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The Influence of the Reverse Early Effect on the Performance of Bandgap References

A. van Staveren, C. J. M. Verhoeven, and A. H. M. van Roermund

***Abstract*—In the literature three key-parameters are commonly used for the design of bipolar bandgap references: E_G (bandgap energy), I_S (saturation current) and X_{TI} (exponent of the temperature behavior of I_S). This paper shows that four key parameters exist: these three and V_{AR}, the reverse Early voltage. This parameter models the influence of the base-width modulation at the base-emitter junction on the collector current. A general expression for the error in the output voltage caused by the reverse Early effect is derived and a comparison is made with other errors.**

I. INTRODUCTION

The design of an electronic circuit starts with the mathematical description of the desired function. Several basic operations can be distinguished in this mathematical description. For accurate prediction and optimal performance of the electronic implemented operation and for minimizing the *systematic* errors, the behavior of these basic blocks must be known extensively. A systematic design theory for each of these basic blocks will provide the required knowledge [1].

An important aspect of a design theory is the identification of the key parameters. This is the minimal set of parameters that has to be known for an accurate design of the circuit.

One of the basic blocks is the bandgap reference. This paper shows that four key parameters exist for the design of bandgap references:

E_G bandgap energy
I_S saturation current
X_{TI} exponent of the temperature behavior of I_S
V_{AR} reverse Early voltage

In the literature [2], [3], and [4] the first three parameters are well known. The fourth parameter V_{AR} is not well known. Its influence is generally accounted for by the fitting of I_S. The drawback of this method is that two different physical phenomena, saturation current and base-width modulation, are modeled with one parameter and this parameter looses thereby its physical meaning. Further, for the transistor parameters for the design of bandgap references a dedicated parameter extraction needs to be done.

This brief describes the influence of the V_{AR} on the behavior of bandgap references, so that the standard extracted parameters of a process can be used.

The starting point is the Gummel and Poon model from which a minimal set of parameters is deduced for the design of bandgap references. The influence of the reverse Early effect on the temperature behavior is calculated. Finally, the error due to the Early effect is compared with other errors, e.g., absolute and matching errors of base-emitter junctions, absolute, and matching errors of resistors and amplifier offset.

II. THE FIRST-ORDER COMPENSATED BANDGAP REFERENCE

The first-order temperature behavior of a first-order compensated bandgap reference should ideally be zero. A first-order compensated

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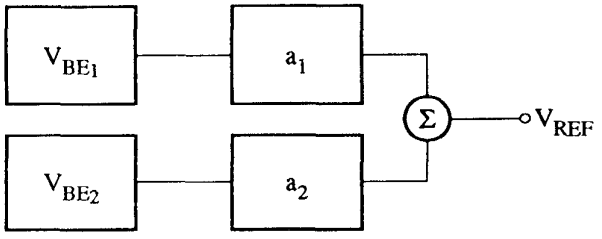

Fig. 1. A linear combination of two base-emitter voltages.

bandgap reference is obtained when a suitable linear combination of at least *two* base-emitter voltages is taken (see Fig. 1).

$$V_{\mathrm{REF}} = a_1 V_{\mathrm{BE1}}(T) + a_2 V_{\mathrm{BE2}}(T) \tag{1}$$

The scaling factors a_1 and a_2, need to be chosen such that the first-order temperature behavior is zero. For accurate design it is favorable to have as few key parameters as possible which describe the relation between the base-emitter voltage and the collector current [5]. In the next section a minimal set of key parameters is derived from the Gummel and Poon model.

III. The Key Parameters of the Gummel and Poon Model

The Gummel and Poon model [6] as used in SPICE [7] is a well known model and often used for circuit design. Therefore this model is used as the basis for the design of bandgap references. A minimum set of key parameters will be derived that describes the relation between the base-emitter voltage and the collector bias current.

