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RESEARCH ARTICLE

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Optimal reset law design based on guaranteed cost control method for Lipschitz nonlinear systems

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Abstract

This article proposes a systematic approach for optimal reset law design of a class of nonlinear systems. By using the guaranteed cost control method, sufficient conditions for the design of optimal reset law are derived in terms of linear matrix inequalities. In an offline design procedure, the reset law is computed that minimizes the upper bound of a quadratic cost function. The proposed method can be implemented for real-time applications even with small sampling time. The simulation results verify the efficacy and effectiveness of the proposed theoretical results.

KEYWORDS

guaranteed cost control, linear matrix inequalities, Lipschitz nonlinearity, reset control systems, reset law

1 | INTRODUCTION

Reset control is a class of hybrid control which is able to overcome the inherent limitations of linear control systems. The basic idea of reset control is to reset all or part of the controller states whenever the reset condition is met. If the reset condition is not met, it is referred to as the base system so that the system response has no jumps. The reset controller has one more degree of freedom than the base system, which can improve the transient response of the system. It was initially introduced by Clegg.[1] A Clegg integrator (CI) consists of an integrator and a reset mechanism which resets its state to zero when its input signal crosses zero. This controller was generalized to first order reset element (FORE) in Reference 2. In the last three decades, various stability results for reset control systems are reported in the literature.[3-10] For instance, H_β stability conditions,[11] reset times-dependent stability,[12] delay-independent and delay-dependent stability,[13,14] and passivity-based approach.[15,16] The L_2 exponential stability problem of the reset control systems with time-varying delay was investigated in Reference 17. In Reference 18, a periodic triggering reset control was presented, where the resetting actions are only occurred at periodically sampling instants. In Reference 19, a broadband phase compensation element named CgLp was presented, which provides a broadband phase lead while maintaining constant gain. In Reference 20, to overcome the overshoot performance limitation, a generalized first order reset element (GFORE) was proposed.

Recently, the design of reset law based on model predictive strategy (MPS) has drawn the attention of researchers,[21-29] whose main idea is to determine the reset law by minimizing a cost function at each sampling time. Based on robust MPS, a reset law for linear systems under Lipschitz uncertainty was designed for the first time in Reference 21. In Reference 22, a model prediction-based framework was provided to determine an appropriate reset law for linear systems with norm-bounded uncertainty. In Reference 23, by using an MPS, an observer-based reset law design for a class of uncertain systems was presented. The problem of discrete-time triggered reset law design based on MPS for linear systems was addressed in Reference 24. A systematic approach to design a reset controller for polytopic linear parameter varying

TABLE 1 List of variables and definitions

Symbol	Definition
L_p	Lipschitz matrix
Λ, Ξ	Reset map matrices
M_r, M	Reset surfaces
$\rho\left(x_p, x_c\right)$	After-reset value
$\mathcal{J}$	Cost function
Q	Weight matrix
t_ρ	Temporal regularization parameter
t_k	Reset times

systems was proposed in Reference 25. In Reference 26, the reset law design procedure of Reference 21 was extended to nonlinear time-delay systems. An MPS was presented to design a reset dynamic output feedback control (DOFC) for linear systems in Reference 27. By employing MPS and genetic algorithm, a robust reset DOFC was designed for a class of uncertain linear systems in Reference 28. The application of Reference 21 to head-positioning systems was provided in Reference 29.

In the articles above, an optimization problem should be solved to determine the reset law. These approaches demand large computational time and may not be useful for real-time applications with small sampling time. This challenge is addressed in this article via an offline design procedure.

The main contribution of this study is to introduce a systematic approach to design an optimal reset law for a class of nonlinear systems, where this design is done in a full offline procedure. To this end, first, the sufficient conditions for the asymptotic stability of the reset control system are derived based on the Lyapunov theorem. Then, the upper bound of the predefined performance index is minimized by the guaranteed cost control (GCC) method. The problem of optimal rest law design is transformed into an linear matrix inequalitie (LMI) optimization problem. By solving the optimization problem offline, the reset law is designed. Therefore, the proposed reset law can be easily implemented on systems with small sampling time. Finally, the proposed method is compared with the existing methods in the literature. The simulation results illustrate the superiority and advantages of our method.

This article is structured as follows: The problem formulation is presented in Section 2. The reset law design is given in Section 3. The simulation results are provided in Section 4. Finally, Section 5 draws the conclusions.

The definitions and symbols of the variables used in this article are summarized in Table 1.

2 | PROBLEM FORMULATION

Consider a class of nonlinear systems described by

$$\begin{cases} \dot{x}_p = Ax_p + Bu + Hf(x_p), \\ y = Cx_p, \end{cases} \tag{1}$$

where $x_p \in \mathbb{R}^{n_p}, u \in \mathbb{R}^{n_u}$, and $y \in \mathbb{R}^{n_y}$ represent the state, the control input, and the output vectors, respectively, $f(x_p)$ denotes the nonlinear dynamics associated with the state vector.

