

A Decentralized Recovery Method for Air Traffic Con- flicts

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by

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Preface

Dear reader,

First of all, thank you for showing an interest in this MSc Thesis. For the past nine months I have had a very enjoyable time working on decentralized air traffic control. As a result, two articles are included in this thesis. One related to a decentralized recovery method for air traffic conflicts and one related to the construction and characterization of a state based solution space to air traffic conflicts.

I would like to extend a big thank you to the people who have helped me in the process of this thesis. First of all I would like to thank my daily supervisor Joost Ellerbroek. His indelible enthusiasm and his ability to turn problems into challenges made our meetings and this thesis overall both enthusing and challenging of the most enjoyable kind. I would also like to say thank you to my supervisor Jacco Hoekstra, whose expertise, fervor for air traffic management, and ability to provide context was very important to the process of this thesis.

As this work brings my time as a student at the Faculty of Aerospace Engineering to and end, I look back at an amazing time.

Wouter Schaberg
Delft, June 2020

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List of Abbreviations

MVP	Modified Voltage Potential
CR	Conflict Resolution
CP	Conflict Prevention
CD	Conflict Detection
CPA	Closest Point of Approach
PZ	Protected Zone
CD&R	Conflict Detection and Resolution
ADS-B	Automatic Dependent Surveillance-Display of Traffic Information
UAV	Unmanned Aerial Vehicle
ASAS	Airborne Separation Assurance System
PASAS	Predictive Airborne Separation Assurance System
VO	Velocity Obstacle
RV	Reachable Velocities
UV	Uncoordinated Velocities
DV	Diverging Velocities
NR	Non-recoverable Velocities
SS	State-based solution space
CC	Collision Cone
SSD	Solution Space Diagram
PSS	Pair-wise Solution Space
DEP	Domino Effect Parameter
CR	Conflict Rate
LoS	Loss of Separation

List of Symbols

D	Distance vector
S_h	Horizontal separation requirement
x	Relative position vector
V	Velocity vector
t	Time
P	Probability
p	Significance test score
α	Heading range
ρ	Traffic density
θ	Angle between initial relative velocity and new relative velocity
ϕ	Angle between initial relative velocity and velocity change
ψ	Angle between velocity change and new relative velocity

Reading Guide

This report consists of the following three parts:

I **Scientific Papers**

The main findings of this thesis are presented in two scientific papers. The first focuses on the proposed decentralized recovery method while the second focuses on a construction and characterization of the state based solution space for conflict resolution. Both papers contain similarities and these similarities have been left that way on purpose to ensure the readability of both papers separately.

II **Appendices**

A side-track of the second paper was an investigation into the possibility to combine expected relative velocities and path deviations to form a conflict resolution strategy. This process and conclusion is presented in an appendix. Also, an overview of the significance tables used for the first paper is presented in the appendices.

III **Preliminary Report**

The MSc thesis is divided into two parts; the preliminary thesis phase and the main thesis phase. The preliminary thesis phase is reported in the preliminary report. Note that this report has been graded already and is included in this report for completeness.

I

Scientific Papers

1

A Decentralized Recovery Method for Air Traffic Conflicts

A Decentralized Recovery Method for Air Traffic Conflicts

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Abstract– In an effort to increase airspace capacity, new methods for the decentralization of air traffic control have been a topic for research since the early nineties. The Modified Voltage Potential is a method to provide decentralized conflict detection and resolution. Through this method, aircraft can fly direct routes while resolving conflicts in a distributed manner. Though successfully implemented in various studies, its behavior in conflicts where aircraft are flying near parallel tracks has raised concern. It was found that problems can arise during the recovery after the conflict resolution, the phase where an aircraft regards the conflict as resolved and reverts to its desired velocity. To improve the Modified Voltage Potential's behavior during recovery, a modification is proposed. Through the use of two criteria, aircraft evaluate whether a conflict is resolved and if they can revert to their initial or desired velocity. Through analysis of a state-based solution space and an experiment, this study finds that the proposed recovery method significantly improves the stability, efficiency, and safety of the MVP method.

I. INTRODUCTION

AIR traffic has been increasing ever since commercial flight was introduced. Every year, new reports are released arguing that the conventional highways in the sky are congested. These congestions are, among others, leading to delays [1]. A primary result of these congestions is that air traffic controllers experience an increasing workload, which is one of

the driving constraints on air space capacity. Therefore, methods to provide (a more) decentralized air traffic control have been a topic for research since the early nineties. One of these research fields is free-flight [2]. In free-flight, pilots can fly direct routes to their destination instead of flying via predefined routes [3].

Free-flight requires the cockpit and/or on-board systems to preserve a required minimal separation with other aircraft. In most free-flight studies, this separation requirement is geometrically described as a flat disk with a radius of five nautical miles and a total height of 2000 feet. This is often called the Protected Zone (PZ). This separation requirement corresponds to the ICAO requirements when surveillance systems are used, such as Automatic Dependent Surveillance - Broadcast (ADS-B) or radar, unless instructed otherwise by the local air traffic service authority [4].

Conflict Detection (CD) is the process of establishing, through propagating current aircraft states, whether this separation requirement will be violated in the near future. The event where two aircraft physically enter each others protected zone is called a Loss of Separation (LoS), or an intrusion. It is called a conflict if two aircraft are expected to intrude each others PZ in the near future. Determining what actions have to be taken by both aircraft to prevent this expected intrusion is called Conflict Resolution (CR). Also, efforts to prevent aircraft from getting into conflict in the first place is called Conflict Prevention (CP).

The Modified Voltage Potential (MVP) is one of the available methods able to provide CD, CR, and CP. In 1994 Martin Eby proposed a voltage potential method to maintain safe separation with other aircraft [5, 6, 7, 8]. This method was later modified and extended by Hoekstra and named the MVP [2]. The MVP is a tried and tested method [9]. However, its behav-

ior when resolving conflicts between aircraft which are flying near-parallel tracks has raised concern. In the studies by Sunil, which used the MVP method as the CD&R method in airspace capacity analysis, the results were influenced by the inefficiency of the resolution provided by the MVP method when aircraft were instructed to fly increasingly parallel trajectories [10]. Additionally, in a study by Balsoriyaan [11] where the MVP method was also used, similar observations were made regarding conflicts with low heading differences. As the heading difference between conflicting aircraft became sufficiently low, aircraft were unable to effectively resolve the conflict and remained 'stuck' together.

The current study investigates the relationship between heading differences and the behavior of the MVP. It then proposes a modification to the MVP method to improve its behavior in these conflicts with low heading differences. First, through using the notion of conflict angles - the heading difference between two conflicting aircraft - the behavior of the MVP method in the velocity vector-space is investigated. In this section, the working principles of the MVP method are explained, and the corresponding problems with low conflict angles are presented. To solve these problems, a modification is proposed in the following section. This modification is then compared to the original MVP method through comparison in a state-based solution space and an experiment.

II. CONFLICT ANGLES AND THE VELOCITY VECTOR-SPACE

In this section, the working principles of the MVP method are detailed first. Additionally, the behavior of the MVP method in conflicts with different conflict angles is investigated using the velocity vector-space. Based on these findings, the problem statement is formulated and a modification is introduced.

i. The Modified Voltage Potential

As stated, the modified voltage potential is an algorithm able to provide conflict detection and the corresponding conflict resolution. The workings of the modified voltage potential are explained through chronological evaluation of the phases of a conflict. Two aircraft being in conflict will be called

the ownship and the intruder. The ownship is the aircraft for which a resolution is calculated, and the intruder is the aircraft to maintain the required separation with. In this explanation, the conflict will be regarded one-sided. This means that the conflict will be evaluated as if only the intruder takes action to maintain the safe separation. What would happen if the intruder were to also take action, will be addressed during the relevant phases.

The modified voltage potential during the three conflict phases:

Conflict Detection – The first phase of a conflict is conflict detection, an example illustration is shown in Figure 1a. Note that in these figures, the trajectories are depicted in the relative frame. In other words, the trajectories are depicted relative to the intruder location. Also, the figures used in this article are not to scale.

When a conflict is detected, it means that a loss of separation is predicted between the two aircraft within a certain look-ahead time (t_l). When using the MVP, this look-ahead time is 300 seconds. The calculated time to loss of separation (t_{LoS}) is thus lower than 300 seconds. If a loss of separation is expected, as can be seen in the example of Figure 1a, the predicted Closest Point of Approach (CPA) of the ownship with respect to the intruder lies within the protected zone of the intruder. In other words, the horizontal distance between the two aircraft at the predicted closest point of approach ($|D_{cpa}|$), is smaller than the required horizontal separation (S_h) of 5 nautical miles. In this figure, the ownship velocity is depicted as V_o , the intruder velocity as V_i , and the initial - conflicting - relative velocity as V_{rel} . Identically, the situation is rotational symmetrical from the perspective of the intruder. Due to this rotational symmetry, the intruder calculates an identical time to loss of separation and closest point of approach.

Conflict Resolution – The required velocity change, or the resolution velocity-vector, provided by the MVP (ΔV_{mvp}) is calculated by directing some velocity change ΔV along the direction of D_{cpa} until the new predicted CPA due to conflict resolution (CPA_{cr}) lies at a distance of 5 NM from the intruder, or on the outer bound of the PZ. This results in the ownship grazing the PZ of the intruder instead of crossing it, see Figure 1b. In

this figure, the new - CR provided - relative velocity is depicted as $\mathbf{V}_{rel,cr}$. Note that this example illustrates a horizontal conflict with a horizontal resolution. The MVP method is able to resolve vertical conflicts in a similar fashion.

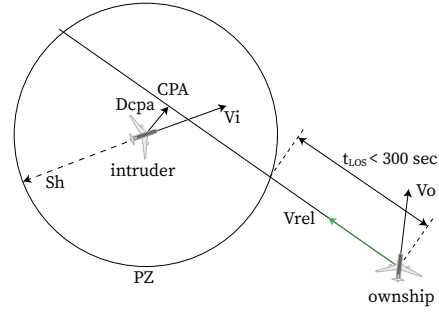
After the resolution vector is calculated, the ownship will adapt this velocity change and retain this resolution velocity until the new CPA has been reached, during this phase they are still said to be *in conflict*. The trajectory flown by the ownship during this phase is called the resolution trajectory. Due to the rotation symmetry of the conflict, the intruder would calculate an identical, but opposite, velocity change ($\Delta \mathbf{V}_{mvp}$). Therefore, it would suffice for both aircraft to implement half of this velocity change to prevent an intrusion.

Due to this rotational symmetry in combination with the direction the velocity change is directed in, conflicts can be resolved implicitly. Since, due to the rotational symmetry, both aircraft resolve the conflict in opposite direction and thus pass each other on the same side, without resolving 'into' each other. The side of the intruder that the ownship passes on is called the coordinated side. This coordination without an explicit line of communication is often referred to as implicit coordination. For more information regarding types of coordination the reader is referred to [12]

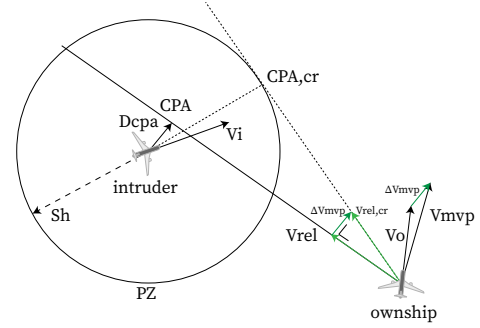
Recovery – When both aircraft have reached CPA, the two aircraft start diverging, and the conflict is considered to be resolved. The ownship and intruder reach their closest point of approach, per definition, at the same time, and therefore consider the conflict as resolved at the same time. Now that the conflict is resolved, recovery can be initiated. Recovery is the phase where aircraft revert to their initial - or desired - velocities because the conflict is resolved, see Figure 1c for an example of this instant.

Multi-aircraft conflicts

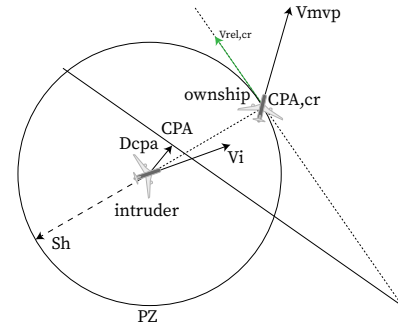
In the case of multi-aircraft conflicts, the case where an aircraft is in conflict with at least two other aircraft at the same time, the modified voltage potential calculates the required velocity changes in a sequential manner. In other words, it calculates the required velocity change for each conflict separately. These separate velocity changes are then summed, through vector-addition, to obtain the overall velocity change. This results in an iterative resolution of the multi-aircraft con-



(a) Phase 1: Conflict Detection. Through nominally extrapolating the current aircraft states of the ownship and the intruder, it is established that a loss of separation will occur within 300 seconds.



(b) Beginning of phase 2: Conflict Resolution. The resolution vector $\Delta \mathbf{V}_{mvp}$ as provided by the MVP method, is to be adapted [2].



(c) Beginning of phase 3: Recovery. The ownship has reached CPA and can start to revert to its initial velocity.

Figure 1: Thee conflict phases when using the MVP method; Conflict Detection, Conflict Resolution, and Recovery.

flict, as the conflicting aircraft trajectories are 'pushed' in a certain direction until all aircraft have an intrusion free trajectory. This is a strong resemblance to physical force-field mechanics.

ii. CPA recovery and conflict angles

When two aircraft in conflict have reached CPA it means that from that moment on they will diverge, given that they retain their resolution provided velocity. However, it does not guarantee that both aircraft will not get in conflict again when reverting to their initial velocity at that time. An example of such a scenario can be seen in Figure 2. In this figure, if both aircraft were to instantaneously revert to their initial velocities at CPA_{cr} , and thus obtain the same relative velocity vector as at the initiation of the conflict, an additional conflict would occur. A conflict which occurs again after the recovery has been initiated will be called a repetitive conflict for the remainder of the current study.

In fact, for all grazing resolution trajectories does instantaneous recovery at CPA_{cr} lead to a repetitive conflict.

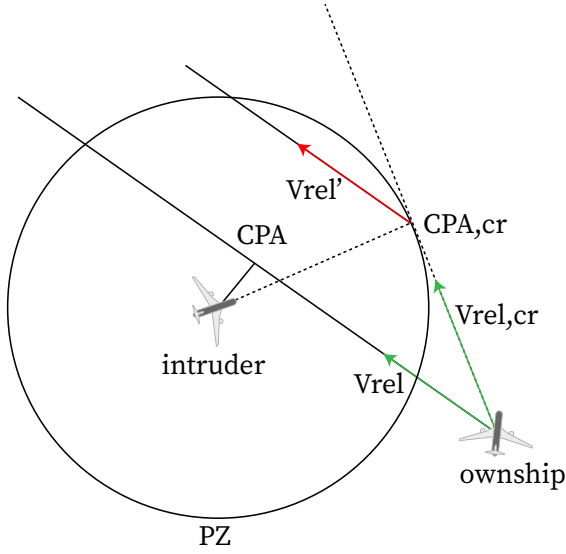


Figure 2: An example of a scenario where instantaneous recovery at CPA_{cr} leads to a repetitive conflict. As can be seen, the initial relative velocity translated to the CPA_{cr} (V'_{rel}) is conflicting.

Lemma 1: Instantaneous recovery to initial velocities at CPA_{cr} always leads to a repetitive conflict for grazing resolution trajectories.

Proof: Reorientation of the conflict as in Figure 3 illustrates that instantaneous recovery at CPA_{cr} will lead to a repetitive conflict as long as there is an angle

$\theta > 0$. In order to have $\theta = 0$, either $|D_{cpa}|$ has to be equal to S_h (in which case there is no conflict to begin with) or the distance between ownship and intruder has to be infinitely large (which is a trivial solution). Therefore, instantaneous recovery to initial velocities at CPA_{cr} always lead to a repetitive conflict for grazing resolution trajectories.

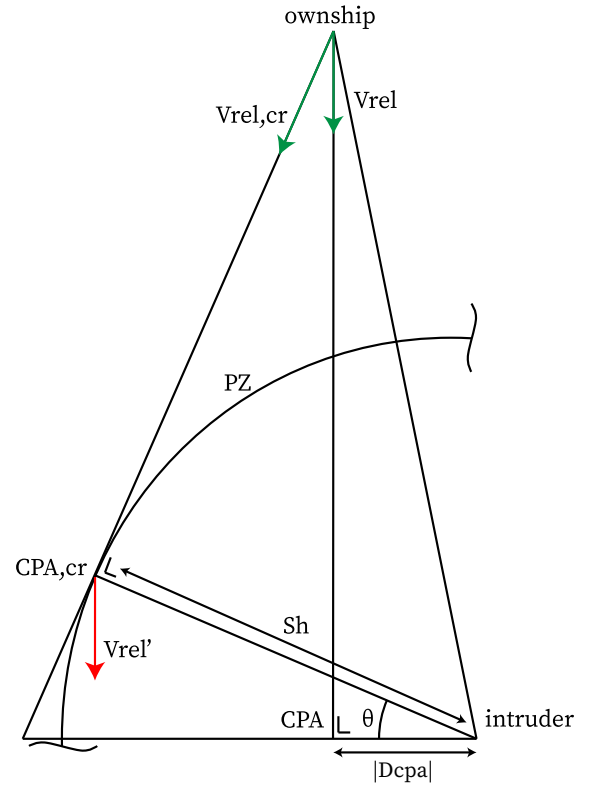


Figure 3: Reorientation of conflict geometry to illustrate that instantaneous recovery to initial velocities at CPA always leads to a repetitive conflict for grazing resolution trajectories.

However, in reality - and most simulations, recovery is not instantaneous due to aircraft dynamics. The time it takes to change the resolution velocity back to the initial velocity may cause both aircraft to have diverted sufficiently before completing recovery to prevent such a repetitive conflict from occurring. For conflicts where the relative velocities are lower, such as conflicts between aircraft flying near-parallel trajectories, the probability that both aircraft have diverted enough before they complete recovery is smaller. This leads to an

increased probability of repetitive conflicts occurring between aircraft flying near-parallel trajectories.

On the other hands, due to the vector-addition in multi-aircraft conflicts, two aircraft might be in a non-grazing resolution. In non-grazing resolution trajectories, CPA based recovery might lead to an unnecessary long resolution trajectory, because the initial relative velocity is conflict-free at some time before reaching the CPA_{cr} . An example of this phenomenon is illustrated in Figure 4. In this figure, the initial relative velocity is conflict free before CPA_{cr} is reached, indicating that both aircraft can start the recovery at some time before reaching CPA_{cr} .

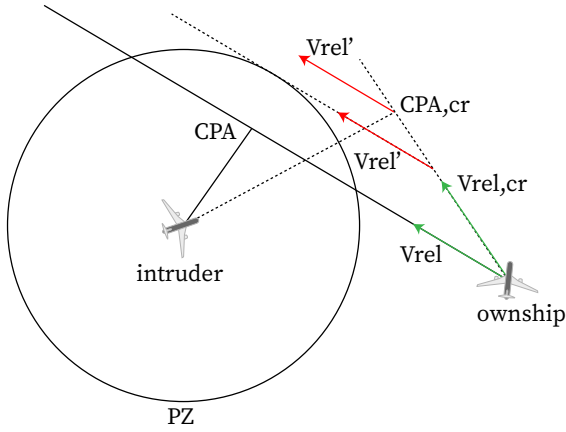


Figure 4: Example of a scenario where CPA recovery leads to a unnecessary long resolution trajectory. At some time before reaching CPA_{cr} is the initial relative velocity conflict-free, as can be seen by the nominally propagated initial relative velocity from CPA_{cr} .

In an effort to generalize this behavior, the set of predicted CPAs in a conflict is analyzed. Since the predicted CPA is the location on the relative trajectory which is closest to the location of the intruder, it can be described as the location along the relative trajectory where the intruder is perpendicular to the current relative trajectory.

Therefore, a predicted CPA forms the right angle in a right triangle between the ownship location, intruder location, and the predicted CPA. Using Thales's theorem, the set of coordinated CPAs between ownship and intruder can therefore be described as a semicircle with a radius of half the distance between ownship and intruder and its center in between the ownship and intruder, as in Figure 5.

The CPAs described by this semicircle can, in combination with the current relative velocity, be divided into two sets; a set at which instantaneous recovery leads to a repetitive conflict, and a set at which it does not. By parallel translation of the current relative trajectory until it is a tangent on the coordinated side of the PZ of the intruder, a set of CPAs is found for which the initial relative velocity is conflict-free. Therefore, the part of the semicircle enclosed by the intersections of this translated relative trajectory contains the CPA locations that do not lead to repetitive conflicts.

For the relative velocity depicted in Figure 5, the CPAs that lead to a repetitive conflict are depicted by the red part of the semicircle, whereas the CPAs that do not lead to a repetitive conflict are depicted by the green part of the semicircle. CPAs that are an intrusion in themselves are indicated by the black part of the semicircle. Indeed, according to Lemma 1, the grazing CPA belongs to the kind that causes a repetitive conflict when the recovery is instantaneous.

The location of a predicted CPA, in the relative frame, is dependent on two variables. The first being the direction of the relative velocity, and the second being the distance between the ownship and intruder. Each direction of the relative velocity therefore has a specific predicted CPA location, and the magnitude of that relative velocity only influences how quickly that predicted CPA is reached, not its relative location. Thus, the set of new relative velocities can be divided into relative velocities which result in a repetitive conflict and ones that do not result in a repetitive conflict. Whether these new relative velocities result in a repetitive conflict is depicted by the red and green shades in Figure 5, red for relative velocities which result in an intrusion or a repetitive conflict, and green for relative velocities which do not. Relative velocities which do not resolve the initial conflict are indicated in grey.

Interestingly, there is a set of relative velocities in the along-track direction of the intruder which also result repetitive conflicts, on the far right side of Figure 5. These repetitive conflicts occur when a new relative velocity leads to both aircraft reaching CPA in a sufficiently short time, that the conflict is not resolved in the sense that the initial relative velocity is conflict-free. Note that there are precisely two relative velocity directions which result in a coordinated CPA location corresponding precisely to the instant where the initial relative velocity is conflict-free. These CPAs lie at

the intersections of the semi-circle and the translated initial relative velocity. For all other CPAs, they are either too soon or too late from a conflict-free recovery standpoint.

Additionally, relative velocities which result in instantaneous separation of the two aircraft cannot be counted as an effective solution to the conflict. If a new relative velocity leads to instantaneous separation of the two aircraft, it causes the time to CPA to be, and remain, equal to zero. In other words, it causes the CPA location to be equal to the current ownship location. Therefore, the recovery is undefined as the aircraft do not know how long they have to sustain that resolution velocity. This leads to a set of solutions to the conflict which are unworkable since the recovery is undefined.

As a whole, the CPA indicates that both aircraft start to diverge. What it does not indicate, however, is whether desired velocities (the velocity an aircraft would fly if it would not be in conflict at that time) are conflict-free and/or if these velocities have not been conflict-free for some time already. Additionally, solutions to the conflict which lead to instantaneous separation of the aircraft cannot be used since the recovery is undefined in such cases. Therefore, the current study proposes a modification to the recovery mechanics of the MVP method. A modification which is based on the best-guess whether the desired velocity is conflict-free. The moment in time where this desired velocity is conflict-free will be called Free to Revert (FTR) for the remainder of the current study.

III. RECOVERY USING THE TWO CRITERIA METHOD

In the proposed recovery method, the ownship decides, during the conflict, whether it can revert to its desired velocity given two potential circumstances; (1) the intruder maintains its current velocity at that time and (2) the intruder reverts to its initial velocity at that time, which is the velocity it had at the initiation of the conflict. Since the ownship has no knowledge of the actual desired velocity of the intruder, the intruder's initial velocity is the best-guess of this desired velocity. The ownship can decide whether to start the recovery given the two potential circumstances on-line, at each time-step and without explicit communication with the intruder. This recovery method will be referred to as the two criteria recovery method for the remainder of

the current study.

Note that this method does not need more state information to be broadcast than the CPA method; it only needs the current position and the current velocity of the intruder to be broadcast. Indeed, the initial velocity of the intruder can be logged by the ownship at the initiation of the conflict. See Table 1 for an overview of the required states and how they are obtained.

Table 1: Ownship and intruder states which have to be known to the ownship to determine if both criteria are met.

A/C	state	symbol	obtained using
Int.	current position w.r.t. ownship	$\mathbf{x}_i$	Continuous Broadcast
Int.	current velocity	$\mathbf{V}_{i,c}$	Continuous Broadcast
Int.	initial velocity	$\mathbf{V}_{i,i}$	Logging at conflict initiation
Own.	current desired velocity	$\mathbf{V}_{o,d}$	Known

i. Criterion no. 1

The desired velocity is conflict-free given that the intruder maintains its current velocity.

If $\mathbf{V}_{i,c}$ is defined as the current velocity of the intruder and $\mathbf{V}_{o,d}$ as the current desired velocity of the ownship, the relative velocity for criterion no. 1 ($\mathbf{V}_{rel,1}$) can be calculated:

$$\mathbf{V}_{rel,1} = \mathbf{V}_{o,d} - \mathbf{V}_{i,c} \quad (1)$$

Using this relative velocity and the relative position of the intruder with respect to the ownship ($\mathbf{x}_i$), the predicted time to Closest Point of Approach for criterion no. 1 ($t_{cpa,1}$) can be calculated if the ownship were to revert to its desired velocity at that time instant:

$$t_{cpa,1} = -\frac{\mathbf{x}_i \cdot \mathbf{V}_{rel,1}}{|\mathbf{V}_{rel,1}|^2} \quad (2)$$

Using this $t_{cpa,1}$, the predicted distance vector between the intruder and ownship at CPA ($\mathbf{D}_{cpa,1}$) can be calculated:

$$\mathbf{D}_{cpa,1} = -\mathbf{x}_i + \mathbf{V}_{rel,1}t_{cpa,1} \quad (3)$$

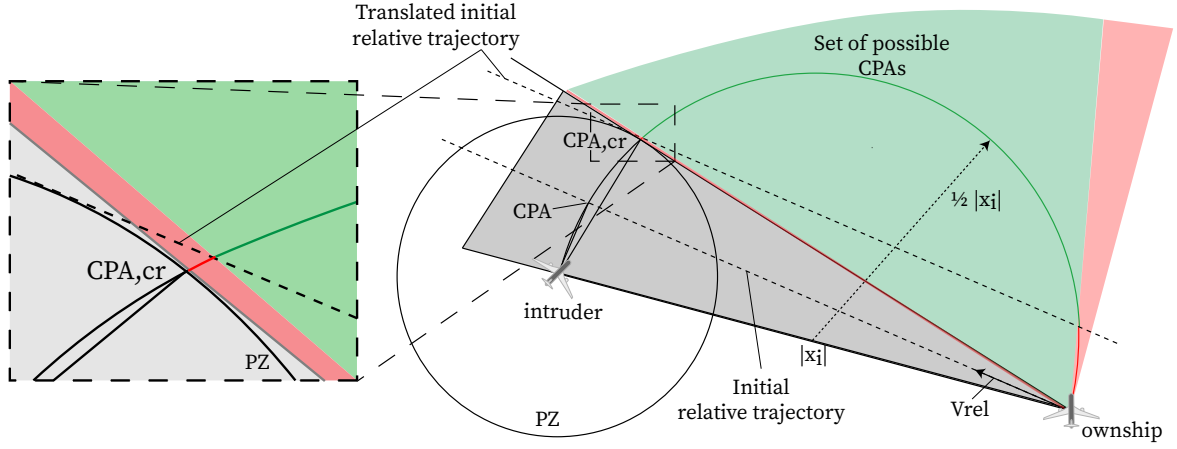


Figure 5: The green part of the semicircle indicates CPAs at which instantaneous recovery is conflict-free. The red part of the semicircle indicates CPAs at which the instantaneous recovery is not conflict-free. The corresponding relative velocity sets are indicated by green and red shades. Relative velocities which do not resolve the initial conflict are indicated in grey.

Criterion no. 1 is met when the magnitude of this predicted distance at CPA vector ($|\mathbf{D}_{cpa,1}|$) is larger than the horizontal separation requirement of 5 NM:

$$\text{Criterion no. 1} = |\mathbf{D}_{cpa,1}| > 5 \text{ NM} \quad (4)$$

In other words, if this criterion is met, the ownship could revert to its desired velocity at that time and not get in conflict with the intruder - given that the intruder maintains its current velocity.

ii. Criterion no. 2

The desired velocity is conflict-free given that the intruder reverts to its initial velocity.

If only criterion no. 1 would be used as recovery criterion, repetitive conflicts would still occur. The nature of these repetitive conflicts are as follows. Nominally, when two aircraft get into conflict, their desired velocity remains identical to their initial velocity. Since the MVP method provides rotational symmetrical velocity changes to the two conflicting aircraft, there is a moment during the resolution where both aircraft simultaneously conclude that their initial velocity is conflict-free given that the other aircraft retains its current velocity. However, if both aircraft were to revert to their initial velocity at that time, a follow-up conflict would occur. Comparable problems have been found by the conflict resolution studies of [13].

To prevent these repetitive conflicts from happening,

an additional check is required based on the expected desired velocity of the intruder. Since the actual desired velocity of the intruder ($\mathbf{V}_{i,d}$) is unknown, the initial velocity of the intruder is the best-guess of this desired velocity:

$$\mathbf{V}_{i,d} = \mathbf{V}_{i,i} \quad (5)$$

Therefore, criterion no. 2 checks whether the desired velocity is conflict-free given that the intruder reverts to its initial velocity at that time. Criterion no. 2 is calculated in a similar fashion as criterion no. 1, the only difference being that the intruder's initial velocity ($\mathbf{V}_{i,i}$) is used instead of its current velocity.

$$\mathbf{V}_{rel,2} = \mathbf{V}_{o,d} - \mathbf{V}_{i,i} \quad (6)$$

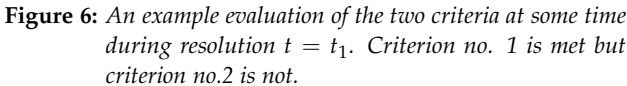
$$t_{cpa,2} = -\frac{\mathbf{x}_i \cdot \mathbf{V}_{rel,2}}{|\mathbf{V}_{rel,2}|^2} \quad (7)$$

$$\mathbf{D}_{cpa,2} = -\mathbf{x}_i + \mathbf{V}_{rel,2} t_{cpa,2} \quad (8)$$

Criterion no. 2 is met when the magnitude of this predicted distance at CPA vector ($|\mathbf{D}_{cpa,2}|$) is larger than the horizontal separation requirement of 5 NM:

$$\text{Criterion no. 2} = |\mathbf{D}_{cpa,2}| > 5 \text{ NM} \quad (9)$$

In other words, if this criterion is met, the ownship could revert to its desired velocity, and the intruder



An example of these two criteria at some time during a conflict resolution ($t = t_1$) can be seen in Figure 6. In this figure, the desired velocity is conflict free only if the intruder sustains its current resolution velocity, and not if it were to revert to its initial velocity. Therefore, criterion no. 1 is met but criterion no. 2 is not and the ownship can thus not initiate the recovery.

When both criteria *are* met, the ownship regards the conflict as resolved and can start the recovery - given that it is not simultaneously in conflict with another aircraft at the time. If this would be the case, the ownship still regards the conflict as resolved. However, it does not initiate recovery until the two criteria are met for all the remaining conflicts it is involved in.

IV. THEORETICAL ANALYSIS

The influence of the proposed method is investigated on two metrics; the theoretical conflict duration and the theoretical path deviation. These comparisons will be performed using a framework. This framework consists of characterizing various conflict solutions according to a metric. First, the framework design is detailed. Then it is used to perform comparisons regarding theoretical conflict duration and theoretical path deviations.

This framework provides insight into how certain solutions to a conflict relate to other solutions to that conflict. Due to the vector addition in multi-aircraft conflicts, the resolution velocity might shift from the initial MVP provided solution to neighboring solutions. Therefore, it is useful to investigate how these neighboring solutions relate. Indeed, a method which deteriorates quickly - from a conflict duration or path deviation standpoint - as the velocity vector is pushed to neighboring solutions, might be less-suited to use in a method which utilized sequential conflict resolution with vector-addition.

As described, the framework consists of characterizing a solution space diagram according to certain metrics. The solution space diagram (SSD) is a diagram in the velocity-vector space illustrating the set of solutions to a conflict. The SSD is not something new, in various types of air traffic management related research has it proven resourceful. Although first used as a workload metric [14, 15, 16] it can, for instance, also be used as a supporting tool in the cockpit [17, 18]. The construction of the solution space diagram, as used in the current study, is detailed first.

The solution space diagram as used in the current study is based on three demarcations to the velocity-vector space; minimum and maximum velocities, the velocity obstacle, and uncoordinated velocities. Why these three demarcations are used will be addressed in the corresponding paragraphs.

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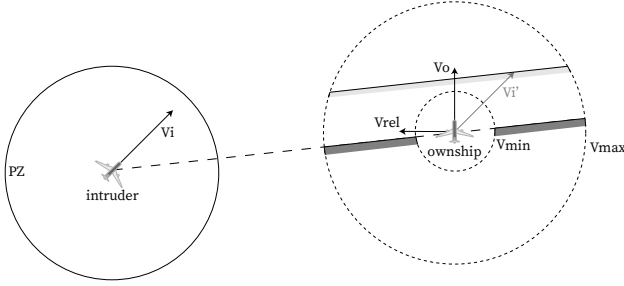


Figure 8: Coordinated and uncoordinated velocities for the ownship. The set of uncoordinated relative velocities is indicated by the dark grey side of the lower border. The set of uncoordinated absolute velocities is indicated by the light grey side of the upper border.

sen; the theoretical conflict duration and the theoretical path deviations.

Note that solutions which result in a CPA at which the initial relative velocity is not yet conflict-free are still included in this solution space. This is for two reasons. The first being that the MVP provides, per definition, solutions within this set. The second being that due to vector addition solutions can be pushed in this set without instigating a recalculation.

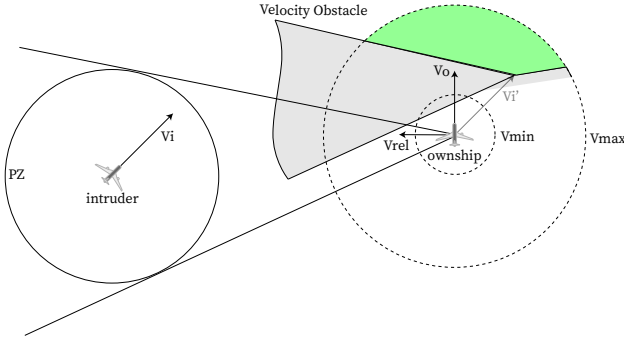


Figure 9: The union of the velocity obstacle (VO) and the coordination related restriction imposed on the reachable velocities. Set of coordinated solutions depicted in green.

ii. Comparison 1 - Conflict duration

For non-grazing resolution trajectories, CPA recovery can lead to an unnecessarily long conflict duration, an example of such a track can be seen in Figure 4. The two criteria recovery, on the other hand, is based on evaluating - at every time step - whether the aircraft

can revert towards its desired velocity. To visualize these differences in conflict duration, the solution space diagram of an example conflict was characterized according to these durations. The example conflict is a conflict where both aircraft have a ground speed of 300 *kts*, the predicted distance at CPA is 2.5 NM, and the time to loss of separation (t_{LOS}) is 300 seconds.

First, conflict duration when using CPA recovery is investigated. This conflict duration is defined as the predicted time to CPA. The equation for predicted time to CPA is restated below for convenience:

$$t_{cpa} = -\frac{\mathbf{x}_i \cdot \mathbf{V}_{rel,cr}}{|\mathbf{V}_{rel,cr}|^2} \quad (11)$$

Figure 10 shows solutions to the conflict in the solution space diagram, characterized according to the resulting predicted time to CPA. In this figure, the MVP provided solution is indicated as well. It was assumed that velocity changes were instantaneous and that the intruder takes no action. Due to the rotation symmetry of the conflict, conflict durations would be the same of both ownship and intruder were to implement half of the required velocity change. This effectively results in both aircraft contributing equally to the required relative velocity ($\mathbf{V}_{rel,cr}$), whilst their relative distance remains the same. Thus, according to the Equation (11), the time to CPA would remain the same if both aircraft were to contribute to the resolution, as long as the same relative velocity is obtained.

Figure 10 shows that for solutions near the apex of the velocity obstacle, the predicted time to CPA goes asymptotically to infinity, because the apex of the velocity obstacle represents a solution where the ownship flies parallel to the intruder with the same ground speed as the intruder. Thus retaining a constant distance between the ownship and the intruder and causing an infinitely long conflict duration.

In addition it can be found that when solutions are chosen on the same radial but further away from the apex of the velocity obstacle, conflict durations decrease linearly. Solutions on the same radial with respect to the apex of the velocity obstacle represent relative velocities with the same direction. Since these solutions have the same relative velocity direction, they therefore have identical predicted CPA locations. As a solution is chosen with the same relative velocity direction, but an increased magnitude, the conflict duration therefore decreases linearly.

Finally, the semicircular behavior of the predicted CPA locations - described in Section ii - can be seen in the solution space characterization. As solutions are chosen radially outward from the legs of the velocity obstacle, distances to CPA decrease in a semicircular fashion, resulting in a lower relative velocity required to maintain the same conflict duration.

The time to FTR, in a conflict where both aircraft want to revert to their initial velocity, is limited by criterion no. 2. If a point along the resolution trajectory with respect to the ownship location as a function of some time t can be described as:

$$\mathbf{V}_{rel,cr} \cdot t$$

And a point along the translated initial relative trajectory as a function of some time u , with respect to the ownship location, as:

$$\left(\frac{S_h}{|\mathbf{D}_{cpa}|} \mathbf{D}_{cpa} - \mathbf{x}_i \right) + \mathbf{V}_{rel} \cdot u$$

Then these two trajectories intersect if a t and u can be found such that:

$$\mathbf{V}_{rel,cr} \cdot t = \left(\frac{S_h}{|\mathbf{D}_{cpa}|} \mathbf{D}_{cpa} - \mathbf{x}_i \right) + \mathbf{V}_{rel} \cdot u \quad (12)$$

Crossing both sides with $\mathbf{V}_{rel}$:

$$(\mathbf{V}_{rel,cr} \cdot t) \times \mathbf{V}_{rel} = \left(\left(\frac{S_h}{|\mathbf{D}_{cpa}|} \mathbf{D}_{cpa} - \mathbf{x}_i \right) + \mathbf{V}_{rel} \cdot u \right) \times \mathbf{V}_{rel} \quad (13)$$

Since $\mathbf{V}_{rel} \times \mathbf{V}_{rel} = 0$:

$$t \cdot (\mathbf{V}_{rel,cr} \times \mathbf{V}_{rel}) = \left(\frac{S_h}{|\mathbf{D}_{cpa}|} \mathbf{D}_{cpa} - \mathbf{x}_i \right) \times \mathbf{V}_{rel} \quad (14)$$

Therefore, the time until criterion no. 2 is met can be calculated as follows:

$$t_{c2} = \frac{\left(\frac{S_h}{|\mathbf{D}_{cpa}|} \mathbf{D}_{cpa} - \mathbf{x}_i \right) \times \mathbf{V}_{rel}}{\mathbf{V}_{rel,cr} \times \mathbf{V}_{rel}} \quad (15)$$

Figure 11 shows solutions to the conflict in the solution space diagram characterized according to the resulting time until criterion no. 2 is met. In this figure, the MVP provided solution is also indicated. As in the

CPA recovery case, conflict durations would remain the same when both aircraft were to contribute to the resolution, as long as they contribute to the same required relative velocity ($\mathbf{V}_{rel,cr}$) whilst their relative distance remains the same.

A difference between CPA and the two criteria recovery is that the majority of the non-grazing solutions are resolved quicker when using the two criteria recovery. As FTR trajectories are shorter than CPA trajectories for relative velocities in the green set of Figure 5, conflict durations with similar relative velocities are shorter as well. It can be seen in Figure 11 that conflict durations still go asymptotically to infinity near the apex of the velocity obstacle. However, the behavior of the conflict durations no longer shows the semicircular behavior of the CPA case, but shows the linear behavior along the direction of the initial relative velocity.

Predominantly, conflict durations are expected to decrease when resolution trajectories are shifted to the non grazing kind when the two criteria recovery is used. This can happen for instance due to multi-aircraft conflict induced vector-addition.

For solutions corresponding to CPAs at which the initial relative trajectory is not yet conflict-free, conflict durations are slightly longer when using the two criteria recovery. This is expected, as the conflict duration when using CPA recovery is, for those solutions, too short.

iii. Comparison 2 - Path deviations

An important metric for a conflict resolution method is the magnitude of the path deviation it imposes on the trajectory of the aircraft. A method that leads to a short path deviation is regarded as more efficient than a method that leads to a long path deviation [21]. Also, large path deviations have a destabilizing effect on the airspace. Since aircraft fly longer distances through the airspace, the probability that they encounter additional conflicts during these trajectories increases [22]. If the velocity change is assumed to be instantaneous and constant, this path deviation (DEV) can be expressed by the product of the imposed velocity change ($|\Delta \mathbf{V}|$) and the amount of time that velocity change has to be sustained by the aircraft (t_{conf}) [23]:

$$DEV = |\Delta \mathbf{V}| \cdot t_{conf} \quad (16)$$

Geometrically, this theoretical path deviation is the

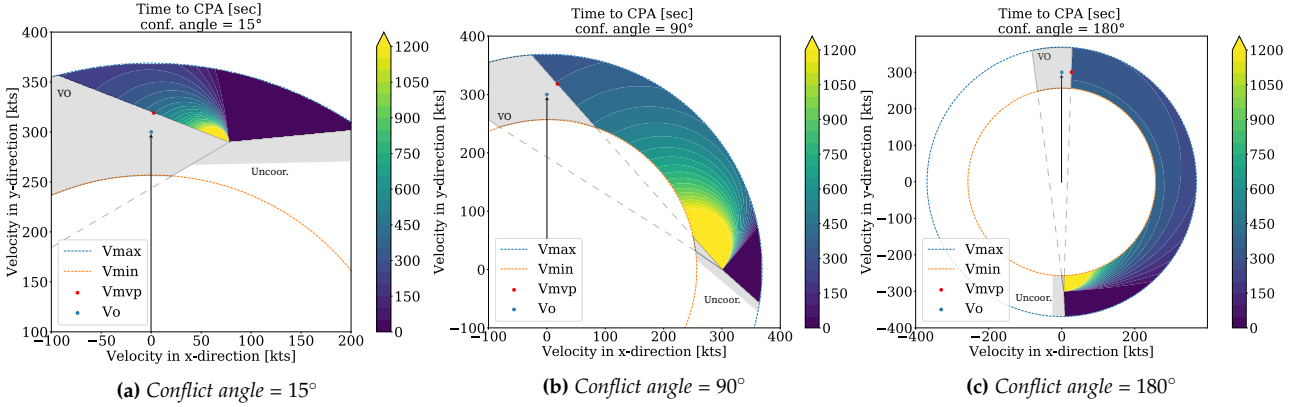


Figure 10: Predicted times to CPA within the solution space diagram of a conflict at time of initiation. Both aircraft have a ground speed of 300 kts and use a look-ahead time of 300 seconds. The predicted distance at CPA is 2.5 NM. The initial velocity of the ownship is indicated in blue and the MVP provided solution in red. The velocity Obstacle (VO) is depicted in grey as well as the border of the uncoordinated solutions.

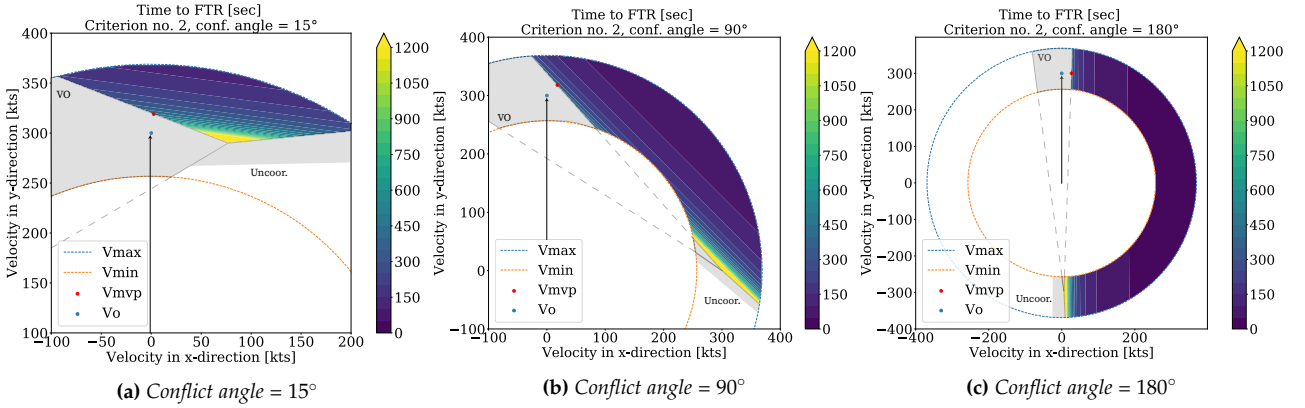


Figure 11: Predicted times to FTR within the solution space diagram of a conflict at time of initiation. Since both aircraft want to revert to their initial velocity at recovery, criterion no. 2 is limiting. Both aircraft have a ground speed of 300 kts and use a look-ahead time of 300 seconds. The predicted distance at CPA is 2.5 NM. The initial velocity of the ownship is indicated in blue and the MVP provided solution in red. The velocity Obstacle (VO) is depicted in grey as well as the border of the uncoordinated solutions.

difference between where an aircraft is once the conflict has been resolved, compared to where it would have been if it would have flown its nominal track for the same duration, this distance is illustrated in Figure 12 for the MVP where $t_{conf} = t_{cpa}$.

The conflict resolution provided by the MVP method as presented by [2] results in minimal theoretical path deviations, both when using CPA and the two criteria recovery. In other words, both when using the two criteria recovery and CPA recovery is the resolution velocity imposed by the MVP optimal with regard to

theoretical path deviations. This proof is established as follows:

Path deviations with CPA recovery

Consider a one-to-one conflict as in Figure 13. The initial relative velocity $\mathbf{V}_{rel}$ is depicted as well as an arbitrarily chosen CR-provided new relative velocity $\mathbf{V}_{rel,cr}$, the difference between the two is the CR-provided $\Delta \mathbf{V}$. In this illustration, a grazing resolution track is provided by the CR. The initial distance vector to predicted CPA ($\mathbf{D}_{cpa}$) as well as the new distance

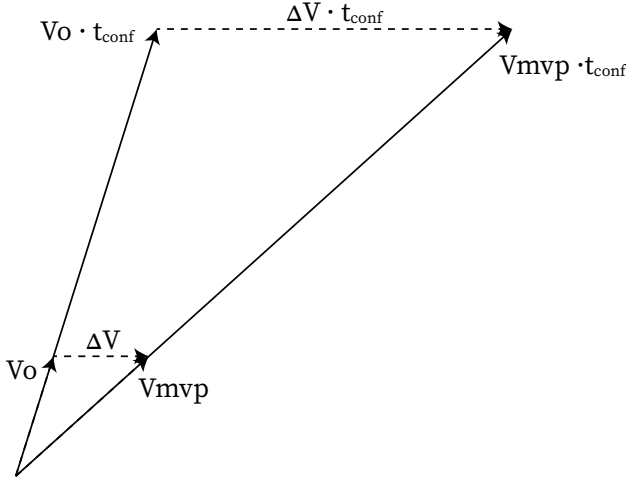


Figure 12: Nominal - no resolution - ownship velocity ($\mathbf{V}_o$) and the MVP provided velocity ($\mathbf{V}_{mv}$). The difference between the two is the MVP provided velocity change ($\Delta \mathbf{V}$). The duration of the conflict is t_{conf} . The resulting theoretical path deviation is the product of $\Delta \mathbf{V}$ and t_{conf} . Adapted from [21].

vector to CPA_{cr} ($\mathbf{D}_{cpa,cr}$) are depicted as well.

