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A TWO-DIMENSIONAL MACROSCOPIC MODEL OF TRAFFIC FLOWS BASED ON KCFD-SCHEMES[1]

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Abstract. *The problem of modeling vehicular traffic flows on city roads and freeways is considered. An original two-dimensional model of congestion traffic flows is developed based on the continuum approach and analogy with the KCFD (kinetically consistent finite-difference) schemes. The character minimum in space and in time is presented. Additional fluxes in two directions are initiated in order to provide smoothing along the road and diffusion across the road. The model is verified by test problems.*

1 INTRODUCTION

The aim of investigations is development of a new two-dimensional mathematical model and computer simulation methods for vehicular traffic flows. At present scientific research in this field is worldwide very actual due to the traffic situation closing to the critical point. During daylight hours traffic on city roads and freeways becomes strongly congested. The average vehicular speed is not much higher than the pedestrian speed and is far from the free-flow velocity. In these conditions jams can take place. Expensive and not popular methods of overcoming this problem connected, for example, with construction of new roads frequently do not lead to a success. It is stipulated not only by permanent increase of the vehicle amount but also by the fact that objective regularities of traffic flows are not taken into account. These regularities can be analyzed using empirical as well as theoretical approaches. The theoretical approach consists in development of adequate mathematical models of traffic flows and numerical solution of corresponding equations.

At present two basic concepts of traffic flow models exist: microscopic models, considering individual vehicles as interacting particles[1-11], and macroscopic models, considering traffic as a compressible fluid-dynamical flow[12-14]. Note, that there are also kinetic (mesoscopic) models, based on Boltzmann type kinetic equations[15-18]. They present an intermediate step between the above two kinds of models.

In spite of the large number of works on the traffic flow simulation and in spite of interesting results having the scientific and practical value the situation is far from the satisfactory completion. There is no model which can account for all aspects of vehicular

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traffic. Further improvement of traffic flow models is urgently required in order to obtain by means of these models concrete results for optimization of traffic on roads.

Traditionally traffic flow models are one-dimensional and describe flows only along one lane. In some cases influence of neighboring lanes (the multilane traffic) can be taken into account, for example, by using sources and sinks of the probability character in right-hand sides of equations[17, 19]. The present paper is devoted to development and verification of a two-dimensional fluid-dynamical model allowing predictions for real geometry of multilane roads.

The new model is constructed by analogy with the original kinetically-consistent finite difference (KCFD) schemes and the corresponding quasi gas dynamic system (QGDS) of equations[20], using the information on the one-particle distribution function behavior. Formerly this approach was successfully applied for viscous compressible and incompressible gas modeling. It draws a parallel with the Lattice Boltzmann schemes and the stabilization technique by E. Oñate[21]. Main properties of the new traffic flow model are the next: the accelerating force and some analogy of the pressure acting along the road and the shift viscosity characterizing transition to the lane with the lower density exist. The acceleration, the equilibrium velocity, the relaxation time and some kind of the velocity across the road (the so-called "lateral velocity") are introduced.

2 BASIC ASSUMPTIONS

In the framework of the paper the case of congestion (synchronized) traffic is studied: the average vehicular speed is far from the free-flow velocity, jams can take place, therefore drivers' strategies are similar: to choose a lane with the lower vehicular density and to move with a speed providing safe traffic. In these conditions and under the condition that distances to be considered are much more lengthy than vehicular sizes the continuum approach can be used. The authors hold the macroscopic theory in which individual vehicles do not appear explicitly. The traffic is viewed as a compressible fluid dynamic flow: the notions of the density $\rho(x;t)$ as a quantity of vehicles per lane in a distance unit and of the flux $W(x;t)$ at an arbitrary location x at an arbitrary instant of time t are introduced. The initial model of this type belongs to M.H. Lighthill and G.B. Witham[12].

One of main assumptions in the basement of the KCFD schemes and the related QGDS of equations is the presence of some additional mass flux to guarantee solution smoothing at the character distance l. For example, in gas dynamics the free path length of a molecule is treated as such a character distance, in porous medium flows as such a size the distance of the order of magnitude equal hundred grains of the rock can be taken.