Assumption 1. The nonlinear function $f(x_p)$ satisfies the following Lipschitz condition locally on a set $\mathbb{D} \subset \mathbb{R}^{n_p}$:

$$\|f(x_p) - f(\tilde{x}_p)\| \le \|L_p(x_p - \tilde{x}_p)\|, \;\; \forall x_p, \tilde{x}_p \in \mathbb{D}, \tag{2}$$

where $f(0) = 0$, and L_p is a Lipschitz constant matrix.

The following reset controller is proposed to control the given plant (1):

$$\begin{cases} \dot{x}_c = A_c x_c + B_c e & x_c \notin M_r \\ x_c^+ = \rho\left(x_p, x_c\right) & x_c \in M_r \\ u = C_c x_c \end{cases}, \tag{3}$$

where $x_c \in \mathbb{R}^{n_c}$ is the state of the reset controller, x_c^+ is the reset value of the controller state, $e = r - y$ with r being the reference signal, and $\rho\left(x_p, x_c\right)$ is a continuous function of the controller and the plant states. The jump set (or reset surface) M_r determines when the reset occurs.

Remark 1. In the reset controller (3), the controller states are reset to the after-reset value $\rho\left(x_p, x_c\right)$ when the reset condition is met. In this article, in order to handle the determination of this function, the reset law is proposed as

$$\rho\left(x_p, x_c\right) = \Lambda x_p\left(t_k\right) + \Xi x_c\left(t_k\right), \tag{4}$$

where t_k are reset times for $k = 1, 2, \ldots$, and $\Lambda \in \mathbb{R}^{n_c \times n_p}$ and $\Xi \in \mathbb{R}^{n_c \times n_c}$ are reset map matrices.

Substituting the controller (3) into (1) gives the following closed-loop system:

$$\begin{cases} \dot{\overline{x}} = \bar{A}\overline{x} + \overline{B}r + \overline{H}f(x_p) & \overline{x} \notin M \\ \overline{x}^+ = A_R \overline{x} & \overline{x} \in M \\ y = \overline{C}\overline{x} \end{cases}, \tag{5}$$

where $\overline{x} = [x_p^{\mathrm{T}} \;\; x_c^{\mathrm{T}}]^{\mathrm{T}}$ and

$$\bar{A} = \begin{bmatrix} A & BC_c \\ -B_c C & A_c \end{bmatrix}, \quad A_R = \begin{bmatrix} I & 0 \\ \Lambda & \Xi \end{bmatrix}, \quad \overline{B} = \begin{bmatrix} 0 \\ B \end{bmatrix}, \quad \overline{H} = \begin{bmatrix} H \\ 0 \end{bmatrix}, \quad \overline{C} = \begin{bmatrix} C & 0 \end{bmatrix}.$$

The reset surface M is defined as the following:[11]

$$M = \left\{\overline{x} \in \mathbb{R}^{n_p + n_c} | e = 0 \quad \varsigma \quad \overline{x}^+ \neq \overline{x}\right\}. \tag{6}$$

Remark 2. In presence of noise, the reset surface M can be modified as[21]

$$M = \left\{\overline{x} \in \mathbb{R}^{n_p + n_c} | |e| < \eta \quad \varsigma \quad \overline{x}^+ \neq \overline{x}\right\}, \tag{7}$$

where η a small positive number.

Remark 3. In real-time applications, the reset condition can be determined based on a discrete-time zero-crossing method. That is:

$$M = \left\{\overline{x} \in \mathbb{R}^{n_p + n_c}, K \in N | e\left((K-1)T_s\right) e\left(KT_s\right) \leq 0 \quad \varsigma \quad \overline{x}^+ \neq \overline{x}\right\}, \tag{8}$$

where T_s is the sampling time.

Assumption 2. In this article, the reference signal r is set to zero, and the regulation problem is addressed.

Remark 4. In order to ensure well-posedness and improve the system performance, the reset actions and reset conditions must be chosen properly; otherwise, beating and deadlock phenomena may occur, which can destroy the well-posedness property of solutions.[30]

Assumption 3. To avoid deadlock and beating phenomena, it is supposed that the after-reset values do not belong to the reset surface M. That is:

$$\text{If } \overline{x}(t_k) \in M \text{ then } \overline{x}(t_k^+) \notin M. \tag{9}$$

The condition (9) ensures the existence and uniqueness of the solutions.

Remark 5. Zenoness is a phenomenon where there are infinite number of resetting actions within a compact time interval. In practice, a common method to avoid this phenomenon is to use temporal regulation. By using this method, a reset action can occur only at least after t_ρ second. The closed-loop reset system (5) with $r = 0$ and temporal regulation is expressed as

$$\begin{cases} \dot{\tau} = 1, & \dot{\overline{x}} = \bar{A}\overline{x} + \overline{H}f(x_p) \quad \overline{x} \notin M \quad \text{or} \quad \tau < t_\rho \\ \tau^+ = 0, & \overline{x}^+ = A_R\overline{x} \quad \overline{x} \in M \quad \text{and} \quad \tau \geq t_\rho \end{cases}, \tag{10}$$

where t_ρ is the minimum time between two consecutive reset actions (i.e., $t_{k+1} - t_k > t_\rho$).

