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Efficient Analysis of Gradient Index Flat Lenses

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Efficient Analysis of Gradient Index Flat Lenses

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Chapter 1

Introduction

1.1 Lens Antennas

Lens antennas have long been an attractive solution for applications requiring high gain, wide-angle beam steering, and multi-beam operation [1, 2]. Lens antennas can support multiple feed locations, each producing a distinct beam direction. This enables passive beam steering as well as the simultaneous generation of multiple beams, making lens antennas particularly suitable for applications such as radar, satellite communications, and imaging arrays [2].

Interest in lens antennas has grown significantly with the emergence of millimeter-wave (mmWave) communication and sensing systems [3, 4]. As the operating frequency increases, the wavelength decreases, allowing electrically large apertures to be realized in physically compact structures. As a result, dielectric lenses that are impractically large at microwave frequencies become much smaller and more suitable for integration into modern systems. Moreover, the high propagation losses at mmWave frequencies require antennas with high directivity, which lens antennas can provide through passive beamforming without the complexity and losses associated with large phased arrays.

The operating principle of a conventional mmWave lens antenna is analogous to that of an optical lens. Figure 1.1 portrays how a homogeneous dielectric material with a curved surface introduces different electrical path lengths through the lens, compensating for the varying distances between the feed and the bottom aperture. Consequently, in transmission, the spherical wave radiated by the feed is transformed into a collimated beam with a planar phase front. Conversely, in reception, an incident plane wave is focused onto the focal region, enabling efficient signal reception. The required phase transformation is therefore achieved through shaping the geometry of a homogeneous dielectric.

Despite these advantages, conventional lens antennas also have important limitations. Their curved geometry leads to relatively bulky and heavy structures, increasing material usage and complicating fabrication. Furthermore, their non-planar profile makes integration with printed circuit boards, integrated circuits, and other planar technologies challenging [4].

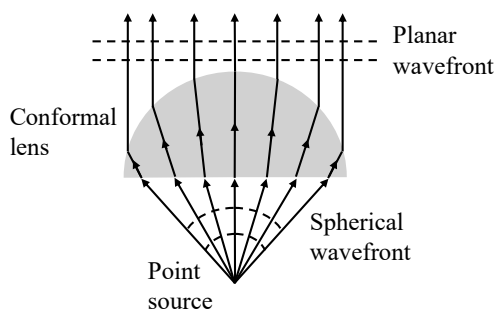


Figure 1.1: The operating principle of a focusing conventional dielectric lens. A spherical wavefront is transformed into a planar wavefront through a difference in dielectric thickness.

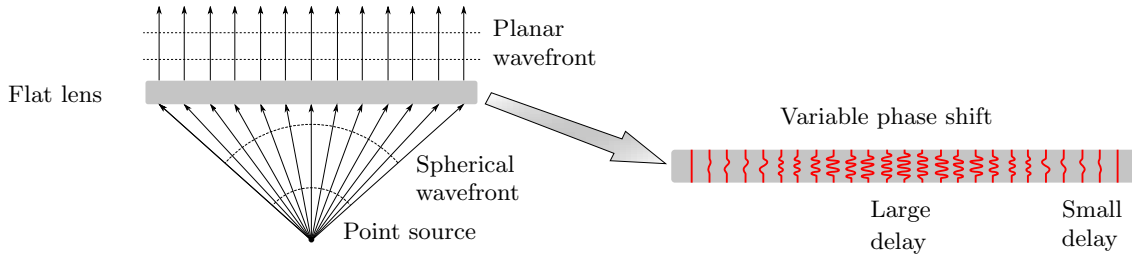


Figure 1.2: The operating principle of a focusing flat lens. A spherical wavefront is transformed into a planar wavefront through controlled, spatially varying material properties.

1.2 Flat Lens Antennas

To overcome the size and integration limitations of conventional dielectric lenses, significant research has focused on the development of flat Gradient-Index (GRIN) lens antennas [2, 5, 6, 7, 8, 9, 10]. By replacing the curved profile of a homogeneous lens with a spatially varying refractive index, flat GRIN lenses achieve the required phase transformation within a planar structure. Figure 1.2 illustrates the effect of a focusing flat lens and how the spherical wavefront is converted to a planar one by introducing variable material properties over the lens.

Compared to conventional curved lenses, flat GRIN lenses are more compact, low profile, and better suited for integration into planar antenna platforms. In addition, they can be fabricated using planar multilayer technologies, making them particularly attractive at mmWave and terahertz frequencies, where the fabrication of curved dielectric lenses becomes increasingly challenging. The spatially varying refractive index also provides an additional degree of freedom for tailoring the phase distribution and realizing more general radiation characteristics [11].

A common aspect in the design of flat lens antennas is the trade-off between thickness and bandwidth [11]. Thin lenses are characterized by a narrow frequency band of operation, due to the use of resonant elements in the metalens design or to phase wrapping [2, 6, 7]. True-time delay (TTD) lenses without phase wrapping can be used to obtain a wider bandwidth [8], at the cost of increased thickness. To reduce the thickness of TTD lenses, materials with a high refractive index can be used [9, 10].

In summary, planar lenses are emerging as a favorable alternative to curved dielectric lenses for mmWave and terahertz wireless systems, due to their ease of integration with electronics and reduced manufacturing complexity.

1.3 Refractive Index Profile Synthesis

The design of flat GRIN lenses generally begins by determining the refractive-index profile throughout the lens. Once this distribution has been obtained, it can be realized using various fabrication techniques, such as 3D-printed dielectric materials with different refractive indices [12], perforated dielectric slabs that form artificial dielectrics (ADLs) [13], or multilayer subwavelength metallic periodic structures with spatially varying densities [14, 15, 16].

Several methods have been proposed to determine the refractive index profile of a GRIN lens. One of the most widely used approaches is transformation optics (TO) [17, 18, 19, 20, 21, 22, 23]. In this technique, a suitable coordinate transformation is applied to achieve the desired lens characteristics, resulting in a corresponding distribution of the permittivity and permeability tensors [24, 25]. However, because the resulting materials are typically anisotropic and thus challenging to fabricate, simplified electromagnetic parameters are often used in practical implementations.

An alternative design strategy, introduced in [26], is based on field transformation (FT). In this approach, the lens is divided into discrete unit cells, each characterized by an unknown permittivity, ϵ . A synthesis procedure is then employed to determine the permittivity of each cell by transforming a prescribed input field distribution into the desired output field at the lens aperture.

In addition to these methods, GRIN lens designs have also been developed by combining ray-tracing techniques with numerical optimization algorithms [27].

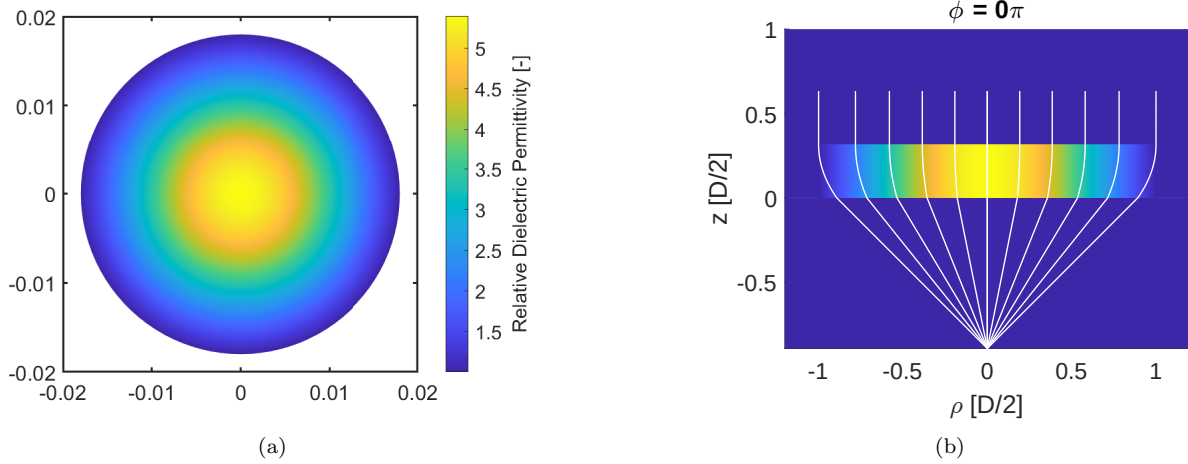


Figure 1.3: (a) The dielectric permittivity distribution and (b) the corresponding collimation ray tracing of an example lens.

Lastly, in [28], a geometrical optics (GO)-based semi-analytical method is proposed for the design of GRIN lenses for several cases, such as collimation. Given a lens diameter, a focal distance, and the lens thickness or maximum refractive index, a permittivity distribution can be computed. An example of this is given in Figure 1.3a, where the distribution is plotted for a collimating lens with a diameter of 36 mm and a focal length of 16 mm. It can be seen that for a collimating lens, fed from the center, the distribution is rotationally symmetric, with the highest permittivity in the center. A ray-tracing method shows the collimating capabilities of this distribution in Figure 1.3b. This refractive index computation method is applied to compute the required distribution for all lenses in this work.

