Role of Friction Estimation in Quadrupedal Locomotion MPC

Friction estimation as a method of slip recovery and prevention in quadrupedal robot

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Thesis report

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Contents

Nomenclature

List of Abbreviations

CoT	Cost of Transport
IMU	Inertial Measurement Unit
MPC	Model Predictive Controller
ODE	Open Dynamics Engine
RL	Reinforcement Learning
ROS	Robot Operating System
ZMP	Zero Moment Point

List of Symbols

α	Angle of Attack
β	Angle of Sideslip
λ	Ground Reaction Force
μ	Coefficient of Friction
Ω	Angular Velocity
ϕ	Roll Angle
ψ	Yaw Angle
q	Pitch Rate
t	Time Step
u	Control Input Vector
x	State Vector

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Introduction

1.1. Problem Background

In the proliferating field of robotics, there has been a trend of a class of robots that has gained significant media coverage in recent years. From advertisements to academic publications, legged robots have captured the attention of the masses with their lifelike movement due to advances in their control systems, hardware design, and perception. This class of robots promises that because of its legs, it can traverse and interact with its environment unlike any other. There have been multiple studies and publications, that investigated the usefulness of the robot in unstructured environments such as offshore oil rigs [1] and underground sewer systems [2].

A typical example of the north star of performance that researchers often show is the Ibex goat that can traverse steep hills without slipping as they have the same morphology as a subclass of legged robots that has shown the most promise with regards to a good balance of stability, and performance, which are quadrupedal robots. However, the current ability of the robot is not yet comparable to that of these animals, as they still lack stability in extremely rugged environments with slippery surfaces [3]. One glaring issue that these robots often face is slipping in these environments, which causes a loss of stability that can usually lead to falling and a failure to recover. Many attempts have been investigated to either prevent or recover the robot from these slip scenarios. There have been multiple studies that focus solely on slip recovery [4] [5].

However, to solve the issue from the root, one must realize that the problem lies in the failure to estimate or measure the coefficient of friction of the surface or the surface normal at the contact point [4]. These wrong estimations make the commanded force that the robot applies to the ground outside the allowed limits modeled by the Coulomb friction cone and cause the robot to slip. If the robot can accurately estimate and apply the proper force, then the robot can prevent slipping from happening. In addition, the extensive effort to create slip recovery strategies, which are often only heuristics to solve the problem, would no longer be needed.

1.2. Difficulty of Estimating Coefficient of Friction

Friction is a notoriously difficult metric to model. The phenomenon not only has a non-linear behavior [6] but also, to measure, it requires a measurement device that can monitor the effects at the site of contact, which is currently not provided by many out-of-box sensors. Friction itself arises from the interaction of two contacting surfaces, which makes it very dependent on the type of materials of the surfaces. To directly measure the frictional interaction of two surfaces, many have attempted to create sensors that track optical markers where contact is made, and extract friction-related characteristics from the movement of the markers [7] [8]. There have been many attempts to introduce some aspect of friction estimation using an optical tactile sensor as a part of the control loop in robotic manipulation [9] [10] [11], but there have been only few successful attempts in robot locomotion [12]. Most of the attempts that have come close to using friction estimation as a part of the loop in locomotion control estimate indirect metrics related to friction such as slip detection [6]. Even in those where the optical friction sensor was used, the robot walked in a slow static gait, such that the center of mass was always above the contact polygon created by the feet in contact with the ground.

1.3. Research Formulation

Given the background to the problem of slip in quadrupedal robot locomotion and how it relates to friction estimation, a study on the importance of friction estimation and the extent of its usefulness in quadrupedal robot locomotion was conducted in this thesis. To create a concrete and objective thesis for the problem a main research objective was formulated:

Research Objective

Can accurate friction estimation improve locomotion?

From the main research objective to conduct the main hypothesis test about the benefits of accurate friction estimation, research questions were formulated to unravel the path of investigation.

Research Question 1

How does an accurate friction estimation improve the locomotion performance of a quadrupedal robot?

Research Question 2

When does accurate friction estimation fail to prevent slip in locomotion?

Research Question 3

Is an explicit slip recovery procedure still required if an accurate friction estimation is present?

With these research questions in mind, the proper environments and tests were designed and set up to conduct a thorough study that will answer these questions

1.4. Structure of the Report

This thesis work is a case study to prove that accurate friction estimation significantly reduces slip occurrence in legged robots, and that slip recovery strategies are no longer needed in most legged robots using optimal control as its locomotion controller when accurate friction estimation is available.

The structure of this thesis is as follows: firstly, the theoretical background of the problem will be defined for legged robot locomotion and a controller that will be used in the experiments using a model predictive controller will be described in detail. Then, the modification to the controller that allows it to prevent and reduce slip occurrence will be shown. This modified controller will then be tested in a controlled experiment that allows comparison with a base controller without modification. Then interesting metrics to measure and compare locomotion performance will be extracted and shown from the results of the experiments. Then, a discussion of the results and the limitations of the modified controller will be shown in the following chapter and finally, the thesis will be closed with a conclusion and future additional work discussion to further expand the research.

Part I

Preliminary Study and Analysis

2

Literature Review

To introduce the readers to the state of the art of quadrupedal robot locomotion, and understand the context of the role that friction plays in it, a short literature review is included in the report in this chapter, before getting into the experiments and analysis of the contribution of this thesis .

2.1. Hierarchical Control Architecture of Quadrupedal Robot

To tackle the most challenging terrains, a control architecture that takes in terrain information, including friction, and plans the motion accordingly is usually utilized in commonly used systems in research such as the HyQ robot[13], ANYmal [14], and LittleDog [15]. The controller mostly consists of a high-level planner that plans a set of possible footsteps across the terrain, a low-lever planner that plans trajectories for the robot's feet and COG, and a low-level controller that tracks the desired trajectory [16].

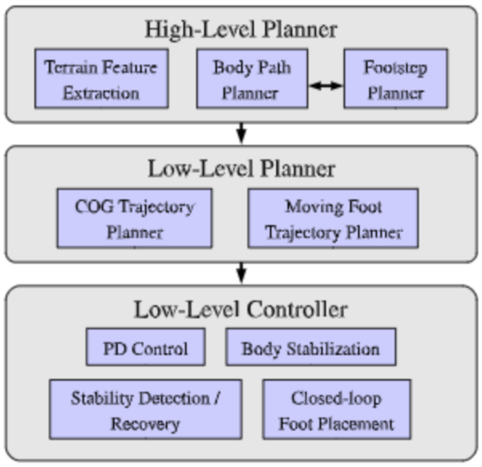

Figure 2.1: Illustration of the Hierarchical Control Structure[15]

In this section, the complexity of controlling locomotion in a quadrupedal robot in a hierarchical control structure is explored which can be divided into: 1. state estimation, localization, and mapping, 2. foothold planning, and lastly 3. Whole-body motion control. The most relevant part of the hierarchical control architecture to this literature review relies on the high-level planner as that is where the terrain feature extraction and state estimation of the robot take place. The most obvious problem of current methods of the high-level planner is that it requires a preconceived map of the environment to produce a cost map that will be used for planning the footstep of the robot [17].

However, it is obvious that in natural environments this map will not always stay accurate due to weather conditions that might have changed the terrain's properties, or sometimes the map does not exist at all. Therefore, exploration and sensing methods that do not rely on an existing map or can add local information to it are needed as a solution for these conditions. There is a need for methods that can measure and anticipate local unanticipated variability in environments so that the robot can re-evaluate its path's feasibility and change it accordingly whenever it is needed.

The feasibility of a footstep is often judged by the stability and non-slip condition that the robot requires. To fulfill these needs, several assumptions and constraints have been utilized in controllers. One of the most important is the friction cone constraint, to keep the robot's motion valid during locomotion concerning the environment.

If the terrain is assumed to be planar, then the stability constraint implemented in the robot would be the ZMP(Zero Moment Point)[18] constraint which would keep the robot's center of mass above the contact polygon created by the contacting feet. But if the terrain is irregular, stability would be determined by the sum of the span of feasible forces that the robot can implement to the ground by the feet before slipping occurs. This constraint is called the Coulomb friction cone constraint and will be discussed in the following section.

2.2. MPC as Low-Level Planner

One of the main trends in quadrupedal locomotion is the use of MPC as the low-level planner [19] [20], to plan the ground reaction forces that the robot needs to take given the constraints of the dynamics and environment that the robot needs to obey. This is due to MPCs ability to handle complex, multi-variable systems with constraints. MPC enables the robot to anticipate future states and make real-time adjustments to its locomotion strategy, thus improving stability and adaptability in dynamic environments.

Model Predictive Controller(MPC) is prevalently used in controlling dynamic systems. The technology itself started in the 60s as a way to control industrial process which requires less processing frequency. However, due to the advent development of microprocessors and computers in general, the ability to perform the optimization process at a high frequency opened the way for the implementation of MPC in robotics systems that require replanning and calculation of control input at frequencies as high as 100 hz or even higher.

