

# INTERNATIONAL COURSE IN HYDRAULIC ENGINEERING

LECTURE NOTES ON  
SEDIMENT TRANSPORT 1

BY  
H.N.C. BREUSERS

DELFT  
1983 - 1984

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### SEDIMENT TRANSPORT I

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## 1. INTRODUCTION

A study of the sediment transport by water is of importance in several aspects of hydraulic engineering:

- fluvial hydraulics: knowledge of sediment transport forms the basis for the design of river-training works, navigation improvement, flood control.
- irrigation: design of stable channels, intakes, settling bassins.
- coastal engineering: prediction of littoral drift, design of coastal protection works and harbours.
- dredging: the suction, transport and deposition of material has many aspects related to the transport of sediments.

The main objective of sediment transport hydraulics is to predict whether an equilibrium condition, erosion (scour) or deposition (silting) will occur and to determine the quantities involved. The rate of sediment transport, expressed as mass, weight or volume per unit time can be determined from measurements or from calculations. Both methods only have a low degree of accuracy so that the sensitivity of the design to possible variations in the calculated transport rates has to be considered.

The main reason for the empirical character of sediment transport knowledge is the complexity of the transport process. The interaction of a turbulent flow, the characteristics of which are only known by empirism, and a boundary consisting of loose sediments cannot be described by simple equations. Most of our knowledge is based therefore on experiments and measurements both in the field and in laboratories.

The following subjects will be discussed:

- the flow characteristics of the water
- the characteristics of the sediments
- their mutual interaction:
  - initiation of motion,
  - transport mechanisms,
  - bed forms, roughness,
  - stable channels,
  - bed material transport - bed load,
  - suspended load,
  - siltation and scour,
- sediment transport measurements
- applications



These lecture notes should be considered as an introduction to the subject. The following general references may be used for further studies:

- |                                 |      |   |
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## 2. PROPERTIES OF THE WATER

Some of the relevant properties of water are:

Property	symbol	dimension	remarks
density	$\rho$	$\text{kg} \cdot \text{m}^{-3}$	-
relative density under water	$\Delta$	- (ratio)	$\Delta = \frac{\rho_s - \rho_w}{\rho_w}$
dynamic viscosity	$\eta$	$\text{kg} \cdot \text{m}^{-1} \cdot \text{s}^{-1}$ or $\text{N} \cdot \text{s} \cdot \text{m}^{-2}$	$\tau = \eta \frac{\partial u}{\partial z}$
kinematic viscosity	$\nu$	$\text{m}^2 \cdot \text{s}^{-1}$	$\nu = \eta / \rho_w$
surface tension	$\sigma$	$\text{kg} \cdot \text{s}^{-2}$ or $\text{N} \cdot \text{m}^{-1}$	

The following S.I. units are used:

mass	(kg)	(kilogram)
length	(m)	(meter)
time	(s)	(second)
force	( $\text{kgm}/\text{s}^2$ ) or (N)	(Newton)
energy	( $\text{kgm}^2/\text{s}^2$ ) or (Nm) or (J)	(Joule)
power	( $\text{kgm}^2/\text{s}^3$ ) or (Nm/s) or (J/s) or (W)	(Watt)
pressure, stress	( $\text{kg}/\text{ms}^2$ ) or ( $\text{N}/\text{m}^2$ ) or Pa	(Pascal)

### 2.1. Density ( $\text{kg}/\text{m}^3$ )

The density of fresh water varies with temperature T:

T:	0	4	12	16	21	32	(°C)
$\rho_w$ :	999.87	1000.0	999.5	999.0	998.0	995.0	( $\text{kg}/\text{m}^3$ )

The variation of the density may be neglected in most sediment transport calculations.

	kg/m <sup>3</sup>
$\rho_w$ fresh water	1000
$\rho_w$ sea water	1026

## 2.2. Viscosity

Dynamic viscosity (Ns/m<sup>2</sup>)

Defined as the factor of proportionality in:

$$\tau = \eta \frac{\partial U}{\partial z}$$

which is valid for laminar flow.

$\partial U / \partial z$  = velocity gradient (s<sup>-1</sup>)

Kinematic viscosity (m<sup>2</sup>/s)

Defined by  $\nu = \eta / \rho_w$

$\eta$  and  $\nu$  are a function of temperature. The influence of temperature is significant.

T	0	5	10	15	20	25	30	35	40	(°C)
$\nu$	1.79	1.52	1.31	1.14	1.01	0.90	0.80	0.72	0.65	. (10 <sup>-6</sup> m <sup>2</sup> /s)

## 2.3. Surface tension

For the surface water/air:  $\sigma = 0.074$  N/m at atmospheric pressure.  
The variation with temperature can be neglected.

#### 2.4. Uniform flow in open channels

The equation of motion for steady, uniform flow is reduced to:

$$\frac{\partial \tau}{\partial z} = \frac{\partial p}{\partial x} \quad \tau = \text{shear stress} \quad p = \text{pressure}$$

or  $\tau(z) = \rho_w g(h - z) \cdot I$

$h$  = water depth

$z$  = distance from the bed

$I$  = hydraulic gradient or slope

The difficulty is now the relation between shear stress and velocity distribution which is necessary to predict this distribution.

For laminar flow the relation is:

$$\tau(z) = \eta \cdot \frac{\partial U(z)}{\partial z}$$

which leads to the parabolic velocity distribution:

$$U(z) = \frac{gI}{2\nu} (h^2 - (h - z)^2)$$

and a mean velocity  $\bar{U} = \frac{gI}{3\nu} \cdot h^2$

For turbulent flow Prandtl gave the following empirical mixing-length expression:

$$\tau(z) = \rho_w l^2 (\partial U(z)/\partial z)^2$$

Near the bed  $\tau(z) \approx \tau_o$ , the bed shear stress:

$$\tau_o = \rho_w g h I$$

and  $l = \kappa z$

$$\kappa = \text{kappa, von Kármán's constant} \approx 0.4 \quad (\text{from measurements})$$

This leads to the logarithmic velocity distribution:

$$U(z) = \kappa^{-1} \sqrt{gh I} \cdot \ln(z/z_o)$$

Define  $u^* = \sqrt{gh I}$  = shear velocity =  $\sqrt{\tau_o/\rho_w}$

and take:  $\kappa = 0.4$

then:  $U(z) = 2.5 u^* \ln (z/z_0)$

$z_0$  = the point where  $U = 0$  according to the logarithmic profile.

$U(z)$  is equal to the mean velocity at  $z \approx 0.4 h$

or  $\bar{U} = 2.5 u^* \ln (0.4 h/z_0)$

or  $\bar{U} = 5.75 u^* \log (0.4 h/z_0)$  ( $\ln \rightarrow \log$  gives factor 2.303)

Although the logarithmic velocity distribution was derived for the area near the bed, it appears from measurements that the logarithmic velocity profile is a good approximation for the full depth of the flow due to a simultaneous decrease in shear stress and mixing-length with  $z$ .

Values of  $z_0$  are found from experiments on smooth and rough boundaries. For smooth boundaries a viscous sublayer exists in which viscous effects predominate. The approximate thickness of this layer is  $\delta \approx 10 \nu/u^*$  (see below) and  $z_0 \approx 0.01 \delta \approx 0.1 \nu/u^*$ . For boundaries with uniform roughness Nikuradse has found:

$$z_0 \approx 0.03 k_s$$

in which  $k_s$  was the size of the sand grains used as roughness. This  $k_s$  is used as a standard roughness for other types of roughness.

#### Smooth boundary

$$z_0 \approx 0.01 \delta$$

$$U(z) = 5.75 u^* \log (100z/\delta)$$

$$\bar{U} = 5.75 u^* \log (40 h/\delta)$$

#### Rough boundary

$$z_0 \approx 0.03 k_s$$

$$U(z) = 5.75 u^* \log (33 z/k_s)$$

$$\bar{U} = 5.75 u^* \log (12 h/k_s)$$

$$\bar{U} = 5.75 u^* \log \left( \frac{12h}{k_s + 0.3\delta} \right)$$

$$\text{or } \bar{U} = (5.75\sqrt{g}) \cdot \sqrt{h I} \cdot \log \left( \frac{12h}{k_s + 0.3\delta} \right)$$

$$\text{or } \boxed{\bar{U} = 18\sqrt{h I} \cdot \log \left( \frac{12h}{k_s + 0.3\delta} \right)} \quad (\text{White - Colebrook})$$

which is the well-known Chézy equation:

$$\boxed{\bar{U} = C\sqrt{h I}}$$

A bed is defined as hydraulically smooth for  $k_s < 0.1\delta$

hydraulically rough for  $k_s > 6\delta$

The transition laminar - turbulent flow is generally given as:

$$Re = \bar{U}h/\nu \approx 600 \text{ for open channels.}$$

The value of  $u^*$  is related to the velocity distribution by:

$$u^* = \frac{1}{5.75} \cdot \frac{\partial U(z)}{\partial (\log z)} = \frac{1}{5.75} \cdot \frac{U(z_2) - U(z_1)}{\log z_2 - \log z_1}$$

but this method gives generally inaccurate results.

### Viscous sublayer $\delta$

In the viscous sublayer viscosity predominates. The velocity distribution therefore follows from  $\tau(z) = \eta \partial U(z)/\partial z$

$$\tau(z) = \tau_0 = \rho_w g h I = \rho_w u^{*2}$$

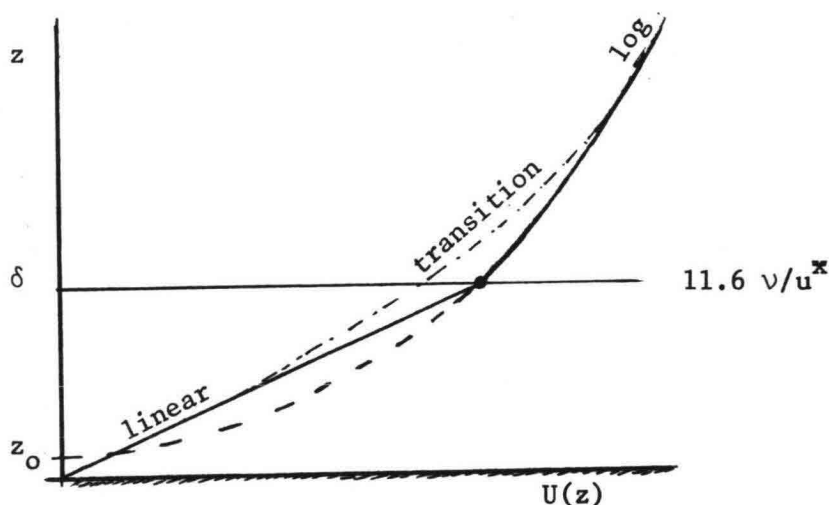
$$\text{or } \frac{U(z)}{u^*} = \frac{u^{*2} z}{\nu}$$

Intersection with the logarithmic velocity distribution gives a "theoretical" value for  $\delta$ :

$$\delta = 11.6 \nu / u^*$$

In fact there is a transition zone from the linear to the logarithmic profile extending from:

$$z = (5 \text{ to } 30) \nu / u^*$$



### Roughness value $k_s$

For uniform sediment  $k_s = D$ .

For graded sediment  $k_s = D_{65}$  to  $D_{90}$ .

For ripples  $k_s = (0.5 \text{ to } 1) h_{\text{ripple}}$ .

Errors in  $k_s$  give the following errors in  $C$ :

$\frac{k_{\text{actual}}}{k_{\text{estimated}}}$	1	2	5	10
$C_{\text{est-act}}$	0	5.5	12.5	18

### 2.5. Turbulence

Turbulence is a random fluctuating velocity field which interacts with and derives its energy from the mean flow field. A turbulent velocity field can only be described by statistical quantities such as r.m.s. values, amplitude distribution, correlations and spectra. The amplitudes are generally normally distributed so that the root-mean-square deviation gives a good idea of the fluctuations.  $\sigma_u = \sqrt{(U - \bar{U})^2}$  where  $U$  = the instantaneous velocity and  $\bar{U}$  the time-averaged value.

A turbulent field has a diffusive character. Gradients of momentum and scalar quantities are rapidly diminished by this diffusive action.

The analogy of turbulent motion with the movements of molecules leads to the analogy given by Boussinesq and the introduction of a eddy-viscosity concept for the apparent turbulent shear stress  $-\rho_w \overline{u'w'}$

$$-\rho_w \overline{u'w'} = \rho_w \epsilon_m \frac{\partial U}{\partial z} \quad (u', w' \text{ are velocity fluctuations in horizontal and vertical direction})$$

so that the total shear stress becomes :

$$\tau = \eta \frac{\partial U}{\partial z} - \rho_w \overline{u'w'} = \rho_w (\nu + \epsilon_m) \frac{\partial U}{\partial z}$$

$\epsilon_m$  = eddy viscosity.

The logarithmic velocity distribution:

$$U(z)/u_*^* = \frac{1}{\kappa} \ln (z/z_0)$$

and the linear shear stress distribution:

$$\tau(z) = \tau(0) (h - z)/h$$

give the following distribution for  $\epsilon_m(z)$

$$\epsilon_m(z) = \kappa u_*^* z (1 - z/h)$$

The average value of  $\epsilon_m(z)$  (averaging over the depth) is therefore:

$$\bar{\epsilon}_m = \frac{1}{6} \kappa u_*^* h.$$

## 2.6. Diffusion

The diffusion of scalar quantities (concentration, heat) is described by analogy with the diffusion of momentum by:

$$N = (D + \varepsilon_c) \partial C / \partial z$$

in which:

$N$  = lateral flux of scalar quantity

$D$  = molecular diffusivity

$\varepsilon_c$  = turbulent diffusion coefficient

$C$  = concentration

The value of  $D$  depends on the properties of the scalar:

heat in water       $D \approx 0.2 \cdot 10^{-6} \text{ m}^2/\text{s}$

salt in water       $D \approx 2 \cdot 10^{-9} \text{ m}^2/\text{s}$

The ratio of  $\varepsilon_c$  to  $\varepsilon_m$  depends also on the properties of the scalar but the value of this ratio is generally of the order one.

## 2.7. Literature

- |                         |   |
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## 2.8. Problems

For all problems  $g = 10 \text{ m/s}^2$        $\nu = 10^{-6} \text{ m}^2/\text{s}$        $\rho_w = 1000 \text{ kg/m}^3$

2.1 Given: a wide open channel has the following characteristics:

depth  $h = 2 \text{ m}$       roughness  $k_s = 1 \text{ mm}$   
 slope  $I = 10^{-5}$

Question: compute  $\bar{U}$ . (Is the bed rough/smooth/transition?).  
 same for  $k_s = 0.05 \text{ mm}$  and  $k_s = 5 \text{ mm}$ .

2.2 Given: wide open channel:

depth  $h = 1.2 \text{ m}$        $k_s = 0.5 \text{ mm}$   
 discharge/m<sup>1</sup>  $q = 0.8 \text{ m}^3/\text{s.m}$

Question: compute slope  $I$ . Is the bed smooth/rough/transition?

2.3 Given: wide open channel:

$k_s = 5 \text{ mm}$   
 $I = 2.10^{-5}$   
 $q = 1.6 \text{ m}^3/\text{s.m}$

Question: compute depth  $h$ .

2.4 Given: measurements in a wide open channel gave the following velocity profile:

$$U(z) = 0.148 \log z/z_0 \quad (U \text{ in m/s, } z \text{ in m.})$$

Questions: 1) compute  $U^*$ .

2) compute  $k_s$  if the velocity at  $z = 0.1 \text{ m}$  was equal to  $0.31 \text{ m/s}$ .

2.5 Given: Velocity measurements in a wide open alluvial channel gave the following results:

at  $z = 0.1 \text{ m}$  :  $U(z) = 0.345 \text{ m/s}$       Sediment size  $D = 150 \mu\text{m}$   
 at  $z = 0.3 \text{ m}$  :  $U(z) = 0.427 \text{ m/s}$ .

Questions: 1) Compute  $U^*$  (from the difference in the  $U(z)$  values, assuming the logarithmic velocity distribution).

2) Compute  $k_s$ .

3) Compare  $k_s$  with  $D$ . Is the bed plane or are bedforms present?

### 3. PROPERTIES OF THE TRANSPORT MATERIAL

Some of the properties of sediment which are often used are:

size  
shape  
density  
fall velocity  
porosity

#### 3.1. Size

A classification of particles according to size is given in table 3.1. This table gives the classification by the American Geophysical Union for clay, silt, sand, gravel, cobbles and boulders.

Various definitions of "diameter" are possible:

sieve diameter  $D$  = diameter of square mesh sieve which will just pass the particle.

sedimentation diameter  $D_s$  = diameter of sphere with same density and same settling velocity in same fluid at same temperature.

nominal diameter  $D_n$  = diameter of sphere with equal volume.

triaxial dimensions  $a, b, c$  ( $a$  = largest,  $c$  = smallest axis)

#### Size determination

boulders, cobbles and gravel: direct measurement

gravel, sand : sieving

fine sand, silt : sedimentation or microscope analysis

##### 3.1.1. Sieving

Sieving can be applied for particles down to  $44\ \mu\text{m}$  but gives good results down to  $74\ \mu\text{m}$ . Sieve sizes (openings) are made in a geometric series with every sieve being  $\sqrt[4]{2}$  larger in size than the preceding. Taking every other size gives a  $\sqrt{2}$  series. For most sands a  $\sqrt{2}$  series gives sufficient results but a  $\sqrt[4]{2}$  series may be necessary for very uniform sands. Some general rules for sieving can be given:

1. Do not overload sieves to avoid clogging. The following maximum residues on individual 8-inch sieves are recommended (after Shergold 1946).

Table 3.1

Major classification of sediment size  
according to H.A. Einstein

Size	Designation	Remark
$D < 0.5\mu\text{m}$	Colloids	Always flocculated
$0.5\mu < D < 5\mu\text{m}$	Clay	Sometimes or partially flocculated
$5\mu < D < 64\mu\text{m}$	Silt	Nonflocculating-individual crystals
$64\mu < D < 2\text{ mm}$	Sand	Rock fragments
$2\text{ mm} < D$	Gravel, boulders	Rock fragments

American Geophysical Union (AGU) grade scale for particle sizes

Size			Class
Millimeters	Microns	Inches	
4,000-2,000		160-80	Very large boulders
2,000-1,000		80-40	Large boulders
1,000-500		40-20	Medium boulders
500-250		20-10	Small boulders
250-130		10-5	Large cobbles
130-64		5-2.5	Small cobbles
64-32		2.5-1.3	Very coarse gravel
32-16		1.3-0.6	Coarse gravel
16-8		0.6-0.3	Medium gravel
8-4		0.3-0.16	Fine gravel
4-2		0.16-0.08	Very fine gravel
2.00-1.00	2,000-1,000		Very coarse sand
1.00-0.50	1,000-500		Coarse sand
0.50-0.25	500-250		Medium sand
0.25-0.125	250-125		Fine sand
0.125-0.062	125-62		Very fine sand
0.062-0.031	62-31		Coarse silt
0.031-0.016	31-16		Medium silt
0.016-0.008	16-8		Fine silt
0.008-0.004	8-4		Very fine silt
0.004-0.002	4-2		Coarse clay
0.0020-0.0010	2-1		Medium clay
0.0010-0.0005	1-0.5		Fine clay
0.0005-0.00025	0.5-0.24		Very fine clay

Sieve opening mm.	U.S. Sieve nr.	Maximum residue in grams		
		2-series	$\sqrt{2}$ -series	$\sqrt[4]{2}$ -series
2.4	8	150	75	38
1.2	16	100	50	25
0.6	30	70	35	18
0.295	50	50	25	12
0.15	100	35	18	9
0.076	200	25	12	6

The total sample size should be about 20 - 50 grams for 8"-inch sieves and fine sand.

2. A sieving time of 10 minutes with a mechanical sieving apparatus should be used
3. For coarse sands and gravel the following minimum size is recommended to obtain a sufficient number of grains in each fraction (see De Vries 1971).

$$\text{Sample size (gram)} > 20 \cdot D_{85}^3 \quad D_{85} \text{ in mm.}$$

Sieve types and series are different in various countries, but are generally based on a  $\sqrt[4]{2}$ -series.

### 3.1.2. Sedimentation

For fine sand and silt a size distribution can be determined by sedimentation. For particles  $< 50 \mu\text{m}$  the Stokes law for the settling velocity is valid; for coarser particles empirical relations have to be used. Various principles are used: sedimentation balances, pipette analysis, visual accumulation tube (fig. 3.1) (for a review see ASCE 1969). Sedimentation gives of course no independent size and shape determination.

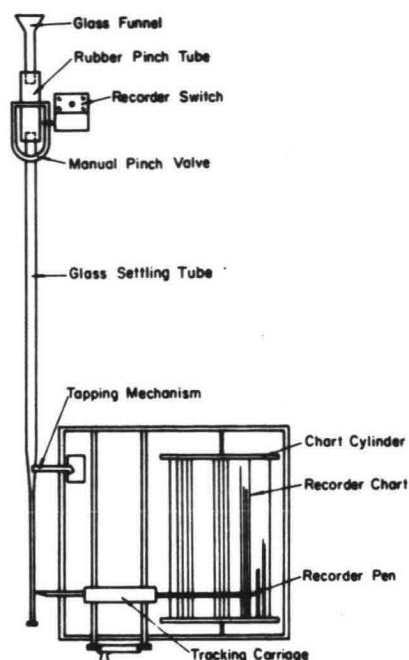


FIG. 3.1 .--SKETCH OF VISUAL ACCUMULATION TUBE AND RECORDING MECHANISM

### 3.1.3. Size distribution

By sieving or sedimentation a size distribution can be obtained which is generally expressed as a "percent by weight" vs "grain size" distribution. The cumulative size distribution of most sediments can be approximated by a log-normal distribution. A log-normal distribution will give a straight line if logarithmic probability paper is used (figure 3.2).

From the cumulative size distribution the mean diameter can be defined:

$$\bar{D} \text{ or } D_m = \frac{\sum p_i D_i}{\sum p_i}$$

in which  $p_i$  : fraction with diameter  $D_i$ .

$D_i$  is the geometric mean of the size fraction limits.

Also the notation  $D_p$  is used which denotes the diameter in a mixture of which  $p\%$  is smaller than  $D_p$ .  $D_{50}$  is also called the median diameter

For a given distribution we can define the geometric mean diameter

$$D_g = (D_{84} \cdot D_{16})^{\frac{1}{2}} \text{ (which is equal to } D_{50} \text{ for a log-normal distribution)}$$

and the geometric standard deviation:

$$\sigma_g = [D_{84}/D_{16}]^{\frac{1}{2}}$$

In geological literature also  $\phi$ -units are used:

$$\phi = -2 \log D \quad (D \text{ in mm})$$

$$\phi (1 \text{ mm}) = 0, \phi (0.5 \text{ mm}) = 1 \text{ etc.}$$

$$\sigma_g \text{ becomes in } \phi\text{-units: } \sigma_\phi = \frac{1}{2}(\phi_{16} - \phi_{84}).$$

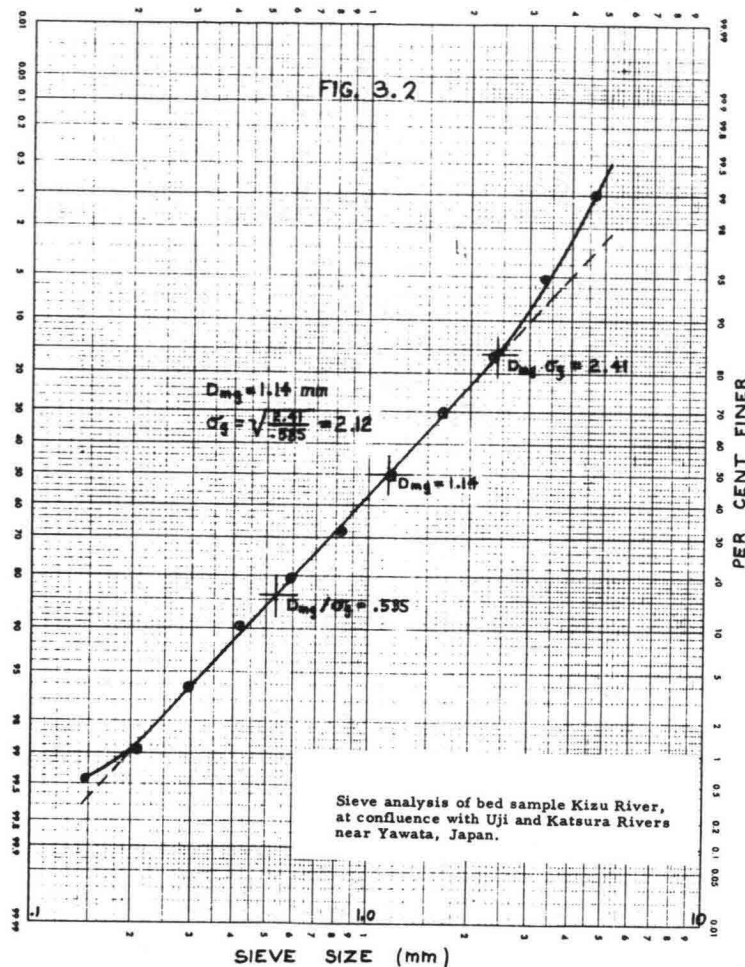


Fig. 3.2 Example of cumulative distribution of sieve diameter on logarithmic probability paper

### 3.2. Shape

Beside of the grain-diameter also the shape is of importance. A flat particle will have a smaller fall velocity and will be more difficult to transport as a rounded particle as bed load.

Several definitions may be used to characterise the shape:

**Sphericity** = ratio of the surface area of a sphere and surface area of the particle at equal volume

**Roundness** = ratio of the average radius of curvature of the edges and the radius of circle inscribed in the maximum projected area of the particle

**Shape factor** =  $s.f = c/\sqrt{ab}$  in which  $a$ ,  $b$  and  $c$  are three mutually perpendicular axes, from which  $a$  is major,  $b$  is intermediate and  $c$  is minor axis.

For spheres  $s.f = 1$ , for natural sands  $s.f \approx 0.7$

Roundness and sphericity are not suited for practice whereas the shape factor gives sufficient results for practical application.

### 3.3. Density

Most sediments originate from disintegration or decomposition of rock.

clay : fragments of feldspars and micas  
 silt : silicas  
 sand : quartz  
 gravel and boulders: fragments of original rock

The density of most sediment particles (< 4 mm) varies between narrow limits. Since quartz is predominant in natural sediments the average density can be assumed to be 2650 kg/m<sup>3</sup> (specific gravity 2.65). Sometimes heavy minerals are present which can be segregated during ripple formation or other modes of transport. Clay minerals range from 2500 - 2700 kg/m<sup>3</sup>.

### 3.4. Fall velocity

The fall velocity of a sediment is an important parameter in studies on suspension and sedimentation of sediments. The fall velocity is defined by the equation giving equilibrium between gravity force and flow resistance:

$$\underbrace{\frac{\pi}{6} \cdot D^3 (\rho_s - \rho_w) g}_{\text{gravity}} = \underbrace{C_D \cdot \frac{1}{2} \rho_w W^2 \cdot \frac{\pi}{4} D^2}_{\text{resistance}}$$

in which  $C_D$  = drag coefficient  
 $W$  = fall velocity

From this relation follows:

$$W = \left( \frac{4}{3} \cdot \frac{gD}{C_D} \cdot \Delta \right)^{\frac{1}{2}}$$

in which  $\Delta = (\rho_s - \rho_w) / \rho_w$

Values of  $C_D$  depend on a Reynold's number  $W.D/\nu$  and the shape of the particle (expressed by  $s.f = c/\sqrt{ab}$ )

For spherical particles and low Reynolds number ( $Re < 1$ ),  $C_D$  can be given by  $C_D = 24/Re$  so that:

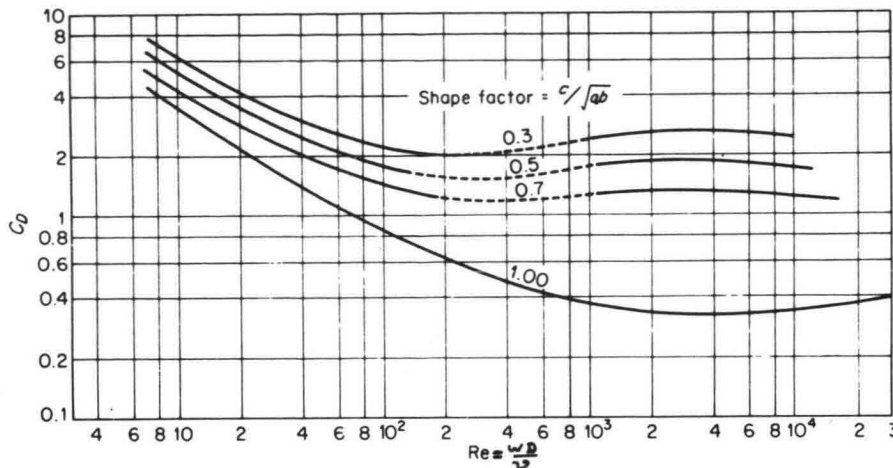
$$W = \frac{\rho_s - \rho_w}{18\mu} gD^2 = \frac{\Delta g D^2}{18\nu} \quad (\text{Stokes law})$$

For large Reynolds numbers  $C_D$  becomes a constant so that  $W$  varies as:

$$(\Delta g D)^{\frac{1}{2}}$$

Therefore  $W$  varies with  $D^{\frac{1}{2}}$  to  $2$ .

Relations between  $C_D$ ,  $Re$  and s.f are given by Albertson (1953) (see Figure 3.3). For natural sands s.f  $\approx 0.7$ . From these relations graphs for  $W$  as a function of grain size, shape and temperature can be obtained (see Figure 3.4).



**Fig.3.3** Drag coefficient vs. Reynolds number for different shape factors. [After ALBERTSON (1953).]

The presence of large number of other particles will decrease the fall velocity of a single particle. A cluster of particles will have a greater velocity however. Therefore care must be taken with experiments on the fall velocity to avoid currents in the fluid that will influence the fall velocity of the particle and the influence of concentration should be considered.

