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Due to Changes in the Bottom Profile

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ON THE GENERATION OF LONG-PERIOD SECOND-ORDER FREE-WAVES DUE TO CHANGES IN THE BOTTOM PROFILE*

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ABSTRACT

At first-order of approximation a sea-state consists of sinusoidal components with periods ranging from a few seconds to about 20 seconds, but a second-order analysis shows the co-existence of long-period components (appearing at the difference frequencies), which may be associated with such phenomena as surf-beats or harbor resonance.

Considering the simple bidimensional case of two regular waves superposed with close frequencies, it is questioned how the accompanying long wave is modified when some irregularity of the bottom profile occurs. Assuming the waterdepth to be constant upstream and downstream the bottom irregularity, the first-order components to behave as deepwater waves, and the second-order long wave to obey shallow-water theory, the problem is solved analytically or numerically in a number of cases. The decomposition of the second-order wave into a locked wave (accompanying the first-order waves and propagating at the group velocity) and free waves is clearly made, and it is shown that transfers of energy may occur between the first-order waves and the second-order free waves.

Last the case of shallower waterdepth (when first-order waves cannot be considered deep-water waves any more) is considered, and some approximate solutions are given.

INTRODUCTION

Phenomena such as slow-drift motion, surf-beats, or harbor resonance, indicate the presence of low-frequency components within a wave system. Correlations between offshore sea-states and seiches in sheltered bays or harbors [1] suggest that such low-frequency waves can escape the wave-system and propagate independently.

It is conjectured here that this low-frequency phenomenon consists of second-order waves (in the wave-amplitude) appearing at the difference-frequencies of the individual components of the wave-spectrum. These second-order waves consist of "locked-waves" (or "bound-waves") accompanying the first-order waves and propagating at the group velocities, and of "free-waves" traveling independently.

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It is with the emission of these second-order free-waves that we are dealing here, when changes in the bottom profile occur. We neglect other phenomena such as higher-order interaction or bottom friction. As a further simplification we restrict ourselves to the case of two-dimensional waves propagating over a cylindrical bottom. Also the derivations presented here are obtained for the case of two regular waves superposed. However they may easily be extended to the case of irregular waves defined by a given spectrum [3].

In a first paragraph we derive the expression of the second-order locked-waves. It appears that we can distinguish different wave-length/waterdepth regimes for the first-order and second-order waves. In particular if the difference-frequency is small enough, there exists a range of waterdepths for which the first-order waves be considered deep-water waves and the second-order ones shallow-water waves. As this assumption considerably simplifies the problem, it will be used throughout most of this paper. Eventhough the consequent results may be restricted in so far as the values of wave-frequencies and waterdepths, they do apply to cases of practical interest. Moreover we may expect some of the results to remain qualitatively valid in other configurations.

The second paragraph is devoted to energy flux considerations. It appears that transfers of energy between first-order waves (through a third-order decrease or increase of their amplitudes) and second-order waves are possible.

In the third paragraph governing equations for the second-order free-waves are established when the bottom profile presents some irregularities. Two cases are considered: undulating bottom for which a resonant effect may occur, and sloping bottom. Some numerical results are presented.

I. SECOND-ORDER LOCKED-WAVES

We make the usual assumptions of perfect fluid and irrotational motion. The flow is described by a velocity potential $\Phi(x, y, t)$ expressed as a power series of a perturbation parameter ε identified with the wave-steepness:

$$\Phi(xyt)=\varepsilon\Phi^{(1)}(xyt)+\varepsilon^2\Phi^{(2)}(xyt)+\cdots \tag{1}$$

The governing equations for Φ are:

-in the fluid: $\qquad \Delta\Phi=0 \qquad -h(x)\leqslant y\leqslant\eta(x, t) \qquad (2)$

-at the bottom: $\qquad \Phi_n=0 \qquad y=-h(x) \qquad (3)$

-at the free-surface $\eta(x, t)$:

$$0=-\rho g\eta-\rho\Phi_t-\rho\frac{\nabla\Phi^2}{2} \tag{4}$$

$$0=\Phi_y-\eta_t-\Phi_x\eta_x \tag{5}$$

Eliminating η one obtains

$$\Phi_{tt}+g\Phi_y+\frac{\partial}{\partial t}(\nabla\Phi)^2+\Phi_x^2\Phi_{xx}+2\Phi_x\Phi_y\Phi_{xy}+\Phi_y^2\Phi_{yy}=0 \tag{6}$$

at $\quad y=\eta(x, t)$

Assuming Φ equal to its Taylor development between $y=0$ and $y=\eta(x, t)$ the free-surface condition may be expressed at $y=0$ to yield:

first-order $\qquad \Phi_{tt}^{(1)}+g\Phi_y^{(1)}=0 \qquad (7)$

second-order $\qquad \Phi_{tt}^{(2)}+g\Phi_y^{(2)}+\eta^{(1)}\dfrac{\partial}{\partial y}(\Phi_{tt}^{(1)}+g\Phi_y^{(1)})+\dfrac{\partial}{\partial t}(\nabla\Phi^{(1)})^2=0 \qquad (8)$

etc.

Obviously other boundary conditions are needed in order to fully determine Φ. We shall consider those later.

Let us write now the first-order potential corresponding to the superposition of two regular waves:

$$\begin{aligned}\varepsilon\Phi^{(1)}=&\frac{a_1 g}{\omega_1}\frac{\cosh k_1(y+h)}{\cosh k_1 h}\sin(k_1x-\omega_1t+\theta_1)\\&+\frac{a_2 g}{w_2}\frac{\cosh k_2(y+h)}{\cosh k_2 h}\sin(k_2x-\omega_2t+\theta_2)\\&\omega_1^2=gk_1\tanh k_1h \qquad \omega_2^2=gk_2\tanh k_2h\end{aligned} \tag{9}$$

where the waterdepth h is assumed to be constant.

Then at first-order the free-surface elevation writes:

$$\varepsilon\eta^{(1)}=a_1\cos(k_1x-\omega_1t+\theta_1)+a_2\cos(k_2x-\omega_2t+\theta_2) \tag{10}$$

From (8) one may see that the second-order potential appears at angular frequencies $2\omega_1$, $2\omega_2$, $\omega_1+\omega_2$ and $\omega_1-\omega_2$. In particular the free-surface equation for the second-order potential at angular frequency $\omega_1-\omega_2$ writes:

$$\Phi_{tt}^{(2)}+g\Phi_y^{(2)}=(P^{(2)}+Q^{(2)})\sin[(k_1-k_2)x+(\omega_1-\omega_2)t+\theta_1-\theta_2] \tag{11}$$

where

$$\varepsilon^2P^{(2)}=-\frac{1}{2}a_1a_2g^2\left[\frac{k_1^2}{\omega_1\cosh^2k_1h}-\frac{k_2^2}{\omega_1\cosh^2k_2h}\right] \tag{12}$$

$$\varepsilon^2Q^{(2)}=-a_1a_2g^2\frac{k_1k_2}{\omega_1\omega_1}(\omega_1-\omega_2)(1+\tanh k_1h\tanh k_2h) \tag{13}$$

so that a solution to (2), (3), (8) is:

$$\Phi^{(2)}=\frac{P^{(2)}+Q^{(2)}}{-(\omega_1-\omega_2)^2+g(k_1-k_2)\tanh(k_1-k_2)h}\frac{\cosh(k_1-k_2)(y+h)}{\cosh(k_1-k_2)h} \tag{14}$$

$$\times\sin[(k_1-k_2)x-(\omega_1-\omega_2)t+\theta_1-\theta_2]$$

From now on we assume $\omega_1>\omega_2$ and we write:

$$\Delta\omega=\omega_1-\omega_2 \qquad \Delta\omega>0$$

$$\Delta k=k_1-k_2 \qquad \Delta k>0$$

From (14) it appears that if $\Delta\omega\ll\omega_1$ the second-order potential decreases much more slowly than the first-order one with the depth. Thus as the wave-system moves from deep water toward shore we can distinguish different configurations:

1. The waterdepth is deep both for the first-order and second-order waves. Then $\Phi^{(2)}$ simplifies into:

$$\varepsilon^2\Phi^{(2)}=-a_1a_2\omega_1e^{\Delta ky}\sin(\Delta kx-\Delta\omega t+\Delta\theta) \tag{15}$$

2. The waterdepth is deep for the first-order waves but intermediate for the second-order ones:

$$\varepsilon^2\Phi^{(2)}=\frac{-2a_1a_2\omega_1\omega_2\Delta\omega}{-\Delta\omega^2+gk\tanh\Delta kh}\frac{\cosh\Delta k(y+h)}{\cosh\Delta kh}\sin(\Delta kx-\Delta\omega t+\Delta\theta) \tag{16}$$

3. If $\Delta k/k_1<0.1$ the waterdepth will become shallow for $\Phi^{(2)}$ while it may still be considered deep for $\Phi^{(1)}$. This is the case for instance of two waves with periods 7.7 and 8 seconds traveling in 50 meters waterdepth. The period of the associated beat is 200 seconds.

