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Scheduling multimodal alternative services for managing infrastructure maintenance possessions in railway networks

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ABSTRACT

Highly utilized railway networks require regular infrastructure maintenance. Different track sections often need to be closed for entire days to carry out engineering works, which makes the regular timetables no longer feasible and thus adjusted railway services and temporary alternative services need to be planned. We introduce the Multimodal Alternative Services for Possessions (MASP) problem to support the planning of alternative services, from the passenger and transport operator points of view, including an adjusted train timetable, bus-bridging services and extra train services. The MASP problem is formulated based on the Service Network Design Problem and the Vehicle Routing Problem. To solve it efficiently, we develop a solution framework that incorporates heuristics based on the column and row generation with mixed-integer linear programming. The developed framework provides the optimized alternative service routes, schedules and passenger flows routing. We demonstrated the performance of the MASP solution framework on the real-life Dutch railway network. The results show that the MASP framework is capable of efficiently generating alternative services to route passenger flows affected by possessions with a very limited increase in the total passenger costs compared to a scenario with no link closures. High computational efficiency is observed even for highly disrupted networks.

1. Introduction

1.1. Background

Railway networks form the backbone of the transport systems in most urban areas worldwide. As part of the public transport system, railways provide high capacity and speed, and they are in general the most efficient mean of transport. In the Netherlands, the national railway operator Netherlands Railways (NS) transported 1.2 million passengers per day in 2015 (NS, 2016) and the Dutch railway transport is the second busiest system in Europe. Highly utilized railway systems require regular maintenance to minimize the effects of failures that can cause traffic disruptions and thus keep a high level of service. However, maintenance involves engineering works that often require closure of tracks or stations that can last from short periods (e.g. one hour) to several days (Looij, 2017). These closures are known as a possession of the railway infrastructure. Possessions are usually planned by the infrastructure manager in cooperation with the engineering companies that execute the works (Looij, 2017). During possessions in railways, the regular timetable may no longer be feasible and therefore a temporarily adjusted timetable is needed to continue serving the passenger demand while minimizing delays, cancellations and short-turnings (Van Aken et al., 2017a,b). Cancellation of a train line happens when a complete corridor is blocked, whereas in short-turning the train runs until the station before a

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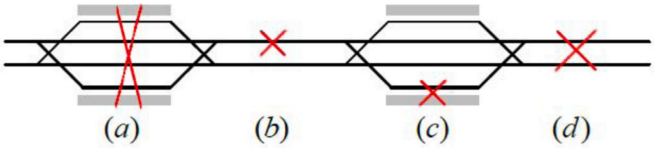

Fig. 1. Possible possessions that can be dealt with on the macroscopic level: (a) complete station closure, (b) partial blockage of open tracks, (c) platform track closure, and (d) complete blockage of open tracks.
Source: Van Aken et al. (2017a,b)

closed track section, where it will turn provided that the station track layout allows for it (Van Aken et al., 2017a). Different possession possibilities are shown in Fig. 1. For example, in the Netherlands, every weekend between 15 to 25 possessions, lasting from multiple hours to the complete weekend, are undertaken. Currently, planners are manually defining adjusted services around the given possessions.

Possessions are usually related to long, planned disruptions due to their inherent characteristics, compared to unplanned disruptions, which usually last for shorter and uncertain periods. Therefore, the design of an alternative timetable for planned disruptions should target the regular timetables. In the Netherlands, the train timetable is based on the basic hour pattern (BHP), which is a clock-face scheduling system (i.e. consistent intervals and departures taking place at the same times during the day). The BHP is usually repeated 18 times each day, with adjustments for peak hours and non-periodic trains such as international services and freight trains. When the BHP is no longer feasible due to possessions, the Alternative Hour Pattern (AHP) takes over the original train timetable.

1.2. Current practice

The alternative services provided by the train operating companies are usually based on two strategies: train short-turning around disrupted areas and bus-bridging services to serve disconnected stations (Shires et al., 2018; Meesit and Andrews, 2019). Railway companies generally subcontract bus services to bus companies, which arrange its operation. These planning problems often results in a complex multi-actor system in which sometimes it is difficult to get a complete picture of the situation. Bus-bridging services are usually scheduled to operate in parallel to the closed sections of a corridor, serving all intermediate stations in the disrupted area (Meesit and Andrews, 2019). These are widely used practices to deal with railway disruptions, both for planned or unplanned disruptions, which are often suboptimal since passenger flows are ignored (Jin et al., 2015). Also, alternative bus services are usually designed ad hoc and thus are unlikely to be an optimal solution for a railway network (Gu et al., 2018). For instance, in the Netherlands this planning is performed manually. Recently, some research has been carried out to develop mathematical models to tackle these specific kinds of planning problems.

Also, currently, in alternative timetables trains operate using originally planned train paths. Train paths are the infrastructure capacity needed to run a train between two places over a given time (European Parliament, 2001), defined by speed, stopping pattern and frequency among other things.

1.3. This paper

The resilience of railway transport systems is defined as the ability to provide effective services in normal conditions, as well as to resist, absorb, accommodate and recover quickly from disruptions (Bešinović, 2020). Bešinović (2020) makes a distinction between two aspects of resilience: proactive and reactive. This research focuses on the preparedness as part of proactive aspect, which involves dealing with actions in advance before certain disruption effects are expected to take place, and contributes to the system resilience.

In this research, we introduce the Multimodal Alternative Services for Possessions (MASP) model to design an optimal adjusted transport plan, from a passenger and transport operator perspective, given a set of possessions in the railway network. We develop a multimodal transport plan that includes (1) the adjustment of the original train services, (2) new extra train services and (3) bus-bridging services, defined in an hourly timetable. We model the railway infrastructure at the macroscopic level. Used possessions are complete closures of links during a complete period. Passenger perspective is addressed by modelling minimal travel costs. As a result of possessions, the regular train timetable has to be adapted, and the disconnected passenger flows, which look for their alternative paths over the network in terms of travel cost, have to be rerouted via bus-bridging services or alternative train services. The timetable adjustment considers four possible measures: train cancellation, rerouting, retiming and short-turning. We test the MASP model in a real-life case study in the Dutch railway network.

The contributions of this research are the following:

- A new mathematical model formulation to design alternative services for managing possessions in railways.
- The first work integrating the train timetable adjustment problem and the bus-bridging problem.
- An extended multimodal approach to the traditional bus-bridging problem including bus-bridging services and extra train services.

- A solution framework to the MASP model combining multi-column and row generation techniques to efficiently manage (1) the number of passenger paths, (2) the number of possible combinations for alternative services and (3) the number of headway constraints.
- A real-life case study in the Dutch railway network.

The remainder of the paper is structured as follows. Section 2 provides an overview of related research carried out in the field of alternative services. Section 3 presents the MASP problem description and mathematical formulation. Section 4 presents the solution framework used to solve the MASP problem. Section 5 gives the experimental analysis on a real-life case study on the Dutch railway network and the discussion of the results is given in Section 6. Finally, the conclusions of the research are presented in Section 7.

2. Literature review

2.1. Planned disruptions in railways

We distinguish between planned and unplanned disruptions, i.e. possessions. Unplanned disruptions comprise those sudden disruptions that are not expected by the train operator (and neither by passengers), such that alternative plans have to be developed in real-time during operations. In practice, unplanned disruptions are evaluated in continuous time. For an extensive review of unplanned disruptions, refer to Cacchiani et al. (2014) and Ghaemi and Goverde (2015). Planned disruptions, by contrast, (1) usually take longer, (2) the duration is generally defined, (3) are known in advance so that alternative plans are designed at the tactical planning level, and (4) passengers are typically aware of the effects in advance and may adapt their travel choice behaviour accordingly. Since planned railway possessions can last from multiple hours to several days to allow for heavy engineering works, it is sufficient to consider an original hourly timetable and then generate an adjusted hourly timetable. Then, such created adjusted periodic timetable can be replicated throughout the possession period.

Shires et al. (2018) reviewed the impacts of planned disruptions in railways on passenger demand. They noted that bus-bridging services are usually an inferior alternative compared to train diversions due to higher costs, but it is often inevitably the alternative chosen to be implemented. Focusing on travel behaviour, the authors make a distinction between immediate behavioural responses and long-term effects. In planned disruptions, passengers are usually aware in advance of their effects and adapt their travel choice behaviour accordingly. Some passengers might move the other means of transport, but most users (60 to 80%) will choose the modified train services (Shires et al., 2018).

2.2. Optimization approaches to timetable adjustments during disruptions

System-based mathematical optimization models have been extensively used in literature. They can provide feasible solutions much faster and more efficiently than with manually generated timetables (Van Aken et al., 2017a). Also, they have proved to be powerful to tackle combinatorial complexity rising from multiple simultaneous disruptions in comparison to topological and simulation approaches (Bešinović, 2020). For solving railway timetable (adjustment) problems, microscopic or macroscopic models can be used. The former consider detailed infrastructure including station layouts, blocks and signalling and rolling stock characteristics, while the latter consider stations as nodes and open tracks as arcs.

Vansteenwegen et al. (2016) proposed a microscopic approach that is only applicable to small networks due to the high complexity of network details. Brucker and Knust (2002)proposed an approach to the train rescheduling problem given a possession consisting of the closure of one track in a double track section, using local search techniques. Furthermore, Arenas (2017) proposed a formulation to adjust the timetable which takes into account the circulation of maintenance trains and temporary speed restrictions, and its microscopic representation guarantees the operational feasibility of the produced timetables.

Veelenturf et al. (2016) proposed a macroscopic real-time rescheduling approach to manage large-scale disruptions, including transitions from and to the original timetable and considering a cyclic timetable. The timetable rescheduling problem considers retiming and cancellation of trains to minimize the deviation from the original timetable.

Louwerse and Huisman (2014) also proposed a similar approach, with a formulation based on event–activity networks. Possible disruption measures to adjust the timetable include delaying trains, cancelling trains and reversing trains at stations adjacent to the blockade.

Van Aken et al. (2017a) introduced an approach to solve the Train Timetable Adjustment Problem (TTAP) at a large scale, which finds, for a given station and open-track maintenance possessions, an alternative periodic timetable that minimizes the deviation from the original timetable. The TTAP extends on Periodic Event Scheduling Problem (PESP) formulation, introduced by Serafini and Ukovich (1989) and further developed by Schrijver and Steenbeek (1993) and Peeters (2003). The model incorporates a row generation algorithm to add station capacity constraints, which proved to be effective for a low number of possessions due to fewer constraints.

Looij et al. (2020) introduced the Train Routing Adjustment Problem (TRAP), which aims at generating feasible and robust train route plans while minimizing passenger dissatisfaction for a given list of possessions in a station area, possible alternative train routes, and an original regular timetable. Possessions may include any combination of track sections, switches, platforms or open track closures. The robustness of a route plan is increased by using an iterative heuristic approach that adds buffer time.

Bešinović et al. (2020) extended the TTAP model from Van Aken et al. (2017a,b) to incorporate freight traffic and to ensure an optimal mix of both passenger and freight trains in the adjusted periodic timetable. The model creates alternative paths to

Table 1
Main characteristics of literature on optimization approaches for timetable adjustment during disruptions. (*) Includes penalties for cancelled trains, shorter trains, routes with a time shift, extra shunting movements, missed transfers, different platform as in the original timetable, etc. (each with different weights). (**) Includes rerouting and cancellation costs. (***) Includes delays, cancellations, non-commercial stop, etc.

Source	Domain	Level of detail	Disruption type	Objective
Arenas (2017)	Unplanned, mainline	Microscopic	Partial closure of open track and station areas	Minimize deviation from original timetable
Brucker and Knust (2002)	Planned, mainline	Microscopic	Partial closure of double track	Minimize passenger delay
Looij et al. (2020)	Planned, mainline	Microscopic	Partial and complete closure of open track and station areas	Minimize passenger dissatisfaction (*)
Louwerse and Huisman (2014)	Unplanned, mainline	Macroscopic	Partial and complete closure of open track	Maximize level of service
Szymula and Bešinović (2020)	Planned, mainline	Macroscopic	Complete closure of links	Minimize passenger and operator costs (**)
Van Aken et al. (2017a,b)	Planned, mainline	Macroscopic	Partial and complete closure of open track and stations	Minimize deviation from original timetable and cancellations
Vansteenwegen et al. (2016)	Planned, mainline	Microscopic	Partial and complete closure of open track	Minimize maintenance conflicts (main) and spreading costs
Veelenturf et al. (2016)	Unplanned, mainline	Macroscopic	Partial and complete closure of open track and station areas	Minimize cancellations and maximize robustness of adjusted timetable
Bešinović et al. (2020)	Planned, mainline	Macroscopic	Partial and complete closure of open track and station areas	Minimize passenger and freight operators inconvenience (***)
This research	Planned, mainline	Macroscopic	Complete closure of links	Minimize passenger cost and operator costs

reroute freight trains over the network while minimizing the potential negative effects such as delay or cancellation of other trains, non-commercial stops, etc.

Szymula and Bešinović (2020) introduced the Railway Network Vulnerability Model (RNVM) to assess the vulnerability of the system by finding the critical combination of links that cause the most adverse consequences to passengers and trains. An interesting feature of the model is the integration of train operations and passenger flows. The model incorporates column and row generation heuristics, as well as both arc-based and path-based formulations that allow for flexible implementations.

In conclusion, the timetable adjustment problems are usually solved using mixed-integer linear programming models. Existing research proves that the mathematical models allow for fast and efficient assessments, and show useful approaches to model flexible train timetables (under disruption effects), passenger flows and train services among others. Table 1 shows the main characteristics of the papers reviewed in Section 2.2, including domain, level of detail, disruption type and objective.

2.3. *Optimization approaches in bus-bridging scheduling*

Some research has been carried out during the last few years on the optimal design of rail-replacement bus services as part of the alternative plans to deal with railway service disruptions. Yet, most of these studies have focused on unplanned disruptions.

Kepaptsoglou and Karlaftis (2009) developed a model to design bus-bridging services in a metro disruption to minimize the total travel time and unsatisfied demand. The model was based on the shortest path algorithm and was solved using a genetic algorithm. They noted that only a fraction of the demand around 35% could be recaptured by surface alternative services in their Athens metro system case study.

Gu et al. (2018) proposed a strategy to flexibly allocate and schedule buses to predefined bridging routes during a metro disruption consisting of closure of several stations between two short-turning stations. They developed a mixed-integer linear programming model and solved it using a heuristic algorithm.