There are many expressions giving the influence of concentration on the fall velocity. Based on systematic experiments, Richardson and Zaki (1954) give a useful expression:

$$W(c)/W(o) = (1 - c)^\alpha \quad 0 \leq c < 0.3$$

$W(c)$  is the fall velocity of a grain in a suspension with concentration by volume  $c$

$W(o)$  is the fall velocity for a single grain

$\alpha$  is a function of Reynolds number  $W.D/v$

$Re < 0.2$	$\alpha = 4.65$
$0.2 < Re < 1$	$\alpha = 4.35.Re^{-0.03}$
$1 < Re < 200$	$\alpha = 4.45.Re^{-0.1}$
$Re > 500$	$\alpha = 2.39$

The coefficient is slightly dependent on particle shape but this can be neglected. For fine sediments this means that a concentration of 1% gives a reduction in fall velocity of 5%.

The fall velocity of a particle in a turbulent fluid can be different from that in a quiescent fluid (see chapter 6.2).



Example:  $D = 0.4 \text{ mm}$      $s.f. = 0.7$      $T = 100^\circ \text{C}$   
 This gives  $W = 5.3 \text{ cm/s}$

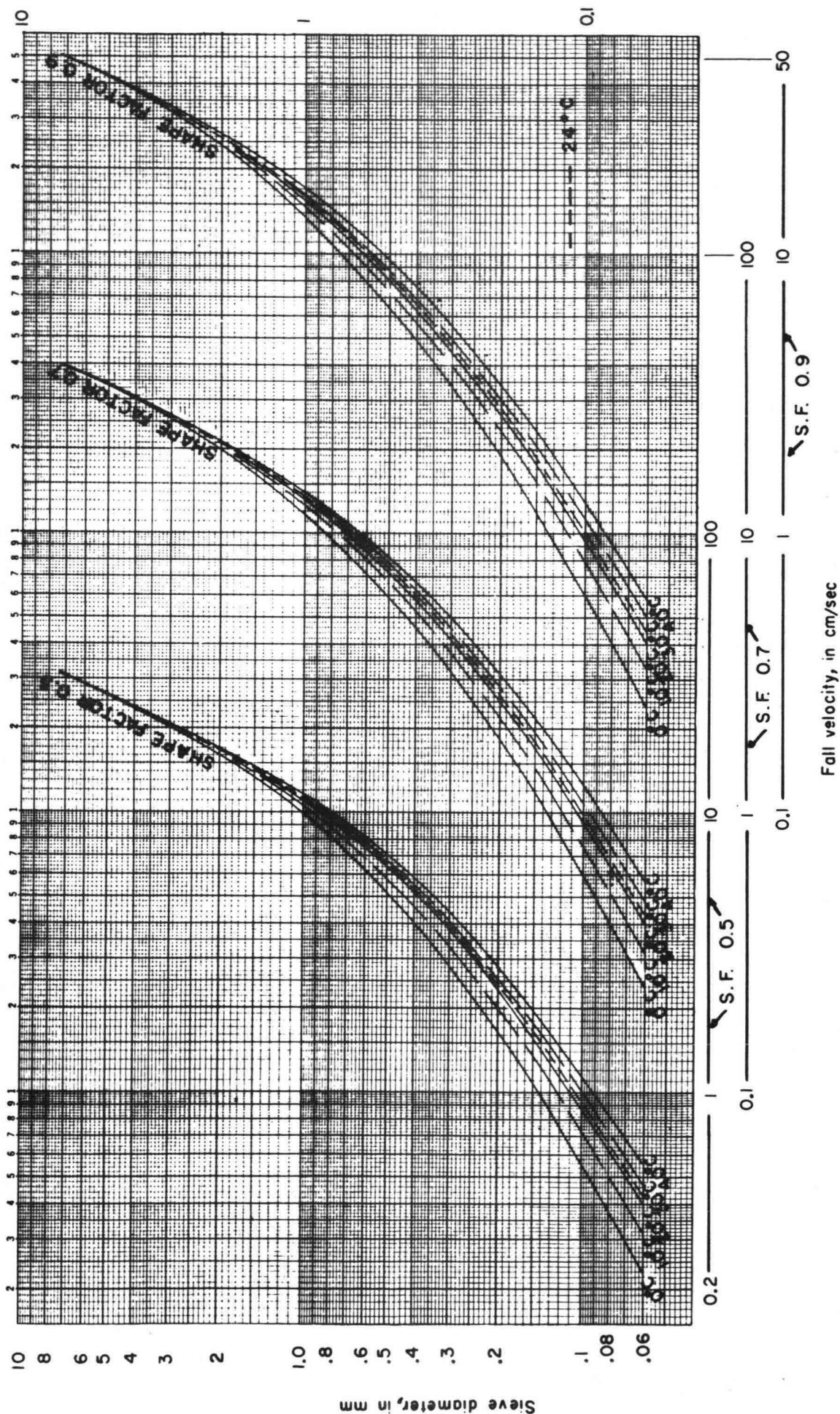


FIG.3.4 RELATION OF SIEVE DIAMETER AND FALL VELOCITY FOR NATURALLY WORN QUARTZ PARTICLES  
 FALLING ALONE IN QUIESCENT DISTILLED WATER OF INFINITE EXTENT

### 3.5. Bulk density and porosity

In estimating the life of a reservoir and similar cases the calculated weight of the sediment transported to the reservoir has to be converted into volume. For this the dry mass per unit volume of sediment in place, bulk density,  $\rho_b$ , has to be estimated.

For instance for air-dried fine sediments 1200-2000 kg/m<sup>3</sup> applies. The same material deposited under continuously submerged conditions may range from 300 - 1000 kg/m<sup>3</sup>. The density will also depend on the grainsize and silt content.

Bulk density,  $\rho_b$  = the mass of dry sedimentary material within a unit of volume (kg/m<sup>3</sup>). The volume taken by the sediment depends on the conditions of settling and may be a function of time due to consolidation. An empirical relation is presented by Lane and Koelzer (1953) for estimating the bulk density of deposits in reservoirs:

$$\rho_{b_T} = \rho_{b_1} + B \log T$$

$$\rho_b = (1 - \epsilon)\rho_s$$

$\epsilon$  = relative pore volume (porosity)

T = time in years

$\rho_{b_1}$  = initial bulk density taken to be the value after one year of consolidation

B = consolidation coefficient

Reservoir operations	sand		silt		clay	
	$\rho_{b_1}$	B	$\rho_{b_1}$	B	$\rho_{b_1}$	B
sediment always submerged or nearly submerged	1500	0	1050	90	500	250
normally a moderate reservoir drawdown	1500	0	1185	45	750	170
normally considerable reservoir drawdown	1500	0	1275	15	950	100
reservoir normally empty	1500	0	1320	0	1250	0

Lane and Koelzer also gave the simple relation  $\rho_{b_1} = 817(P + 2)^{0.13}$  in which  $P$  = percentage of sand.

Lara and Pemberton (1963) analysed 1316 samples and gave somewhat different values of  $\rho_{b_1}$  (in  $\text{kg}/\text{m}^3$ ). The following size classification was used:

clay: material  $< 4\mu\text{m}$

silt: material 4 to  $62.5\mu\text{m}$

sand: material  $> 62.5\mu\text{m}$

Type	Reservoir operation	$\rho_{b_1}$		
		clay	silt	sand
I	Sediment always submerged or nearly submerged	420	1120	1550
II	Normally moderate to considerable reservoir drawdown	560	1135	1550
III	Reservoir normally empty	640	1150	1550
IV	River-bed sediments	960	1170	1550

The r.m.s. deviation for the correlation was  $200\text{ kg}/\text{m}^3$  which means that considerable deviations are possible.

Example: A sediment in a type I reservoir contains 20% clay, 45% silt and 35% sand. The density of the sediment will then be

$$\rho_{b_1} = 0.20 \times 420 + 0.45 \times 1120 + 0.35 \times 1550 = 1130\text{ kg}/\text{m}^3$$

Murthy and Banerjee (1976) analysed 832 samples from Indian reservoirs with type II operation. The following values of  $\rho_{b_1}$  were obtained:

sand:  $1506\text{ kg}/\text{m}^3$     silt:  $866\text{ kg}/\text{m}^3$     clay:  $561\text{ kg}/\text{m}^3$

The results cannot be compared directly with Lara and Pemberton because the division between sand and silt was taken at  $20\mu\text{m}$ .

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## 4. INITIATION OF PARTICLE MOTION

### 4.1. Introduction

The equilibrium of a particle on the bed of a stream is disturbed if the resultant effect of the disturbing forces (drag force, lift force, viscous forces on the particle surface) becomes greater than the stabilising forces as gravity and cohesion. Cohesion is only important for sediments in the clay and silt range or fine sands with an appreciable silt content. The acting forces have to be expressed in known quantities such as velocities or bottom shear stress. They will have a strongly fluctuating character so that the initiation of motion also has a statistical aspect.

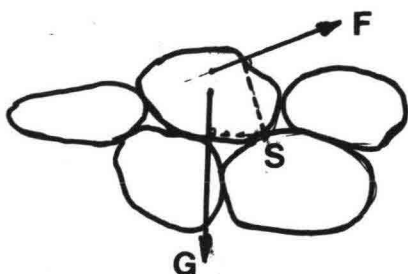
Theoretical work on the initiation of a motion has started with work by Brahms (1753) who gave a sixth power relation between flow velocity and the necessary weight of a stone and by Dubuat (1779, 1786) who introduced the concept of bottom shear stress and did some experiments on particle movement. Most of the older relations have the form:

$$U_{\text{bottom, crit}} = (4 - 5)\sqrt{D} \quad (D \text{ in m, } U \text{ in m/s})$$

As the "bottom" is not well defined the use of this type of formula is limited.

### 4.2. Theory

White (1940) gave a thorough discussion on the equilibrium of a grain on the bed of a stream.



The disturbing force  $F$  (resultant of drag and lift forces) will be proportional to the bottom shear stress  $\tau_o$  and the particle surface area ( $D^2$ ).

The stabilizing gravity force is proportional to  $(\rho_s - \rho_w)gD^3$ . Taking the moment with respect to the turning point  $S$  gives the equation:

$$\alpha_1 \tau_o D^2 \geq \alpha_2 (\rho_s - \rho_w) g D^3$$

$$\text{or: } \tau_o \geq C(\rho_s - \rho_w) g D$$

The factor C will depend on the flow condition near the bed, particle shape, the position of the particle relative to other particles etc. The flow condition near the bed can be described by the ratio of grain size to thickness of the viscous sublayer which ratio is proportional to  $U_{cr}^x D/\nu = Re^x$ , a Reynolds number based on grain size and shear velocity.

All other theoretical considerations based for example on drag force due to velocity will give the same result that:

$$\psi_{cr} = U_{cr}^{x2} / \Delta g D = f(Re^x)$$

### 4.3. Experiments

The relation:

$$\psi_{cr} = \frac{\tau_{cr}}{(\rho_s - \rho_w) g D} = \frac{U_{cr}^{x2}}{\Delta g D} = f \frac{(U_{cr}^x D)}{\nu} = f(Re^x)$$

has been investigated by many authors especially by Shields (1936) who did systematic tests and compared his results with results from other investigations (see figure 4.1). The difficulty in all tests is the definition of "initiation" of motion. It is the movement of the first particle or of a large number of grains? Shields correlated the rate of sediment transport with  $\tau_o$  and defined  $\tau_{cr}$  by extrapolating to zero material transport.

For large  $Re^x$  (rough bed) it can be seen that  $U_{cr}^x$  varies with  $\sqrt{D}$  (figure 4.2). For equal values of  $h/D$  and therefore equal values of  $\bar{U}/U^x$  it follows that  $\bar{U}_{cr} \sim \sqrt{D}$  and that the critical velocity of a stone is proportional to the 1/6 power of the weight of the stone (or stone weight proportional to  $\bar{U}^6$ ).

### 4.4. Influence of various factors

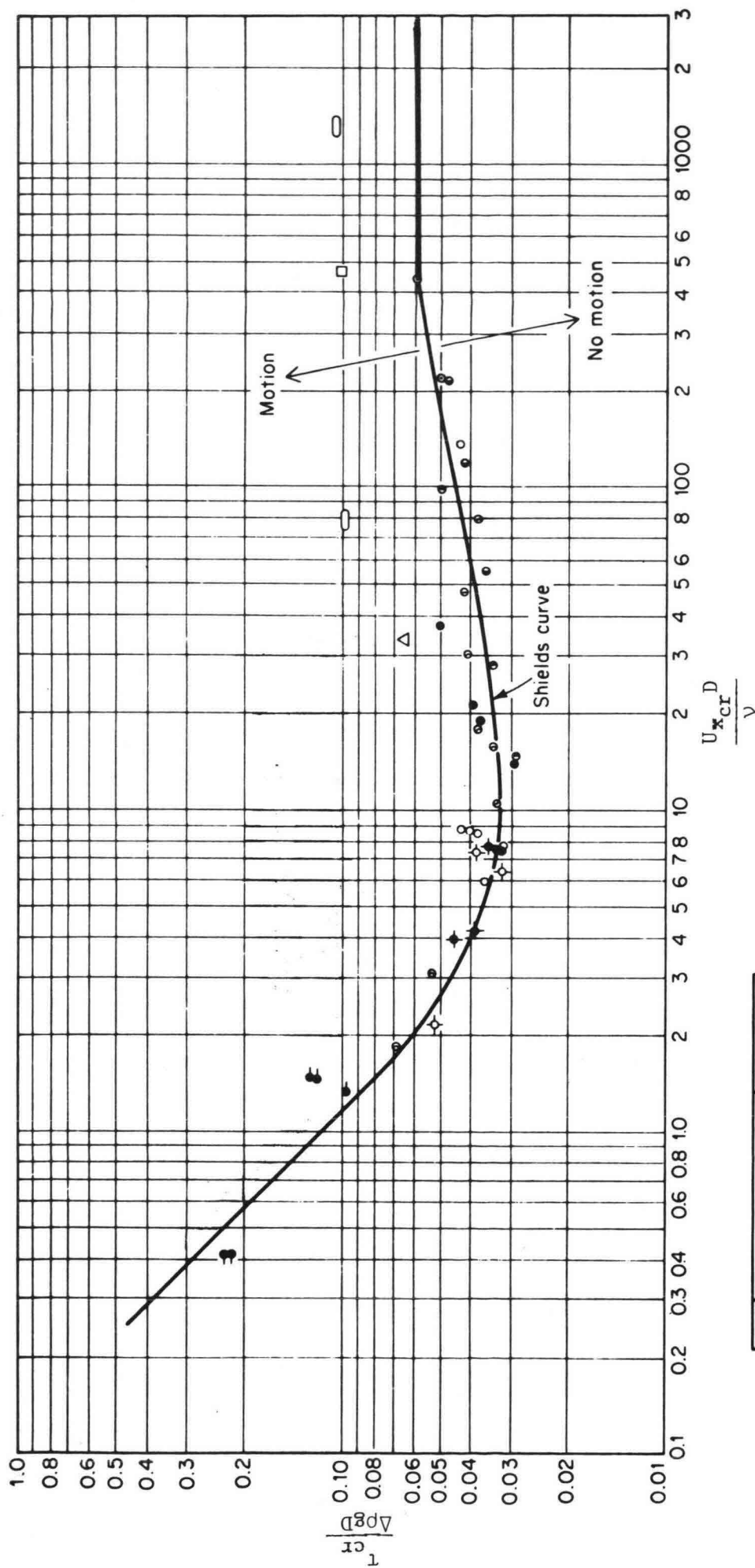
#### 4.4.1. Effect of criterion

It is clear that the critical value of  $\tau_o$  will depend on the criterion for initiation of motion. To get an objective criterion Neill (1968, 1969) proposed the dimensionless parameter :

$$N = n D^3 / U^x$$

in which n is the number of grains displaced per unit area and unit time. Shields graph corresponds roughly with a N-value of  $15 \cdot 10^{-6}$  for coarse material. For designs of bottom protections etc. a much lower criterion





**Fig.4.1** Shields' diagram; dimensionless critical shear stress vs. shear Reynolds number. [After VANONI (1964).]

Sym	Description	$\gamma_s, g/cm^3$
○	Amber	1.06
●	Lignite	1.27
●	Granite	2.7
●	Barite	4.25
●	Sand (Casey)	2.65
◇	Sand (Kramer)	2.65
◆	Sand (U.S.W.E.S)	2.65
○	Sand (Gilbert)	2.65

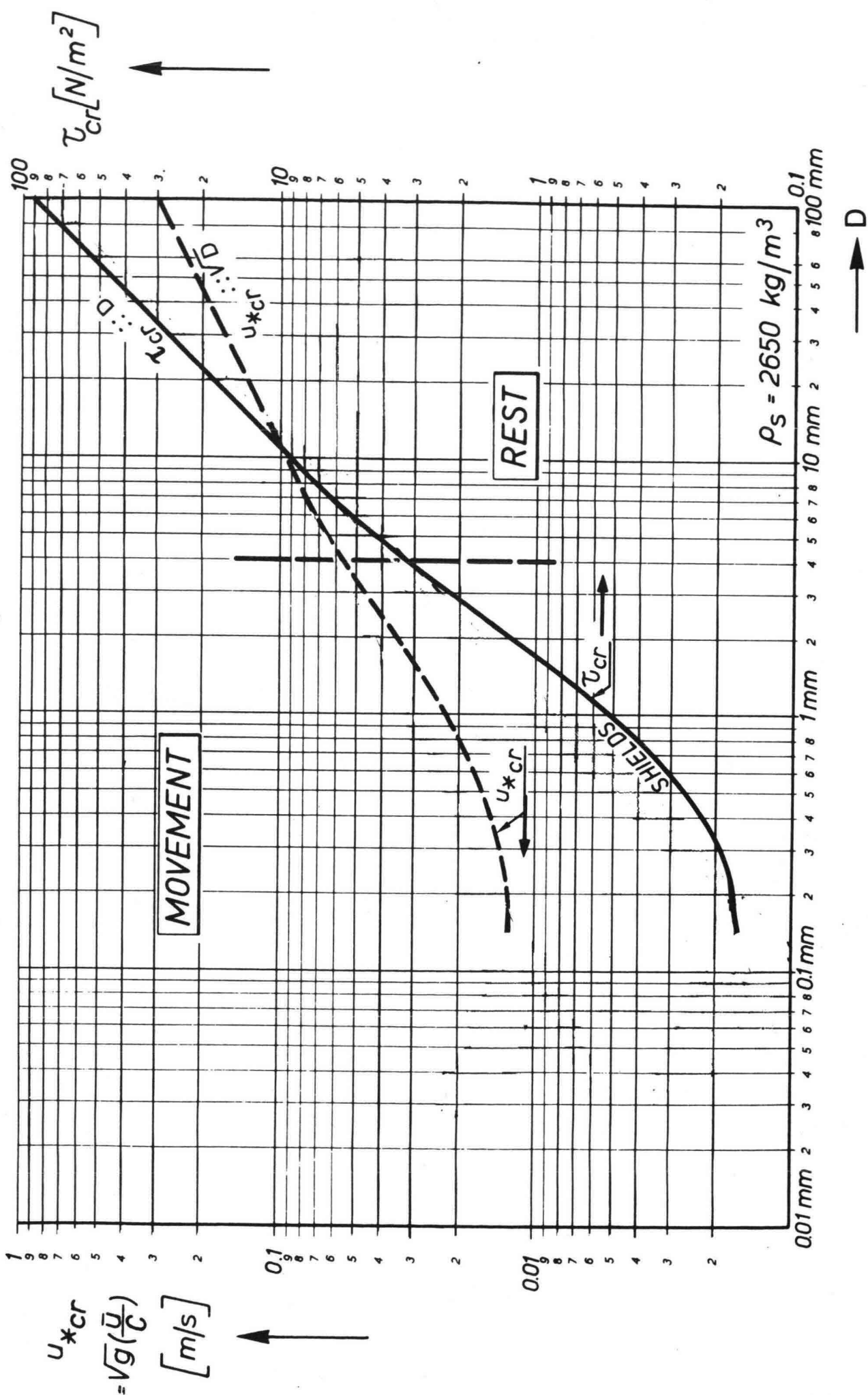
Fully developed turbulent velocity profile

Turbulent boundary layer

●	Sand (Vanoni)	2.65
●	Glass beads (Vanoni)	2.49
□	Sand (White)	2.61
□	Sand in air (White)	2.10
△	Steel shot (White)	7.9



Curves are derived from Shield's curve, Fig. 4.1 for  $\rho_s = 2650 \text{ kg/m}^3$  and  $\nu = 10^{-6} \text{ m}^2/\text{s}$  (20°C)



CRITICAL SHEAR STRESS AND CRITICAL SHEAR VELOCITY  
AS FUNCTION OF GRAIN SIZE FOR  $\rho_s = 2650 \text{ kg/m}^3$  (SAND)

should be used (for instance  $N = 10^{-6}$ ). Also Paintal (1971) has measured very low rates of transport with coarse material down to  $\psi = 0.02$ , thus well below the Shields value (see Figure 4.3).

#### 4.4.2 Effect of particle shape

Shields experiments were done with several types of material and systematic influence of shape could not be observed. Tests at the Delft Hydraulics Laboratory with coarse material showed that the critical value of  $\psi$  is the same for various shapes (spheres, cubes, broken stones etc.) if the nominal diameter  $D_n$  is used for comparison.

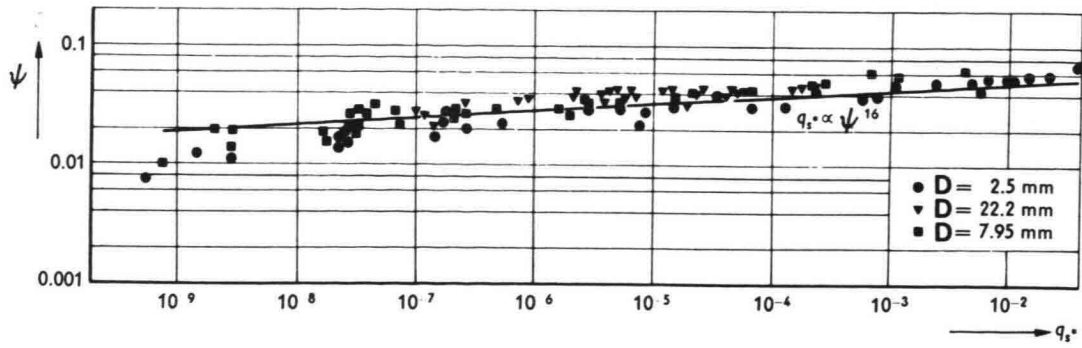
#### 4.4.3 Effect of gradation

It will be clear that a wide gradation will have an influence on  $\tau_{cr}$ . In practice however the gradation has an influence for  $D_{95}/D_5 > 5$  only (Knoroz, 1971), because the larger grains are more exposed and smaller grains are shielded by the larger ones. Therefore  $D_{50}$  is a good measure for most samples. For the effect of a gradation also see Eguisaroff (1965).

For a wide particle gradation the effect of armoring will occur which means that fine particles are eroded and an armor layer of coarse particles is formed, which prevents the bed from further scour. This effect is very important in degradation downstream of dams (Livesey, 1963, Gessler 1970). In that case  $D_{85}$  to  $D_{95}$  can be taken as a representative value for the mixture.

#### 4.4.4 Effect of $h/D$

For small values of  $h/D$  (waterdepth/particle diameter) a deviation from Shields graph is possible because  $\tau_o$  is not representative in that case for the turbulent flow structure. The turbulence structure near the bed in an infinite fluid is completely defined by bed shear stress ( $\tau_o$ ) and roughness ( $k_s$ ) but for small values of  $h/D$  also the waterdepth gives a limitation on size and frequency of the large eddies. Also the ratio of eddy duration and the time necessary to accelerate a particle becomes small so that an influence of  $h/D$  may be expected (more stability with smaller  $h/D$ ). Experiments have indeed shown that  $\psi_{cr}$  increases with decreasing  $h/D$  (Ashida 1973).

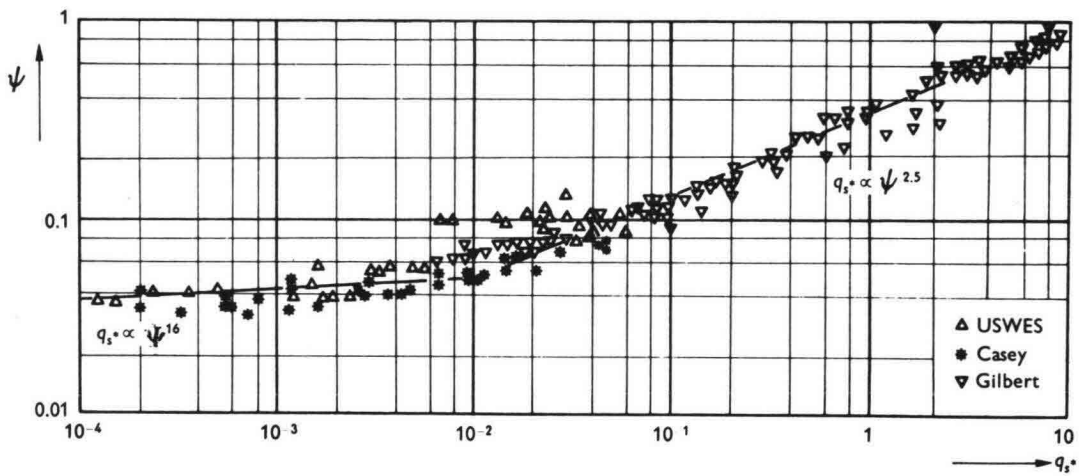


Variation of bed load transport at low shear values.

Débit de charriage à tension de frottement faible.

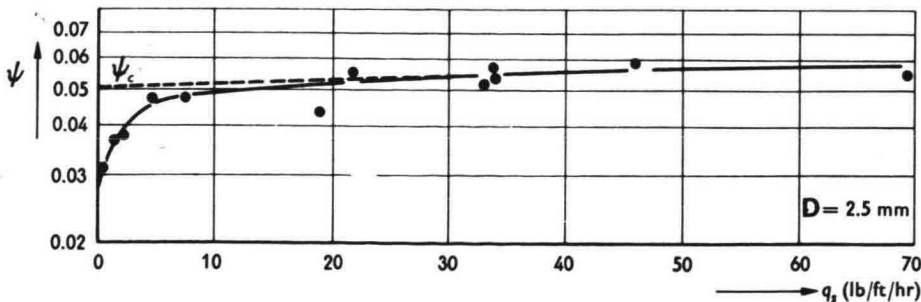
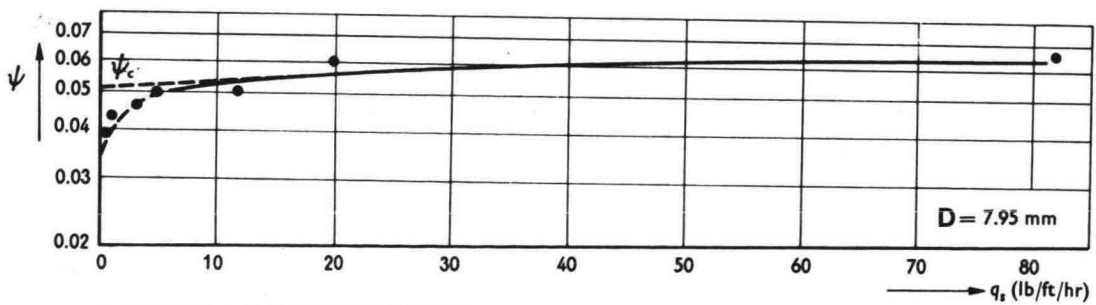
$$q_s^* = \frac{q_s}{(\Delta g D^3)^{1/2}}$$

$$q_s = \text{sed.tr.}/\text{m.s}$$



Variation of bed load transport at high shear values.

Débit de charriage à tension de frottement élevée.



Determination of critical shear stress.

Détermination de la tension de frottement critique.

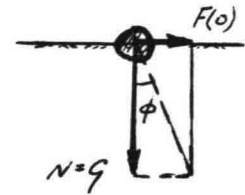
Figure 4.3 Measurements by Paintal

#### 4.4.5 Influence of bed slope

For a particle on a slope the value of  $\tau_{cr}$  will be reduced. For a horizontal bed the relation

$$F(o) = G \tan \phi$$

is valid, in which  $\phi$  is an angle characteristic for the particle stability.



For a bed slope in the flow direction with angle  $\alpha$  the following stability condition holds:

$$F(\alpha) + G \sin \alpha = N \tan \phi =$$

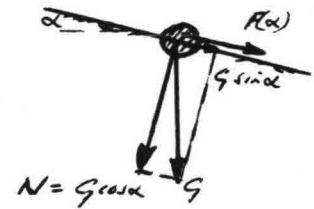
$$G \cos \alpha \tan \phi$$

$$F(\alpha) = G \cos \alpha \tan \phi - G \sin \alpha$$

$$\frac{F(\alpha)}{F(o)} = \frac{G \cos \alpha \tan \phi - G \sin \alpha}{G \tan \phi}$$

$$= \frac{\cos \alpha \sin \phi - \sin \alpha \cos \phi}{\sin \phi}$$

$$k(\alpha) = \frac{F(\alpha)}{F(o)} = \frac{\sin(\phi - \alpha)}{\sin \phi} \quad (\text{given by Schoklitsch in 1914!})$$



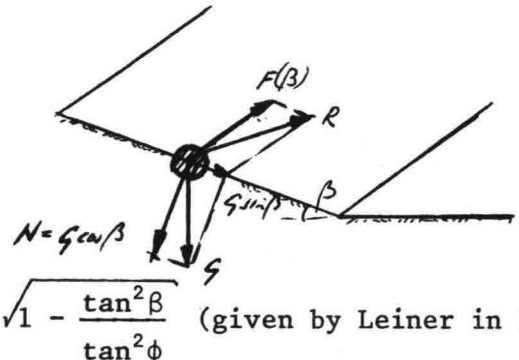
For a side slope with angle  $\beta$

Stability condition

$$R = \sqrt{F(\beta)^2 + G^2 \sin^2 \beta} = G \cos \beta \tan \phi$$

$$F(\beta) = \sqrt{G^2 \cos^2 \beta \tan^2 \phi - G^2 \sin^2 \beta}$$

$$k(\beta) = \frac{F(\beta)}{F(o)} = \sqrt{\frac{\cos^2 \beta \tan^2 \phi - \sin^2 \beta}{\tan^2 \phi}} = \cos \beta \sqrt{1 - \frac{\tan^2 \beta}{\tan^2 \phi}} \quad (\text{given by Leiner in 1912!})$$



For a combination of longitudinal and side slope the reduction factor  $k(\alpha, \beta)$  becomes  $k(\alpha, \beta) = k(\alpha) \cdot k(\beta)$ .