Analogous to QGDS in the right-hand side of the continuity equation the additional flux $W_{\rho x}$ is introduced to provide smoothing along the road:

$$W_{\rho x} = \frac{\tau}{2} \frac{\partial}{\partial x} \left(\rho u^2 + P \right) \tag{1}$$

where P is the force connecting with acceleration or deceleration of vehicles:

$$P = \nu_x \frac{\rho^{\beta_x}}{\beta_x} \tag{2}$$

ν_x and β_x are phenomenological constants. In the system of traffic equations this force fulfills functions of the gas-kinetic pressure in equations of gas dynamics.

As in QGDS the character minimum scale in time is also introduced in order to justify the continuum approach. As such a character time interval the time of crossing the desired point by several vehicles can be used. By the order of magnitude τ equals:

$$\tau \approx \frac{\delta(u)}{u} \tag{3}$$

where $\delta(u)$ is the character distance between vehicles at the velocity *u*. It should be noted that τ is conservative enough and does not strongly vary at varying the velocity. Moreover, by the order of magnitude it is close to the character time of the lateral shift τ^*:

$$\tau^* = \frac{\Delta - h}{V} \tag{4}$$

where Δ is the lane width, *h* is the character vehicle width, *V* is some lateral velocity (the mean velocity of transition from one to another lane).

Development of the 2D traffic flow model is the question of the day. The problem of its construction consists in the fact that ordinary extension of a 1D model for the 2D case is not possible due to non-equivalence of flows along and across the road. If to compare with fluid dynamics one can conclude that in fluid dynamics the Knudsen number *Kn ~ 0.001* but in traffic flows *Kn ~ 0.1* what explains the difficulties.

Let us introduce the diffusion flux $W_{\rho y}$ connected with transition of vehicles to neighboring lanes:

$$W_{\rho y} = \frac{\tau}{2}\left(\frac{\partial \rho V^2}{\partial y} + \nu_y \rho^{\beta_y} \frac{\partial \rho}{\partial y} \right) \tag{5}$$

where *V* is the lateral velocity.

Smoothing terms are also introduced in the momentum equation. These terms like in QGDS are connected with additional transportation of vehicles by fluxes (1), (5):

$$W_u = \frac{\tau}{2}\left(\frac{\partial}{\partial x}\left(\rho u^3 + Pu \right) \right) \tag{6}$$

$$W_V = \frac{\tau}{2}\left(\frac{\partial \rho u V^2}{\partial y} + \nu_y \rho^{\beta_y} \frac{\partial \rho u}{\partial y} \right) \tag{7}$$

Traffic flow equations differ from gas dynamic equations by presence of terms describing the human will. Drivers usually try to achieve the maximum or the optimal velocity in the given conditions, i.e. vehicle can be accelerated or decelerated by desires of drivers. In literature there are different models of the acceleration based mainly on considering the situation “follow-the-leader”. After the paper[13] the next expression for the acceleration can be written out:

$$a = \frac{\left[u_{eq}(x) - u\right]}{T} \tag{8}$$

where u_{eq} is the equilibrium velocity depending on the density and optimal in the given conditions, T is the relaxation time. These functions can be defined differently, for example, in the next manner:

$$u_{eq} = u_f \times \left(1 - \frac{\rho}{\rho_{jam}}\right) \tag{9}$$

$$T = t_0 \left(1 + \frac{r\rho}{\rho_{jam} - r\rho}\right) \tag{10}$$

Thus the acceleration, the equilibrium velocity and the relaxation time are determined traditionally. However the notion of the lateral velocity is perfectly new. In the current version of the traffic flow model the lateral velocity is considered as a constant by the absolute value but it changes the direction depending on the density gradient across the road. This reflects the tendency of transition to the lane with the lower density.