1.4 Objective of the Thesis

Simulating an array-fed flat-lens antenna is not trivial, as using commercial software is impractical in this case. A general-purpose simulator would need huge computational resources to mesh the electrically large structure with a gradient index and the feed array. Although several techniques have been devised to analyze flat lenses, a combined tool that enables the design of flat lenses is not available. To avoid lengthy simulation times for each design adjustment in such lenses, the development of in-house simulation tools is necessary.

Currently, several asymptotic and full-wave methods exist, each computing a different aspect of the lens's behavior for design optimization. This includes ray tracing in inhomogeneous media [29], transmission line models and physical optics (PO) [11], extending it to modeling artificial dielectric behavior [30, 31, 32, 33].

This work attempts to develop a lens antenna simulation framework that combines these methods and introduces new approaches to fill gaps between existing ones. This GRIN lens analysis is incorporated in a user-friendly tool to enable performance assessment of such lenses, in terms of radiation patterns and efficiencies.

1.5 Outline of the Thesis

In Chapter 2, the base model for analyzing center-fed GRIN lenses is introduced, in which ray tracing, a Delaunay-triangulation-based ray tube theorem, a transmission line model, and GO and PO optics are combined. Chapter 3 presents an in-depth validation of this model, comparing its results with those of the commercial full-wave simulator CST. The model is further extended in Chapter 4 to accommodate off-axis feeds in the analysis. This further discusses the validation and limitations of the current model. Thereupon, the implementation of this work's model into the graphical user interface (GUI) is shown. Subsequently, Chapter 5 first presents the modeling of the anisotropic behavior of square-patch ADLs, followed by the introduction of a lens optimization technique based on the ray-traced output angles. Lastly, Chapter 6 presents the conclusion and suggestions for future work.



Chapter 2

Analysis of Center-Fed GRIN Lenses

In this Chapter, the modeling approach for analyzing GRIN lens antennas is presented. First, the geometry under consideration is described. Then, the procedure for ray tracing in non-homogeneous media is explained. Subsequently, the transmission line model for propagation along the ray path is discussed. Lastly, the procedure for finding the far-field radiation patterns and the relevant figures of merit is defined.

The side-view cross-section of the geometry under consideration is shown in Figure 2.1. It consists of a flat lens, which has N layers with spatially varying index of refraction $n_i(x, y)$ and thickness t_i , with $i \in [1, N]$. The lens has a focal length F and diameter D . The feed is assumed to be placed at the focal distance, with a possible displacement Δx . In this chapter, Δx is assumed to be zero. In Chapter 4, displaced feeds are studied.

2.1 Ray Tracing

The first step in obtaining the far-field patterns of a GRIN lens is to perform a ray-tracing analysis. This assumes rays that are orthogonal to the local wavefront are infinitely thin and travel in straight lines in homogeneous media. At the boundaries of two media, light rays can be reflected or refracted according to Snell's law. In a slowly varying inhomogeneous medium, as is assumed in GRIN lenses, the rays can be curved. Additionally, no diffraction is taken into account. This simplification is valid for the geometrical optics approximation but may limit the model precision near sharp features or small-scale structures.

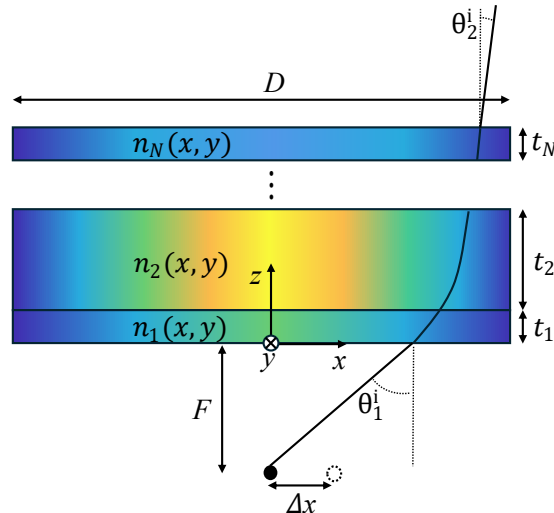


Figure 2.1: A schematic representation of the geometry of the GRIN lenses under analysis.



The electric field at a point $\mathbf{r} \equiv (x, y, z)$ can be expressed as

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_0(\mathbf{r})e^{-jk_0S(\mathbf{r})}, \quad (2.1)$$

where k_0 is the free-space wavenumber, $\mathbf{E}_0$ is a complex vector amplitude, and $S(\mathbf{r})$ is the Eikonal function representing a scalar field whose gradient gives the local direction of rays and whose value represents the optical path length accumulated by the ray from the source point [29, 34, 35]. It can be demonstrated that substituting this wave solution into Maxwell's equations, and assuming a small wavelength, results in the Eikonal equation:

$$(\nabla S)^2 = n^2. \quad (2.2)$$

The Eikonal equation can be described by the following system of ordinary differential equations (ODEs):

$$n \frac{dx}{ds} = p_x \quad (2.3a)$$

$$n \frac{dy}{ds} = p_y \quad (2.3b)$$

$$n \frac{dz}{ds} = p_z \quad (2.3c)$$

$$\frac{dp_x}{ds} = \frac{\delta n}{\delta x} \quad (2.3d)$$

$$\frac{dp_y}{ds} = \frac{\delta n}{\delta y} \quad (2.3e)$$

$$\frac{dp_z}{ds} = \frac{\delta n}{\delta z}, \quad (2.3f)$$

where $p_x = \partial S / \partial x$, $p_y = \partial S / \partial y$, and $p_z = \partial S / \partial z$ are the x -, y -, and z -components of the gradient ∇S , respectively, s is the ray path, and n is the refractive index. These equations have initial values given by [29, 35]

$$x = x_0 \quad (2.4a)$$

$$y = y_0 \quad (2.4b)$$

$$z = z_0 \quad (2.4c)$$

$$p_x = n(x_0, y_0, z_0) \sin(\theta) \cos(\phi) \quad (2.4d)$$

$$p_y = n(x_0, y_0, z_0) \sin(\theta) \sin(\phi) \quad (2.4e)$$

$$p_z = n(x_0, y_0, z_0) \cos(\theta). \quad (2.4f)$$

$$(2.4g)$$

Given a spatially varying index of refraction, this set of equations is solved using MATLAB's function `ode45`, resulting in the ray path in x , y , and z throughout the lens. Additionally, the propagation angles at each point along the ray are found from Equations (2.3d)-(2.3f).

The ODE solver starts from the bottom interface of the lens and runs until the ray intersects a certain boundary. In the simple case of a lens consisting of one slab, a boundary is crossed in two cases: when the z -coordinate of the ray equals the z -coordinate of the top of the lens or when the ρ -coordinate of the ray equals the radius of the lens, with $\rho = (x^2 + y^2)^{1/2}$. This would entail that the ray exits from the top or the side of the lens, respectively. Furthermore, the solver requires a smooth analytical function for the refractive index. This is because the refractive index function has to be defined and differentiable everywhere.

The rays originate from a fixed focal point and are propagated toward the bottom surface of the lens on a grid uniformly sampled in ρ and ϕ , with N_ρ and N_ϕ points, respectively. At the interface between the surrounding medium and the lens, Snell's law is applied to determine the direction of propagation inside

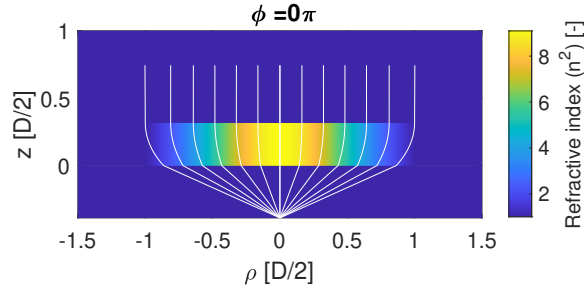


Figure 2.2: The rays found by the ray-tracing algorithm for a lens with a diameter of 36 mm, a height of 5.7 mm, and a focal length of 7 mm.

the lens. From here, the ODE solver finds the ray path in the lens until a boundary is encountered. At this boundary, Snell's law is again applied. The ray tracing and Snell's law are then repeated until the top of the lens is reached.

As an illustrative example, the ray-tracing result for a single-layer collimating lens is shown in Figure 2.2. The ratio of focal length to diameter (F/D) is 0.19, with a thickness equal to 0.15 times the diameter and a maximum refractive index of 3. Here, the application of Snell's law at the lens bottom and top surfaces can be seen, as well as the curved paths of rays inside the lens due to the varying refractive index.

2.2 Optical Path Length

The optical path length (OPL) of a ray is defined as the integral of the refractive index along the ray path c :

$$\text{OPL} = \int_c n(x, y, z) dl. \quad (2.5)$$

Writing (2.5) in differential form, one can use the ODE solver to simultaneously find the OPL with the ray paths, as

$$\frac{d\text{OPL}}{ds} = n. \quad (2.6)$$

The OPL represents the equivalent distance that a wave would propagate in free space while accumulating the same phase as along the traced ray. Multiplying the OPL by the free-space wavenumber, the phase progression of the forward-traveling wave along the ray can be determined as $\phi_{\text{ray}} = \text{OPL} \cdot k_0$. In this ray-tracing phase computation, it is assumed that only the forward-traveling wave is considered, neglecting any internal reflections. By performing this phase calculation for all rays reaching the aperture, a first estimate of the aperture phase front can be obtained.

