

# COMPUTATIONAL MODELLING OF ACOUSTIC SCATTERING FROM A CYLINDRICAL DUCT WITH A ROTOR INTO UNIFORM MEAN FLOW

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**Abstract.** *The paper presents a computational study of acoustic scattering from a cylindrical duct into uniform flow. The sound source is a fan inside the duct responsible for generating a spinning acoustic mode. The sound pressure scattering behaviour as function of the fan tip speed and mean flow velocity, as well as the acoustic modes propagation inside the duct are investigated. A direct collocation boundary integral equation method (BIEM) is employed for calculating the acoustic propagation in incompressible and compressible flows. The BIEM is based on the equations of linearised acoustics with uniform inflow and has the advantage of taking into account wake effects.*

## 1 INTRODUCTION

Duct acoustics is of special importance both in aerospace and built environment applications. Aeroengine noise generation is one of the most popular subjects in aeroacoustics. In a simplified form, an aeroengine can be modelled as a duct with a rotor inside and the interest lies in examining sound propagation inside the duct as well as its radiation in the atmosphere. The rotor consists a sound source, which depending on the angular speed of the blades, propagates or decays exponentially in time.

One could describe sound pressure as a sequence of harmonics or spinning modes, which fluctuate as function of the distance from the rotor. The pressure is associated with the rotor angular velocity  $\Omega = 2\pi\nu$ , where  $\nu$  is the angular frequency. Since it is a periodic function, it can be written in a Fourier series<sup>1</sup>:

$$P(\theta, t) = \sum_{n=0}^{\infty} A_n \cos(nB(\theta - \Omega t) + \Phi_n), \quad (1)$$

where  $B$  is the number of blades;  $n$  is the  $n$ th harmonic;  $\theta$  is the angular coordinate of pressure and  $A_n$  and  $\Phi_n$  are amplitude and phase variables, respectively. Equation 1

implies that the pressure distribution is a superposition of patterns, which are generated at a specific time,  $t$ , and at different angles,  $\theta$ . For the sound to be radiated in the field, the acoustic pressure should be ‘strong’<sup>1</sup> enough to propagate through a cylindrical duct and then to be released in the atmosphere. The ‘cut-off’ frequency is different for each mode of each harmonic as the wavelength is different depending on the mode number. If the mode frequency is less than the ‘cut-off’ frequency, then sound attenuates and does not radiate. Hence, it becomes apparent that there are certain modes that are of great interest as they generate a radiation field around the duct.

Sound radiation from an unflanged cylindrical duct was first calculated using the Wiener-Hopf technique by Levine and Schwinger<sup>2</sup>. Their work concentrated on non-spinning modes. They obtained an explicit solution for the sound radiation of an unflanged circular pipe assuming that only plane waves can propagate in the pipe. Because of the complexity of the Wiener-Hopf method, simpler approximate methods were found. Tyler and Sofrin<sup>1</sup> in their study of duct propagation and radiation, proposed a formula for calculating the acoustic radiation field due to propagation modes from a semi-infinite flanged duct with no flow. They used the Kirchhoff approximation, in which an estimated acoustic source strength at the duct face is inserted in the radiation integral. This study still remains as one of the most classical and detailed works on compressor noise. Among the researchers who have studied ducted fan noise, is Dunn et. al<sup>3,4,5</sup> who have combined the linearised equations of acoustics with the Helmholtz integral equation and developed computer codes for the prediction of the sound field around engines. The method is valid for a wide range of inflow Mach numbers and for engines with liners fitted. It is an indirect boundary element method where the surface unknown variables are related to the jumps of the acoustic pressure and its radial derivative across the duct wall. The fan is modelled as a superposition of monopoles or dipoles placed at the corresponding blade location.

Hamdi and Ville<sup>6</sup> introduced a new variational formulation based on integral equations in order to solve Helmholtz’s equation. More recently<sup>7</sup> the investigation was extended by discretising this formulation using boundary finite elements. The latter leads to symmetrical matrices of a smaller range compared to those obtained by classical finite element methods that require the discretisation of the fluid domain. The method was valid for finite length ducts with arbitrary shapes and was validated against experimental data. Hwang<sup>8</sup> introduced a rather different computational method for computing the Helmholtz integral equation for acoustic radiation and scattering problems. Unlike previous studies, Hwang’s method allowed the surface integral to be integrated directly and globally. The accuracy of the numerical integration was increased by using high-order Gaussian quadrature formula. To date, there are many studies of sound radiation from ducts, pipes or similar shape bodies but most of them do not include flow effects. Moreover, methods have been developed which are applicable to axisymmetric bodies<sup>9,10,11,12,13</sup> like ducts, which simplify the implementation of the integral formulation.