The mechanism of work of an MPC is that at each time step, it receives an estimation of the state of the robot and then it determines the optimal sequence of control actions by solving a constrained optimization problem, which minimizes the cost over the prediction horizon. This process relies on an internal model of the system and is influenced by the current state of the system. The MPC will then apply the first calculated step of the control action sequence disregarding the rest, and then repeats this process again in the next time steps [21].

The lower section of the image below provides a detailed view of the reference trajectory along with the predicted outputs of the plant. Using its internal prediction model, the MPC controller forecasts these outputs over a specified prediction horizon p. Meanwhile, the upper section illustrates the sequence of control actions planned by the MPC, highlighting the first action, which is directly applied to the plant. The control horizon represents the number of planned control moves, which may be shorter than the prediction horizon [21].

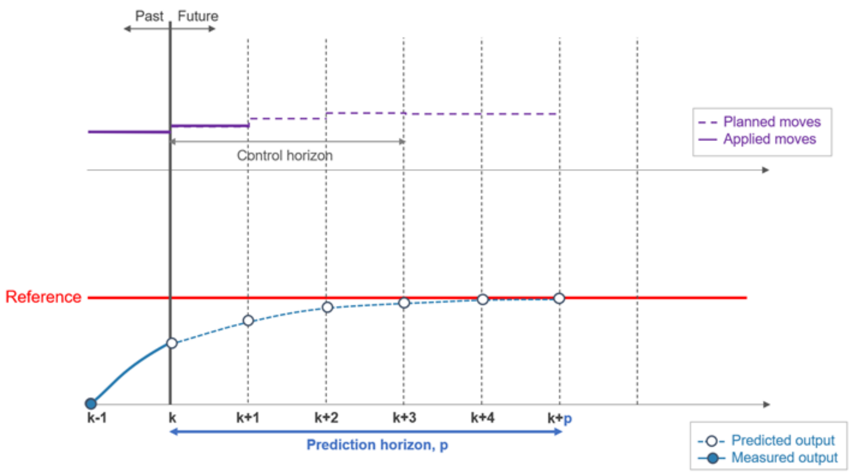

Figure 2.2: Visualization of MPC[21]

The mathematical formulation involves the following components:

1. Prediction Model

The system dynamics are typically represented by a discrete-time linear model:

$$\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k + \mathbf{B}\mathbf{u}_k \tag{2.1}$$

where:

- $\mathbf{x}_k$ is the state vector at time step k,
- $\mathbf{u}_k$ is the control input vector at time step k,
- **A** and **B** are matrices representing the system dynamics.

2. Cost Function

The objective is to minimize a cost function over a finite prediction horizon N, typically including terms for tracking error and control effort:

$$J = \sum_{i=0}^{N-1} \left((\mathbf{x}_{k+i} - \mathbf{x}_{\text{ref},k+i})^T \mathbf{Q} (\mathbf{x}_{k+i} - \mathbf{x}_{\text{ref},k+i}) + \mathbf{u}_{k+i}^T \mathbf{R} \mathbf{u}_{k+i} \right)$$

where:

- $\mathbf{x}_{\text{ref},k+1}$ is the reference trajectory that the robot has to follow from the global planner
- **Q** is a positive semi-definite matrix that penalizes deviations from the desired state,
- **R** is a positive definite matrix that penalizes big changes in the control effort.

3. Constraints

The optimization is subject to constraints on both the states and inputs, and other constraints relating to the dynamics and limits of the robot:

$$\mathbf{x}_{k+i} \in \mathcal{X}, \quad \mathbf{u}_{k+i} \in \mathcal{U} \tag{2.3}$$

where $\mathcal{X}$ and $\mathcal{U}$ represent feasible sets for the states and inputs, respectively.

4. Optimization Problem

At each time step k, the MPC problem is formulated as:

$$\min_{\{\mathbf{u}_k, \mathbf{u}_{k+1}, \ldots, \mathbf{u}_{k+N-1}\}} J \tag{2.4}$$

subject to the system dynamics (Equation 2.1) and constraints (Equation 2.3). The solution to this optimization problem yields the optimal control sequence, from which only the first control input $\mathbf{u}_k$ is applied to the system. The process is then repeated at the next time step with updated state information.

One of the most important constraints of an MPC optimization problem in quadrupedal robot locomotion relates to the frictional force interaction between the robot's feet and the ground, which prevents the robot from applying ground reaction forces that will cause it to slip. Which will be covered in the next section.

2.3. The Role of Friction

Most of the current systems of quadrupedal robots assume a point-foot condition, whereas the contacts of the feet are defined by a three-dimensional point vector in space where the ground reaction forces are measured. Ground reaction forces are the main driver of the motion of the center of mass of the robot, as shown in the dynamic equations in section 2.1. So in total, these robots have four feet that are in contact with the ground during static conditions and less during locomotion. As you can see, the forces generated by legged robots cannot be arbitrary, as they can only push on the ground and can only exist if friction interaction exists between the foot and the ground.

These ground reaction forces are usually constrained during the optimization of the control input trajectory so that the robot does not experience slippage. So it is inherent that the friction interaction of the robot's feet with the ground requires a dependable model. Friction in itself is a highly nonlinear phenomenon that involves many factors such as material properties, microscopic surface geometry, etc [22]. The unit of measurement of friction is represented by a unitless measure called the coefficient of friction μ, which increases in value with the roughness of a surface. The type of μthat concerns the kind of interaction that the robot's feet has with the ground is called as static coefficient of friction, which scales the friction force between two objects when neither of the objects is moving.

In static friction interaction between two surfaces, when there is an applied force that opposes the frictional force that reaches a certain limit, slipping will occur [22]. This limit, is often modelled with someting called as the Coulomb fiction force in legged robots [23].

$$\mu\lambda_z \geq \sqrt{{\lambda_x}^2 + {\lambda_y}^2} \tag{2.5}$$

Coulomb's law is described such that the magnitude of the tangential force should not exceed the friction coefficient μ times the magnitude of the normal force to avoid slipping [23], because most of the common controllers of current legged robots assume a non-slip condition, perhaps due to the problem being highly non-linear and difficult to model[6].

Very few attempts have been made to model the slip conditions and apply them in the locomotion for the robot to recover from and continue moving [6]. In this example, the controller involved a Markov decision process to capture the nonlinearity. However, to keep the problem of friction in the robot linear, most robots keep a constraint to keep ground reaction forces within the Coulomb Friction cone.

This constraint can be illustrated by a cone that prevents the ground reaction force from going beyond the surface area of the cone constraint, to prevent slipping. This friction cone is always normal to the surface.

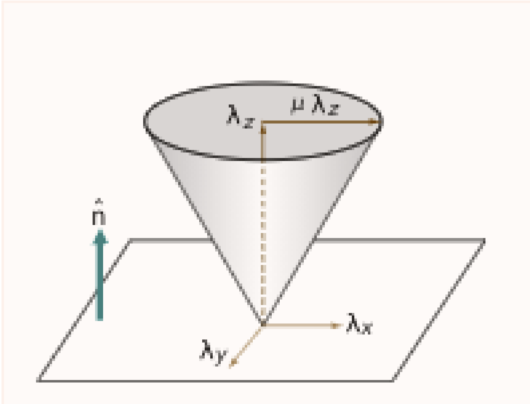

Figure 2.3: Illustration of the Coulomb Law[23]

As can be seen, the constraint relies heavily on the knowledge of the terrain: its slope (normal vector) and coefficient of friction. These properties vary wildly in different environments that legged robots may encounter, and they may even vary locally in the same environment. For example, if the robot is to be deployed in sewers on the same floor, the robot may encounter a puddle of water, broken concrete, and slippery mold all within the proximity of a single step. This reality makes slippage a great risk in legged robot deployments, as the previous ideal assumptions of the environment are no longer usable. The ZMP approach that is often deployed breaks down once the robot meets an environment that is non-planar, and is replaced by the sum of the contact friction cones instead plus the gravitational wrench cone, as demonstrated in a paper by Dai et al [24]. With the importance of surface and frictional interactions, approaches to estimating features of the terrain surface during this non-ideal situation have been explored and will be discussed in the following chapter.

2.4. State of the Art

With the basics and main components of robot locomotion covered, the state of the art of how current robots incorporate friction estimation needs to also be considered and expanded upon by this research. As mentioned before, most of the time quadrupedal robots are fed with a global map of the environment with a set coefficient of friction at the time of mapping. However, surface conditions may vary and change locally once the robot arrives in a specific area. Few attempts have been done to estimate surface features that measure friction and stability to aid the robot's locomotion.

Many techniques can be used to estimate surface features implemented in quadrupedal robots. However, current literature mostly focuses on classifying terrain classes, but that might be limiting for deployment in complex terrains with high variability in surface properties [25] [26]. Most of them rely on the proprioception of the robot and force-torque sensors which are subject to significant error and noise, especially in unstructured terrains, where each foot are most likely in different configurations to one another.

