wibbens, 2015); this property will be described in Section 2.3. Dimensions also play an important role; an increase in width can have a positive relation to ball velocity for off-centre hits (Miller, 2006). And a larger contact surface improves handling control (Wibbens, 2015).

2.2.5. Coefficient of Restitution

All properties mentioned above influence the coefficient of restitution (COR), which can be used to describe the velocities of colliding masses. When simplifying the ball and the stick to point masses described by a collision model, the COR can be used to describe the velocities after collision by the following formula:

$$e = \frac{v_2 - V_2}{v_1 + V_1} \tag{2.2}$$

Where the v describes the ball velocity and V the stick velocity, before (1) and after (2) collision. And the collision model is described by:

$$M_{\text{e}}V_1 + mv_1 = M_{\text{e}}V_2 + mv_2 \tag{2.3}$$

Where M_{e} is the effective mass of the stick and m is the mass of the ball. Sticks with a lower bending stiffness cause many vibrations; thereby, energy is lost and the COR is decreased. However, an elastic and softer impact surface can decrease energy loss and increase the COR. Additionally, a heavier stick can increase the COR and the influence is even larger with more mass close to the location of impact (increasing M_{e}).

An important factor for the value of the COR is the location of impact, because the energy loss due to vibration and twisting is minimised at the sweet spot. In addition, the effective mass is higher than for off-centre hits (Cross, 2013).

2.3. Field hockey stick properties over the years

Field hockey sticks were originally made of wood. These sticks had a low stiffness and low power, and therefore changes in material were made to obtain higher ball velocities. Aluminium sticks were used for a short period, but they provided dangerous situations for other players and were prohibited by the FIH. This development was followed by a new material; carbon and glass reinforced polymers. The composite materials provided high power and the control and feel was better than for the aluminium sticks (Carlisle, 2011 & Wibbens, 2015). The composite stick is hollow, with a support structure inside (see Figure 2.2). Looking at the figure, it becomes clear that the moment of inertia of the stick changes when the backhand side is used for striking.

The shape of the stick also developed over the years. The head shape has become smaller than the original size (Figure 2.3), mainly due to developments of the field played on (Carlisle, 2011).

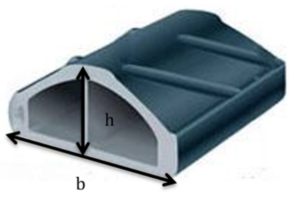


Figure 2.2: Cross section of a field hockey stick (Carlisle, 2011)

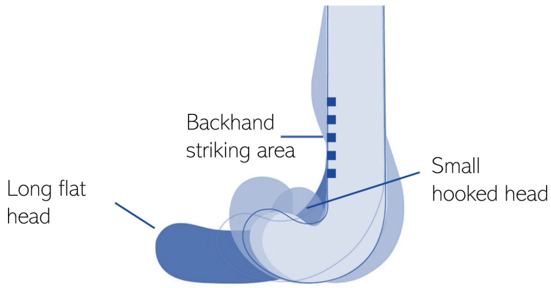


Figure 2.3: Development in head shape, from a long flat shape to a small hooked shape (Carlisle, 2011)

Recent developments in stick shape are shown in the bow of the stick, which can be located more to the head of the stick (low bow) or more towards the handle (high bow) (Wibbens, 2015), see Figure 2.4. The bow allows for more control and is more powerful in a drag flick.

2.4. Conclusion

The goal of this report is to identify interesting stick properties and design an adaptable field hockey stick; not all parameters discussed above are taken into account in the following chapters. The scope is limited to an adaptation of the stick while keeping the outer properties constant. So, the properties discussed about shape and dimensions are outside the scope of this project and the next chapters will focus on stiffness, damping and mass.

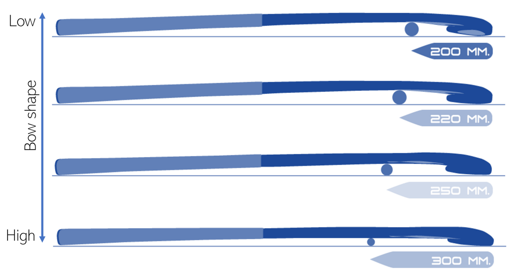


Figure 2.4: Range in bow shapes, where the maximum curvature can be more towards the head or more towards handle of the stick (Solo Hockey / ClubColors, n.d.)

3
Experimental Analysis

3.1. Introduction

This chapter experimentally analyses the influence of the stick properties on the stick behaviour, the goal of this analysis is to define what stick property has the greatest influence on the stick behaviour. The first step in this analysis is a numerical and comparable description of each stick, which is done in the first experiment and defines the mass, stiffness and damping properties of each stick. In the second step, the influence of these measured properties is tested by measuring the travelled ball distance after a stick-ball collision, this step concludes which property has the greatest influence.

The experiments consist of measurements of a set of hockey sticks, shown in Figure 3.1 and numbered 1 to 6 from top to bottom; Table 3.1 shows the properties of each stick. Four field hockey sticks and two indoor hockey sticks are included; the indoor sticks are used to include a wider range of stiffness and damping properties, allowing better analysis. The set of sticks includes four carbon fibre sticks and two wooden sticks. The carbon percentage differs over the sticks and is unknown for most sticks; besides, one of the wooden sticks also contains some carbon, but at a very low percentage and is therefore classified as wooden. The sticks have all been used for several years and therefore show wear.

Figure 3.1: Measured sticks - a set of used field (F) and indoor (I) hockey sticks made of carbon (C) or wood (W)

Label	Type	Mass (kg)	Length (m)	COM (m)
1FC	F - carbon	0.558	0.930	0.542
2FC	F - carbon	0.566	0.930	0.545
3FC	F - carbon	0.497	0.925	0.532
4IC	I - carbon	0.476	0.935	0.510
5IW	I - wood	0.566	0.930	0.500
6FW	F - wood	0.560	0.920	0.551

Table 3.1: Properties of the measured sticks (F = field, I = indoor) (COM = centre of mass location measured from the handle downwards)

3.2. Stick property measurements (EXP1)

3.2.1. Introduction

The literature on field hockey sticks is scarce, and the information provided by the manufacturers of the sticks is incomplete and not comparable between different brands; therefore, an experimental analysis is needed to obtain knowledge of the stick properties. Mainly numerical values of the linear stiffness and damping of field hockey sticks are unknown; this experiment measures these values by applying a force disturbance and measuring the displacement of the stick.

3.2.2. Method

The Proprio is used for this experiment; this is a linear hydraulic manipulator, originally used to perform measurements on the human shoulder (Van Der Helm et al., 2002). The Proprio has a piston that performs a force perturbation while measuring the displacement. For this experiment, the Proprio applies a force perturbation to the tip of the stick.

The measurement setup is shown in Figure 3.2. The stick tip and the piston are fixed to each other and remain clamped during the experiment; therefore, the measured stiffness and damping properties are bending properties and not local properties of the stick. The piston applies a force perturba-

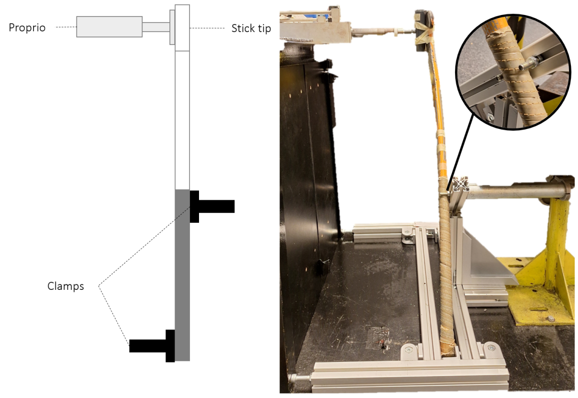


Figure 3.2: Schematic overview and picture of the measurement setup (EXP1)

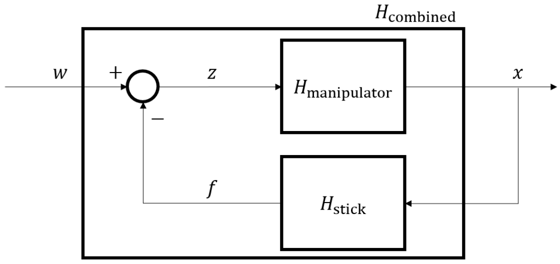


Figure 3.3: Block scheme of the total system from input (w) to output (x), described by H_{combined} which can be split into $H_{\text{manipulator}}$ and H_{stick}

tion to the stick and measures the displacement of the tip; the disturbance is a multisine pushing on the stick over a frequency of 0.5 to $100Hz$ for 30 seconds.

Each measurement is repeated eight times for each stick. The values obtained by the Proprio consist of the applied disturbance force w, the measured displacement x, the measured reaction force on the piston f and the difference between the perturbation force and the measured force z. The average values of the data over the eight trials are used to obtain the stick properties.

The total system has a force disturbance as input and a displacement as output, shown in Figure 3.3; however, this includes both the behaviour of the manipulator and the stick. As the focus is on stick behaviour only, H_{stick} is identified as shown in Figure 3.3; in which H_{combined} is split into $H_{\text{manipulator}}$ and H_{stick}. This figure shows that H_{stick} can be described as:

$$H_{\text{stick}} = \frac{-G_{\text{wf}}}{G_{\text{wx}}} = \frac{force}{displacement} \tag{3.1}$$

Where G_{wf} and G_{wx} describe the force and displacement, but the noise is minimized by normalising it to the input signal w.

With the average data over the eight repetitions, a gain and phase plot of H_{stick} are created for each stick.

By describing H_{stick} as a linear function, it can be fit to the averaged data to obtain the properties of the sticks; where the linear function is described as:

$$H_{\text{stick, fit}}(\omega) = \frac{1}{m(j\omega)^2 + cj\omega + k} \tag{3.2}$$

Where m is the mass, c is the bending damping and k is the bending stiffness; and $j\omega$ represents the frequency. By fitting this linear system to the measured data, the mass, stiffness and damping properties of each stick are determined. The mass in this formula describes the mass effecting the bending of the stick, which is a part of the total stick mass depending on the clamp position and stick properties.

The fit of the linear function $H_{\text{stick, fit}}$ to the measured H_{stick} is obtained by optimising an error function. Several error functions are used to find the best fit for each stick; a manual visual fit is performed for sticks two and three, where there was a high noise that made it difficult to fit. The error functions used are (e_a, e_b or e_c):

$$e_a = |log(\frac{H_{\text{stick}}}{H_{\text{fit}}})| \tag{3.3}$$

$$e_b = \sqrt{\frac{coherence}{1 + frequency}}|log(\frac{H_{\text{stick}}}{H_{\text{fit}}})| \tag{3.4}$$

$$e_c = \sqrt{\frac{1}{frequency}}|log(\frac{H_{\text{stick}}}{H_{\text{fit}}})| \tag{3.5}$$

Where e_a is the standard error function, e_b has a higher weight for measurements with a high coherence and low frequency and e_c has a lower weight for measurements with a high frequency.

3.2.3. Results

The results of the experiment are shown in Figures 3.4 to 3.9, which show the fit of the linear function to the measured data and the error function used to obtain the fit.

Figure 3.10 gives the numerical values for the mass, damping and stiffness of each stick. A more extensive explanation of the code used to obtain the results can be found in Appendix A.

3.2.4. Discussion

The measurements show expected results, as the indoor and wooden sticks (sticks 4 - 6) have a lower stiffness than the carbon field sticks (sticks

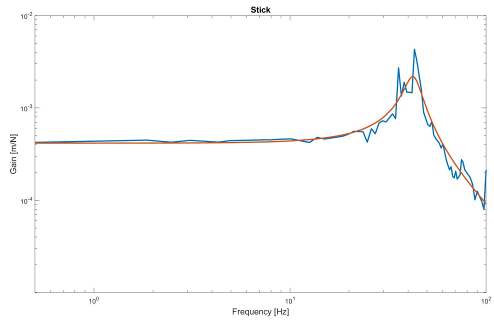


Figure 3.4: Gain plot of measured data (blue) and identified fit (red) stick 1FC (e_b)

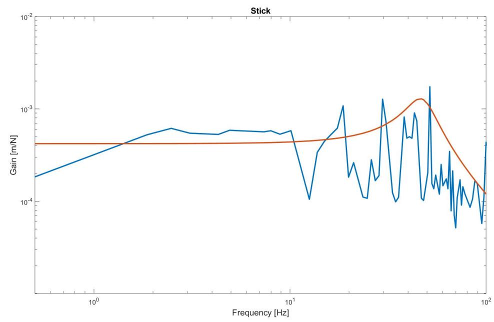


Figure 3.5: Gain plot of measured data (blue) and identified fit (red) stick 2FC (e_a + manual fitting)

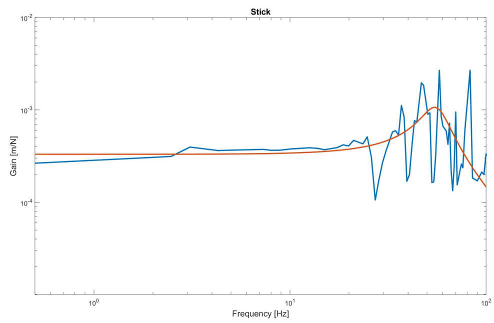


Figure 3.6: Gain plot of measured data (blue) and identified fit (red) stick 3FC (e_a + manual fitting)

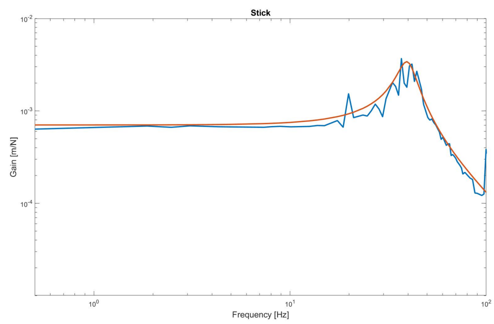


Figure 3.7: Gain plot of measured data (blue) and identified fit (red) stick 4IC (e_a)

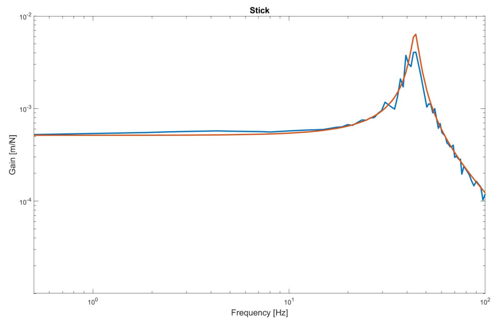


Figure 3.8: Gain plot of measured data (blue) and identified fit (red) stick 5IW (e_c)

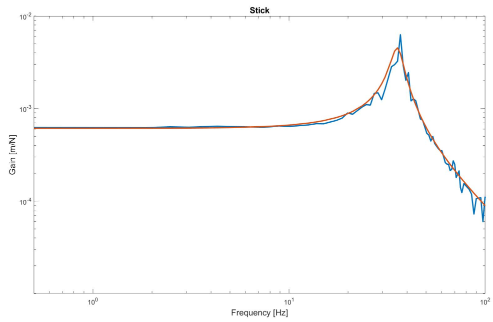


Figure 3.9: Gain plot of measured data (blue) and identified fit (red) stick 6FW (e_c)

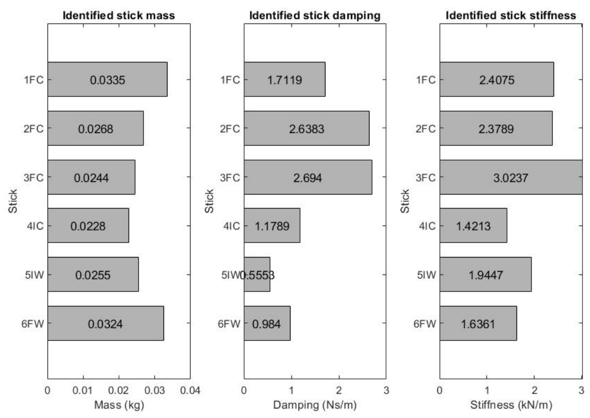


Figure 3.10: Identified stick parameters

1 - 3). The damping values were more difficult to have a numerical expectation of, as available knowledge is scarce; nevertheless, it was expected that beginner sticks, with a low stiffness, would have a high bending. However, the ratio between stiffness and damping seems to be quite constant over the set of measured sticks and does not seem to be influenced by the stiffness of the stick. Sticks 3 and 4 were expected to have a lower mass than the other sticks, because they have a lower total mass than the other four sticks and a high centre of mass.

The signal-to-noise ratio was low for high-frequency measurements, especially for the stiffer sticks; therefore, the results would improve if a logarithmic disturbance was used, with more measurement points at the higher frequencies. However, a more thorough understanding of the Proprio is needed to adjust the input signal, which is beyond the scope of this project.

3.3. Stick - ball collision measurements (EXP2)

3.3.1. Introduction

From the background knowledge, it became clear that the mass, stiffness and damping of a stick have an influence on stick performance; however, the size of their effect is unclear. This experiment determines which property has the greatest influence on the stick performance and would therefore be most effective to be adjusted in the stick design.

3.3.2. Method

For this experiment, the distance travelled by the ball after a stick-ball collision is measured for sticks with different mass, centre of mass, damping and stiffness. The setup shown in Figure 3.11 is used to perform these experiments.

For each measurement, the stick is clamped into the setup and it is rotated to a vertical upward position up to the upper bar. From this position, it is released and it falls towards the ball. The ball travels forward and the travelled distance is measured. Each measurement is repeated three times.

Three types of experiments are performed for the different variables to be measured.

The first experiment (EXP2.1) uses the six sticks introduced earlier; however, weights are added to give all sticks an equal mass. Therefore, the sticks only differ in their stiffness and damping properties in this experiment; and the influence of the stiffness and damping properties is measured. The influence of the location of the centre of mass is minimised, since the distance between the end of the handle and the centre of mass of each stick does not show large differences between all sticks after the weights were added (Table 3.2 - left column).

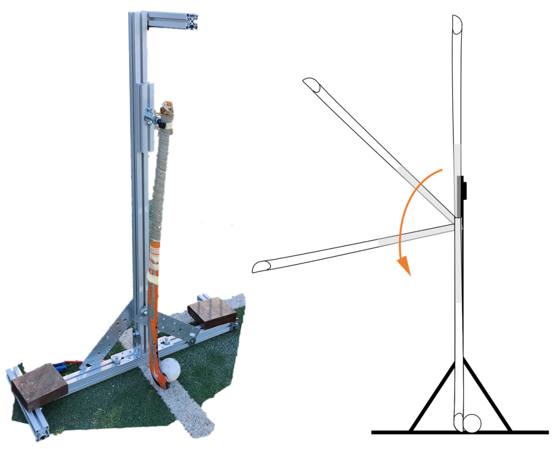


Figure 3.11: Picture and schematic overview of the measurement setup (EXP2)

For the second experiment (EXP2.2), only sticks 4 - 6 are used, since they show only small differences in their stiffness and damping properties. These sticks are measured without added mass, with an addition of $69g$ and with an addition of $159g$. This mass is added to the centre of mass location of the stick, and therefore the centre of mass does not change over the measurements. These experiments show the influence of the total mass on stick performance. The stick properties of this experiment are shown in Table 3.2 - middle column.

The last experiment (EXP2.3) uses the same sticks as in the previous experiment and measures the influence of the centre of mass location of the sticks. The influence of lowering the centre of mass is measured by adding the weight ($159g$) from the previous experiment to the stick head instead of the stick centre of mass. The difference between the high and low centre of mass is evaluated (Table 3.2 - right column). Figure 3.12 shows the location of the added mass; it is taped onto the stick, and therefore added just above the location of ball impact to not change the contact surface between ball and stick.

The values of the moment of inertia of the tested sticks in Table 3.2 are calculated by the parallel axis theorem with the following formula (Vallery and Schwab, 2021):

$$I = \frac{1}{12}ML^2 + Mb^2 \tag{3.6}$$

Where M is the total stick mass, L is the total stick length and b is the distance from the end of handle

	EXP2.1		EXP2.2						EXP2.3			
	m (kg)	I (kgm^2)	m_1 (kg)	I_1 (kgm^2)	m_2 (kg)	I_2 (kgm^2)	m_3 (kg)	I_1 (kgm^2)	m_h (kg)	I_h (kgm^2)	m_l (kg)	I_l (kgm^2)
1FC	0.566	0.21										
2FC	0.566	0.21										
3FC	0.566	0.20										
4IC	0.566	0.19	0.476	0.16	0.566	0.19	0.635	0.21	0.635	0.21	0.635	0.28
5IW	0.566	0.18	0.566	0.18	0.635	0.20	0.725	0.23	0.725	0.23	0.725	0.30
6FW	0.566	0.21	0.560	0.21	0.629	0.24	0.719	0.27	0.719	0.27	0.719	0.33

Table 3.2: Stick properties EXP2. For EXP2.1 all sticks have an equal mass and the influence of stiffness and damping is measured. For EXP2.2 only sticks 4-6 are measured, mass is added from measurement 1 to 3. For EXP2.3 only sticks 4-6 are measured, with a high (h) COM and low (l) COM.