#### 4.4.6 Influence of pore water-flow

It might be expected that an inflow or outflow of water from a sand bed has an influence on the stability of the sand particles. The pore-water flow may be caused by a ground-water table lower or higher than the river water level. It has been shown by Oldenziel and Brink (1974) however, that the influence is very limited. For hydraulic gradients up to  $\pm 0.3$  only a factor of 2 in the transport rate was observed. In view of the strong variation of transport rate with  $\psi$  near incipient motion this means only a few percent variation in  $\psi_{cr}$  and can be neglected (inflow of water had

a somewhat stabilising tendency). There is one exception however. Harrison and Clayton have shown that a seepage into the bed for a flow carrying fine silt particles gives an enormous increase in stability due to the formation of a plastered bed layer.

#### 4.5 Cohesive sediments

A cohesive character of a soil will increase the resistance against erosion. Empirical data on critical mean velocities are given by Lane 1953.

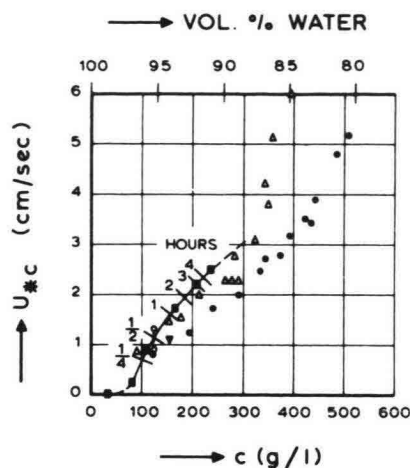
material	loose	moderately compact	compact
sandy clay	0.45 m/s	0.9 m/s	1.25 m/s
clay	0.35 m/s	0.8 m/s	1.20 m/s
lean clayey soil	0.30 m/s	0.7 m/s	1.05 m/s

Several authors have tried to correlate critical shear stress with mechanical properties of the soil (siltcontent, plasticity index, vane shear strength) (see Smerdon and Beasley (1959), Carlson and Enger (1960), Partheniades (1965, 1970). From the data given it appears that for cohesive soils with  $D_{50} = 10 - 100 \mu$  a critical shear velocity  $U_{cr}^*$  of 3 - 4,5 cm/s is possible.

There is some tendency for an increase of  $U_{cr}^*$  with vane shear strength and plasticity index.

For very recently deposited sediments (silt in estuaries) Migniot (1968) and Partheniades (1970) give relations between  $U_{cr}^*$ , vane shear strength and dry weight of the sediments. Minimum values are in the order of  $U_{cr}^* = 1.0$  cm/s (consolidation period of some days) to 3.0 cm/s for consolidation periods of some weeks. For an example see Figure 4.4 taken from Terwindt and Breusers (1972).

For an exact determination of a critical shear stress of a cohesive soil a special test for each soil will be necessary. Raudkivi (1974) and Arulanandan (1975) have shown that the erosion resistance of clay depends very much on the type of clay mineral and the chemical composition (salts) of the pore water and the eroding fluid.



MUD :	SAND IN %	THICKNESS OF MUD LAYER IN cm
△, LA VILLAIN " (MIGNIOT, 1968)	8	12
●, MAHURY " (MIGNIOT, 1968)	2	12
—■— I	37	2
▼ II	7	20
○ III	2	2

Fig. 4.4 Critical shear velocity ( $U_{*cr}$ ) in relation to mud concentration ( $c$ ).

#### 4.6 Stability of stones

The stability of stones on dams or in revetments is discussed by several authors. Taking a "safe" value for the Shields parameter  $\psi = 0.03$  and  $k_s \approx 2D$  (in view of the large roughness of stones) the following relation is obtained:

$$\frac{\bar{U}_{cr}}{\sqrt{\Delta g D}} = 1.0 \log \frac{6h}{D}$$

Isbash (1935) neglects the influence of  $h/D$  and gives the empirical relation for the stability of a stone in a bed:

$$\bar{U}_{cr} = 1.2 \sqrt{2\Delta g D} = 1.7 \sqrt{\Delta g D}$$

For a stone on the top of a dam the critical velocity is reduced:

$$\bar{U}_{cr} = 0.86 \sqrt{2\Delta g D} = 1.2 \sqrt{\Delta g D}$$

Goncharov (see Shamov 1959) gives the following relations:

$$\frac{\bar{U}_{cr}}{\sqrt{\Delta g D}} = 0.75 \log \frac{8.8h}{D} \quad \text{for absolute rest of a stone}$$

$$\text{and } \frac{\bar{U}_{cr}}{\sqrt{\Delta g D}} = 1.07 \log \frac{8.8h}{D} \quad \text{for the critical condition.}$$

Levi (see Shamov 1959) gives the empirical relation:

$$\frac{\bar{U}_{cr}}{\sqrt{\Delta g D}} = 1.4 \left(\frac{h}{D}\right)^{0.2}$$

Maynard (1978) gives the empirical expression:

$$\frac{D_{50}}{h} = 0.22 Fr^3 \quad Fr = \frac{\bar{U}}{\sqrt{gh}}$$

This can be converted into (taking  $\Delta = 1.65$ ):

$$\frac{\bar{U}_{cr}}{\sqrt{\Delta g D}} = 1.28 \left(\frac{h}{D}\right)^{1/6}$$

All relations are compared in Fig. 4.5

The formulas given do not take into account the influence of turbulence generated by constructions for example dams.

In that case the critical velocity has to be reduced with a factor

$$\alpha = \frac{1.45}{1+3r}$$

in which  $r$  is the relative turbulence intensity and a value  $r = 0.15$  has been assumed in uniform flow over a rough bed.

Just downstream of a hydraulic jump (stilling basin) values of  $r$  in the order of 0.3 to 0.35 can be expected. This gives a value for  $\alpha$  of about

$$\alpha = 0.7$$

This agrees with the design graphs given by Cox (1958).

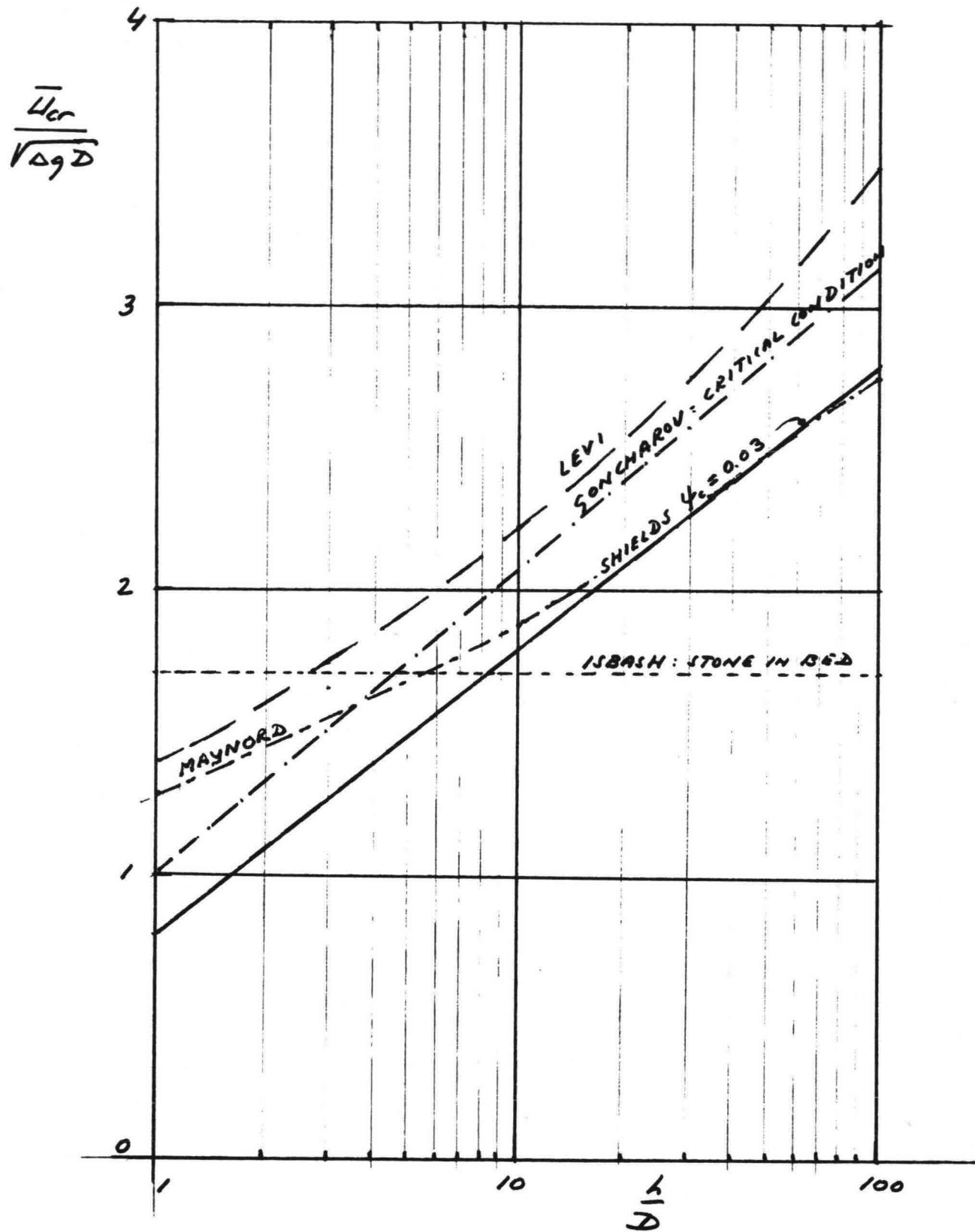


FIG 4.5 CRITICAL VELOCITIES FOR STONES



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#### 4.8. Problems

Use Shields curve and  $k_s = D$  unless otherwise specified.

- 4.1 Given: A wide open channel excavated in uniform material ( $\rho_s = 2650 \text{ kg/m}^3$ ) with  $D = 2 \text{ mm}$  has a slope  $I = 0.5 \cdot 10^{-3}$  and a depth of  $h = 2 \text{ m}$ .

Question: Is the channel bed stable?

- 4.2 Given: A wide open channel has a depth of  $h = 1.7 \text{ m}$ , a mean velocity  $\bar{U} = 2.5 \text{ m/s}$ .

Question: What is the minimum size of the bed material to obtain a stable bed?  $\rho_s = 2650 \text{ kg/m}^3$

- 4.3 Given: A wide open channel has a slope  $I = 10^{-5}$  and bed material  $D = 0.2 \text{ mm}$ . No bedforms are present.

Question: What is the maximum discharge  $/\text{m}^1$  without movement of bed material. ( $\rho_s = 2650 \text{ kg/m}^3$ )?

- 4.4 Given: A wide open channel is excavated in uniform material with ( $\rho_s = 2650 \text{ kg/m}^3$ ) and  $D = 3 \text{ mm}$  under a slope  $I = 10^{-4}$ .

Question: What is the permissible discharge  $/\text{m}^1$ ?

- 4.5 Given: The bottom of a wide open channel with a depth of  $4 \text{ m}$  is protected with stones with a mass of  $30 \text{ kg}$ .  $\rho_s = 2800 \text{ kg/m}^3$ .

Question: What is the critical mean velocity for this bottom protection, using  $\psi_{cr} = 0.03$  and the nominal diameter as the representative size.

- 4.6 Given: Experiments are designed to check Shields curve, using a wide flume (neglect side-wall effects). The water depth for the experiments is  $0.6 \text{ m}$ .

Question: If uniform flow is required (water surface slope = bed slope), what is the required slope of the channel bed and discharge  $/\text{m}^1$  for: a) an experiment with uniform sand  $k_s = D = 200 \text{ }\mu\text{m}$ ;  
b) an experiment with uniform gravel  $k_s = D = 4 \text{ mm}$ .

$$\rho_s = 2650 \text{ kg/m}^3$$

## 5 TRANSPORT MECHANISM, BED FORMS, ALLUVIAL ROUGHNESS

### 5.1 Introduction

For turbulent flow over a rigid bed a description of the flow structure could be given only by empirical methods. Bottom shear stress, waterdepth and bed roughness were the most important parameters. Description of particle motion under the action of the flow is also largely empirical sothat it is not difficult to understand why there is only a limited theoretical basis for the relation between flow and sediment transport.

Most of the existing knowledge is obtained from experiments and general physical arguments. For the initiation of motion a reasonable picture was obtained in this way. At greater values of the bed-shear stress sediment transport will increase and deformation of the bed will occur. As the deformation is also time-dependent and nature is always unsteady, an equilibrium situation will be hardly found in practice.

### 5.2 Transport mechanism

According to the mechanism of transport two major modes may be distinguished:

1. Bed load            - movement of particles in contact with the bed by rolling, sliding and jumping
2. Suspended load - movement of particles in the flow. The settling tendency of the particle is continuously compensated by the diffusive action of the turbulent flow field.

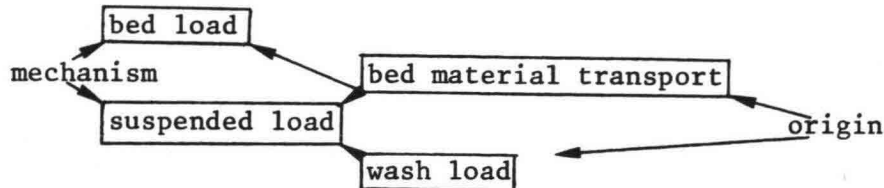
A sharp distinction is not possible. A general criterion for the beginning of suspended load is a ratio of shear velocity and fall velocity  $U^*/W \sim 1.5$ . Sometimes also saltation load is mentioned. This is the mode where particles bounce from one position to another. This is only important for particle movement in air. The maximum particle elevation of a particle moving in water is in the order of 2-3 times the diameter sothat this mode of transport can be considered as bed load.

According to the origin of the transported material a distinction is made as follows:

- A. Bed-material transport            This transport has its origin in the bed, which means that the transport is determined by the bed and flow conditions (can consist of bed load and suspended load).

### B. Wash load

Transport of particles not or in small quantities in the bed. The material is supplied by external sources (erosion) and no direct relationship with the local conditions exists (can only be transported as suspended load, generally fine material  $< 50 \mu\text{m}$ ). It can have influence on turbulence and viscosity and therefore have some influence on the flow.



Wash load is not important for changes in the bed of a river but only for sedimentation in reservoirs etc.

### 5.3 Bed Forms

Much literature exists on the classification and dimensions of bedforms, mainly in the form of empirical relations. Bed forms are of interest in practice for several reasons.

- Bed forms determine the roughness of a stream. A change in bed form can give changes in friction factor of 4 and more.
- Navigation is limited by the maximum bed level and depends therefore on the height of the bed deformation.
- Bed forms and sediment transport have a mutual influence.

A generally accepted classification is the following:

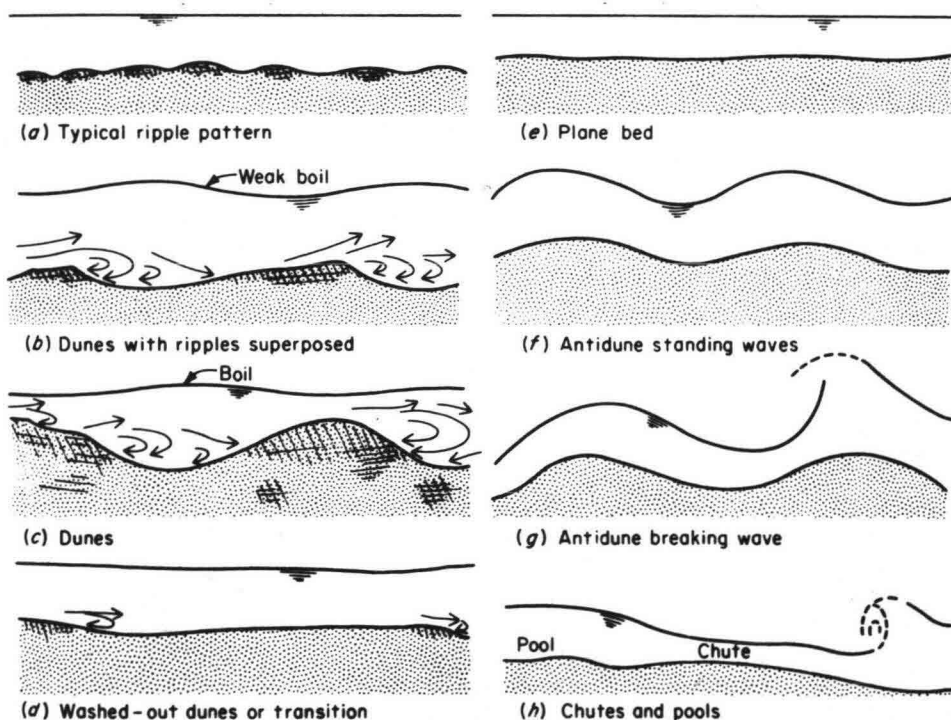
A. Lower flow regime (Froude number  $Fr = \bar{U}/\sqrt{gh} < 0.4$  to 1; no sharp transition).

- A.1 flat bed At values of the bed shear stress just above the critical, sediment transport without deformation of the bed is possible. Grains are transported by rolling and bouncing.
- A.2 ripples For sediment sizes  $< 0.6 \text{ mm}$  and increasing bed shear stress small regular waves appear with wavelenghts in the order of 5-10 cm and heights in the order of 1 cm. They become gradually irregular and three-dimensional in character.
- A.3 dunes For all sediment sizes and increasing shear stress dunes are developed. Dunes are more two-dimensional than ripples and have

much greater wavelengths and heights. The crests of the waves are perpendicular to the flow, the form is more or less triangular with a gentle slope along which the particles are transported and a steep downstream slope where particles are deposited. The angle of this slope is roughly the angle of repose of the material.

### B. Upperflow regime ( $Fr > 0.4$ to $1.0$ )

- B.1 plane bed** As the velocity is further increased, the dunes are flattened, gradually disappear and the bed becomes flat. Sediment transport rates are high.
- B.2 antidunes** A further increase in velocity to Froude numbers around 1.0 causes the water surface to become instable. Interaction of surface waves and the bed (sediment transport is maximum under the troughs of the surface waves) gives a bed form called antidunes. They can travel upstream and occur in trains of 4 to 20. Antidunes and surface waves grow in amplitude and often break in a way similar to ocean waves.
- B.3 chute and pools** At still higher velocities chutes and pools are formed. For an illustration of the bed forms see figure 5.1 (Simons and Richardson 1968).

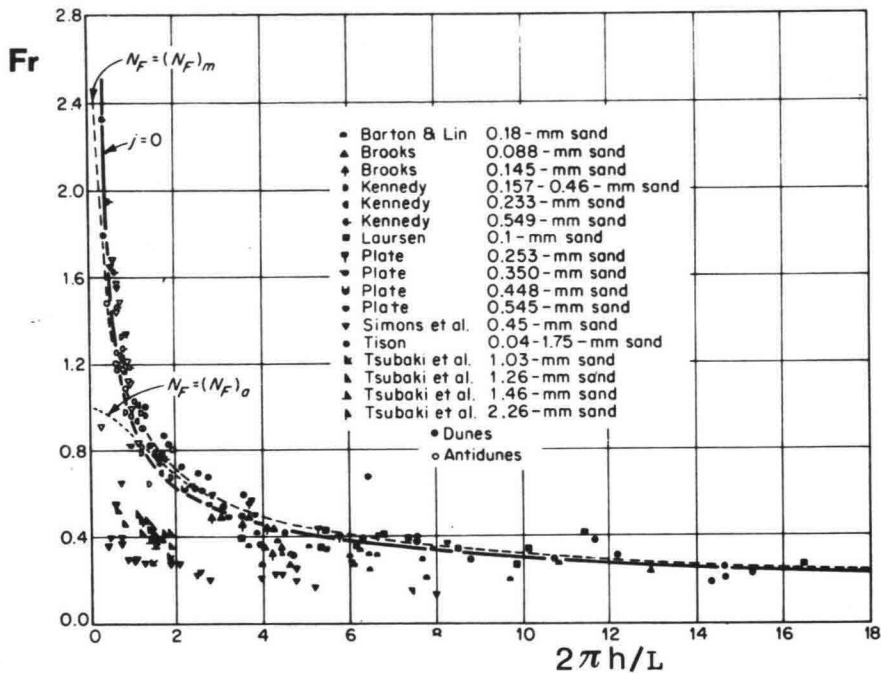


**Fig. 5.1** Idealized bedforms in alluvial channels. [After SIMONS *et al.* (1961).]

### 5.4 Classification Criteria

Several authors have tried to develop theoretical explanations for the origin of ripples and dunes (see for example Exner (1925) who discusses the growth of an initial instability on a sand bed.)

Other authors have assumed potential flow to predict the reaction of the main-flow on variations in bed level (Kennedy, 1963). The result of Kennedy's work is a relation between the wavelength  $L$  of the bed deformation and the Froude number (see figure 5.2).  $Fr = \frac{\bar{U}}{\sqrt{gh}}$



**Fig. 5.2** Comparison of predicted and observed bedform regions. [After KENNEDY (1963).]

Results of the theoretical models are not very convincing so that we have to rely again on empirical correlations. The first classification was given by Liu (1957) who proposed  $U^*/W$  vs  $U^*D/v$  as a criterion for ripple formation. This diagram was extended by Simons (1966) for other bed forms (see fig. 5.3).



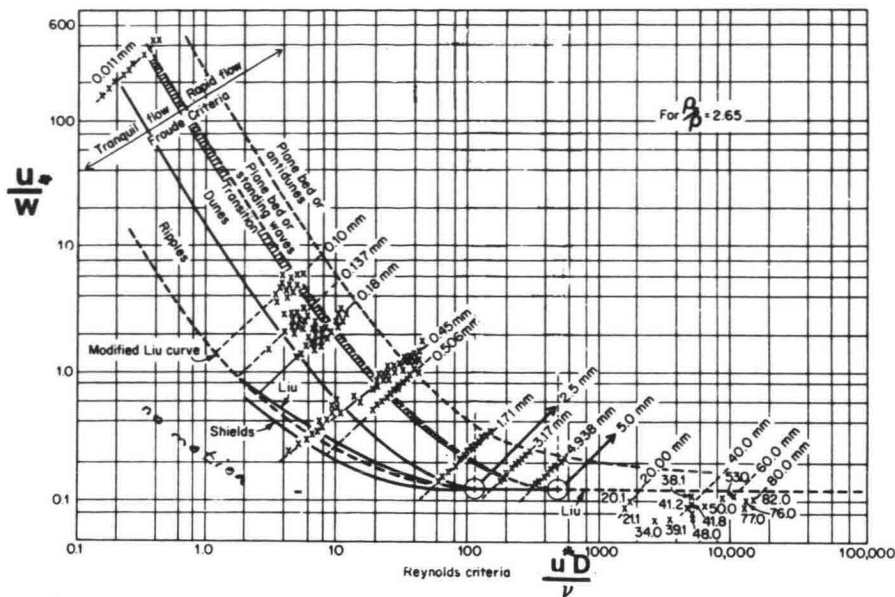


Fig. 5.3 Criteria for bedforms. [After SIMONS *et al.* (1961a).]

Simons *et al.* 1963 gave a diagram based on grainsize and streampower ( $\tau_0 \bar{U}$ ), see figure 5.4

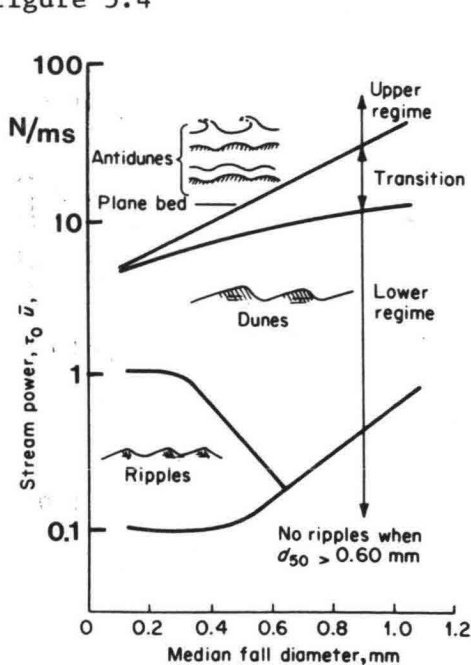


Fig. 5.4 Relation of bedforms to stream power and grain diameter. [After SIMONS *et al.* (1963a).]

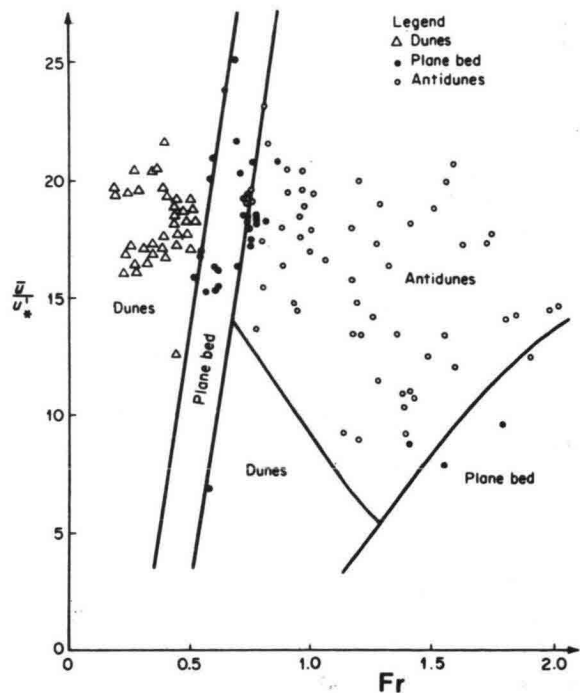
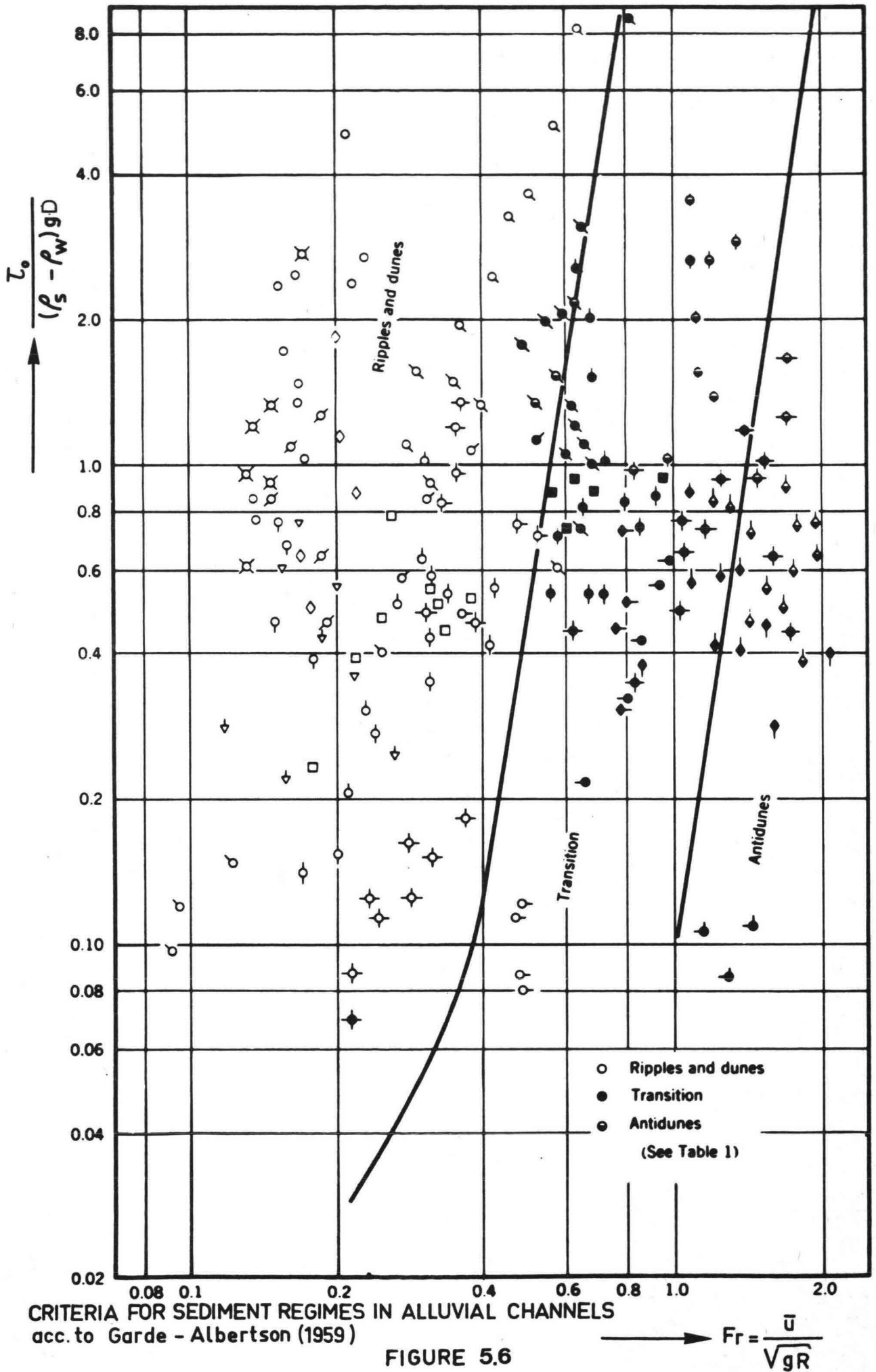


Fig. 5.5 Stability diagram; the bedforms are indicated. [After ENGELUND *et al.* (1966).]