3 | RESET CONTROLLER DESIGN

In general, the design of reset controllers consists of two main steps. In the first step, a suitable linear controller (A_c, B_c, C_c) is designed. In this article, we design it using pole-placement method such that the base system is asymptotically stable. The second step is to design the reset law. The aim of the present study is to design the reset law which guarantees the closed-loop system stability and improves transient performance of the response.

3.1 | Reset law design

In this part, sufficient conditions for reset law design are derived in terms of LMIs.

In order to conclude our results, the following proposition is borrowed from the literature.

Proposition 1. *Let $V : \mathbb{R}^n \to \mathbb{R}$ be a positive-definite, continuously differentiable, radially unbounded function such that*

$$\begin{cases} \dot{V}(\bar{x}) < 0 & \bar{x} \notin M, \quad (11) \\ \Delta V(\bar{x}) := V(\bar{x}^+) - V(\bar{x}) \leq 0 & \bar{x} \in M. \quad (12) \end{cases}$$

Then, the closed-loop reset system (5) is asymptotically stable.[11]

Now, the problem of finding a reset law for system (5) is transformed into an equivalent LMI problem. Then, by solving the problem, the reset map matrices Λ and Ξ are designed.

Theorem 1. *Consider the reset control system (5). If there exist symmetric matrices $X_{11} \in \mathbb{R}^{n_p \times n_p}$, $X_{22} \in \mathbb{R}^{n_c \times n_c}$, matrices $X_{12} \in \mathbb{R}^{n_p \times n_c}$, $\mathcal{L}_1 \in \mathbb{R}^{n_p \times n_c}$, $\mathcal{L}_2 \in \mathbb{R}^{n_c \times n_c}$, and a positive scalar μ such that the following LMIs hold:*

$$\begin{bmatrix} \Phi_{11} & \Phi_{12} & \mu H & X_{11}L_p^{\mathrm{T}} \\ * & \Phi_{22} & 0 & X_{12}^{\mathrm{T}}L_p^{\mathrm{T}} \\ * & * & -\mu I & 0 \\ * & * & * & -\mu I \end{bmatrix} < 0,$$

$$\begin{aligned} \Phi_{11} &= AX_{11} + BC_cX_{12}^{\mathrm{T}} + X_{11}A^{\mathrm{T}} + X_{12}C_c^{\mathrm{T}}B^{\mathrm{T}}, \\ \Phi_{12} &= AX_{12} + BC_cX_{22} - X_{11}C^{\mathrm{T}}B_c^{\mathrm{T}} + X_{12}A_c^{\mathrm{T}}, \\ \Phi_{22} &= -B_cCX_{12} + A_cX_{22} - X_{12}^{\mathrm{T}}C^{\mathrm{T}}B_c^{\mathrm{T}} + X_{22}A_c^{\mathrm{T}}, \end{aligned} \tag{13}$$

$$\begin{bmatrix} -X_{11} & -X_{12} & X_{11} & \mathcal{L}_1 \\ * & -X_{22} & X_{12}^{\mathrm{T}} & \mathcal{L}_2 \\ * & * & -X_{11} & -X_{12} \\ * & * & * & -X_{22} \end{bmatrix} \leq 0, \tag{14}$$

the reset map matrices Λ *and* Ξ *are given by*

$$\begin{bmatrix}\Lambda & \Xi\end{bmatrix} = \begin{bmatrix}\mathcal{L}_1^{\mathrm{T}} & \mathcal{L}_2^{\mathrm{T}}\end{bmatrix} X^{-1}. \tag{15}$$

Then, the reset law is obtained as

$$x_c(t_k^+) = \Lambda x_p(t_k) + \Xi x_c(t_k). \tag{16}$$

Proof. Consider a Lyapunov function candidate

$$V(\bar{x}) = \bar{x}(t)^{\mathrm{T}} P \bar{x}(t). \tag{17}$$

The time-derivative of $V(\bar{x})$ along the trajectories of (5) yields

$$\dot{V}(\bar{x}) = \bar{x}^{\mathrm{T}}(t)(P\bar{A} + \bar{A}^{\mathrm{T}}P)\bar{x}(t) + \bar{x}^{\mathrm{T}}(t)(P\overline{H})f(x_p) + f^{\mathrm{T}}(x_p)(\overline{H}^{\mathrm{T}}P)\bar{x}(t). \tag{18}$$

The Lipschitz condition predefined in Assumption 1 gives the following inequality:

$$0 \leq \mu^{-1} x_p^{\mathrm{T}}(t) L_p^{\mathrm{T}} L_p x_p(t) - \mu^{-1} f^{\mathrm{T}}(x_p) f(x_p), \tag{19}$$

where μ is an arbitrary positive scalar.

