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Reciprocity and representation theorems for rotational seismology

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SUMMARY

Recently, there has been an increasing interest in employing rotational motion measurements for seismic source inversion, structural imaging and ambient noise analysis. We derive reciprocity and representation theorems for rotational motion. The representations express the rotational motion inside an inhomogeneous anisotropic earth in terms of translational and rotational motion at the surface. The theorems contribute to the theoretical basis for rotational seismology methodology, such as determining the moment tensor of earthquake sources.

Key words: Computational seismology; Rotational seismology; Theoretical seismology.

1 INTRODUCTION

Measurements of the seismic wave field are traditionally restricted to three mutually perpendicular components of the particle velocity (also called translational motion). Observational studies by Igel *et al.* (2007), Lin *et al.* (2011) and others have demonstrated the potential of additionally measuring three components of the rotational motion. Recently, researchers have been exploring the advantages of rotational seismology in source localization and inversion (Bernauer *et al.* 2014; Donner *et al.* 2016; Li & van der Baan 2017b; Ichinose *et al.* 2021), structural imaging (Bernauer *et al.* 2009; Abreu *et al.* 2023), ambient noise analysis (Hadziioannou *et al.* 2012; Paitz *et al.* 2019) and exploration geophysics (Li & van der Baan 2017a; Schmelzbach *et al.* 2018).

Reciprocity and representation theorems for translational motion (de Hoop 1966; Aki & Richards 1980; Fokkema & van den Berg 1993) have been employed as a theoretical basis for the development of methodologies for seismic imaging, inverse scattering, source characterization, seismic holography, multiple elimination, Green's function retrieval, etc. Given the current interest in rotational seismology, it is opportune to derive reciprocity and representation theorems for rotational motion. An important step in this direction has been made by Li & van der Baan (2017b). In their derivation they assume the medium is homogeneous and isotropic. In this paper we derive several forms of reciprocity and representation theorems for translational and rotational motion in an inhomogeneous anisotropic earth. These theorems complement the theoretical basis for the development of methodologies for rotational seismology.

2 RECIPROCITY THEOREMS FOR ROTATIONAL SEISMOLOGY

The rotational motion-rate vector in an inhomogeneous anisotropic medium is defined as $\dot{\mathbf{\Omega}} = \frac{1}{2}\nabla \times \mathbf{v}$, where $\mathbf{v}$ is the particle velocity vector. This definition holds in the space–time $(\mathbf{x}, t)$ domain as well as in the space–frequency $(\mathbf{x}, \omega)$ domain. In the following, all expressions are in the space–frequency domain. Note that for the special case of a homogeneous isotropic medium, $\dot{\mathbf{\Omega}}$ would represent the S-wave part of $\mathbf{v}$.

Consider a spatial domain $\mathbb{D}$ enclosed by boundary $\partial\mathbb{D}$ with outward pointing normal vector $\mathbf{n}$. A reciprocity theorem for elastic wave fields in a homogeneous isotropic medium in this domain reads

$$\oint_{\partial\mathbb{D}} \rho\Big[c_P^2\{\mathbf{v}_A\nabla\cdot\mathbf{v}_B - \mathbf{v}_B\nabla\cdot\mathbf{v}_A\} + c_S^2\{\mathbf{v}_A\times\nabla\times\mathbf{v}_B - \mathbf{v}_B\times\nabla\times\mathbf{v}_A\}\Big]\cdot\mathbf{n}d^2\mathbf{x} = i\omega\int_{\mathbb{D}}\{\mathbf{v}_A\cdot\mathbf{f}_B - \mathbf{v}_B\cdot\mathbf{f}_A\}d^3\mathbf{x}. \quad (1)$$

Here $\mathbf{v}(\mathbf{x},\omega)$ is the particle velocity vector, $\mathbf{f}(\mathbf{x},\omega)$ the force source vector, c_P and c_S are the P- and S-propagation velocities, ρ is the mass density and i the imaginary unit. Upper-case subscripts A and B denote two independent states, which can be physical or mathematical wave fields (or a combination thereof), emitted by different sources. Eq. (1) is a slightly modified form of a theorem formulated by Knopoff (1956). Because it explicitly contains $\nabla\times\mathbf{v} = 2\dot{\mathbf{\Omega}}$ in both states, Li & van der Baan (2017b) used this as the starting point for deriving representations for rotational seismology. A limitation is that eq. (1) was derived from the elastic wave equation for a homogeneous isotropic medium. Here we show that its derivation can be generalized for an inhomogeneous anisotropic medium in $\mathbb{D}$, with only some restrictions on the medium parameters at the boundary $\partial\mathbb{D}$.

