INTERNATIONAL INSTITUTE FOR HYDRAULIC AND ENVIRONMENTAL ENGINEERING

SCALE MODELS IN HYDRAULIC ENGINEERING

by

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Contents

Scale models in hydraulic engineering

1. Introduction
1.1. General

Hydraulic engineering works are in many cases so complex that their design requires much care. Before the work is carried out it has to be sure that the effects anticipated will indeed be reached and that negative effects on the environment are reduced as much as possible.

This is directly clear for works effecting large areas (e.g. high dams in rivers). It holds, however, also for works that only seem to have a restricted area of influence.

This implies that the behaviour of the structure itself and its influence on the environment has to be forecasted. This prediction is carried out by means of *models* of various sorts and sophistication.

Two different types of models can be distinguished at once
(i) *Mathematical models* can be used if the problem can be described mathematically with sufficient details. In most cases nowadays computers are used to solve these problems.
(ii) *Scale models* are used if the physical phenomena can be reproduced with sufficient similarity by reducing the length dimensions of the real problem area.

These two types of models are used for different types of problems. In some cases both types of models can be used; then an adequate selection has to be made. In some other cases a combination of the two model types is used to arrive at the forecast anticipated.

These lecture notes deal mainly with *scale models*. The relation with *mathematical models* is discussed in general in Subsection 1.3.3. The intention is to present a general outline on the theory and application of these scale models.

1.2. Principle of modelling

The use of a model for the design of a hydraulic engineering work
in fact implies making a detour (Fig.1.1)

```
┌─────────────────┐  solving   ┌─────────────────┐
│ Model problem   │──────────▶ │ Model solution  │
└─────────────────┘            └─────────────────┘
         ▲                               │
      modelling    interpretation        ▼
┌─────────────────┐            ┌─────────────────┐
│ Prototype problem│           │ Prototype solution│
└─────────────────┘            └─────────────────┘
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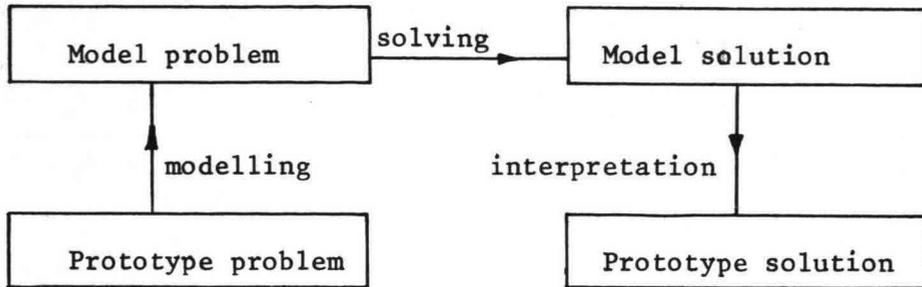
Fig.1.1.Modelling is making a detour.

There are three interrelated phases:

Modelling   implies the reproduction of the real problem area
(the 'prototype') in a suitable way. It requires of
course knowledge of the phenomena involved to arrive at
a useful model

Solving   of the technical problem in the model leads to a solution
which in principle only holds for the model. It requires
engineering capability because the model is not solving
the problem itself!

Interpretation of the model solution is necessary to arrive at the
solution valid for the prototype. The more reliable the
model is the easier is this interpretation.

It will be clear that these three phases are closely related, especially
modelling and interpretation. In fact the modelling has to be carried
out considering the possibilities of interpretation.

The reliability of the forecasts obtained by means of the model is
served largely if the accuracy is tested in some way. This implies
in fact verification of the model (hindcasting).

A clear distinction has to be made between two stages:

(i) *Calibration* of the model implies adjusting the model by means of prototype measurements in such a way that the model data fit the prototype data sufficiently. The model is then reproducing a specific, known, situation in the prototype.

(ii) *Verification* of the model implies hindcasting of *another* known situation without adjusting the model anymore. In fact verification is a must because calibration alone is not a sufficient guarantee for reliability.

Both calibration and verification stress the need of prototype data. These data form an essential element in the whole set up of the forecasting.

In some cases calibration and verification are not possible because the prototype does not yet exist (e.g. design of weirs and locks). It is then advisable to carry out measurements on existing similar structures. Experience in modelling is gained if the model results are later checked in the prototype.

It has to be stressed that from the above given considerations it follows that modelstudies cannot *replace* (expensive!) prototype measurements. On the contrary modelstudies *require* prototype data.

1.3. Scale models in hydraulic engineering
1.3.1. Principle of scaling

The principle of the use of scale models consists of the possibility to reproduce the real problem (the 'prototype') on a smaller scale in such a way that the phenomena in the scale model are *similar* in model and prototype.

This similarity regards various aspects: (i) *geometric similarity* (ii) *kinematic similarity* (iii) *dynamic similarity* etc.

The relation between model and prototype is found by defining for each parameter a *scale* (or scale factor).

The scale of a parameter is defined by the ratio between the prototype value and the model value of this parameter.

For instance:

$$\text{length scale} = n_L = \frac{L_p}{L_m} = \frac{\text{length in prototype}}{\text{length in model}} \qquad (1-1)$$

By defining the scales in this way (and not in the opposite way) the scales usually are larger than unity. This facilitates mental arithmetic.

Dimensionless parameters such as Reynolds number and Froude number consist of a number of individual parameters. For these dimensionless parameters also scales can be defined.
For instance:

$$\text{Re} = \frac{uL}{\nu}$$

Simple arithmetic shows

$$n_{Re} = \frac{Re_p}{Re_m} = \frac{u_p \cdot L_p}{\nu_p} \cdot \frac{\nu_m}{u_m \cdot L_m} = \frac{n_u \cdot n_L}{n_\nu} \qquad (1-2)$$

Reynolds number can be expressed as the ratio between the force due to inertia and the force due to (viscous) friction.
Or

$$\frac{(\rho L^3)(u^2/L)}{(\eta u/L)(L^2)} = \frac{\rho u L}{\eta} = \frac{uL}{\nu} = \text{Re} \qquad (1-3)$$

Note that the celerity du/dt of the fluid has the order of magnitude $du/dt \sim u/(L/u) = u^2/L$. This is used in Eq. (1-3)

Now, if in a certain problem the two forces together defining Re are of (almost) equal importance then their ratio has to be in the scale model the same as in the prototype.

This is an example of *dynamic similarity*: $n_{Re} = 1$
Or the condition

$$n_{Re} = n_u \cdot n_L \cdot n_\nu^{-1} = 1 \qquad (1-4)$$

This is an example of a *scale condition*: it is a requirement for similarity. In principle deviations from a scale condition are possible albeit that then to a certain extent *scale effects* are generated.

In some cases scale relations can be derived which *must* be fulfilled. An example is formed by the relation which follows for a periodic surface wave with wavelength λ, wave period τ and wave celerity c. From

$$\lambda = c \cdot \tau \qquad (1-5)$$

it can be easily deduced that

$$n_\lambda = n_c \cdot n_\tau \qquad (1-6)$$

Equation (1-6) cannot but be fulfilled. This is because Eq. (1-5) in fact defines the celerity c and of course the definition of c has to be made in the same way in model and prototype.

The deduction of Eq.(1-6) from Eq.(1-5) applies a simple rule:

> *The scale of a product of parameters is equal to the product of the scales of these parameters.*

Also in Eq.(1-2) this rule has been applied.

In practice relations for scales have many times to be deduced for a *sum* of parameters. A simple example is formed by a steady flow with free surface, for which friction can be neglected.
Here Bernoulli law holds

$$H = h + s \qquad (1-7)$$

This gives a link between the **total** head (H), the piezometric head (h) and the velocityhead ($s = u^2/2g$)

A suitable form of the scale relation following from Eq.(1-9) requires some algebra

$$n_H = \frac{H_p}{H_m} = \frac{h_p + s_p}{h_m + s_m} = \frac{n_h + s_p/h_m}{1 + s_m/h_m} = \frac{n_h + n_s(s_m/h_m)}{1 + s_m/h_m} \tag{1-8}$$

Now two cases can be distinguished from Eq. (1-8)

$n_h = n_s$ leads to $n_H = n_h = n_s$. Thus all lengths present in Eq.(1-7) have the same scale

$n_h \neq n_s$ leads to the case that

$$n_H = f\{n_h, \; n_s \; \text{and} \; s_m/h_m\} \tag{1-9}$$

Equation (1-9) expresses that a scale (here n_H) is not only a function of other scales (here n_h and n_s) but also of a parameter (here s_m/h_m). This means that *scale effects* are present: in this case n_H will vary in space.

It is clear that in this case scale effects can be avoided by selecting $n_h = n_s$.
Geometric similarity will require that the scale of the piezometric head (n_h) is equal to the scale of depth (n_a). Moreover it can easily be shown as $n_g = 1$ that

$$n_s = n_{u^2} = n_u^2 \tag{1-10}$$

Thus the condition for absence of scale effects is, using the condition for geometric similarity ($n_h = n_a$):

$$n_a = n_u^2 \quad \text{or} \quad n_u = \sqrt{n_a} \tag{1-11}$$

This can also be derived in the following way. Free surface flow for which friction can be neglected, is characterized by Froude number; which can be derived by stating that in model and protoype the ratio between the force due to inertia and the force due to gravity has to be the same.

Hence

$$\frac{(\rho L^3) \cdot (u^2/L)}{gL^3} = \frac{u^2}{gL} = Fr^2 \qquad (1-12)$$

The Froude number is here defined as $Fr = u/\sqrt{gL}$ or $Fr = u/\sqrt{ga}$ expressing the ratio of the flow velocity and the celerity ($c = \sqrt{ga}$) of a surface wave in stagnant water. In literature also the definition $Fr = u^2/ga$ appears.

It can easily be shown that the condition $n_{Fr} = 1$, necessary for free surface flow leads to Eq. (1-11).

Two important remarks have to be made at this stage:

(i) Considering only in general the forces active, like in Eq. (1-12), only leads to the main condition for the proper similarity (Eq. (1-11)). However, determining the main condition from the basic hydrodynamic equation (here Eq. (1-7)) gives as a by-product also insight into the magnitude of the scale effects present if the main condition is not fulfilled (see Eq. (1-8)).

(ii) Consider now a case of free surface flow for which viscosity is also present. Here both Reynolds condition and Froude condition have to be fulfilled at the same time. If the same fluid is used in model and prototype ($n_\nu = 1$) this leads to

$$
\begin{aligned}
n_{Re} &= 1 & &\rightarrow & n_u &= n_L^{-1} \\
&&&&&& (1-13) \\
n_{Fr} &= 1 & &\rightarrow & n_u &= n_L^{1/2}
\end{aligned}
$$

The two conditions of Eq. (1-13) can only be fulfilled at the same time if the *trivial solution* $n_u = n_L = 1$ is selected.

These two remarks indicate the key role played by scale effects in the procedure of scaling. The presence of more than one condition at the same time forces to deviate to a certain extent from the scale conditions. This in principle leads to scale effects and in practice a compromise has to be found. Therefore insight into the magnitude of these scale effects is necessary. Or, in other words the better the mathematical description of the flow problem is, the more reliable the scale model can be.

The principle of scaling has been discussed here by means of some simple examples. A more thorough treatment of the scale relations following from various hydraulic phenomena is given in Chapter 3. The deduction of scales for various problems in hydraulic engineering (i.e. finding the most adequate compromise has been discussed in Chapter 4. Of course also practical considerations play a role in these cases.

1.3.2. Examples of scale models

A large variety of scale models is possible. Here at this stage only some examples will be given. For the correct understanding it has to be mentioned that sometimes scale models are *distorted* (e.g. $n_L \neq n_a$).

The need for distortion can be explained as follows. Suppose a river reach of 5 km has to be studied. The lenght scale might be of the order of $n_L = 500$ depending on the problem to be studied and the space available.

For $n_L = n_a = 500$ (*undistorted*) a depth of say 2 m in prototype would result in $a_m = 0.4$ cm. Fulfilling Froude condition in this case leads to $n_u = \sqrt{n_a} \approx 22$. With $u_p = 1$ m/s this leads to $Re_p = (u_p \cdot a_p/\nu) = 2 \times 10^6$ and to $Re_m \approx 200$. This would mean laminar flow in the model.

In selecting $n_a < n_L$ both a_m and u_m become larger without the need of more space. The possibility of using distortion will be discussed in more detail in Chapter 3. It can be stated already that distortion is possible if the vertical accelerations can be neglected with respect to the acceleration due to gravity (thus the presence of a hydrostatic pressure distribution is a prerequisite for distortion).

Figure 1.2 shows model and prototype of a section of a tunnel. The aim of the study is to find the proper procedure for sinking the section.
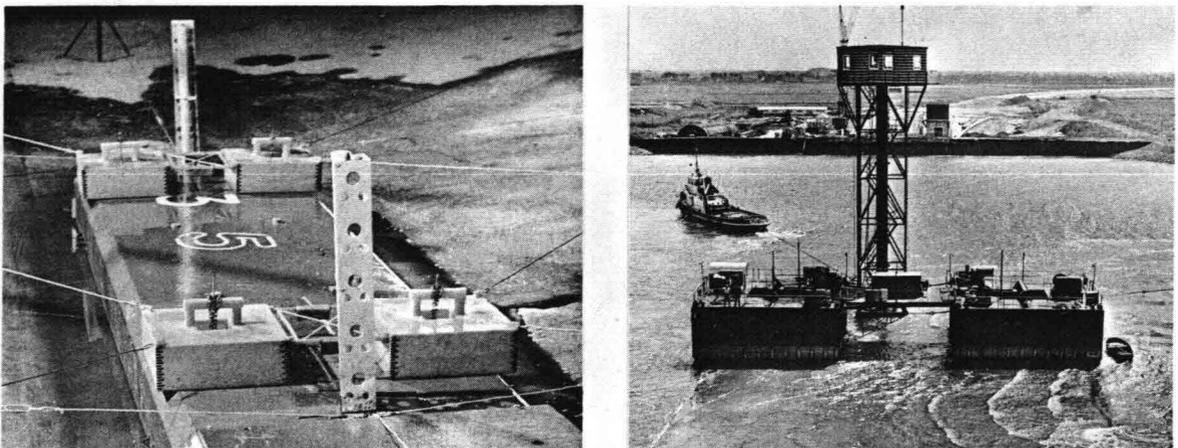
Fig. 1.2 Sinking tunnel section: scale model and prototype
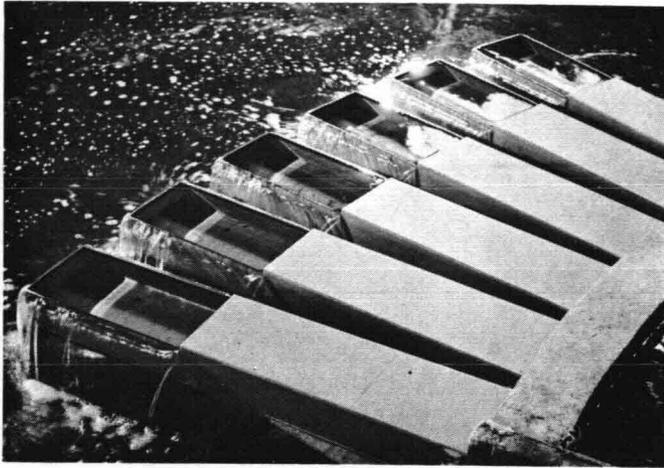
Fig. 1.3. Cooling-water outlet

It is intended to measure the forces to be delivered by the tugboats during the operation. Obviously in the case of Fig. 1.2 distortion is not allowed.

Figure 1.3. shows a scale model of a cooling-water outlet. The ships sailing in the canal or river receiving the cooling-water should not be hindered (undistorted model).

Figure 1.4. demonstrates a model of a navigation lock to be constructed between an area with salt water and one with fresh water. To reduce salt-intrusion into the fresh-water channel a complex filling and emptying system is necessary. Obviously this has to be an undistorted model in which the density differences between fresh water and salt water are reproduced, in order to reproduce the currents correctly. In this case density differences influence the currents present ('density currents').
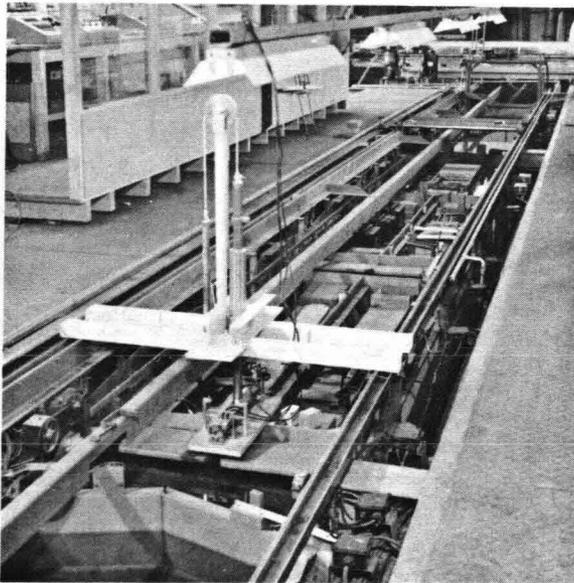
Fig. 1.4. Scale model of the Kreekrak navigation lock.

Although also between cold water and warm water small density differences are present, it is not necessary always to reproduce these differences.

For example the flow pattern of Fig. 1.3. will be hardly effected by the small density differences present. Thus water with a constant density can then be applied.

The pressure fluctuations in a turbulent flow or the time-dependent forces due to surface waves may cause vibrations in a hydraulic structure (gates, valves etc.). The elastic properties of the structure now also play a role in the magnitude of these vibrations and hence in the stresses in the structure.
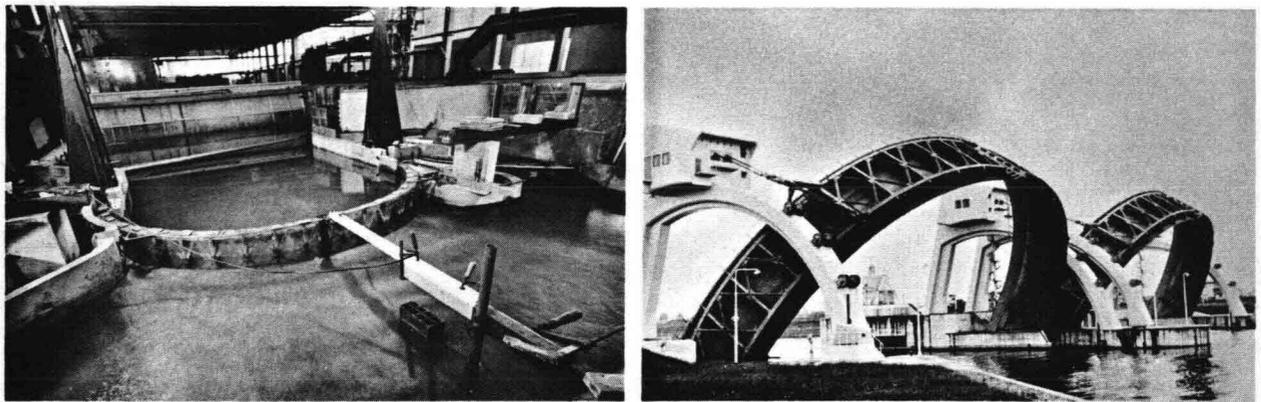
Fig. 1.5. Example of elastical similar model (model and prototype).

This means that also the elastic properties of the structure have to be reproduced correctly. Fig. 1.5. gives an example of such a scale-model study. It regards the visor gate near Hagestein in the Lower-Rhine.

Also for morphological problems in rivers and coastal areas scale models are applied. In these cases watermovement and sediment-movement have to reproduced correctly to simulate bedlevel and bedlevel changes correctly.
Figure 1.6. gives a picture of a mobile-bed river-model of the Waal River, the mainbranch of the Rhine River in the Netherlands.

Fig. 1.6. Mobile-bed river-model

1.3.3 Scale models versus mathematical models

There is a large similarity between the use of scale models and mathematical models. In fact in Section 1.2 it was not necessary to indicate which one of the two model types was discussed. The general philosophy is exactly the same.

Starting from the physical knowledge of the problems involved the model can be constructed. A mathematical model requires a relatively detailed mathematical description of the physical problem. For a scale model this is rather a desire than a must. It can be stated that the better the mathematical description of the problem is the more insight can be obtained in possible scale effects. This can be demonstrated by the case of steady flow with free surface for which Bernoulli law holds.

As will be shown in Section 2.2. dimensional considerations lead to the knowledge that Froude number has to be the same in the scale model as in the prototype. No insight is obtained what happens if $n_{Fr} \neq 1$ is selected. This insight, however, is gained if the relevant mathematical description is used. This is the case in Subsection 1.3.1. starting from Bernoulli law. The result is an equation (Eq. (1.8)) from which scale effects can be estimated.

It is logical that in some cases the mathematical description is such that both scale models and mathematical models are possible. An example is formed by a system of tidal channals. The selection is then based on a number of criteria which vary in time. The prediction of the hydraulic changes due to the construction of the Enclosure dyke of the Zuydersea was made by means of a mathematical model (Lorentz, 1926). In the late fourties the Delft Hydraulics Laboratory constructed a scale model for the delta in the South-Western part of the Netherlands (Fig. 1.8)
The improvement of the measuring systems and control systems during the last decades made gradual improvement of this model possible. A similar development was present for the electrical analogue made for the Dutch delta. Gradually the analogue became the special purpose hybride computer (Fig. 1.7) which it is present.