For this proof, a change of reference frame is used. Through the use of two angles in combination with the initial relative velocity, all solutions can be expressed. In Figure 13, θ is the inner angle between the initial relative velocity vector ($\mathbf{V}_{rel}$) and the CR-provided relative velocity vector ($\mathbf{V}_{rel,cr}$). The angle ϕ is the angle between the initial relative velocity vector ($\mathbf{V}_{rel}$) and the CR-provided $\Delta \mathbf{V}$. For completeness, the angle ψ is the resulting inner angle between the CR-provided relative velocity vector ($\mathbf{V}_{rel,cr}$) and the CR-provided $\Delta \mathbf{V}$. Angle ψ can be calculated using θ and ϕ in combination with the triangle property. Therefore, using θ and ϕ in combination with the known $\mathbf{V}_{rel}$, all solutions to a conflict can be expressed.

In order to prove that the MVP method provides solutions that result in minimal theoretical path deviations, it has to be established that directing $\Delta \mathbf{V}$ along $\mathbf{D}_{cpa}$ results in minimal theoretical path deviations. Similarly, using the angles of Figure 13, it has to be established that when ϕ is a right angle, theoretical path deviations are minimal.

First an expression for the first term of Equation (16) is derived. By using the law of sines:

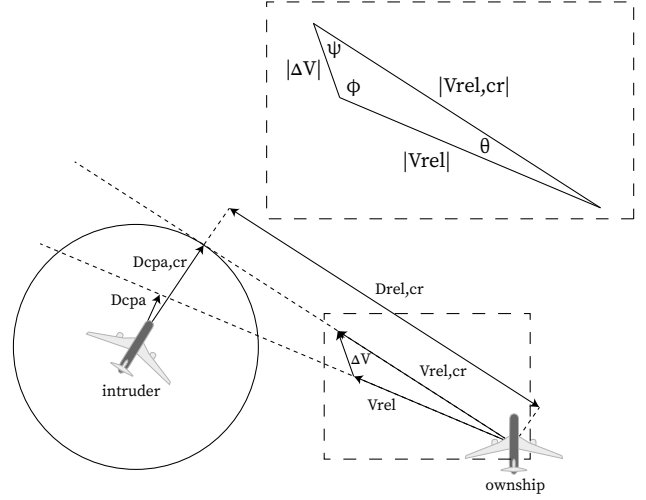


Figure 13: Example conflict illustrating the angles as required for the proof that the original MVP method leads to minimal path deviations.

$$\frac{|\Delta \mathbf{V}|}{\sin \theta} = \frac{|\mathbf{V}_{rel}|}{\sin \psi} \quad (17)$$

Rewriting gives:

$$|\Delta \mathbf{V}| = \frac{|\mathbf{V}_{rel}| \sin \theta}{\sin \psi} \quad (18)$$

Now an expression for the second term is derived. The predicted time to CPA ($t_{cpa,cr}$) can be expressed as the relative distance to CPA_{cr} ($|\mathbf{D}_{rel,cr}|$) divided by the relative velocity along that track ($|\mathbf{V}_{rel,cr}|$):

$$t_{cpa,cr} = \frac{|\mathbf{D}_{rel,cr}|}{|\mathbf{V}_{rel,cr}|} \quad (19)$$

Using the law of sines to relate $\mathbf{V}_{rel,cr}$ to $\mathbf{V}_{rel}$:

$$\frac{|\mathbf{V}_{rel,cr}|}{\sin \phi} = \frac{|\mathbf{V}_{rel}|}{\sin \psi} \quad (20)$$

Rewriting:

$$|\mathbf{V}_{rel,cr}| = \frac{|\mathbf{V}_{rel}| \sin \phi}{\sin \psi} \quad (21)$$

Substituting eq. (21) in eq. (19):

$$t_{cpa,cr} = \frac{|\mathbf{D}_{rel,cr}| \sin \psi}{|\mathbf{V}_{rel}| \sin \phi} \quad (22)$$

Now, substituting eq. (22) and eq. (18) in eq. (16):

$$DEV = \frac{|\mathbf{V}_{rel}| \sin \theta}{\sin \psi} \cdot \frac{|\mathbf{D}_{rel,cr}| \sin \psi}{|\mathbf{V}_{rel}| \sin \phi} \quad (23)$$

Rewriting:

$$DEV = \frac{|\mathbf{D}_{rel,cr}| \sin \theta}{\sin \phi} \quad (24)$$

An expression for the relative distance to the closest point of approach ($|\mathbf{D}_{rel,cr}|$) can be found by knowing that the closest point of approach is the right angle in a right triangle between ownship location, intruder location, and closest point of approach:

$$|\mathbf{D}_{rel,cr}| = |\mathbf{x}_i| \cos \left(\theta + \arcsin \left(\frac{|\mathbf{D}_{cpa}|}{|\mathbf{x}_i|} \right) \right) \quad (25)$$

Substituting Equation (25) in Equation (24) then gives:

$$DEV = \frac{\left(|\mathbf{x}_i| \cos \left(\theta + \arcsin \left(\frac{|\mathbf{D}_{cpa}|}{|\mathbf{x}_i|} \right) \right) \right) \sin \theta}{\sin \phi} \quad (26)$$

Which is minimal, for some θ , if $\phi = \frac{1}{2}\pi$. Setting $\phi = \frac{1}{2}\pi$ gives an expression for the minimal available path deviation as a function of a chosen θ :

$$DEV_{min}(\theta) = \left(|\mathbf{x}_i| \cos \left(\theta + \arcsin \left(\frac{|\mathbf{D}_{cpa}|}{|\mathbf{x}_i|} \right) \right) \right) \sin \theta \quad (27)$$

Taking the derivative with respect to θ :

$$\frac{\delta}{\delta \theta} DEV_{min}(\theta) = |\mathbf{x}_i| \cos \left(\arcsin \left(\frac{|\mathbf{D}_{cpa}|}{|\mathbf{x}_i|} \right) + 2\theta \right) \quad (28)$$

Which, when $|\mathbf{D}_{cpa}| > 0$, and $|\mathbf{x}_i| \geq |\mathbf{D}_{cpa}|$ is positive for:

$$-\frac{1}{4}\pi < \theta + \frac{1}{2} \arcsin \left(\frac{|\mathbf{D}_{cpa}|}{|\mathbf{x}_i|} \right) < \frac{1}{4}\pi$$

An example construction of this set of θ 's is depicted in Figure 14.

Thus, in conclusion, the most nearby solution with a minimal path deviation has the characteristic that $\Delta \mathbf{V}$ is directed along $\phi = \frac{1}{2}\pi$ and that θ is minimal.

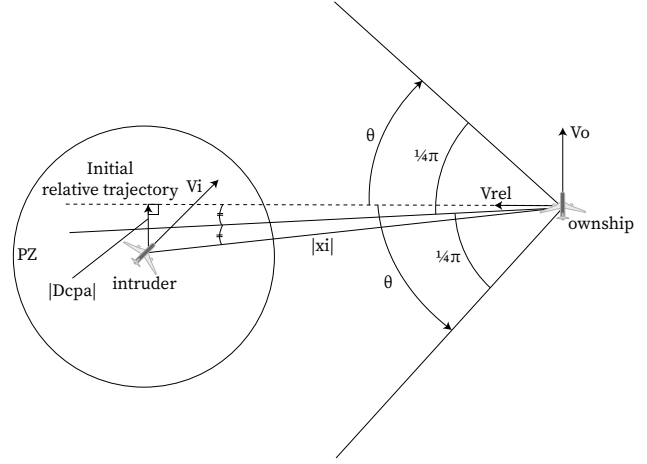


Figure 14: Example illustration of the set of θ 's on which the derivative of DEV_{min} with respect to θ is increasing.

The minimal θ is the θ which provides a grazing resolution trajectory. An expression for this θ_{min} has been found by Hoekstra [2]:

$$\theta_{min} = \arcsin \left(\frac{S_h}{|\mathbf{x}_i|} \right) - \arcsin \left(\frac{|\mathbf{D}_{cpa}|}{|\mathbf{x}_i|} \right) \quad (29)$$

The behavior of path deviations in the solution space diagram is visualized for three types of conflict in Figure 15. For all three illustrated conflict angles does the MVP method provide solutions with minimal path deviations when CPA recovery is used. It was assumed that velocity changes were instantaneous and that the intruder takes no action. Due to the rotation symmetry of the conflict, path deviations would be the halved if both ownship and intruder were to implement half of the velocity change, because the duration this velocity change has to be sustained remains the same. Relating this to the angles-based reference frame, it would result in both aircraft implementing half of the required θ , whilst they direct $\Delta \mathbf{V}$ along the same ϕ .

Since solutions near the apex of the velocity obstacle asymptotically lead to infinity long conflict durations, theoretical path deviations become infinity long as well. Theoretical path deviations remain minimal along $\phi = 90^\circ$ as θ is increased. This is expected, as the optimality of the MVP method with respect to theoretical path deviations is not dependent on the specific value of the horizontal separation requirement.

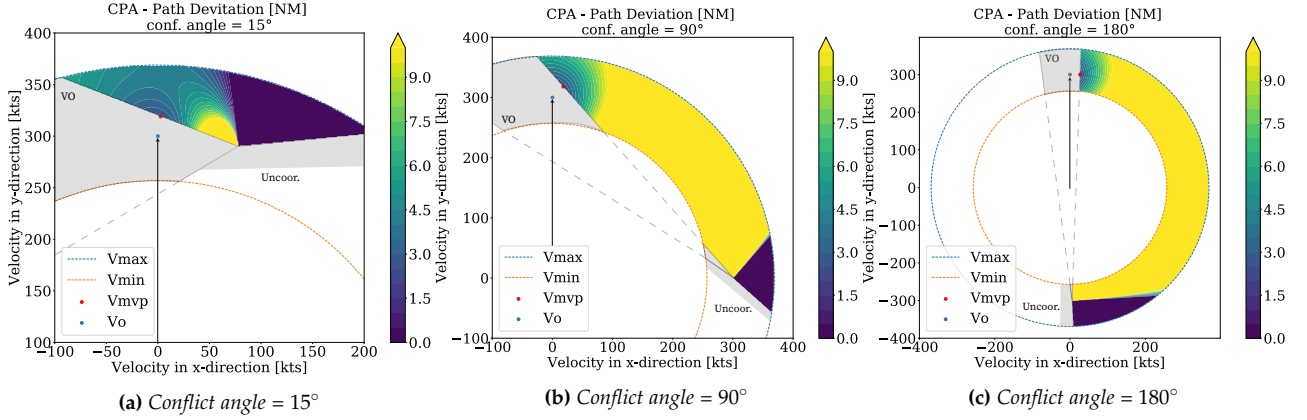


Figure 15: Theoretical path deviation when using CPA recovery within the solution space of a one-to-one conflict at time of initiation. Both aircraft have a ground speed of 300 kts and use a look-ahead time of 300 seconds. The predicted distance at CPA is 2.5 NM. The initial velocity of the ownship is indicated in blue and the MVP provided solution in red. The velocity Obstacle (VO) is depicted in grey as well as the border of the uncoordinated solutions.

Path deviations with the two criteria recovery method

Take a one-to-one conflict as in Figure 16. The $\mathbf{V}_{rel,cr}$ is again arbitrarily chosen, and the difference between the two is the CR provided $\Delta \mathbf{V}$. The expected horizontal intrusion is depicted by iH . For this proof, it is assumed that both aircraft want to revert to their initial velocities at recovery. Therefore, criterion no. 2 is the limiting criterion.

In order to prove that the MVP method provides solutions resulting in minimal theoretical deviations when using the two criteria recovery, it has to be established that directing $\Delta \mathbf{V}$ along $\mathbf{D}_{cpa}$ results in minimal theoretical deviations when the velocity change is sustained until criterion no. 2 is met. Similarly, using the angles of Figure 16, it has to be established that when ϕ is a right angle, theoretical path deviations are minimal.

The expression for the first term of Equation (16) is similar to the one in the CPA related proof, and can therefore be reused:

$$|\Delta \mathbf{V}| = \frac{|\mathbf{V}_{rel}| \sin \theta}{\sin \psi}$$

Now an expression for the second term is derived. The conflict duration (t_{conf}) can be expressed as the relative distance to the point where criterion no. 2 is met ($|\mathbf{D}_{c2}|$) divided by the relative velocity along that track ($|\mathbf{V}_{rel,cr}|$):

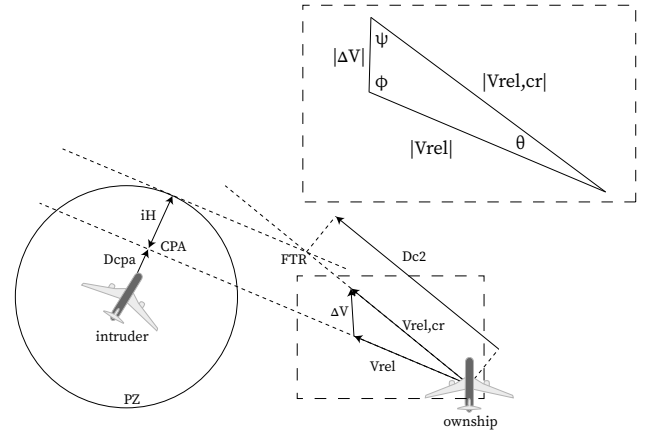


Figure 16: Example conflict illustrating the angles as required for the proof that the MVP method with the two criteria recovery method leads to minimal path deviations.

$$t_{c2} = \frac{|\mathbf{D}_{c2}|}{|\mathbf{V}_{rel,cr}|} \quad (30)$$

Substituting Equation (21) in Equation (30):

$$t_{c2} = \frac{|\mathbf{D}_{c2}| \sin \psi}{|\mathbf{V}_{rel}| \sin \phi} \quad (31)$$

Now, Substituting Equation (18) and Equation (31) in Equation (16):

$$DEV_{c2} = \frac{|\mathbf{D}_{c2}| \sin \theta}{\sin \phi} \quad (32)$$

Using Figure 16 it can be seen that:

$$iH = |\mathbf{D}_{c2}| \sin \theta, \quad (33)$$

Substituting and rewriting:

$$DEV_{c2} = \frac{iH}{\sin \phi} \quad (34)$$

Which is minimal for:

$$\phi = \frac{1}{2}\pi + n\pi, \quad n \in \mathbb{Z} \quad \text{as required.} \quad (35)$$

Interestingly, as can be seen in Equation (34), DEV_{c2} is independent of θ . Therefore, theoretical path deviations are constant if $\Delta \mathbf{V}$ is increased along some angle ϕ . Meaning that non-grazing solutions have identical path deviations as grazing solutions, as long as $\Delta \mathbf{V}$ is directed along the same angle ϕ . Also, if $\phi = \frac{1}{2}\pi$, path deviations are equal to the expected horizontal intrusion (iH), independent of θ .

This behavior is illustrated in Figure 17. For all three illustrated conflict angles does the MVP method provide solutions with minimal path deviations when the two criteria recovery method is used. Similar as in the path deviations when using CPA recovery; if both aircraft were to equally contribute to the resolution, path deviations would be halved since they would implement half of the required $\Delta \mathbf{V}$. Relating this to the angle-based reference frame, they would account for half of the required θ value. Interestingly, when they would both use a $\phi = \frac{1}{2}\pi$, they would both have a path deviation of half the horizontal intrusion - as long as they implement at least half of the required θ , as can be seen from Equations (33) and (34).

V. EXPERIMENT DESIGN

To investigate the emergent performance of the proposed recovery method, an experiment is performed. The design of this experiment is detailed in this section.

i. Simulation environment

This experiment is performed in the python-based BlueSky open-source air traffic simulator [24]. In BlueSky, aircraft are modeled using point-mass dynamics. Aircraft properties, kinematic performance,

Table 2: Performance characteristics of the B737-400 model at FL100 in BlueSky.

Characteristic	value
Speed	257 - 369 kts
Acceleration and Deceleration	1.0 kts s ⁻²
Turn rate	1.067 - 1.531° s ⁻¹

dynamic performances, and utility libraries are gathered from the OpenAP library [25]. More information on BlueSky and its validation can be found in [9].

The simulations are performed using the B737-400 model. All aircraft in the experiment fly at flight level 100. In BlueSky, the B737-400 has a minimum and maximum ground speed of 257 and 369 kts respectively at FL100. Therefore, the instructed ground speeds are distributed uniformly between 286 and 342 kts respectively, resulting in a mean groundspeed ($\bar{V}$) of 314 kts. This range results in all aircraft having at least 33% of their ground speed range available for acceleration and deceleration if required by CR. The performance characteristics of the B737-400 model at FL100 in BlueSky can be seen in Table 2. Note that, due to conflict resolution, an aircraft might not have been flying their nominal velocity when encountering another conflict, resulting in a different available ground speed range.

The simulation area is defined as a two-dimensional circular sector. Aircraft are only generated within this simulation area. Therefore, aircraft that fly at the border of this simulation area experience fewer conflicts and perceive a lower traffic density. To prevent this effect from skewing the results, a measurement area was defined within the simulation area. Only data from aircraft flying in the measurement area is logged. A rule of thumb is that the margin between the measurement area and the simulation area should at least be the average look-ahead distance ($\bar{V} \cdot t_l$) [22]. Since the B737-400 is used with an average instructed speed of 314 kts, and the look-ahead time t_l is 300 seconds, a minimal margin of 26.2 NM is required. To be on the safe side, a margin of 50 NM is used.

The simulation area has a radius of 400 NM, resulting in a simulation area of 502655 NM². The measurement area is a two-dimensional circle within this simulation area with a radius of 350 NM, resulting in an measurement area of 384845 NM². Aircraft are spawned on the outer edge of the simulation area and are given a waypoint on the outer edge of the simula-

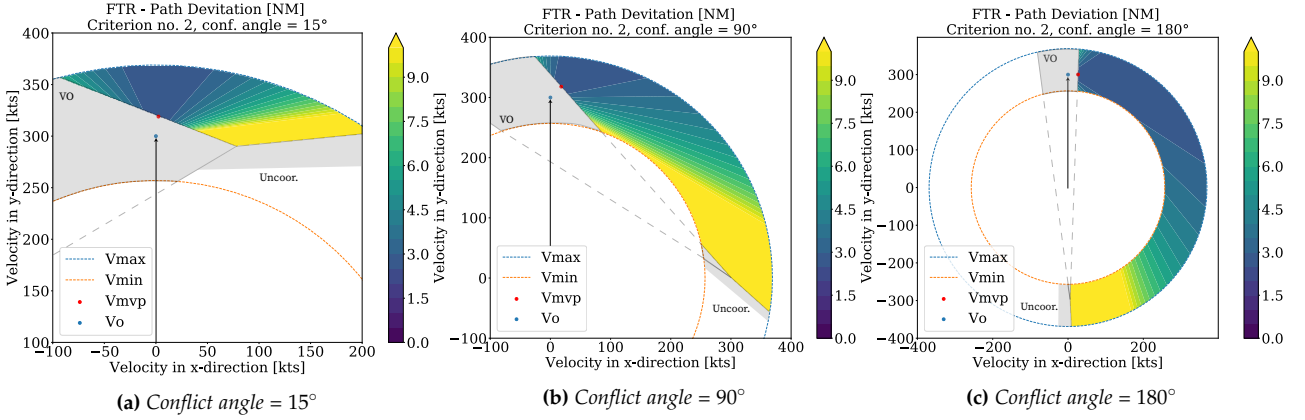


Figure 17: Theoretical path deviation when using the two criteria recovery method within the solution space of a one-to-one conflict at time of initiation. Since both aircraft want to revert to their initial velocity at recovery, criterion no. 2 is limiting. Both aircraft have a ground speed of 300 kts and use a look-ahead time of 300 seconds. The predicted distance at CPA is 2.5 NM. The initial velocity of the ownship is indicated in blue and the MVP provided solution in red. The velocity Obstacle (VO) is depicted in grey as well as the border of the uncoordinated solutions.

tion area. This results in the simulated aircraft flying through the measurement area, see Figure 18. It is possible that an aircraft trajectory does not cross the measurement area, in which case the data of that flight is not logged. No wind or turbulence is simulated in the simulations. Note that wind would have an effect on the outcome of the experiment, since it influences resolution trajectories. However, the influence of these effects is not the subject of the current study.

ii. Independent variables

Three independent variables are used in the current study; allowed heading range (α), traffic density, and recovery type.

Scenarios with varying allowed heading ranges are used to investigate the effect of using the two criteria recovery compared to using CPA recovery when different conflict angles are dominant. Six heading ranges are used in this experiment; 11.25°, 22.5°, 45°, 90°, 180°, and 360°. For all heading ranges except 360°, aircraft fly in eastern direction. The relationship between α and the allowed heading range can be seen in Table 3. The heading distribution of spawning aircraft is uniform within the defined range. For more information on airspace structuring and heading ranges α , the reader is referred to [22].

According to a study by Magill, the average western traffic density of 1996 was 10 to 12 aircraft per 10,000

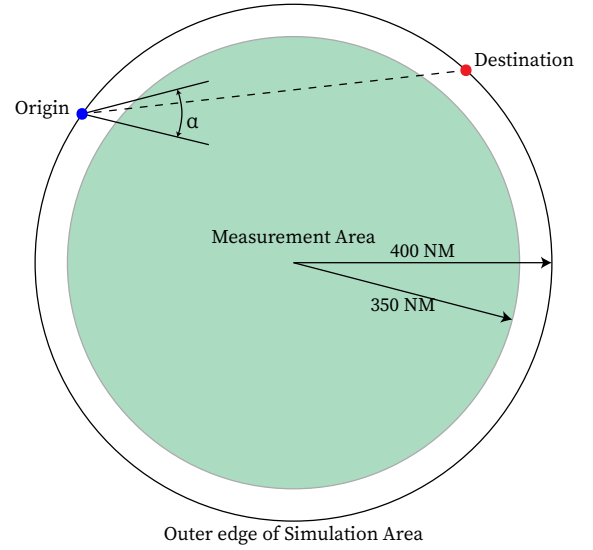


Figure 18: Simulation Area and Measurement Area (in green). An example trajectory is shown.

NM² at FL100 [26]. Therefore, three traffic densities are used; 5.0, 7.5, and 10.0 aircraft per 10,000 NM². Which correspond to traffic densities of $\approx$ 50%, 75%, and 100% of the average western traffic density of 1996.

The last independent variable is the type of CR. 3 types of CR were investigated; no CR to form the baseline experiments, MVP with CPA recovery, and

Table 3: Allowed heading range α with corresponding minimum and maximum spawn headings.

α	Min. and max. spawn heading
360°	0° - 360°
180°	0° - 180°
90°	45° - 135°
45°	67.5° - 112.5°
22.5°	78.75° - 101.25°
11.25°	84.375° - 95.625°

MVP with the two criteria recovery. Each combination of dependent variables is repeated 20 times, resulting in a total number of $6 \times 3 \times 3 \times 20 = 1080$ simulations.

iii. Scenario generation

Based on the spawn heading range a feasible set of spawn locations along the edge of the simulation area is defined. Within this set, the spawn locations are uniformly generated. Each aircraft is given a waypoint along the outer edge of the simulation area within the allowed heading range. Finally, each aircraft is given an instructed ground speed as described in Section i.

To maintain the required density in the simulation area a spawn rate (Ω) is used [27]. The required spawn rate can be calculated using

$$\Omega = \frac{\bar{V}}{\bar{D}} N_{inst} \quad (36)$$

where N_{inst} is the required instantaneous number of aircraft in the simulation area and $\bar{D}$ is the mean distance flown in the experiment, see Tables 4 and 5 for the number of instantaneous aircraft and the mean distance flown respectively. Once aircraft have reached their waypoint, the flight data is logged and the aircraft is deleted from the simulation area.

Each experiment starts with a two-hour density build-up phase in which no data is logged yet. This ensures that the required density is achieved when the data logging starts. The duration of the data logging is 4 hours per experiment, resulting in a total simulation time of 6 hours per experiment. Aircraft are spawning at all times throughout the simulation time.

Table 4: Experiment densities and corresponding number of instantaneous aircraft.

Label	Density	N_{inst}
A	5.0 AC/10,000 NM ²	251
B	7.5 AC/10,000 NM ²	377
C	10.0 AC/10,000 NM ²	503

Table 5: Mean distance flown $\bar{D}$ for scenarios with different heading range α values. Obtained by Monte-Carlo simulations.

α	$\bar{D}$ [NM]
360°	457.6
180°	368.6
90°	355.1
45°	371.3
22.5°	403.6
11.25°	429.9

iv. Dependent variables

The dependent variables of this experiment are divided into three categories; stability, efficiency, and safety [10]. These categories are detailed in the following sections.

iv.1 Stability

Airspace stability is often related to conflict chains due to CR maneuvers. When resolutions lead to secondary conflicts, conflict chains can propagate throughout the airspace. It is beneficial when CD&R algorithms result in an as low as possible number of conflict chains. Bilimoria et al. devised a metric to quantify these conflict chains, the Domino Effect Parameter (DEP) [28]. It is defined as the relative increase of the conflict count when conflict resolution is on ($CC_{cr,on}$) with respect to the conflict count when conflict resolution is off ($CC_{cr,off}$):

$$DEP = \frac{CC_{cr,on}}{CC_{cr,off}} - 1 \quad (37)$$

Even though Bilimoria et al. calculated the DEP using the total number of aircraft which have been in conflict, the current study calculates the DEP using conflict occurrences [22]. Both the DEP when counting all conflict occurrences and the DEP when only counting unique conflict pairs is used. Comparing these two DEP calculations provides insight into the number

Table 6: Overview of the dependent variables

Category	Variable
Stability	Domino Effect Parameter
Efficiency	Average extra distance flown
Safety	Average conflict duration
Safety	Number of LoS
Safety	Percentage of aircraft with multi-ac conflicts

of repetitive conflicts whilst simultaneously providing information regarding airspace stability.

A conflict is counted if at least one of the two conflicting aircraft is inside the measurement area, or the CPA of the conflict lies inside the measurement area [22].

iv.2 Efficiency

The efficiency metric used in the current study is the extra distance flown (D_{ext}) between spawn location and waypoint compared to the nominal experiment (D_{nr}). Due to conflict resolution aircraft are expected to fly longer distances before reaching their waypoint (D_{cr}). When an aircraft is deleted from the simulation area, its distance flown is logged. This allows for comparison.

$$D_{ext} = \frac{D_{cr}}{D_{nr}} - 1 \quad (38)$$

iv.3 Safety

The safety of both methods is evaluated based on average conflict durations, the total number of LoS per experiment, and the percentage of aircraft that is, at one point, involved in a multi-aircraft conflict. Throughout the experiment, the amount of time between the initiation of the conflict and the final recovery is logged. When two aircraft are in a repetitive conflict, the conflict duration is defined as the sum of the individual conflict durations. The number of LoS is the total number of LoS occurrences during the data logging phase in the measurement area.

?? shows an overview of the dependent variables used in this experiment.

v. Hypotheses

The two criteria recovery method is designed to mitigate repetitive conflicts between aircraft. Therefore it is hypothesized that the DEP when counting all conflict occurrences is lower when using the two criteria recovery compared to CPA recovery.

This difference is hypothesized to be larger in the scenarios with lower heading ranges, due to the lower relative velocities.

Additionally, it is hypothesized that the conflict durations will decrease. Both because there are less repetitive conflicts and because non-grazing resolution trajectories are resolved quicker when using the two criteria recovery.

Finally, it is hypothesized that the percentage of aircraft with multi-aircraft conflicts will decrease for all heading ranges and all densities.

VI. RESULTS

The results of the experiment are presented in this section. A discussion of these results can be found in the next section.

Not all obtained samples are normally distributed. Therefore, the results are presented through the use of the box and whisker plot. To indicate the significance found between the different samples, a non-parametric test was used (Wilcoxon signed-rank test [29]). Differences between samples are regarded as significant when the null-hypothesis is rejected with a $p < 0.05$. If differences are regarded as significant, a green box will be displayed below the samples in the box and whisker plot. If the differences are not regarded as significant, a red box will be displayed below the samples. In the box and whisker plots, the label CPA and FTR stand for MVP resolution with CPA and two criteria recovery respectively. The addition of A, B, and C stand for the traffic density.

i. Stability

The first stability metric used in the current study is the DEP. Both the DEP when counting all conflict occurrences, as well as the DEP when only counting unique conflict pairs was investigated.

Figure 19 shows the distribution of DEP values when counting all conflict occurrences. All differences between recovery methods can be regarded as significant,

as can be seen by the green boxes below the samples. The y-axis is logarithmic to improve the readability of the chart.

The results show that median DEP values are lower when using the two criteria recovery method compared to when using CPA recovery in all cases. The DEP values when using two criteria recovery method remain below 0.4 for all experiments. These DEP values for the two criteria recovery method were more dependent on the traffic density than on the instructed heading range. On the other hand, The DEP values when using CPA recovery increase significantly as α decreases. From a median DEP of 0.12 ($\alpha = 360^\circ$, $\rho = 5.0$ AC/10,000 NM^2) to 14.36 ($\alpha = 11.25^\circ$, $\rho = 10.0$ AC/10,000 NM^2).

The results that two criteria based recovery leads to a significant reduction in the DEP when counting all conflict occurrences suggest that the number of repetitive conflicts was reduced. That the biggest reduction was found in scenarios with a lower heading range, and thus a lower average relative velocity, might be due to more repetitive conflicts being present in these scenarios when CPA recovery is used.

Relating this to the hypotheses that (1) the DEP when counting all conflict occurrences is lower when using the two criteria recovery compared to CPA recovery, and that (2) this difference is higher in the scenarios with lower heading ranges, leads to an acceptance of both these hypotheses.

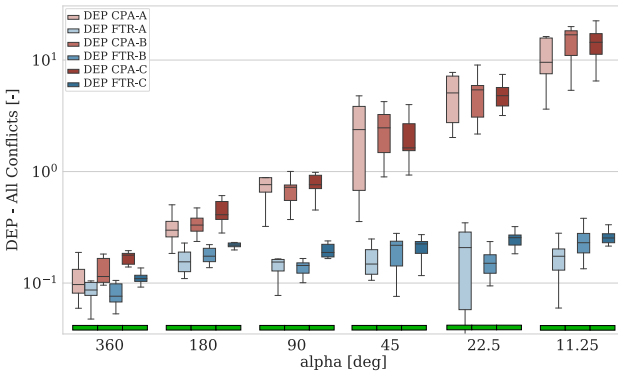


Figure 19: DEP when counting all conflict occurrences.

When, instead of counting all conflict occurrences, only the unique conflict pairs are counted, the differences between the CPA and two criteria recovery become less pronounced, see Figure 20. The DEP values for the two criteria recovery show similar behavior

when comparing between all conflict occurrences and unique conflict pairs. However, the DEP values when using CPA recovery decrease when only counting unique pairs.

Even though all but one median DEP values are lower when using the two criteria recovery in the unique conflict pair case, only at $\alpha = 360^\circ$ and 11.25° are significant improvements found for all densities ($p \leq 0.0284$).

Decreasing the DEP when counting all conflict occurrences was not one of the aims of two criteria recovery method. However, a significant increase in these DEP values when using this two criteria recovery would be a disadvantage of the method. Therefore, its influence on these DEP values was investigated.

Since the immediate effects of repetitive conflicts are no longer evident when only counting unique conflict pairs, the differences between CPA and two criteria recovery become smaller and less significant overall. More consistent differences were found in both the lower and higher heading range experiments.

For higher heading ranges this could be contributed to a decrease in path deviations for non grazing resolution trajectories. Since an increase in path deviations is expected to lead to an increase in the DEP, the expected reduction of path deviations for a non-grazing solution might have been the cause for the reduction in the DEP when using two criteria recovery. For the lower heading range, the decrease in DEP might suggest some destabilizing effects of repetitive conflicts.

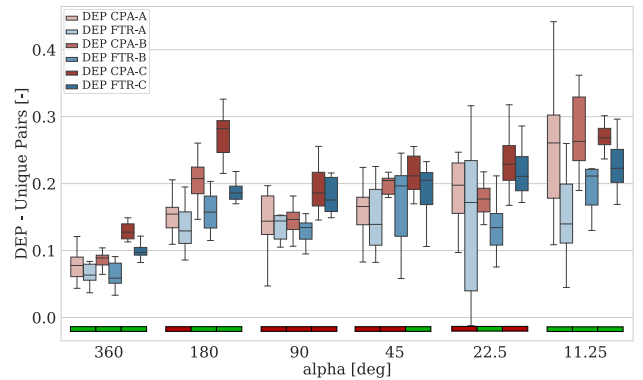


Figure 20: DEP when counting unique conflict pairs.

ii. Efficiency

The efficiency metric used in this experiment is the extra distance flown compared to the nominal experiment. Figure 21 shows that the two criteria recovery method lead to a reduction in extra distance flown compared to CPA recovery for $\alpha = 360^\circ$ and 180° , $p \leq 0.04698$. For the simulations with an α value of 90° or lower, significant differences were only found for $\rho = 10.0$ AC/10,000 NM².

These results might suggest the presence of multi-aircraft conflicts and thus non-grazing resolution trajectories in the experiments with higher heading ranges. These non-grazing resolution trajectories could have been resolved quicker when using the two criteria recovery method, in turn reducing the total path deviations.

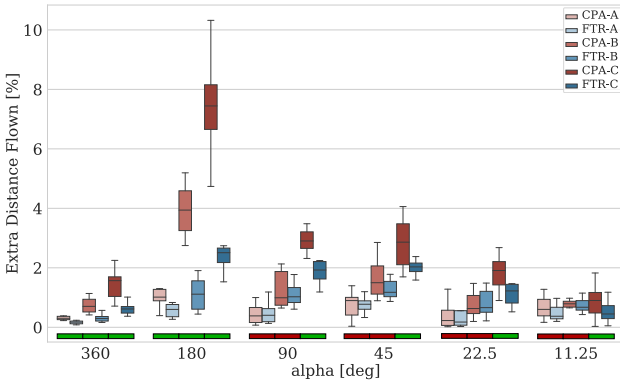


Figure 21: Percentage increase in distance flown compared to nominal experiment.

iii. Safety

Three safety metrics were used; the average conflict duration, the number of loss of separation occurrences, and the percentage of aircraft involved in multi-aircraft conflicts.

Figure 22 shows the average conflict duration within the experiments. For all scenarios did the two criteria based recovery result in a significantly shorter average conflict duration ($p \leq 9.1335e - 5$). Also confirming the suggestion that experiments with higher heading ranges contained more non-grazing resolution trajectories. These non-grazing resolution trajectories could have been resolved considerably quicker when using the two criteria recovery method.

The result that average conflict durations also decreased in the experiments with lower heading ranges might suggest that the sum of the individual conflict durations in repetitive conflicts is higher than when a conflict is resolved in one resolution, as in the two criteria recovery method. As a whole, the hypothesis that the conflict durations will decrease when using 2 criteria recovery compared to CPA recovery is accepted.

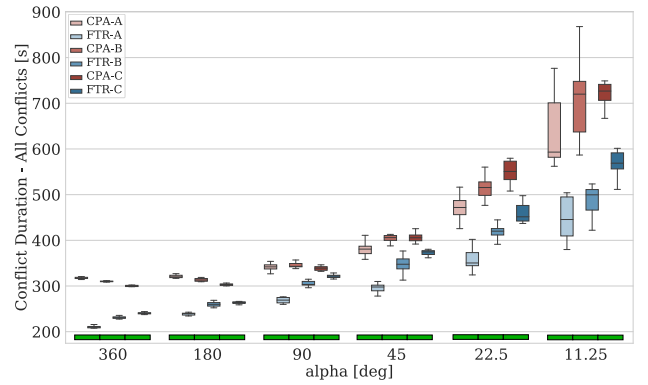


Figure 22: Mean conflict duration when counting all conflict occurrences. For repetitive conflicts the total conflict duration is the sum of the separate conflict durations.

Figure 23 shows significant mitigation of LoS occurrences in the two criteria based recovery. Note that the Wilcoxon signed-rank test was not used for these samples since the data-set contains samples filled with only zeros. Only at ($\alpha = 360^\circ, \rho = 10.0$ AC/10,000 NM²) and ($\alpha = 180^\circ, \rho = 7.5$ AC/10,000 NM²) were LoS occurrences observed when using the two criteria recovery method. A combination of high relative velocities and conflict repetitions is a dangerous one since the aircraft might find themselves close together during the recovery. This might cause the two aircraft to not be able to implement the velocity change in time to prevent the intrusion corresponding to the repetitive conflict.

Increasing the probability that the desired relative velocity is conflict-free, through the two criteria recovery method, might have decreased the number of conflicts with a high relative velocity in combination with a low time to LoS, in turn decreasing the number of intrusions.

Logically, higher traffic densities lead to a higher number of loss of separation occurrences, as more conflicts can be expected in such an airspace.

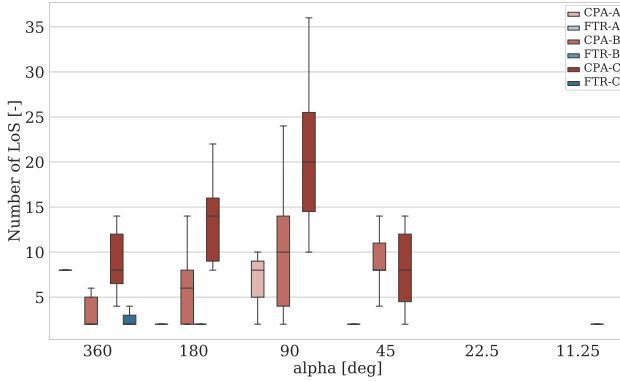


Figure 23: Number of LoS occurrences.

In Figure 24 the distribution of the percentage of aircraft involved in multi-aircraft conflicts is presented. The y-axis is logarithmic to improve the readability of the chart. Looking at the results, the two criteria recovery method lead to a significant reduction in the percentage of aircraft involved in multi-aircraft conflicts for $\alpha = 360^\circ$, 180° , and 90° . For $\alpha = 45^\circ$ and 11.25° is the only significant decrease found for $\rho = 7.5$ and $10.0 \text{ AC}/10,000 \text{ NM}^2$.

Two effects might have contributed to this reduction. The first being the reduction of total conflict occurrences. More repetitive conflicts increase the probability that during one of the repetitions a secondary conflict is initiated. The second effect this decrease might be contributed to is a decrease in conflict durations for non-grazing trajectories. If a conflict is resolved quicker, the probability that a second conflict is initiated during the duration of the initial conflict is smaller.

The hypothesis that the percentage of aircraft with multi-aircraft conflicts will decrease for all heading ranges and all densities when using two criteria recovery compared to CPA recovery has failed to be accepted due to lacking statistical support.

VII. DISCUSSION

The current study investigated whether a modification to the recovery of the MVP method could resolve the repetitive conflicts of aircraft flying near-parallel tracks. First, some aspects of the theoretical analysis will be discussed, then the results of the experiment will be clarified further and recommendations for further re-

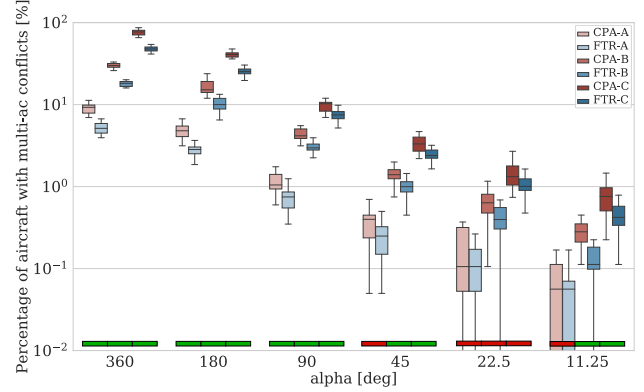


Figure 24: Percentage of aircraft involved in multi-aircraft conflicts.

search are made.

i. Discussion on the Theoretical Analysis

Path Deviations

It was shown that the MVP method provides solutions to conflicts that result in minimal theoretical path deviations. Even though solutions are available which result in smaller velocity changes, the product of the velocity change and the amount of time this velocity change has to be sustained is minimal when this velocity change is perpendicular to the initial relative velocity between the aircraft. This holds both when the CPA is used as recovery as when the two criteria are used as recovery in two aircraft conflicts.

These theoretical path deviations are different from path deviations as measured in an experiment. Theoretical path deviations measure the deviation from the nominal trajectory whereas in experiments the extra distance flown before reaching the exit waypoint is measured. An example of this discrepancy is resolving a conflict by only increasing the ground speed. In the theoretical path deviations sense this would count as a deviation, since at the end of the conflict the aircraft is at another location compared to where it would have been nominally. In an experiment however, this would not have been registered as an extra distance flown.

In the used method to derive a velocity change resulting in a minimal theoretical path deviation, the effect of aircraft dynamics was neglected. In reality, velocity changes are not instantaneous, and the aircraft dynamics influence these path deviations.

Solution Space Size

Solutions that result in an instantaneous separation of the ownship and intruder have a defined recovery point when using the two criteria recovery and do not when using CPA recovery. Therefore, when using the two criteria recovery, the effective size of the applicable solution space is increased compared to when using CPA recovery.

ii. Discussion of the experiment results

Influence of simulation parameters

It is important to note that the results found in the current study are, to a certain degree, dependent on the specific parameters set for the experiment. The results for the DEP when counting all conflict pairs, the number of conflicts involved in multi-aircraft conflicts, the average conflict duration, and the number of LoS show differences of significance and therefore it can be expected that these trends hold when subjected to different situations. However, for the extra distances flown, the initial route length is of importance. Extra distances flown of routes that are short compared to a path deviation imposed by a conflict resolution maneuver are highly dependent on their initial route length. The distribution of these nominal route lengths influences the differences between recovery methods.

In the experiment, aircraft were given a waypoint to fly towards. This made it possible to compare the distances flown. However, because of these waypoints, desired velocities were for the most part not equal to the initial velocities during conflict resolution, as they change over time. Whether or not waypoints are used has, therefore, effect on the results of both CPA and the two criteria recovery.

Influence of conflict counting

In the current study, an measurement area within the simulation area was used. The choice was made to not spawn any aircraft within the measurement area, as this would lead to an unrealistic type of conflicts with short times to LoS. This effect would lead to conflicts that would not be representative of realistic situations. Similarly, no waypoints were located in the measurement area as this could lead to unrealistically hefty maneuvers as aircraft try to reach their waypoint

after resolving a conflict. As a result, all aircraft were spawned outside the measurement area and all aircraft had waypoints outside the measurement area. Undoubtedly, some conflicts had neither aircraft nor the CPA in the measurement area when they got into conflict. If these conflicts were of the repetitive kind and occurred near the spawn location of the aircraft, it could be that these repetitions were mitigated when using the two criteria recovery, causing the conflict to not be counted since the repetitions did not occur in the measurement area any longer. Even though the number of such conflicts is expected to be small since neither aircraft nor the CPA had to lie in the measurement area, it most likely had an influence to some extent on the results.

ii.1 Failure modes of the Two Criteria Method

Two failure modes of the two criteria based recovery are identified and discussed here. One is related to a possible extensive discrepancy between desired and initial velocities, and the other is related to simultaneous criteria acceptance combined with desired velocities unequal to initial or current velocities.

Coordination related failure modes

The two criteria recovery leads to a repetitive conflict when the ownship has a desired velocity which results in the two criteria being met on the uncoordinated side of the intruder. An example of this phenomenon is illustrated in Figure 25. For this failure mode to occur, a substantial difference between the desired velocity and resolution velocity is required. Such a difference could occur close to a waypoint for instance. A proper method to solve this failure mode would be to check whether both criteria are met on the coordinated side of the intruder. The effect of this modification on the behavior of the proposed method should be investigated.

Simultaneous criteria acceptance combined with desired velocities unequal to initial or current velocities

If criteria no. 1 and 2 are met for both the ownship and intruder at the same time, they will simultaneously begin the recovery. If both or one of the two aircraft has a desired velocity unequal to its current or initial velocity, a conflict repetition could occur, see Figure 26.

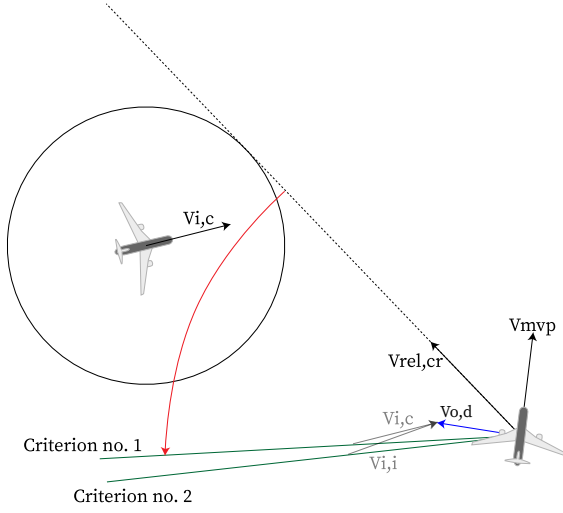


Figure 25: Example of coordination related failure mode. If the ownship were to revert to its desired velocity, a conflict repetition would occur.

This is one of the intrinsic problems with airspace where the desired state is not broadcast. However, using the two criteria method, this phenomenon only occurs when both criteria are met for both aircraft at the same time.

iii. Recommendations

In the experiments performed, the two criteria method either performed better than the CPA-based method, or no significant difference was found. However, certain behavior of the two criteria method is still unknown, and different experiments are therefore required to further investigate its behavior.

Firstly, the two criteria method decides whether an aircraft can revert based on two criteria. Through the combination of these two criteria, the best guess is formed regarding the desired state of the intruder. Further research should be conducted regarding this best guess. Indeed, in airspace where aircraft are flying towards a waypoint, the assumption that the intruder will revert to its exact initial velocity is most realistic in en-route flight where waypoints are located far in front of the aircraft. The validity of this assumption in other flight phases should be investigated, and possible revoked.

Also, the design of the current experiment should be adjusted. Investigating the use of multiple types

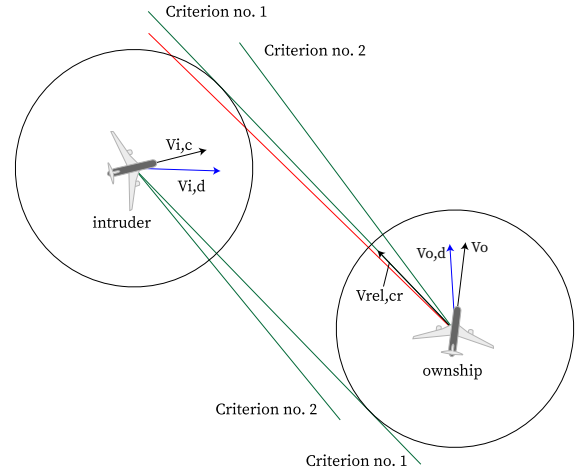


Figure 26: Example of simultaneous criteria acceptance combined with desired velocities unequal to initial or current velocities. The desired velocity of the intruder is depicted as $V_{i,d}$. If both aircraft revert to their desired velocity, the conflicting red relative trajectory is obtained.

of aircraft, with a different ground speed distribution could provide insight regarding the reliance of the results. Additionally, the effect of introducing wind should be investigated as well, as this changes the resolution trajectories.

If aircraft were to broadcast their desired state as well, no assumptions would have to be made regarding this desired state. If this would be the case, more progress towards a guarantee regarding conflict repetitions could be made. However, when broadcasting the desired state, many other methodologies become interesting as well, such as optimization methods. Nonetheless, these effects on the behavior of the method could be investigated.

The method could also be extended towards 3D CD&R. By not only considering the horizontal separation requirement but also including the vertical separation requirement in the two criteria the method could also be used in 3D airspace.