Jin et al. (2015) proposed an approach to develop efficient alternative services using a column generation procedure to generate candidate bus routes and a path-based multi-commodity network flow-based procedure to identify the most effective combination of candidate routes, aiming at reducing the average travel delay. The authors noticed that the optimal solution may include non-intuitive bus routes and that the solution is sensitive to the time-of-day demand.

Deng et al. (2018) proposed a route generation method with station capacity and resource constraints for urban rail transit disruptions to generate feasible routes between Origin–Destination (OD) pairs and taking into account passenger route choice behaviour. Feasible routes for each affected OD pair include urban rail transit transfer, direct bus-bridging, and indirect bus-bridging.

Luo et al. (2019) introduced an optimization model for the development of efficient bus-bridging services to minimize the negative effects of disruptions in urban rail transit. This work focuses on route selection and bus allocation, whereas the generation

Table 2
Main characteristics of literature on optimization approaches for bus-bridging services. (*) Candidate set routes are given, only does route selection.

Source	Domain	Objectives	Solution method
Deng et al. (2018)	Unplanned, Urban transit	Minimal passenger cost	Route generation, k-short
Gu et al. (2018)	Unplanned, Urban transit	Minimal bus-bridging time and passenger delay	Heuristics (Weight Shortest Processing Time)
Jin et al. (2015)	Unplanned, Urban transit	Minimal passenger travel time	Column generation
Kepaptsoglou and Karlaftis (2009)	Unplanned, Urban transit	Minimal passenger travel time and unsatisfied demand	Shortest path, genetic algorithms
Luo et al. (2019)	Unplanned, Urban transit	Minimal passenger delay and unsatisfied demand	- (*)
Liang et al. (2019)	Unplanned, Urban transit	Minimal passenger cost and operational cost	Column generation
Meesit and Andrews (2019)	Planned, mainline/ commuter	Minimal passenger delay and operational cost	Discrete-event simulation, genetic algorithms
Van Der Hurk et al. (2016)	Planned, Urban transit	Minimal passenger delay	Path reduction method
Wang et al. (2019)	Unplanned, Urban transit	Minimal waiting time, stranded passengers and dispatched buses	(@LNon-dominated Sorting) genetic algorithm
This research	Planned, mainline	Minimal passenger cost and operator cost	Column and row generation

of a candidate set of bus-bridging routes is an input of the model. The model takes into account the travel demand and capacity of existing buses, which are derived from historical smart card data. Short-turning stations are selected based on the availability of crossovers for train turn-back.

Wang et al. (2019) incorporated dynamic passenger flows to the bus-bridging problem, with a multi-objective optimization model to minimize total waiting time, the number of stranded passengers and dispatched vehicles with constraints such as fleet size and vehicle capacity. Trains short-turn around the disrupted areas in the stations where crossovers are available. A Non-Dominated Genetic Algorithm is used to find the Pareto set of solutions.

Liang et al. (2019) introduced a path-based multi-commodity flow formulation to bus-bridging service design with a robust counterpart to incorporate bus travel time uncertainty and solved it using a column generation algorithm. The model formulation incorporates the capacity of regular bus transit lines to avoid designing redundant bus-bridging lines. Schedules of train services in the non-disrupted parts of the system are assumed to remain unchanged. The focus is on minimizing total passenger and operational costs by optimizing passenger flow and bus-bridging line frequency.

Meesit and Andrews (2019) focused on the optimization of bus-bridging services during planned disruptions. This work introduces a model to find optimal bus-bridging services integrated with the simulation of train short-turning as an alternative service plan to minimize the impact of possessions. Train services in the non-disrupted areas are assumed to follow the original timetable. The model is based on a stochastic discrete-event simulation technique and uses a genetic algorithm to minimize passenger delays and the cost of operations. The overall approach proved to be effective but computationally expensive.

Lastly, Van Der Hurk et al. (2016) proposed a model for planned disruptions to select bus-bridging lines and frequencies under budget constraints. The objective is to minimize the passenger inconvenience cost, which depends on the frequency and includes assumptions on passenger choice behaviour. The approach features a path reduction process that helps to increase computational efficiency. Trains short-turn around the disrupted sections and their frequency can be adjusted in the undisrupted areas.

Most of these studies aimed at minimizing passenger delay (Jin et al., 2015; Kepaptsoglou and Karlaftis, 2009), and some of them also operational costs (Borndorfer et al., 2007; Meesit and Andrews, 2019) or bus-bridging time (Gu et al., 2018). Kepaptsoglou and Karlaftis (2009) argue that the focus of bridging bus routes is on maximizing efficiency (partly to build passenger trust) and that minimizing cost is less important, and thus they neglect it. Therefore, their approach is to maximize traveller welfare in terms of capacity and social welfare. Additionally, Kepaptsoglou and Karlaftis (2009) and Van Der Hurk et al. (2016) agree on setting a minimum frequency restriction. The main characteristics of the papers reviewed in Section 2.3, i.e. domain, objectives and solution approach, are summarized in Table 2.

2.4. Research gap

Tables 1 and 2 summarize papers on the train timetable adjustment and the bus-bridging problem, respectively. Given there is no overlap between the two problems, papers were split into two separate tables. On one hand, some scientific research has been carried out to date in the field of bus-bridging services for planned disruptions (e.g. Van Der Hurk et al., 2016; Meesit and Andrews, 2019), while significantly more research has been carried out in recent years focusing on bus-bridging services in unplanned disruptions (e.g. Kepaptsoglou and Karlaftis, 2009; Jin et al., 2015; Gu et al., 2018; Van Der Hurk et al., 2016). Also, most of the papers focus on urban railway environment. The literature on bus-bridging generally does not look at the train timetable adjustment in detail and assume that trains short-turn around the disrupted areas and continue operating in the non-disrupted areas with the regular timetable.

On the other hand, the current research on adjusting timetables during disruptions lacks scheduling alternative services, commonly focuses only on train services and does not consider passenger demand explicitly. Microscopic modelling is typically applied at smaller areas, like stations and smaller corridors (e.g., Arenas, 2017; Looij et al., 2020) due to their high computation demand, while macroscopic is used for network level problems and are most commonly used for railway problems (e.g., Louwerse and Huisman, 2014; Van Aken et al., 2017a,b). In addition, some authors combined microscopic and macroscopic modelling to guarantee operational feasibility for nominal scheduling problems (e.g. Caimi, 2009; Bešinović et al., 2016; Burggraeve and Vansteenwegen, 2017). In this paper, we follow a commonly used macroscopic approach to address network-wide modelling of a railway system. In particular, we combine timetabling adjustment and scheduling extra multimodal services, i.e. extra trains and bus-bridging services, while considering passenger demand.

Finally, despite the fact that bus-bridging services is a widely used practice to manage passenger flows during disruptions in railway networks, most implementations are not designed by means of optimization approaches. Such modelling can be particularly beneficial for dense railway passenger networks such as Switzerland and the Netherlands. Therefore, there is a clear and relevant gap to fill in both from the scientific and societal point of view.

3. Multimodal Alternative Services for Possessions (MASP) problem

3.1. Problem description

This research focuses on the situations in which train services cancelled due to possessions are replaced by alternative services which are coordinated with the adjusted train timetable at the macroscopic level. To tackle this problem, we introduce the Multimodal Alternative Services for Possessions (MASP) problem. It is assumed that a possession lasts for the complete time period, e.g. one hour, or one day. Consequently, the transition from the original service to the adjusted service and the other way around is not considered. Therefore, our approach in this paper focuses on the design of a (steady) alternative hourly timetable, i.e. alternative hour pattern. The MASP problem is organized in three modelling levels: (1) the network infrastructure, (2) the transport service network including the original train services, extra train services and bus-bridging services, and (3) the passenger paths. The notation used in the MASP problem, containing sets, parameters and decision variables is given in Appendix.

The inputs of the MASP problem framework are classified in four elements: (1) infrastructure network configuration, which includes the railway and road network, with their sets of nodes and arcs; (2) train timetable, which includes the regular timetable; (3) passenger demand, which includes the passenger Origin–Destination matrix; and (4) possessions set, which includes all possessions that are scheduled simultaneously. The output of the model is a multimodal adjusted timetable including the routes of the original train services, extra train services and bus-bridging services, and passenger paths and costs.

The railway infrastructure network is modelled using an undirected graph $G^T = (N, A^T)$, where N is the set of nodes and A^T is the set of arcs. Nodes represent stations and major junctions, whereas arcs represent open track sections between nodes. Each arc has a capacity CAP_{ij}, which represents the maximum amount of trains that can be operated in the link during a certain period of time. We define A^d as the set of possessed links. A specific possession is modelled as a complete closure of a link using the input parameter v_{ij}, equal to 1 if the link is closed (i.e. $CAP_{ij} = 0$) and equal to 0 if the link is available. Only complete link closures are considered. Also, each arc has an associated travel cost c_{ij}.

Transport services can be either trains $t \in T$ or buses $b \in B$, such that the sets are disjoint and $T \cup B = M$, where M is the set of multimodal transport services. The train service network part of the MASP problem is modelled by routing train services $t \in T$ over the railway infrastructure network G^T. A train service route consists of its traversed arcs and nodes, connecting one origin and one destination node. The subsets A^t and N^t represent the arcs and nodes over which each train route is originally scheduled, in the order of traversing. The origin and destination nodes $N(O_t)$ and $N(D_t)$ are used to model the terminals of each train service. Each train service is represented by the binary decision variable x^t_{ij}, modelling the service flow of train $t \in T$ on arc $(i,j) \in A^T$, equal to 1 if the train is using the corresponding arc and otherwise equal to zero. Therefore, a train service $t \in T$ is defined by a set of arcs $(i,j) \in A^T$ with $x^t_{ij} = 1$. Also, we use the decision variables o^t_i and d^t_i to model the origin and destination, respectively, of train t at node i. The parameter s^t represents the number of seats available for passengers in train service t, and the parameter LF represents the average load factor for all trains $t \in T$, meant to capture the variability of demand for each service. Finally, the parameter $h^{t,u}_{ij}$ represents the minimum headway time between two successive trains $t, u \in T$ on arc $(i,j) \in A^T$.

Four measures are considered for the original train services timetable adjustment given a set of blockages: rerouting, short-turning, cancellation and retiming of trains. We distinguish different types of trains depending on whether they can be rerouted ($T^{RR} \subset T$) or short-turned ($T^{ST} \subset T$). In practice, long distance trains such as freight or international trains may be rerouted in case of disruptions as their main objective is to reach their destination while the exact route is less relevant. For passenger local and intercity trains, by contrast, it is more important to operate on their predefined route in order to serve the existing passenger demand during a disruption as much as possible. In that case, rerouting is not allowed and trains can only be short-turned (Szymula

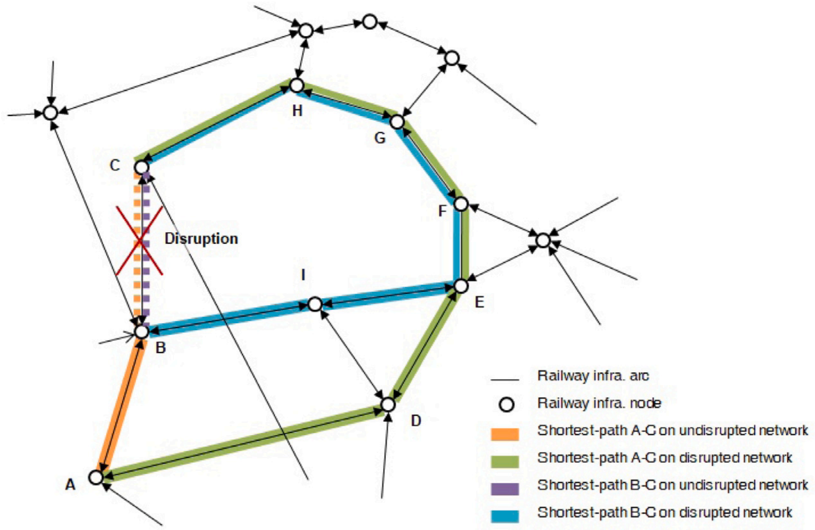

Fig. 2. Example of a short-turned train and an extra train service on an alternative route.

and Bešinović, 2020). Also, we introduce a third subset which represents the extra train services the MASP problem can generate and add on top of the regular timetable ($T^{EXT} \subset T$). All these sets are disjoint and verify $T^{RR} \cup T^{ST} \cup T^{EXT} = T$.

In order to model feasible railway operations in the disrupted network, trains arrival and departure times and thus their retimings are considered. The model assumes an input feasible train timetable which includes trains T^{RR} and T^{ST}. The parameters $T^t_{A,i}$ and $T^t_{D,i}$ represent the arrival and departure or through times, respectively, of train $t \in T^{RR} \cup T^{ST}$ at node $i \in N$ of the input timetable. The parameter t^t_i captures the minimal dwell time of train $t \in T$ at node $i \in N$, and the parameter τ^t_{ij} represents the minimum running time between nodes $i, j \in N$ for train $t \in T$. The decision variables $T^t_{\bar{A},i}$ and $T^t_{\bar{D},i}$ represent the retimed arrival and departure of train $t \in T$ at node i, respectively. Note that the subset of trains T^{EXT} are inserted into the adjusted input timetable.

Adding extra train services $t \in T^{EXT}$ provides greater flexibility to the model to allocate capacity over the service network to optimally route demand from their respective origins and destinations. The potential advantages of adding extra train services include the additional transport capacity to passengers, direct train services for affected OD pairs and shorter travel time compared to alternatives combining short-turned trains and transfer to bus-bridging services (and possibly transfer back to rail). We assume that extra train services and rerouted services can collect passengers at all stops along their routes.

Extra train services are formulated as a Vehicle Routing Problem with time windows, which are bounded by the parameters a_i (lower bound) and b_i (upper bound). Also, these extra train services are generated with the following conditions, which resemble those of regular train services. First, we generate one pair of trains T^{EXT-P} at a time (so that in the end $T^{EXT} = \bigcup T^{EXT-P}$) such that the origin of the train in one direction is the destination of the opposite one, and the other way around, so that the train supply is balanced at both route ends. Secondly, both the origin and the destination have to belong to a set of specific 'hub' stations $N^h \subset N$. The idea is to reproduce realistic operations reflecting the fact that rolling stock and crew are based at specific 'hub' stations, from where our services can originate and terminate. Additionally, we define the parameter D_{max} as an upper bound to the route length.