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<sup>1</sup>The term ‘strong’ means that the driving frequency of the harmonic is above a critical value, the ‘cut-off’ frequency. In this case, the sound will be transmitted through the duct.

Apart from BEMs, there are also analytical expressions for the prediction of sound. In Chapman<sup>14</sup> the ray structure of a spinning acoustic mode propagating inside a semi-infinite circular cylindrical duct was calculated, thereby determining the field radiated from the end of the duct. Inside the duct, the rays are helices striking the rim of the end-face of the duct; these rays produce cones called ‘Keller cones’ of diffracted rays. The cones determine the structure of the radiated field. Hocter<sup>15</sup> extended Chapman’s work by using the analytical expressions for the mode angles to further derive three formulae for the calculation of the sound radiated from a cylindrical duct and compared the directivity patterns. Firstly, he considered the behaviour of the Kirchhoff approximation, which is simple to use and does not require any extensive computation. The U-approximation, which is another analytical approximation, performs very well and is remarkably accurate for non-spinning modes. The only drawback of this method is a slight discontinuity at the sideline. In conclusion, the exact Wiener-Hopf solution for radiation from a cylindrical pipe is very complicated and numerically intensive. Peake<sup>16</sup> investigated the radiation properties of an asymmetric cylinder. The results of his analysis suggested that scarfing can be used to modify the radiation directivity.

In recent years Keith and Peake extended Peake’s work to high-wavenumber acoustic radiation from a thin-walled axisymmetric cylinder<sup>17</sup> and from a thin-walled scarfed cylinder<sup>18</sup>. Both studies are based on the Geometrical Theory of Diffraction and on uniform asymptotics. As far as scarfing is concerned, the preliminary indications show that scarfing appears to decrease the radiation directed downwards at the expense of an increase in radiation directed upwards. The scarfed cylinder is used as a model for a novel technique of noise reduction in modern aeroengines in which the intake is diverted a little upwards in order to radiate noise away from the ground.

Many boundary element methods have been focused on the direct derivation of the acoustic variables and the acoustic mode propagation inside the duct<sup>19</sup>. The acoustic field is actually split into two fields, one inside the duct and another outside. The two boundary value problems are related through the duct’s inlet and outlet surfaces. Most of the research has been done on axisymmetric problems for the sake of mathematical and geometrical simplicity. An attempts to extend axisymmetric problems to non-symmetric ones has been presented with success in relation to acoustic propagation from ducts<sup>20</sup>.

In this paper, we present a BIE for a finite duct, which is based on the linearised equations of acoustics in conjunction with a uniform flow field. This technique has the advantage of solving a simple asymmetric problem, where recent developments in the Gaussian quadrature are used to numerically deal with the singularities in Green’s function and its normal derivative<sup>13</sup>. The problem in question is an asymmetric problem for a finite duct in uniform flow with open both ends, which is mainly focused on the pressure radiation field around the duct. We investigated how the sound is propagating in the near and the far field when a rotor is spinning inside the duct. The computations were validated against analytical results for a spinning rotor in uniform flow.

### 1.1 Integral representation of the incident field

The sound pressure field of a rotating rotor inside a duct in forward flight is investigated. The analysis is carried out by replacing the normal pressure distribution over the rotor with a distribution of acoustic doublets acting at the rotor disc while in uniform motion. The sound field is obtained by integration over the rotor disc. The spinning propeller is simulated by a uniformly moving concentrated force and the acoustic pressure is calculated for an observer in the field moving along with the duct. The incident pressure for the disc source<sup>21</sup>, is given by:

$$P = im \int_0^{2\pi} \frac{e^{i(k\sigma+m\theta_1)}}{4\pi^2 r_{rotor}^2 S} d\theta_1, \quad (2)$$

where  $m$  is the azimuthal order,  $\theta_1$  is the angle on the disk source and the source co-ordinates are:

$$x_1 = x_1, \quad y_1 = r_{rotor} \sin \theta_1, \quad z_1 = r_{rotor} \cos \theta_1, \quad (3)$$

where  $r_{rotor}$  is the rotor radius and the subscript 1 denotes the source position.