Currently, both criteria are evaluated without regard for time to LoS. If reverting to the desired velocity causes an intrusion - for one of the criteria - at *any* time, the criterion is not passed. It could be interesting to investigate the behavior of the method when the criteria are extended with a time-bound requirement. For instance, evaluating if reverting to the desired velocity causes an intrusion - for one of the criteria - within a

certain time.

The method could be tested in a different context as well. Scenarios such as corridors where aircraft have to overtake each other experience many problems when using CPA recovery, the two criteria based recovery might provide improvement.

VIII. CONCLUSION

This article presented the findings of a study into the behavior of an alternative recovery method, a method based on two criteria. Theoretical behavior was analyzed using characterization of a solution space diagram and emergent behavior was analyzed using an experiment. The two criteria method was implemented in combination with MVP provided conflict resolution and compared to the original MVP method which uses Closest Point of Approach recovery. Theoretical analysis showed that the MVP method provides resolutions with a minimal theoretical path deviation, both when using CPA and the two criteria recovery. For most non-grazing resolution trajectories does the two criteria recovery method provide lower conflict durations and shorter path deviations. The results of the experiment provide the following conclusions:

The two criteria based recovery method leads to significantly fewer conflicts when all conflict occurrences are counted. This can be explained by the recovery criteria which check whether recovery is likely to lead to a repetitive conflict.

The two criteria based recovery method lead to shorter average conflict durations, as it decreases the conflict duration of non-grazing resolution tracks. Also, a reduction in loss of separation occurrences was found when using the two criteria based recovery method.

The two criteria based recovery method has shown promising initial results. Further research should be conducted to gain more insight regarding its behavior in different scenarios and in combination with with different CR methods.

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2

A Characterization of State-Based Solutions for Air Traffic Conflicts

A Characterization of State-Based Solutions for Air Traffic Conflicts

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Abstract– In decentralized conflict detection and resolution, the cockpit and on-board systems are responsible for maintaining safe separation with other aircraft. If this safe separation is expected to be violated in the future it is called a conflict. The process of detecting and subsequently resolving these conflicts is called conflict detection and resolution. In most situations, there are many maneuvers - or solutions - available to a conflicting aircraft that resolve the conflict. However, the implications of these various solutions remain largely unknown. The current study investigates these possible maneuvers through constructing a state-based solution space, and how they are affected by aircraft dynamics. These solutions are based on current states, without regard for flight intent. Within this state-based solution space, the solutions are investigated with regard to conflict duration and imposed path deviation from the nominal trajectory. Through derivations, solutions resulting in minimal path deviations are found.

I. INTRODUCTION

As air traffic density increases and air traffic routes congest, the workload on air traffic controllers increases. Methods that could aid in the reduction of this workload are therefore a prolific research field. A way to reduce this workload is having on-board systems - either fully automated or not - aid in the managing of the airspace. One of the air traffic control tasks that could be moved to on-board systems - during certain flight phases - is conflict resolution [1, 2]. An air traffic

conflict is an event where two aircraft are expected to pass at a distance, within a certain look-ahead time, lower than the separation requirement imposed by the local Air Traffic Service. During en-route flight, for instance, this separation requirement is, in most parts of the world, defined as a 3-dimensional disk around the aircraft with a height of 2000 feet and a radius of 5 nautical miles, often called the Protected Zone (PZ). If the two aircraft pass at a distance lower than this separation requirement, it is called an intrusion. The state-based solution space is a method to represent sets of aircraft velocities - or resolution maneuvers - which, in the event of a conflict, are solutions to the conflict [3]. In other words, it represents sets of aircraft velocities which result in passing the other aircraft at - at least - the horizontal and vertical separation requirement. This information can be used in automation [4] as well as in-cockpit displays that convey these solutions to the crew [5].

The current study provides an overview of demarcations to the horizontal - or two-dimensional - velocity-vector space to obtain a state-based solution space. This solution space is subsequently characterized according to conflict duration and path deviation. First, the chain of events which comprise an air traffic conflict is explained as well as some important concepts and points.

Then, based on this chain of events, the state-based solution space is constructed and defined using the proposed demarcations. Based on a proposed reference frame, the corresponding limitations on the available maneuvers, when using the proposed state-based solution space, are derived.

The set of solutions in the state-based solution space is then characterized according to two primary conflict resolution metrics; conflict duration and path deviations from the nominal trajectory. Finally, a simple

aircraft model is used to visualize how aircraft dynamics change the shape of the state-based solution space and how they change the conflict durations and path deviations.

II. DECENTRALIZED CONFLICT DETECTION & RESOLUTION

The chain of events that comprise an air traffic conflict and its resolution can be divided into three; conflict detection, conflict resolution, and recovery. These three phases, along with corresponding properties, are described using the notion of an ownship and an intruder. The ownship is the conflicting aircraft for which a solution is found, and the intruder is the other conflicting aircraft which PZ is to be 'avoided'.

Detection

If two aircraft are in conflict, it means that the two aircraft are expected to reach a distance lower than the horizontal separation requirement (S_h) within some time in the future - they are thus expected to have an intrusion within some time in the future. This expected distance is often obtained through the propagation of the current aircraft states. Multiple propagation methods are available, such as nominal state propagation or probabilistic state propagation [6].

The current study assumes nominal state propagation, indicating that the current aircraft states are propagated nominally though time to obtain expected future states, without accounting for uncertainty. In many decentralized conflict detection and resolution studies, a 'look-ahead' time of 300 seconds is used [4]. Meaning that aircraft states are propagated up to 300 seconds into the future to determine if the separation requirement between aircraft will be violated within those 300 seconds, if this is the case, those aircraft are said to be in conflict.

An example illustration of conflict detection can be found in Figure 1. Note that the figures used in the current study are not necessarily to scale. In this figure, the expected intrusion is illustrated both in the absolute and in the relative velocities frame. Absolute velocities being the velocities in a frame that is at absolute rest. In this figure, both the absolute and relative velocities are propagated nominally through time to find the time until expected intrusion (t_{int}). If both aircraft were to take no action, the ownship would cross

through the PZ of the intruder, thus violating the separation requirement. The expected minimal distance between the two aircraft during such a trajectory is called the distance at closest point of approach ($|D_{cpa}|$).

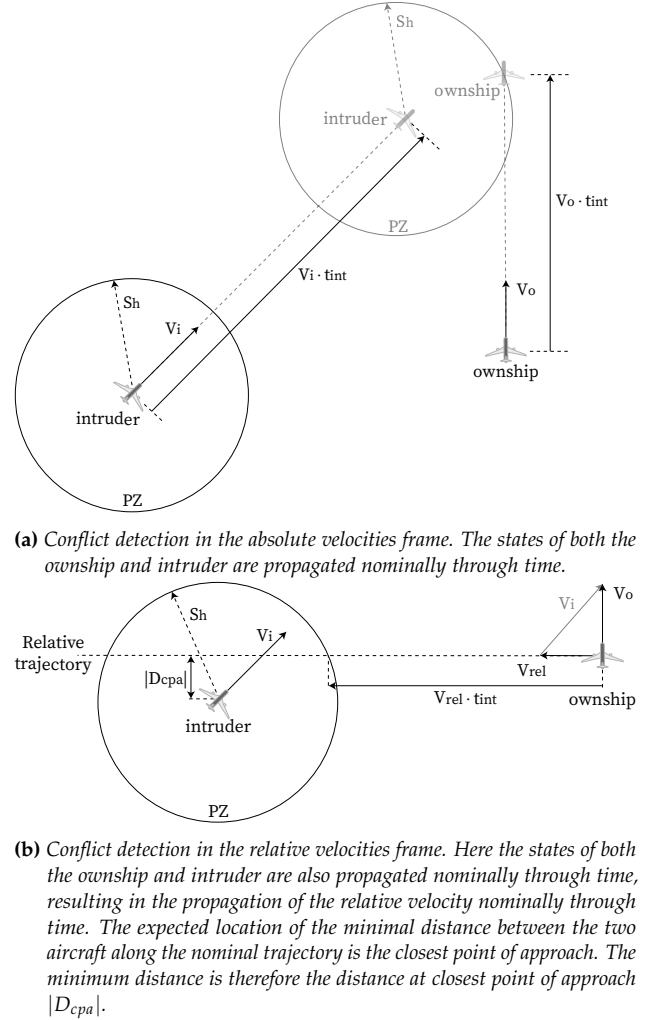


Figure 1: Example illustration of a conflict detection, both in the absolute and relative velocities frame. For conflict detection, an intrusion is expected within 300 seconds, meaning that $t_{int} \leq 300$ sec. Note that in this case only the PZ of the intruder is drawn. The figure could be constructed in similar fashion when drawing the PZ of the ownship. The horizontal separation requirement is indicated by S_h , the ownship and intruder velocities by V_o and V_i respectively, and the relative velocity is indicated by V_{rel} .

Resolution

Conflict resolutions has to be performed when a conflict is detected. One or both of the two aircraft need to implement a velocity change which results in the new expected minimal distance (D_{cpa}) to be larger than - or equal to - the separation requirement. Such a new velocity for a conflicting aircraft, which thus prevents the expected intrusion, is called a solution. An example illustration of such a solution velocity is illustrated in Figure 2. In this figure, an arbitrary solution velocity is depicted which results in the ownship not crossing the PZ of the intruder but instead merely grazing it. This results in the distance at closest point of approach corresponding to this solution ($D_{cpa,s}$) to be equal to the horizontal separation requirement (S_h). This solution is to be sustained by the ownship until the corresponding closest point of approach is reached.

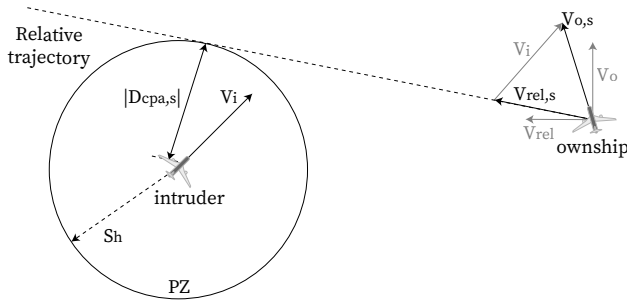


Figure 2: Example illustration of conflict resolution. The ownship chose a, in this case arbitrary, solution $V_{o,s}$ which results in the relative trajectory - the nominal propagation of $V_{rel,s}$ - not crossing the PZ of the intruder, thus preventing the initially expected intrusion.

Recovery

A commonly used instant to initiate the recovery is the closest point of approach. Once both aircraft have reached the closest point of approach corresponding to the solution velocities, they will start diverging given that they retain their current solution velocities. The instant they start diverging is the start of the recovery. The recovery is the phase where both - if having implemented a solution velocity - can omit this solution velocity and revert to their initial velocity. An example illustration of the initiation of the recovery phase can be seen in Figure 3. In this figure, both aircraft have reached closest point of approach and therefore the

resolution phase has ended and the recovery phase can start. The recovery phase is completed when both aircraft have reverted to their initial velocity, which thus concludes the conflict.

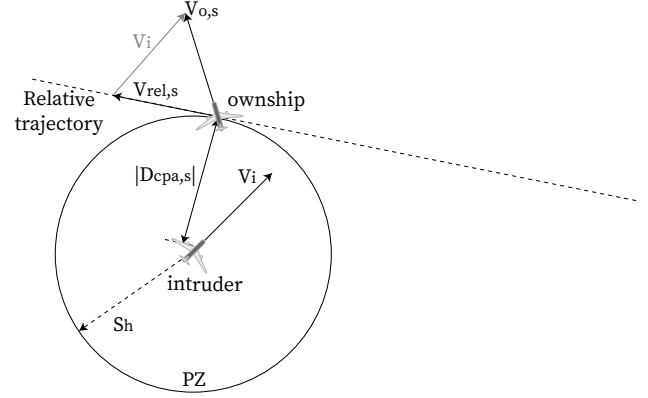


Figure 3: Example illustration of the initiation of the recovery phase. Since both aircraft have reached closest point of approach, the recovery phase can start. During this phase, both aircraft - if having implemented a solution velocity - can omit this solution velocity and revert to their initial velocity. Once this has happened, the recovery phase is completed and the conflict is said to be resolved.

The location of a predicted closest point of approach, in the relative frame, is dependent on two variables. The first being the direction of the relative velocity, and the second being the distance between the ownship and intruder (x_i). These two variables define the relative trajectory of the ownship with respect to the intruder. Since the predicted CPA is the location on the relative trajectory which is closest to the location of the intruder, it can be described as the location along the relative trajectory where the intruder is perpendicular to the current relative trajectory, such as in Figure 3. The predicted time to closest point of approach (t_{cpa}), as a function of some relative velocity, can therefore be calculated using [7]:

$$t_{cpa} = \min\left\{\frac{-\mathbf{x}_i \cdot \mathbf{V}_{rel}}{|\mathbf{V}_{rel}|^2}, 0\right\} \quad (1)$$

Therefore, the set of predicted closest points of approach form a right angle in a right triangle between the ownship location, intruder location, and the predicted closest point of approach. Using Thales's theorem, the set of predicted closest points of approach

between ownship and intruder can thus be described as a circle with a radius of half the distance between ownship and intruder and its center in between the ownship and intruder, as in Figure 4.

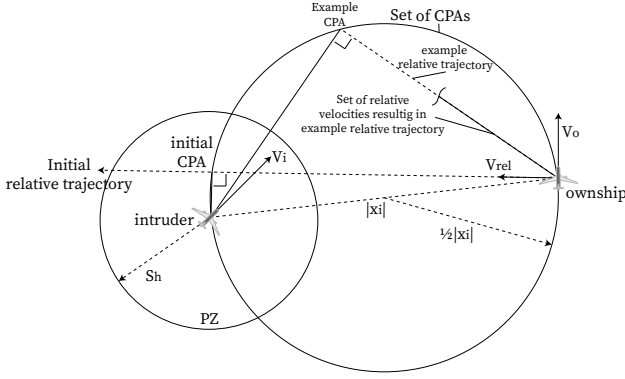


Figure 4: Example illustration of the set of closest points of approach (CPAs). As explained, this set is described as a circle with a radius of half the distance between ownship and intruder and its center in between the ownship and intruder. The expected initial relative trajectory is drawn with the corresponding expected initial CPA. An example trajectory is drawn as well. Note that this example trajectory would be a solution since it does not cross the PZ of the intruder. Indeed, all relative velocities along the indicated set of relative velocities would result in this relative trajectory being flown, with the only difference being the relative speed. Therefore, all the relative velocities along the indicated set result in the same CPA location in the relative frame.

Therefore, each direction of the relative velocity has, for a given distance between ownship and intruder, a specific predicted CPA location in the relative frame, and the magnitude of that relative velocity only influences how quickly that predicted CPA is reached, not its location in the relative frame.

III. CONSTRUCTION OF THE STATE-BASED SOLUTION SPACE

As stated in the previous section, in most situations there are many solution velocities available to the aircraft. This section makes an effort to construct this state-based solution space by combining multiple sets of aircraft velocities which bound the state-based solution space. The sets of velocities this study will

consider are reachable velocities, velocities which lie in the velocity obstacle, uncoordinated velocities, and non-recoverable velocities. These sets will also be indicated through mathematical set notations. These set notations impose criteria on some potential solution velocity $V_{o,s}$.

Reachable Velocities

The ownship has, as a function of the altitude, a minimum, and maximum ground speed. Logically, only solutions to a conflict within the minimum (V_{min}) and maximum (V_{max}) ground speed are worth considering. These velocities are called Reachable Velocities (RV) [4]. The representation of these reachable velocities in the two-dimensional velocity-vector space is illustrated, through an example, in Figure 5. Note that the ownship also has a PZ with a radius of 5 NM, but for the clarity of the illustrations, this PZ is not drawn in the figures of the current study.

$$RV = \{V_{o,s} \mid V_{min} \leq |V_{o,s}| \leq V_{max}\} \quad (2)$$

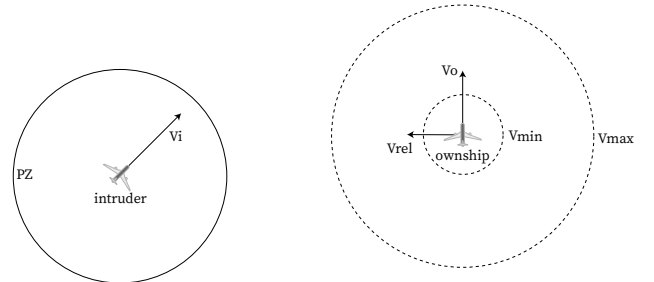


Figure 5: Reachable velocities in the two dimensional velocity-vector space. The minimum and maximum ground speeds of the ownship are depicted by V_{min} and V_{max} respectively.

Velocity Obstacle

The Velocity Obstacle (VO) is a method to represent sets of velocities leading to an expected intrusion through using obstacles in the velocity-vector space. The method was formalized by Fiorini and Shiller [8, 9] who used velocity obstacles for motion planning in a dynamic environment.

A velocity obstacle can be defined in two steps. First of all, the Collision Cone (CC) represents the set of relative velocities for an aircraft which result in an expected intrusion. In the horizontal conflicts considered

in the current study, this set of relative velocities can be described as the relative velocities which lie between the tangents to the protected zone of the intruder. If the relative velocity lies between these tangents, the relative trajectory will cross the PZ of the intruder. An example construction of the collision cone can be seen in Figure 6.

Through the collision cone, the set of relative velocities for the ownship is represented which result in an expected intrusion. However, it is useful to represent this set of intruding relative velocities in a set of intruding absolute velocities for the ownship. This is exactly what the velocity obstacle does. Through translating the collision cone by the intruder velocity ($\mathbf{V}_i$), the set of intruding absolute velocities for the ownship is represented in the form of an obstacle, as illustrated in Figure 6. Mathematically, the set of velocities represented by the velocity obstacle can be described as the set of velocities which result in a distance at closest point of approach which is lower than the horizontal separation requirement. Or, the current distance between ownship and intruder minus the time to closest point of approach multiplied with the relative velocity is lower than the horizontal separation requirement:

$$VO = \{\mathbf{V}_{o,s} \mid |\mathbf{x}_i - t_{cpa,s} \mathbf{V}_{rel,s}| < S_h\}$$

Or, if substituting Equation (1):

$$VO = \{\mathbf{V}_{o,s} \mid |\mathbf{x}_i - \min\left\{\frac{-\mathbf{x}_i \cdot \mathbf{V}_{rel,s}}{|\mathbf{V}_{rel,s}|^2}, 0\right\} \cdot \mathbf{V}_{rel,s}| < S_h\} \quad (3)$$

Where:

$$\mathbf{V}_{rel,s} = \mathbf{V}_{o,s} - \mathbf{V}_i$$

Uncoordinated Velocities

To resolve conflicts in a decentralized manner, some level of coordination between the conflicting aircraft is useful. The authors of [10] differentiate between three types of coordination; explicit coordination, implicit coordination, and no coordination. Explicit coordination entails an explicit line of communication between the two aircraft, in which - for instance - resolution maneuvers can be negotiated. Whereas implicit coordination entails imposing a restriction on the solution space based on a common rule-set, such that both aircraft know from what set of velocities to choose without explicitly communicating.

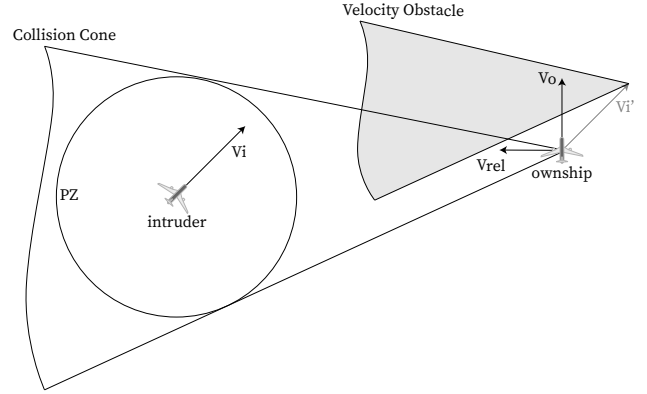


Figure 6: Example conflict with corresponding collision cone and velocity obstacle. As indicated, the velocity obstacle is the collision cone translated by the intruder velocity ($\mathbf{V}_i$).

An often used type of implicit coordination - in velocity obstacle based conflict resolution - is choosing solutions in the direction of the closest leg of the velocity obstacle. Due to the rotational symmetry of the conflict, this results in both aircraft passing each other on the same side, without resolving 'into' each other [11]. Figure 7a shows these sides of the velocity obstacle in green. In this figure, the velocity obstacles are constructed for both the ownship and the intruder, with the center axis of the velocity obstacle depicted by a black line. Indeed, if both of the aircraft in this figure would choose a solution on the nearest leg of their velocity obstacle, both aircraft would successfully pass each other on the same side.

The reason why this is a common rule-set is because it was found that such solutions lead to lower path deviations [12, 13, 11], as they allow for solutions with smaller velocity changes than solutions on the furthest leg of the velocity obstacle. This ruleset is used, among others, by the reciprocal collision avoidance studies by Van Den Berg et al. [14, 15] and the free-flight studies by Hoekstra [7].

To generalize this implicit coordination in the velocity-vector space, the current study relates this rule-set to the relative bearing and relative velocities of the ownship and intruder. The ruleset is adhered to as long as both the ownship and intruder choose solutions that result in their relative velocity remaining on the same side of the relative bearing between the ownship and the intruder. This, similarly, results in both aircraft

passing each other on the same side and allowing for solutions with smaller velocity changes compared to solutions that result in their relative velocity crossing to the other side of the relative bearing, as in Figure 7b. In this figure, the set of (implicitly) coordinated relative velocities - for both aircraft - is indicated by the border with the darker shade. Translating this set of coordinated relative velocities to absolute velocities, through using the velocity vector of the opposite aircraft, results in the set of absolute coordinated velocities for both aircraft. This set of absolute coordinated velocities - for both aircraft - is indicated by the green area in the figure. All velocities in the green area result in the relative velocity remaining on the same side of the relative bearing between the two aircraft.

Mathematically, a method to check if a velocity is coordinated is to compare the cross product between the initial relative velocity and the bearing to the cross product between the relative velocity corresponding to another velocity and the bearing. If these two cross products have opposite signs or one of the cross products is equal to 0, then the relative velocity corresponding to the other velocity lies on, or on the other side of, the relative bearing. Thus, a way to represent uncoordinated velocities (UV) is:

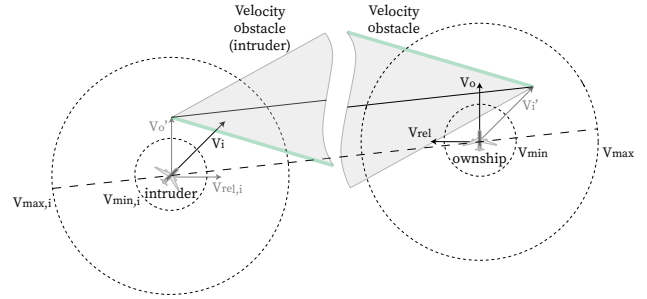
$$UV = \{ \mathbf{V}_{o,s} \mid (\mathbf{x}_i \times \mathbf{V}_{rel}) \cdot (\mathbf{x}_i \times \mathbf{V}_{rel,s}) < 0 \} \quad (4)$$

Where:

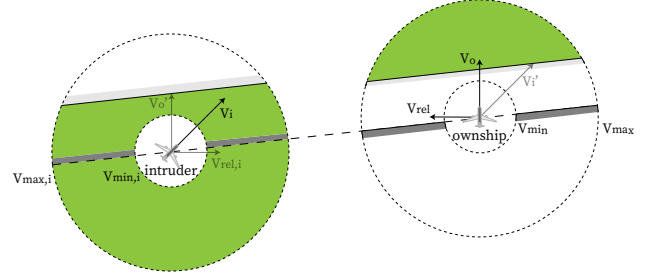
$$\begin{aligned} \mathbf{V}_{rel} &= \mathbf{V}_o - \mathbf{V}_i \\ \mathbf{V}_{rel,s} &= \mathbf{V}_{o,s} - \mathbf{V}_i \end{aligned}$$

Diverging Velocities

Since, in the current study, the closest point of approach is used as an indication that the recovery can begin, velocities which result in instantaneous divergence of the two aircraft cannot be used as solutions. Solutions that result in instantaneous divergence of the two aircraft result in both aircraft immediately being at CPA, in other words, the CPA being equal to the current ownship location, as in Equation (1). This results in an unworkable solution as the ownship does not know when to initiate the recovery. An example construction of the set of converging velocities is illustrated in Figure 8. Diverging relative velocities have the property that they form an obtuse angle with the distance vector to the other aircraft, similar they form an obtuse angle with the relative bearing of the other aircraft. This set



(a) The velocity obstacles for both the ownship and the intruder, with the center-line of the velocity obstacles indicated with a black line. The closest leg of the velocity obstacle is, for both aircraft, indicated by green. Indeed, if both aircraft choose a solution on this leg, they will pass each other on the same side. The relative velocity from the perspective of the intruder is indicated by $\mathbf{V}_{rel,i}$, and its minimum and maximum velocities are indicated by V_{min} and V_{max} respectively.



(b) The sets of coordinated absolute velocities for both the ownship and the intruder indicated in green. These sets are obtained by translating the relative bearing by the velocity of the other aircraft. Indeed, if both aircraft pick a solution within the coordinated absolute velocities, they will pass each other on the same side. The same notations are valid as in Figure 7a.

Figure 7: Example construction of the set of implicitly coordinated velocities to a conflict. The rule-set used in the current study is that aircraft must resolve in the direction of the nearest leg of the velocity obstacle. Or, more generally, they must keep their relative velocity on the same side of the relative bearing as the current relative velocity.

can be translated to the absolute velocities by translation of a line perpendicular to relative bearing with the other intruder velocity. Therefore, the set of diverging velocities can be described as velocities which result in the dot product between relative velocity and distance vector to the intruder being negative.

$$DV = \{ \mathbf{V}_{o,s} \mid \mathbf{V}_{rel,s} \cdot \mathbf{x}_i < 0 \} \quad (5)$$

Where:

$$\mathbf{V}_{rel,s} = \mathbf{V}_{o,s} - \mathbf{V}_i$$

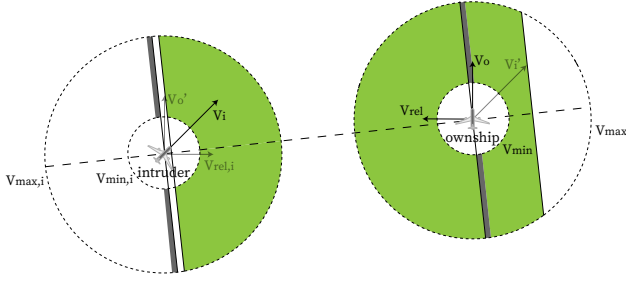


Figure 8: Example construction of the set of diverging velocities to a conflict. The set of absolute velocities that result in convergence of the two aircraft is indicated in green. The border of this set is a line, perpendicular to the relative bearing of the other aircraft, translated by the velocity of the other aircraft.

Non-Recoverable Velocities

Some solutions have the characteristic that, once ownship and intruder have reached the closest point of approach - and thus want to initiate the recovery, the initial velocities are not yet conflict-free. If both aircraft revert to their initial velocity, the initial relative velocity is obtained. Through propagating this initial relative velocity through time, it can be checked whether this initial relative velocity causes an expected intrusion or not. In some cases, it will. These solutions will be called non-recoverable solutions for the remainder of the current study. Since these solutions are not able to successfully complete all three steps of the conflict - detection, resolution, and recovery - they will not be regarded as solutions in the current study.

This set of predicted CPAs can be investigated for CPAs at which the initial relative velocity is conflict-free and CPAs at which the initial relative velocity is not conflict-free. This is done by propagating the initial relative velocity through time from the CPA. By parallel translation of the initial relative velocity to the tangents of the protected zone of the intruder, the set of predicted CPAs is thus divided into two sets, CPAs at which the initial relative velocity is conflict-free and CPAs at which the initial relative velocity is not conflict-free. CPAs at which the initial relative velocity is conflict-free will be called recoverable CPAs and CPAs at which the initial relative velocity is not conflict-free will be called non-recoverable CPAs.

Relative velocities for the ownship corresponding to recoverable CPAs are depicted as recoverable relative velocities in Figure 9, and are indicated by the darker

green shade. When these recoverable relative velocities are translated by the intruder velocity, the set of absolute recoverable solutions is found. This set is indicated by the lighter green shade.

Mathematically, the set of non-recoverable solutions can be defined as solution which result in a relative CPA location from which the initial relative velocity cannot be propagated through time without violating the horizontal separation requirement:

$$NR = \{ \mathbf{V}_{o,s} \mid |\mathbf{x}_{i,cpa} - \min\{ \frac{-\mathbf{x}_{i,cpa} \cdot \mathbf{V}_{rel}}{|\mathbf{V}_{rel}|^2}, 0 \} \cdot \mathbf{V}_{rel}| < S_h \} \quad (6)$$

With $\mathbf{x}_{i,cpa}$ being the distance vector from the ownship to intruder when both aircraft are at CPA:

$$\mathbf{x}_{i,cpa} = \mathbf{x}_i - \min\{ \frac{-\mathbf{x}_i \cdot \mathbf{V}_{rel,s}}{|\mathbf{V}_{rel,s}|^2}, 0 \} \cdot \mathbf{V}_{rel,s}$$

Where:

$$\begin{aligned} \mathbf{V}_{rel} &= \mathbf{V}_o - \mathbf{V}_i \\ \mathbf{V}_{rel,s} &= \mathbf{V}_{o,s} - \mathbf{V}_i \end{aligned}$$

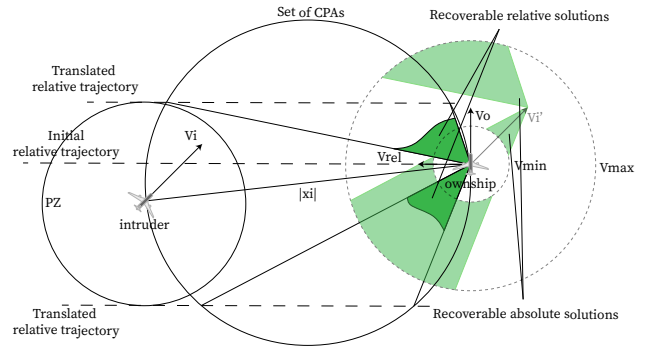


Figure 9: The set of possible CPAs for a conflict. Relative velocities corresponding to recoverable CPAs are depicted as recoverable relative velocities and indicated by the darker green shade. When these recoverable relative solutions are translated by the intruder velocity, the set of absolute recoverable solutions is found. This set is indicated by the lighter green shade.

i. Solutions Within the Velocity-Vector Space

Excluding ($\setminus$) the union ($\cup$) of the velocity obstacle, the uncoordinated solutions, diverging velocities, and the non-recoverable velocities from the reachable velocities results in the state based solution space as considered

in the current study. In mathematical set notation this state based solution space (SS) is defined as:

$$SS = \{RV \setminus (VO \cup UV \cup DV \cup NR)\} \quad (7)$$

An example of this set is illustrated in green in Figure 10. These reachable, coordinated, converging, and recoverable solution velocities are the state-based solution space that will be characterized in the current study.

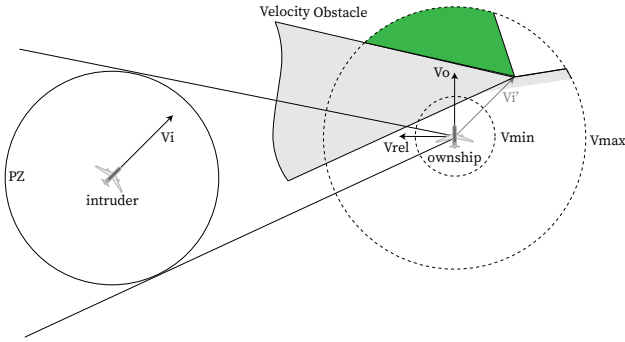


Figure 10: The union of the velocity obstacle, the uncoordinated solutions, and the non-recoverable solutions excluded from the reachable velocities results in the state based solution space as considered in the current study. The state based solution space is indicated in green.

IV. THE BOUNDS OF MANEUVERS IN THE STATE-BASED SOLUTION SPACE

To further characterize the bounds of the state-based solution space, a change of reference frame is used. Through the combination of two angles in combination with the known relative velocity vector, all velocity changes for the ownship can be described. These angles will be described using the example of Figure 11.

Define θ as the inner angle between the initial relative velocity and the relative velocity corresponding to some chosen solution. Note that θ is defined as positive in the coordinated direction - the direction towards the closest leg of the velocity obstacle.

Define ϕ as the inner angle between the initial relative velocity and the velocity change ΔV corresponding to some chosen velocity.

For completeness, define ψ as the inner angle between the velocity change ΔV and the relative velocity

corresponding to some chosen solution.

Thus, all velocity changes ΔV can be described through the combination of θ , ϕ , and the known relative velocity vector V_{rel} . This reference frame allows for more straight-forward derivations.

i. Minimum and maximum θ

Using this reference frame, an expression for the minimum and maximum θ can be found. To do so, a distinction between the minimal θ with respect to the velocity obstacle and the minimal θ with respect to the recoverable solutions is described first. As stated in Section III, not all relative trajectories between ownship and intruder lead to closest points of approach where the initial relative velocity is conflict-free, in some cases the propagation of the initial relative velocity at closest point of approach leads to an expected intrusion. In this regard, two expressions for a minimal θ can thus be found. One that (1) adheres to the velocity obstacle, and thus ensures a solution which prevents an intrusion, without considering the trajectory during recovery. And one that (2) adheres to the recoverable solutions, ensuring an intrusion free solution both during the resolution and during the recovery.

The minimal θ corresponding to the velocity obstacle has been expressed by Hoekstra [7]:

$$\theta_{min} = \arcsin\left(\frac{S_h}{|x_i|}\right) - \arcsin\left(\frac{|D_{cpa}|}{|x_i|}\right) \quad (8)$$

On the other hand, the minimal θ for a relative trajectory with a CPA at which the initial relative trajectory is conflict-free ($\theta_{min,rec}$), can be expressed by using Thales's theorem and is:

$$\theta_{min,rec} = \frac{1}{2} \left(\arcsin\left(|D_{cpa}| \left(\frac{1}{|x_i|} - \frac{1}{S_h}\right) + 1\right) + \arccos\left(\frac{|D_{cpa}|}{|x_i|}\right) \right) \quad (9)$$

Note that both expressions are functions of the initially expected distance at CPA $|D_{cpa}|$, the initial distance between ownship and intruder $|x_i|$, and the horizontal separation requirement S_h only. An example illustration of the difference between the two expressions can be seen in Figure 12. In this figure, the minimum θ for both cases are depicted both with and without translating the angles to the velocity obstacle.

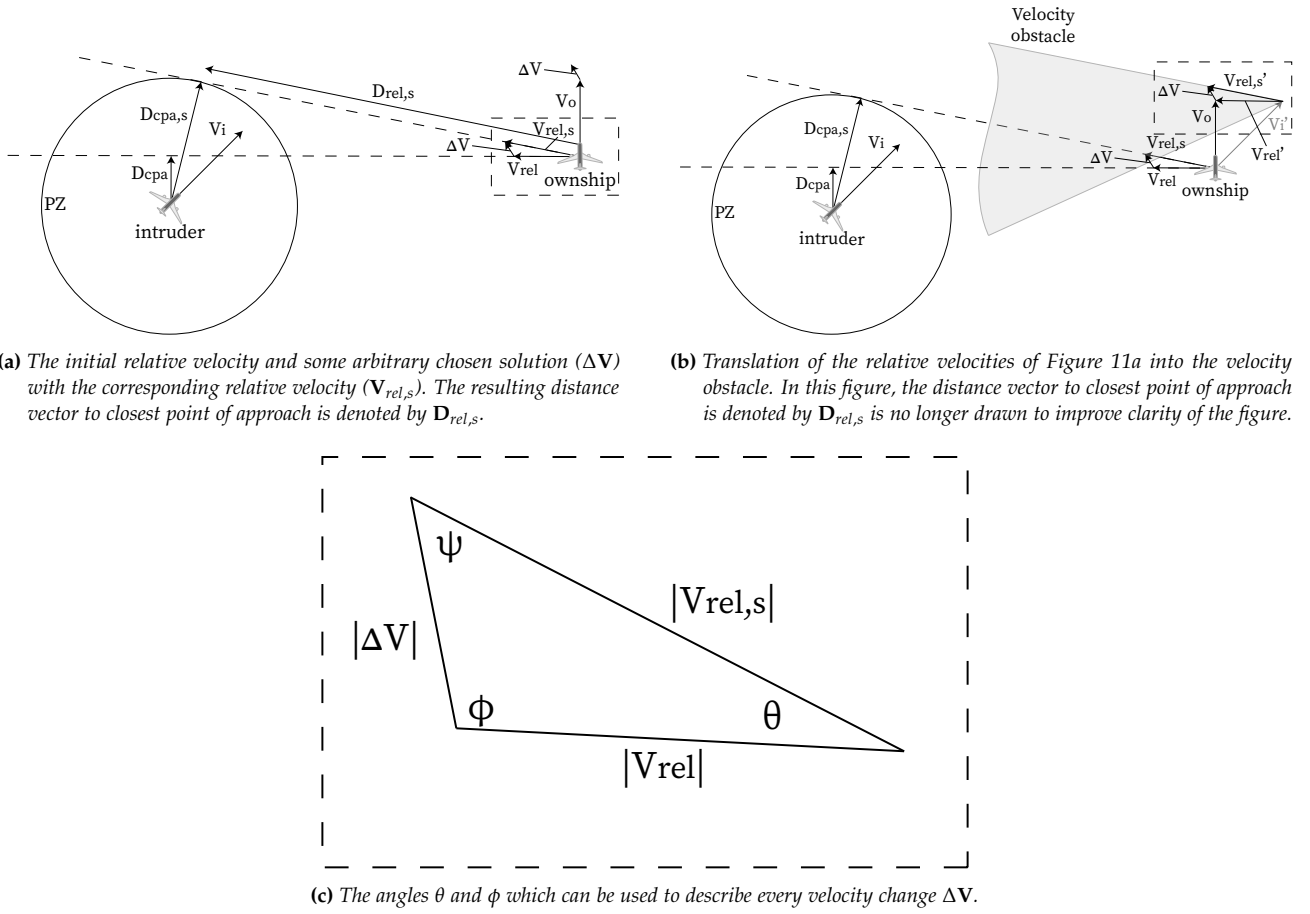


Figure 11: Illustration of the angles θ and ϕ which can be used to describe every velocity change ΔV .

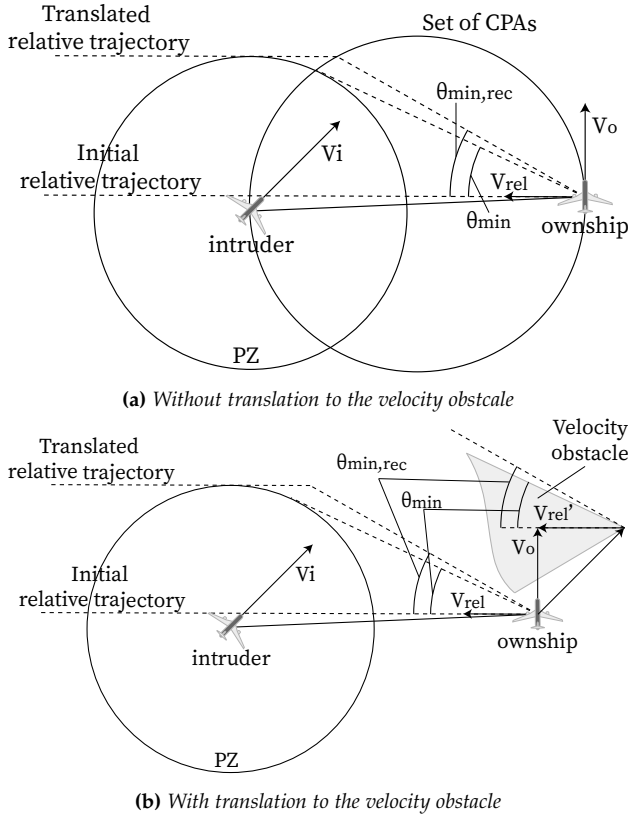


Figure 12: The minimum θ for adhering to the velocity obstacle, and thus ensures a solution which prevents an intrusion without considering the trajectory during recovery (θ_{min}). And the minimum θ which adheres to the recoverable solutions, ensuring a intrusion free solution both during the resolution and during the recovery ($\theta_{min,rec}$).

The maximum θ is, when disregarding whether a solution is recoverable or not, bound by θ values which will result in instantaneous divergence of the two aircraft. Because, if this is the case, both aircraft are continuously at closest point of approach and therefore do not know when to initiate the recovery. This is the case when the angle between the relative velocity vector corresponding to some solution and the relative bearing of the intruder is obtuse:

$$\theta_{max} = \frac{1}{2}\pi - \arcsin\left(\frac{|\mathbf{D}_{cpa}|}{|\mathbf{x}_i|}\right) \quad (10)$$

The maximal θ for a recoverable solution ($\theta_{max,rec}$), can be expressed by using Thales's theorem and is:

$$\theta_{max,rec} = \frac{1}{2} \left(\arccos\left(|\mathbf{D}_{cpa}| \left(\frac{1}{|\mathbf{x}_i|} - \frac{1}{S_h}\right) + 1\right) + \arcsin\left(\frac{|\mathbf{D}_{cpa}|}{|\mathbf{x}_i|}\right) + \pi \right) \quad (11)$$

An example illustration of the difference between the two expressions can be seen in Figure 13. In this figure, the maximum θ for both cases is depicted both with and without translating the angles to the velocity obstacle.

ii. Minimum and maximum ϕ

The minimum value for ϕ is always equal to 0. As a ϕ equal to zero results in the magnitude of the relative velocity corresponding to the solution ($|\mathbf{V}_{rel,s}|$) being equal to zero. Thus, the ownship is flying parallel to the intruder with the same ground speed, see Figure 11.

The maximum value for ϕ is dictated by the minimum and maximum ground speeds of the ownship. It must be chosen such that the resulting solution adheres to the reachable velocities as in Equation (2).

iii. Intruder Action

In this section and the remainder of the current study, it is assumed that the intruder takes no action. In other words, it is assumed that the intruder maintains its current velocity. If the conflict were to be resolved cooperatively, it would suffice for both aircraft to implement at least half of the required θ_{min} . They are not required to choose the same ϕ value since this value is not of influence to the relative trajectory. This works because of the rotational symmetry of the conflict, making it possible to let both aircraft contribute to half of the required relative velocity direction change θ . However, this is not incorporated in the derivations of the current study, as it is assumed that the intruder takes no action.

V. CHARACTERIZATION OF SOLUTIONS - INSTANTANEOUS VELOCITY CHANGE

In this section, the solutions in the state-based solution space - as defined in Equation (7) - are investigated. First, the conflict durations of certain solutions are

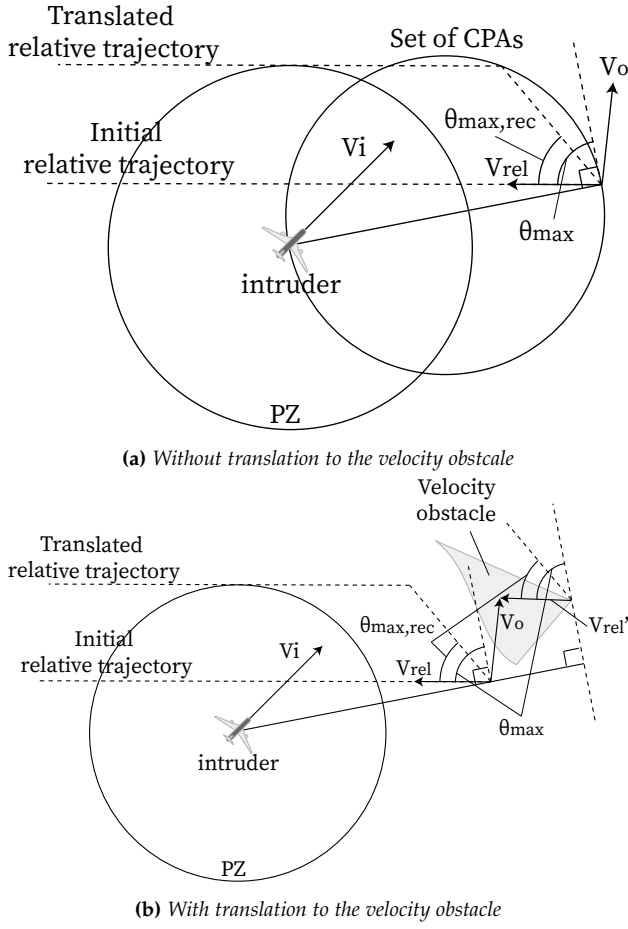


Figure 13: The maximum θ for adhering to converging solutions, and thus ensuring a solution with an expected time to closest point of approach larger than 0. And the maximum θ when adhering to the recoverable solutions, ensuring an intrusion free solution both during the resolution and during the recovery ($\theta_{max,rec}$). In both figures is the ownship aircraft not illustrated to improve the clarity of the figure.

investigated, then these conflict durations are used to investigate expected path deviations.

Three example conflicts are used to visualize the characterization of the state-based solution space. These figures are included for each characterization. In all three conflicts do the ownship and intruder fly at an initial ground speed of 300 *kts*. The ownship has a minimum and maximum velocity of 257 and 369 *kts* respectively. The aircraft have a time to intrusion (t_{int}) of 300 seconds and the horizontal separation re-

quirement (S_h) is 5 NM. The initial expected distance at CPA ($|D_{cpa}|$) is 2.5 NM. The parameter that varies between the three conflicts is the conflict angle (the heading difference between the ownship and intruder), it is set at 15, 90, and 180 degrees. These parameters are illustrated in Figure 14.

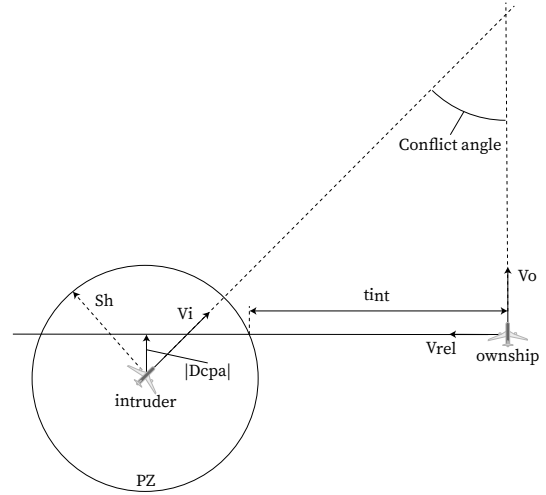


Figure 14: The parameters defining the conflict geometry used in the current study.

i. Conflict Duration

The conflict duration is defined as the time it takes for both aircraft to reach the new closest point of approach. If we let $\mathbf{x}_i$ be the distance vector between ownship and intruder and $\mathbf{V}_{rel}$ the relative velocity between ownship and intruder, then the predicted time to closest point of approach for some solution can be calculated as:

$$t_{cpa,s} = -\frac{\mathbf{x}_i \cdot \mathbf{V}_{rel,s}}{|\mathbf{V}_{rel,s}|^2}$$

The time to closest point of approach can also be expressed by using the reference frame of Section IV. In which case the time to closest point of approach is first defined as the relative distance to the closest point of approach ($|D_{rel,s}|$) divided by the relative velocity along that trajectory ($|\mathbf{V}_{rel,s}|$) [16]:

$$t_{cpa,s} = \frac{|D_{rel,s}|}{|\mathbf{V}_{rel,s}|} \quad (12)$$

These vectors can also be seen in, for instance, Figure 11a. Then, using the law of sines to relate $\mathbf{V}_{rel,s}$ to

V_{rel} :

$$\frac{|V_{rel,s}|}{\sin \phi} = \frac{|V_{rel}|}{\sin \psi} \quad (13)$$

Rewriting, and knowing that $\psi = \pi - \theta - \phi$:

$$|V_{rel,s}| = \frac{|V_{rel}| \sin \phi}{\sin(\pi - \theta - \phi)} \quad (14)$$

An expression for the relative distance to the closest point of approach ($|D_{rel,s}|$) can be found by knowing that the closest point of approach is the right angle in a right triangle between ownship location, intruder location, and closest point of approach:

$$|D_{rel,s}| = |x_i| \cos \left(\theta + \arcsin \left(\frac{|D_{cpa}|}{|x_i|} \right) \right) \quad (15)$$

Substituting Equations (14) and (15) in Equation (12) and rewriting gives:

$$t_{cpa,s} = \frac{|x_i| \cos \left(\theta + \arcsin \left(\frac{|D_{cpa}|}{|x_i|} \right) \right) \sin(\theta + \phi)}{|V_{rel}| \sin \phi} \quad (16)$$

In Figure 15, the solutions in the state-based solution space are characterized according to their predicted time to CPA. The optimal solution, in this case the solution with the lowest time to CPA, is found experimentally and indicated in orange. The figure shows that for solutions near the apex (the tip) of the velocity obstacle, the predicted time to CPA goes asymptotically to infinity because the apex of the velocity obstacle represents a solution where the ownship flies parallel to the intruder with the same ground speed as the intruder. Thus retaining a constant distance between the ownship and the intruder and causing an infinitely long conflict duration.