3 THE SYSTEM OF TRAFFIC FLOW EQUATIONS

Summarizing the aforesaid assumptions in the framework of the proposed model of multilane traffic one can write out the system of equations:

$$\frac{\partial\rho}{\partial t} + \frac{\partial\rho u}{\partial x} + \frac{\partial\rho V}{\partial y} = \frac{\partial}{\partial x}\frac{\tau}{2}\frac{\partial(\rho u^2 + P)}{\partial x} + \frac{\partial}{\partial x}\tau\frac{\partial\rho u V}{\partial y} + \frac{\partial}{\partial y}\frac{\tau}{2}\frac{\partial\rho V^2}{\partial y} + \frac{\partial}{\partial y}\frac{\tau}{2}\nu_y\rho^{\beta_y}\frac{\partial\rho}{\partial y} \tag{11}$$

$$\begin{aligned} \frac{\partial\rho u}{\partial t} + \frac{\partial\rho u^2}{\partial x} + \frac{\partial\rho u V}{\partial y} &= f - grad\,P + \frac{\partial}{\partial x}\frac{\tau}{2}\frac{\partial(\rho u^3 + Pu)}{\partial x} + \\ &+ \frac{\partial}{\partial x}\tau\frac{\partial\rho u^2 V}{\partial y} + \frac{\partial}{\partial y}\frac{\tau}{2}\frac{\partial\rho u V^2}{\partial y} + \frac{\partial}{\partial y}\frac{\tau}{2}\nu_y\rho^{\beta_y}\frac{\partial\rho u}{\partial y} \end{aligned} \tag{12}$$

Here $f = a\rho$, the acceleration a is calculated as (8) accounting for (9) and (10), the “pressure” P is defined as (2). The following values of parameters are used:

$$\begin{gathered} \nu_x = 60, \quad \nu_y = 60, \quad \beta_x = 2, \quad \beta_y = 0, \ \tau = 10^{-3}, \quad \Delta t = 10^{-7}, \\ t_0 = 50, \quad r = 0.95, \quad \rho_{jam} = 120, \quad u_f = 90 \end{gathered} \tag{13}$$

One of advantages of this model is simplicity of its numerical implementation basing on approximations by conservative finite-difference schemes. Dissipative terms in right-hand sides of the equations open additional computational opportunities. The authors have substantial background in development of algorithms (including parallel algorithms for multiprocessor systems with distributed memory) for implementation of such KCFD-like models[22, 23].

Note that the above advantage makes possible successful application of the 2D model for simulation of flows on roads consisting of a large number of lanes, while known 1D multilane traffic models are very difficult to apply in cases when, for example, the number of lanes is more than three due to complicity and high costs of their numerical implementations.

4 TEST PREDICTIONS

For the new model implementation computational algorithms and codes have been developed based on explicit and semi-implicit numerical methods. The model was verified by solving some test problems. First of all a number of quasi one dimensional flows was simulated in order to compare results obtained by using the new model with results obtained on the basis of currently existent one dimensional models. Note that in computations the unit of the velocity measurement is km/hour, consequently the length along the road is measured in km and the unit of the density is the number of vehicles on km of a lane.

Fig .1 and 2 show profiles of the density and the velocity correspondently in the midsection along the road at different time moments. As the initial condition for the density the congestion flow density equal *100* was assigned before the point $x = 3.5\ km$ and the jam density equal *120* – after this point. One can see that in this case vehicles decelerate and stop and the jam moves back to the beginning of the road.

Results depicted in Fig. 3 and 4 are more interesting. Fig. 3 shows the density profile when the initial density is rather a high step. First the compaction moves to the beginning of the road (to the origin). In the domain $x < 2\ km$ the flow becomes slower and the density increases. At the same time the density increases in the free flow domain ($x > 4\ km$) decreasing the density of the step. When the step density is low enough this compaction shifts to the right and the flow density becomes even on this section of the road.

Fig. 4 illustrates the analogous situation but the initial density is low. In this case the step moves forward from the outset.

Results of the above quasi one dimensional tests were compared with results by D. Helbing and M. Treiber[14] in a qualitative sense. A good agreement was achieved.

In Fig. 5 results of 2D flow modeling are presented. The road consists of three lanes. For graphic interpretation of results the density is averaged over each lane at every time step. At the initial moment there is a jam on the second part of the right lane. During the movement vehicles begin gradually to transfer to left lanes. Consequently the second part of the whole road falls within the essentially congested phase. Then the domain of the high density shifts backward to the beginning of the road.

