Full paper for review for the '4th International Symposium on Dynamic Traffic Assignment', Massachusetts, USA, June 2012

DTA2012 Symposium: Probability in traffic: a challenge for modelling

S.C. Calvert*, H. Taale, M. Snelder & S.P. Hoogendoorn

(All with the Delft University of Technology, The Netherlands. Calvert & Snelder are also with TNO, Netherlands organisation for applied scientific research) *presenting and corresponding author

Abstract

In the past decade an increase in research regarding stochasticity and probability in traffic modelling has occurred. The realisation has grown that simple presumptions and basic stochastic elements are insufficient to give accurate modelling results in many cases. This paper puts forward a strong argument for the further development and application of probabilistic models and argues that a realisation must arise of the detrimental effects of blindly applying non-probabilistic models to traffic where probability is rife. This is performed by the demonstration that deterministic and simple stochastic models will, in many cases, produce substantially biased results where variability is present in traffic. Prior to this demonstration, recent developments in probabilistic modelling are discussed.

While the case for probabilistic modelling is strong in theory, the application of such modelling approaches is only possible with sufficiently developed models. However there are still certain challenges to be addressed in probabilistic modelling before a widespread implementation is likely. Remaining challenges for probabilistic approaches are therefore discussed and it is shown that computational efficiency, correlations between variables, and data gathering and processing all remain difficulties that have yet to be fully overcome.

1. Introduction

Since traffic modelling became a mainstream area of scientific research halfway through the last century, continuous developments have taken place in order to improve performance and eradicate shortcomings of models. Since the turn of the century an increase in research regarding stochasticity and probability in traffic modelling has occurred. The realisation that simple presumptions and basic stochastic elements are insufficient to give accurate modelling results has grown. Tampere & Viti [1] remarked on this and included questions relating the reliability of dynamic modelling and the lack of most current models to properly consider stochastic elements.

Van Lint et al [2] experimentally demonstrated the importance of not ignoring variations in traffic by showing biases in results that occurred by not considering stochastic variations. In other recent works, Sumalee et al [3], Jabari & Liu [4] and Szeto et al [5], among others, expanded on initial contributions from Chen et al [6], Clark & Watling [7], Boel & Mihaylova [8] and others to incorporate probability in models. Many of these contributions propose elaborate analytical solutions for the application of probability in modelling. However, most remain incomplete from the point of view of practical widespread implementation. Tampere & Viti [1] and Jabari & Liu [4] also argue that randomness is often applied in an imperfect and incomplete fashion. This may be through merely adding stochastic 'noise' or presuming an inaccurate distribution, for example. For many of the prosed models this remains the case, which can lead to the misrepresentation of reality.

In this paper a concise review of the current state of affairs in the area of probabilistic and stochastic modelling is given, as well as their shortcomings of these models (section 2). The case for the necessity of probabilistic modelling under certain conditions is argued, and demonstrations are given

and discussed for two cases in which non-stochastic deterministic approaches are shown to be inferior compared to a probabilistic approach (section 3). Challenges for further future development of probabilistic models are also discussed (section 4).

2. State-of-art on probability modelling

In this section a short description is given of both traffic modelling and, more specifically, stochastic and probabilistic modelling in general. The distinction made between probabilistic and stochastic is given, and is followed by an overview of the latest developments in probabilistic traffic modelling along with a discussion of each type.

2.1 Traffic modelling in general

Various types of traffic models exist, each with their specific purposes and applications. A well accepted differentiation is based on the level of detail and differentiates between *macroscopic*, *mesoscopic* and *microscopic* models [9]. Another categorisation focuses on the deterministic level of the model. This indicates the extent to which a model incorporates variation in its calculations and distinguishes between *deterministic* and *stochastic* models [9]. Within these categories further differentiation can be made, also between the categories further differentiation is possible. In this paper the main focus will be on probabilistic models, which falls in the category of stochastic models. And while microscopic models are also mentioned, the focus will be on macroscopic probabilistic models.

Macroscopic traffic models do not consider individual vehicles, but rather the collective behaviour of vehicles and are therefore more readily applied to larger networks. In essence the vast majority of *macroscopic traffic models* are deterministic. Deterministic traffic models presume that no stochastic variability is present in traffic, while stochastic traffic models do presume certain levels of variations. A distinction in macroscopic models is generally made between first order models and higher order models. Lighthill and Whitham [10] were among the first to propose a first order approach based on fluid dynamics from the field of continuum mechanics. This group of models makes use of the law of conservation, combined with a fundamental relation between the main traffic quantities, density, volume and speed, and makes use of the numerical Godunov scheme to solve the model equations. Later Daganzo proposed an extention to the LWR-model in the form of the Cell Transmission Model (CTM) [11, 12]. In this work shockwaves are automatically incorporated in the applicable equations, which avoid the necessity of considering shockwaves as an external case.