Extra trains are routed through the set of arcs $A^{EXT} \subset A^T$. Hence, we introduce a preprocessing step to select a subset of candidate arcs A^{EXT} from the railway network A^T that can be used to route extra trains. This allows on one hand to efficiently generate extra trains over the network without necessarily considering all arcs A^T, and on the other hand it prevents generating extra trains in areas of the network G^T not affected by railway possessions. The subset is defined on the basis of the railway network, the input set of possessions the passenger OD pairs and the shortest alternative train paths. The pseudocode and the details of the preprocessing for extra trains are in Appendix, Algorithm 2.

Fig. 2 gives an illustrative example. There is a possession between stations B and C and we consider an OD pair from station A to C, which in undisrupted conditions would use the path from A to C via station B (orange path). We consider another OD pair path from station B to C (purple path). Due to the possession, the shortest path of OD pair A-C is the path from A to C via nodes D, E, F, G and H (green path), whereas the shortest path of OD pair B-C is the path from B to C via I, E, F, G and H (blue path). The union of the arcs contained in each of these shortest paths forms the subset A^{EXT}.

The bus-bridging services are modelled as a service network design (SND) problem with management and coordination of multiple fleets (Andersen et al., 2009). An SND is concerned with the planning of operations including the selection, routing and scheduling of services, and the routing of the demand through the physical and service network. The main decisions of the SND problem to be made are (1) the determination of the service network and (2) the routing of the demand (selecting routes, frequency or schedule). In this research, we use an extension of the standard formulation for the service network design problem to capture additional features of bus operations such as asset balancing and asset restrictions. Asset balancing ensures that at each node the

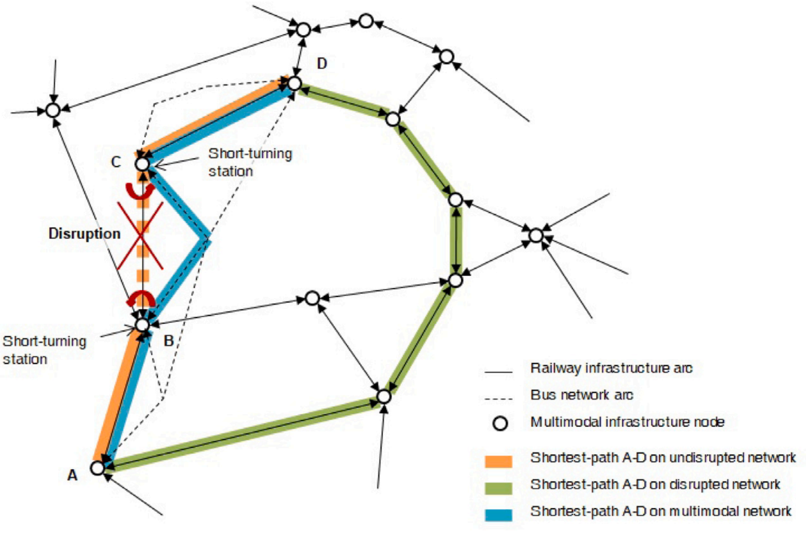

Fig. 3. Example of candidate bus-bridging arcs for a given possession between stations A and B, at the infrastructure network modelling level.

number of vehicles leaving is equal to the number of vehicles entering. Asset restriction may relate to an upper bound of the fleet size or frequency lower and upper bound.

The infrastructure of the road network is formed by intersections, junctions, streets and roads. We define the bus network, on top of the road network, which connects all nodes $n \in N$ with the shortest path between them using the road network. The set of shortest paths forms the set of arcs $(i,j) \in A^R$ of our bus network. Therefore, the bus network is modelled similarly to the railway network, using an undirected graph $G^R = (N, A^R)$, where N is the set of nodes and A^R is the set of arcs. Nodes represent train stations and arcs represent road shortest paths between nodes. Arc capacity is assumed to be unbounded since the interaction with road traffic other than bus-bridging services is not considered assuming that the road infrastructure provides sufficient capacity to run our bus-bridging services. Additionally, we introduce the multimodal network $G^M = (N, A^R \cup A^T \setminus A^d)$. It should be noted that the disrupted railway network dominates the road network. This is made under two assumptions: (1) in any arc the train cost is always lower than the bus cost (hence the model prefers to use the option with lower cost) and (2) duplication of modes, i.e. train and bus on the same arc, is not allowed. Consequently, one would expect the shortest path costs in the multimodal network to be equal or greater than the shortest path costs in the undisrupted railway network.

Several steps are needed to incorporate bus-bridging services. It is required first to make a set of potential routes for bus services. Note that bus-bridging services are initially undefined. Therefore, the additional decision that has to be made is bus network design. Bus routes consist of its traversed arcs and nodes (which can be more than two nodes), are bidirectional (due to asset balancing) and are assumed to have a fixed supply from origin to destination (i.e. the same number of departures per hour at each stop served by the route). Hence, we do not consider bus route branches. Given a set of possessions, the number of possible bus-bridging routes is of the order ***n!***, where ***n*** is the number of nodes in the railway network. Consequently, enumerating all possible bus lines may become impractical. In fact, it should be noted that possessions are often localized and thus, the number of ODs in a large-scale network affected by the possession is rather limited. As a result, the number of potential bus-bridging routes dramatically decreases. We introduce a preprocessing step to select a subset of arcs A^r from the road network A^R that can be used to route bus-bridging services. This subset is defined on the basis of the input set of possessions and the shortest paths of passenger ODs affected by the respective possessions. The pseudocode and the details of the preprocessing for bus-bridging services are shown in Appendix, Algorithm 3.

Fig. 3 gives an illustrative example. There is a possession between stations B and C and we consider an OD pair from station A to D, which in undisrupted conditions would use the path from A to D via stations B and C (orange path). Due to the possession, the shortest path of OD pair A–D using the railway network is the path from A to D via the nodes E, F, G and H (green path). However, if we consider the multimodal network, a path from A to D via B and C using the bus network arc between B and C (blue path) appears to have a lower cost than the green path. The road arcs contained in the shortest path in the multimodal network are added to the subset A^r of candidate bus arcs. It can be noticed that many different candidate arcs are possible, particularly when involving nodes different from B and C. Also, it is interesting to note that with this logic, in a scenario with one single possession, the resulting bus routes will likely be shuttle routes between two nodes, i.e. with no intermediate stops. However, if the set of possessed links includes more than one link, then the resulting bus routes can consist of multiple stops.

After selecting a set of candidate bus arcs, it is needed to route bus-bridging services. Routing is performed in a similar way as train services. Bus services $b \in B$ are routed over the bus network $G^r = (N^r, A^r)$. A bus service route consists of its traversed arcs and nodes, connecting one origin and one destination node. The subsets A^b and N^b represent the arcs and nodes over which each bus service is scheduled along, in the order of traversing. The origin and destination nodes $N(O_b)$ and $N(D_b)$ are used to model

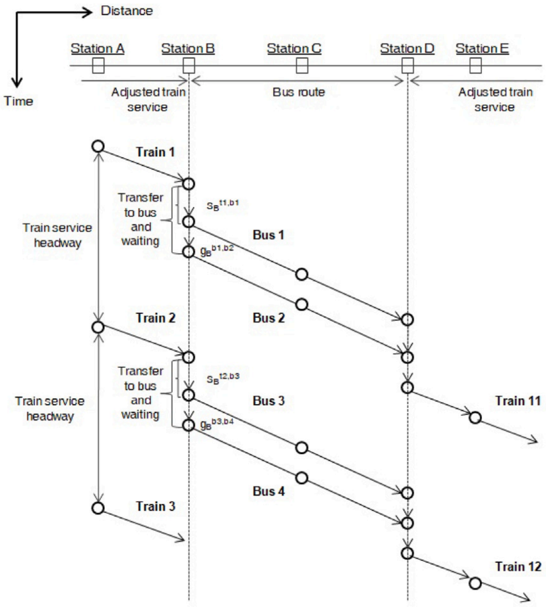

Fig. 4. Time–space network with one bus-bridging route.

the terminals of each bus service. Each bus service is represented by the binary decision variable x_{ij}^{b}, modelling the service flow of bus $b \in B$ on arc $(i,j) \in A^r$, equal to 1 if the bus service is using the corresponding arc and otherwise equal to zero. Therefore, a bus service $b \in B$ is defined by a set of arcs $(i,j) \in A^r$ with $x_{ij}^{b} = 1$. Also, we use the decision variables o_i^b and d_i^b to model the origin and destination, respectively, of bus b at node i. The bus-bridging services connect at the transfer nodes N^{tf} with the train services operating in the disrupted network. The parameter s^b represents the number of seats available for passengers in bus b, and the parameter LF represents the average load factor for all buses $b \in B$, meant to capture the variability of demand for each service.

After selecting possible bus routes, it is needed to set a timetable for buses. It is often assumed in the literature that urban transit operates at very high frequencies and that therefore bus-bridging routes may also operate at fixed frequencies, disregarding a specific timetable for their operation. However, in this research we focus on mainline railway, where the frequencies are generally lower and sometimes irregular so that users tend to adhere to a published timetable. Therefore, we consider that the generated bus-bridging services have to be coordinated with the adjusted train timetables and offer seamless transfers for passengers. This means that a batch of buses would depart from the origin transfer station shortly after the arrival of a train at that station, and the next batch of buses would depart after the arrival of the next train in the timetable. For this, we only consider connections with short-turned trains $t \in T^{ST}$, assuming that the rest of train services, i.e. rerouted trains $t \in T^{RR}$ and extra trains $t \in T^{EXT}$, are not relevant for connections with bus-bridging services and hence they do not need to be synchronized. The total number of bus departures is defined by the capacity required based on the passenger flows using bus arcs $(i,j) \in A^r$. Additionally, the number of bus departures on a link has to be equal in both directions. Also, a bus frequency upper bound U_{ij} is imposed, which may depend on the passenger demand on the link.

Bus timetables are modelled in a similar approach as train timetables. Nevertheless, some additional considerations that apply in real situations have to be taken into account, such as the limited number of bus holding points at train stations. This is represented by the parameter $g_i^{b,c}$, which represents the minimum headway between the departures of bus b and c at node i, which depends on its capacity. However, for simplicity, this condition only applies for bus departures associated with the same train arrival, which means that overlap in the temporal dimension of departures of buses associated with different train arrivals is allowed. Furthermore, transfers from railway to bus and vice versa are modelled using the parameter $s_i^{t,b}$, which represents the minimum transfer time at node i between train $t \in T^{ST}$ and bus $b \in B$. This coordination only applies to the first bus service following the arrival of a train at the transfer node. This also means that we have to ensure that there is at least one bus service departure following the arrival of a train. For simplicity, the synchronization of transfers is only enforced for connections from train arrival to bus departures at a transfer node.

The time–space network presented in Fig. 4 gives an illustrative example of how the bus-bridging services could be organized between two short-turning stations B and D. Note that the bus services are not equally spaced in time. The number of bus services following the arrival of one train at Station B is defined by the required capacity to transport all OD flows in the arc from Station B to D through C, which is equal to two buses for both train services in our example. The headway between the pairs of buses 1 & 2 and 3 & 4 relates to the bus holding points limited capacity.

Finally, the passenger network is modelled by routing passenger flows $k \in K$ on their corresponding paths $p \in P^k$ over the network. A passenger path p is a sequence of nodes $n \in N$ and arcs A^p between the origin and destination of passenger flow $k \in K$.

We use the parameter δ^p_{ij}, which is equal to 1 if arc ij is included in path p, and 0 otherwise. The parameter d_k represents the demand of $k \in K$ during a certain time period. The passengers are routed according to the train capacity in the service network. The assigned passenger flows are represented by the decision variable f^k_p as the share of demand of the OD pair $k \in K$ over path $p \in P^k$. Passenger flows are routed based on their shortest path over the network using the travel times c^k_p of path p for flow k.

3.2. Mathematical formulation

The mathematical formulation of the MASP problem is based on the multi-commodity flow problem. Transport services are modelled in arc-based formulation and the passenger flow in path-based formulation, following the approach by Szymula and Bešinović (2020). The classical model is extended to incorporate the transport service adjustment in a disrupted network and the timetable adjustment.