Another example of 2D flow modeling is illustrated by Fig. 6. It is supposed that from some time moment there is an opportunity for vehicles to move on the local widening left to the road in the interval $x \in [2.24, 4.78]$. Initially the flow density on the road is rather high and there are no vehicles in this widening. Then the flow shifts to the left occupying the widening, a high density wave appears across the road originating from x = 4.78 and moves back to the line x = 2.24. But after the line x = 4.78 the density decreases. As a result the traffic capacity at the section x = 4.78 drops greatly in comparison with the initial state when the flow was synchronized and the movement over the local widening was not allowed.

5 CONCLUSIONS

- The model proposed in the paper is the first attempt to describe multilane traffic in the 2D statement using the continuum approach.
- In the future the model will be specified and complicated by accounting for the variable lateral velocity, entrances/exits, real road geometry, statistical data and probability functions concerning the human will.
- New efficient computational algorithms (including parallel algorithms) will be developed based on finite-difference schemes of a high order of approximation.
- Usage of developed model, algorithms and codes will allow to predict congested traffic and jams and to elaborate recommendations on their avoiding and dissolving, to analyze the influence of geometric road conditions, road standards and handling regimes on the capacity of traffic networks and to solve many other problems of traffic flow dynamics.

REFERENCES

[1] R. Herman and R.B. Potts, "Single lane traffic theory and experiment", In: *Proc. of Symp. on Theory of Traffic Flow*, Ed. R. Herman, Elsevier, 120-146 (1959).
[2] D.C. Gazis, R. Herman and R.B. Potts, "Car following theory of steady state traffic flow", *Operations Research*, **7**, 499-505 (1959).
[3] R.W. Rothery, R. Silver, R. Herman and C. Torner, "Analysis of experiments on single lane bus flow", *Operations Research*, **12**, p. 913 (1964).
[4] K. Nagel, *High-speed microsimulations of traffic flow*, Thesis, University Cologne, Germany (1995).
[5] I. Progogine and F.C. Andrews, "A Boltzmann like approach for traffic flow", *Operations Research,* **8**, #789 (1960).
[6] I. Prigogine and R. Herman, *Kinetic theory of vehicular traffic*, Elsevier, Amsterdam (1971).
[7] A. Reuschel,"Fahrzeugbewegungen in der Kolonne begleichformig beschleunigtem oder verzogertem Leitfahrzeug", *Zeit. d. Oster. Ing. u. Architekt. Vereines*, **4**, #193, 50-62 and 73-77 (1950).
[8] L.A. Pipes, "An operational analysis of traffic dynamic", *J. Appl. Phys.,* **24**, 271-281 (1953).

[9] M. Bando, K. Hasebe, A. Nakayama, A. Shibata and Y. Sugiyama, "Phenomenological study of dynamical model of traffic flow", *J. Physique I France*, **5**, 1389-1399 (1995).
[10] K. Nagel and M. Schreckenberg, "A cellular automaton model for freeway traffic", *J. Phys. I France,* **2**, #2221 (1992).
[11] S.L. Paveri-Fontana, "On Boltzmann like treatments for traffic flow", *Transportation Research*, **9**, #225 (1975).
[12] M.H. Lighthill and G.B. Witham, "On kinematic waves: A theory of traffic flow on long crowded roads", In: *Proc. Royal Soc. Ser. A*, **229**, 317-345 (1955).
[13] A.T. Chronopoulos and G. Wang, "Parallel solution of a traffic flow simulation problem", *Parallel Computing*, **22**, 1965-1983 (1997).
[14] D. Helbing and M. Treiber, "Numerical simulation of macroscopic traffic equations", *Computing in Science & Engineering*, September/October, 89-99 (1999).
[15] D. Helbing, "Gas-kinetic derivation of Navier-Stokes-like traffic equations", *Phys. Rev. E*, **53**, #2366 (1996).
[16] D. Helbing, "Structure and instability of high-density equations for traffic flow", *Phys. Rev. E*, **57**, #6176 (1998).
[17] A. Klar and R. Wegener, "A hierarchy of models for multilane vehicular traffic I: Modeling", *SIAM J. Appl. Math.,* **59**, 983-1001 (1999).
[18] A. Aw, A. Klar, T. Materne and M. Rascle, "Derivation of continuum traffic flow models from microscopic follow-the-leader models", *SIAM J. Appl. Math.,* **63**, 259-278 (2002).
[19] D.Helbing, "Modeling multi-lane traffic flow with queuing effects", *Physica A,* **242,** 175-194 (1997).
[20] B.N. Chetverushkin, "Application of the kinetic schemes for simulation of viscous gas dynamics problems", *CFD Journal (Jap. Sos. of CFD),* **10 (3)**, 363-371 (2001).
[21] E. Oñate, "Derivation of stabilized equations for numerical solution of advective-diffusive transport and fluid flow problems", *Comput. Math. Appl. Mech. Eng.,* **151**, 233-265 (1998).
[22] B.N. Chetverushkin, N.G. Churbanova and M.A. Trapeznikova, "Parallel simulation of low Mach number flows based on the quasi gas dynamic model with pressure decomposition", In: *Proc. of Par CFD 2004 Conf.*, Elsevier, Amsterdam, 297-304 (2005).
[23] Yu. N. Karamzin, M.A. Trapeznikova, B.N. Chetverushkin and N.G. Churbanova, "A two-dimensional model of traffic flows", *Matematicheskoe modelirovanie* (2006) – in Russian, to appear.