Eventhough this case is limited to very small values of $\Delta k/k_1$ and to a narrow range of waterdepth it provides an easily handalable frame when one considers the modification of the wave-system over a bottom irregularity: first-order waves remain unperturbed while second-order low-frequency ones are governed by shallow-water equations. This is the case that we consider in paragraph III.

II. ENERGY FLUX CONSIDERATIONS

Be $F(x)$ the time-average of energy-flux at abscissa x. It writes:

$$F(x)=\overline{\int_{-h}^{\eta(x,t)}-\rho\Phi_t\Phi_x dy} \tag{17}$$

where the bar denotes the time-average.

$F(x)$ may be developped as a power series in ε:

$$F(x)=\varepsilon^2F^{(2)}(x)+\varepsilon^3F^{(3)}(x)+\varepsilon^4F^{(4)}(x)+ \tag{18}$$

For two regular waves traveling together in deep water $F^{(2)}$ writes:

$$F^{(2)}(x)=\overline{\int_{-\infty}^{0}-\rho\Phi_t^{(1)}\Phi_x^{(1)}dy}$$

$$\varepsilon^2 F^{(2)}(x)=\frac{1}{4}\rho g^2\left(\frac{a_1^2}{\omega_1}+\frac{a_2^2}{\omega_2}\right) \tag{19}$$

As a result of the time-averaging it is easy to show that if $\omega_1 \neq 2\omega_2$ (we still assume $\omega_1>\omega_2$) $F^{(3)}$ is zero.

Our intention here is to establish a relationship between $F(x)$ and the waterdepth. That means that we have to carry the derivation up to fourth-order in ε, and thus develop the velocity potential up to third-order. We assume the waterdepth to be deep for the first-order waves and intermediate for the second-order ones.

As a matter of fact $F^{(4)}(x)$ writes:

$$\begin{aligned}F^{(4)}(x)=&\overline{\int_{-h}^{0}-\rho(\Phi_t^{(1)}\Phi_x^{(3)}+\Phi_t^{(2)}\Phi_x^{(2)}+\Phi_t^{(3)}\Phi_x^{(1)})dy}\\&\overline{-\rho\eta^{(1)}(\Phi_t^{(1)}\Phi_x^{(2)}+\Phi_t^{(2)}\Phi_x^{(1)})|_{y=0}}-\overline{\frac{1}{2}\rho\eta^{(1)2}(\Phi_t^{(1)}\Phi_{xy}^{(1)}+\Phi_{ty}^{(1)}\Phi_x^{(1)}|_{y=0}}\\&\overline{-\rho\eta^{(2)}(\Phi_t^{(1)}\Phi_x^{(1)})|_{y=0}}+\left[\overline{\int_{-h}^{0}-\rho\Phi_t^{(1)}\Phi_x^{(1)}dy}\right]^{(4)}\end{aligned} \tag{20}$$

where

$$\eta^{(2)}=-\frac{1}{g}\left(\eta^{(1)}\Phi_{ty}^{(1)}+\frac{\nabla\Phi^{(1)2}}{2}+\Phi_t^{(2)}\right)\bigg|_{y=0} \tag{21}$$

Considerations on the Third-Order Potential

Due to the time-averaging we need only take account of the components appearing at pulsations ω_1 and ω_2.

It is wellknown that the third-order approximation of the velocity potential appears as a correction to the wave-number so that:

$$\begin{aligned}\Phi=&\frac{a_1 g}{\omega_1}e^{(k_1+\varepsilon^2k_1^{(2)})y}\sin\left[(k_1+\varepsilon^2k_1^{(2)})x-\omega_1t+\theta_1\right]\\&+\frac{a_2 g}{\omega_2}e^{(k_2+\varepsilon^2k_2^{(2)})y}\sin\left[(k_2+\varepsilon^2k_2^{(2)})x-\omega_2t+\theta_2\right]\\&-\frac{2a_1a_2\omega_1\omega_2\Delta\omega}{-\Delta\omega^2+g\Delta k\tanh\Delta kh}\,\frac{\cosh\Delta k(y+h)}{\cosh\Delta kh}\sin(\Delta kx-\Delta\omega t+\Delta\theta)+O(\varepsilon^3)\end{aligned} \tag{22}$$

where the other third-order terms appear at pulsations different from ω_1 or ω_2. (Under our assumption of deepwater approximation for $\Phi^{(1)}$, $\Phi^{(2)}$ appears only at the difference-frequency $\Delta\omega$.)

The complete derivation of $k_1^{(1)}$ and $k_2^{(2)}$ is a tedious task the result of which may be found in [2]. However we need not explicit them as:

$$\overline{\int_{-h}^{0} -\rho\Phi_t^{(1)}\Phi_x^{(3)}dy} = \frac{1}{4}\rho g^2\left[\frac{a_1^2k_1^{(2)}}{\omega_1 k_1} + \frac{a_2^2k_2^{(2)}}{\omega_2 k_2}\right] \tag{23}$$

and

$$\overline{\int_{-h}^{0} -\rho\Phi_t^{(1)}\Phi_x^{(1)}dy} = \frac{1}{4}\rho g\left[\frac{a_1^2\omega_1}{k_1+\varepsilon^2k_1^{(2)}} + \frac{a_2^2\omega_2}{k_2+\varepsilon^2k_2^{(2)}}\right] \tag{24}$$

so that:

$$\overline{\int_{-h}^{0} -\rho\Phi_t^{(1)}\Phi_x^{(1)}dy^{(4)}} = \frac{1}{4}\rho g\left[\frac{a_1^2\omega_1k_1^{(2)}}{k_1^2} + \frac{a_2^2\omega_2k_2^{(2)}}{k_2^2}\right] \tag{25}$$

and expressions (23) and (25) cancel each other. Since $\Phi_t^{(3)}$ equals zero it appears that $F^{(4)}(x)$ does not depend on the third-order potential (under our assumptions of constant waterdepth and deep-water approximation of the first-order waves).

As we are interested in the relationship between $F^{(4)}$ and the waterdepth we carry out the computation of the energy flux only for those terms where $\Phi^{(2)}$ appears, which yields:

$$F^{(4)}(x) = F_h^{(4)}(x) + F_2^{(4)}(x) \qquad \frac{\partial F_2^{(4)}}{\partial h} = 0 \tag{26}$$

$$\begin{aligned}\varepsilon^4 F_h^{(4)}(x) = {}& \frac{1}{4}\rho B^{(2)}(h)^2 \Delta\omega\left[\frac{\Delta kh}{\cosh^2\Delta kh} + \tanh \Delta kh\right]\\ &+ \rho a_1 a_2 B^{(2)}(h)\Delta\omega(\omega_1+\omega_2)\end{aligned} \tag{27}$$

where

$$B^{(2)}(h) = -\frac{2a_1a_2\omega_1\omega_2\Delta\omega}{-\Delta\omega^2 + g\Delta k \tanh \Delta kh} \tag{28}$$

What is going to happen now if the waves encounter a bottom irregularity through which the waterdepth changes from h_L to h_R? The change in value of $F_h^{(4)}$ is an indication that something has happened, that is the second-order waves have diffracted.