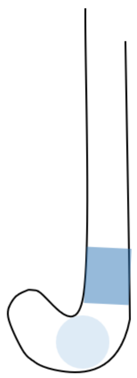


Figure 3.12: Weight is added at the location highlighted in dark blue, just above the hit location highlighted in light blue (EXP2.3)

of the stick (rotation point) to the centre of mass location.

3.3.3. Results

Figure 3.13 shows the results of the experiments performed. For the measurements of EXP2.1, the figure does not show a logical relationship between stiffness or damping and the ball distance travelled.

The results of EXP2.2 show an increase in ball distance as the mass increases. The measurements of EXP2.3 show a small increase in ball distance for sticks four and six when the centre of mass is lowered, stick five shows a smaller distance.

3.3.4. Discussion

The experiments were carried out outside in January; therefore, the temperature was very low and increased slightly during the experiments, which could have influenced the results. The low temperature influenced the stiffness and damping properties of the sticks and the low temperature of the field influenced the ball velocity. It would have been better to perform the experiments in an environment with a constant temperature of 20 degrees Celsius.

An unexpected result is shown in the first experiment (EXP2.1). The stiffer sticks do not show an increase in ball displacement; however, this can also be due to the stiffness of the measurement setup. If the stiff sticks had a higher stiffness than the setup, it could be that the setup deformed and therefore a lower ball displacement was achieved. Another explanation can be the location of the sweet spot; the background information described that the location of the hit influences the stick behaviour, and the best behaviour is obtained for a hit at the sweet spot. It could be that the stiffer sticks are designed for more experienced players and therefore have a smaller sweet spot location than the less stiff sticks. With a smaller sweet spot, it is possible that the measurements did not perform an optimal hit, and therefore the ball displacement was lower than expected. Similar research described this as a difficulty in obtaining a ”clean” impact (Allen et al., 2012). However, despite the uncertainty in the description of the unexpected result, it can be concluded that the stiffness and damping do not have a large influence on the ball displacement and/or are more difficult to control and need a more experienced player to show good stick performance.

An increase in mass shows an increase in ball displacement (EXP2.2), which was expected by reviewing the collision model. However, the influence of lowering the centre of mass (EXP2.3) is not as expected. A collision with a higher effective mass should give a higher ball velocity, but stick five does not show an increase and stick four and six only show a small increase. A reason for this could be that the mass distribution over the stick changed and thereby influenced the deformation properties of the stick. The weight was added as a point mass, and it may be better to have a smoother mass distribution over the stick.

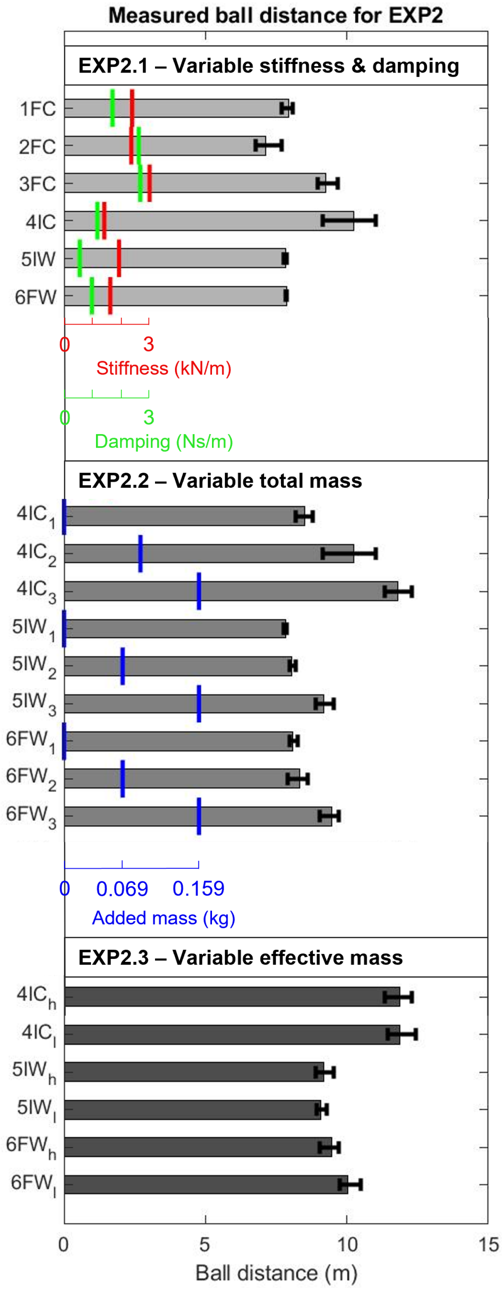


Figure 3.13: Measured ball distance in meters for EXP2. The upper measurements show the influence of stiffness (in red) and damping (in green) on the ball distance (EXP2.1). The middle measurements show the influence of an increasing mass over measurements 1 to 3 (in blue) for stick 4, 5 and 6 (EXP2.2). The lowest measurements show the influence of lowering the centre of mass of sticks 4, 5 and 6 on the ball distance (EXP2.3).

4 Theoretical Analysis

4.1. Introduction

In addition to the previous chapter, this chapter answers the same question - *which property has the greatest influence on stick performance?*; however, it is done theoretically. By developing a mathematical model of the stick and the ball, the dynamics of the collision can be described and their influence is analysed.

Several models are discussed in the following sections, from which one model is used to describe the influence of the stick properties on the stick performance. The codes used to obtain the modelled results can be found in Appendices B.3 - B.5.

4.2. Triple mass-spring-damper model

4.2.1. Model setup

To model the stiffness and damping properties, a mass-spring-damper model is used to simplify the dynamics. The model consists of the human, the stick and the ball, all modelled as point masses connected by springs and dampers, see Figure 4.1.

The model is simplified by the following assumptions: it is a linear motion, gravity is neglected, the initial position of all masses is zero, the human mass is the effective mass of the human body when playing hockey - assumed to be 50% of the total body mass, the stick part also includes the arm mass as this mass is fixed to the stick.

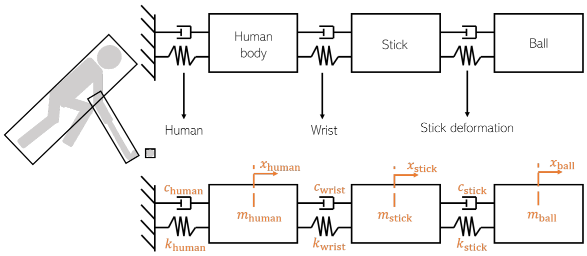


Figure 4.1: Setup of the triple mass-spring-damper system, including the human, stick and ball connected by springs and dampers representing the human, wrist and stick

4.2.2. Formulas

The free-body diagrams of the human, stick and ball are shown in Figure 4.2. This leads to the following equations of motion:

$$m_h\ddot{x}_h = -F_{c,h} - F_{k,h} + F_{c,w} + F_{k,w} \tag{4.1}$$

$$\ddot{x}_h = \frac{1}{m_h}(-c_h\dot{x}_h - k_h x_h + c_w(\dot{x}_s - \dot{x}_h) + k_w(x_s - x_h)) \tag{4.2}$$

$$m_s\ddot{x}_s = -F_{c,w} - F_{k,w} + F_{c,s} + F_{k,s} \tag{4.3}$$

$$\ddot{x}_s = \frac{1}{m_s}(-c_w(\dot{x}_s - \dot{x}_h) - k_w(x_s - x_h) + c_s(\dot{x}_b - \dot{x}_s) + k_s(x_b - x_s)) \tag{4.4}$$

$$m_b\ddot{x}_b = -F_{c,s} - F_{k,s} \tag{4.5}$$

$$\ddot{x}_b = \frac{1}{m_b}(-c_s(\dot{x}_b - \dot{x}_s) - k_s(x_b - x_s)) \tag{4.6}$$

Where subscript h describes the human, subscript s the stick and subscript b the ball; and subscript w describes the wrist joint. All equations contain a damper force, defined by subscript c and a spring force, defined by subscript k. The damper constant is defined as c, in Ns/m; and the spring constant is defined as k, in N/m.

These equations are used to simulate the ball displacement after a hit and when stopping the ball.

4.2.3. Simulations

Since the combination of the player and the stick is important for the overall performance, the human

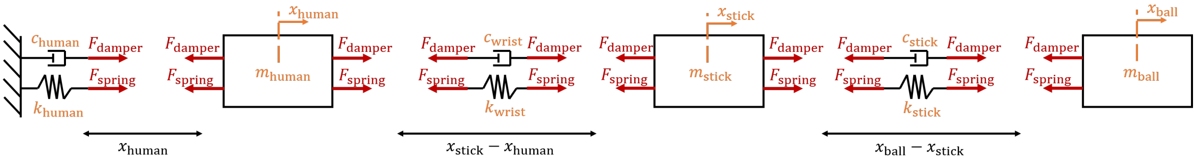


Figure 4.2: Free-body diagram of the human, stick and ball, where spring and damper forces are applied

			Experienced player + advanced stick	Beginner + advanced stick	Beginner + beginner stick
Stopping	Player	Stiffness	HIGH	LOW	LOW
		Damping	HIGH	LOW	LOW
	Stick	Stiffness	HIGH	HIGH	LOW
		Damping	LOW	LOW	HIGH
Striking	Player	Stiffness	LOW	LOW	LOW
		Damping	LOW	LOW	LOW
	Stick	Stiffness	HIGH	HIGH	LOW
		Damping	LOW	LOW	HIGH

Figure 4.3: The combinations of players and sticks. The experienced player can adapt its properties during the game, the beginners is not able to adapt to different actions - striking versus stopping

is included in this model. Performance is modelled by stopping the ball and striking the ball for several combinations of players and sticks.

Players are modelled as an *advanced player*, having adaptive properties between striking and stopping a ball, or as a *beginner*, not being able to adapt their properties between stopping and striking a ball. The sticks are modelled as *advanced* sticks, with high stiffness and low damping, or *beginner* sticks, with high damping and low stiffness.

It is assumed that an *experienced* player with an *advanced* stick shows the best performance; a *beginner* with an *advanced* stick is not able to control the ball when stopping, but develops a high ball velocity when striking; and a *beginner* with a *beginner* stick can properly control the ball when stopping, but has a low power when hitting a ball. This assumption can be tested by the model. The player and stick combinations are shown in Figure 4.3. These combinations are simulated for striking and stopping, using the parameters in Table 4.1.

	Value	Sources
Constants		
m_{h} (kg)	37.00	Krishnan et al., 2016 & Chapter 3 & FIH, 2023
m_{s} (kg)	0.537 + 4.71	
m_{b} (kg)	0.16	
Initial velocities - stopping		
$\dot{x}_{\text{h},0}$ (m/s)	0	FIH, 2023
$\dot{x}_{\text{s},0}$ (m/s)	0	
$\dot{x}_{\text{b},0}$ (m/s)	-22.22	
Initial velocities - striking		
$\dot{x}_{\text{h},0}$ (m/s)	22.22	FIH, 2023
$\dot{x}_{\text{s},0}$ (m/s)	22.22	
$\dot{x}_{\text{b},0}$ (m/s)	0	
Variables - human		
$k_{\text{h, high}}$ (N/m)	1 083.09	De Vlugt et al., 2003 & De Vlugt et al., 2006 & Diefenbach and Lipps, 2019
$k_{\text{h, low}}$ (N/m)	495.77	
$c_{\text{h, high}}$ (Ns/m)	54.15	
$c_{\text{h, low}}$ (Ns/m)	24.79	
Variables - wrist		
$k_{\text{w, high}}$ (N/m)	116.12	De Vlugt et al., 2006 & Peaden and Charles, 2014
$k_{\text{w, low}}$ (N/m)	13.93	
$c_{\text{w, high}}$ (Ns/m)	2.32	
$c_{\text{w, low}}$ (Ns/m)	0.14	
Variables - stick		
$k_{\text{s, high}}$ (N/m)	3 000	Chapter 3 - EXP1
$k_{\text{s, low}}$ (N/m)	1 400	
$c_{\text{s, high}}$ (Ns/m)	2.7	
$c_{\text{s, low}}$ (Ns/m)	0.5	

Table 4.1: Values of the parameters used for the simulations of the triple mass-spring-damper model

4.2.4. Results

The results of the simulations are shown in Figures 4.4 and 4.5. It can be seen that the properties of the player do not influence the performance, as the lines of different players with the same stick overlap.

4.2.5. Discussion

It was assumed that a beginner with an advanced stick would not be able to control the ball when stopping the ball and that a beginner with a beginner stick would not be able to produce high ball

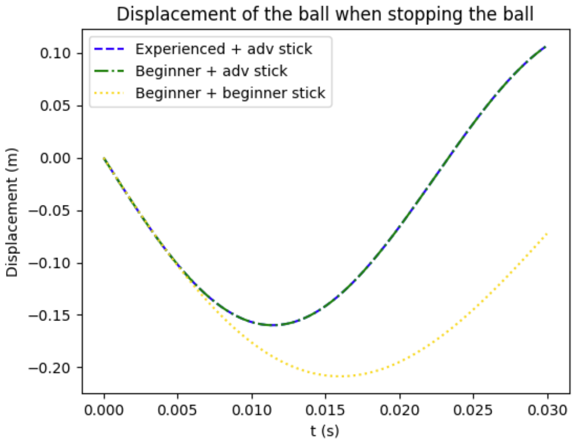


Figure 4.4: Displacement of the ball when stopping a ball - modelled as a triple mass-spring-damper system (for different combinations of *beginner* versus *experienced* player; and *beginner* versus *advanced* stick)

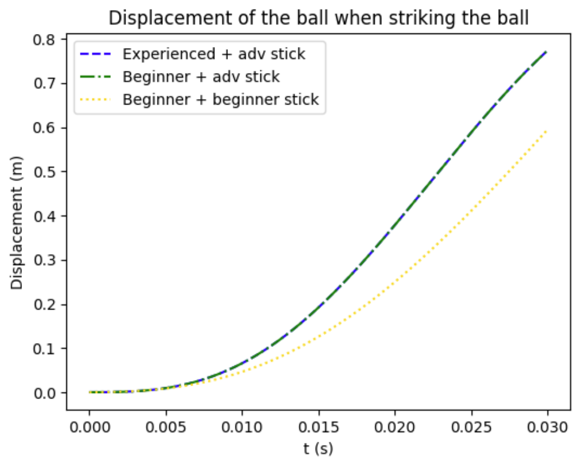


Figure 4.5: Displacement of the ball when striking a ball - modelled as a triple mass-spring-damper system (for different combinations of *beginner* versus *experienced* player; and *beginner* versus *advanced* stick)

velocity when striking the ball. However, the results do not show any difference between the experienced player and the beginner, the stick properties are dominant for the performance shown by these results. Since it is clear that player properties do have an influence on overall performance, this model does not show correct results and these results will not be used.

4.3. Double mass-spring-damper model

4.3.1. Model setup

The previous model did not show the correct output, as the human behaviour did not influence the performance. Therefore, in this model the human behaviour is simplified to only one spring-damper system instead of two, assuming that the wrist joint is fully stiff during the motion. Also, it is assumed that during the stick-ball contact, the ball and stick always touch each other. Therefore, this model contains two parts: the human and the stick handle, defined by $human*$; and the stick tip and the ball, defined by $ball*$. The model setup is shown in Figure 4.6.

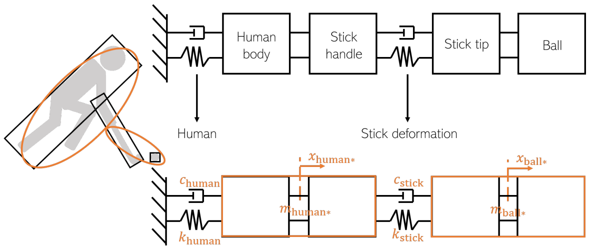


Figure 4.6: Setup of a double mass-spring-damper system, where $human*$ consists of the human plus the stick handle and $ball*$ consists of the ball plus the stick tip. The parts are connected by springs and dampers representing the human and the stick

The following assumptions are made: it is a linear motion, gravity is neglected, the initial position of all masses is zero, the human mass is the effective mass of the human body when playing hockey - assumed to be 50% of the total body mass, the stick part also includes the arm mass as this mass is fixed to the stick. These are the same as in the previous model.

4.3.2. Formulas

The free-body diagram of this model is shown in Figure 4.7. Which leads to the following equations of motion:

$$m_{\mathrm{h}^*}\ddot{x}_{\mathrm{h}^*} = -F_{\mathrm{c,\,h}} - F_{\mathrm{k,\,h}} + F_{\mathrm{c,\,s}} + F_{\mathrm{k,\,s}} \tag{4.7}$$

$$\ddot{x}_{\mathrm{h}^*} = \frac{1}{m_{\mathrm{h}^*}}(-c_{\mathrm{h}}\dot{x}_{\mathrm{h}^*} - k_{\mathrm{h}}x_{\mathrm{h}^*} + c_{\mathrm{s}}(\dot{x}_{\mathrm{b}^*} - \dot{x}_{\mathrm{h}^*}) + k_{\mathrm{s}}(x_{\mathrm{b}^*} - x_{\mathrm{h}^*})) \tag{4.8}$$

$$m_{\mathrm{b}^*}\ddot{x}_{\mathrm{b}^*} = -F_{\mathrm{c,\,s}} - F_{\mathrm{k,\,s}} \tag{4.9}$$

$$\ddot{x}_{\mathrm{b}^*} = \frac{1}{m_{\mathrm{b}^*}}(-c_{\mathrm{s}}(\dot{x}_{\mathrm{b}^*} - \dot{x}_{\mathrm{h}^*}) - k_{\mathrm{s}}(x_{\mathrm{b}^*} - x_{\mathrm{h}^*})) \tag{4.10}$$

Where subscript $h*$ describes the human plus the handle of the stick and subscript $b*$ the ball plus the stick tip; and subscript h describes the human properties and subscript s the stick properties. All equations contain a damper force, defined by subscript c and a spring force, defined by subscript k. The damper constant is defined as c, in Ns/m; and the spring constant is defined as k, in N/m.

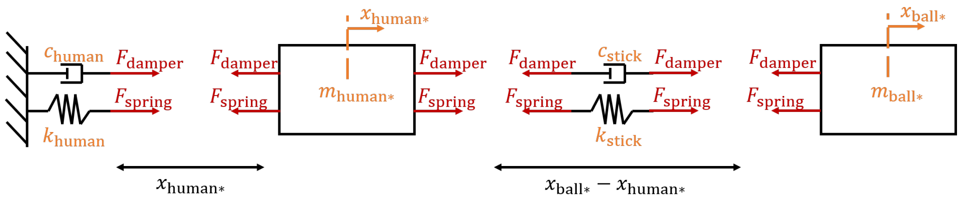


Figure 4.7: Free-body diagram of the $human*$ and $ball*$, consisting of the human plus the stick handle and the ball plus the stick tip, respectively

	Value	Sources
	Constants	
m_{h^*} (kg)	37.00 + 0.269 + 4.71	Krishnan et al., 2016 & Chapter 3 & FIH, 2023
m_{b^*} (kg)	0.16 + 0.269	
	Initial velocities - stopping	
$\dot{x}_{h^*,0}$ (m/s)	0	FIH, 2023
$\dot{x}_{b^*,0}$ (m/s)	-22.22	
	Initial velocities - striking	
$\dot{x}_{h^*,0}$ (m/s)	22.22	FIH, 2023
$\dot{x}_{b^*,0}$ (m/s)	0	
	Variables - human	
$k_{h,\ high}$ (N/m)	1 083.09	De Vlugt et al., 2003 & De Vlugt et al., 2006 & Diefenbach and Lipps, 2019
$k_{h,\ low}$ (N/m)	495.77	
$c_{h,\ high}$ (Ns/m)	54.15	
$c_{h,\ low}$ (Ns/m)	24.79	
	Variables - stick	
$k_{s,\ high}$ (N/m)	3 000	Chapter 3 - EXP1
$k_{s,\ low}$ (N/m)	1 400	
$c_{s,\ high}$ (Ns/m)	2.7	
$c_{s,\ low}$ (Ns/m)	0.5	

Table 4.2: Values of the parameters used for the simulations of the double mass-spring-damper model

4.3.3. Simulations

The same conditions as in the previous model are simulated. The player can be an *experienced* player (adapting its properties between stopping and striking a ball) or a *beginner* (not able to adapt its properties). And the stick can be an *advanced* stick (high stiffness, low damping) or a *beginner* stick (low stiffness, high damping). The combinations of players and sticks are described in Figure 4.3. The model simulates striking and stopping a ball, using the parameters in Table 4.2.

4.3.4. Results

The results of the simulations are shown in Figures 4.8 and 4.9. The results do not shown any differences between the players, only differences between the sticks are shown.