This is in accordance with Equation (16). As solutions go towards the apex of the velocity obstacle, ϕ goes to zero. Or, taking the limit of $t_{cpa,s}$ as ϕ goes to 0 from the right is equal to infinity as the denominator of Equation (16) goes to zero first:

$$\lim_{\phi \rightarrow 0^+} t_{cpa,s} = \infty \quad (17)$$

Additionally, as can be seen in the expression, the time to closest point of approach is inversely proportional to the magnitude of the initial relative velocity between the two aircraft.

ii. Path Deviations

If the velocity change is assumed to be instantaneous and constant, this path deviation (DEV) can be expressed by the product of the chosen velocity change ($|\Delta V|$) and the amount of time that velocity change has to be sustained (t_{cpa}) [11]:

$$DEV = |\Delta V| \cdot t_{cpa,s} \quad (18)$$

Geometrically, this path deviation is the difference between where an aircraft is located once the conflict has been resolved, compared to where it would have been located if it would have flown its nominal track for the same duration. This distance when using CPA recovery is illustrated in Figure 16.

First, the solution corresponding to a minimal path deviation is derived, then the solution space is characterized according to path deviations.

Minimal path deviation

An expression for the required velocity change for the ownship ΔV which minimizes the theoretical path deviation is found in this section. First an expression for the first term of Equation (18) is derived. By using the law of sines:

$$\frac{|\Delta V|}{\sin \theta} = \frac{|V_{rel}|}{\sin \psi} \quad (19)$$

Rewriting gives:

$$|\Delta V| = \frac{|V_{rel}| \sin \theta}{\sin \psi} \quad (20)$$

To find a useful expression for $t_{cpa,s}$, substitute a rewritten eq. (14) in eq. (12):

$$t_{cpa,s} = \frac{|D_{rel,s}| \sin \psi}{|V_{rel}| \sin \phi} \quad (21)$$

Now, substituting eq. (21) and eq. (20) in eq. (18):

$$DEV = \frac{|V_{rel}| \sin \theta}{\sin \psi} \cdot \frac{|D_{rel,s}| \sin \psi}{|V_{rel}| \sin \phi} \quad (22)$$

Rewriting:

$$DEV = \frac{|D_{rel,s}| \sin \theta}{\sin \phi} \quad (23)$$

Substituting Equation (15) then gives:

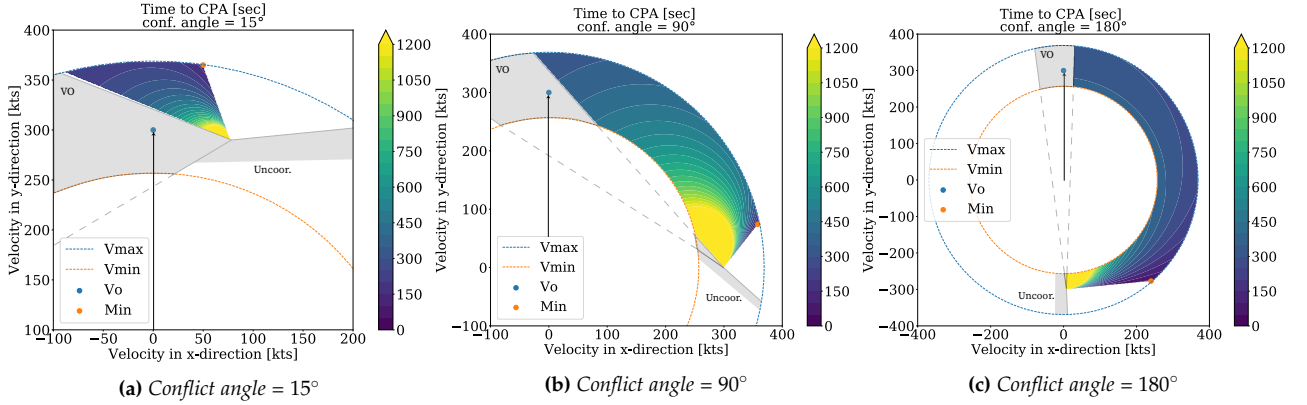


Figure 15: Predicted times to CPA within the solution space diagram of a conflict at time of initiation. Both aircraft have a ground speed of 300 kts and use a look-ahead time of 300 seconds. The predicted distance at CPA is 2.5 NM. The initial velocity of the ownship is indicated in blue, the overall solution with a minimum path deviation is indicated in orange. The velocity obstacle (VO) is depicted in grey as well as the border of the uncoordinated solutions. Improper solutions are not characterized and thus remain white.

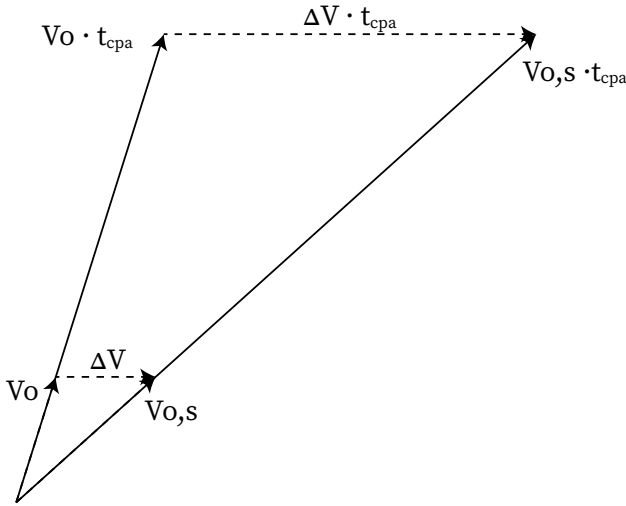


Figure 16: Nominal - no resolution - ownship velocity (V_o) and some solution velocity ($V_{o,s}$). The difference between the two is the conflict resolution provided velocity change (ΔV). The duration of the conflict is the time to CPA (t_{cpa}). The resulting theoretical path deviation is the product of ΔV and t_{cpa} . Adapted from [13].

$$DEV = \frac{\left(|x_i| \cos \left(\theta + \arcsin \left(\frac{|D_{cpa}|}{|x_i|} \right) \right) \right) \sin \theta}{\sin \phi} \quad (24)$$

Which is minimal, for some θ , if $\phi = \frac{1}{2}\pi$. Setting

$\phi = \frac{1}{2}\pi$ gives an expression for the minimal available path deviation as a function of a chosen θ :

$$DEV_{min}(\theta) = \left(|x_i| \cos \left(\theta + \arcsin \left(\frac{|D_{cpa}|}{|x_i|} \right) \right) \right) \sin \theta \quad (25)$$

Taking the derivative with respect to θ :

$$\frac{\delta}{\delta \theta} DEV_{min}(\theta) = |x_i| \cos \left(\arcsin \left(\frac{|D_{cpa}|}{|x_i|} \right) + 2\theta \right) \quad (26)$$

Which, when $|D_{cpa}| > 0$, and $|x_i| \geq |D_{cpa}|$ is positive for:

$$-\frac{1}{4}\pi < \theta + \frac{1}{2} \arcsin \left(\frac{|D_{cpa}|}{|x_i|} \right) < \frac{1}{4}\pi$$

An example construction of this set of θ 's is depicted in Figure 17.

Thus, in conclusion, the most nearby solution with a minimal path deviation has the characteristic that ΔV is directed along $\phi = \frac{1}{2}\pi$ and that θ is minimal.

If a solution with a higher than required θ value is chosen, the corresponding minimal path deviation is still found by directing ΔV along $\phi = \frac{1}{2}\pi$.

This behavior is visualized for the three example conflicts in Figure 18. Since solutions near the apex of the velocity obstacle asymptotically lead to infinity

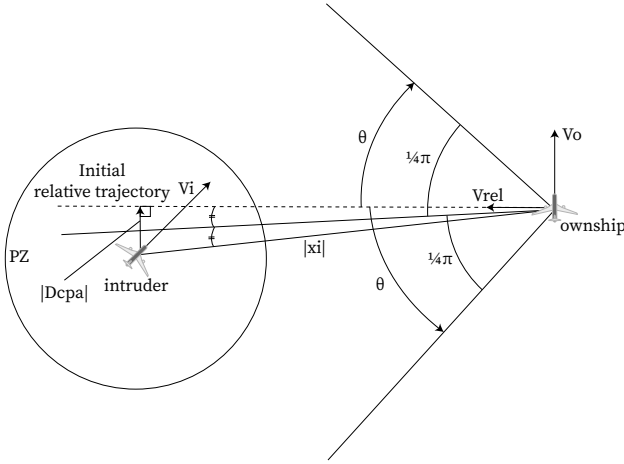


Figure 17: Example illustration of the set of θ 's on which the derivative of DEV_{min} with respect to θ is increasing.

long conflict durations, theoretical path deviations become infinity long as well. This is in accordance with Equation (24), which also states that as ϕ goes to zero from the right, the path deviation goes to infinity:

$$\lim_{\phi \rightarrow 0^+} DEV = \infty \quad (27)$$

The $\phi = \frac{1}{2}\pi$, corresponding to the optimal solution, and the other derived angles, are constructed in the state-based solution space of the conflict with a conflict angle of 15 degrees, see Figure 19.

Relating this to the behavior found for the conflict durations, it can be stated that, for the conflicts used in the current study, resolving conflicts quicker does not necessarily lead to shorter path deviations, since the corresponding higher velocity changes for those solutions might outweigh the shorter conflict durations. Additionally, resolving conflicts with a minimal $|\Delta V|$ does also not necessarily minimize path deviations, as the longer conflict duration of such a solution might outweigh the benefits of the minimal $|\Delta V|$.

An existing velocity obstacle based conflict resolution method which directs ΔV along D_{cpa} - and thus along $\phi = \frac{1}{2}\pi$ - is the Modified Voltage Potential (MVP) by Hoekstra [7]. This method sequentially resolves conflicts and uses vector addition to obtain overall solutions in multi-aircraft conflicts. Therefore, the MVP results in theoretical minimal path deviations, when assuming instantaneous velocity changes, in two aircraft conflicts. However, in multi-aircraft conflicts, this can

Table 1: Used aircraft dynamic performance characteristics

Performance Characteristic	Value
Maximum turn rate	1.5 deg/sec
Maximum acceleration / deceleration	1.0 kts/sec

no longer be guaranteed due to the vector addition.

A key difference between the MVP and the state-based solution space as presented in the current study is that the MVP does not check if solutions are recoverable. In other words, it uses Equation (8) as the minimal θ instead of Equation (9). Therefore, problems during the recovery might arise.

VI. CHARACTERIZATION OF SOLUTION - ACCOUNTED FOR AIRCRAFT DYNAMICS

Velocity changes are rarely instantaneous. The time and distance flown by the ownship before acquiring the solution velocity have, therefore, influence on the outcome of the conflict. This section incorporates a simple dynamic model to account for dynamics in the state-based solution space. During this entire section, it is assumed that the intruder maintains its initial velocity. The same example conflicts are used as in the instantaneous velocity change section.

i. Dynamic model

The dynamic model assumes a constant and instantaneous turn rate and acceleration/deceleration. The maximum turn rate corresponds to a rate half turn - 1.5 degree per second - and the maximum acceleration/deceleration is set to 1.0 kts per second, these values can be seen in Table 1.

ii. Behavior of VO and Non-Recoverable Solutions

In this section, the same example conflicts as in the previous section are characterized. The set of velocities which were a solution in the case where instantaneous velocity change was assumed but which are no longer solutions when accounting for aircraft dynamics are illustrated by the red shaded zone in the figures of this section. Both the set of non-recoverable solutions grows as well as the velocity obstacle.

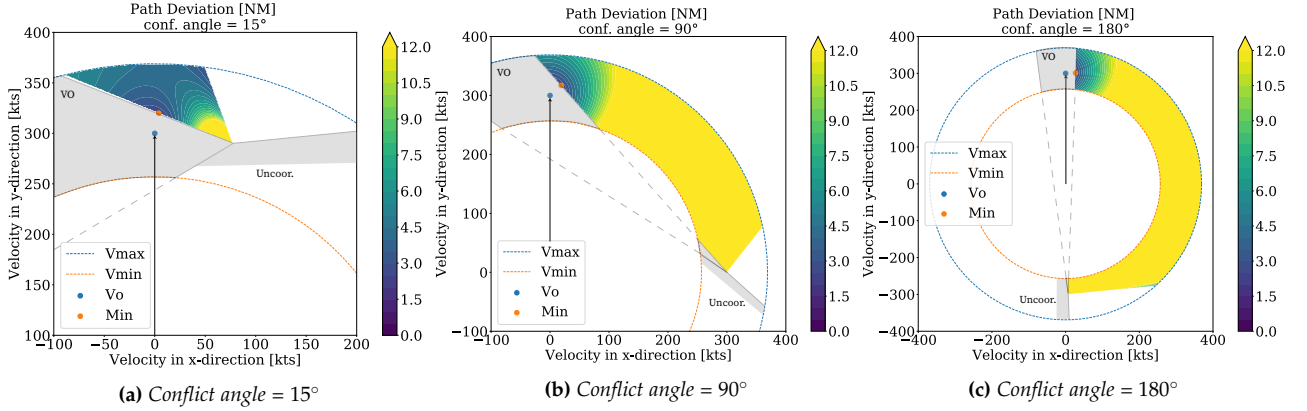


Figure 18: Theoretical path deviation when using CPA recovery within the state based solution space of a one-to-one conflict. Both aircraft have a ground speed of 300 kts and use a look-ahead time of 300 seconds. The predicted distance at CPA is 2.5 NM. The initial velocity of the ownship is indicated in blue, the overall solution with a minimum path deviation - found experimentally - is indicated in orange. The velocity obstacle (VO) is depicted in grey as well as the border of the uncoordinated solutions. Improper solutions are not characterized and thus remain white.

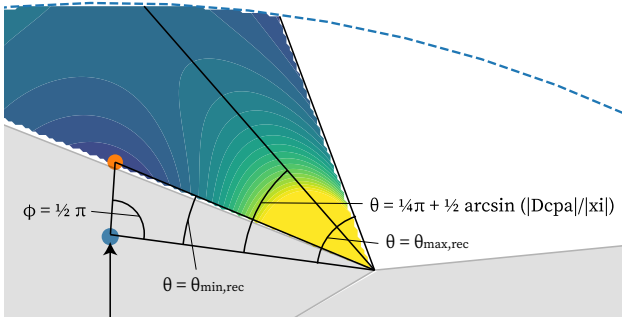


Figure 19: Example construction of the angles used in the derivation of the current study. The figure is a zoomed in subsection of Figure 18a.

The set of non-recoverable solutions grows as the distance between the ownship and intruder decreases. An example of this phenomenon is illustrated in Figure 20. In this figure, both the ownship and intruder do not change their velocity over time. As the ownship closes in on the intruder, the circle representing the set of closest points of approach shrinks. Subsequently, the area encapsulated by the intersections of the circle and the parallel translated relative trajectory shrinks as well. Logically, as the distance between the ownship and the intruder decreases, the size of the velocity obstacle increases because the angle between the tangents to the intruder's PZ increases [11, 17]. Thus, if the

ownship is on a relative trajectory within the tangents to the PZ of the intruder, both velocity obstacle legs turn away from its current velocity in the state-based solution space. Corresponding, the minimal θ for a recoverable solution increases through time.

The phenomenon that the edges of the state-based solution space - when accounting for dynamics - become curved, can, therefore, be explained by the fact that solutions on the edge of but further away from the apex of the velocity obstacle, require the velocity of the ownship to remain longer within the velocity obstacle, as the ownship is acquiring the target solution. Thus also decreasing the distance between ownship and intruder more during this maneuver. This also causes the angle between the tangents to the PZ of the intruder to increase. Therefore, velocities on the edge of but further away from the apex of the velocity obstacle experience more of this effective 'enlargement' of the velocity obstacle.

The set of velocities which were recoverable solutions in the instantaneous velocity case, but become non-recoverable when accounting for aircraft dynamics, are also indicated by a red shade. Interestingly, as the distance between two conflicting aircraft gets sufficiently low, it might be possible that there are no recoverable solutions available. This can occur, for instance, in conflicts where the heading difference is particularly low. Investigating the domain of Equation (9), it can be found that there are no recoverable

solutions available when:

$$|\mathbf{x}_i| < \frac{|\mathbf{D}_{cpa}| S_h}{|\mathbf{D}_{cpa}| - 2S_h} \quad (28)$$

If this is the case, a recoverable solution is not available. If a recoverable solution is required, either both aircraft should divert until this criterion is met or an alternative recovery method should be used.

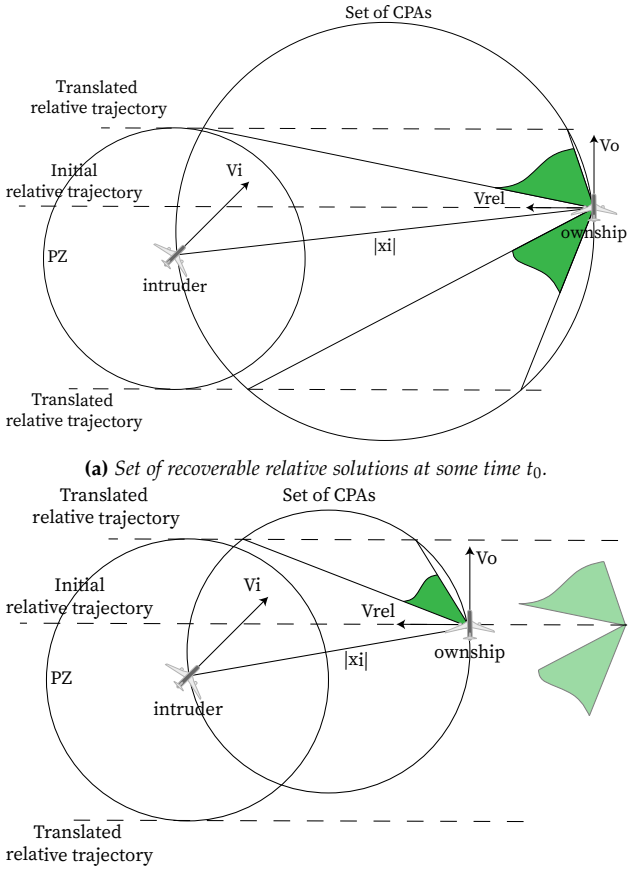


Figure 20: The set of recoverable relative solutions changes over time. As the ownship closes in on the intruder, the circle representing the set of CPAs shrinks. Subsequently, the set of recoverable relative solutions shrinks as well.

iii. Conflict Duration

The first step is generating the trajectory of the ownship and intruder for the duration that the ownship is

changing its velocity to some solution velocity. When this velocity is acquired, and the ownship and intruder are on a new location, the remaining time to closest point of approach is calculated. The total time to closest point of approach is then equal to the time it took to acquire the chosen solution velocity plus the remaining time to closest point of approach once that solution is acquired.

What this looks like for certain solutions in the state-based solution space can be seen in Figure 21. For comparison purposes, the location of the optimal solution when assuming instantaneous velocity changes is indicated as well. The conflict durations of the optimal solutions for the three conflict, both for the instantaneous velocity change case as for the dynamic case, can be seen in Table 2.

As can be expected, when accounting for dynamics, the solutions corresponding to minimal conflict durations also require flying maximum ground speeds.

For the conflict with a conflict angle of 180 degrees, the set of non-recoverable solutions has become recoverable. Indicating that the path traveled by the ownship before acquiring those solutions was a recoverable resolution in itself.

Table 2: Minimal conflict duration both when assuming instantaneous velocity changes and when accounting for the aircraft dynamics as in Table 1.

Conflict Angle [deg]	Inst. Minimum [sec]	Dyn. Minimum [sec]
15	127	195
90	98	108
180	38	127

iv. Path Deviation

Similar to the steps taken for the conflict duration, the path deviation is calculated by summing the path deviation for the duration that the ownship is changing its velocity to the solution velocity and the path deviation for the remaining time to closest point of approach. If $\Delta \mathbf{V}(t)$ is the difference between nominal and current velocity at each time t , and t_{cpa} the total duration of the conflict; then the path deviation is calculated as [13]:

$$DEV = \int_0^{t_{cpa}} \Delta \mathbf{V}(t) dt \quad (29)$$

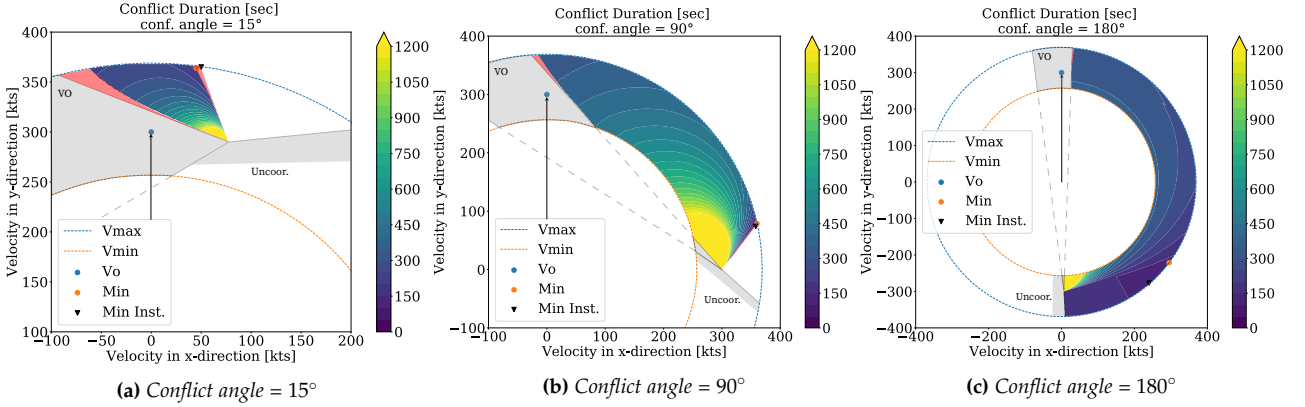


Figure 21: Predicted conflict durations within the state based solution space of a conflict at time of initiation, accounted for aircraft dynamics such as in Table 1. Both aircraft have a ground speed of 300 kts and use a look-ahead time of 300 seconds. The predicted distance at CPA is 2.5 NM. The initial velocity of the ownship is indicated in blue, the overall solution with a minimum path deviation is indicated in orange, and the minimal solution of the instantaneous velocity change case is indicated by the black triangle. The velocity obstacle (VO) is depicted in grey as well as the border of the uncoordinated solutions. Non-recoverable solutions are not characterized and thus remain white. The set of velocities which were solutions in the instantaneous velocity change case, but are no longer solutions when accounting for dynamics is indicated by the red shade.

What this looks like for certain solutions in the state-based solution space can be seen in Figure 22. Here the location of the optimum solution - when assuming instantaneous velocity changes - is indicated as well. The path deviations corresponding to the optimal solutions of the three conflicts, both for the instantaneous velocity change case as for the dynamic case, can be seen in Table 3.

Table 3: Minimal available path deviation both when assuming instantaneous velocity changes and when accounting for the aircraft dynamics as in Table 1.

Conflict Angle [deg]	Inst. Minimum [NM]	Dyn. Minimum [NM]
15	2.53	2.51
90	2.51	2.53
180	2.53	2.52

Both the location and the value of the optimal solution have changed little, indicating that dynamics have little effect on the location and value of the optimal solution. This is as expected as the optimal solutions lies close to the initial velocity.

VII. DISCUSSION AND RECOMMENDATION

The current study provided an overview of demarcations to the horizontal velocity vector space to obtain a state-based solution space. Through a reference frame based on two angles, some characteristics of this state-based solution space were defined. Then, the state-based solution space was characterized according to conflict durations and path deviations.

A solution which minimizes path deviations was derived. This corresponds, to a large extent, to the Modified Voltage Potential as proposed by Hoekstra [7]. However, the Modified Voltage Potential does not check whether solutions are recoverable, which might cause some problems to arise during the recovery.

The behavior and shape of the state-based solution space are susceptible to the maneuvers initiated by the intruder and should, therefore, be investigated. How this behavior would influence the derivations of the current study is to be investigated as well.

All conflicts in the current study were conflicts with a time to intrusion of 300 seconds. In more high-density airspace, secondary conflicts can be expected. These secondary conflicts can have lower times to intrusion. However, the optimality of directing velocity changes perpendicular to the initial relative velocity, with regard to path deviations, is independent of the time to

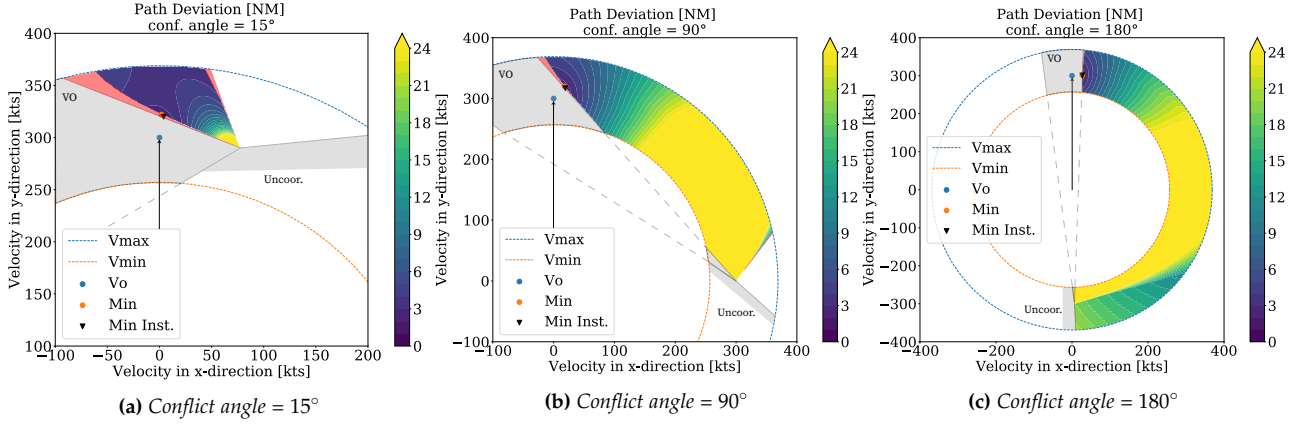


Figure 22: Predicted path deviations within the state based solution space of a conflict at time of initiation, accounted for aircraft dynamics such as in Table 1. Both aircraft have a ground speed of 300 kts and use a look-ahead time of 300 seconds. The predicted distance at CPA is 2.5 NM. The initial velocity of the ownship is indicated in blue, the overall solution with a minimum path deviation is indicated in orange, and the minimal solution of the instantaneous velocity change case is indicated by the black triangle. The velocity obstacle (VO) is depicted in grey as well as the border of the uncoordinated solutions. Non-recoverable solutions are not characterized and thus remain white. The set of velocities which were solutions in the instantaneous velocity change case, but are no longer solutions when accounting for dynamics is indicated by the red shade.

intrusion.

Different, more extensive, aircraft models should be used to investigate the behavior of the state-based solution space when accounting for aircraft dynamics.

The current study did not incorporate the dynamics of the maneuver back to the initial velocity - once CPA had been reached - into the conflict duration or path deviation. How this would change the solutions within the state-based solution space would be interesting.

VIII. CONCLUSION

An overview of the demarcations to the horizontal velocity-vector space was proposed to obtain a state-based solution space. Solutions in this state-based solution space ensure a coordinated, intrusion free, and recoverable conflict resolution trajectory.

Then, the behavior of solutions in the state-based solution space with respect to conflict durations and path deviations was investigated. Solutions within the state-based solution space with minimal path deviations were defined.

The simple aircraft dynamics used in this study have a remarkably small effect on both the location and the values of the experimentally found optimal solutions. Only for conflict durations was a noteworthy difference

observed, because its optimal solutions required more severe velocity changes and thus experienced more effect of the aircraft dynamics.

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II

Appendices

1

Appendix A: Scaling Path Deviations with the Expected Horizontal Relative Velocity

Appendix A: Scaling Path Deviations with the Expected Horizontal Relative Velocity

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I. INTRODUCTION

THIS appendix is a side-track of the article about the analysis of the state-based solution space. In this appendix, an effort to formulate a conflict resolution method that improves airspace stability is reported. Firstly, the notion of airspace stability is touched upon. Then, the corresponding concepts used to formulate the conflict resolution method are described. Finally, the state-based solution space is characterized according to this proposed method, and conclusions are drawn.

Airspace stability is a notion first used by the researchers K. Bilimoria [1] and M. Jardin [2]. When decentralized or distributed control strategies are used to maintain safe separation between aircraft, emergent behavior starts to occur. As aircraft deviate from their nominal trajectories in order to prevent a loss of separation, they might encounter a conflict with another aircraft. This, in turn, requires both aircraft to change trajectories and so on. The propagation of such conflict-chains through the airspace is captured by a metric proposed by K. Bilimoria [1]. This metric is called the Domino Effect Parameter and consists of comparing the number of conflicts in a scenario when all aircraft fly straight lines to their destination without resolving conflicts, and when all aircraft do resolve the conflicts while flying towards their destination. As the domino effect parameter gets higher, the airspace is said to have shown more unstable behavior.

In the literature, various parameters have been linked to airspace stability. This appendix will use two of these parameters to propose a conflict resolution method which could be beneficial to airspace stability.

The first parameter is the path deviations as a function of conflict resolution. Both K. Bilimoria and M.

Jardin have both related these path deviations to the airspace stability. Other research has also used the notion of these path deviations to investigate conflict resolution methods [3, 4], and even airspace structuring [5]. The conclusions are clear; lower path deviations lead to less destabilization.

The second parameter is the expected horizontal relative velocity. The study by M. Jardin used the expected horizontal relative velocity to formulate the chance of two aircraft being in conflict [2]. The author used this notion, in combination with other model parameters, to derive semi-empirical models that could be used to derive a maximum capacity of an airspace. These models were later extensively expanded in the studies by E. Sunil [5]. The models by both authors were in accordance; lower expected horizontal relative velocities decrease the chance of two aircraft getting into conflict.

Therefore, this appendix will investigate whether a conflict resolution method can be formulated which minimizes the path deviations *scaled* with the expected horizontal relative velocity during that path deviation. It is based on an assumption that a path deviation in which the expected horizontal relative velocity is low is better, from a stability standpoint, than a path deviation in which the expected horizontal relative velocity is high.

This appendix assumes no knowledge of the ground speed distribution in the airspace. Therefore, for derivation purposes, it is assumed that the mean ground speed in the airspace is equal to the ground speed of the aircraft for which a conflict resolution strategy is investigated - the ownship. Therefore, the derivations are based on flight headings only.

In airspace where flight headings are distributed uni-

formly between 0 and 360 degrees, each flight heading results in the same expected horizontal relative velocity - using the assumption stated above. Therefore, a method that scales path deviations with expected horizontal relative velocities is only relevant in an airspace where these flight headings are not distributed uniformly between 0 and 360 degrees.

A type of airspace where this is the case is a layered airspace [6, 7, 5]. In a layered airspace, the airspace is divided into vertical segments (layers) of a certain height in which a certain range of flight headings is allowed. In such a layer, the overall expected horizontal relative velocity can thus be reduced. This appendix will use the symbol α as used by E. Sunil [5] to denote the range of flight headings allowed.

First, an expression for the expected horizontal relative velocity in the layer as a function of some chosen heading is derived. This expression will then be used to scale the path deviations of the solutions within the state-based solution space. The same example conflicts and aircraft dynamics are used as in the main article.

i. Average Relative Velocity in a Layer

First, an expression is used for the horizontal relative velocity ($V_{r,h}$) between two randomly selected aircraft [7]:

$$V_{r,h} = 2V_0 \sin\left(\frac{|\Delta\psi|}{2}\right) \quad (1)$$

Where $|\Delta\psi|$ is the absolute difference in heading between the two aircraft and V_0 is the ground speed of the two aircraft. Now, let $|\omega|$ be the absolute heading difference between a solution heading and the mean heading within a layer. Three scenarios will be used to formulate the distribution of absolute heading differences between the ownship and any other aircraft.

1. *The ownship chooses a solution heading with $|\omega| = 0$:* In this case, the ownship chooses a solution with a heading equal to the mean heading of the airspace, see Figure 1a. Therefore, the distribution of expected absolute heading differences between the ownship and any other aircraft is uniformly distributed. The maximum expected heading difference is $\frac{\alpha}{2}$ since no aircraft are expected to fly outside heading range α . The probability of any absolute heading difference - below the

maximum heading difference - is equal to $\frac{2}{\alpha}$ since the absolute heading difference is used.

2. *The ownship chooses a solution heading within heading range α with $|\omega| \neq 0$:* In this case, the distribution of absolute heading differences between the ownship and any other aircraft is no longer uniform. Instead, heading differences between $\frac{1}{2}\alpha$ and $|\omega|$ have a twice as high probability of occurring compared to other heading differences, see Figure 1b.

3. *The ownship chooses a solution heading outside the heading range α , $|\omega| > \frac{\alpha}{2}$:* In this case, the distribution of absolute heading differences between the ownship and any other aircraft is uniform again. Indeed, only heading differences larger than $|\omega| - \frac{1}{2}\alpha$ and smaller than $|\omega| + \frac{1}{2}\alpha$ have a chance of occurring, see Figure 1c.

As a whole, the Probability (P) of any absolute heading difference occurring as a function the $|\omega|$ of a chosen solution heading can be expressed as:

$$P(|\Delta\psi|) = \begin{cases} \frac{2}{\alpha}, & \text{if } 0 \leq |\Delta\psi| \leq \frac{\alpha}{2} - |\omega| \\ \frac{1}{\alpha}, & \text{if } \frac{\alpha}{2} - |\omega| < |\Delta\psi| \leq \frac{\alpha}{2} + |\omega| \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

The expected relative velocity between the ownship and any other aircraft can then be calculated using [7]:

$$\mathbb{E}(V_{r,h}) = \int V_{r,h}(|\Delta\psi|) P(|\Delta\psi|) \quad (3)$$

Substituting Equations (1) and (2) in Equation (3) gives:

$$\mathbb{E}(V_{r,h}) = \begin{cases} \int_0^{\frac{\alpha}{2}-|\omega|} 2V_0 \sin\left(\frac{\Delta\psi}{2}\right) \frac{2}{\alpha} d\Delta\psi \\ + \int_{\frac{\alpha}{2}-|\omega|}^{\frac{1}{2}\alpha+|\omega|} 2V_0 \sin\left(\frac{\Delta\psi}{2}\right) \frac{1}{\alpha} d\Delta\psi, & \text{if } |\omega| < \frac{\alpha}{2} \\ \int_{-\frac{1}{2}\alpha+|\omega|}^{\frac{1}{2}\alpha+|\omega|} 2V_0 \sin\left(\frac{\Delta\psi}{2}\right) \frac{1}{\alpha} d\Delta\psi, & \text{if } |\omega| \geq \frac{\alpha}{2} \end{cases} \quad (4)$$

Which results in:

$$\mathbb{E}(V_{r,h}) = \begin{cases} -\frac{8V_0(\cos(\frac{\alpha}{4})\cos(\frac{\omega}{2})-1)}{\alpha}, & \text{if } |\omega| < \frac{\alpha}{2} \\ \frac{8V_0\sin(\frac{\alpha}{4})\sin(\frac{\omega}{2})}{\alpha}, & \text{if } |\omega| \geq \frac{\alpha}{2} \end{cases} \quad (5)$$

This equation provides the expected horizontal relative velocity between the ownship and any other aircraft in the layer. Using this expression, the ownship

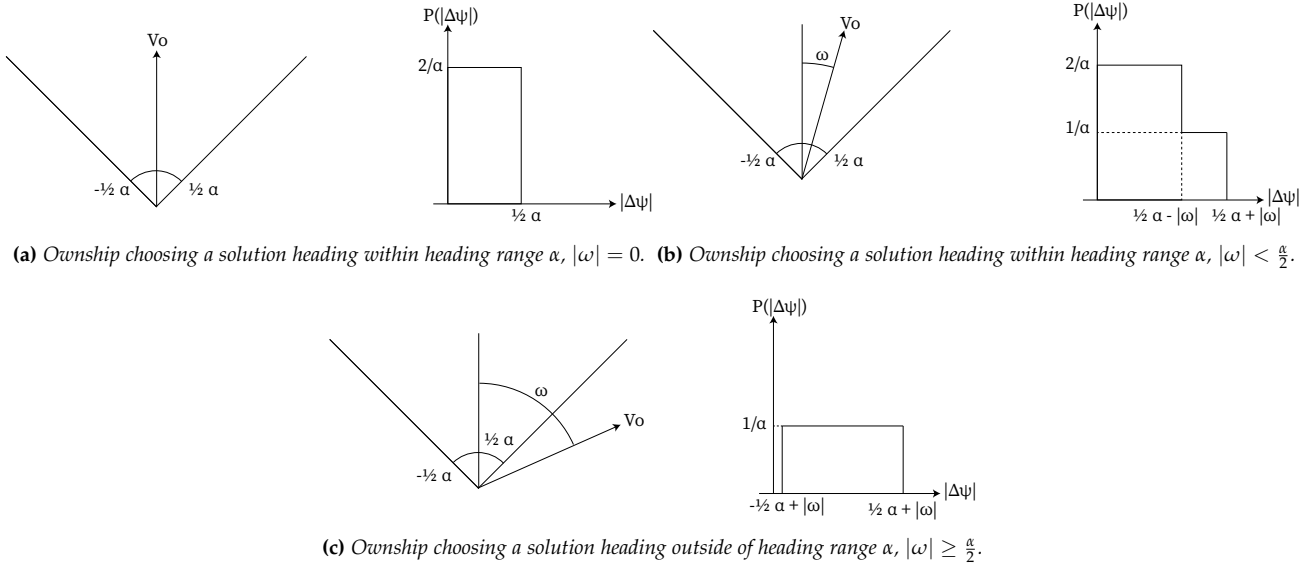


Figure 1: Probability Density Functions for the three used scenarios. The velocity vector of the ownship is denoted by V_o .

can make choices that result in a low expected horizontal relative velocity. To illustrate this, the solution space will be characterized according to the dimensionless $\mathbb{E}(V_{r,h})/V_0$ for a conflict in an airspace with heading range $\alpha = 180^\circ$ and where the mean ground speed V_0 is assumed to be 300 kts, equal to the initial ground speed of the ownship. Figure 2 shows the state-based solution space when the ownship is initially flying at $|\omega| = 90^\circ$ in a heading range $\alpha = 180^\circ$ layer. These values are chosen because they should lead to a relatively strong incentive to choose solutions more in the direction of the mean flight heading of the airspace compared so smaller heading ranges and smaller $|\omega|$ values. Logically, as can be seen in the solution space, choosing solutions with lower $|\omega|$ values result in trajectories with a lower $\mathbb{E}(V_{r,h})/V_0$ value.

When the path deviations of the main article are scaled with the dimensionless expected horizontal relative velocity, as in Figure 3, it can be seen that the optimal values remain very close to the optimal values of the path deviation DEV . In fact, the addition of $\mathbb{E}(V_{r,h})/V_0$ has a, to some extent, neglectable effect on the location of the optimal solution.

ii. Accounting for Simple Aircraft Dynamics

As stated in the introduction, the same simple aircraft dynamics are used as in the main article. When accounting for aircraft dynamics, the mean expected horizontal relative velocity of the maneuver is used ($\mathbb{E}(V_{r,h})$). Let t_{cpa} be the time to closest point of approach counted from initiation of the conflict. The ownship first has to accelerate/decelerate and change heading to obtain the chosen solution velocity, then this solution velocity has to be sustained until closest point of approach is reached. Thus, the mean expected horizontal relative velocity during the conflict can be calculated as:

$$\mathbb{E}(V_{r,h}) = \frac{1}{t_{cpa}} \int_0^{t_{cpa}} \mathbb{E}(V_{r,h}(t)) dt \quad (6)$$

Figure 4 shows these mean expected horizontal relative velocities for solutions in the state-based solution space. Logically, solutions corresponding to lower $|\omega|$ values result in lower mean expected horizontal relative velocities. For solutions with a heading towards the mean heading within the layer, lower ground speeds result in lower mean expected horizontal relative velocities, because this results in a higher remaining time to closest point of approach, which thus reduces the average expected horizontal relative velocity even further.

When the path deviation of the main article is scaled

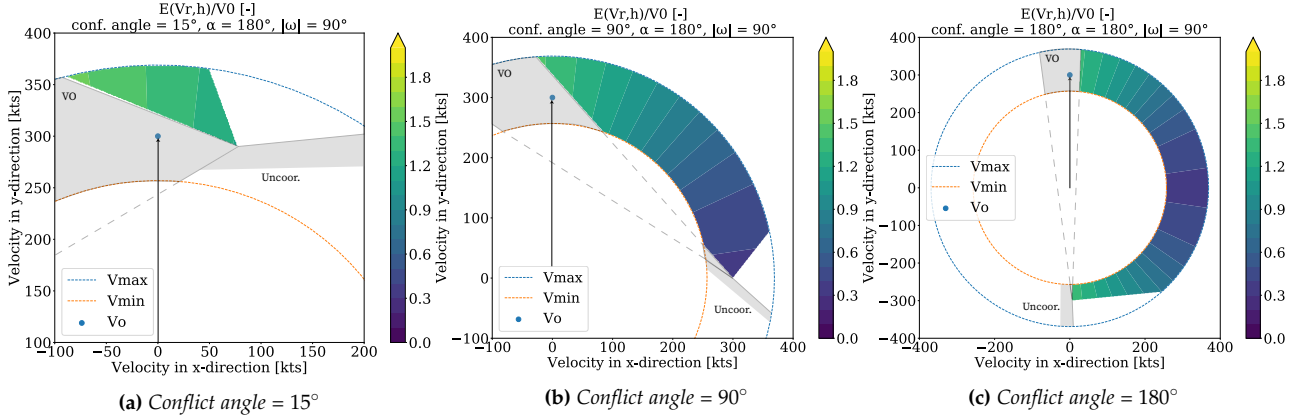


Figure 2: Expected horizontal relative velocity within the state based solution space of a one-to-one conflict. Both aircraft have a ground speed of 300 kts and use a look-ahead time of 300 seconds. The predicted distance at CPA is 2.5 NM. The initial velocity of the ownship is indicated in blue. The velocity obstacle (VO) is depicted in grey as well as the border of the uncoordinated solutions. Non-recoverable solutions are not characterized and thus remain white.

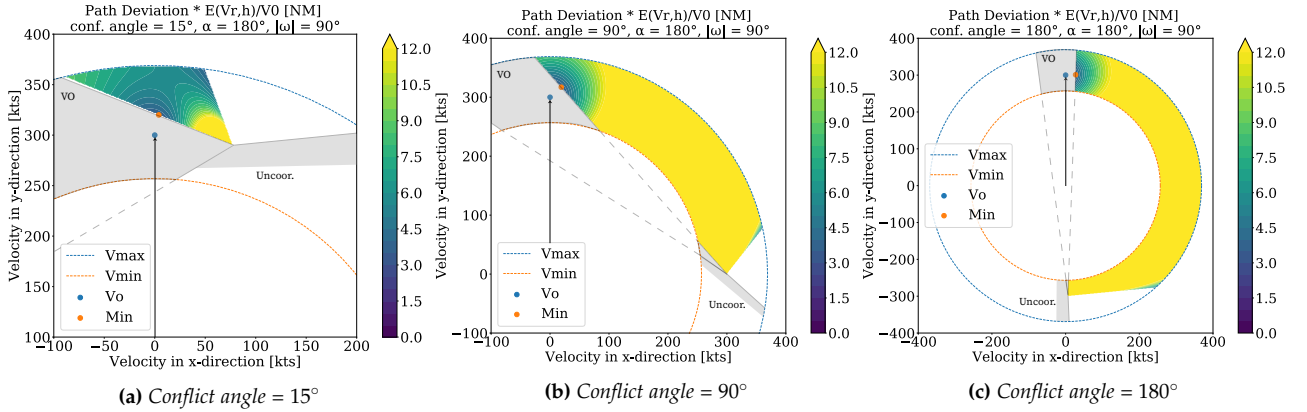


Figure 3: Expected path deviations scaled with the dimensionless expected horizontal relative velocity. The CPA is used as end-of-conflict indication. Both aircraft have a ground speed of 300 kts and use a look-ahead time of 300 seconds. The predicted distance at CPA is 2.5 NM. The initial velocity of the ownship is indicated in blue, and the overall solution with a minimum path deviation is indicated in orange. The velocity obstacle (VO) is depicted in grey as well as the border of the uncoordinated solutions. Non-recoverable solutions are not characterized and thus remain white.

with the dimensionless expected relative velocity of each solution, as in

$$DEV_{scaled} = \int_0^{t_{cpa}} \Delta \mathbf{V}(t) \cdot \frac{\mathbb{E}(\mathbf{V}_{r,h}(t))}{V_0} dt \quad (7)$$

the behavior of Figure 5 is found. The scaled path deviations of the optimal solutions for the three conflicts, both for the instantaneous velocity change case as for the dynamic case, can be seen in Table 1. For all conflicts, the locations of the optimal solutions have shifted little, indicating that dynamics

have little effect on the location and value of the optimal solution. Therefore, also when accounting for dynamics, the solution corresponding to the minimal scaled path deviation is remarkably close to the solution corresponding to the minimal unscaled path deviation. Also confirming that, in fact, the scaling of path deviations has a - to some extent - neglectable effect on the location of the optimal solution.