Fig. 1.7 DELTAR: analogue-digital model for the Netherlands' Delta
(by courtesy of Rijkswaterstaat)
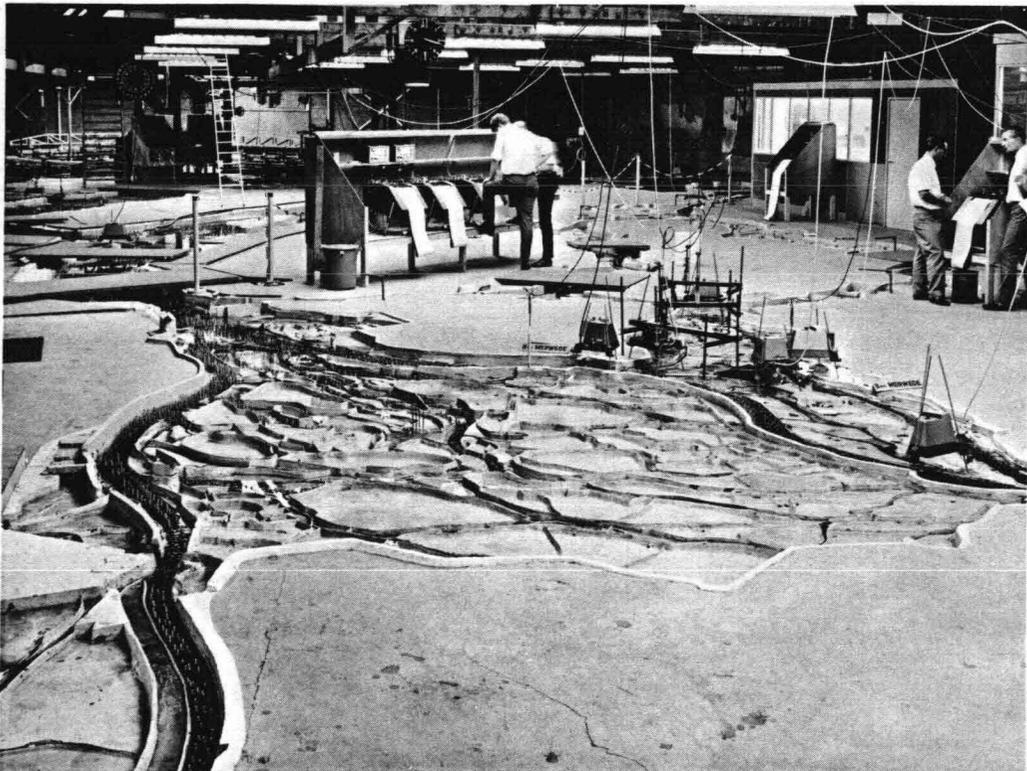
Fig. 1.8 Former scale model of the Netherlands' Delta (Delft Hydraulics
Laboratory)

Table 1. Various types of models for hydraulic network.

type of model		results obtained by	remarks
mathematical model	digital	computations	The physical phenomena and the geometry are described mathematically. Numerical solution by means of a digital computer.
	analog/digital (hybride)	computations/ measurements	Solution by means of a (special) hybrid computer. Example: DELTAR of Rijkswaterstaat (Fig. 1.7).
analog model	electric analogue	measurements/ computations	The equations are 'translated' electrically. In fact electric tensions and currents are measured. Originally the DELTAR had this character.
	hydraulic analogue	measurements	This analogue (Curtet-mod.) consists of reservoirs reproducing storage and diaphragms in between for the resistance. Only possible if inertia is negligible. Hardly used. No geometric affinity.
scale models	strongly distorted	measurements	Due to strong distortion geometric similarity reduces to geometric affinity. Example: former scale model of Dutch delta (Fig. 1.8).
	hardly or not distorted	measurements	Due to $n_L \simeq n_a$ there is geometric similarity. Application for hydraulic networks restricted.

In Table 1 an outline has been given of model types possible for the study of unsteady flow in open-channel networks. The table shows a gradual transition between the pure mathematical model based on (numerical) *computations* and the scale model in which the results are obtained via *measurements*.

Not for all hydraulic problems such a large selection of possibilities is present. The computational methods are still not powerful enough to solve problems like the ones shown in Subsection 1.3.2. (Figs. 1.1 ... 1.6). On the other hand waterhammer problems for instance are not very suitable to be studied with scale models; here mathematical models are superior.

An interesting development is the combined use of a mathematical model and a scale model. It can be stated that this is the result of two circumstances

(i) Mathematical models are possible for relatively simple flow problems. Little detail is obtained.

(ii) Both model types reproduce a certain area. Boundaries have to be supplied with boundary conditions. These boundary conditions have to be *independent*. This means that any change in the in the geometry of the model area may only have a negligible effect on the boundary conditions.

It is now possible to make a mathematical model of a large area (having independent boundaries) which delivers dependent boundary conditions for a scale model of a smaller area. In the small area much more detail information is obtained than the mathematical model can provide.

2. Aids to experimental research

2.1. General

Experimental research in the field of hydraulic engineering requires tools. The tools are an experimental set-up from which specific characteristics (waterlevels, pressures, flowvelocities etc.) are measured by means of suitable instruments.

Some general remarks are made in this Chapter on the aids to experimental research in the field of hydraulic engineering. These remarks have a wider sense than for the fruitful application for scale-models only. Many aspects apply also to the set-up of a good field survey for the prototype.

This holds specifically to Par. 2.2. and Par. 2.3. where respectively dimensional analysis and statistics are treated. The discussion on instruments (Par. 2.4.) is limited to laboratory instruments.

2.2. Dimensional analysis

The scales of the various parameters can be determined from the mutual relations of these parameters expressed in the dimensionless products (e.g. Reynolds number, Froude number etc.). First of all the technique of dimensional analysis will be treated and then the advantages and disadvantages will be discussed.

If a physical phenomenon is described by n parameters p_i with $i = 1, \ldots,$ n and if m elementary quantities are involved in the problem, then $n - m$ dimensionless products can be derived.

In hydraulic engineering there usually are 3 elementary quantities: *mass*, *length* and *time*; this means $m = 3$. The derivation below shall be restricted to this case.

Any of the $n - 3$ dimensionless products (Π) can be composed with

$$\Pi = p_1^{k_1} \cdot p_2^{k_2} \cdot p_3^{k_3} \cdot \ldots \cdot p_n^{k_n} \tag{2-1}$$

If p_i has the dimensions $\left[M^{\alpha_i}\ L^{\beta_i}\ T^{\gamma_i} \right]$ then the dimension of Π can be expressed by

$$\left[\Pi \right] = \left[M^{\alpha_1}\ L^{\beta_1}\ T^{\gamma_1} \right]^{k_1} \left[M^{\alpha_2}\ L^{\beta_2}\ T^{\gamma_2} \right]^{k_2} \cdot \ldots \cdot \left[M^{\alpha_n}\ L^{\beta_n}\ T^{\gamma_n} \right]^{k_n} \qquad (2-2)$$

Or, written slightly differently:

$$\left[\Pi \right] = \left[M^{(\alpha_1 k_1 + \alpha_2 k_2 + \ldots + \alpha_n k_n)} \right] \left[L^{(\beta_1 k_1 + \beta_2 k_2 + \ldots + \beta_n k_n)} \right] \left[T^{(\gamma_1 k_1 + \gamma_2 k_2 + \ldots + \gamma_n k_n)} \right] \quad (2-3)$$

This implies that Π can only be dimensionless if

$$\alpha_1 k_1 + \alpha_2 k_2 + \ldots + \alpha_n k_n = 0$$
$$\beta_1 k_1 + \beta_2 k_2 + \ldots + \beta_n k_n = 0 \qquad\qquad\qquad (2-4)$$
$$\gamma_1 k_1 + \gamma_2 k_2 + \ldots + \gamma_n k_n = 0$$

From Eq. (2-4) the coefficients α_i, β_i and γ_i are known because it is known which parameters p_i are involved. The problem is now to derive n exponents k_i from Eq. (2-4). It has to be remarked that if m elementary quantities are involved then m equations to determine k_i are present.

Apparently in the case of Eq. (2-4) it is possible to select freely n-3 values for k_i and then the remaining 3 values are fixed by Eq. (2-4).

It is advisable to determine k_i in a systematic way. Here a method is used treated extensively by Langhaar (1957). This will be done by means of an example to avoid too much abstraction.
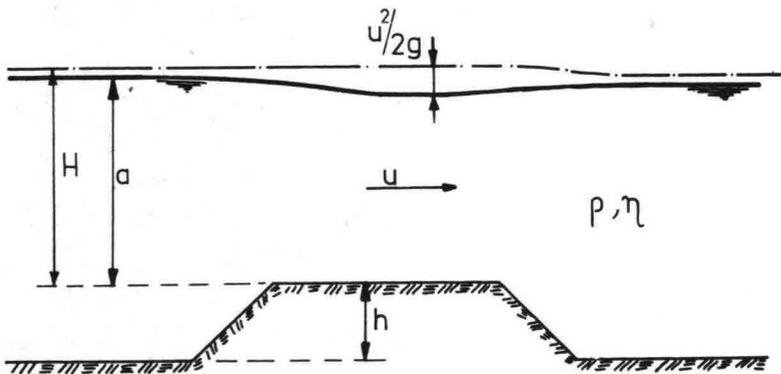
Consider the flow over a sill (Fig. 2.1). A number of physical parameters are involved in this problem. They will be put in a special order for reasons that become clear later.

Fig. 2.1 Flow over a sill

Group 1 dependent parameters   →   H
Group 2 parameters that can be
         changed in the experiment   →   a, u, ρ, h
Group 3 all other parameters   →   g and η

The exponents α_i, β_i and γ_i are now ordered systematically.

GROUP	1	2				3	
PARAMETER	H	a	u	ρ	h	g	η
M	0	0	0	1	0	0	1	← α_i
L	1	1	1	-3	1	1	-1	← β_i
T	0	0	-1	0	0	-2	-1	← γ_i

Table 2.1.

In this case Eq. (2-4) read

$$
\begin{aligned}
k_4 \qquad\qquad\quad + k_7 &= 0 \\
k_1 + k_2 + k_3 - 3k_4 + k_5 + k_6 - k_7 &= 0 \\
- k_3 \qquad\qquad -2k_6 - k_7 &= 0
\end{aligned}
$$

(2-5)

Elimination of k_5, k_6 and k_7 leads to

$$
\begin{aligned}
k_5 &= \quad - k_1 - k_2 - \tfrac{1}{2} k_3 + \tfrac{3}{2} k_4 \\
k_6 &= \qquad\qquad -\tfrac{1}{2} k_3 + \tfrac{1}{2} k_4 \\
k_7 &= \quad - k_4
\end{aligned}
$$

(2-6)

The four dimensionless products can be selected easily from the matrix of solutions given in Table 2.2.

Table 2.2. is composed by means of the following procedure

(i)   The first four k-values are selected in such a way that one value becomes equals to one and the other three equals to zero.

(ii) The other k-values are obtained by using the rows of Eq. (2-6) as columns for Table 2.2.

k_i	k_1	k_2	k_3	k_4	k_5	k_6	k_7
parameter	H	a	u	ρ	h	g	η
Π_1	1	0	0	0	-1	0	0
Π_2	0	1	0	0	-1	0	0
Π_3	0	0	1	0	$-\frac{1}{2}$	$-\frac{1}{2}$	0
Π_4	0	0	0	1	$\frac{3}{2}$	$\frac{1}{2}$	-1

Tabel 2.2.

The following Π - values result from Tabel 2.2.

$$\Pi_1 = H/h$$
$$\Pi_2 = a/h$$
$$\Pi_3 = u/\sqrt{gh}$$
$$\Pi_4 = \rho g^{1/2} \eta^{-1} h^{3/2}$$

(2-7)

Naturally the four Π - values can be used to obtain other dimensionless products:

For instance

$$\Pi_5 = \Pi_3 \cdot \Pi^{-1/2} = u/\sqrt{ga}$$

This dimensionless product has the character of a Froude number.

And also as

$$\Pi_4 = \nu^{-1} \cdot (\Pi_3^{-1} \cdot u) \cdot h = \nu^{-1} (\Pi_3^{-1} \cdot u) \, a. \Pi_2^{-1}$$

it can be stated

$$\Pi_6 = \Pi_4 \cdot \Pi_3 \cdot \Pi_2 = ua/\nu$$

which has the character of a Reynolds number.

Hence for the determination of scales the following Π - values can be used

Π_1 and Π_2 → geometric similarity

Π_5 → Froude number ⎤ dynamic

Π_6 → Reynolds number ⎦ similarity

For the determination of scales it is not important whether $\Pi_1...\Pi_4$ or Π_1, Π_2, Π_5 and Π_6 are used.

This is not always the case when the results of a dimensional analysis are used. It has to be recalled that dimensional analysis is also used for the presentation of experimental data. Here the danger of *spurious corre-lations* is always present. To explain this a simple example will be given here. For more details reference is made to Benson (1965).

A stochastic variable x is characterized by its mean $(\bar{x})$ and standard deviation (σ) or relative standard deviation $(r = \sigma/\bar{x})$.

For two stochastic variables (x and y) also the correlationcoefficient r_{xy} is of importance. Spurious correlation can now be created if x is corre-lated with the stochastic variable z composed from x and y.

For instance if x and y are not correlated $(r_{xy} = 0)$. Then r_{xz} with $z = x/y$ can show a significant correlation! For this case it can be shown that

$$r_{xz} \approx \frac{r_x}{(r_x^2 + r_y^2)^{1/2}} \qquad (2-8)$$

The value of r_{xz} is thus directly dependent on the *ratio* between r_x and r_y.

For instance

$r_x = r_y$ → $r_{xz} = 0.71$

$r_x = 2r_y$ → $r_{xz} = 0.89$

$r_x = 3r_y$ → $r_{xz} = 0.95$

It is clear that this correlation is *spurious*. To avoid that wrong conclusions are drawn from statistical analysis of experimental data by means of dimension-less products, it is of importance that the parameters present in the problem and having a strong stochastic character appear only in *one* dimensionless pro-duct. This is why in Table 2.1. the parameters are grouped in a special order. Note that by this procedure in the example of Fig. 2.1. only in one of the resulting Π_1 through Π_4 values the dependent variable H appears.

Remarks:

(i) Related to spurious correlations are cases in which results of experimental data are plotted in such a way that nice conclusions seem possible which, however, are only obvious. The following example explains this statement.

In alluvial channels the bedroughness e.g. expressed by the Chézy-value (C), may vary considerably. If in such a channel with constant width C values are measured under a variety of flow velocities then the question is how to plot the data, preferably with dimensionless parameters. It seems logical here to plot $C/\sqrt{g}$ versus $Fr = u/\sqrt{ga}$ as the bed material is the same for all these data. Most probably this will give a nice lineair relationship $C/\sqrt{g} \sim Fr$. This can be seen as follows.

From Chézy equation

$$u = C\sqrt{ai} \qquad\qquad (2-9)$$

it yields

$$\frac{u}{\sqrt{ga}} = \frac{C}{\sqrt{g}} i^{1/2} \qquad\qquad (2-10)$$

Or

$$C/\sqrt{g} = (i^{-1/2})Fr \qquad\qquad (2-11)$$

For all these data uniform flow is present. Thus as the bed slope in this case is a constant $C/\sqrt{g}$ is proportional to Froude number. Most likely values of $C/\sqrt{g}$ that do not fit Eq. (2-11) contain a C-value computed wrongly!

(ii) It has to be noted that dimensional analysis does not give *insight* into the physical phenomena. It only puts the available notion on the physical phenomena into a certain *order*. The example of Fig. 2.1 demonstrates this. If the modelling of the sill is based on the four dimensionless products of Eq. (2-7) then with $n_\nu = 1$ only the trivial solution $n_L = 1$ results. This inattractive solution can only be avoided if the physical notion is used that in the prototype the influence of the viscosity is negligible. This is also the case in the scale model if this is not too small. Hence $n_{Re} \neq 1$ is possible.

(iii) Besides Froude number and Reynolds number a large number of dimension-
less products are in use. The following examples are given here.
Weber number consists of the ratio between the force due to inertia
and the force due to surface tension (σ)

$$\frac{(\rho L^3)(u^2/L)}{\sigma L} = \frac{\rho u^2 L}{\sigma} = We \qquad (2-12)$$

Note that σ is a force per unit of *length*.

Cauchy number is the ratio between the force due to inertia and the
elastic force (E is modulus of elasticity)

$$\frac{(\rho L^3)(u^2/L)}{EL^2} = \frac{\rho u^2}{E} = Ca \qquad (2-13)$$

2.3. Statistics

In experimental research the measured value of a parameter will differ from
the 'real' value. The errors may be *systematic* or *stochastic*. Systematic errors
can usualy be determined from a proper calibration of the instruments. The
stochastic errors will mainly be discussed here.

Statistical methods have to be applied for the processing of the measured
data. The following general problems may come foreward
(i)   *Design of experimental set-up*

There is a known theoretical relation

$$\underline{z} = f\{\underline{x}, \underline{y}\} \qquad (2-14)$$

The values of $\underline{x}$ and $\underline{y}$ are measured and $\underline{z}$ is deduced by using Eq. (2-14).
In the design it is important to know the desirable accuracy by which
$\underline{x}$ and $\underline{y}$ have to be measured in order to obtain $\underline{z}$ with a required accu-
racy.
(ii) *Data processing*

In a certain given experimental set-up Eq. (2-14) holds. How is $\underline{z}$ deter-
mined, together with its accuracy? In many cases stochastic observations
$x(t)$ have to be processed (spectral analysis).
(iii) *Risc-analysis*

Related to the cases above is to find the probability distribution $P\{\underline{z}\}$
from $P\{\underline{x}\}$ and $P\{\underline{y}\}$ provided the relation of Eq. (2-14) is given.

These problems in fact are solved by applying regression analysis in which the properties of $\underline{z}$ are determined. In many cases the equations like Eq. (2-14) are non-linear. This may create substantial statistical complications.

Here the discussion will be restricted to the *propagation of errors*. Two cases will be considered.

- *computergeneration of P{$\underline{z}$}*

There exists a known relation $\underline{z} = f(\underline{x}, \underline{y})$. Moreover the probability distributions P{$\underline{x}$} and P{$\underline{y}$} are given. As $0 < P\{\underline{x}\} < 1$, a random number in the interval $(0,1)$ leads to a value $\underline{x}$. Similarly a $\underline{y}$-value can be determined. Hence one $\underline{z}$-value can be deduced. Repeating the procedure leads to P{$\underline{z}$}. Provided a computer-program is available, this is a quick procedure.

- *Analytical determination*

If $\underline{x}$ and $\underline{y}$ are normally distributed with known standarddeviations σ_x and σ_y respectively then an analytical treatment is possible.
The standarddeviation σ_z of z can then *approximatively* be found from the *quadratic rule for error-propagation*

$$\sigma_z^2 \approx \sigma_x^2 \left[\frac{\partial f}{\partial x}\right]^2 + \sigma_y^2 \left[\frac{\partial f}{\partial y}\right]^2 + \text{covariance term} \tag{2-15}$$

The rule is especially attractive if $\underline{x}$ and $\underline{y}$ are independent ($\rightarrow$ cov $= 0$).

The analytical procedure is attractive for the design of experiments, provided σ_x and σ_y can be estimated beforehand.
Equation (2-15) obtains a simple form if $z = f(x, y)$ has an exponential shape.
For instance

$$z = a \, x^b \, y^c \tag{2-16}$$

in which a, b and c are constants and x and y not correlated.

Application of Eq. (2-15) to Eq. (2-16) yields

$$\sigma_z^2 = \sigma_x^2 \{abx^{b-1}y^c\}^2 + \sigma_y^2 \{acx^b y^{c-1}\}^2 \tag{2-17}$$

Or

$$\frac{\sigma_z^2}{z^2} = b^2 \frac{\sigma_x^2}{x^2} + c^2 \frac{\sigma_y^2}{y^2} \qquad (2\text{-}18)$$

Defining the relative error r_x with

$$r_x = \frac{\sigma_x}{\bar{x}} \qquad (2\text{-}19)$$

leads to the simple result

$$r_z^2 = b^2 r_x^2 + c^2 r_y^2 \qquad (2\text{-}20)$$

It should be noted that the function values in Eq. (2-15) are based on the mean value (expectation) of the variables $\underline{x}$ and $\underline{y}$ respectively.

Examples:

The dragcoefficient C_D on a body under water has to be determined for a range of Reynolds numbers.
The physical relationship is

$$F = C_D \cdot \tfrac{1}{2} \rho u^2 \cdot A \quad \text{with} \quad C_D = C_D \text{ (Re)} \qquad (2\text{-}21)$$

in which F is the force acting on the body with characteristic dimension A in the velocity field characterized by the velocity u. The required value of C_D is found from

$$C_D = \frac{F}{\tfrac{1}{2}\rho u^2 \cdot A} \qquad (2\text{-}22)$$

The quadratic error-propagation follows from Eq. (2-22)

$$r_{C_D}^2 = 4 r_u^2 + r_F^2 \qquad (2\text{-}23)$$

if the logical assumption is made that r_ρ and r_A can be neglected with respect to other terms. The values of r_u and r_F depend directly on the instrumentation available.

If the experiments are carried out in a scale model then $n_{Re} = 1$ is the relevant scale condition because $C_D = f(Re)$. The lengthscale n_L of the model can

then be selected in such a way that dynamic similarity is present and r_{CD} is sufficiently small according to Eq. (2-23).

The following example regards a V-notch (Thomson weir) to measure (small) discharges. The discharge formula for such a weir is

$$Q = m \cdot \tan \tfrac{1}{2} \phi \cdot H^{5/2} \qquad (2-24)$$

in which H is the head of the upstream waterlevel above the lowest point of the V and ϕ is the top-angle of the V. The discharge coefficient amounts to $m \approx 1.4 \; m^{1/2}/s$. Usually $\phi = 90^0$ is taken. If the error in ϕ can be neglected then the relative error of Q follows from

$$r_Q^2 = r_m^2 + (^5/_2)^2 r_H^2 \qquad (2-25)$$

This equation can be used to determine r_Q if r_H is known as well as r_m (from calibrations).