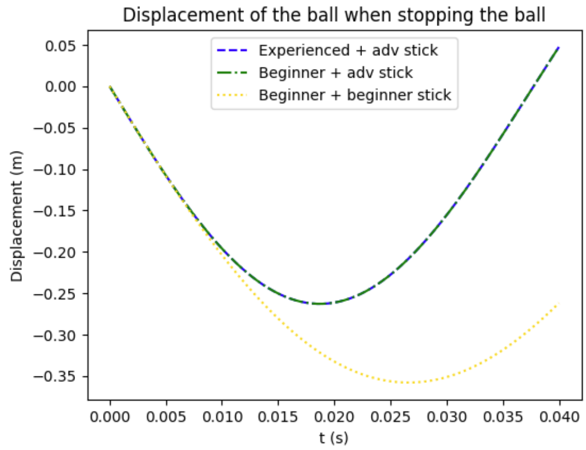


Figure 4.8: Displacement of the ball when stopping a ball - modelled as a double mass-spring-damper system (for different combinations of *beginner* versus *experienced* player; and *beginner* versus *advanced* stick)

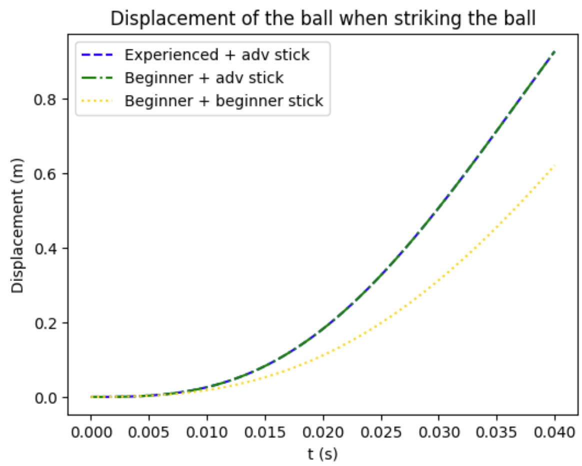


Figure 4.9: Displacement of the ball when striking a ball - modelled as a double mass-spring-damper system (for different combinations of *beginner* versus *experienced* player; and *beginner* versus *advanced* stick)

4.3.5. Discussion

The results of the simulations do not show any differences between beginner and experienced players, therefore the model is not valid. It can be concluded that the simplifications of the human behaviour compared to the previous model, did not influence the quality of the results. These models show that the stiffness and damping properties of a stick should improve the performance of all players; however, real observations of stick-ball interaction of different players show that this is not the case for inexperienced players. Therefore, it is interesting to identify which stick property does help inexperienced players to improve their performance. To identify the next step in the development of a mathematical model, several sticks were tested on the hockey field. This helped identify influential properties of the stick; the focus in the previous models was mainly on the stiffness and damping properties of the stick. The tests opened up a broader perspective and showed that the mass and coefficient of restitution could also be used in the development of the model. This was used as the basis for the development of the next model.

4.4. Collision model

4.4.1. Model setup

This model describes the stick-ball interaction as a simple collision, without taking into account the human. Figure 4.10 shows the situation of the stick-ball interaction to be described in a model. The stick moves towards a ball, the two objects collide and they both move forward in the same direction. To describe this situation, the collision model shown in Figure 4.11 is used. The stick and ball are both simplified to be a point mass with a linear velocity. The collision model helps simplifying the complex interaction between the ball and the stick (Cross, 2013), by only taking into account the velocities before and after collision; this is sufficient for this application, since the main focus is on the stick performance which is reflected by the outgoing ball velocity.

4.4.2. Formulas

The collision dynamics can be described by the following formula (Cross, 2013):

$$M_e V_1 + m v_1 = M_e V_2 + m v_2 \tag{4.11}$$

The stick is described by the capital M and V; the ball is described by lower case m and v. Subscript 1 describes the dynamics before the collision and subscript 2 describes the behaviour after collision. When simplifying the stick-ball collision to a collision of two point masses, only the effective mass of the stick should be taken into account, which is denoted by M_e (Cross, 2013). Since the model describes the situation of hitting a ball, the ball has no initial velocity and the formula can be simplified

Figure 4.10: Stick hitting a ball - before and after collision

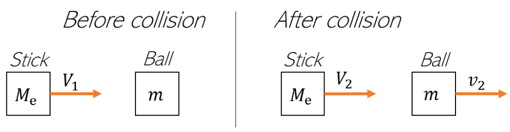


Figure 4.11: Collision model of a stick hitting a ball

to:

$$M_e V_1 = M_e V_2 + m v_2 \tag{4.12}$$

So far, only the mass and the velocity have an influence on the behaviour; however, also the deflective stiffness and damping properties of the stick need to be taken into account. These properties influence the stick behaviour during the collision. When the stick and ball collide, the stick deforms and energy is dissipated; a higher stiffness decreases the deformation and a higher damping increases the energy dissipation. The energy needed for stick deformation is stored as potential energy, and this elastic deformation can be transferred into kinetic energy again; however, a part is also lost in deformation and vibrations in the stick. The exact energy transfer during collision is hard to describe, as it depends on many factors. Therefore, it can be simplified into one factor, the coefficient of restitution (COR), e. This coefficient describes the ratio of relative energy between the masses before and after collision:

$$e = \frac{v_2 - V_2}{v_1 + V_1} \tag{4.13}$$

For this situation it holds that:

$$e = \frac{v_2 - V_2}{V_1} \tag{4.14}$$

The COR describes the efficiency of the energy transfer from the stick to the ball; a higher COR describes a higher transfer of energy (Cross, 2013). The stick stiffness positively influences the COR and the damping has a negative influence.

By implementing the COR into the collision formula, the final ball velocity can be described as:

$$v_2 = \frac{1+e}{1+\frac{m}{M_e}} V_1 \tag{4.15}$$

4.4.3. Simulations

The situations to be simulated are similar to the experiments done in the Chapter 3; the focus is on the influence of damping, stiffness, mass and centre of mass of the stick. Therefore, the first simulation (SIM1) shows the influence of an increasing COR on the outgoing ball velocity, which describes the influence of stick stiffness and damping. The second simulation (SIM2) determines the influence of an increasing total mass on the outgoing ball velocity. And the third simulation (SIM3) evaluates the increase in effective mass, by lowering the centre of mass, on the outgoing ball velocity.

The parameters used in the simulation are described in Table 4.3. The mass and centre of mass of the stick are determined by taking the average values and the range of the total stick mass and the centre of mass of the six sticks used in the experiments.

The COR of the sticks is unknown, so a realistic value and range for the COR need to be determined. A realistic range of COR over different sticks is determined by Allen et al. (Allen et al., 2012), where other research is analysed and tests are performed. For the second and third simulations, the average COR of this range is used.

As stated by Cross (Cross, 2013), not the total mass, but the effective mass of the stick should be used to calculate the outgoing ball velocity. The effective mass is calculated as follows (which is a simplification of equation 2.1):

$$M_e = \frac{I}{S^2} \tag{4.16}$$

with

$$I = \frac{1}{12} ML^2 + Mb^2 \tag{4.17}$$

Where I is the moment of inertia about an axis through the end of the handle and S is the distance from the impact location to the end of the handle (rotation point); M is the total stick mass, L is the total length of the stick and b is the distance from the end of the handle to the centre of mass location of the stick.

	Value	Source
$V_1(m/s)$	22.2	FIH, 2023
$m(kg)$	0.16	FIH, 2023
$L(m)$	0.928	Table 3.1
First simulation		
$M(kg)$	0.537	Table 3.1
e	$0.26-0.35$	Allen et al., 2012
$b(m)$	0.530	Table 3.1
Second simulation		
$M(kg)$	$0.476-0.566$	Table 3.1
e	0.31	Allen et al., 2012
$b(m)$	0.530	Table 3.1
Third simulation		
$M(kg)$	0.566	Table 3.1
e	0.31	Allen et al., 2012
$b(m)$	$0.530-0.585$	Table 3.1

Table 4.3: Parameters used in the simulations of the collision model

For the three simulations, the outgoing ball velocity is calculated using Formula 4.15 with the parameters listed in Table 4.3, where the effective mass is calculated by Formula 4.16. The results plot the relative increase of the parameter (M, e and b) to the relative increase in the outgoing ball velocity (v_2). Hence, all simulations show the influence of a realistic increase of the parameter on the outgoing ball velocity, since all ranges in parameters are taken from real stick measurements.

4.4.4. Results

The results of the simulations are shown in Figure 4.12. All properties show a positive relationship with the outgoing ball velocity. The increase in ball velocity is greatest for the increase in total mass; but the differences between the three variables are small.

4.4.5. Discussion

The results of the simulations show the biggest increase in outgoing ball velocity by an increase in total stick mass, which was expected from the experiments (Chapter 3).

The effect of the stiffness and damping properties, represented by the COR, is bigger than expected by the experiments. However, the range of COR over different sticks cannot only be ascribed to changes in stick stiffness and damping. Furthermore, research shows that for a hit at the centre of the stick, at the sweet spot, the stick stiffness has almost no effect on the COR (Allen et al., 2015 & Miller, 2006). Therefore, the effect shown

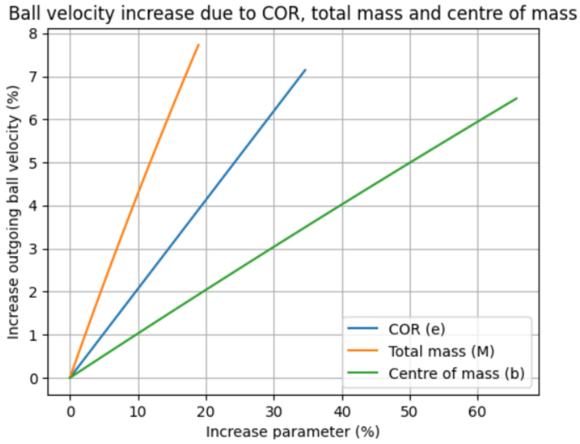


Figure 4.12: Simulation of the collision model showing the influence of the stick COR (SIM1), total stick mass (SIM2) and centre of mass location (SIM3) on outgoing ball velocity

in the simulations is too large to describe changes in stiffness and damping only.

The effect of lowering the centre of mass is greater than what was expected from the measurements. However, when comparing it to literature (Allen et al., 2012 & Cross, 2013), it can be assumed that the centre of mass has a large influence on the outgoing ball velocity. Because lowering the centre of mass increases the moment of inertia (Taraborrelli et al., 2019), which positively influences the outgoing ball velocity. Thereby, it can be concluded that the model output is valid.

4.5. Conclusion

Three models are developed, two mass-spring-damper models and a collision model. Only the collision model shows valid results and is therefore used to evaluate the stick properties.

The analysis of the stick properties takes into account the results of the collision model, combined with the experimental results of Chapter 3. In this analysis, the influence of total stick mass, effective stick mass and bending stiffness and damping on stick performance was measured and modelled. To perform these measurements, the first step was a numerical quantification of the stick stiffness and damping (EXP1). For the simulations, a realistic range of the COR was found in literature (Allen et al., 2012) to represent changes in stick stiffness and damping. Stick performance was measured and simulated by a stick hitting a ball, measuring the output distance and velocity of the ball.

Combining the results of the measurements and the simulations, and comparing this with the available literature, it can be concluded that the mass has the greatest influence on the stick performance for centre hits; where the centre of mass has a crucial role in determining the effective mass of the collision between the stick and the ball. The influence of the stiffness and damping properties can be reflected by the COR; however, this influence is almost zero for centre hits and only becomes influential for off-centre hits (Allen et al., 2015 & Miller, 2006). Although the influence of lowering the centre of mass of the stick seems to have a very small impact concluding from the experiments (EXP2.3), it is assumed to be influential concluding from the simulations (SIM3) and literature (Cross, 2013). However, the experiment shows that changes in the centre of mass location are small when adding little weight; which is important to take into account when reflecting the impact of this stick property.

The performance is only reflected by striking a ball in this analysis; however, in the introduction (Chapter 1), performance is defined as the combination of striking power and stopping control. The results of these experiments are also useful when taking into account stopping control, because the reverse action takes place. Instead of zero ball velocity and a non-zero stick velocity, the ball has a non-zero velocity and the stick has zero velocity for stopping a ball. For striking, a high energy transfer and high velocities post collision are desired; whereas for stopping a low energy transfer and near-zero velocities are desired. Therefore, the desired values of the analysed stick properties are opposite for stopping and striking. Moreover, it should be taken into account that a low total mass is desired for good control. So, it can be concluded that for overall stick performance, stick mass, mainly its effective mass, has the greatest impact, followed by stiffness and damping.

5

Adaptive Stick Design

5.1. Introduction

The previous section concluded that the desired mass, stiffness and damping values of a field hockey stick are opposite for striking a ball and stopping a ball. To optimise stick performance, it would be interesting to develop a stick that can change its properties between striking and stopping a ball. Since the focus so far has been on the internal properties of the stick (mass, stiffness and damping), the outer properties (e.g., bow shape, length, head shape, etc.) are not taken into account and the focus will be on a mechanism that fits inside a regular field hockey stick. Therefore, the following design goal is formulated: *design a mechanism that can adapt properties of a field hockey stick*.

5.2. Design requirements

Since the performance of the stick is perceived by the human playing with the stick, the design requirements originate from user requirements. These user requirements are translated into product aspects, which can be seen as the design requirements with a specified value, see Table 5.1.

5.3. Concepts

5.3.1. Introduction

From the defined design requirements, several concepts are developed. The first design decision to be made is the type of trigger to which the mechanism will react to adapt its properties. Since it has to adapt between hitting and stopping a ball, the differences between these actions are reviewed and an overview is shown in Figure 5.1. The mechanism has to work fully mechanically and it is therefore desirable if the trigger shows a change in force or moment. Moreover, following from the requirements, it is not preferred that the player is involved in controlling the adaptive stick behaviour. Therefore, angular acceleration is chosen as the trigger for the adaptive behaviour. The angular acceleration of the stick creates a centrifugal force, a reaction to this centrifugal force should activate the stick to adapt between stopping and striking.

	STOPPING	STRIKING
Hand position	Middle	Top
Relative hand position	Wide	Small
Angular acceleration	0	>0
Hand grip	Firm grip	Relaxed grip

Figure 5.1: Overview of differences between stopping and striking a ball to identify triggers for the mechanism to adapt its properties. The difference in angular acceleration will be used in the design.

5.3.2. Concepts: adapting mass

It is not desirable or possible to adapt the total stick mass during the game; however, it is possible to adapt the effective stick mass by changing the centre of mass of the stick (Formula 4.16). In the hollow stick frame, a movable mass can be installed. The centrifugal force, when hitting, will project the mass to the head of the stick; when the centrifugal force decreases to zero again, the spring force will return the mass to its original position. A simple overview of this mechanism is shown in Figure 5.2. Another similar concept is shown in Figure 5.3, where two masses are used. These masses are placed away from the centre line; hereby, not only the effective mass is adjusted, but the addition of mass also changes the rotational moment of inertia of the stick head.

5.3.3. Concepts: adapting stiffness

Stiffness adaptation can be done in several ways (Overweel, 2024), some interesting mechanisms are applied in this hockey stick design. All concepts need a force to initiate the adaptation, which has to be the centrifugal force when hitting a ball. The concept shown in Figure 5.4 applies laminar jamming with mechanical interference (Caro and

	User req.	Product aspect	Value	Type
1	High power when striking the ball and high control when stopping the ball	a) Adaptive properties between striking and stopping b) Striking: high mass / high stiffness / low damping - Stopping: low mass / low stiffness / high damping c) Immediate response during the game d) No active control of the player is needed		f s s c
2	Feel as a normal stick	a) Dimensions and weight should stay within regular range b) The mechanism should fit inside a regular stick c) No maintenance needed, robust design	$0.470 - 0.570kg$ mass, $0.930m$ length Max diameter $51mm$	c c s
3	It should be allowed to play with the stick	a) The stick cannot contain any metal b) Dimensions and weight should comply with the FIH rules c) The maximum outgoing ball velocity after striking is 98% of the initial stick velocity	Max $0.737kg$, $1050mm$ length, $51mm$ diameter	c c c

Table 5.1: Design requirements: product aspects follow from user requirements (f = functional, s = specification, c = constraint)

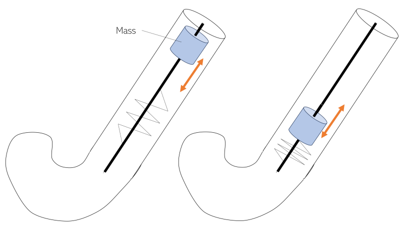


Figure 5.2: Concept 1a - Single mass adaptation (L: stopping, R: striking)

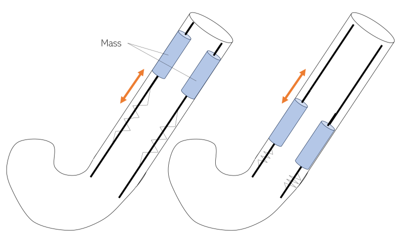


Figure 5.3: Concept 1b - Double mass adaptation (L: stopping, R: striking)

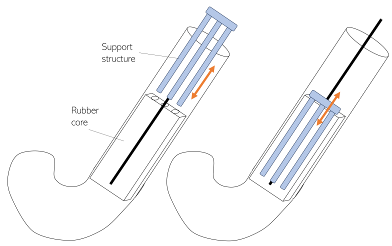


Figure 5.4: Concept 2a - Laminar jamming with mechanical interference (L: stopping, R: striking)

Carmichael, 2024). The rubber core facilitates good control when stopping a ball; the stiff interfering structure blocks the flexibility of the rubber core and thereby increases the stiffness. The second concept in Figure 5.5 uses sliding-layer laminates (Caro and Carmichael, 2024). The green parts consist of a rigid material and the blue parts of a soft material. When the layers of the same material overlap, the mechanism is flexible; when they do not overlap, it is stiff. The last concept, shown in Figure 5.6, changes the bending stiffness by increasing the moment of inertia of the internal structure. The structure supports the head when striking the ball and it returns upward when there is no angular acceleration.

5.3.4. Concepts: adapting damping

By adding a core to the stick, the damping increases. This is applied in the concept shown in Figure 5.7, the core will remain in the head of the stick and will be lifted upward when a centrifugal force acts on the counter weights.

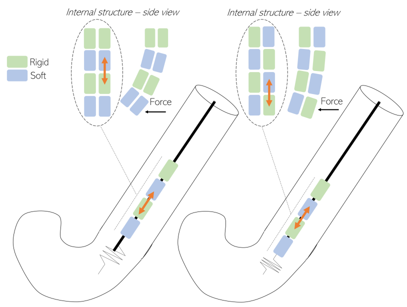


Figure 5.5: Concept 2b - Laminar jamming with sliding laminates (L: stopping, R: striking)

	Mass		Stiffness			Damping
	1a	1b	2a	2b	2c	3a
PA1a	++	++	+	+	+	+
PA1b	0	0	0	0	0	0
PA1c	++	++	++	++	++	++
PA1d	++	++	++	++	++	++
PA2a	+	+	+	+	+	+
PA2b	++	++	+	+	++	++
PA2c	++	++	-	-	++	++
PA3a	++	++	++	++	++	++
PA3b	++	++	+	+	++	++
PA3c	0	0	0	0	0	0

Table 5.2: Comparison of the concepts based on the product aspects (PA) listed in Table 5.1 - with scores from very bad (--), bad (-), neutral (0), good (+) to very good (++)

5.3.5. Evaluation

The designs are compared to the requirements listed in Table 5.1, the overview of the evaluation is shown in Table 5.2. Where the biggest differences are shown for product aspect 1a, 2b, 2c and 3b. Concerning product aspect 2a, the designs that adapt the effective mass were shown to be most effective based on the analysis in Chapters 3 and 4. For product aspects 2b and 2c, the concepts applying laminar jamming require multiple particles that should interact correctly with each other and the applied mechanisms are not developed especially for small scale, therefore maintenance and dimensions can become a problem or these designs. Therefore, these two designs also score less for product aspect 3b. Based on this analysis, the concepts adapting the effective mass will be further developed and combined with the other promising concepts; the next section describes the final design.

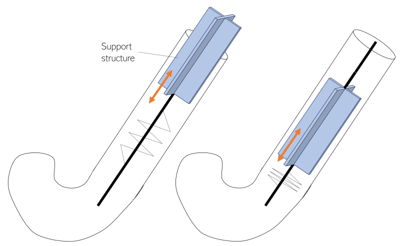


Figure 5.6: Concept 2c - Increasing moment of inertia (L: stopping, R: striking)

5.4. Final design

5.4.1. Overall design

The final design combines several of the mechanisms described in the previous section. The main purpose of the mechanism is to increase the effective mass when striking the ball by lowering the centre of mass, since this property was shown to be most influential from the analysis performed. An overview of the design is shown in Figure 5.8.