The objective of the MASP problem is twofold. First, it minimizes the travel time of passengers. Second, it minimizes the operational costs of the extra train services T^{EXT} and the bus-bridging services B, which are defined by the unit of distance travelled with the parameters C^{EXT} and C^B respectively. The parameter d_{ij} represents the distance between nodes i and j on the multimodal network. Note that we do not include the operating costs of the train services already included in the original input timetable. Furthermore, in order to solve the multi-objective problem, a weight λ is incorporated to the objective function. This allows to vary the relative importance of the different terms. Therefore, the objective function for the MASP problem is defined as follows:

$$min \quad \lambda \sum_{k\in K}\sum_{p\in P^k} d_k f^k_p c^k_p + (1-\lambda)\Big[\sum_{t\in T^{EXT}}\sum_{(i,j)\in A^{EXT}} c^{EXT} d_{ij} x^t_{ij} + \sum_{b\in B}\sum_{(i,j)\in A^r} c^B d_{ij} x^b_{ij}\Big] \tag{1}$$

The constraints related to the arc-based formulation of train routing are presented as:

$$\sum_{j\in N} x^t_{ij} - \sum_{j\in N} x^t_{ji} = \begin{cases} -o^t_i, & \text{if node } i \text{ is a starting node} \\ d^t_i, & \text{if node } i \text{ is an ending node} \\ 0, & \text{otherwise} \end{cases} \qquad \forall t\in T, i \in N^t \tag{2}$$

$$\sum_{i\in N^t} o^t_i = \sum_{i\in N^t} d^t_i \qquad \forall t \in T \tag{3}$$

$$o^t_i = 1 \qquad \forall t\in T^{RR}\cup T^{ST}, i = N(O_t) \tag{4}$$

$$d^t_i = 1 \qquad \forall t\in T^{RR}\cup T^{ST}, i = N(D_t) \tag{5}$$

$$o^t_i = 0 \qquad \forall t\in T^{RR}, i \neq N(O_t), N(D_t) \tag{6}$$

$$d^t_i = 0 \qquad \forall t\in T^{RR}, i \neq N(O_t), N(D_t) \tag{7}$$

$$o^t_j \geq v_{ij} \qquad \forall t\in T^{ST}, i \in N^t, (i,j)\in A^t \tag{8}$$

$$d^t_i \geq v_{ij} \qquad \forall t\in T^{ST}, j \in N^t, (i,j)\in A^t \tag{9}$$

$$\sum_{i\in N^t} o^t_i \leq \sum_{(i,j)\in A^t} v_{ij} + 1 \qquad \forall t\in T^{RR}\cup T^{ST} \tag{10}$$

$$\sum_{i\in N^t} d^t_j \leq \sum_{(i,j)\in A^t} v_{ij} + 1 \qquad \forall t\in T^{RR}\cup T^{ST} \tag{11}$$

$$\sum_{i\in N^h} o^t_i = 1 \qquad \forall t\in T^{EXT} \tag{12}$$

$$\sum_{i\in N^h} d^t_i = 1 \qquad \forall t\in T^{EXT} \tag{13}$$

$$o^t_i + d^t_i \leq 1 \qquad \forall t\in T^{EXT}, i\in N^{EXT} \tag{14}$$

$$\sum_{j\in N^{EXT}} x^t_{ij} \leq 1 \qquad \forall t\in T^{EXT}, i\in N^{EXT} \tag{15}$$

$$\sum_{(i,j)\in A^{EXT}} d_{ij} x^t_{ij} \leq D_{max} \qquad \forall t\in T^{EXT} \tag{16}$$

$$o^t_i = d^u_i \qquad \forall t\in T^{EXT-P}, u\in T^{EXT-P}, t\neq u, i\in N^h \tag{17}$$

$$d^t_i = o^u_i \qquad \forall t\in T^{EXT-P}, u\in T^{EXT-P}, t\neq u, i\in N^h \tag{18}$$

$$\sum_{t\in T} x^t_{ij} \leq CAP_{ij}(1-v_{ij}) \qquad \forall (i,j)\in A^t \tag{19}$$

$$x^t_{ij}, o^t_i, d^t_i \in \{0,1\} \qquad \forall t\in T, n\in N^t \tag{20}$$

Eq. (2) ensures the flow continuity and that train services are only allowed to start and end at origin or destination nodes o^t_i and d^t_i respectively. Eq. (3) ensures that there are always as many origins and destinations on each train route. Eqs. (4) and (5) ensure that the originally scheduled origins (destinations) at the terminals of the transport services are always kept as sources (sinks) for the service flows in order to create the original train services in the undisrupted state. Eqs. (6) and (7) prevent rerouted trains from being short-turned. Eqs. (8) and (9) connect the short-turn location selection to the possession of a link. If a link is possessed, trains are forced to short-turn at the station right next to the possessed link. These constraints deal with short-turning due to possessed links in the original train route only. Eqs. (10) and (11) ensure that originally scheduled trains are only short-turning according to the number of possessed links at the original route, in order to prevent unnecessary short-turning, e.g. on an undisrupted part of the network. Additionally, they allow the existence of at least one origin and destination to represent the undisrupted state. Eqs. (12) and (13) ensure that at most one origin and destination can be selected, respectively, among the set of hub nodes for routing extra trains over the network. Eq. (14) ensures that a node cannot be selected as both origin and destination for a specific extra train service simultaneously. Eq. (15) ensures that each node can only be visited at most once by each extra train service. Eq. (16) imposes an upper bound to each extra train service route length to avoid unrealistically long routes. Eqs. (17) and (18) enforce the route symmetry of both extra train services in the same train pair by setting the origin of one of them equal to the destination of the other one, and the other way around. Eq. (19) guarantees that the train flows on each link do not exceed the link capacity. Finally, Eq. (20) limits the range of the decision variables of the problem.

The input train timetable is modelled with the constraints shown below (Eq. (21)–(31)) in order to allow creating a feasible alternative timetable for the disrupted network.

$$T^t_{\bar{D},i} + \tau^t_{ij} \le T^t_{\bar{A},j} + M(1 - x^t_{ij}) \qquad \forall t \in T^{RR}, (i,j) \in A^T \tag{21}$$

$$T^t_{\bar{D},i} + \tau^t_{ij} \le T^t_{\bar{A},j} + M(1 - x^t_{ij}) \qquad \forall t \in T^{EXT}, (i,j) \in A^{EXT} \tag{22}$$

$$T^t_{\bar{D},i} + \tau^t_{ij} x^t_{ij} \le T^t_{\bar{A},j} \qquad \forall t \in T^{ST}, (i,j) \in A^t \tag{23}$$

$$T^t_{\bar{D},i} \ge \sum_{j \in N} T^t_{D,i} x^t_{ij} \qquad \forall t \in T \setminus T^{EXT}, i \in N^t \tag{24}$$

$$T^t_{\bar{D},i} - T^t_{\bar{A},i} \ge t^t_i - M(1 - x^t_{ij}) \qquad \forall t \in T^{RR}, (i,j) \in A^T \tag{25}$$

$$T^t_{\bar{D},i} - T^t_{\bar{A},i} \ge t^t_i - M(1 - x^t_{ij}) \qquad \forall t \in T^{EXT}, (i,j) \in A^{EXT} \tag{26}$$

$$T^t_{\bar{D},i} - T^t_{\bar{A},i} \ge t^t_i x^t_{ij} \qquad \forall t \in T^{ST}, (i,j) \in A^t \tag{27}$$

$$T^t_{\bar{D},i} \ge T^u_{\bar{D},i} + h^{t,u}_{ij} - M(1 - x^t_{ij}) - M(1 - x^u_{ij}) \qquad \forall t \in T, u \in T, i \in N^t, t \ne u, (i,j) \in A^t \tag{28}$$

$$a_i \le T^t_{\bar{D},i} \le b_i \qquad \forall t \in T^{EXT}, i = N(O_t) \tag{29}$$

$$T^t_{\bar{A},i} - T^t_{\bar{D},j} \le T_t \qquad \forall t \in T^{EXT}, i = N(O_t), j = N(D_t) \tag{30}$$

$$T^t_{\bar{A},i}, T^t_{\bar{D},i} \ge 0 \qquad \forall t \in T, i \in N^t \tag{31}$$

Eqs. (21)–(23) set the minimum running times for a specific train between two stations for rerouting, extra and short-turning train, respectively, on the arcs used by that train. Eq. (24) ensures that the actual departure cannot be rescheduled earlier than the originally scheduled one. Eqs. (25)–(27) constrain the dwelling times for each rerouted, extra and short-turning train, respectively, on the arcs used by that specific train. Eq. (28) ensures the minimum headway times between successive trains. Eq. (29) imposes a lower and an upper bound to the time windows for the time variables at the origin node. Eq. (30) bounds the duration of the train route. Finally, Eq. (31) guarantees the non-negativity of all time events.

The constraints related to the arc-based formulation of bus-bridging routing are presented as follows:

$$\sum_{j \in N^r} x^b_{ij} - \sum_{j \in N^r} x^b_{ji} = \begin{cases} -o^b_i, & \text{if node } i \text{ is a starting node} \\ d^b_i, & \text{if node } i \text{ is an ending node} \\ 0, & \text{otherwise} \end{cases} \qquad \forall b \in B, i \in N^r \tag{32}$$

$$\sum_{i \in N^r} o^b_i = 1 \qquad \forall b \in B \tag{33}$$

$$\sum_{i \in N^r} d^b_i = 1 \qquad \forall b \in B \tag{34}$$

$$\sum_{j \in N^r} x^b_{ij} \le 1 \qquad \forall b \in B, i \in N^r \tag{35}$$

$$\sum_{b \in B} x^b_{ij} \le U_{ij} \qquad \forall (i,j) \in A^r \tag{36}$$

$$\sum_{b \in B} x^b_{ij} \le U_{ij} v_{ij} \qquad \forall (i,j) \in A^T \tag{37}$$

$$\sum_{b \in B} x^b_{ij} = \sum_{b \in B} x^b_{ji} \qquad \forall (i,j) \in A^r \tag{38}$$

$$x^b_{ij}, o^b_i, d^b_i \in \{0,1\} \qquad \forall b \in B, i \in N^r \tag{39}$$

Eq. (32) ensures the flow continuity and that bus services are only allowed to start and end at origin or destination nodes o_i^b and d_i^b respectively. Eqs. (33) and (34) ensure that at most one origin and destination can be selected, respectively, for routing bus services over the network. Eq. (35) ensures that each node can only be visited at most once by one bus service. Eq. (36) imposes an upper bound to the number of bus frequencies that can be operated each hour on each direction of a road arc. Eq. (38) enforces the number of bus departures on a link to be equal in both directions. Finally, Eq. (39) limit the range of the decision variables of the problem.

Furthermore, bus-bridging schedules are also modelled similarly to railway services:

$$T_{D,i}^b + \tau_{ij}^b \le T_{A,j}^b \qquad \forall b \in B, (i,j) \in A^r \tag{40}$$

$$T_{D,i}^b - T_{A,i}^b \ge t_i^b \qquad \forall b \in B, (i,j) \in A^r \tag{41}$$

$$T_{D,i}^b \ge T_{D,i}^c + g_i^{b,c} - M(1-x_{ij}^b) - M(1-x_{ij}^c) \qquad \forall b \in B, c \in B, i \in N^r, b \neq c, (i,j) \in A^r \tag{42}$$

$$T_{D,i}^b \ge T_{A,i}^t + s_i^{t,b} - M(1-x_{ji}^b) - M(1-x_{ij}^t) \qquad \forall b \in B, t \in T^{ST}, i \in N^{tf}, (i,j) \in A^M \tag{43}$$

$$T_{A,i}^b, T_{D,i}^b \ge 0 \qquad \forall b \in B, i \in N^b \tag{44}$$

Eq. (40) sets the minimum running times for a specific bus service between two stations. Eq. (41) constrains the dwelling times for each bus-bridging service. Eq. (42) ensures the minimum headway times between successive bus departures, reflecting the capacity of bus holding points. Eq. (43) regulates the departure time of a bus at a transfer node, which is synchronized with the arrival of a short-turned train. Finally, Eq. (44) guarantees the non-negativity of all time events.

Passengers are routed on the service network using alternative services. The constraints related to the path-based modelling of passenger flows are as follows:

$$\sum_{k \in K} \sum_{p \in P^k} \delta_{ij}^p d_k f_p^k \le \sum_{m \in M} LFs^m x_{ij}^m \qquad \forall (i,j) \in A^M \tag{45}$$

$$\sum_{p \in P^k} f_p^k = 1 \qquad \forall k \in K \tag{46}$$

$$f_p^k = [0,1] \qquad \forall k \in K, p \in P^k \tag{47}$$

Eq. (45) models the arc capacity, given by the transport service flow, which bounds the total passenger flow from all paths using this arc. Eq. (46) ensures that all passenger demand from origin to destination has to be transported. Eq. (47) restricts the range of the decision variables.

4. Solution framework

In large-scale network problems, the number of constraints in arc-based formulation and the number of decision variables in path-based formulation substantially increase and therefore the size of the model matrices can be large enough to result in computational challenges. Considering all passenger paths in the model at once may render the problem impossible to solve (e.g. Bouillet et al., 2007; Gentile et al., 2016), and this holds also when considering all headway constraints. In fact, it is observed that many of the possible paths will not be used in the final solution (Szymula and Bešinović, 2020). Similar holds for extra train services and bus-bridging services. Although the corresponding preprocessing by Algorithms 2 and 3 help reducing the number of additional decision variables and constraints associated with these alternative services, the number of remaining possible combinations may be still quite large to be managed efficiently. And thus, most combinations will likely not be used in the final solution. Finally, regarding headway constraints, the input timetable used is feasible and therefore it already satisfies all the headway constraints. Consequently, only a limited number of headway constraints, resulting from the timetable adjustment and extra train services generation, will actually need to be added to the model.

Therefore, we propose a new solution framework consisting of a combination of multi-column and row generation approaches. First, the MASP framework has two column generation loops over the initial problem for (1) the generation of new paths, on the decision variables f_p^k, to identify and add iteratively sets of candidate passenger paths that can bring an improvement in the objective function, and (2) the generation of alternative services, which include extra train services $t \in T^{EXT}$ and bus-bridging services $b \in B$. For both column generation based heuristics, we use the duality principles of mathematical optimization to generate new decision variables. The column generation approach has been applied by several authors, such as Borndorfer et al. (2007), Jin et al. (2015) and Liang et al. (2019). Second, for the headway constraints we propose a row generation algorithm to generate efficiently the associated infrastructure constraints that prevent timetable conflicts. The row generation approach has been applied by Szymula and Bešinović (2020) and Van Aken et al. (2017a) in a similar fashion. Fig. 5 shows the MASP solution framework, and the steps are described in detail subsequently.

In the initialization, for the incorporation of bus-bridging services, we initially assign an upper bound to the capacity of the arcs in the bus network A^r (defined in Algorithm 3), since we do not know in advance the necessary number of bus departures. The resulting passenger flows on these arcs will be transformed afterwards to the actual number of buses running in those arcs (step 7).

Firstly, the generation of passenger paths is performed using the column generation approach, which consists of two sub-problems. The first is the Restricted Master Problem (RMP), i.e. restricted MASP, which contains only a subset of all decision variables which are beneficial to the solution. The second is the Pricing Problem, which we use to determine if any column needs to be added to the RMP in order to improve our objective function. Therefore, the algorithm starts with an initial relaxed RMP, and the results from this problem are plugged into the Pricing Problem.