If such a weir is calibrated then the discharge coefficient m is according to Eq. (2-24) to be applied from

$$m = \frac{Q}{H^{5/2} \tan \tfrac{1}{2}\phi} \qquad (2-26)$$

For this case the rule for quadratic error-propagation gives

$$r_m^2 = r_Q^2 + (-^5/_2)^2 r_H^2 \qquad (2-27)$$

The apparent discrepancy between Eqs. (2-25) and (2-27) can be explained as follows

- In the *calibration phase* the values of Q and H are measured and m and r_m are determined from the observations. This leads to Eqs. (2-26) and (2-27).
- In the *application phase* the value of H is measured, while m and r_m are taken from the calibration. The Eqs. (2-24) and (2-25) are used to determine Q and r_Q.

It has to be recalled that a similar situation is present for the regression analysis between two parameters $\underline{x}$ and $\underline{y}$. The analysis gives two lines and it depends on the *use* whether $\underline{y} = f(\underline{x})$ or $\underline{x} = f(\underline{y})$ is applied.

2.4. Instrumentation and control systems

Instruments are required to obtain the necessary information from scale models (flow velocities, pressures, bedlevels etc.). The accuracy of the model results depend to a large extent on the quality of the instrumentation.

Moreover a scalemodel has boundaries for which boundary conditions have to be introduced. This asks for a control system.

Both instrumentation and control systems with different degree of sophistication are used. An adequate selection is necessary for an optimal investigation by means of a scale model. Therefore some remarks are made here regarding these topics.

In principle the system of Fig. 2.2 applies when a physical property is measured.

property ⟶ [sensor] ⟶ signal ⟶ [indicator] ⟶ presentation
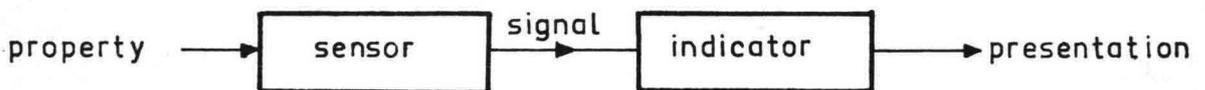
Fig. 2.2 Principle of measurements

The physical property is by means of a sensor translated into an adequate signal. The indicator translates the signal in such a way that a proper presentation is obtained.

For hydraulic scale models this may regard

- waterlevels and differences in waterlevels
- flowvelocities and discharges
- pressures and forces

- concentrations (e.g. of salt or sediment)
- temperatures

The proof that the presentation (reading) is a reliable measure of the physical property is found by *calibrating* the instrument. In this way the errors (both systematic and stochastic ones) caused by the instrument become known. Usually systematic errors can easily be taken into consideration. The random errors may be determinated for the overall accuracy that can be expected from an experimental study.

In many cases the random error is known as a percentage of the reading at the full scale of the instrument. Hence the range of the instrument has to be selected properly in order to quarantee sufficient accuracy for small readings.

The availability of certain instruments may influence the selection of the model scales.

Example

The design of a navigation lock is tested in order to guarantee that the forces acting on the ships during filling and emptying are not too large. In a particular case a maximum force of 20 kN is allowed and a relative error of 5% is accepted. The available laboratory equipment for measuring these forces has a range of $0 < F < 400$ mN with a relative error of 4% of the maximum value. Which length scale is advisable?

First of all it has to be realized that during filling and emptying of the lock vertical accelerations take place that are not negligible with respect to g. Hence all accelerations should have a scale equal to $n_g = 1$. This is the case in an undistorted model fulfilling Froude condition.
From

$$\vec{F} = m \cdot \vec{a} \tag{2-28}$$

then follows, using water in the model ($n_\rho = 1$)

$$n_F = n_m = n_\rho \cdot n_L^3 = n_L^3 \tag{2-29}$$

Having an absolute error of 4% x 400 mN = 16 mN, the relative error in
the model will be also just 5% (as acceptable in the prototype) if the
maximum force in the model amounts to 20 x 16 mN.
Thus

$$n_F = \frac{20 \times 10^3}{20 \times 16 \times 10^{-3}} = 62500 = n_L^3 \qquad (2\text{-}30)$$

This means that for $n_L \approx 40$ the requirement is just fulfilled. For $n_L > 40$
the model is not accurate enough. For $n_L < 40$ the model may be too large (and
too expensive) and the instrument may become inadequate.
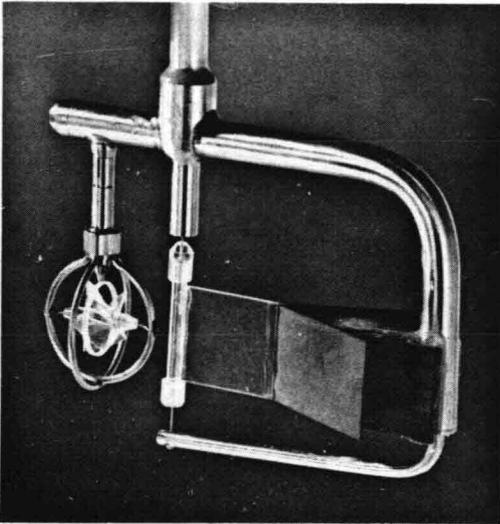
Fig. 2.3. Mini current meter.

Some special attention will be
paid to the measurement of flow
velocities, because this may
also influence the selection of
scales. Especially small veloci-
ties are difficult to measure.
The current meter of Figure 2.3.
is able to measure a *velocity
vector* (magnitude and direction
are measured independently). A
velocity > 2 cm/s can be mea-
sured. The propellor has a dia-
meter of only 15 mm.

Suppose this current meter is used in an undistorted model, fulfilling
Froude condition and with $n_L = 50$. The minimum prototype velocity
which can be measured in the model then amounts to $2\sqrt{50} \approx 14$ cm/s.
The maximum power available to the sensor with diameter D amounts to
$(\frac{1}{2}\rho u^2)(\pi D^2 u)$. In this example this is only 3 μW. From this very small
power the sensor may of course only take a small portion. A similar
problem is present for this instrument as far as measuring the
direction is concerned. The pressure differences are so small that
the flow cannot stear the vane properly. Therefore the vane is
steered automatically in the required neutral position: i.e. power is
introduced from outside.

For an available instrument (e.g. such as the one in Fig. 2.3.) the charac-
teristics with respect to accuracy and minimum measurable velocity may set
limits to model scales when a certain accuracy for the prototype is required.

For instance in a distorted tidal model the maximum depth scale to be accepted is via the Froude condition to be applied linked to the maximum velocity scale. The latter is determined by (i) prototype velocities and their required accuracy and (ii) measurable velocities and their available accuracy.

Scale models have boundaries at which boundary conditions have to be introduced. This may simply mean the control of a constant discharge and a constant waterlevel e.g. by means of a tailgate. Tidal models may require electronic control systems to guarantee sufficient accuracy of the results.

Obviously the accuracy in the forecasts that can be obtained by means of a scale-model study depends on
- accuracy of prototype data
- accuracy of control systems
- accuracy of measuring systems.

The accuracy that can be reached with a certain scale model is tested by reproducing a known situation of the past (*verification tests*)

3. Reproduction of hydraulic phenomena

3.1. Introduction

In this Chapter various hydraulic phenomena are discussed in order to arrive at the relevant scale relations. In practical problems more than one hydraulic aspect can be present at the same time. Therefore in order to arrive at a suitable compromise for scaling it is not sufficient to derive for each individual hydraulic phenomenon the relevant scale relation, but it is also necessary to study the magnitude of scale effects.

In this respect it is attractive to divide *scale relations* into two groups

scale laws are relations between scales of parameters that *must be* fulfilled. Usually these scale laws come from hydraulic equations containing definitions.

scale conditions are relations between scales of parameters that *have to be* fulfilled in order to avoid scale effects. Deviation from scale conditions is possible but the degree of deviation has to be selected with reference to the acceptability of scale effects.

Examples

(i)   The *scale law* $n_L = n_u \cdot n_t$ cannot but be fulfilled. The underlying equation $L = u \cdot t$ in fact defines the velocity u.

(ii)   *Froude condition* ($n_u = \sqrt{n_a}$) is following from the requirement to have dynamic similarity for free surface flow with respect to the influence of gravity. It is possible to deviate from this condition albeit that then scale effects are present. In principle Eq. (1-8) makes it possible to determine the scale effects once $n_{Fr} \neq 1$ is selected for a certain problem.

(iii)   By means of Chézy equation

$$u = C\sqrt{ai} \qquad (3-1)$$

a definition of C is present. Hence the scale relation following from Eq. (3-1) is a *scale law*

$$n_u = n_C \cdot n_a^{1/2} \cdot n_i^{1/2} \qquad (3-2)$$

(iv) *Reynolds condition* ($n_{Re} = 1$) is of importance if viscous effects have to be reproduced. In hydraulic engineering in many cases the flow is highly turbulent. These cases allow to accept $n_{Re} \neq 1$ and this gives the possibility to arrive at a solution for model scales other than $n_L = 1$ as was the case in Eq. (1-13).

3.2. Free surface flow

3.2.1. Derivations

In the first place the flow in a system of open channels with rigid boundaries and with a homogeneous fluid will be discussed. In this case a hydrostatic pressure distribution can be assumed and the problem can be treated considering only one space dimension (x). The dependent variables are the depth (a) and the flowvelocity (u).

The equations of motion and continuity read

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = -\frac{1}{\rho} \frac{\partial p}{\partial x} - \frac{gu|u|}{C^2 a} \tag{3-3}$$

and

$$\frac{\partial a}{\partial t} + u \frac{\partial a}{\partial x} + a \frac{\partial u}{\partial x} = 0 \tag{3-4}$$

With $p = \rho g a + \rho g z$ Eq. (3-3) becomes

$$\frac{1}{g} \frac{\partial u}{\partial t} + \frac{\partial}{\partial x} \left[\frac{u^2}{2g} \right] = -\frac{\partial a}{\partial x} - \frac{\partial z}{\partial x} - \frac{u|u|}{C^2 a} \tag{3-5}$$

The Eqs. (3-4) and (3-5) can be used to derive the scale relations relevant to this problem. The terms of these equations may be considered as parameters. Hence in order to avoid scale effects each term of an equation has to have the same scale.

This leads to four scale relations:

(i)   The *kinematic condition*

$$n_L = n_u \cdot n_t \tag{3-6}$$

   This follows for instance from the first two terms of Eq. (3-4).

From

$$n_{(\partial a/\partial t)} = n_{(u\partial a/\partial x)} \tag{3-7}$$

follows

$$n_a \cdot n_t^{-1} = n_u \cdot n_a \cdot n_L^{-1} \tag{3-8}$$

which results in Eq. (3-6)

(ii) *Geometric condition*

$$n_a = n_z \tag{3-9}$$

easily to derive from the terms $\partial a/\partial x$ and $\partial z/\partial x$ of Eq. (3-5).

(iii) *Froude condition*

$$n_u = \sqrt{n_a} \tag{3-10}$$

which can be derived here from the terms $\partial\{u^2/2g\}/\partial x$ and $\partial a/\partial x$.

(iv) *Roughness condition*

$$n_C^2 = \frac{n_L}{n_a} = r \tag{3-11}$$

This condition follows for instance from the requirement that $\partial\{u^2/2g\}/\partial x$ and $u|u|/\{C^2 a\}$ have to be reproduced on the same scale.

Remarks

- Note that in this case it is *not* necessary to select $n_x = n_L$ equal to n_a. Hence a distortion $r = n_L/n_a > 1$ is possible. This is possible because a hydrostatic pressure distribution was postulated. In other words vertical accelerations are supposed to be negligible with respect to the acceleration of gravity. This is only the case if the flowlines are not curved in the vertical plane.

- For simplicity reasons the convective term $\partial\{u^2/2g\}/\partial x$ has been used here to derive Froude condition. For small Froude numbers this term can be neglected with respect to $\partial a/\partial x$ as can be seen as follows:

$$\frac{\partial}{\partial x}\left[\frac{u^2}{2g}\right] + \frac{\partial a}{\partial x} = \frac{\partial}{\partial x}\left[\frac{u^2}{2g} + a\right] = \frac{\partial}{\partial x}\left[a\{\tfrac{1}{2} Fr^2 + 1\}\right] \approx \frac{\partial a}{\partial x} \qquad (3\text{-}12)$$

if $Fr^2 \ll 1$.

In this case, however, still Froude condition is present as can be seen from a consideration of the terms $g^{-1}\partial u/\partial t$ and $\partial a/\partial x$.

This gives the scale relation

$$n_u \cdot n_t^{-1} = n_a \cdot n_L^{-1} \qquad (3\text{-}13)$$

which in combination with Eq. (3-6) again leads to $n_u = \sqrt{n_a}$. Similarly also still roughness condition holds for the case in which $Fr \ll 1$.

A special case of the flow problem described by Eqs. (3-4) and (3-5) is *steady uniform flow*, described by Chézy equation:

Then $\partial q/\partial x = 0$ thus q = constant and with $i = -\partial z/\partial x$ Eqs. (3-5) gives

$$u = C\sqrt{ai} \qquad (3\text{-}14)$$

This leads directly to

$$n_u^2 = n_C^2 \cdot n_a \cdot n_i \qquad (3\text{-}15)$$

The slope i is equal to the difference in piezometric head (Δh) divided by a length (L).

Hence

$$n_i = n_{\Delta h}/n_L \qquad (3\text{-}16)$$

Both Eq. (3-15) and (3-16) must be fulfilled. Elimination of n_i yields

$$n_u^2 = n_a \cdot \frac{n_C^2 \cdot n_a}{n_L} \cdot \frac{n_{\Delta h}}{n_a} \qquad (3\text{-}17)$$

This implies that if (i) Froude condition is fulfilled, thus $n_u^2 = n_a$ and if (ii) roughness condition is fulfilled, thus $n_C^2 \cdot n_a/n_L = 1$, then automatically $n_{\Delta h} = n_a$ or, differences in piezometric head are reproduced on the vertical scale.

This also implies that if Froude condition and/or roughness condition is not fulfilled, then scale effects ($n_{\Delta h} \neq n_a$) are present.

Many flow problems in seas, estuaries and rivers have to be treated as
two dimensional in the horizontal plane (still hydrostatic pressure dis-
tribution). For simplicity reasons a stationary two-dimensional flow will
be considered here, using the basic equations along and perpendicular to
the flow lines:

and

$$\frac{\partial}{\partial s}\left[\frac{u^2}{2g}\right] = -\frac{\partial h}{\partial s} - \frac{u|u|}{C^2 a} \qquad (3-18)$$

$$\frac{u^2}{gr} + \frac{\partial h}{\partial n} = 0 \qquad (3-19)$$

in which h denotes the piezometric head and r the local radius of curvature.
For kinematic similarity it is required that $n_r = n_L$. This leads again to
Froude condition and *roughness condition* (Bíjker *et al*, 1957).
Note that in this case distortion is possible (hydrostatic pressure distribution).

However, a *two-dimensional flow pattern in a vertical plane* cannot be distorted
if the flowlines are curved. Considering again a stationary flow (e.g. the
one over a sharp crested weir) leads to the following equations:

$$u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} \qquad (3-20)$$

$$u\frac{\partial w}{\partial x} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g \qquad (3-21)$$

In this case the velocity vector $\vec{u}$ has no component in y-direction (v = 0).

For similarity of the flowpattern the components u and w should have the same scale.
Hence the left hand terms of Eqs. (3-20) and (3-21) lead to the requirement
$n_x = n_z$, or $n_L = n_a$ (no distortion).

3.2.2. Artificial roughness

In order to fulfil the roughness condition the scale model has to be equiped
with artificial roughness. For rigid bed models the following cases can be
considered.

● *Case (i): Undistorted, hydraulic rough*

An undistorted model requires $n_C^2 = 1$; with

$$C = 18 \log\left[\frac{12a}{k + \delta/3.5}\right] \qquad (3-22)$$

This means for a hydraulic rough case (k >> δ)

$$C = 18 \log 12a/k \tag{3-23}$$

It is not difficult to see that $n_C = 1$ is reached for

$$n_k = n_a \tag{3-24}$$

• *Case (ii): Undistorted, hydraulic smooth*

For this case still $n_C = 1$ holds but now $\delta \gg k$ and the result is the requirement

$$n_\delta = n_a \tag{3-25}$$

This is difficult to obtain. The expression for the thickness (δ) of the viscous sublayer reads

$$\delta = \frac{11.6\nu}{u_*} \tag{3-26}$$

or

$$n_\delta = n_\nu \cdot n_{u_*} \tag{3-27}$$

For $n_\nu = 1$ (water in the model) and with $u/u_* = C/\sqrt{g}$ this leads to

$$n_\delta \approx n_a^{-1/2} \tag{3-28}$$

if Froude condition is fulfilled.

It is easy to see that Eqs. (3-25) and (3-28) can only be fulfilled for the trivial solution $n_a = 1$.

In fact the practical case of $n_a > 1$ implies that the model automatically is too rough. This asks for a model that is not too small.

This also is the reason that scale models of ships should not be too small. Otherwise the forces due to (viscous) friction are relatively too large with respect to other forces such as the one due to the propellor.

In the transition between hydraulic rough and hydraulic smooth both k and δ play a role. Now $k + \delta/3.5$ has to be reproduced according to n_a. The relatively

too large value of δ_m can now be compensated by selecting k_m relatively too small.

● *Case (iii): Normally distorted models*

If the distortion is restricted (say $n_L/n_a <$ 4 to 5) then still roughness elements can be placed at the rigid bed to fulfil the roughness condition. In most cases the flow will be hydraulic rough. It is handy to apply the formula of Strickler here

$$C = 25(a/k)^{1/6} \qquad (3-29)$$

Thus the condition resulting from $n_C^2 = r$ becomes

$$r = \frac{(a_p/k_p)^{1/3}}{(a_m/k_m)^{1/3}} \qquad (3-30)$$

Thus

$$n_k = n_a \cdot r^{-3} \qquad (3-31)$$

which leads automatically to $n_k = n_a$ for an undistorted model.
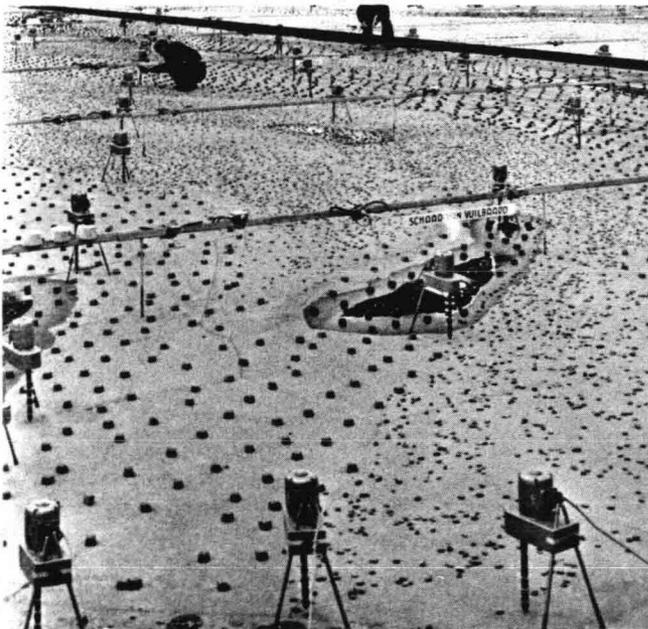
Figure 3.1 shows a detail of a tidal model (r = 4) in which artificial roughness is obtained from concrete cubes and gravel. Both the size of the roughness elements and their spacing determine the roughness obtained.

Special (flume) tests may be required to determine the k-value of such kind of artificial roughness.

Fig. 3.1 Roughness elements

• *Case (iv): Strongly distorted models*

For large values of the distortion the roughness elements become so large that they are not small anymore with respect to the depth. The flow in the model then becomes unrealistic. For these cases resistance elements consisting of bars are used in the fluid. In fact the shear stress that has to be delivered is replaced by a resistance of the elements in the entire fluid body. The number of bars can be estimated als follows.
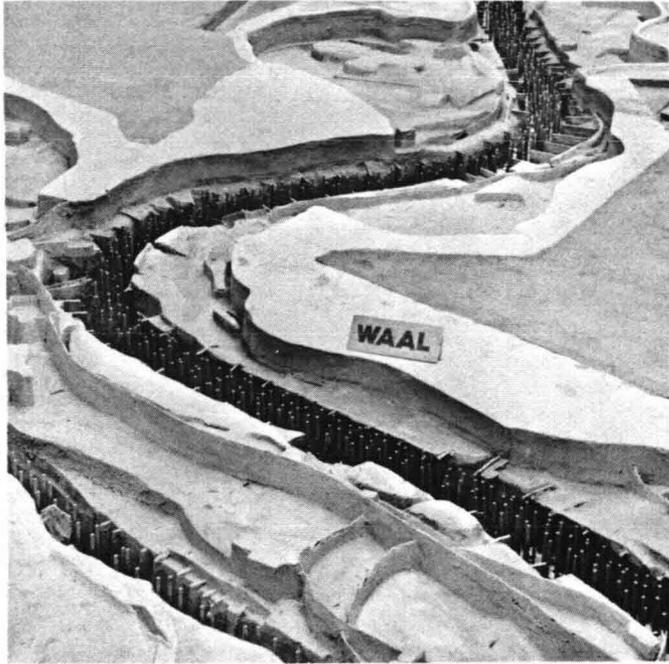
Suppose the model should have a Chézy roughness $C_0^2 = u^2/ai$. This is equivalent to a shear stress:

$$\tau_0 = \rho g u^2 C_0^{-2} \qquad (3-32)$$

Fig. 3.2 Resistance elements

The existing shear stress of the (usually concrete) bed amounts to

$$\tau_1 = \rho g u^2 C_1^{-2} \qquad (3-33)$$

Each bar exposed to the flow over the total depth acts with a force F on the fluid

$$F = \tfrac{1}{2}\rho C_D u^2 Da \qquad (3-34)$$

in which D denotes the diameter of the bar. If n bars are placed on the unit area of the model then the equivalent shear stress τ_2 due to the bars amounts to

$$\tau_2 = \tfrac{1}{2}n\rho C_D u^2 Da \qquad (3-35)$$

From $\tau_0 = \tau_1 + \tau_2$ it follows easily that

$$n = \frac{2g}{C_D aD} \left[\frac{1}{C_0^2} - \frac{1}{C_1^2} \right] \tag{3-36}$$

Remarks

(i)   For round bars C_D may vary as the separation point is not fixed and hence $C_D = C_D(Re)$. Round bars, however, have the advantage that the resistance does not depend on the flow direction.