## 2 THEORETICAL ASPECTS

The wave equation in uniform irrotational flow in the  $x$  direction is given by<sup>22</sup>:

$$\nabla^2 \phi + k^2 \phi - 2ikM \frac{\partial \phi}{\partial x} - M^2 \frac{\partial^2 \phi}{\partial x^2} = 0. \quad (4)$$

Let the undisturbed mean flow be specified by mean velocity  $u$  in the  $x$  direction. The acoustic velocity and the pressure perturbations are given by

$$\mathbf{v} = -\nabla \phi, \quad (5)$$

and

$$p = \rho \frac{D\phi}{Dt}, \quad (6)$$

where  $\rho$  is the mean density and  $D\phi/Dt$  is the material derivative,

$$\frac{D\phi}{Dt} = i\omega\phi + u \frac{\partial \phi}{\partial x}. \quad (7)$$

Let us consider a finite duct with a spinning rotor in uniform mean flow. Figure 1 shows the duct and the position of the rotor.

The sound field radiation from a duct inlet can be decomposed into a set of acoustic modes. Each of these modes has a specific directivity pattern that varies with frequency. Investigation of the important modes propagating and radiating in the domain is of interest for understanding the noise generation mechanisms. The pressure fluctuation at any position  $\theta$ , is a sum of harmonics and can be expanded in Fourier series,

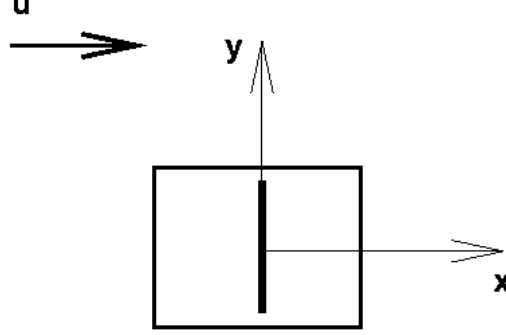


Figure 1: Side view of a cylindrical duct with a rotor inside.

$$p(\theta, t) = \sum_{n=1}^{\infty} \sum_{m=-\infty}^{\infty} p_{mn}, \quad (8)$$

where  $p_{mn}$  is:

$$p_{mn} = A_{mn} \cos(m\theta - n\omega t + \phi_{mn}). \quad (9)$$

Equation (8) can be written as a sinusoidal azimuthal variation and a Bessel function radial variation. Thus, the total pressure is made up of modes of the form

$$p_{mn}(r, \theta) = \sum_m \sum_n A_{mn} J_m(k_{mn}r) e^{im\theta}, \quad (10)$$

where  $m$  is the circumferential mode and  $A_{mn}$  is the amplitude or the mode coefficient. The latter can be found by applying the Hankel transform<sup>23</sup>, where  $k_{m1}r, k_{m2}r, k_{m3}r, \dots$  are the positive zeros of:

$$J'_m(k_{mn}r), \quad (11)$$

where  $J'_m$  is the first derivative of the Bessel function of order  $m$  and  $r$  is the duct radius. For describing the mode, a system of cylindrical co-ordinates  $(r, \theta, x)$  is used (Figure 2). An acoustic mode denoted by  $A_{mn}$  can be written as

$$p = p_0 e^{i(-\omega t + m\theta + k_x x)} J_m(k_r r). \quad (12)$$

In Equation 12,  $p$  denotes the pressure;  $t$  is the time; and the modal parameters are:  $\omega$  the frequency,  $m$  the azimuthal order,  $k_x$  the axial wavenumber and  $k_r$  the radial wavenumber. The duct wall is assumed to be hard, so from the boundary condition  $\partial p / \partial r = 0$  we obtain  $J'(k_r r_{duct}) = 0$ . The axial wavenumber is given by

$$k_x^2 = k^2 - k_r^2, \quad (13)$$

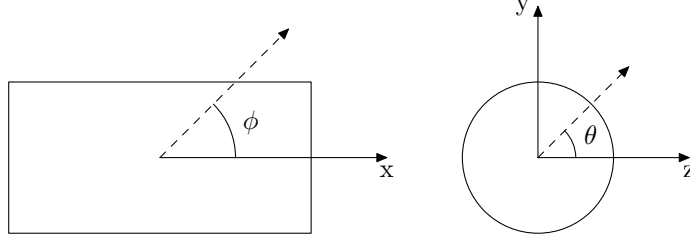


Figure 2: Geometry of a cylindrical duct.

where  $k = \omega/c$ , is the free space wavenumber. This equation is very important as it determines which modes propagate and which modes attenuate for a given frequency. When  $k < k_r$ ,  $k_x$  is imaginary and the corresponding frequency is called ‘cut-off’ frequency, while for  $k > k_r$  it is called ‘cut-on’. The noise source in the duct is assumed to be a rotor of radius  $r_{rotor}$  which spins with angular velocity  $\Omega$  ( $\omega = m\Omega$ ) and tip speed (Mach number),  $M_t = \Omega r_{rotor}/c$ . The speed with which the azimuthally varying pressure can propagate inside the duct is  $M_{duct} = \omega r_{duct}/c$ . A specific mode of order  $m$  can propagate (i.e. to be ‘cut-on’) if the condition  $M_t > M_{duct}$  is satisfied.