Higher order traffic models make use of multiple differential equations to describe traffic flow. One of the first higher order models to be proposed was by Payne [13] in which the LWR-model was extended with a dynamic speed equation. This addition solved a number of difficulties with the original first order models, which occurred at the boundaries of traffic states. Such a difficulty is the inability to create start-stop waves, as a first order model presumes instantaneous speed correction from vehicles. Despite the improvements, higher order models received a fair amount of criticism, partly due to the explicit level of complexity in solving them. And while methods have been developed to perform the task of solving the equations [14], the greater level of complexity makes completely understanding the mathematical properties of these models a rigorous task [15], which can lead to instability in their implementation [12]. However further developments by Aw & Rascle [16] and Zhang [17] eradicated many deficiencies, such as the violation of the anisotropic character of traffic [18], and opened the door for further developments. As the majority of applied macroscopic traffic models make use of first order theory, or an adaptation of, and these models are easier to understand our focus will lie with the first order approaches.

Microscopic traffic models consider the individual vehicles and their interaction with the surrounding infrastructure and other vehicles. Often they will make use of longitudinal and latitudinal rules in these interactions. The rules often include a certain degree of variability corresponding to the variability that is seen between different drivers in different vehicles. Even for a single driver in the

same vehicle, actions may vary from situation to situation as in reality. Modelling individual vehicles is generally performed based on the principle of *car-following* [9, 19]. The vast majority of microscopic models make use of equations related to the distance, speed and acceleration of predecessors. Hoogendoorn [9] defined three main types of car-following model: *safe-distance, stimulus-response, and psycho-spacing car-following models*. Of these, the psycho-spacing variant is most the commonly applied. This was first introduced by Wiedemann [20], in which a distinction is made between driving behaviour under unconfined conditions and those under confined conditions. This led to a much greater reality in the manner that vehicles related to other vehicles at various downstream distances. As with many car-following models since, Wiedemann made use of lane-changing and overtaking actions in his model, based partly on gap acceptance. It has become common place in micro-simulation that both car-following parameters as well as gap acceptance parameters contain stochastic elements [21, 22]. These variations are aimed to correspond to the differences between drivers, and may be applied as simple random stochastics or from a validated distribution of probabilities. In the rest of this paper, the focus will predominately lie with macroscopic models, rather than microscopic models.

2.2 Stochastic and probabilistic models

Both macroscopic and microscopic models can be stochastic. Application of stochasticity in traffic models entails the inclusion of variability in the manner in which traffic is modelled. Contrary to deterministic models, in which one set situation is modelled, variables in stochastic models may vary due to stochastic effects. Although this adds complexity, it represents the real world to a better extent. In this research we have chosen to explicitly make a distinction between stochastic models in general and probabilistic models, as the terms stochastic and probabilistic are often used identically, while they are not synonymous [23]. Stochastic is defined as the inclusion of variability, while probabilistic is defined as the occurrence of deterministic states with given probabilities. To this extent, we define *probabilistic models* as a subcategory of stochastic models in which the chance of certain values is the result of a probability. This probability is directly derived from proven theory and/or empirical observation of the considered traffic system. An empirical distribution of traffic demand in a peak period derived from years of data is an example of probabilistic random variable, while a Gaussian distribution closely representing the traffic demand in a peak period would be a stochastic random variable, according to the definition.

In macroscopic models, stochasticity is often incorporated by means of traffic assignment or route choice. In deterministic assignment, vehicles are designated a route depending on the shortest cost, often the travel time. In the stochastic traffic assignment certain stochastic variation is added so that traffic makes use of multiple routes according to a variability cost function. In these models, traffic propagation is not considered stochastically. Including 'complete' stochasticity in macroscopic models, in which a wide range of variables are varied, is generally performed in two ways: by means of *repetitive simulations*, and secondly by including *variation in the model core*. Both of these methods are now described and discussed.

2.3 Probabilistic modelling through repetitive simulation

The method of repetitive simulations has been widely applied in various sciences to help describe probabilistic and stochastic systems [2, 6, 24]. The method, commonly known as Monte Carlo simulation, presumes predefined probabilities for each of the input variables, indicating the probability of occurrence and the corresponding value [25]. From each variable, random values are sampled which are applied simultaneously to model for all the input variables. The probability of certain values occurring is directly related to the predefined probabilities. The outcome of the simulation is recorded as a single entry of the results' distribution. By repeating this process many times with different input samples, a complete distribution of the results is constructed. The application of Monte Carlo simulation has been widely applied, mainly due to its relative simplicity and effectiveness. However, the method has its drawbacks. Main concerns in traffic

modelling in the past have been the computational load of the method [3, 6, 24] and the presence of correlation between input variables. As one may imagine, performing hundreds or even thousands of simulations is time consuming, but will often be necessary to achieve a required level of accuracy, especially when large numbers of input variables are applied.

Solutions for these difficulties have been offered in various forms. The use of intelligent sampling methods to reduce the variance from sampling and therefore the required number of simulations has been applied [2, 26, 27]. This also helps towards reducing possible sampling errors, which may cause discrepancies in results [3]. However, it has been shown that despite low initial biases for this solution, an amplification may occur leading to high standard error in the final results [7]. A more brute force solution to computational load lies in ever more powerful computers, which have the capability of performing many more calculations at an ever greater pace [6].