(ii) Square bars or in general angular bars have just opposite characteristics. The value of C_D is rather constant. Angular bars placed *at random* are attractive. The resistance is then not dependent on the flow direction.

(iii) The expression of Eq. (3-36) can only give an estimate of n. Bars placed rather close to each other have a mutual influence. Special tests are then required to find the resistance of a group of bars.

(iv) A varying C-value with respect to the waterlevel can be obtained by taking bars of different lengths.

(v)   For movable bed models the bed roughness is mainly formed by the bedform (ripples and dunes). Sometimes, however, some extra roughness is needed to fulfil the roughness condition. This can be achieved by placing a small number of bars in this (normally distorted) model.

3.2.3. Earth rotation

In tidal models (i.e. fulfilling Froude condition) the scale of horizontal accelerations should be r^{-1}.

This can be seen from

$$n_{\partial u / \partial t} = \frac{n_u}{n_t} = \frac{n_u^2}{n_L} = \frac{n_a}{n_L} = r^{-1} \tag{3-37}$$

This is not the case for the acceleration (a_g) due to earth rotation (geostrophic acceleration).

The relevant expression is

$$a_g = 2u \, \omega \sin \phi \tag{3-38}$$

in which

$$\omega = \text{angular velocity} = \frac{2\pi}{24 * 60 * 60} = 0.73 * 10^{-4} \text{rad/s} \tag{3-39}$$

and ϕ is the geographical latitude ($\phi = 52^0$ in the Netherlands).

Hence if the model is placed at the same latitude as the prototype then $n_{ag} = n_u$, that means a factor n_t *too small*.

There are a number of measures possible to give a proper reproduction of a_g and hence to avoid scale effects in this respect.

(i) The model can be constructed on a rotating disc. For models larger than some meters this becomes very expensive due to technical implications.

(ii) Another possibility is according to Schoemaker (1958). This is by using the *Magnus effect* to reproduce the geostrophic acceleration.
A rotating cylinder with diameter D acts on a fluid with flow velocity u with a force F perpendicular to the current

$$F = \alpha\pi\rho DauU \tag{3-40}$$

in which $U = \pi Dn$ is the velocity of the circumference of the cylinder which rotates with n revolutions per second. Theoretically α should be equal to unity but in practice $\alpha = 0.3$ to 0.5.
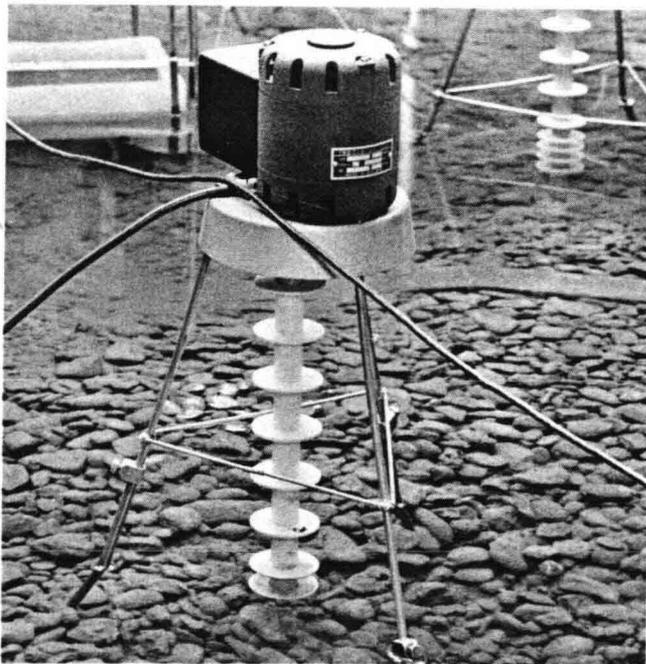
Fig. 3.3 Reproduction of geostrophic acceleration.

Figure 3.3 shows the realization in a tidal model. The horizontal discs are applied to surpress secondary circulations.
The application is clarified by means of the following example.

Example:

A tidal estuary ($a_p = 20$ m) has to be reproduced in a scale model. The following scales are selected: $n_L = 300$; $n_a = 100$; $n_u = 10$ and $n_t = 30$.
It is assumed that model and prototype are situated at the same latitude ($\phi_p = \phi_m$). The prototype velocity is $u_p = 1$ m/s.

From Eq. (3-40) it follows

$$n_{a_g} = n_u \cdot n_\omega = n_u \cdot n_t^{-1} = \tfrac{1}{3} \qquad (3-41)$$

For $\phi_p = 52^0$ the value of a_{gp} becomes

$$a_{gp} = 2 * 1 * 0.73 * 10^{-4} * \sin 52^0 = 1.15 * 10^{-4} \text{ m}^2/\text{s}$$

On 1 m² in the model ($a_m = 0.2$ m) a force F_{gm} is required

$$F_{gm} = a_m\rho. \{a_{gp}/n_{ag}\} = 0.2 * 10^3 \{1.15 * 10^{-4} * 3\} = 69 \text{ mN}$$

Selecting D = 2 cm; n = 12.5 Hz and α = 0.4 yields with

$$F_c = \alpha\pi^2\rho D^2 \text{nau} \qquad (3-42)$$

a force F_c by each cylinder of

$$F_c = 0.4 * 10 * 10^3 * (4 * 10^{-4}) * 12.5 * 0.2 * 0.1 = 0.4 \text{ N}$$

Hence there are $0.069/0.4 \approx 0.17$ cylinders required per m².
Of course the cylinders also contribute to the hydraulic resistance
to the flow.

3.3. Surface waves

3.3.1. Wave propagation

The need to repoduce periodic gravity waves occurs frequently in coastal
problems. The starting point for considerations on scaling can be the
general expression for the celerity c (c.f. Lamb, 1945, p. 459)

$$c = \sqrt{\left[\frac{g\lambda}{2\pi} + \frac{2\sigma\pi}{\rho\lambda}\right] \tanh \frac{2\pi a}{\lambda}} \qquad (3-43)$$

Here λ denotes the wave length and σ the surface tension (for air/water
σ = 74 mN/m (= 74 mPam)).

In the prototype ususually waves not influenced by σ are present. This has
then to be the case also in the scale model.

If $2\pi\sigma/\rho\lambda$ is accepted for the scale model to be not more than 2% of the term $g\lambda/2\pi$ then the following inequality holds,

$$\frac{2\pi\sigma/\rho\lambda}{g\lambda / 2\pi} < 0.02 \qquad (3-44)$$

or

$$(0.02)^{-1} \frac{(2\pi)^2}{\rho g} * \sigma < \lambda^2 \qquad (3-45)$$

For $\sigma = 74$ mPa.m this yields $\lambda > 0.12$ m.

Hence for practical cases the formula for c simplifies to

$$c = \sqrt{\frac{g\lambda}{2\pi} \tanh \frac{2\pi a}{\lambda}} \qquad (3-46)$$

It is possible now to distinguish three cases:

(i)   *Relatively deep water*

For this case $a > \frac{1}{2}\lambda$ leading to $\tanh 2\pi a/\lambda \to 1$
Or

$$c = \sqrt{g\lambda/2\pi} \qquad \text{or} \qquad n_c = \sqrt{n_\lambda} \qquad (3-47)$$

(ii) *Relatively shallow water*

Now $a < \lambda/20$ which gives $\tanh 2\pi a/\lambda \to 2\pi a/\lambda$
This results in

$$c = \sqrt{ga} \qquad \text{or} \qquad n_c = \sqrt{n_a} \qquad (3-48)$$

(iii) *Intermediate range*

For $0.05 < a/\lambda < 0.5$ it can be noted that scale effects can only be avoided if the argument of the hyperbolic tangent is equal in model and prototype. Thus $n_a = n_\lambda$ is a necessity.
But then also, considering Eq. (3-46)

$$n_c = \sqrt{n_\lambda} = \sqrt{n_a} \qquad (3-49)$$

Note that in all three cases no link as yet is available with the length scale (n_L) of the model.

In the general case Eq. (3-49) and the scale law

$$n_\lambda = n_c \cdot n_\tau \qquad (3\text{-}50)$$

lead to *three* equations between the scales of *four* parameters (*viz.* a, τ, c and λ). Hence one scale (e.g. the depth scale) can then be selected and the other three scales can be deduced from the available three equations. The length scale of the model is then also fixed if no distortion is allowed. When distortion is possible then there is some freedom in selecting n_L.

3.3.2. Wave deformation

• *Reflection of a wave* against a slope making an angle α with the horizontal depends on the magnitude of α. Hence the reflection is reproduced correctly for $n_\alpha = 1$ which implies an undistorted model. Therefore breakwaters in wave flumes have to be studied by means of an undistorted model. However, a sloping beach albeit that waves are partly reflected may be distorted as usually this effect is small and refraction is dominant.

• *Refraction of waves* is due to the bedtopography as the celerity depends on the depth (Fig. 3.4.). In other words as long as the celerity is reproduced correctly, no scale effects in refraction have to be feared.
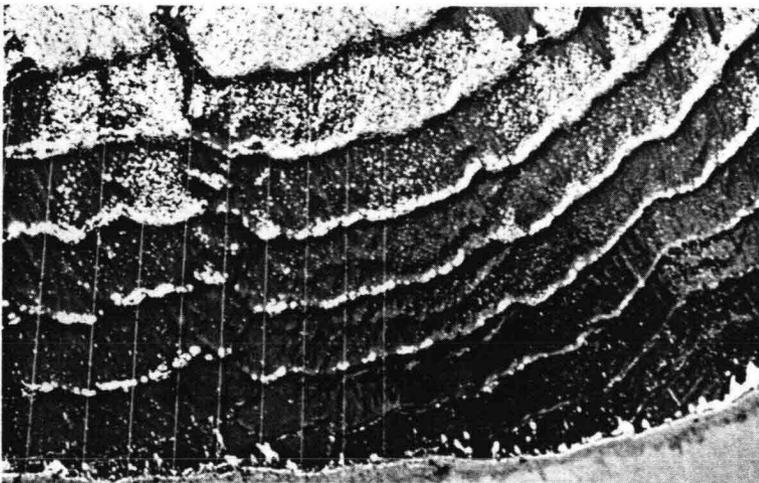
Fig. 3.4. Wave refraction

Thus a sufficient condition is

$$n_\tau = \sqrt{n_a} \qquad (3\text{-}51)$$

because in that case Eqs. (3-50) and (3-51) lead by eliminating n_τ to

$$n_\lambda = n_c \cdot n_a^{1/2} \qquad (3\text{-}52)$$

From Eq. (3-46) follows in general

$$n_c = n_\lambda^{1/2} \cdot n_{\tanh}^{1/2} \qquad (3\text{-}53)$$

Eliminating n_c from Eq. (3-52) and (3-53) gives

$$n_\lambda^{1/2} = n_a^{1/2} \cdot n_{tanh}^{1/2} \qquad\qquad (3-54)$$

Equation (3-54) is fulfilled if $n_\lambda = n_a$. Apparently no requirement is set for the length scale and hence distortion is possible.

- *Wave refraction due to currents* is also possible. A correct reproduction can be obtained (c.f. Bijker, 1967) if the current is reproduced according to the Froude condition. Also here no condition for n_L is present, hence distortion is possible.

In both cases no requirement in reproducing refraction is set for the wave-height n_H. Hence the waveheight in the model can be selected freely, of course as long as the waves become not too steep.

- *Diffraction of waves* is the deformation of waves due to the presence of an obstacle, for instance a breakwater (Fig. 3.5.).

The diffraction pattern in a given horizontal geometry varies with the length of the incoming wave. Hence the wavelength has to be reproduced on the horizontal scale (n_L).

With the requirement of Eq. (3-49) this leads to the conclusion that diffraction is correctly reproduced in
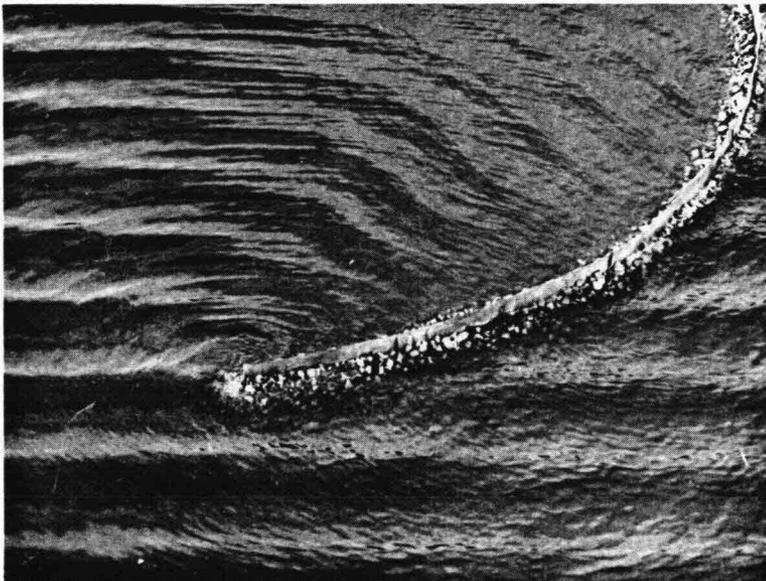
Fig. 3.5. Wave diffraction

an *undistorted model*.

Remark

In coastal engineering problems waves are usually deformed due to a combination of the above mentioned single causes. The effect on the selection of scales will be further discussed in Chapter 4.

3.3.3. Orbital velocity and wave height

The orbital velocity (v) and the wave height (H) are linked by

$$v(z) = \frac{\omega \, H \, \cosh \, kz}{2 \, \sinh \, ka} \cdot \cos(\omega t - kx) \qquad (3-55)$$

in which $k = 2\pi/\lambda$ is the wave number and $\omega = 2\pi/\tau$ is the wave frequency.

Scale effects are avoided if $n_{kh} = 1$ or $n_\lambda = n_a$. The scale of the maximum orbital velocity is then restricted to the scales of ω and H only

$$n_{v_0} = n_\omega \cdot n_H = n_\tau^{-1} \cdot n_H \qquad (3-56)$$

With $\lambda = c \cdot \tau$ it yields $n_\tau = n_a^{1/2}$
Hence

$$n_v = n_H \cdot n_a^{-1/2} \qquad (3-57)$$

For $n_H = n_a$ this results in $n_v = \sqrt{n_a}$. Hence reproduction of the wave height on the vertical scale is one of the requirements to reproduce the orbital velocity according to Froude condition.

3.4. Hydro elasticity

In some hydraulic structures (e.g. gates and valves) the elastic properties of the structure and the watermovement interact and determine together the movements (and stresses) of the structure. Now not only the watermovement has to be similar (Froude condition and/or Reynolds condition) but also the response of the structure due to any external load (by turbulence and/or waves) requires similarity.

The scale conditions can be derived by the following example. For a more complete treatment reference can be made to literature (see Prins, 1969; Kolkman, 1970, 1976).

For a simple mass-spring system the forces are linked approximately by a linear differential equation for the displacement (a). See Fig. 3.6.

$$M \frac{d^2a}{dt^2} + R \frac{da}{dt} + ca = F \qquad\qquad (3\text{-}58)$$
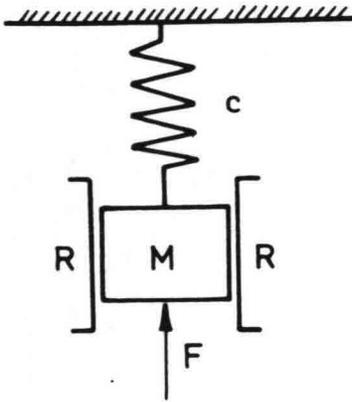
The terms represent the inertia force, the resistance force, the spring force and the external force respectively.

No scale effects are present if all terms of Eq. (3-58) have the same scale.

Hence with

$$n_t = n_\omega^{-1} \qquad\qquad (3\text{-}59)$$

Fig. 3.6. Definition sketch

in which ω is the angular frequency,

$$n_M * n_\omega^2 = n_R * n_\omega = n_c = n_F * n_a^{-1} \qquad\qquad (3\text{-}60)$$

Thus

$$n_M = \frac{n_F}{n_\omega^2 \cdot n_a} = \frac{n_F}{n_\omega^2 \cdot n_L} \qquad\qquad (3\text{-}61)$$

$$n_R = \frac{n_F}{n_\omega \cdot n_a} = \frac{n_F}{n_\omega \cdot n_L} \qquad\qquad (3\text{-}62)$$

$$n_c = \frac{n_F}{n_a} = \frac{n_F}{n_L} \qquad\qquad (3\text{-}63)$$

A logical requirement is that $n_a = n_L$ (geometric similarity). Note that Eqs. (3-61) ... (3-63) are *scale conditions* for the three parameters M, R and c in Eq. (3-58) with respect to the force F.

A forced oscillation of the system of Fig. 3.6. can be studied mathematically by applying for the external force a periodic function

$$F = F_0 \cos \omega t \qquad\qquad (3\text{-}64)$$

The solution reads

$$a = \alpha \frac{F_0}{c} \cos(\omega t - \phi) \qquad\qquad (3\text{-}65)$$

in which

$$\alpha = \frac{1}{\sqrt{\{1 - \omega^2/\omega_0^2\} + \{R^2\omega^2/M^2\omega_0^4\}}} \qquad (3\text{-}66)$$

The parameter $\omega_0 = \sqrt{c/M}$ represents the resonance frequency.

Logically $n_F = n_{F_0}$. Hence if the conditions for n_M, n_R and n_c are fulfilled then the following results are obtained.

(i)   Fulfilling the conditions in Eqs. (3-61) and (3-63) lead with $\omega_0 = \sqrt{c/M}$ to $n_\omega = n_{\omega_0}$.

(ii) This together with fulfilling Eq. (3-62) and in using Eq. (3-65) yields $n_\alpha = 1$.

(iii) But then applying Eq. (3-63) to Eq. (3-65) the result is $n_a = n_L$ and this ofcourse is a logical requirement.

Hence if the three scale conditions for M, R and c are fulfilled then the amplitude of the oscillation and the resonance frequency of the structure have correct scales.

Now the stresses in the structure are to be considered.
According to *Hooke's law* the relative deformation (ε) is proportional to the main stress (σ) with

$$\varepsilon = E^{-1}\sigma \qquad (3\text{-}67)$$

in which E is the modulus of elasticity.
Hence

$$n_\varepsilon = n_E^{-1} \cdot n_\sigma = n_E^{-1} \cdot n_F \cdot n_L^{-2} \qquad (3\text{-}68)$$

has to be fulfilled.
Thus as $n_\varepsilon = 1$ is necessary for geometric similarity

$$n_E = n_F \, n_L^{-2} \qquad (3\text{-}69)$$

For the modulus of shear (G) it can be derived similarly:

$$n_E = n_G \qquad (3\text{-}70)$$

For the combination of the requirements for the water movement and properties of the structure two cases have to be considered.

● *Without free surface flow* the above given conditions can be combined with the condition for the pressure (p)

$$n_p = n_\rho \cdot n_u^2 \qquad (3-71)$$

Combination of Eqs. (3-69) and (3-71) yields *Cauchy condition*

$$n_{Ca} = n_\rho \cdot n_u^2 \cdot n_E^{-1} = 1 \qquad (3-72)$$

Hence *Cauchy number*

$$Ca = \frac{\rho u^2}{E} \qquad (3-73)$$

has to be the same in model and prototype.

This case is relatively simple because if $n_\rho = 1$ and $n_E = 1$ is selected (same fluid and same material in model and prototype) then $n_u = 1$ results. This is a realistic possibility if Reynolds condition is not applicable (viscous effects negligible). Note that Froude condition does not apply in this case.

● *Free surface flow* agitating a hydraulic structure leads to a more complex situation because now *Cauchy condition* and *Froude condition* have to be fulfilled at the same time leading to the requirement (see Eq.(3-72)):

$$n_\rho \cdot n_L = n_E \qquad (3-74)$$

When water is used in the model $n_\rho = 1$. But then this should also hold for the structure because both the structure and part of the water (*virtual mass*) vibrate. Hence the requirements are

$$n_\rho = 1 \qquad (3-75)$$

and

$$n_E = n_L \qquad (3-76)$$

It is difficult to find a material for the model that fulfils the conditions expressed in Eqs. (3-75) and (3-76).

As far as Eq. (3-76) is concerned *trovidur* can reproduce steel and concrete as follows from Table 3.1.

	E	ρ
	GPa	kg/m³
steel	210	7850
concrete	21	2500
trovidure	3.5	1400

Table 3.1. Properties of some materials

According to these data trovidure can reproduce steel if n_L = 60. Reproduction of concrete is correct if n_L = 6 is selected. In both cases, however, the condition for the density, expressed in Eq. (3-75) is not fulfilled. Extra mass (e.g. lead) has to be added locally to the model structure to ensure an average n_ρ = 1. This should of course not change the overall elasticity of the structure.