Far away from the disturbance we may assume that the emitted waves consist of free-waves traveling upstream and downstream with wave-numbers $k_{\Delta L}$ and $k_{\Delta R}$ given by:

$$\begin{aligned}\Delta\omega^2 &= gk_{\Delta L}\tanh k_{\Delta L}h_L\\ \Delta\omega^2 &= gk_{\Delta R}\tanh k_{\Delta R}h_R\end{aligned} \tag{29}$$

The corresponding velocity potentials write:

$$\Phi_{FL}^{(2)} = A_{FL}^{(2)} \frac{\cosh k_{\Delta L}(y+h_L)}{\cosh k_{\Delta L} h_L} \sin(k_{\Delta L} x + \Delta\omega t + \varphi_{\Delta L})$$
$$\Phi_{FR}^{(2)} = A_{FR}^{(2)} \frac{\cosh k_{\Delta R}(y+h_R)}{\cosh k_{\Delta R} h_R} \sin(k_{\Delta R} x - \Delta\omega t + \varphi_{\Delta R}) \quad (30)$$

To carry out the expression of the fourth-order energy flux associated to $\Phi+\Phi_{FL}^{(2)}$ or $\Phi+\Phi_{FR}^{(2)}$ one realizes that, due to the occurrence in $\Phi_{FL}^{(2)}$, $\Phi_{FR}^{(2)}$ and $\Phi^{(2)}$ of same pulsations but different wave-numbers, one has to consider the x-average of $F^{(4)}$ as well, in which case it writes:

upstream:

$$F_L^{(4)} = F_2^{(4)} + F_{hL}^{(4)} - \frac{1}{4}\rho A_{FL}^{(2)2} \Delta\omega \left[\frac{k_{\Delta L} h_L}{\cosh^2 k_{\Delta L} h_L} + \tanh k_{\Delta L} h_L \right] \quad (31)$$

downstream:

$$F_R^{(4)} = F_2^{(4)} + F_{hR}^{(4)} + \frac{1}{4}\rho A_{FR}^{(2)2} \Delta\omega \left[\frac{k_{\Delta R} h_R}{\cosh^2 k_{\Delta R} h_R} + \tanh k_{\Delta R} h_R \right] \quad (32)$$

so that we may expect the extra-terms to compensate for the difference between $F_{hL}^{(4)}$ and $F_{hR}^{(4)}$.

If $h_R < h_L$ that seems a likely possibility since $F_h^{(4)}$ is a decreasing function of h. However an inconsistency appears if $h_R > h_L$.

This inconsistency stems from the fact that we have omitted the third-order waves which occur from interaction between the first-order waves and the second-order perturbations of the free-surface kinematics in the vicinity of the bottom irregularity. Third-order free-waves at pulsations ω_1 and ω_2 are emitted which result into an increase or a decrease of the first-order wave amplitudes. Only through this process can we equal the energy fluxes upstream and downstream the bottom irregularity. (This should also remove the inconsistency of having expressed $F^{(4)}$ as an x-average.

As a consequence we may conceive that eventhough $h_R = h_L$, variations in the bottom profile may cause emission of second-order free-waves, the energy which they carry away being compensated by a third-order decrease of the wave-amplitudes.

III. APPLICATION OF SHALLOW-WATER THEORY

In this paragraph we make the assumption of deepwater waves for the first-order and shallow-water waves for the second-order, that is:

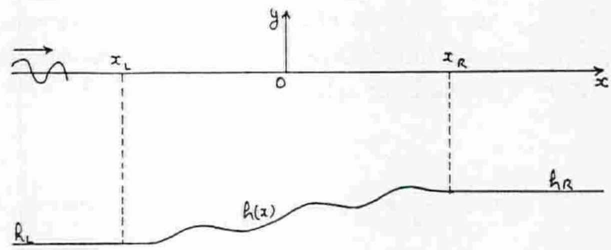
$$\text{both} \quad k_1 h \gg 1 \quad \text{and} \quad \Delta k h \ll 1$$

Practically it is sufficient that $k_1 h \geqslant 3$ and $\Delta k h \leqslant 0.3$.

It is easy to draw the consequence that the emitted free-waves are shallow-water waves as well:

$$(k_\Delta h)^2 = \frac{\Delta\omega^2 h}{g} = \frac{(\omega_1^2-\omega_2^2)h^2 g^{-2}}{(\omega_1+\omega_2)^2 h g^{-1}} \simeq \frac{(\Delta kh)^2}{4k_1 h} \tag{33}$$

so that $\quad k_\Delta < \Delta k$



The waterdepth is assumed to be constant for $x<x_L$ or $x>x_R$ with corresponding values h_L and h_R.

As a consequence of our assumptions the first-order waves are unaffected by the change in waterdepth, so that we need only consider the diffraction problem for the second-order waves.

Since locked-waves and free-waves appear at the same pulsation $\Delta\omega$ we make use of complex notation (from now on we drop the ε^2)

$$\Phi^{(2)} = \mathrm{Re}\,\{\varphi(x,y)e^{-i\Delta\omega t}\} \tag{34}$$

$$\varphi = \varphi_L + \varphi_F \tag{35}$$

The problem in φ writes:

$$\begin{cases} \varphi_{xx}+\varphi_{yy}=0 & -h(x)\leqslant y\leqslant 0 \\ -\Delta\omega^2\varphi + g\varphi_y = i2a_1a_2\omega_1\omega_2\Delta\omega e^{i\Delta kx} & y=0 \\ h_x\varphi_x+\varphi_y=0 & y=-h(x) \\ \varphi=\varphi_L(h_R)+\varphi_{FR} & x>x_R \\ \varphi=\varphi_L(h_L)+\varphi_{FL} & x<x_L \end{cases} \tag{36}$$

where

$$\begin{aligned} \varphi_L(h) &= -iB(h)\frac{\cosh \Delta k(y+h)}{\cosh \Delta kh}e^{i\Delta kx} \\ \varphi_{FR} &= A_{FR}\frac{\cosh k_{\Delta R}(y+h_R)}{\cosh k_{\Delta R}h_R}e^{ik_{\Delta R}x} \qquad A_{FR}\in C \\ \varphi_{FL} &= A_{FL}\frac{\cosh k_{\Delta L}(y+h_L)}{\cosh k_{\Delta L}h_L}e^{ik_{\Delta L}x} \qquad A_{FL}\in C \end{aligned} \tag{37}$$

As a matter of fact the decomposition (35) makes sense only for $x>x_R$ or $x<x_L$. In the interval $[x_L\ x_R]$ we can arbitrarily decompose φ into

two components so that they match φ_F and φ_L at x_L and x_R.

For instance we can take:

$$\varphi_{L1} = -iB(h(x)) \frac{\cosh \Delta k(y+h(x))}{\cosh \Delta k h(x)} e^{i\Delta k x} \tag{38}$$

In this case φ_L satisfies the non-homogeneous free-surface condition but not any more Laplace condition nor of course the bottom condition.

If $h_R = h_L = h_0$ we can take:

$$\varphi_{L2} = -iB(h_0) \frac{\cosh \Delta k(y+h_0)}{\cosh \Delta k h_0} e^{i\Delta k x} \tag{39}$$

Here only the bottom condition remains to be fulfilled.

In the following we shall make use of either one of the potentials φ_{L1} or φ_{L2}. The problem in φ_F writes:

$$\begin{cases} \varphi_{Fxx} + \varphi_{Fyy} = -\varphi_{Lxx} - \varphi_{Lyy} & -h(x) \leqslant y \leqslant 0 \\ -\Delta\omega^2 \varphi_F + g\varphi_{Fy} = 0 & y = 0 \\ h_x \varphi_{Fx} + \varphi_{Fy} = -h_x \varphi_{Lx} - \varphi_{Ly} & y = -h(x) \\ \varphi_{Fx} = ik_{\Delta R} \varphi_F & x > x_R \\ \varphi_{Fx} = -ik_{\Delta L} \varphi_F & x < x_L \end{cases} \tag{40}$$

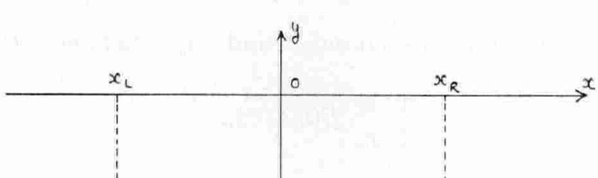
Application of linear shallow-water theory yields the following equation in φ_F:

$$\frac{\Delta\omega^2}{g} \varphi_F(x, 0) + \frac{\partial}{\partial x} [h(x) \varphi_{Fx}(x, 0)]$$
$$= -h_x \varphi_{Lx}(x, -h) - \varphi_{Ly}(x, -h) - \int_{-h}^{0} \Delta \varphi_L dy \tag{41}$$

with

$$\begin{aligned} \varphi_{Fx} &= ik_{\Delta R} \varphi_F & x > x_R \\ \varphi_{Fx} &= -ik_{\Delta L} \varphi_F & x < x_L \end{aligned}$$

