

On the vibro-acoustics of piles in layered media

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Introduction. The aim of the present study is the development of a model for the prediction of the high-frequency pile-soil-water dynamics induced by offshore pile driving. The ground motion and the generated sound field due to pile driving depends on the source parameters (driving method, input energy of the hydraulic hammer etc.), the soil-fluid-pile interaction and the propagation of waves in the surrounding media. Steel monopiles are nowadays widely used as foundation of offshore wind turbines and are driven into place with the help of hydraulic hammers as shown in Fig.1. The installation process is accompanied with high sound pressures in the surrounding fluid which are considered to be harmful for the marine ecosystem. In this study, a linear semi-analytical formulation of the coupled vibro-acoustic problem of the complete system is addressed. The model is similar to, but significantly more advanced than, the one presented by the authors in [1]. The advance is associated with a three-dimensional description of the soil adopted in the new model. The pile is described by a thin shell theory whereas both fluid and soil are treated as three-dimensional continua. Fluid is assumed to be present only at the exterior of the pile.

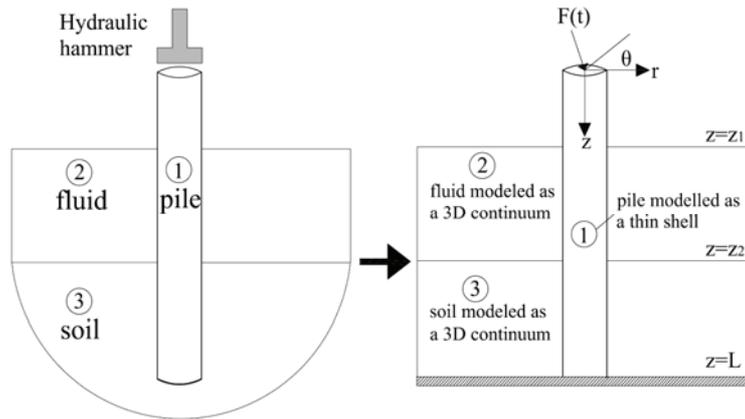


Fig.1 Geometry of the model and definition of the adopted coordinate system

The solution of the system of coupled partial differential equations is based on the dynamic substructuring technique according to which the total system is divided into two sub-systems: the shell structure and the soil-fluid layered medium. The linearity of the model allows for the representation of the response of each subsystem in the form of a superposition over appropriate eigenfunctions. The completeness of the modal sum for the layered soil-fluid domain is guaranteed by the introduction of a rigid boundary at a certain depth as shown in Fig.1.

Model description. The pile is described by an appropriate thin shell theory which includes the effects of shear deformation and rotational inertia [2]. The governing equation describing the shell vibrations in the time domain reads:

$$\mathbf{L} \mathbf{u}_p(z, \theta, t) + \mathbf{I}_m \mathbf{u}_p(z, \theta, t) = -H(z - z_2) \boldsymbol{\sigma}_s(R, \theta, z, t) + [H(z - z_1) - H(z - z_2)] \mathbf{p}_f(R, \theta, z, t) + \mathbf{f}_e(z, \theta, t). \quad (1)$$

In the equation above, $\mathbf{u}_p(z, \theta, t)$ is the displacement vector of the mid-surface of the shell. The terms \mathbf{L} and \mathbf{I}_m are the stiffness and modified inertia matrix operators of the shell, respectively, which are based on the applied thin shell theory. Material dissipation is accounted for by means of a frequency independent complex-valued elasticity modulus of the steel in the frequency domain. The term $\boldsymbol{\sigma}_s(R, \theta, z, t)$ represents the boundary traction vector that takes into account the reaction of the soil

surrounding the shell at $z_2 \leq z \leq L$. The term $\mathbf{p}_f(R, \theta, z, t)$ represents the fluid pressure exerted at the outer surface of the shell at $z_1 \leq z \leq z_2$. The functions $H(z - z_i)$ are the Heaviside step functions which are used here to account for the fact that the soil and the fluid are in contact with different segments of the shell.