Two weights are added in the stick, these weights can move over the length of the stick. The movement is controlled by the centrifugal force when striking a ball and the equilibrium is remained by a spring-damper system. The movement of the two weights towards the head of the stick lowers the centre of mass location of the stick and thereby increases the effective mass of the

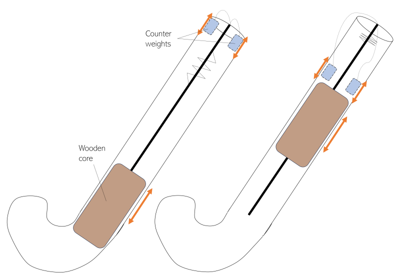


Figure 5.7: Concept 3a - Solid vs hollow (L: stopping, R: striking)

stick when striking a ball. Taking into account the mass range of the measured sticks in Chapter 3, the weights both have a mass of 50 grams, so the total increase in stick head mass is 100 grams for striking.

In addition to a change in effective mass, the shape of the weights also improves the stick performance. The shape supports the internal structure of the stick and thereby decreases bending of the lower part of the stick. Moreover, the shape still allows the stick to be partly hollow, improving the local stiffness properties when striking a ball.

The regular internal structure of a field hockey stick is taken into account. The stick is hollow with one support structure at the centre line; which makes it impossible to place the weights at the centre of the stick. Therefore, two weights are needed to maintain the regular internal structure of the stick.

The movement of the weights is directed by grooves inside the stick (Figure 5.8). The weights are fixed inside these grooves and can only move over the length of the stick. The movement of the weights has to comply with the stick motion; therefore, the dynamics of the striking motion are taken into account when determining the parameters of the spring damper system. This is described in the next section.

The dimensions of the weights are limited to the stick dimensions. The maximum diameter of the stick is defined by the International Hockey Federation and is limited to $51mm$; therefore, the maximum height of the weights is assumed to be $18mm$ mm, taking into account the thickness of the stick and the circular properties of the stick.

The weights have to be small, heavy and adaptable to a specific shape. Concrete can be used in this application, due to its high density and adjustability to specific shapes. It has to be covered by a plastic to make it resistant to high impacts and to increase its lifetime. With the specified height of $18mm$, a width of $10mm$ and a density of $2400kg/m^3$ (Dorf, 2018), the concrete weights have a length of $117mm$.

5.4.2. Spring-damper system specifications

The adaptation mechanism is initiated by the centrifugal force (Hibbeler, 2017). The free-body diagram of the weight is shown in Figure 5.9. The

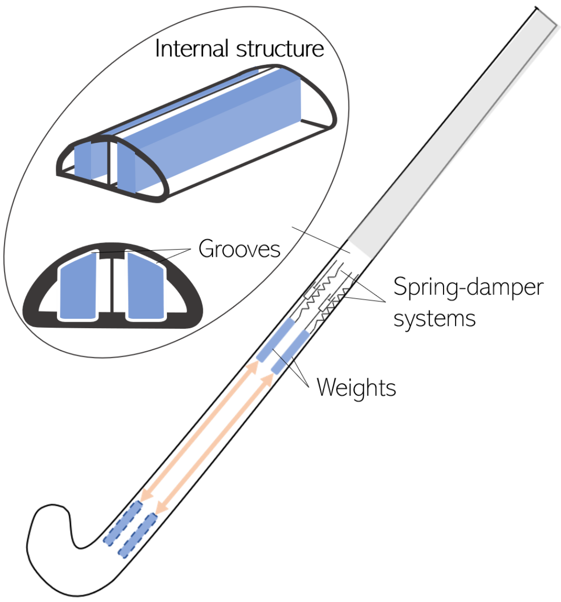


Figure 5.8: Final Design: two internal weights can move over the length of the stick. The motion is guided by the grooves and a spring-damper system.

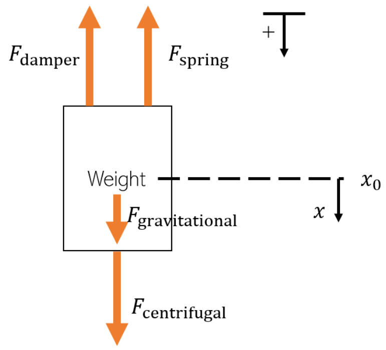


Figure 5.9: Free-body diagram of the weight

forces acting on the weight are:

$$F_{\text{centrifugal}} = m\omega^2 x \tag{5.1}$$
$$F_{\text{gravitational}} = mg \tag{5.2}$$
$$F_{\text{spring}} = k(x - x_0) \tag{5.3}$$
$$F_{\text{damper}} = c\dot{x} \tag{5.4}$$

By which the motion of the weight can be described as follows:

$$m\ddot{x} = m\omega^2 x + mg - k(x - x_0) - c\dot{x} \tag{5.5}$$

Where m is the mass of the weight, x is the distance from the centre of rotation to the position of the weight, ω is the rotational velocity at the location of the weight, and k and c are, respectively, the spring and damper constants of the spring-

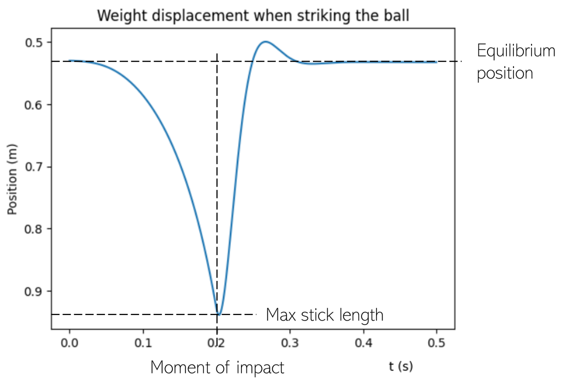


Figure 5.10: Movement of the weight during a hit: the weight moves from the centre of mass location to the head of the stick. The centre of mass location is the equilibrium position of the weight. The end of the handle of the stick is defined as 0.

Symbol	Description	Value
$V_{<0.2\text{s}}\ (m/s)$	Linear stick head velocity first 0.20 seconds	$\frac{V_{max}}{0.2sec}t$
$V_{>0.2\text{s}}\ (m/s)$	Linear stick head velocity after 0.20 seconds	0
$V_{\text{max}}\ (m/s)$	Maximum linear stick head velocity	45.6
$L\ (m)$	Total stick length	0.93
$k\ (N/m)$	Spring stiffness	200
$c\ (Ns/m)$	Damping coefficient	4
$m\ (kg)$	Mass of the weight	0.050
$x_0\ (m)$	Initial position of the weight	L_{COM}
$L_{\text{COM}}\ (m)$	Distance from top to COM	0.53

Table 5.3: Parameters to obtain desired behaviour of the moving weights

damper system. When the object's neutral position is at the stick's centre of mass, it can be assumed that $x_0 = 0.53m$ (average of the centre of mass locations of the sticks in Table 3.1). The angular velocity of the stick head is the linear velocity divided by the total stick length ($\omega = V/L$), where the linear stick head velocity for a hit is approximately $45.6m/s$ (Rai et al., 2002) and the average stick length is $0.93m$ (Table 3.1).

The average stick hit has a duration of 0.20 seconds (Rai et al., 2002) and it is desired that the weights are at their maximum displacement at the moment of impact. The weights should return to their original position when the angular acceleration decreases to zero; this should be a smooth movement without oscillatory behaviour around the equilibrium. Therefore, the values of the spring damper systems should be tuned to obtain the desired behaviour. The tuning is done by solving the following differential equation:

$$\ddot{x} = (\frac{V}{L})^2 x + g - \frac{k}{m}(x - x_0) - \frac{c}{m}\dot{x} \tag{5.6}$$

Where the stick velocity V increases to $45.6m/s$ in the first 0.20 seconds and then becomes zero again. The desired behaviour of the weights is a movement to the head of the stick in 0.20 seconds and a smooth return to the equilibrium position. The desired output as shown in Figure 5.10 is obtained using the parameters shown in Table 5.3, the entire code used can be found in Appendix B.6.

The calculations show that the desired spring and damper properties are constants of $200N/m$ and $4Ns/m$, respectively, with a spring displacement of $0.4m$ when the maximum gravitational force is exerted. Moreover, the spring-damper system cannot contain any metal, should fit inside a hockey stick and should not require maintenance; these requirements are quite specific, and therefore it is not possible to use a general off-the-shelve spring-damper system. The proposed spring-damper system is shown in Figure 5.11, which contains a plastic composite spring and a shock-absorbing foam that functions as a damper. The damper is placed inside the spiral spring; the spring is active over the full length of the system, while the damper is activated when the weight moves into the material. The properties of the plastic material of the spring should be chosen to fit the spring requirements; an example of a plastic composite spring is shown by the Lee plastic spring, which is a lightweight and low-maintenance spring (Lee Spring, n.d.); another example is shown by a slinky, made of plastic and providing a large spring elongation (Owls Hollow, n.d.). When the weight returns to its equilibrium position, by the pulling force of the spring, the shock-absorber dampens the movement and thereby minimises vibrations of the weight around its equilibrium position; an example of a shock absorbing material is the Sorbothane viscoelastic polymer, build out of several layers and with robust properties (Sorbothane, Inc, n.d.). Another optional spring-damper configuration is by using a cord spring (Kalsi Cords, 2020) instead of a spiral spring. For this setup, the damping foam will be cylindrically shaped and the spring cord fits inside the foam cylinder, as shown in Figure 5.12. The exact dimensions of the spring-damper system should still be determined; however, these calculations and the drawing show the feasibility of the system. The spring-damper system is located above the weights and, therefore, the system can span over the full length of the handle if needed, maximising the available space for the system.

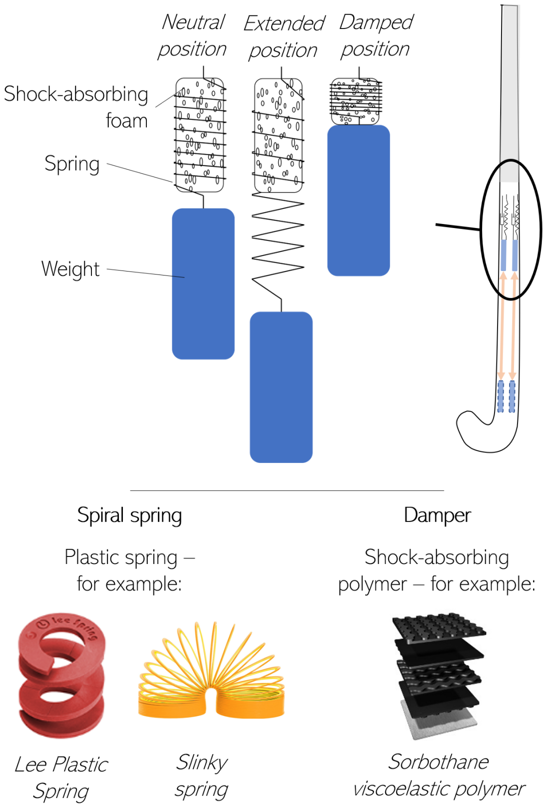


Figure 5.11: The spring-damper system of the moving weights inside the stick, consisting of a spiral plastic composite spring with a core of a shock-absorbing foam. Examples of the spring and damper materials are shown in the bottom figures (Lee Spring - Lee Spring, n.d., Slinky - Owls Hollow, n.d. & Sorbothane - Sorbothane, Inc, n.d.)

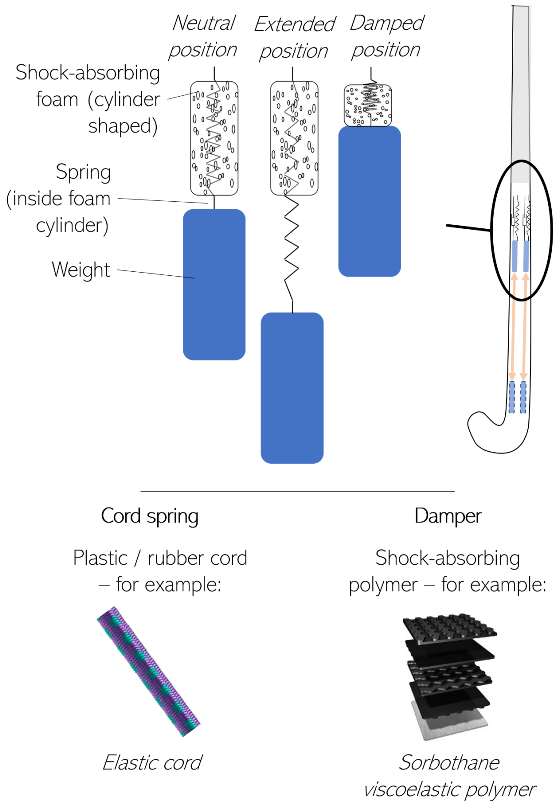


Figure 5.12: The spring-damper system of the moving weights inside the stick, consisting of a rubber/plastic cord spring inside a shock-absorbing foam. Examples of the spring and damper materials are shown in the bottom figures (Cord spring - Kalsi Cords, 2020 & Sorbothane - Sorbothane, Inc, n.d.)

The design will need a quite specific spring-damper system, as it should provide a large elongation over the stick length, it should be robust and low in maintenance, and it cannot contain any metal parts. The spring-damper designs described above show several possibilities for this system; where the plastic composite spring is very robust and the slinky and elastic cord provide large elongation. However, tests and a more extensive material analysis are needed to decide which system would be best for this application.

5.4.3. Expected improvements

The mechanism is expected to improve overall stick performance by facilitating a duality in stick behaviour. When stopping the ball and handling the stick, the stick has a high centre of mass, which allows for easy handling control. Moreover, the stick bends slightly when receiving a ball, making it easier to reduce the kinetic energy of the stick and the ball.

When the stick is accelerated to hit a ball, the two weights move towards the head of the stick and thereby lower the centre of mass of the stick. This creates a higher effective mass and thereby increases the outgoing ball velocity.

The effect of lowering the centre of mass location on the effective mass and thereby the outgoing ball velocity is shown in Figure 5.13. This figure shows the effect of the position of the weights on the outgoing ball velocity, where the outgoing ball velocity is calculated by the collision model described in Chapter 4 by Formula 4.15 with the ball displacement described in Figure 5.10 (for the used code see Appendix B.6). An increase of approximately 7% of the outgoing ball velocity is expected when the weights are maximally displaced. The figure shows that the design also increases the outgoing ball velocity when the weights are not maximally displaced, so also for other hits than the optimal, regular hit (for example a fast hit), the mechanism will improve performance. Since the centrifugal force is dominant in the movement of the stick and the gravitational force is negligible, the improvements are valid not only for hitting (where the stick is lifted from the ground) but for all striking movements by which the stick is angularly accelerated.

However; when the centre of mass is lowered due to the displacement of the weights, the required moment delivered by the muscles also increases. When assuming an average acceleration from zero to 45.6 m/s in 0.20 seconds and neglecting the gravity, the required moment can be calculated as follows:

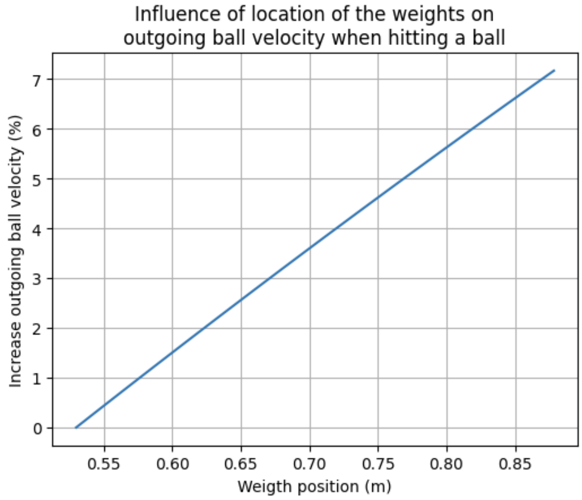


Figure 5.13: The effect of the position of the weights on the outgoing ball velocity, when the weights move from the equilibrium position (stick's centre of mass) to the head of the stick

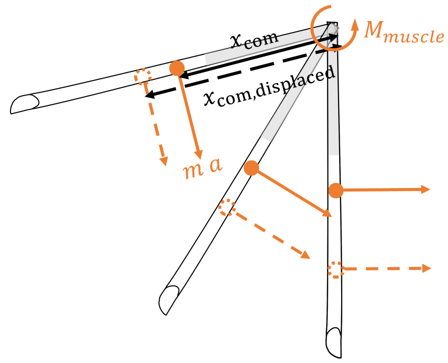


Figure 5.14: The required muscle moment increases as the centre of mass of the sticks moves towards the head of the stick. A regular stick has a constant centre of mass position (illustrated by the solid lines), the adaptive stick design lowers the centre of mass position during the swing (illustrated by the dotted lines)

$$Moment = m_{\text{stick, total}} a_{\text{average}} x_{\text{com}} \quad (5.7)$$

Where the total stick mass $m_{\text{stick, total}}$ is multiplied by the average stick acceleration a_{average} and the moment arm between the top end of the stick and the centre of mass of the stick. For a normal stick, the centre of mass location is constant; for a stick with the adaptive design the centre of mass moves and thereby the moment arm increases. This influence is explained in Figure 5.14. When the adaptive stick is compared to a normal stick, the increase in the average moment needed to accelerate the stick is 2% (for the Python calculations see Appendix B.6); which is small compared to the increase in ball velocity.

Moreover, in addition to the increase in outgoing ball velocity due to the increase in effective mass, the design also decreases the deflection of the stick by the supporting shape of the weights. This will minimise the energy loss due to deformation of the stick and thereby increases the outgoing ball velocity.

6 Discussion

6.1. Limits

For this project, the behaviour of the stick is simplified to stopping and striking a ball. However, as described in the background section (Chapter 2), hockey includes many more actions for which the stick is used. Since this is a first step in the development of an adaptive field hockey stick, stopping and striking are used to simplify the the stick performance as these two actions show the largest contradiction in desired stick behaviour. Moreover, for the measuring and modelling parts (Chapter 3 & 4), only striking is taken into account for the analysis of the stick performance. For the modelling section, the two models consisting of a mass-spring-damper system modelled both striking and stopping, but these systems were found to be invalid. For the final model, based on the collision model, only striking is simulated. However, it would also be interesting to model the stopping dynamics, to identify the stick and ball behaviour when stopping a ball. In contrast, the measurement setup used for the measurements with the Proprio simulates stopping a ball, as the clamps simulate a wide grip used when stopping a ball. This setup is used to be able to clamp the stick correctly to obtain values of the stick properties; however, it would be interesting to create a measurement setup that simulates the stick behaviour when striking a ball, where the clamps simulate a small grip of two hands at the end of the grip. Moreover, for the identification measurements with the Proprio, only the bending properties are measured since the stick and piston were clamped during the experiment. Therefore, in the distance measurements (EXP2.1) only the influence of bending stiffness and damping could be measured, and local properties are not taken into account.

Another simplification of the stick behaviour is that only the stick properties are taken into account, and the human is not implemented in the measurements and mathematical model. The combination of the player and the stick determines the overall performance, and therefore the human has an important role. However, the scope is limited to the stick properties; this limit is valid for this project because the motivation of this project, as described in the introduction, is to make it easier for inexperienced adults to start playing hockey. For this target group, it can be assumed that the human properties have a limited effect on the overall performance and the stick properties are most important. However, when further developing this design, it would be interesting to measure and model the human influence on the stick behaviour, as this will show interesting information to take into account for the design. The human stiffness and damping properties could be measured with the Proprio; for example, the participant can hold the stick with a wide grip (simulating stopping) and a small grip (simulating striking) and try to minimise the displacement when the stick tip is connected to the piston. These values could be applied in a model in which human stiffness and damping properties are also taken into account. Also for the stick performance measurements, it would be interesting to see how the stiffness, damping and mass influence the stick performance when a player strikes and stops a ball.

The analysis and design process only take into account the stiffness, damping and mass properties of the stick. However, there are several other interesting properties identified in the background section; but these were beyond the scope of this project. An interesting stick property to be further analysed is the sweet-spot; this location has a large impact on the stick behaviour. And since this project aims to facilitate easier participation of inexperienced adults, the sweet spot plays an important role in the overall performance of inexperienced players. It would be interesting to identify possibilities to increase this sweet spot area.

Some small improvements could improve the performed measurements. Firstly, the stick measurements were performed on a set of six used sticks; however, it would be interesting to extend this set by including some new high-performance sticks. These sticks can broaden the range of measured properties, which improves the identification of the stick properties. Secondly, for future measurements with the Proprio, it is advised to use a logarithmic disturbance, since the signal-to-noise ratio is low for stiff sticks at high frequency.

Lastly, the design has some limits to take into account. The design thus far has only been developed based on drawings and calculations. For the next steps, it is important to develop prototypes and test the design. Another important development of the design is the manufacturability of the mechanism inside the stick. Since available information on the manufacturing process is scarce, it may be interesting to get in contact with manufacturers and discuss possibilities.