The Froude number has to be considered as an important parameter as well. This was done by several authors (Garde-Albertson, figure 5.6) and by Engelund (figure 5.5).  $U^*$  is the value of  $U^*$  computed from  $\bar{U}$  by assuming the grainsize as bed roughness and taking  $k_s = 2.5 D_{50}$  and  $h = h'$



It must be borne in mind that the transition in practical conditions from one bed form to another may show an important phase lag with changes in flow condition.

Raudkivi (1967) has measured the shear stress distribution on a dune profile. The maximum shear stress on the upper part of the ripple had about the same value as for a horizontal bed with the same mean velocity and grain roughness. Behind the steep downstream face of the dune an eddy develops. Around the reattachment point the flow is very turbulent so that particles are transported in bursts.

### 5.5 Alluvial roughness

The bed forms discussed in par. 5.3 all have their specific roughness. For a flat bed without transport it can be assumed that the roughness is in the order of the grainsize (for example  $D_{65}$  or  $D_{90}$ ). For flows over ripples and dunes the total resistance consists of two parts:

the roughness of the grains and the form drag of the bed forms. The roughness of a dune bed is much greater than that of a flat bed and the corresponding friction factor is also much larger. Dunes generally give the maximum roughness of a flow.

A flat bed with sediment transport (B.1) can have a friction factor slightly different from that of a flat bed without transport. The presence of antidunes does not appreciably change the magnitude of the effective roughness of the bed if compared with a flat bed. If the waves break however, the friction factor will be increased due to the energy dissipation in wave breaking.

It cannot be expected in general that the friction factor of an alluvial channel is constant. Experiments have shown that the friction factor can vary by a factor 5 or more. This is demonstrated in figure 5.8 and 5.9 where changes in bed form give a great difference in bed roughness.

Figure 5.9 shows that the same value of  $\tau_o$  can occur for different values of  $\bar{U}$  (take for example  $\tau_o = 0.1 \text{ lbs/ft}^2$ ). Due to phase lags between bed form (and roughness) and flow condition rivers very often exhibit hysteresis effects in discharge-stage relations (not to be confused with the hysteresis during a flood wave).

Prediction methods for the roughness of an alluvial stream generally divide the total shear  $\tau_o$  or friction factor ( $C$  or  $\lambda$ ) into two parts, one for the grain roughness (surface drag) denoted by  $\tau_o'$  or  $C'$  or  $\lambda'$  and one for the form drag ( $\tau_o''$ ,  $C''$  or  $\lambda''$ ).

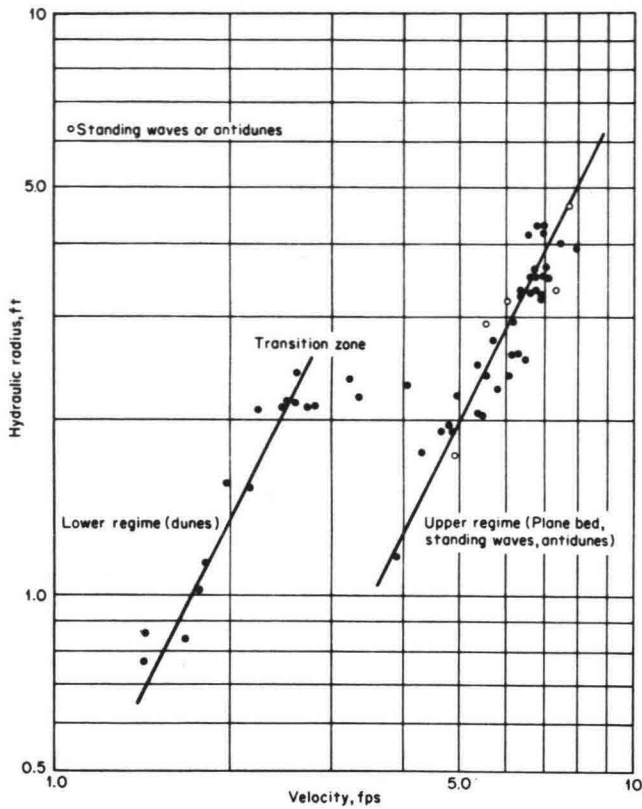


Fig. 5.8 Relation of hydraulic radius to velocity for Rio Grande near Bernalillo. [After NORDIN (1964).]

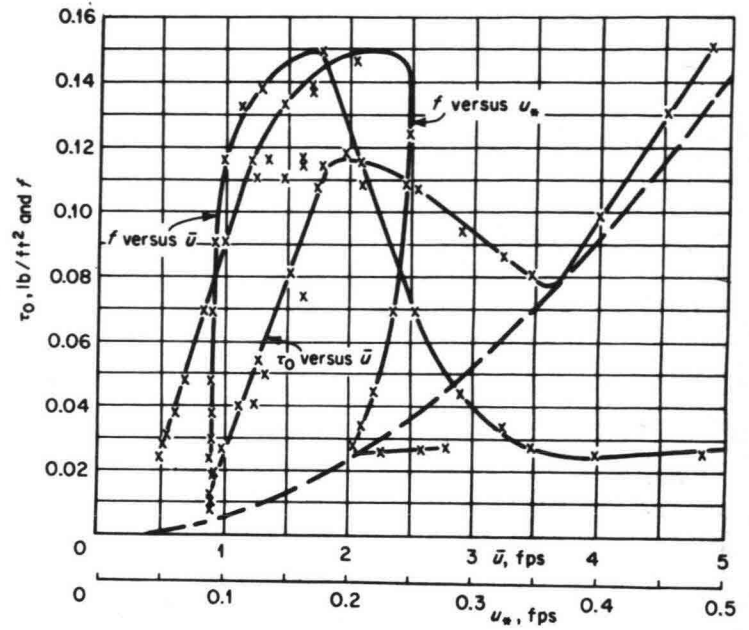


Fig. 5.9 Flow resistance due to bedforms. [After RAUDKIVI (1967).]

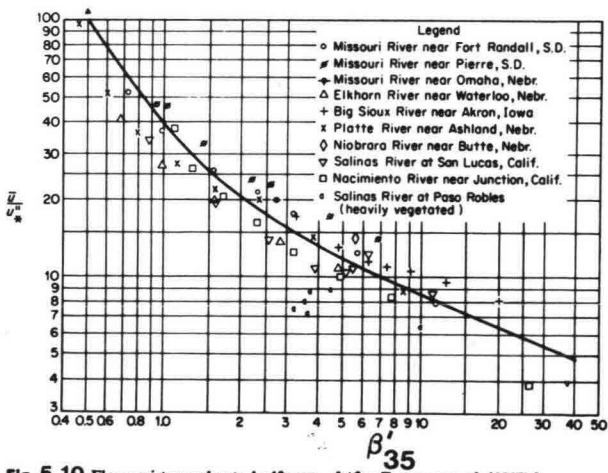


Fig. 5.10 Flow resistance due to bedforms. [After EINSTEIN *et al.* (1952).]

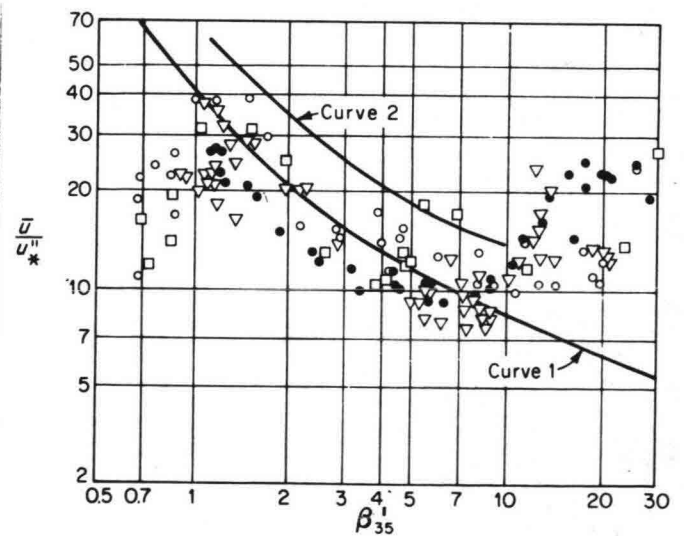


Fig. 5.11 Flow resistance due to bedforms; curve 1—river data, curve 2—flume data. [After SIMONS *et al.* (1966).]

By definition:

$$\tau_o = \tau_o' + \tau_o'' \quad C^{-2} = C'^{-2} + C''^{-2} \quad \lambda = \lambda' + \lambda''$$

$$\lambda \text{ is defined by } I = \text{slope} = \lambda \cdot \frac{1}{4R} \cdot \frac{U^2}{2g} \quad \lambda = \frac{8g}{C^2}$$

Several procedures are given in literature.

### 1. Einstein-Barbarossa (1952)

E.B. divide the hydraulic radius  $R$  in two parts:  $R'$  and  $R''$ , where  $R' + R'' = R$  and  $R'/R'' = \tau_o'/\tau_o''$ .  $U_x''$  is computed by taking  $k_s = D_{65}$  in the Chézy relation and  $\beta'_{35}$  is computed from:

$$\beta'_{35} = \frac{\Delta g D_{35}}{(U_x'')^2} = \frac{\Delta D_{35}}{h' I}$$

With the diagram given in figure 5.10 the value of  $\bar{U}/U_x''$  is found by trial and error. For larger values of  $\beta'_{35}$  ( $> 7$ ) deviations are observed for river data (see figure 5.11).

### Procedure

- If  $I$  and  $h$  are given and  $\bar{U}$  has to be known: guess  $h'$ , compute  $\beta'_{35}$ ,  $U_x'$  and  $\bar{U}$  and with fig. 5.10:  $\bar{U}/U_x''$ . Compute  $h''$  from  $U_x''$  and  $h = h' + h''$ . If  $h$  is not correct, estimate a new value for  $h'$  and repeat until  $h = h' + h''$ . Then use the last value of  $\bar{U}$ .
- If  $q$  and  $h$  are given and  $I$  or  $C$  has to be computed: estimate  $h'$  and compute  $\beta'_{35}$  and  $\bar{U}/U_x''$ . From  $U_x'$  and  $U_x''$  a new value of  $h'$  can be obtained. Repeat until  $U_x'$  remains constant. Then compute

$$U_x = (U_x'^2 + U_x''^2)^{\frac{1}{2}}, \quad I \text{ and } C.$$

### 2. Engelund and Hansen (1967)

E. and H. give an expression for  $f''$  of the form:

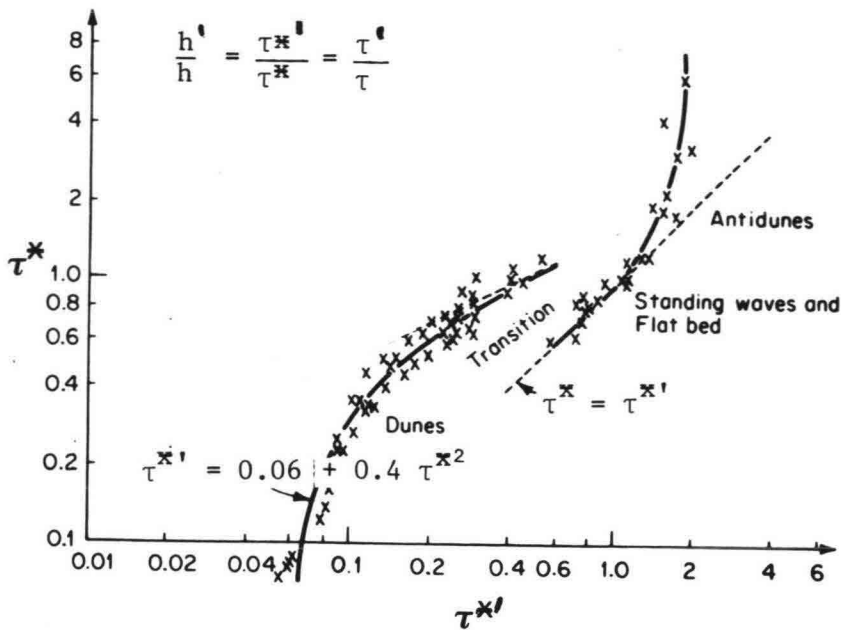
$$f'' = \alpha H^2 / h.L \quad H = \text{dune height} \quad L = \text{dune length} \quad h = \text{water depth}$$

$$\text{where } f = \tau / (\frac{1}{2} \rho U^2) = 2g / C^2 = \frac{1}{4} \lambda$$

and introduce the dimensionless parameters:

$$\tau^* = \tau / \rho g \Delta D_{50} \quad \tau^{*'} = \tau' / \rho g \Delta D_{50} \quad \tau' = \rho U_x'^2 \quad \bar{U}/U_x'' = 5.75 \log 4.8 h' / D_{50}$$

Engelund concludes that  $\tau^*$  is a function of  $\tau^{*'}$  only (see figure 5.12).



**Fig. 5.12** Flow resistance  
*et al. (1967).*

[After ENGELUND]

#### Procedure

- a) If  $I$  and  $h$  are known and  $\bar{U}$  has to be computed:  
Compute  $\tau$ ,  $\tau^*$  and with fig. 5.12  $\tau'^*$ . This gives  $\tau'$ ,  $U_x'$  and  $h'$ .  
Then compute  $\bar{U}$ .
- b) If  $h$  and  $q$  are given and  $I$  or  $C$  has to be computed:  
Guess  $h'$ , compute  $U_x'$ ,  $\tau_x'$  and with fig. 5.12  $\tau^*$  and then  $\tau$ . Compute  $h'$  from  $h'/h = \tau'/\tau$ ; if different from first value, repeat calculation until  $h'$  is constant. Then compute  $U_x$ ,  $I$  and  $C$ .

#### 3. White, Paris, Bettess (1980)

WPB give an empirical relation between:

$$F_{fg} = \frac{U_x}{(\Delta g D)^{1/2}}, \quad D_{gr} = D \cdot \left(\frac{\Delta g}{v^2}\right)^{1/3}$$

$$\text{and } F_{gr} = \frac{U_x^n}{(\Delta g D)^{1/2}} \left\{ \frac{\bar{U}}{5.64 \log(10h/D)} \right\}^{1-n} \quad (a)$$

where the characteristic diameter is  $D = D_{35}$ .

The relation is given by (see Fig. 5.13):

$$\frac{F_{gr} - A}{F_{fg} - A} = 1.0 - 0.76 \left\{ 1.0 - \exp \left[ -(\log D_{gr})^{1.7} \right] \right\} \quad (b)$$

where  $A$  and  $n$  are functions of  $D_{gr}$ :

$$n = 0 \quad \text{and} \quad A = 0.17 \quad \text{for } D_{gr} \geq 60$$

$$\left. \begin{aligned} n &= 1.0 - 0.56 \log D_{gr} \\ A &= 0.23 D_{gr}^{-\frac{1}{2}} + 0.14 \end{aligned} \right\} \quad 1 \leq D_{gr} < 60$$

(see also Fig. 6.8)

#### Procedure:

- a) If  $h$  and  $I$  are given and  $\bar{U}$  has to be computed:

Compute  $U_{*}$ ,  $D_{gr}$ ,  $n$ ,  $A$  and  $F_{fg}$ . Compute  $F_{gr}$  using Fig. 5.13 or formula (b).

Compute  $\bar{U}$  from expression (a).

- b) If  $\bar{U}$  and  $h$  are given and  $I$  or  $C$  has to be known:

Estimate a value for  $C$ , compute  $U_{*}$  and follow procedure a).

If the computed value of  $\bar{U}$  is not correct, repeat the procedure using the new value of  $C$ .

#### 4. Paris (1980)

Paris gives an empirical relation between  $C/C'$  and  $\chi = \psi/\psi_{cr}$ :

$$\frac{C}{C'} = 1.0 - 0.47 \log \chi + 0.12 (\log \chi)^2$$

$$C' = 17.7 \log (10h_{cr}/D_{35})$$

$$h_{cr} = \chi^{-1} \cdot h$$

The expression is valid for  $\chi \geq 1$ . If  $\chi \leq 1$  then  $C = C'$ .

$\psi_{cr}$  is computed using the expressions given by White ( $\psi_{cr} = A^2$ ) where  $\psi_{cr}$  is a function of  $D_{gr}$  (computed from  $D_{35}$ ).

$$\psi_{cr} = 0.029 \quad \text{for } D_{gr} \geq 60$$

$$\psi_{cr} = (0.23 D_{gr}^{-\frac{1}{2}} + 0.14)^2 \quad \text{for } 1 \leq D_{gr} < 60$$

#### Procedure

- a) If  $h$  and  $I$  are given and  $\bar{U}$  has to be known:

Compute  $\psi$ ,  $\psi_{cr}$ ,  $\chi$ ,  $h_{cr}$ ,  $C'$  and  $C$ . Then compute  $\bar{U}$  using the Chezy relation.

- b) If  $\bar{U}$  and  $h$  are given and  $I$  or  $C$  has to be known:

Estimate a value for  $C$ , compute  $I$  and follow the procedure under a).

If  $C$  is not correct, repeat the procedure using the new value of  $C$ .

### Comparison of various prediction methods

White c.s. and Paris have compared various methods with data (1432 flume tests and 263 river data). The result was for  $\alpha = C_{\text{calc.}}/C_{\text{observed}}$

Method	$0.7 < \alpha < 1.4$	$0.89 < \alpha < 1.11$
Einstein	21%	-
Engelund	83%	38%
White c.s.	89%	48%
Paris	89%	46%

The last three relations seem to give a reasonable result and can be used to give a first estimate. If more accurate data are required, observations in the field for the specific situation are necessary. Observations on dune-bed rivers have shown that dune height and resistance increase with increasing water temperature (Vanoni, 1975).



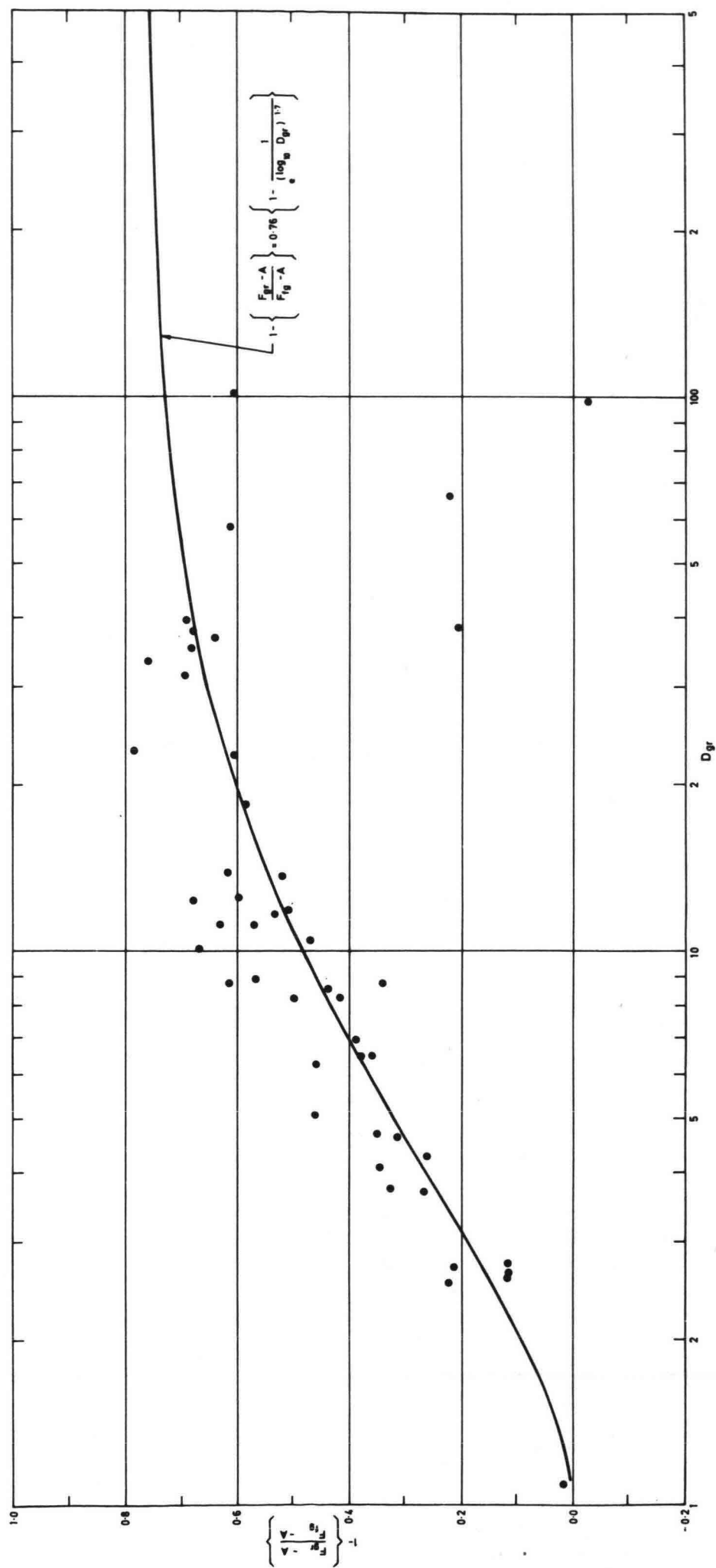


Fig. 5.13 Shear relationship based on  $D_{35}$  of the parent material, (New method)

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5.7 Problems  $\nu = 10^{-6} \text{ m}^2/\text{s}$  s.f. = 0.7

5.1 Given: depth  $h = 2 \text{ m}$  grainsize  $D = 150 \text{ }\mu\text{m}$  (uniform)  
 $T = 20^\circ \text{ C}$  chezy  $C = 63 \text{ m}^{1/2}/\text{s}$   
 mean vel.  $\bar{U} = 0.6 \text{ m/s}$

Question: What bedform can be expected according to:

- a) Simons - Liu (Fig. 5.3)
- b) Simons - stream power (Fig. 5.4)
- c) Engelund (Fig. 5.5) (take  $h' = 0.5h$ )
- d) Garde - Albertson (Fig. 5.6)

5.2 Same questions as in 5.1

for  $h = 0.5 \text{ m}$   $D = 1 \text{ mm}$   $C = 42 \text{ m}^{1/2}/\text{s}$   
 $\bar{U} = 1.2 \text{ m/s}$   $T = 20^\circ \text{ C}$

5.3 Given: depth  $h = 2 \text{ m}$ , grainsize  $D = 0.5 \text{ mm}$  (uniform)  
 Slope  $I = 2 \cdot 10^{-4}$   $T = 20^\circ \text{ C}$

Question: Compute  $\bar{U}$  using the methods of Engelund-Hansen, White c.s. and Paris.

For the Engelund-Hansen method assume that dunes are present.

5.4 Same question for slope  $I = 10^{-4}$ .

5.5 Given: depth  $h = 2 \text{ m}$ , grainsize  $D = 0.2 \text{ mm}$  (uniform)  
 $\bar{U} = 0.7 \text{ m/s}$   $T = 20^\circ \text{ C}$

Question: Compute  $C$  using the methods of Engelund-Hansen and Paris.

## 6 BED MATERIAL TRANSPORT

Bed material transport can be divided in bed load and suspended load. Both modes of transport have an influence on processes of erosion and deposition. Many relations between sediment transport and flow conditions are based on the bed shear stress. It has been shown that the bed-shear stress may be divided in a form drag and a grain roughness. It will be clear that the form drag does not contribute to the transport but that only the grain roughness will be of importance. Measurements of water depth and slope give the total bed shear stress, so that most transport relations require a reduction of the total bed shear stress to a value which is relevant for the transport.

This reduction factor is called the ripple factor  $\mu$ . Theoretically one should expect:  $\mu = \frac{\lambda'}{\lambda} = \left(\frac{C}{C'}\right)^2$ .

Many authors use  $\mu$  as a closing term, however, so that various expressions are given. This manipulation with the bed shear stress has led several authors to use the mean velocity  $\bar{U}$  instead of  $\tau_0$  as the important factor for the sediment transport. The problem then is that the same value of  $\bar{U}$  in different water depths will give different sediment transport rates, so that again some correction is necessary.

### 6.1 Bed load

Because several authors use some type of a physical model to predict a sediment transport relation it is not surprising that most formulas may be expressed as relations between dimensionless groups. The most common are a group related to the transport:

$$\Phi = S / [D^{3/2} (g\Delta)^{1/2}]$$

$S$  = transport in  $m^3/ms$       transport = volume of grains

For conversion to total volume,  $S$  has to be divided by  $(1 - \epsilon)$

in which  $\epsilon$  = porosity. As a first estimate, take  $\epsilon = 0.4$

$$\Delta = (\rho_s - \rho_w) / \rho_w$$

$D$  = grainsize

and a group related to the flow:

$$\psi = U^{\star 2} / \Delta g D \quad \psi' = \psi \cdot \mu = \text{effective value of } \psi$$

(the parameter used by Shields for the initiation of motion).

Some of the relations given in literature are the following:

### 1. Du Bois (1879)

Du Bois gave a simple model in which layers of sediment move relative to each other. The number of layers was proportional to  $\tau_o/\tau_{cr}$ .

The resulting expression is of the form:

$$S = \text{const. } \tau_o (\tau_o - \tau_{cr})$$

Although the physical model is not very convincing, it has been found that the form of the relation can be used to describe experiments in a reasonable way.

### 2. Kalinske (1947)

Kalinske assumed that grains are transported in a layer with thickness  $D$  with an instantaneous grain velocity  $U_g$  equal to:

$$U_g = b(U_o - U_{cr})$$

$U_o$  = instantaneous fluid velocity at grain level

$U_{cr}$  = critical fluid velocity to start grain movement.

For  $U_o$  a normal distribution is assumed:

$$f(U_o) = \frac{1}{\sigma\sqrt{2\pi}} \exp. \left[ -\frac{(U_o - \bar{U}_o)^2}{2\sigma^2} \right]$$

$\sigma$  = r.m.s. value of velocity fluctuations.

Taking the number of grains per unit area  $p/(\pi/4D^2)$  and using  $\bar{U}_g$  then the mean rate of particle movement, by dry weight per unit width and time is:

$$T_b = \frac{2}{3} \rho_s g D \bar{U}_g \cdot p \quad p = 0.35$$

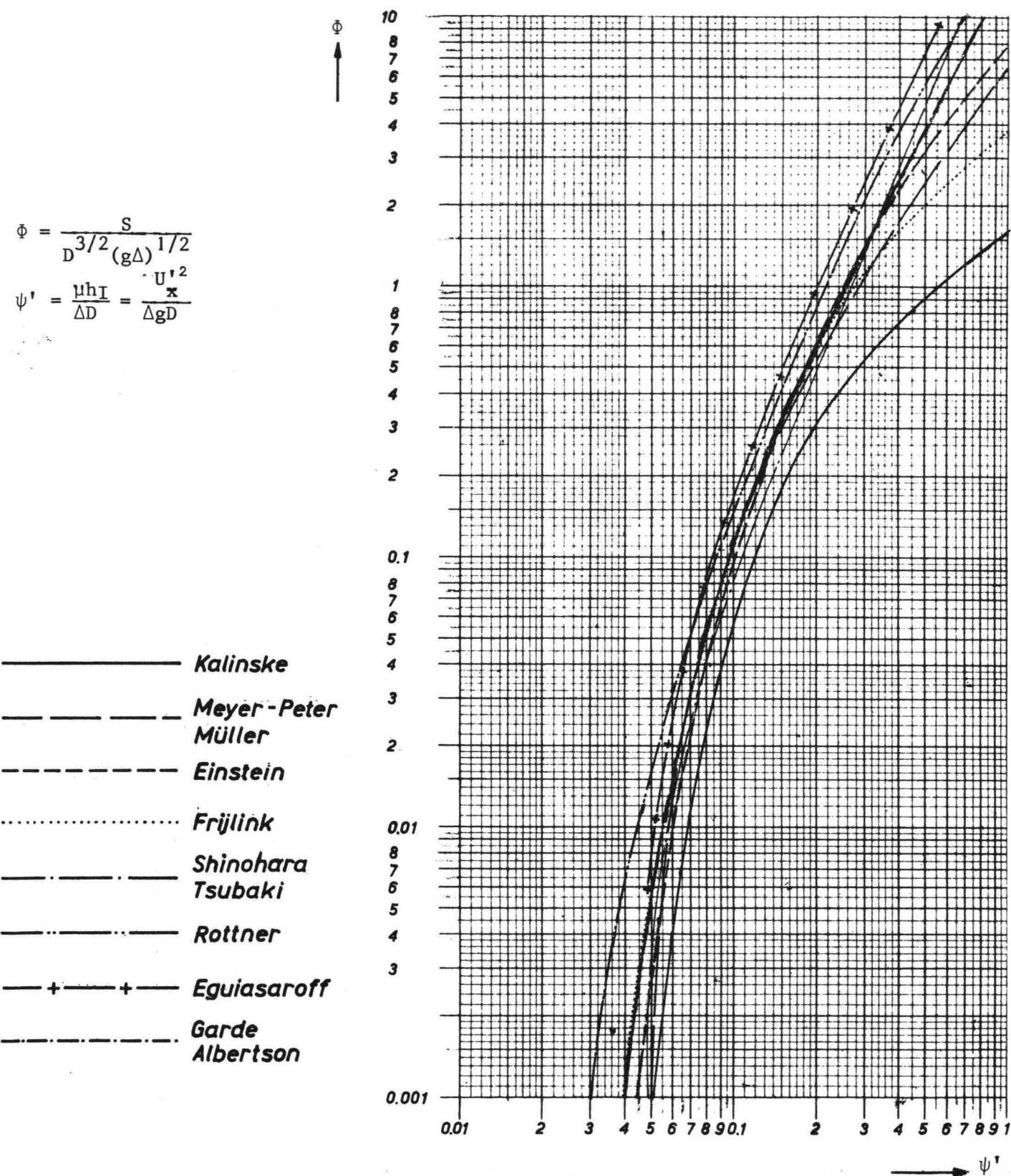
$$\text{where } \bar{U}_g = b \int_{U_c}^{\infty} (U_o - U_{cr}) f(U_o) dU_o \quad b = 1.0$$

The resulting expression may be made dimensionless with the parameters  $\Phi$  and  $\psi$  with the result:

$$\Phi = 2.5\psi^{\frac{1}{2}} \left\{ \frac{r}{\sqrt{2\pi}} \exp. \left[ -\frac{1}{2r^2} \left( \sqrt{\frac{0.12}{\psi}} - 1 \right)^2 \right] - \left( \sqrt{\frac{0.12}{\psi}} - 1 \right) \frac{1}{2\sqrt{\pi}} \operatorname{erf} \left[ \frac{1}{r\sqrt{2}} \left( \sqrt{\frac{0.12}{\psi}} - 1 \right) \right] \right\} \quad (\text{see figure 6.1})$$

in which  $r = \sigma / \bar{U}_o \quad r = 0.17$

Kalinske did not reduce the bed shear stress, so the relation is valid for plane beds only, so  $\psi = \psi'$



COMPARISON OF BED-LOAD TRANSPORT EQUATIONS

FIGURE 6.1

### 3. Meyer - Peter and Müller

M.P.M. have performed a large number of experiments in a wide flume with coarse sands. The resulting empirical expression may be written in  $\Phi$  and  $\psi'$  units as:

$$\Phi = (4\psi' - 0.188)^{3/2} \quad (\text{figure 6.1}).$$

By comparison of results with flat beds and dune beds the ripple factor is found:

$$\mu = (C/C')^{3/2} \quad (\text{theoretical exponent } 2). \quad C^1 = 18 \log \frac{12h}{D_{90}}$$

For a mixture M.P.M. take:

$$D_m = \bar{D} = \Sigma p.D / \Sigma p \quad \text{as the relevant parameter for the value of } \psi' \text{ and } \Phi ;$$

$$D = D_{90} \quad \text{for the grain roughness.}$$

### 4. Einstein (1950)

Einstein gave a complicated statistical description of the grain transport process in which the exchange probability of a grain is related to flow conditions. The resulting expression is given in figure 6.1 in a graphical form.