The above inequality can be rewritten as below:

$$0 \leq \mu^{-1} \bar{x}^{\mathrm{T}}(t) \overline{L}_p^{\mathrm{T}} \overline{L}_p \bar{x}(t) - \mu^{-1} f^{\mathrm{T}}(x_p) f(x_p), \tag{20}$$

where

$$\overline{L}_p = \begin{bmatrix} L_p & 0 \\ 0 & 0 \end{bmatrix}.$$

Incorporating (18) and (20) results in

$$\begin{aligned}\dot{V}(\bar{x}) \leq\ & \bar{x}^{\mathrm{T}}(t)(P\bar{A} + \bar{A}^{\mathrm{T}}P)\bar{x}(t) + \bar{x}^{\mathrm{T}}(t)(P\overline{H})f(x_p) + f^{\mathrm{T}}(x_p)(\overline{H}^{\mathrm{T}}P)\bar{x}(t) \\ & + \mu^{-1}\bar{x}^{\mathrm{T}}(t)\overline{L}_p^{\mathrm{T}}\overline{L}_p\bar{x}(t) - \mu^{-1} f^{\mathrm{T}}(x_p) f(x_p).\end{aligned} \tag{21}$$

The inequality (21) further implies that $\dot{V}(\bar{x}) \leq \nu^{\mathrm{T}}(t)\Psi\nu(t)$, where $\nu(t) = \begin{bmatrix}\bar{x}^{\mathrm{T}}(t) & f^{\mathrm{T}}(x_p)\end{bmatrix}^{\mathrm{T}}$ and

$$\Psi = \begin{bmatrix} P\bar{A} + \bar{A}^{\mathrm{T}}P + \mu^{-1}\overline{L}_p^{\mathrm{T}}\overline{L}_p & P\overline{H} \\ * & -\mu^{-1}I \end{bmatrix}. \tag{22}$$

Note that $\Psi < 0$ ensures $\dot{V}(\bar{x}) < 0$.

By applying the Schur complement to $\Psi < 0$, we have

$$\begin{bmatrix} P\bar{A} + \bar{A}^{\mathrm{T}}P & P\overline{H} & \overline{L}_p^{\mathrm{T}} \\ * & -\mu^{-1}I & 0 \\ * & * & -\mu I \end{bmatrix} < 0. \tag{23}$$

Pre- and post-multiplying (23) by diag(P^{-1}, μI, I) yields

$$\begin{bmatrix} \bar{A}P^{-1} + P^{-1}\bar{A}^{\mathrm{T}} & \mu\overline{H} & P^{-1}\overline{L}_p^{\mathrm{T}} \\ * & -\mu I & 0 \\ * & * & -\mu I \end{bmatrix} < 0. \tag{24}$$

By letting $X = P^{-1}$ and partitioning X as

$$X = \begin{bmatrix} X_{11_{(n_p \times n_p)}} & X_{12_{(n_p \times n_c)}} \\ * & X_{22_{(n_c \times n_c)}} \end{bmatrix}, \tag{25}$$

the inequality (24) is equivalent to (13).

Now, consider the change of value of $V(\overline{x})$ at reset times. From (12), it yields that

$$\Delta V(\overline{x}) = \overline{x}^{\mathrm{T}}(t)\underbrace{\left(A_R^{\mathrm{T}} P A_R - P\right)}_{\Omega}\overline{x}(t) \leq 0. \tag{26}$$

Note that $\Omega < 0$ implies $\Delta V(\overline{x}) < 0$.

Multiplying both sides of $\Omega < 0$ by P^{-1}, gives

$$P^{-1}A_R^{\mathrm{T}} P A_R P^{-1} - P^{-1} \leq 0. \tag{27}$$

By applying the Schur complement to (27) and substituting $X = P^{-1}$, one can have

$$\begin{bmatrix} -X & XA_R^{\mathrm{T}} \\ * & -X \end{bmatrix} \leq 0. \tag{28}$$

The matrix XA_R^{T} is computed as

$$XA_R^{\mathrm{T}} = \begin{bmatrix} X_{11} & X_{11}\Lambda^{\mathrm{T}} + X_{12}\Xi^{\mathrm{T}} \\ X_{12}^{\mathrm{T}} & X_{12}^{\mathrm{T}}\Lambda^{\mathrm{T}} + X_{22}\Xi^{\mathrm{T}} \end{bmatrix}. \tag{29}$$

Now, to avoid any nonlinearities, the change-of-variable technique is used. Therefore, we define the following new variables:

$$\begin{cases} \mathcal{L}_1 = X_{11}\Lambda^{\mathrm{T}} + X_{12}\Xi^{\mathrm{T}}, & (30) \\ \mathcal{L}_2 = X_{12}^{\mathrm{T}}\Lambda^{\mathrm{T}} + X_{22}\Xi^{\mathrm{T}}. & (31) \end{cases}$$

By substituting (29)–(31) into (28), one can get LMI (14).

After solving LMIs (13) and (14), the reset map matrices Λ and Ξ are found by solving (30) and (31).