2.3 Refractive Index Profile Polynomial Fit

As described in Section 2.3, a sampled refractive index profile is computed using semi-analytical equations in [28, 36]. As the ODE solver requires an analytical expression for the permittivity, a polynomial fit has been performed on the sampled profiles. Figure 2.3a shows the effect of the polynomial fit order, with the difference between the polynomial and the desired pattern plotted as a function of ρ . Logically, as the order increases, the error decreases. As the permittivity is assumed to be symmetric, only even-order polynomials are considered. The effect on the output angles for different-order fits is shown in Figure 2.3b. Here, it is observed that a slight error in dielectric permittivity already affects the output angle by a few degrees. It was chosen to use at least an eighth-order fit, which is a good compromise between computational speed and accuracy, with less than 1° variation in the rays' output angle.

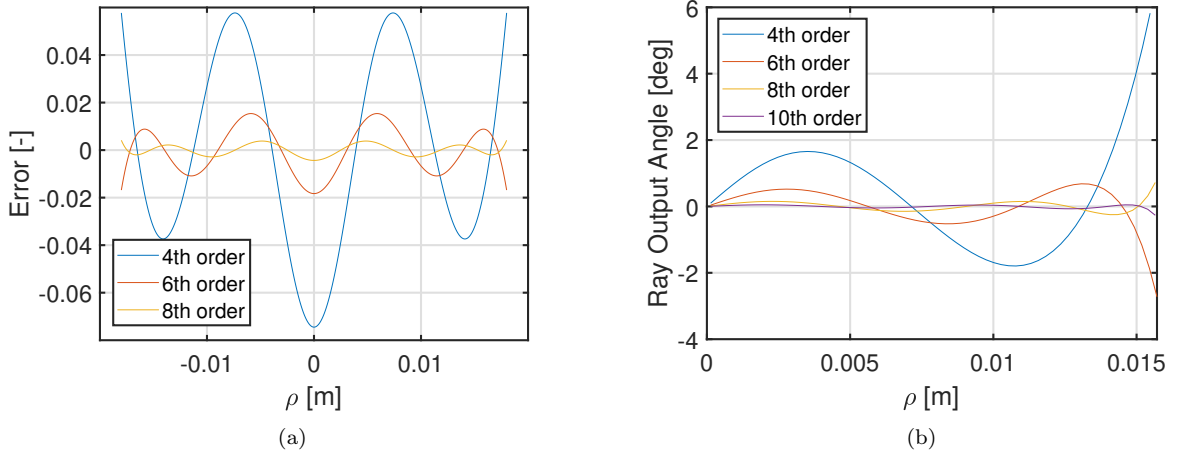


Figure 2.3: The influence of the order of polynomial fits to the relative dielectric desired permittivity in (a) difference in permittivity and (b) ray output angles.

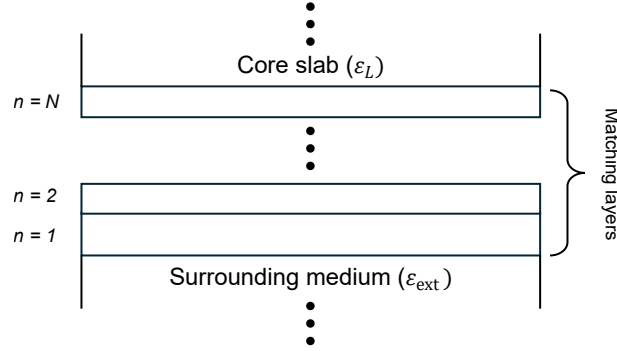


Figure 2.4: A schematic representation of the matching layer stackup.

2.4 Matching Layers

As the dielectric permittivity at the center of the lens can become large, especially for thin lenses, significant reflections are expected. To reduce the reflections, matching layers can be added. A simple-to-implement matching layer is a single-section quarter-wave transformer, whose intrinsic impedance is

$$Z_{\text{ml}} = \sqrt{Z_{\text{ext}} Z_L}, \quad (2.7)$$

where Z_{ext} is the impedance of the medium outside the lens and Z_L is the impedance of the core layer, varying as a function of x and y . The condition leads to $\epsilon_{\text{ml}} = \sqrt{\epsilon_{\text{ext}} \epsilon_L}$, where ϵ_{ext} is the relative permittivity in the medium outside the lens and ϵ_L is the permittivity of the medium in the lens. The length of the slab is

$$l = \lambda_{\text{ml}}/4 = \lambda_0/(4\sqrt{\epsilon_{\text{ml}}}),$$

with λ_0 the free-space wavelength for a specific calculation frequency f_0 [37]. The dielectric permittivity is to be matched this way over the entire lens. Extending this formula to N matching layers, this becomes

$$\epsilon_n = \sqrt[N+1]{(\epsilon_{\text{ext}})^{N-n+1} (\epsilon_L)^n}, \quad (2.8)$$

for the n -th matching layer, counting from the surrounding medium, and N is the number of matching layers. A schematic representation is portrayed in Figure 2.4.

These proposed matching layers are an extension of quarter-wavelength transformers and provide a bandwidth similar to that of Chebyshev transformers if the Chebyshev passband ripple is chosen to match that of the proposed transformer. In Fig. 2.5, the reflection coefficient of a 4-section impedance transformer

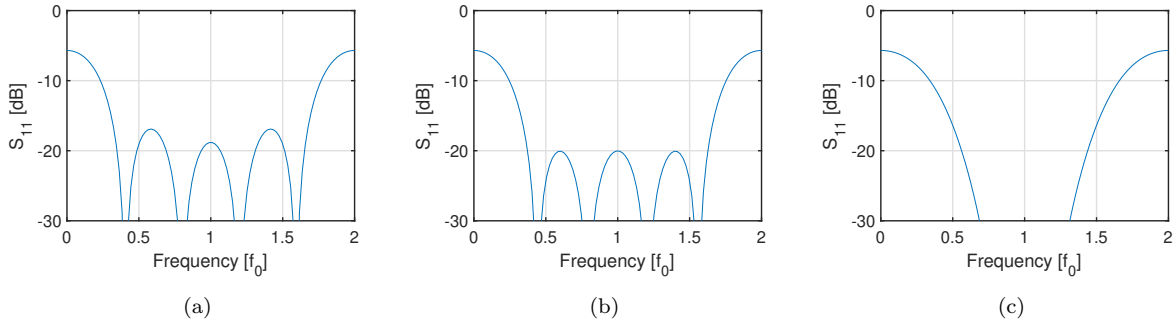


Figure 2.5: The reflection coefficient under boresight plane wave illumination over different frequencies for (a) the proposed quarter-wavelength transformer, (b) the proposed Chebyshev transformer, and (c) a binomial transformer.

is given, if the matching layers are infinitely extended and illuminated with a boresight plane wave. Furthermore, it illustrates a Chebyshev transformer with a passband ripple of -20 dB and a binomial transformer. Alternatively, to implement matching layers, a tapered transformer could be used [37].

Although the permittivity of the matching layer varies with space, its thickness remains a design parameter. The thickness is set to $\lambda_{ml}/4$ at the center of the lens. This is for two reasons. Firstly, for non-phaseswrapping and non-scanning lenses, the dielectric permittivity is usually highest at the center of the lens, leading to the highest reflections. Secondly, feeds typically do not illuminate the lens with constant power across the aperture; instead, they direct the most power to the lens's center. To prevent the amount of power reflected, we fit the thicknesses to the center.

2.5 Optimization of Lens Thickness

The refractive index profile presented in Section 2.3 assumes that there are no matching layers present around the lens. The matching layers affect the ray propagation; therefore, the lens must be adjusted. An option is to compensate for this additional bending by changing the thickness of the lens core, multiplying it by a compensation factor $h_{comp} < 1$. This was done manually in [36] through trial and error to match the rays' output angle as closely as possible. A code to automatically compute the compensation factor is developed in this project. For an arbitrary ϕ -cut of the lens, the ray tracing is performed, and the output angles of each ray are computed. The output angle is denoted as θ_2^i in Figure 2.1.

For a collimating lens, this angle should be as close to zero as possible across the entire lens aperture. This procedure is first performed for the original height of the core slab, and the average output angle is computed as a measure of the collimation performance. Subsequently, the compensation factor is decreased in steps of 0.1, and the average output angle is recalculated at each step. This process is repeated until the average output angle increases again, indicating that the optimum lies between the last and the third-last evaluated compensation factors. Here, it is assumed that only one local optimum exists.

Let us take the example given in Fig. 2.6. This figure shows the output angles of the rays over ρ , for different compensation factors, showing the different iterations. Figure 2.6a depicts the first iteration, where, as the average increases for $h_{comp} = 0.7$, the optimum is found to be somewhere between 0.9 and 0.7. Subsequently, the same procedure is repeated, but with a step size five times smaller and only running from 0.9 to 0.7. This iterates four times, finding an optimal value of h_{comp} accurate to 0.0012. The steps taken are shown in Figures 2.6a to 2.6d.

Although this optimization still leaves the edge rays in this case with an output angle of -1.5° , it is a significant improvement over the -10° without compensation. Furthermore, the highest power is usually in the center, where the rays have a smaller output angle. For a more accurate output angle compensation, other strategies could still be applied, such as the one explained in Chapter 5.2.

This algorithm currently targets collimating lenses but can be generalized to other designs with different desired phase fronts. By subtracting the desired output angles from the computed output angles, the same optimization procedure can be applied by minimizing the average deviation.