The Betti–Rayleigh reciprocity theorem for elastic wave fields in an inhomogeneous anisotropic medium in $\mathbb{D}$ reads (de Hoop 1966; Aki & Richards 1980)

$$\oint_{\partial\mathbb{D}}\{v_{i,A}\tau_{ij,B} - v_{i,B}\tau_{ij,A}\}n_j d^2\mathbf{x} = -\int_{\mathbb{D}}\{v_{i,A}f_{i,B} - v_{i,B}f_{i,A}\}d^3\mathbf{x}. \quad (2)$$

Here v_i, τ_{ij}, f_i and n_j are components of the particle velocity vector $\mathbf{v}$, stress tensor $\boldsymbol{\tau}$, force source vector $\mathbf{f}$ and normal vector $\mathbf{n}$, respectively. Einstein's summation convention applies to repeated lower-case subscripts. Our aim is to recast eq. (2) into the form of eq. (1). We start by expressing the stress at the boundary $\partial\mathbb{D}$ in terms of the particle velocity. Because the boundary integral in eq. (1) contains the isotropic velocities c_P and c_S, for our derivation we assume that the medium is isotropic in a vanishingly thin shell around $\partial\mathbb{D}$. Hence, we use the isotropic stress–velocity relation $\tau_{ij} = -\frac{1}{i\omega}\{\lambda\delta_{ij}\partial_k v_k + \mu(\partial_j v_i + \partial_i v_j)\}$, where $\lambda(\mathbf{x})$ and $\mu(\mathbf{x})$ are the Lamé parameters in the thin shell around $\partial\mathbb{D}$. Substituting this into eq. (2) for both states, we obtain after some manipulations

$$\oint_{\partial\mathbb{D}}\Big[\lambda\{\mathbf{v}_A\nabla\cdot\mathbf{v}_B - \mathbf{v}_B\nabla\cdot\mathbf{v}_A\} + 2\mu\{(\mathbf{v}_A\cdot\nabla)\mathbf{v}_B - (\mathbf{v}_B\cdot\nabla)\mathbf{v}_A\}$$
$$+\mu\{\mathbf{v}_A\times\nabla\times\mathbf{v}_B - \mathbf{v}_B\times\nabla\times\mathbf{v}_A\}\Big]\cdot\mathbf{n}d^2\mathbf{x}$$
$$= i\omega\int_{\mathbb{D}}\{\mathbf{v}_A\cdot\mathbf{f}_B - \mathbf{v}_B\cdot\mathbf{f}_A\}d^3\mathbf{x}. \quad (3)$$

For a more detailed derivation see the online material. This expression has the form of eq. (1), except for the second term on the left-hand side. This term can be reorganized into the form of the first term, using the theorem of Gauss, if we assume μ is constant along the boundary $\partial\mathbb{D}$ (see the online material for details). Using $\lambda + 2\mu = \rho c_P^2$ and $\mu = \rho c_S^2$, we thus obtain eq. (1), this time for an inhomogeneous anisotropic medium in $\mathbb{D}$; only in a vanishingly thin shell around the boundary $\partial\mathbb{D}$ the medium is assumed to be isotropic, with μ constant along $\partial\mathbb{D}$.