For the determination of the ripple factor  $\mu$  a graphical procedure is given by Einstein.<sup>\*)</sup> He used  $D = D_{35}$  as the relevant parameter for the transport and  $D = D_{65}$  for the roughness. The correlation is not valid for large rates of transport because there the transport varies with the first power of velocity  $\bar{U}$  only.

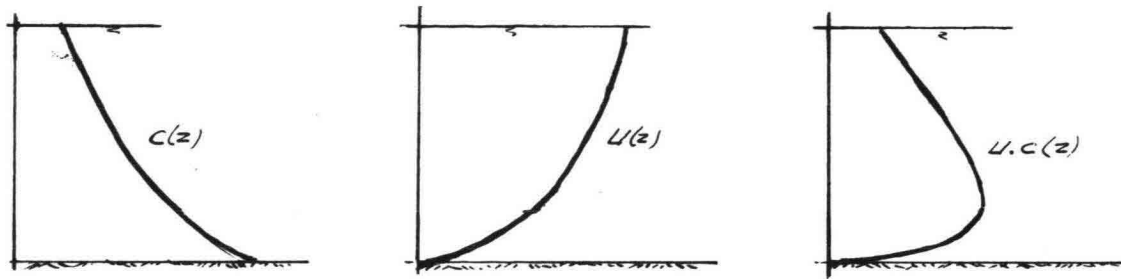
<sup>\*)</sup> See par. 5.5

The relations given were for bed-load. In most conditions a predominant contribution of suspended load will be present. The final accuracy of the bed-material discharge will depend therefore mostly on the accuracy of the suspended load determination.

### 6.2 Suspended load

Suspended load can be determined from measurements of  $U(z)$  and  $C(z)$  and integration of:

$$S = \int_0^h C(z) U(z) dz$$



In most cases estimates based on theoretical expressions will be necessary. The basic equation describing the concentration distribution in uniform steady flow is:

$$W \cdot C + \epsilon_s \cdot \frac{\partial C}{\partial z} = 0$$

The first term  $W \cdot C$  ( $W$  = fall velocity;  $C$  = volume concentration of sediments) represents the settling tendency of the flow. The second term represents the diffusive action of the turbulence.  $\epsilon_s$  is the turbulent diffusion coefficient. An explanation for this term is the following. Water packets moving upward carry a larger amount of grains than packets moving downward because there is a concentration gradient. Although there is no net transport of water there will be a net vertical transport due to this exchange of water packets, which will be proportional to the local value of the concentration gradient.

If it is assumed that the diffusion coefficient for sediment is equal to the coefficient to the exchange of momentum, then:

$$\epsilon_s = \epsilon_m = \kappa \cdot U^* z (1 - z/h)$$

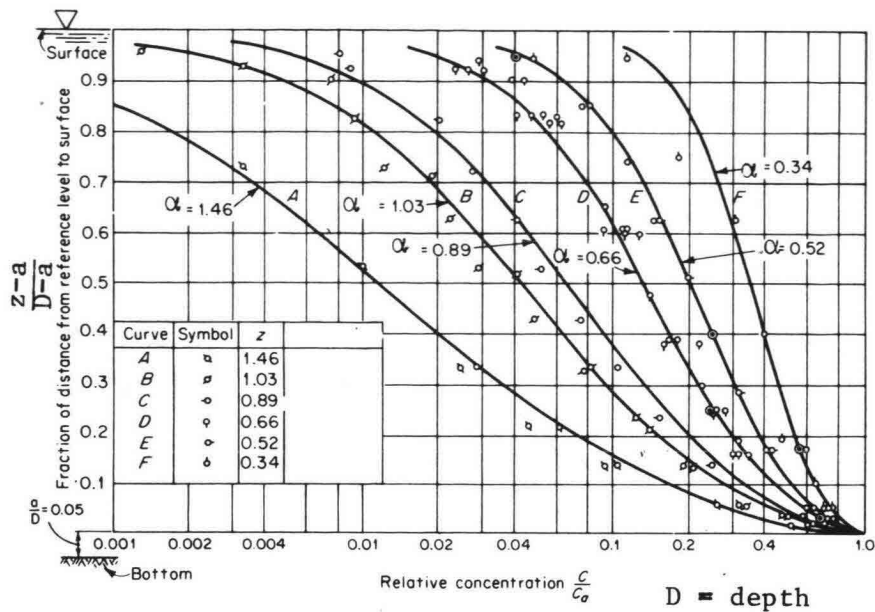
The resulting equation may be integrated and gives:

$$\frac{C(z)}{C(a)} = \left( \frac{h-z}{z} \cdot \frac{a}{h-a} \right)^\alpha$$

with  $\alpha = W/\kappa U^*$ .  $a$  is a reference level where  $C = C(a)$ .

For a graphical presentation see figure 6.2





**Fig. 6.2** Distribution of suspended sediment; comparison of experimental data with Eq. (8.35). [After VANONI (1946).]

From this figure and the analytical expression the following rough criteria may be given:

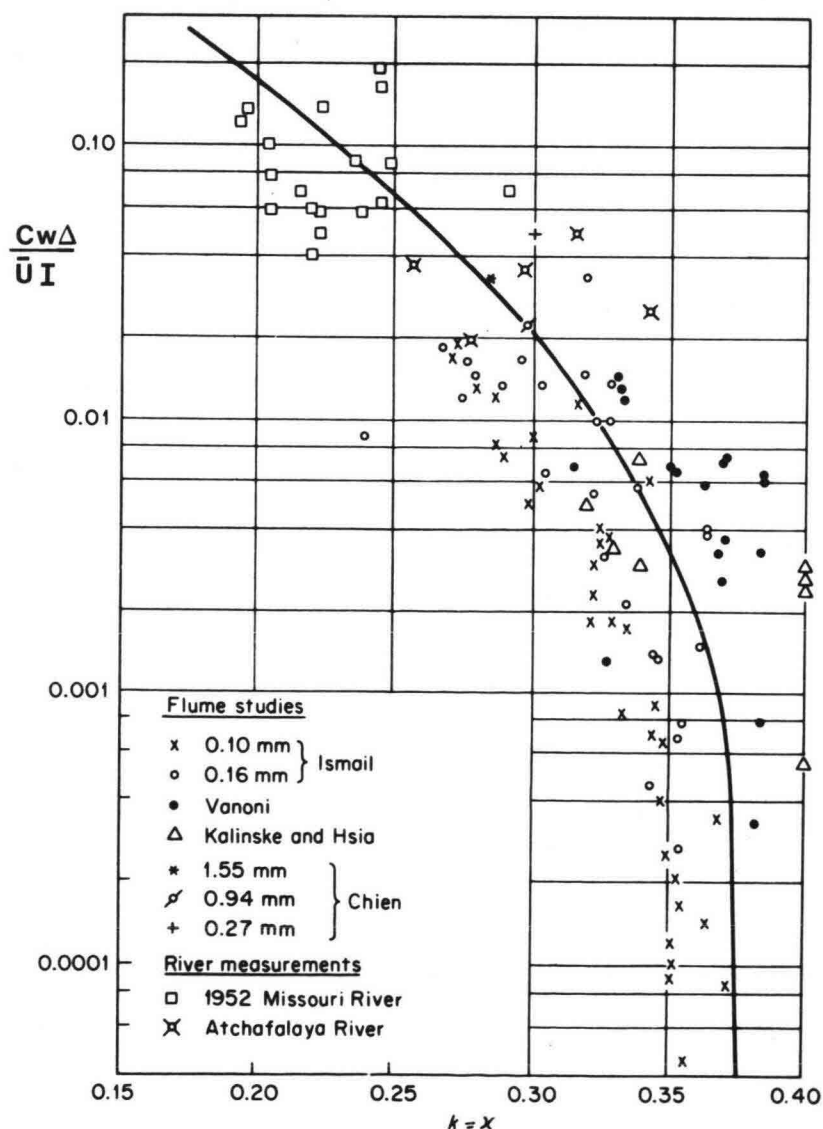
$W/\kappa U^*$	$U^*/W$	description
1.6	1.5	some suspension
0.8	3	concentration at surface > 0
0.25	10	fully developed suspension
0.06	40	almost uniform concentration

The last criterion shows that particles < 50  $\mu\text{m}$  ( $W < 0.2 \text{ cm/s}$ ) are uniformly distributed for  $U^* > 8 \text{ cm/s}$  or  $\bar{U} > 1 - 1.5 \text{ m/s}$ .

Although the basic equation is very simple, some critical remarks have to be made:

1. The term  $W.C.$  should be  $(1 - C).C.W$  to account for the presence of the particles (see Hunt 1954). This correction is not important for  $C \ll 1$ .
2. The fall velocity is changed by the presence of other particles (see chapter 3) and by the turbulent movements of the water. Symmetric vertical velocity fluctuations give a-symmetric drag force for non-Stokes particles. Therefore, although the mean value of the vertical velocity is zero, there will be a resultant vertical force which will reduce the settling velocity.

3. The expression for  $\varepsilon_s$  gives  $\varepsilon_s = 0$  for  $z = 0$  or  $\partial C/\partial z = \infty$  at  $z = 0$  which is not very real.
4. The value of  $C = C(a)$  is not given. Several assumptions are made in the literature. Einstein (1950) divides the computed bed-load by a layer with thickness  $2D$  and by the velocity in this layer. ( $11.6 U_*'$ ) The value of  $C(a)$  is one of the problems to be solved in sediment transport.
5. The velocity distribution is influenced by the presence of particles. The weight of the particles suppresses the vertical velocity fluctuations and gives a decrease in the momentum diffusion coefficient. This is similar to a decrease in the value of  $\kappa$ . In fact several expressions have been given in which  $\kappa$  decreases with the power to keep the sediment in suspension:  $C.W.\Delta/U_*I$  (see figure 6.3).  
Velocity profiles become less "full" by this effect. Care should be taken in the application of this correlation because the determination of  $\kappa$  from velocity profiles or concentration profiles is not very accurate. For literature see Einstein and Ning Chien (1954) and Ippen (1971).



**Fig. 6.3** Effect of suspended load on the  $k$  value. [After EINSTEIN et al. (1954).]

6. The assumption  $\epsilon_s = \epsilon_m$  has also some objections. It is not necessary that the diffusion of particles is equal to that of momentum.

Measurements by Coleman (1970) show indeed that  $\epsilon_s$ -values derived from concentration profiles give some differences with values of  $\epsilon_s$  obtained from:

$$\epsilon_s = \epsilon_m = \kappa U^* z (1 - z/h).$$

It is a reasonable assumption, however. Differences between  $\epsilon_s$  and  $\epsilon_m$  are generally put in  $\kappa$  which is often used as a closing factor. If  $U(z)$  and  $C(z)$  are known, integration will give the suspended load. The integration cannot be performed analytically. Graphs are presented by Einstein (see Graf 1971, p. 189-195).

### 6.3 Total bed-material load

The total bed-material load of a stream can be determined by adding the bed-load and the suspended load. This is done in the Einstein (1950) procedure. This procedure was modified by Colby (1955, 1961). Also Toffaletti (1969) gives a procedure which is especially adapted for computer programming.

Besides these "adding" procedures several direct empirical relations are proposed in literature:

1. Shinohara and Tsubaki (1959) gave an empirical relation:

$$\Phi = 25(\psi')^{1.3} (\psi' - 0.038) \quad (\text{see figure 6.1})$$

The corresponding ripple factor  $\mu = (C/C')$ .

2. Garde and Albertson (1961) gave a graphical relation of  $\Phi/\psi$  with  $\bar{U}/U^*$  as the third variable. The resulting  $\Phi - \psi$  relation is almost identical with Shinohara (figure 6.1).
3. Colby (1964) has given a graphical relation between total load, mean velocity  $\bar{U}$ , flow depth and grain-size with correction factors for temperature and silt content (see figures 6.4 and 6.5).

4. Engelund and Hansen (1967) gave an empirical relation of the form:

$$\Phi = 0.1 f^{-1} \psi^{2.5} \quad \text{with } f = \tau / (\frac{1}{2} \rho \bar{U}^2) = 2g/C^2$$

The formula is based on measurements with  $D_{50} < 1 \text{ mm}$  and gave good results in comparison with sediment transport measurements in rivers. At all values of  $\psi$ , the sediment rate increases with the fifth power of the velocity.

5. Ackers and White (1973) define the parameters:

$$F_{gr} = \frac{U^{*n} \cdot (U^{*'})^{1-n}}{(\Delta g D)^{\frac{1}{2}}} \quad U^{*'} = \frac{\bar{U}}{5.64 \log(10 h/D)}$$

$$D_{gr} = D \cdot \left(\frac{\Delta g}{v^2}\right)^{1/3} \quad (\text{dimensionless grain size})$$

$$G_{gr} = \frac{S}{UD} \cdot \left(\frac{U^{*'}}{\bar{U}}\right)^n \quad (\text{transport parameters})$$

The relation between the transport parameter  $G_{gr}$  and the sediment mobility number  $F_{gr}$  is given as:

$$G_{gr} = C \left(\frac{F_{gr}}{A} - 1\right)^m$$

in which  $C$ ,  $A$ ,  $m$  and  $n$  are functions of the dimensionless grain size  $D_{gr}$  (see figure 6.8).

For coarse materials ( $D_{gr} > 60$ )  $n = 0$  and  $U^{*} = U^{*'}$  so that the parameters  $F_{gr}$  and  $G_{gr}$  are reduced to a more simple form.

For a modest range of particle sizes ( $D_{84}/D_{16} < 5$ ) Ackers and White suggest to take  $D = D_{35}$ . For a wider gradation a fraction by fraction computation is suggested, using a corrected value for  $A$ :

$$A' = A \cdot \left(\frac{D_i}{D_{50}}\right)^{-0.2} \quad \text{in which}$$

$D_i$  = average size of the fraction (Ackers and White, 1980)

#### NOTE

It must be noted that due to the strong variation of sediment transport with velocity, predictions of total sediment load will not be very accurate. Differences of a factor 10 between various formulas or between computations and measurements are no exception (see figure 6.6 and 6.7).

#### 6.4 Comparison of relations

White, Milli and Crabbe (1975) have made a comparison of 8 of the most widely used transport relations (a.o. Meyer-Peter Müller, Einstein, Engelund and Hansen and Ackers and White) with 840 flume data and 260 field experiments in natural water courses. If the percentage of all data with a ratio  $R$  of calculated to observed transport in the range

$\frac{1}{2} < R < 2$  is taken then the following result is obtained:

Ackers and White	68%
Engelund and Hansen	63%
Einstein	46%

It is not surprising that Ackers and White give relatively good results in view of the large number of tuning parameters (C, A, m and n are all functions of grain size). It is surprising however that the far more simple formula of Engelund and Hansen gives such a good result.

Application of formulas remains a matter of experience. For each situation a comparison with field measurements and an adjustment of the formulas remains necessary for reliable results.

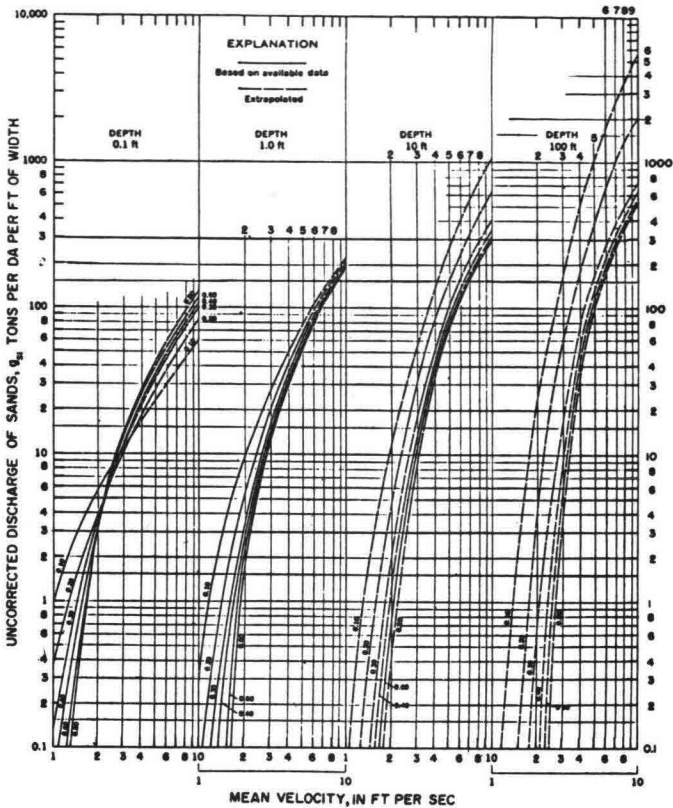
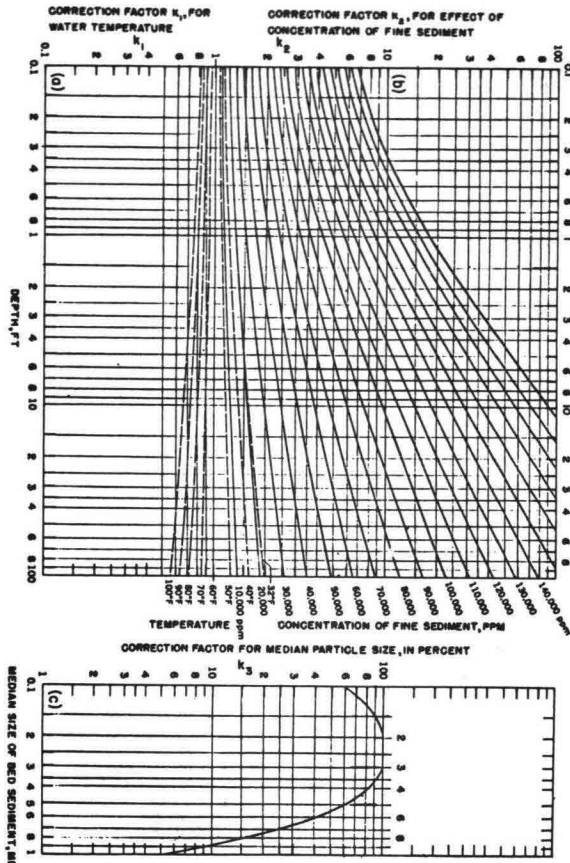


FIG. 6.4 —COLBY'S (2-H.14) RELATIONSHIP FOR DISCHARGE OF SANDS IN TERMS OF MEAN VELOCITY FOR 6 MEDIAN SIZES OF BED SANDS, 4 DEPTHS OF FLOW, AND WATER TEMPERATURE OF 60° F

FIG. 6.5 —COLBY'S (2-H.14) CORRECTION FACTORS FOR EFFECT OF WATER TEMPERATURE, CONCENTRATION OF FINE SEDIMENT AND SEDIMENT SIZE TO BE APPLIED TO UNCORRECTED DISCHARGE OF SAND GIVEN BY FIG. 2-H.10



Colby correction factor  $k = [1 + (k_1 k_2 - 1) \cdot 0.01 k_3]$

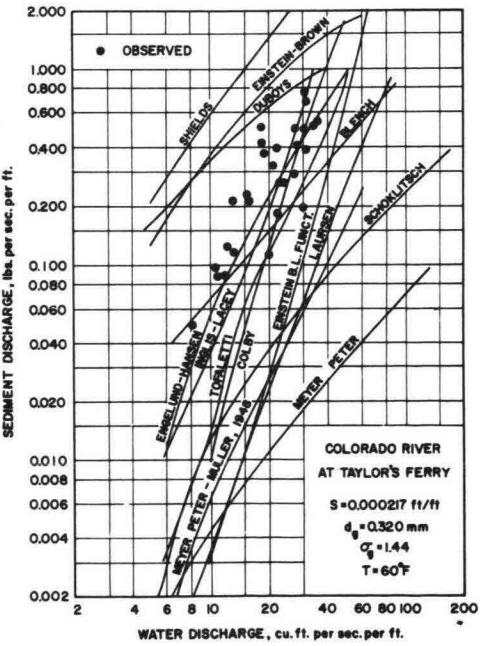


FIG. 6.6 —SEDIMENT DISCHARGE AS FUNCTION OF WATER DISCHARGE FOR COLORADO RIVER AT TAYLOR'S FERRY OBTAINED FROM OBSERVATIONS AND CALCULATIONS BY SEVERAL FORMULAS

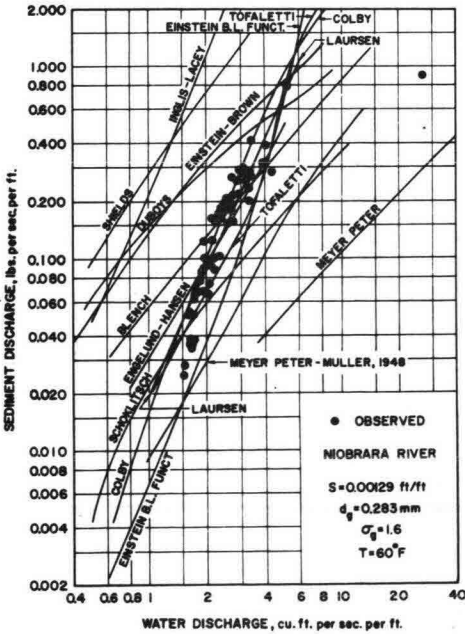
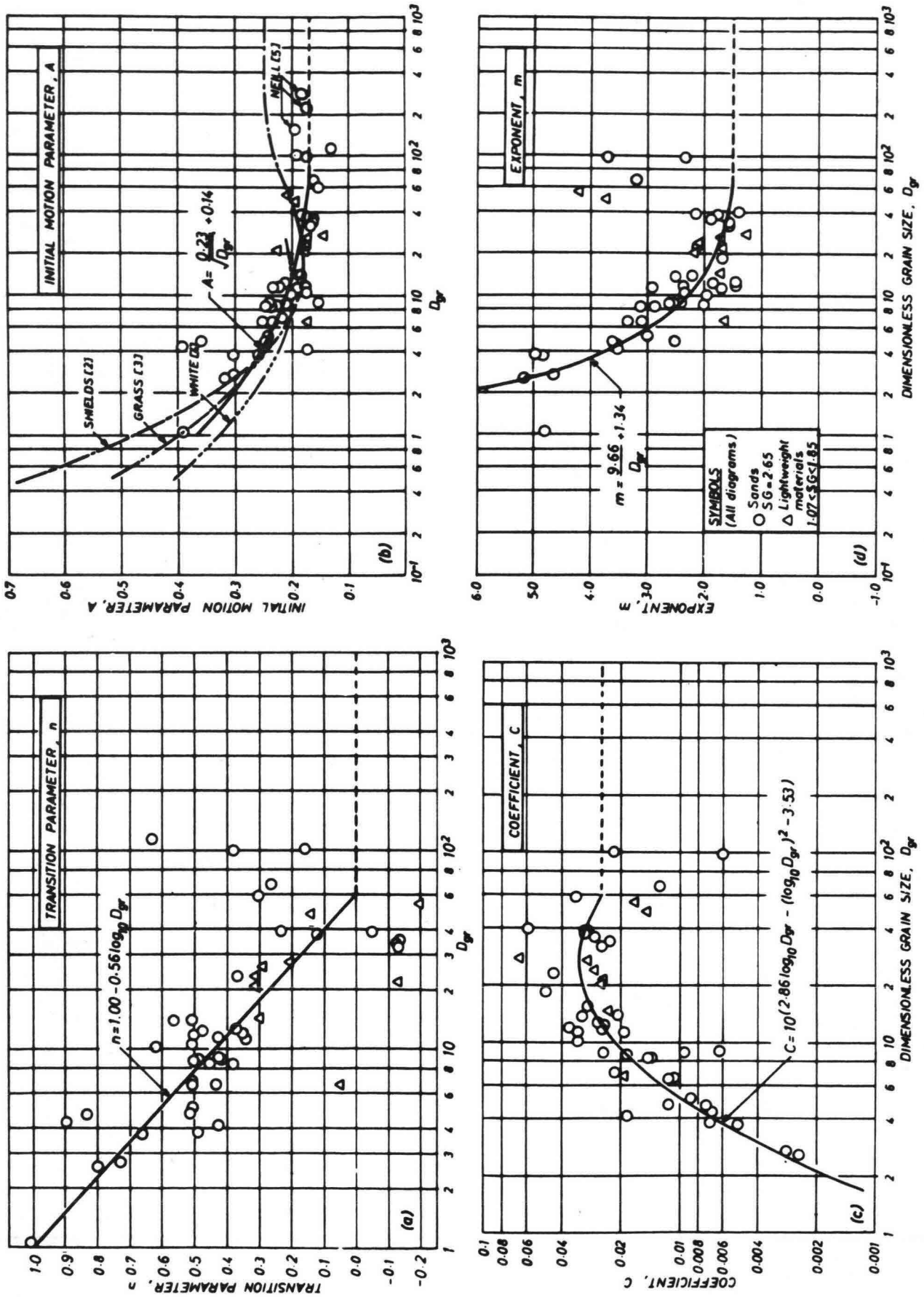


FIG. 6.7 —SEDIMENT DISCHARGE AS FUNCTION OF WATER DISCHARGE FOR NIOBRARA RIVER NEAR CODY, NEB. OBTAINED FROM OBSERVATIONS AND CALCULATIONS BY SEVERAL FORMULAS



Coefficients in General Sediment Transport Function (Ackers and White)

fig. 6.8

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## 6.5 Problems

6.1 Given: A wide river with  $h = 3$  m,  $\bar{U} = 1.2$  m/s,  $I = 1/6000$

$$T = 20^\circ \text{ C} \quad D_m = 2 \text{ mm} \quad D_{90} = 3 \text{ mm}.$$

Question: Compute the bed load with the Meyer-Peter-Müller method.

6.2 Given: A wide river with depth  $h = 2$  m, width  $B = 80$  m,  $\bar{U} = 1.1$  m/s,

$$I = 8.10^{-4}, \quad D_m = 0.6 \text{ mm}, \quad D_{90} = 1.5 \text{ mm}, \quad \epsilon = 0.4.$$

Question: Compute the annual bulk transport using the M.P.M. method.

6.3 Given: A wide river has the following characteristics:

$$\text{depth } h = 3.2 \text{ m}$$

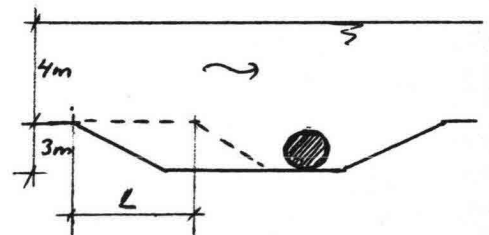
$$\text{Slope } I = 0.5.10^{-4}$$

$$\text{bed mat.: } D_m = 0.5 \text{ mm} \quad D_{90} = 1 \text{ mm}$$

$$\text{Temp.: } T = 20^\circ \text{ C}$$

- Questions:
- 1) Determine the critical shear stress  $\tau_{cr}$  of the bed material and the bottom shear stress  $\tau_o$ . Is there transport?
  - 2) Which type of bottom configuration is present according to Simons-Liu (Fig. 5.3)? Use  $D_m$  to obtain the fall velocity.
  - 3) If the mean velocity  $\bar{U} = 0.66$  m/s, what is the bed roughness  $k_s$ ?  
Is the bed hydraulically smooth or rough?
  - 4) Compute the ripple factor  $\mu$  according to M.P.M.
  - 5) Compute  $\tau' = \mu \tau_o$  and the bed load/m' according to M.P.M.
  - 6) Will there be transport in suspension?

6.4 Given: In a wide river a trench is made for a pipe line crossing. The depth of the trench is 3 m. Because it takes some time to lay the pipe and the river transports bed load, some storage has to be provided.



$$\text{River data: } \bar{U} = 1.2 \text{ m/s} \quad h = 4 \text{ m} \quad C = 45 \text{ m}^{1/2}/\text{s}$$

$$D_m = 1 \text{ mm} \quad D_{90} = 2 \text{ mm} \quad \epsilon = 0.4.$$

The relation of M.P.M. is valid.