Correlation between input variables may be considered prior to simulation at the sampling stage [6]. Variables with dependencies may have probabilities which rely on the values sampled from other variables. In this way correlation between two or more variables is included and allows for a realistic simulation. However calculating non-bias outcomes in situations in which correlations are more complex and, furthermore, have dependencies on variables in the model, becomes much more difficult [24]. In many approaches the extent of bias is presumed to be limited and therefore little attention is spent on this difficulty.

2.4 Probabilistic modelling through probability in the model core

Modelling probability in the core of traffic models considers multiple stochastic factors in model equations and therefore eradicates the need for multiple simulations. Therefore the method is sometimes also known as the *one-shot* method. A number of different approaches have been proposed in which variability is incorporated in the core of a traffic model. Distinction may be made between those methods that propose an analytical or numerical approach extension to traffic propagation in which stochastic variables are included, and those which consider stochastic effects by bringing stochasticity into the fundamental relations.

An analytical approach to probability in the model core, or simply one shot, probabilistic traffic modelling has proven an extremely difficult undertaking. Clark & Watling [7] proposed a method for travel time reliability based on day-to-day variations in the travel demand matrix. Their framework computes a total travel time distribution based on the multivariate moments of a link flow vector. This was successfully demonstrated, however the method only considers a single random variable, namely the traffic demand, and therefore has limited difficulties with correlation. Others propose a more numerical approach to analytically incorporating stochasticity in the model core. Recent developments include Sumalee et al [3], who proposed a stochastic cell transmission model (S-CTM) which makes use of fives operational modes depending on the states of traffic flow. Each mode incorporates a set of probabilistic conditions to describe probability in each mode. Others who proposed using multiple functions as dictated by the traffic state, include Munoz et al [28] and Sun [29]. A main reason for considering multiple traffic states is the avoidance of nonlinearity in the fundamental relation, which is difficult to quantify otherwise. More recently Jabari and Liu [4] argued that presuming non-linearity, while being mathematically beneficial, may lead to inconsistency with the original deterministic dynamics. Therefore Jabari and Liu [4] propose to include stochasticity as a function of the uncertainty in the driver gap choice, represented by the random vehicle headway. In doing so, they argue that non-linearity is avoided in continuous time as all traffic dynamics may be derived to the longitudinal car following behaviour. Boel and Mihaylova [8] similarly proposed an extension to the CTM with stochastic elements. Rather than reconstructing the CTM as piece-wise structure based on traffic states, they defined the sending and receiving functions from the CTM as random variables in which the dynamics of the average speed in each cell is stochastically varied. The purpose was to incorporate stochasticity in the heart of the model at link level, which may propagate through an entire network through cell interaction. However, as their approach only considers a single stochastic scenario at a time, repetitive simulations are required to compose a probability distribution of the outcomes.

Stochasticity can also be included in (macroscopic) traffic models by means of a stochastic fundamental diagram. Li et al [30] make a strong argument that a simple, but effective manner of probabilistic modelling is to make use of a probabilistic fundamental diagram. Such a diagram is constructed through a flux function obtained from random elements observed from speed-density data. Kim and Zhang [31] also previously described stochasticity in the fundamental diagram by defining the growth and delay of perturbations from random fluctuations in both the gap time and transitions between traffic states. In their work they closely examined fluctuations in car following to derive their defined *gap time*.

Advances in approaches bringing probability to the core of a model have generally been performed as extensions of existing methods. This has the obvious advantage that sound theory may be further elaborated on. The extension of the cell transmission model (CTM) is therefore a logical one. While disadvantages of applying such non-linear approaches are brought forward [4], the question remains to which extent this has a detrimental effect on the outcomes. Jabri and Liu [4] argue that most models are nonlinear and therefore handle traffic propagation inconsistently, and that stochastic variables are often applied as mere white noise. While possibly guaranteeing consistency when avoiding nonlinearity, it must be realised that random stochastics in traffic will not adhere to set analytical formulations, as they result from human behaviour. Therefore marginal errors, if any, may not be of great significance, and we must remember that a model is merely a representation of reality and not visa versa.

The majority of the presented methods, while applying stochasticity, do this based on presumptions of random variables. In many cases, certain random distributions may be acceptable, however many random variables do not align to a set form when empirically challenged. To this degree the random variables are not pure probabilistic, according to the definition used in this paper¹, as the random variables do not always accurately correspond to real-life probabilities. A further major difficulty that is only partially addressed is that of dependence between random variables [2, 3, 6]. These correlations are often presumed non-existent for the ease of modelling [3], or are simplified by means of presumptions or transformations [4, 7]. While some research does consider correlations between random variables, these models often are restricted to less elaborate modelling approaches.

Some advances have been recently made in stochastic core modelling, as shown. The majority of these models are developed for very specific purposes with possibilities for larger scale implementation. However the, sometimes complex, formulations may provide difficulty for implementation of methods in a complete macroscopic or mesoscopic framework. To the knowledge of the authors, no model has yet been developed that is capable of matching the accuracies of the computationally heavy repetitive simulation through a one-shot approach on a comprehensive network.

3. Application area of probabilistic models

Often there is a specific and sometimes urgent need to use probabilistic models. This is argued in many of the papers presented here thus far. The application of simple stochastic or deterministic models in some cases may be unintentionally deceiving policy-makers with biased results. However, it is not always apparent when probabilistic models should be applied and what the extent is of errors made by applying non-probabilistic or non-stochastic models.