In order to use eq. (1) as a basis for backpropagation, we replace the quantities in state A by their complex-conjugates (denoted by asterisks), which is allowed when the medium in $\mathbb{D}$ is lossless. Using $\dot{\boldsymbol{\Omega}} = \frac{1}{2}\nabla\times\mathbf{v}$ and defining the cubic dilatation-rate as $\dot{\Theta} = \nabla\cdot\mathbf{v}$ we thus obtain

$$\oint_{\partial\mathbb{D}}\rho\Big[c_P^2\{\mathbf{v}_A^*\dot{\Theta}_B - \mathbf{v}_B\dot{\Theta}_A^*\} + 2c_S^2\{\mathbf{v}_A^*\times\dot{\boldsymbol{\Omega}}_B - \mathbf{v}_B\times\dot{\boldsymbol{\Omega}}_A^*\}\Big]\cdot\mathbf{n}d^2\mathbf{x}$$
$$= i\omega\int_{\mathbb{D}}\{\mathbf{v}_A^*\cdot\mathbf{f}_B + \mathbf{v}_B\cdot\mathbf{f}_A^*\}d^3\mathbf{x}. \quad (4)$$

3 REPRESENTATION THEOREMS FOR ROTATIONAL SEISMOLOGY

A representation theorem is obtained by choosing for one of the states in a reciprocity theorem a Green's state (Gangi 1970). Hence, for state A, we replace $\mathbf{v}_A$ by the Green's velocity vector $\mathbf{G}_{v,f_n}(\mathbf{x},\mathbf{x}_A,\omega)$, defined as the response at $\mathbf{x}$ to a unit force source in the x_n-direction at $\mathbf{x}_A$ in $\mathbb{D}$, i.e., $f_{i,A}(\mathbf{x},\omega) = \delta_{in}\delta(\mathbf{x}-\mathbf{x}_A)$. Moreover, we replace $\dot{\boldsymbol{\Omega}}_A$ and $\dot{\Theta}_A$ by the Green's rotational motion-rate vector and cubic dilatation-rate, defined as $\mathbf{G}_{\dot{\Omega},f_n}(\mathbf{x},\mathbf{x}_A,\omega) = \frac{1}{2}\nabla\times\mathbf{G}_{v,f_n}(\mathbf{x},\mathbf{x}_A,\omega)$ and $G_{\dot{\Theta},f_n}(\mathbf{x},\mathbf{x}_A,\omega) = \nabla\cdot\mathbf{G}_{v,f_n}(\mathbf{x},\mathbf{x}_A,\omega)$. As usual, the second subscript of a Green's function refers to the source type at $\mathbf{x}_A$ (here a force f_n), whereas the first subscript (v, $\dot{\Omega}$ or $\dot{\Theta}$) refers to the type of response at $\mathbf{x}$. For state B we take the actual physical state and drop the subscripts B. Choosing the source distribution $\mathbf{f}(\mathbf{x},\omega)$ of the actual state outside $\mathbb{D}$, we thus obtain from eq. (4)

$$v_n(\mathbf{x}_A,\omega) = \frac{1}{i\omega}\oint_{\partial\mathbb{D}}\rho\Big[c_P^2\{\mathbf{G}_{v,f_n}^*\dot{\Theta} - G_{\dot{\Theta},f_n}^*\mathbf{v}\}$$
$$+2c_S^2\{\mathbf{G}_{v,f_n}^*\times\dot{\boldsymbol{\Omega}} + \mathbf{G}_{\dot{\Omega},f_n}^*\times\mathbf{v}\}\Big]\cdot\mathbf{n}d^2\mathbf{x}. \quad (5)$$

This is a representation of the particle velocity component $v_n(\mathbf{x}_A,\omega)$ at $\mathbf{x}_A$ in $\mathbb{D}$, expressed in terms of the wave fields $\mathbf{v}(\mathbf{x},\omega)$, $\dot{\boldsymbol{\Omega}}(\mathbf{x},\omega)$ and $\dot{\Theta}(\mathbf{x},\omega)$ at $\partial\mathbb{D}$. Next, we derive a representation of the rotational motion-rate component $\dot{\Omega}_h(\mathbf{x}_A,\omega)$. Using the subscript notation for the curl-operator, this component is defined as $\dot{\Omega}_h(\mathbf{x}_A,\omega) = \frac{1}{2}\epsilon_{hmn}\partial_{m,A}v_n(\mathbf{x}_A,\omega)$, where ϵ_{hmn} is the Levi-Civita symbol and where $\partial_{m,A}$ denotes differentiation with respect to $x_{m,A}$. Applying the operator $\frac{1}{2}\epsilon_{hmn}\partial_{m,A}$ to both sides of eq. (5), interchanging the order of integration (over $\mathbf{x}$) and differentiation (with respect to $x_{m,A}$), yields