6.2. Limitations

This project includes some limitations, which should be taken into account when interpreting the conclusions drawn from this research.

First of all, due to the timing of the project, the stick performance experiments measuring striking distance (Chapter 3 - EXP2), were performed in January. Therefore, the temperature was very low and the field was frozen when the experiments were performed. The temperature could have influenced the stick properties and the ball distance after the hit; and the temperature slightly increased over the measurements. It is advised to redo the measurements when the temperature is more constant and around 20 degrees Celsius. Additionally, it is advised to do more trials per stick, to obtain better measurement results. Moreover, the measurement setup can be improved by making it stiffer to reduce noise in the measurements.

In addition, limitations are shown in the analysis of the stiffness and damping properties of the stick. Firstly, for the measurements performed on the influence of the stiffness and damping properties on stick performance (Chapter 3 - EXP2.1), only the influence of the combination of stiffness and damping could be measured. It would be interesting to do measurements where only the stiffness is increased and where only the damping is increased. However, since these are predefined stick properties, is was not possible to perform these measurements. Therefore, the influence of these properties was simulated by mathematical models; however, the mass-spring-damper systems did not show valid results and the influence of the stiffness and damping is modelled by the coefficient of restitution. Consequently, the influence of the separate stiffness and damping properties could not be properly analysed. Moreover, the COR only shows a rough estimation of the influence of stiffness and damping; however, the real dynamics of the stick stiffness and damping are not included in the model. It would be interesting to further develop the model, where the stiffness and damping properties are represented more accurately.

The proposed design is still very simplified and should be developed further. First of all, it should become more detailed by defining materials, dimensions and masses of the parts. As all parts need to be lightweight, robust, small and no metal can be used, a more extensive material research is needed in order to find the optimal materials for the weights and the spring-damper system. Moreover, the mass of the added weights is based on the masses of the set of measured sticks, but a further analysis by testing and modelling is needed to find the optimal mass of the weights. Lastly, simplifications should be taken out of the design; for example, friction between the weights and the stick shell are not taken into account, which will have an important role in the movement of the weights and thereby the improvement on stick performance.

7 Conclusion

The objective of this project was *to identify the properties that influence stick performance and to design a field hockey stick with adaptable properties to improve stick performance*, where performance is defined as *the ability of the stick to develop a high ball velocity when hitting a ball, and to provide proper control when stopping the ball*.

Literature on field hockey sticks is scarce; therefore, a literature research into similar hand-held sports equipment is performed. Hereby, stiffness, damping and mass were identified as interesting properties; these properties are present locally, at the impact location, and over the full stick length, as bending properties originating from the moment due to the force at impact location. To be able to classify the sticks based on these properties, measurements were required to obtain numerical values of the bending stiffness and damping properties of a set of six sticks. These measurements were performed using the Proprio, which applies a disturbance force and measures the displacement.

Measurements with the six sticks and simulations with a mathematical model showed the influence of the total mass, effective mass and bending stiffness and damping properties on the stick performance. It could be concluded that the mass, mainly the centre of mass location, has the biggest influence; and that the desired properties are opposite for striking power and stopping control.

This showed the opportunity for an adaptive stick design, for which the stick properties change between striking and stopping a ball. Concluding from a design process, a stick with an adaptive effective mass is proposed. This stick contains a mechanism that reacts to angular acceleration when the striking a ball; the resulting centrifugal force acts on two weights inside the stick. These weights can move over the length of the stick and will move toward the stick head when striking. This increases the effective stick mass by lowering the centre of mass location. When the angular acceleration decreases to zero again, the weights return to their original position by a spring-damper system. An increase of 7% in ball velocity after hitting the ball is expected, compared to the original effective mass.

The design requires further development, the next steps should include a prototyping phase in which the design develops and becomes more specified. It is also important to take into account the player, as this report only focusses on the stick. In addition, a broader research into other stick properties may result in interesting insights.

To conclude, this report shows interesting information on the stiffness, damping and mass properties of field hockey sticks, obtained by several measurements and a mathematical model. This research is used to develop an adaptive stick design improving stick performance; this is proven theoretically, and now needs to be further developed and tested to show measurable improvements. Such an adaptive stick could encourage inexperienced adults to start playing hockey; participating in a team sport is beneficial for their mental health and overall well-being.

Bibliography

Allen, T., Choppin, S., & Knudson, D. (2015). A review of tennis racket performance parameters. *Sports Engineering*, *19*(1), 1–11. https://doi.org/10.1007/s12283-014-0167-x

Allen, T., Foster, L., Carré, M., & Choppin, S. (2012). Characterising the impact performance of field hockey sticks. *Sports Engineering*, *15*(4), 221–226. https://doi.org/10.1007/s12283-012-0099-2

Andersen, M. H., Ottesen, L., & Thing, L. F. (2019). The social and psychological health outcomes of team sport participation in adults: An integrative review of research [PMID: 30113260]. *Scandinavian Journal of Public Health*, *47*(8), 832–850. https://doi.org/10.1177/1403494818791405

Betzler, N. F., Monk, S. A., Wallace, E. S., & Otto, S. R. (2012). Effects of golf shaft stiffness on strain, clubhead presentation and wrist kinematics. *Sports Biomechanics*, *11*(2), 223–238. https://doi.org/10.1080/14763141.2012.681796

Carlisle, G. N. (2011, August). Stiffness variation in hockey sticks and the impact on stick performance. http://etheses.bham.ac.uk/3716/

Caro, F., & Carmichael, M. (2024). A review of mechanisms to vary the stiffness of laminar jamming structures and their applications in robotics. *Actuators*, *13*. https://doi.org/10.3390/act13020064

Covill, D., Farr, J., Katz, T., & White, D. (2009, January). *Use of static stiffness behaviour to characterise field hockey sticks (P185)* (Vol. 2). Springer. https://doi.org/10.1007/978-2-287-99056-4\{_}29

Cross, R. (2013). Impact of sports balls with striking implements. *Sports Engineering*, *17*(1), 3–22. https://doi.org/10.1007/s12283-013-0132-0

De Vlugt, E., Schouten, A. C., & Van Der Helm, F. C. (2006). Quantification of intrinsic and reflexive properties during multijoint arm posture. *Journal of Neuroscience Methods*, *155*(2), 328–349. https://doi.org/10.1016/j.jneumeth.2006.01.022

De Vlugt, E., Schouten, A. C., Van Der Helm, F. C., Teerhuis, P. C., & Brouwn, G. G. (2003). A force-controlled planar haptic device for movement control analysis of the human arm. *Journal of Neuroscience Methods*, *129*(2), 151–168. https://doi.org/10.1016/s0165-0270(03)00203-6

Diefenbach, B. J., & Lipps, D. B. (2019). Postural differences in shoulder dynamics during pushing and pulling. *Journal of Biomechanics*, *85*, 67–73. https://doi.org/10.1016/j.jbiomech.2019.01.005

Dorf, R. C. (2018, October). *The Engineering Handbook* (2nd ed.). CRC Press. https://doi.org/10.1201/9781315220338

FIH. (2023). *Rules of hockey* (tech. rep.). The International Hockey Federation. https://www.fih.hockey/static-assets/pdf/fih-rules-of-hockey-June23-update.pdf

Hibbeler, R. C. (2017). *Engineering Mechanics Dynamics* (14th ed.). Pearson.

Iulian-Doru, T., & Maria, T. (2013). Leisure sports activities impact on adults personal development and quality of life [The 3rd World Conference on Psychology, Counseling and Guidance, WCPCG-2012]. *Procedia - Social and Behavioral Sciences*, *84*, 1090–1094. https://doi.org/https://doi.org/10.1016/j.sbspro.2013.06.705

Kalsi Cords. (2020, October). Kalsi Cords | 5mm Purple & Green Round Elastic Bungee Shock Cord. https://www.kalsicords.co.uk/product/5mm-purple-green-round-elastic-bungee-shock-cord/

Kawazoe, Y. (2002). Mechanism of High-Tech Tennis Rackets performance. *Theoretical and applied mechanics Japan*, *51*, 177–187. https://doi.org/10.11345/nctam.51.177

KNHB. (n.d.). Trimhockey - KNHB. https://www.knhb.nl/kenniscentrum/hockeysoorten/trimhockey

KNHB. (2023). *De hockeystick* (tech. rep.). Koninklijke Nederlandse Hockey Bond. Retrieved June 4, 2024, from https://hockey.nl/de-sport/materiaal/de-hockeystick/

KNLTB. (2024). *Tennisspelregels* (tech. rep.). Koninklijke Nederlandse Lawn Tennis Bond. www.knltb.nl

Krishnan, R. H., Devanandh, V., Brahma, A. K., & Pugazhenthi, S. (2016). Estimation of mass moment of inertia of human body, when bending forward, for the design of a self-transfer robotic facility. *Journal of En-*

gineering Science and Technology, *11*(2), 166–176.

Lee Spring. (n.d.). Plastic Composite Compression Springs from Lee Spring. https://www.leespring.com/plastic-springs-available

Miller, S. (2006). Modern tennis rackets, balls, and surfaces. *British Journal of Sports Medicine*, *40*(5), 401–405. https://doi.org/10.1136/bjsm.2005.023283

Nicholls, R. L., Elliott, B. C., & Miller, K. (2004). Impact injuries in baseball: Prevalence, aetiology and the role of equipment performance. *Sports Medicine*, *34*(1), 17–25. https://doi.org/10.2165/00007256-200434010-00003

Nicholls, R. L., Miller, K., & Elliott, B. C. (2005). Numerical analysis of maximal bat performance in baseball. *Journal of Biomechanics*, *39*(6), 1001–1009. https://doi.org/10.1016/j.jbiomech.2005.02.020

Overweel, M. R. (2024). Evaluation of the suitability of adaptive stiffness and/or damping mechanisms for application in a hockey stick - A systematic review. *(unpublished)*.

Owls Hollow. (n.d.). Slinky plastic version. https://www.owlshollow.com/slinky-plastic-version.html

Peaden, A. W., & Charles, S. K. (2014). Dynamics of wrist and forearm rotations. *Journal of Biomechanics*, *47*(11), 2779–2785. https://doi.org/10.1016/j.jbiomech.2014.01.053

Rai, R., Bhangu, G. S., Mohanty, S. K., & Goel, A. (2002). Kinematic and Temporal Evaluation of Swings, Stick Length and their Interaction in Field Hockey. *Medicine & Science in Sports & Exercise*, *34*(5), 18. https://doi.org/10.1097/00005768-200205001-01716

Solo Hockey / ClubColors. (n.d.). Solo Hockey – For teamplayers. https://solohockey.nl/

Sorbothane, Inc. (n.d.). Shock absorbing foam. https://www.sorbothane.com/technical-data/articles/shock-absorbing-foam/

Taraborrelli, L., Grant, R., Sullivan, M., Choppin, S., Spurr, J., Haake, S., & Allen, T. (2019). Recommendations for estimating the moments of inertia of a tennis racket. *Sports Engineering*, *22*(1), 1–9. https://doi.org/10.1007/s12283-019-0303-8

Vallery, H., & Schwab, A. L. (2021). *Advanced Dynamics* (3rd ed.). Stichting Newton-Euler.

Van Der Helm, F. C. T., Schouten, A. C., De Vlugt, E., & Brouwn, G. G. (2002). Identification of intrinsic and reflexive components of human arm dynamics during postural control. *Journal of Neuroscience Methods*, *119*(1), 1–14. https://doi.org/10.1016/s0165-0270(02)00147-4

Wibbens, B. P. (2015). Design of Field Hockey Sticks. *TU Delft repository*. https://repository.tudelft.nl/islandora/object/uuid:9f19bba6-a7fc-4281-98b5-648f95fdedbe?collection=research

A
Proprio data processing

MatLab is used to process the raw data of the measurements, the entire code can be found in Appendix B.2, this code is based on the available codes of the Proprio. The code first averages the data of the eight trials; this data is used to obtain the transfer functions of the combined system, the manipulator and the stick. The first step is taking the Fourier transform of the data as follows:

$$W = fft(w) \tag{A.1}$$

$$W = W(2 : N/2 + 1) \tag{A.2}$$

The same is done for F, X, and Z. Where w, f, x and z are the disturbance force, reaction force, displacement and difference between the disturbance and reaction force signals, respectively. N equals 2^{16}, which is equal to one repetition of a full multisine signal. W takes the Fourier transform of the signal and thereby translates the signal to the frequency domain. Only the frequencies within the Nyquist frequency are taken to be further analysed. The signals are multiplied by the conjugate of the input signal W, this is done to minimise errors by normalising it around the errorless input signal.

$$G_{\text{ww}} = conj(W)W \tag{A.3}$$

$$G_{\text{wf}} = conj(W)F \tag{A.4}$$

$$G_{\text{wx}} = conj(W)X \tag{A.5}$$

$$G_{\text{wz}} = conj(W)Z \tag{A.6}$$

$$G_{\text{xx}} = conj(X)X \tag{A.7}$$

These signals are averaged over a number of bands to minimise noise, which gives mG_{ww}, mG_{wf}, mG_{wx}, mG_{wz} and mG_{xx}. The number of bands increases from four for the low frequencies to 64 for the high frequencies; because the transfer functions are plotted over a logarithmic scale, and therefore the high frequencies have more measurement points than the low frequencies. These averaged signals are taken to calculate the final transfer functions and coherence.

$$H_{\text{stick}} = \frac{-mG_{\text{wx}}.}{mG_{\text{wf}}} \tag{A.8}$$

$$H_{\text{manipulator}} = \frac{mG_{\text{wx}}.}{mG_{\text{wz}}} \tag{A.9}$$

$$H_{\text{cobined}} = \frac{mG_{\text{wx}}.}{mG_{\text{ww}}} \tag{A.10}$$

$$Coherence = \frac{|mG_{\text{wx}}^2|}{mG_{\text{ww}}.mG_{\text{xx}}} \tag{A.11}$$

The gain, phase and coherence plot of each stick are shown in Figures A.1 to A.6.

To determine the numerical values for the stiffness, damping and mass of the hockey sticks, a linear-mass-spring-damper model is fit to the measured data. This is done by minimizing an error function as described in Chapter 3.

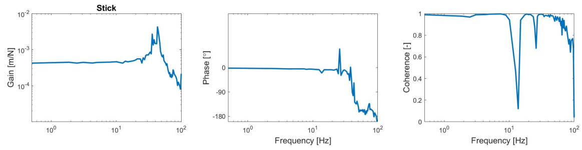


Figure A.1: Gain, phase and coherence plot of stick 1FC

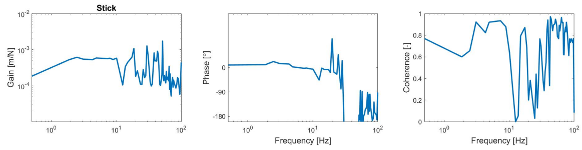


Figure A.2: Gain, phase and coherence plot of stick 2FC

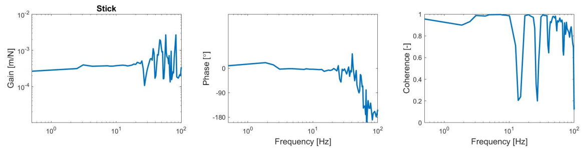


Figure A.3: Gain, phase and coherence plot of stick 3FC

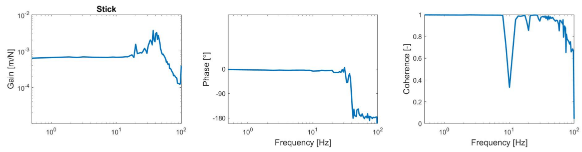


Figure A.4: Gain, phase and coherence plot of stick 4IC

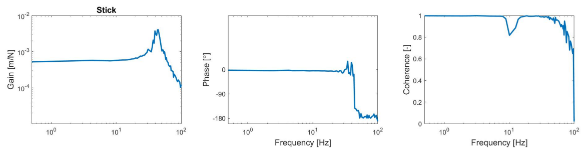


Figure A.5: Gain, phase and coherence plot of stick 5IW

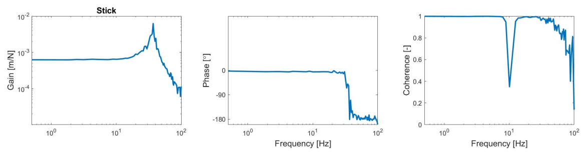


Figure A.6: Gain, phase and coherence plot of stick 6FW

B

MatLab and Python codes

B.1. Introduction

This appendix includes the codes used to process measurement data (B.2 - measurements of the stiffness and damping properties by using the Proprio) and to obtain modelling results (B.3 - calculations of the triple mass-spring-damper model, B.4 - calculations of the double mass-spring-damper model, B.5 - calculations of the collision model and B.6 - calculations of the design results). Both MatLab (B.2) and Python (B.3 - B.6) are used.

B.2. Proprio measurements (EXP1)