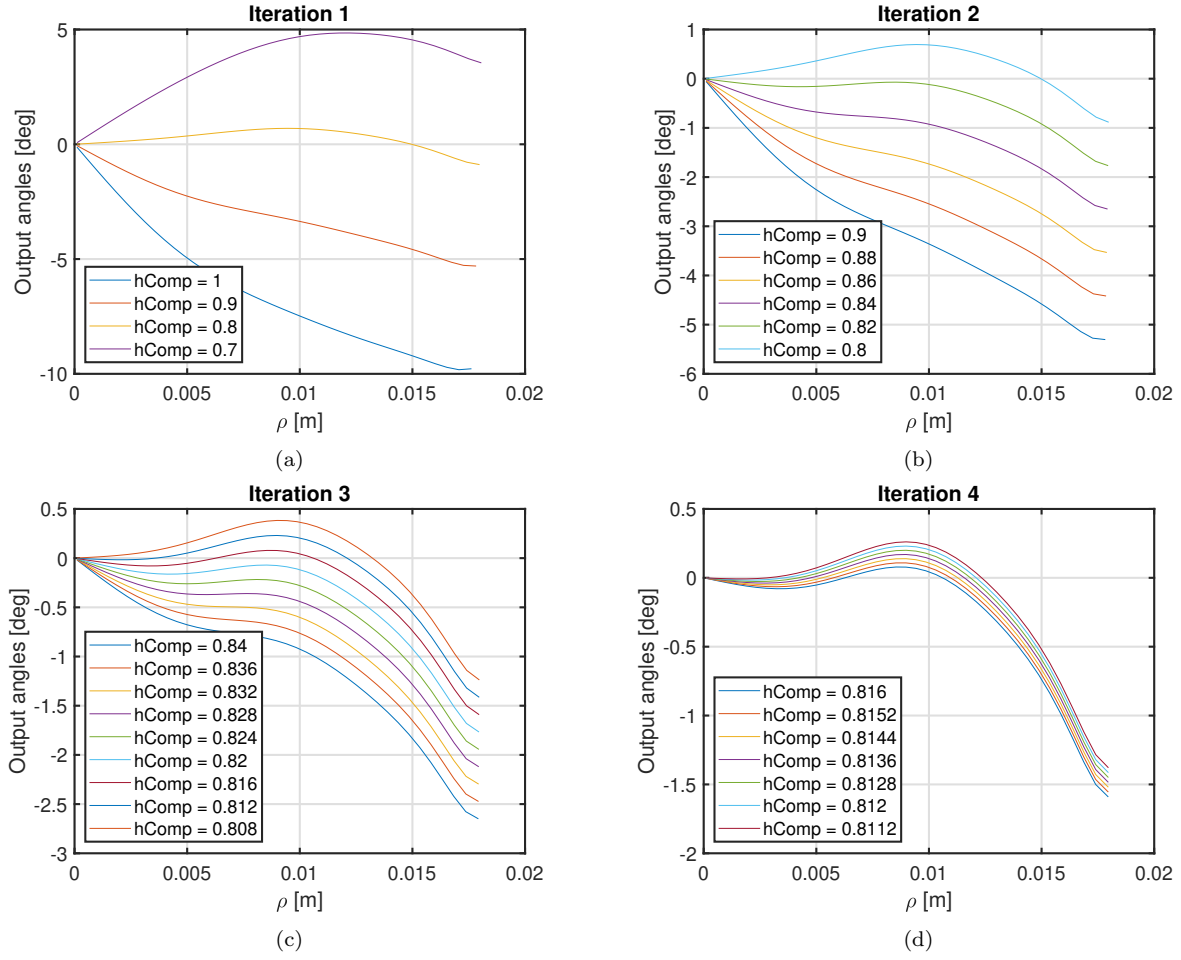


Figure 2.6: Output angles of the rays for different height compensation factors h_{comp} , considered by the height compensation algorithm, cascaded in four iterations.

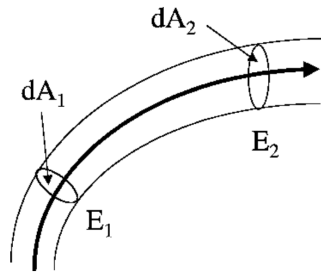


Figure 2.7: Cross-sectional area of a ray tube.

2.6 Ray Tube Theorem

With the amplitude of the electric field at the bottom of the lens assumed to be known, the literature suggests several methods for determining the field transmitted through the lens. One is the ray tube theorem. Here, a differential ray tube is considered rather than a single ray. As the medium is inhomogeneous, the cross-sectional area of the ray tube is expected to change [38, 39]. This is depicted in Figure 2.7[39], where the cross-sections dA_1 and dA_2 differ in size and orientation.

Conservation of Energy

Following the conservation of energy, the power flux through both differential areas must be constant. The following should hold:

$$p_1(dA_1) = p_2(dA_2), \quad (2.9)$$

where p_1 and p_2 refer to the power densities associated to the ray at points 1 and 2. Assuming local plane waves, this can be rewritten as [39]

$$|E_2| = |E_1| \sqrt{\frac{dA_2}{dA_1}}, \quad (2.10)$$

where E_1 and E_2 are the electric fields at the surfaces dA_1 and dA_2 , respectively, as shown in Figure 2.7. A downside to the ray tube theorem is that for each ray, often some guide rays around the center ray are traced as well, to estimate the tube's cross-sectional area and its divergence.

Delaunay Triangulation

To implement the ray tube theorem, a Delaunay-triangulation [40] approach is used. This version is based on the principle that the small wavelengths would already require dense ray sampling. This would remove the need for guide rays, as the closely spaced surrounding rays could fulfill that function, thereby reducing the computation time. The Delaunay triangulation is applied to the rays at the output plane, which is the top of the lens. This draws a set of triangles, using the ray points as vertices for each triangle (Figure 2.9). On the input plane, the same vertices for the triangles are applied. Therefore, the same area attribution is present on the top and bottom planes for each vertex. The choice to define the triangulation on the top aperture of the lens is convenient because the resulting area discretization is later used to evaluate the radiation patterns.

Each vertex represents a ray and can be part of multiple triangles. As there are three vertices per triangle, the area attributed to the i -th ray can be computed as

$$dA_i = \frac{1}{3} \sum_{n=0}^N A_{t,i}^n, \quad (2.11)$$

with N the number of triangles that the ray point is part of, and $A_{t,i}^n$ the area of the n -th triangle sharing the ray vertex. This way, an estimate of the area on the lens's top that belongs to a given ray is obtained, and the same holds for the corresponding area on the lens's bottom.

As the ray tubes cross the surfaces under an angle, the angles of incidence θ_1^i and θ_2^i , as shown in Figure 2.1, need to be taken into account as well. It is assumed that the sampling of the rays is small enough that the phase front is locally flat. The amplitude of the i -th ray after the lens can therefore be written as

$$|E_2^i| = |E_1^i| \sqrt{\frac{\sum_{n=0}^N A_{t,1}^{i,n} \cos(\theta_2^i)}{\sum_{n=0}^N A_{t,2}^{i,n} \cos(\theta_1^i)}}. \quad (2.12)$$

The ratio between $|E_2^i|$ and $|E_1^i|$ is denoted as the divergence factor.

The method described above can be further illustrated with an example. In Figure 2.8, a lens is shown, with dielectric permittivity designed to collimate the rays and with the feed located at the center. Counting from the center, there are 5 rays in ρ ($N_\rho = 5$) that are launched evenly spaced in ρ at the bottom of the lens. This is repeated 12 times equidistantly in ϕ as well ($N_\phi = 12$). From the ray paths exiting the lens on the top, the ray points at the top of the lens are taken, and a Delaunay triangulation is performed, which is drawn in Figure 2.9b. Here, one can see that each ray point has become a vertex of multiple triangles. For one specific ray, the attributed area is highlighted in red and is obtained by connecting the centers of all the triangles surrounding the ray point. Within the red curve, a third of the area of each triangle is enclosed. In Figure 2.9a, the corresponding triangulation at the bottom of the lens is shown. One can observe that the area associated with the top also resembles the area now associated with the bottom, but scaled down, resulting in a higher magnitude. The resulting divergence factor for each ray is illustrated in Figure 2.10.

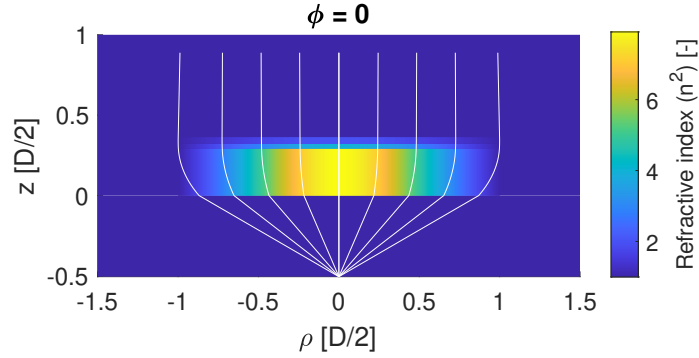


Figure 2.8: A collimating, center-fed ray-tracing example with $N_\rho = 5$, counting from the center, and $N_\phi = 12$, with two matching layers on top.