$$\dot{\Omega}_h(\mathbf{x}_A,\omega) = \frac{1}{i\omega}\oint_{\partial\mathbb{D}}\rho\Big[c_P^2\{\mathbf{G}_{v,\dot{\Omega}_h}^*\dot{\Theta} - G_{\dot{\Theta},\dot{\Omega}_h}^*\mathbf{v}\}$$
$$+2c_S^2\{\mathbf{G}_{v,\dot{\Omega}_h}^*\times\dot{\boldsymbol{\Omega}} + \mathbf{G}_{\dot{\Omega},\dot{\Omega}_h}^*\times\mathbf{v}\}\Big]\cdot\mathbf{n}d^2\mathbf{x}, \quad (6)$$

where $\mathbf{G}_{\dot{\Upsilon},\dot{\Omega}_h}(\mathbf{x},\mathbf{x}_A,\omega) = \frac{1}{2}\epsilon_{hmn}\partial_{m,A}\mathbf{G}_{\dot{\Upsilon},f_n}(\mathbf{x},\mathbf{x}_A,\omega)$, with subscript $\dot{\Upsilon}$ standing for v, $\dot{\Omega}$ or $\dot{\Theta}$. Note that here the operator $\frac{1}{2}\epsilon_{hmn}\partial_{m,A}$ transforms the force-source of the Green's function into a rotational motion source (which for the special case of a homogeneous isotropic medium would correspond to a S-wave source). A representation of the cubic dilatation-rate $\dot{\Theta}(\mathbf{x}_A,\omega)$ can be derived in a similar way by applying the operator $\partial_{n,A}$ to both sides of eq. (5), but this is beyond the scope of this paper.

The representations of eqs (5) and (6) are exact. In practice, however, measurements are not available on a closed boundary but, say, on a horizontal boundary $\partial\mathbb{D}_0$ (with upwards pointing normal vector $\mathbf{n} = (0,0,-1)$), see Fig. 1. We define a second horizontal boundary $\partial\mathbb{D}_1$ between $\mathbf{x}_A$ and the source distribution $\mathbf{f}(\mathbf{x},\omega)$ of the actual state. The boundaries $\partial\mathbb{D}_0$ and $\partial\mathbb{D}_1$ (together with a cylindrical boundary with a vertical axis through $\mathbf{x}_A$ and infinite radius) form the closed boundary $\partial\mathbb{D}$. Because measurements are available only on $\partial\mathbb{D}_0$, in practice we neglect the integral over $\partial\mathbb{D}_1$ and approximate eq. (6) by an integral over $\partial\mathbb{D}_0$. This is a suitable approximation for backpropagation of the wave field from $\partial\mathbb{D}_0$ to $\mathbf{x}_A$. It is illustrated in Fig. 1(a), where the downwards pointing arrows represent the complex conjugated (i.e., backpropagating) Green's functions. These Green's functions are defined in the inhomogeneous anisotropic medium in $\mathbb{D}$, but for simplicity they are visualized by straight rays. Since the integral over $\partial\mathbb{D}_1$ is neglected, evanescent waves are ignored, internal multiples are erroneously handled and the recovered primary wave field at $\mathbf{x}_A$ contains small amplitude errors, proportional to the amplitudes of internal multiples (Wapenaar & Haimé 1990). For a weakly scattering medium these approximations are acceptable and of the same order as those of the standard elastodynamic Kirchhoff–Helmholtz integral for backpropagation (Kuo & Dai 1984; Hokstad 2000).

