UNIVERSAL STATISTICAL PROPERTIES OF INERTIAL-PARTICLE TRAJECTORIES IN THREE-DIMENSIONAL, HOMOGENEOUS, ISOTROPIC, FLUID TURBULENCE

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Abstract We obtain new universal statistical properties of heavy-particle trajectories in three-dimensional, statistically steady, homogeneous, and isotropic turbulent flows by direct numerical simulations. We show that the probability distribution functions (PDFs) $P(\phi)$, of the angle ϕ between the Eulerian velocity $\mathbf{u}$ and the particle velocity $\mathbf{v}$, at a point and time, scales as $P(\phi) \sim \phi^{-\gamma}$, with a new universal exponent $\gamma \simeq 4$. The PDFs of the trajectory curvature κ and modulus θ of the torsion ϑ scale, respectively, as $P(\kappa) \sim \kappa^{-h_\kappa}$, as $\kappa \to \infty$, and $P(\theta) \sim \theta^{-h_\theta}$, as $\theta \to \infty$, with exponents $h_\kappa \simeq 2.5$ and $h_\theta \simeq 3$ that do not depend on the Stokes number St. We also show that γ, h_κ and h_θ can be obtained by using simple stochastic models. We show that the number $N_{\rm I}(t, {\rm St})$ of points (up until time t), at which ϑ changes sign, is such that $n_{\rm I}({\rm St}) \equiv \lim_{t\to\infty} \frac{N_{\rm I}(t,{\rm St})}{t} \sim {\rm St}^{-\Delta}$, with $\Delta \simeq 0.4$ a universal exponent.

INTRODUCTION

The elucidation of the statistical properties of inertial particles in turbulent flows is an important problem of great interest [1, 2]. We study the statistical properties of the geometries of heavy-inertial-particle trajectories; such inertial-particle-trajectory statistics have not received much attention hitherto in homogeneous, isotropic, three-dimensional (3D) fluid turbulence.

RESULTS AND CONCLUSIONS

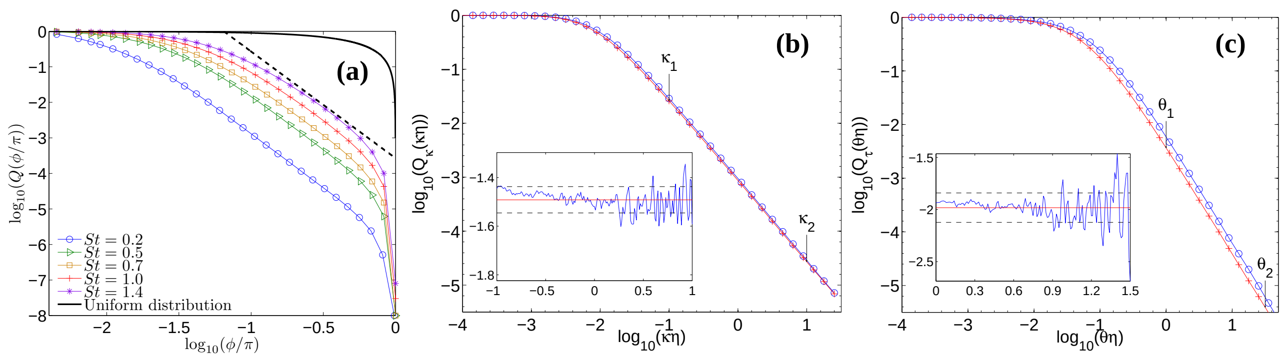

Figure 1. Cumulative PDFs of (a) the angle ϕ between $\mathbf{u}$ and $\mathbf{v}$ ($Q(\alpha) \equiv P(\phi \geq \alpha)$), for ${\rm St} = 0.2$ (blue circles), ${\rm St} = 0.5$ (green triangles), ${\rm St} = 0.7$ (brown squares), ${\rm St} = 1.0$ (red pluses), and ${\rm St} = 1.4$ (purple stars); the slope of the black dashed line is -3, (b) the curvature κ and (c) the magnitude of the torsion θ of the trajectories of heavy inertial particles, for $St = 0.2$ (in blue) and 1.0 (in red), obtained using rank order method. Inset: the values of the local slope of the tail, for $St = 1.0$.

Our direct-numerical-simulation (DNS) studies of these statistical properties yield new and universal scaling exponents that characterize heavy-particle trajectories. We calculate the probability distribution functions (PDFs) of the angle ϕ between the Eulerian velocity $\mathbf{u}(\mathbf{x}, t)$, at the point $\mathbf{x}$ and time t, and the velocity $\mathbf{v}$ of an inertial particle at this point and time, PDFs of the curvature κ and torsion ϑ of inertial-particle trajectories, and several joint PDFs. In particular, we find that the PDF $P(\phi)$ shows a power-law region in which $P(\phi) \sim \phi^{-\gamma}$, with an exponent $\gamma \simeq 4$, which has never been considered so far; the extent of this power-law regime decreases as St increases Fig. 1 (a); we find good power-law fits if $0 < {\rm St} \lesssim 0.7$; in this range γ is universal, in as much as it does not depend on St and the fluid Reynolds number Re (given our error bars). The PDFs of κ Fig. 1 (b) and $\theta = |\vartheta|$ Fig. 1 (c) show power-law tails for large κ and θ, respectively, with power-law exponents h_κ and h_θ that are also universal. We calculate the number of points, per unit time, at which the torsion ϑ changes sign along a particle trajectory Fig. 2 ; this number $n_I({\rm St}) \sim {\rm St}^{-\Delta}$, as ${\rm St} \to 0$, with $\Delta \simeq 0.4$ another universal exponent. We show how simple stochastic models can be used to obtain the exponents γ, h_κ, and h_θ; however, the evaluation of Δ requires the velocity field from the Navier-Stokes equation [3].

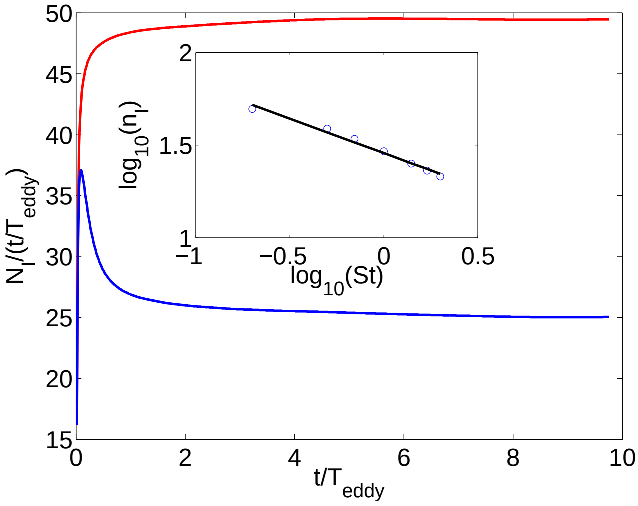

Figure 2. Number of inflection points per unit time as a function of dimensionless time t/T_{eddy}, for $St = 0.2$, (red curve), and $St = 1.4$, (blue curve); the inset shows the plot of the number of inflection points per unit time n_I, as a function of St.

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