This code is based on the available codes of the Proprio.

```
clear all;
clc;

TestNumber = 'Test2'; %Test, Test2
StickNumber = 'Stick1'; %Stick1, Stick2, Stick3, Stick4, Stick5, Stick6

%%%%%%%%%%%%%%%%%%%%%%
%%%%%MEASURED_DATA%%%%
%%%%%%%%%%%%%%%%%%%%%%

%Analyses the data and plots the results

set(0,'DefaultLineLineWidth',2)

N=2^16; % 1 repitition of the full MS signal

FileA = sprintf('C:↙
\\Users\\mover\\Documents\\Master\\Y3\\MasterThesis\\2_Measuring\\Analysis\\Data_An↙
alysis\\%s\\%s\\b0k0a', TestNumber, StickNumber);
FileB = sprintf('C:↙
\\Users\\mover\\Documents\\Master\\Y3\\MasterThesis\\2_Measuring\\Analysis\\Data_An↙
alysis\\%s\\%s\\b0k0b', TestNumber, StickNumber);
FileC = sprintf('C:↙
\\Users\\mover\\Documents\\Master\\Y3\\MasterThesis\\2_Measuring\\Analysis\\Data_An↙
alysis\\%s\\%s\\b0k0c', TestNumber, StickNumber);
FileD = sprintf('C:↙
\\Users\\mover\\Documents\\Master\\Y3\\MasterThesis\\2_Measuring\\Analysis\\Data_An↙
alysis\\%s\\%s\\b0k0d', TestNumber, StickNumber);
FileE = sprintf('C:↙
\\Users\\mover\\Documents\\Master\\Y3\\MasterThesis\\2_Measuring\\Analysis\\Data_An↙
alysis\\%s\\%s\\b0k0e', TestNumber, StickNumber);
FileF = sprintf('C:↙
\\Users\\mover\\Documents\\Master\\Y3\\MasterThesis\\2_Measuring\\Analysis\\Data_An↙
alysis\\%s\\%s\\b0k0f', TestNumber, StickNumber);
FileG = sprintf('C:↙
\\Users\\mover\\Documents\\Master\\Y3\\MasterThesis\\2_Measuring\\Analysis\\Data_An↙
alysis\\%s\\%s\\b0k0g', TestNumber, StickNumber);
FileH = sprintf('C:↙
\\Users\\mover\\Documents\\Master\\Y3\\MasterThesis\\2_Measuring\\Analysis\\Data_An↙
alysis\\%s\\%s\\b0k0h', TestNumber, StickNumber);

data_a = load(FileA);
data_b = load(FileB);
data_c = load(FileC);
data_d = load(FileD);
data_e = load(FileE);
data_f = load(FileF);
data_g = load(FileG);
data_h = load(FileH);

data = load('C:↙
\Users\mover\Documents\Master\Y3\MasterThesis\2_Measuring\Analysis\Data_Analysis\te↙
st_freq_negabs');
```

```
time = data.t;

% Gemiddelde over de 4 trials
mean_w = mean([data_a.w, data_b.w, data_c.w, data_d.w, data_e.w, data_f.w, data_g.↙
w, data_h.w], 2);          % Averaging rows
mean_fc = mean([data_a.fc, data_b.fc, data_c.fc, data_d.fc, data_e.fc, data_f.fc, ↙
data_g.fc, data_h.fc], 2);     % Averaging rows
mean_x = mean([data_a.x, data_b.x, data_c.x, data_d.x, data_e.x, data_f.x, data_g.↙
x, data_h.x], 2);          % Averaging rows

% Taking 1 full MS period
%Created signal is: w=[zeros(20,1);xtmp;xtmp(1:(Ntot-N-40));zeros(20,1)];
%--> we only want the xtmp part, so 20*8 to 20*8 + N
mean_w = mean_w(160:160+N);
mean_fc = mean_fc(160:160+N);
mean_x = mean_x(160:160+N);

matrix = [mean_w, mean_fc, mean_x];

%t=matrix(1,:)';
w=matrix(:,1);
fc=matrix(:,2);
x=matrix(:,3);
z=w+fc;
clear matrix

% The signal looks like a white noise, so a window is added
% When the signal behaves like a proper multisine, this is not needed anymore
N_window = length(w);
window = hann(N_window); % ones(1,N_window)'

% Optionally: normalise around 0, to avoid a power at 0Hz (currently the fit↙
becomes worse by doing this)
% w = w - mean(w);
% fc = fc - mean(fc);
% x = x - mean(x);
% z = z - mean(z);

W=fft(w.*window);   W=W(2:N/2+1);
F=fft(fc.*window);  F=F(2:N/2+1);
X=fft(x.*window);   X=X(2:N/2+1);
Z=fft(z.*window);   Z=Z(2:N/2+1);

Gww=conj(W).*W;
Gwf=conj(W).*F;
Gwx=conj(W).*X;
Gwz=conj(W).*Z;
Gxx=conj(X).*X;
clear W X F Z
fr=(1:(N/2))/(N/2)*2500;

% NrBands=16;
% tem=zeros(NrBands,N/2/NrBands);
```

```
% tem(:)=fr;         mfr=mean(tem).';
% tem(:)=Gww;        mGww=mean(tem).';
% tem(:)=Gwf;        mGwf=mean(tem).';
% tem(:)=Gwx;        mGwx=mean(tem).';
% tem(:)=Gwz;        mGwz=mean(tem).';
% tem(:)=Gxx;        mGxx=mean(tem).';
% %clear fr Gww Gwf Gwx Gwz Gxx
% mn=find(mGww(1:800)>0.05*mean(mGww(1:800)));

%Increasing the number of bands for higher frequencies
Group1 = 1:12;
Group2 = 13:108;
Group3 = 109:1004;
Group4 = 1005:10028;
Group5 = 10029:32684;

NrBands1=4;
NrBands2=8;
NrBands3=16;
NrBands4=32;
NrBands5=64;

N1= 2*length(fr(Group1));
tem=zeros(NrBands1,N1/2/NrBands1);
tem(:)=fr(Group1);       mfr1=mean(tem).';
tem(:)=Gww(Group1);      mGww1=mean(tem).';
tem(:)=Gwf(Group1);      mGwf1=mean(tem).';
tem(:)=Gwx(Group1);      mGwx1=mean(tem).';
tem(:)=Gwz(Group1);      mGwz1=mean(tem).';
tem(:)=Gxx(Group1);      mGxx1=mean(tem).';

N2= 2*length(fr(Group2));
tem=zeros(NrBands2,N2/2/NrBands2);
tem(:)=fr(Group2);       mfr2=mean(tem).';
tem(:)=Gww(Group2);      mGww2=mean(tem).';
tem(:)=Gwf(Group2);      mGwf2=mean(tem).';
tem(:)=Gwx(Group2);      mGwx2=mean(tem).';
tem(:)=Gwz(Group2);      mGwz2=mean(tem).';
tem(:)=Gxx(Group2);      mGxx2=mean(tem).';

N3= 2*length(fr(Group3));
tem=zeros(NrBands3,N3/2/NrBands3);
tem(:)=fr(Group3);       mfr3=mean(tem).';
tem(:)=Gww(Group3);      mGww3=mean(tem).';
tem(:)=Gwf(Group3);      mGwf3=mean(tem).';
tem(:)=Gwx(Group3);      mGwx3=mean(tem).';
tem(:)=Gwz(Group3);      mGwz3=mean(tem).';
tem(:)=Gxx(Group3);      mGxx3=mean(tem).';

N4= 2*length(fr(Group4));
tem=zeros(NrBands4,N4/2/NrBands4);
tem(:)=fr(Group4);       mfr4=mean(tem).';
tem(:)=Gww(Group4);      mGww4=mean(tem).';
```

```
tem(:)=Gwf(Group4);     mGwf4=mean(tem).';
tem(:)=Gwx(Group4);     mGwx4=mean(tem).';
tem(:)=Gwz(Group4);     mGwz4=mean(tem).';
tem(:)=Gxx(Group4);     mGxx4=mean(tem).';

N5= 2*length(fr(Group5));
tem=zeros(NrBands5,N5/2/NrBands5);
tem(:)=fr(Group5);      mfr5=mean(tem).';
tem(:)=Gww(Group5);     mGww5=mean(tem).';
tem(:)=Gwf(Group5);     mGwf5=mean(tem).';
tem(:)=Gwx(Group5);     mGwx5=mean(tem).';
tem(:)=Gwz(Group5);     mGwz5=mean(tem).';
tem(:)=Gxx(Group5);     mGxx5=mean(tem).';
clear fr Gww Gwf Gwx Gwz Gxx
clear Group1 Group2 Group3 Group4 Group5
clear N1 N2 N3 N4 N5
clear NrBands1 NrBands2 NrBands3 NrBands4 NrBands5

mfr = [mfr1; mfr2; mfr3; mfr4; mfr5];
mGww = [mGww1; mGww2; mGww3; mGww4; mGww5];
mGwf = [mGwf1; mGwf2; mGwf3; mGwf4; mGwf5];
mGwx = [mGwx1; mGwx2; mGwx3; mGwx4; mGwx5];
mGwz = [mGwz1; mGwz2; mGwz3; mGwz4; mGwz5];
mGxx = [mGxx1; mGxx2; mGxx3; mGxx4; mGxx5];
clear mfr1 mfr2 mfr3 mfr4 mfr5
clear mGww1 mGww2 mGww3 mGww4 mGww5
clear mGwf1 mGwf2 mGwf3 mGwf4 mGwf5
clear mGwx1 mGwx2 mGwx3 mGwx4 mGwx5
clear mGwz1 mGwz2 mGwz3 mGwz4 mGwz5
clear mGxx1 mGxx2 mGxx3 mGxx4 mGxx5

clear Gww Gwf Gwx Gwz Gxx
mn=find(mGww(1:355)>0.05*mean(mGww(1:355)));

Hstick=-mGwx./mGwf;
Hmanip=mGwx./mGwz;
Htot=mGwx./mGww;
Cohwx=abs(mGwx.^2)./(mGww.*mGxx);

%Save figures and values in general folder
foldername = fullfile('Processed_Data',TestNumber);
filename_1 = 'Bode_1';
filename_2 = 'Bode_2';
filename_3 = 'Fit';
filename_4 = 'p_sol';

figure(1)
set(1,'Position',[20 60 1450 300]);
subplot(131),loglog(mfr(mn),abs(Hstick(mn))), hold on
axis([0.5 100 1e-5 1e-2]), ylabel('Gain [m/N]','FontSize',13)
title('Stick','FontSize',13)
subplot(132),semilogx(mfr(mn),unwrap(angle(Hstick(mn)))*180/pi), hold on %unwrap
(angle(Hstick(mn))) can give a strange output
```

```
axis([0.5 100 -200 200]), set(gca,'YTick',[-180 -90 0]), ylabel('Phase
[\circ]','FontSize',13)
xlabel('Frequency [Hz]','FontSize',13)
subplot(133),semilogx(mfr(mn),Cohwx(mn)), hold on
axis([0.5 100 0 1]), ylabel('Coherence [-]','FontSize',13)
xlabel('Frequency [Hz]','FontSize',13)
orient landscape
%saveas(figure(1), fullfile(foldername, [StickNumber, '_', filename_1, '.jpg']));

figure(2)
set(2,'Position',[20 60 1200 700]);
subplot(331),loglog(mfr(mn),abs(Hstick(mn))), hold on
axis([0.5 100 1e-5 1e-2]), ylabel('Gain [m/N]','FontSize',13)
title('Stick','FontSize',13)
subplot(334),semilogx(mfr(mn),unwrap(angle(Hstick(mn)))*180/pi), hold on %unwrap
(angle(Hstick(mn)))
axis([0.5 100 -200 200]), set(gca,'YTick',[-180 -90 0]), ylabel('Phase
[\circ]','FontSize',13)
xlabel('Frequency [Hz]','FontSize',13)
subplot(332),loglog(mfr(mn),abs(Hmanip(mn))), hold on
axis([0.5 100 1e-5 1e-2])
title('Manipulator','FontSize',13)
subplot(335),semilogx(mfr(mn),(angle(Hmanip(mn)))*180/pi), hold on
axis([0.5 100 -200 200]), set(gca,'YTick',[-180 -90 0])
xlabel('Frequency [Hz]','FontSize',13)
subplot(333),loglog(mfr(mn),abs(Htot(mn))), hold on
axis([0.5 100 1e-5 1e-2])
title('Combined','FontSize',13)
subplot(336),semilogx(mfr(mn),unwrap(angle(Htot(mn)))*180/pi)
hold on, axis([0.5 100 -200 200]), set(gca,'YTick',[-180 -90 0])
subplot(339),semilogx(mfr(mn),Cohwx(mn)), hold on
axis([0.5 100 0 1]), ylabel('Coherence [-]','FontSize',13)
xlabel('Frequency [Hz]','FontSize',13)
orient landscape
%saveas(figure(2), fullfile(foldername, [StickNumber, '_', filename_2, '.jpg']));
%%

%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%DATA_FIT%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%

% p = [m b k].';
% m, mass [N*s^2/m]
% b, damping [N*s/m]
% k, stiffness [N/m]

Hstick_mn= Hstick(mn);
mfr_mn = mfr(mn);
Cohwx_mn = Cohwx(mn);

% Initial values for the optimisation
p_start = [1,10,1500];
p_min = [0,0,500];
```

```
p_max = [5,50,6000];

%optimize
options=optimset('Display','iter','MaxIter',1000,'TolFun',1e-10,'DiffMinChange',1e-↙
15, 'TolX', 1e-10, 'maxfuneval', 10000);            %Extra specifications for↙
lsqnonlin
[psol, ~, e, ~, ~, ~, J]=lsqnonlin('Error_mod_MCK',p_start,p_min,p_max,options,↙
Hstick_mn,mfr_mn,Cohwx_mn);                              %Performs the data fir

[error, H_fit] = Error_mod_MCK(psol, Hstick_mn, mfr_mn, Cohwx_mn);

%Manual fitting
% m = 0.0268;
% c = 2.6383;
% k = 2378.9;
% s=2*pi*1j*mfr_mn;
% s2=-(4*pi^2*mfr_mn.^2);    % = s.^2 !!
% H_manfit = 1./(m*s2+c.*s+k);
%
% psol_manfit = [m, c, k];

figure(3)
set(3,'Position',[20 60 1200 700]);
loglog(mfr_mn, abs(Hstick_mn)); hold on;
loglog(mfr_mn, abs(H_fit));
%loglog(mfr_mn, abs(H_manfit));
axis([0.5 100 1e-5 1e-2]), ylabel('Gain [m/N]','FontSize',13), xlabel('Frequency↙
[Hz]','FontSize',13)
title('Stick','FontSize',13)
orient landscape
%saveas(figure(3), fullfile(foldername, [StickNumber, '_', filename_3, '.jpg']));
%saveas(figure(3), fullfile(foldername, [StickNumber, '_', filename_3, '_manfit',↙
'.jpg']));

display(psol)
%mean(error)

%save(fullfile(foldername, [StickNumber, '_', filename_4]), 'psol');
%save(fullfile(foldername, [StickNumber, '_', filename_4, '_manfit']),↙
'psol_manfit');
```

```
function [e, Hest] = Error_mod_MCK(p, frf, fvec, coh)
    M = p(1);
    C = p(2);
    K = p(3);

    s=2*pi*1j*fvec;
    s2=-(4*pi^2*fvec.^2);     % = s.^2

    Hest= 1./(M*s2+C.*s+K);

    %e = abs(log(frf./Hest)); %%a
    %e = sqrt(coh./(1+fvec)).*abs(log(frf./Hest)); %%b
    e = sqrt(1./fvec).*abs(log(frf./Hest)); %%c

end
```

B.3. Triple mass-spring-damper model

Setup

The motion of the human-stick-ball model is simulated as a triple mass spring damper model with 3 DOF, and is described by 3 equations of motion.

In [1]:
```
from scipy.integrate import odeint
from pylab import figure, plot, xlabel, ylabel, grid, legend, title
from matplotlib.font_manager import FontProperties
import numpy as np

import matplotlib.pyplot as plt
```

Parameters and initial conditions

Defined for 6 conditions, being combinatations of one of the following two actions:

_1 = stopping the ball

_2 = striking the ball

And one of the following three player + stick combinations:

_a = experienced player + advanced stick

_b = inexperienced player + advanced stick

_c = inexperienced player + beginner stick

In [2]:
```
###Defining values
## Parameter values
# Masses:
mass_human = 74
mass_stick = 0.537
mass_ball = 0.16

m_human = 0.5*mass_human
m_stick = mass_stick + 2*(0.03+0.0018)*mass_human
m_ball = mass_ball

## Initial conditions
# Positions
x_human_0 = 0
x_stick_0 = 0
x_ball_0 = 0
# Velocities - condition 1 = stopping
dx_human_0_1 = 0.0
dx_stick_0_1 = 0.0
dx_ball_0_1 = -22.22
# Velocities - condition 2 = striking
dx_human_0_2 = 22.22
dx_stick_0_2 = 22.22
dx_ball_0_2 = 0.0

#Constants - condition 1 = stopping
# Spring constants
k_human_1a = 1083.09
k_human_1b = 495.77
k_human_1c = 495.77

k_wrist_1a = 116.12
k_wrist_1b = 13.93
k_wrist_1c = 13.93

k_stick_1a = 3000
k_stick_1b = 3000
k_stick_1c = 1400

# Damper constants
c_human_1a = 54.15
c_human_1b = 24.79
c_human_1c = 24.79

c_wrist_1a = 2.32
c_wrist_1b = 0.14
c_wrist_1c = 0.14

c_stick_1a = 0.5
c_stick_1b = 0.5
c_stick_1c = 2.7

#Constants - condition 2 = striking
# Spring constants
k_human_2a = 495.77
k_human_2b = 495.77
k_human_2c = 495.77

k_wrist_2a = 13.93
k_wrist_2b = 13.93
k_wrist_2c = 13.93

k_stick_2a = 3000
k_stick_2b = 3000
k_stick_2c = 1400

# Damper constants
c_human_2a = 24.79
c_human_2b = 24.79
c_human_2c = 24.79

c_wrist_2a = 0.14
c_wrist_2b = 0.14
c_wrist_2c = 0.14

c_stick_2a = 0.5
c_stick_2b = 0.5
c_stick_2c = 2.7
```

In [3]:
```
#Defining the equations of motion

def eom(q, t, p):
    """
    q :  generalised coordinates + derivatives:
        q = [x_human, x_stick, x_ball, dx_human, dx_stick, dx_ball]
```

```
    t :  time
    p :  parameters:
        p = [m_huamn, m_stick, m_ball, c_human, c_wrist, c_stick, k_human, k_wrist, k_stick]"""
    x_human, x_stick, x_ball, dx_human, dx_stick, dx_ball = q
    m_human, m_stick, m_ball, c_human, c_wrist, c_stick, k_human, k_wrist, k_stick = p

    # Define f = (x_human', x_stick', x_ball', dx_human', dx_stick', dx_ball'):
    f = [dx_human,
    dx_stick,
    dx_ball,
    (- c_human * dx_human  - k_human * x_human + c_wrist * (dx_stick - dx_human) + k_wrist * (x_stick - x_human)) / m_human,
    (- c_wrist * (dx_stick - dx_human) - k_wrist * (x_stick - x_human) + c_stick * (dx_ball - dx_stick) + k_stick * (x_ball - x_stick)) / m_stick,
    (- c_stick * (dx_ball - dx_stick) - k_stick * (x_ball - x_stick)) / m_ball]
    return f
```

Motion in time

Defined for the 6 defined conditions

By using odeint

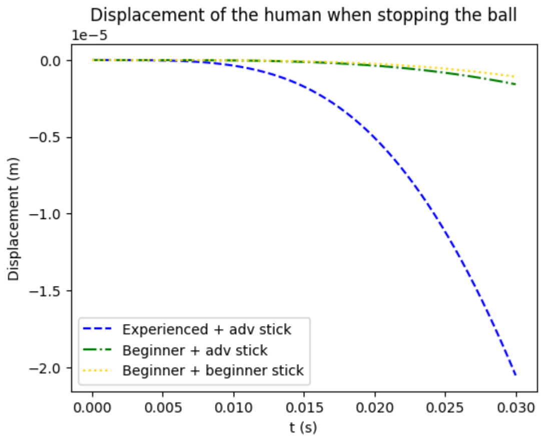
In [4]:
```
#Solve the ODE / EOM
# ODE solver parameters
abserr = 1.0e-8
relerr = 1.0e-6
stoptime = 0.03
numpoints = 250

# Time samples for the output of the ODE solver.
t = [stoptime * float(i) / (numpoints - 1) for i in range(numpoints)]

# Parameters and initial conditions:
# Stopping
params_1 = [
    [m_human, m_stick, m_ball, c_human_1a, c_wrist_1a, c_stick_1a, k_human_1a, k_wrist_1a, k_stick_1a],
    [m_human, m_stick, m_ball, c_human_1b, c_wrist_1b, c_stick_1b, k_human_1b, k_wrist_1b, k_stick_1b],
    [m_human, m_stick, m_ball, c_human_1c, c_wrist_1c, c_stick_1c, k_human_1c, k_wrist_1c, k_stick_1c],
]

q0_1 = [x_human_0, x_stick_0, x_ball_0, dx_human_0_1, dx_stick_0_1, dx_ball_0_1]

#Striking
params_2 = [
    [m_human, m_stick, m_ball, c_human_2a, c_wrist_2a, c_stick_2a, k_human_2a, k_wrist_2a, k_stick_2a],
    [m_human, m_stick, m_ball, c_human_2b, c_wrist_2b, c_stick_2b, k_human_2b, k_wrist_2b, k_stick_2b],
    [m_human, m_stick, m_ball, c_human_2c, c_wrist_2c, c_stick_2c, k_human_2c, k_wrist_2c, k_stick_2c],
]

q0_2 = [x_human_0, x_stick_0, x_ball_0, dx_human_0_2, dx_stick_0_2, dx_ball_0_2]

#Results from solving the EOM
#Stopping
x_human_1 = []
x_stick_1 = []
x_ball_1 = []

for p_1 in params_1:
    result = odeint(eom, q0_1, t, args=(p_1,), atol=abserr, rtol=relerr)
    x_human_1.append(result[:, 0])
    x_stick_1.append(result[:, 1])
    x_ball_1.append(result[:, 2])

#Striking
x_human_2 = []
x_stick_2 = []
x_ball_2 = []

for p_2 in params_2:
    result = odeint(eom, q0_2, t, args=(p_2,), atol=abserr, rtol=relerr)
    x_human_2.append(result[:, 0])
    x_stick_2.append(result[:, 1])
    x_ball_2.append(result[:, 2])
```

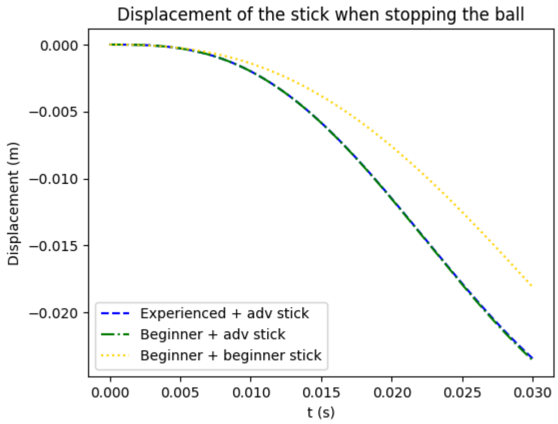
Plotting the displacement over time

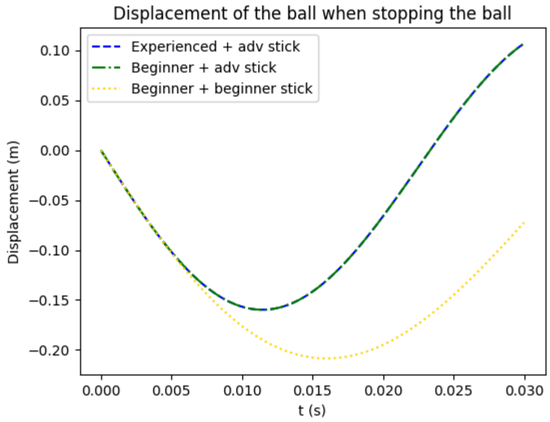
In [5]:
```
#Plot
#Stopping the ball, for the 3 defined conditions
figure(1, figsize=(6, 4.5))
xlabel('t (s)')
ylabel('Displacement (m)')
plot(t, x_human_1[0], 'b', linestyle='--', label = 'Experienced + adv stick')
plot(t, x_human_1[1], 'g', linestyle='-.', label = 'Beginner + adv stick')
plot(t, x_human_1[2], 'gold', linestyle=':', label = 'Beginner + beginner stick')
title('Displacement of the human when stopping the ball')
legend()

figure(2, figsize=(6, 4.5))
xlabel('t (s)')
ylabel('Displacement (m)')
plot(t, x_stick_1[0], 'b', linestyle='--', label = 'Experienced + adv stick')
plot(t, x_stick_1[1], 'g', linestyle='-.', label = 'Beginner + adv stick')
plot(t, x_stick_1[2], 'gold', linestyle=':', label = 'Beginner + beginner stick')
title('Displacement of the stick when stopping the ball')
legend()

figure(3, figsize=(6, 4.5))
xlabel('t (s)')
ylabel('Displacement (m)')
plot(t, x_ball_1[0], 'b', linestyle='--', label = 'Experienced + adv stick')
plot(t, x_ball_1[1], 'g', linestyle='-.', label = 'Beginner + adv stick')
plot(t, x_ball_1[2], 'gold', linestyle=':', label = 'Beginner + beginner stick')
title('Displacement of the ball when stopping the ball')
legend()

#Striking the ball, for the 3 defined conditions
figure(4, figsize=(6, 4.5))
xlabel('t (s)')
ylabel('Displacement (m)')
plot(t, x_human_2[0], 'b', linestyle='--', label = 'Experienced + adv stick')
plot(t, x_human_2[1], 'g', linestyle='-.', label = 'Beginner + adv stick')
plot(t, x_human_2[2], 'gold', linestyle=':', label = 'Beginner + beginner stick')
title("Displacement of the human when striking the ball")
legend()

figure(5, figsize=(6, 4.5))
```