There were also those which fused extra sensors [27] at the feet to increase the dimension of the data which showed increased accuracy in classifying surfaces. However, These offline techniques may not generalize to all surface conditions which is why online estimation techniques that estimate surface properties in a regression manner may be more useful for locomotion. Specifically, the friction cone, the slip-preventing constraint in the locomotion control, is of high interest.

Current online techniques rely on limiting assumptions, such as the robot's feet need to be under the same slope conditions [28] or that certain probability distributions are expected from the surface feature [29], which may be impossible when deployed in nature. Most of the techniques are also time-consuming and can be costly in actual deployment. For example in a paper by Lin et Al [30] surface normal and friction estimation was done by touching multiple points on the surface before it can confidently guess the posterior probability distribution.

Another crucial determining limitation to the estimation of friction cone in a quadrupedal robot is that oftentimes, robots do not have information about the coefficient of friction nor the inclination information of the ground when they face an unknown environment. Often an assumption is made that the robot will not experience slip(friction is infinity) or an arbitrary value is given and corrected once the robot experiences slip [4],[6].

There are also limitations in the resolution of the sensors being used as they often do not capture the complete image of the reaction between the ground surface and the feet. Most approaches estimate contact conditions on the feet based on proprioception that may not be accurate enough in harsh conditions [4] and are prone to drift. To create a good analogy, it's like having a numb foot that has been injected with anesthesia and trying to feel the ground condition by judging how your knees and elbows feel.

Some attempts to mitigate this are the design of new feet for legged robots that are adaptable, soft, and can capture tactile information about the ground directly [31],[32]. However, the sensors implemented in these feet also suffer from the same limitations that were mentioned as they are often instrumented with force-torque sensors and IMUs [33]. A shift to newer types of sensors can be seen, especially those that produce a richer image such as the TekScan [34] and the TacTip [8]. However, even with these newer sensors, few are focused on measuring coefficient of friction to improve locomotion.

One promising implementation of the Tactip in a legged robot is done by Stone et Al [12], where the robot detected the edge of a table and followed the contour during locomotion. However, this approach only yields binary information regarding the environment, where it helps the robot to determine where and where not to place its feet. However, when deployed in a very complex environment, it would result in an environment where the robot would not be able to place its feet anywhere because all possible footholds would be deemed as "infeasible." In conclusion, current applications of haptic sensing use the high dimensional data from a haptic sensor to estimate a low dimensional feature about the surface. Future endeavors to produce a better, and faster representation of the surface friction would better benefit the legged locomotion body of research, especially on the frictional side as it shows a lacking body of research in legged robots.

2.5. Chapter Conclusion

Legged robot locomotion is a challenging task, as it includes many variables to be controlled and monitored that can be defined in many different ways. However, the most crucial part of generating a stable walking or running motion is the contact interaction of the feet with the ground. Specifically, what force vector can the robot generate, and at what foot position to produce a motion that propels the robot forward, while keeping its center of mass stable?

With that in mind, it can be seen that knowledge of the environment's surface plays an important role as it affects the contact dynamics of the feet with the ground and creates different reaction forces depending on the friction and shape of the surface. This leads to one of the most important constraints that legged robots face, which is the friction constraint, often modeled by the Coulomb friction cone. The constraint states that the magnitude of the tangential ground reaction force should not exceed the friction coefficient times the magnitude of the normal force to avoid slipping. However, accurate estimates of the normal vector and coefficient of friction are not always available. Even if it is available, it can vary locally depending on the texture and composition of the surface.

This is why reliable methods to measure and estimate friction and include them in the control loop are important parts of legged robot locomotion to keep the robot from slipping.

3
Preliminary Work

3.1. Methodology

As mentioned in the first section, the research objective of this thesis is to prove the hypothesis that ”accurate friction estimation in a quadrupedal robot that has an MPC locomotion controller is a sufficient method of slip prevention that omits the need for explicit slip recovery”, as it would prevent the robot from slipping entirely.

To prove or disprove this hypothesis a statistical t-test will be performed on an experiment that involves a quadrupedal robot in locomotion with an accurate coefficient of friction estimation/measurement being compared to one that does not have a coefficient of friction estimation/measurement in the loop. The latter is the most common scenario with quadrupedal robots that are currently on the market. Usually in commercial quadrupedal robots, the coefficient of friction is set manually by the operator and held constant throughout the locomotion task as they do not have a force sensor or friction sensor settled on the feet of the robot[35].

3.2. Experiment Setup

The author has chosen to construct a simulation environment to perform the mentioned experiment in subsection 3.1. To set up a simulation environment that is as close to the real-life experiment there needs to be three components to be set up.

1. A robot simulator with a quadrupedal robot model
2. A simulated environment with a physics engine that allows different sets of coefficients of friction to allow for the evaluation of performance
3. An MPC locomotion controller for the Quadrupedal robot
4. A simulated friction sensor to measure the coefficient of friction of the environment
5. Extra simulated sensors to measure the performance of the model predictive controller

Each component is required to reflect the behavior of the robot, in reality, such that an analogous and same trend of result to a real real-life experiment can be expected. The breakdown of the decisions in constructing each of the components is further explained in the following subsections

3.2.1. Simulator and Physics Engine

Gazebo was chosen as the simulation environment as it is the most ubiquitous simulator in robotics and has the most integration with ROS and its existing controllers and sensor packages. In this particular simulation environment spirit quadrupedal robot from Ghost Robotics was used. The robot is then placed on a flat world with a uniform coefficient of friction. The coefficient of friction is modifiable, such that we can view the performance of the controller when faced with different levels of slipperiness. In the simulator, the frictional properties follow Coulomb friction, as mentioned in the previous chapter as this can be set in the physics engine, which in this case was using ODE(Open Dynamics Engine) [36].

3.2.2. Model Predictive Controller

An open-source model predictive controller was chosen for the robot. The particular controller was created by the Robomechanics lab from Carnegie Mellon University called Quad-SDK [37]. The open-source nature of the project allows for low-level modification to how the controller works. The modification that was done to the existing controller was to update the coefficient of friction that was being used as the constraint of the optimization problem with measurements from the simulated friction sensor.

3.2.3. Simulated Sensor

The simulated sensor was made by using Gazebo's sensor plugin. By extending the sensor plugin, the simulated sensor can measure the coefficient of friction that was set in the world perfectly. To make a more realistic behavior of the sensor and expose the new controller to a real-world-like situation, Gaussian noise was added to the measurement.

Algorithm 1: Simulated Friction Sensor

Data: $feet$, $groundContacts$, $frictionMap$
for *each* $foot$ *in* $feet$ **do**
  if $foot$ *is touching the ground* **then**
    $bodyContacts \leftarrow$ get body in contacts with $foot$;
    $frictionValues \leftarrow$ [ ];
    for *each* $contact$ *in* $bodyContacts$ **do**
      $friction \leftarrow$ access coefficient of friction from $frictionMap$;
      $frictionValues$.append($friction$);
    end
    $avgFriction \leftarrow$ mean($frictionValues$);
    $noisyFriction \leftarrow avgFriction + \mathcal{N}(\text{avgFriction}, \sigma^2)$ `// Add Gaussian noise`
  end
end

3.2.4. Extra Sensors

Lastly, another simulated sensor was made to measure the performance of a controller, a slip detector. The slip detector allows for measurement of the frequency of slip occurring during the locomotion of the quadrupedal robot. This slip detector was made by measuring the force vector produced by the robot's foot at contact. If the force vector lies on the friction cone, then the contact is considered to be slipping.

3.3. Algorithms used

The focus of this thesis is to improve the locomotion of a quadrupedal robot, with a measured value of coefficient of friction from a simulated friction sensor. The driver algorithm of locomotion in the robot is the non-linear model predictive controller that generates the input force to be applied by the robot's end effectors at every timestep. As mentioned in the literature review section before, a model predictive controller is a receding horizon optimal controller. In short, it is a controller that runs an optimization problem of a nonlinear objective function, run at a set frequency for a limited time horizon. This controller predicts the input that the robot needs to apply to its end effectors based on its current state and a nonlinear model that predicts its future states if it follows the action plan that was calculated in the optimization problem.