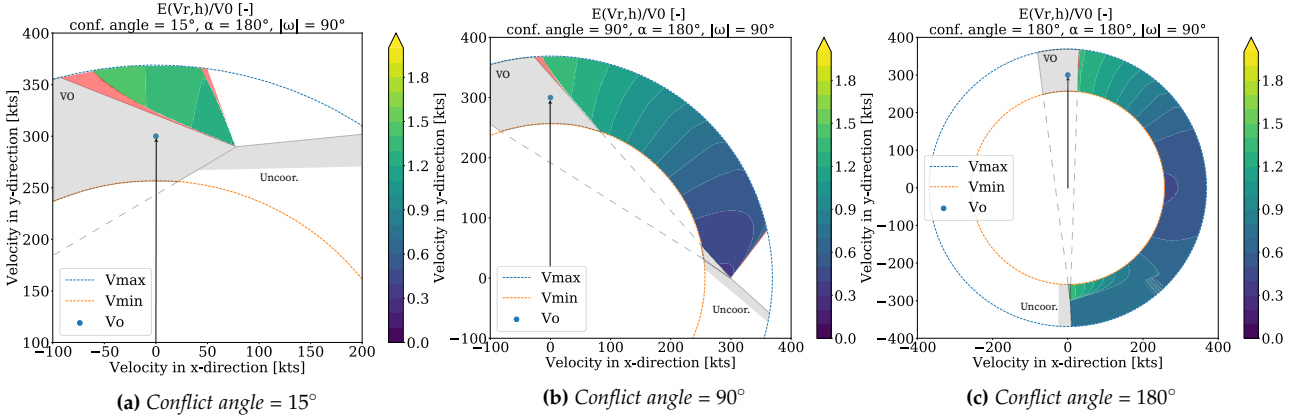


Figure 4: Mean expected horizontal relative velocity - for the duration of the conflict - within the state based solution space of a conflict, accounted for simple aircraft dynamics such as in the main article. Both aircraft have a ground speed of 300 kts and use a look-ahead time of 300 seconds. The predicted distance at CPA is 2.5 NM. The initial velocity of the ownship is indicated in blue, the overall solution with a minimum path deviation is indicated in orange, and the minimal solution of the instantaneous velocity change case is indicated by the black triangle. The velocity obstacle (VO) is depicted in grey as well as the border of the uncoordinated solutions. Non-recoverable solutions are not characterized and thus remain white. The set of velocities which were solutions in the instantaneous velocity change case, but are no longer solutions when accounting for dynamics is indicated by the red shade.

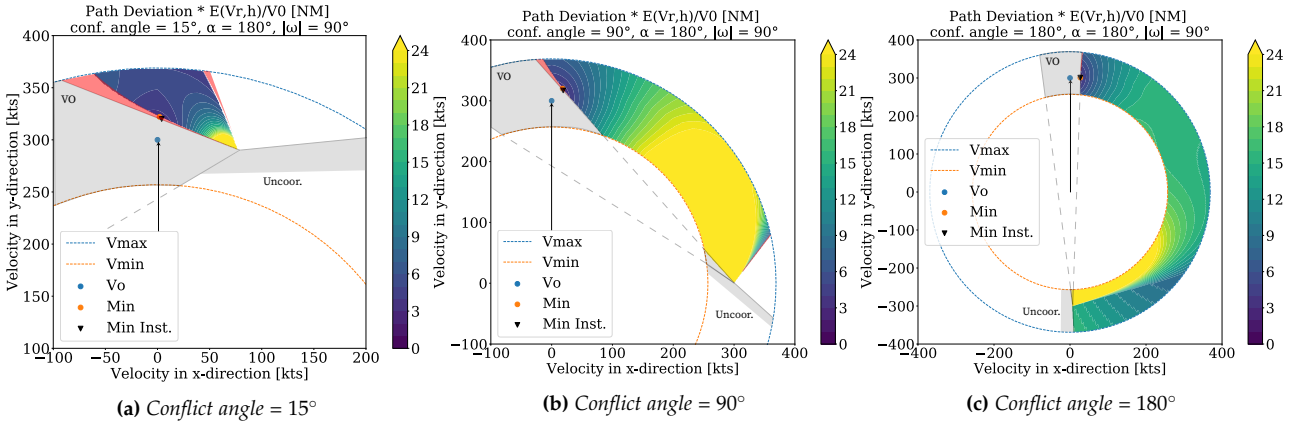


Figure 5: Predicted path deviations scaled with the expected horizontal relative velocity within the state based solution space of a conflict, accounted for simple aircraft dynamics such as in the main article. Both aircraft have a ground speed of 300 kts and use a look-ahead time of 300 seconds. The predicted distance at CPA is 2.5 NM. The initial velocity of the ownship is indicated in blue, the overall solution with a minimum path deviation is indicated in orange, and the minimal solution of the instantaneous velocity change case is indicated by the black triangle. The velocity obstacle (VO) is depicted in grey as well as the border of the uncoordinated solutions. Non-recoverable solutions are not characterized and thus remain white. The set of velocities which were solutions in the instantaneous velocity change case, but are no longer solutions when accounting for dynamics is indicated by the red shade.

II. CONCLUSION

Since the effect of scaling path deviations with expected horizontal relative velocities had such a neglectable

effect on the location of the optimal solutions - both with and without accounting for aircraft dynamics, this method was not investigated further for the time being. However, differentiating between different kinds

Table 1: Minimal path deviation, scaled with the dimensionless expected horizontal relative velocity, both when assuming instantaneous velocity changes and when accounting for the aircraft dynamics as in the main article.

Conflict Angle [deg]	Inst. Minimum [NM]	Dyn. Minimum [NM]
15	3.46	3.45
90	3.35	3.39
180	3.33	3.32

of path deviations, whether or not based on expected horizontal relative velocities, can prove interesting from a stability standpoint. It is therefore recommended to investigate conflict resolution methods that minimize path deviations scaled with other model parameters.

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2

Significance Tables

To determine whether the differences between the non-normally distributed samples obtained with CPA recovery and two criteria recovery are significant, the non-parametric Wilcoxon signed-rank test was used. This is a paired difference test to determine whether the population mean ranks differ. The null-hypothesis is that the two dependent samples were selected from populations that have the same distribution. If this null-hypothesis is rejected with a $p < 0.05$, the differences between the samples is assumed to be significant. In the current study, the choice is made to not apply a multiple comparisons correction. This choice was made because no hypothesis was particularly subjected to the influence of single false-positives between two samples. Therefore, there was no need to reduce the *power of the test* (β) by applying a correction to the Wilcoxon signed-rank test as-is.

Table 2.1: p-values for the Wilcoxon signed-rank test when comparing the samples for the DEP when counting all conflict occurrences.

Density	360°	180°	90°	45°	22.5°	11.25°
A	0.028	0.007	0.005	0.005	0.005	0.005
B	0.005	0.005	0.005	0.005	0.005	0.005
C	0.005	0.005	0.005	0.005	0.005	0.005

Table 2.2: p-values for the Wilcoxon signed-rank test when comparing the samples for the DEP when counting unique conflict pairs.

Density	360°	180°	90°	45°	22.5°	11.25°
A	0.012	0.173	0.721	0.859	0.139	0.011
B	0.005	0.007	0.114	0.441	0.028	0.012
C	0.008	0.005	0.110	0.037	0.508	0.028

Table 2.3: p-values for the Wilcoxon signed-rank test when comparing the samples for the percentage increase in distance flown compared to the nominal experiment.

Density	360°	180°	90°	45°	22.5°	11.25°
A	0.007	0.114	0.007	0.114	0.005	0.176
B	0.012	0.005	0.074	0.074	0.037	0.007
C	0.005	0.005	0.203	0.721	0.799	0.007

Table 2.4: p-values for the Wilcoxon signed-rank test when comparing the samples for mean conflict durations when counting all conflict occurrences.

Density	360°	180°	90°	45°	22.5°	11.25°
A	0.000	0.000	0.000	0.000	0.000	0.000
B	0.000	0.000	0.000	0.000	0.000	0.000
C	0.000	0.000	0.000	0.000	0.000	0.000

Table 2.5: p-values for the Wilcoxon signed-rank test when comparing the samples for the percentage of aircraft involved in multi-aircraft conflicts.

Density	360°	180°	90°	45°	22.5°	11.25°
A	0.000	0.000	0.004	0.187	0.534	0.274
B	0.000	0.000	0.001	0.002	0.312	0.008
C	0.000	0.000	0.000	0.000	0.174	0.008

III

Preliminary Report

Thesis framework

1.1. Introduction

At the beginning of the 1990s, various reports started addressing the increasing costs and delays due to limitations of the current Air Traffic Management system. A report by the Air Transport Association of America estimated that these limitations contributed to losses as high as 3.5 billion dollars per year in 1994 (Air Transport Association of America, 1994). As a result, various airlines started to investigate alternative ATM systems, to find one that would cope better with the increasing traffic density and would increase the efficiency of the airspace. The idea of having pilots fly direct routes towards their destination, instead of flying through the predefined layouts, quickly got a hold on the aviation industry. As a result, the Radio Technical Commission for Aeronautics (RTCA) convened a committee to investigate this concept (RTCA, 1995). This concept was subsequently named free-flight.

A recurring definition in this research is self-separation. Self-separation is the capability to maintain safe separation from other aircraft without the need for instructions from air traffic control. The separation requirement is geometrically described in most parts of the world as a flat disk around the aircraft with a radius of 5 nautical miles and a total height of 2000 feet. This is often called the Protected Zone (PZ). By propagating state information, such as altitude, speed, and heading, two aircraft are said to be in conflict when a Loss-of-Separation (LoS) is expected in the future. Most research focuses on three elements to achieve this self-separation; conflict detection (CD), conflict prevention (CP), and conflict resolution (CR).

Eby (1994) proposed a method to determine collision-free routes for aircraft without the use of fixed routes. His self-separation method is based on using modified electrostatic equations to construct conflict resolution strategies. The method proposed by Eby is based on the approach of horizontally "pushing away" the trajectories of both conflicting aircraft until these do no longer cross the PZ of the other. This is where the analogy to potential fields lies. Hoekstra et al. (2002) later modified this potential field based method and called it the Modified Voltage Potential (MVP). The adaptation consisted of pushing away the Closest Point of Approach (CPA) until it no longer lies within the PZ. The mechanics of the method by Hoekstra et al. will be explained in section 2.2.

The MVP makes use of implicitly coordinated CR. Implicitly coordinated CR entails CR methods which make use of a set of predefined rules. This ensures that the CR will be successful without communication between the two aircraft. As described before, the rule-set of the MVP consists of pushing away the CPA. Indeed, when both aircraft follow this rule, the solution is implicitly coordi-

nated.

One aspect of CD, CR, and CP, which will be recurring during this thesis, is conflict angles. The conflict angle is the difference in heading between two conflicting aircraft. Sunil (2019) performed interesting research making use of the MVP and conflict angles. Sunil described a way to analyze and model capacity for decentralized air traffic control. One aspect of this research was analyzing the effect of subdividing the airspace into layers of equal heights. Each layer has an allowed flight heading. This way, each layer consists of aircraft flying a defined range of headings. To analyze what the effect of this subdivision is on the airspace capacity, the impact on the conflict count and conflict resolution behavior had to be investigated. Sunil found that decreasing the allowed heading ranges per layer did not necessarily increase the capacity per layer. He argued that reducing the heading range per layer does not lead to an increase in capacity of the layered airspace that uses the modified voltage potential as CR because the safety benefits of lowering the heading range per layer are counteracted by the drop in efficiency caused by longer resolution trajectories.

As a whole, the (emergent) effects of conflict angles on the behavior of the MVP are not yet fully investigated. In addition, the effects these angles have on the behavior of the MVP method and characteristics of airspace structure, leave room for further research and potential improvements. This thesis will address some of the questions regarding the effect of conflict angles on the behavior of the MVP and address potential solutions, of which the objectives will be described in the next section.

1.2. Objective of this Thesis

To define the scope of this thesis, the following research objective is defined:

Investigate the effects of conflict angles on airspace stability and conflict resolution behavior in general when the Modified Voltage Potential is used as CR. Additionally, Propose an adequate set of solutions to counter any negative effects and investigate the effects these solutions have on airspace metrics and conflict resolution behavior.

To ensure that the research objective will be completed, the following activities are defined:

Part 1: Investigate the effects of conflict angles on airspace stability and conflict resolution behavior in general when the Modified Voltage Potential is used as CR.

This first aim of the research objective is split into the following research activities:

Research Activity 1

Define a set of metrics to characterize the MVP performance and airspace stability under varying conflict angles.

To investigate which effects conflict angles have on the performance of the MVP under varying conflict angles, a set of metrics is to be defined, which describes the differences in performance best. The results of these metrics are the main drivers for changes in the algorithm proposed in part 2 of the thesis. The set of metrics must provide insights into the performance of both small scale and large scale simulations.

1. Which set of metrics describe the effects of conflict angles on the performance of the MVP and airspace stability best?

Research Activity 2

Set up experiment scenarios to evaluate the MVP performance under varying conflict angles.

The metrics described in research activity 1 are used to describe the performance of the MVP algorithm in various experiments. These experiments must describe the performance as precise as possible, with as little disturbing factors as possible. Both many-to-many, as well as one-to-one conflict scenarios, are to be used in this phase because they both can shed light on the problem in different ways. many-to-many experiments, for instance, provide more insight regarding the emergent behavior of the algorithm. one-to-one conflict experiments, however, offer the chance to gain more insight regarding the direct behavior of the algorithm.

1. What is the most appropriate setup to a one-to-one conflict angle experiment?
2. What is the most appropriate setup to a many-to-many conflict angle experiment?

Research Activity 3

Run the defined scenarios to evaluate the MVP performance under varying conflict angles.

The defined scenarios in activity 2 can subsequently be simulated to evaluate the MVP performance under varying conflict angles. If during the simulation of the scenarios, it is found that certain behavior starts to emerge, additional scenarios will be defined to investigate this behavior further. This is why this research activity alternates with research activity 2.

Research Activity 4

Explain the performance of the experiments using the metrics and the MVP mechanics.

After completion of activity 3, the results of the experiments are then to be interpreted. It is important to determine which distinctive results are to be contributed to conflict heading changes, and which are not to be contributed to conflict heading changes. These unique results then have to be explained using the mechanics of the MVP algorithm and the metrics, whilst keeping in mind the effects of simulation characteristics and the way the MVP was implemented in the simulation platform.

1. Which metrics and mechanics of the MVP algorithm contribute to which performance results in its behavior?

Part 2: Propose a set of solutions to counter any negative effects and investigate the effects of these solutions on airspace metrics and conflict resolution behavior.

Research Activity 5

Define the solution space.

A solution space has to be defined, which contains the solutions which can be used for modifications to the algorithm. Any algorithm modification proposed in this thesis will be based on this solution space.

1. How can the solution space be defined?

Research Activity 6

Characterize the solutions within the solution space.

The solutions space defined in activity 5 contains all the possible solutions while maintaining the coordination and pair-wise approach of the MVP. These solutions have to be characterized based on their effects on the resolution maneuver.

1. How can the solutions within the solution space be characterized?

Research Activity 7

Propose an adequate set of methods to counter the found negative effects making use of the solution space.

The characterization of solutions within the solution space defined in research activity 6 can be used to propose adequate solutions for the negative performance characteristics found in research activity 4.

1. Which possible implementation of solutions within the solution space is adequate concerning the found negative performance characteristics under varying conflict angles?

Research Activity 8

Implement these methods in BlueSky.

Each selected modification to the MVP algorithm found in research activity 7 has to be implemented in BlueSky. This is necessary to compare the behavior of the modified algorithms with the MVP. Because this implementation can be performed in different ways, it has to be established which method of implementation is adequate. It has to be ensured that the comparison is based on the fundamental mechanics of the methods, not on the implementation strategies.

1. How might different ways of implementation influence the performance of both the original and modified MVP algorithms?

Research Activity 9

Set up simulation scenario's to compare the MVP performance to the modified algorithms.

A set of simulation scenarios is required to compare the MVP performance to the performance of the modified algorithms. These scenarios are not necessarily the same as the ones in activity 3. The main goal of these scenarios is to explain the performance difference between the modified algorithms and the MVP as complete as possible. To achieve this, both large scale, as well as small scale scenarios, are to be used. This because they both can shed light on the differences in different ways. Large scale experiments, for instance, give more insight into the emergent behavior differences. Small scale experiments, however, offer the chance to gain more insight regarding direct performance differences of the modified algorithms and the MVP.

1. What is the most appropriate setup for a small scale conflict angle experiment to describe the differences between the modified algorithms and the MVP as complete as possible?
2. What is the most appropriate setup for a large scale conflict angle experiment to describe the differences between the modified algorithms and the MVP as complete as possible?

Research Activity 10

Run the defined scenarios to compare the MVP performance to the proposed methods.

The described scenarios in activity 9 can subsequently be simulated to compare the MVP performance to the proposed methods. If, during the simulation of the scenarios, it is found that particular behavior starts to emerge, additional scenarios will be defined to investigate this behavior further.

Research Activity 11

Compare the modified CR algorithms to the classical MVP algorithm.

Finally, using the results of the described experiments, the proposed methods can be compared to the MVP.

1. In what way do the various proposed methods compare to the MVP?

1.3. Approach of this Research

To meet the research objective, all research activities have to be completed. In this section, the sequence of research activities is explained.

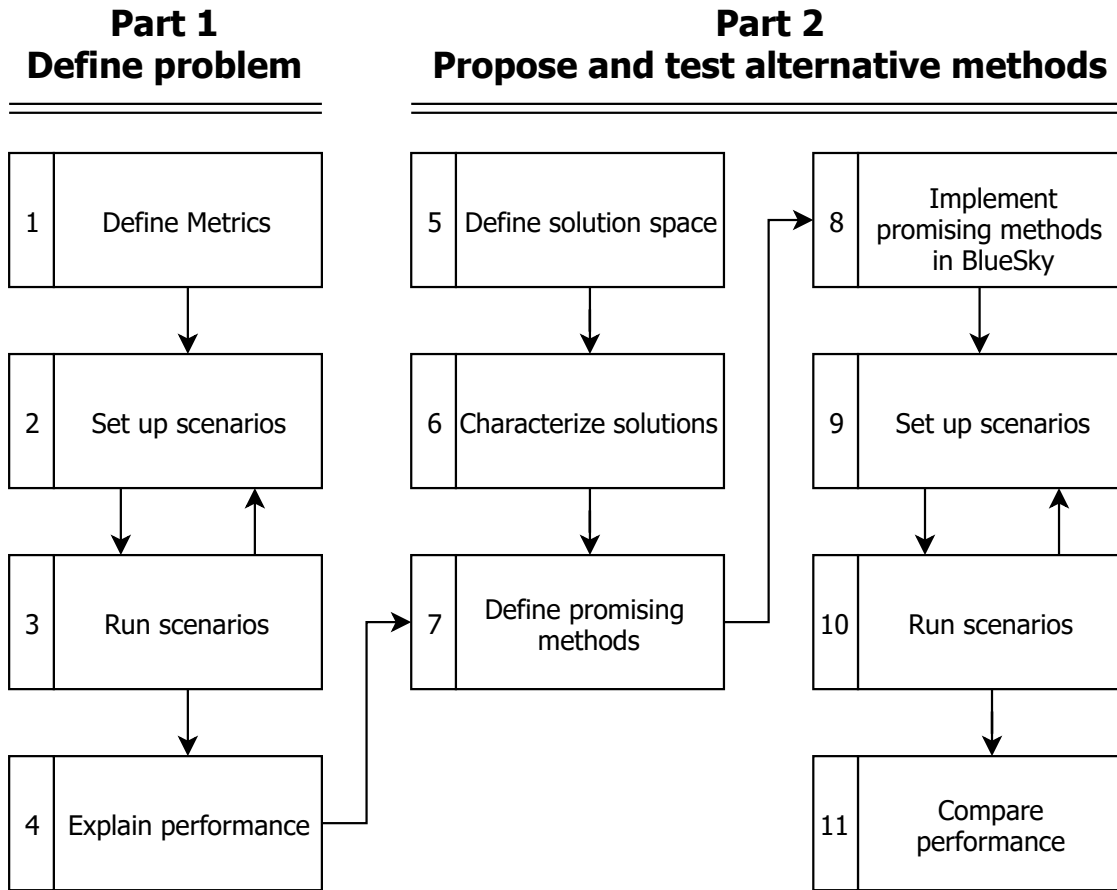


Figure 1.1: A flowchart of the research approach of this thesis.

1.4. Scope of this Research

In order to limit the scope of this thesis, additional demarcations are presented in the following sections. Note that these demarcations can be revoked later in the thesis if this is required.

1.4.1. Definitive scope demarcation

The following assumptions mark the considerations under which the results are obtained.

The airspace is assumed to be unmanaged

As this research is focused on investigating the performance of decentralized CD&R methods, all aircraft in the experiments are supposed to fly towards their destination in a straight line, unless the CD&R method dictates otherwise. No additional airspace manager directing routes will be present.

The proposed methods will use horizontal resolution maneuvers.

To isolate the effects of the conflict angles between the aircraft on the CR performance, only hor-

horizontal resolution maneuvers will be considered. This choice was made to keep this research in line within studies where vertical maneuvers are not possible (such as in the layer related research), or vertical maneuvers are excluded since pilots do not prefer these vertical resolution maneuvers (Rantanen et al., 2006).

Conflicts are resolved pair-wise

A characteristic of the MVP algorithm is that conflicts are resolved pair-wise. This means that resolution strategies are calculated for each conflict pair individually. If an aircraft is in conflict with two aircraft at the same time, both conflicts will be treated separately. The resulting resolution strategies are then summed to obtain the overall resolution strategy.

Multi aircraft conflicts are resolved using an equally weighted summing-up technique.

As described in the assumption above, multi-aircraft collisions are resolved by summing the pair-wise obtained resolution strategies. The MVP algorithm treats each resolution strategy in a multi-aircraft conflict separately, not assigning more weight to resolve one conflict over another. Any propositions for modified algorithms will follow the same equally weighted summing-up technique.

The proposed adapted CR methods will use implicit coordination.

Determining resolution maneuvers can either be done explicitly, implicitly or uncoordinated. Implicit coordination means that both aircraft will decide upon their resolution maneuver based on a common set of rules deposited by Jenie et al. (2017). Explicit negotiation between the aircraft is not required for safe passing of the resolution.

The proposed adapted CR methods will be tactical.

Tactical conflict resolution entails the set of CR methods that do not use intent information to calculate the resolution strategies and thus focuses on mid-range actions. This also has an influence on the look-ahead time. A look ahead time of 5 minutes will be used. This ensures that the CR method will be tactical and prevents any unnecessary resolution strategies for conflicts in the far future. Furthermore, according to McCallie et al. (2011), the coverage of ADS-B indicates a maximum look-ahead time of five minutes.

The method will assume perfect ADS-B data

Finally, the data broadcast between the aircraft is assumed to be perfect. No errors in the broadcasted state or location will be assumed as well as no latency in the broadcast signal. This then implies that all the required information is available continuously. A study by Langejan et al. (2016) has shown that this assumption does not have problematic effects on the performance of the CD and CR methods used in this thesis.

1.4.2. Preliminary scope demarcation

The preliminary scope demarcation is a type of demarcation that might be relaxed if required or if time is left within the thesis duration.

Only en-route traffic is considered

All traffic used in this research will be en-route and flying in cruise. This because if decentralized air traffic control using the self-separation as described in this research would become a reality, it would first be implemented in en-route airspace. Additionally, when flying en-route, aircraft have sufficient room to maneuver in order to resolve conflicts. Furthermore, the effects of airspace structuring on the behavior of CR is not the focus of this thesis.

No wind influence will be simulated

The effect of wind on the resolution strategies and their performance is not the main focus of this thesis. However, if there is time left, it costs very little extra effort to implement it in experiments. This is because BlueSky has a fully functional wind simulating module.

Each experiment will consist of one aircraft type.

Initially, each experiment will consist of one type of aircraft (B737). This ensures that all aircraft have identical performance limits. However, using a mix of aircraft types or another aircraft type is very little work to achieve in BlueSky.

A step-wise recalculation approach with $\Delta t = 1\text{ s}$ is used.

In BlueSky, conflict detection and resolution maneuver calculation is performed at a certain time interval. The default of this time interval is one second. This means that if a conflict is detected, new resolution maneuvers are calculated each second until the trajectories are conflict-free. When the aircraft have passed the closest point of approach (CPA), they resume towards their destination. This is further explained in section 2.2. Initially, this approach will also be used in the experiments in this thesis for both the original MVP as any adaptations to the algorithm. If time is left, variations regarding the implementation of the MVP algorithm and any adaptations into BlueSky will be investigated.

1.5. Structure of this Preliminary Report

This preliminary report continues with a literature study, in which the core definitions are explained as well as the development and mechanics of the methods used in this thesis. Next the small scale experiment is described, including the metrics. This small scale experiment is already performed in this preliminary phase, and therefore the results will be discussed and interpreted as well (*Research activity 1, 2, 3, and 4*). Then the large scale experiment is defined, including the metrics used in this experiment (*Research activity 1 and 2*). Finally the solution space is defined, including the characterization of the various solutions in it (*Research activity 5 and 6*). Research activities 7,8,9,10, and 11 are not part of this preliminary report, as they are either dependent on results or not yet relevant in this phase. As an appendix, a preliminary method is explained and tested. This preliminary method provides valuable lessons for the remainder of this thesis.

2

Literature Study

This chapter gives an overview of the existing literature regarding the concepts used in this thesis. The first section will explain the differences in conflict detection, resolution, and prevention. Then the modified voltage potential, the self-separation algorithm at the root of this theses, is explained. Since proposed methods will be based on the solution space, this concept will be explained as well. Velocity obstacles are an essential concept to understand this solution space, and will be explained as well. Then some of the research regarding the layered airspace is explained. This research is the reason more attention to conflict angles in combination with the MVP was given. It is also the motivation for one of the assumptions, and thus it will be substantiated further.

2.1. CD, CR, and CP

In this section, the concepts of conflict detection (CD), conflict resolution (CR), and conflict prevention (CP) are explained.

2.1.1. Conflict Detection

Conflict detection is the act of propagating aircraft state of intent into the future to determine whether a LoS will occur. If a LoS will occur, the two aircraft are said to be in conflict. How accurate a model is able to predict the future is a vital indication of how accurate it will be able to detect and resolve future conflicts. That is why the extrapolation method used by a method, is one of the most particular differences between conflict detection methods. Kuchar and Yang (2000) have identified three extrapolation methods; nominal, worst case, and probabilistic.

Nominal state extrapolation is projecting the current aircraft states along a single trajectory into the future. Whilst extrapolating these states into the future, there is no direct consideration for uncertainties (Valenzuela and Rivas, 2009). The worst-case approach is based on the assumption that an aircraft will perform any set of maneuvers, and if any of these maneuvers could cause a conflict, a conflict is predicted (Shewchun et al., 1997). This other extreme of dynamic modeling, however, can prove to be quite unworkable. Because, if there even is a minimum chance of a conflict in the future, a conflict is raised. This leads to a high amount of false positives (Ayhan et al., 2018). Finally, the probabilistic approach is a combination of the two. Since both the worst case and the nominal extrapolation are subsets of the probabilistic approach (Kuchar and Yang, 2000). In the probabilistic approach, decisions can be made based on the likelihood of a conflict. This offers possibilities to address considerations such as safety and false alarm rates. However, research conducted by Pritchett and Hansman (1997) indicated that pilot confidence is negatively influenced by the use of probabilistic conflict detection. This due to the fact that these probability-based systems might be

difficult to convey to operators.

Another subdivision to be made within conflict detection is pair-wise conflict detection and global conflict detection. In pairwise conflict detection, multiple conflicts are detected sequentially in pairs. Algorithms using this pair-wise approach usually consider two only aircraft at a time, the ownship and the intruder. Each aircraft within the conflict detection space is treated sequentially as the intruder to determine whether a conflict is present (Goodloe et al., 2013). Global conflict detection, however, determines whether a conflict is present in a global manner. Meaning that the entire conflict detection space is treated as one space, and the conflict detection is based on finding conflicts within the set of all intruder trajectories.

2.1.2. Conflict Prevention

Conflict prevention is about propagating aircraft states into the future in order to prevent conflicts from arising in the first place. A conflict prevention system consists of algorithms that sense traffic aircraft and characterize ranges of maneuvers for the ownship that are either conflict-free or that lead to conflict (Narkawicz et al., 2012). this way the pilots can prevent conflict from arising.

A common method to implement conflict prevention is by making use of a display in the cockpit. In this method, the display would indicate traffic information. A display of this kind is often referred to as a CDTI (Cockpit Display of Traffic Information) (Hoekstra et al., 2002).

2.1.3. Conflict Resolution Approaches

Conflict resolution is, in the broadest sense, propagating current intruder states into the future to find trajectories for the ownship which would not lead to a loss of separation. This can be achieved by a variety of methods. A recent effort was made by Jenie et al. (2017) to propose a taxonomy of CD&R approaches for Unmanned Aerial Vehicles (UAVs). The authors categorized various CD&R approaches based on various characteristics. These characteristics are:

1. Type of airspace surveillance
2. Type of coordination
3. Type of avoidance maneuver
4. Type of autonomy

Type of airspace surveillance

The first of the characteristics are not based on the conflict resolution part of the approach, but on the conflict detection part. The authors define three types of surveillance; centralized-dependent, distributed-dependent, and independent surveillance. Surveillance is the use of systems to determine the position of aircraft.

Centralized-dependent surveillance entails surveillance systems where the aircraft obtains surveillance data from a station-network. This surveillance data can contain both static obstacles as well as moving obstacles such as other aircraft.

Decentralized-dependent surveillance is a type of surveillance where the aircraft obtain their surveillance data from other aircraft. This requires all the aircraft in the system to broadcast their flight data

periodically. This type of surveillance can be found in manned flight due to the ADS-B.

Independent surveillance means that the aircraft obtain the surveillance data themselves through a sensor onboard. This type of surveillance is most popular in UAV studies.

Type of Coordination

This characteristic is based on the conflict resolution part of the approach. The avoidance between two aircraft is based on one of three types of coordination; explicitly coordinated avoidance, implicitly coordinated avoidance and uncoordinated avoidance.

Explicitly coordinated avoidance is avoidance based on explicit coordination between the two aircraft. Using this explicit communication, a resolution can be negotiated for both aircraft.

Implicitly coordinated avoidance is based on both aircraft maneuvering according to a common set of strategies. Thus, using implicit coordination, resolution maneuvers can be found without the need for a direct line of communication between the aircraft.

Finally, uncoordinated avoidance is avoidance in which each aircraft chooses its resolution maneuver based on its own preference. This type of avoidance is rarely used since it can lead to dangerous and complex situations.

Types of Avoidance maneuver

The type of avoidance maneuvers is also part of the conflict resolution approach. Jenie et al. differentiate between three types of maneuvers: strategic, tactical, and escape maneuvers.

Strategic maneuvers are a long-range type of maneuver. Essentially, it aims to avoid conflicts altogether. This makes it related to the CP described above. Through, for instance, moving waypoints, the flight plan is changed and less conflicts are found.

A tactical maneuver is a type of maneuver which aims to maintain the separation between two aircraft on a mid-range basis. The resolution maneuver is chosen in such a way that the flight path deviation is as small as possible. In manned flight, this type of maneuvers can be found through the Airborne Separation Assurance System (ASAS).

Finally, an escape maneuver is the most aggressive type of maneuver of the three. It aims to avoid a collision on a more short-range to mid-range basis. No regard for the path deviation is given. The number one priority is the avoidance of the collision, and often the maximum performance limit of the aircraft is used.

Type of Autonomy

The last characteristic of CD&R approaches described by Jenie et al. is the type of autonomy. When the resolution strategy has been decided, it can be conducted based on two different ways: manual avoidance and autonomous avoidance.

Manual avoidance means that the operator of the aircraft conducts the avoidance maneuver. This means that the final decision for avoidance maneuvers is given to the operator. In autonomous avoidance, an autonomous system onboard has the final choice for avoidance maneuvers. Autonomous avoidance is not implemented in commercial manned flights due to safety regulations.

Altogether, these characterizations of CD&R approaches by Jeniet et al. provides a solid framework. Throughout this thesis, these characteristics will be used to provide context for various approaches and methods.

2.2. MVP

In the 1990s, Eby (1994) developed a CR method based on potential field analogy. Although this analogy had been around for a few years at the time, Eby (1994) was the first to develop a feasible self-organized method based on the analogy. Eby, too, argued that fixed routes limited the efficiency and the airspace capacity in comparison to paths that are not predefined or constrained. He proposed a technique for computationally determining conflict-free trajectories for multiple aircraft. This algorithm was later extended and adapted by Hoekstra et al. (2002). The authors then named it the Modified Voltage Potential (MVP).

The MVP is a CD, CR, and CP method. Considering the CD, it uses nominal state propagation. The type of airspace surveillance is distributed-dependent, as it makes use of the broadcasted state information through the ADS-B. It makes use of implicitly coordinated avoidance vectors, as the algorithm forms the common set of rules. No explicit communication is required between the aircraft in order to resolve the conflicts. The avoidance maneuvers it produces are regarded as tactical, as they aim to maintain the separation between two aircraft on a mid-range basis while keeping the path deviation as small as possible. It is mid-range as the look-ahead time of the CD is 5 minutes. Finally, since the pilots are required to execute the formulated resolution strategy the type of autonomy is manual.

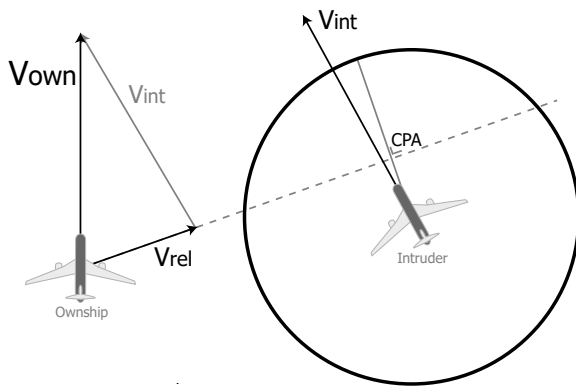
As stated before, the CR algorithm is based on a potential-field analogy. Going through the steps of calculating the avoidance strategy as proposed by Hoekstra et al. (2002), the procedure is as follows:

1. If a conflict is detected, it indicates that within the look-ahead time, a LoS will occur. In other words, within the look-ahead time, a Closest Point of Approach (CPA) will lie within the PZ of another aircraft, see fig. 2.1a.
2. Now the distance at CPA (D_{cpa}) can be calculated. The horizontal intrusion (D_i) equals the horizontal separation requirement S_h minus the D_{cpa} , see fig. 2.1b.
3. Following the potential field analogy, the relative velocity V_{rel} between the ownship and the intruder has to change so that this CPA does no longer lies within the PZ of the intruder. However, if the CPA is 'pushed out' of the PZ directed along (D_{cpa}), the new CPA CPA^* would still lie within the PZ, see fig. 2.1c. Instead, the new CPA has to lie on the edge of PZ, so that the ownship grazes the PZ of the intruder and no LoS occurs, see fig. 2.1d.
4. To achieve this grazing of the PZ, the amount of 'pushing out' has to be corrected, indicated by $D_{i,corr}$ as in eq. (2.1), see fig. 2.1e. The required change in velocity is then calculated using eq. (2.2). This results in the correct solution of the conflict shown in fig. 2.1f.

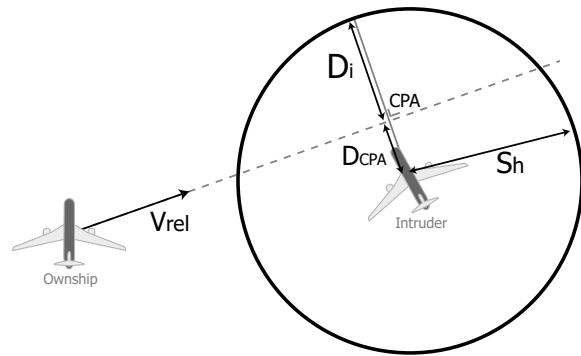
$$\frac{D_i}{D_{i,corr}} = \left| \cos \left(\arcsin \left(\frac{S_h}{D_{o,i}} \right) - \arcsin \left(\frac{D_{cpa}}{D_{o,i}} \right) \right) \right| \quad (2.1)$$

Where: $D_{o,i}$ = The distance between ownship and intruder.

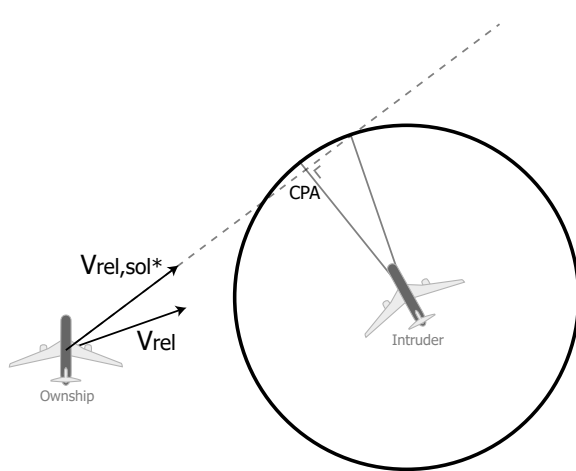
$$\Delta \vec{V}_{MVP} = \frac{D_{i,corr}}{t_{cpa}} \cdot \frac{\vec{D}_{cpa}}{D_{cpa}} \quad (2.2)$$



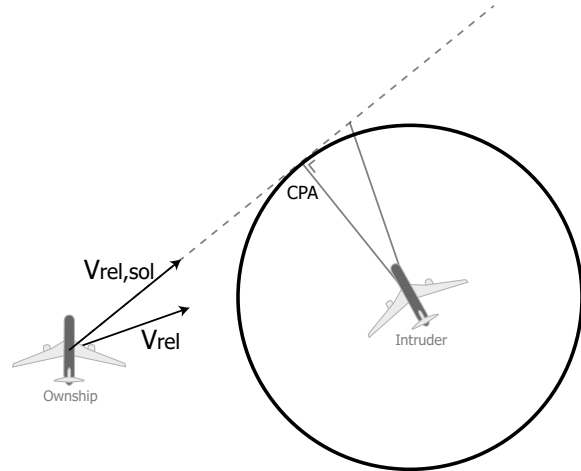
(a) Conflict is detected since CPA lies within PZ.



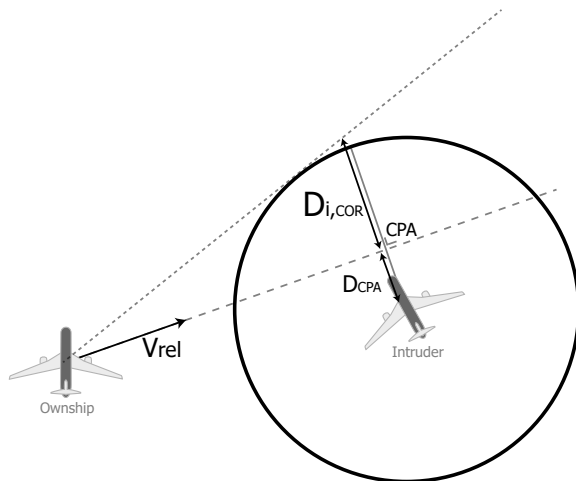
(b) Distance at CPA (D_{cpa}) can be calculated. The horizontal intrusion (D_i) equals the horizontal separation requirement S_h minus the D_{cpa} .



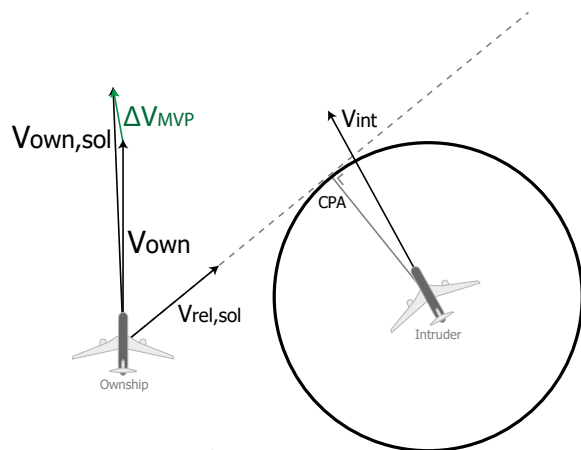
(c) If the CPA is 'pushed out' of the PZ directed along (D_{cpa}), the new CPA CPA^* would still lie within the PZ.



(d) The new CPA has to lie on the edge of PZ, so that the ownship grazes the PZ of the intruder and no LoS occurs.



(e) The amount of 'pushing out' has to be corrected, indicated by $D_{i,corr}$.



(f) The correct resolution vector $\Delta \vec{V}_{MVP}$ of the MVP algorithm, this results in the solution velocity vector $\vec{V}_{own,sol}$.

Figure 2.1: The steps for constructing the MVP resolution maneuver.

These steps can be performed for each conflict separately if a multi-aircraft conflict is present. These separate avoidance vectors can then be summed to achieve the total avoidance vector. Thus the multi-aircraft conflicts are solved pair-wise. This summing of the avoidance vectors is an essential part in retaining the potential field analogy.

When this resolution trajectory has been found, the pilot has to fly this trajectory until the (new) CPA has passed. When the CPA has been passed, the pilot can change its trajectory towards its destination again. If the aircraft was flying without destination, the pilot could return to its initial heading and velocity. In a multi-aircraft conflict, the pilot must wait until the CPA has passed for each conflicting aircraft before changing its trajectory towards its destination. Note that, in order to implement a layer of safety, a margin can be added to the intrusion vector to try further to mitigate the chances of LoS. When both aircraft use the MVP for CD&R, they will both have to take equal measures to resolve the conflict. This is because the avoidance vector being calculated is opposite identical for both aircraft in the conflict.

Hoekstra et al. added a method to prevent the pilot from initiating conflicts as well, which is fundamental to CP, through a module that calculates no-go zones. These no-go zones are indicated on the heading, vertical speed, and speed scale. Using this, the pilots can prevent themselves from initiating conflicts. Placing both the CD&R and the CP of the MVP in a broader system, ASAS and PASAS come into scope. The Airborne Separation Assurance System (ASAS) is a system with the goals of providing situational awareness, spacing, delegating separation, and full responsibility for self-separation. The MVP can be seen as part of the ASAS. The Predictive ASAS (PASAS), as described by Hoekstra, is a module that predicts maneuvers which will lead to conflicts. In conclusion, using the ASAS and PASAS, the MVP is a CD, CR, and CP method. The MVP will form one of the cornerstones of this thesis. More information regarding the validation of the MVP can be found in (Maas et al., 2016; Sunil et al., 2017b).

2.3. Velocity Obstacles

A concept used in Conflict Detection and Resolution is Velocity Obstacles (VOs). It was first investigated by Tychonievich et al. (1989), who used the term Avoidance Cones. They described a path planner that uses these avoidance cones to find a collision-free path through a field of moving obstacles. This method relies on choosing velocity vectors for the vehicle, which results in a collision-free path. It was suggested choosing the best candidate speed vector based on the target point, thus choosing the solution vector, which is the closest (in Euclidean, straight-line distance) to the target point. Fiorini and Shiller (1993, 1998) further investigated motion planning in dynamic environments using this method and gave it the definitive name; Velocity Obstacles.

In this section, the working principles of velocity obstacles are explained, as well as how these were used in other experiments. Velocity obstacles are of interest to goals of this thesis since they provide methods which can be used to define solution spaces. Placing the mechanics of the MVP in a vector-space context using velocity obstacles provides the bigger picture.

2.3.1. The working principles of velocity obstacles

The fundamental building blocks of velocity obstacles are made up of both the location and velocity of obstacles. When these two states are known, one can construct the velocity obstacle. In order to construct a velocity obstacle, it is useful to first construct a collision cone (CC). Consider two air-

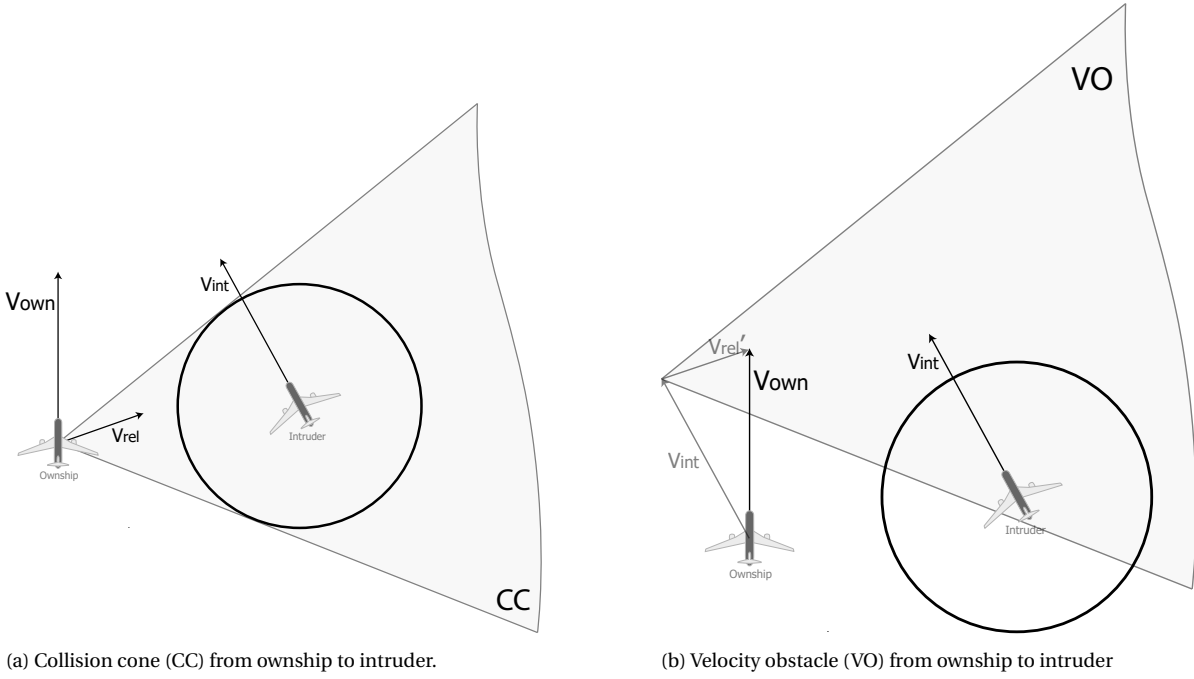


Figure 2.2: Collision cone (CC) and velocity obstacle (VO) from ownship to intruder.

craft, the ownship and the intruder. The collision cone from the ownship to the intruder is the set of relative velocities from the ownship to the intruder, which result in a LoS. This set can geometrically be described as the set of velocities for the ownship where the relative velocities from the ownship to the intruder lie between the two tangents from the ownship to the PZ of the intruder, see fig. 2.2a. When the relative velocity from the ownship to the intruder is not within this collision cone, no LoS will occur given that object the intruder retains its current PZ shape and velocity.

The Velocity Obstacle can be constructed by moving the apex of the Collision Cone by $\vec{V}_{int}$. This now results in a set of velocities for the ownship which results in a LoS with the intruder. The shapes of the collision cone and the velocity obstacle are identical. In general, the Velocity Obstacle groups the velocities of the ownship into avoiding and colliding velocities. Additionally, velocities on the boundary of the velocity obstacle result in the ownship grazing the PZ of the intruder. In the case of multiple intruders, the union of each separate velocity obstacle results in the combined velocity obstacle for the ownship. Using Velocity Obstacles results in a more intuitive representation of the forbidden velocities than Collision cones provide. This is because one can see the required change in velocity directly in a velocity obstacle, whereas, in a collision cone, one can only see the required change in relative velocity, see fig. 2.2b. In a VO the relative velocity can also be indicated by translating $\vec{V}_{rel}$ by $\vec{V}_{int}$ resulting in $\vec{V}_{rel}'$. Because of this, the effect a change in velocity has on the relative velocity remains visible when using VOs.

In the case of multi-object avoidance, and thus a combined velocity obstacle, it can prove useful to modify these velocity obstacles. The most common modification is only representing velocities that result in imminent collisions. Imminent collisions are collisions that occur within some time horizon. This time horizon can be set based on system dynamics, obstacle trajectories, and computational rate (Fiorini and Shiller, 1998).

When the velocity of an aircraft lies within a velocity obstacle of another aircraft, any velocity change which results in the new velocity lying outside of the velocity obstacle is counted as a solution —

choosing which solution to choose is the subsequent step. Various methods have various strategies for choosing these solutions. These will be explained in the next subsection through explaining experiments that have worked with velocity obstacles.