Although we have achieved our goal (i.e. deriving a representation in terms of translational and rotational motion for an inhomogeneous anisotropic medium), eq. (6) (with $\partial\mathbb{D}$ replaced by $\partial\mathbb{D}_0$) is still rather complex. This expression simplifies significantly when the medium at and above $\partial\mathbb{D}_0$ is homogeneous and isotropic. At and above $\partial\mathbb{D}_0$ we express the particle velocity as $\mathbf{v} = -\frac{c_P^2}{\omega^2}\nabla\dot{\Theta} + \frac{2c_S^2}{\omega^2}\nabla\times\dot{\boldsymbol{\Omega}}$, see the online material for details. Note that here $\dot{\Theta}$ and $\dot{\boldsymbol{\Omega}}$ are scaled versions of P- and S-wave potentials. We substitute this expression, and a similar expression for $\mathbf{G}_{v,\dot{\Omega}_h}$, into the right-hand side of eq. (6). Using the fact that the actual wave field and the Green's functions are upwards propagating at $\partial\mathbb{D}_0$ (and hence the complex conjugated Green's functions are downwards propagating at $\partial\mathbb{D}_0$), we can use one-way wave equations for P- and S-waves at $\partial\mathbb{D}_0$ to simplify the right-hand side of

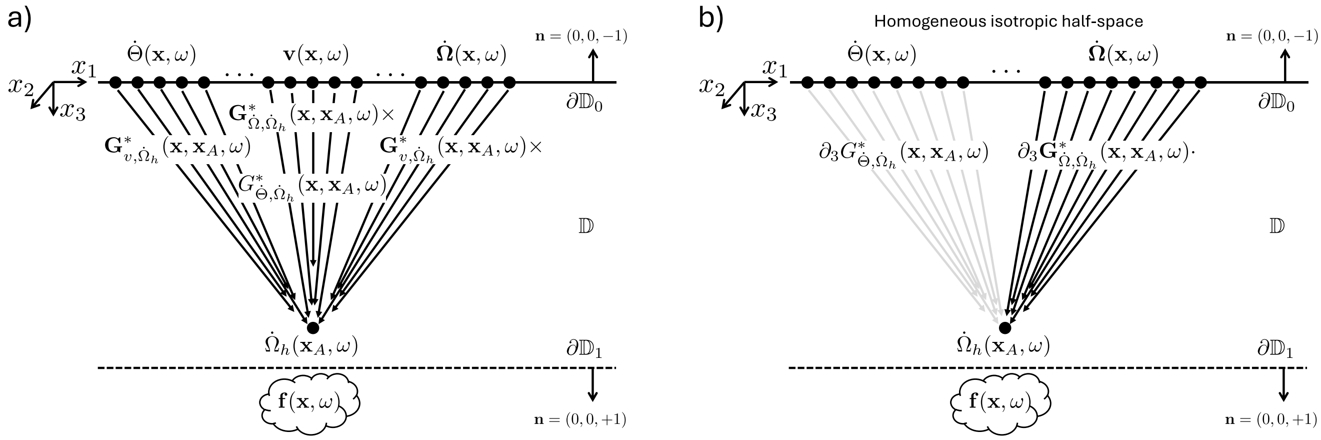

Figure 1. (a) Illustration of eq. (6) (with $\partial\mathbb{D}$ replaced by $\partial\mathbb{D}_0$) for backpropagation. (b) Illustration of eqs (7) and (8). The amplitude of the "converted" Green's function $G_{\dot{\Theta},\dot{\Omega}_h}(\mathbf{x},\mathbf{x}_A,\omega)$ (light-grey) is one order of magnitude lower than that of the "non-converted" Green's function $\mathbf{G}_{\dot{\Omega},\dot{\Omega}_h}(\mathbf{x},\mathbf{x}_A,\omega)$.