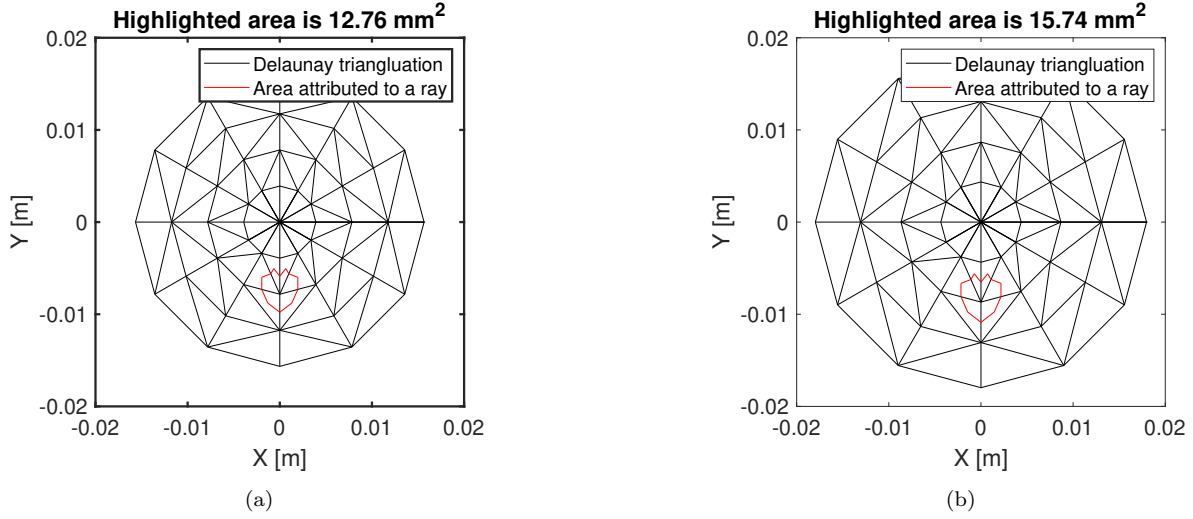


Figure 2.9: An example of the Delaunay triangulation on the (a) bottom and (b) top of the lens, for a center-fed collimating lens, with the area attributed to a ray highlighted for both planes.

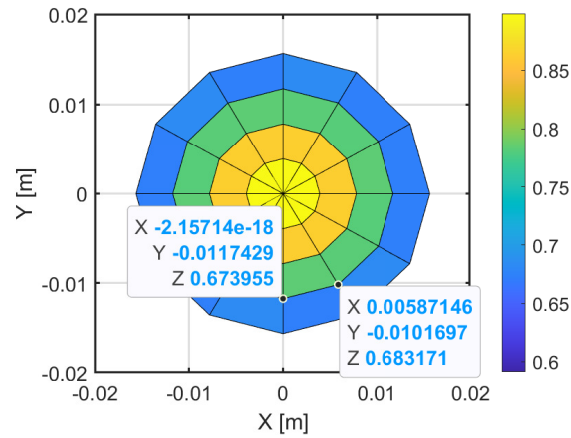


Figure 2.10: The electric field divergence factor at the bottom of the lens, for $N_\phi = 12$.

It can be noted that the triangulation is not fully rotationally symmetric, although the geometry is. However, for well-sampled raytracing, the variation in amplitude along ϕ is small, even for a small number of rays ($N_\phi = 12$), as can be seen in Figure 2.10. This variation does decrease with a higher

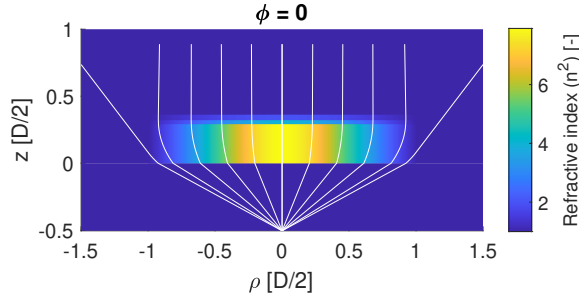


Figure 2.11: Raytracing example ($N_\rho = 6$), where rays exit the sides of the lens, poorly functioning as guide rays.

number of rays. To ensure a suitable Delaunay triangulation pattern and half-wavelength sampling, we choose

$$N_\rho = \max \left[6, \left\lceil 1.25 \frac{D/2}{\lambda_0 \sqrt{\epsilon_{r,\text{out}}}/2} \right\rceil + 1 \right], \quad (2.13)$$

where the second term computes the number of rays required to have at least half-wavelength spacing in the cross-section in the top medium. A safety factor of 1.25 is applied to account for possible non-uniform spacing of the rays at the lens exit. The $\lceil \cdot \rceil$ operator denotes that the value is rounded up to the nearest integer. The first term, with value 6, is used to ensure a well-sampled Delaunay triangulation pattern.

A similar approach is applied to compute N_ϕ ,

$$N_\phi = \max \left[16, 4 \left\lceil \frac{1}{4} \left(\frac{\pi D}{\lambda_0 \sqrt{\epsilon_{r,\text{out}}}/2} + 1 \right) \right\rceil \right], \quad (2.14)$$

where the half-wavelength sampling at the outer ring is ensured. The number is rounded up to a multiple of four, to ensure that there are samples at each $\pi/2$ -cut.

Limitations of the Ray Tube Theorem

The proposed method has limits, since the assumption that rays can serve as each other's guide rays is entirely dependent on the sampling of the rays. As can be seen in Fig. 2.11, which depicts another lens with slightly different ray sampling, the outermost rays follow a completely different path than their neighbors. One can conclude that these rays would not suffice as guide rays. Hence, the method requires a much denser sampling of the rays in these cases. Such poorly sampled situations might also occur when certain rays are designed to bend inward, whilst a section adjacent to them is designed to bend outward. To avoid poorly sampled situations, it is decided to only take into account rays that exit the lens top. A binary search is performed to find the outermost ray that satisfies this condition.

Other problems tend to arise in the presence of critical angles and caustics, which are rays that cross each other, as shown in Figure 2.12. The four outermost rays enter the matching layers at an angle above the critical angle, causing total internal reflection. It is therefore impossible to track the ray paths further, and these rays are consequently excluded from the Delaunay triangulation. The fifth ray from the edge is located further inward than the sixth ray, leading to the formation of caustics. When one ray crosses another, the corresponding vertex may be assigned to a different triangle, causing some areas to be counted twice. This can be seen in the corresponding Delaunay triangulation in Figure 2.13, where the sequence of the outermost rings at the bottom in Figure 2.13a is not the same as the one at the top surface in Figure 2.13b, leading to overlapping triangles. This small overlap will result in a larger total area at the bottom than in reality, leading to a computed amplitude loss that is too high and violating energy conservation.

Despite the mentioned limitations, it is assumed that the tool will be used to design mainly collimating lenses, in which neither of these two effects should occur.

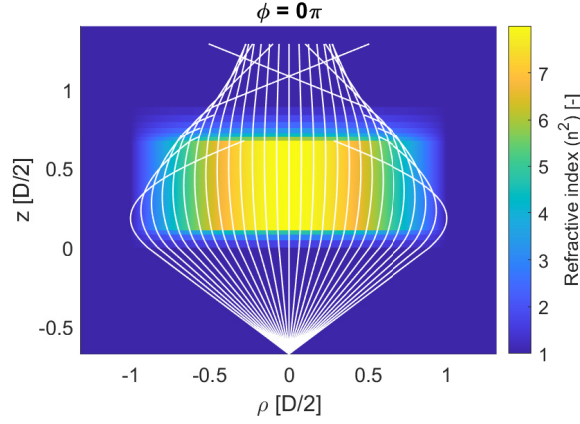


Figure 2.12: The ray tracing of a lens, with caustics and critical angles appearing, $N_\rho = 15$.

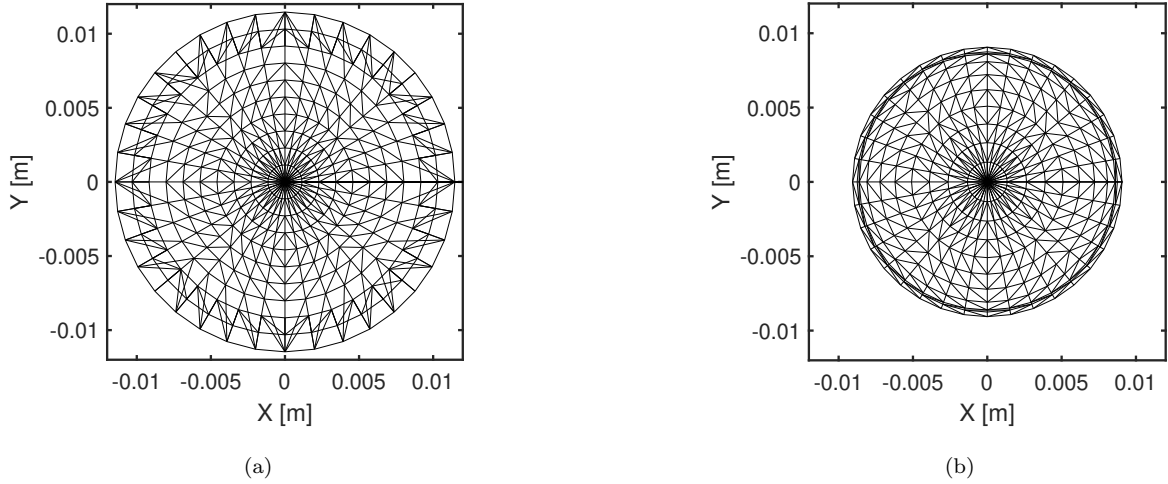


Figure 2.13: The Delaunay triangulations of a lens, where caustics and critical angles are present, for the (a) bottom plane and (b) top plane.