2.3.2. Experiments working with velocity obstacles

In this subsection, some research into the use of VOs is discussed. Three types of research are used in this subsection; a small-scale experiment, a medium-sized experiment, and a large-scale experiment. These experiments were performed by respectively, Ellerbroek et al. (2013b), Jenie et al. (2015), and Balasooriyan et al. (2017). Interestingly, the research by Ellerbroek et al. also investigated the influence of conflict angles on certain CR behavior, which will be discussed in section 2.4.3.

Small-scale experiment

Ellerbroek et al. performed fast-time simulations to analyze the emergent properties of airborne separation. The resolution maneuvers in these simulations are based on VOs. In the experiment to be discussed here, the authors defined three coordination rules; optimal, rules of the air, and a combination of both.

The optimal solution is defined as the solution for which the required change in velocity is as small as possible. Geometrically this can be described as the solution found by perpendicular moving the velocity to the nearest VO leg. The rules of the air, however, are divided into three rules. Firstly, aircraft from the right have the right of way. The other aircraft has to remain clear by passing behind that aircraft. Secondly, when two aircraft are approaching head-on, they are both required to alter heading to the right. Finally, an aircraft that is being overtaken has the right of way, and the other aircraft has to alter heading to the right in order to remain clear. In essence, the rules of the air dictate to which VO leg each aircraft should change their velocity vector if they should change it. In the experiments by Ellerbroek et al. this change in velocity is performed in an optimal fashion, thus moving the velocity perpendicular to the required VO leg.

The combined solution is required when both VO legs are equally far away with respect to the current velocity. This can also be described as a conflict with $D_{cpa} = 0$ NM, see figs. 2.1 and 2.2a. In this case, the optimal solution is not defined. To solve this, the optimal solution method is supplemented with the rules of the air to determine to which VO leg the velocity vector should move.

Mid-scale experiment

The mid-scale experiment using VOs is the experiment performed by Jenie et al. (2015). Although this experiment considers multi-aircraft scenarios, it should be stated that the method developed by the authors does not resolve multi-aircraft conflicts simultaneously. The authors argue that multiple encounters are very unlikely to happen in a random simulation setup. The method developed by Jenie et al. makes, like Ellerbroek et al., use of the rules of the air in combination with VOs. The reason why, however, is different from the one argued by Ellerbroek et al..

The reason Jenie et al. combines the VO methods with the rules of the air is due to a well-known problem with VO methods, described as the reciprocating dance. This reciprocating dance is described by Snape et al. (2011) and occurs when both aircraft are taking measures to resolve the conflict. Jenie et al. combines the VO method with the rules of the air in order to counter this effect. Using the rules of the air in combination with the VO method, this CR uses implicitly coordinated maneuvers.

The rules of the air used by the authors differentiate between four sectors; the same path, right converging, left converging, and head-on. These are, in essence, the same encounter-types as used by Ellerbroek et al.. However, the heading-space used by both methods is different. Where Ellerbroek et al. defines head-on encounters as encounters with a relative heading of 160 to 180 degrees, Jenie et al. use a relative heading of 135 to 180 degrees for head-on encounters. Jenie et al. define left and right crossing encounters as encounters with a relative heading between 45 and 135 degrees, Ellerbroek et al. uses relative headings between 45 and 160 degrees. The rules of the air used by both authors, however, are exactly the same.

The algorithm described by Jenie et al. consists of three modules; turn, maintain, and mission. An aircraft flying in the mission mode is changing its trajectory towards its destination if the trajectory was not directed to the destination in the first place. In maintain mode, the aircraft remains flying its current heading and speed. Thus, the aircraft which has the right of way is put in the maintain mode. The other aircraft, which does not have the right of way, is put in the turn mode in order to generate a CR maneuver.

The turn module does not work in the same VO manner as the one described by Ellerbroek et al.. Where most VO methods choose a solution on the leg of the VO, the method described by Jenie et al. uses a minimum avoidance turning rate. This minimum avoidance turning rate is defined as the required turning rate, at constant velocity, for the resolving aircraft, which resolves the conflict. When the conflict is resolved, both aircraft are put back in mission mode.

Large-scale experiment

The large-scale experiment is also part of a thesis report and conducted by Balasooriyan et al. (2017). The authors developed a multi-aircraft conflict resolution method using the Solution Space Diagram (SSD) as a horizontal conflict resolution method in an automated sense. The solution space diagram is obtained by constructing the union of the velocity obstacles of the surrounding aircraft. Using this SSD, conflicts are not solved pairwise but globally. The authors defined eight different coordination rule-sets to use in the SSD.

The eight different rule-sets are; shortest way out (corresponding to the approach described by Ellerbroek et al. (2013b)), turning right, heading only, speed only, resolve towards the target heading, adhere to the rules of the air, resolve optimal sequentially, and resolve towards target heading sequentially. The last three of these rule-sets are based on a priority rule. The authors differentiate between primary and secondary VOs by defining VOs of aircraft within a 40 NM range as primary, and VOs of aircraft further away than 40 nm as secondary. Interestingly, this method encompasses CP since resolution maneuvers are found, which do not result in a new conflict.

This method was tested in a large scale experiment consisting of a simulation area of $455,625 \text{ NM}^2$. Within this simulation area, aircraft densities of 2.5, 5.0, or 7.5 AC per $10,000 \text{ NM}^2$ are generated. These aircraft fly, on average, 1,800 seconds and have an airspeed chosen uniformly between 450 and 500 kts. The authors tested each rule-set in each density 50 times. To provide context to the results, each combination of rule-set and density was compared to the MVP. The MVP performed better in all of the major metrics.

2.4. Airspace Capacity and the layered airspace

A part of ATM research where the - not necessarily negative - influence of conflict angles becomes visible, is research into airspace structure and airspace capacity. First, some findings and results of

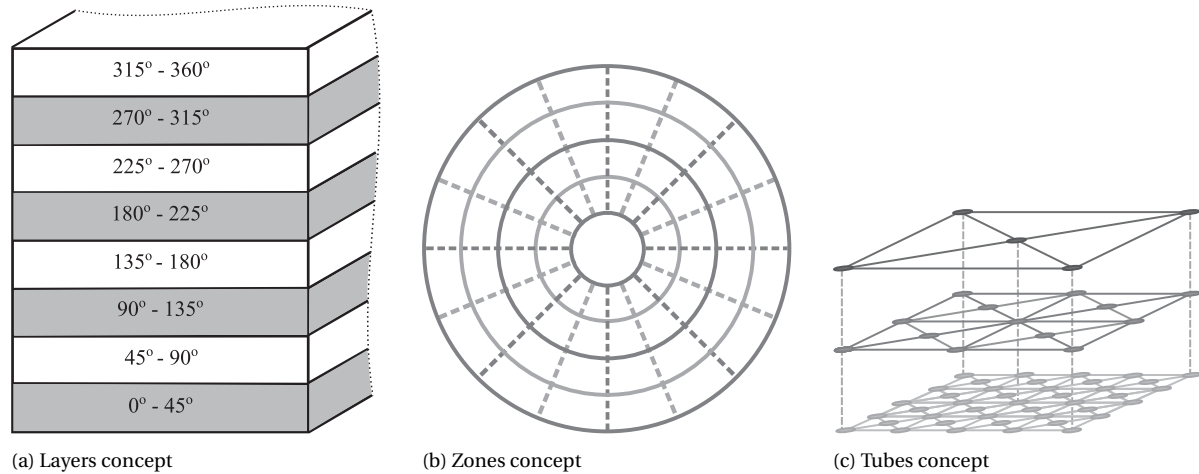


Figure 2.3: Airspace structures of increasing structure. (modified from citeSunil2019)

the metropolis project, a research initiative funded through the seventh framework program of the European Commission, are discussed. Then, more attention will be given to airspace capacity in an analytic sense.

2.4.1. Metropolis

Research regarding the aforementioned Metropolis project conducted by Sunil et al. (2015) focused on investigating the influence of airspace structure on capacity, complexity, safety, and efficiency for high-density airspace. Four airspace concepts were compared, full mix, layers, zones, and tubes. The full mix concept is fundamentally unstructured airspace. Aircraft do not have any restrictions regarding the path of their flight. Layers, on the other hand, do impose restrictions on flight paths. The layer concept consists of vertically stacked bands in which aircraft are only allowed to fly within a certain heading range. One of the fundamental ideas on which this concept is based, is the notion that reducing relative velocities between aircraft reduces the probability of conflicts arising. The heading range per layer in the Metropolis research is 45 degrees, see fig. 2.3a.

An airspace concept of even more structure is the zones concept. In this concept, aircraft are also segmented based on travel direction. However, in contrast to the layered structure, this structure is based on city layout and not based on vertically stacked bands, see fig. 2.3b. Finally, the concept of maximum structuring is the tubes concept. This concept divides the airspace in predefined routes connected at nodes. Using these predefined routes, an algorithm tries to find a conflict-free route before departure. This means that the tubes concept, in contrast to other concepts, uses time-based separation, see fig. 2.3c.

In the first three concepts, MVP was used as CD&R. However, certain restrictions were imposed on resolution strategies in different airspace concepts. In the full mix concept, heading, speed, and vertical resolutions were allowed. In the layers concept, only horizontal resolutions were allowed, since vertical resolutions would result in aircraft entering a forbidden layer based on their heading. Finally, in the zones concept, only speed and vertical resolutions were allowed since heading resolutions would result in aircraft entering a forbidden zone based on their heading.

The authors concluded that the layers concept resulted in the best balance between safety and efficiency. In terms of safety, the layers concept had the least amount of intrusions, even at increasing densities. It is argued that this reduction of intrusions is indeed due to the reduced relative veloci-

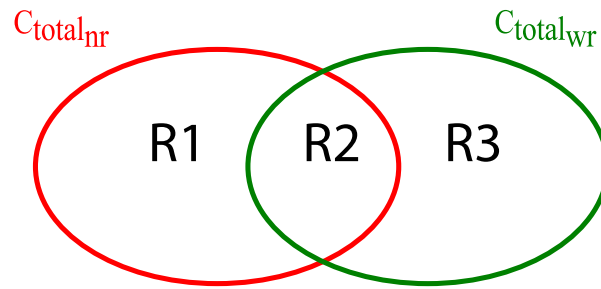


Figure 2.4: Three distinct regions of the two overlapping sets describe the effect of conflict resolution on the Domino Effect (R3). (Modified from Krozel et al. (2000)).

ties in the layers concept. In other words, when a group of aircraft has low relative velocities, they will generally have fewer conflicts, as aircraft will meet fewer aircraft within a unit time.

As Sunil et al. argues, the CD&R implementation into each airspace structure was not optimized for the concept or density. One way this could be noticed is through fact that there were still many conflicts and intrusions for each concept. Based on these findings, it makes sense to investigate the effects of conflict angles on the behavior of the MVP and try to optimize it for use in an airspace concept with lower conflict angles, such as the layers concept.

2.4.2. Airspace Capacity

More theoretical research into the capacity of an airspace has been performed as well. Sunil (2019) used airspace stability to define a maximum theoretical capacity of decentralized airspace concepts. Sunil argued that when an airspace reaches maximum capacity, each conflict leads to another conflict, which leads to another conflict. This effect results in all aircraft constantly being in conflict with other aircraft. The work regarding airspace capacity in the sense of stability offers important insights regarding CD&R behavior, especially so when the layered airspace is included. Because of this, the major findings of various capacity research will be discussed here, as well as the metric to define airspace stability.

The Domino Effect Parameter

As stated above, airspace stability is related to conflict chains as an effect of CR maneuvers. At a high enough density, such a conflict chain is able to propagate throughout the entire airspace. Bilimoria et al. (2000); Krozel et al. (2000) provided a metric to quantify this stability, the Domino Effect Parameter (DEP). The DEP is a measure of the amount of destabilizing conflicts, see fig. 2.4.

$$DEP = \frac{CC_{CR/ON} - CC_{CR/OFF}}{CC_{CR/OFF}} = \frac{CC_{CR/ON}}{CC_{CR/OFF}} - 1 \quad (2.3)$$

When the DEP is 0, it indicates that there is a similar number of conflicts when CR is on and off. The destabilizing effect can be described as aircraft drawn into conflict as a result of other aircraft resolving their own conflicts. The stabilizing effect, however, can be described as aircraft that had conflicts in their nominal trajectories but did not experience this conflict when CR was set to ON due to prior deviations of the other aircraft as a result of prior conflict. Indeed, CR can have both destabilizing and stabilizing effects on an airspace. The net result of these effects is captured in by the DEP (Bilimoria et al., 2000).

As stated by Sunil (2019), the DEP is related to both the safety and the efficiency of an airspace. Safety in the sense of conflict chains occurring due to CR being ON, and efficiency in the sense of

aircraft deviating from their optimal path due to these conflict chains. That is why the DEP is an important metric to analyze the airspace capacity.

Conflict angles and airspace capacity

Hoekstra (2001); Hoekstra et al. (2000) described the global conflict rate, the number of conflicts in an airspace, as a function of the total number of aircraft N as in Equation (2.4).

$$CR_{global} = \frac{1}{2}N(N-1)p_{2_{global}} \quad (2.4)$$

With:

N = total number of aircraft in an airspace.

p_2 = average conflict probability any given aircraft pair in the airspace.

In words, the amount of conflicts at a given time equals the number of aircraft combinations in the airspace multiplied with the chance of any two aircraft being in conflict. Hoekstra et al. (2016) later modified this expression to reflect the global conflict rate within a layer. This results in the expression seen in Equation (2.5).

$$CR_{layer} = \frac{1}{2}N\left(\frac{N}{L} - 1\right)p_{2_{layer}} \quad (2.5)$$

Jardin (2004, 2005) used this same relation in combination with other parameters to propose an expression for p_2 . In order to do so, Jardin considered the observation time T_{tot} , the total airspace area A , the average time an aircraft spends in the airspace T_{FL} , the lateral separation minimum R , and the average ground speed V . He argued that the conflict rate is proportional to the area 'searched' by an aircraft divided by the total area of the airspace, as in Equation (2.6).

$$CR_{global} \sim \frac{\bar{V} \cdot R \cdot T_{FL}}{A \cdot T_{tot}} \quad (2.6)$$

Datta and Oliver (1991); Graham and Orr (1969); Sunil et al. (2018) also stated that conflict probability between two aircraft in a horizontal airspace can be determined by dividing the area searched for conflicts at some time A_c by the total airspace area A_{total} . Sunil et al. (2018) expressed it as:

$$p_{2d} = \frac{A_c}{A_{total}} = \frac{2S_h E(V_{r,h}) t_l}{A_{total}} \quad (2.7)$$

To find an expression for the expected relative velocity $E(V_{r,h})$, the assumption is made that all aircraft are flying at equal speed. Using this assumption, the expected horizontal relative velocity $V_{r,h}$ is written as:

$$V_{r,h} = 2V_o \sin\left(\frac{|\Delta\psi|}{2}\right) \quad (2.8)$$

Where: $|\Delta\psi|$ = the absolute heading difference between the ownship and the intruder.

Using a PDF of the absolute heading difference as defined by (Dimitrios, 2009; Sunil et al., 2018):

$$P(|\Delta\psi|) = \frac{2}{\alpha} \left(1 - \frac{|\Delta\psi|}{\alpha}\right) \quad (2.9)$$

By combining eqs. (2.8) and (2.9) the expression for $E(V_{r,h})$ can be found by integrating over $|\Delta\psi|$, (Hoekstra et al., 2016):

$$E(V_{r,h}) = \frac{8V_o}{\alpha} \left(1 - \frac{2}{\alpha} \sin \frac{\alpha}{2}\right) \quad (2.10)$$

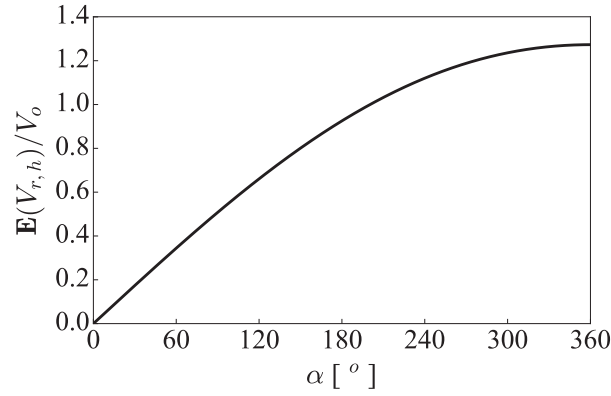


Figure 2.5: The expected horizontal relative velocity $E(V_{r,h})$ as a function of the heading range per flight level α as described in eq. (2.10) (Sunil et al., 2018).

Using eqs. (2.7) and (2.10), it becomes visible that lowering the permitted heading ranges lowers the expected horizontal relative velocities and, finally thus lowers the conflict probability. To visualize this reduction in average relative velocities $E(V_{r,h})$ was plotted in fig. 2.5.

Combining eqs. (2.5), (2.7) and (2.10) the expression for the number instantaneous conflicts between cruising aircraft in a layered airspace is found (Sunil et al., 2018):

$$C_{inst,LAY} = N_{inst} \left(\frac{N_{inst}}{L} - 1 \right) \left(\frac{8S_h V_o t_l}{\alpha A_{total}} \right) \left(1 - \frac{2}{\alpha} \sin \frac{\alpha}{2} \right) \quad (2.11)$$

Sunil (2019) took this further by deriving an expression for the DEP in a layered airspace. To start, the number of instantaneous conflicts is determined as in (Hoekstra et al., 2016; Sunil, 2019), see eq. (2.5). However, Sunil introduced the aircraft density ρ as the ratio between the amount of instantaneous aircraft and the airspace area to rewrite eq. (2.5):

$$C_{inst,nr,LAY} = \frac{P_{LAY} \rho^2 A_{total}^2}{2L} \quad (2.12)$$

The approach Sunil took was to define the total amount of conflicts as the integral of the number of instantaneous conflicts over the analysis time T and dividing by the average conflict time t_c :

$$C_{total,nr,LAY} = \frac{p_{LAY} \rho^2 T A_{total}^2}{2L t_c} \quad (2.13)$$

Capacity with Conflict Resolution

When CR is used within an airspace, this conflict count as in eq. (2.13) is no longer valid. The extra distance aircraft are going to fly due to CR $D_{cdr,LAY}$ has to be included in the expression. Therefore, the conflict count per unit distance flown $C'_{1,nr,LAY}$ has to be found. This conflict count per unit distance flown is given by eq. (2.14) (Sunil, 2019). Note that the subscripts nr, wr, and cdr indicate no resolution, with resolution, and due to conflict detection and resolution respectively.

$$C'_{1,nr,LAY} = \frac{C_{total,LAY}}{N_{total,nr} D_{nr}} \quad (2.14)$$

With: D_{nr} as the average distance flown without conflict resolution

The assumption (Sunil, 2019) used is that the total amount of aircraft in the airspace during an analysis time interval is not affected by tactical resolution maneuvers. Therefore, an expression for $N_{total,nr}$ is found:

$$N_{total,nr} = N_{total,wr} = \frac{TV_0 N_{inst}}{D_{nr}} + N_{inst} = \rho A_{total} \left(\frac{TV_0}{D_{nr}} + 1 \right) \quad (2.15)$$

As described by Sunil, eq. (2.15) can be explained as the amount of aircraft at the start of the analysis interval plus the number of aircraft starting their flight during this same analysis interval. Substituting eqs. (2.13) and (2.15) into eq. (2.14) gives:

$$C'_{1,wr_{LAY}} = \frac{p_{LAY} \rho T A_{total}}{2Lt_c(TV_0 + D_{nr})} \quad (2.16)$$

Using Equation (2.16), Sunil could express the total amount of conflicts within an airspace with CR. By taking into account the average extra distance flown because of CR for a single aircraft $D_{cdr_{LAY}}$, the total amount of conflicts within an airspace with CR is defined as Equation (2.17).

$$\begin{aligned} C_{total,wr_{LAY}} &= \frac{D_{nr} C'_{1,wr_{LAY}}}{1 - D_{cdr_{LAY}} C'_{1,wr_{LAY}}} N_{total_{wr}} \\ &= \frac{p_{LAY} \rho^2 T A_{total}^2 (TV_0 + D_{nr})}{2Lt_c(TV_0 + D_{nr}) - p_{LAY} \rho T A_{total} D_{cdr_{LAY}}} \end{aligned} \quad (2.17)$$

In conclusion, substituting Equations (2.13) and (2.17) in Equation (2.3):

$$DEP_{lay} = \frac{2Lt_c(TV_0 + D_{nr})}{2Lt_c(TV_0 + D_{nr}) - p_{lay} \rho T A_{total} D_{cdr_{lay}}} - 1 \quad (2.18)$$

Since the research of Sunil (2019) was aimed at modeling capacity, this equation was rewritten using the CAMDA capacity definition (Sunil, 2019) into Equation (2.19).

$$\rho_{max_{LAY}} = \frac{2Lt_c(TV_0 + D_{nr})}{p_{LAY} T A_{total} D_{cdr_{LAY}}} \quad (2.19)$$

These derivations and expressions are of interest to this thesis in the following ways:

- To increase the capacity of a layered airspace, according to the equations by (Sunil, 2019), the average extra distance flown due to CR has to be reduced.
- The low relative velocities in a layered airspace are providing safety in the sense of conflict counts. Propositions for CR methods should try to retain this advantage.

Fast time simulations

Using these expressions, Sunil performed fast time simulations so investigate the effect of different layer configurations on the airspace capacity. The MVP was used as CR. The total conflict count, both with and without CR, decreased for decreasing α . In other words, implementing more layers, and thus reducing the average V_{rel} , reduced the total amount of conflicts in the airspace. Interestingly, Sunil found, the maximum theoretical capacity did not increase as α was decreased below 180 degrees. Similarly, the DEP did not decrease at high densities for lower values of α .

Sunil explained this phenomenon using eq. (2.19). The argument was made that this had to do with ρ_{max} being inversely proportional to both p_{LAY} and $D_{cdr_{LAY}}$. The results obtained by Sunil showed that, indeed, p_{LAY} decreased for lower values of α . However, the benefits of this decrease in p_{LAY} were counteracted by an increase in $D_{cdr_{LAY}}$ for lower values of α . It was argued that due to the low conflict angles for low values of α , increased lengths of the resolution path occurred. As a whole, Sunil concluded, lowering α does not increase the capacity of an airspace which uses MVP since efficiency drop due to the longer CR trajectories outweighs the safety benefits by lowering α .

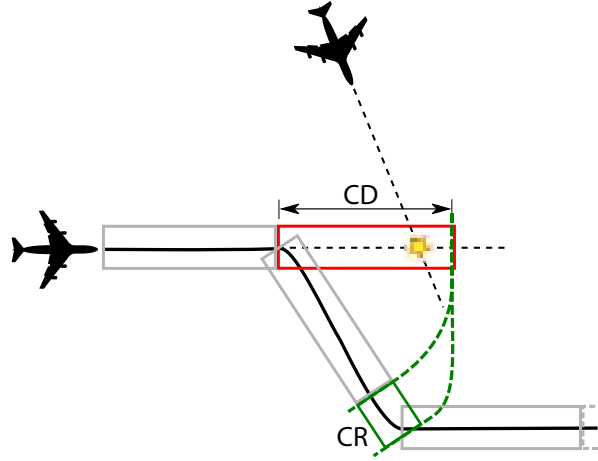


Figure 2.6: As conflicts are resolved, the extra distance searched increases due to conflict detection (CD) in red, and conflict resolution (CR) in green. The solution depicted is an arbitrary one. (Adapted from Sunil et al. (2017a))

2.4.3. Path deviations and extra distance searched

In this subsection, more attention to path deviations and extra distance searched due to using the MVP as CR is given. As described in section 2.4.2, the main effect path deviations have on conflict counts, are due to the extra distance searched for conflicts (Jardin, 2004, 2005).

Sunil et al. (2017a) provided more insight into this extra distance searched when the MVP is used as CR. It was argued that the total extra distance searched per conflict k_{cdr} consists of two components:

1. k_{cd} , the extra distance searched due to conflict detection.
When a conflict is detected, there is an area of airspace searched for conflicts which is not flown through. This is because, when a conflict is detected, the airspace searched for conflicts after the point where the resolution maneuver is initiated is not flown through, see the red part in fig. 2.6.
2. k_{cr} , the extra distance searched due to conflict resolution.
This extra distance can be defined as the distance searched while performing the resolution maneuver minus the distance that would have been searched in the same time while flying the average no conflict speed, i.e., eq. (2.20). This is indicated by the green part in fig. 2.6.

$$k_{cr}(\theta, D_{cpa}) = |\vec{V}_{o,sol}|t_{cpa} - |\vec{V}_o|t_{cpa} \quad (2.20)$$

Sunil et al. calculated the weighted average of the extra distance searched due to conflict resolution. The authors did this by integrating eq. (2.20) over all D_{cpa} and θ . In this paper, the airspace was assumed to be unrestricted, so no heading limitations were imposed on the aircraft. Therefore, the distribution of D_{cpa} was assumed to be uniform. Subsequently, the distribution of θ is triangular as θ is between 0 and 180 degrees. The result of this integration can be seen in fig. 2.7. It should be noted, the authors argue, that these extra distances searched due to conflict resolution are small compared to the extra distances searched due to conflict detection. This could be seen through the result that reducing the look-ahead time had a much larger influence on k_{cdr} than reducing the separation minimum did. In fact, when reducing the look-ahead time by 50%, the extra distance searched reduced by approximately 50% as well. Also, through example, using the values to compute fig. 2.7, the extra distances searched due to conflict detection is 41.676 NM. This is logical, the authors argue, since the MVP provides shortest path deviations (Sunil et al., 2017a).

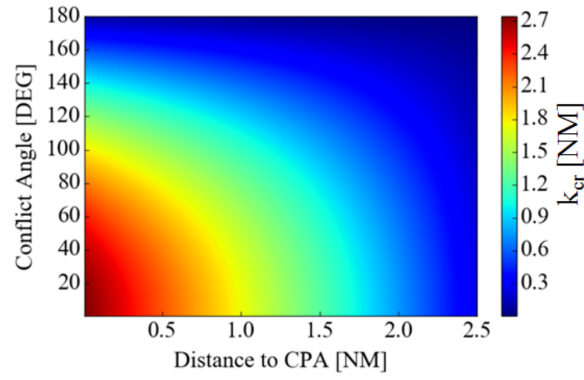


Figure 2.7: The resulting extra distance being searched as a result of the MVP provided resolution $S_h = 2.5$ NM, $t_l = 5$ min, and $V_{nr} = 500$ kts (Sunil et al., 2017a).

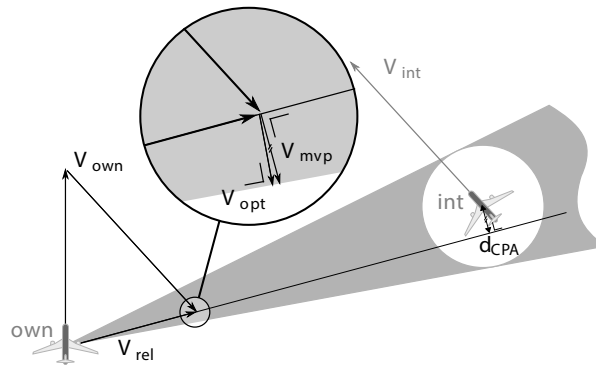


Figure 2.8: The optimal solution in a geometrical sense (V_{opt}) and the MVP provided solution (V_{mvp}) out of the forbidden zone. (Ellerbroek et al., 2013a)

Additionally, using similar derivations as in Jardin (2004, 2005); Sunil et al. (2018); Tra et al. (2017), the expression for the DEP and maximum theoretical capacity ρ_{max} were rewritten to reflect the influence of k_{cdr} , see eq. (2.21).

$$DEP = \frac{\rho - \frac{1}{A_{total}}}{\frac{(V_o T + L)(2T - t_l)}{2k_{cdr} \cdot p_s \cdot S_h \cdot V_o \cdot T^2} - \left(\rho - \frac{1}{A_{total}} \right)} \quad (2.21)$$

$$\rho_{max} = \frac{(V_o T + L)(2T - t_l)}{2k_{cdr} \cdot p_s \cdot S_h \cdot V_o \cdot T^2} \quad (2.22)$$

To gain additional insight into the effect the CR has on path deviations, the research by Ellerbroek et al. (2013a) is used. Under varying conflict geometries, the path deviation was determined for two aircraft conflicts. Note that, in these simulations, Ellerbroek et al. did not use the MVP for CR, but the shortest way out technique in a VO based CR method, called optimal by the authors. These two methods are, in two-aircraft conflicts, very similar, see fig. 2.8. Therefore, the results obtained by the authors are of interest to this thesis.

The way Ellerbroek et al. calculated the path deviations was as the difference between the resolution trajectory and the original trajectory, see eq. (2.23).

$$DEV = \int_{t_0}^{t_1} |\vec{V}_{sol}(t) - \vec{V}_{orig}(t)| dt = \int_{t_0}^{t_1} |\Delta \vec{V}_{sol}(t)| dt \quad (2.23)$$

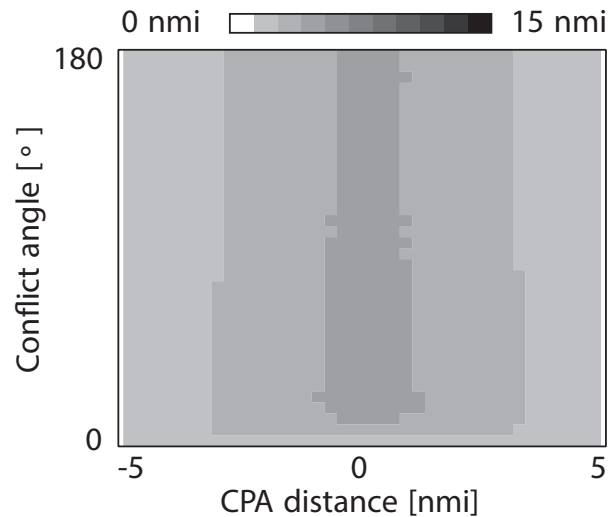


Figure 2.9: Mean path deviations for optimal method. (Ellerbroek et al., 2013a).

The results of the path deviation when using the optimal method can be seen in fig. 2.9. It should be noted that in the simulation by Ellerbroek et al., not all conflicts were resolved cooperatively. Because the decision process by the pilots was modeled as well, some pilots refrained from taking action on time. In which case, the other aircraft solved the (largely) conflict on its own.

Looking at fig. 2.9, the path deviation for the optimal method shows no distinctive increase for varying conflict angles. A slight increase in path deviation can be seen for small absolute values of the CPA distance, from approximately 1.5 NM at higher CPA distances to approximately 4.5 NM at the lower CPA distances.

2.5. Summary

To summarize some aspects from the literature study which are important for the transition into the research activities:

- The inefficiency of the MVP in small conflict angle scenarios is one of the main conflict angle related problems with the algorithm, according to Sunil (2019).
- The low relative velocities in a layered airspace are providing safety in the sense of conflict counts (Hoekstra, 2001; Hoekstra et al., 2000; Jardin, 2004, 2005).. Propositions for CR methods should try to retain this advantage.
- To increase the capacity of a layered airspace, according to the equations by (Sunil, 2019), the average extra distance flown due to CR has to be reduced.
- Relating the mechanics of the MVP to a vector-space related method, and combining this with the various VO research that has been conducted throughout the years (Balasooriyan et al., 2017; Ellerbroek et al., 2013b; Jenie et al., 2015), provides a solid base to investigate problems and propose alternatives using a solution space.

3

One-to-one Experiment

In this chapter, the one-to-one experiment is explained and performed. First, the goal of the experiment is restated. Then, the simulation characteristics, scenario generation, and variables of the experiment are discussed. This experiment has already been performed in this phase of the thesis. Therefore the results and discussion of these results are concluding this chapter.

3.1. Experiment Goal

The goal of the one-to-one experiment is to obtain a wide range of insights into the mechanics of the MVP. These insights are important for two reasons. First, they are required to explain any emergent behavior of the MVP found in the large scale experiment. Without fully understanding the mechanics of the MVP, it becomes hard to explain these emergent behaviors undisputedly. Secondly, to propose meaningful modifications to the algorithm, both the mechanical shortcomings and advantages of the algorithm have to be known. For corresponding research activity steps, see Figure 3.1.

3.2. Simulation characteristics

This section explains the simulation characteristics of the one-to-one experiment. The one-to-one experiment is performed in Python. The CD and the MVP implementation are discussed in the following subsections.

3.2.1. Conflict Detection

As well as in the large scale experiment, the nominal state-based CD was used in this experiment. The look-ahead time was set to 5 minutes, and the horizontal and vertical separation was set to 5 NM and 1000 ft, respectively.

In line with the assumptions, the CD module assumes perfect ADS-B data and, therefore, perfect knowledge of the intruder states.

3.2.2. Conflict Resolution

In this experiment, the CR was tactical avoidance using the MVP. In line with the scope, only horizontal resolution maneuvers were calculated by the CR module.

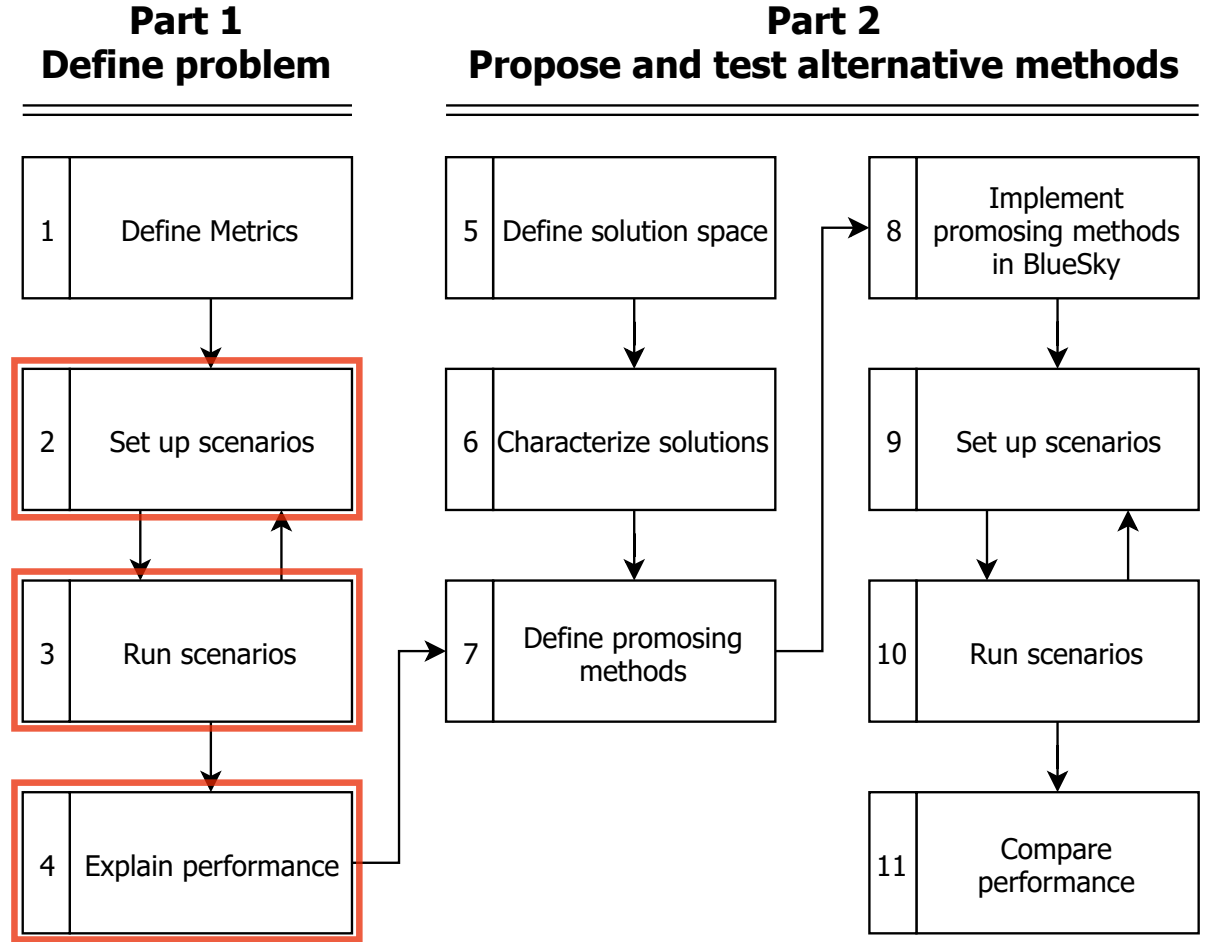


Figure 3.1: Corresponding thesis phases for Chapter 3.

In this experiment, no step-wise recalculation method was used. Instead, at the time of conflict detection, the CR module calculates the resolution maneuver for a cooperative solution. These resolution maneuvers are then performed by both the ownship and the intruder. In other words, the initial resolution maneuver is performed instantaneously.

This cooperative solution is achieved by calculating half of the $\Delta \vec{V}_{MVP}$. Since the MVP calculates opposite identical $\Delta \vec{V}_{MVP}$ for both the ownship and the intruder, see fig. 3.2a, the conflict is solved when both aircraft implement half of their calculated $\Delta \vec{V}_{MVP}$, see fig. 3.2b. The speed changes are assumed to be instantaneous, and both aircraft are assumed to fly paths parallel to their original trajectory once they have reached CPA.

3.3. Scenario generation

The one-to-one experiment consists of two aircraft, the ownship, and an intruder, both flying at identical speeds. The ownship is placed at lat/lon (0,0) with a heading of 0 degrees. The placement of the intruder depends on three input variables; time to LoS t_{los} , Conflict angle, and distance at CPA D_{cpa} . These three input variables define the location of the intruder, see fig. 3.3.

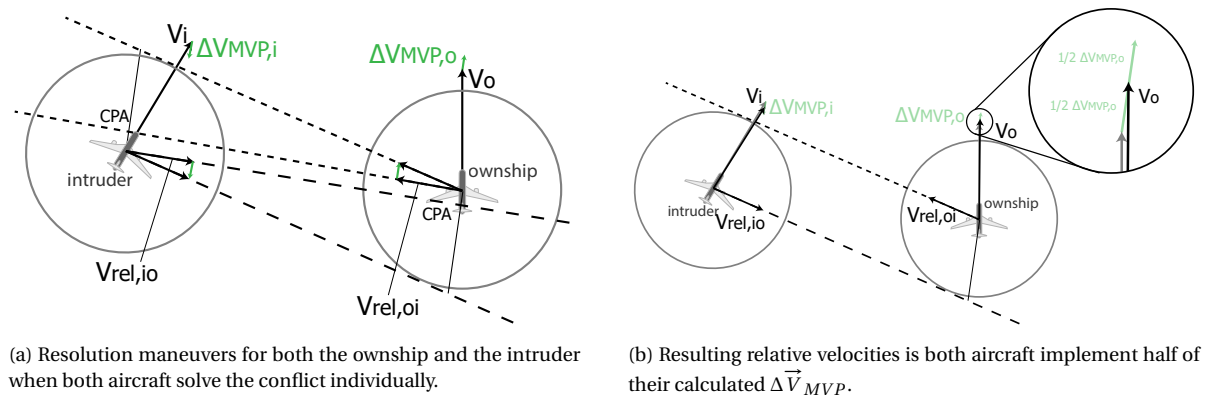


Figure 3.2: MVP resolution maneuvers for conflicting aircraft. As illustrated, when both aircraft implement half of the resolution maneuver, separation is maintained.

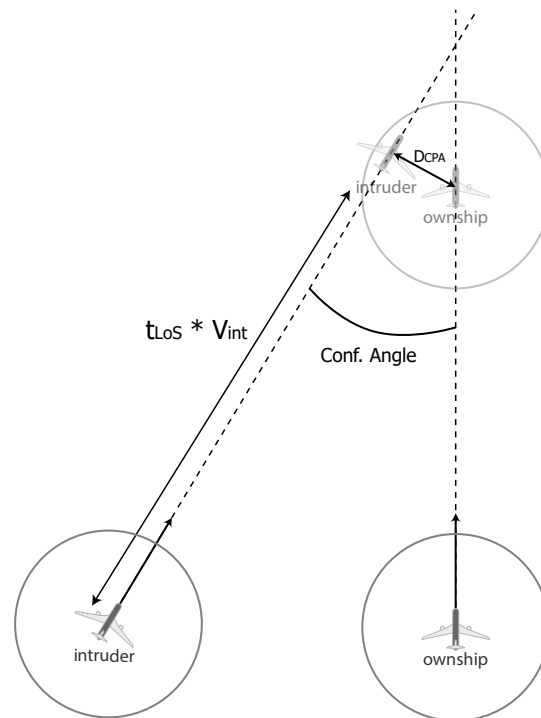


Figure 3.3: Conflict generation for one-to-one experiment

3.4. Independent and dependent variables

The independent variables in this experiment are the following:

- three velocities for ownship and intruder; 200, 300, and 400 kts.
- 750 conflict angles equally spaced between 15 and 90 degrees.
- 500 D_{cpa} values equally spaced between 0 and 5 NM.

The dependent variables are used to both explain the performance and the mechanics of the MVP and to use in the calculation of derivatives of these variables. This experiment has the following dependent variables:

- Pre-resolution $t_{cpa,nr}$
- Resolution $t_{cpa,cr}$
- Magnitude and direction of $\Delta \vec{V}_{MVP}$.

3.5. Results and discussion

First the results regarding pre resolution time to CPA and the resolution time to CPA are discussed. Then, the route deviations, heading changes, and required ground speed changes are detailed.

In each experiment, three different speeds are tested for the ownship and the intruder. As can be seen in the results, these speeds have little effect on the distribution of the dependent variables. This is because the conflict is essentially being scaled as these velocities change. As the results are being discussed, and the distribution of the decision variables is detailed, it becomes clear why the change of velocities does not change this distribution notably.

3.5.1. Pre resolution time to CPA $t_{cpa,nr}$

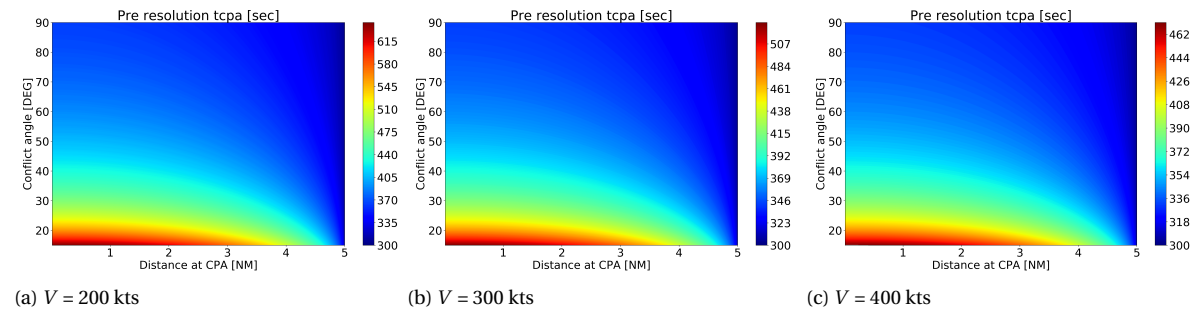


Figure 3.4: Pre conflict $t_{cpa,nr}$ for varying values of V .

As can be seen in fig. 3.4, notable differences can be seen between varying conflict geometries.

Effect of D_{cpa}

The lower the D_{cpa} , the longer the aircraft has to fly within the PZ of the intruder to reach the $t_{cpa,nr}$. Since the time to intrusion t_{LoS} is an input variable of the experiment setup, decreasing $t_{cpa,nr}$ values are found for increasing D_{cpa} values.

Effect of conflict angle

Lower conflict angles result in lower relative velocities. As a result of these lower relative velocities, the ownship spends more time between t_{LoS} and $t_{cpa,nr}$. Again, since the time to intrusion t_{LoS} is an input variable of the experiment setup, lower conflict angles thus result in higher $t_{cpa,nr}$ values.

3.5.2. Resolution time to CPA $t_{cpa,cr}$

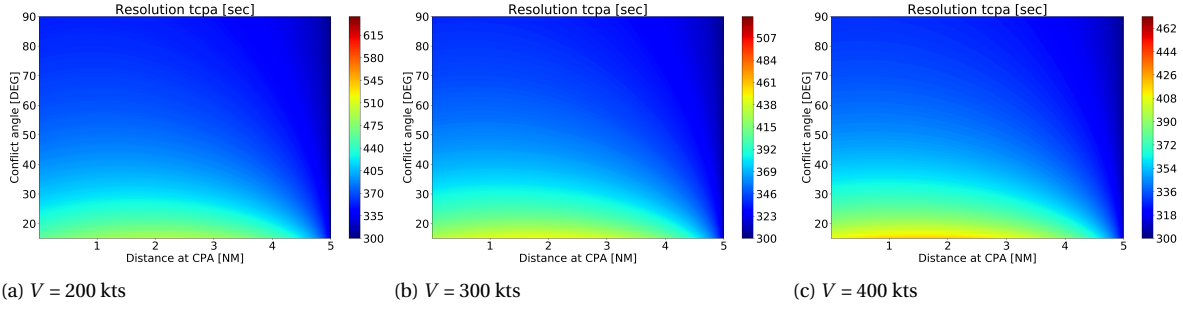


Figure 3.5: Resolution $t_{cpa,cr}$ for varying values of V .

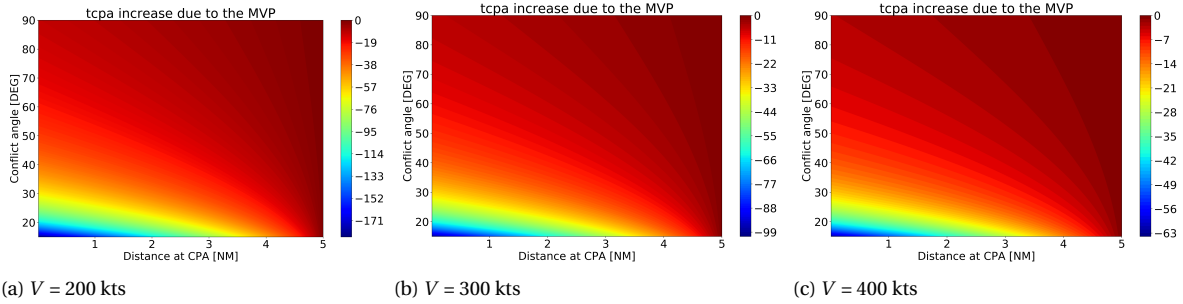


Figure 3.6: Difference between time to CPA for resolution $t_{cpa,cr}$ and time to CPA without resolution $t_{cpa,nr}$ in seconds for varying values of V .

The time to CPA of the resolution maneuver $t_{cpa,cr}$, see fig. 3.5, is best explained using the difference between $t_{cpa,nr}$ and $t_{cpa,cr}$ seen in fig. 3.6.

Effect of D_{cpa}

At lower values of the D_{cpa} , the magnitude of $\Delta \vec{V}_{MVP}$ increases, see the restatement of eq. (2.2) below. This increase leads, in turn, to an increase in $|\Delta V_{rel}|$. Subsequently, larger $|\Delta V_{rel}|$ values lead to larger $\Delta t_{cpa,cr}$ values, see eq. (3.1). Thus, as can be seen in fig. 3.6, larger t_{cpa} reductions are found for lower D_{cpa} values.

$$\Delta \vec{V}_{MVP} = \frac{D_{i,corr}}{t_{cpa}} \cdot \frac{\vec{D}_{cpa}}{D_{cpa}}$$

$$t_{cpa} = \frac{D_{rel}}{|\vec{V}_{rel}|} \quad (3.1)$$

Lemma 1:

Resolutions provided by the MVP always increase the relative velocity $|\vec{V}_{rel,cr}|$ between the ownship and the intruder i.e. $|\vec{V}_{rel,cr}| \geq |\vec{V}_{rel,nr}|$.