3.5. Morphological processes

3.5.1. Transport by currents

In many river-, and estuarine problems the mobility of the bed plays a role and therefore reproduction of the morphological processes in a scale model is of importance.

Here the treatment will be restricted to *cohesionless sediment* and in the first place the morphological processes due to currents will be discussed.

The starting point can be that many formulae describing the transport of bed-material due to a current show a unique relationship between two parameters

$$Transportparameter: \quad X = \frac{s}{D^{3/2} \sqrt{g\Delta}} \tag{3-77}$$

$$Flowparameter \quad : \quad Y = \frac{\Delta D}{\mu \ ai} \tag{3-78}$$

in which
$\Delta = (\rho_s - \rho)/\rho$ is the relative density of the bedmaterial
μ = ripple factor

In general the relation between X and Y reads

$$X = f\{Y\} \tag{3-79}$$

without specifying at this stage this function.

A condition for similarity of the transport reads

$$n_Y = 1 \qquad (3-80)$$

because as $X = f\{Y\}$ is a unique function this leads automatically to

$$n_X = 1 \quad \text{or} \quad n_s = n_D^{3/2} \cdot n_\Delta \qquad (3-81)$$

Equation (3-81) implies that if similarity in grain sorting is present (n_D = constant in the model) then the *transport scale* (n_s) is a constant in the model (no scale effects).

The equation of continuity for the sediment reads

$$\frac{\partial z}{\partial t} + \frac{\partial s_1}{\partial x} + \frac{\partial s_2}{\partial y} = 0 \qquad (3-82)$$

if s_1 and s_2 are the components in x and y direction respectively of the transportvector s.

The time scale (n_{tm}) for the morphological process follows directly from Eq. (3-82)

$$n_{tm} = \frac{n_L \cdot n_a}{n_s} = \frac{n_L^2 \cdot n_a}{n_S} \qquad (3-83)$$

Hence if n_s is a constant in the model then also n_{tm} is.

This situation is reached if $n_Y = 1$ is selected or

$$n_{\Delta D} = n_{\mu\, ai} \qquad (3-84)$$

Together with Chézy equation this leads to a requirement for the velocity scale

$$n_u^2 = n_\Delta \cdot n_D \cdot n_C^2 \cdot n_\mu^{-1} \qquad (3-85)$$

The value of n_u obtained in this way is called the *ideal velocity scale* because its selection in principle leads to a situation in which no scale effects are present. In practice as will be shown in Section 4.2.2; n_u is determined from Eq. (3-85) by selecting a certain bedmaterial (which fixes n_Δ and n_D) and by estimating C_m and μ_m from experimental evidence.

Remarks

(i)   In principle now two different conditions for n_u have been derived.

- The Froude condition for reproducing the watermovement with the free-surface flow.

- The conditions following from the concept of the ideal velocity scale i.e. reproducing the sediment movement without scale effects.

In practice this may very well lead to two *different* values of n_u. This will be discussed below.

(ii) Neglecting for the time being the influence of n_C and n_μ means that Eq. (3-85) in principle reads

$$n_u^2 \approx n_\Delta \cdot n_D \qquad (3\text{-}86)$$

Or, if sand is used in the model ($n_\Delta = 1$) this implies

$$n_u^2 = 0(n_D) \qquad (3\text{-}87)$$

In other words if $n_D = n_a$ is selected then the two conditions mentioned under (i) are fulfilled at the same time. This, however, is only possible if gravel is present in the prototype. If the bedmaterial is smaller than say $D_m = 0.1$ to 0.2 mm then the transportmechanism in the model may be different from the one in the prototype (this can also be concluded from Shields' curve).

(iii) Two ways are open to overcome this difficulty:

- Accept $n_u^2 < n_a$ thus fulfil Eq. (3-85) as a correct reproduction of the morphological process is essential. The consequence is (assuming that the roughness condition is fulfilled) that the slopes in the model become too steep as can be seen from Eq. (3-17). This gives an error that can be corrected by *tilting* the model as will be discussed later.

- Select a light bedmaterial in the model ($n_\Delta > 1$) in order to fulfil Eq. (3-85) without a value of D_m that is too small.

These two ways are often used in a combination.

If suspended load prevails then it seems logical to promote similarity with respect to the concentration distribution. This leads to the condition

$$n_{W/\kappa u_*} = 1 \qquad (3\text{-}88)$$

or with $n_K = 1$

$$n_W = n_{u_*} \tag{3-89}$$

From the general equation for the fallvelocity W:

$$W = \sqrt{\frac{4}{3} \frac{g}{C_D} * \Delta D} \tag{3-90}$$

two ranges can be distinguished by means of $Re = WD/\nu$

- *Stokes range* $C_D = 24/Re$ valid for $Re < 1$

- *Newton range* $C_D \approx 1$ valid for $Re > 10^3$

It can be shown that in principle the scale condition following from Eq. (3-89) for the Newton range is the same as the one given in Eq. (3-84).

For the Stokes range, however, there is a distinct difference between Eqs. (3-84) and (3-89). It seems logical to work with Eq. (3-89) for this case. Details can be found from Zwamborn (1966).

The scale conditions found here for the morphological processes do not give information on the magnitude of possible scale effects. This is logical because Eqs. (3-81) and (3-84) follow from dimensional considerations.
Scale effects can be studied if specific transport formulae are used.

Two examples will be given here

(i) *Meyer-Peter and Mueller formula*

The Meyer-Peter & Mueller formula (1948) reads

$$X = \alpha\{Y^{-1} - \beta\}^{3/2} \tag{3-91}$$

in which α and β are constants. With $\beta = 0.047$ it can easily be shown that

$$n_X^{2/3} = \frac{1 - 0.047\ Y_p}{n_Y - 0.047\ Y_p} \tag{3-92}$$

Equation (3-92) has been reproduced graphically in Fig. 3.7.
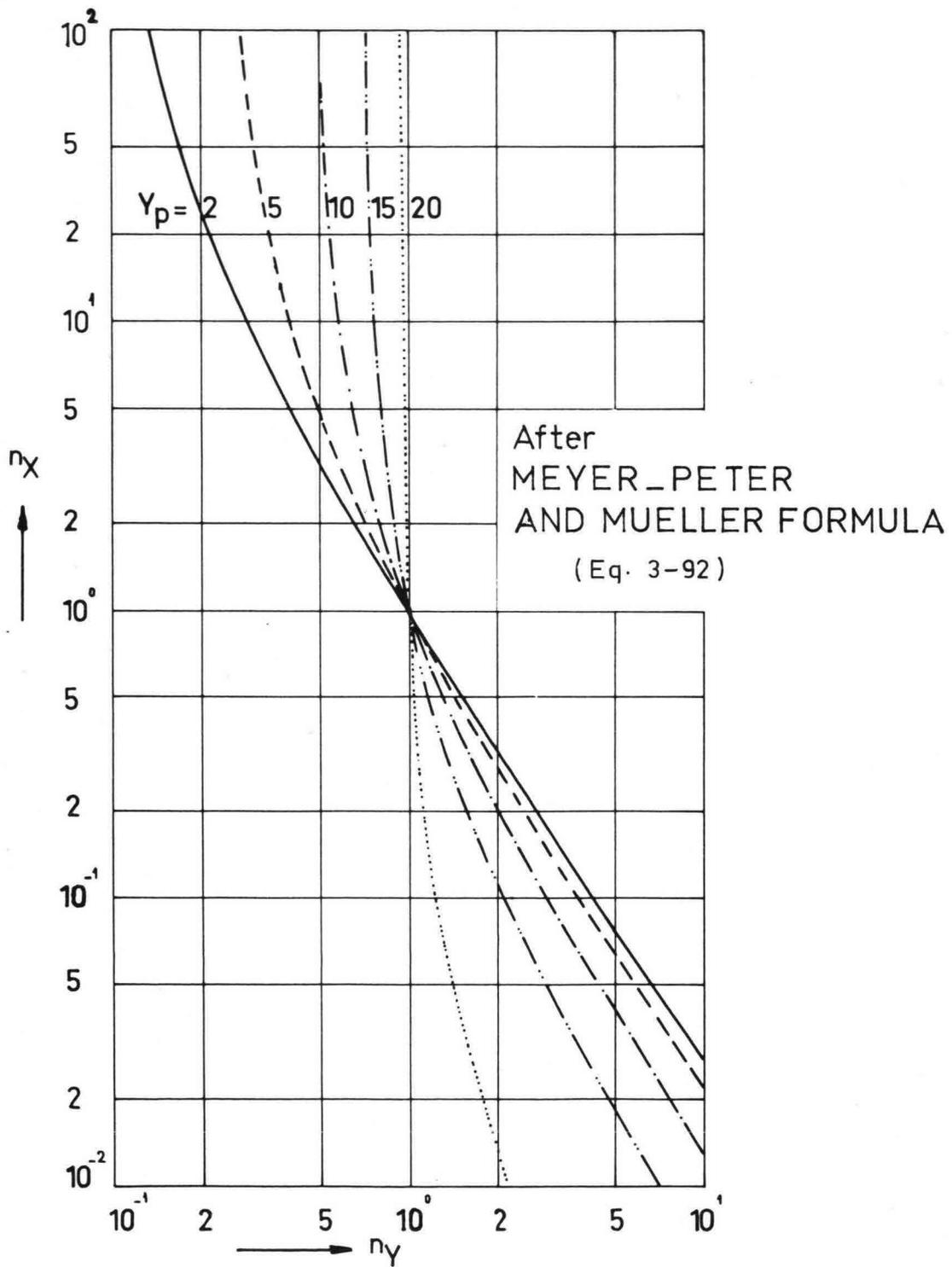
Fig. 3.7 Possible scale effects (M-P & M formula)

Logically $n_Y = 1$ leads to $n_X = 1$ independent of Y_p. Moreover for very intensive transport (exceptional situation for $Y^{-1} \gg \beta$) the following result is obtained

$$n_X = n_Y^{-3/2} \qquad (3-93)$$

This conditions is less strict than $n_X = n_Y = 1$.

For the M.P & M formula the equation for the ideal velocity scale can be simplified.
Here

$$\mu = \{C/C_{90}\}^{3/2} \qquad (3-94)$$

Hence

$$n_u^2 = n_\Delta \cdot n_D \cdot n_C^{1/2} \cdot n_{C_{90}}^{3/2} \qquad (3-95)$$

(ii) *Engelund-Hansen formula*

The Engelund-Hansen (1967) formula can be written as

$$X = 0.084 \ Y^{-5/2} \qquad (3-96)$$

This gives the scale condition

$$n_X = n_Y^{-5/2} \qquad (3-97)$$

This condition is also less strict than $n_X = n_Y = 1$. According to the E-H formula

$$\mu = \{C/g\}^{2/5} \qquad (3-98)$$

Hence in this case

$$n_u = n_\Delta^{2/5} \cdot n_D^{1/5} \cdot n_C^{4/5} \cdot n_s^{1/5} \qquad (3-99)$$

This gives some freedom in the selection of n_u via n_s provided both in prototype and model the E-H-formula describes the sediment transport properly.

Remark

Note that the Eqs. (3-85), (3-95) and (3-99) express the velocity scale in terms of n_C. The estimation of n_C will be further discussed in Sub-section 4.2.2.

3.5.2. Transport by waves and currents

Morphological processes in which both waves and currents are present logically require more conditions than if only a current is involved. Bijker (1967) derived a formula starting by looking at the resulting shear stress due to waves and currents. In its present form this transport formula can be written (see Massie, 1978) for bedload as

$$s = \frac{D u \sqrt{g}}{C} \exp\left[\frac{-0.27 \quad \Delta D \quad C^2}{\mu u^2 \left\{1 + \frac{1}{2}\left[\xi \frac{v_0}{u}\right]^2\right\}}\right] \tag{3-100}$$

in which

v_0 = orbital velocity near the bed
μ = ripple-factor = $\{C/C_{90}\}^{3/2}$

Further

$$\xi = \frac{C\sqrt{f_w}}{\sqrt{2g}} \tag{3-101}$$

in which the friction factor f_w is an experimental function of a_0/r (Swart, 1974). Here a_0 is the maximum waterdisplacement at the bed due to the orbital motion and r stands for a roughness height.

The situation without scale effects appears if $n_{f_w} = 1$. Thus $n_\xi = n_C$. From Eq. (3-100) follows then the requirement if absence of scale effects is anticipated

$$n_{v_0} = n_\xi^{-1} \cdot n_u = n_u \cdot n_C^{-1} \tag{3-102}$$

Moreover if

$$n_u^2 = n_\Delta \cdot n_D \cdot n_C^2 \cdot n_\mu^{-1} \tag{3-103}$$

(which is equal to Eq. (3-85) for current only) then

$$n_s = n_D \cdot n_u \cdot n_C^{-1} \tag{3-104}$$

Note that this ideal situation (i.e. the one without scale effects) is reached if n_u and n_{v_0} do not have the same scale:

$$n_u = n_C \sqrt{n_\Delta \cdot n_D \cdot n_\mu^{-1}} \tag{3-105}$$

$$n_{v_0} = \sqrt{n_\Delta \cdot n_D \cdot n_\mu^{-1}} \tag{3-106}$$

Usually $n_C > 1$ (requiring according to the roughness condition a distorted model). Hence $n_{v_0} < n_u$ is required. This can be accomplished by selecting a relatively large waveheight (H) in the model: $n_H < n_a$.

The following additional remarks can be given

- As fine sand is present in many coastal problems, these models are difficult to scale because conditions for *roughness, currents* and *waves* are not easily fulfilled at the same time.

- The use of bedmaterial in the model lighter than sand is restricted because *liquefaction* may occur. Hence for scale models on coastal morphology a sub-stantical deviation from the Froude condition can be expected.

- It has to be noted that the ideal conditions given in Eqs. (3-105) and (3-106) do not consider the specific shapes for $\xi = f\{a_0/r\}$ and Eq. (3-100). A detailed analysis may give some release to these strict conditions as was the case for current only in the previous Sub-section when the M.P. & M. and E-H formulae were used.

3.6. Density currents and dispersion
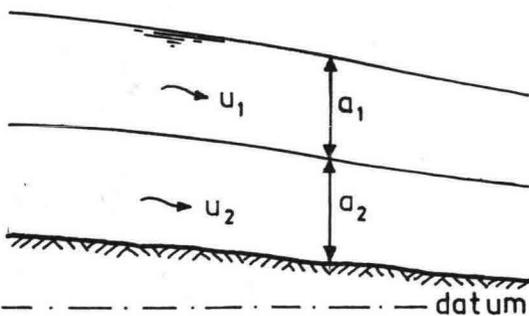
Fig.3.8 Definition sketch

The watermovement may be influenced by small differences in density. In tidal estuaries for instance the difference in density of the salt (sea) water and the fresh (river) water influence the flow field.

Knowledge on the current pattern may be important for navigation. Also sedimentation

induced by density currents is an important hydraulic engineering aspect. Scaling of these phenomena is again based on theoretical and experimental evidence gathered in this respect.

As an example the scaling of a two-layer system (stratified flow) in a tidal estuary will be considered here (Fig. 3.8). If $a_{1,2}$ are the thicknesses of the upper layer and the lower layer respectively then the basic equations for a wide estuary read

Continuity

$$\frac{\partial a_1}{\partial t} + \frac{\partial (a_1 u_1)}{\partial x} = 0 \tag{3-107}$$

$$\frac{\partial a_2}{\partial t} + \frac{\partial (a_2 u_2)}{\partial x} = 0 \tag{3-108}$$

Motion

$$\frac{\partial u_1}{\partial t} + u_1 \frac{\partial u_1}{\partial x} + g \frac{\partial (a_1 + a_2)}{\partial x} = -\frac{\tau_i}{\rho_1 a_1} \tag{3-109}$$

$$\frac{\partial u_2}{\partial t} + u_2 \frac{\partial u_2}{\partial x} + g \frac{\partial (a_1 + a_2)}{\partial x} - \frac{\Delta \rho}{\rho_2} g \frac{\partial a_1}{\partial x} = -\frac{\tau_b - \tau_i}{\rho_2 a_2} \tag{3-110}$$

For a saltwater-freshwater system in an estuary $\Delta \rho / \rho = O(10^{-2})$. For instance $\rho_1 = 1000$ kg/m^3 and $\rho_2 = 1025$ kg/m^3.

Derivation of the scale relations indicated previously brings forward the following:

(i)   *Geometric similarity*

$$n_a = n_{a_1} = n_{a_2} \tag{3-111}$$

(ii)   *Froude conditions*

$$n_u = \sqrt{n_a} \quad \text{or} \quad n_{Fr} = 1 \tag{3-112}$$

Moreover as can be derived from the third and the fourth term of Eq. (3-110)

$$n_{\Delta \rho / \rho} = 1 \tag{3-113}$$

In fact Eqs. (3-112) and (3-113) express that besides the 'normal'
Froude number (Fr = $u/\sqrt{ga}$) also the *densimetric* or internal Froude number

$$Fr_i = \frac{u}{\sqrt{ga\ \Delta\rho/\rho}} \qquad (3-114)$$

should be equal in model and prototype.

As fresh water is usually reproduced by fresh water ($n_{\rho_1} = 1$) this implies
that $\Delta\rho$ is to be equal in model and prototype ($n_{\Delta\rho} = 1$).

(iii) *Roughness conditions*

It has been shown above that for this problem *two* Froude conditions are of
importance (Eqs. 3-112, 3-114). Similarly *two* roughness conditions apply.
This is logical as the bedroughness (τ_b) and the roughness at the interface
(τ_i) have to be reproduced properly.
Using the Darcy-Weisbach (dimensionless) roughness coefficient (f) the
shearstresses can be expressed as follows.

$$\tau_i = \tfrac{1}{8}\ f_i\ \rho(u_1 - u_2)\ |u_1 - u_2| \qquad (3-115)$$

and

$$\tau_b = \tfrac{1}{8}\ f_b\ \rho\ u_2 |u_2| \qquad (3-116)$$

Considering Eqs. (3-109) and (3-110) again and using Froude condition the
following roughness conditions can be derived:

- *For the surface layer:*

$$\text{distortion} = r = \frac{n_L}{n_a} = n_{f_i}^{-1} \qquad (3-117)$$

- *For the bottom layer*

$$\text{distortion} = r = \frac{n_L}{n_a} = n_{f_b}^{-1} = n_C^2 \qquad (3-118)$$

Compared to open-channel flow with a homogeneous fluid as discussed in Section
3.2 there are now two additions:

- The densimetric Froude number has now also to be equal in model and prototype.
  This is accomplished by selecting $n_{\Delta\rho} = 1$.
- The distortion has now according to Eq. (3-118) not only to be equal to the
  square of n_C for the bed but also Eq. (3-117) has to be fulfilled.

The extra requirement for the roughness, expressed in Eq. (3-117), fixes in fact the distortion and C_m has to be selected accordingly. The present insight into f_i under various circumstances does not yet allow an accurate determination of the distortion.

A related topic regards the dispersion of dissolved matter and heat. Here the situation will be discussed in which the concentrations are so small that the resulting density currents may be neglected.

The concentration changes in time and place due to *convection* by the main stream and by the *turbulent diffusion*. The molecular diffusion can usually be neglected.
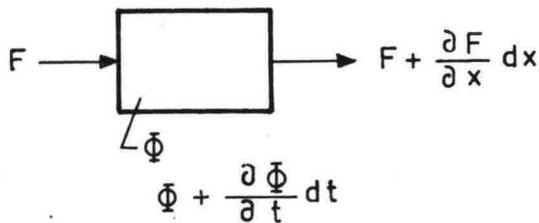
Consider the most simple case of a one-dimensional problem. The conservation law for the momentary concentration (Φ) and the flux of dissolved matter or heat (F) can be deduced using Fig. 3.9.

Fig. 3.9 Definition sketch

$$\frac{\partial \Phi}{\partial t} + \frac{\partial F}{\partial x} = 0 \qquad (3-119)$$

The flux is equal to the product of the momentary velocity (U) and the momentary concentration:

$$F = U \times \Phi \qquad (3-120)$$

The usual definitions for turbulent flow can now be applied

$$U = u + u' \qquad (3-121)$$

$$\Phi = \phi + \phi' \qquad (3-122)$$

in which u and ϕ are time-averages over a period θ, small enough to ensure $\overline{u'} \approx 0$ and $\overline{\phi'} \approx 0$ and large enough to get rid of the turbulent fluctuations. Now a similar definition can be used for the flux F

$$F = f + f' \qquad (3-123)$$

A time-averaging of Eq. (3-123) gives

$$\overline{F} = \overline{U * \Phi} = \overline{(u + u')(\phi + \phi')} = u\phi + \overline{u'\phi'} \tag{3-124}$$

because according to the definitions $\overline{u'\phi} \approx 0$ and $\overline{u\phi'} \approx 0$.

Taking now a time-average of Eq. (3-119) gives

$$\frac{\partial\phi}{\partial t} + \frac{\partial}{\partial x} \{u\phi + \overline{u'\phi'}\} = 0 \tag{3-125}$$

Now a Fickian diffusion is assumed i.e. $\overline{u'\phi'} = -K \, \partial\phi/\partial x$. For a homogeneous flowfield (u = const) this leads to

$$\frac{\partial\phi}{\partial t} - K \frac{\partial^2\phi}{\partial x^2} + u \frac{\partial\phi}{\partial x} = 0 \tag{3-126}$$

If more space dimensions have to be considered then in a similar way the (turbulent) diffusion can be defined. The diffusion coefficient K becomes now a tensor K_{ij} and the equation for the time-averaged flux in the x-direction becomes (denoting the x,y and z direction with 1, 2 and 3 respectively):

$$\overline{F}_1 = u_1\phi - K_{11} \frac{\partial\phi}{\partial x} - K_{12} \frac{\partial\phi}{\partial y} - K_{13} \frac{\partial\phi}{\partial z} \tag{3-127}$$

If the axes are taken along the main axes of the tensor then $K_{ij} = 0$ for $i \neq j$.