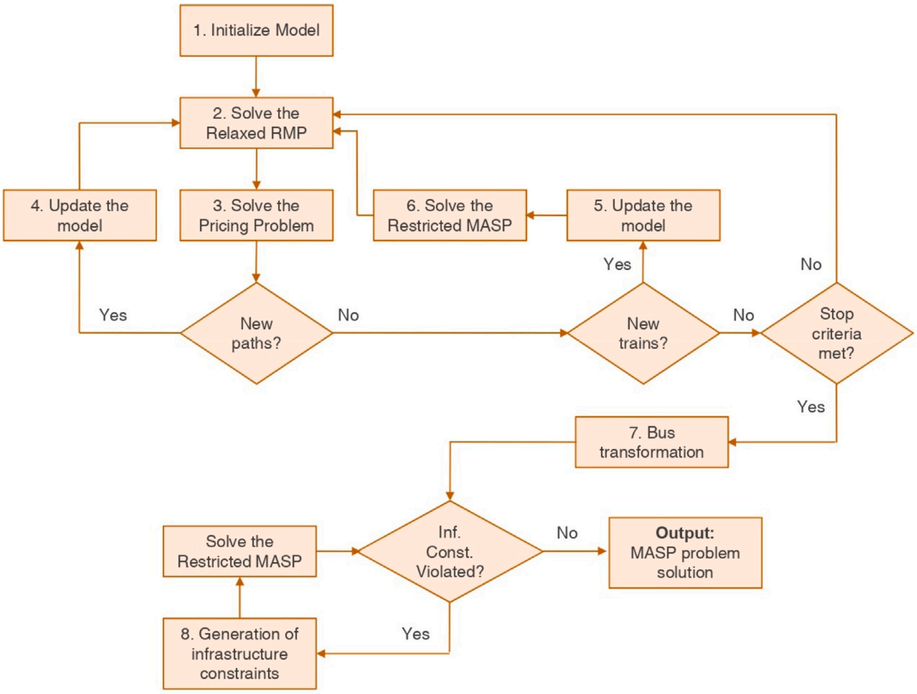

Fig. 5. MASP problem solution framework.

1. **Initialization of the Model.** An initial set of columns, i.e. decision variables f_p^k, is generated assigning each passenger OD pair to their shortest path on the multimodal network A^M, taking into account capacity constraints.
2. **Relaxed RMP.** In order to perform the pricing, the problem needs to be modified and consider a relaxed version of RMP. In particular, Eqs. (20) and (39) have to be relaxed:

$$x_{ij}^m, o_i^m, d_i^m \in [0,1] \qquad \forall m \in M, n \in N^m \tag{48}$$

Also, due to a capacitated multi-commodity flow problem formulation, the initial solution may be infeasible since some passenger OD pairs cannot be assigned to their shortest paths due to lack of capacity in the network. Therefore, in order to absorb this infeasibility, we use slack variables s_{ij}, which are incorporated in the objective function (Eq. (49)) and in the bundle constraints (Eq. (50)) as follows:

$$min \lambda \sum_{k\in K}\sum_{p\in P^k} d_k f_p^k c_p^k + (1-\lambda)\Big[\sum_{t\in T^{EXT}}\sum_{(i,j)\in A^{EXT}} c^{EXT} d_{ij} x_{ij}^t + \sum_{b\in B}\sum_{(i,j)\in A^r} c^B d_{ij} x_{ij}^b\Big] + \sum_{(i,j)\in A} M s_{ij} \tag{49}$$

$$\sum_{k\in K}\sum_{p\in P^k} \delta_{ij}^p d_k f_p^k - s_{ij} \le \sum_{m\in M} LF s^m x_{ij}^m \qquad \forall (i,j) \in A^M \tag{50}$$

$$s_{ij} \ge 0 \qquad \forall (i,j) \in A^M \tag{51}$$

Eq. (51) guarantees the non-negativity of the slack variables, which are defined as:

$$s_{ij} = max\Big\{0, \sum_{k\in K}\sum_{p\in P^k} \delta_{ij}^p d_k f_p^k - \sum_{m\in M} LF s^m x_{ij}^m\Big\} \qquad \forall (i,j) \in A^M \tag{52}$$

Besides, the bus-bridging services, from which initially we only know the arcs A^r that may be used to route and schedule them, are represented by allocating a certain amount of passenger capacity, represented by the parameter s^b to the arcs A^r. In other words, a fictitious bus b is allocated to each arc with capacity equal to s^b. As a result, Eqs. (32)–(38) and (40)–(44) are not used at this moment and are replaced by the following:

$$x_{ij}^1 = 1 \qquad \forall (i,j) \in A^r \tag{53}$$

Then, we solve the relaxed RMP, which is then defined by Eqs. (2)–(19), (21)–(28), (31), (39) and (46)–(53). We use the dual variables from the solution to the RMP to solve the pricing problem. Here, π_{ij} are the dual variables of the bundle constraints (Eq. (45)), for all arcs $(i,j) \in A^M$, and σ_k are the dual variables of the passenger related constraints, for all OD pairs $k \in K$ (Eq. (46)).

3. **Pricing Problem.** The pricing problem, for each OD pair $k \in K$, is equivalent to a shortest path problem with modified costs, which have the form $c_{ij} - \pi_{ij}$. These costs are only evaluated in those arcs which actually have train services, i.e. $\forall (i,j) \in A^T$ such that $\sum_{t \in T} x^t_{ij} > 0$, to prevent passenger flows from being routed in arcs with no train flows. Therefore, the objective function of the pricing problem for each $p \in P^k$ and $k \in K$ is:

$$min\Big[d_k\Big[\sum_{(i,j)\in A}(c_{ij} - \pi_{ij}) \times \delta^p_{ij} - \sigma_k/d_k\Big]\Big] \qquad \forall k \in K \tag{54}$$

For every passenger OD pair $k \in K$, we check if:

$$c^k_p = \sum_{(i,j)\in A}(c_{ij} - \pi_{ij}) \times \delta^p_{ij} < \sigma_k/d_k \qquad \forall k \in K \tag{55}$$

where c^k_p is the cost of the new shortest path p.

4. **Update the RMP.** If the inequality (55) is true for any commodity k, we add the corresponding paths p to the set of the RMP variables and the algorithm moves back to step 2. Otherwise, no paths are added and then, the algorithm moves directly to generating alternative services in step 5.

Secondly, to generate alternative services, including extra train services and bus-bridging services, we again use a column generation approach. Note that regarding the pricing problem, we address train services and bus services differently. For the former, we use the economic interpretation of the dual variables π_{ij}, which is the marginal value of adding one unit of capacity on arc $(i,j) \in A^M$ or the contribution to the objective function of using the capacity of the arc. This is possible since these services are generated on arcs that already have capacity and thus can yield dual variables π_{ij} different from zero. For the latter, instead, given that bus-bridging services are generated on arcs that initially have zero capacity, the associated dual variables are null. Therefore, we cannot use the pricing problem and we adopt a different approach for the transformation from bus-bridging routes to its services, as explained in step 7.

For the generation of extra train services, we use the output dual variables from the last iteration of the generation of passenger paths to identify which arcs of the network are beneficial to have increased capacity, which is defined by the number of transport services. The extra train services have to comply with the requirements and specifications described in Section 3.1. Particularly, extra train services are generated on a subset of the arcs of the disrupted railway network A^{EXT} (defined in Algorithm 2), which means that we only take into account the dual variables of the arcs contained in this subset of A^M, and these trains can only have origin or destination in a subset of stations N^h. If the sum of these dual variables is higher than a parameter D (i.e. $\sum_{(i,j)\in A^{EXT}} \pi_{ij} < -D$), then algorithm moves to step 5 and generates a pair of extra train services in the current iteration. We assume that a maximum of n^{EXT} pairs of extra train services can be generated for each scenario.

Furthermore, it should be noted that each transport service $m \in M$ is defined by its associated variables x^m_{ij}, o^m_i, d^m_i, $T^m_{\bar{A},i}$, and $T^m_{\bar{D},i}$. Therefore, the generation of one transport service implies the generation of a set of decision variables (columns).

5. **Update the model.** If a pair of extra train services have been generated, then the associated decision variables are added to the set of the RMP decision variables and the algorithm moves forward to step 6. Otherwise, if no extra train services have been generated, the algorithm moves on to check the stopping criteria.
6. **Solve the Restricted RMP.** The model runs now in the restricted form (i.e. with binary variables) to find the optimal routing of the new train service over the network, according to the respective constraints (Eqs. (2) and (12)–(18)). In order to deal with possible fractional solutions before solving the restricted MASP, there may be various approaches taken, e.g. rounding off to integers as in Szymula and Bešinović (2020). The resulting new train services are stored in the input timetable together with the original train services. This means that in the follow-up iterations, from a modelling perspective, the already generated train services will be treated as the trains that can be short-turned (although they will never be short-turned since they are predefined to run on undisrupted parts of the network), i.e. governed by Eqs. (4), (5) and (8), (9), instead of Eqs. (12)–(18). This prevents such train services to be modified in subsequent iterations. The extended input timetable is hence feed into the relaxed RMP in step 2 incorporating the time window constraints for extra train services (Eqs. (29) and (30)).

Then, the algorithm moves to the stopping criteria check, which includes the primal feasibility (Eqs. (56)–(58)), complementary slackness (Eqs. (59) and (60)) and dual feasibility (Eq. (61)), which all have to be satisfied in order to yield the best MASP solution.

$$\sum_{k\in K}\sum_{p\in P^k}\delta^p_{ij} d_k f^k_p \le \sum_{m\in M} LFs^m x^m_{ij} \qquad \forall (i,j) \in A^M \tag{56}$$

$$\sum_{p\in P^k} f^k_p = 1 \qquad \forall k \in K \tag{57}$$

$$f^k_p \ge 0 \qquad \forall k \in K, p \in P^k \tag{58}$$

$$\pi_{ij} \times \Big[\sum_{k\in K}\sum_{p\in P^k}\delta^p_{ij} d_k f^k_p - \sum_{m\in M} LFs^m x^m_{ij}\Big] = 0 \qquad \forall (i,j) \in A^M \tag{59}$$

$$\sigma_k \times \Big[\sum_{p\in P^k} f^k_p - 1\Big] = 0 \qquad \forall k \in K \tag{60}$$

$$d_k\Big[\sum_{(i,j)\in A}(c_{ij}-\pi_{ij})\times\delta_{ij}^p-\sigma_k/d_k\Big]\geq 0 \qquad \forall k\in K, p\in P^k \tag{61}$$

If all conditions are satisfied, we obtain all passenger flows routed in the network for the given set of transport services. Then, the algorithm moves to step 7. Otherwise, some of the slack variables s_{ij} will still have a non-zero value, and more alternative services could be added. Then, we go back to step 2.

7. **Transformation of bus-bridging Services.** We transform the passenger flow on arcs A^r into the actual number of bus-bridging services running on these arcs. First, we use a realistic parameter s^b, which represents the number of seats on each bus, to obtain the necessary number of bus departures on each arc to cover the assigned passenger demand (per hour). Second, in order to balance the number of bus departures in both directions on each link, we assign the maximum among the ideal number of bus departures in each direction. Third, we ensure that each arrival of a short-turned train service $t\in T^{ST}$ at a transfer node $n\in N^{tf}$ connects directly with at least one bus-bridging service departure. Fourth, it is needed to match the number of train arrivals and the number of bus departures. On one hand, to increase the number of bus departures (due to limited passenger demand), we assign additional buses until we have enough bus departures at the node. On the other hand, in case of a higher number of bus departures, then the surplus of bus departures is spread among the train arrivals already served. As a result, we may have two or more sequential bus departures following the arrival of a train. The output includes all the resulting pairings of short-turned train arrivals $t\in T^{ST}$ and bus departures $b\in B$ at the transfer nodes N^{tf}. This process is detailed in the pseudocode in Algorithm 1. Now the relaxed MRP is solved using Eqs. (32)–(44), i.e. the arc-based formulation of bus-bridging services and bus timetable, instead of Eq. (53), to determine the timetable for bus-bridging services.

Algorithm 1 Bus Transformation Process

Input: bus-bridging arcs A^r, passenger flows on each arc A^r, transfer nodes N^{tf}, train arrivals at nodes N^{tf} and all parameters
Output: pairings of short-turned train arrivals $t\in T^{ST}$ and bus departures $b\in B$ at the transfer nodes N^{tf}
for All bus-bridging arcs A^r **do**
    Estimate the ideal number of bus service departures by dividing the passenger flow on the arc by the number of seats on each bus s^b
    Round up to integer the resulting number of buses
    Take the minimum between the resulting number of buses and the upper bound U_{ij}
    Balance the number of bus departures in both directions by assigning the maximum value among both directions
end for
for all transfer nodes N^{tf} **do**
    Retrieve all the arrivals of the short-turned trains T^{ST}
end for
for All bus-bridging arcs A^r **do**
    if the number of short-turned train arrivals is higher than the number of calculated bus departures **then**
        Increase the number of bus departures so that it equals the number of train arrivals
    else
        Spread all bus departures as evenly as possible among train arrivals
    end if
end for

Lastly, by using the row generation approach, we generate only those infrastructure constraints that are violated and hence significantly decreasing the number of constraints of this type. This step is performed after obtaining the output timetable resulting from the generation of passenger paths and alternative services. The generation of headway constraints focuses on guaranteeing the feasibility of the timetable solution and therefore it does not affect the demand side of the problem, which is fully addressed after generating passenger paths and alternative services. As a result, it is not necessary to generate additional paths and alternative services after generating headway constraints.

8. **Generation of Infrastructure Constraints.** Check if the infrastructure constraints between consecutive trains on the same arc are violated (Eq. (28)). If yes, the associated constraints are added and then the restricted MASP is solved again. This process is repeated until no more infrastructure constraints need to be incorporated into the model. The output is the optimized solution of the MASP problem with a feasible timetable.

5. Computational experiments

5.1. Experiments setup

We demonstrate the performance of the MASP framework on the Eastern part of the Dutch passenger railway network, which is shown in Fig. 6. The graph representation of the network consists of 255 nodes (stations) and 546 directed arcs (infrastructure track sections). We use an input timetable from a basic hour pattern on a weekday of 2019, consisting of 329 trains.