In the following paragraph, two experimental cases are given to demonstrate area's in which deterministic modelling has shortcomings and a probabilistic or stochastic approach is required. Thereafter the application horizon for the probabilistic approach is discussed.

¹ Probabilistic: The occurrence of deterministic states with given probabilities

3.1 Experimental demonstrations

To demonstrate potential situations in which modelling, without consideration of variation in traffic quantities, can lead to biased results, two small scale experimental cases are considered. These cases each have a focus on a specific contribution of probabilistic modelling. The goal of the experiments is to show that considering variations as probabilities gives significantly different results than by considering a single deterministic run. In each case, the capacity of the road sections is varied according to a probable real-life distribution. Variations in capacity are applied to all road sections as a blanket factor, which may represent the reduction in operational capacity from i.e. weather conditions, luminance conditions, etc. The applied distributions are logarithm functions and are shown in Figure 1. To avoid the necessity to derive correlation between capacity and demand variation, only the capacity is varied, which is more than sufficient to give an indication of the effects of modelling traffic variability. In each case, use is made of dynamic macroscopic traffic assignment model INDY, which is based on the link transmission model, as developed by Yperman [32]. The model is applied to a section of the Amsterdam network, as shown in figure 2.



Fig.1 Capacity factor functions for model input: case 1 (left) and case 2 (right)



Fig.2 Network used for the experimental cases in INDY, representing the south ring of Amsterdam

The outcomes of the experiments are analysed using the *total experienced delay* on the entire network compared to free-flow conditions, and are expressed in *lost vehicle hours*. For case 1, the averaged travel time over route AB (see Fig. 2) are also analysed. Other result indicators may also be used, such as the travel over other specified trajectories or the average network speed, among others. For the demonstration here, it is not of great importance which is chosen, merely that the network can be evaluated. The mean average and the median of the distributed results are

compared with that of a single model run for the median situation, which represents a deterministic model run.

In general, the *total experienced delay* T_{lost} is defined as:

$$T_{lost} = \sum_{veh=1}^{\infty} (tt_{scen.veh} - tt_{ff.veh})$$
(1)

Where

 $\begin{array}{ll} \mbox{veh} &= \mbox{vehicles} \\ \mbox{tt}_{scen.veh} &= \mbox{travel time in the scenario} \\ \mbox{tt}_{ff.veh} &= \mbox{travel time in free flow} \end{array}$

In the macroscopic model, where vehicles are not modelled individually, the *total experienced delay* T_{lost} is calculated by:

$$T_{lost} = \sum_{t=1}^{\infty} \sum_{link=1}^{\infty} \left(q_{link.t} \cdot \frac{l_{link}}{v_{\text{ff}.link} - v_{link.t}} \right)$$
(2)

Where

t	= time
$\mathbf{q}_{link,t}$	= traffic flow on link at time t
l _{link}	= length of link
V _{link.t}	= cell speed on link at time t
$V_{\rm ff.link}$	= cell speed on link in free-flow

The averaged travel time over route AB is the average of all travel times during the simulation on the route, and is defined as:

$$TT_{AB} = \sum_{t=1}^{n} \frac{\sum_{link=1}^{\infty} \left(\frac{l_{linkAB}}{v_{linkAB.t}} \right)}{n}$$
(3)

Where

 $\begin{array}{ll} \mathsf{TT}_{\mathsf{AB}} &= \text{travel time between origin A and destination B} \\ \mathsf{I}_{\mathsf{linkAB}} &= \mathsf{length of a link, situated between origin A and destination B} \\ \mathsf{v}_{\mathsf{linkAB.t}} &= \mathsf{cell speed on link at time t} \\ \mathsf{n} &= \mathsf{number of time steps} \end{array}$

Case setups

In the **first experimental case** a near-critical level of traffic flow is present on the network. This could represent a situation in a busy peak hour period on a well-designed network which nicely meets the extreme level of demand. In the reference scenario, the capacities are set to the median value of all possible capacity values corresponding to the capacity distribution; this is the 'average' situation. The stochastic scenario takes a sample from the capacity distribution (Fig. 3a) and applies these values to the network. This is iterated for 40 simulations and is performed for Latin hypercube and systematic sampling, to verify that the sampling method is not leading. Both systematic and Latin hypercube sampling are both advanced sampling methods that systematically sample from ordered subselections. For more information on these methods see [33, 34]. These methods are chosen, as they represent the input distributions in the outcomes much better than for simple random sampling for a low number of samples [33-35].

The **second experimental case** considers the event that the variability of the capacity is extensive. This may be the case in a period in which extreme weather is present in varying severity over an extended period of time. The capacity distribution as in figure 3b is applied, which shows a greater variation in capacity value compared to case 1. Again for the reference scenario, the capacities are set to the median value of the all possible capacity values corresponding to the capacity distribution. The stochastic scenario takes a sample from the capacity distribution and applies it to the network, which is repeated for 40 iterations. This is also performed for Latin hypercube and systematic sampling.

Results of experimental cases

The results from the experimental cases are shown in the form of histograms, as well as the numerical values for each sampling method. The outcome of the median input value, which is used to represent the deterministic case, is also given.