### 3 INTEGRAL FORMULATION

The differential equation in the time domain that gives the velocity potential for a body in inviscid compressible flow is written<sup>24</sup>

$$\nabla^2 \phi - \frac{1}{c^2} \frac{\partial \phi}{\partial t^2} = \Sigma, \quad (14)$$

where  $\Sigma$  represents the non-linear terms arising in transonic flow regimes. These terms are not included in the present work. Assuming that the problem is linear we obtain

$$\nabla^2 \phi - \frac{1}{c^2} \frac{\partial \phi}{\partial t^2} = 0. \quad (15)$$

Converting (4) to an integral equation, a formulation both for compressible and incompressible potential flows can be introduced<sup>24</sup>. Additionally, the formulation is valid for bodies which produce lift including the wake surface behind the trailing edge.

The boundary integral formulation is given by<sup>25</sup>:

$$\begin{aligned} \phi(\mathbf{x}, t) = & \int_{\Gamma} \left[ \frac{\partial \phi}{\partial \hat{\mathbf{n}}} G - \phi \frac{\partial G}{\partial \hat{\mathbf{n}}} + G \frac{\partial \phi}{\partial t} \left( \frac{\partial \theta}{\partial \hat{\mathbf{n}}} + 2 \frac{\mathbf{u} \cdot \mathbf{n}}{c^2} \right) \right]^\theta d\Gamma \\ & - \frac{1}{c^2} \int_{\Gamma} \left[ G \phi \frac{\partial}{\partial t} [\mathbf{u} \cdot \mathbf{n} (\mathbf{u} \cdot \nabla \theta - 1)] \right]^\theta d\Gamma - \int_{\Gamma_w} \Delta \phi \frac{\partial G}{\partial \mathbf{n}} d\Gamma_w, \end{aligned} \quad (16)$$

where

$$\frac{\partial}{\partial \hat{\mathbf{n}}} = \frac{\partial}{\partial \mathbf{n}} - \frac{1}{c^2} \mathbf{u} \cdot \mathbf{n} \cdot \mathbf{u} \cdot \nabla \quad (17)$$

and  $[\dots]^\theta$  denotes evaluation at time  $\tau = t - \theta$ , where  $\theta$  is the time elapsed for the sound to travel from the source to the observer;  $\mathbf{x}$  denotes position and  $\phi(\mathbf{x}, t)$  is the value of the velocity potential at  $\mathbf{x}$ . The above equation is the full boundary integral equation in a compressible flow based on the potential formulation. In the present paper the equation is simplified by eliminating the second integral term because the velocity of the body is constant and does not involve any relative motion with respect to the free motion of the body. The reduced equation is written as

$$\phi(\mathbf{x}, t) = \int_{\Gamma} \left[ \frac{\partial \phi}{\partial \hat{\mathbf{n}}} G - \phi \frac{\partial G}{\partial \hat{\mathbf{n}}} + G \frac{\partial \phi}{\partial t} \left( \frac{\partial \theta}{\partial \hat{\mathbf{n}}} + 2 \frac{\mathbf{u} \cdot \hat{\mathbf{n}}}{c^2} \right) \right]^\theta d\Gamma - \int_{\Gamma_W} \left[ \Delta \phi \frac{\partial G}{\partial \mathbf{n}} \right]^\theta d\Gamma_W. \quad (18)$$

The total velocity potential is obtained by using the integral equation including both steady and unsteady terms. When the observer is located on the boundary  $\Gamma$ , then 18 can be used to evaluate the velocity potential,  $\phi$  on  $\Gamma$ .