The model predictive controller components consist of an objective function to be optimized and its constraints. In quadrupedal robots that use an MPC, the objective function to be optimized is a quadratic function of the error in the state and the input of the system. Mathematically defined as follows:

$$\min_{\mathbf{u}} \sum_{k=0}^{N-1} \left(\mathbf{x}_{k+1}^T \mathbf{Q} \mathbf{x}_{k+1} + \mathbf{u}_k^T \mathbf{R} \mathbf{u}_k \right) \tag{3.1}$$

This objective function is then optimized under a set of constraints, which includes the dynamics of the robot as the equality constraint, and the joint limits and coulomb friction constraint as the inequality constraints. Mathematically formulated as such:

Subject to:

$$\mathbf{x}_{k+1} = f(\mathbf{x}_k, \mathbf{u}_k) \quad \forall k = 0, \dots, N-1 \tag{3.2}$$
$$\mathbf{x}_0 = \mathbf{x}_{\text{init}} \tag{3.3}$$
$$\mathbf{x}_N = \mathbf{x}_{\text{goal}} \tag{3.4}$$
$$\mathbf{u}_{\text{min}} \leq \mathbf{u}_k \leq \mathbf{u}_{\text{max}} \quad \forall k = 0, \dots, N-1 \tag{3.5}$$
$$\mathbf{x}_{\text{min}} \leq \mathbf{x}_k \leq \mathbf{x}_{\text{max}} \quad \forall k = 0, \dots, N-1 \tag{3.6}$$
$$\|\mathbf{u}_k - \mathbf{u}_{k-1}\| \leq \Delta u_{\text{max}} \quad \forall k = 1, \dots, N-1 \tag{3.7}$$
$$h(\mathbf{x}_k, \mathbf{u}_k) \leq 0 \quad \forall k = 0, \dots, N-1 \tag{3.8}$$
$$\mathbf{q}_{\text{min}} \leq \mathbf{q}_k \leq \mathbf{q}_{\text{max}} \quad \forall k = 0, \dots, N-1 \tag{3.9}$$
$$|\mathbf{F}_k| \leq \mu \mathbf{N}_k \quad \forall k = 0, \dots, N-1 \tag{3.10}$$

Definitions:

- $\mathbf{x}_k$: State vector at time step k
- $\mathbf{u}_k$: Control input vector at time step k
- N: Prediction horizon
- $\mathbf{Q}$: State cost matrix
- $\mathbf{R}$: Control input cost matrix
- $f(\mathbf{x}_k, \mathbf{u}_k)$: System dynamics
- $\mathbf{x}_{\text{init}}$: Initial state
- $\mathbf{x}_{\text{goal}}$: Goal state
- $\mathbf{u}_{\text{min}}, \mathbf{u}_{\text{max}}$: Control input bounds
- $\mathbf{x}_{\text{min}}, \mathbf{x}_{\text{max}}$: State bounds
- Δu_{max}: Maximum change in control input
- $h(\mathbf{x}_k, \mathbf{u}_k)$: Additional constraints (e.g., obstacle avoidance)
- $\mathbf{q}_k$: Joint positions at time step k
- $\mathbf{q}_{\text{min}}, \mathbf{q}_{\text{max}}$: Joint position limits
- $\mathbf{F}_k$: Ground reaction forces at time step k
- μ: Coefficient of friction (Coulomb friction)
- $\mathbf{N}_k$: Normal forces at time step k

This defines the locomotion of a quadrupedal robot as an optimal control problem to be solved at every time iteration for a set sliding time window often called the receding horizon of an MPC.

3.3.1. Novel Approach of MPC with Friction Sensor

In this thesis, the novel approach of the MPC when an accurate friction sensor is present, is to update the coulomb friction cone constraint with new measurements of the coefficient of friction at every start of the optimization process. This allows for the coefficient of friction to be taken into account in the MPC solution. Rather than a constant coefficient of friction, the coefficient of friction is updated in the constraints and becomes a new variable in the optimization problem.

Using this modification in the MPC should allow for better performance in the locomotion of the robot. To evaluate the effects, a structured and systematic experiment was performed, to allow for a consistent comparison of the results.

3.4. Experiment

To evaluate the effects of the MPC modification, the simulated robot with the simulated sensor was put in the simulation environment described in the previous section with a set coefficient of friction. The robot was spawned in the same position at the edge of the world with a starting velocity of 0. Then, a constant command of forward velocity is sent for 10 seconds, and the robot performs the commanded input. This experiment was observed in a set of different velocities while keeping the coefficient of friction constant. The coefficient of friction that was chosen was acquired from observation of using the base controller, by lowering the coefficient of friction to when the robot starts to slip significantly using the slip detector.

Using this approach, it was also possible to do a limitation study. At what velocity does the modified controller start to degrade and doesn't improve the locomotion anymore?

3.4.1. First Glance at Results

Initially, to see whether the experiment had any merit, the responses in both of the controllers were observed graphically. Standard measurements for a controller which are response time, settling time, and steady-state error were observed as a preliminary investigation of the result of the modified controller. The two experiments that will be observed graphically are where the robot was sent a 0-meter-per-second velocity command and a 5-meter-per-second velocity command.

The result from the 0 meters per second velocity command is the most telling out of the two because it showed that the base controller could not stay stationary without slipping or replanning its steps, while the modified controller adjusted for a short period in the beginning, but allowed itself to gain stability and stayed stationary for the rest of the test.

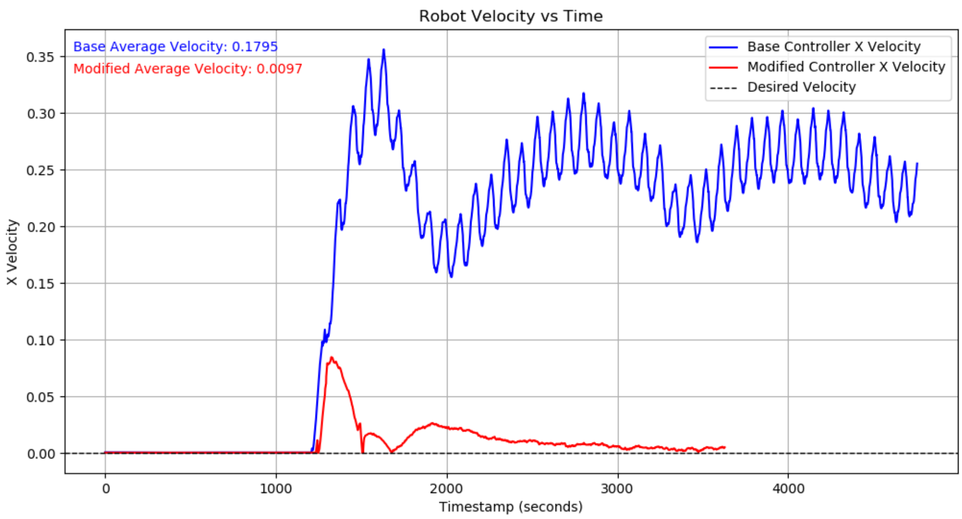

Figure 3.1: Comparison of Controller Performance given 0 m/s Velocity Command

An observation of the x velocity of the robot's front left foot when it was walking shows an even more apparent difference in the effectiveness of the modified controller in regaining stability, as it only needed

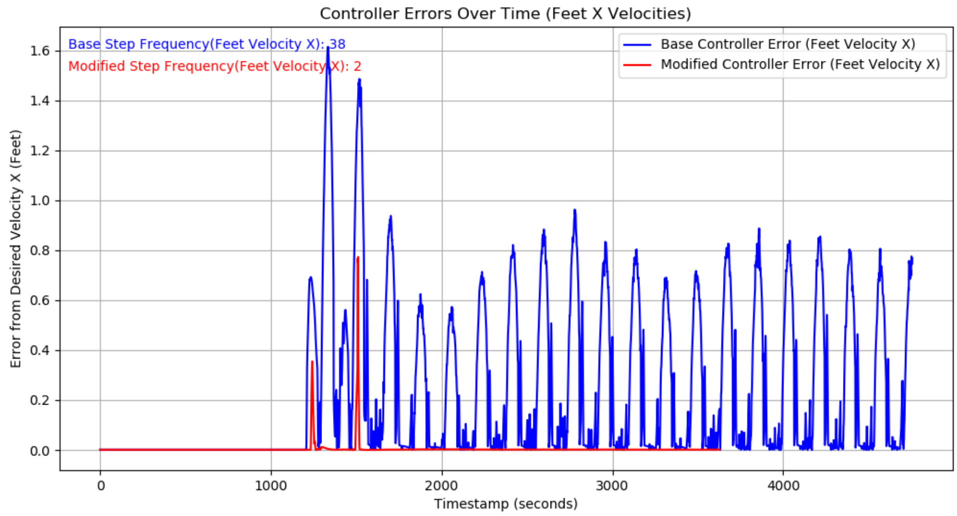

Figure 3.2: Comparison of Foot Velocity given 0 m/s Velocity Command

It can be seen that the performance of the controller when modified has improved in comparison to the base controller by a noticeable amount. Then, when given a constant velocity command it also showed this improvement in the performance.