The used data is the General Transit Feed Specification (GTFS) data. It consists of the operated lines, routes and the scheduled arrival and departure times. For trains not originating in the considered network, i.e. entering or leaving the case study network, train routes were modified to operate only within the considered boundaries by deleting all stops outside the case study network for each train route. The timetable includes three passenger train types: local, intercity and international trains. We assume that local and intercity trains can be short-turned and that trains dwell only at the originally scheduled stops. International trains can

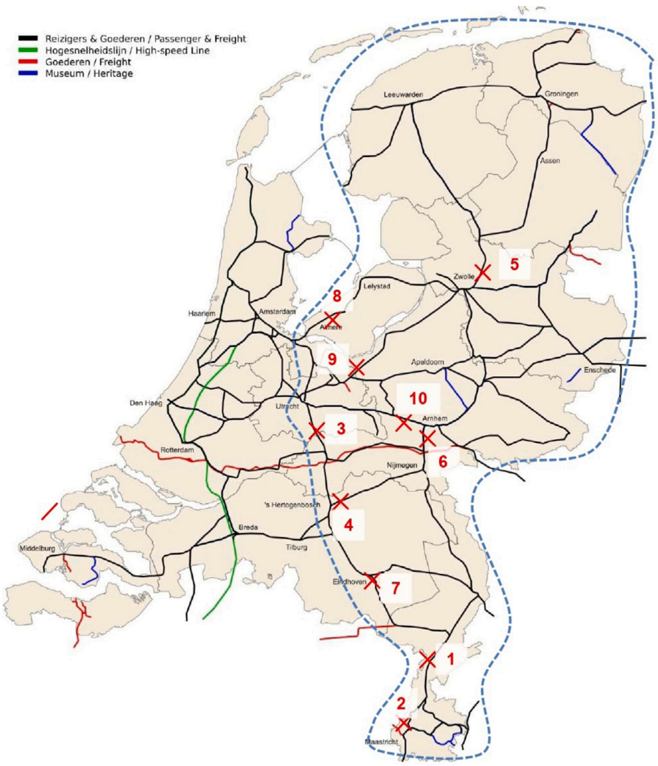

Fig. 6. Dutch railway network, with the case study network marked in dashed blue (adapted from Wikipedia). Only passenger railway lines (black lines) are used. Considered possessions are shown on the map (and also given in Table 8). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Table 3
Input parameters for experiments.

Parameter	Value	Parameter	Value
λ	0.001	U_{ij}	15
M	100,000	LF	1
s^t	500	D_{max}	200
s^b	65	c^{EXT}	1
$h_{ij}^{t,m}$	3 min	c^B	1
$g_i^{b,c}$	3 min	n^{EXT}	1
$s_i^{t,b}$	3 min	D	0

be rerouted but need to reach their destination. The input passenger demand is based on real demand data from the Dutch railway network. Table 3 defines the default model parameters used in the experiments. We assume the cost parameters c^{EXT} and c^B to be equal to 1, so that the respective objective function terms represent the number of vehicle-kilometres required to operate the alternative services.

A number of experiments have been carried out to test the capabilities of the MASP framework, which include functionality tests (Section 5.2), demand sensitivity analysis (Section 5.3) and multiple possessions scenario analysis (Section 5.4). All runs have been performed using IBM ILOG CPLEX 12.8 on an Intel Core i7-7500U CPU with 8 GB RAM.

5.2. Experiment 1: Generation of alternative services

To test the functionality of the MASP solution framework, we consider a single possession in the network between Geldermalsen (Gdm) and Culemborg (Cl), a 7.8 kilometres double-track link located on the route between Utrecht (Ut) and Hertogenbosch (Ht) (see #3 in Fig. 6). We perform a number of runs with different components of the MASP framework. As the base scenarios, we use a scenario without possessions (Scenario 1) and with the possession but no alternative services generated (Scenario 2). Then, we run: with the generation of bus-bridging services (Scenario 3), with the generation of extra train services (Scenario 4) and with the generation of both bus-bridging services and extra train services (Scenario 5), and finally, we compare the results. We assume that if extra train services are considered, then a maximum of one pair may be generated. Also, we consider 50% of the complete passenger demand, which is a total of 89,468 passengers, and rerouting is not considered.

Table 4
Results experiment 1.

	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5
	Undisrupted	Disrupted	Disrupted with bus-bridging	Disrupted with Extra Trains	Disrupted with bus-bridging + Extra Trains
Passenger costs (pax-km)	2,384,200	2,523,300	2,302,100	2,508,700	2,297,400
Transported passenger costs (pax-km)	2,228,000	2,179,879	2,048,826	2,246,984	2,105,200
Spillover demand costs (pax-km)	156,200	343,421	253,274	261,716	192,200
Short-turned trains	0	20	20	20	20
Operating costs bus-bridging services (veh-km)	0	0	450	0	450
Number of bus links	0	0	1	0	1
Total number of bus departures	0	0	30	0	30
Operating costs extra train services (veh-km)	0	0	0	373.4	338.8
Number of extra train services	0	0	0	2	2

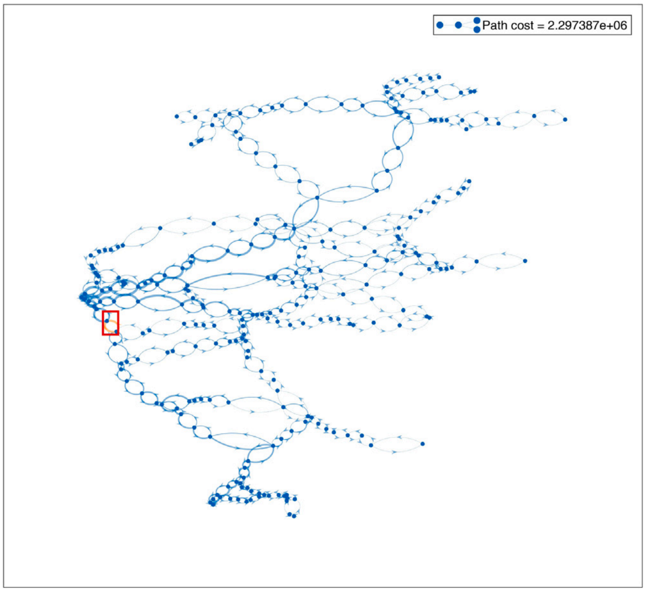

Fig. 7. Directed graph with passenger flows in blue. Weight of arcs represents passenger volume. Red square shows the location of the possession (Experiment 1, Scenario 5). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

The main results are shown in Table 4, including the passenger costs (in passenger-kilometres), the number of short-turned train services around the disrupted areas, the operating costs of bus-bridging services (in vehicle-kilometres), the number of bus-bridging links, the total number of bus departures (per hour), the operating costs of extra train services (in vehicle-kilometres) and finally the number of extra train services. The results show that 37 OD pairs out of 711 are directly impacted by this possession (5.9% of the total passenger demand), which means that their (desired) shortest path is no longer feasible and passengers are forced to follow an alternative shortest path with higher costs. The passenger costs, associated with the complete passenger flows including spillover demand costs, may show a counterintuitive decrease in some scenarios. Thus, we split passenger costs in transported passenger cost and spillover demand cost, i.e. associated to the non-transported demand computed as $\sum_{(i,j)\in A^m} s_{ij}c_{ij}$.[1] Note that due to the possession, the spillover demand increases in Scenarios 2 to 5, meaning that the number of transported passengers over the network decreases. In particular, the spillover demand is smaller in Scenarios 3–5, under 261,716 pax-km, than in Scenario 2 with no alternative services, 343,421 pax-km. Scenarios 3 and 4 have a similar spillover cost of around 255,000 pax-km; however, Scenario 3 has significantly smaller costs for transported passengers showing that passengers are travelling faster (i.e. over shorter routes), 2,048,826 versus 2,246,984 pax-km. Most importantly, Scenario 5 with the spillover cost of 192,200 shows that including both bus-bridging and extra trains provides the best service to passengers among all scenarios. A total of 20 train services are short-turned (Scenarios 2–5) and the bus-bridging links are deployed at full capacity (15 departures per hour in each direction, Scenarios 3 and 5). It is also noticeable that the addition of both bus-bridging services and extra train services in Scenario 5 leads to aggregated benefits in terms of passenger costs. Furthermore, when comparing Scenarios 4 and 5, there is a slight modification of the origins and destinations of the routes of the extra train services leading to a shorter line in Scenario 5.

[1] We allow a maximum of one pair of extra trains to be generated, which may lead to having some non-transported demand.

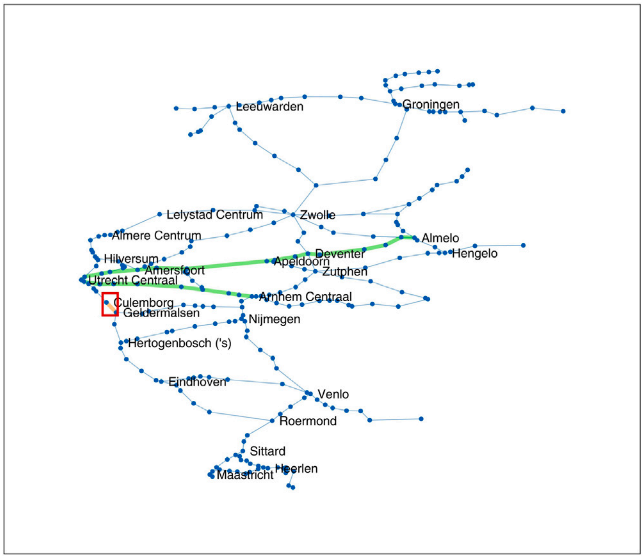

Fig. 8. Undirected graph with bus links in orange and extra train service in green. Red square shows the location of the possession (Experiment 1, Scenario 5). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Fig. 7 shows the passenger flows in the network in Scenario 5. Nodes represent stations in the network while arcs represent the single-direction infrastructure lines connecting the stations. The flow volumes are highlighted with bold lines on the corresponding arc. The disrupted arcs are shown in red, which in this case overlap with the deployed bus arcs.

Fig. 8 visualizes the extra services in the network including bus-bridging (orange line) and train (green line). Buses are used on the disrupted link only. Extra train service runs from Almelo to Nijmegen via Amersfoort and Utrecht (169 km). The representation shows that the generated train service provides, in part of its route (particularly in the southern part, between Utrecht and Nijmegen), an alternative route for some of the affected OD pairs. These passenger flows heading south are rerouted via Arnhem and Nijmegen with the extra train service, from where they can continue their trips with regular train services. The northern part of the route, between Almelo and Utrecht, provides additional capacity in the busy corridor between Amersfoort and Utrecht. This section of the route seemingly may not have a direct relation with the disrupted link, but it is a result of the conditions under which extra trains can be generated, the network configuration and especially the demand patterns. Furthermore, the model tends to stretch the route as much as possible to capture as many passengers as possible within the restriction of 200 kilometres.

Fig. 9 shows the time–distance diagram of the corridor between Utrecht and Geldermalsen (heading south) in Scenario 5. In particular, trains are running between Ut and Cl (blue lines) only due to the possession, whereas bus-bridging services are servicing the disrupted link Cl–Gdm (orange). It also shows that bus-bridging services depart from the transfer node a few minutes after the arrival of a train. Finally, Fig. 10 shows the time-distance diagram of the corridor between Utrecht and Amersfoort, showing the northbound extra train service in the timetable in Scenario 5.

The computational results of the MASP framework are shown in Table 5, presenting the number of iterations for the generation of passenger paths (CG Paths), extra train services (CG Trains) and headway constraints (RG). The framework finds a solution in a limited number of iterations, i.e. at most 5 CG paths iterations were needed. Also, incorporating extra train services in the MASP problem results in (1) an additional iteration of generating passenger paths following the generation of a pair of extra trains, which is a consequence of adding more capacity to the network and thus additional attractive paths for affected passenger flows, and (2) up to two RG iterations to find a feasible timetable. In addition, the generation of passenger paths within CG Paths iterations leads to adding between 138 and 263 new columns, while the generation of one extra train service involves adding a total of 876 new columns. Note that without the preprocessing for extra train services in Algorithm 2, the required number of columns would equal 1,566.

In conclusion, based on the results, including the routing and scheduling of the generated bus-bridging services and extra train services, and the coordination of these with the regular train services in undisrupted areas, we can confirm that the model behaves as expected and shows additional benefits of multimodal alternative services.

5.3. Experiment 2: Demand sensitivity analysis

The second experiment is used to explore the effects of varying levels of demand in the outputs of the MASP problem. We vary the input demand from 25% to 100% with increments of 25 percentage points. The complete demand is 178,936 passengers. For this, we consider the same settings as in experiment 1 (see Table 3) and the same possession between Geldermalsen and Culemborg.

Table 5
Computational results experiment 1.

	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5
	Undisrupted	Disrupted	Disrupted with bus-bridging	Disrupted with Extra Trains	Disrupted with bus-bridging + Extra Trains
Iterations CG Paths	4	4	3	5	5
Generated columns CG Paths	263	226	138	226	169
Iterations CG Trains	0	0	0	1	1
Generated columns CG Trains	0	0	0	876	876
Iterations RG	0	0	0	1	2

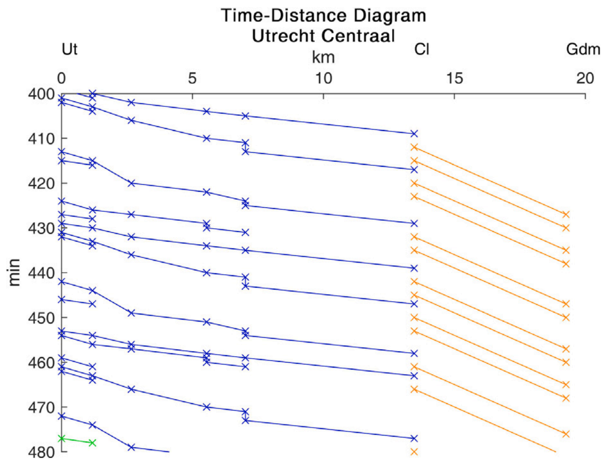

Fig. 9. Time–distance diagram Utrecht Centraal–Geldermalsen. Blue lines represent trains and orange lines represent bus-bridging services (Experiment 1, Scenario 5). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

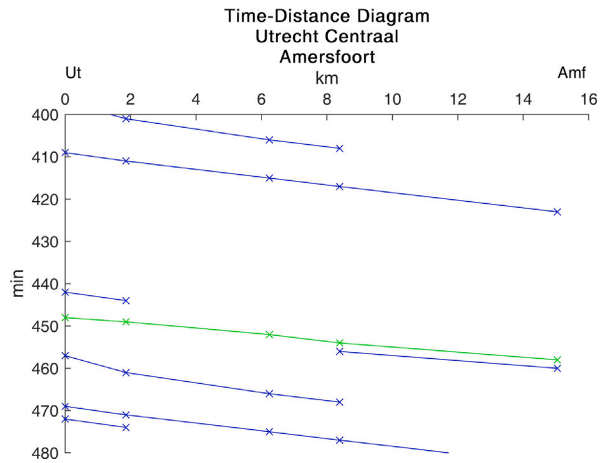

Fig. 10. Time–distance diagram Utrecht Centraal–Amersfoort Centraal, with extra train service depicted in green (Experiment 1, Scenario 5). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

It should be noted that the passenger demand levels are varied based on the same demand patterns, which means that the relative passenger affection due to the closure of the link remains constant (37 OD pairs directly impacted).