Question: How large should  $L$  be to provide sufficient storage for 2 days?

6.5 Given: A wide open channel,  $h = 3$  m,  $\bar{U} = 1.2$  m/s,  $I = 10^{-4}$ .

Transported material  $D = 150 \mu\text{m}$  (uniform)

The concentration at  $z = 0.5$  m is  $250 \text{ mg/l}$ .

Question: Compute the concentration at  $z = 0.25$  m and  $z = 2$  m.

6.6 Given: A wide river with  $h = 2 \text{ m}$   $I = 1.5 \cdot 10^{-4}$   $\bar{U} = 0.9 \text{ m/s}$   
sediment uniform  $D = 0.2 \text{ mm}$   $\epsilon = 0.4$ .

Question: What is the bulk transport/ $\text{m}^3 \cdot \text{day}$  using the Engelund-Hansen method.

6.7 Same question for:

$h = 3 \text{ m}$   $I = 10^{-4}$   $\bar{U} = 0.8 \text{ m/s}$   $D = 0.15 \text{ mm}$  (uniform)

6.8 Use the data of 6.6 and compute the total-load with the method of Ackers-White (as bulk load/ $\text{m}^3 \cdot \text{day}$ ).

6.9 Use the data of 6.1 and the method of Ackers-White to compute the transport.  $D_{35} = 1.5 \text{ mm}$ .

6.10 Given: Sediment size  $D = 150 \text{ }\mu\text{m}$  (uniform) and shear velocity  $U^* = 0.05 \text{ m/s}$ .

Question: Compute fall velocity (Fig. 3.4), critical shear stress (Fig. 4.2) bedform according to Simons (Fig. 5.3) and the degree of suspension (table page 6.6).

6.11 Same question for  $D = 2 \text{ mm}$ . and  $U^* = 0.05 \text{ m/s}$ .

## 7. STABLE CHANNELS

### 7.1 Introduction

For the design of stable channels two approaches can be distinguished:

1. The tractive force theory (Lane)
2. The regime theory (originating in India. Developed by Kennedy, Lindley, Lacey, Inglis, Blench).

Both methods are used to design stable channels. A stable channel in alluvial material is one in which scour of banks and changes in alignment do not occur. Deposition on or scour of the bed is not objectionable in general, provided there is equilibrium over a long period.

The stability of a channel depends on the properties of the excavated material (grain-size, cohesion), of the flow (discharge, silt content, transported material) and the design variables such as profile, shape, slope. Sediments introduced in the channel must be conveyed in view of the definition given above.

### 7.2 Tractive force theory

This approach is specially suited if the flow transports very little or no sediment. The design is then based on a limiting velocity or critical shear stress of the bed material.

For uniform cohesionless material, Shields' graph may be used to compute  $\tau_{cr}$ . In practice materials will have a wide gradation and will have some cohesion due to the silt content. For these materials, Lane's (1953) design curves are recommended (see figure 7.1).

It must be noted, however, that the large values of  $\tau_{cr}$  as compared with Shields values are due to the fact that  $\tau_{cr}$ -values are based on actual channel roughness, including irregularities, bedforms, whereas Shields' graph is based on a flat bed. When there is some bed load, the problem is more complicated and calculations should be made to check the transport capacity of the channels.

Bank stability will depend on the characteristics of the bed material and the side slopes of the channel. From experiments and calculations it appeared that for trapezoidal channels with side slopes 1:1 to 1:2 the following values can be given for the shear stress:

$$\begin{aligned}\tau_o, \text{ horizontal part of the profile} &\approx \rho g h I \\ \tau_o, \text{ side slopes} &\approx \frac{3}{4} \rho g h I\end{aligned}$$

For non-cohesive materials the reduction of  $\tau_{cr}$  due to the side slope was given in chapter 4:

$$\frac{\tau_{cr}(\beta)}{\tau_{cr}(0)} = \cos\beta \sqrt{1 - \left(\frac{\tan\beta}{\tan\phi}\right)^2} \quad (\text{see figure 7.2})$$

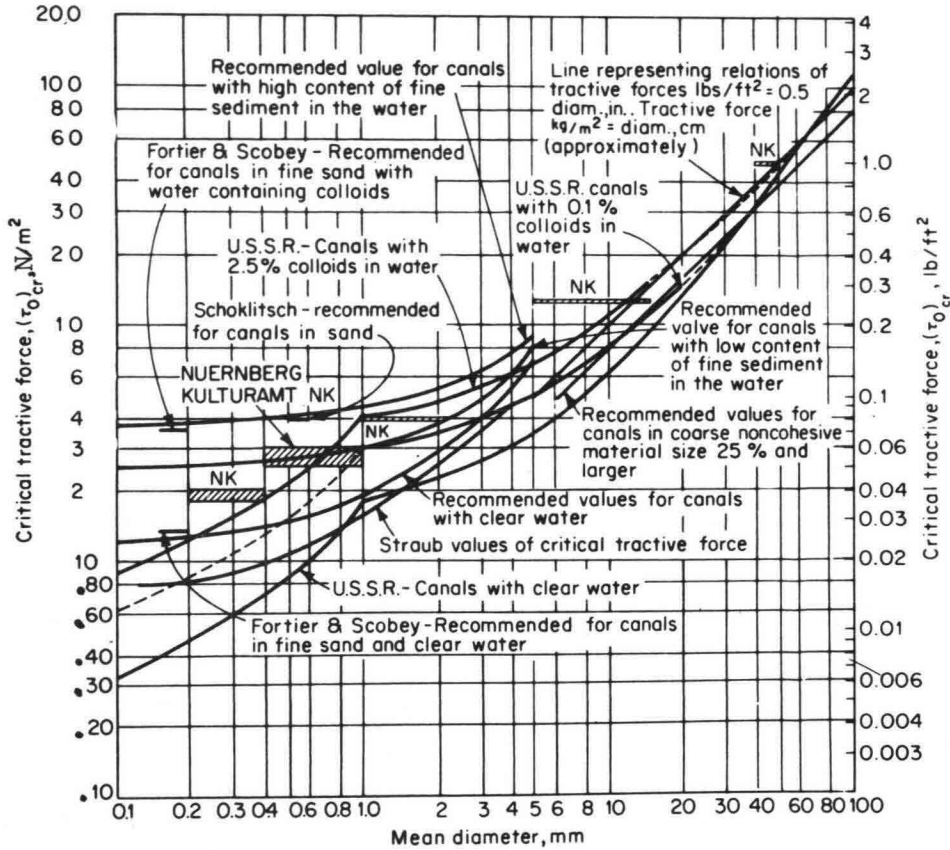
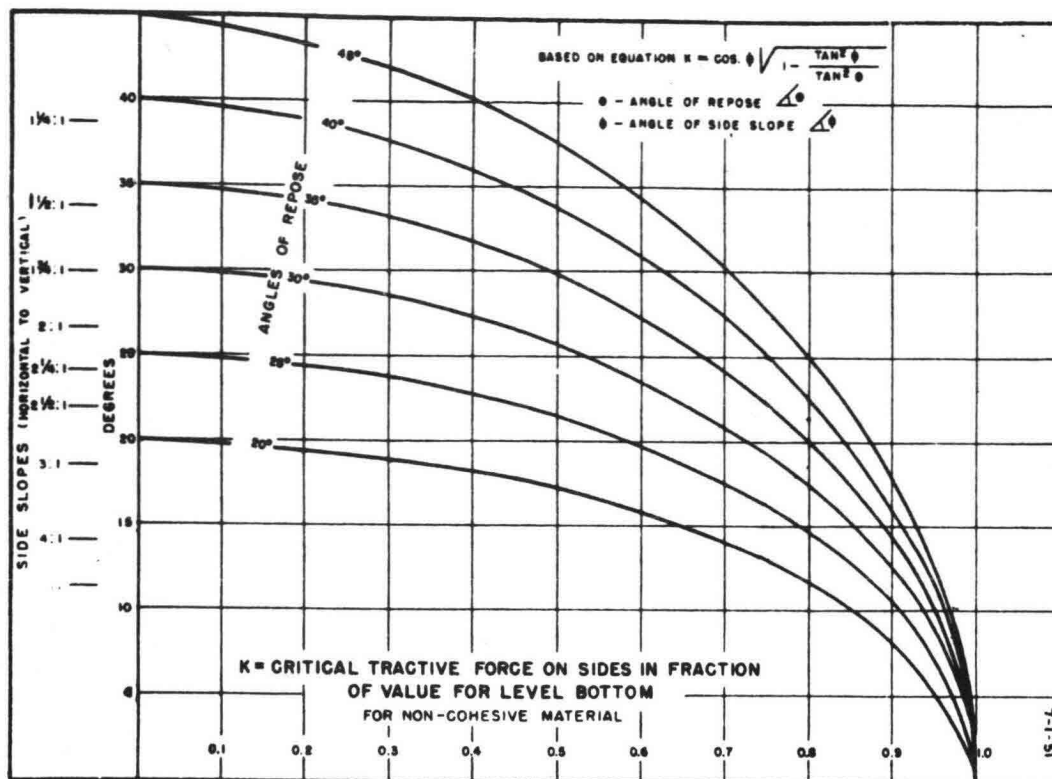


Fig. 7.1 Critical shear stress as function of grain diameter. [After LANE (1953).]

Assuming  $\tau_{side}/\tau_{horizontal} = \frac{3}{4}$  and  $\phi = 30 - 40^\circ$  it can be seen that a side slope of 1:2 to 1:3 is necessary. For practical values of  $\phi$  see figure 7.3

A theoretical stable profile for which at all points the same critical conditions occur is found in the following way. Assume that the local value of the shear stress  $\tau(y)$  is proportional to the local water depth  $h(y)$  and acts on a length of  $1/\cos\beta(y)$ . Then  $\tau(y) = \tau_{max} \cdot h(y) \cdot h_{max}^{-1} \cos\beta(y)$  in which  $h_{max}$  is the maximum depth with corresponding  $\tau_{max}$ . With the reduction formula for  $\tau_{cr}(\beta)$  the following theoretical sinus-profile results:

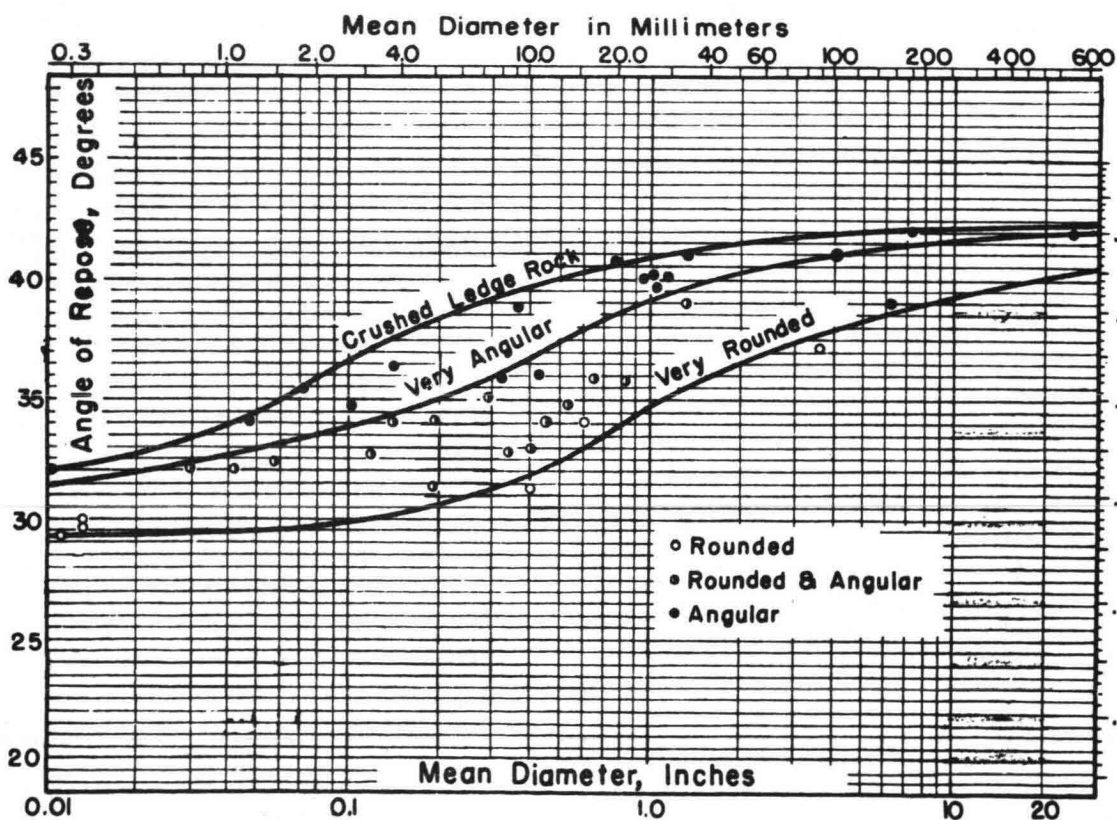
$$h(y)/h_{max} = \sin(y/h_{max} \cdot \tan\phi). \quad y = \text{horizontal coordinate}$$



- K = Critical tractive force on sides in fraction of value for level bottom for non-cohesive material.

Acc. to Lane (1955)

FIG. 7.2



ANGLE OF REPOSE OF NON-COHESIVE MATERIAL

Acc. to Simons and Albertson (1960)

FIG. 7.3

The width of the channel is then  $B = \pi \cdot h_{\max} / \tan \phi$  but may be extended of course with a horizontal section. It is clear that the sinusoidal profile is difficult to construct and will be approximated by a trapezoidal profile in practice.

Lane (1953) gives some reduction factors to account for the sinuosity of channels.

Channel type	$\tau_{cr} / \tau_{cr \text{ straight}}$	$U_{cr} / U_{cr \text{ straight}}$
straight	1.00	1.00
slightly sinuous	0.90	0.95
moderately sinuous	0.75	0.87
very sinuous	0.60	0.78

Also local effects of contractions, bridges etc. should be considered.

In applying Shields' graph for coarse material ( $\psi_{cr} \approx 0.06$ ) for the stability of stones on revetments and banks care should be taken with the criterion (general movements). For a safe design without movement of the stones a value  $\psi_{cr} = 0.03$  is recommended.

### 7.3 Regime theory

With the tractive force theory designs of channels can be made. Another approach to the problem is to study successful alluvial channels. Numerous studies of man-made and natural alluvial channels have given empirical relations between depth, width, velocity, discharge, sediment transport and material characteristics. These techniques are referred to as "regime theory". Usually three equations are presented: (1) a flow formula which gives the required slope (2,3) formulas for channel depth and width. Regime theory originated in India where extensive canal systems were built. One of the disadvantages of the regime theory is that results are related to a specific area, so that application to other areas can give errors.

"Regime" can be defined as a situation in which a channel will not change on a long-term average. Short term changes will occur with changes in discharge or sediment transport.

Important contributions were given by Kennedy, Lindley, Lacey and Blench (1957). Some of the results of Blench are given here. Blench gave three equations:

$$(1) F_b = U^2/h \quad F_b = \text{bedfactor (ft.s-units)}$$

$$(2) F_s = U^3 \cdot h/A \quad F_s = \text{side factor } A = \text{area of cross section}$$

$$(3) I = \frac{F_b^{5/6} F_s^{1/12} \nu^{1/4}}{3.63Q^{1/6} \cdot g(1 + c/2330)}$$

$I$  = slope

$\nu$  = viscosity of water-sand mixture

$c$  = sediment concentration in p.p.m. by weight

From these relations the following equations are derived:

$$\bar{B} = A/h = \sqrt{(F_b/F_s) \cdot Q}$$

$$h = \sqrt[3]{(F_s/F_b^2)Q} \quad \longrightarrow \quad h = \sqrt[3]{q^2/F_b} \quad q = Q/\bar{B}$$

$$\bar{U} = \sqrt[5]{F_b \cdot F_s \cdot Q}$$

Blench suggests:  $F_b = 1.9\sqrt{D} (1+0.012 c)$

with  $D$  in mm, sand range only and  $F_s = 0.1, 0.2, 0.3$  for loam with very slight, medium and high cohesion. For practical applications tests in similar channels seem necessary.

Simons and Albertson (1960) have analysed a large number of Indian and American canals. The results presented are valid for sediment concentrations  $< 500$  p.p.m. and grain sizes  $0.1 < D < 7.5$  mm. From figure 7.4 to 7.8 width and depth can be selected. Curves E should be used for channels with high sediment load only. From the graphs  $A = P \cdot R$  can be computed and also  $\bar{U} = Q/A$ . Values of depth, average width and top width can be adjusted as required to maintain these values of hydraulic radius and wetted perimeter. If the bank is non-cohesive, the side slope must not exceed the value for the angle of repose given in figure 7.3. For a good design values  $5-10^\circ$  lower than the angles given should be taken. Figures 7.9 to 7.11 can be used to estimate three values of the slope  $S$  (depth =  $D$  in Simons notation,  $W$  = width,  $R$  = hydraulic radius). The designer must now "invoke his engineering judgement, guided by these slopes to arrive at the design slope".

It will be clear that with the regime theory only rough estimates of channel dimensions will be obtained. Experience in a specific area will be of equal importance.



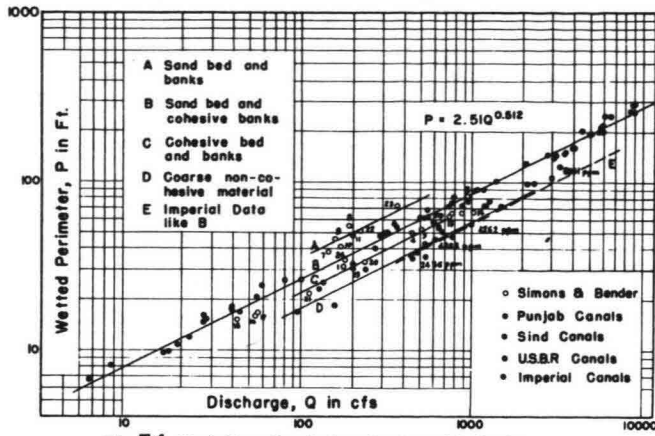


Fig. 7.4 Variation of wetted perimeter with discharge for regime channels.

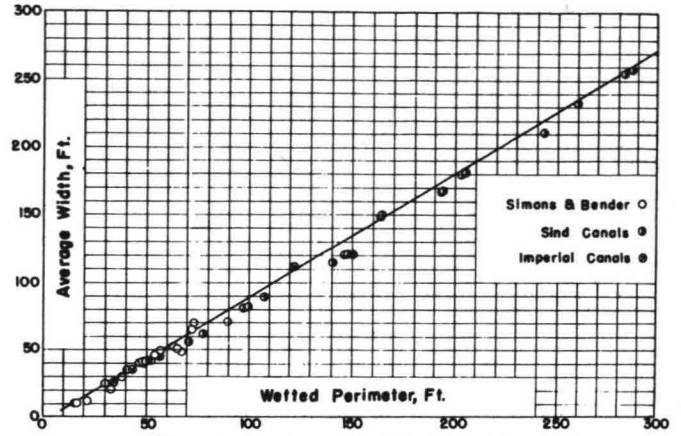


Fig. 7.5 Variation of average width with wetted perimeter for regime channels.

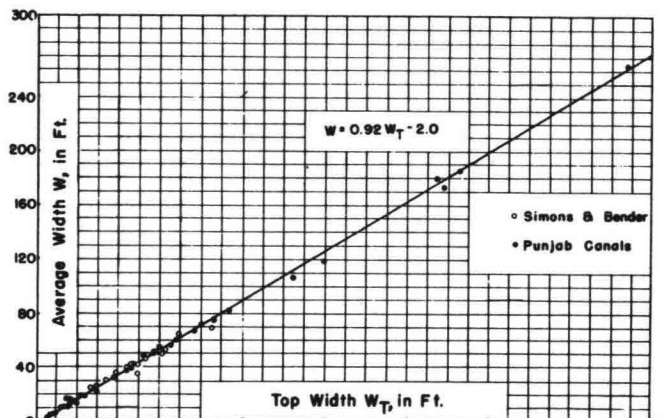


Fig. 7.6 Variation of top width with average width for regime channels.

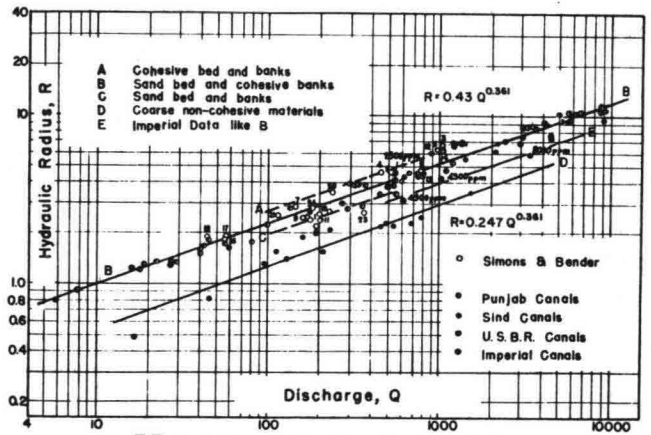


Fig. 7.7 Variation of hydraulic radius with discharge for regime channels.

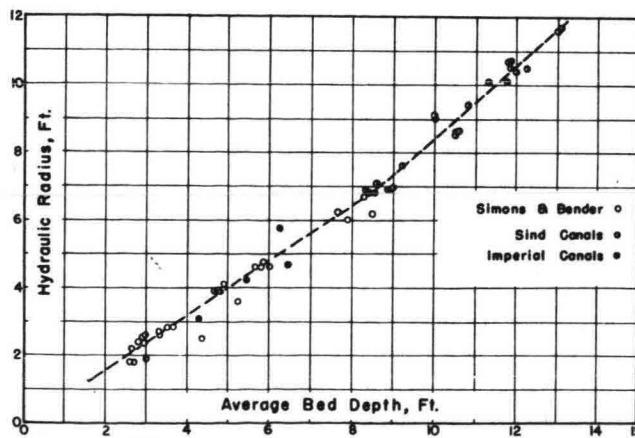


Fig. 7.8 Variation of average bed depth with hydraulic radius for regime channels.



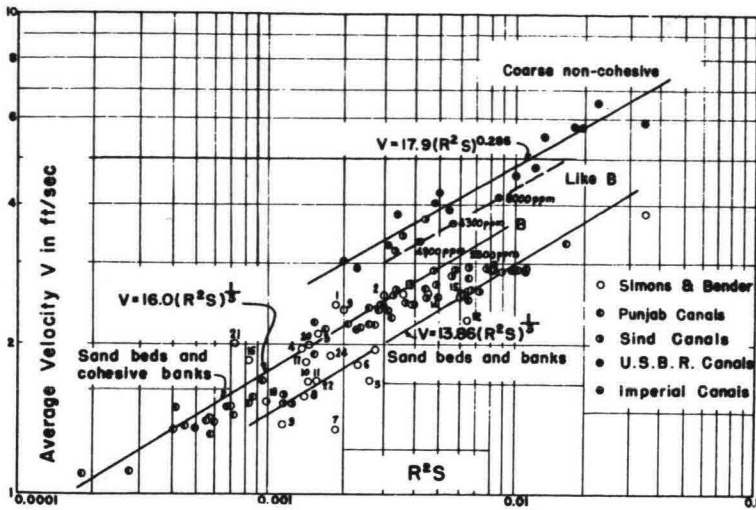


Fig. 7.9 Variation of mean velocity with  $R^2 S$  for regime channels. (Lacey type slope relation.)

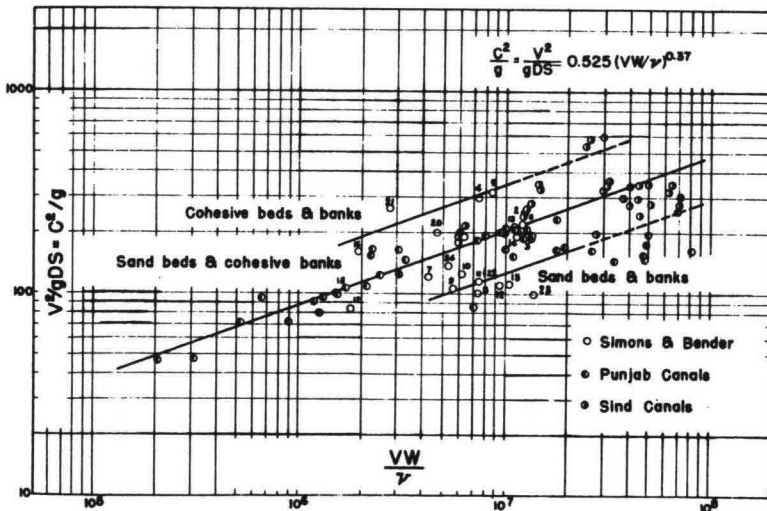


Fig. 7.10 Variation of  $\frac{V^2}{gDS}$  with  $\frac{VW}{v}$  for regime channels. (Blench-King type slope relation.)

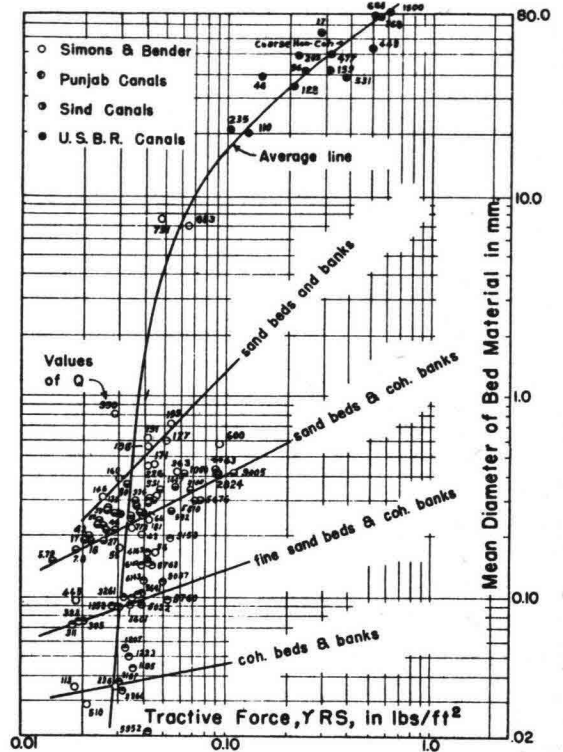


Fig. 7.11 Variation of tractive force with mean size of bed material for regime channel. (Tractive-force type slope relation.)

$S$  = slope

$W$  = average width

$D$  = average depth

$R$  = hydraulic radius

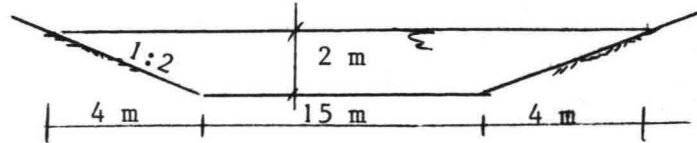
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- |                        |   |
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## 7.6 Problems

$$\rho_s = 2650 \text{ kg/m}^3$$

- 7.1 Given: A channel is excavated in very rounded material with  $D = 5 \text{ cm}$ . Design depth is 2 m, side slopes 1 vert:2 hor.



Question: What are the permissible slope and discharge, using Shields curve for  $\tau_{cr}$ ? Which part of the channel is critical? (slopes or horizontal part).

- 7.2 Given: A channel is designed in coarse very angular stones.

$D_{50} = 2 \text{ cm}$   $D_{90} = k_s = 4 \text{ cm}$  for a discharge of  $95 \text{ m}^3/\text{s}$ . The required depth in the middle section is 3 m. The side slopes are 1:2 (1 vert, 2 hor.) and the width of the horizontal part is 20 m.

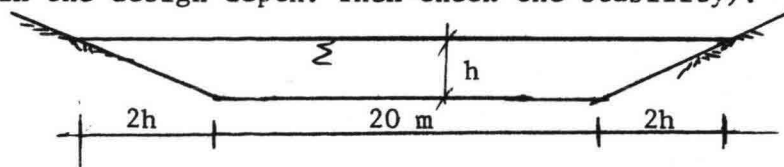
Questions: What is the necessary slope of the channel (uniform flow) to transport this discharge. Is the channel stable in this condition if  $\psi_{cr} = 0.03$ ?

- 7.3 Given: The side slope of a channel has to be protected with very rounded gravel. Channel data:  
depth (horizontal part)  $h = 2.5 \text{ m}$   
side slope 1:3 (1 vert, 3 hor.)  
channel slope  $I = 4.10^{-4}$

Question: What is the minimum stone size assuming uniform material? Shields diagram can be used for  $\tau_{cr}$ .

- 7.4 Given: A channel is made in a layer of coarse rounded gravel ( $D_{50} = 0.04 \text{ m}$ ,  $k_s = D_{90} = 0.08 \text{ m}$ ) for a discharge of  $100 \text{ m}^3/\text{s}$ . The slope of the channel in the flow direction is equal to  $3.5.10^{-4}$ , the bottom width is 20 m and the side slopes are 1:2 (1 vert, 2 hor.)

Question: What is the water depth for this discharge and is the gravel stable both on the horizontal part and on the side slopes? Shields curve can be used for  $\tau_{cr}$ . (Compute the discharge for various depth and interpolate to obtain the design depth. Then check the stability).



7.5 Given: A stable channel has to be designed for  
 $Q = 56 \text{ m}^3/\text{s} = 2000 \text{ cfs}$ . The type is B (sand bed, cohesive  
banks).

Questions: What are the dimensions and slope of channel using the Simons-  
Albertson method (Figs. 7.4 to 7.10).

## 8. RIVER BED VARIATIONS, AGGRADATION AND DEGRADATION

A natural river will never be in an exact equilibrium condition. Variations in discharge can give variations in bed level, roughness etc. Also changes in the regime of a river may give deviations from an equilibrium state. If the sediment discharge  $S$  entering a river reach is greater than the equilibrium value  $S_e$  aggradation will occur until a new equilibrium is approached. Some examples:

- aggradation upstream from a reservoir,
- tributary channel bringing heavy sediment load to a main channel giving local aggradation,
- river regulations eliminating floods which formerly periodically cleared the channel of accumulated sediment.