Case 1

Fig.3 Network delay for case 1. Sampled as systematic (a-left) and as Latin hypercube (b-right) sampling

Sampling method	Median Network delay (vehicle hours)	Average Network delay (vehicle hours)
Latin Hypercube	12164	17990
Systematic	12166	17986
Median input	9113	9113

Table 1 Network delay of case 1 in lost vehicle hours



Fig.4 Averaged travel times on route AB (see fig. X) for case 1. Sampled as systematic (left) and as Latin hypercube (right) sampling

Sampling method	Median Travel times (minutes)	Average Travel times (minutes)
Latin Hypercube	20.23	23.98
Systematic	20.23	23.95
Median input	18.16	18.16

Table 2 Averages travel times for case 1 on route AB (see fig. X)

Case 2



Fig.5 Network delay of case 2. Sampled as systematic (left) and as Latin hypercube (right) sampling

Sampling method	Median Network delay (vehicle hours)	Average Network delay (vehicle hours)
Latin Hypercube	12136	17845
Systematic	12164	18481
Median input	12359	12359

Table 3 Network delay of case 2 in lost vehicle hours

The results of case 1 show that, depending on the sampled capacity value, a skewed distribution is produced with an average of a little under 18000 lost vehicle hours in the network (Fig. 3 and Table 1). This is considerably higher than the deterministic situation, modelled with the input median, which produced a little over 9000 lost vehicle hours. In the probabilistic case, the near-critical level of traffic flow on capacity will be breached in many cases in which the capacity is marginally below the critical level of traffic flow. And while this may not happen in the majority of cases, when it does, widespread congestion can occur in the network and a higher number of lost vehicle hours are registered. The average capacity remains above that of the critical traffic demand. Because the 'average' situation, as modelled in a deterministic approach, does not trigger widespread congestion, the number of lost vehicle hours is significantly lower which gives a misleading outcome. When considering the travel times over route AB, a similar outcome is obtained (Fig. 4 and Table 2). The deterministic value (18.2 minutes) is set very close to the left side of the distribution, while travel times well above these 18 minutes are recorded in many cases.

The results of case 2 show similar distributions to that of the first case. The average of the repetitive simulations lies just over 18000 lost vehicle hours, while the deterministic run produces just over

12000 lost vehicles hours (Fig. 5 and Table 3). For this experiment a larger variation is applied to the input capacity variable. This results in a slightly larger spread of lost vehicle hours for the probabilistic approach. The deterministic approach also shows a greater number of lost vehicle hours due to a greater average capacity drop in the input, which allows the traffic demand to exceed capacity to a greater extent. As capacity in this case is low enough to become critical, the difference between the probabilistic and deterministic outcomes is smaller, while the average probabilistic outcomes are only slightly higher for case 2 than in case 1. This further show the sensitivity of the deterministic approach to small changes in the input, while these are easily represented by the probabilistic approach.

By considering a complete distribution of probable input values, a complete distribution of outcomes can be considered for the probabilistic approach. In the model, a small deterioration in road capacity has an amplified effect on the experienced traffic delay, a characteristic that is not picked up by the deterministic approach. We have therefore demonstrated a major deficiency of deterministic and simple stochastic models. The inability to consider anything other than an average situation and the sensitively to variations in 'real' input variables, by presuming single values rather than distributions, leads to a considerable chance of model results giving unreliable and biased outcomes.

3.2 Application horizon

While the need for a greater element of probability and consideration of variability has been shown, this does not apply for all applications in which traffic models are required. In many cases deterministic models will work just as well. It is therefore necessary to evaluate under which conditions variability should be considered, while considering potential drawbacks of including variation in traffic².

In general the main advantages of using deterministic models are the relatively short calculation time and the limited amount of input data required. The advantages of using probabilistic models are an increased accuracy with consideration of numerous situations, as demonstrated in the experimental cases, and the possibility of giving results with a reliability score. It is easy to see that a probabilistic model will always be preferred if it can be just as easily applied as a deterministic model, however this is not the case. It is therefore necessary to review the goals and requirements of a model analysis before performing calculations. This is a step that is too often omitted in practice, mainly due to practical issues or understandable unawareness from the viewpoint of the user. Considering the aforementioned advantages of the models, a concise overview of conditions under which both model types should be used, is given in Table 4.

Probabilistic modelling	Deterministic modelling
Applicable for	Applicable for
Large variation in input variables	Limited variation in input variables
Distribution of input variable is reliable and can	Distribution is unreliable and cannot be easily
be easily determined	determined
Variation in input variables has an amplified	Variation in input variables has a limited or linear
effect on model outcome	effect on model outcome
Congested network with high congestion	Uncongested network or congested with low
volatility	congestion volatility
Comprehensive overview of network	General indication of network performance
performance	

 Table 4
 Application horizon for probabilistic models versus deterministic models.

² For deterministic, one may also read deterministic and simple stochastic, and for probabilistic, one also may read probabilistic and fully stochastic.