The Green's function  $G$  represents the potential field associated with a uniformly moving acoustic source. For compressible potential flow in the positive  $x$  direction the Green's function is<sup>21</sup>:

$$G_{3D} = -\frac{e^{ik\sigma}}{4\pi S} \quad (19)$$

where

$$\begin{aligned} S &= \sqrt{(x - x_1)^2 + \beta^2 [(y - y_1)^2 + (z - z_1)^2]}, \\ \beta &= \sqrt{1 - M^2}, \\ \sigma &= \frac{M(x - x_1) + S}{\beta^2}, \\ M &= \frac{u}{c}, \end{aligned} \quad (20)$$

is the flow Mach number.

Once  $\phi$  is obtained, the same equation is used to get the velocity potential,  $\phi(\mathbf{x}, t)$  anywhere in the field in terms of the values of  $\phi$  and  $\partial \phi / \partial \mathbf{n}$  on the surface of the body and  $\Delta \phi$  on the wake. With  $\phi$  known on the surface, the acoustic pressure can be calculated by 6.

#### 4 SINGULARITIES

The velocity potential on the surface is calculated on grid nodes and this poses numerical problems (singularities). These singularities have logarithmic identity due to Green's function: the Green's function is of the form  $1/S$  and the first derivative is of the form  $1/S^2$ , which in the integral equation are both improper integrals. Since the integration over the surface is carried out numerically, to overcome the singularities problem a generalised Gaussian quadrature<sup>26</sup> is employed, which integrates exactly the logarithmic

singularities on any of the nodes of the elements using a single formula. All the integral equations of the boundary element method are evaluated using either (i) a regular Gaussian quadrature or (ii) a combination of regular and logarithmic Gaussian quadratures. The regular Gaussian quadrature is used to evaluate most of the integrals along the boundary, and the combined scheme is employed for integrals that are singular. All the integrals containing non-singular functions are evaluated using a regular form of Gaussian quadrature. The advantage of this scheme is that the same set of quadrature points can be used for constant, linear or higher order elements. This is very convenient from a computational point of view as there is no need for selection when it comes to different types of singularities.

## 5 NUMERICAL VALIDATION

An analytical formula for the far pressure field generated by a propeller in forward flight is available<sup>21</sup>:

$$P_{thr} = -\frac{m\mathbf{i}^{m+1}}{2\pi S_0 r_{rotor}^2} e^{-ik(S_0+Mx)/\beta^2} J_m\left(\frac{kyr_{rotor}}{S_0}\right), \quad (21)$$

where  $S_0 = \sqrt{x^2 + \beta^2 y^2}$  and  $J_m$  is the Bessel function of first kind and order  $m$ . Figure 3 shows the results both for the computed and analytical pressure in the far field ( $R/r = 9$ ) around a propeller in forward flight. Analytical and computed results agree very well for a wide range of mean flow Mach numbers.

Validation was also obtained for an acoustic source placed inside the boundary so as to obtain the surface pressure and the sound radiated from the surface being identical to the sound radiated directly from the source. Figure 4 shows the error of the unsteady pressure for  $M = 0.3$  for different number of nodes around the circle; there is very good agreement between computed and theoretical results.

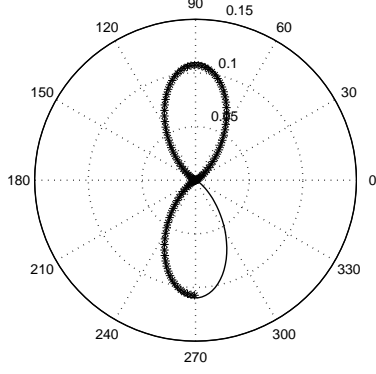
## 6 RESULTS

Figures 6, 7 and 8 show the pressure contour plots around a duct with a rotor inside (Figure 1) for different mean flow Mach numbers and frequencies. Three tip Mach numbers were chosen; 0.9, 1.2 and 1.5. The azimuthal order is  $m = 6$ . For all the above speeds there are ‘cut-on’ modes and, therefore, the sound propagation in the outer field becomes more apparent. Figures 6, 7 and 8 show that the pressure distribution is symmetrical about the  $x$  axis, due to the flow in the streamwise direction. In the  $y$  direction the velocity is mirrored and the sound radiation becomes identical above and below the  $x$  axis of the duct.