2.7 Polarization Tracking

The procedures described in the previous sections allow us to estimate the phase and amplitude of the electric field propagated through the lens. However, the ray bending in the lens can also change the orientation of the electric field vector. There are general rotation matrices for rotations about the x -, y -, and z -axes, but it is difficult to determine the amount of rotation about each axis. Therefore, Rodrigues' rotation is performed on the vector. This approach is chosen due to its simplicity and its ability to determine the axis of rotation associated with the ray bending, assuming that the bending can be approximated as a rotation about a single axis. For this purpose, the propagation vectors at the input and output of the lens are required. The rotation axis is then found as

$$\hat{\mathbf{u}} = \frac{\mathbf{k}_1 \times \mathbf{k}_2}{|\mathbf{k}_1 \times \mathbf{k}_2|}, \quad (2.15)$$

where $\mathbf{k}_1$ and $\mathbf{k}_2$ are the propagation vectors of the ray at the input and output plane, respectively. The angle of rotation around this axis is then found by

$$\alpha = \arccos(\mathbf{k}_1 \cdot \mathbf{k}_2). \quad (2.16)$$

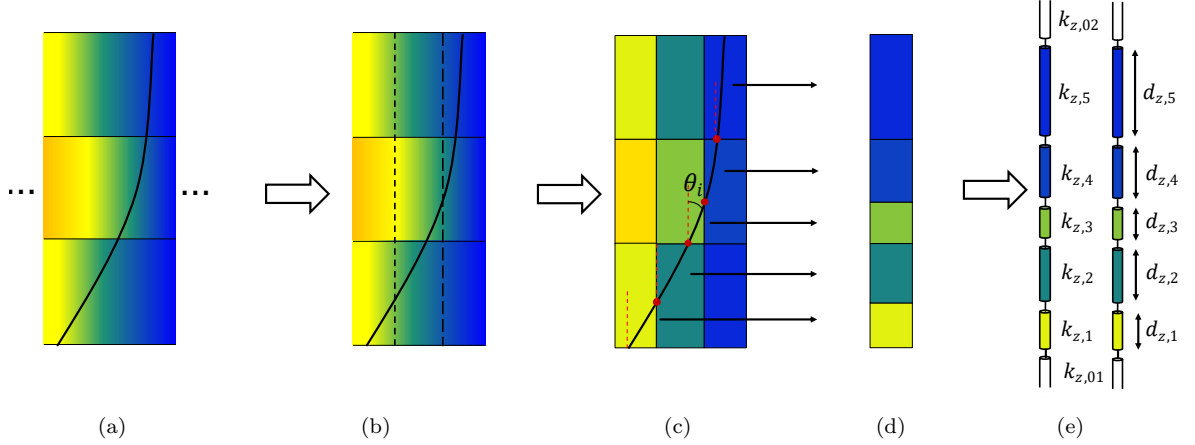


Figure 2.14: In a section of a one-matching-layer ideal inhomogeneous lens, a ray is segmented by the unit cells and matching layers, resulting in a unit cell stack and a transmission line model.

The rotated field can be found using Rodrigues' rotation [41]:

$$\mathbf{E}_2 = \mathbf{E}_1 \cos \alpha + (\hat{\mathbf{u}} \times \mathbf{E}_1) \sin \alpha + (\hat{\mathbf{u}} \cdot \mathbf{E}_1) (1 - \cos \alpha) . \quad (2.17)$$

It is important to note that this rotation only holds when the ray bends around a single axis. For collimating lenses with a centered feed, this is true. A more rigorous approach based on the transport equation can be used in more general cases [42]. This is described in Section 4.1.

2.8 Transmission Line Model

As stated previously, reflections occur at each layer interface and should be accounted for. Since a layer can exhibit multiple internal reflections, calculating the Fresnel coefficients at the interfaces alone is insufficient to determine the transmitted power. Therefore, the multilayer lens structure is modeled as a transmission line.

Figure 2.14 illustrates the procedure step by step. First, each ray is ray-traced through a lens composed of potentially multiple ideal gradient-index layers, as shown in Figure 2.14a. Figure 2.14b depicts how this ideal gradient index is horizontally subdivided into equally spaced unit cells (UCs). These UCs are assigned a uniform dielectric permittivity, set to the value at the center of the UC, shown in Figure 2.14c. This figure also shows how the ray is segmented by the UC and matching-layer boundaries, as indicated by the red dots. These segments are then combine into a unit-cell stack, assumed to be periodic, which serves as the basis for the transmission-line model, as depicted in Figure 2.14d. Each segment i has the height of the part of the UC it traversed ($d_{z,i}$), the dielectric permittivity of the middle of the UC, and an angle θ_i . The angle is computed as the angle of the straight line between the points at which it enters and leaves the unit cell. This stack is then converted to the transmission line, as illustrated in Figure 2.14e.

Using the thickness, angle, and dielectric permittivity, one can obtain the ABCD matrix for a single segment for both TE and TM. The ABCD matrix for a lossless transmission line section is given by

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_i = \begin{bmatrix} \cos(k_{z,i}d_{z,i}) & jZ_{c,i} \sin(k_{z,i}d_{z,i}) \\ jY_{c,i} \sin(k_{z,i}d_{z,i}) & \cos(k_{z,i}d_{z,i}) \end{bmatrix} , \quad (2.18)$$

where $k_{z,i} = k \cos \theta_i$ and $d_{z,i}$ is the height of the i -th transmission line segment. k is the wavenumber in the medium. The characteristic impedances are defined as

$$Z_{c,i}^{TE} = 1/Y_{c,i}^{TE} = kZ_0/(\sqrt{\epsilon_{r,i}}k_{z,i}) \quad (2.19)$$

for transverse electric (TE) modes, and

$$Z_{c,i}^{TM} = 1/Y_{c,i}^{TM} = k_{z,i}Z_0/(\sqrt{\varepsilon_{r,i}}k) \quad (2.20)$$

for transverse magnetic (TM) modes, with Z_0 being the characteristic impedance of free space. As the equations show, the ABCD matrix differs for TE and TM. Hence, the electric field is split into TE and TM modes. The field at the bottom of the lens is projected to the TE and TM unit vectors. The ray tracing, the transmission line, and Rodrigues' rotation are performed separately for these projections. After propagation through the lens, the fields can be summed.

By multiplying the ABCD matrices for each segment, the total ABCD matrix for the entire unit-cell path the ray has propagated through is obtained. Because the input and output angles of the lens differ, the normalization impedances above and below the lens also differ. Therefore, the S-parameters can be computed with the formulas given in [43]:

$$\begin{aligned} S_{11} &= \frac{AZ_{02} + B - CZ_{01}^*Z_{02} + DZ_{01}^*}{AZ_{02} + B + CZ_{01}Z_{02} + DZ_{01}} \\ S_{12} &= \frac{2(AD - BC)\sqrt{R_{01}R_{02}}}{AZ_{02} + B + CZ_{01}Z_{02} + DZ_{01}} \\ S_{21} &= \frac{2\sqrt{R_{01}R_{02}}}{AZ_{02} + B + CZ_{01}Z_{02} + DZ_{01}} \\ S_{22} &= \frac{-AZ_{02}^* + B - CZ_{01}^*Z_{02}^* + DZ_{01}^*}{AZ_{02} + B + CZ_{01}Z_{02} + DZ_{01}} \end{aligned} \quad (2.21)$$

In these equations, the $\{\cdot\}^*$ operator denotes complex conjugation, and Z_{01} and Z_{02} are the (complex) normalization impedances below and above the lens, respectively.

The phase progression through the lens can be computed by taking the phase of S_{12} . This includes the effects of multiple reflections, as opposed to simply taking the optical path length multiplied by the free-space wavenumber. However, S_{12} as computed in (2.21) only takes into account the vertical movement of the ray, not the horizontal movement. Therefore, a phase compensation of

$$k_0 d_\rho \sin \theta n(x, y, z, \theta), \quad (2.22)$$

with d_ρ the horizontal displacement, is added to include the horizontal phase progression. This total phase is thus taken into account as the lens phase contribution.

The same transmission line model can be easily applied to ADLs, where the UC stack methodology can also be applied. In that case, the S-parameters can be computed using the equations in [30, 31, 32, 33].

2.9 Far-Field Computation Through Equivalence Theorem

The combination of ray tracing, the Delaunay triangulation ray-tube theorem, Rodrigues' rotation, and the transmission-line model yields an estimate of the electric field at the top aperture of the lens. Using the equivalence theorem, the far field can then be computed from the aperture electric field using physical optics. This field can be converted into equivalent currents on the aperture. To determine the far field, a Fourier transform of these currents over the aperture is performed. In general, integrating over a non-uniform grid can be cumbersome. However, the Delaunay triangulation method from Section 2.6 has already provided the required differential areas.