Proof:

From the mechanics of the MVP described in section 2.2 follows that $\vec{V}_{MVP}$ is chosen perpendicular to the relative velocity between the ownship and the intruder $\vec{V}_{rel,nr}$, i.e. $\Delta \vec{V}_{MVP} \perp \vec{V}_{rel,nr}$. Using $\vec{V}_{rel,cr} = \vec{V}_{rel,nr} + \Delta \vec{V}_{MVP}$ proves that $|V_{rel,cr}| \geq |V_{rel,nr}|$.

Effect of conflict angle

To characterize the effect of the conflict angle on the difference between $t_{cpa,nr}$ and $t_{cpa,cr}$, both the behavior of D_{rel} and $|V_{rel}|$ from eq. (3.1) are investigated.

Behavior of D_{rel}

The MVP pushes the CPA outwards until it no longer lies within the PZ, using the erratum it then ensures that the new CPA does not lie in the PZ. However, this new CPA does not lie in the direction of $D_{CPA,nr}$. Indeed, a new CPA is located at the point where the distance between the intruder and the new relative path is the smallest. These CPA points have the property that they form the right angle of a right triangle between the ownship, the intruder and the new CPA, see fig. 3.8. Using trigonometry the following expression for $D_{rel,cr}$ is found:

$$D_{rel,cr} = \frac{1}{2}r \sqrt{\frac{4r^2 - d^2}{r^2}} \sqrt{\frac{D_{rel,nr}^2 + i^2}{D_{rel,nr}^2}} - r \sqrt{\frac{D_{rel,nr}^2 + i^2}{D_{rel,nr}^2}} \sin \left(\frac{d + 4r \tan^{-1} \left(\frac{i}{D_{rel,nr}} \right) - \pi r}{2r} \right) \quad (3.2)$$

Where:

$$r = \frac{1}{2}D_{o,i}$$

$$d = D_{cpa}$$

$$i = \text{intrusion } (D_i)$$

Since it is too hard to analyze the behavior of $D_{rel,cr}$ through this equation, a contour plot will be used to analyze the behavior, see fig. 3.7. It can be seen that, as expected, at low conflict angles, $D_{rel,cr}$ becomes significantly smaller than $D_{rel,nr}$ since low conflict angles lead to small distances between ownship and intruder. Additionally, as expected when looking at fig. 3.8, lower D_{cpa} values lead to lower values for $D_{rel,cr}/D_{rel,nr}$. Note that the conflict angle axis of fig. 3.7 does not equal that of the figures above.

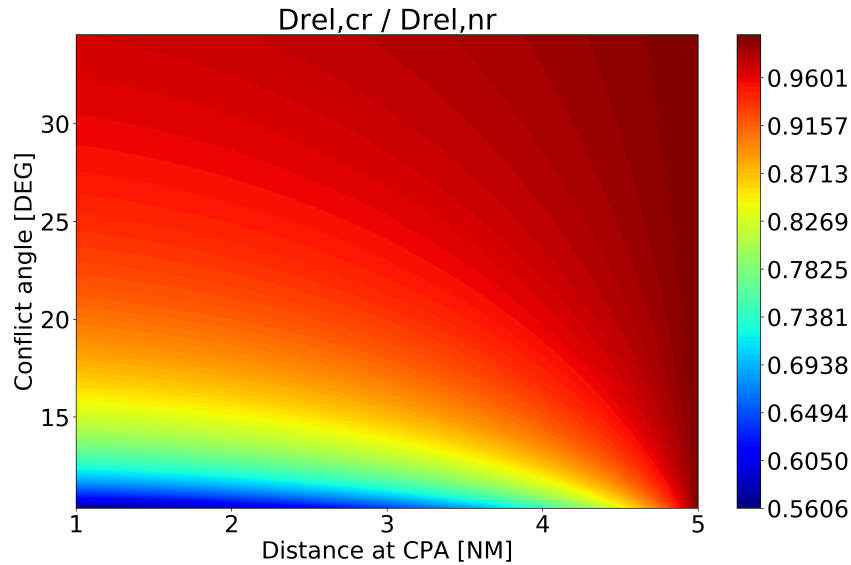


Figure 3.7: Contour plot of the ratio $D_{rel,cr}$ to $D_{rel,nr}$. V_o and $V_i = 300kts$, $S_h = 5NM$, and $t_{LoS} = 300sec$. Contour plot is generated using the MVP erratum for corrected intrusion calculation.

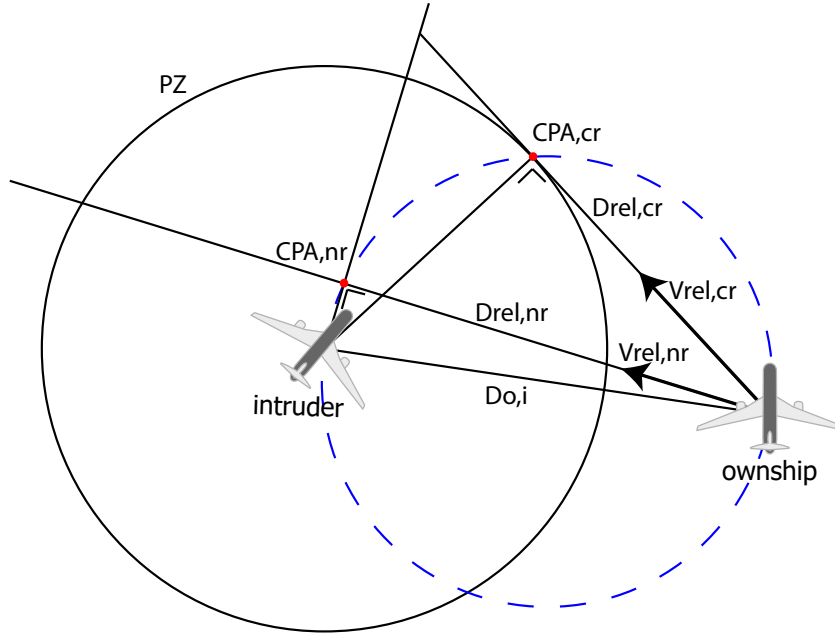


Figure 3.8: The right triangles formed by CPA locations, ownship and intruder. Due to the right angle property of the CPA, the set of possible CPA locations is described by a semicircle with a radius of half the distance between ownship and intruder and its center in between the ownship and intruder.

Behavior of $|V_{rel}|$

Since the MVP provides new relative velocities $V_{rel,cr}$ by projecting $V_{rel,nr}$ perpendicular to the tangent between the ownship and the intruder PZ, it can be stated that $|V_{rel,cr}| > |V_{rel,nr}|$ (see lemma 1). However, the relative increase is dependent on the initial relative velocity, as can be seen in fig. 3.10. Using trigonometry the following equation for $|V_{rel,cr}|$ is found:

$$|V_{rel,cr}| = \frac{2R}{\sqrt{4R^2 - d^2}} \cos \left(\frac{\arcsin\left(\frac{d}{2R}\right)}{\sqrt{1 + \frac{i^2}{4R^2 - d^2}}} \right) \quad (3.3)$$

With:

$$R = \frac{1}{2}|V_{rel,nr}|$$

$$d = D_{cpa}$$

$$i = \text{intrusion } D_i$$

The behavior of this equation is investigated through the contour plot in fig. 3.9. As can be seen, the influence of conflict angles and D_{cpa} is most dominant for very low conflict angles and very low D_{cpa} values. Indeed, for conflict scenarios with a conflict angle of more than 20 degrees or a D_{cpa} of more than 2 nautical miles, the relative increase of $|V_{rel,cr}|$ with respect to $|V_{rel,nr}|$ is less than 9 percent. Therefore, it can be stated that the decrease in $t_{cpa,cr}$ with respect to $t_{cpa,nr}$ is mostly due to the decrease of D_{rel} .

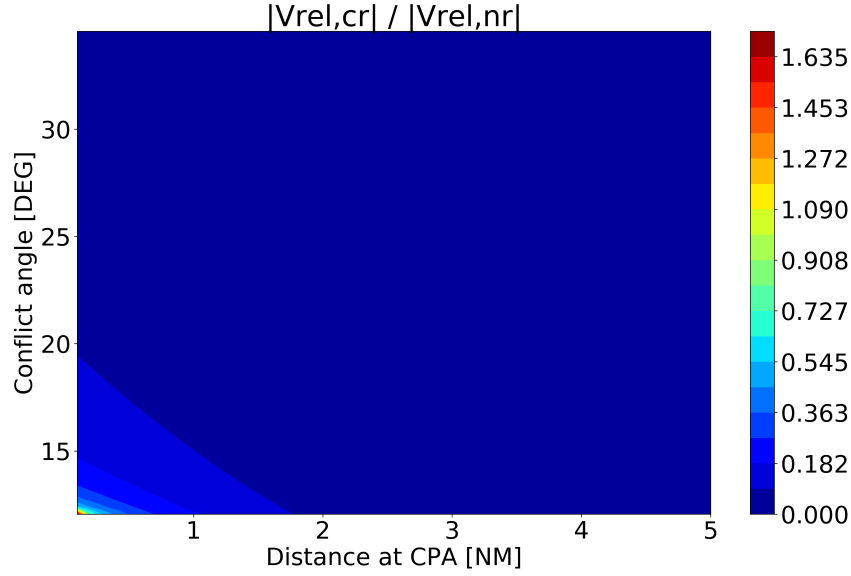


Figure 3.9: Contour plot of the ratio $|V_{rel,cr}|$ to $|V_{rel,nr}|$. V_o and $V_i = 300kts$, $S_h = 5NM$, and $t_{LoS} = 300sec$. Contour plot is generated using the MVP erratum for corrected intrusion calculation.

3.5.3. Route deviation

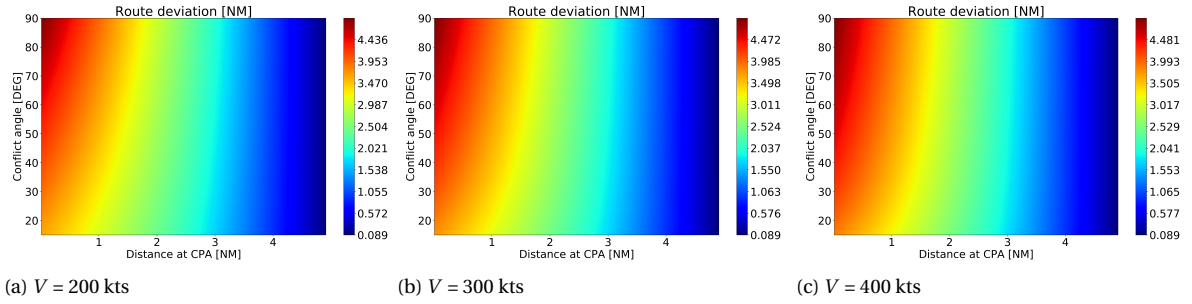


Figure 3.11: Route deviation in nautical miles for varying values of V .

The route deviation is calculated by integrating eq. (2.23) (Ellerbroek et al., 2013a) over the time to CPA due to CR $t_{cpa,cr}$. A visualization of this deviation can be seen in fig. 3.12. The equation is restated below for convenience. The results can be seen in fig. 3.11.

$$DEV = \int_{t_0}^{t_1} |\Delta \vec{V}_{sol}(t)| dt = |\Delta \vec{V}_{MVP}| \cdot t_{cpa,cr}$$

Effect of D_{cpa}

Since the magnitude of $\Delta \vec{V}_{MVP}$ is increasing with decreasing $D_{cpa,nr}$, the route deviation increases as well with decreasing $D_{cpa,nr}$.

Effect of Conflict angle

The effect of conflict angles on route deviation can be explained by rewriting and investigating equation eq. (2.23):

$$DEV = |\Delta \vec{V}_{MVP}| \cdot t_{cpa,cr} \quad (3.4)$$

Using eq. (2.2) gives:

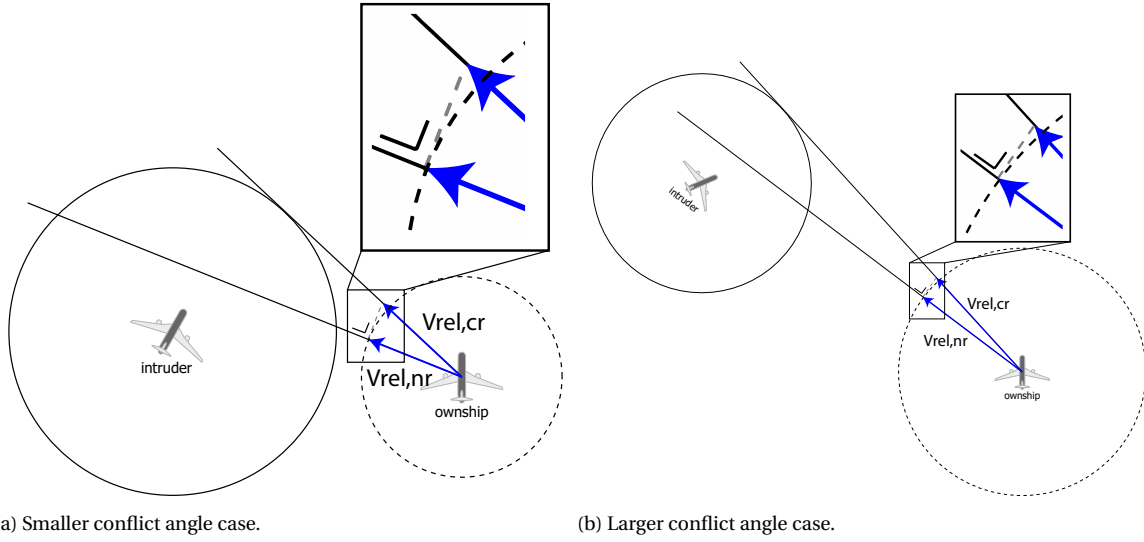


Figure 3.10: Relative increase of $|V_{rel,cr}|$ with respect to $|V_{rel,nr}|$ for both a small and large conflict angle scenario. Equal magnitudes for $|V_{rel,nr}|$ indicated by dotted circle around the ownship.

$$DEV = \frac{(S_h - D_{cpa,nr})}{t_{cpa,nr}} \cdot t_{cpa,cr} \quad (3.5)$$

$$DEV = (S_h - D_{cpa,nr}) \left(\frac{t_{cpa,cr}}{t_{cpa,nr}} \right) \quad (3.6)$$

Using eq. (3.6), it can be stated that the deviation, in the sense of Ellerbroek's equation, can be expressed as the corrected intrusion multiplied with $\frac{t_{cpa,cr}}{t_{cpa,nr}}$. Looking at fig. 3.13a, it can be seen that the corrected intrusion is mostly independent of the conflict angle, except the slight increases at lower conflict angles. This is due to the erratum used in the MVP calculation. Therefore, at lower conflict angles, the corrected intrusion is larger than the actual intrusion. Looking at fig. 3.13b, the behavior of $\frac{t_{cpa,cr}}{t_{cpa,nr}}$ can be explained using similar reasoning as in section 3.5.2. The product of these two effects results in the plots in fig. 3.11. Indeed, mainly due to the effects of $\frac{D_{cpa,cr}}{D_{cpa,nr}}$, route deviation decreases for decreasing conflict angles.

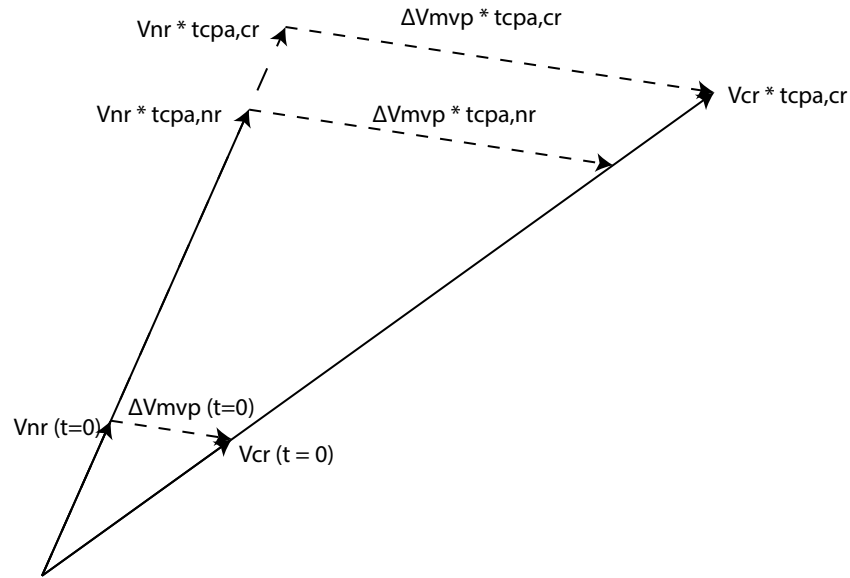


Figure 3.12: Example construction of deviation vector, not to scale. In the case where both aircraft apply half the required velocity changes, similar reasoning applies with the use of $\frac{1}{2}\Delta V_{mvp}$.

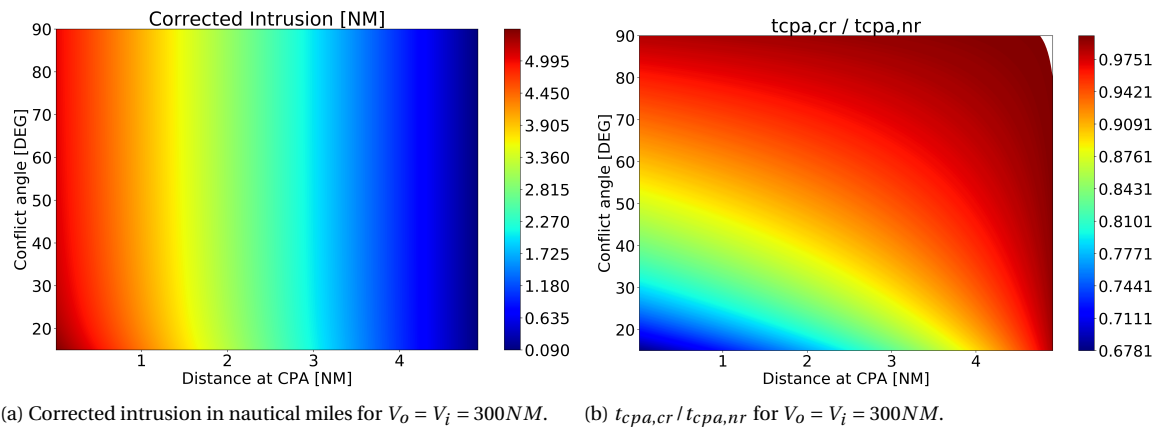


Figure 3.13: The corrected intrusion and the ration

3.5.4. Heading change

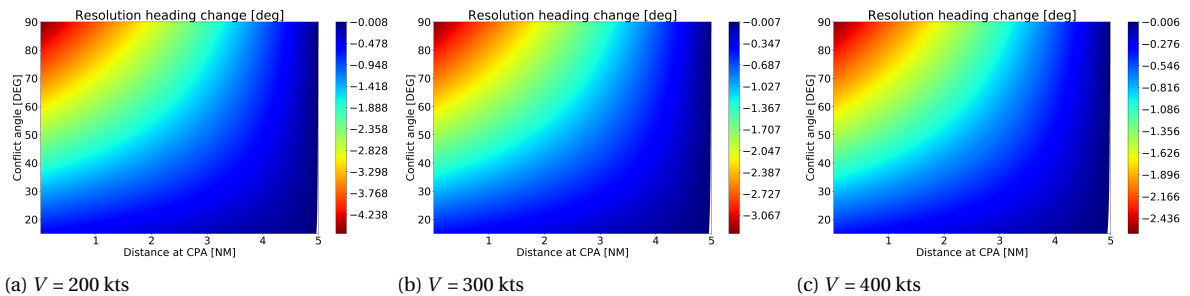


Figure 3.14: Required heading change due to CR in degrees for varying values of V .

The required heading change due to CR of the ownship can be seen in fig. 3.14.

Effect of D_{cpa}

As D_{cpa} decreases, the magnitude of $\Delta \vec{V}_{MVP}$ decreases as well. Therefore, at lower values of D_{cpa} , less heading changes are required.

Effect of conflict angle

The direction of $\Delta \vec{V}_{MVP}$ is perpendicular to the direction of $\vec{V}_{rel,nr}$, as described in section 2.2. Therefore, the more $\vec{V}_{rel,nr}$ is in line with $\vec{V}_o$, the more heading changes are required due to the MVP. It can be seen in figs. 3.2a and 3.3 that as conflict angles increase, the difference in directions of $\vec{V}_{rel,nr}$ and $\vec{V}_o$ decrease. Therefore, at larger conflict angles, more heading changes are required.

It should be noted that, since $t_{cpa,nr}$ also has an effect on the magnitude of $\Delta \vec{V}_{MVP}$, the effect described above is increased, as an increase in conflict angles lead to an decrease in $t_{cpa,nr}$ which leads to an increase in $|\Delta \vec{V}_{MVP}|$. This increased effect results in the curved lobes of fig. 3.14.

3.5.5. Ground speed change

In fig. 3.15, the required ground speed changes can be seen for the ownship.

Effect of D_{cpa}

Identical as in the case with the heading changes, as D_{cpa} decreases, the magnitude of $\Delta \vec{V}_{MVP}$ increases as well. This increase results in a increase in the required ground speed change.

Effect of Conflict Angle

Opposite of the conflict angle effect described for the heading change case, more ground speed change is required as $\Delta \vec{V}_{MVP}$ is in the same direction of $\vec{V}_o$. This happens when $\vec{V}_{rel,nr}$ is perpendicular to $\vec{V}_o$. Therefore, at lower conflict angles, more required ground speed change is expected. However, as can be seen in fig. 3.15, this is not visible in the results.

This can be explained by comparing the phenomenon described above with the effects of the $t_{cpa,nr}$. While at low conflict angles $\vec{V}_{rel,nr}$ is more perpendicular to $\vec{V}_o$ than at high conflict angles, the increase in $t_{cpa,nr}$ at low conflict angles counters this effect. The higher $t_{cpa,nr}$ values decrease the magnitude of $\Delta \vec{V}_{MVP}$, and thus the required ground speed change. Identically, lower $t_{cpa,nr}$ values increase the magnitude of $|\Delta \vec{V}_{MVP}|$, and thus the required ground speed change. This combined effect results in the vertical curved contour lines of fig. 3.15.

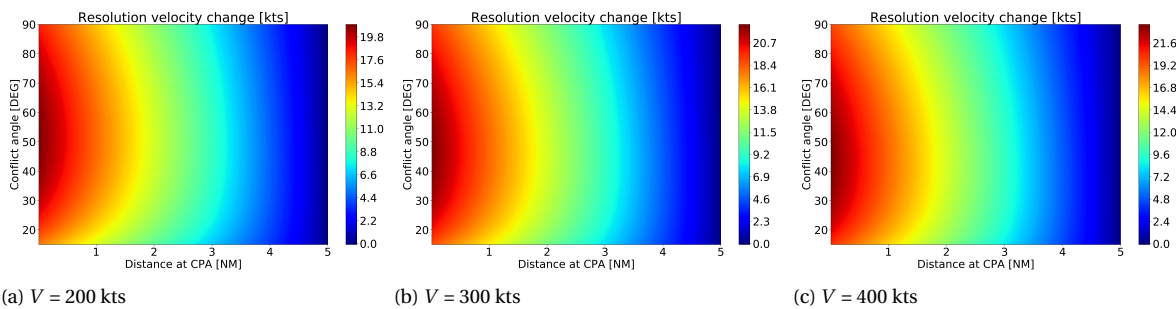


Figure 3.15: Required ground speed change due to CR for varying values of V .

3.6. Conclusion

The one-to-one experiment has provided information of two kinds, firstly the conclusions from the results valid for two-aircraft conflicts:

- There is, mathematically speaking, no direct reason to assume that small conflict angle resolutions provided by the MVP lead to larger route deviations than large conflict angle resolutions. If any, lower conflict angles lead to smaller deviations. The large scale experiment is required to investigate if these route deviations do (emergently) occur for multi-aircraft conflicts.
- It can be seen that, at low conflict angles, $t_{cpa,cr}$ is larger than at higher conflict angles. However, the decrease in time to CPA with and without CR is the largest for small conflict angles. This is an important distinction. To determine what the strengths and weaknesses are of the MVP, attention should be directed to the *changes* the MVP brings about on these metrics, and not only on the resulting metrics for small conflict angles conflict resolutions. Conflicts with different conflict angles have different *intrinsic* characteristics, and when looking at these results of these metrics, these *intrinsic* characteristics have to be kept in mind.

Secondly, it provides experimental context as well as mathematical tools to interpret the results from the large scale experiment. Some results in the large scale experiment will inevitably differ from the results obtained in this one-to-one experiment. These differences are interesting and also an essential part of understanding the MVP behavior. Meaningful propositions for alternative methods cannot be made without the results and lessons learned from both experiments.

4

Many-to-many Experiment Proposal

In this chapter, the many-to-many experiment to extensively investigate the emergent behavior of the MVP in small conflict angle scenarios is proposed. As described before, it is inevitable that the large-scale experiment shows different performance than the large-scale experiment and the calculations. This emergent behavior is essential to understanding the MVP and to be able to propose adaptations. First, the goal of this many-to-many experiment is detailed. Then the data which will be logged is discussed. This data will be used for metrics as well as insights into the emergent mechanics of the MVP in multi-aircraft, small conflict angle scenarios.

4.1. Experiment Goal

As stated in section 1.2, the goal of this large-scale experiment is to gain insight into the emergent behavior of tactical CR using the MVP. Using these insights, propositions regarding improvements to the algorithm are to be made at the beginning of the main phase of this thesis. For corresponding research activity steps, see Figure 4.1.

The many-to-many experiment is set up in such a way, that the experiment resembles a piece of en-route airspace. This piece of en-route airspace is configured in such a way, that varying conflict angles are dominant. To generate scenarios in which different conflict angles are dominant, different flight heading ranges are used. This is achieved by choosing the destinations of the aircraft in such a way that the required heading to fly to the destination falls within a specified band. This way, a maximum conflict angle when CR is OFF can be achieved. It should be noted that these maximum conflict angles can increase when CR is on due to previous maneuvers of the aircraft. These bands are the following:

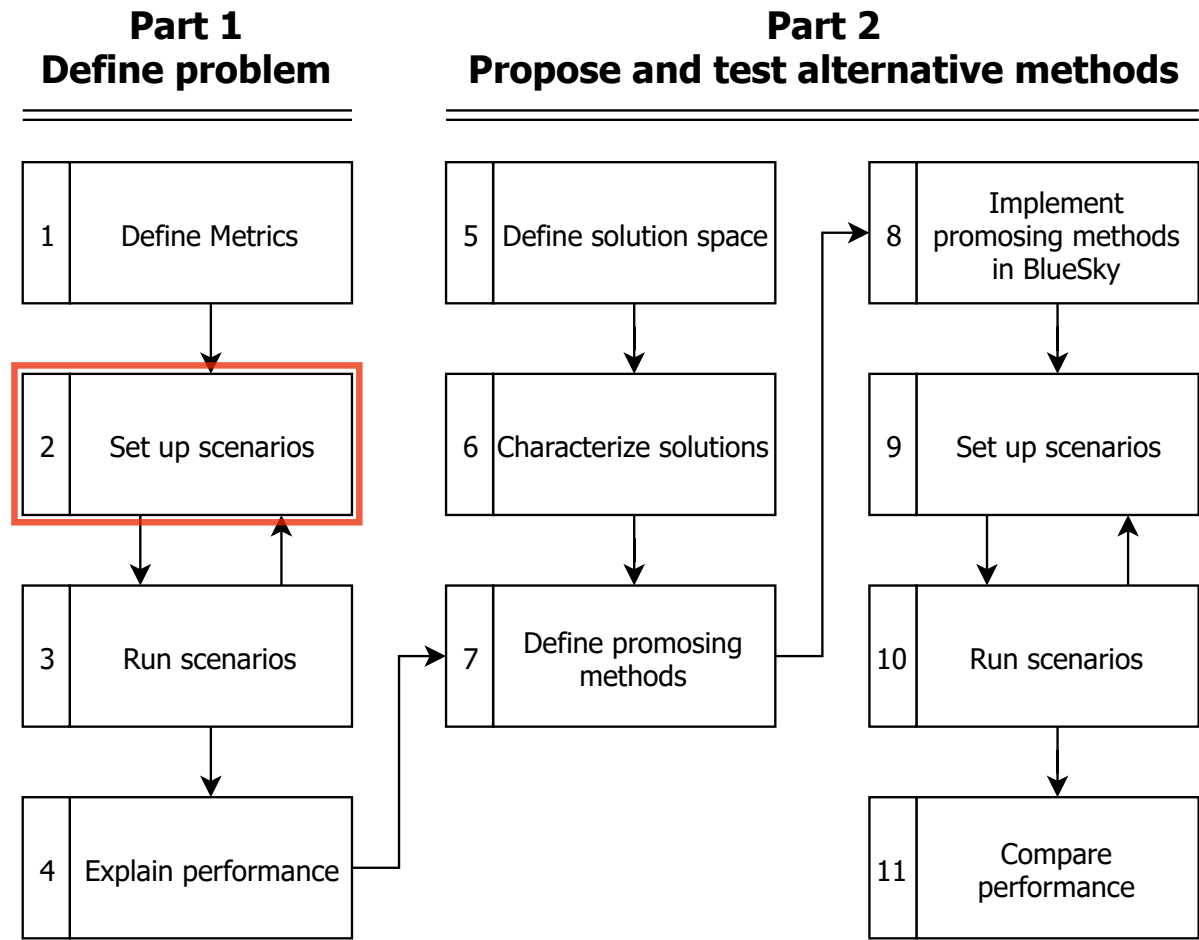


Figure 4.1: Corresponding thesis phases for Chapter 4.

No.	Max. conflict angle (CR OFF)	Heading Band
1	5	[87.5, 92.5]
2	7.5	[86.25, 93.75]
3	10	[85, 95]
4	15	[82.5, 97.5]
5	20	[80, 100]
6	25	[77.5, 102.5]
7	30	[75, 105]
8	35	[72.5, 107.5]
9	40	[70, 110]
10	45	[67.5, 112.5]

Table 4.1: Maximum conflict angles when CR is OFF and corresponding heading band.

4.2. Simulation characteristics

In this section, the used simulation platform, conflict detection module, and used conflict resolution module are explained.

4.2.1. Platform

As stated in section 1.4, the simulation platform used in this thesis is the BlueSky open-source ATM simulator (Hoekstra and Ellerbroek, 2016) written in Python. This open-source ATM simulator was developed at the TU Delft and has conflict detection and resolution algorithms implemented as well as a wide range of data logging functions. The aircraft are simulated using an aircraft performance model based on point-mass dynamics. For more information regarding the validation of BlueSky and the used aircraft performance models, the reader is directed to (Metz et al., 2016). In this experiment, all traffic was simulated as a Boeing 737 model.

4.2.2. Conflict detection

The CD method used in this thesis is state-based, as indicated in chapter 1. Additionally, the look-ahead time was set to 5 minutes. Both the horizontal and vertical separation was set to 5 NM and 1000 ft, respectively.

Furthermore, the CD module assumes perfect knowledge of the ADS-B data and thus perfect knowledge of the aircraft states. This, too, is in line with the assumptions stated in section 1.4 and the research performed by Langejan et al. (2016).

4.2.3. Conflict resolution

The conflict resolution in this experiment is tactical CR using the MVP. The full explanation of the MVP can be found in section 2.2 or (Hoekstra, 2001).

In line with the thesis scope, the resolution maneuvers will be restricted to combined heading and speed resolutions. This is for two reasons. Firstly, conflict angles have the most effect on the horizontal resolution strategies when using the MVP, thus using horizontal resolution strategies provides the most insight. Secondly, small conflict angles are found to have a negative effect mostly on horizontal resolution strategies in previous research (Sunil, 2019).

Because in this experiment, scenarios are created with different conflict angle ranges, it should be noted that these ranges might be violated due to previous CR maneuvers. For example, if all the aircraft in the scenario are flying a heading between 30 and 40 degrees to reach their exit waypoint, it might occur that an aircraft will have to fly a heading outside of this range in order to reach its exit waypoint due to previous CR maneuvers. Additionally, MVP resolution instructions take precedence over the constraints imposed on the routing. In other words, if the MVP instructs an aircraft to fly a heading outside of the scenario heading range, this is allowed.

As stated in section 1.4, a step-wise recalculation method with $\Delta t = 1s$ is used. This means that at each second, all aircraft perform CD. If a conflict is detected, a resolution maneuver is calculated using the MVP module. This maneuver is then initiated by the aircraft instantaneously and simulated using a timestep of 0.05 seconds. If at the next second, the CD module still detects a conflict, a new resolution maneuver is calculated and initiated. This is repeated until there is no longer a conflict, at which point the current trajectory is followed until CPA. After CPA, both aircraft are free to change their heading and velocity to the desired state.

4.3. Scenario generation

This section explains the scenario generation. This consists of setting up the experiment area in BlueSky and defining the simulation procedure. Since certain dimensions of the experiment area

are related to choices made in the simulation procedure, the area dimensions are explained after the simulation procedure.

4.3.1. Experiment area

A horizontal sector was generated as the environment of the experiment, see fig. 4.4. The complete sector is called the simulation area. To minimize the error of the flat-earth approximation used by BlueSky, the simulation area is placed at Lat/Lon (0,0). Aircraft are only generated within this simulation area. Therefore, aircraft flying near the border of this area experience fewer conflicts than aircraft in the center of the area, as well as a lower aircraft density. To solve this issue, a logging area is defined within this simulation area. General flight data, as well as conflict-related data, is logged of the flights in the logging area throughout.

More information regarding the sizing of the experiment area and the route generation is provided in section 4.3.2. The reason for this, is that certain information from section 4.3.2 is required for the sizing of the experiment area.

4.3.2. Simulation procedure

The scenario itself was generated using a Python script. This scenario generation was performed off-line and then loaded into BlueSky. The basics of which entail spawning aircraft, making them fly a certain heading through a waypoint and subsequently deleting them when they have reached their waypoint. These three steps, among other information, are detailed in this section

The aircraft used for this experiment is the B737. In BlueSky, this aircraft has a minimum ground speed of 257 kts and a maximum ground speed of 369 kts at FL100. Note that this can be stated due to the assumption that no wind is present. To incorporate variation in the aircraft velocity, a uniform distribution between 286 kts and 342 kts is chosen to determine the instructed ground speeds for each aircraft. These values are chosen so that the average velocity is the mean velocity of the velocity range, resulting in all aircraft having sufficient room to accelerate and decelerate to resolve conflicts, see fig. 4.3.

To keep the simulation times low, an average flight distance of in the x-direction of 300 NM is the chosen. The average euclidean flight distance can therefore be calculated using eq. (4.1). Using the average ground speed of 314 kts, an average flight time can be calculated as well.

$$\overline{D} = \frac{300}{\cos\left(\frac{1}{4}\text{Heading Band}\right)} \quad (4.1)$$

Since five different densities were tested, the aircraft spawn rate Ω by Sunil (2019) was used, see eq. (4.2). This equation calculates the spawn rate by dividing the average velocity of the aircraft $\overline{V}$ by the average distance flown by the aircraft $\overline{D}$ and multiplying this with the desired instantaneous amount of aircraft at each time in the scenario N_{inst} . Using this spawn rate, various densities can be build up in the simulation.

To guarantee a smooth and stable as possible experiment, the scenarios consisted of two phases; the density buildup phase and the data logging phase, see fig. 4.2. During the density buildup phase, the required density for the scenario is build up using the spawn rate in eq. (4.2). During the data logging phase, the data of the aircraft in the tracked aircraft list is logged.

It should be noted that this data logging phase consists of two sub-phases. In the first hour, new aircraft are allowed in the tracked aircraft list. During the second hour, no new aircraft are allowed in the tracked aircraft list. Similar to the arguments made by Sunil (2019), this is to prevent short flights from skewing the results. Skewed results would occur if the data logging would instantaneously stop while tracking an aircraft in the tracked aircraft list. Traffic is spawning at a semi-constant rate throughout all phases.

Aircraft spawn locations are distributed uniformly throughout the spawn area. A instructed heading and matching waypoint are generated uniformly within the corresponding heading band. Finally, the instructed flight speed is generated uniformly from the set of instructed ground speeds.

The logging area will be defined during the post-processing using density data. The largest possible area with an average density equal to the required density is selected as logging area. Additionally, since the uniform distributed spawning locations might result in a LoS and/or conflicts at the time of spawning, the spawn area is not included in the logging area. Finally, since, as a result of prior conflict resolution, aircraft might engage in high heading changes to revert back to their waypoint heading when nearing the right edge of the simulation area, a margin of 25 nautical miles is chosen between the end of the logging area and the right end of the simulation area.

$$\Omega = \frac{\bar{V}}{D} \cdot N_{inst} \quad (4.2)$$

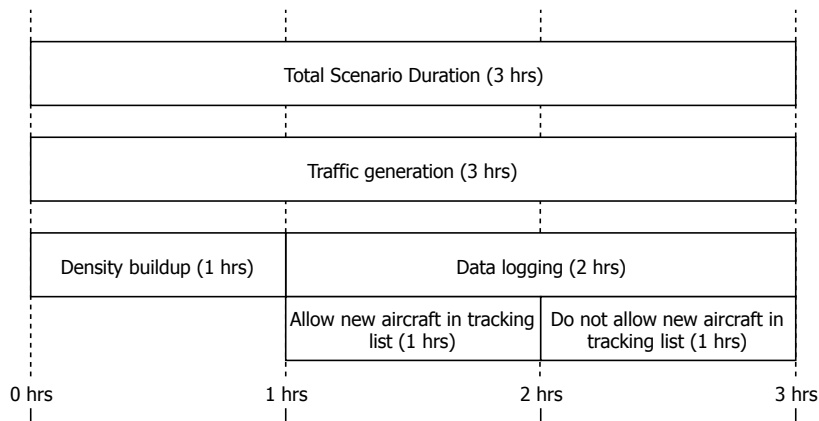


Figure 4.2: The three scenario phases

4.4. Independent and dependent variables

In this section, both the independent and dependent variables of this many-to-many fast time simulation are discussed.

4.4.1. Independent variables

The independent variables used in this experiment are the following:

- 2 CR setting, 'CR OFF' and 'CR ON'
- 10 heading ranges, see table 4.1
- 5 densities; 5, 10, 20, 30, and 50 A/C per 10,000 NM².

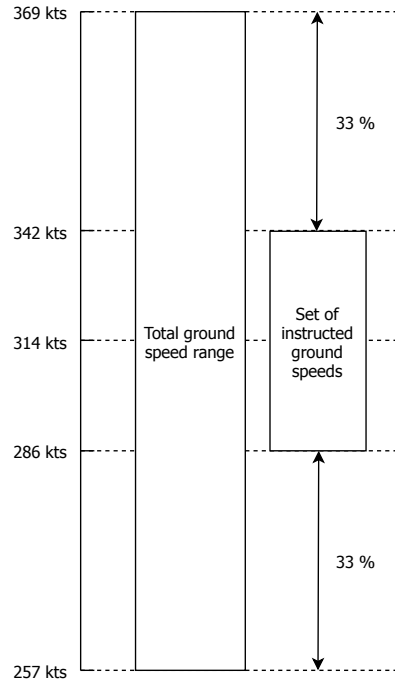


Figure 4.3: Instructed B737 speed distribution for the scenarios.

Using the rule of thumb that the number of repetitions should be equal to at least the number of steps taken in the main independent variable Sunil (2019), each heading range scenario is generated ten times. This results in 1000 simulation runs.

4.4.2. Dependent variables

The initial characteristics of conflicts and their resolution are being logged. These characteristics can be divided in initial detection characteristics and resolution characteristics. The initial detection characteristics are:

- Conflict angle
- Time to CPA before resolution $t_{cpa,nr}$
- Distance between conflicting aircraft D_{conf}
- Distance at CPA D_{cpa}
- Time until LoS t_{los}
- Ownship and intruder location at start of conflict
- Ownship and intruder heading at start of conflict

The initial resolution characteristics are the characteristics of the first resolution maneuver calculated by the MVP CR module. The fact that it is the first resolution maneuver is important since the CR module recalculates a CR maneuver every second until the CPA lies outside of the PZ. Therefore, the first resolution maneuver calculated by the module is the one providing the most insight into the conflict. These initial resolution maneuver characteristics are:

- Magnitude of ΔV_{mvp}

- Time to CPA due to resolution maneuver $t_{cpa,cr}$
- Required heading change $\Delta\Psi_{cr}$
- Required speed change ΔV_{cr}

When a aircraft reaches its destination, certain data is logged as well before deleting the aircraft. This data is:

- Distance flown
- Time flown

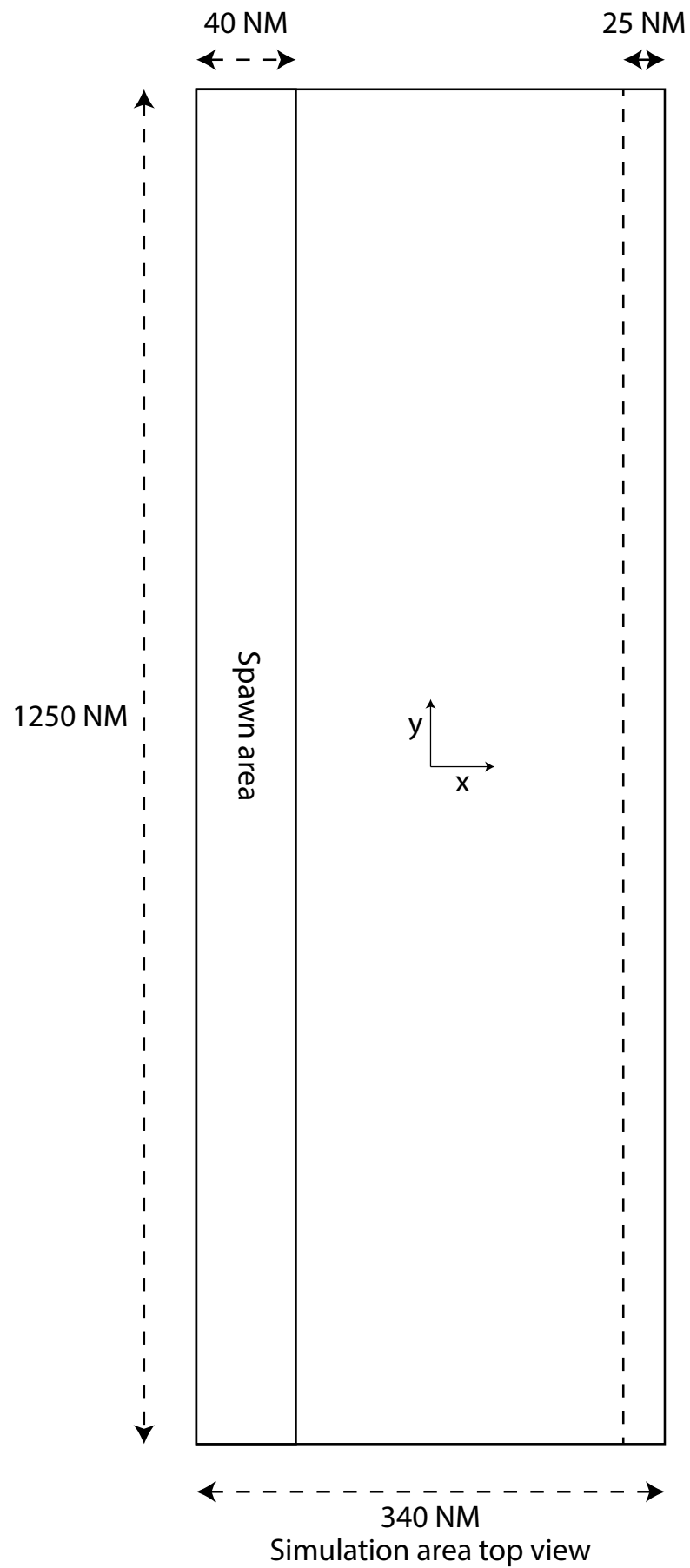


Figure 4.4: Experiment Area representing a piece of en-route airspace

5

Pairwise Solution Space

In this chapter the pairwise solution space (PSS) is defined. The PSS consists of the set of velocities for the ownship which result in a LoS free passing of the intruder in coordinated fashion. First the PSS is defined, then the characteristics of the solutions within the PSS are explained. These insights are essential when proposing alternative methods. For corresponding research activity steps, see Figure 5.1.

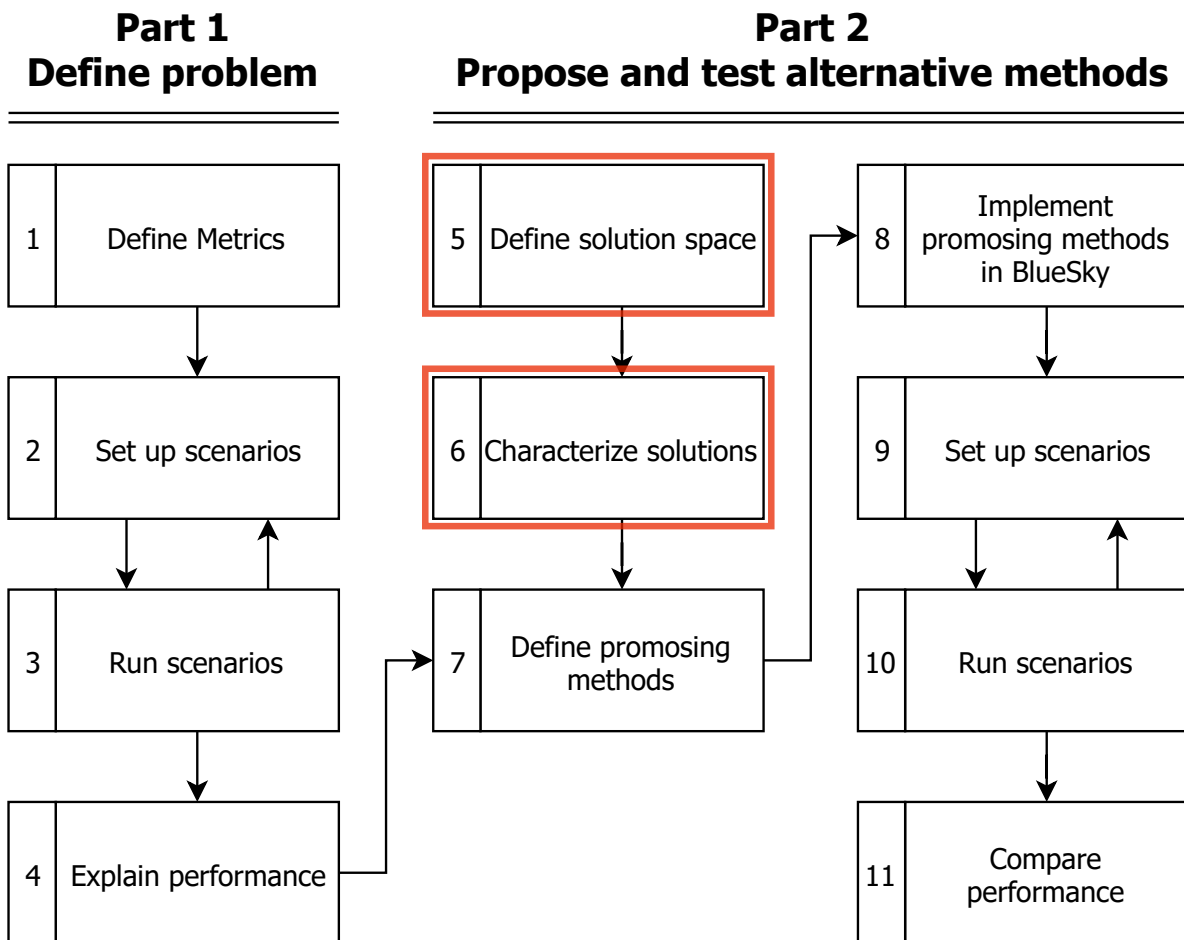


Figure 5.1: Corresponding thesis phases for Chapter 5.