Variations in the input variables lead to a primary source of variation in the model results. When these variations are relatively large, the results from model runs will also show greater variations. When the level of variation is small, there is no need to apply a probabilistic approach and a deterministic approach suffices. The same is valid when the variation in input variables has a direct linear correlation with the outcome of a model, as in both cases a similar 'average' situation will result from both a distributed input and a mean or median input. In an uncongested network this will often be the case, as traffic can propagate at (near) desired speeds without too much disruption, resulting in a stable model output. Furthermore, it goes almost without saying that when probability distributions or functions cannot be accurately constructed, one should apply a known variable in deterministic model rather than applying inaccurate presumptions of a distribution function. Finally the main application of probabilistic modelling should be to give an accurate and comprehensive overview of traffic on network under a wide variety of conditions. If one is merely interested in a general indication of network performance then a deterministic model again suffices.

4. Challenges for further development

Though research on pure probabilistic modelling is gaining momentum, a number of significant challenges remain for the further development of probabilistic macroscopic modelling. And while many of these challenges have been addressed individually or in part in research, a further challenge remains in bringing each part together to form a complete and operational probabilistic model. The main challenges discussed here are:

- 1. Computational efficiency
- 2. Correlation between multiple variables
- 3. Data gathering and processing (as input and for calibration)

We might add a fourth in the form of the implementation, however this has already been discussed in part in the previous section, and does not explicitly affect the core workings of the model. Therefore we limit ourselves to the first three.

4.1 Computational efficiency

Consideration of computational efficiency applies to the computational load of a model on the applied hardware, but also the speed at which calculations can be made as a consequence of the applied calculations. Macroscopic models in their application are almost always applied to larger networks and therefore demand computational power, which is severely compromised by including probability. The computational load of models in general has been seen as a problem in the past [3, 6, 24] However nowadays this problem is diminishing with the increase in computational power of hardware [24]. Nevertheless the possibilities of increased computational power seem to always be tested to the limit as advancement of modelling techniques continually demand greater computational power [36]. For both probabilistic methods mentioned in this paper: repetitive simulations and one-shot analytical solutions, there are difficulties relating to scientific advancement in terms of the computational efficiency.

Repetitive, or rather Monte Carlo, simulation techniques have applied greater computing power to tackle the lack of applied variables and the complexity of the variables functions [24]. Greater numbers of random variables are considered in the input, and model, in an attempt to describe the traffic system to a more realistic extent. This however means that correlation between considered variables becomes of greater importance as the effect of correlation becomes greater as one considers larger numbers of dependant events. As described in the following paragraph, determining correlation functions is hard enough, however calculating them also leads to a greater demand of hardware resources.

The analytical approach to probabilistic modelling may even hold a greater challenge to computational possibilities. Even now they can still be time-consuming due to multiple intricate equations that need to be solved [3]. As solutions for probabilistic approaches emerge, the complexity level of mathematical algorithms with multiple differential equations remains high [24]. In this, a simple rule that the more elaborate the solution, the greater the computational load, is

evident. Recent developments should be lauded, but come in many cases with such drawbacks. The challenge for researchers in this field is therefore: not only to develop elegant solutions for probabilistic modelling, but to do this in a manner that allows easy and efficient application in computational terms. Furthermore, with a greater efficiency, comes a larger network that can be calculated, shorter calculation times, and a greater robustness of the model.

4.2 Correlation between multiple variables

When applying probabilistic modelling it is a necessity to consider multiple random variables as both input and in the model itself, depending on the applied approach. In the simplest terms, one has at least the traffic demand and supply as input variables, however these may consist of many other variables, such as weather effects, general randomness in demand, and others. These all have some level of dependence which cannot be ignored [24]. Also in the core of the model, dependencies are present between values of random variables. In deterministic modelling, one has only to consider single values, which relate directly to one another. Within random variables, not every permutation will be possible in conjunction with another from a separate random variable. A simple example of this is a speed of 100 km/h which will never occur simultaneously with a traffic density of 40 veh/hr/lane, while both may be present as part of the probability of their random variables. A limited number of solutions have been proposed to deal with correlations in [24, 37], however many of the approaches are complex or may only deal with specific dependant relations. While offering some sort of solutions, a difficulty remains and is connected to the challenges from the previous paragraph, in that the applicableness of the methods in an operational model may be cumbersome due to their complexity. To this extent there remains a challenge to develop a global approach to consider correlation between random variables in a manner that can be easily implemented and that does not substantially detract from the efficiency of the model.

4.3 Data gathering and processing

Probabilistic models by definition work with a wide variety of possible values for the considered random variables. The outcomes of these models will often be given as a distribution, and the input will often encompass an even greater spread of data points. In some cases input for probabilistic models will be explicitly applied from empirically collected data, and other cases will be applied from an empirically derived or presumed analytical function. In either case there is a need for large amounts of data to form a generically valid distribution or to validate the presumed function. The specific type of data depends heavily on the manner in which an approach is applied. However for approaches which try to include multiple variations of traffic influencing variables, such as weather conditions, gathering and processing the required data is not a trivial task. If we consider weather and even the effects of snow, it must be pointed out that a great number of permutations are possible. One can distinguish between snowfall and lying snow on the road surface, between the first snowfall of the year and snow two weeks later when drivers have already become accustomed to the conditions. Also various combinations of weather conditions can be considered, such as strong winds, poor luminance, and low sunshine, all in combination with snow. Each situation needs consideration to be able to determine specific causation of events and correlations between the events. This requires years of data, and even then this may be insufficient. This challenge obviously applies for many other variables, besides the weather. And once sufficient data has been gathered, it still needs to be processed. The principal difficulty of this, is processing the data in such a way that dependencies between variables are correctly reflected in the random variables, or as a correlation function. To address these issues the application or development of concise methodologies is required, which will allow for an efficient and comprehensive data processing and result in accurate distributions.