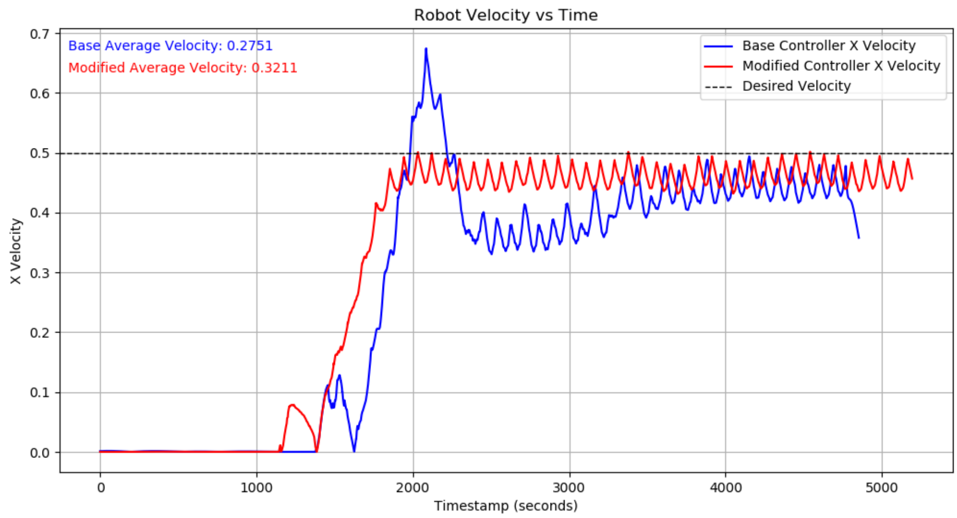

Figure 3.3: Comparison of Center of Mass Velocity given 5 m/s Velocity Command

The robot not only improved in its ability to respond promptly, but it also improved in the time it took to settle into the setpoint velocity in the command.

Lastly, it can also be seen that the steady state error and oscillation were significantly reduced in the modified controller, as the optimization problem prevents the robot from slipping and provides input that obeys the updated Coulomb friction law due to the low coefficient of friction. However, in locomotion, steady-state error will always exist as is seen on the graph, because of the nature of walking that oscillates the center of mass back and forth.

Another interesting observation is to observe the stability of the steps that the robot takes to perform the constant velocity command. It can be a sign that the robot struggles to achieve stability and reach the commanded velocity if it is seen to take inconsistent steps with large spikes in velocity. Imagine a person trying to hold his or her footing when walking on a section of slippery floor, as they will struggle to search for the necessary foothold to regain stability and continue on their initial trajectory or motion.

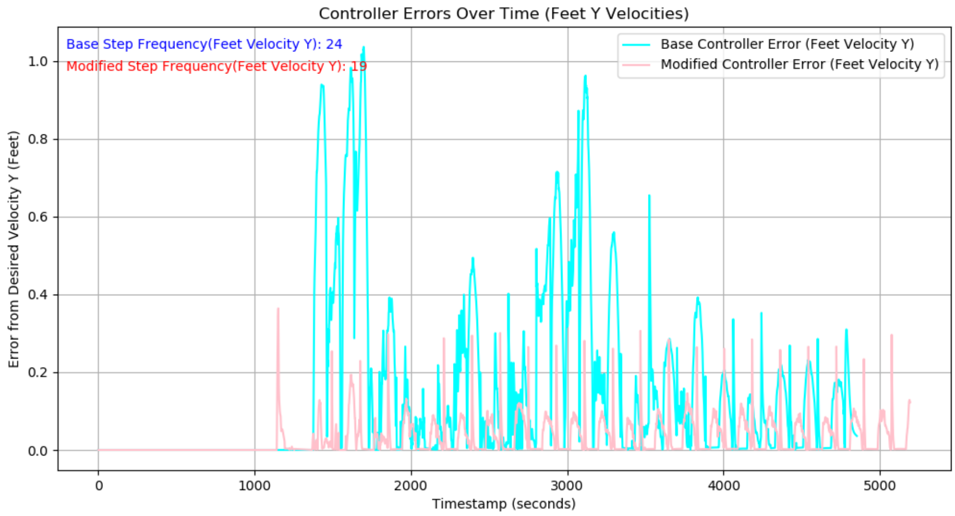

Figure 3.4: Comparison of Frequency of Steps Taken by the Robot

As can be seen from the figure, the velocity in the y directions of the feet shows the stability of the footholds that the robot produces when taking a velocity command, which is in the lateral direction of the robot's body. It can be seen that the velocities in the modified controller have constant peaks, while the base controller shows large spikes, especially at the beginning of the locomotion. This signifies the moment when the robot tries to compensate for the lack of friction on the ground, and it underestimates how slippery the ground is.

These short observations show an insight into what kind of effect the modified controller has on the locomotion of the quadruped. However, these observations are not enough to conclude that the controller has truly improved the performance in locomotion of the quadrupedal robot. Another set of experiments was conducted more systematically to perform a proper t-hypothesis test, to disprove or approve the hypothesis that "Accurate friction estimation improves locomotion in a quadrupedal robot, and omits the need for slip mitigation and recovery."

3.4.2. Metrics Collection for Hypothesis Test

To perform the hypothesis test, a metric in the experiment will be chosen to be compared with the same metric in a control experiment. This way, a hypothesis can be tested and seen if the improvement creates an effect in the performance of the robot's locomotion. Several metrics were chosen to represent the performance of locomotion in quadrupedal robots. The first category of metric that relates to performance locomotion is the energy-related metric. The second category is directly related to the contact conditions of the feet, which is the frequency of slip. The list below shows the metrics that were chosen for the experiment.

- Cost of Trasport
- Total Energy Consumption
- Total Slip Frequency

The two metrics that were chosen related to energy are total energy consumption and cost of transport. The cost of transport, in particular, is a widely used value to measure the efficiency of a legged robot's locomotion [38]. The cost of transport is derived as the energy consumed during locomotion, divided by the weight of the robot multiplied by the distance traveled. The equation below defines the cost of transport.

$$\text{COT} = \frac{E}{m \cdot g \cdot d}$$

where:

- E is the total energy consumed (in joules),
- m is the mass of the body (in kilograms),
- g is the gravitational acceleration (in m/s^2),
- d is the distance traveled (in meters).

The energy consumption is calculated using the total energy consumed in each joint by the robot during locomotion. The individual joint energy consumption in each joint is calculated by integrating the instantaneous joint power (watts) over time. Instantaneous joint power is the product of the joint velocity in radians per second and the joint effort in torque. The instantaneous power can be defined in the equation:

$$P(t) = \omega(t) \cdot \tau(t)$$

where:

- $\omega(t)$ is the joint velocity(in rad/s),
- $\tau(t)$ is the joint torque(in N/s).

Then integrating this value over a discrete time interval gives the total energy equation:

$$E = \sum_{i=1}^{n} P(t_i)\Delta t$$

Then, lastly, a friction-related metric was chosen to show the direct effects of friction sensors in MPC on the contact conditions of the feet and the ground. This metric is the number of slips that occurred. The number of slips is calculated using the simulated friction sensor, and checking that the feet that are in contact with the grounds are within the Coulomb friction cone defined in the previous section. In this simulated detector, the slip is a Boolean function that returns true only if the contact force is at the wall of the Coulomb friction cone.

$$\text{slip}(F, \mu, N) = \begin{cases} 1 & \text{if } \|F\| = \mu \cdot |N| \\ 0 & \text{otherwise} \end{cases}$$

where,

- F is the contact force.
- N is the normal force.
- μ is the coefficient of friction.

The normal force, in this case, is the z component of the contact force vector. With the ability to detect slips, the total occurrence of slips was recorded for every experiment trial.

3.5. Preliminary Work Conclusion

In this chapter, we detailed the preliminary work conducted to investigate the role of friction estimation in improving quadrupedal locomotion. The methodology is centered on a hypothesis that accurate friction estimation can significantly enhance robot stability and negate the need for explicit slip recovery strategies. The experimental setup involved a simulated environment using Gazebo, incorporating a Model Predictive Controller (MPC) and a novel approach to dynamically updating friction constraints.

The employed algorithms were outlined, focusing on the integration of real-time friction data into the MPC framework. Through systematic experiments comparing the base and modified controllers, we collected key performance metrics: Cost of Transport, Total Energy Consumption, and Total Slip Frequency. Initial observations indicated improvements in stability and slip prevention with the modified controller.

This foundation sets the stage for a comprehensive statistical analysis in the following chapters, aiming to validate our hypothesis and quantify the benefits of accurate friction estimation in quadrupedal robot locomotionThese metrics were chosen because they were observed to be the closest gauge for measuring the effects that a controller might cause to a legged robot's locomotion.

Part II

Verification and Validation

4 Hypothesis Test

4.1. Data Collection and Aggregation

As mentioned in the previous chapter, data was collected for each controller at varying velocity levels of 0,0.5,0.75,1.0, and 1.25. Then, to represent them with metrics at the different velocity levels, they are measured and averaged across all the trials at each level. Then, hypothesis tests were performed on the metrics at each velocity level using a t-test.

4.2. Hypothesis Test Methodology

Since there were five different velocity levels for each of the metrics, the hypothesis test was carried out at all levels, and a majority decision was taken from all the results on whether to prove or disprove the hypothesis.