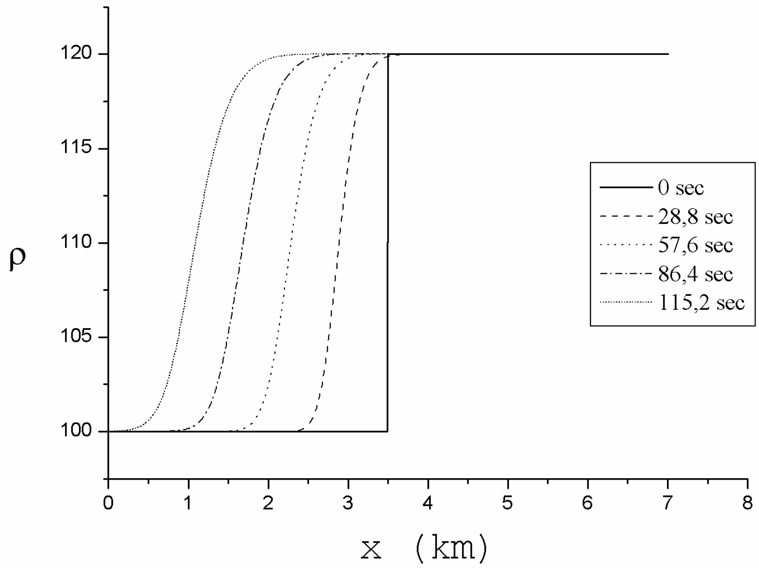

Figure 1: Time evolution of the density profile in the jam spreading problem

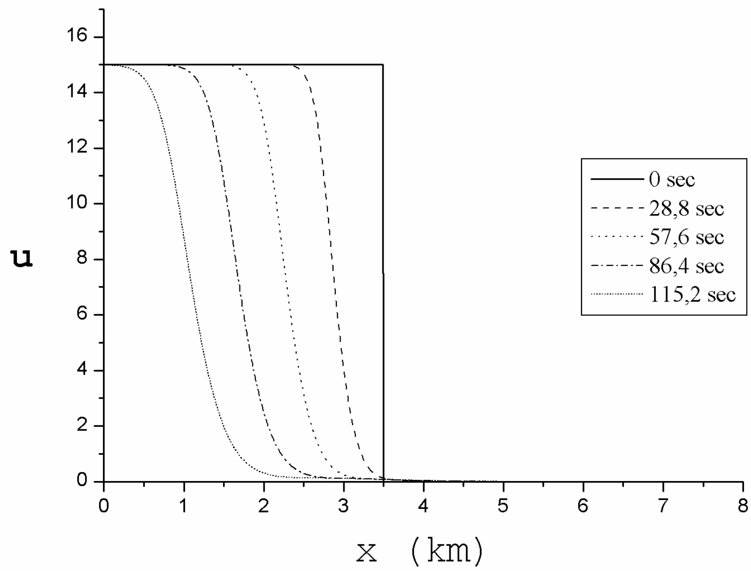

Figure2: Time evolution of the velocity profile in the jam spreading problem