Some examples of degradation ( $S < S_e$ ):

- degradation downstream of dams,
- canals in fine material carrying clear water,
- realigned channels with increased slope.

For calculations on non-steady or non-uniform conditions it is necessary to introduce a sediment transport relation, for example of the form  $s = a.v^b$ . For the transport relations given in Ch. 6 values of  $b$  in the order of 3 - 7 are found (high values for low transport rates), with  $b = 4$  to 5 for high sediment rates and fine material. (Engelund  $b = 5$ , Shinohara  $b = 4.6$ ).

Application is shown in the following example.

What will be the reaction of a river to a local decrease in width?

Suppose  $Q_1 = Q_0$  (continuity)  $S_0 = S_1$  (after some time continuity of sediment transport)  $S = s.B$   $B$  = width.

$C_1 = C_0$  (Chézy value, index 1 = new situation; index 0 = old situation).

From  $S = B.s = B.av^b = B^{1-b}.aQ^b.h^{-b}$  ( $v = QB^{-1}h^{-1}$ ) it follows that:

$$\frac{h_1}{h_0} = \left(\frac{B_0}{B_1}\right)^{\frac{b-1}{b}}$$

From the Chézy formula it follows that:

$$B_1.h_1^{3/2}.I_1^{1/2} = B_0.h_0^{3/2}.I_0^{1/2}$$

with the relation for  $h_1/h_0$  it follows that:

$$\frac{I_1}{I_0} = \left(\frac{B_1}{B_0}\right)^{1 - \frac{3}{b}}$$

It can be seen from the equation for  $h_1/h_0$  that for a large value of  $b$  and values of  $B_0/B_1$  not far from 1, that:

$$h_1/h_0 \approx B_0/B_1$$

or the decrease in width is compensated by an increase in depth. The slope will always decrease.

This rough approximation only gives a first estimate. For accurate values detailed computations or model studies will be necessary.

### Computation on river-bed variations

The reaction of a river to a change in its regime (meander cut-off, dam) can be computed with the equations of motion and continuity of water and sediment. In most cases non-steadiness of the flow may be neglected so that the following equations are valid for constant width:

(1)	$U \frac{\partial U}{\partial x} + g \frac{\partial h}{\partial x} + g \frac{\partial z}{\partial x} = -g \frac{U \cdot  U }{C^2 R}$	$h$ = waterdepth $z$ = bed level from reference datum $C$ = Chézy value $R$ = hydraulic radius
	$C = \text{constant}$	
(2)	$U \cdot h = q = \text{constant}$	$q$ = river discharge/m'
(3)	$s = f(u)$	$s$ = sediment discharge/m'
(4)	$\frac{\partial z}{\partial t} + \frac{\partial s}{\partial x} = 0$	

Solving these equations requires numerical techniques (see Vreugdenhil and De Vries 1973, De Vries 1973, De Vries 1975).

As a first approximation it may be assumed that the flow is uniform and two-dimensional so that equation (1) reduces to:

$$(5) \quad \frac{\partial z}{\partial x} = \frac{U^2}{C^2 h} = \frac{U^3}{C^2 q} \quad (q = U \cdot h) \quad I_0 = \frac{U_0^3}{C^2 q}$$

or 
$$\frac{\partial^2 z}{\partial x^2} = -3 \frac{U^2}{C^2 q} \cdot \frac{\partial U}{\partial x}$$

Combination with (3) and (4) gives  $(\frac{\partial s}{\partial x} = \frac{ds}{dU} \cdot \frac{\partial U}{\partial x})$ :

$$(6) \quad \frac{\partial z}{\partial t} - k \frac{\partial^2 z}{\partial x^2} = 0$$

$$\text{in which } k = \frac{1}{3} \frac{C^2 q \cdot (ds/dU)}{U^2} = \frac{1}{3} \frac{U \cdot (ds/dU)}{I_0} \cdot \left(\frac{U_0}{U}\right)^3$$

in which  $I$  = slope and index 0 refers to the original, uniform situation.

After linearisation (possible for  $U/U_0 \approx 1$ )

$$k = \frac{1}{3} \frac{U_0 (ds/dU)}{I_0}$$

or for  $s = au^b$  :  $k = \frac{1}{3} b \cdot \frac{s}{I_0}$

The equation (6) is a parabolic one (diffusion equation) for which solutions are known. This will be applied to compute the reaction of a river to a sudden decrease in sediment discharge  $s$  which will give a decrease in bed and water level of  $z_0$  after a long time.

Introduce  $z' = z(x, 0) - z(x, t)$

with boundary conditions:

$$z'(x, 0) = 0$$

$$z'(0, t) = z_0$$

The solution of (6) is given by:

$$\frac{z'}{z_0} = \operatorname{erfc}(x/2\sqrt{kt})$$

in which  $\operatorname{erfc}$  is the function:  $\operatorname{erfc}(\alpha) = \frac{2}{\sqrt{\pi}} \int_{\alpha}^{\infty} e^{-\xi^2} d\xi$

From this solution one can compute for which time  $t_{50}$  at  $x = x_0$ , 50% of the final lowering of the bed ( $z_0$ ) has been reached.

This is the case for  $\operatorname{erfc}(x_0/2\sqrt{k \cdot t_{50}}) \approx 0.5$

$$\text{or } x_0 \approx 1.0 \cdot \sqrt{k \cdot t_{50}} \quad (\text{see tables for } \operatorname{erfc}(\alpha))$$

$$\text{or } t_{50} \approx x_0^2/k$$

From comparison with the solution of the full equation it appears that this solution is valid for:  $x_0 \geq 2 I_0^{-1} \cdot h$ .

Suppose  $s = 10^4 \text{ m}^3/\text{year}$  (river with  $B = 100 \text{ m}$   $S = 10^6 \text{ m}^3/\text{year}$ )

$$I = 2 \cdot 10^{-4}$$

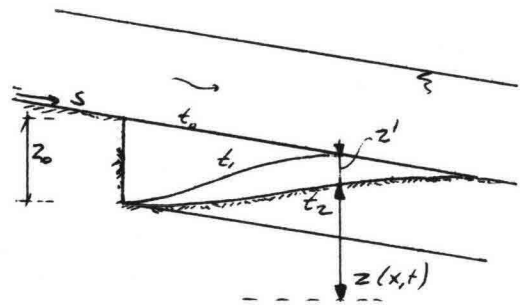
$$b = 5$$

$$h = 3 \text{ m}$$

Then the solution is valid for  $x > 30 \text{ km}$

$$k = \frac{5}{3} \cdot \frac{10^4}{2 \cdot 10^{-4}} = 0.83 \cdot 10^8 \text{ [m}^2/\text{year]}$$

$$\text{or } t_{50} = \frac{(30 \cdot 10^3)^2}{83 \cdot 10^6} \approx 10 \text{ years}$$



This means that at 30 km from the discontinuity 50% of the expected change in bed level will be reached in 10 years.

This means that the reaction of this river to change in regime is relatively slow.

For a more complete treatment of these problems see the references cited above.



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## 9. LOCAL SCOUR

### 9.1 INTRODUCTION

Local scour is caused by local disturbances of the flow and sediment transport field. Examples are: scour around (bridge) piers and abutments and scour downstream of dams. In all these cases a local increase in mean velocity and/or turbulence intensity gives an increase in local transport capacity. From the equation of continuity:

$$\frac{\partial h}{\partial t} = \frac{\partial S}{\partial x} \quad (h = \text{depth}, S = \text{transport})$$

it follows that scour will occur. The scouring continues until the local depth has increased so much that the velocities are reduced sufficiently to bring

$$\frac{\partial S}{\partial x} \text{ to zero.}$$

S can remain positive of course so that a dynamic equilibrium is obtained, for example for a pile in a sediment transporting river.

There are too many examples of failure of constructions due to local scour to neglect the phenomenon. The effects of local scour can be overcome by an increase in construction depth (bridge piers) or diminished by a bottom protection.

The following subjects will be discussed:

- scour around (bridge) piers,
- scour downstream of constructions (dams, weirs),
- scour around abutments and spur dikes,
- model investigations,
- protection.

## 9.2 SCOUR AROUND BRIDGE PIERS

Scour around bridge piers is due to a combination of three effects:

- local scour near the bridge pier caused by the disturbance of the flow field around the pier,
- a lowering of the river bed in the cross section of the bridge due to the contraction of the river profile at that section,
- a general lowering of the river bed in the river around the bridge site due to degradation or non-uniform river bed changes during floods.

These last two aspects, together with practical experience for the situation of a bridge in the flood plain, are discussed in an excellent way by C.R. Neill (1977).

The local scour near the bridge pier is discussed in detail in a review article by Breusers, Nicollet and Shen (1977).

The dominant feature of the flow near a pier is the large-scale eddy structure which can be composed of the horseshoe-vortex system, the wake-vortex system and trailing vortices. Vortex filaments, transverse to the flow in a two-dimensional undisturbed velocity field, are concentrated by the presence of a blunt-nosed pier to form the horseshoe-vortex system. The mechanism by which the concentration is accomplished is the pressure field induced by the pier. If the pressure field is sufficiently strong, it causes a three-dimensional separation of the boundary layers which, in turn, rolls up ahead of the pier to form the horseshoe-vortex system.

A pier developing such a vortex system is called a blunt-nosed pier (Fig. 9.1).

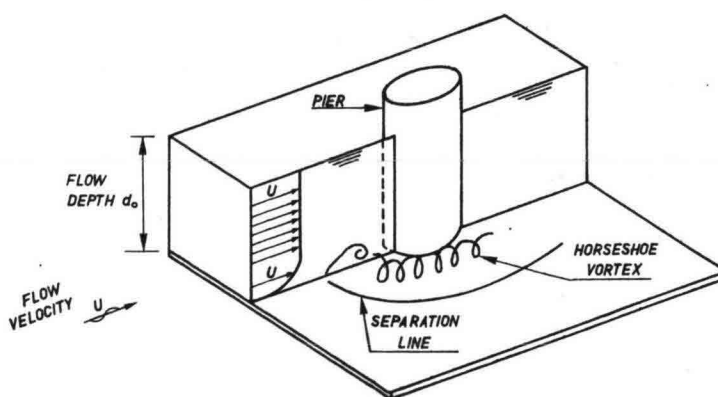


Fig. 9.1



Scour around piers starts at a velocity equal to about  $0.5 \bar{U}_c$  ( $\bar{U}_c$  is the critical mean velocity for beginning of motion of sediment). Scour then increases with velocity until  $\bar{U} \approx \bar{U}_c$  and remains practically constant thereafter. For  $\bar{U} > \bar{U}_c$  variations in scour depth due to approaching bed forms occur but the average depth is constant due to an equilibrium between sediment discharged into the scouring hole and the amount of eroded sediment (see Fig. 9.3, taken from Chabert and Engeldinger).

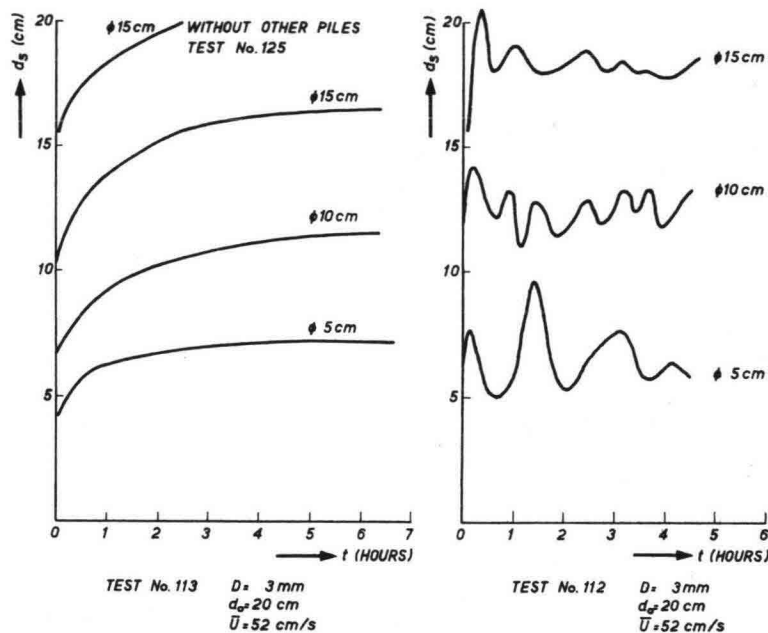


Fig. 9.3 Scour as a function of time  $\bar{U} < \bar{U}_c$ .

Scour as a function of time  $\bar{U} > \bar{U}_c$ .

Scour depth  $d_s$  increases with the initial river water depth  $d_0$  until the water depth/pile diameter ratio becomes larger than 2. For larger ratios scour depth  $d_s$  only depends on the pile diameter  $b$ . Grain size of the bed material has a relatively minor influence, but cohesive sediments give a smaller scour depth. Important factors are the pile shape and the angle of attack for longer piers (Fig. 9.4, 9.5).

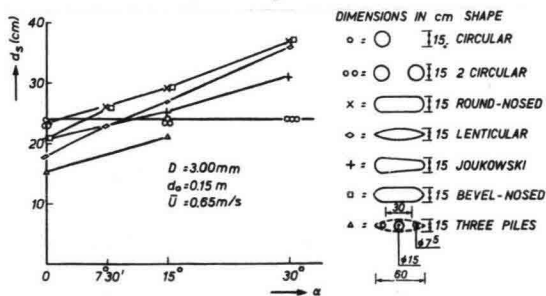


Fig. 9.4

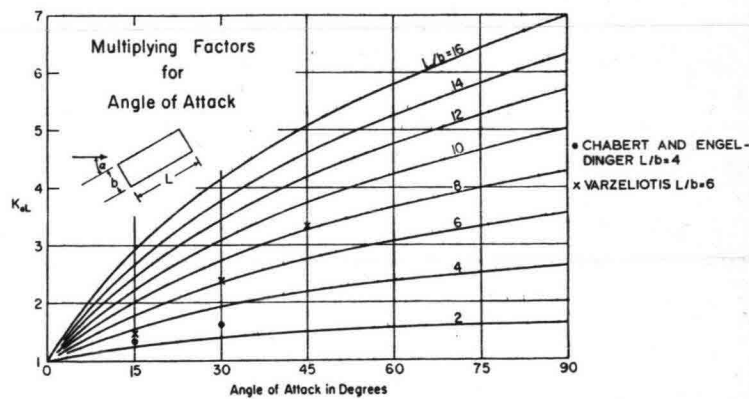


Fig. 9.5

Observations on railway bridges in India showed that scour depth could be related also to the Lacey regime depth

$$d_s \approx d_{r,3} = 0,473 (Q/f)^{1/3}$$

$Q$  = discharge ( $m^3/s$ )

$f$  = silt factor =  $1,76 D_{50}^{1/2}$        $D_{50}$  in mm.

For practical application the following design relation is given by Breusers, Nicollet and Shen (1977):

$$\frac{d_s}{b} = f_1\left(\frac{\bar{U}}{\bar{U}_c}\right) \cdot f_2\left(\frac{d_o}{b}\right) \cdot f_3(\text{shape}) \cdot f_4\left(\alpha, \frac{\ell}{b}\right)$$

$d_s$  = scour depth (below original river bed)

$b$  = width of pier

$\bar{U}$  = mean velocity

$\bar{U}_c$  = critical mean velocity for beginning of motion

$d_o$  = water depth

$\alpha$  = angle of attack

$\ell$  = length of pier

$$\begin{aligned} f_1\left(\frac{\bar{U}}{\bar{U}_c}\right) &= 0 && \text{for } \frac{\bar{U}}{\bar{U}_c} \leq 0.5 \\ &= 2 \left(\frac{\bar{U}}{\bar{U}_c} - 0.5\right) && \text{for } 0.5 \leq \frac{\bar{U}}{\bar{U}_c} \leq 1 \\ &= 1.0 && \text{for } \frac{\bar{U}}{\bar{U}_c} \geq 1.0 \end{aligned}$$

For most practical situations:  $\bar{U}/\bar{U}_c > 1.0$  so that  $f_1(\bar{U}/\bar{U}_c) = 1.0$

$f_2(d_o/b)$  is given as:

$$f_2(d_o/b) = 2.0 \tanh(d_o/b)$$

in which

$$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

For large  $d_o/b$ :  $f_2 \approx 2$

$f_3(\text{shape}) = 1.0$  for circular and round-nosed piers

$= 0.75$  for stream-lined piers

$= 1.3$  for rectangular piers

For  $f_4(\alpha, \ell/b)$ , see Figure 9.5.

If the river bed around the pier is protected with a revetment, then it should be placed at or below the lowest river bed level to avoid an extra obstruction. The stone size should be designed for a velocity 2.0 times the approach velocity  $\bar{U}$  to account for the increase in velocities near the pier. For a preliminary design the following relation can be used:

$$\bar{U} = 0.42 \sqrt{2g\Delta D}$$

$D$  = stone size.

### 9.3 SCOUR DOWNSTREAM OF CONSTRUCTIONS

The construction of a dam or a weir in a river changes the transport conditions and causes local scour. In literature several approaches can be found:

#### 9.3.1 Relations for the equilibrium scour depth downstream of weirs etc.

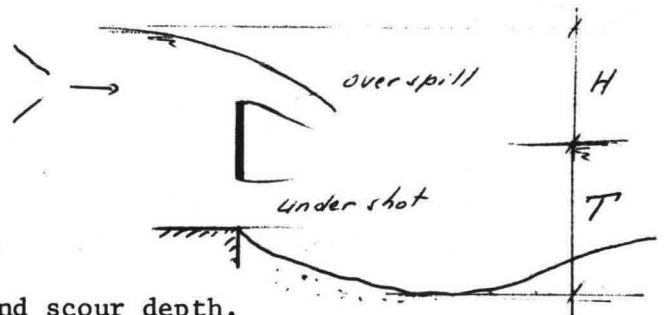
These relations were derived mainly for coarse material ( $d > 1$  mm). Examples are the relations given by Eggenberger and Müller [1]

overspill:  $T = 22.8 H^{0.5} q^{0.6} D_{90}^{-0.4}$

undershot:  $T = 10 H^{0.5} q^{0.6} D_{50}^{-0.4}$

$T$  and  $H$  in m,  $q$  in  $\text{m}^2/\text{s}$ ,  $D_{90}$  in mm

$T$  = sum of downstream waterdepth and scour depth.



or by Kotoulas [2]

$$T = 1.9 g^{-0.35} H^{0.35} q^{0.7} D_{95}^{-0.4}$$

$D_{95}$  in m



### 9.3.2 Relations for rivers with fine sand bed, based on the regime theory (Blench, 1957)

The starting point is the regime depth  $d_r$  for example the Lacey expression:

$$d_{r,3} = 0.473 (Q/f)^{1/3} \quad (\text{m or ft-units})$$

$$Q = \text{total discharge} \quad (\text{m}^3/\text{s} \text{ or } \text{ft}^3/\text{s})$$

or if the flow is limited in width:

$$d_{r,2} = 1.34 q^{2/3} .f^{-1/3} \quad (\text{m-units})$$

$$q = \text{discharge per m'} \quad (\text{m}^2/\text{s})$$

$$f = \text{siltfactor, sometimes given as } 1.76D^{0.5} \quad D \text{ in mm}$$

The total scoured depth T (sum of original waterdepth and scoured depth) is than taken as a multiple of the regime depth:

for scour near bridge piers	$T = 2 d_r$
for scour at nose of spur dikes and guide banks	$T = 2 \text{ to } 2.75 d_r$
for flow perpendicular to banks	$T = 2.25 d_r$
downstream of barrages with hydraulic jump on the stilling-basin floor	$T = 1.75 \text{ to } 2.25 d_r$

### 9.3.3 Time-dependent relations for scour in fine-sand estuaries

For several practical problems, the equilibrium scour depth is not of interest because the situation in which scour occurs is only of a temporary character. Examples are closure works in tidal channels in which scour has to be considered only during the construction phases.

Interpretation of model tests requires in this case the knowledge of the time scale of the scouring process. The Delft Hydraulics Laboratory developed relations based on a large number of tests (see Breusers (1967), Vinjé (1967), van der Meulen and Vinjé (1975)).

During the closing of an estuary situations will occur with a greatly reduced cross section whereas the tidal discharges remain very large. This means that the mean velocity in the closing gap and the turbulence strongly increase which gives an increase in scour depth. Especially methods in which an estuary is closed from the sides (for example with caissons) will have an enormous scouring potential (see Figures 9.6 and 9.7). The scour depths can be reduced by making bottom protections on both sides of the closing gap but scour will always occur. If the sand bed is loosely packed, flow slides can be triggered by the local scour.



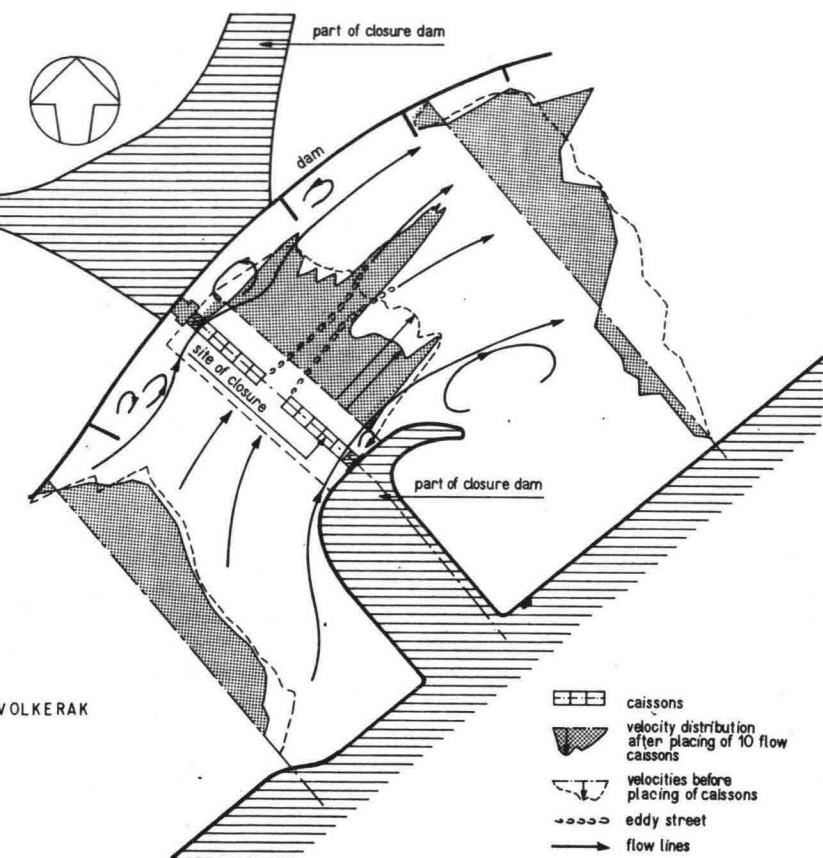


Fig.9.6 Velocity distribution in a closure gap.

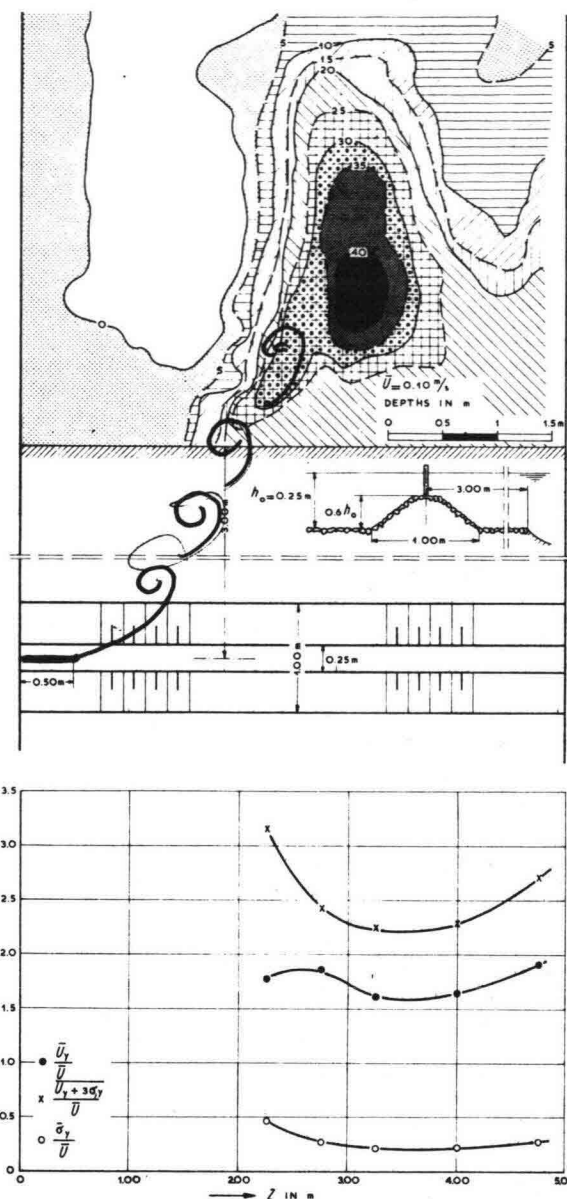


Fig.9.7 Scouring-pattern after 10 hours model. Dam-height/water-depth = 0.6.

The general conclusion of the studies by the Delft Hydraulics Laboratory on local scour was that for a given flow field, independent of the bed material the scour could be expressed as a unique function of time:

$$\frac{h_{\max}}{h_0} = f\left(\frac{t}{t_1}\right)$$

$h_{\max}$  = scour depth (measured from the original bed level)

$h_0$  = original water depth

$t_1$  = time to reach  $h_{\max} = h_0$

The independence of bed material and flow velocity is shown in Fig. 9.8.

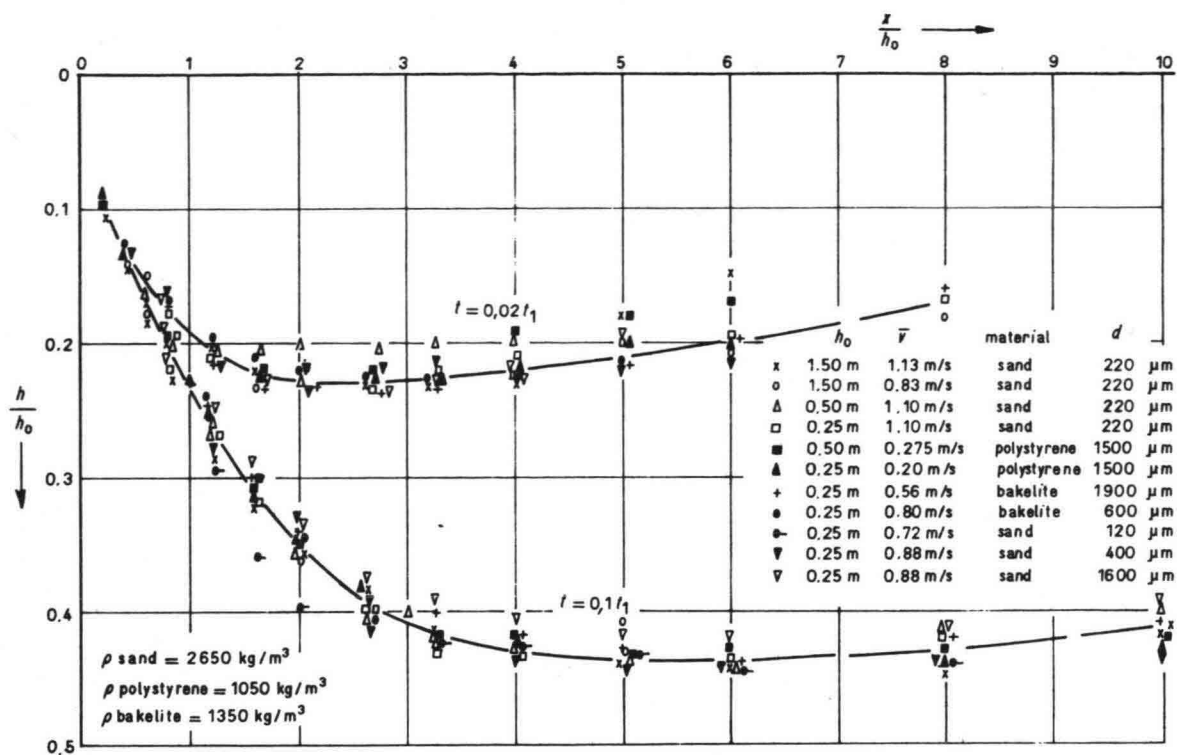


Fig. 9.8 Comparison of scour profiles for scour downstream of a rough horizontal bed.

For two-dimensional scour it was found that:

$$\frac{h_{\max}}{h_0} = \left(\frac{t}{t_1}\right)^{0.38}$$

but for other (three-dimensional) situations other relations apply (see for example Figure 9.9). These figures also show that the relationships are independent of bed material and waterdepth for a given geometry.