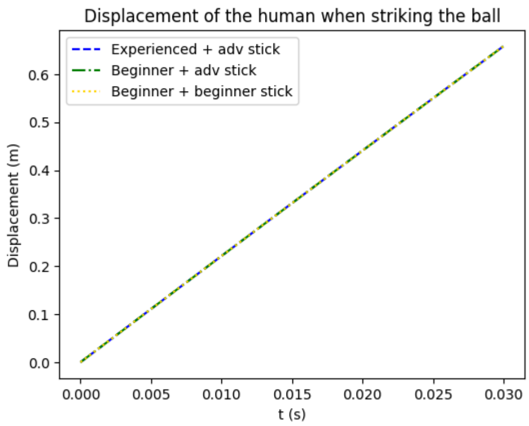
```
xlabel('t (s)')
ylabel('Displacement (m)')
plot(t, x_stick_2[0], 'b', linestyle='--', label = 'Experienced + adv stick')
plot(t, x_stick_2[1], 'g', linestyle='-.', label = 'Beginner + adv stick')
plot(t, x_stick_2[2], 'gold', linestyle=':', label = 'Beginner + beginner stick')
title("Displacement of the stick when striking the ball")
legend()

figure(6, figsize=(6, 4.5))
xlabel('t (s)')
ylabel('Displacement (m)')
plot(t, x_ball_2[0], 'b', linestyle='--', label = 'Experienced + adv stick')
plot(t, x_ball_2[1], 'g', linestyle='-.', label = 'Beginner + adv stick')
plot(t, x_ball_2[2], 'gold', linestyle=':', label = 'Beginner + beginner stick')
title("Displacement of the ball when striking the ball")
legend()
```

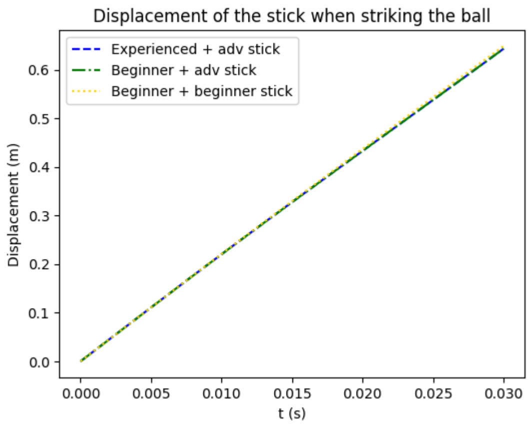
Out[5]: <matplotlib.legend.Legend at 0x1b386ed3c20>

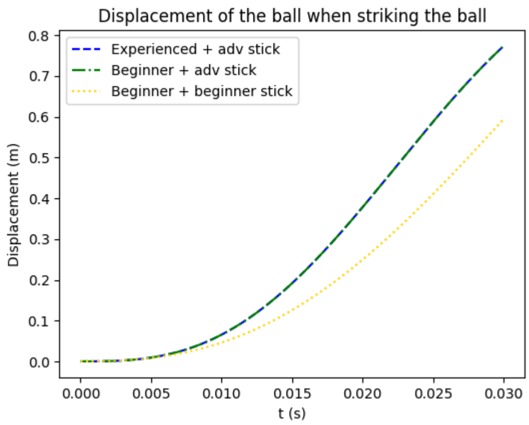







Displacement of the human when striking the ball
Experienced + adv stick
Beginner + adv stick
Beginner + beginner stick
Displacement (m)
t (s)


Displacement of the stick when striking the ball
Experienced + adv stick
Beginner + adv stick
Beginner + beginner stick
Displacement (m)
t (s)


Displacement of the ball when striking the ball
Experienced + adv stick
Beginner + adv stick
Beginner + beginner stick
Displacement (m)
t (s)

B.4. Double mass-spring-damper model

Setup

The motion of the human-stick-ball model is simulated as a double mass spring damper model with 2 DOF, and is described by 2 equations of motion.

In [1]:
```
from scipy.integrate import odeint
from pylab import figure, plot, xlabel, ylabel, grid, legend, title
from matplotlib.font_manager import FontProperties
import numpy as np

import matplotlib.pyplot as plt
```

Parameters and initial conditions

Defined for 6 conditions, being combinatations of one of the following two actions:

_1 = stopping the ball

_2 = striking the ball

And one of the following three player + stick combinations:

_a = experienced player + advanced stick

_b = inexperienced player + advanced stick

_c = inexperienced player + beginner stick

In [2]:
```
#Defining values
# Parameter values
# Masses:
mass_human = 74
mass_stick = 0.537
mass_ball = 0.16

m_human = 0.5*mass_human + 0.5*mass_stick + 2*(0.03+0.0018)*mass_human
m_ball = mass_ball + 0.5*mass_stick

## Initial conditions
# Positions
x_human_0 = 0
x_ball_0 = 0
# Velocities - condition 1 = stopping
dx_human_0_1 = 0.0
dx_ball_0_1 = -22.22
# Velocities - condition 2 = striking
dx_human_0_2 = 22.22
dx_ball_0_2 = 0.0

### Constants - condition 1 = stopping
## Spring constants
# Human stiffness (higher stiffness: higher frequency of displacement)
k_human_1a = 1083.09
k_human_1b = 495.77
k_human_1c = 495.77

# Stick stiffness (higher stiffness: smaller amplitude of oscillations)
k_stick_1a = 3000
k_stick_1b = 3000
k_stick_1c = 1400

## Damper constants
# Human damping (Higher damping: smaller amplitude of displacement)
c_human_1a = 54.15
c_human_1b = 24.79
c_human_1c = 24.79

# Stick damping (higher damping: faster reduction of oscillations)
c_stick_1a = 0.5
c_stick_1b = 0.5
c_stick_1c = 2.7

### Constants - condition 2 = striking
# Spring constants
# Human stiffness (higher stiffness: higher frequency of displacement)
k_human_2a = 495.77
k_human_2b = 495.77
k_human_2c = 495.77

# Stick stiffness (higher stiffness: smaller amplitude of oscillations)
k_stick_2a = 3000
k_stick_2b = 3000
k_stick_2c = 1400

# Damper constants
# Human damping (Higher damping: smaller amplitude of displacement)
c_human_2a = 24.79
c_human_2b = 24.79
c_human_2c = 24.79

# Stick damping (higher damping: faster reduction of oscillations)
c_stick_2a = 0.5
```

```
c_stick_2b = 0.5
c_stick_2c = 2.7
```

In [3]:
```
#Defining the equations of motion

def eom(q, t, p):
    """
    q :  generalised coordinates + derivatives:
        q = [x_human, x_ball, dx_human, dx_ball]
    t :  time
    p :  parameters:
        p = [m_human, m_ball, c_human, c_stick, k_human, k_stick]"""
    x_human, x_ball, dx_human, dx_ball = q
    m_human, m_ball, c_human, c_stick, k_human, k_stick = p

    # Define f = (x_human', x_ball', dx_human', dx_ball'):
    f = [dx_human,
         dx_ball,
         (- c_human * dx_human  - k_human * x_human + c_stick * (dx_ball - dx_human) + k_stick * (x_ball - x_human)) / m_human,
         (- c_stick * (dx_ball - dx_human) - k_stick * (x_ball - x_human)) / m_ball]
    return f
```

Motion in time

Defined for the 6 defined conditions

By using odeint

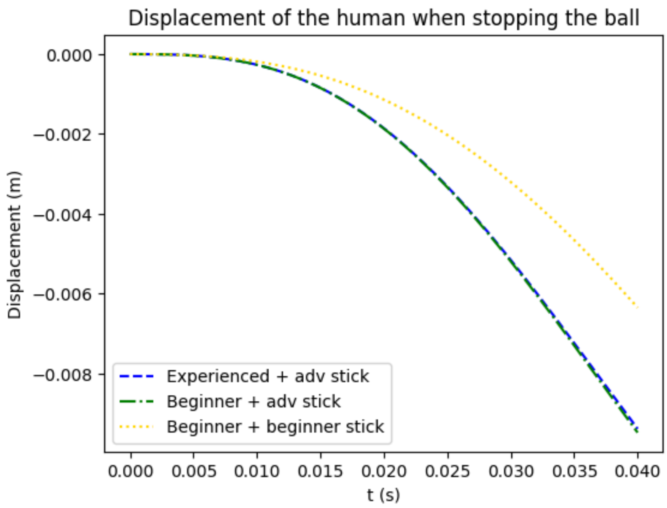
In [4]:
```
#Solve the ODE / EOM
# ODE solver parameters
abserr = 1.0e-8
relerr = 1.0e-6
stoptime = 0.04
numpoints = 250

# Time samples for the output of the ODE solver.
t = [stoptime * float(i) / (numpoints - 1) for i in range(numpoints)]

# Parameters and initial conditions:
#Stopping
params_1 = [
    [m_human, m_ball, c_human_1a, c_stick_1a, k_human_1a, k_stick_1a],
    [m_human, m_ball, c_human_1b, c_stick_1b, k_human_1b, k_stick_1b],
    [m_human, m_ball, c_human_1c, c_stick_1c, k_human_1c, k_stick_1c],
]
q0_1 = [x_human_0, x_ball_0, dx_human_0_1, dx_ball_0_1]

#Striking
params_2 = [
    [m_human, m_ball, c_human_2a, c_stick_2a, k_human_2a, k_stick_2a],
    [m_human, m_ball, c_human_2b, c_stick_2b, k_human_2b, k_stick_2b],
    [m_human, m_ball, c_human_2c, c_stick_2c, k_human_2c, k_stick_2c],
]

q0_2 = [x_human_0, x_ball_0, dx_human_0_2, dx_ball_0_2]

#Results from solving the EOM
#Stopping
x_human_1 = []
x_ball_1 = []

for p_1 in params_1:
    result = odeint(eom, q0_1, t, args=(p_1,), atol=abserr, rtol=relerr)
    x_human_1.append(result[:, 0])
    x_ball_1.append(result[:, 1])

#Striking
x_human_2 = []
x_ball_2 = []

for p_2 in params_2:
    result = odeint(eom, q0_2, t, args=(p_2,), atol=abserr, rtol=relerr)
    x_human_2.append(result[:, 0])
    x_ball_2.append(result[:, 1])
```

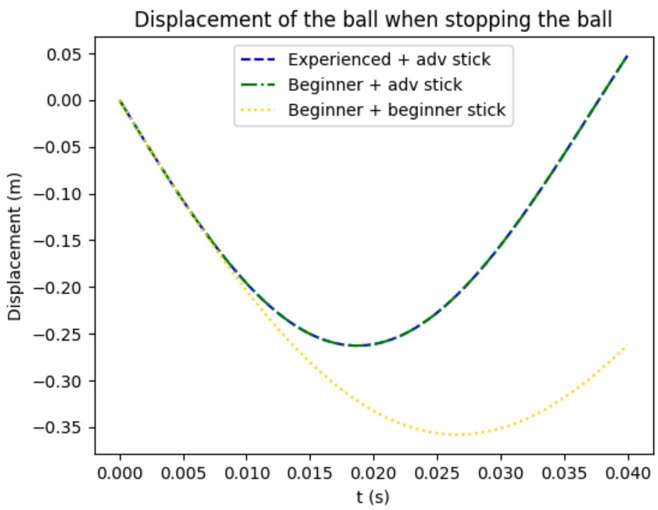
Plotting the displacement over time

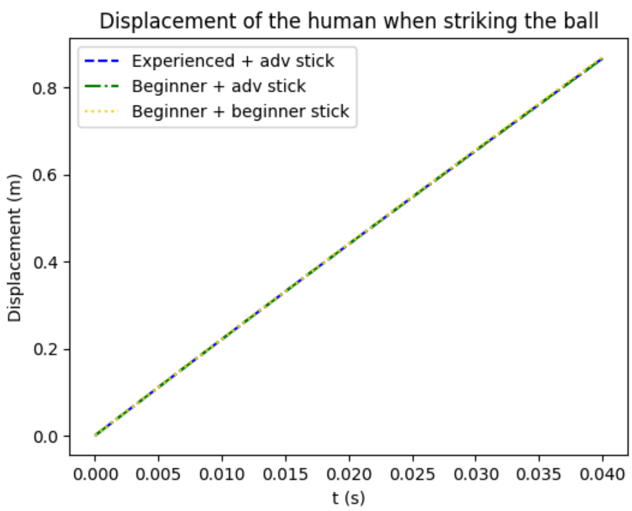
In [5]:
```
#Plot
#Stopping the ball, for the 3 defined conditions
figure(1, figsize=(6, 4.5))
xlabel('t (s)')
ylabel('Displacement (m)')
plot(t, x_human_1[0], 'b', linestyle='--', label = 'Experienced + adv stick')
plot(t, x_human_1[1], 'g', linestyle='-.', label = 'Beginner + adv stick')
plot(t, x_human_1[2], 'gold', linestyle=':', label = 'Beginner + beginner stick')
title('Displacement of the human when stopping the ball')
legend()

figure(2, figsize=(6, 4.5))
xlabel('t (s)')
ylabel('Displacement (m)')
plot(t, x_ball_1[0], 'b', linestyle='--', label = 'Experienced + adv stick')
plot(t, x_ball_1[1], 'g', linestyle='-.', label = 'Beginner + adv stick')
plot(t, x_ball_1[2], 'gold', linestyle=':', label = 'Beginner + beginner stick')
title('Displacement of the ball when stopping the ball')
```

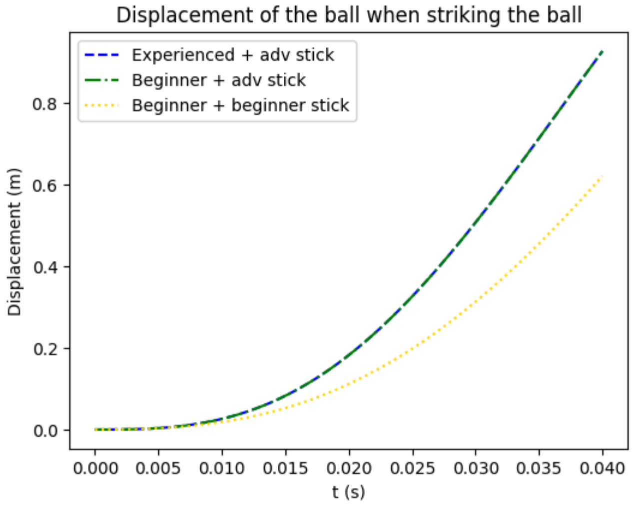
```
legend()

#Striking the ball, for the 3 defined conditions
figure(3, figsize=(6, 4.5))
xlabel('t (s)')
ylabel('Displacement (m)')
plot(t, x_human_2[0], 'b', linestyle='--', label = 'Experienced + adv stick')
plot(t, x_human_2[1], 'g', linestyle='-.', label = 'Beginner + adv stick')
plot(t, x_human_2[2], 'gold', linestyle=':', label = 'Beginner + beginner stick')
title("Displacement of the human when striking the ball")
legend()

figure(4, figsize=(6, 4.5))
xlabel('t (s)')
ylabel('Displacement (m)')
plot(t, x_ball_2[0], 'b', linestyle='--', label = 'Experienced + adv stick')
plot(t, x_ball_2[1], 'g', linestyle='-.', label = 'Beginner + adv stick')
plot(t, x_ball_2[2], 'gold', linestyle=':', label = 'Beginner + beginner stick')
title("Displacement of the ball when striking the ball")
legend()
```

Out[5]: <matplotlib.legend.Legend at 0x2c9c79b6b10>








Displacement of the ball when striking the ball
Experienced + adv stick
Beginner + adv stick
Beginner + beginner stick
Displacement (m)
0.0
0.2
0.4
0.6
0.8
0.000
0.005
0.010
0.015
0.020
0.025
0.030
0.035
0.040
t (s)

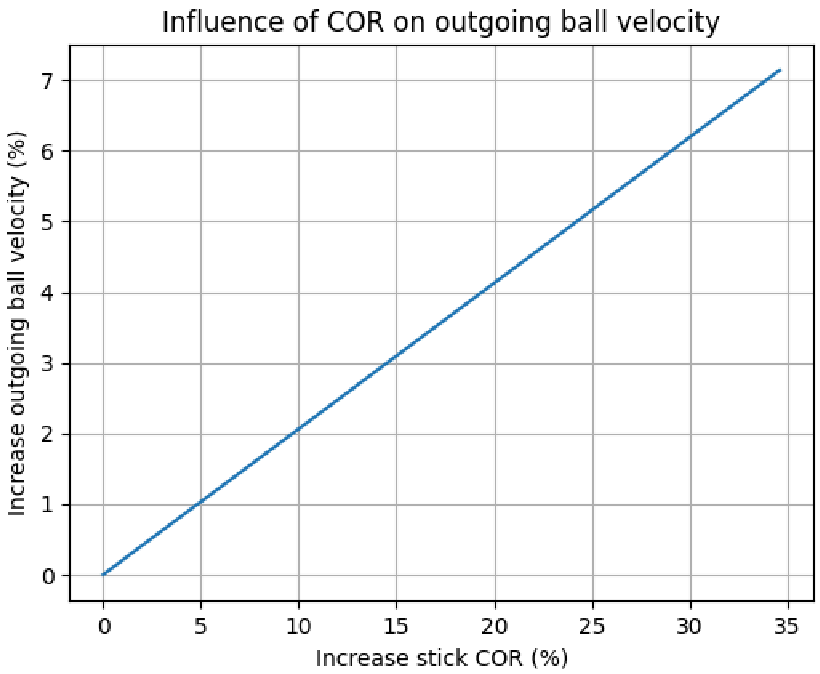
B.5. Collision model

Setup

In [1]:
```
from pylab import figure, plot, xlabel, ylabel, grid, legend, title
from matplotlib.font_manager import FontProperties
import numpy as np
import matplotlib.pyplot as plt

#Parameters
##Stick mass
M_list = [0.558, 0.566, 0.497, 0.476, 0.566, 0.560]
M_added_min = 0
M_added_max = max(M_list)-min(M_list)
M_avg = (sum(M_list) / len(M_list))

##Ball mass
m = 0.16

#Initial stick velocity
V1 = 80/3.6

#Average values
X_com_avg = (0.542 + 0.545 + 0.532 + 0.510 + 0.500 + 0.551)/6
L_stick_avg = (0.930 + 0.930 + 0.925 + 0.935 + 0.930 + 0.920)/6
X_added_avg = X_com_avg

#COR
e_max = 0.35
e_min = e_max-0.09
e_list = [e_min, e_max]
e_avg = sum(e_list) / len(e_list)

def eq(M_tot, M_added, e, X_added):
    M_original = M_tot - M_added

    X_com = (M_original * X_com_avg + M_added * X_added)/M_tot

    I = (1/12)*M_tot*(L_stick_avg**2) + M_tot*(X_com**2)

    M_e = I/(L_stick_avg**2)

    v2 = ((1+e)/(1+(m/M_e)))*V1
    return v2
```