Equations (30) and (31) can rewritten as follows:

$$\begin{bmatrix} \mathcal{L}_1 \\ \mathcal{L}_2 \end{bmatrix} = X \begin{bmatrix} \Lambda^{\mathrm{T}} \\ \Xi^{\mathrm{T}} \end{bmatrix}. \tag{32}$$

By simple calculation, we can get (15).

When the reset condition is met, the after-reset values are obtained from (16). ■

Remark 6. Since $X > 0$, therefore it is nonsingular. This ensures that Equation (32) has a unique non-trivial solution.

Remark 7. To implement the proposed control scheme, the base controller needs to have internal dynamics such that the base system is stable. That is:

$$\begin{cases} \dot{x}_c = A_c x_c + B_c e, \\ u = C_c x_c + D_c e. \end{cases} \tag{33}$$

In the next theorem, we design the optimal reset law (i.e., the reset map matrices Λ and Ξ) to minimize a cost function by using the GCC. This optimization problem is expressed by a set of LMIs.

Theorem 2. *Consider the reset control system (5). If there exist symmetric matrices $X_{11} \in \mathbb{R}^{n_p \times n_p}$, $X_{22} \in \mathbb{R}^{n_c \times n_c}$, matrices $X_{12} \in \mathbb{R}^{n_p \times n_c}$, $\mathcal{L}_1 \in \mathbb{R}^{n_p \times n_c}$, $\mathcal{L}_2 \in \mathbb{R}^{n_c \times n_c}$, and positive scalars μ and δ such that the following LMIs hold:*

$$\min \ \delta$$

subject to

$$\begin{bmatrix} \Phi_{11} & \Phi_{12} & \mu H & X_{11} L_p^{\mathrm{T}} & X_{11} & X_{12} \\ * & \Phi_{22} & 0 & X_{12}^{\mathrm{T}} L_p^{\mathrm{T}} & X_{12}^{\mathrm{T}} & X_{22} \\ * & * & -\mu I & 0 & 0 & 0 \\ * & * & * & -\mu I & 0 & 0 \\ * & * & * & * & -\hat{Q}_{11} & -\hat{Q}_{12} \\ * & * & * & * & * & -\hat{Q}_{22} \end{bmatrix} < 0, \tag{34}$$

$$\begin{bmatrix} -X_{11} & -X_{12} & X_{11} & \mathcal{L}_1 \\ * & -X_{22} & X_{12}^{\mathrm{T}} & \mathcal{L}_2 \\ * & * & -X_{11} & -X_{12} \\ * & * & * & -X_{22} \end{bmatrix} \leq 0, \tag{35}$$

$$\begin{bmatrix} -\delta & x_p^{\mathrm{T}}(0) & x_c^{\mathrm{T}}(0) \\ * & -X_{11} & -X_{12} \\ * & * & -X_{22} \end{bmatrix} \leq 0. \tag{36}$$

Then, the reset map matrices and reset law are obtained from (15) and (16), respectively.

Proof. The reset law is designed by minimizing the following cost function:

$$\mathcal{J} = \int_0^{\infty} \bar{x}^{\mathrm{T}}(t)\, Q \bar{x}(t)\, \mathrm{d}t, \tag{37}$$

where the weight matrix $Q \in \mathbb{R}^{(n_p+n_c)\times(n_p+n_c)}$ is a positive definite symmetric matrix. Q^{-1} can be partitioned as

$$Q^{-1} = \begin{bmatrix} \hat{Q}_{11} & \hat{Q}_{12} \\ * & \hat{Q}_{22} \end{bmatrix}. \tag{38}$$

Consider the following upper bound to ensure the negativity of the derivative of the Lyapunov function:

$$\dot{V}(\bar{x}) < -\bar{x}^{\mathrm{T}}(t) Q \bar{x}(t). \tag{39}$$

From (18) and (39), we can get

$$\bar{x}^{\mathrm{T}}(t)(P\bar{A} + \bar{A}^{\mathrm{T}}P + Q)\bar{x}(t) + \bar{x}^{\mathrm{T}}(t)(P\overline{H})f(x_p) + f^{\mathrm{T}}(x_p)(\overline{H}^{\mathrm{T}}P)\bar{x}(t) < 0. \tag{40}$$

By adding the inequality (20) to (40), we have

$$\dot{V}(\bar{x}) < \bar{x}^{\mathrm{T}}(t)(P\bar{A} + \bar{A}^{\mathrm{T}}P + Q)\bar{x}(t) + \bar{x}^{\mathrm{T}}(t)(P\overline{H})f(x_p) + f^{\mathrm{T}}(x_p)(\overline{H}^{\mathrm{T}}P)\bar{x}(t) + \mu^{-1}\bar{x}^{\mathrm{T}}(t)\overline{L}_p^{\mathrm{T}}\overline{L}_p\bar{x}(t) - \mu^{-1}f^{\mathrm{T}}(x_p)f(x_p). \tag{41}$$

The inequality (41) can be rewritten as

$$\dot{V}(\bar{x}) < \nu^{\mathrm{T}}(t)\Pi\nu(t), \tag{42}$$

where

$$\Pi = \begin{bmatrix} P\bar{A} + \bar{A}^{\mathrm{T}}P + \mu^{-1}\overline{L}_p^{\mathrm{T}}\overline{L}_p + Q & P\overline{H} \\ * & -\mu^{-1}I \end{bmatrix}. \tag{43}$$

We deduce that $\dot{V}(\bar{x}) < 0$ holds if $\Pi < 0$ is fulfilled.