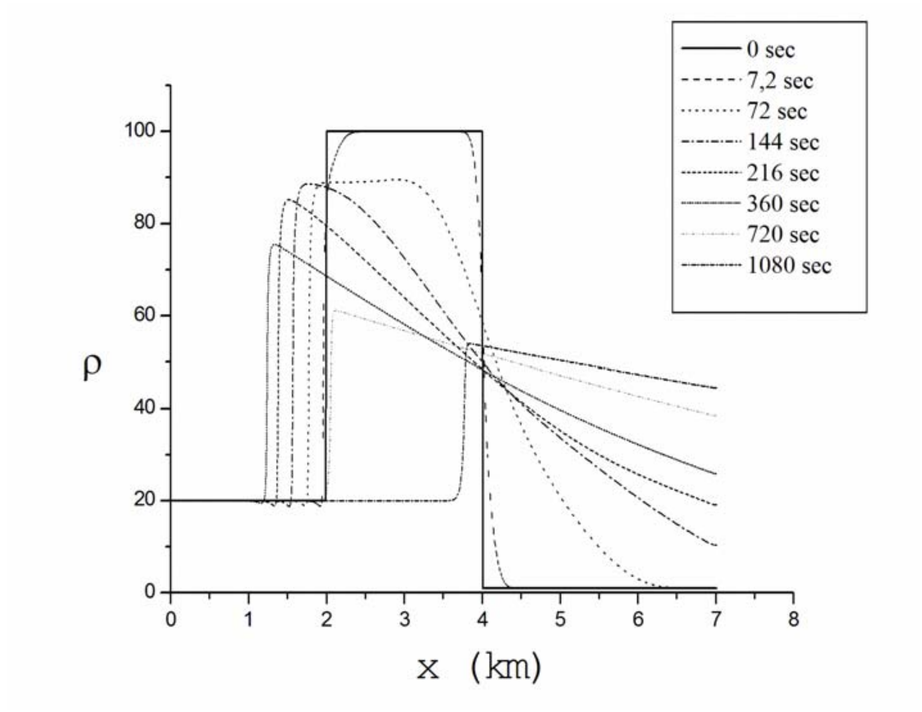

Figure 3: Time evolution of the density profile in the high step spreading problem

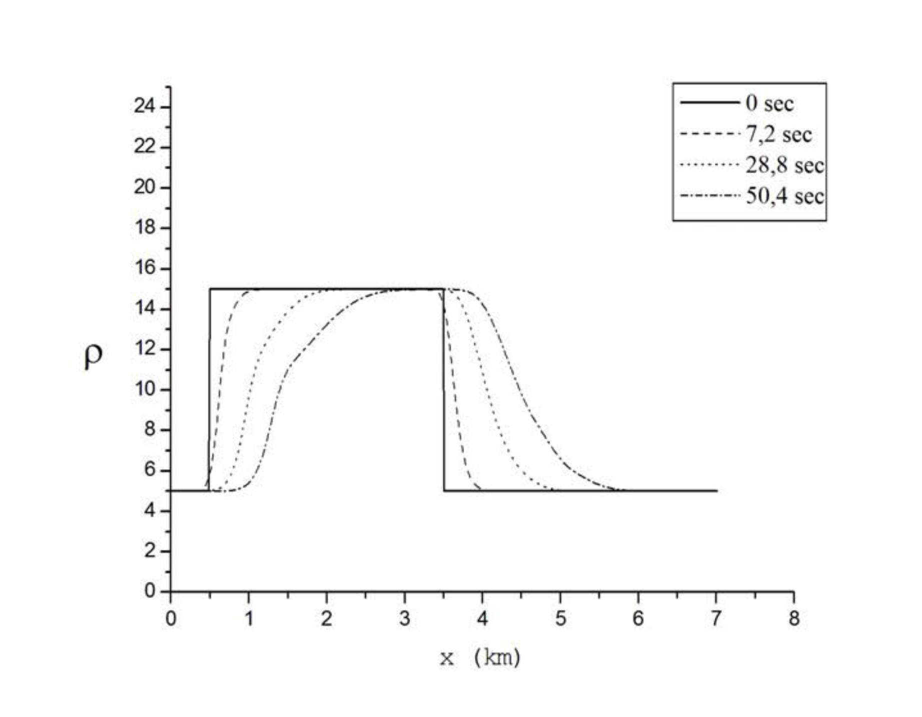

Figure 4: Time evolution of the density profile in the low step spreading problem