The fluid is treated as a three-dimensional inviscid compressible medium with a pressure release boundary describing the sea surface. The motion of the fluid is fully characterized by a velocity potential $\varphi_f(r, \theta, z, t)$:

$$\nabla^2 \varphi_f(r, \theta, z, t) - \frac{1}{c_f^2} \frac{\partial^2 \varphi_f(r, \theta, z, t)}{\partial t^2} = 0 \quad (2)$$

where c_f is the sound speed in the exterior fluid domain. The soil is described as a three-dimensional perfectly elastic continuum able to support both dilatational and shear waves and is terminated at a certain depth with a rigid boundary. The body waves generated at the pile tip are not accounted for in the framework of this model. These waves are not expected to contribute to the acoustic field significantly since they will mainly consist of shear and compressional body waves with a spherical front spreading outwards and thus they will experience large attenuation [3] at the frequency range of interest. The motion of the soil medium is described by the following set of linear equations:

$$\mu_s \cdot \nabla^2 \mathbf{u} + (\lambda_s + \mu_s) \cdot \nabla \nabla \cdot \mathbf{u} = \rho_s \cdot \frac{\partial^2 \mathbf{u}}{\partial t^2} \quad (3)$$

$\lambda_s = E_s \nu_s / [(1 + \nu_s) \cdot (1 - 2\nu_s)]$ and $\mu_s = E_s / 2 \cdot (1 + \nu_s)$ being the Lamé constants for the soil. The constants E_s and ν_s correspond to the Young's modulus and the density of the soil respectively. The solution for the soil domain can be found using the Lamb's decomposition and introduction of potentials. The response of the coupled system is sought for in the form of a modal expansion with respect to the *in vacuo* shell modes and to those of the soil-fluid domain. The analytical approach is based on the following steps: **i)** solution of the eigenvalue problem of the shell without the presence of the fluid and the soil; **ii)** solution of the eigenvalue problem of the soil-fluid domain; **iii)** solution of the coupled system of equations resulting from the substitution of the obtained solutions for the shell and the soil-fluid domains into the interface conditions. For the shell *in vacuo* the response can be represented as follows:

$$u_{j,p}(z, \theta, t) = \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} A_{nm} U_{jnm,p}(z) \cos\left(\delta_{j\theta} \frac{\pi}{2} - n\theta\right) \exp(i\Omega_{nm} t), \quad (4)$$

where $n = 0, 1, 2, \dots, \infty$ is the circumferential order and $m = 1, 2, \dots, \infty$ is the axial order. The functions $U_{jnm,p}(z)$ with $j = z, \theta, r$ describe the axial distribution for the axial, circumferential and radial displacement fields respectively. A_{nm} are the undetermined shell modal factors. For the exterior soil-fluid domain the response can be represented as:

$$u_{j,s,f}(r, \theta, z, t) = \sum_{n=0}^{\infty} \sum_{k=1}^{\infty} C_{nk} U_{jnk,s,f}(z) \begin{cases} \cos(n\theta) \\ \sin(n\theta) \end{cases} R_{nk}(r) \exp(i\omega t). \quad (5)$$

The subscripts s and f refer to the soil and fluid respectively. The eigenfunctions $U_{jnk,s,f}(z)$ along the vertical coordinate are obtained by satisfying the set of boundary and interface conditions at $z = z_q$ with $q = 1, 2, \dots, l$ corresponding to the different layers. The index k is used here to reflect the different modes along the vertical coordinate. In accordance with Fig.1, the following set of boundary and interface conditions is imposed. At $z = z_1$ the fluid pressure is set equal to zero. At $z = z_2$ the normal stress and displacement are continuous whereas the shear stress of the soil is set equal to zero (inviscid fluid). Finally, at $z = L$ the displacements of the soil are set equal to zero. The functions $R_{nk}(r)$ correspond to the Hankel functions of the second type and of different order depending on the circumferential index n , which appropriately describe the radial dependence of the field. By enforcing force equilibrium and displacement compatibility at the interface between the shell and the exterior

domain, the original system of coupled partial differential equations is reduced to a system of coupled algebraic equations which can be solved with high accuracy. To achieve this, the orthogonality condition for the shell structure *in vacuo* and the one of the exterior soil-fluid domain are used to relate the unknown sets of modal coefficients (A_{mm} , C_{nk}) and to solve the coupled problem.