III-1. Undulating Bottom



In this section we assume the bottom profile given by:

$$h(x)=h_0-\alpha\sin\lambda x \qquad \lambda>0 \tag{42}$$

$$x_L=-x_R \tag{43}$$

Furthermore we assume $\alpha\ll h_0$ and $\lambda\sim\Delta k$ so that $|h_x|\ll 1$.
Since $h_R=h_L$ we take $\varphi_L=\varphi_{L2}$

$$\varphi_L=-iB(h_0)\frac{\cosh\Delta k(y+h_0)}{\cosh\Delta kh_0}e^{i\Delta kx} \tag{44}$$

$$\begin{aligned}&\varphi_{Lx}(x,-h(x))\simeq B(h_0)\Delta ke^{i\Delta kx}\\&\varphi_{Ly}(x,h(x))\simeq -iB(h_0)\Delta k^2\alpha\sin\lambda xe^{i\Delta kx}\end{aligned} \tag{45}$$

Neglecting the term $h_x\varphi_{Fx}$ in (41) we obtain the approximate equation:

$$\varphi_{Fxx}+k_{\Delta 0}^2\varphi_F=(f\cos\lambda x+ig\sin\lambda x)e^{i\Delta kx} \tag{46}$$

where

$$\begin{aligned}&f=\frac{\alpha}{h_0}B(h_0)\Delta k\lambda\\&g=\frac{\alpha}{h_0}B(h_0)\Delta k^2\end{aligned} \tag{47}$$

The general solution of which writes:

$$\varphi_F=\frac{q_0}{k_{\Delta 0}^2-\mu_1^2}e^{i\mu_1x}+\frac{q_2}{k_{\Delta 0}^2-\mu_2^2}e^{i\mu_2x}+r_+e^{ik_{\Delta 0}x}+r_-e^{-ik_{\Delta 0}x}$$

$$q_1=\frac{1}{2}(f+g) \quad q_2=\frac{1}{2}(f-g) \quad \mu_1=\lambda+\Delta k \quad \mu_2=-\lambda+\Delta k$$

which upon identification with (37) in x_L and x_R yields:

$$\begin{aligned}&A_{FR}=-i\left[\frac{q_1}{k_{\Delta 0}(\mu_1-k_{\Delta 0})}\sin x_R(\mu_1-k_{\Delta 0})+\frac{q_2}{k_{\Delta 0}(\mu_2-k_{\Delta 0})}\sin x_R(\mu_2-k_{\Delta 0})\right]\\&A_{FL}=-i\left[\frac{q_1}{k_{\Delta 0}(\mu_1+k_{\Delta 0})}\sin x_R(\mu_1+k_{\Delta 0})+\frac{q_2}{k_{\Delta 0}(\mu_2+k_{\Delta 0})}\sin x_R(\mu_2+k_{\Delta 0})\right]\end{aligned} \tag{48}$$

For $\lambda=\Delta k\pm k_{\Delta 0}$ we obtain a large amplification due to a resonant effect.

If $\lambda=\Delta k-k_{\Delta 0}$, A_{FL} remains bounded and $|A_{FR}|$ behaves as

$$\frac{1}{2}\frac{\alpha}{h_0}B(h_0)\Delta kx_R \tag{49}$$

so that the amplitude of the downstream free-wave increases linearly with x_R.

On the other hand if $\lambda = \Delta k + k_{\Delta 0}$ one obtains the same resonant effect for the reflected free-wave.

Bottom of General Shape

Let the waterdepth be described as:

$$h(x) = h_0 - \alpha(x) \tag{50}$$

where we still assume

$$h(x) = h_0 \quad \text{for } |x| > x_R \qquad x_L = -x_R$$
$$|\alpha| \ll h_0 \qquad |\alpha_x| \ll 1$$

(50) may be re-written:

$$h - h_0 = -\frac{1}{\pi}\int_0^\infty d\lambda \left\{ \cos \lambda x \int_{-x_R}^{x_R} \alpha(\nu) \cos \lambda\nu d\nu + \sin \lambda x \int_{-x_R}^{x_R} \alpha(\nu) \sin \lambda\nu d\nu \right\} \tag{51}$$

If previously we had considered the bottom profile given by:

$$h(x) = h_0 - \alpha \sin(\lambda x + \delta)$$

we would have obtain for the potential amplitude of the transmitted free-wave:

$$A_{FR} = -i\alpha \left[\frac{q_1/\alpha}{k_{\Delta 0}(\mu_1 - k_{\Delta 0})} \sin x_R(\mu_1 - k_{\Delta 0}) + \frac{q_2/\alpha}{k_{\Delta 0}(\mu_2 - k_{\Delta 0})} \sin x_R(\mu_2 - k_{\Delta 0}) \right] \tag{52}$$

or

$$A_{FR} = \alpha a_{FR}(\lambda, \delta) \tag{53}$$

Taking advantage of linearity we may write the amplitude of the downstream potential as

$$\tilde{A}_{FR} = \frac{1}{\pi}\int_0^\infty d\lambda \left\{ a_{FR}\left(\lambda, \frac{\pi}{2}\right) \int_{-x_R}^{x_R} \alpha(\nu) \cos \lambda\nu d\nu + a_{FR}(\lambda, 0) \int_{-x_R}^{x_R} \alpha(\nu) \sin \lambda\nu d\nu \right\} \tag{54}$$

When x_R increases it is possible to derive the asymptotic behavior of this expression. One finally obtains:

$$\tilde{A}_{FR} \simeq \frac{1}{2} \frac{B(h_0)}{h_0} \Delta k \int_{-x_R}^{x_R} \alpha(\nu) e^{i\nu(\Delta k - k_{\Delta 0})} d\nu \tag{55}$$

as x_R increases.

If we now assume the bottom profile $\alpha(x)$ to be described as a zero mean, stationary signal with correlation length small as compared to x_R, we obtain, squaring the modulus of $\tilde{A}_{FR}$:

$$\tilde{A}_{FR}\tilde{A}_{FR}^* \simeq \frac{1}{4}\frac{B^2(h_0)}{h_0^2}\Delta k^2 \int_{-x_R}^{x_R}\int_{-x_R}^{x_R} \alpha(\mu)\alpha(\nu)e^{i(\mu-\nu)(\Delta k - k_{\Delta 0})}d\mu d\nu$$

so that:

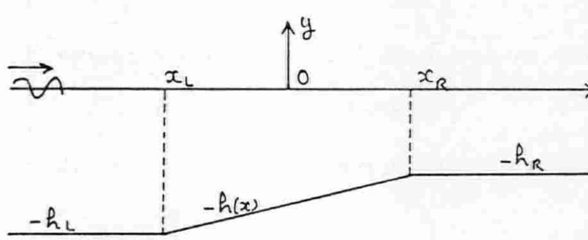
$$\tilde{A}_{FR}\tilde{A}_{FR}^* \simeq \frac{\pi}{2}\frac{B^2(h_0)}{h_0^2}\Delta k^2 x_R S_\alpha(\Delta k - k_{\Delta 0}) \tag{56}$$

where S_α is the power spectrum of $\alpha(x)$ (Fourier transform of its covariance).

Thus in this case A_{FR} behaves as $\sqrt{x_R}$ for large values of x_R.

Corresponding expressions for the reflected free-waves may be obtained by replacing $\Delta k - k_{\Delta 0}$ by $\Delta k + k_{\Delta 0}$ in (55) and (56).