Applying the Schur complement to $\Pi < 0$ yields

$$\begin{bmatrix} P\bar{A} + \bar{A}^{\mathrm{T}}P & P\overline{H} & \overline{L}_p^{\mathrm{T}} & I \\ * & -\mu^{-1}I & 0 & 0 \\ * & * & -\mu I & 0 \\ * & * & * & -Q^{-1} \end{bmatrix} < 0. \tag{44}$$

By performing a congruence transformation with $\mathrm{diag}(P^{-1}, \mu I, I, I)$ on the inequality (44), where $X = P^{-1}$, one can get LMI (34).

Integrating (39) from 0 to ∞ can result in

$$V\left(\bar{x}(\infty)\right) - V\left(\bar{x}(0)\right) < -\int_0^{\infty} \bar{x}^{\mathrm{T}}(t)Q\bar{x}(t)\mathrm{d}t = -\mathcal{J}. \tag{45}$$

The asymptotic stability of the closed-loop system implies that $V(\overline{X}(\infty)) = 0$, therefore

$$\mathcal{J} < \bar{x}^{\mathrm{T}}(0)P\bar{x}(0) \leq \delta. \tag{46}$$

By minimization of δ, the upper bound of the quadratic performance index $\mathcal{J}$ is found. By applying the Schur complement to (46), we obtain LMI (36), which completes the proof. ■

Remark 8. The weight matrix can be tuned based on the designer's experience. For the sake of simplicity, Q can be selected in the diagonal form (i.e., when $\hat{Q}_{12} = 0$).

Remark 9. The quadratic performance index $\mathcal{J}$ in (37) can be substituted by a performance index in terms of error $e(t)$ or control input $u(t)$, which does not change the framework of the reset law design of this study.

Remark 10. The overall design procedures of the proposed reset controller and the reset controller introduced in Reference 21 are demonstrated in Figure 1A,B, respectively. The red dashed line and the blue dashed line show the online and offline calculations, respectively. As shown in Figure 1, unlike conventional methods like in Reference 21 which need to solve optimization problem at every reset time, the LMI optimization method is solved offline and only once in our proposed method.