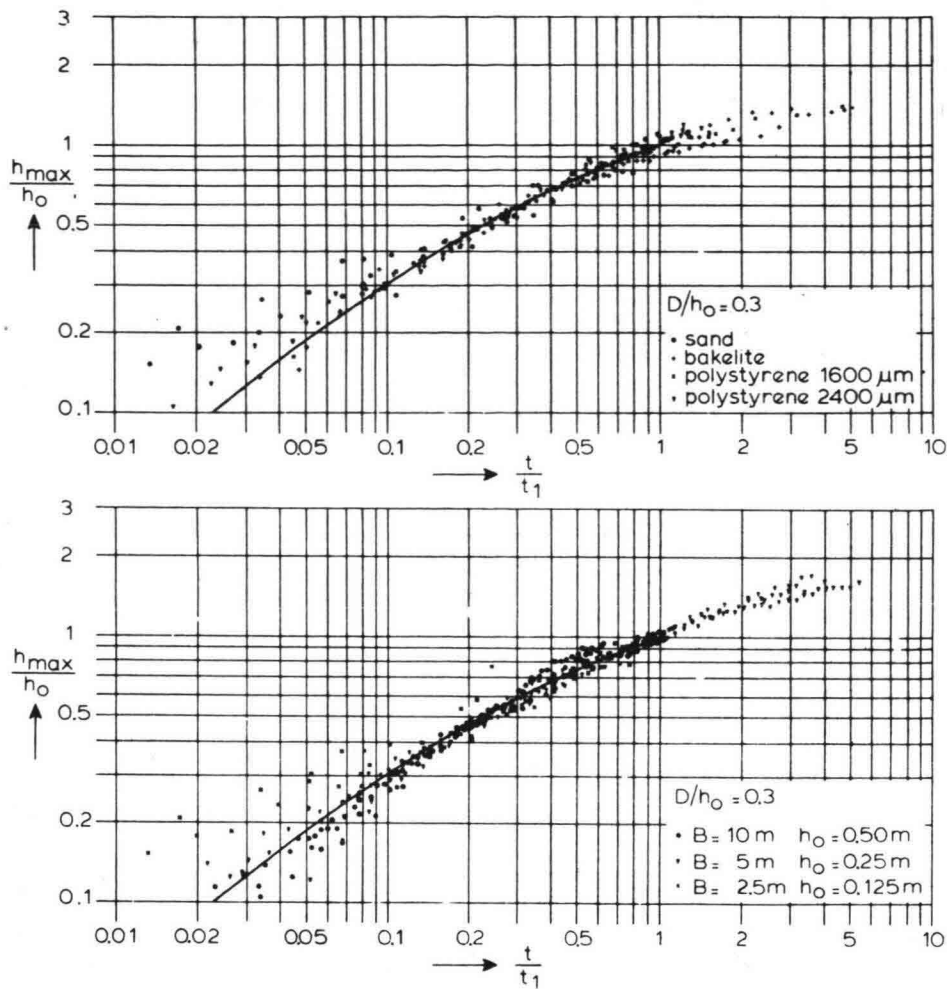


Fig. 9.9 Scour downstream of a dam (see Fig. 9.7)

Important is the time scale of the process, or the scale of  $t_1$ . For all tests, both two and three-dimensional, the following relation was valid:

$$\eta_{t_1} = \eta_{\Delta}^{1.7} \cdot \eta_h^{2.0} \cdot \eta_{(\alpha \bar{U} - \bar{U}_{cr})}^{-4.3}$$

$\eta$  = scale factor (prototype/model)

$\Delta$  =  $(\rho_s - \rho_w)/\rho_w$

$h$  = waterdepth

$\alpha$  = factor, depending on flow field and turbulence. For uniform, two-dimensional flow  $\alpha = 1.5$ , whereas for very turbulent three-dimensional flow situations  $\alpha$  can be as high as 6 - 8 (see Van der Meulen and Vinjé, 1975)

$\bar{U}$  = mean velocity at the end of the bed protection

$\bar{U}_{cr}$  = critical mean velocity for beginning of motion.

This relation has been proven to be valid also for predictions of scour under prototype conditions.

The time scale  $\eta_{t_1}$  for the scouring process is different from the hydraulic time scale  $\eta_t = \eta_L \cdot \eta_U^{-1}$ .

All relations given above were for cohesionless materials. In fact only a limited number of experiments have been performed for cohesive soils. The scouring resistance of clay is of course larger than for sand. No general relations can be given however.

#### 9.4. SCOUR AROUND ABUTMENTS AND SPUR DIKES

For these types of constructions no general design rules may be given, except the general relations of the regime theory. The actual scour depends too much on the geometry of the construction and the flow field. Some references are given below:

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Proc. ASCE 99 (HY12), 1973.

C.R. NEILL (ed.). Guide to bridge hydraulics.  
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M.A. GILL. Erosion of sand beds around spur dikes.  
Proc. ASCE 98 (HY9), p. 1587/1602, 1972.

L. VEIGA DA CUNHA. Erosões localizadas junto de obstáculos salientes de margens.  
Diss. Lisboa, 1971.

#### 9.5. MODEL INVESTIGATIONS

For model studies on the equilibrium scour depth the following scale laws have to be considered:

- a. undistorted model  $\eta_L = \eta_h$
- b. Froude law  $\eta_{\bar{U}} = \eta_h^{\frac{1}{2}}$  in view of the necessary reproduction of the free surface.
- c.  $\eta_{u_x} = \eta_{u_{xcr}}$  to obtain a correct reproduction of the equilibrium conditions in the scour hole.

The third law reduces to the simple law:

$$\eta_D = \eta_L$$

if the bed material in the prototype is so coarse that the model material is larger than 1 mm. If the model material becomes finer, deviations from this simple relation occur due to the influence of viscosity (Shields curve).

If the material in the prototype is already fine, one cannot fulfill all scale relations using sand in the model, so that materials with a lower density have to be used. In that case also the time scale of local scour can be of importance (see par. 9.3.3).

#### 9.6. PROTECTION AGAINST SCOUR

Scour can be reduced by streamlining the construction (bridge piers), making guide walls (abutments) or by stilling basins (spillways). If the resulting scour is not acceptable a bottom protection has to be constructed. Except for the circular bridge piers no general design rules can be given because the necessary protection depends too much on the actual geometry, the composition of the bed etc. A minimum requirement is of course that the upper part of the protection is stable against the flow and that the filter construction is sufficient to prevent leaking of sand through the protection. Special care has to be given to the end of the protection where undermining has to be avoided.

Both stability and filter construction are discussed in the lecture notes on "Revetments". See also Par. 4.6.

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| BREUSERS, H.N.C.,<br>NICOLLET, G.,<br>SHEN, H.W. | 1977 | Local scour around cylindrical piers.<br>Journal of Hydraulic Research <u>15</u> (3), p. 211-252.                               |

## 10. MEASUREMENT TECHNIQUES

### 10.1 Introduction

Besides computations on sediment transport also measurements are necessary for verification of theories, good description of a river and of consequences of changes in regime. The existing techniques can be divided into two groups:

- measurement with samplers
- tracer techniques.

### 10.2 Measurements with samplers

Due to the difference in mechanism of bed-load and suspended load there are different samplers for each type of transport.

#### 1. Bed-load samplers

Variations in bed-load transport and the influence of bed forms will give large variations in results with samplers. For a significant value of the bed load a large number of observations has to be taken. Bedforms can give variations in local transport rate of  $0 - 2 \bar{s}$  (zero in the trough of a sand wave and  $2 \bar{s}$  at the crest of the wave). It is therefore advised to take many samples at various locations instead of taking long sampling times.

Further problems with bed-load samplers are:

1. It is difficult to give the equipment a correct vertical and horizontal alignment with the bed.
2. The meter should be calibrated to determine the efficiency which is also a function of the amount of material caught.
3. It should be avoided that the sampler collects bed material during the lowering of the instrument.
4. A sampler disturbs the flow field. Scour or a decrease of velocity in the sampler can occur

Most samplers are of the box or basket type and consist of a pervious container. Water and sediment enter the sampler; the sediment is caught. As an example figure 10.1 is given. This sampler is used extensively on Dutch and other rivers for  $D > 0.4 \text{ mm}$ . The Helley-Smith sampler (Benedict 1979) is similar.

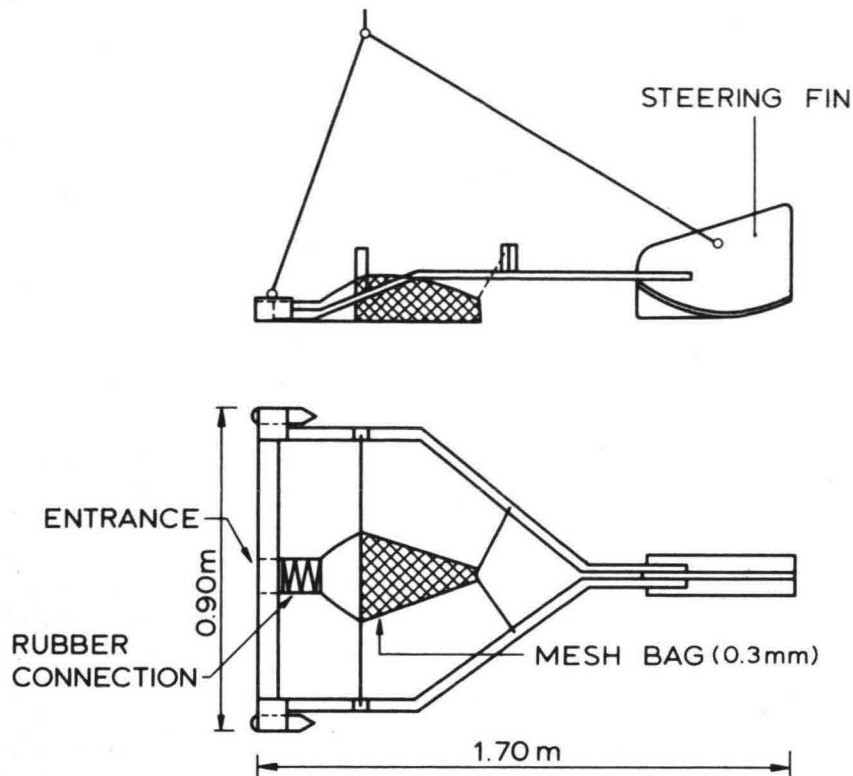


Fig. 10.1 Arnhem sampler (BTMA)

## 2. Suspended load samplers

The amount of material caught depends on the hydraulic coefficient ( $U$  inside/ $U$  outside) and the efficiency (% of material caught) of the instrument. Both factors have to be determined by calibration. The following points should be considered:

1. Suspended load shows large fluctuations so that repeated sampling is necessary.
2. Suspended sediment sampling also includes part of the wash load.
3. If only concentration is measured also velocity profiles have to be measured.

There are two types of samplers:

- a) concentration samplers: a value of water is sampled at a certain level or as an average over the depth. (Nansen bottle, mouse trap, depth integrating samplers).



b) suspended-load samplers: suspended-load (U.C.) is measured at a point or by integration over the depth.

For an example see figure 10.2 (Delft bottle). The bottle acts as a sand trap so that most of the material is caught. Another type of suspended load sampler is the US-P61, where a bottle is filled with a velocity approximately equal to the flow velocity (Fig. 10.3).

- 1 STRAIGHT NOZZLE
- 2 DIFFUSOR CONE
- 3 FLOW CHANNEL
- 4 SAMPLING CHAMBERS
- 5 WATER OUTLET
- 6 HINGE

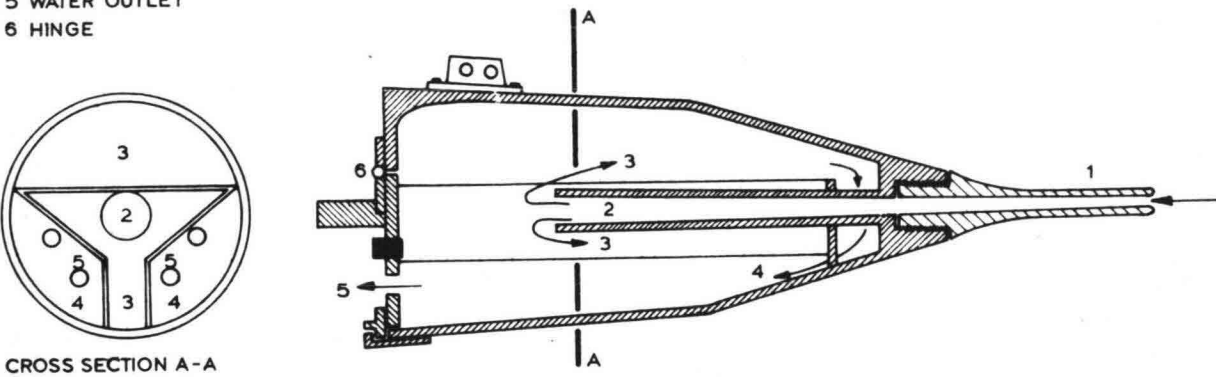
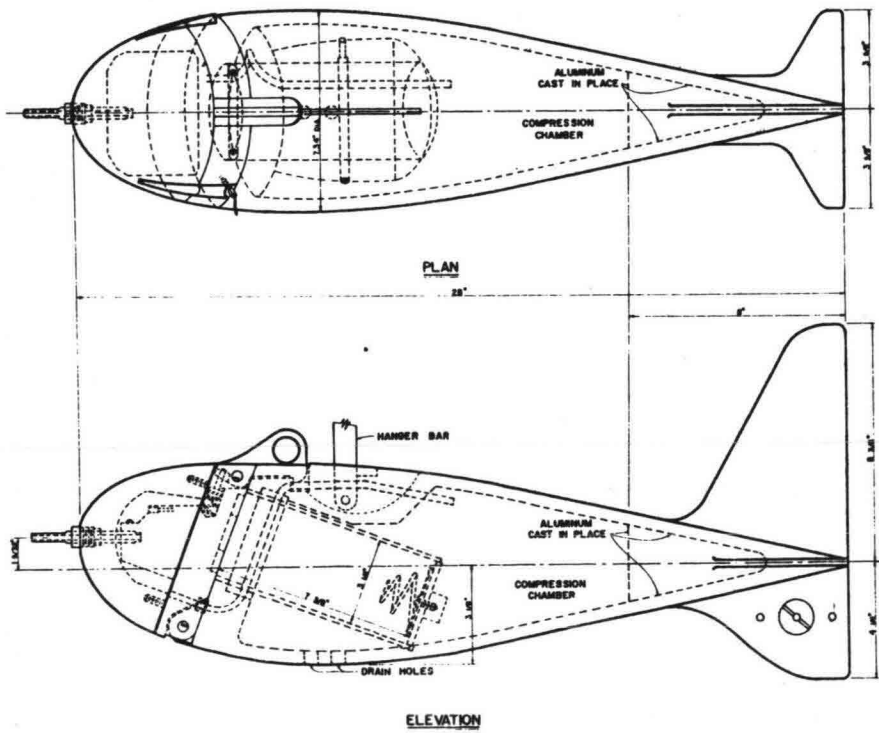


Fig. 10.2 Delft bottle

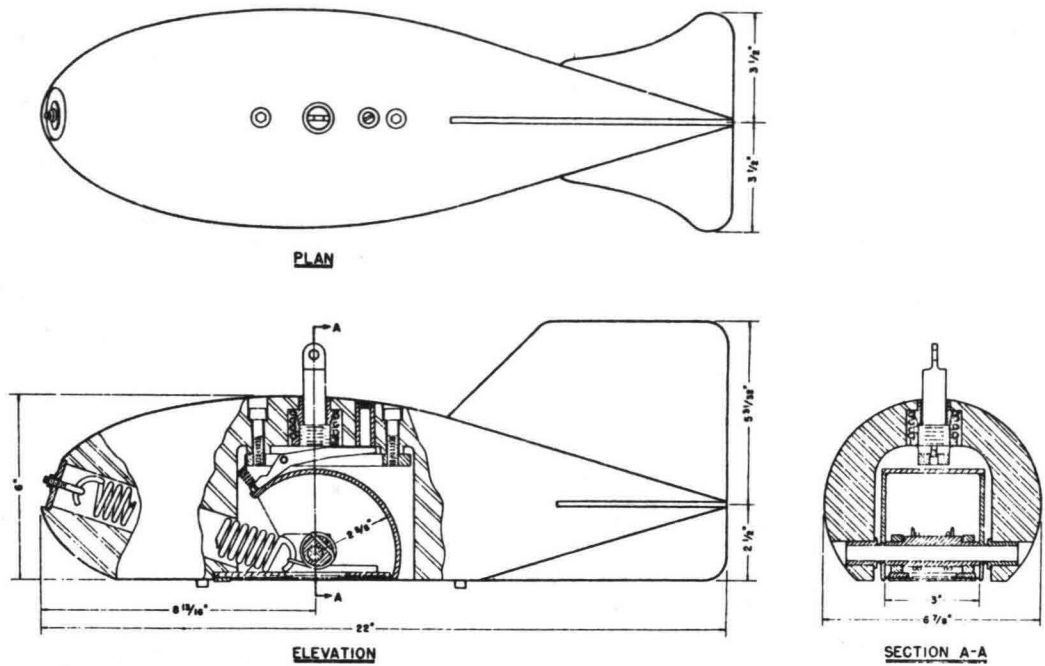


—US P-61 POINT-INTEGRATING SUSPENDED-SEDIMENT SAMPLER

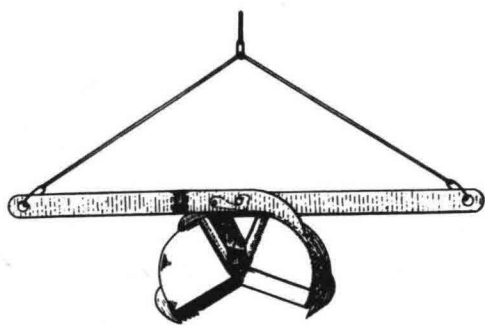
Fig. 10.3

### 3. Bed-material samplers

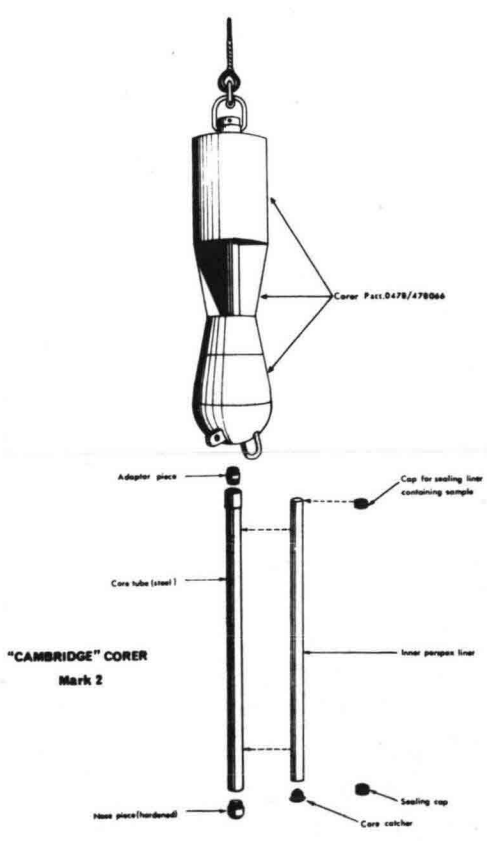
Numerous devices are described in literature to collect bed-samples. Several types may be distinguished: grabs, corers etc. For some examples, see Fig. 10.4



—US BM-54 BED MATERIAL SAMPLER



"VAN VEEN"  
GRAB



"CAMBRIDGE" CORER  
Mark 2

Fig.10.4 Bed-material samplers.

### 10.3 Tracer techniques

For sediment transport measurements also tracers can be used. Grains are marked so that their transport characteristics are not changed, are added to the flow in small quantities and their displacements are determined. From their displacements the transport can be computed.

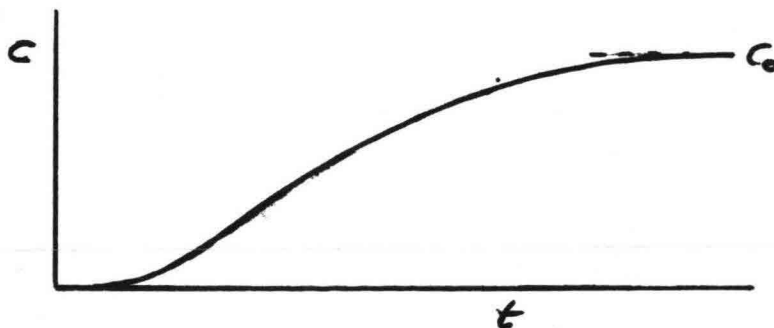
Several types of tracers are used:

1. Fluorescent (luminofores). Marked grains can be detected after sampling under U.V. light. Different types can be used simultaneously.
2. Radioactive. Natural sand is provided with a coating with radioactive material. Disadvantage: public health is important because relatively large quantities are necessary to remain above the back-ground level of radioactivity. Advantage: detection in situ.
3. Activation analysis. Particles are marked and radiated after sampling. Difficult procedure; applied for silt.

Several techniques for the interpretation are used:

1. Constant injection method. A constant amount of tracer material (rate  $\tau$ ) is distributed over the profile and injected during a long time-interval. At a downstream cross section samples are collected and concentration as a function of time is determined. After some time the concentration becomes constant =  $C_0$ . Then the rate of transport can be computed from the relation:

$$s = \tau / C_0$$



2. Point-injection method. At a certain time an amount of tracer material is injected. At several downstream locations concentration is determined as a function of distance. From the displacement of the centre of gravity of the concentration-distance curves the average transport velocity can be computed. Multiplication with the effective depth of transportation  $\delta$  gives the rate of transport. The effective depth is of the order of half the height of the bed forms and can also be determined by sampling

in the bed. Both methods are relatively inaccurate.

Problems with these techniques are the length of the measuring interval, the fact that the external conditions have to be constant and the large number of observations. Some of these restrictions can be diminished by applying "dispersion methods". The data are compared with a theoretical dispersion model (see de Vries, 1966).

For a review of existing measuring techniques see also Jansen (1979). Development in this field is slow. For some recent developments see Anon. (1976).

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## 11. SEDIMENT TRANSPORT IN PIPES

### 11.1 Introduction

Sediment transport in pipes is of importance in the field of sewage transport and dredging. In both cases the purpose of the system is to transport solids without deposits at minimum head losses. Important aspects of this way of transport is the prediction of head losses and of minimum (or critical) velocities to avoid deposition.

In pipe transport several modes of transport can be distinguished:

1. for fine sediments and high velocities a pseudo-homogeneous suspension is formed. The criterion for a fully developed suspension was  $U^x/W > 10$ . For  $\bar{U}/U^x \approx 20$  it follows that  $\bar{U}/W > 200$ . This means in practice ( $\bar{U} = 3-4$  m/s) a value of  $W < 2$  cm/s or  $D < 150$   $\mu$ m.
2. heterogeneous suspension. For smaller velocities and coarser material a heterogeneous suspension is formed with a strongly non-uniform concentration distribution.
3. Sliding bed regime. For very coarse material all sediments will be transported sliding along the pipe wall. The criterion for beginning of suspension was  $U^x/W > 1.5$ . It follows then for  $\bar{U}/W > 30$  or  $W > 10$  cm/s,  $D > 1$  mm.

Thus:

$D < 150$   $\mu$ m    pseudo-homogeneous suspension  
 $150$   $\mu$ m  $< D < 1$  mm    heterogeneous suspension  
 $D > 1$  mm    sliding bed

In practice transition zones between the various regimes will be found.

Typical head-loss curves for pipe-line transport of sediments are given in figure 11.1. For large velocities losses approach the loss for clear water, but at the critical velocity where deposition starts head-losses strongly increase. The difference between head-losses for the mixture and clear water increases linearly with concentration  $c$ .

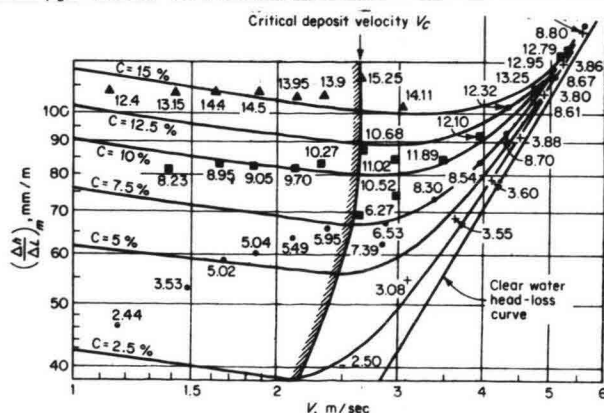


Fig. 11.1 Head loss vs. velocity relationship with equiconcentration lines, for sand graded to 0.44 mm. [After CONDOLIOS et al. (1963).]

### 11.2 Critical velocity

The critical velocity is defined as the velocity where stationary deposits are formed at the bottom of the pipe, or by deposition from the suspension or because part of the sliding material comes to rest. This condition is very critical for the operation of the pipe line because around the critical velocity head losses are at a minimum and a decrease in velocity will give an increase in head loss.

A large number of data and formulas are given in literature. The best known are the results by Durand (1953) (see figure 11.2, where

$$F_L = \bar{U}_{crit} / \sqrt{2g\Delta(2a)} \text{ and } a = \text{pipe radius}).$$

From experiments it follows

that the critical velocity  $\bar{U}_{crit}$  increases with  $(2a)^{\frac{1}{2}}$ , slightly increases with

transport concentration  $C$  (up to volume concentrations of 15%) and increases with grain size  $D$  up to  $D \approx 1$  mm.

The values given by Durand are somewhat pessimistic for fine sand in large diameters; here and increase with a smaller exponent ( $1/4$  to  $1/3$  instead of  $1/2$ ) gives better results. For more information see the literature.

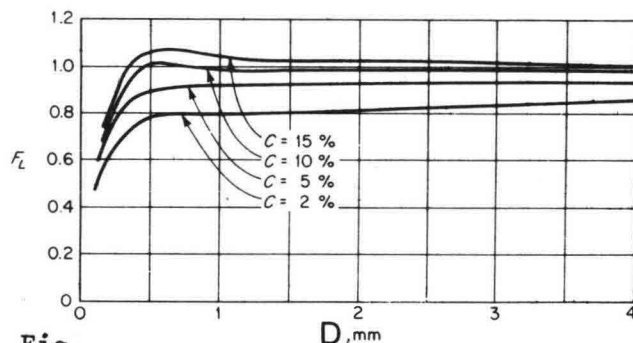


Fig.

11.2  $F_L$  value vs. particle diameter, concentration as parameter.  
[After DURAND et al. (1956).]

### 11.3 Head losses

Most (empirical) relations for the head losses in sand-water mixtures have the form:

$$I_m = I_w + C.f(\bar{U}, 2a, \text{grain characteristics})$$

$I_m$  = hydraulic gradient for the sand-water-mixture

$I_w$  = hydraulic gradient for clear water

$$I_w = \frac{\lambda}{2a} \cdot \frac{\bar{U}^2}{2g}$$

For  $\lambda$  smooth-pipe values can be used, because the sand polishes the surface of the pipe. For  $I_m$  various relations are proposed for example:

Durand (1953):

$$\phi = \frac{I_m - I_w}{C \cdot I_w} = 176 \left[ \frac{g \cdot 2a}{U^2} \cdot \frac{W}{\sqrt{gD}} \right]^{3/2}$$

(see figure 11.3,  $v_{ss} \rightarrow W$   $D \rightarrow 2a$   $d \rightarrow D = \text{grainsize}$ )

Führbörter (1961):

$$\frac{I_m - I_w}{C} = \frac{S_{kt}}{U}$$

in which  $S_{kt}$  is a parameter depending on grain size

(see figure 11.4,  $d \rightarrow \text{grainsize}$ ,  $D \rightarrow \text{pipe diameter}$ ).

From experience with dredge pipe-lines it appears that the relation given by Führbörter is suited for fine sand and large pipe diameter and that Durand's equation is good for smaller pipe diameters and coarse sand.

For further information see literature (par. 11.4).

FLOW OF SOLID-LIQUID MIXTURES IN PIPES

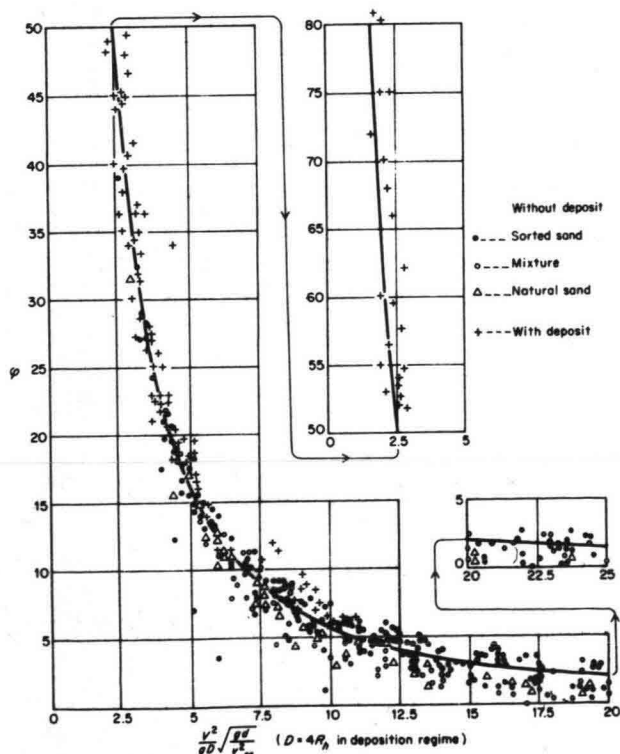


Fig 11.3 Head loss as given by the Durand-Condolios relation for sand ( $s = 2.65$ ); SOGREAH data are used. [After GIBERT (1960).]

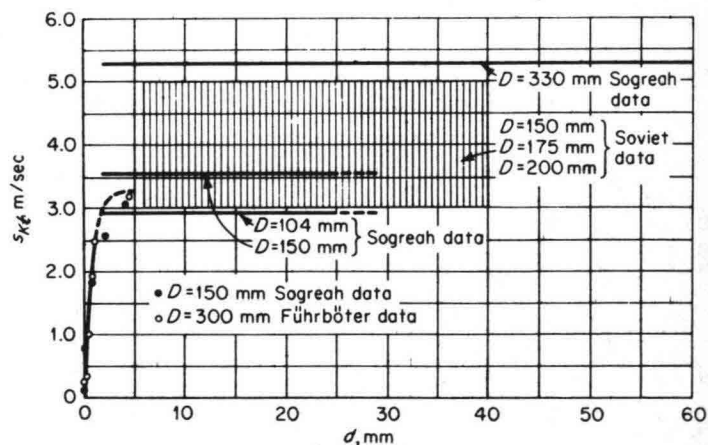


Fig.11.4 Relationship of the sediment constant  $s_k$ . [After FÜHRBÖTER (1961).]



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