5. Conclusions

In this paper the case for probabilistic approaches in macroscopic traffic modelling is argued. This begins with a description of current practices in traffic flow modelling, and more importantly, in stochastic and probabilistic traffic flow modelling. It is shown that currently two main avenues of probabilistic models are utilised: repetitive Monte Carlo simulation, and the analytical consideration of probability in the core of a model. Current and recent research developments on both of these approaches are discussed. While classically, the Monte Carlo approach has been applied, the advancement of various analytical approaches has increased, with a number extensions of deterministic models being proposed.

Too often probabilistic models are not considered in practice, either for application or the necessity for development. Focussing on deterministic or simple stochastic models has the danger of closing ones eyes to inaccuracies caused by an incorrect choice of modelling approach. To demonstrate this, two experimental cases are given in which the application of a deterministic approach is shown to yield substantially biased results in comparison to a probabilistic approach. While probabilistic models can be seen as more 'complete' than deterministic models, their application is not recommended in every situation. A short investigation is therefore performed on the application horizon of probabilistic models.

While the case for probabilistic modelling is strong in theory, the application of such modelling approaches is only possible with sufficiently developed models. However there are still certain challenges to be addressed in probabilistic modelling before a widespread implementation is likely. These refer to computational efficiency of the models, both as computational load and efficiency in applied algorithms. Overcomplicated analytical approaches, while possibly being sound, may negatively contribute to this inefficiency. Furthermore the matter of correlation between variables poses an interesting challenge. This is the case for both correlations between input variables in the models, and also between random variables for analytical approaches. Also collecting and processing empirical data as probability input or for calibration requires attention and is not as trivial as may seem.

To conclude, there is a necessity, but also many challenges for the scientific and consultancy worlds to further the development and application of probabilistic modelling in traffic analysis. A realisation must arise of the detrimental effects of blindly applying non-probabilistic models where probability is rife. It is the joint responsibility of both worlds to address this and make further developments in this area of research possible.

Acknowledgements

This research is jointly funded by *TNO*, Netherlands Organisation for Applied Scientific Research, and *TrafficQuest*, a joint collaboration between TNO, Delft University of Technology, and Rijkswaterstaat, highway agency of the Dutch Ministry of Infrastructure and Environment.

References

- [1] C. Tampère and F. Viti, "Dynamic Traffic Assignment under Equilibrium and Non-equilibrium: Do We Need a Paradigm Shift?," in *3rd International Symposium on Dynamic Traffic Assignment*, 2010.
- [2] J. W. C. van Lint, O. Miete, H. Taale, and S. P. Hoogendoorn, "A systematic framework for the assessment of traffic measures and policies on the reliability of traffic operations and travel time," *91th meeting of the Transportation Research Board*, 2012.
- [3] A. Sumalee, R. Zhong, T. Pan, and W. Szeto, "Stochastic cell transmission model (SCTM): A stochastic dynamic traffic model for traffic state surveillance and assignment,"
 Transportation Research Part B: Methodological, vol. 45, pp. 507-533, 2011.
- [4] S. E. Jabari and H. X. Liu, "A stochastic model of traffic flow: Theoretical foundations," *Transportation Research Part B: Methodological*, vol. 46, pp. 156-174, 2012.