eq. (6). This yields (ignoring evanescent waves)

$$\dot{\Omega}_h(\mathbf{x}_A,\omega) \approx \frac{2}{i\omega^3}\int_{\partial\mathbb{D}_0} \rho\Big[c_P^4(\partial_3 G^*_{\dot{\Theta},\dot{\Omega}_h})\dot{\Theta} + 4c_S^4(\partial_3 \mathbf{G}^*_{\dot{\Omega},\dot{\Omega}_h})\cdot\dot{\mathbf{\Omega}}\Big]d^2\mathbf{x}, \tag{7}$$

see the online material for a detailed derivation. This expression is illustrated in Fig. 1(b) (a similar expression was previously derived in a somewhat different way for P- and S-wave potentials by Wapenaar & Haimé 1990). Note that the Green's function $G_{\dot{\Theta},\dot{\Omega}_h}(\mathbf{x},\mathbf{x}_A,\omega)$ stands for the cubic dilatation-rate at $\mathbf{x}$ in response to a rotational motion source at $\mathbf{x}_A$. The amplitude of this "converted" Green's function is one order of magnitude lower than that of the "non-converted" Green's function $\mathbf{G}_{\dot{\Omega},\dot{\Omega}_h}(\mathbf{x},\mathbf{x}_A,\omega)$ (and in a homogeneous isotropic medium it would completely vanish). Hence, in a weakly scattering medium we can ignore the term containing $G_{\dot{\Theta},\dot{\Omega}_h}(\mathbf{x},\mathbf{x}_A,\omega)$, which leaves

$$\dot{\Omega}_h(\mathbf{x}_A,\omega) \approx \frac{8}{i\omega^3}\int_{\partial\mathbb{D}_0} \rho c_S^4\{\partial_3 \mathbf{G}^*_{\dot{\Omega},\dot{\Omega}_h}(\mathbf{x},\mathbf{x}_A,\omega)\}\cdot\dot{\mathbf{\Omega}}(\mathbf{x},\omega)d^2\mathbf{x}. \tag{8}$$

This very simple Rayleigh-type integral formulates backpropagation of rotational motion-rate measurements $\dot{\mathbf{\Omega}}(\mathbf{x},\omega)$ from the acquisition boundary $\partial\mathbb{D}_0$, through a weakly scattering inhomogeneous anistropic medium, towards sources below $\partial\mathbb{D}_0$.

4 CONCLUSIONS

We have derived reciprocity and representation theorems for the translational and rotational components of a seismic wave field. Eq. (6) (with $\partial\mathbb{D}$ replaced by $\partial\mathbb{D}_0$) is an expression for backpropagation of measurements at the boundary $\partial\mathbb{D}_0$ towards real or secondary sources in the inhomogeneous anisotropic medium below $\partial\mathbb{D}_0$. The medium is assumed to be isotropic in a vanishingly thin shell around $\partial\mathbb{D}_0$, with μ constant along $\partial\mathbb{D}_0$. This representation does not rely on a specific propagation direction of the wave field at $\partial\mathbb{D}_0$, hence, the medium above $\partial\mathbb{D}_0$ can also be inhomogeneous and anisotropic, or $\partial\mathbb{D}_0$ can be a free surface. This generality comes with complexity. When the medium above $\partial\mathbb{D}_0$ is homogeneous and isotropic, the wave fields at $\partial\mathbb{D}_0$ propagate upwards, which leads to the significantly more simple representation of eq. (7) or, when the medium below $\partial\mathbb{D}_0$ is only weakly scattering, to the very simple Rayleigh-type integral of eq. (8).

The derived reciprocity and representation theorems contribute to the theoretical basis for rotational seismology methodology. The representations of eqs (6)–(8) can be used to generate virtual rotational motion sensors inside the medium, closer to the area of interest than the physical sensors at the surface. Together with virtual translational motion sensors, these can be used to improve the determination of the moment tensor of earthquake sources (Donner *et al.* 2016; Li & van der Baan 2017a; Ichinose *et al.* 2021) or to improve the efficiency of (local) structural imaging (Bernauer *et al.* 2009). Note that by rotating the configurations of Fig. 1 by 90 degrees, similar representations can be formulated for measurements in a vertical borehole.

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SUPPORTING INFORMATION

Supplementary data are available at *GJIRAS* online.

GJI-SupportingMaterial.pdf

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DATA AVAILABILITY

No data have been used for this study.

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