As will be shown in Chapter 3, there are different possible configurations of the lens and a possible metal separation plane, with an opening of the same size as the lens. Below, the equivalence theorem is applied to the configuration in which the metal plate is positioned at the height of the lens top surface and extends infinitely in the lateral directions. Figure 2.15 portrays the configuration with the lens, top plate, and a generic feed illuminating the lens.

As the Green's function is difficult to determine for this configuration, the equivalence theorem is applied. First, a surface S , enclosing a volume V , is applied around the configuration, which is portrayed in Figure

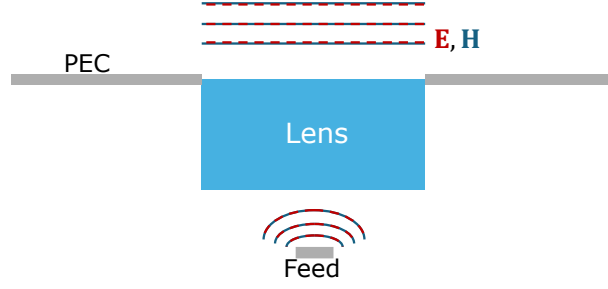


Figure 2.15: A schematic representation of a center cross-section of the top-plate configuration, with a feed radiating.

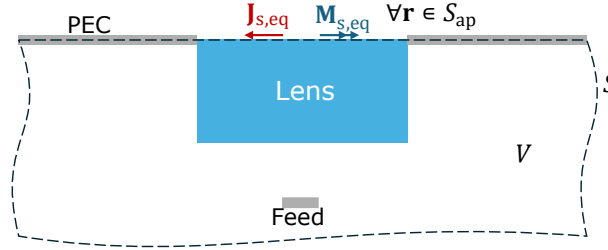


Figure 2.16: A schematic representation of the equivalent problem where the fields inside volume V , enclosed by surface S , are set to zero, and the resulting equivalent surface currents are shown.

2.16. This volume V contains the entire bottom half-space; thus, it is infinitely extended to the sides and the bottom.

The fields inside V are set to be zero, and the fields in the top half-space, outside V , due to the equivalent currents on S , must be identical to those in the original situation to maintain equivalence.

$$\begin{aligned} \mathbf{E}_{\text{eq}} &= 0; \mathbf{H}_{\text{eq}} = 0 & \forall \mathbf{r} \in V \\ \mathbf{E}_{\text{eq}} &= \mathbf{E}; \mathbf{H}_{\text{eq}} = \mathbf{H} & \forall \mathbf{r} \notin V. \end{aligned} \quad (2.23)$$

The currents on the surface can be computed, following the boundary conditions:

$$\mathbf{J}_{s,\text{eq}} = \hat{\mathbf{z}} \times (\mathbf{H}_{\text{eq}}(\mathbf{r}_s^+) - \mathbf{H}_{\text{eq}}(\mathbf{r}_s^-)) \quad (2.24a)$$

$$\mathbf{M}_{s,\text{eq}} = -\hat{\mathbf{z}} \times (\mathbf{E}_{\text{eq}}(\mathbf{r}_s^+) - \mathbf{E}_{\text{eq}}(\mathbf{r}_s^-)), \quad (2.24b)$$

where $\mathbf{r}_s^+$ and $\mathbf{r}_s^-$ are locations on the surface boundary just above and below it, respectively.

As the bottom half-space is infinitely extended, the currents on the bottom and side surfaces are neglected. On the PEC surface, the tangential electric field is zero, causing the equivalent magnetic current to vanish. Additionally, from the image theorem, the electric surface current density cancels out with its image. Therefore, only on the lens aperture surface S_{ap} equivalent magnetic and electric currents are present, $\mathbf{M}_{\text{eq}}$ and $\mathbf{J}_{\text{eq}}$, respectively, as depicted in Figure 2.16.

The volume V is then filled with PEC, as portrayed in Figure 2.17. Again, the electric currents cancel out due to the image theorem, leaving only $\mathbf{M}_{s,\text{eq}}$ on the aperture, which can be expressed as

$$\mathbf{M}_{s,\text{eq}} = -\hat{\mathbf{z}} \times (\mathbf{E}_{\text{eq}}(\mathbf{r}_s^+) - \mathbf{E}_{\text{eq}}(\mathbf{r}_s^-)) = -\hat{\mathbf{z}} \times (\mathbf{E}_{\text{ap}} - 0) = -\hat{\mathbf{z}} \times \mathbf{E}_{\text{ap}}. \quad (2.25)$$

Figure 2.18 shows that, applying the image theorem, the PEC can be removed, resulting in free space everywhere and only magnetic currents on S_{ap} , with double amplitude.

Using the free-space spectral Green's functions for electric fields from magnetic currents $\bar{\bar{G}}_{\text{EM,fs}}$ and the Fourier transform of the aperture equivalent current distribution $\mathbf{M}_{s,\text{eq}}$, indicated as $\tilde{\mathbf{M}}_{s,\text{eq}}$, the electric far field can be computed according to

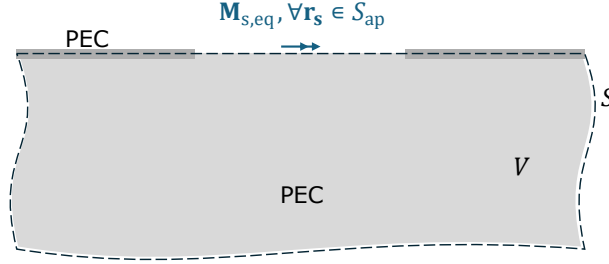


Figure 2.17: A schematic representation of the equivalent problem where volume V , enclosed by surface S , is filled with PEC, and the resulting equivalent surface currents are shown.

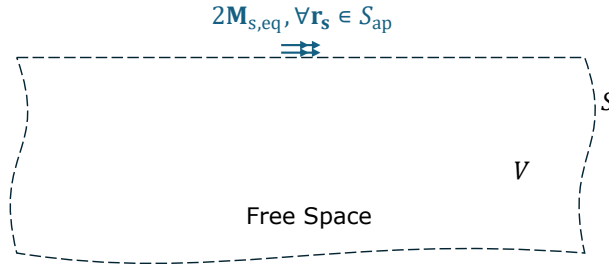


Figure 2.18: A schematic representation of the equivalent problem where the PEC is removed from volume V , enclosed by surface S , and the resulting equivalent surface currents are shown.

$$\mathbf{E}_{\text{FF}} \approx j k_z \bar{\bar{G}}_{\text{EM,fs}}(k_{xs}, k_{ys}, z, z') 2\tilde{\mathbf{M}}_{s,\text{eq}}(k_{xs}, k_{ys}) \frac{e^{-jk_0 R_{\text{FF}}}}{2\pi R_{\text{FF}}} . \quad (2.26)$$

A similar approach can be applied to configurations with a bottom plate and the one without. This yields a similar result, but the magnetic currents on the rest of the top surface must also be taken into account, not just those on the lens top aperture. As the model only computes the currents on that aperture, the currents on the rest are neglected; these configurations inherently cannot be simulated with full accuracy.

2.10 Figures of Merit

To assess the performance of the flat lens system, some figures of merit are defined. These include spillover efficiency, amplitude taper loss (ATL), phase error loss (PEL), reflection efficiency, and directivity.

Spillover Efficiency

The spillover efficiency is the fraction of radiated power that effectively illuminates the lens. It is determined by

$$\eta_{\text{SO}} = \frac{\int_0^{2\pi} \int_0^{\theta_0} |E_{\text{feed}}(\theta)|^2 \sin(\theta) d\theta d\phi}{\int_0^{2\pi} \int_0^{\pi} |E_{\text{feed}}(\theta)|^2 \sin(\theta) d\theta d\phi} , \quad (2.27)$$

where θ_0 is the subtended angle. The subtended angle is taken to be the angle of the last ray exiting the lens at the top.

Phase and Amplitude Taper Efficiency

Assuming the aperture S_{ap} at the top of the lens is in the x - y plane, and the maximum of the radiation pattern is in boresight, the efficiency due to PEL can be computed as [44]

$$\eta_{\text{PEL}} = \frac{\left| \iint_{S_{\text{ap}}} E_{\text{ap}}(x, y) dS \right|^2}{\left(\iint_{S_{\text{ap}}} |E_{\text{ap}}(x, y)| dS \right)^2} \quad (2.28)$$

and the efficiency due to ATL as

$$\eta_{\text{ATL}} = \frac{\left(\iint_{S_{\text{ap}}} |E_{\text{ap}}(x, y)| dS \right)^2}{A \iint_{S_{\text{ap}}} |E_{\text{ap}}(x, y)|^2 dS}, \quad (2.29)$$

where E_{ap} is the field atop the lens and A is taken to be the sum of the rays' attributed areas on the top of the lens. The multiplication of the η_{ATL} and η_{PEL} gives the taper efficiency η_{tap} .

Reflection Efficiency

The reflection efficiency is determined by

$$\eta_{\text{refl}} = 1 - \frac{\iint_{S_{\text{bot}}} |E_{\text{bot}}(x, y)|^2 |S_{11}(x, y)|^2 dS}{\iint_{S_{\text{bot}}} |E_{\text{bot}}(x, y)|^2 dS}, \quad (2.30)$$

where $|E_{\text{bot}}|$ is the magnitude of the electric field at the bottom surface of the lens S_{bot} , and S_{11} is the reflection at the bottom of the lens for each ray, defined with the transmission line model.