5.1. Setting up the Pairwise Solution Space

The construction of the PSS is divided into four steps; imposing performance limits, drawing the velocity obstacle, excluding conflict avoiding velocities and finally excluding uncoordinated resolution maneuvers. These four steps as well as their reasoning are described in the following subsections. To visualize the construction of the PSS, an example construction of the PSS for the ownship in the conflict of fig. 5.2 will be performed. Since only horizontal resolution maneuvers are considered, the vector space in $\mathbb{R}^2$ is used for construction of the PSS.

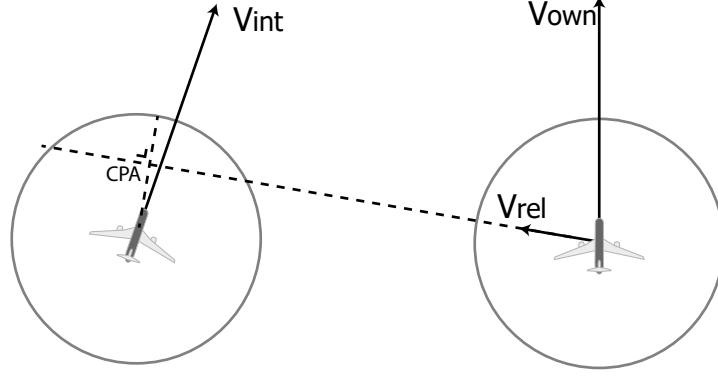


Figure 5.2: Example conflict used for construction of the solution space.

5.1.1. Step 1: Performance Limits

The first step of creating the PSS is imposing the performance limits. In this phase of the thesis, only static performance limits are taken into account. Dynamic performance limits, such as maximum acceleration and deceleration, are not taken into account. The static performance limits used are the minimum ground speed V_{min} and maximum ground speed V_{max} of the aircraft. These two limits can be drawn as two circles in the vector space around the aircraft, limiting the possible new velocities $V_{o,new}$, see fig. 5.4a. The velocities which fall inside of the performance limits are called the Reachable Velocities RV_o , see eq. (5.1).

$$RV_o = \left\{ V_{o,new} \in \mathbb{R}^2 : \left| \vec{V}_{o,new} \right| \geq V_{o,min} \wedge \left| \vec{V}_{o,new} \right| \leq V_{o,max} \right\} \quad (5.1)$$

5.1.2. Step 2: Velocity obstacle

Step two is introducing the VO from the ownship to the intruder $VO_{o,i}$ to the vector space. This VO indicates the forbidden velocities of the ownship because these would lead to a LoS. This VO is constructed by moving the apex of the CC by V_i , as described in section 2.3. Mathematically, it can be described as the set of velocities for which the relative velocity from the ownship to the intruder, $V_{o,new} - V_i$, lies within the tangents from the ownship to the PZ of the intruder. This is expressed in eq. (5.2). The resulting $VO_{o,i}$ can be seen in fig. 5.4b.

$$VO_{o,i} = \left\{ \vec{V}_{o,new} \in \mathbb{R}^2 : \vec{V}_{o,new} - \vec{V}_i \in \text{TANGENTS}_{o,i} \right\} \quad (5.2)$$

5.1.3. Step 3: Diverging Velocities

Velocity vectors which result in divergence of the two aircraft are not regarded solutions in this thesis. These Diverging Velocities $DV_{o,i}$ are defined as velocities which result in a t_{cpa} of exactly 0 seconds. In other words, velocities which result in instantaneous further separation of the two aircraft. This instantaneous separation can mathematically be described as the dot product of the dis-

tance vector from ownship to intruder $\vec{D}_{o,i}$ with the relative velocity of the ownship to the intruder $(\vec{V}_{o,new} - \vec{V}_i)$ being bigger than 0, see eq. (5.3).

$$DV_{o,i} = \left\{ \vec{V}_{o,new} \in \mathbb{R}^2 \quad : \quad \vec{D}_{o,i} \cdot (\vec{V}_{o,new} - \vec{V}_i) \geq 0 \right\} \quad (5.3)$$

How to draw this set in the vector-space, can be explained by rewriting eq. (5.3):

$$\frac{d}{dt} (|\vec{D}_{o,i}|) = \vec{D}_{o,i} \cdot \vec{V}_{rel,o,i}$$

Writing out the dot product gives:

$$\frac{d}{dt} (|\vec{D}_{o,i}|) = D_{o,i,x} \cdot V_{rel,o,i,x} + D_{o,i,y} \cdot V_{rel,o,i,y}$$

Using that, for avoidance velocities, $\frac{d}{dt} (|\vec{D}_{o,i}|) \geq 0$, the expression for avoidance velocities becomes:

$$D_{o,i,x} \cdot V_{rel,o,i,x} + D_{o,i,y} \cdot V_{rel,o,i,y} \geq 0$$

$$\frac{D_{o,i,x}}{D_{o,i,y}} \geq -\frac{V_{rel,o,i,y}}{V_{rel,o,i,x}}$$

Using this expression, it can be stated that the set of diverging velocities can be described as the set of ownship velocities which result in the relative velocity being perpendicular to the bearing of the intruder aircraft, as in fig. 5.4c.

5.1.4. Step 4: Uncoordinated resolution maneuvers

Since the implicit coordination of the MVP is to be retained, uncoordinated solutions are also not a part of the PSS. These are solutions in which the dot product of the initial distance at CPA vector $\vec{D}_{cpa}$ and the new distance at CPA vector, $(\vec{D}_{o,i} + (\vec{V}_{o,new} - \vec{V}_i) \cdot t_{cpa,new})$, is negative. In other words, the angle between the old and new distance at CPA vector is obtuse. Geometrically, it can also be described as the ownship choosing a resolution velocity not in the direction of the nearest $VO_{o,i}$ leg. The uncoordinated velocities $UV_{o,i}$ can be seen in fig. 5.4d.

$$UV_{o,i} = \left\{ \vec{V}_{o,new} \in \mathbb{R}^2 \quad : \quad (\vec{D}_{o,i} + (\vec{V}_{o,new} - \vec{V}_i) \cdot t_{cpa,new}) \cdot \vec{D}_{cpa} < 0 \right\} \quad (5.4)$$

5.1.5. Step 5: Improper solutions

An additional requirement of a CR maneuver is that, once the CPA has been reached, both aircraft can revert back to their original velocity and heading. Solutions which do not meet this requirement are named improper solutions (IS), see eq. (5.5). These solutions are harder to define geometrically since they are a function of $t_{cpa,new}$, $\vec{V}_{o,nr}$, and $VO_{o,i}(t_{cpa,new})$ and therefore not drawn in fig. 5.4. However, an example of improper solutions can be seen in fig. 5.3. The scenario in this figure is the Small angle scenario from table 5.1 which will be explained in the next section.

$$IS_{o,i} = \left\{ \vec{V}_{o,new} \in \mathbb{R}^2 \quad : \quad \vec{V}_{o,new} \cdot t_{cpa,new} \Rightarrow \vec{V}_{o,nr} \in VO_{o,i}(t_{cpa,new}) \right\} \quad (5.5)$$

Conflict	Conflict Angle	t_{LoS}	Ground speeds
Small angle	15 degrees	300 seconds	300 kts
Medium angle	50 degrees	300 seconds	300 kts
Large angle	80 degrees	300 seconds	300 kts

Table 5.1: Characteristics of the used example conflicts.

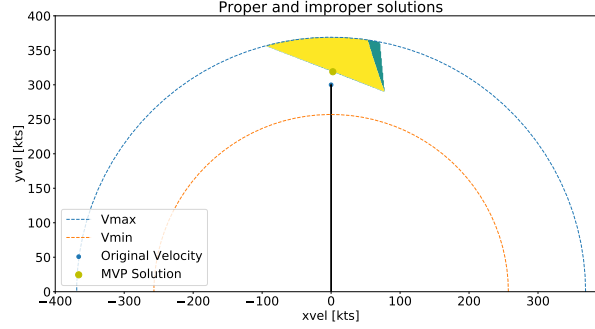


Figure 5.3: Proper (Yellow) and improper (Violet) solutions.

5.1.6. Additional restrictions

In this part of the research, no additional restrictions are imposed on the PSS. For instance, one might impose additional restrictions regarding allowed heading ranges, or restrictions related to velocity and heading changes. These additional restrictions are considered when proposing alternative methods. However, in the preliminary part of the thesis, these are not yet addressed.

5.1.7. Pairwise solution space

The velocities which are left are considered the pairwise solution space. This set can be described as the set of reachable velocities RV_o which are not in the velocity obstacle $VO_{o,i}$, not in the diverging velocities $DV_{o,i}$, not in the uncoordinated velocities $UV_{o,i}$ and not improper $IS_{o,i}$.

$$PSS_{o,i} = RV_o \setminus (VO_{o,i} \cup DV_{o,i} \cup UV_{o,i} \cup IS_{o,i}) \quad (5.6)$$

5.2. Characteristics of the pairwise solution space

Now that the PSS is defined, it is interesting to investigate the characteristics of the different solutions within the PSS. To provide visual examples, three conflict scenarios are used; a small conflict angle scenario, a medium conflict angle scenario, and a large conflict angle scenario. The characteristics of these conflicts can be seen in table 5.1. During the construction of the PSS, the subscript *new* will be used. This subscript indicates velocities or times to CPA for a chosen new solution in the PSS.

Using the mathematical expressions and visualization, four characteristics of various solutions within the PSS are investigated; relative velocities, time to CPA, distance at CPA, and extra distance flown (deviation).

5.2.1. Relative velocities

The relative velocities as a result of various solutions in the PSS can be seen in figs. 5.5 and 5.6. In fig. 5.6 the gradient of $\vec{V}_{rel}$ with respect to $\vec{V}_{o,new}$ is indicated by red arrows. This gradient is

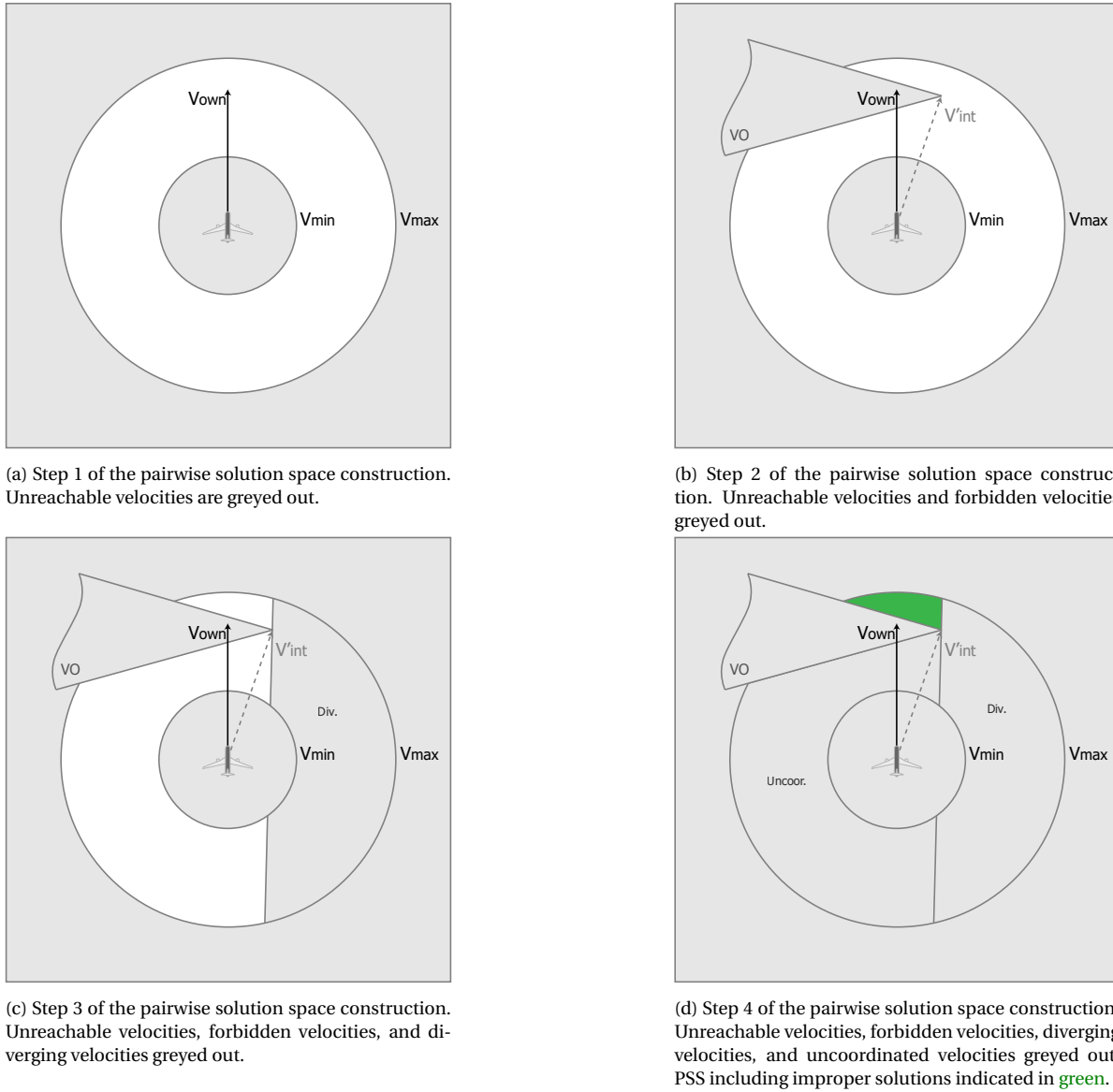


Figure 5.4: Construction steps of the pairwise solution space

calculated by taking the partial derivative of $\vec{V}_{rel}$ with respect to $\vec{V}_{o,new}$, see eq. (5.7).

$$\nabla \vec{V}_{rel}(\vec{V}_{o,new}) = \begin{bmatrix} \frac{\partial \vec{V}_{rel}}{\partial V_{o,new,x}}(\vec{V}_{o,new}) \\ \frac{\partial \vec{V}_{rel}}{\partial V_{o,new,y}}(\vec{V}_{o,new}) \end{bmatrix} \quad (5.7)$$

Looking at the results the following conclusion can be drawn:

- For each conflict scenario, solutions of increasing $|\vec{V}_{rel}|$ are obtained by choosing solutions with increasing distance to the apex of the VO, as expected. Subsequently, the gradients flow radially outward from the apex of the VO.

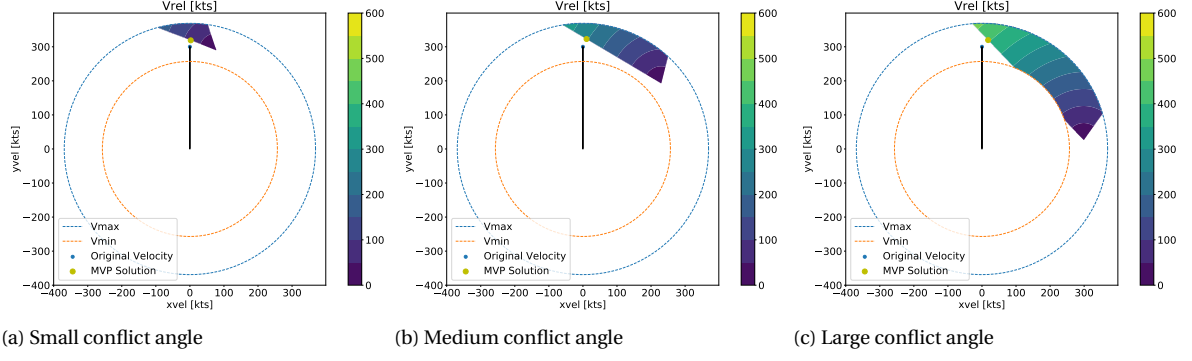
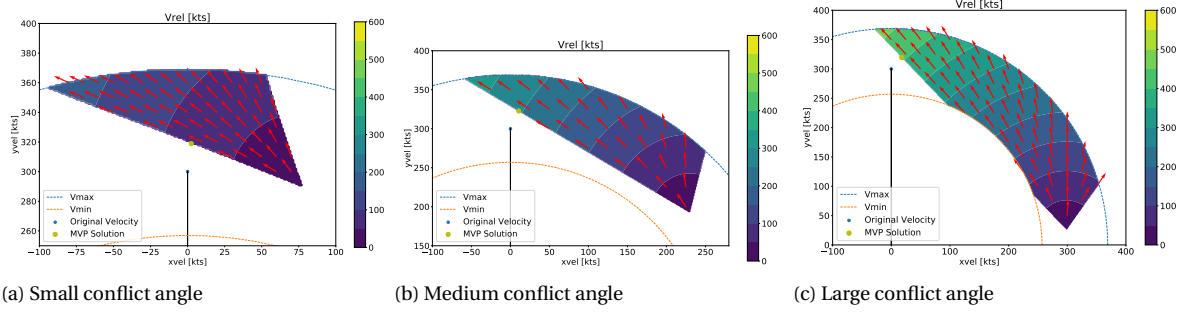


Figure 5.5: Resulting relative velocities for various solutions within the PSS.

Figure 5.6: Resulting relative velocities for various solutions within the PSS, zoomed in on the PSS. Red arrows indicate gradient of V_{rel} with respect to $V_{o,new}$.

5.2.2. Time to CPA

The time to CPA t_{cpa} as a result of various solutions in the PSS can be seen in figs. 5.7 and 5.8. In fig. 5.8 the gradient of t_{cpa} with respect to $\vec{V}_{o,new}$ is indicated by red arrows. This gradient is calculated by taking the partial derivative of t_{cpa} with respect to $\vec{V}_{o,new}$, see eq. (5.8).

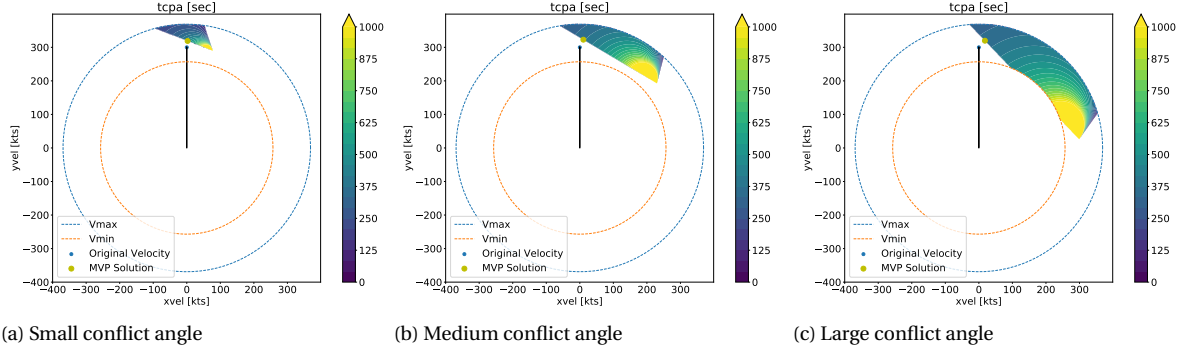
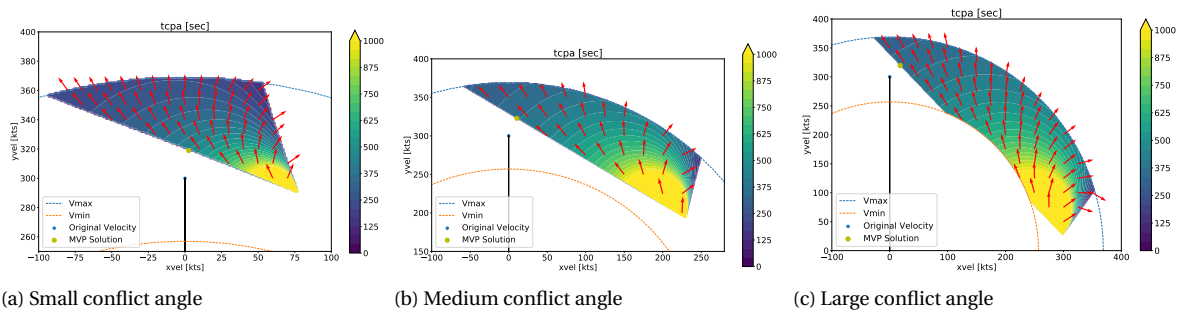
$$\nabla t_{cpa}(\vec{V}_{o,new}) = \begin{bmatrix} \frac{\partial t_{cpa}}{\partial V_{o,new,x}}(\vec{V}_{o,new}) \\ \frac{\partial t_{cpa}}{\partial V_{o,new,y}}(\vec{V}_{o,new}) \end{bmatrix} \quad (5.8)$$

Looking at the results the following conclusion can be drawn:

- Solution of decreasing t_{cpa} are also found by choosing solutions further away from the apex of the VO. However, unlike the case with $|\vec{V}_{rel}|$, the gradients do not necessarily flow radially away from the apex of the VO. Instead, especially at low conflict angles, a more efficient t_{cpa} reduction might be found by choosing solutions not on the leg of the VO.
- Geometrically, the gradient can also be interpreted as an arrow pointing in the direction of where the CPA is going to be. Thus, naturally, increasing the $\vec{V}_{rel}$ in the direction of the CPA decreases the t_{cpa} most efficiently.

Interestingly, the largest reduction in t_{cpa} is not necessarily obtained by the largest increase in $|\vec{V}_{rel}|$. This can be explained by looking at eq. (3.1), restated below for convenience. Choosing $V_{o,new}$ also has influence on the distance from the ownship to the new CPA D_{rel} . These combined effect results in fig. 5.8.

$$t_{cpa} = \frac{D_{rel}}{|\vec{V}_{rel}|}$$

Figure 5.7: Resulting time to CPA t_{cpa} for various solutions within the PSS.Figure 5.8: Resulting time to CPA t_{cpa} for various solutions within the PSS, zoomed in on the PSS. Red arrows indicate gradient of t_{cpa} with respect to $V_{o,new}$.

5.2.3. Distance at CPA

The distance at CPA D_{cpa} as a result of various solutions in the PSS can be seen in figs. 5.9 and 5.10. In fig. 5.10 the gradient of D_{cpa} with respect to $\vec{V}_{o,new}$ is indicated by red arrows. This gradient is calculated by taking the partial derivative of D_{cpa} with respect to $\vec{V}_{o,new}$, see eq. (5.9).

$$\nabla D_{cpa}(\vec{V}_{o,new}) = \begin{bmatrix} \frac{\partial D_{cpa}}{\partial V_{o,new,x}} \\ \frac{\partial D_{cpa}}{\partial V_{o,new,y}} \end{bmatrix} \left(\vec{V}_{o,new} \right) \quad (5.9)$$

Looking at the results the following conclusion can be drawn:

- As expected when looking at fig. 2.2, solutions on the same radial with respect to the apex of the VO have the same distance at CPA D_{cpa} . Therefore, as can be seen by the gradients, the biggest increase in D_{cpa} is achieved by choosing a $V_{o,new}$ perpendicular to the current radial with respect to the apex of the VO.

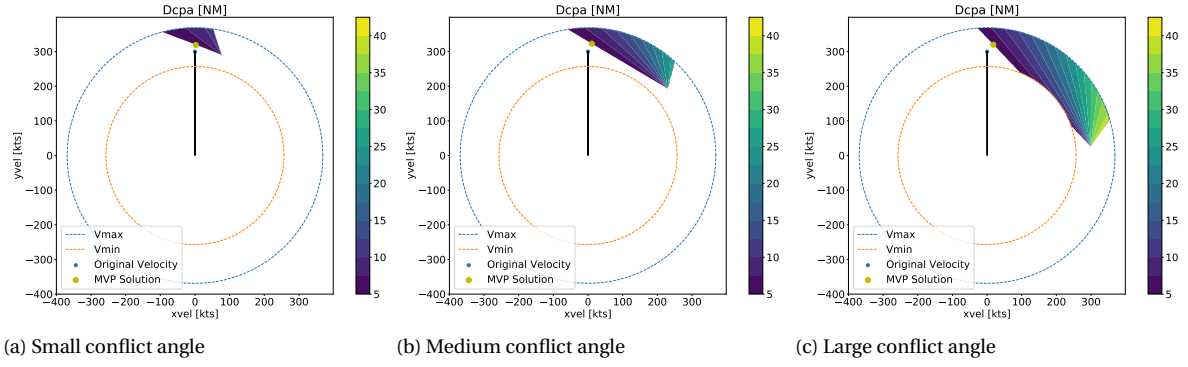


Figure 5.9: Resulting distance at CPA D_{cpa} for various solutions within the PSS.

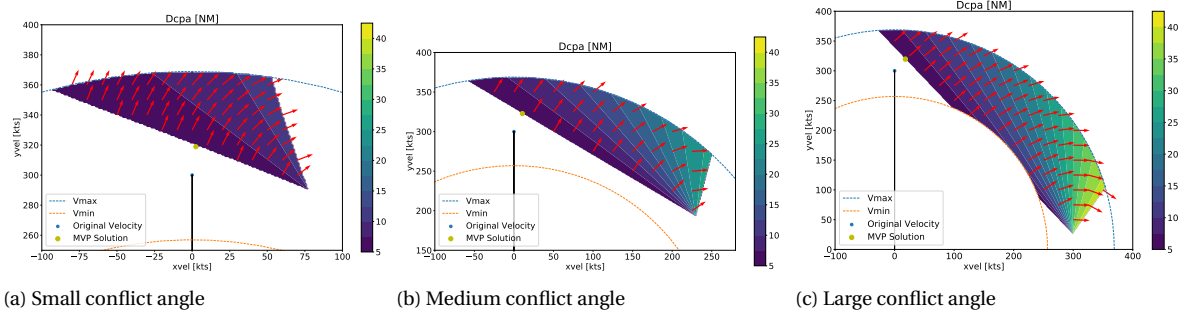


Figure 5.10: Resulting distance at CPA D_{cpa} for various solutions within the PSS, zoomed in on the PSS.

5.2.4. Extra distance flown

The extra distance flown as a result of various solutions in the PSS can be seen in figs. 5.11 and 5.12. This extra distance flown is calculated using eq. (3.4), restated below for convenience.

$$DEV = |\Delta \vec{V}_{MVP}| \cdot t_{cpa,cr}$$

Looking at the results, the following conclusion can be drawn:

- Sunil et al. (2017a) argued that the MVP chooses minimum path deviations solutions. This is confirmed by the results in the graph. For each of the conflict scenarios, the MVP chooses the solution with the lowest deviation. In other words, although the MVP does not propose solutions with the lowest $|\Delta V|$, it does propose solutions with the lowest $|\Delta V| \cdot t_{cpa,new}$.

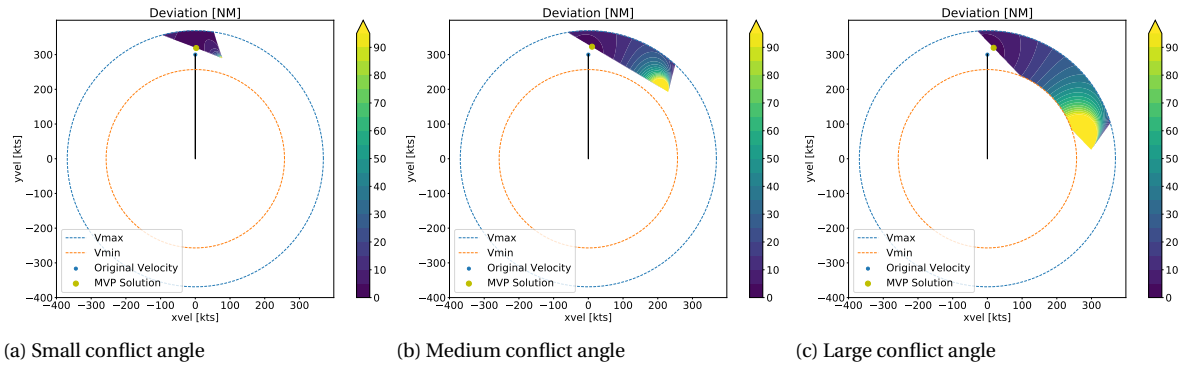


Figure 5.11: Resulting deviations flown for various solutions within the PSS.

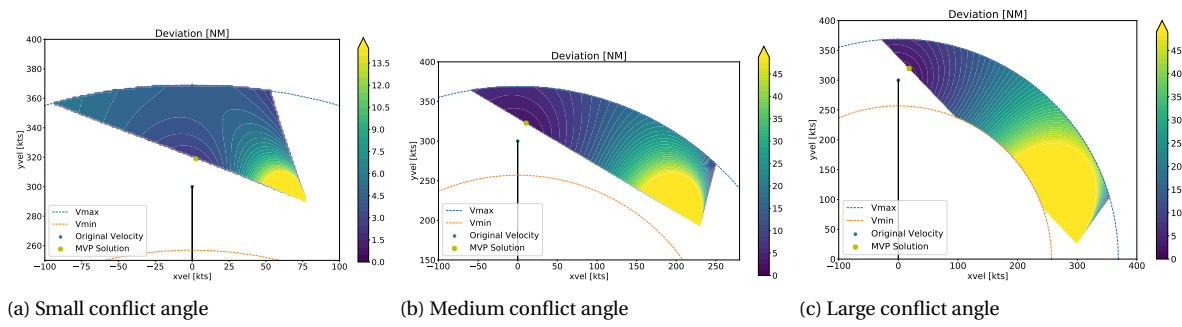


Figure 5.12: Resulting deviations flown for various solutions within the PSS, zoomed in on the PSS.

5.3. One-step and multi-step solutions

The information provided by the PSS can be used in both one-step solutions and multi-step solutions. One-step solutions entail choosing a single solution in the PSS, as does the MVP. In a multi-step solution approach, different solutions are chosen as a function of time. To encapture the possibilities of multi-step approaches, the behavior of the PSS over time has to be investigated. This will be one of the focal points while moving into the main phase of the thesis.

6

Conclusion

The free flight air traffic management concept recommends a decentralized self separation approach to cope with the increasing congestion of the airspace. In a decentralized airspace, each aircraft is responsible for sustaining a safe separation between itself and the other aircraft. An early algorithm which provides this self organized method, is the Modified Voltage Potential. It is a method based on a force field analogy and entails 'pushing away' conflicting trajectories between aircraft. It is a tried and trusted method, however, various studies have addressed problems related to small conflict angles when the Modified Voltage Potential is used. These problems, as well as potential solutions to these problems, are the subject of this thesis.

The investigation into these problems is divided into two approaches; investigating the conflict angle related performance in two-aircraft conflict scenarios and investigating the emergent conflict angle related performance in a large scale experiment. This two-aircraft conflict angle scenario was supplemented with mathematical derivations regarding the performance of the MVP. Based on this two-aircraft conflict scenario, no conclusive problems were found with the algorithm in small conflict angle scenarios. These derivations and calculations, however, provide essential insight into the mechanics of the MVP and provide tools to explain the difference in performance for the large scale experiment.

Parallel to the investigation into the conflict angle related problems, a framework was established to propose alternative resolution strategies. This framework contains a pair-wise solution space. A large part of the behavior of the PSS for single-step solutions has been evaluated. However, multi-step solutions have to be investigated as well. This requires knowledge on how the PSS changes over time. Moving into the main phase of the thesis, this will be one of the focal points.

As this document is the transition of the preliminary phase of the thesis into the main phase of the thesis, the following activities are to be performed. First, the behavior of the PSS over time is investigated while the many-to-many conflict experiment is running. Once the many-to-many conflict experiment has completed, its performance has to be explained using the mechanics of the MVP and the results of the one-to-one conflict experiment. Once these two activities are completed, multiple promising single-step and/or multi-step methods have to be defined. Once these methods have been defined they are implemented into the open-source air traffic simulator BlueSky. Using BlueSky, the behavior of the proposed methods is compared to the behavior of the MVP.

Regarding the behavior of the MVP in both one-to-one conflict scenarios and many-to-many conflict scenarios, a lot is still unknown. For instance, how pairwise summation of resolution maneu-

vers influences behavior, what different conflict geometries do with the behavior of the MVP, precisely what behavior emerges in large-scale experiments, and which adaptations might provide improvement, are all either being researched right now or yet to be researched. This thesis will not only pursue adaptations to the MVP, but also, in any case, provide valuable insights into the behavior of the MVP and develop a framework for future adaptations with different motives.

A

Preliminary Experiments and Propositions

In the preliminary phase of this thesis, initial experiments and propositions were formulated. First the preliminary experiments are explained, then the preliminary propositions and the related findings. This appendix concludes with the lessons learned from these preliminary experiments and propositions.

A.1. Preliminary Experiment

To gain some preliminary insight into the behavior of the MVP algorithm in shallow angle conflicts, a preliminary experiment was set up. This experiment is designed to investigate how certain metrics change when conflict angles in a scenario differ. Different scenarios with different conflict angle distributions are compared. Scenarios with maximum conflict angles of 5, 10, 15, ..., 85, and 90 degrees (if aircraft were to fly with CR off) are compared. Of course, conflicts with higher conflict angles might occur due to multi-aircraft conflicts when CR is on. A more detailed explanation of the experiment and how the metrics are obtained can be seen in Appendix A. The results for the Domino Effect Parameter (DEP) (Krozel et al., 2000), average conflict duration, and the percentage increase in distance flown due to CR being on, can be seen in Figures A.1 to A.3.

As can be seen in Figure A.1, there is no clear indication that different conflict angles result in an increase or decrease in DEP. If any, a slight increase in DEP can be seen when the maximum conflict angles increase. It should be noted that the results are quite noisy due to the low amount of data points (12) per maximum conflict angle. On the whole, no definite conclusions can be drawn based on the DEP results at this time.

Figure A.2, however, shows a clear trend. The average conflict duration increases significantly when the maximum conflict angles decrease. From an average of about 200 seconds at 90 degrees to about 435 seconds at 5 degrees. This change makes sense intuitively since, at low conflict angles, the relative speeds are low, resulting in a longer conflict duration.

Additionally, Figure A.3 shows an increase in distance flown for higher conflict angles. From about 0% at 5 degrees up to approximately 0.1% at 90 degrees. At this time, no apparent reason is pinpointed to explain this increase in distance flown. This choice is because multiple arguments are to be made both in favor and against this increase in distance flown. Additionally, configuration influences these effects as well.

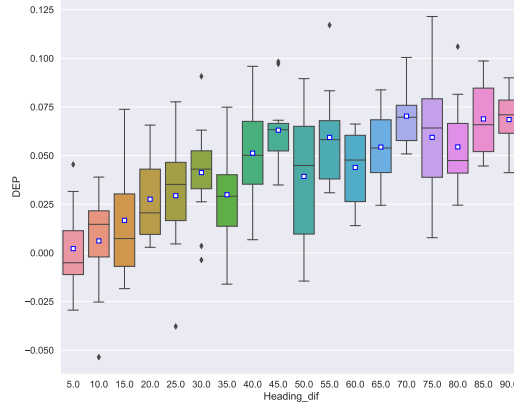


Figure A.1: Degree of Stability in each scenario configuration. Mean values indicated by blue squares.

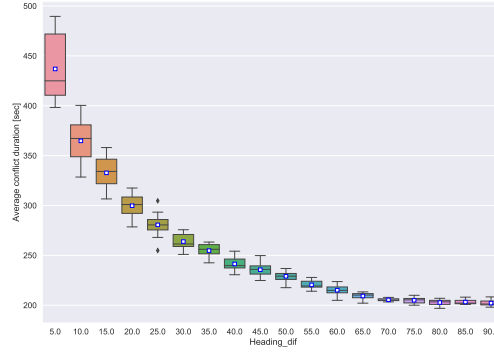


Figure A.2: Average conflict duration in each scenario configuration. Mean values indicated by blue squares.

A.2. Preliminary Proposition

In this chapter one of the initial modifications to the MVP is explained and tested. This modification was implemented in very early stages of the thesis, therefore not all the findings of the chapters above were of influence on this modification. Additionally, the experiment used is in-elaborate compared to the large scale experiment proposal of chapter 4. However, effects observed in the behavior of this proposal provide valuable lessons for future proposals.

First the proposed method will be explained, then the experiment used to test it, and finally the results of this experiment.

A.2.1. Proposal

A method is defined to solve the long conflict duration, as seen in both literature (Sunil, 2019) and preliminary experiments. The currently proposed method is an adaptation to the Modified Velocity Potential algorithm by Hoekstra (2001). To resolve these shallow angle conflicts quicker, the relative velocity between the two aircraft is increased. While maintaining the symmetry of the solution for both aircraft, and the praised simplicity of the MVP method, this increase of the relative velocity is obtained through using an angular rotation of the 'push away' point with respect to the center of the Protected Zone. This angular rotation is denoted by ϕ and is defined positive in the direction of V_{rel} .

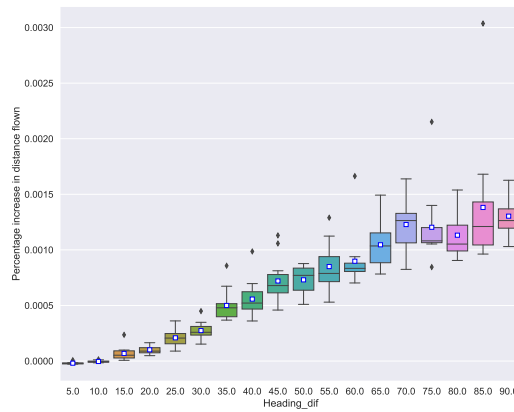


Figure A.3: Percentage extra distance flown due to CR being on in each scenario configuration. Mean values indicated by blue squares.

This is illustrated in Figure A.4. By making ϕ more positive or more negative, the new conflict-free relative velocity becomes higher or lower. When both aircraft use the same value for ϕ , the ΔV_{MVP} vectors are opposite identical for both aircraft.

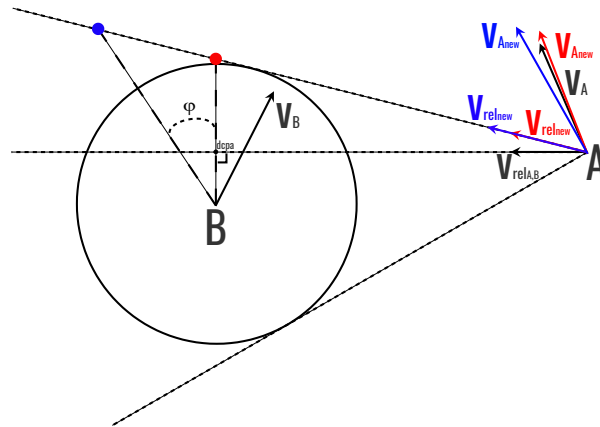


Figure A.4: Mechanics of the proposed phi-method in blue. Red is the Hoekstra (2001) method.

A.2.2. Preliminary results

The effect of this rotation can be seen both in calculations and preliminary simulation results. As expected, the calculations show that for higher values of ϕ , the time to the closest point of approach is shorter and vice versa. The magnitude of this increase or decrease depends on the geometry of the conflict. A preliminary experiment to gain some insight into the emergent behavior of this adaptation was formulated. A number of 600 aircraft (B737) are placed in the airspace with at least 18 nautical miles in between them. This guarantees that no instantaneous conflicts are present at $t = 0$. They each have a waypoint making them fly a heading between 67.5 and 112.5 degrees for 732 NM. This to ensure that most conflicts have a heading difference of no more than 45 degrees. Again, conflicts with higher conflict angles might occur due to multi-aircraft conflicts. The results can be seen in Figures A.5 to A.7.

As can be seen in Figure A.5, both an increase or decrease of ϕ adds to a significant increase to the

Domino Effect Parameter in this case. The reasons for this can be explained using two phenomena:

1. Highly negative ϕ : For very negative values of ϕ , conflicts are solved so slow, that a type of swarming starts to develop. The relative velocities are set to such low values due to the highly negative ϕ that aircraft get stuck together, fly past their destinations, and destabilize the entire airspace.
2. High absolute ϕ : For very high absolute values of ϕ , the resolution maneuvers initiated by the ϕ adapted MVP require severe heading changes, this results in destabilization of the airspace. This due to higher absolute values of ϕ resulting in conflicts being resolved with more severe heading changes, which in turn result in new conflicts with neighboring aircraft. This triggers a chain of events destabilizing the airspace.

In Figure A.6, it can be seen that higher values of ϕ reduce the average conflict duration in this experiment. This means that these small conflict angle conflicts are resolved quicker. Additionally, Figure A.7 shows an increase distance flown for higher absolute values of ϕ . Both increase and decrease can be explained using the same reasons discussed for the DEP values.

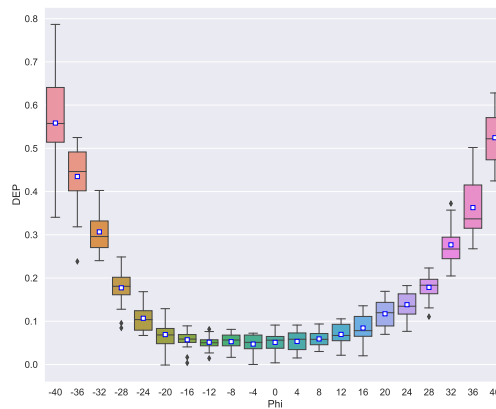


Figure A.5: Degree of Stability as a function of ϕ . Mean values indicated by blue squares.

A.3. Lessons learned

The following lessons were learned from these preliminary proposals and experiments:

- From observations it was clear that effects near the edge of a simulation area play an important role. The main experiment should mitigate these effects as well as possible.
- Resolution strategies which make use of big heading changes destabilize a high density airspace in which a look-ahead time of five minutes is used. When the propositions are made in the beginning of the main phase, this effect should be taken into account.
- The DEP does, indeed, seem to be related to the extra distance flown. This is extra distance flown is, therefore, an important parameter and should play a significant role in the proposition of alternative methods.

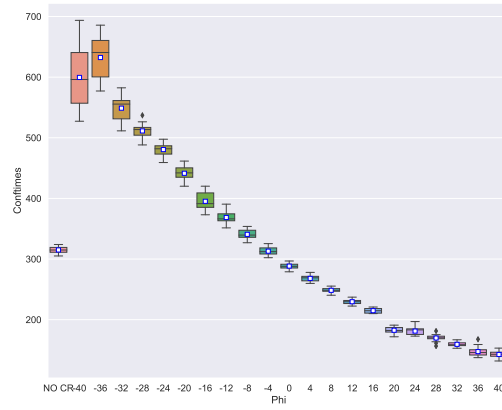


Figure A.6: Average duration per conflict as a function of ϕ . Mean values indicated by blue squares.

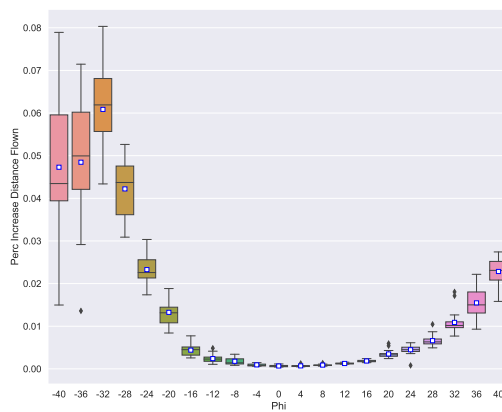


Figure A.7: Percentage increase in distance flown due to CR as a function of ϕ . Mean values indicated by blue squares.

B

Gantt chart

Thesis

Preliminary Phase

- Literature Study
- Shallow Angle Conflict + Bluesky Orientation
- Preliminary Experiments
- Formulate and perform small scale experiment
- Formulate large scale experiment
- Formulate Pairwise Solution Space
- Characterize Pairwise Solution Space
- Finalize Draft
- Hand in Draft
- Holiday

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Main Phase

- Run large scale experiment
- Interpret results
- Write research methodologies
- Hand in research methodologies
- Define alternative CR methods
- Implement alternative CR methods in BlueSky
- Define experiment for comparison
- Run experiment for comparison
- Compare performance
- Reiterate if required
- Finish writing draft
- Hand in thesis draft
- Finish writing thesis
- Hand in thesis final
- Write paper
- Prepare presentation
- Defence

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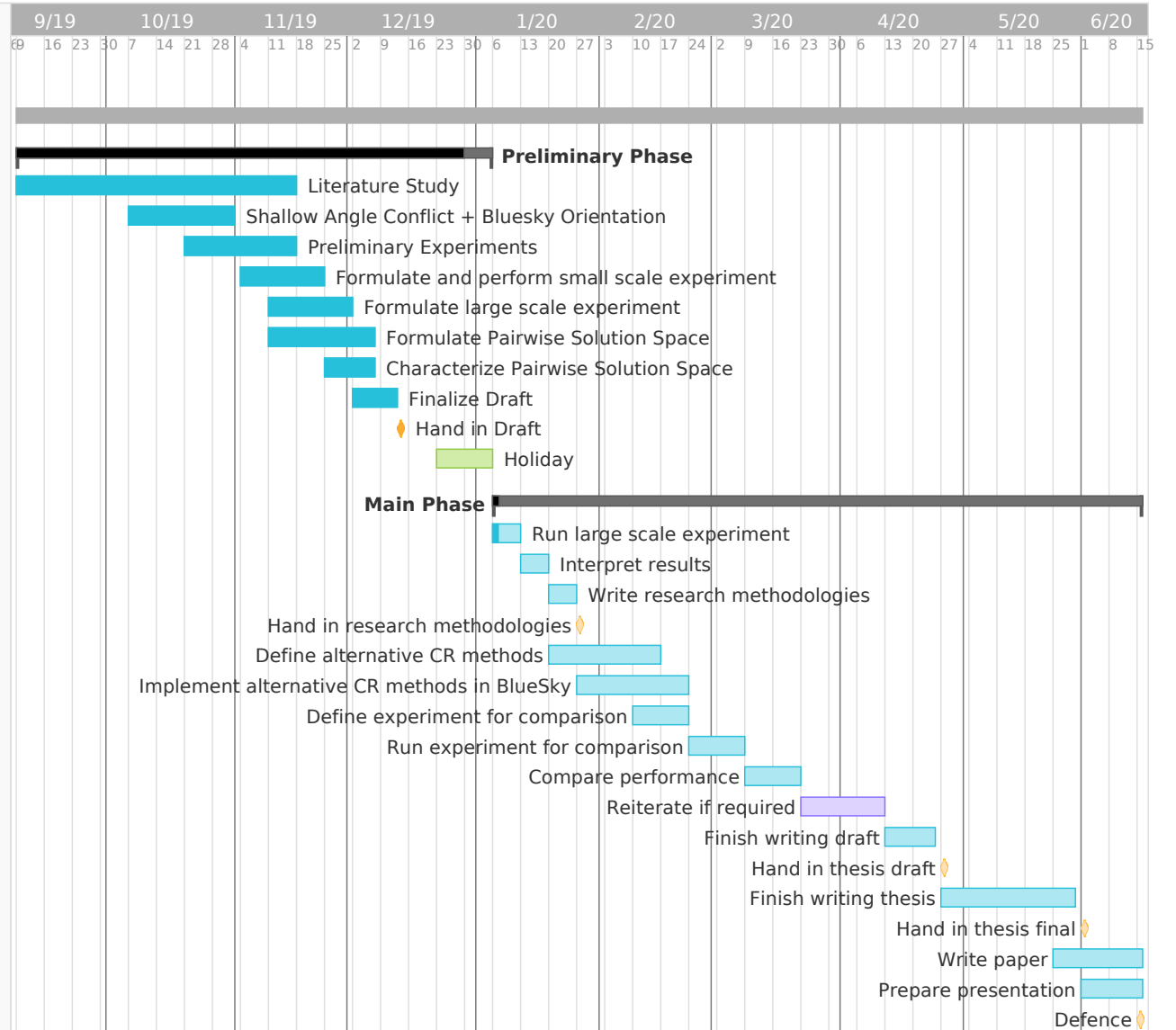
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