The bulk resistors do not need to be taken into account because it is possible to make their influence negligible. The Gummel and Poon model is reduced further to the effects that are relevant for the forward-biased transistor. The leakage currents are ignored too, because in modern IC processes these leakage currents are negligibly small [8]. It is assumed that the transistor is biased far from high-level injection. The relevant part of the Gummel and Poon model that remains is given in the following equation

$$I_C(T) = I_S \exp\left[\left(\frac{T}{T_{\mathrm{nom}}} - 1\right)\frac{E_G}{qV_T}\right]\left(\frac{T}{T_{\mathrm{nom}}}\right)^{X_{TI}} \times \left[1 - \frac{V_{\mathrm{BC}}}{V_{AF}} - \frac{V_{\mathrm{BE}}}{V_{AR}}\right]\cdot\left[\exp\left(\frac{V_{\mathrm{BE}}}{N_F V_T}\right) - 1\right]. \tag{2}$$

In this model the following parameters are used (Table I).

A further reduction is obtained when the transistor is biased such that $V_{\mathrm{BC}} \approx 0$ V. In that case the forward Early effect, modeled with V_{AF}, can be ignored. For a given temperature V_T is known. T_{nom} is generally just the center of the temperature range for which the bandgap reference has to be designed. Finally, N_F equals 1. Thus for an accurate design of first-order compensated bandgap references, *four* parameters needs to be known accurately in order to describe the relation between the base-emitter voltage and collector current accurately (Table II). These parameters are the key parameters for the design of bandgap references. When other models are used the corresponding key parameters are found.

In the following sections the reverse Early effect is explained and the influence on the output voltage of a first-order compensated bandgap reference is derived.

IV. The Reverse Early Effect

Just like the base-collector junction, the base-emitter junction influences the effective base width and thus the actual collector current. This influence is modeled by the reverse Early effect. The reverse Early voltage V_{AR} is the model parameter.

The *forward* Early voltage V_{AF} can easily be in the order of 80 V. By biasing-circuit design the base-collector voltage is $\approx$0 V and the forward Early effect can be ignored. The *reverse* Early effect on the contrary, causes a serious problem. Because the doping level of the base is much lower than the doping level of the emitter, the variation of the depletion layer is predominantly in the base region, in contrast with the base-collector junction where most of the widening of the depletion layer takes place in the collector region. The reverse Early voltage can easily be of the order of only several volts, e.g., 4 V, which gives a reduction of the collector current of approximately [see (2)]

$$\frac{\Delta I_C}{I_C} = 0.15. \tag{3}$$

For accurate circuit design the reverse Early voltage has to be taken into account.

V. The Influence of the Reverse Early Effect

In this section a general expression is derived for the error due to the reverse Early voltage in the output voltage of a first-order compensated bandgap reference. Starting point is (2) with $N_F = 1$ and $I_C \gg I_S$. Rewriting for V_{BE} gives

$$V_{\mathrm{BE}} = V_T \ln\left[\left(1 - \frac{V_{BE}}{V_{AR}}\right)^{-1}\frac{I_C(T)}{I_S(T)}\right]. \tag{4}$$

A suitable linear combination of two base-emitter voltages yields the reference voltage. Substitution of (4) in (1) results in an expression for the reference voltage V'_{REF}, in which the effect of the V_{AR} is included

$$V'_{\mathrm{REF}} = a_1 V_{\mathrm{BE1}} + a_2 V_{\mathrm{BE2}} = \sum_{i=1}^{2} a_i V_{\mathrm{BE}_i} \tag{5}$$

$$= \sum_{i=1}^{2} a_i V_T \ln\left[\left(1 - \frac{V_{\mathrm{BE}_i}}{V_{AR_i}}\right)^{-1}\frac{I_{C_i}(T)}{I_{S_i}(T)}\right] \tag{6}$$

$$= \sum_{i=1}^{2} a_i V_T \ln\left[\frac{I_{C_i}(T)}{I_{S_i}(T)}\right] - \sum_{i=1}^{2} a_i V_T \ln\left(1 - \frac{V_{\mathrm{BE}_i}}{V_{AR_i}}\right) \tag{7}$$

$$= V_{\mathrm{REF}} + V_{\mathrm{ERROR}}. \tag{8}$$

where V_{REF} is the specified reference voltage and V_{ERROR} the difference between the specified and the actual reference voltage V'_{REF} due to the V_{AR}. The error in the reference voltage is given by

$$V_{\mathrm{ERROR}} = -\sum_{i=1}^{2} a_i V_T \ln\left(1 - \frac{V_{\mathrm{BE}_i}}{V_{AR_i}}\right). \tag{9}$$