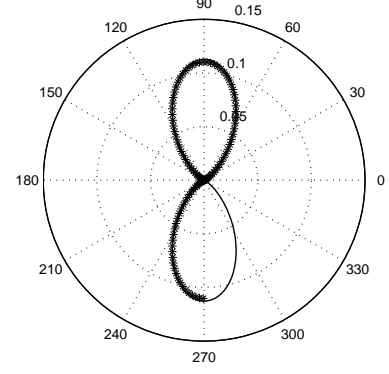
Comparing Figures 8a, 6a and 7a with Figures 8b, 6b and 7b the shielding effect can be detected according to which a higher amplitude region occurs at the front side of the engine compared to the rear. The frequency perceived by the observer is the same with the one of the source as the observer is moving along with the duct at the same speed.



$$R/r = 6, ka = 9$$

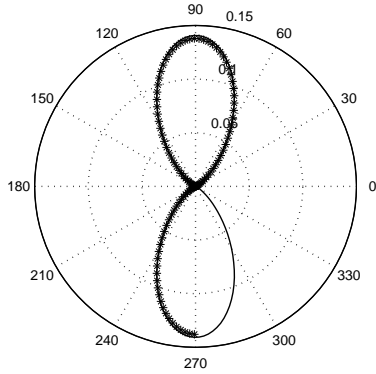


$$M = 0.0$$

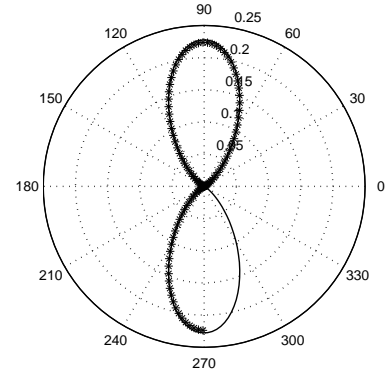


$$M = 0.1$$

$$R/r = 6, ka = 9$$



$$M = 0.3$$



$$M = 0.5$$

Figure 3: Sound pressure directivities in the far field of a rotor inside a duct for various mean flow Mach numbers; — analytical, \* computed results.

The streamwise component of the wavenumber  $k_x$  changes by a factor which is dependent on the flow Mach number. The results also reveal the dominant role of the mean flow in the sound radiation, which becomes more important at higher Mach numbers as the Prandtl-Glauert factor,  $\beta = (1 - M^2)^{1/2}$ , which is involved in the pressure expression, decreases with the Mach number. This results in changing the streamwise component of the wave number thus causing a stronger sound field for an observer standing in the front of the duct compared to the rear.

The cylindrical duct has an elliptical cross section (Figure 5) with a curved edge, which prevents a non-uniform sound-radiation pattern arising from sharp edges.

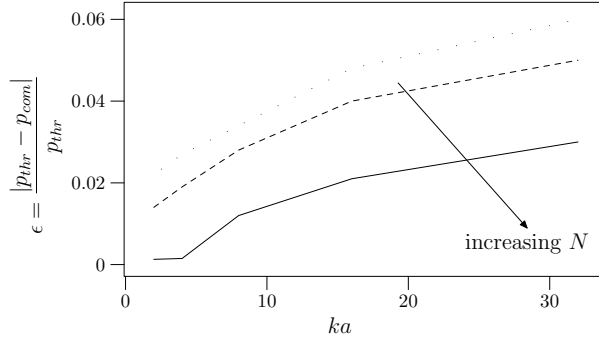


Figure 4: Error in unsteady pressure around a circle for  $N = 200$ ,  $N = 300$  and  $N = 400$ .

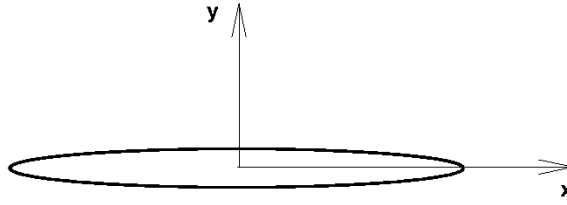


Figure 5: Duct geometry generator.

At higher frequencies, where more modes propagate, the difference in the radiation field can be noticed; compare Figure 7 with Figure 8. Also the Mach number effects become more important at high frequencies; compare Figure 7 ( $M = 0.1$ ) with Figure 7 ( $M = 0.6$ ).