III-2. Sloping Bottom



We now assume $h_R \neq h_L$. In this case φ_F is added to the locked-wave potential defined as:

$$\varphi_{L_1} = -iB(h(x))\frac{\cosh \Delta k(y+h(x))}{\cosh \Delta k h(x)}e^{i\Delta k x} \tag{38}$$

Still assuming that shallow-water theory applies, and given that:

$$\begin{aligned} &\varphi_{L_1}=0 \qquad y=-h(x)\\ &\varphi_{L_1 x}=(\Delta kB - ih_x B_h)e^{i\Delta kx}\\ &\Delta\varphi_{L_1}=[2\Delta kh_x B_h - i(h_{xx}B_h + h_x^2 B_{hh})]e^{i\Delta kx} \end{aligned} \tag{57}$$

we obtain the following equation in φ_F:

$$\begin{aligned} &h\varphi_{Fxx} + h_x\varphi_{Fx} + \frac{\Delta\omega^2}{g}\varphi_F\\ &\quad = \{-\Delta kh_x(B+2hB_h) + i(hh_{xx}B_h + h_x^2 B_h + hh_x^2 B_{hh})\}e^{i\Delta kx}\\ &\varphi_{Fx} = ik_{\Delta R}\varphi_F \qquad x = x_R + \Delta x\\ &\varphi_{Fx} = -ik_{\Delta L}\varphi_F \qquad x = x_L - \Delta x \end{aligned} \tag{58}$$

where we have made no hypothesis as far as the smallness of h_x (other than those inherent to shallow-water theory). Note that the radiation condition is set some distance from x_L or x_R due to the discontinuity of φ_{L1x}.

Equation (58) is solved by finite differences.

(Obviously, due to the numerical scheme employed, it would have been just as simple to solve directly for $\varphi_L + \varphi_F$. However we have prefered to do so by similarity to paragraph III–1).

Numerical Results

Since numerous parameters are involved numerical results are given only as an illustration.

Two wave periods are selected: 8/1.04 and 8 seconds so that the period of the associated beat is 200 seconds. It follows that the first-order waves are deep-water waves for values of the waterdepth larger than 40 meters, whereas the locked second-order wave is shallow for h smaller than 60 meters.

Both wave-amplitudes are assumed to be unity (one meter).

Figure 1 shows the potential amplitude of the second-order locked wave (eq. (14)) and its proposed approximation (16). Underneath the energy-flux variation from $h=-\infty$ is represented.

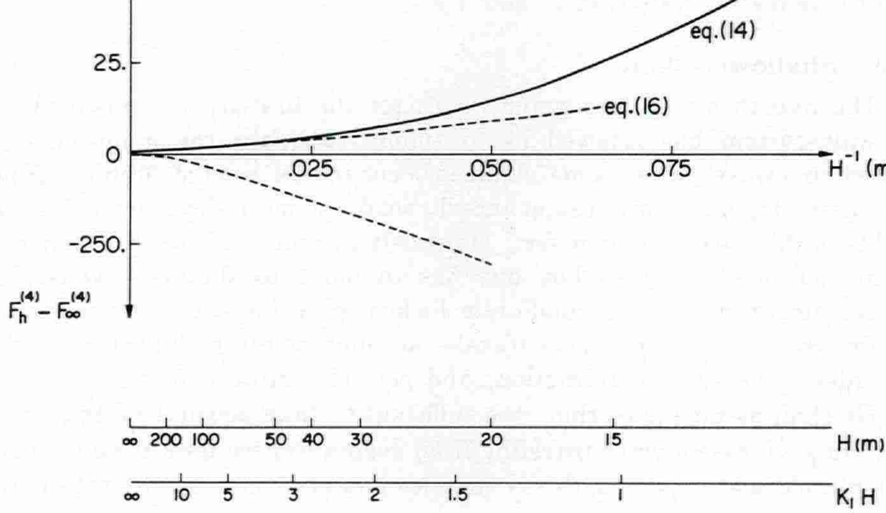


Figure 1. Variations of the potential amplitude of the second-order locked-wave, and of the energy flux, with the waterdepth

In order to check the numerical scheme it was first applied to the case of the undulating bottom, and its results compared to the analytical ones. Moreover the numerical scheme was run repeatedly for both second-order locked potentials (φ_{L1} and φ_{L2}). Corresponding results are shown on figures 2 and 3.

Next the case of sloping bottom is considered. The waterdepth variation is supposed to be from 60 to 30 meters and different lengths of the slope are considered.

Corresponding potential amplitudes of the transmitted and reflected free-waves are shown on figure 4. It appears that the transmitted wave always exceeds the reflected one. They turn out to be decreasing functions of the slope, with some undulations superposed, the "wave-numbers" of the oscillations being apparently equal to $\Delta k-k_{\Delta R}$ for the transmitted wave and $\Delta k+k_{\Delta L}$ for the reflected wave.

For large values of the slope one obtains solutions close to those calculated for the case of a step-like change in waterdepth (see Appendix), eventhough in the limiting case shallow-water theory cannot be applied any more. This gives us some confort as far as the domain of validity of the proposed model.

Figure 5 provides an illustration of the building-up of φ_F for a slope length equal to 2000 meters.

The case of superposed bottom undulation is illustrated by figures 6 and 7. In this case the waterdepth is described as:

$$h=h_L-\delta(x+x_R)+\alpha \sin 2\pi \frac{x_R}{x} \tag{59}$$

where α takes the values 0, 2, and 4 meters.

III–3. Shallower Water

The hypothesis of deep-water waves for the first-order components of the wave-system has allowed us to simply calculate the amplitudes of emitted free-waves when some changes occur in the bottom profile. However these amplitudes are disappointedly small, some millimeters for wave-heights of the order of one meter. Obviously in order that the phenomenon become physically appreciable, one has to move to shallower water, so that the amplitude of the second-order locked wave increases substantially. The drawback is that we have to take account of the modification of the first-order wave system: refraction, and possibly diffraction.

We shall assume here that it is sufficient to take account of refraction only. In such case a wave traveling from deep water exhibits some changes in amplitude and wave-length, so that its potential may be described by:

$$\varepsilon\Phi^{(1)}=\frac{a(h)g}{\omega}\frac{\cosh k(h)(y+h)}{\cosh k(h)h}\sin\left(\int_{-\infty}^{x} k(h)ds-\omega t\right) \tag{60}$$

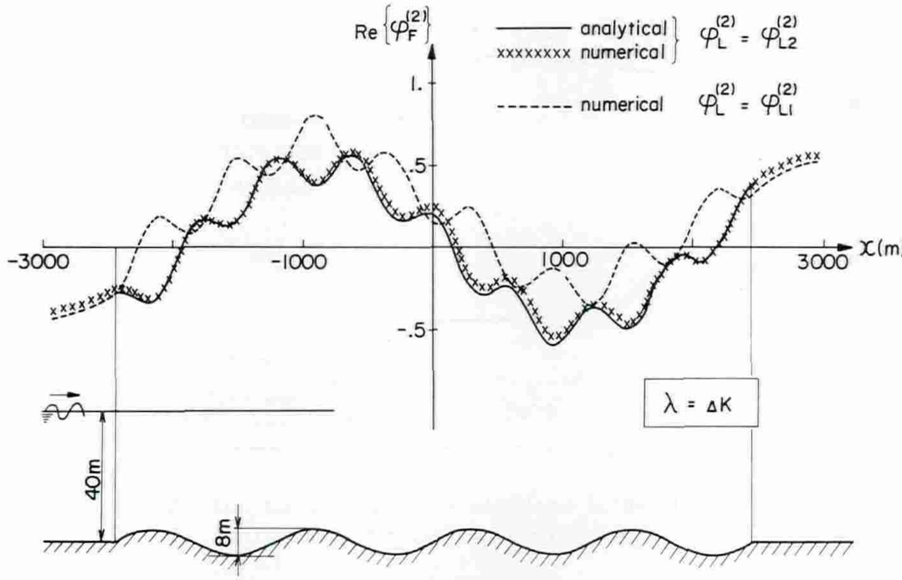


Figure 2. Illustration of the generation of the second-order free-waves on an undulating bottom.