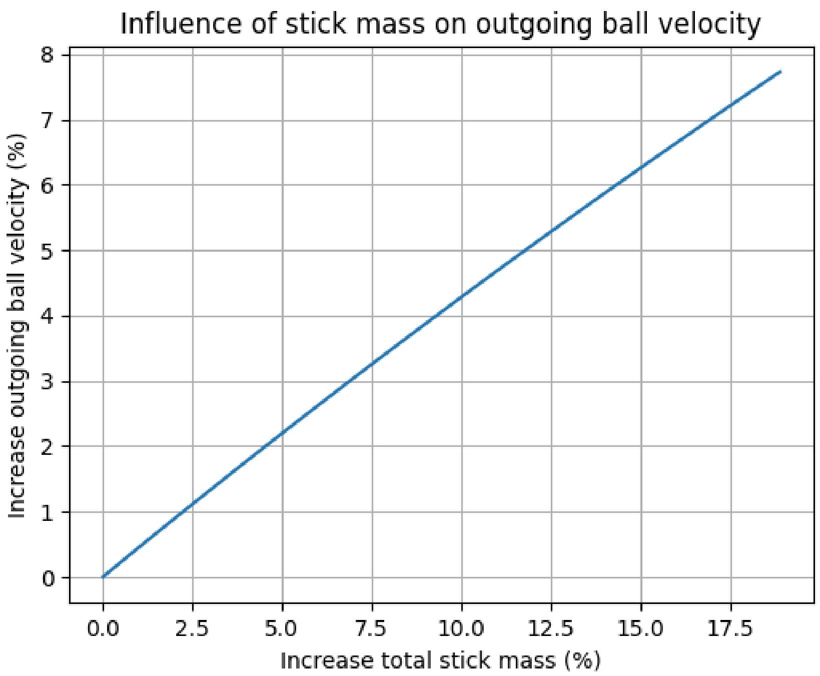
Analysing COR

In [2]:
```
######TEST 1
e_range = np.linspace(e_min, e_max, 100)

# Berekenen van v2 voor elke waarde van M
v2_1 = eq(M_avg, M_added_min, e_range, X_added_avg)

figure(1, figsize=(6, 4.5))
xlabel('Increase stick COR (%)')
ylabel('Increase outgoing ball velocity (%)')
plot(((e_range/min(e_range))*100-100), ((v2_1/min(v2_1))*100-100))
title("Influence of COR on outgoing ball velocity")
grid(True)
```



Analysing mass

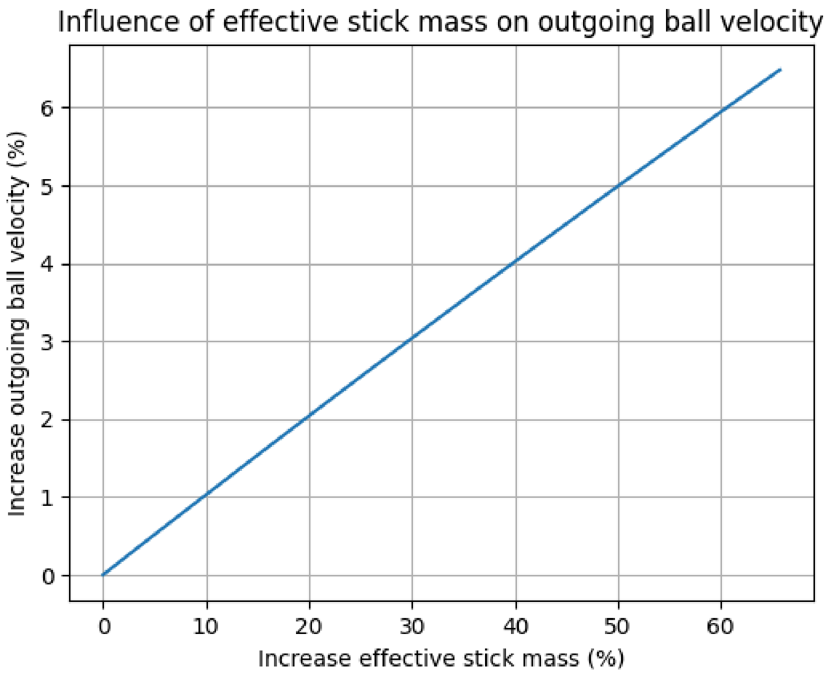
In [3]:
```
######TEST 2
M_tot_range = np.linspace(min(M_list), max(M_list), 100)

# Berekenen van v2 voor elke waarde van M
v2_2 = eq(M_tot_range, M_added_min, e_avg, X_com_avg)

figure(2, figsize=(6, 4.5))
xlabel('Increase total stick mass (%)')
ylabel('Increase outgoing ball velocity (%)')
plot((((M_tot_range)/(min(M_tot_range)))*100-100), ((v2_2/min(v2_2))*100-100))
title("Influence of stick mass on outgoing ball velocity")
grid(True)
```



Analysing effective mass by lowering COM

In [4]:
```
######TEST 3
X_com_range = np.linspace(X_com_avg, (L_stick_avg-0.05), 100)

# Berekenen van v2 voor elke waarde van M_e
v2_3 = eq(max(M_list), M_added_max, e_avg, X_com_range)

figure(3, figsize=(6, 4.5))
xlabel('Increase effective stick mass (%)')
ylabel('Increase outgoing ball velocity (%)')
plot(((X_com_range/min(X_com_range))*100-100), ((v2_3/min(v2_3))*100-100))
title("Influence of effective stick mass on outgoing ball velocity")
grid(True)

X_com_res = ((max(M_list)-M_added_max) * X_com_avg + M_added_max * (L_stick_avg-0.05))/(max(M_list))
display(X_com_res)
```

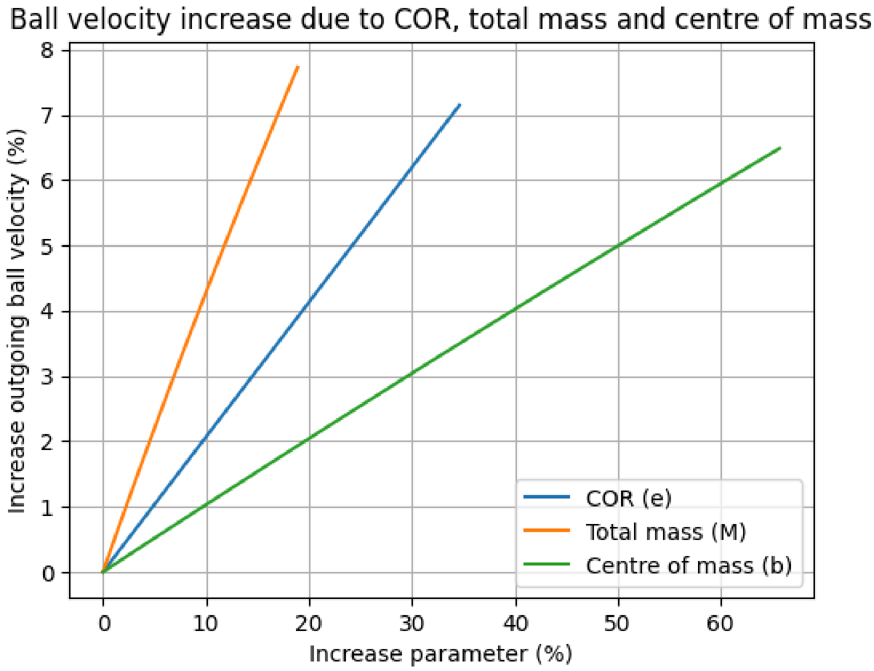
```
0.5853886925795054
```



Comparison

In [5]:
```
figure(4, figsize=(6, 4.5))
xlabel('Increase parameter (%)')
ylabel('Increase outgoing ball velocity (%)')
plot(((e_range/min(e_range))*100-100), ((v2_1/min(v2_1))*100-100), label = 'COR (e)')
plot((((M_tot_range)/(min(M_tot_range)))*100-100), ((v2_2/min(v2_2))*100-100), label = 'Total mass (M)')
plot(((X_com_range/min(X_com_range))*100-100), ((v2_3/min(v2_3))*100-100), label = 'Centre of mass (b)')
title("Ball velocity increase due to COR, total mass and centre of mass")
grid(True)
legend()
```

Out[5]:
```
<matplotlib.legend.Legend at 0x26fff59c290>
```


Ball velocity increase due to COR, total mass and centre of mass
Increase outgoing ball velocity (%)
Increase parameter (%)
COR (e)
Total mass (M)
Centre of mass (b)
0
1
2
3
4
5
6
7
8
0
10
20
30
40
50
60

In [ ]:

B.6. Design calculations

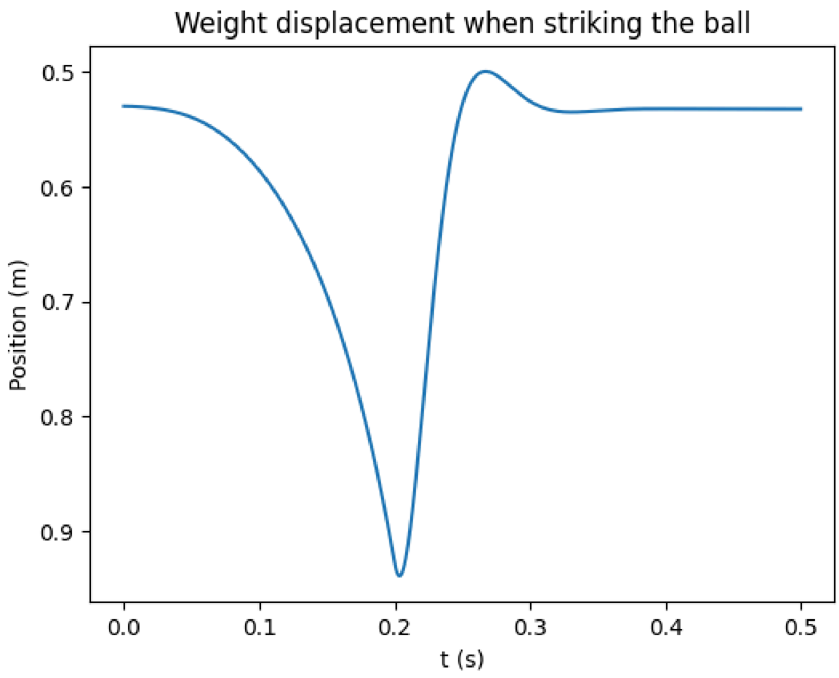
Movement of the weights

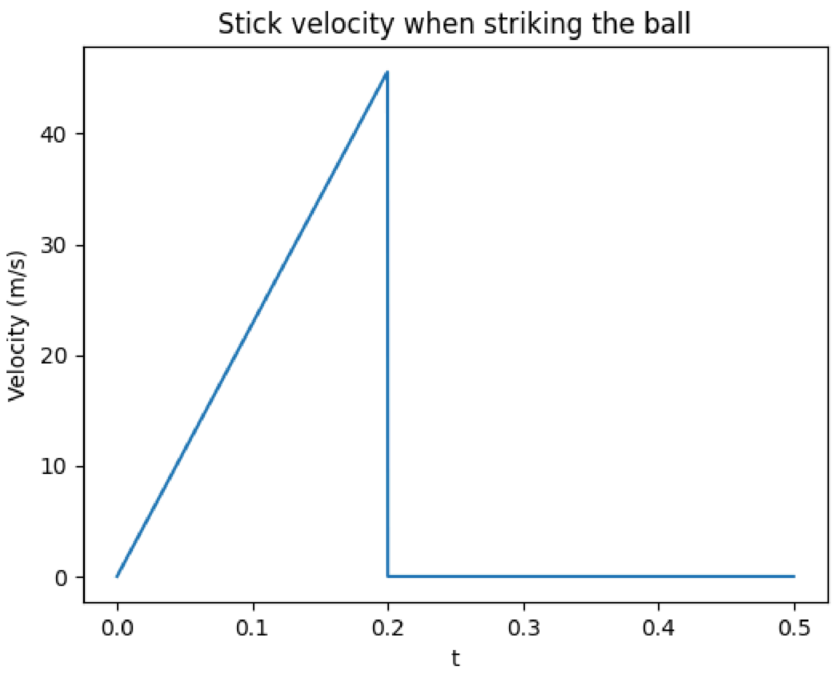
In [1]:
```
from scipy.integrate import odeint
from pylab import figure, plot, xlabel, ylabel, grid, legend, title
from matplotlib.font_manager import FontProperties
import numpy as np
import math

def eom(q, t):
    """
    q :  generalised coordinate + derivative:
        q = [x, dx]
    t :  time"""

    x, dx = q

    if t > 0 and t <= hit_time:
        V_stick = (V_stick_max / hit_time) * t
    else:
        V_stick = 0

    f = [dx,
         ((V_stick/L_stick)**2) * x + g - (k/m_weight) * (x-L_com) - (c/m_weight) * dx]

    return f

#Solve the ODE / EOM
# ODE solver parameters
abserr = 1.0e-8
relerr = 1.0e-6
stoptime = 0.5
numpoints = 2500

# Time samples for the output of the ODE solver.
t = [stoptime * float(i) / (numpoints - 1) for i in range(numpoints)]

#Parameters
hit_time = 0.20
V_stick_max = 45.6
L_stick = 0.93
L_com = 0.53
g = 9.81
m_weight = 0.100/2

#To be tuned: stiffness and damping
k = 200
c = 250 * math.sqrt(m_weight/k)
display(c)

#Initial conditions
x_0 = L_com
dx_0 = 0
q0 = [x_0, dx_0]

#Plot the displacement and the velocity of the weights during the hit
result = odeint(eom, q0, t, atol=abserr, rtol=relerr)

x_weight = result[:, 0]
v_weight = result[:, 1]

V_stick_values = [(V_stick_max / hit_time) * t_i if t_i <= hit_time else 0 for t_i in t]

import matplotlib.pyplot as plt
fig, ax = plt.subplots(figsize=(6, 4.5))

figure(1, figsize=(6, 4.5))
ax.invert_yaxis()
xlabel('t (s)')
ylabel('Position (m)')
plot(t, x_weight)
title("Weight displacement when striking the ball")
```

```
figure(2, figsize=(6, 4.5))
xlabel('t')
ylabel('Velocity (m/s)')
plot(t, V_stick_values)
title("Stick velocity when striking the ball")
```

3.952847075210474

Out[1]: Text(0.5, 1.0, 'Stick velocity when striking the ball')





Dimensions of the weights

In [2]:
```
def dimensions(mass, width, thickness):
    height = mass / (density * width * thickness)

    return height

density = 2400
m = m_weight
d = 0.051 - 0.01
r = d/2
```

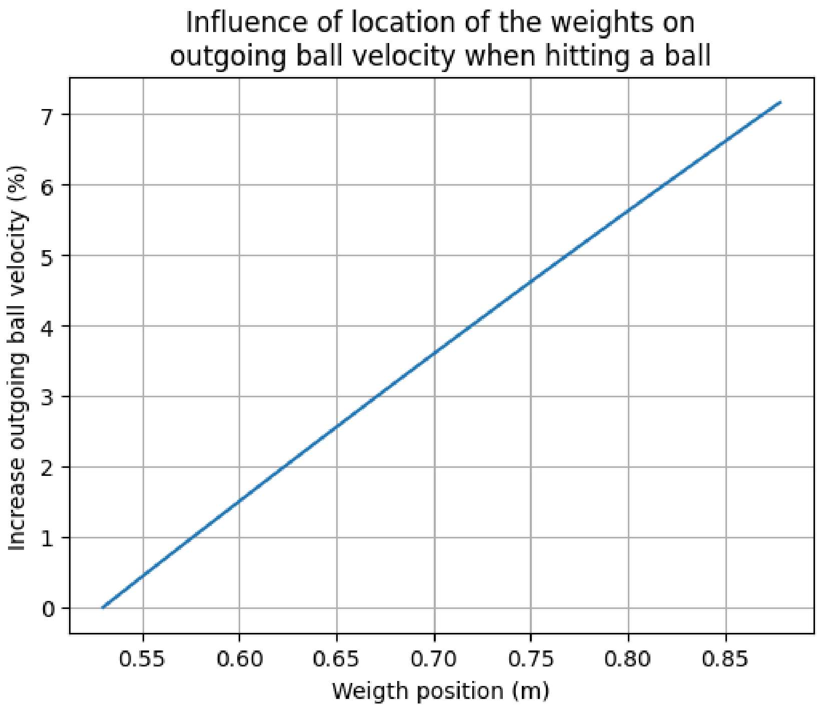
```
w = (1/2)*math.sqrt(3)*r
th = 0.01

h = dimensions(m, w, th)
display(h)
display(w)
```

```
0.11734761568894836
0.017753520777580988
```

Expected effects

In [3]:
```
##Expected increased ball velocity

#Parameters
##Stick mass
M_list = [0.558, 0.566, 0.497, 0.476, 0.566, 0.560]

##Ball mass
m = 0.16

#Initial stick velocity
V1 = V_stick_max

#Average values
X_com_avg = (0.542 + 0.545 + 0.532 + 0.510 + 0.500 + 0.551)/6
L_stick_avg = (0.930 + 0.930 + 0.925 + 0.935 + 0.930 + 0.920)/6
X_added_avg = X_com_avg

#COR
e_max = 0.35
e_min = e_max-0.09
e_list = [e_min, e_max]
e_avg = sum(e_list) / len(e_list)

#Formula outgoing ball velocity
def eq(M_tot, M_added, e, X_added):
    M_original = M_tot - M_added

    X_com = (M_original * X_com_avg + M_added * X_added)/M_tot

    I = (1/12)*M_tot*(L_stick_avg**2) + M_tot*(X_com**2)

    M_e = I/(L_stick_avg**2)

    v2 = ((1+e)/(1+(m/M_e)))*V1
    return v2, X_com

X_com_range = np.linspace(X_com_avg, (L_stick_avg-0.05), 100)

M_added_design = 0.100

# Berekenen van v2 voor elke waarde van M_e
result = eq(max(M_list), M_added_design, e_avg, X_com_range)
v2_design = result[0]
X_com_design = result[1]

figure(3, figsize=(6, 4.5))
xlabel('Weigth position (m)')
ylabel('Increase outgoing ball velocity (%)')
plot(X_com_range, ((v2_design/min(v2_design))*100-100))
title("Influence of location of the weights on\noutgoing ball velocity when hitting a ball")
grid(True)

max(v2_design)
```

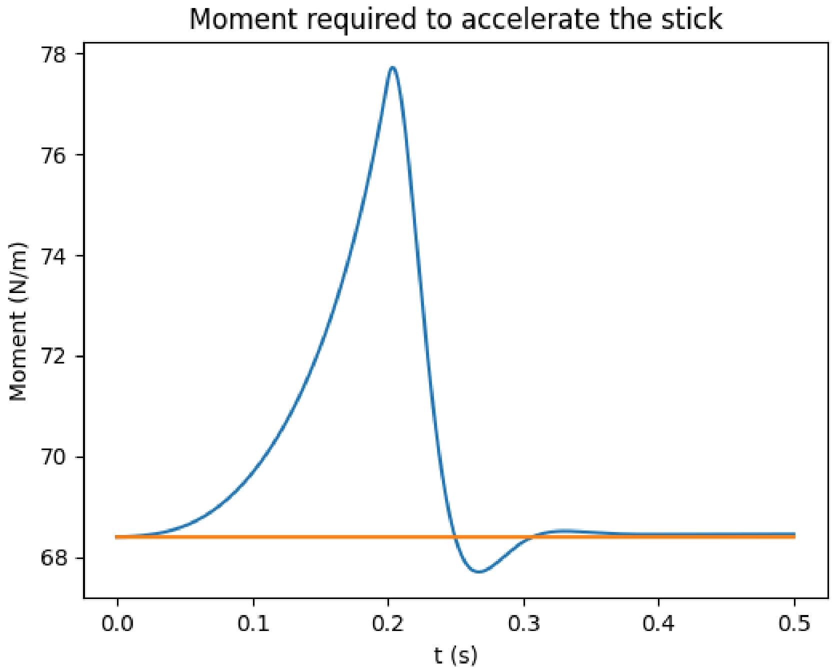
Out[3]:
```
np.float64(37.71933361387106)
```



In [4]:

```
##Expected increased required muscle torque
result_2 = eq(max(M_list), M_added_design, e_avg, x_weight)
X_com_design_2 = result_2[1]

a_avg = V_stick_max/hit_time
Moment_added = a_avg*max(M_list)*X_com_design_2
Moment_original = [(a_avg*max(M_list)*X_com_avg)]*len(t)

figure(4, figsize=(6, 4.5))
xlabel('t (s)')
ylabel('Moment (N/m)')
plot(t, Moment_added)
plot(t, Moment_original)
title("Moment required to accelerate the stick")

Increase_torque_max = (max(Moment_added) - max(Moment_original))/max(Moment_original)
display(Increase_torque_max)

Moment_added_avg = sum(Moment_added)/len(Moment_added)
Moment_original_avg = sum(Moment_original)/len(Moment_original)
Increase_torque_avg = (Moment_added_avg - Moment_original_avg)/Moment_original_avg
display(Increase_torque_avg)

Impulse_added = Moment_added_avg * hit_time
Impulse_original = Moment_original_avg * hit_time
Increase_impulse = (Impulse_added - Impulse_original)/Impulse_original
display(Increase_impulse)
```

```
np.float64(0.13635489445721255)
np.float64(0.020192282257333348)
np.float64(0.020192282257333372)
```



Moment required to accelerate the stick
Moment (N/m)
t (s)
78
76
74
72
70
68
0.0
0.1
0.2
0.3
0.4
0.5