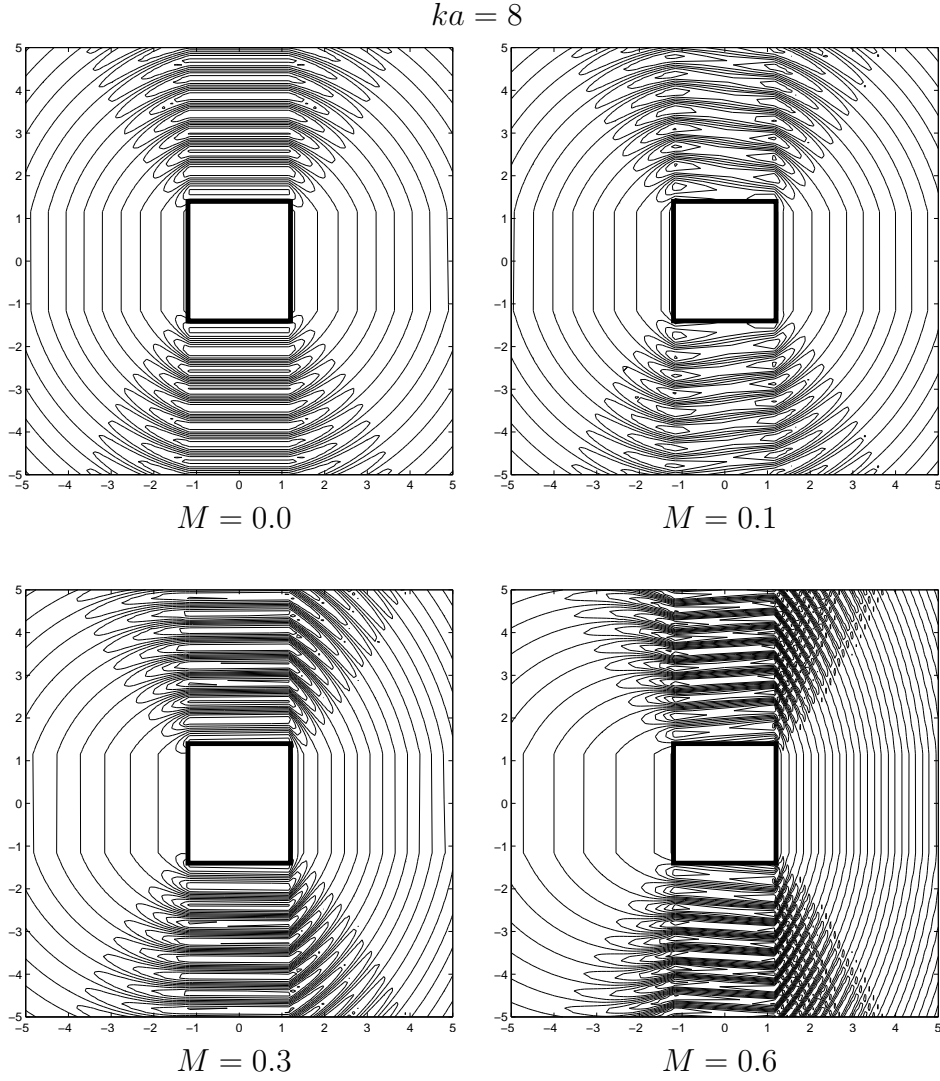


Figure 6: Total acoustic pressure (real part) around a duct with a rotor; contour levels  $\pm 10^{-1}, \pm 10^{-2}, \pm 10^{-3}$ ; solid and dashed lines denote positive and negative values, respectively.