Then the differential equation becomes

$$\underbrace{\frac{\partial\phi}{\partial t}}_{(a)} - \underbrace{\frac{\partial}{\partial x}\{K_{11}\frac{\partial\phi}{\partial x}\}}_{(b)} - \underbrace{\frac{\partial}{\partial y}\{K_{22}\frac{\partial\phi}{\partial y}\}}_{(c)} - \underbrace{\frac{\partial}{\partial z}\{K_{33}\frac{\partial\phi}{\partial z}\}}_{(d)} + \underbrace{u_1\frac{\partial\phi}{\partial x}}_{(e)} + \underbrace{u_2\frac{\partial\phi}{\partial y}}_{(f)} + \underbrace{u_3\frac{\partial\phi}{\partial z}}_{(g)} = 0$$
$$\tag{3-128}$$

if K_{ii} are not supposed to be constant in the flow field.

If velocity gradients are present then these gradients also contribute to the variation of $\phi(x_i, t)$. In this case equations similar to Eq. (3-126) appear in which K_{ii} is a *dispersion* tensor, responsible for all effects besides the pure *convection* by the main stream.

It can be shown quite easily that the various elements of the vector K_{ii} are only reproduced on the same scale if an *undistorted* model is used.

For instance from the terms (b) and (e) follows the relation

$$n_{K_{11}} = n_u \cdot n_L \tag{3-129}$$

From the terms (c) and (f) follows

$$n_{K_{22}} = n_u \cdot n_L \tag{3-130}$$

and from the terms (d) and (g) follows

$$n_{K_{33}} = n_u \cdot n_a \tag{3-131}$$

Remarks

(i) For the steady state case for both flow and concentration ($\partial\phi/\partial t = o$) there seems to be no condition for n_u. This is of course only true if fulfilling of the Froude condition is not required for the reproduction of the flow field.

(ii) The above given reasoning does not mean that dispersion cannot be reproduced in a distorted model. In a relatively narrow stream Eq. (3-126) can discribe the longitudinal dispersion (Fischer, 1967). In such a case distortion obviously is possible. In a distorted (tidal) model longitudinal and lateral dispersion has been reproduced correctly for a case in which the contribution of the vertical velocity distribution to the total dispersion was of minor importance.

4. Design of scale models

4.1 Introduction

After having given the *analytical* treatment of the various scale conditions for reproducing hydraulic phenomena, now the *synthetic* approach will be discussed.

Conflicting scale conditions will have to be fulfilled at the same time. Obviously the compromise to be selected will contain some scale effects. It is necessary for a specific hydraulic engineering problem to weigh the relevance of the scale conditions involved and optimize with respect to scale effects that are likely to be present.

In finding this compromise it is of importance to keep an eye on the *aim* of the model study during the whole process of scaling. This is directly related to the question what *accuracy* the model should have.

This is of course related to the accuracy of the available prototype data. Boundary conditions have to be introduced in the model and this implies that the prototype circumstances have to be schematized into a number of *designconditions*. In fact the entire problem (of which the scale model is only a part) has to be analysed properly in order to get optimal results.

Moreover laboratory facilities are of importance to the design of scale models. The requirements differ from case to case. Important aspects of the facilities are for instance:
- available space and discharges
- available instrumentation and control systems
- available experience.

These aspects do not *dictate* the scale model that will be constructed. This can be demonstrated by two examples:
(i)   The available instruments for measuring flow velocities in a tidal model are too inaccurate for the velocity scale that is adopted in the first approach. Two ways are open
   - Construct or purchase better instruments
   - Make the model larger i.e. increase the flow velocities.

(ii) For a particular scale model the staff of the laboratory lacks experience. Again two ways are open:
- Reject the scale-model study because of the lack of experience
- Accept the scale-model study to gain experience.

Studies with scale models still require from the researcher a sound approach using a suitable combination of *science and art*. Consequently common sense plays an important role.

In the following sections three groups of models will be discussed. Problems in inland waters (Section 4.2) are treated separately from the ones in coastal waters (Section 4.3) where tides and waves may be important. In the third group (Section 4.4) some examples are given of scale models of hydraulic structures.

4.2. Problems in inland waters

4.2.1. Rigid-bed models

Whenever possible rigid-bed models should be applied because they are less complicated and less expensive than mobile-bed models. In a number of cases this is possible. Leaving apart the problems dealing with hydraulic structures it can be said that the compromise in scaling has to be found here for the velocity scale, the distortion and the artificial roughness.

It seems logical to select for all these models the velocity scale according to the Froude condition. This is, however, not always the case as can be seen as follows.

Example

Consider the case of the design of a scale model in order to determine the effective area of a lake that is used for cooling water. The problem is to find the optimal location of intake and outlet for the cooling water.

The lake will be characterized by large horizontal and small vertical dimensions. Hence distortion is a must. From the character of the problem it appears that areas with small flow velocities are present. Hence *Reynolds condition* cannot be neglected here. It can be stated that *Froude condition* is of less importance here. This is because of the fact that Froude number is small in this case. Hence the factor $s_m/a_m = \frac{1}{2} Fr_m^2$ is small also. As $n_{Re} = 1$ and $n_{Fr} = 1$ cannnot be fulfilled at the same time a good compromise seems possible with

- $n_u < n_a^{1/2}$ (deviation from Froude condition)
- $n_L > n_a$ (apply distortion)
- $n_C^2 = n_L/n_a$ (fulfil roughness condition).

Navigability of rivers and canals can be tested by means of manoeuvring tests with model-ships. In this case an undistorted model is a must. Moreover Froude condition and roughness condition have to be fulfilled. The model-ship should not be too small, otherwise the influence of the viscosity on the skin friction of the model-ship becomes relatively too large.

Special attention has to be paid to the steering method to be selected (Hoekstra, 1965). There are a number of possibilities.
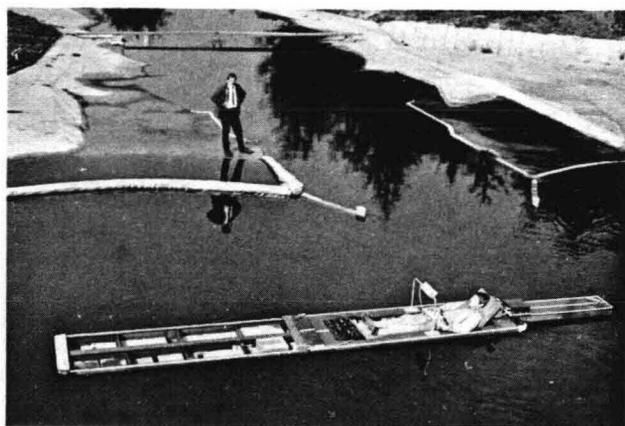
Fig. 4.1 Helmsman aboard ($n_L = 25$)

(i) *Steering with a helmsman aboard*
Figure 4.1 shows a scale model of barges steered with a helmsman aboard. The helmsman has an almost correct view on the surroundings. In this case the power for the propellors is taken from batteries aboard the barges. Obviously the model has to be large in this case because the barges have to carry the pilot and the batteries. At one point the model-pilot has a better information than his collegue in the prototype. This is due to the fact that the human eyes are capable to get a 3-D-view at distances smaller than some 40 m. Consequently the model-pilot will have to reduce his information by only using one eye.

(ii) *Remote controlled steering from the banks.*
There are two versions of this system. The pilot may get his information on the position of the ship through direct observation or via a display connected with a TV-camera aboard the ship. In the first case the model-polot gets relatively to much information on the situation. In the second case he gets relatively too little information.

(iii) *Electronic steering.*
In this case the electronic pilot steers the ship along a predescribed course. The variation of the propeller speed and the rudder angle are measured.
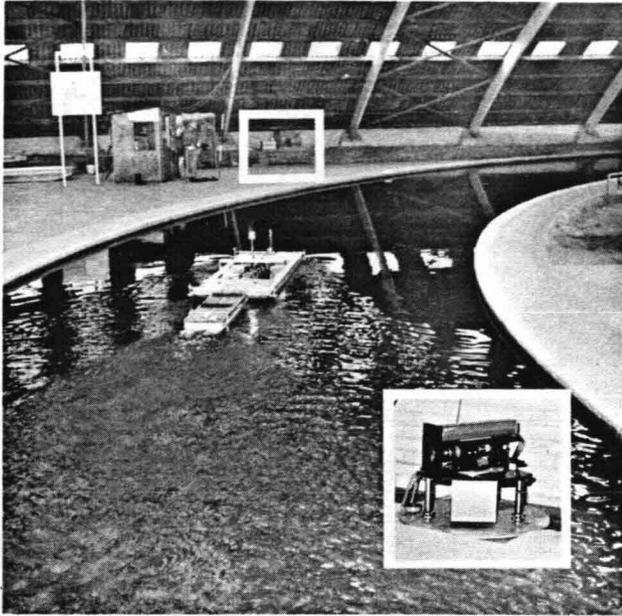
Fig. 4.2 Laser guidance system

Figure 4.2 shows such a model. The location of the model-ship is measured by means of two laser beams generated ashore and reflected at small mirrors at the ship. The control system takes care of the action of propellor and rudder required for the correct course of the ship. The capability of the prototype pilot can be programmed. Naturally the reliability of such an electronic steering is increased by prototype tests.

In the last case the course of the ship is known as it is directly measured by means of the laser beams. The first two methods require a technique to establish the course of the ship. Photographic techniques have been used in this respect. Figure 4.3 shows a birdseye view of the barges of Fig. 4.1 while crossing in a scale model the Lower Rhine near Wijk bij Duurstede on their course along the Amsterdam-Rhine canal.
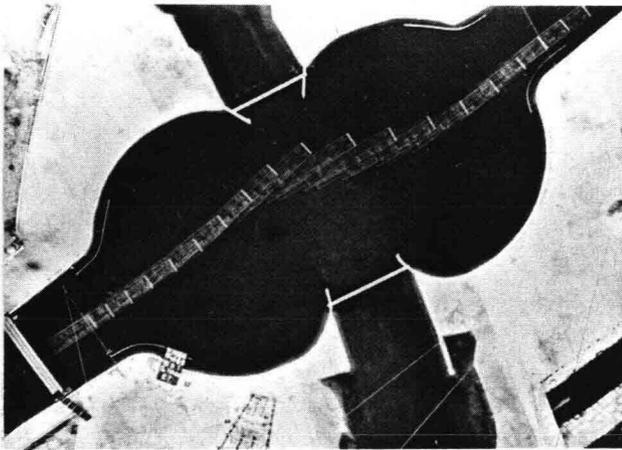
Fig. 4.3 Photographic locationing

As will be discussed below mobile-bed river models usually will require distortion. This creates a conflict in cases in which manoeuvrability has to be studied on a mobile bed.

The only solution here is to use two models:

(i)   An undistorted fixed bed model to test the manoeuvrability and

(ii)   A distorted model with mobile bed to study morphological changes if the geometry is changed.

An example of such models is given in Fig. 4.1 and 4.3. They regard a study on the improvement of the navigability of the crossing: Amsterdam-Rhine canal and Lower-Rhine near wijk bij Duurstede.

Figure 4.1 (and also Fig. 4.3) show the undistorted navigation model ($n_L = 25$)

The morphological model was distorted $(n_L = 75; n_a = 25)$.

4.2.2. Mobile-bed models

4.2.2.1. General

In many problems in inland waters the mobility of the bed cannot be neglected. Here the intention will be focussed on mobile-bed river models. Remarks on local scour near hydraulic structures will be discussed in Section 4.4.

Before in Subsection 4.2.2.3. the actual scaling of the model will be discussed, in Subsection 4.2.2.2. the *tilting* of such models will be treated. This is a measure to overcome scale effects arising from the fact that the velocity scale is deviating from the one according to the Froude condition. Special attention to the roughness scale is given in Subsection 4.2.2.4.

The reproduction of the river regime will be discussed in Subsection 4.2.2.5.

4.2.2.2. Tilted models

The application of the ideal velocity scale for the reproduction of the morphological process most likely implies a deviation from the Froude condition. It will be discussed here what the consequences are, moreover the surpressing of scale effects will be treated.

Starting with Chézy equation and with the notion that a slope (i) is the ratio of a difference in waterlevel (Δh) over a certain length (L) it can be stated

$$n_u^2 = n_C^2 \cdot n_{ai} = n_a \cdot \frac{n_C^2 \cdot n_a}{n_L} \cdot \frac{n_{\Delta h}}{n_a} \tag{4-1}$$

This leads to the conclusion that $n_{\Delta h} = n_a$ provided
- The *roughness condition* is fulfilled ($n_C^2 = n_L/n_a$)
- The *Froude condition* is fulfilled ($n_u^2 = n_a$)

The selection $n_u^2 = n_a$ may not be possible, however, because the application of the *ideal velocity scale* has priority to reproduce the morphological process.

Usually $n_u^2 < n_a$ will follow from the concept of the ideal velocity scale. As in many rivers the velocity head will be much smaller than the waterdepth this will as such not create large scale effects (c.f. Eq. (1-8)).

More serious is the effect that according to Eq. (4-1) the selection $n_u^2 < n_a$ will (if still $n_C^2 = n_L/n_a$ is adopted) lead to $n_{\Delta h} < n_a$. This will imply that the slope in the model is too steep.

To see what the consequences are it is in the first place possible to look at a river with *vertical banks*. Also in this case the waterlevel slope will be too steep. However, due to the mobility of the bed the bedslope will follow the waterslope and hence the depth will be reproduced correctly. Both bedlevels and waterlevels will not be correct with respect to a *horizontal* datumlevel.

However, for sloping banks, especially in relatively narrow rivers this will be different. If the waterlevel is reproduced correctly in the centre of the model then upstream waterlevels will be too high and the downstream waterlevels will be too low.

Fig. 4.4 Errors in waterlevels
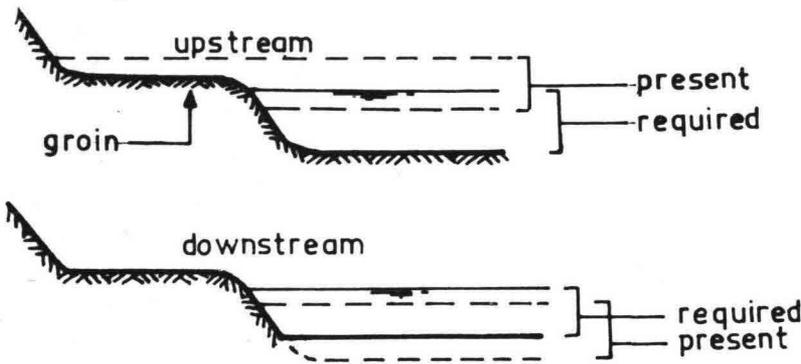
Now the *waterdepth* will not be reproduced correctly as can be deduced from Fig. 4.4. In the upstream part of the model not only the waterlevel but consequently also the width at the watersurface will be reproduced incorrectly. The width will be too large and hence the depth will be too small.

Downstream an opposite effect will be present. The waterlevel is too low here, hence the width at the watersurface is too small and consequently the depth is too large.

This inconvenience can to a large extent be overcome by means of *tilting* of the model.
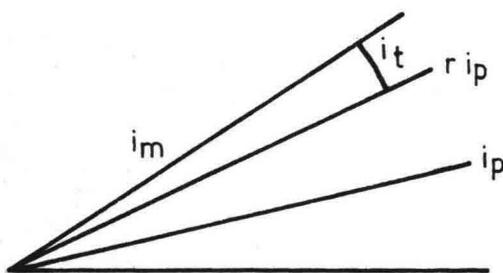
Fig. 4.5 Tilted model

Figure 4.5 indicates the principle. The prototype slope (i_p) should be reproduced as ri_p if r is the distortion selected. However, due to the selection of $n_u^2 < n_a$ a model slope $i_m > ri_p$ will be present.

This inconvenience can be overcome by using for the model a *sloping datum level*.

If the fixed banks or the groins are built with respect to a datum level with a slope $i_t = i_m - ri_p$ then the waterlevels are situated correctly with respect to the banks and hence the waterdepths will be reproduced correctly as well.

The required tilt i_t can quite easily be deduced.

$$i_t = i_m - ri_p \tag{4-2}$$

hence

$$i_t = i_p \left[\frac{n_C^2 \cdot n_a}{n_u^2} - r \right] \tag{4-3}$$

Note that logically $i_t = 0$ follows from Eq. (4-3) if $n_u^2 = n_a$ and $n_C^2 = r$ are selected.

The problem is that in Eq. (4-3) i_p is known, n_u^2 together with n_a and r are selected but n_C^2 is unknown. In fact n_C^2 can only be determined accurately from measurements in prototype *and in the model*. The tilt i_t has to be established, however, *before* the construction of the model.

The only possibility is to *estimate* the model roughness (C_{me}) before the con-struction of the model. It is quite possible that the actual roughness of the model (C_{ma}) will be different from C_{me}. Consequently still errors in the water-level are possible.

The maximum error (Δh_{max}) present at the ends of the modelreach with length L_m is:

$$\Delta h_{max} = \tfrac{1}{2} L_m \{ i_{ma} - i_{me} \} \tag{4-4}$$

if i_{ma} and i_{me} are the *actual* and the *estimated* waterlevel slopes in the model

The following deduction using $i = u^2/(C^2 a)$ can now be given

or

$$\Delta h_{max} = \tfrac{1}{2} L_m \left[\left\{ \frac{u^2}{C_a^2 a} \right\}_m - \left\{ \frac{u^2}{C_e^2 a} \right\}_m \right]$$

$$\Delta h_{max} = \tfrac{1}{2} \frac{L_m \cdot u_m^2}{C_{me}^2 \cdot a_m} \left[\frac{C_{me}^2}{C_{ma}^2} - 1 \right]$$

Thus

$$\frac{\Delta h_{max}}{a_m} = \frac{1}{2a_m} \frac{L_p \cdot u_p^2}{C_p^2 \cdot a_p} \frac{n_C^2 \cdot n_a}{n_L \cdot n_u^2} \left[\frac{C_{me}^2}{C_{ma}^2} - 1 \right] \tag{4-5}$$

Hence

$$\frac{\Delta h_{max}}{a_m} = \frac{1}{2} \left[\frac{L_p \cdot i_p}{a_p} \right] \left[\frac{n_C^2 \cdot n_a}{n_L} \right] \left[\frac{n_a}{n_u^2} \right] \left[\frac{C_{me}^2}{C_{ma}^2} - 1 \right] \qquad (4-6)$$

The relative maximum error $\Delta h_{max}/a_m$ is thus dependent on four factors:

(i) The parameter $L_p i_p/a_p$ characterizing the river involved. Note that $L_p i_p/a_p = 1$ for a river reach with length L_p for which the difference in waterlevel is just equal to the depth.

(ii) The factor $n_C^2 \cdot n_a/n_L$ which is equals unity when the roughness condition is fulfilled.

(iii) The factor n_a/n_u^2 indicating the degree at which the Froude condition is (not) fulfilled.

(iv) The last factor becomes equals zero if the roughness of the model is estimated perfectly beforehand.

Equation (4-6) indicates that in spite of tilting the model errors in the waterlevels may be present as the ideal situation $C_{ma} = C_{me}$ can never be reached. The equation also indicates that the possible error becomes larger if a longer reach of the prototype is to be reproduced. Hence the length of mobile-bed river-models has to be restricted.

Not all models can be tilted. Two examples can be given
• Models of *tidal rivers* have to reproduce ebb and flood discharges. Tilting for the ebb-flow would mean increase of the errors for the flood-flow. Hence tilting is not applicable.
• A river flowing around an island cannot be reproduced in a tilted model unless the two branches which enclose the island are of equal length.

For a model without tilt the maximum error in the waterlevel is found from

$$\Delta h_{max} = \frac{1}{2} L_m \{i_m - r i_p\} \qquad (4-7)$$

This can be expressed as

$$\frac{\Delta h_{max}}{a_m} = \frac{1}{2} \left[\frac{L_p \cdot i_p}{a_p} \right] \left[\frac{n_a}{n_u^2} \left\{ \frac{n_C^2 \cdot n_a}{n_L} \right\} - 1 \right] \qquad (4-8)$$

4.2.2.3. Scaling of the model

A number of requirements have to be fulfilled at the same time. It is therefore
advisable to carry out the scaling in a logical way. A possible way is the one
given below for a constant discharge used in the model. The number of available
bedmaterials for the model are usually restricted. To avoid too much computational
work the bedmaterial is selected at an early stage. The criteria in the selec-
tion of the scales are both of a theoretical and of a practical nature.
The analysis is based on two assumptions:

(i)   All required prototype data are known

(ii) The estimated roughness of the model (C_{me}) is known from experience and/or
      literature. This topic will be given further attention in Subsection
      4.2.2.4.

Step 1: Select n_a on the required accuracy. Usually for a mobile-bed model
      $\bar{a}_m$ < about 0.1 m has to be avoided if a reasonable accuracy is required.

Step 2: Make a tentative guess about n_L. The concept of the ideal velocity
scale leads to

$$n_S = n_\Delta^{1/2} \cdot n_D^{3/2} \cdot n_L \quad (4-9)$$

Fig. 4.6 Selecting adequate bedmaterial

For a specific bedmaterial in the model the right-hand side of Eq. (4-9) is a known constant. Hence for the various available bedmaterials the lines in Fig. 4.6 can be drawn.

Each available bedmaterial is presented by one point in this figure (n_Δ and n_D being known). Now a first selection on the acceptability
of certain available bedmaterials can be made on considering limits for
S_m. These limits can be formulated for n_S as S_p is known. There are two
limits

  • *Lower limit:* In this case S_m becomes so large that it is to difficult
                to handle the model technically.