- [5] W. Szeto, Y. Jiang, and A. Sumalee, "A Cell-Based Model for Multi-class Doubly Stochastic Dynamic Traffic Assignment," *Computer-Aided Civil and Infrastructure Engineering*, 2011.
- [6] A. Chen, H. Yang, H. K. Lo, and W. H. Tang, "Capacity reliability of a road network: an assessment methodology and numerical results," *Transportation Research Part B: Methodological*, vol. 36, pp. 225-252, 2002.
- [7] S. Clark and D. Watling, "Modelling network travel time reliability under stochastic demand," *Transportation Research Part B: Methodological*, vol. 39, pp. 119-140, 2005.
- [8] R. Boel and L. Mihaylova, "A compositional stochastic model for real time freeway traffic simulation," *Transportation Research Part B: Methodological*, vol. 40, pp. 319-334, 2006.
- [9] S. P. Hoogendoorn and P. H. L. Bovy, "State-of-the-art of vehicular traffic flow modelling," *Proceedings of the Institution of Mechanical Engineers Part I-Journal of Systems and Control Engineering*, vol. 215, pp. 283-303, 2001.
- [10] M. J. Lighthill and G. B. Whitham, "On Kinematic Waves .2. A Theory of Traffic Flow on Long Crowded Roads," *Proceedings of the Royal Society of London Series a-Mathematical and Physical Sciences*, vol. 229, pp. 317-345, 1955.
- [11] C. F. Daganzo, "The cell transmission model: A dynamic representation of highway traffic consistent with the hydrodynamic theory," *Transportation Research Part B: Methodological*, vol. 28, pp. 269-287, 1994.
- [12] C. F. Daganzo, "The cell transmission model, part II: network traffic," *Transportation Research Part B: Methodological,* vol. 29, pp. 79-93, 1995.
- [13] H. J. Payne, "Models of freeway traffic and control," *Mathematical models of public systems,* 1971.
- [14] M. Papageorgiou, "Some remarks on macroscopic traffic flow modelling," *Transportation Research Part a-Policy and Practice,* vol. 32, pp. 323-329, Jun 1998.
- [15] S. P. Hoogendoorn and P. H. L. Bovy, "Modeling multiple user-class traffic," *Transportation Research Record: Journal of the Transportation Research Board*, vol. 1644, pp. 57-69, 1998.
- [16] A. Aw and M. Rascle, "Resurrection of" second order" models of traffic flow," *Siam Journal on Applied Mathematics*, pp. 916-938, 2000.
- [17] H. M. Zhang, "A non-equilibrium traffic model devoid of gas-like behavior," *Transportation Research Part B: Methodological,* vol. 36, pp. 275-290, 2002.
- [18] J. P. Lebacque, S. Mammar, and H. H. Salem, "Generic second order traffic flow modelling," in Transportation and Traffic Theory: Proceedings of the 17th International Symposium on Transportation and Traffic Theory, London, UK, 2007.
- [19] M. Brackstone and M. McDonald, "Car-following: a historical review," *Transportation Research Part F: Traffic Psychology and Behaviour*, vol. 2, pp. 181-196, 1999.
- [20] R. Wiedemann, "Simulation des Straßenverkehrsflußes," University of Karlsrühe1974.
- [21] E. Brockfeld, R. D. Kühne, and P. Wagner, "Calibration and validation of microscopic traffic flow models," *Transportation Research Record: Journal of the Transportation Research Board*, vol. 1876, pp. 62-70, 2004.
- [22] L. Chu, H. X. Liu, J. S. Oh, and W. Recker, "A calibration procedure for microscopic traffic simulation," 2003, pp. 1574-1579 vol. 2.
- [23] Attoh-Okine N.O. and A. B.M., *Applied Research in Uncertainty Modelling and Analysis* vol. International Series in Intelligent Technologies.: Springer Science, Germany., 2004.
- [24] C. Chang, Y. Tung, and J. Yang, "Monte Carlo Simulation for Correlated Variables with Marginal Distributions," *Journal of Hydraulic Engineering*, vol. 120, pp. 313-331, 1994.
- [25] A. Ang and W. Tang, *Probability Concepts in Engineering*: John Wiley & Sons, Inc, 2007.
- [26] S. Hess, K. E. Train, and J. W. Polak, "On the use of a Modified Latin Hypercube Sampling (MLHS) method in the estimation of a Mixed Logit model for vehicle choice," *Transportation Research Part B: Methodological*, vol. 40, pp. 147-163, 2006.
- [27] N. Jonnalagadda, J. Freedman, W. A. Davidson, and J. D. Hunt, "Development of microsimulation activity-based model for San Francisco: destination and mode choice

models," *Transportation Research Record: Journal of the Transportation Research Board,* vol. 1777, pp. 25-35, 2001.

- [28] L. Muñoz, X. Sun, R. Horowitz, and L. Alvarez, "Traffic density estimation with the cell transmission model," 2003, pp. 3750-3755 vol. 5.
- [29] X. Sun, L. Muñoz, and R. Horowitz, "Highway traffic state estimation using improved mixture Kalman filters for effective ramp metering control," 2003, pp. 6333-6338 Vol. 6.
- [30] J. Li, Q. Y. Chen, H. Wang, and D. Ni, "Analysis of LWR model with fundamental diagram subject to uncertainties," presented at the The 88th Transportation Research Board Annual Meeting, Washington DC, 2009.
- [31] T. Kim and H. Zhang, "A stochastic wave propagation model," *Transportation Research Part B: Methodological*, vol. 42, pp. 619-634, 2008.
- [32] I. Yperman, "The Link Transmission Model for dynamic network loading," *Ph.D. Thesis, Katholieke Universiteit Leuven.*, 2007.
- [33] R. L. Iman and W. Conover, "Small sample sensitivity analysis techniques for computer models. with an application to risk assessment," *Communications in statistics-theory and methods,* vol. 9, pp. 1749-1842, 1980.
- [34] M. D. McKay, R. J. Beckman, and W. Conover, "A comparison of three methods for selecting values of input variables in the analysis of output from a computer code," *Technometrics*, pp. 239-245, 1979.
- [35] K. Black, Business statistics: Contemporary decision making: Wiley, 2009.
- [36] M. C. J. Bliemer and H. Taale, "Route generation and dynamic traffic assignment for large networks," in *Proceedings of the First International Symposium on Dynamic Traffic Assignment (DTA 2006)*, Leeds, UK, 2006, pp. 90-99.
- [37] K. Berdica, "An introduction to road vulnerability: what has been done, is done and should be done," *Transport Policy*, vol. 9, pp. 117-127, 2002.