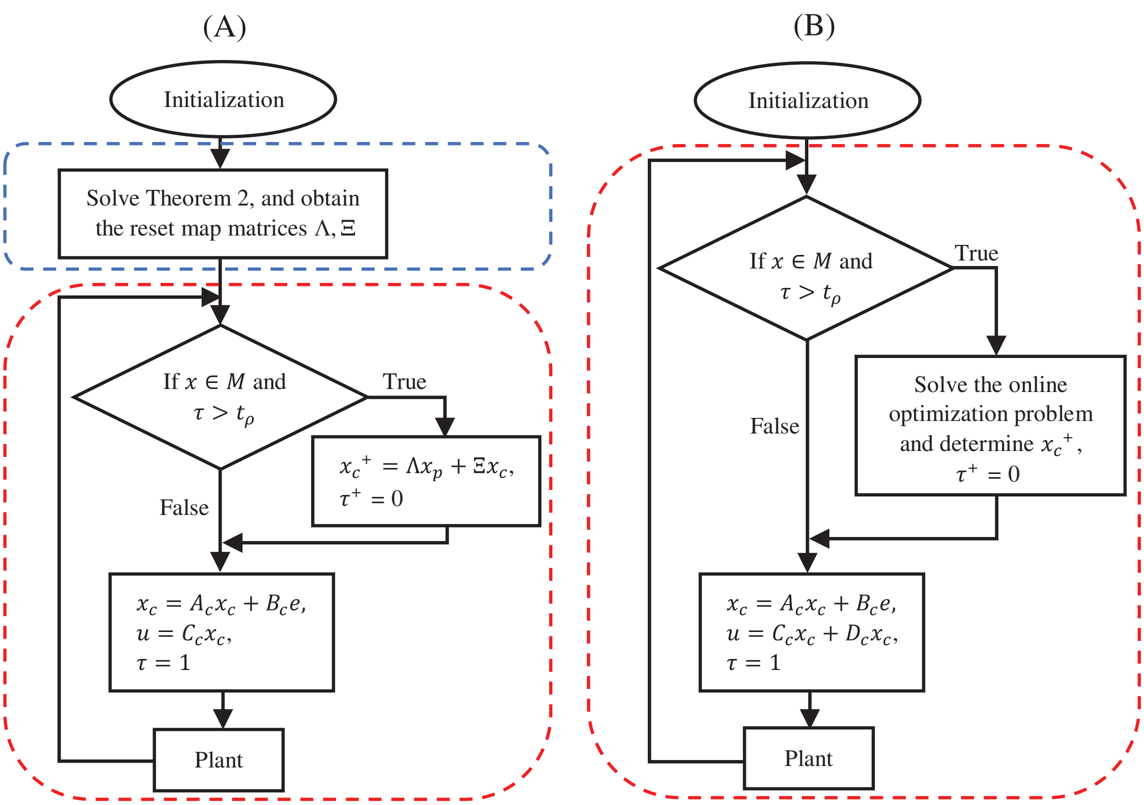

FIGURE 1 The flowchart of the design procedure: (A) Our proposed method (B) the method of Reference 21

4 | SIMULATION RESULTS

In this section, two examples are given to demonstrate the validity of the proposed reset law design. Also, the results are compared with the method introduced in Reference 21.

Example 1. Consider the following nonlinear system:[21]

$$\dot{x}_p(t) = \begin{bmatrix} -8 & 1 \\ 0 & 0 \end{bmatrix} x_p(t) + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u(t) + \begin{bmatrix} 0.5 \\ -1.5 \end{bmatrix} \frac{x_{p_2}(t)}{1 + x_{p_1}^2(t)},$$
$$y(t) = \begin{bmatrix} 64 & 0 \end{bmatrix} x_p(t).$$

A first-order base controller is selected as[21]

$$A_c = -2, \quad B_c = 1, \quad C_c = 1.$$

The weight matrix Q is chosen as diag(100,100, 50).

By solving the optimization problem of Theorem 2, the reset map matrices are obtained as

$$\Lambda = \begin{bmatrix} 4.2935 & -1.4893 \end{bmatrix}, \quad \Xi = 0.0337.$$

Let the initial conditions be $x_p(0) = [0.5 \ -0.5]^T$ and $x_c(0) = -1$. The temporal regularization parameter t_ρ is set to 0.1 s.

The system responses and control effort for Example 1 are presented in Figure 2. The reset action occurs when $y(t) = 0$ (i.e., $x_{p_1}(t) = 0$). As can be seen from the red dash-dotted line in Figure 2, the controller state is reset at times 0.108 and 0.796 s. It is evident from this figure, the transient performance is improved compared with the base system. In Table 2,

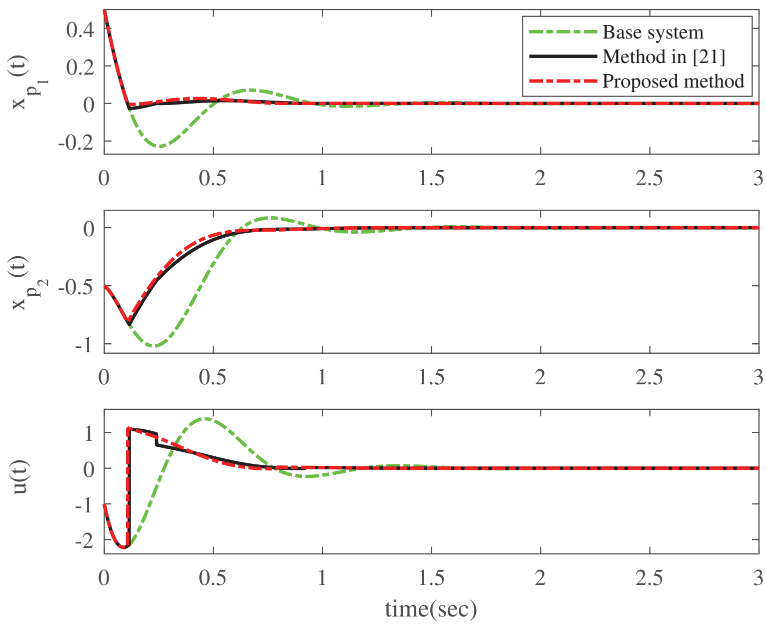

FIGURE 2 The simulation results of Example 1

TABLE 2 Comparison of the cost function $\mathcal{J}$ in Example 1

	Base system	Method in Reference 21	Proposed method
$\mathcal{J}$	96.8332	43.2081	42.3957

TABLE 3 Elapsed time to solve the online optimization problem of Reference 21 in Example 1

	First reset	Second reset	Third reset
Elapsed time	0.6069 s	0.1376 s	0.1164 s

the values of the cost function $\mathcal{J}$ are compared with the method in Reference 21. This table shows that the performance of the system is slightly improved by using the proposed method.