This equation can be simplified because $V_{\mathrm{BE}_i} \ll V_{AR_i}$ to

$$V_{\mathrm{ERROR}} = \sum_{i=1}^{2} a_i V_T \frac{V_{\mathrm{BE}_i}}{V_{AR_i}}. \tag{10}$$

As a good matching between the transistors used in the linear combination is important, the V_{AR_i} can assumed to be equal

$$V_{\mathrm{ERROR}} = V_T \frac{V'_{\mathrm{REF}}}{V_{AR}} = V_T \frac{V_{\mathrm{REF}} + V_{\mathrm{ERROR}}}{V_{AR}}. \tag{11}$$

Solving for V_{ERROR} with the assumption $V_{AR} \gg V_T$ gives

$$V_{\mathrm{ERROR}} = \frac{V_T}{V_{AR}} V_{\mathrm{REF}}. \tag{12}$$

This error voltage is independent of a_1 and a_2 and thus the influence of the reverse Early effect cannot be compensated for by taking a

TABLE I
THE MODEL PARAMETERS OF THE RELEVANT PART OF THE GUMMEL AND POON MODEL

parameter	explanation	parameter	explanation
V_{BE}	the base-emitter voltage	V_{BC}	the base-collector voltage
I_S	the saturation current	X_{TI}	the exponent of temperature dependency I_S
V_{AF}	the forward Early voltage	V_{AR}	the reverse Early voltage
V_T	the thermal voltage	N_F	the forward-emission coefficient
E_G	the bandgap energy	T_{nom}	the nominal temperature

TABLE II
THE FOUR KEY PARAMETERS FOR THE DESIGN OF BANDGAP REFERENCES

parameter	explanation	parameter	explanation
E_G	the bandgap energy	I_S	the saturation current
X_{TI}	the exponent of the temperature behavior of I_S	V_{AR}	the reverse Early voltage

TABLE III
THE INFLUENCE OF THE V_{AR} ON THE TEMPERATURE BEHAVIOR

Case	Mean error
$V_{AR} = \infty$, no influence of the reverse Early effect	19 ppm/K
$V_{AR} = 4V$ and not taken into account	30 ppm/K
$V_{AR} = 4V$ and taken into account	20 ppm/K

suitable linear combination. An explicit compensation is necessary. This can be done, for instance, by taking the effect into account in the equations describing the temperature behavior of $V_{\rm BE}(T)$ and thus implicitly in the expression for a_1 and a_2 [9].

Example: The resulting temperature dependencies of a first-order compensated bandgap reference, in three situations is calculated. First for a bandgap reference with a very high V_{AR}, such that it is negligible. Second for a bandgap reference with a V_{AR} of only 4 V and not taken into account and third for a bandgap reference for which this V_{AR} is taken into account. The resulting temperature dependencies for the temperature range of 250 K–350 K with $T_{\rm nom} = 300$ K are given in Table III.

Due to the omission of the V_{AR} the mean error increases from 19 to 30 ppm/K. The omission of V_{AR} results in an additional first-order term of $\approx$ 21 ppm/K. As the 19 ppm/K is due to an error with a second-order temperature behavior, the effect of the additional first-order error of 21 ppm/K [see (12)] is halved.

VI. COMPARISON TO OTHER ERRORS

For the discussion of other errors in the output voltage of the bandgap reference the following model is used

$$\Delta V_{\rm REF} = (\Delta a_1)(\Delta V_{\rm BE1})(\Delta +)(\Delta a_2)(\Delta V_{\rm BE2}) \qquad (13)$$

with $(\Delta +)$ the error due to a nonideal addition. Three error sources can be distinguished.