The main results are shown in Table 6 presenting the passenger costs (in passenger-kilometres), the number of affected passengers, the percentage of affected passengers, the number of short-turned train services around the disrupted areas, the operating costs of bus-bridging services (in vehicle-kilometres), the number of bus-bridging links, the total number of bus departures (per hour), the operating costs of extra train services (in vehicle-kilometres) and finally the number of extra train services. Clearly, the total passenger cost and the number of affected passengers is increasing with the increasing demand. The results show the same bus-bridging service deployment on each scenario, which already operates at full capacity of 30 departures for a demand level equivalent

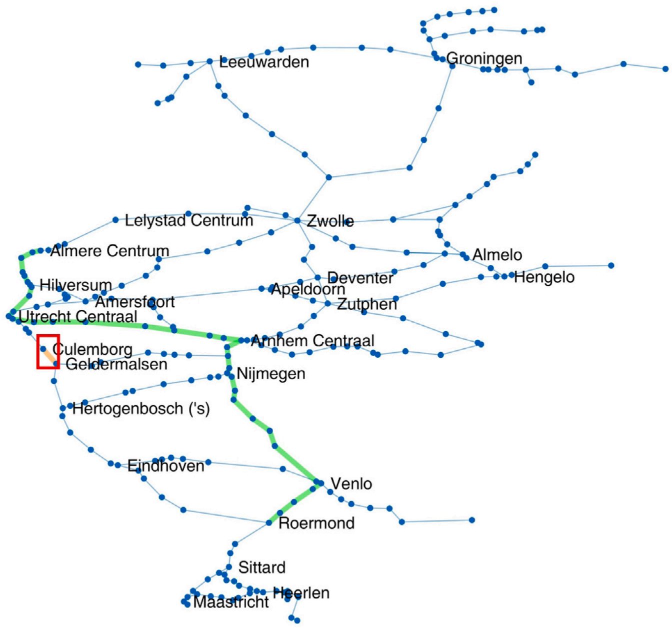

Fig. 11. Undirected graph with bus links in orange and extra train service in green (Experiment 2, Scenario 4). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Table 6
Results experiment 2.

	Scenario 1	Scenario 2	Scenario 3	Scenario 4
	25% Demand	50% Demand	75% Demand	Full demand
Passenger Related				
Passenger costs (pax-km)	1,139,000	2,297,400	3,483,500	4,687,700
Number of affected passengers	2,648	5,289	7,927	10,578
Percentage of affected passengers	5.9%	5.9%	5.9%	5.9%
Transport Services Related				
Short-turned trains	20	20	20	20
Operating costs bus-bridging services (veh-km)	450	450	450	450
Number of bus links	1	1	1	1
Total number of bus departures	30	30	30	30
Operating costs extra train services (veh-km)	338.4	350.4	362.4	381.4
Number of extra train services	2	2	2	2

Table 7
Computational results experiment 2.

	Scenario 1	Scenario 2	Scenario 3	Scenario 4
	25% of the demand	50% of the demand	75% of the demand	Full demand
CPU time (min)	31	33	34	36
Iterations CG Paths	2	3	3	6
Iterations CG Trains	1	1	1	1
Iterations RG	2	2	2	2

to 25% of the demand (15 departures per hour on each direction). Concerning the extra train services, we observe the very similar resulting route in Scenarios 1–3, with trains running along the route from Almelo to Nijmegen via Utrecht (as in Fig. 8). Instead, Scenario 4 shows a significantly different route, from Almere to Hilversum, Utrecht, Arnhem, Nijmegen, and further south to Venlo and Roermond (Fig. 11). In Venlo and Roermond, passenger flows can connect again to the mainline Utrecht–Eindhoven–Maastricht. It should be noted however that all routes share the part of the route between Utrecht and Arnhem, which is probably the most intuitive alternative route heading south, given the link closure on the corridor from Utrecht to Geldermalsen and Hertogenbosch ('s). Also, with increasing the demand, the route length slightly increases.

The computational results are shown in Table 7, which presents the CPU times, the number of iterations for the generation of passenger paths, extra train services and headway constraints. Both the CPU time and the number of iterations for the generation of passenger paths tend to increase with higher demand, ranging from 31 to 36 min. The higher number of iterations needed seems

Table 8
Input possessions for experiments.

Possession	Link ID	Variants with 1 poss.	Variants with 3 poss.	Variants with 5 poss.
(1) Roermond–Echt	3112/13018		B3	C1, C3
(2) Maastricht–Bunde	8216/14058		B1	C1
(3) Geldermalsen–Culemborg	9986/10240	A1	B1	C1, C3
(4) Hertogenbosch ('s)–Hertogenbosch ('s) Oost	10803/23503		B1	C1
(5) Zwolle–Meppel	11010/11264		B2	C1, C3
(6) Arnhem Centraal–Arnhem Zuid	18188/20982	A2	B2	C2
(7) Eindhoven–Geldrop	27490/52128		B3	C2, C3
(8) Almere Centrum–Almere Parkwijk	30978/31232			C2
(9) Amersfoort Centraal–Amersfoort Schothorst	32022/32256	A3	B3	C2, C3
(10) Oosterbeck–Wolfheze	54530/54784		B2	C2

Table 9
Results experiment 3.

	Variants A1–A3	Variants B1–B3	Variants C1–C3
Passenger Related			
Passenger costs (pax-km)	2,355,700	2,317,200	2,281,867
Number of affected OD pairs	42	89	138
Number of affected passengers	6,768	14,927	20,627
Percentage of affected passengers	7.6%	16.7%	23.1%
Transport Services Related			
Short-turned trains	15.7	36.0	62.0
Operating costs bus-bridging services (veh-km)	429	847	1,709
Number of bus links	1.3	2.7	5.0
Average number of departures per link	29.0	27.5	27.5
Total number of bus departures	38.7	73.3	137.5
Operating costs extra train services (veh-km)	339.6	357.0	323.6
Number of extra train services	2	2	2

reasonable since with higher demand levels, the passenger flow capacity constraints on the OD pairs shortest paths play a major role and additional alternative passenger paths are needed.

In conclusion, based on these results, the MASP framework can efficiently cope with different demand levels, which makes it very helpful to further analyse different passenger demand responses to a set of possessions in the railway network.

5.4. Experiment 3: Multiple possessions scenario analysis

We explore the effects of multiple possessions in the MASP problem solutions to investigate the impacts of a significantly disrupted network on passenger flows, the potential of alternative services to reduce passenger costs and lastly possible interdependencies between simultaneous possessions. For this, we create a series of variants with different numbers of simultaneous possessions in the network. In particular, we create 3 variants with 1 possession, 3 variants with 3 possessions and 3 variants with 5 possessions. The list of possessions considered and their associated variants are shown in Table 8 and their location shown in Fig. 6. Also, the variants are created in such a way that the possessions in variants Ax are included in the subset of possessions in variants Bx, and additionally the latter are a subset of the possessions in variants Cx. We assume the demand level of 50% of the complete demand.

The average results for variants A1–A3, B1–B3 and C1–C3 are shown in Table 9, which presents the passenger costs (in passenger-kilometres), the number of affected OD pairs, the number of affected passengers, the number of short-turned train services around the disrupted areas, the operating costs of bus-bridging services (in vehicle-kilometres), the number of bus-bridging links, the total number of bus departures (per hour), the operating costs of extra train services (in vehicle-kilometres) and finally the number of extra train services. Despite the increase in the number of link closures from variants Ax to Cx, the passenger costs remain at very similar levels of magnitude, which suggests that the alternative services perform quite well in routing the affected passengers over the network. With regards to bus-bridging services, one could expect the number of links to be equal to the number of possession (i.e. each possession has a direct replacement link with bus-bridging services), but our results show that from an optimal point of view it is not always the case. In some cases, there are additional possible bus links that are used, especially when the possession is located next to a railway node where multiple railway lines converge. For example, in Variant C2, the model assigned 6 bus-bridging links (see Fig. 13), and it is also the case in B3 and C3. In some other cases, bus-bridging services around the link closure are not even deployed since rerouting passengers using both regular train services over undisrupted areas and extra train services is sufficient. One example of this is the possession between Hertogenbosch ('s) and Hertogenbosch ('s) Oost in variant B1. In relation to the extra train services, there is no evidence that a more disrupted network may lead to either shorter or longer routes. However, it seems logical that it is more difficult to schedule a long route for an extra train service over a very disrupted network, and thus the length may tend to be shorter in these cases. Besides, there are some interesting points to note. First, non-symmetric extra train routes are observed, which means that the associated train services take different routes in each direction of the route. Fig. 14 shows the case

Table 10
Computational results experiment 3.

	Variants A1–A3	Variants B1–B3	Variants C1–C3
Computational time (min)	44	48	53
Iterations CG Paths	5.3	5.3	4.0
Iterations CG Trains	1.0	1.0	1.0
Iterations RG	2.0	2.3	4.0

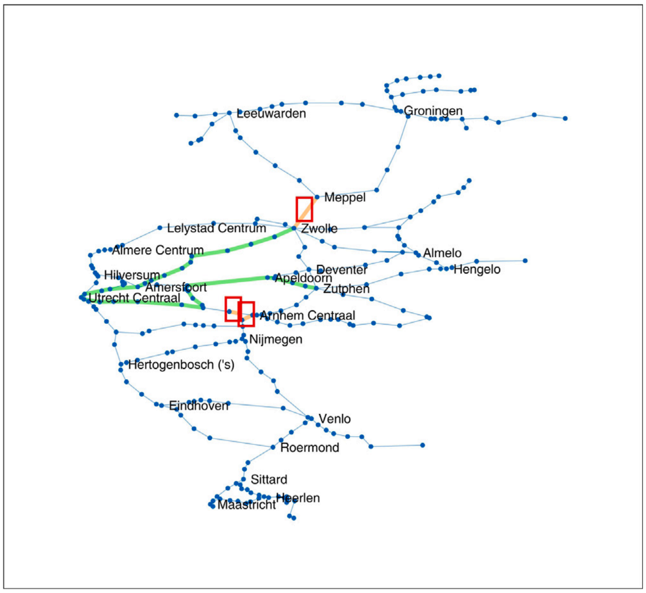

Fig. 12. Undirected graph with bus links in orange and extra train service in green (Experiment 3, Variant B2). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

for C3, in which the extra train services running between Zwolle and Arnhem via Utrecht take different routes between Hilversum and Zwolle. Second, all routes of the extra train services serve Utrecht central station, which is the most important railway node in the Netherlands. This shows that the routing of extra train service is highly dependent on the passenger demand flows.

The alternative services generated in Experiments B2 and C2 are shown in Fig. 12 and Fig. 13 respectively. The locations of the associated possessions are marked in the figures with a red square. It should be noted that in both experiments we have two possessions next to Arnhem, but these are located on different corridors, one on the corridor to Utrecht and the other one on the corridor to Nijmegen.

The computational results are shown in Table 10 including the CPU times and the number of iterations for the generation of passenger paths (CG Paths), extra train services (CG Trains) and headway constraints (RG). It is observed that the average computational time to solve the MASP problem increases with the number of disrupted links in the network, which is a result of more computationally restricted problem variants. Nevertheless, there is no evidence that a more disrupted network results in a higher need for iterations to generate passenger paths. By contrast, Variants Cx have clearly resulted in a significantly higher number of iterations for the generation of headway constraints. This increase could be explained by more adjustments required in a more disrupted railway network in order to generate a conflict-free solution. Therefore, the increased need for row generation iterations is found to be the main cause of a higher computational time.

Finally, we evaluated the number of generated passenger paths through iterations of the MASP framework. In these experiments, we considered a total of 711 passenger OD pairs, and each of these is initially assigned to its shortest path. Table 11 shows the generated passenger paths results and Fig. 15 visualizes the number of additional passenger paths generated at each column generation iteration (CG paths and CG trains combined, ordered according to the algorithm progress) for each of the variants analysed. Depending on the variant, a total of new passenger paths ranging approximately from 150 to 350 have been generated (around 20% to 50% of the initial number of paths). In general, a certain number of additional iterations are performed for the generation of passenger paths after the generation of a pair of extra train services, which on average account for 6% of the total of new paths, supporting our previous findings in Section 5.2. Additionally, we observe that on average the number of paths generated tends to decrease in highly disrupted networks. Looking at Fig. 15, most new paths are generated in the first two iterations.

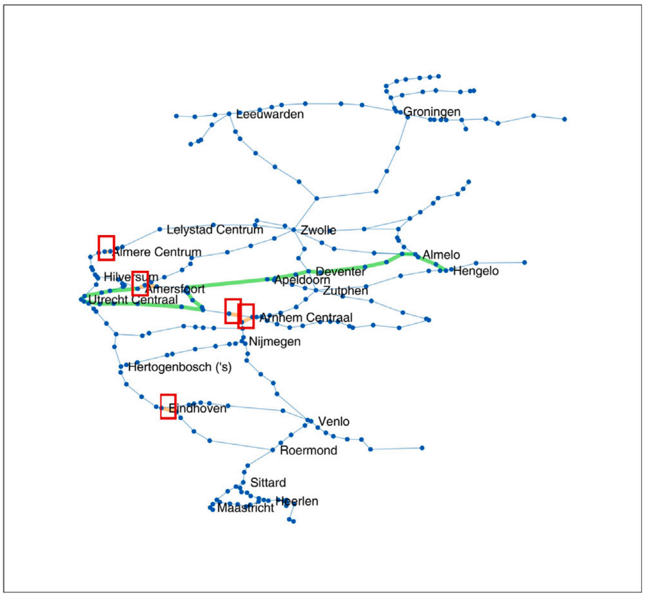

Fig. 13. Undirected graph with bus links in orange and extra train service in green (Experiment 3, Variant C2). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

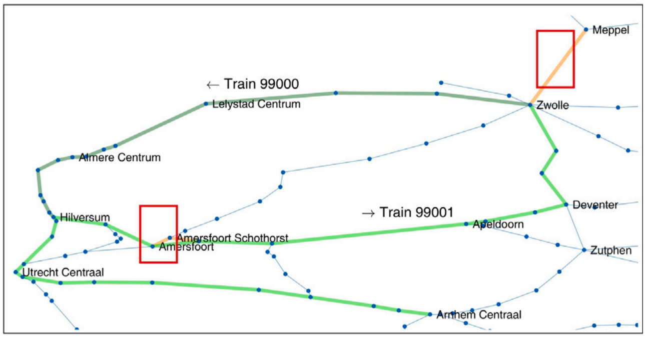

Fig. 14. Detail of Variant C3. Extra train services taking non-symmetric routes between Hilversum and Zwolle (highlighted in green and dark green; green is also common route from Arnhem Centraal to Hilversum). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Table 11
Generation of passenger paths results.