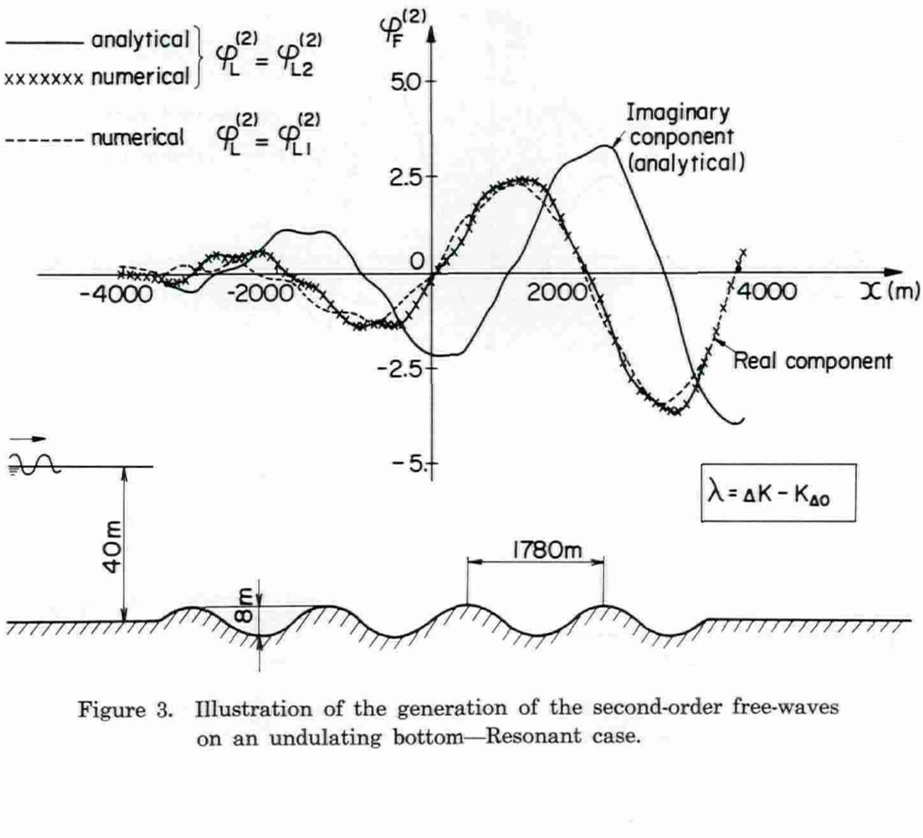


Figure 3. Illustration of the generation of the second-order free-waves on an undulating bottom—Resonant case.

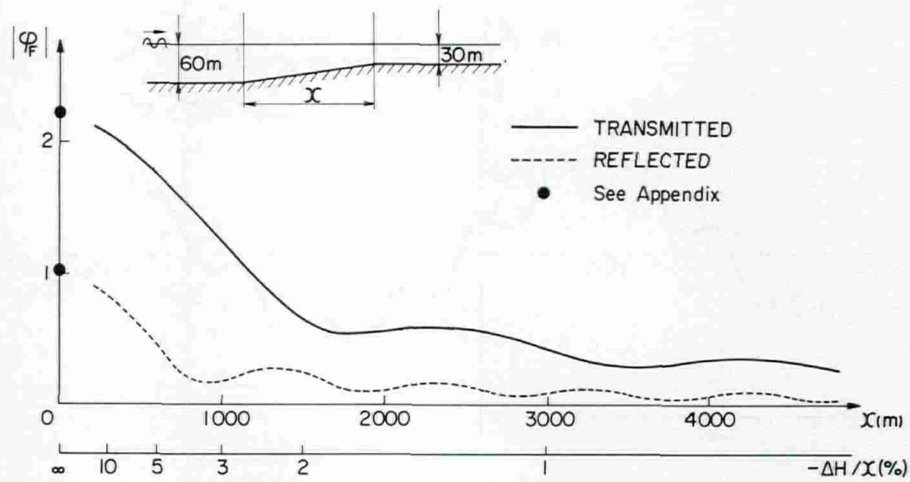


Figure 4. Potential amplitudes of the transmitted and reflected free-waves due to a sloping bottom.

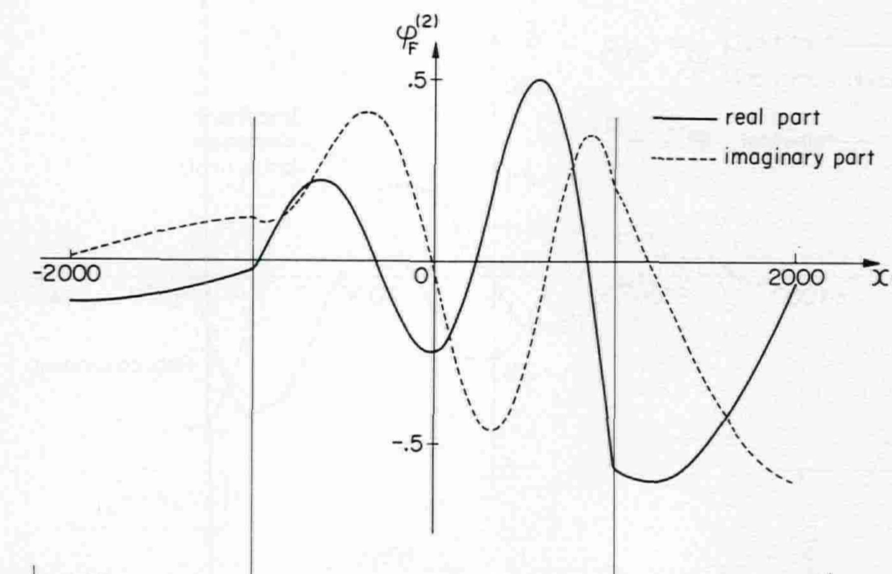


Figure 5. Illustration of the generation of the second-order free-wave on a sloping bottom.

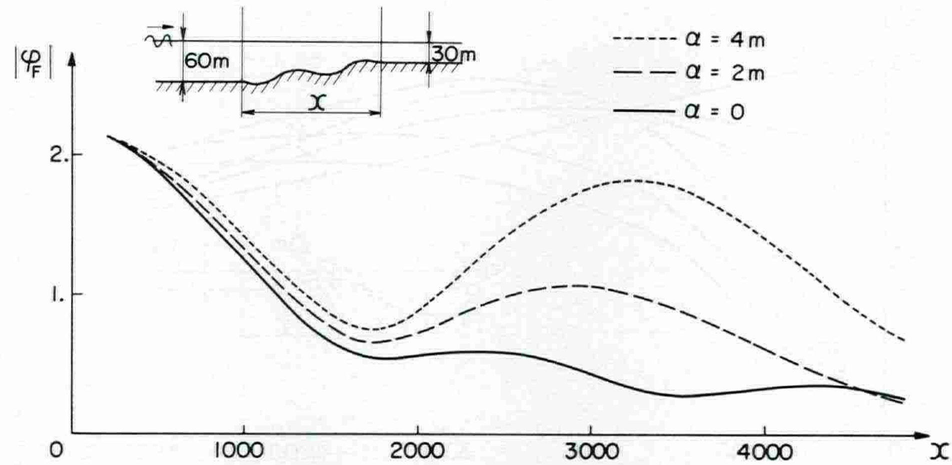


Figure 6. Combination of undulating and sloping bottom. Potential amplitude of the transmitted free-wave.

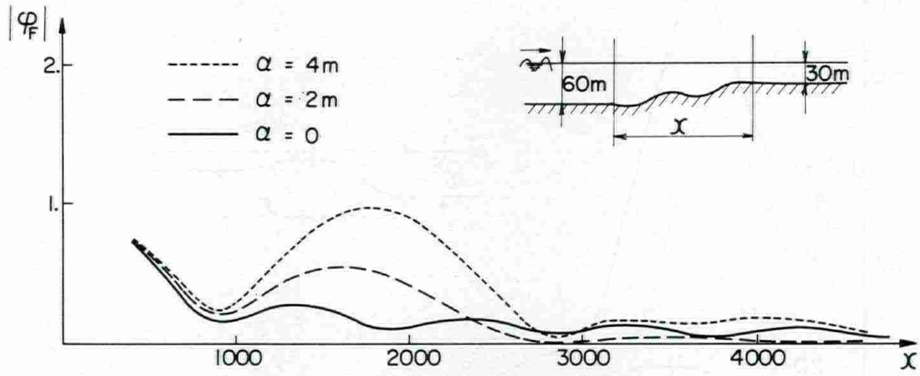


Figure 7. Combination of undulating and sloping bottom. Potential amplitude of the reflected free-wave.

where

$$\omega^2 = gk(h) \tanh k(h)h \tag{61}$$

$$a(h) = a(\infty)\sqrt{\frac{C_G(\infty)}{C_G(h)}} = a(\infty)\sqrt{\frac{\cosh^2 kh}{kh + \sinh kh \cosh kh}} \tag{62}$$

C_G being the group velocity.