In Reference 21, the reset law is determined by solving an online optimization problem at reset times, which leads to large computational time. As a result, it may not be useful for systems with small sampling time. As shown in Figure 2, three reset actions occur using method in Reference 21. The computation times to solve the online optimization problem of Reference 21 are listed in Table 3. The sampling time was selected as 0.005 s in Reference 21. According to this table, the computation times are longer than the sampling time, so it cannot be implemented in practice. But in the proposed method, the computational complexity does not affect the real-time control efficiency, because the LMI optimization problem is solved offline. The simulations have been performed on a computer with Intel Core i7 processor running at 2.2 GHz with 8GB RAM. Also, the optimization problems are solved by using YALMIP interface[31] and Mosek solver.[32]

Example 2. Consider a well-mixed continuous stirred tank reactor in which the following isothermal, liquid-phase, multi-component chemical reaction $A \leftrightarrows B \rightarrow C$ is being carried out.[21,33] The system dynamics are described by

$$\dot{x}_p(t) = \begin{bmatrix} -4 & 0.8796 & 0 \\ 3 & -3.6388 & 0 \\ 0 & 1.7592 & -1 \end{bmatrix} x_p(t) + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 0.5 \\ -1.5 \\ 1 \end{bmatrix} x_{p_2}^2(t),$$

$$y(t) = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} x_p(t).$$

The first-order base linear controller is selected as follows:

$$A_c = -1, \quad B_c = 6, \quad C_c = 1.$$

The weight matrix Q is chosen as diag(0.01, 0.01,100, 0.005), and the time regularization parameter t_ρ is 0.1 s.

After solving the optimization problem of Theorem 2, the reset map matrices are obtained as follows:

$$\Lambda = \begin{bmatrix} -1.0481 & -0.8554 & 1.0318 \end{bmatrix}, \quad \Xi = 0.0042.$$

The initial conditions are set to $x_p(0) = [1\,1\,1]^{\mathrm{T}}$ and $x_c(0) = 1$. The simulation results are depicted in Figure 3. As can be seen from the red dash-dotted line in Figure 3, the reset actions occur at 1.538, 2.658, 3.728, and 4.8 s. It is obvious that the control signal is suddenly changed at these times. From this figure, we see that the system responses are similar to the results presented in Reference 21. According to Table 4, the proposed method has more satisfactory performance than the method in Reference 21.

Similar to the previous example, the computational times to solve the online optimization problem of Reference 21 are given in Table 5. As can be seen, the computational times are much larger than the sampling time of 0.005 s. Therefore, the method of Reference 21 cannot be implemented in practice for systems with small sampling time, while the proposed method does not have this problem because the reset law is designed offline.

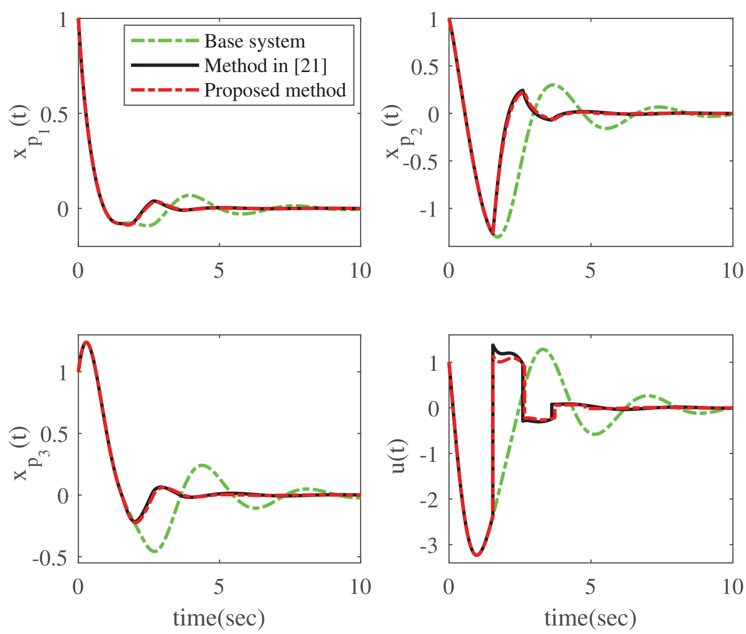

FIGURE 3 The simulation results of Example 2

TABLE 4 Comparison of the cost function $\mathcal{J}$ in Example 2

	Base system	Method in Reference 21	Proposed method
$\mathcal{J}$	137.4570	113.2658	112.7995

TABLE 5 Elapsed time to solve the online optimization problem of Reference 21 in Example 2

	First reset	Second reset	Third reset
Elapsed time	0.6338 s	0.1421 s	0.1209 s

5 | CONCLUSION

In recent years, the method of reset law design based on MPS has been widely reported in the literature.[21-29] In order to design the reset law, this method needs to solve an optimization problem at reset times, which leads to large computational time and often cannot be solved within a time sample. In this study, a systematic approach is proposed to design a reset law for Lipschitz nonlinear systems. By utilizing LMI tools, the problem of optimal reset law design is successfully transformed into a set of LMIs. By solving the LMI optimization problem, the reset law based on GCC is designed. The proposed method minimizes the upper bound of the predefined quadratic performance index. Although the performance level of our method is better than the results in Reference 21, the main advantage of the proposed method is that the reset law is designed offline. Hence, it can be useful for real-time applications with small sampling time.

CONFLICT OF INTEREST

The authors declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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