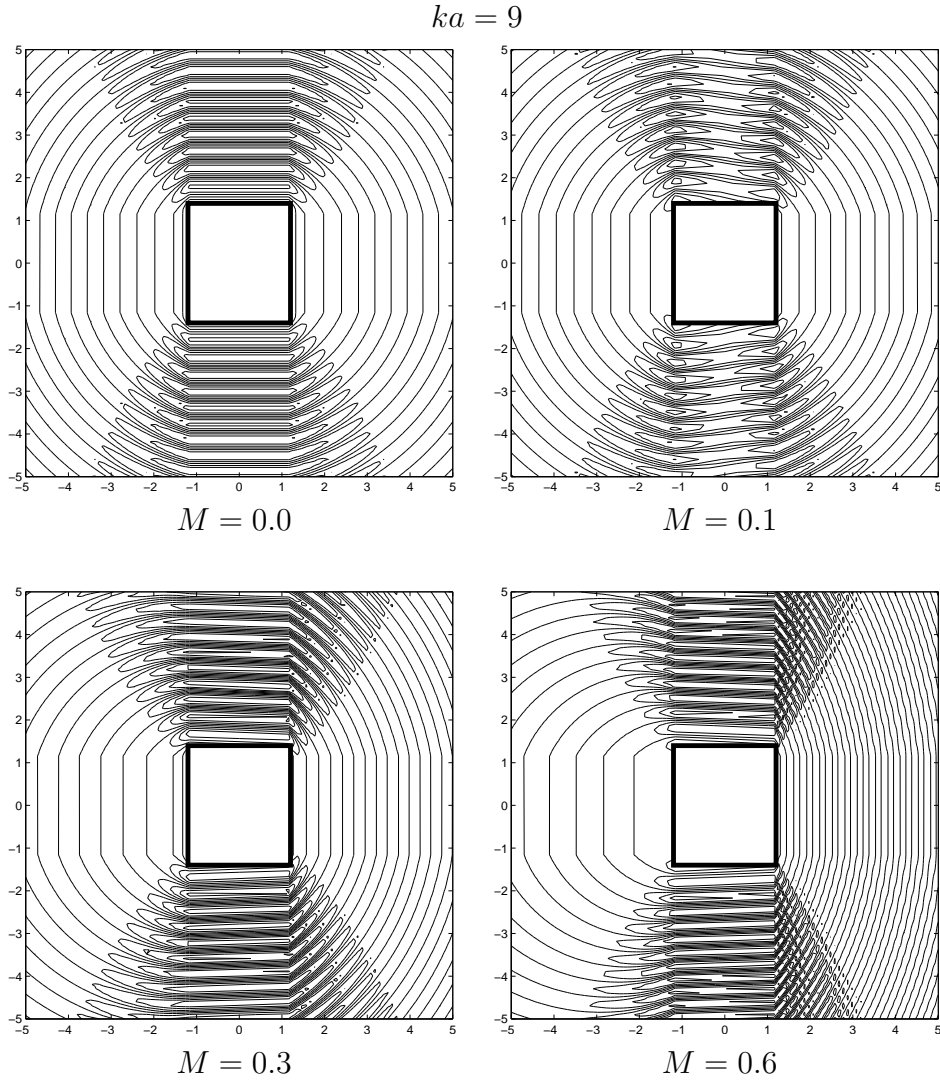


Figure 7: Total acoustic pressure (real part) around a duct with a rotor; contour levels  $\pm 10^{-1}, \pm 10^{-2}, \pm 10^{-3}$ ; solid and dashed lines denote positive and negative values, respectively.

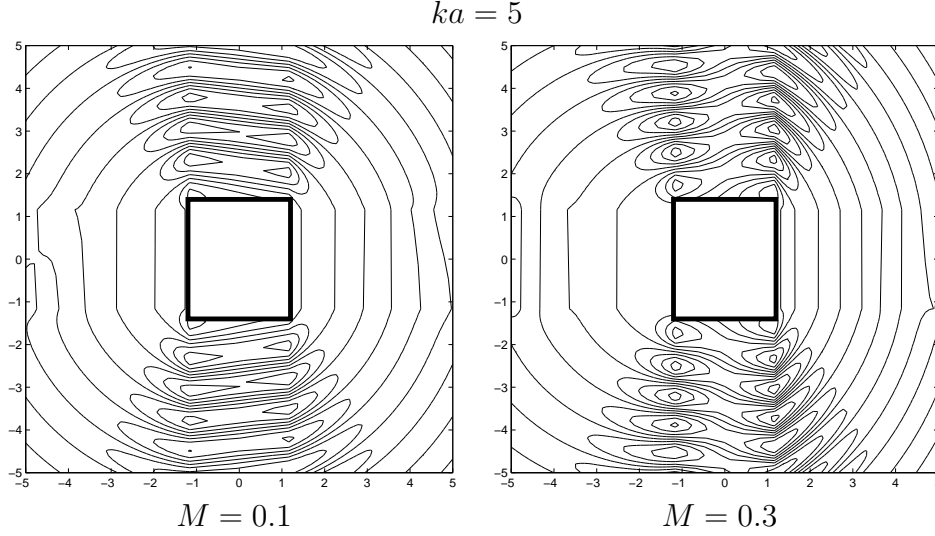


Figure 8: Total acoustic pressure (real part) around a duct with a rotor; contour levels  $\pm 10^{-1}, \pm 10^{-2}, \pm 10^{-3}$ ; solid and dashed lines denote positive and negative values, respectively.

## 7 CONCLUSIONS

A study of the sound field generated by a rotor inside a cylindrical duct using a boundary integral scheme based on the linearised potential flow equations, was presented. The method can be used both for axisymmetric and asymmetric geometries. The computational efficiency is improved by treating numerical singularities using recent developments in Gaussian quadrature. Validation was performed against analytical results. The effects of the mean flow velocity and frequencies on the sound radiation were discussed for different mean flow and rotor tip Mach numbers.

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