Since this expression is only a zero-order approximation in h_x, we shall not consider the x-dependence of a and k in the derivation of the free-surface equation (8). Then the second-order locked potential is obtained

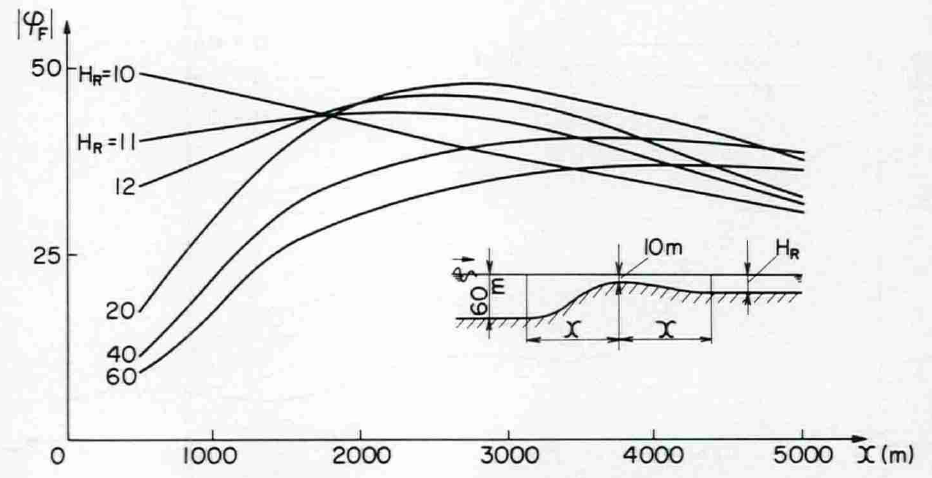


Figure 8. Potential amplitude of the transmitted free-wave due to a bump, with variable downstream waterdepth.

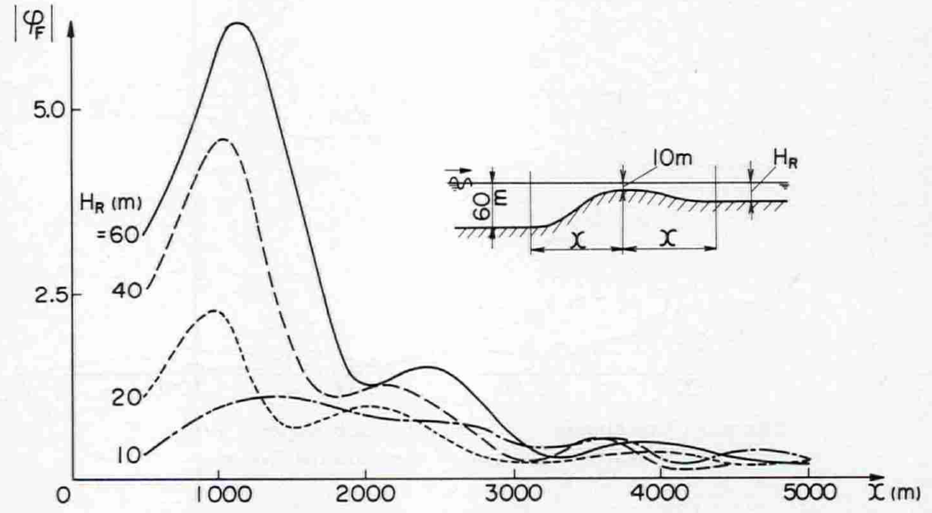


Figure 9. Potential amplitude of the reflected free-wave due to a bump, with variable downstream waterdepth.

from (14) where one takes account of the changes in amplitudes, wavenumbers, and phase angles of the first-order waves.

As a numerical application we consider single bumps of sinusoidal shapes with constant waterdepth upstream and downstream. First the dependence upon the downstream waterdepth is illustrated (figures 8 and 9) with upstream waterdepth 60 meters and waterdepth at top of the bump 10 meters (still for the same 8/1.04 and 8 seconds waves with unit amplitude). For a downstream waterdepth equal to 10 meters we obtain again that the transmitted free-wave decreases in amplitude with increasing bump-length. For downstream waterdepths larger than 10 meters we obtain the interesting result that the maximum amplitude occurs for a non-zero bump-length, so that the corresponding slope is rather mild, which is consistent with the hypotheses. As before we observe that the reflected free-wave is much smaller than the transmitted free-wave (figure 9).

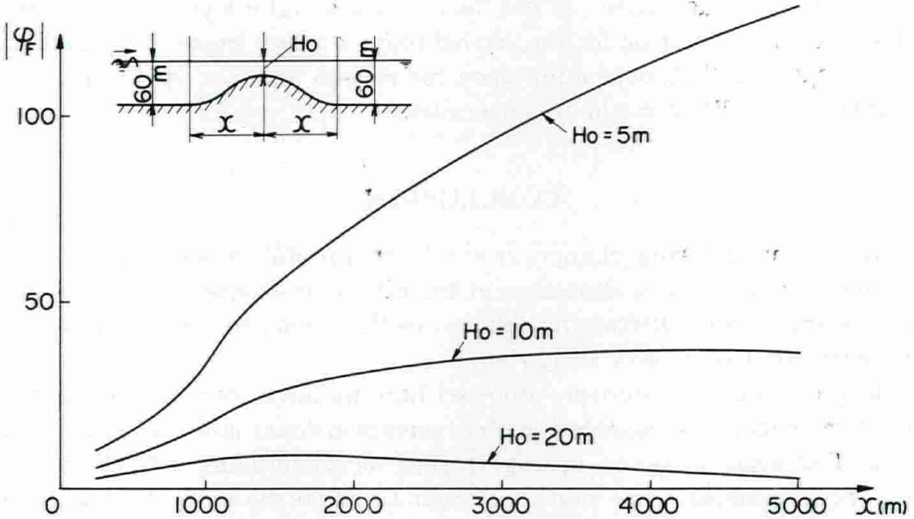


Figure 10. Potential amplitude of the transmitted free-wave due to a bump, as a function of the bump height, for upstream and downstream waterdepths equal to 60 meters.

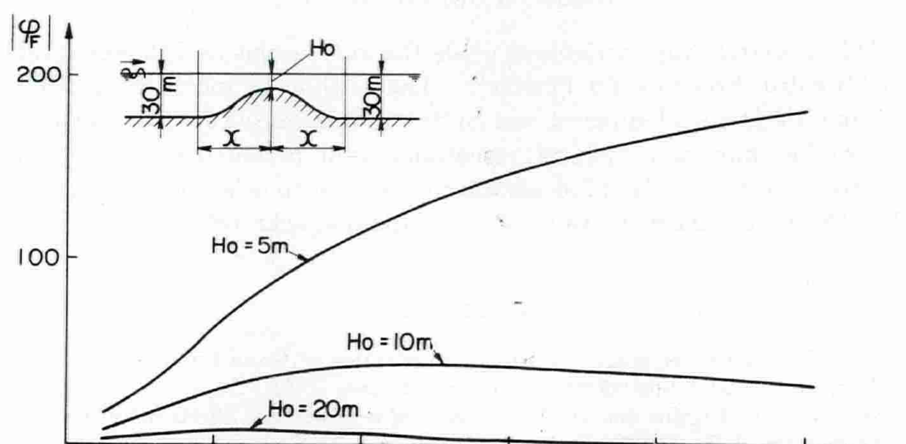


Figure 11. Potential amplitude of the transmitted free-wave due to a bump, as a function of the bump height, for upstream and downstream waterdepths equal to 30 meters.

Next the effect of clearance at the top of the bump is investigated. Corresponding results are plotted on figures 10 and 11, for equal upstream and downstream waterdepths.

As compared to those of the previous paragraph, the amplitudes of the generated free-waves appear to be physically appreciable (10 to 20 cm for 1 m wave-amplitude).

A next step would be to treat the case of a beach. However in such case more difficulties arise, as one has to set a waterdepth for breaking, and a boundary condition for the second-order waves. Some computational runs have shown that, depending upon the chosen answers, surf-beats quite variable in amplitude could be generated.

CONCLUSION

When encountering changes in the bottom profile, a wave-system emits second-order free-waves, appearing at the difference-frequencies of its individual components. Refracting differently these long free-waves may enter apparently sheltered bays or harbors.

In the simplistic approach proposed here we have been able to quantify this phenomenon, by considering the two-dimensional problem, and assuming the first-order waves to be unperturbed, or undergoing refraction only. More work remains to be done, in order to solve the case of a beach and to include three-dimensional effects, as edge-waves are likely to appear along the shore.