1) **Errors due to the base-emitter voltages.** Spread in the base-emitter voltage is caused by the variation in the collector-current density. This variation can be caused by either

   —spread in the emitter area or
   —spread in the collector bias current

   For the modern processes, the spread in an emitter of 2 μm $*$ 4 μm is only in the order of 10% (the σ-value). This results in an error, in the relation $V_{\rm BE} - I_C$, comparable to an error caused by the omission of V_{AR} [see (3)]. The spread in emitter area reduces when larger emitters are used.
   The collector current is mostly determined by a PTAT voltage and a resistor. The accuracy of the PTAT voltage relies on the matching of two base-emitter junctions. This can be an order of magnitude better than the absolute value of the emitter area and thus negligible. The resistor converting the PTAT voltage to the collector bias current has an absolute error in the range of about 1 to 10%.
2) **Errors due to the scalers.** The scaling ratios are set by ratios of impedances. These impedances can be the feedback networks of negative-feedback amplifiers [1]. The accuracy is determined by the matching of those impedances and is in the order of 0.1 to 1%.
   Errors can also occur due to an input offset voltage of the amplifier. For a bipolar input stage this can be negligibly small. For an MOS input stage, to the contrary, this can be a reasonably large error [10]. So, for bandgap references with low systematic errors, bipolar input stages are preferable.
3) **Errors due to the summation.** The summing can be done either in the voltage or current domain. In the voltage domain an input offset voltage may influence the temperature behavior as an input offset voltage of the scaler does. Summation in

the current domain can be done more accurately as input bias currents can generally be kept small.

From these errors the absolute value of the resistor determining the collector bias current and the absolute size of the emitter area, in the case of a minimal sized transistor, are the dominant error sources. The error caused by the omission of the V_{AR} is comparable to these errors. So, taking the V_{AR} into account when designing bandgap references reduces the systematic error.

Finally, errors due to stress in the chip occur [10]. This stress can be caused by packaging or thermal shocks and alter the behavior of the devices. As this error source is not at circuit-design level but at layout and technology level, it is not treated here.

VII. Conclusion

This brief shows that there are *four* key parameters for the design of first-order compensated bandgap references: E_G, I_S, X_{TI}, and V_{AR}. Three already known from the literature but the fourth, V_{AR}, is a rather unknown parameter. The influence of the V_{AR} on the temperature behavior of a first-order compensated bandgap reference is considerable. The V_{AR} has to be taking into account to reduce systematic errors.

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Minimization of the 0–1 Linear Programming Problem Under Linear Constraints by Using Neural Networks: Synthesis and Analysis

M. Aourid and B. Kaminska

***Abstract*—In this brief, we propose a new design: a Boolean Neural Network (BNN) for the 0–1 linear programming problem under inequalities constraints by using the connection between concave programming and integer programming problems. This connection is based on the concavity and penalty function methods. The general objective function obtained, which combines the objective function and constraints is fixed as the energy of the system. The simulation results for the new BNN show that the system converge rapidly within a few neural time constant.**

I. Introduction

In combinatorial optimization problem, Artificial Neural Network (ANN) have turned out to be power fuel in finding good approximate solutions. A wide variety of combinatorial optimization problems by neural network has been published, [1], [2] for the traveling salesman problem, [3] for the knapsack problem, [4] for the set covering problem, [5], [6] and for the job-shop scheduling, [7] for 0–1 quadratic problem. The basic idea is to derive an appropriate energy function which contains the constraints and the objective function, and then to find minima of the energy function using neural network. The drawback with some earlier methods is that, the energy function is associated to several parameters that must be fixed randomly before the network is run, or the network computation must be associated to other methods to help it to find a feasible solution. Although, many applications of neural networks in combinatorial optimization has been proposed, its use to 0–1 linear programming has not been exploited very much. Recently, Ohlsson *et al.* [3] used an ANN to solve the multidimensional knapsack problem where the minima of the energy function has been approximated by using mean field theory (MFT) techniques.

In this brief, a new ANN for solving 0–1 linear problem subject to inequality constraints is introduced. To design the network an energy function is derived. First, by using the equivalence between concave and integer programming problems and second, by incorporating the inequality constraints into the objective function. These energy is associated with two parameters (α and μ) that must be chosen appropriately in order to ensure the convergence and stability of the network. The solutions found by our network are all feasible solutions.

The paper is organized as follows. In Section II, we give a formulation of the 0–1 linear problem and describe how to guarantee the equivalence between the original problem and the real concave problem. In Section III, the energy function for designing the BNN is introduced, then the synthesis and the analysis are performed. In Section IV, we give a method on how to choose μ. Section V provides some simulation results to illustrate the performance of the network. Finally, the conclusion follows in Section VI.

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