4.2.1. Data Collection

Data for CoT, energy consumption, and slip data were collected from multiple ROS bag files, representing different velocity levels. These metrics were calculated for both the base and friction-aware controllers. For each velocity level, the average and standard deviation of the metrics were calculated, providing a basis for comparison.

4.2.2. Paired T-Test

The paired t-test was used to determine whether the differences in the means between the two controllers were statistically significant. For each metric at each velocity level, the null hypothesis (H_0) states that there is no difference in performance between the two controllers, while the alternative hypothesis (H_1) suggests a significant difference.

The test statistic for the paired t-test is calculated as:

$$t = \frac{\bar{d}}{s_d/\sqrt{n}}$$

Where:

- $\bar{d}$ is the mean of the differences between paired observations.
- s_d is the standard deviation of the differences.
- n is the number of paired observations.

The p-value obtained from the t test is compared with the significance level set at 0.05. If the p-value is less than alpha, the null hypothesis is rejected, indicating a significant difference in the metric between the two controllers.

4.2.3. Bonferroni Correction

Given that multiple hypothesis tests were performed (one for each velocity level across multiple metrics), the Bonferroni correction was applied to account for the increased risk of Type I errors (false positives). The correction adjusts the alpha level by dividing it by the number of tests performed:

$$\alpha_{\text{adjusted}} = \frac{\alpha}{m}$$

Where m is the number of tests. This provides a conservative approach to the hypothesis test to ensure that the overall significance level is maintained in all tests.

4.2.4. Conclusion

The methodology used in this study assessed the performance of the friction-aware controller relative to the base controller. The combination of paired t tests and Bonferroni correction provided convincing evidence of the friction-aware controller's superior performance in reducing CoT, energy consumption, and slips across various velocity levels.

4.3. Results Discussion

The results of the experiments showed the effectiveness of the friction-aware controller compared to the base controller across different metrics: Cost of Transport (CoT), total energy consumption, and the number of slips. The paired t-tests conducted at various velocity levels reveal significant differences in performance at most of the velocity levels.

4.3.1. Cost of Transport (CoT)

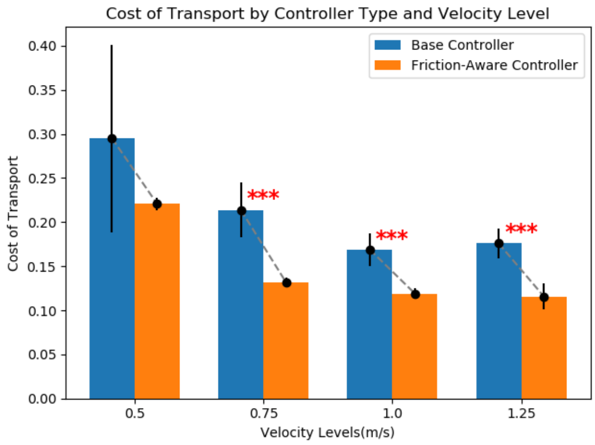

Figure 4.1: Cost of Transport Bar Plots

The friction-aware controller consistently demonstrated a lower CoT in most velocity levels. For instance, at a velocity of 1.25 m/s, the average CoT for the modified controller was 0.1157 J, significantly lower than the base controller's 0.1758 J. This reduction in CoT suggests that the friction-aware controller is more efficient in energy usage relative to the work done, particularly at higher velocities. The significant reduction in CoT across multiple velocity levels indicates the friction-aware controller's capacity to better manage energy use by minimizing unnecessary power expenditure during movement.

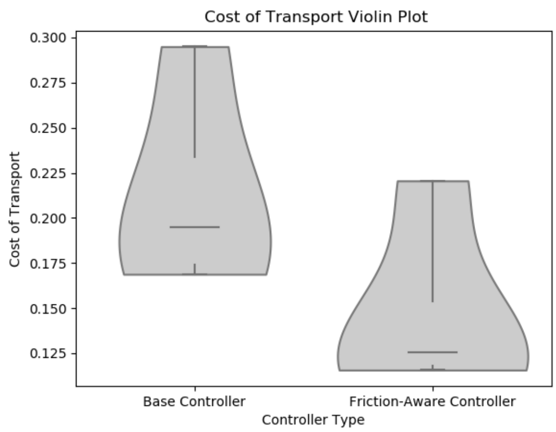

Figure 4.2: Violin Plot of COT

The violin plot shows that the Friction-Aware Controller has a consistently lower median and distribution in COT compared to the Base Controller. The distribution is narrower, indicating less variability.

4.3.2. Total Energy Consumption

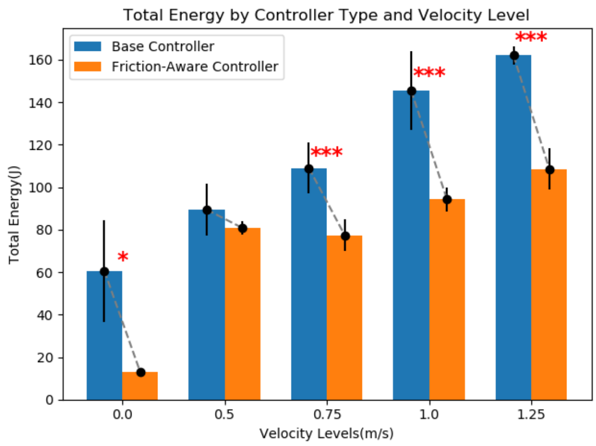

Figure 4.3: Bar Plot of Energy

The total energy consumed by the robot also showed a marked improvement with the friction-aware controller. At the 1.25 m/s velocity, the total energy consumption was substantially lower with the friction-aware controller (108.58) compared to the base controller (162.03). This pattern was consistent across other velocities, such as at 0.75 m/s, where the friction-aware controller consumed significantly less energy. The consistent reduction in energy consumption suggests that the friction-aware controller optimizes power consumption, likely by reducing slip and enhancing traction control, leading to more efficient locomotion.

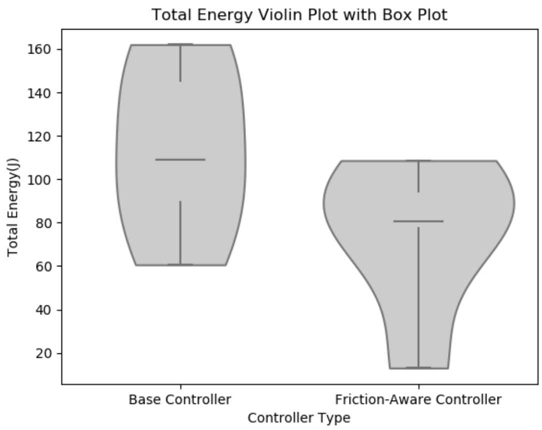

Figure 4.4: Violin Plot of Energy

It can be seen again, in the violin plot of the energy consumption of the friction-aware controller shows less variability as it is more dense around the median than in other values in the distribution.

4.3.3. Slips

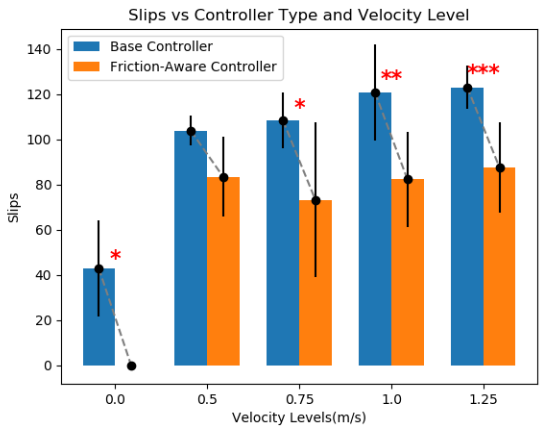

Figure 4.5: Bar Plot of Slips

Slips, which directly relate to the robot's traction and stability, were also significantly reduced with the friction-aware controller. The base controller showed higher slip rates, particularly at higher velocities. For example, at 1.25 m/s, the base controller recorded an average of 123 slips, while the friction-aware controller recorded only 87. This reduction indicates that the friction-aware controller provides better stability and control, reducing the likelihood of slipping and thereby improving the robot's overall performance on challenging terrains.

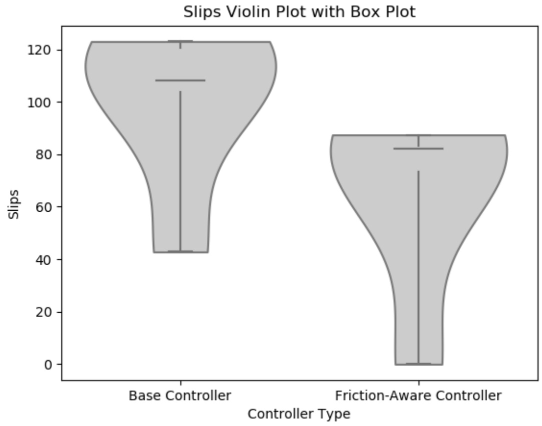

Figure 4.6: Violin Plot of Slips

An interesting finding in the violin plot shows that the variability of the result actually stayed the same or even increased in the friction aware controller.