Numerical Results. In this section some preliminary results are presented for a pile with length $L=32.4$ m, diameter $D=0.92$ m and thickness $t=0.02$ m. The material parameters of the pile are $E_p=2.1 \cdot 10^5$ Mpa, $\nu_p=0.28$ and $\rho_p=7850$ kg m⁻³. The soil consists of a single layer with $E_s=100$ Mpa, $\nu_s=0.40$ and $\rho_s=1600$ kg m⁻³ in the case of a *soft soil* and $E_s=500$ Mpa, $\nu_s=0.40$ and $\rho_s=1800$ kg m⁻³ in the case of a *stiff soil*. The water is modelled as an inviscid fluid with $c_f=1500$ ms⁻¹ and $\rho_f=1000$ kg m⁻³. In accordance with Fig.1, $z_1=6$ m and $z_2=13.2$ m. The external load corresponds to a hammer input energy of 90 kJ. The maximum force amplitude equals 12 MN and the duration of the main pulse is equal to 5 ms. In Fig.2, the generated wave field is shown for the (outer to the shell) soil-fluid domain.

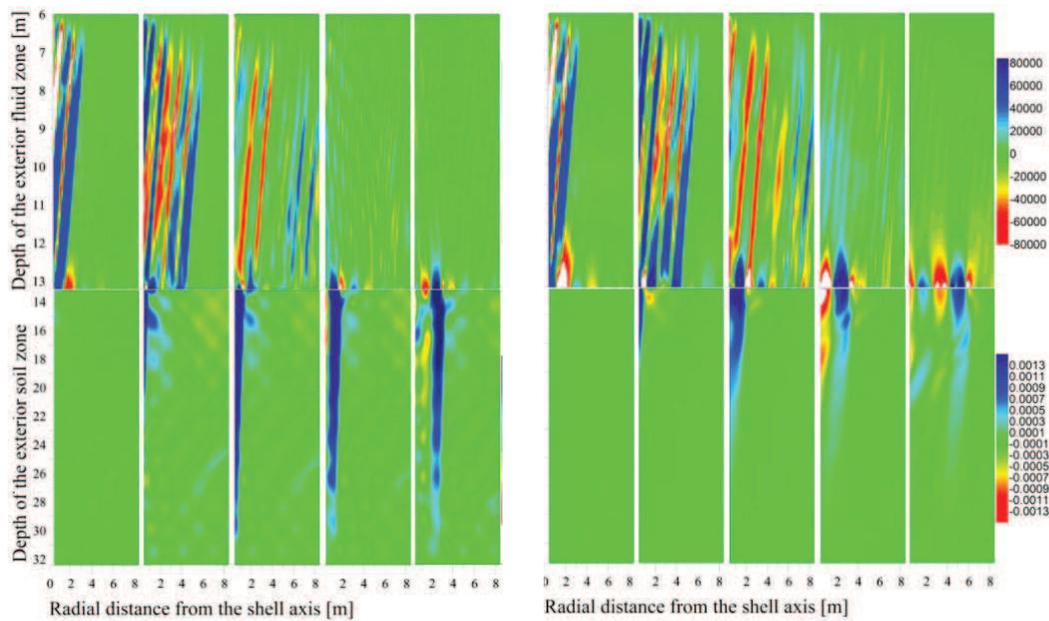


Fig. 1.2 Pressures in the fluid (top) and vertical displacements in the soil (bottom) for successive time moments after the impact in the case of a *soft soil* (left) and a *stiff soil* (right). In both figures from left to right: $t=4$ ms; $t=6$ ms; $t=10$ ms; $t=15$ ms; $t=25$ ms after the impact of the hammer.

As can be seen, the response in the fluid region is dominated by pressure *Mach cones* due to the supersonic compressional waves in the pile generated by the impact hammer. These cones are formed with an angle of about $\sin^{-1}(c_f/c_p) \approx \sin^{-1}(1500/5000) \approx 17^\circ$ to the vertical. The soil response is dominated by shear waves with almost vertical polarization. *Scholte* waves are generated at the soil-fluid interface. These waves propagate with a velocity slightly lower than that of the shear waves in the soil and their energy is localized in a restricted zone close to the interface. The penetration depth of the *Scholte* waves into the soil depends on soil elasticity. For stiffer soils, a larger portion of the input energy is localized in the fluid.

References

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