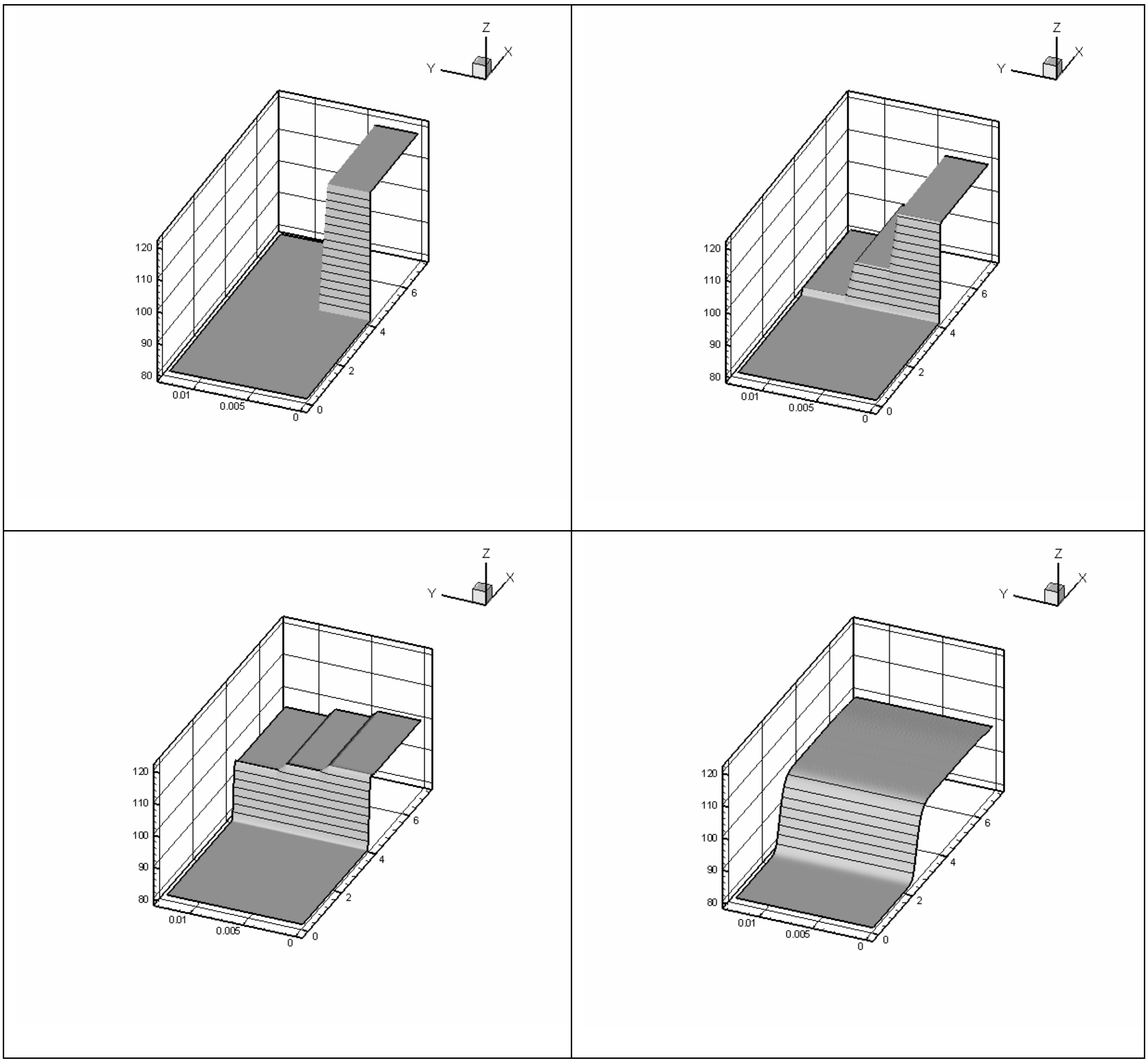

Figure 5: Time evolution of the density surface in the 2D problem