	Variants A1–A3	Variants B1–B3	Variants C1–C3
Initial number of paths generated	127.0	125.3	112.7
Number of paths generated after CG Trains	16.3	9.0	9.3
Total number of paths generated	225.7	221.3	171.3

6. Discussion

We carried out three experiments to evaluate the performance of the MASP solution framework The results give interesting insights into the design of the bus-bridging service network, the design of extra train services and other practical aspects.

With regards to the design of the bus-bridging service networks, the experiments carried out show that the selected bus-bridging links are often a direct replacement of a closed rail link, especially in open track sections. Nevertheless, when the possession is located next to a node of the network with convergence of more than two links, we may also observe bus-bridging arcs between nodes not directly connected by rail (in the undisrupted state). This proves the convenience of taking into account the entire set of possible bus-bridging arcs around the railway nodes next to a possession.

Regarding the generation of extra train services, the experiments show the importance of selecting a subset of railway arcs beforehand to ensure that extra train services actually target those passenger flows directly affected by a link closure. Also, the

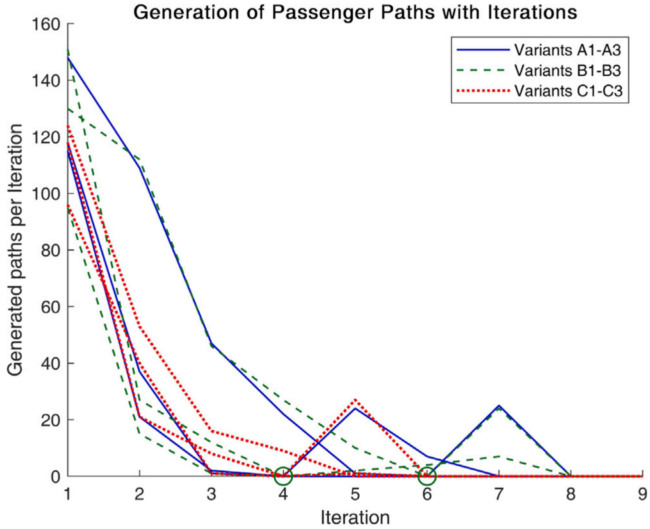

Fig. 15. Generation of Passenger Paths per Iteration (CG Paths and CG Trains combined). Circles on x-axis indicate iteration for CG Trains. Note that circles overlap on iterations 4 and 6.

preprocessing in Algorithm 2 has positive effects from the computational point of view as for each extra train service we use significantly less associated decision variables. Furthermore, another interesting feature of the MASP framework is that the generated pairs of extra train services do not necessarily have to follow symmetric routes, which means that trains in opposite directions can take different routes as long as their origins and destinations are symmetric. This feature gives more flexibility to the MASP framework to cope with non symmetric demand patterns. This is observed in some of the examples presented, such as variants B3 and C3 in Experiment 3.

When looking at the solution framework, we often observe in the experiments additional iterations for the generation of passenger paths right after the generation of extra train services (see Fig. 5, from step 6 to 2). This shows that extra train services may lead to new alternative attractive paths for passenger flows.

From a practical point of view, our results show that the potential of bus-bridging services to provide an alternative for the affected passenger flows due to a link closure may be rather limited in mainline railways. This is typically due to the limited capacity of bus links available (in the experiments, 1,000 passengers per link), while undisrupted train flows on the same link can be tens of thousands of passengers. Therefore, even for lower demands, this may result in the bus-bridging services operating at full capacity. This supports our reasoning that combining the bus-bridging alternative with extra train services may be an appropriate solution to overcome the limitations of bus-bridging services. Also, another possibility to reduce the negative effects of the bus-bridging capacity limits is to incorporate the k-shortest path algorithm to generate more bus links in the preprocessing described in Algorithm 3, in order to increase the overall capacity of bus-bridging services.

7. Conclusions

In this paper, we addressed the problem of planning temporary alternative services during infrastructure maintenance possessions in railway networks. For this, we developed the Multimodal Alternative Services for Possessions (MASP) framework to design optimized alternative services for a given set of link closures, including passenger routing, adjusted train timetable, bus-bridging services and extra train services. We used the principles of the Service Network Design Problem and the Vehicle Routing Problem, mixed-integer linear programming and heuristics based on the multi-column generation in order to efficiently add passenger paths and extra train services, as well as the row generation to include infrastructure constraints.

The application on a part of the Dutch passenger railway network showed the promising potential of alternative services to minimize the inconveniences to passengers affected by railway possessions, especially when both bus-bridging services and extra train services are considered. We have shown the flexibility of the MASP framework to design different combinations of bus-bridging routes and extra train service routes. The experiments also showed the computational efficiency of the developed framework even for highly disrupted networks.

Possible further research directions have been identified. First, to guarantee operational feasibility of the solutions, a microscopic modelling counterpart could be considered (e.g. Looij et al., 2020) which would lead to more advanced solution frameworks (such as Bešinović et al., 2016; Burggraeve and Vansteenwegen, 2017). Second, from the operation of transport services point of view, taking into account the capacity of both short-turning train stations and bus holding points at transfer stations could provide designing more realistic alternative services. Third, with respect to the modelling of passenger flows, detailed transfers of passengers between transport services in the MASP problem can be modelled. In this way, transfer and waiting times could be incorporated in passenger travel times. Fourth, passenger demand could be modelled to specific transport services to allow for more realistic passenger flows routing over the network and capturing preferences in arrival and departure times, which would lead to extending the current solution framework. Fifth, passenger choice behaviour may be taken into account also for a more realistic passenger flows routing. Finally, from an infrastructure point of view, it would be interesting to extend the model to incorporate partial track closures, such as one track in double-track sections or two tracks in four-track sections.

The developed solution framework can be used as a decision support tool by railway planners as part of the planning of possessions in the railway network. The experiments carried out show that the framework is useful in providing the optimal decisions and guidelines to design alternative services. Consequently, this research is a convenient contribution to manage the increasing need for railway infrastructure maintenance and construction works from the passenger and transport operator perspective and to more resilient railway systems.

CRediT authorship contribution statement

Jacob Trepat Borecka: Conceptualization, Methodology, Data curation, Formal analysis, Investigation, Visualization, Writing - original draft. **Nikola Bešinović:** Conceptualization, Methodology, Writing - review & editing.

Appendix

Table 12 gives the notation used in the MASP problem.

The pseudocodes for generating extra trains and bus-bridging services are given in Algorithm 2 and 3 respectively.

In Algorithm 2, for each OD pair, we evaluate the shortest path cost in both the undisrupted ($c^k_{o,ud}$) and the disrupted railway network ($c^k_{o,d}$) in order to identify those passenger OD pairs that are actually affected by possessions. The shortest path costs are calculated as the sum of the individual arc costs c_{ij} along the respective shortest path. If the shortest path in the disrupted network is greater than the shortest path in the undisrupted network, it means that the OD pair is affected by the possessions. Then, for each of the affected OD pairs, we add the arcs included in the alternative shortest path (in $G^T = (N, A^T \setminus A^d)$, where A^d is the set of disrupted arcs) to the subset A^{EXT}. It could also be the case that a possession disconnected a set of nodes from the rest of the network and that consequently the shortest path in the railway network for a specific OD pair is infeasible. In this case, no arcs can be added to the subset A^{EXT}. Additionally, we check whether all the hub nodes are reachable using the arcs in the subset A^{EXT}. If they are not, then we calculate the shortest path from the hub node to the closest node in the subset N^{EXT} (associated with the arcs A^{EXT}) and add the arcs and nodes contained in the shortest path to the subsets A^{EXT} and N^{EXT} respectively so that the hub node is reachable. Besides, for all links we check whether we have added the associated arcs in both directions or not. If a link is only added in one direction, then we add the associated return link to the subset A^{EXT}. This is meant to facilitate the routing of trains in both directions. Finally, we delete duplicated train arcs. The resulting subset represents the arcs that can be used to route extra train services.

Algorithm 2 Extra Trains Arcs Subset Generation Preprocessing

Input: $G^T = (N, A^T), A^d, N^h$ OD pairs and all parameters
Output: subset of extra train arcs A^{EXT} and extra train nodes N^{EXT}
for All ODs pairs K **do**
  Calculate the shortest path cost $c^k_{o,ud}$ in the undisrupted railway network $G^T = (N, A^T)$
  Calculate the shortest path cost $c^k_{o,d}$ in the disrupted railway network $G^T = (N, A^T \setminus A^d)$
  if $c^k_{o,ud} < c^k_{o,d}$ **then**
    Add train arcs from the shortest path to the subset A^{EXT}
  else if the shortest path in the disrupted railway network $G^T = (N, A^T \setminus A^d)$ is infeasible **then**
    No arcs are added to the subset A^{EXT}
  end if
end for
for all nodes in the subset N^h **do**
  if the node is not reachable using the arcs in the subset A^{EXT} **then**
    Calculate the shortest path from the hub node to the closest node in the subset N^{EXT} and add arcs and nodes contained in the shortest path to the subsets A^{EXT} and N^{EXT} respectively
  end if
end for
for all arcs in the subset A^{EXT} **do**
  if the link is added only in one direction in the subset A^{EXT} **then**
    Add the return arc
  end if
end for
Delete duplicated train arcs

In Algorithm 3, for each OD pair, we evaluate the shortest path cost in both the undisrupted ($c^k_{o,ud}$) and the disrupted railway network ($c^k_{o,d}$) in order to identify those passenger OD pairs that are actually affected by possessions. If the shortest path in the disrupted network is greater than the shortest path in the undisrupted network or it is simply infeasible, it means that the OD pair is affected by the possessions. Then, for each of the ODs affected by possessions, we check if the use of road arcs A^R (i.e. by using the multimodal network G^M) can bring a reduction in their disrupted shortest path cost $c^k_{o,d}$. The shortest path cost in the multimodal network is defined as $c^k_{o,m}$. If true, we add the arc to our subset A^r. The resulting subset represents the arcs that can be used to route bus-bridging services.

Algorithm 3 Bus-bridging Arcs Subset Generation Preprocessing

Input: $G^T = (N, A^T), G^M = (N, A^T \setminus A^d \cup A^R), A^d, A^p$ and all parameters
Output: subset of bus-bridging arcs A^r and nodes N^r
for All ODs pairs K **do**
  Calculate the shortest path cost $c^k_{o,ud}$ in the undisrupted railway network $G^T = (N, A^T)$
  Calculate the shortest path cost $c^k_{o,d}$ in the disrupted railway network $G^T = (N, A^T \setminus A^d)$
  if $c^k_{o,ud} < c^k_{o,d}$ or $c^k_{o,d}$ is infeasible **then**
    Calculate the shortest path cost $c^k_{o,m}$ in the multimodal network $G^M = (N, A^T \setminus A^d \cup A^R)$
    if $c^k_{o,m} < c^k_{o,d}$ **then**
      Add road arcs contained in $c^k_{o,m}$ to the subset A^r, in both directions
    end if
  end if
end for
Delete duplicated bus arcs

Table 12
Notation used in the MASP problem.

Sets Passengers		**Parameters Passengers**	
K	set of OD pairs	c_{ij}	travel cost on arc (i,j)
P	set of passenger paths	c^k_p	cost of path p for passenger of OD pair k
P^k	set of passenger paths of OD pair k	$c^k_{o,m}$	shortest path cost for passengerof OD pair k in the multimodal network
A	set of arcs	$s^{t,b}_i$	transfer time in node i between train t andbus b
A^p	set of arcs in path p	d_k	demand of OD pair k
$N(O_k)$	origin node of OD pair k	δ^p_{ij}	1, if arc (i,j) is included in path p, 0otherwise
$N(D_k)$	destination node of OD pair k	v_{ij}	1, if arc (i,j) is disrupted, 0 otherwise
Sets Transport Services		**Parameters Transport Services**	
N^m	set of nodes on transport service$m$ route	s^m	number of seats in transport mode m
$N(O_t)$	origin node of train route t	t^t_i	dwell time in node i for train t
$N(D_t)$	destination node of train route t	τ^t_{ij}	minimum running time on link (i,j) for train t
N^h	set of hub nodes	M	large constant
N^{tf}	set of transfer stations train-bus	U_{ij}	upper bound of bus frequencies per hour on arc (i,j)
A^m	set of arcs on transport service m	LF	average load factor
M	set of transport services	$T^m_{D,i}$	scheduled departure time of transportservice m at node i
T	set of train services	$T^m_{A,i}$	scheduled arrival time of transport service$m$ at node i
T^{RR}	subset of trains, which can bererouted	a_i	time window lower bound
T^{ST}	subset of trains, which can beshort-turned	b_i	time window upper bound
T^{EXT}	subset of extra trains	**Parameters Network**	
T^{EXT-P}	set of extra train pairs	CAP_{ij}	capacity in arc (i,j)
D_{max}	maximum route length for extra trains	$h^{t,m}_{ij}$	minimum headway between train t and$m$ on arc (i,j)
B	set of bus services	$g^{b,c}_i$	minimum headway between bus b and$c$ at node i
Sets Network		**Decision Variables Passengers**	
G^T	infrastructure railway network	f^k_p	share of demand of OD pair k transported over path p

(*continued on next page*)

Table 12 (*continued*).

G^R	infrastructure bus network	**Decision Variables Transport Services**	
G^M	infrastructure multimodal network	o_i^m	1 if node i is origin oftransport service m, 0 otherwise
N	set of nodes	d_i^m	1 if node i is destination oftransport service m, 0 otherwise
A^T	set of railway arcs	x_{ij}^m	transport service flow from i to j oftransport service m
A^d	set of disrupted railway arcs	$T_{\bar{D},i}^m$	actual departure time of transport service$m$ at node i
A^R	set of road arcs	$T_{\bar{A},i}^m$	actual arrival time of transport service m atnode i

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