4.3.4. Statistical Significance

The statistical analysis revealed that the improvements in CoT, energy consumption, and slips with the friction-aware controller were statistically significant at various velocity levels. For instance, the p-values for CoT at 1.25 m/s and total energy consumption at 1.0 m/s were both well below the significance level ($p < 0.05$), leading to the rejection of the null hypothesis. This consistent rejection across different metrics and velocities strengthens the case for the friction-aware controller as a superior control strategy for reducing energy consumption and enhancing stability.

The first hypothesis test that was done was without the bon-ferroni correction. Marked by stars, these indicate where the null hypothesis is rejected, suggesting a significant difference between the base and friction-aware controllers, where the p-value of the paired t-test is less than the alpha at 0.05. 11 out of 14 of the results.

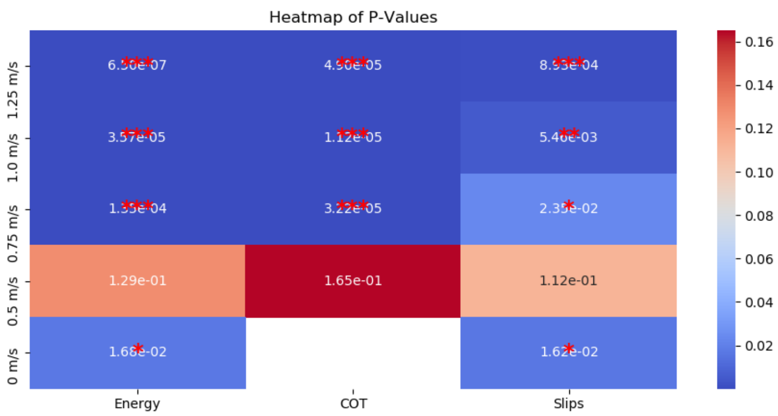

Figure 4.7: Heatmap of P-Values

What is interesting about the findings is that at the 0.5 m/s velocity level, across the whole metrics, the difference is not statistically significant, although it already showed a reduction in all of the metrics. Also, an interesting trend is that, the differences get more significant at higher velocity levels above 0.75 m/s. This is probably because the robot shows more faults in the locomotion at higher velocity levels, such that the friction-aware controller plays more of a role at preventing slips.

Then, a Bon-Ferroni correction is done to the hypothesis test, by dividing the alpha by the number of velocity levels that were performed in the experiment. So the alpha becomes 0.01. After the correction, there is a reduction in the amount of tests where the null hypothesis is not rejected.

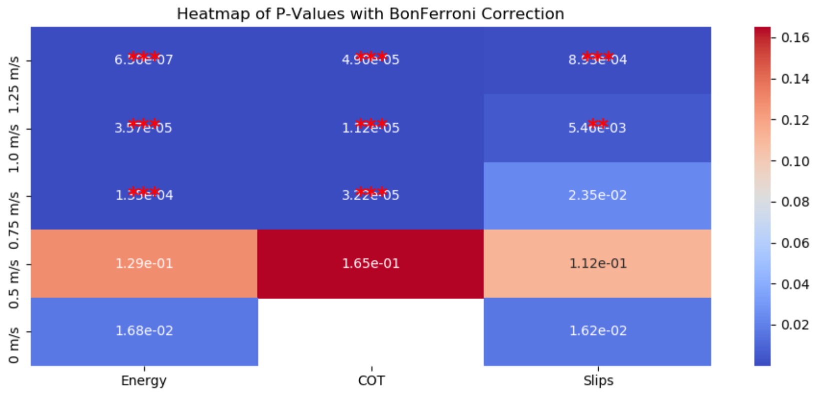

Figure 4.8: Heatmap of P-Values with BonFerroni Correction

It can be seen, that the the p values are below the alpha on 8 out of 14 of the results. This shows that even at a more conservative evaluation of the hypothesis test, the majority of the experiments still shows statistically significant difference.

4.3.5. Conclusion

The analysis confirms that the friction-aware controller offers substantial benefits over the base controller, particularly in terms of reducing energy consumption and improving stability through fewer slips. With and without the Bonferroni correction, the majority of the experiments showed statistically significant improvements in controller performance. These findings are critical for applications where energy efficiency and reliable traction are paramount, such as in autonomous exploration. Future experiments could explore the integration of the friction-aware controller in various terrain geometries and non-homogeneous friction conditions to further validate its robustness and effectiveness.

Part III

Closure

5
Conclusion

5.1. Conclusion

The primary objective of this thesis was to investigate the role of accurate friction estimation in improving quadrupedal robot locomotion, with a specific focus on its potential to reduce or eliminate the need for explicit slip recovery strategies. Through a combination of literature review, preliminary work, and experimental validation, we sought to determine whether the integration of friction estimation within the Model Predictive Controller (MPC) framework could lead to significant enhancements in the robot's performance.

5.1.1. Summary of Findings

Literature Review: The review of existing work highlighted the challenges posed by unpredictable terrain and the limitations of current friction models. The necessity for real-time friction estimation became apparent as a critical factor in preventing slips and improving overall stability.

Preliminary Work: We developed a simulation environment to evaluate the modified MPC that incorporated friction estimation. Initial observations indicated promising improvements in stability and energy efficiency, setting the stage for a more rigorous statistical analysis.

Experimental Results: The experiments conducted across different velocity levels provided clear evidence of the benefits of the friction-aware controller. The bar and violin plots illustrated significant reductions in the Cost of Transport (CoT), total energy consumption, and slip frequency, especially at higher velocities. The statistical analysis, supported by paired t-tests and a Bonferroni correction, confirmed that these improvements were not only consistent but also statistically significant.

5.1.2. Discussion

The results of this study suggest that integrating friction estimation into the control architecture of quadrupedal robots can lead to substantial improvements in locomotion efficiency and stability. The friction-aware controller consistently outperformed the base controller across all key metrics, indicating that it is a viable solution for real-world applications where unpredictable terrain poses a significant challenge.

The reduction in the Cost of Transport and energy consumption observed in the experiments points to a more efficient use of resources, which is critical in scenarios where energy conservation is paramount. Moreover, the significant decrease in slip frequency highlights the controller's ability to maintain stability, even in challenging conditions.

5.2. Research Questions

The research questions posed in Chapter 1 are repeated and answered below for convenience.

Research Question 1

How does an accurate friction estimation improve the locomotion performance of a quadrupedal robot?

Accurate friction estimation improves locomotion performance by enabling the robot to dynamically adjust its gait and contact forces based on real-time terrain feedback. This leads to more stable and energy-efficient movement, as evidenced by lower total energy consumption and reduced cost of transport in the experiments conducted. Additionally, the reduction in slip occurrences further underscores the enhancement in stability, allowing the robot to navigate challenging terrains with greater performance.

Research Question 2

When does accurate friction estimation fail to prevent slip in locomotion?

From the results of the experiments, it seemed like the accurate friction estimation did not improve significantly the locomotion at a moving velocity below 0.75 m/s.Thus, while friction estimation is crucial, it must be complemented by other strategies in extreme conditions, where the robot must move at a low velocity.

Research Question 3

Is an explicit slip recovery procedure still required if an accurate friction estimation is present?

The friction-aware controller was able to adapt to changing friction conditions more effectively, resulting in fewer slips overall. This implies that accurate friction estimation can mitigate the need for additional slip recovery mechanisms, thereby simplifying the control architecture and improving overall system robustness. Reflect on research questions.

5.2.1. Closing Remarks

In conclusion, this thesis has demonstrated that accurate friction estimation can significantly enhance the performance of quadrupedal robots, making them more robust and efficient in navigating complex terrains. The findings suggest that future research should focus on further refining friction estimation techniques and exploring their integration into various legged robot platforms. By doing so, we can continue to push the boundaries of what legged robots can achieve, paving the way for their deployment in increasingly demanding environments.

6 Recommendations

6.1. Recommendations for Future Work

While this thesis has demonstrated the effectiveness of a friction-aware controller using simulated friction estimation, several avenues for future work could further enhance the control architecture and its real-world applicability.