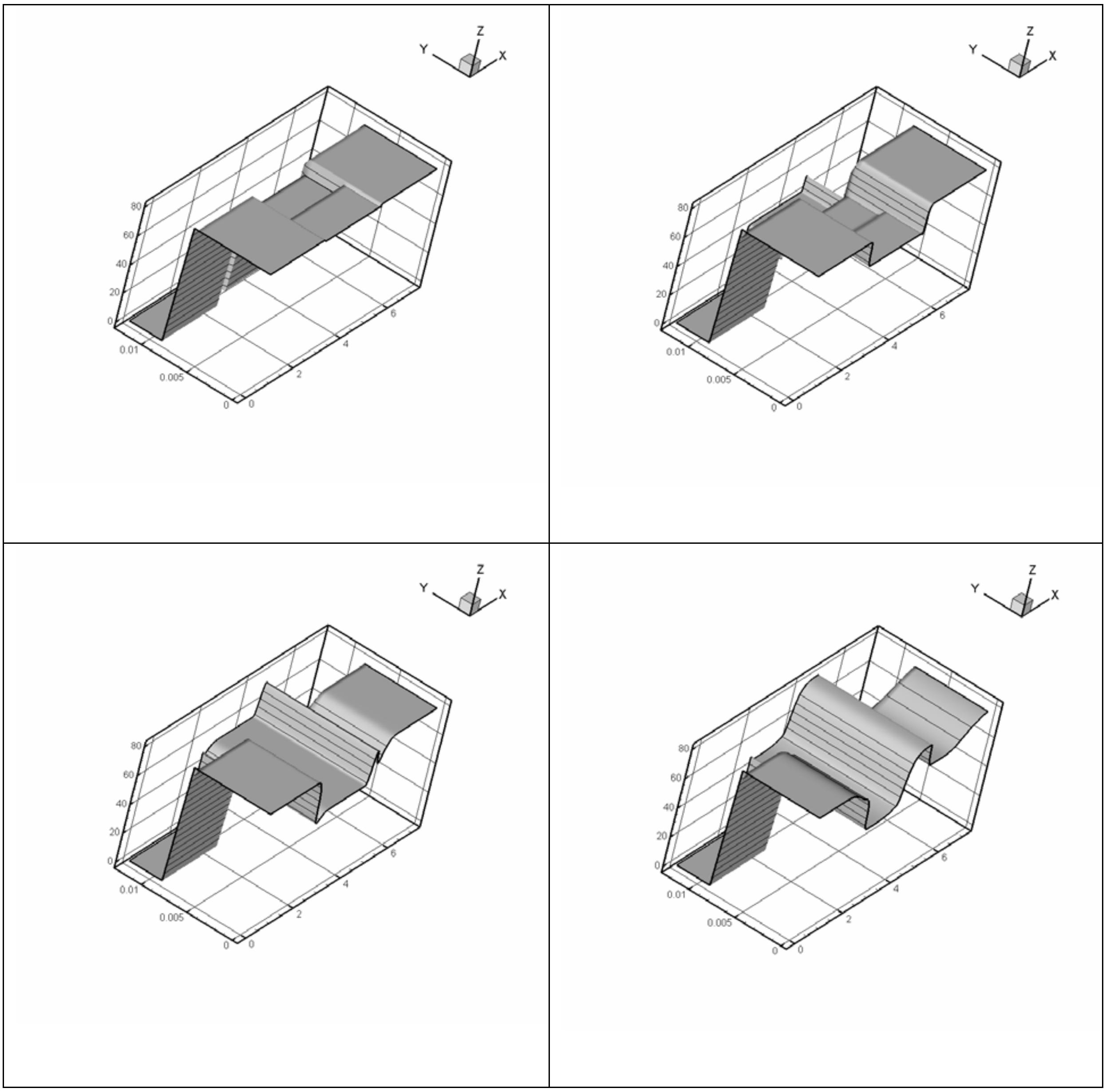

Figure 6: Time evolution of the density surface in the problem with the local widening