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REFERENCES

1) N. E. Ottesen-Hansen et al.: "Correct Reproduction of Group-Induced Long Waves" Proc. 17th Coastal Engineering Conference, Sydney 1980.

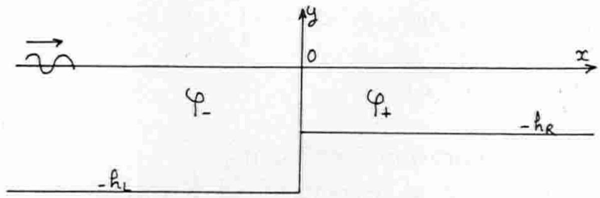
2) M. S. Longuet-Higgins and O. M. Phillips: "Phase Velocity Effects in Tertiary Wave Interaction" J. Fl. Mech. vol. 12, p. 333–336.

3) M. S. Longuet-Higgins and R. W. Stewart: "Radiation Stress and Mass Transport in Gravity Waves with Applications to Surf-Beats" J. Fl. Mech. vol. 13, p. 481–504.

APPENDIX

Diffration of the Second-order Potential on a Step

In this section we assume the waterdepth to be deep for the first-order waves and intermediate for the second-order ones.



We match at $x=0$ analytical expressions of the potential for $x\leqslant 0$ and $x\geqslant 0$.

$$x\leqslant 0: \quad \varphi_- = \varphi_L(h_L) + \sum_{i=1}^{N} a_i f_i \qquad a_i \in C$$

$$f_0 = \frac{\cosh k_{0L}(y+h_L)}{\cosh k_{0L}h_L} e^{-ik_{0L}x}$$

$$f_i = \cos k_{iL}(y+h_L) e^{k_{iL}x} \qquad i\geqslant 1$$

where

$$\Delta\omega^2 = gk_{0L} \tanh k_{0L}h_L$$

$$\Delta\omega^2 = -gk_{iL} \tan k_{iL}h_L \qquad i\geqslant 1$$

$$\varphi_L(h_L) = -iB(h_L) \frac{\cosh \Delta k(y+h_L)}{\cosh \Delta k h_L} e^{i\Delta kx}$$

$$x\geqslant 0 \quad \varphi_+ = \varphi_L(h_R) + \sum_{i=0}^{N} b_h g_i \qquad b_i \in C$$

$$g_0 = \frac{\cosh k_{0R}(y+h_R)}{\cosh k_{0R}h_R} e^{ik_{0R}x}$$

$$g_i = \cos k_{iR}(y+h_R) e^{-k_{iR}x} \qquad i\geqslant 1$$

where

$$\Delta\omega^2 = gk_{0R} \tanh k_{0R}h_R$$

$$\Delta\omega^2 = -gk_{iR} \tan k_{iR}h_R \qquad i\geqslant 1$$

$$\varphi_L(h_R) = -iB(h_R) \frac{\cosh \Delta k(y+h_R)}{\cosh \Delta k h_R} e^{i\Delta kx}$$

At $x=0$ the matching conditions write:

$$\varphi_- = \varphi_+ \qquad -h_R \leqslant y \leqslant 0$$

$$\varphi_{-x}=\varphi_{+x} \qquad -h_R\leqslant y\leqslant 0$$
$$\varphi_{-x}=0 \qquad -h_L\leqslant y\leqslant h_R$$

where we assume $h_L>h_R$.

Thus the set of unknown coefficients $(a_0\cdots a_N,\ b_0\cdots b_N)$ minimizes:

$$F=\alpha\int_{-h_R}^{0}(\varphi_- -\varphi_+)(\varphi_-^* -\varphi_+^*)dy+\beta\int_{-h_R}^{0}(\varphi_{-x}-\varphi_{+x})(\varphi_{-x}^* -\varphi_{+x}^*)dy$$
$$+\gamma\int_{-h_L}^{-h_R}\varphi_{-x}\varphi_{-x}^* dy$$

where α, β, γ are ponderation coefficients.

Expressing that partial derivatives of F with respect to $(a_0\cdots b_N)$ are zero one obtains a set of $2N+2$ linear equations which are solved by classical techniques. (In the numerical resolution some accuracy problems occurred, apparently due to the small values of k_{0L}, k_{0R}, as compared to k_{iL}, k_{iR} $(i\geqslant 1)$. This disagreeableness could be effectively overcome by imposing a further constraint on the second derivative of φ.)

Numerical Results

Again we consider two waves of periods 8/1.04 and 8 seconds, in waterdepths of 500, 200, 100, 60, 40, and 30 meters.

The different components of the second-order waves are illustrated on figure 12.

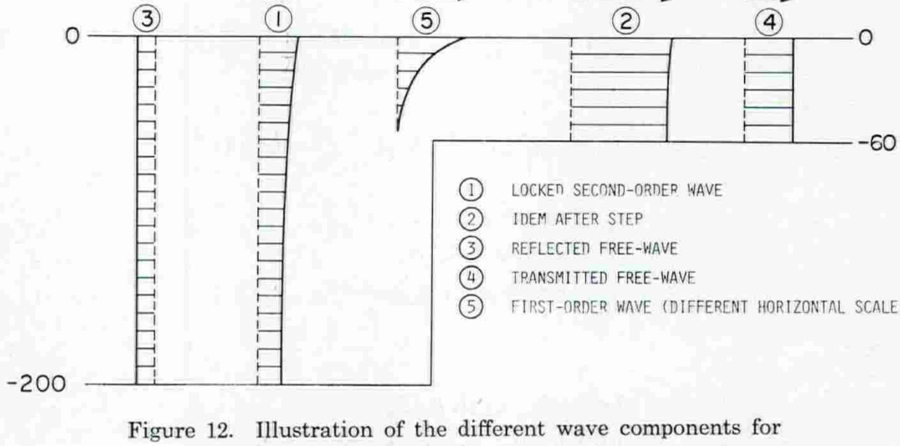


Figure 12. Illustration of the different wave components for a step-like change in waterdepth.

The Table 1 shows the obtained numerical results concerning the amplitudes at the free-surface of the reflected and transmitted potentials, and the amount of energy which is carried away by the free-waves, as compared to the loss in energy-flux due to the change in waterdepth.

Table 1. Potential amplitudes of the different second-order component waves, together with the amount of energy flux transferred to the free-waves

h_L	h_R	$B(h_L)$	$B(h_R)$	$\lvert\varphi_F(h_L)\rvert$ $(y=0)$	$\lvert\varphi_F(h_R)\rvert$ $(y=0)$	$F_F/\Delta F$ $(\times 100)$
500	200	0.83	1.06	0.13	0.20	2.2
〃	100	〃	1.77	0.30	0.80	4.1
〃	60	〃	2.87	0.42	1.79	6,3
〃	40	〃	4.38	0.48	3.25	9.2
〃	30	〃	6.02	0.41	5.00	12.3
200	100	1.06	1.77	0.29	0.48	2.2
〃	60	〃	2.87	0.52	1.38	4.6
〃	40	〃	4.38	0.68	2.75	7.4
〃	30	〃	6.02	0.71	4.39	10.2
100	60	1.77	2.87	0.43	0.71	2.3
〃	40	〃	4.38	0.77	1.92	4.9
〃	30	〃	6.02	0.97	3.39	7.7
60	40	2.87	4.38	0.62	0.92	2.5
〃	30	〃	6.02	1.01	2.22	5.1
40	30	4.38	6.02	0.81	0.91	2.5

Nomenclature:

h_L: waterdepth before the step
h_R: waterdepth after the step
$B(h_L)$: potential amplitude of the locked-wave before the step
$B(h_R)$: potential amplitude of the locked-wave after the step
$\lvert\varphi_F(h_L)\rvert_{y=0}$: potential amplitude of the reflected free-wave
$\lvert\varphi_F(h_R)\rvert_{y=0}$: potential amplitude of the transmitted free-wave
$F_F/\Delta F$: energy flux transferred to the free-waves, as a ratio of the total loss.

In all cases the transmitted free-wave appears to be larger than the reflected free-wave, its amplitude being roughly equal to the difference in amplitude of the locked-wave before and after the step.

It appears that only a small fraction of the loss in energy flux is transferred to the free-waves.

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