- **Integration of Real Friction Sensors:** Future research should focus on incorporating real friction sensors into the robot's feet. This integration would provide direct measurements of the coefficient of friction in real-time, thereby improving the accuracy and responsiveness of the control system. By closing the loop with real-world friction data, the controller could adapt more effectively to varying terrain conditions, further reducing the need for slip recovery strategies.
- **Hardware Implementation and Field Testing:** The current findings are based on simulations and controlled experiments. Implementing the friction-aware controller on a physical robot and conducting field tests across diverse terrains would be a crucial next step. These tests would validate the controller's performance in real-world scenarios and help identify any practical challenges that may arise during hardware integration.
- **Extension to Other Robotic Platforms:** While the focus of this thesis was on quadrupedal robots, the principles of friction-aware control could be extended to other types of legged robots, such as bipedal or hexapodal systems. Future work could explore the adaptability and performance of friction-aware controllers across different robotic platforms.
- **Exploration of Advanced Control Algorithms:** Further research could investigate the integration of more advanced control algorithms, such as reinforcement learning, to enhance the robot's adaptability to dynamic environments. These algorithms could leverage friction data to optimize the robot's locomotion strategy in real-time, potentially leading to even greater performance improvements.
- **Long-Term Adaptation and Learning:** Future work could also explore the robot's ability to learn and adapt its friction estimation over time. By incorporating machine learning techniques, the robot could develop a deeper understanding of different terrain types and their associated friction characteristics, leading to continuous improvement in locomotion efficiency and stability.

References

[1] David Anisi et al. *Robot Automation in Oil and Gas Facilities*. 2010.

[2] Hutter et Al. *ANYmal - A Highly Mobile and Dynamic Quadrupedal Robot*. 2016.

[3] C. Dario Bellicoso et al. "Advances in real-world applications for legged robots". In: *Journal of Field Robotics* 35.8 (Dec. 2018), pp. 1311–1326. DOI: `10.1002/rob.21839`.

[4] Michele Focchi et al. *Slip Detection and Recovery for Quadruped Robots*. Tech. rep. URL: `http://www.youtube.com/watch?v=cNZPRsrwumQ`.

[5] Wen Ruey Chang et al. "Gait adaptation on surfaces with different degrees of slipperiness". In: *Applied Ergonomics* 59 (Mar. 2017), pp. 333–341. DOI: `10.1016/j.apergo.2016.09.008`.

[6] Fabian Jenelten et al. "Dynamic Locomotion on Slippery Ground". In: *IEEE Robotics and Automation Letters* 4.4 (July 2019), pp. 4170–4176. DOI: `10.1109/lra.2019.2931284`.

[7] Elliott Donlon et al. "GelSlim: A High-Resolution, Compact, Robust, and Calibrated Tactile-sensing Finger". In: (Mar. 2018). URL: `http://arxiv.org/abs/1803.00628`.

[8] Benjamin Ward-Cherrier et al. "The TacTip Family: Soft Optical Tactile Sensors with 3D-Printed Biomimetic Morphologies". In: *Soft Robotics* 5.2 (Apr. 2018), pp. 216–227. DOI: `10.1089/soro.2017.0052`.

[9] Siyuan Dong et al. "Maintaining Grasps within Slipping Bound by Monitoring Incipient Slip". In: (Oct. 2018). URL: `http://arxiv.org/abs/1810.13381`.

[10] Zhanat Kappassov et al. "Touch driven controller and tactile features for physical interactions". In: *Robotics and Autonomous Systems* 123 (Jan. 2020). DOI: `10.1016/j.robot.2019.103332`.

[11] Elizabeth A. Stone et al. "Learning to Live Life on the Edge: Online Learning for Data-Efficient Tactile Contour Following". In: (Sept. 2019). URL: `http://arxiv.org/abs/1909.05808`.

[12] Elizabeth A Stone et al. *Walking on TacTip toes: A tactile sensing foot for walking robots*. 2020.

[13] C. Semini et al. "Design of HyQ -A hydraulically and electrically actuated quadruped robot". In: *Proceedings of the Institution of Mechanical Engineers. Part I: Journal of Systems and Control Engineering* 225.6 (2011), pp. 831–849. DOI: `10.1177/0959651811402275`.

[14] M. Hutter et al. "ANYmal - toward legged robots for harsh environments". In: *Advanced Robotics* 31.17 (Sept. 2017), pp. 918–931. DOI: `10.1080/01691864.2017.1378591`.

[15] Jongwoo Lee. *Hierarchical controller for highly dynamic locomotion utilizing pattern modulation and implementation on the MIT Cheetah robot*. Tech. rep. 2013.

[16] Zico Kolter et al. *A Control Architecture for Quadruped Locomotion Over Rough Terrain*. IEEE Xplore, 2008.

[17] Joey Chestnutt et al. "Footstep Planning for the Honda ASIMO Humanoid". In: (2005).

[18] Alexander W Winkler et al. "Fast Trajectory Optimization for Legged Robots using Vertex-based ZMP Constraints". In: (May 2017). URL: `http://arxiv.org/abs/1705.10313`.

[19] Gerardo Bledt et al. *MIT Cheetah 3: Design and Control of a Robust, Dynamic Quadruped Robot*. 2018. DOI: `10.0/Linux-x86{\_}64`.

[20] Marko Bjelonic et al. *Whole-Body MPC and Online Gait Sequence Generation for Wheeled-Legged Robots*. Tech. rep. 2020.

[21] MathWorks. *What is Model Predictive Control (MPC)?* 2024. URL: `https://www.mathworks.com/help/mpc/gs/what-is-mpc.html`.

[22] Bo N. J. Persson. *Sliding Friction*. NanoScience and Technology. Springer Berlin Heidelberg, 1998. DOI: `10.1007/978-3-662-03646-4`.

[23] Trinkle et Al. *On Dynamic Multi-Rigid-Body Contact Problems with Coulomb Friction*. Tech. rep. 2006.

[24] Hongkai Dai et al. *Planning robust walking motion on uneven terrain via convex optimization*. Tech. rep.

[25] X. Alice Wu et al. “Tactile Sensing and Terrain-Based Gait Control for Small Legged Robots”. In: *IEEE Transactions on Robotics* 36.1 (Feb. 2020), pp. 15–27. DOI: `10.1109/TRO.2019.2935336`.

[26] Eric Coyle et al. *A COMPARISON OF CLASSIFIER PERFORMANCE FOR VIBRATION-BASED TERRAIN CLASSIFICATION*. Tech. rep. 2008.

[27] Xiaofeng Guo et al. *Soft Foot Sensor Design and Terrain Classification for Dynamic Legged Locomotion*. 2020. DOI: `10.0/Linux-x86{\_}64`.

[28] Christian Gehring et al. “Dynamic trotting on slopes for quadrupedal robots”. In: *IEEE International Conference on Intelligent Robots and Systems*. Vol. 2015-December. Institute of Electrical and Electronics Engineers Inc., Dec. 2015, pp. 5129–5135. DOI: `10.1109/IROS.2015.7354099`.

[29] Vojtěch Vojtěchˇ et al. *Touching without vision: terrain perception in sensory deprived environments*. Tech. rep. 2016. URL: `http://www.tradr-project.eu`.

[30] Hsiu-Chin Lin et al. “Contact Surface Estimation via Haptic Perception”. In: (Mar. 2020). URL: `http://arxiv.org/abs/2003.02227`.

[31] M J Pollayil et al. *SoftFoot-Q: A Novel Adaptive Foot for Quadrupeds*. Tech. rep. 2020.

[32] Cristina Piazza et al. “Toward an adaptive foot for natural walking”. In: *IEEE-RAS International Conference on Humanoid Robots*. IEEE Computer Society, Dec. 2016, pp. 1204–1210. DOI: `10.1109/HUMANOIDS.2016.7803423`.

[33] J Rogelio Guadarrama-Olvera et al. *Enhancing Biped Locomotion on Unknown Terrain Using Tactile Feedback*. 2018. DOI: `10.0/Linux-x86{\_}64`. URL: `www.ics.ei.tum.de.`.

[34] Jacob J. Shill et al. “Terrain identification on a one-legged hopping robot using high-resolution pressure images”. In: *Proceedings - IEEE International Conference on Robotics and Automation*. Institute of Electrical and Electronics Engineers Inc., Sept. 2014, pp. 4723–4728. DOI: `10.1109/ICRA.2014.6907550`.

[35] Zhang Cong et al. “Contact Force Estimation Method of Legged-Robot and Its Application in Impedance Control”. In: *IEEE Access* 8 (2020), pp. 161175–161187. DOI: `10.1109/ACCESS.2020.3021080`.

[36] *Open Dynamics Engine*. URL: `http://ode.org`.

[37] Joseph Norby et al. *Quad-SDK: Full Stack Software Framework for Agile Quadrupedal Locomotion*. Tech. rep. URL: `https://github.com/robomechanics/quad-sdk.`.

[38] Sangbae Kim et al. “Design of Dynamic Legged Robots”. In: *Foundations and Trends in Robotics* 5.2 (2017), pp. 117–190. DOI: `10.1561/2300000044`.

A

Algorithms

Algorithm 2: Simulated Friction Sensor

```
Data: feet, groundContacts, frictionMap
for each foot in feet do
    if foot is touching the ground then
        bodyContacts ← get body in contacts with foot;
        frictionValues ← [ ];
        for each contact in bodyContacts do
            friction ← access coefficient of friction from frictionMap;
            frictionValues.append(friction);
        end
        avgFriction ← mean(frictionValues);
        noisyFriction ← avgFriction + N(avgFriction, σ²) // Add Gaussian noise
    end
end
```