  • *Upper limit:* Here S_m is so small that the morphological process in the
                model is too slow. This means that the morphological time
                scale according to Eq. (3-83) becomes too small.

Step 3: Estimate the roughness of the model (C_{me}) from available experience, flume tests or literature. For sand much information is given by Guy *et al* (1966).

Compute now

$$n_u = n_\Delta \cdot n_D \cdot n_{Ce}^2 \cdot n_\mu^{-1} \tag{4-10}$$

The value of n_μ can be found from the transportformula applicable in the particular case.

For instance

Meyer-Peter and Mueller formula

$$n_u^2 = n_\Delta \cdot n_D \cdot n_{Ce}^{1/2} \cdot n_{C_{90}}^{3/2} \tag{4-11}$$

Engelund-Hansen formula

$$n_u^2 = n_\Delta \cdot n_D \cdot n_{Ce}^{8/5} \tag{4-12}$$

Step 4: Select distortion and tilt (if any) based on

$$i_t = i_p \left[\frac{n_{Ce}^2 \cdot n_a}{n_u^2} - r \right] \tag{4-13}$$

This infact means the determination of n_L and i_t. The roughness condition has to be fulfilled as good as possible. Usually $r \leqslant 3$ in order to reproduce shoals and troughs adequately. It is generally attractive to select for n_a and n_L round numbers; this is easy to work with.

Step 5: Compute possible errors in the waterlevels due to possible deviation of the actual possible model roughness and the value of C_{me}. The maximum error (Δh_{max}) can be found from

$$\frac{\Delta h}{a_m} = \tfrac{1}{2} \left[\frac{L_p \cdot i_p}{a_p} \right] \left[\frac{n_C^2 \cdot n_a}{n_L} \right] \left[\frac{n_a}{n_u^2} \right] \left[\frac{C_{me}^2}{C_{ma}^2} - 1 \right] \tag{4-14}$$

To get an idea of the last term of the right-hand side of Eq. (4-14) it can be assumed $C_{ma} = C_{me} \pm \sigma_C$. A Taylor expansion for $\sigma_C^2 \ll C_{me}^2$ gives then

$$\frac{C_{me}^2}{C_{ma}^2} - 1 \approx \mp 2 \frac{\sigma_C}{C_{me}} \tag{4-15}$$

Step 6: Compute the morphological timescale n_{tm} according to

$$n_{tm} = \frac{n_L^2 \cdot n_a}{n_S} \qquad (4-16)$$

Step 7: Carry out a number of tests to find the most suitable set of model
scales. Some possibilities are:
- is the flow turbulent?
- is u_m larger than u_{cr} (Shields' graph)?
- is n_{tm} acceptable? (see below)
- is the error in the waterlevel acceptable?

Remarks

(i)   The above given approach to a consistent set of model scales can be com-
       puterized. The starting point is then an input of data from available
       bedmaterials and series of possibilities for n_L and n_a together with
       relevant prototype data. The tests in Step 7 are ment to carry out the
       selection.

(ii)   The selection of the morphological time scale is related to the purpose of
       the model. If the model is ment to study time depending processes (erosion
       and sedimentation) then a small value of n_{tm} is attractive. The model is
       then slow enough to provide the time for the necessary soundings. However,
       a quick model (large n_{tm}) is possible if only future equilibrium situations
       are anticipated.

(iii) The selection of the criteria in Step 7 is of great importance. They have
       to be formulated in such a way that two extremes are avoided
       • If the criteria are too strong then no set of model scales is accepted
       • If the criteria are too weak then a too large number of sets are accepted.
         The results then become unsurveyable.

For a restricted number of acceptable sets the cost for construction and operation
of the model (including the cost of bedmaterial) can be estimated leading to a
final choise.

In the above given analysis it has been assumed implicitely that both in prototype
and model the bedmaterial is (nearly) uniform. However, at least in the prototype
the non-uniformity of the bedmaterial may play a role. For instance at bifurcations
grainsorting takes place. Using uniform material in the model would mean here that
n_D varies over the three branches. Consequently also the depthscale will be different.

This has led in the past to use non-uniform bedmaterial in the model as well (n_D is the same for each size fraction). An example is the model of the bifurcation of the Rhine near Pannerden (see de Vries and van der Zwaard, 1975). There is a complication here as it is not easy to define *the* grainsize distribution of the bedmaterial in the prototype.

4.2.2.4. Roughness scale

Some special attention is given here to the roughness scale because n_C plays an important role in the determination of the scales.

In the previous Subsection n_C is determined by:

- Determination of C_p from direct observations
- Determination of C_m from flume studies and/or literature i.e. for a selected value of D_m, Δ_m and a_m the value of C_m is determined experimentally for various values of u_m.

There is, however, a group of researchers that is following a slightly different line (e.g. Yalin, 1971 and Giese, 1978). They base the determination of n_C on roughness-predictors e.g. usually a mainly experimental relationship between the roughness of an alluvial channel and hydraulic characteristics

It is the writers' impression, however, that the presently available roughness predictors have a low accuracy that may cause inaccurate results if they are used automatically for the estimation of n_C.

This statement is illustrated in Fig. 4.7. A series of field data of the Magdalena R. and tributaries (Columbia) was used to compare with the forecast of C by means of two often used roughness predictors.

(i)   The *Engelund-Hansen (1967) predictor*.
(ii) The *Alam-Kennedy (1969) predictor*. According to this predictor the roughness predictor C' of the bed for flat bed with transport has to be determined with the Lovera-Kennedy (1969) graph. This experimental graph based mainly on flumes and small rivers is not at least adequate for fairly large rivers where a/D_{50} is much larger.
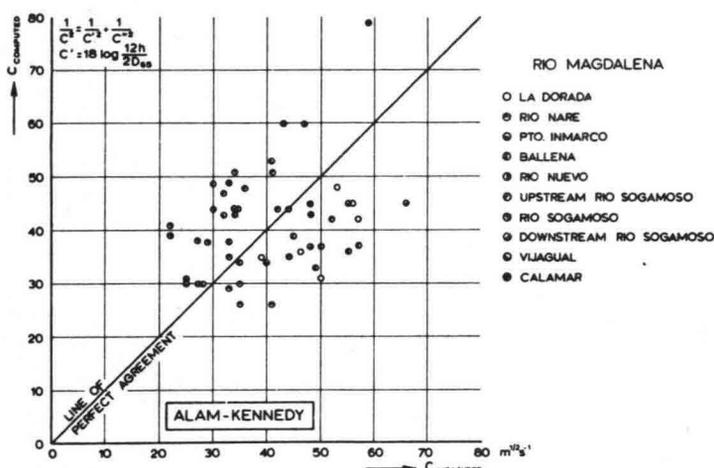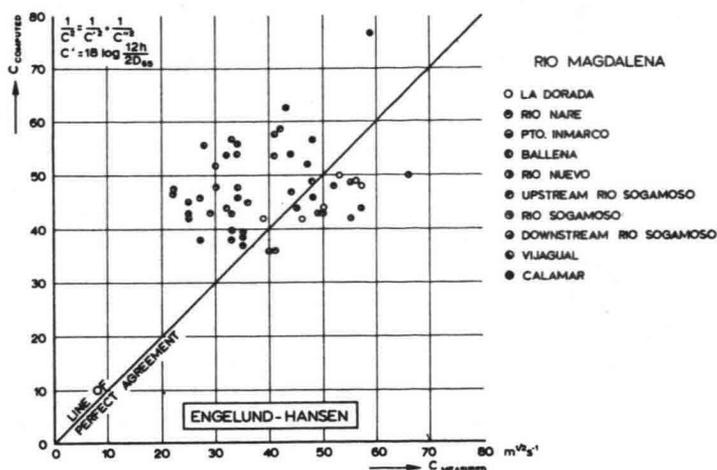
Fig. 4.7 Roughness Magdalena River
(after Jansen, 1979)

Both predictions show that a large scatter is present. Hence a large inaccuracy of n_C may be present. It is therefore advised to be very careful in this respect.

4.2.2.5. Schematization of regime

Sofar the scaling of a mobile-bed river-model has been based on a constant discharge. A constant discharge, however, is obviously not able to reproduce the time depending bedlevel variations due to the riverregime. Hence a constant discharge can only be applied if these variations are small or not important to the problem studied. The selection of such a constant discharge should be based on the problem involved. Morphological computations have been proposed in this respect (Prins and de Vries, 1971).

If a varying discharge has to be applied then a logical sequence of constant discharges has to be selected. For each discharge the scale selection has to be repeated. Obviously n_D, n_a, n_L, n_Δ and also i_t has to be the same for each discharge. In principle it is possible to vary n_u with Q.

4.2.2.6. Scale models and mathematical models

Morphlogical problems in rivers can attractively be solved if two available tools, scale models and mathematical models are applied in the field where they have their strength. For the characteristics and applicability of mathematical morphological models reference can be made to Jansen (1979) and de Vries (1981).

The two model types are complementary to one another as can be seen from the following short characterization:

- *Scale models* can give a good reproduction of the bedlevel variations in space. There are restrictions with respect to the length of the riverreach that can be reproduced. The introduction of the entire regime is not easy.
- *Mathematical models*, however, have opposite characteristics. Restrictions with respect to the length of the reach to be reproduced are hardly present. It is no problem to introduce the regime. A serious restriction as yet (1981) is that only the average depth across the width can be introduced. In other words little detail is attained.

The solution is obvious. Mathematical models can be used to compute roughly the variations over a long reach. Scale models can be used for some parts of the entire reach for which detailed information is required (see also de Vries and van der Zwaard, 1975).

4.3. Coastal and estuarine problems

4.3.1. Rigid-bed models

Many problems along coasts and in estuaries can be tackled by means of scale models. Here also holds the remark that whenever possible a rigid-bed model should be applied. There is a large variety of possibilities. Some examples will be given here with the exception of models for hydraulic structures; those are treated in Section 4.4. Reference can be made to Keulegan (1966), Ippen (1968), Novak & Cabelka (1981), Kobus (1980) and Sharp (1981). The problems discussed here are characterized by the presence of (tidal) currents, short waves and possibly density currents.

- *Wave penetration in harbours along coasts*

  In many cases refraction and diffraction is present at the same time. Hence the model has to be undistorted The wave heights in the model are studied as a function of the incoming waves. These incoming waves may be regular ones (i.e. monochromatic waves) as can be seen in Fig. 4.8.
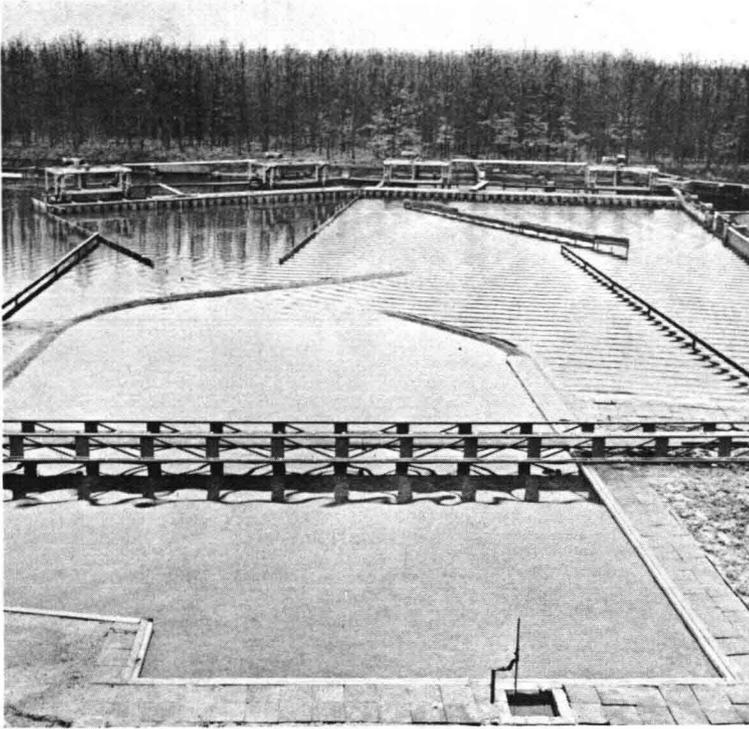
Fig. 4.8 Wave penetration test
          Sekondi harbour (Ghana); $n_L = 70$

There is, however, a tendency to use *irregular waves* (i.e. to reproduce the wave spectrum). The wave generator required is more expensive than the one for regular waves. Reference can be made to Graveson *et al* (1975).

The objective of these tests is mainly that the lay out of the harbour is made in such a way that save navigation is guarantied. Requirements are set for the waves that meet the moored vessels in the harbour. The models are usually built in a permanent facility (wave basin).

In order to measure the wave height inside the harbour accurately it may be neccessary to select $n_H < n_L$. This does not influence the results as long as no breaking waves have to be reproduced from the prototype. Of course the waves should not break in the model if they do not in the prototype.

Wave penetration may be influenced by the tidal movement. This regards both the influence due to the variations of the waterlevel (vertical tide) and due to the tidal currents (horizontal tide).

However, it is not necessary to introduce the complete tidal cycle in the scale model. Important phases during the tidal cycle can be reproduced by steady flow. If these flow currents are strongly influenced by storage effects, e.g. due to filling or emptying of the harbour basins, then these effects have to be introduced in the scale model as well. In this way a correct reproduction of *stream refraction* can be attained.

The scaling can be influenced by the type of wave-generator applied. If e.g. the wave generator only can reproduce distinct wave periods (τ_m)

then this fixes with given τ_p the scale n_τ. The relation between n_L and n_τ then fixes the length scale to be applied.
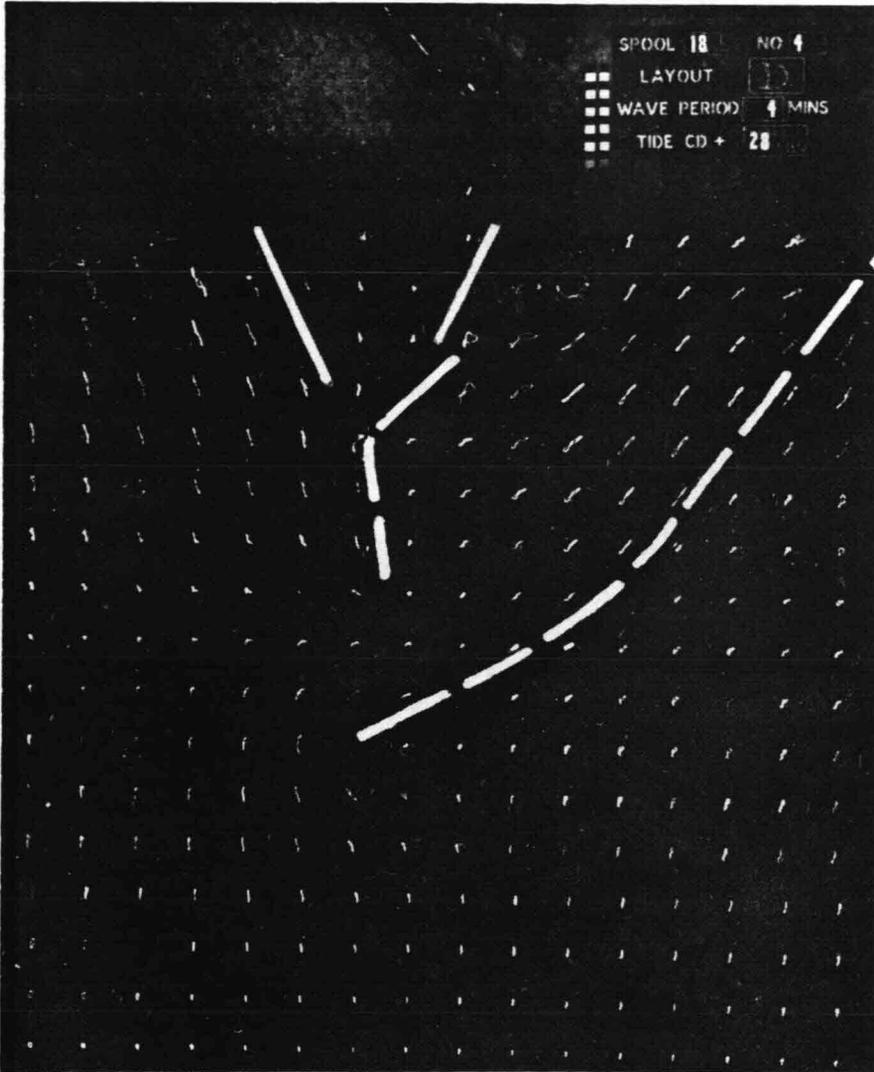
Fig. 4.9   Starry sky model (after Stewart, 1965)

Seiches may create resonance in harbour basins. They are of a meteorological influence (caused by local small depressions). These waves having periods of some minutes can be investigated by a *starry sky model*. The (distorted) model is placed under a dark cieling in which at regular intervals a large number of small lights are placed. The camera in the cieling takes during one wave period a photograph of the light reflected at the water surface. The photographs can be interpreted towards the flow velocities at various places and during the wave period. For details reference can be made to Stewart (1965),

see also Fig. 4.9.

This type of scale model is not frequently used as in many cases a mathematical model can be used adequately.

Problems with tidal waves can be studied in tidal models. Here it is of importance whether density currents can be expected due to the presence of a (fresh) upper discharge, together with the (salt) sea water. If the fresh water influence can be neglected for the problem then all salt water in the prototype can be reproduced by fresh water making $n_\rho \simeq 1$. This is a great advantage as the operation of a scale model with salt water requires much attention to rusting.

The cost of a tidal model depend largely on the accuracy required, which again is also depending on the accuracy of the prototype data available.
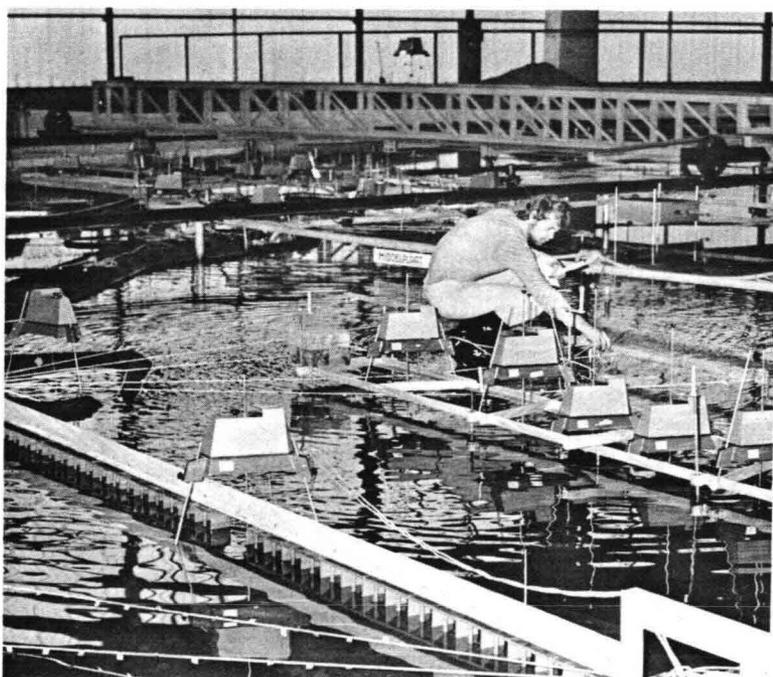
Fig. 4.10   Scale model of the Oosterschelde

Figure 4.10 shows the scale model of the Oosterschelde ($n_L = 400$; $n_a = 100$). The salt water of the prototype is reproduced by fresh water (no density currents).
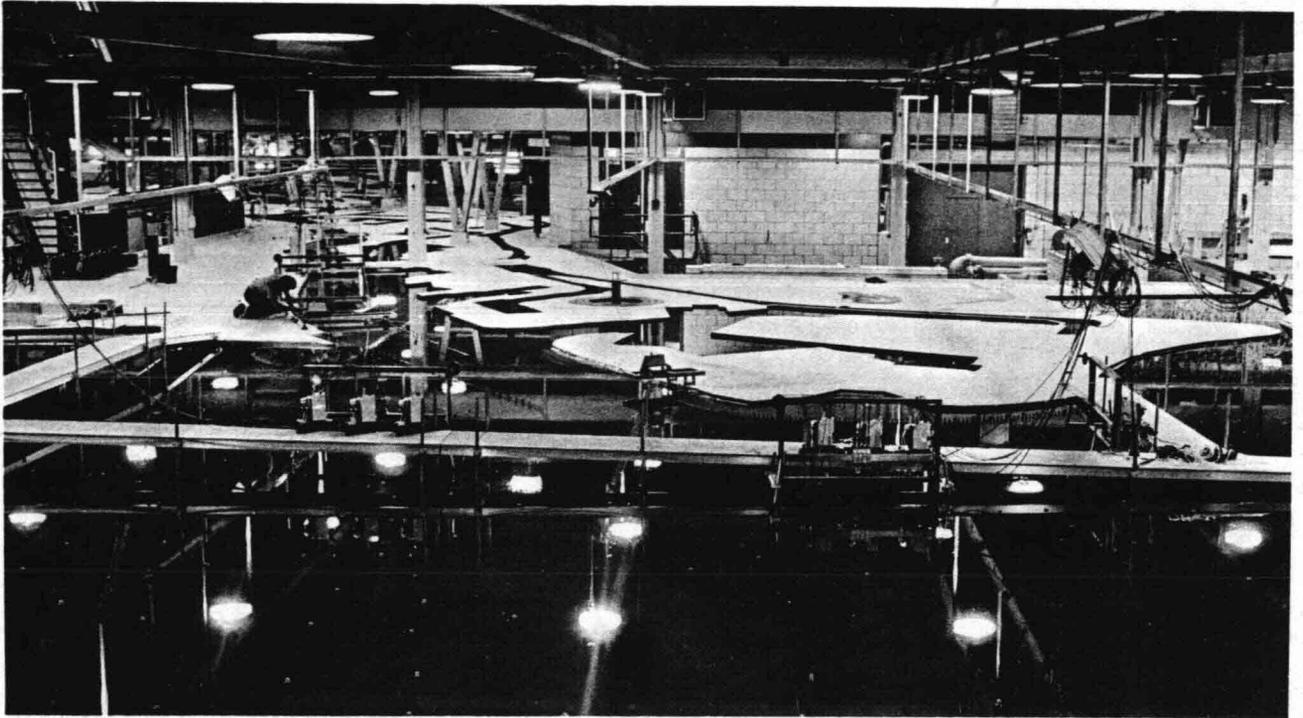
Fig. 4.11   Europort model   (n_L = 640; n_a = 64)

In Figure 4.11 the Europort model is shown. In this case the density
differences are of importance; hence $n_{\Delta\rho}$ = 1 is used.
This model covers a large area in which the harbour of Rotterdam is the
central part. This scale model is used for the complex watermanagement
problems in this part of the Netherlands.

4.3.2. Mobile-bed coastal-models

4.3.2.1. General

In mobile-bed coastal-models the reproduction of the morphological processes
is the central problem. The non-linear relation between flow velocities and
sediment transport plays a major   role.

An additional complication is that the natural conditions inducing the
morphological processes vary considerably in time and place. In many cases
a combination of the following aspects is present.

• Tidal currents vary throughout the year.

• River discharges vary in time.

• Waves of various periods and directions, together with different
  wave heights are present.

Consequently the natural conditions require a good deal of schematization
before they can be applied as boundary conditions for the scale model.
This implies that calibration of the model is a must. In this calibration
test at the same time scaling and schematization of the boundery conditions
are tested.

4.3.2.2. Scaling

In sub-section 3.5.2. it has been indicated that according to the condition
for the ideal velocity scale the following conditions have to be fulfilled
at the same time

Scale for current velocity

$$n_u = n_C \sqrt{n_\Delta \cdot n_D \cdot n_\mu^{-1}} \qquad (4-16)$$

Scale for maximum orbital velocity

$$n_{v_o} = \sqrt{n_\Delta \cdot n_D \cdot n_\mu^{-1}} \qquad (4-17)$$

Apparently Eqs. 4-16 and 4-17 connot be fulfilled at the same time, together
with $n_u = n_v$ as $n_C > 1$. Moreover the use of light bed material ($n_\Delta > 1$) is
restricted *as liquefaction* made be present in the model. Hence scale effects
are present most likely. The scaling therefore is based on minimizing scale
effects in n_s (and hence in the bed levels).

By means of the Bijker formula it is possible to deduce

$$n_s = f\{n_u, n_{v_o}, v_p \text{ and } a_p\} \qquad (4-18)$$

once a bed material has been selected for the model. It is now possible to
indicate ranges for v_p and a_p which are of primary importance for the

prototype area considered. The selection of n_u and n_v (thus n_H) is now based on the consideration that n_s has to be as constant as possible for the ranges of v_p and a_p adopted. Therefore the specific shape of $\xi = f\{a_o/r\}$ has to be used. For instance the one found in Massie (1978). The details of this procedure go beyond the scope of these lecture notes (see Bijker, 1967).

Obviously C_m has to be known here as well as C_p. However, for known values of H_m, D_m and a_m the value of C_m is a result. Hence C_m will generally not fulfil the roughness condition. Additional roughness elements (rods) may be required locally where curved flowlines are present.

A possible procedure is to select a length scale and adopt n_a and n_H provisionally. Tests give an indication of the beach profile in the model. This has to be matched with the beach profile in the prototype. The depth scale is then fixed considering that part of the beach slope that is expected of largest importance to the problem. It is practically impossible to attain geometric similarity for the entire beach slope.

4.3.2.2. Example

As an example the approach to the port of Abidjan (Ivory Coast) is given. The approach from the ocean is through the Canal de Vridi. This (artificial) channel links the ocean and the large lagoon along which the port of Abidjan is located.
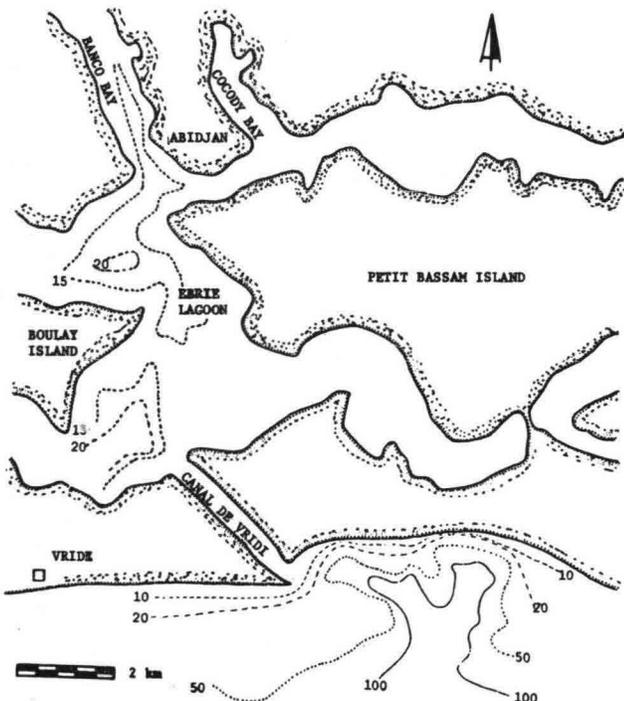
The single breakwater mouth of the channel at the ocean side is situated to the west of a deep hole in the sea bed. There is an easterly littoral current due to waves (dominant period $\tau_p = 12$ s). The sediment along the head of the breakwater is transported by the waves; the tidal currents due to the filling or emptying of the lagoon have a strong influence on the sediment transport. The intention of the design is that the sediment after passing the breakwater is trapped in the deep hole (Fig. 4.12).

Fig. 4.12   Canal de Vridi

Three times the Delft Hydraulics Laboratory carried out studies with scale
models on this problem. The original design was tested in 1933. In 1950 the
canal was completed. In 1960 - 1962 and around 1970 new modeltests are carried
out due to changing situations in the prototype (requirements for more draft
in the channel and hence in the mouth of the channel). The details on the scales
given below regard the 1970 model.

Before the scales can be discussed some remarks have to be made on the
boundary conditions to be applied to the scale model:

(i)   Using a *mathematical model* for the channel and the long lagoon the
       ebb and flood currents are determined in the channel.

(ii)   The littoral current present in the prototype has to be reproduced
       in the model. The littoral current was computed according to Eagleson
       (1965).

(iii) The littoral transport near the breakwater will change gradually in time
       as the coastline to the west of the breakwater will gradually show accre-
       tion. The littoral transport was cumputed according to Pelnard-Considère
       (1956).

The model scales were determined along the lines summarized in the previous
sub-section. The following results were obtained:

Geometric scales: $n_L = 150$; $n_a = 60$

Sediment   : $n_\Delta = 1$; $2_D = 2\frac{1}{2}$ ($D_p = 0.5$ mm ; $D_m = 0.2$ mm)

Waves   : $n_\tau = 1.55$ ($\tau_p = 12$ s ; $\tau_m = 1.55$ s)
                      $n_H = 30$ ($H_p = 1.5$ m ; $H_m = 0.05$ m)
                      $n_{v_o} = 3.5$

Currents   : $n_u = n_{v_o} = 3.5$.

Note that n_u and n_{v_o} differ substantially from the Froude condition.
The value of $n_{v_o} = 3.5$ can be reached by selecting $n_H < n_a$,

Reproducing ebb flow and flood flow by steady alternating currents,
it could be computed that 1 year in prototype was reproduced in 8 hours.
Besides the variations due to the tide also the difference due to the
dry season and wet season (influencing the upstream discharge from the
lagoon) had to be considered. This led to a number of flow situations to
be introduced after each other to take into consideration both tides and
upstream discharge.
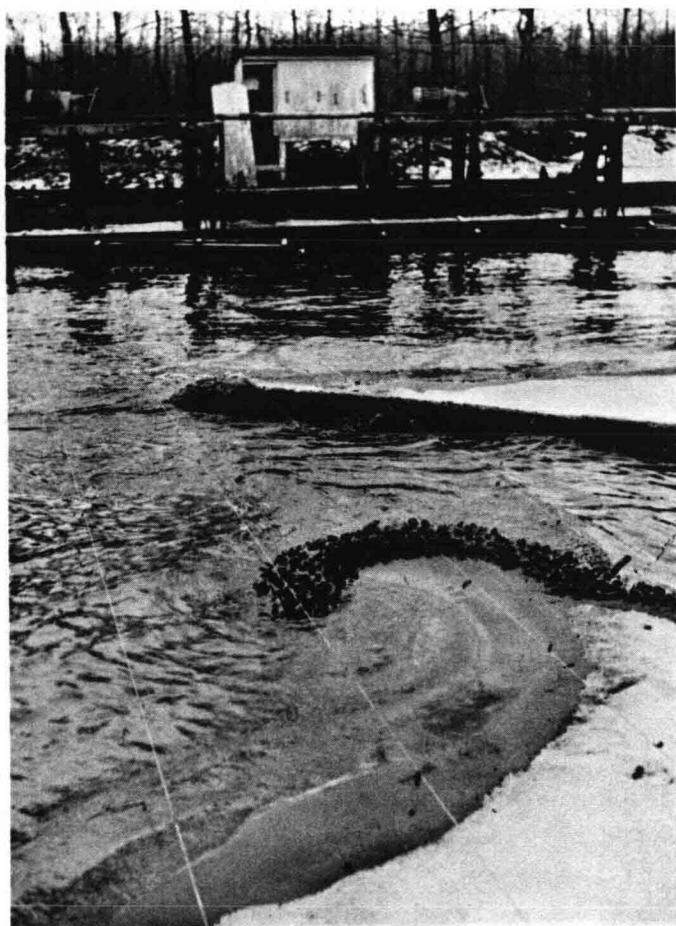
Figure 4.13 shows the scale
model used for this problem.
A number of tests were carried
out for the in 1970 existing
situation.
Each test was characterized
by a certain schematization
of the flow and the wave con-
ditons in the prototype. Only
after having obtained a good
reproduction of the morpholo-
gical situation in the proto-
type, tests ware carried out to
forcast the morphological changes
due to deepening of the channel
in order to obtain more depth
for navigation.

Fig. 4.13 Harbour entrance Abidjan

5.5. Hydraulic structures

4.4.1. General

Many aspects in the design of hydraulic structures cannot be treated in
a quantitative sense by means of computations. Therefore scale models are
used. A large variety of examples are available. In the following subsections
a selection is made.

The following remarks can be made in general.

(i)   The watermovement at hydraulic structures usely is of a threedimensional
      nature. Then the vertical accelerations cannot be neglected with respect
      to the acceleration of gravity. Hence distortion only exceptionally
      is applied in scale models for hydraulic structures.

(ii) For a hydraulic structure in some cases only a section of the
      structure is studied. This is possible if the watermovement perpen-
      dicular to a vertical plane is negligible. Examples are: breakwaters,
      weirs etc. The sections are then tested in flumes.

(iii) The impossibility of distortion leads to the circumstance that scale
      effects due to viscosity can appear rather easily. This implies that
      the general warning has to be given that the model should not be too
      small.

4.4.2. Influence of currents

Examples of scale models of structures in which currents are the dominant
hydraulic factors are represented in Figs. 1.2...1.6. If free surface flow
is present then Froude condition is a must. It is impossible to fulfil at
the same time Reynolds condition (See Eq. 1.13) unless in the scale model a
fluid with $n_\nu = n_L^{3/2}$ is used. This is not easy to obtain.

Also for flow without a free surface similar problems can arise.
Consider the flow through an orifice in a pipe. The difference in pressure
over the orifice (Δp), the density of the fluid (ρ) and the diameter (D)
of the orifice are characteristic parameters in the problem as long as the
fluid is considered ideal.

By dimensional reasoning the following deduction can be given.

$$Q = f\{\Delta p, \rho, D\} \tag{166}$$

or as an approximation:

$$Q = K(\Delta p)^{k_1} \cdot (\rho)^{k_2} \cdot (D)^{k_3} \tag{167}$$

in which K is a constant for an *ideal* fluid.

$$\left[Q\right] = \left[L^3 T^{-1}\right] = \left[\{ML^{-1}T^{-2}\}^{k_1}\right]\left[\{ML^{-3}\}^{k_2}\right]\left[L^{k_3}\right] \tag{4.19}$$

Thus

$$k_1 + k_2 = 0$$
$$-k_1 - 3k_2 + k_3 = 3 \tag{4.20}$$
$$- 2k_1 = -1$$

or $k_1 = \tfrac{1}{2}$; $k_2 = -\tfrac{1}{2}$ and $k_3 = 2$ $\tag{4.21}$

Therefore

$$Q = KD^2\sqrt{\Delta p/\rho} = KD^2\sqrt{\Delta H/g} \tag{4.22}$$

in which ΔH is the difference in head over the orifice.

With the equation of continuety this leads to the requirement that

$$n_u = n_H^{\frac{1}{2}} = n_L^{\frac{1}{2}} \tag{4.23}$$

For the *real* fluid it can be expected that

$$K = K \{Re\} \text{ with } Re = uD/\nu \tag{4.24}$$

Complete similarity requires

$$n_u = n_\nu/n_L \tag{4.25}$$

It is obvious that also Eqs. (4.23) and (4.25) are contradictory for $n_\nu = 1$.

In some cases the model has to be elastical similar. Figure 4.14 gives an example. This regards an emergency weir normally situated in a hole in the bottom of a supply canal. The weir can be inflated if for instance one of the dikes along the canal collapses. Figure 4.14 gives three stages during inflation. The length scale of this model was $n_L = 12\tfrac{1}{2}$. The correct reproduction of the elasticity of the weir is essential.
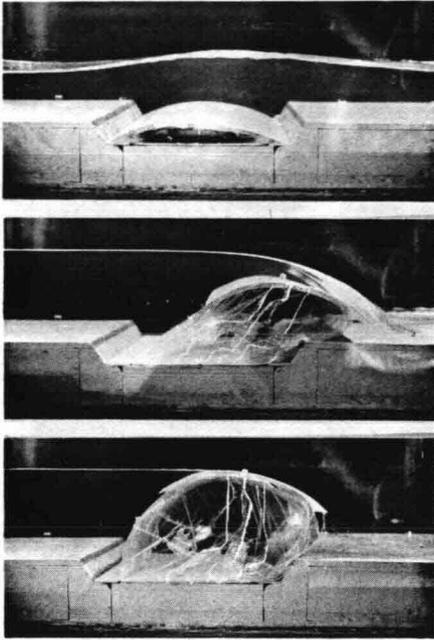
Fig. 4.14   Inflatable weir

The presence of a mobile bed restricts for the scale models for hydraulic structures the selection of the length scale. For these cases Froude condition together with the condition for ideal velocity scale have to be fulfilled at the same time. For fine sand the conditions $n_u = n_W$ and $n_u = n_L^{\frac{1}{2}}$ have to be fulfilled simultaneously.
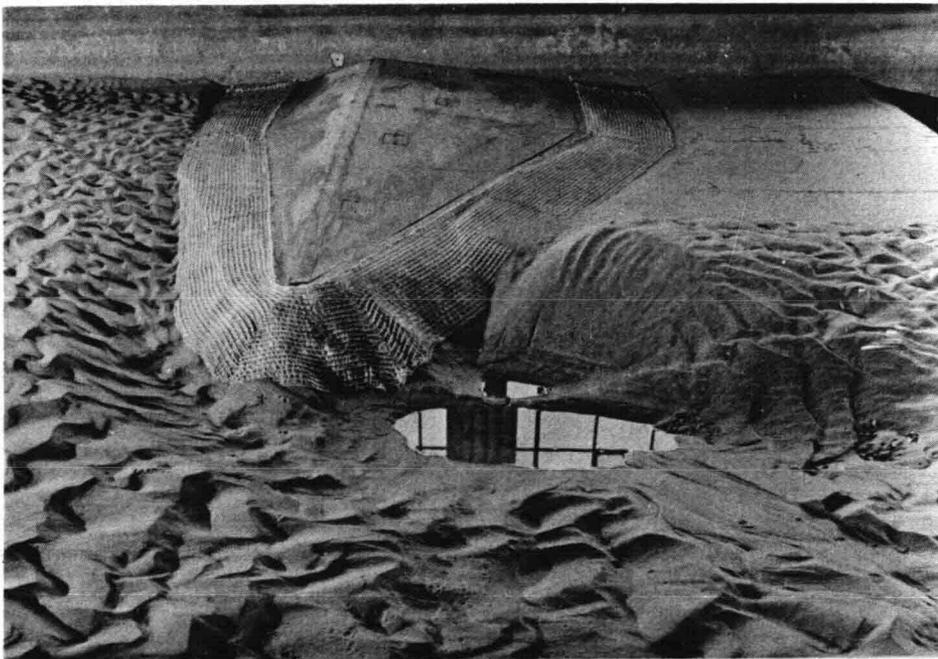
Fig. 4.15   Spur-dike head, initial design

Figures 4.15 and 4.16 show tests for the design of a spurdike. The main problem for local erosion is always the stability of the structure proper. Figure 4.15 shows a serious scouring hole if the spurdike head is built perpendicular to the current.

Figure 4.16 shows the improvement if the head is changed.

Fig. 4.16 Spurdike head, final design.

4.4.3. Influence of waves

In coastal engineering the structures are in many cases subject to wave attack. Two examples will be given here. Breakwater design requires test facilities. The dimensions of these facilities determine to a large extent the scales that can be used in a certain case.

Sections of breakwaters can be tested in waveflumes of restricted width (order of magnitude 1 m). It becomes customary to apply random waves for these tests i.e. reproducing the wave spectrum of the prototype.

These tests are not sufficient for the design of the entire breakwater. Parts of the breakwater (e.g. the head) that are subject of wave
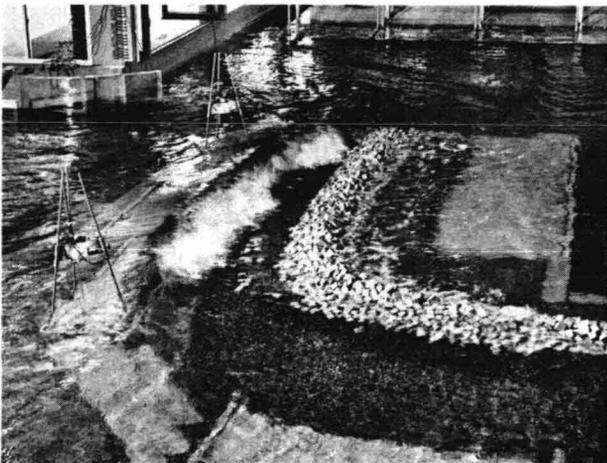
Fig. 4.17   Breakwater design

attack under an angle have to be studied in wider wave flumes or wave basins. Figure 4.17 shows such an experiment in the wind-wave flume of the Delft Hydraulics Laboratory. This flume is 8 m wide and is provided with a random-wave generator. Wind can be blown over the hundred meter long water surface to give the waves the required asymetrical shape.

Fig. 4.18   Cooling water outlet

Figure 4.18 shows other experiments in the same test facility. Here it regards the design of a cooling water outlet in the sea. For the design the forces acting on the structure have to be known. The geometric scale is here completely determined by the size of the test-facilities and the size of the prototype. The instrumentation then has to be set up in such a way that the forces are measured with sufficient accuracy.

Main Symbols		Dimension
a	waterdepth	$[L]$
c	celerity	$[LT^{-1}]$
	spring constant	$[MT^{-2}]$
C	Chézy coefficient	$[L^{\frac{1}{2}}T^{-1}]$
Ca	Cauchy number $= \rho u^2/E$	–
C_D	drag coefficient	–
D	grain diameter	$[L]$
E	modulus of elasticity	$[ML^{-1}T^{-2}]$
F	force	$[MLT^{-2}]$
Fr	Froude number $= u/\sqrt{ga}$	–
g	acceleration of gravity	$[LT^{-2}]$
G	modulus of shear	$[ML^{-1}T^{-2}]$
h	piezometric head	$[L]$
H	energy head	$[L]$
	wave height	$[L]$
i	(energy) slope	–
k	wave number $= 2\pi/\lambda$	$[L^{-1}]$
L	length	$[L]$
m	index for model	–
M	mass	$[M]$
n	scale(factor) $n_x = x_p/x_m$	–
p	index for prototype	–
	pressure	$[ML^{-1}T^{-2}]$
r	radius of curvature	$[L]$
	distortion $= n_L/n_a$	–
Re	Reynolds number $= uL/\nu$	–
s	velocity head $= u^2/2g$	–
	sediment transport per unit width and time	$[L^2T^{-1}]$
S	sediment transport per unit time	$[L^3T^{-1}]$
t	time	$[T]$
u	flow velocity	$[LT^{-1}]$
u_x	shear velocity	$[LT^{-1}]$
v	orbital velocity	$[LT^{-1}]$
W	fall velocity	$[LT^{-1}]$
We	Weber number $= \rho uL/\sigma$	–
x	horizontal co-ordinate	$[L]$
X	dimensionless transport parameter	–
Y	dimensionless flow parameter	–
z	horizontal co-ordinate	–
$z_{(b)}$	bedlevel	–

δ	thickness viscous sublayer	$[L]$
Δ	relative density = $(\rho_s - \rho)/\rho$	–
η	dynamic viscosity	$[ML^{-1}T^{-1}]$
λ	wave length	$[L]$
μ	ripple factor	–
ρ	density of water	$[ML^{-3}]$
ρ_s	density of sediment	$[ML^{-3}]$
τ	wave period	$[T]$
	shear stress	$[ML^{-1}T^{-2}]$
ν	kinematic viscosity	$[L^2T^{-1}]$
ω	(wave) frequency = $2\pi/\tau$	$[T^{-1}]$
ω_o	resonance frequency	$[T^{-1}]$
σ	stress	$[M]$
	surface tension	$[MT^{-2}]$

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