Directivity

Lastly, the directivity can be derived from the far-field pattern. This is given by

$$\text{Dir}(\theta, \phi) = \frac{4\pi U(\theta, \phi)}{\int_0^{2\pi} \int_0^{\pi/2} U(\theta, \phi) d\theta d\phi}, \quad (2.31)$$

where U is the radiation intensity, given by

$$U(\theta, \phi) = \frac{E_{\text{FF}}^2(\theta, \phi) r^2}{2\zeta}. \quad (2.32)$$

The surface integral for the directivity is computed over only the top hemisphere in this case. The maximum gain can then be computed using the spillover efficiency and reflection efficiency, according to

$$G = \max(\text{Dir}(\theta, \phi)) \eta_{\text{SO}} \eta_{\text{refl}}. \quad (2.33)$$



Chapter 3

Model Validation

All methods have been implemented in a MATLAB tool for flat-lens analysis. Validation is performed by comparing its results with full-wave CST simulations.

A 36-mm-diameter collimating lens is studied at 70 GHz. The lens is illuminated by a 2-mm-long dipole with a 1-mm feed gap, excited by a delta-gap source, as shown in Figure 3.1a. The dipole is first simulated without the lens to obtain its far-field radiation pattern. This pattern is then exported to a file, which is read by the MATLAB analysis tool and used to illuminate the lens. MATLAB's `interp2` function is used to interpolate the primary electric field of the dipole onto the ray-tracing grid. The field exported from CST is defined on a spherical surface with a radius of 1 m. It is first back-propagated to a spherical surface centered at the phase center with a radius equal to the focal distance (the focal sphere). This assumes that the lens is located in the far field of the source. The amplitude and phase of the field at the bottom surface of the lens are then obtained from the field on the focal sphere. The field amplitude is scaled by the cosine of the angle, while the phase is adjusted according to the additional propagation distance from the focal sphere to the lens. The incident field at the bottom surface of the lens is thus given by

$$\mathbf{E}_{\text{bot}}(\theta, \phi) = \mathbf{E}_{\text{feed}}^{\text{interp}}(\theta, \phi) \frac{e^{jk_0(1-F)}}{F} \cos \theta e^{jk_0 \text{OPL}_{\text{FtoLens}}}, \quad (3.1)$$

where the first spherical-wave term accounts for the phase and amplitude adjustment from the far-field sphere to the focal sphere, the cosine term accounts for the amplitude scaling from the focal sphere to the lens, and the final exponential term accounts for the phase accumulated along that path.

The total phase from the feed to the top aperture of the lens is obtained by combining the phase accumulated from the feed to the focal sphere, the optical path length (OPL) from the focal sphere to the bottom of the lens, and the phase of the S_{12} -parameter from the transmission through the lens.

In the model proposed in this work, only rays exiting through the top surface of the lens are considered in the far-field calculation. In contrast, CST also accounts for fields that propagate around the lens, which can affect the radiation pattern. Therefore, three different simulation setups are considered in CST:

- A simulation with a metal plate at the bottom of the lens, with an opening of the same size as the lens (see Figure 3.1a). In this case, most of the spillover field is not considered in the far-field calculation, although some power can exit the lens through the sides. Some diffraction from the metal plate edges is expected.

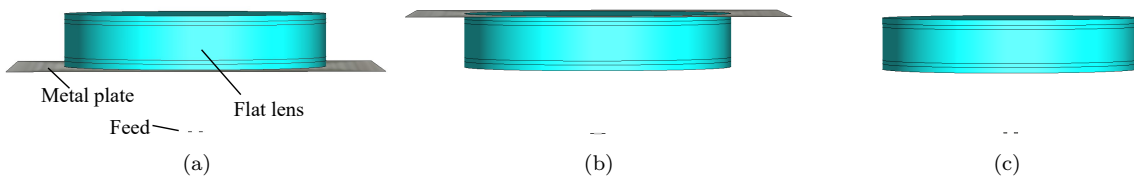


Figure 3.1: Different plate configurations, with (a) the plate at the bottom of the lens, (b) the plate at the top of the lens, and (c) no plate present.

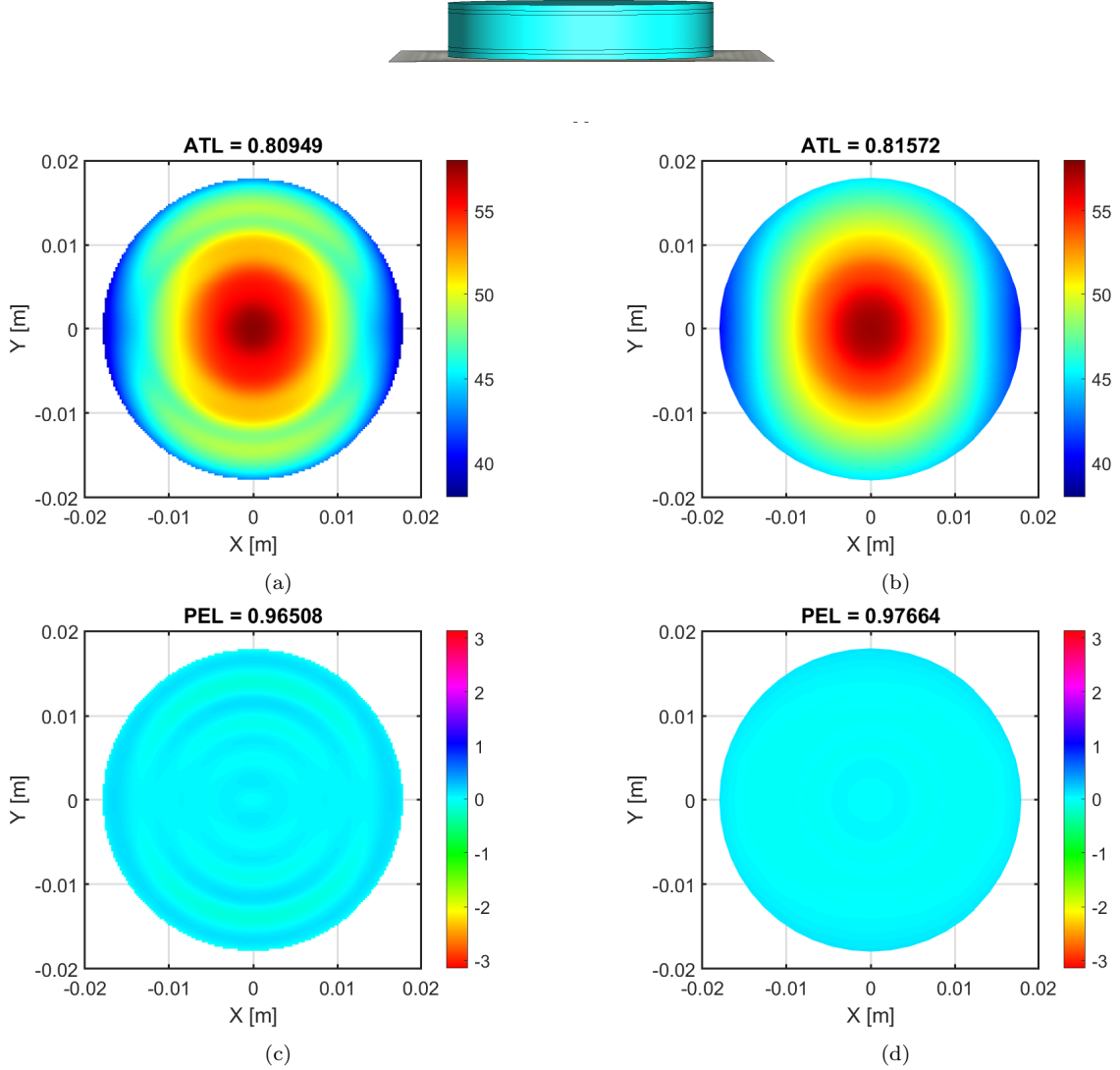


Figure 3.2: Comparison between CST (with bottom metal plate) and the proposed simulation method for the amplitude and phase of the x -oriented field at the aperture for a lens with two matching layers atop and below, 36 mm diameter, 5.7 mm height, and $n_{\text{side}} = 1$: amplitude of E-field from (a) CST and (b) the proposed method; phase of E-field from (c) CST and (d) the proposed method.

- A simulation with a metal plate at the top of the lens, with an opening of the same size as the lens (see Figure 3.1b). In this case, the spillover field is not considered in the far-field calculation. Some diffraction from the metal plate edges is expected.
- A simulation with only the feed and the lens (see Figure 3.1c). In this case, the field that does not illuminate the lens ends up directly in the far-field radiation patterns, mostly in the sidelobes.

3.1 Matching Layers, Bottom Metal Plate, $n_{\text{side}} = 1$

The first simulation is a lens with two matching layers atop the core and two below it. This lens is 36 mm in diameter and 5.7 mm in height, with vacuum as background material. Using the algorithm described in Section 2.3, the refractive index distribution is computed for this geometry. Here, the minimum refractive index, or n_{side} , is set to 1. A separating plane from a perfect electric conductor (PEC) is placed at the bottom of the lens to consider only rays that illuminate the lens for the far-field computation in CST.

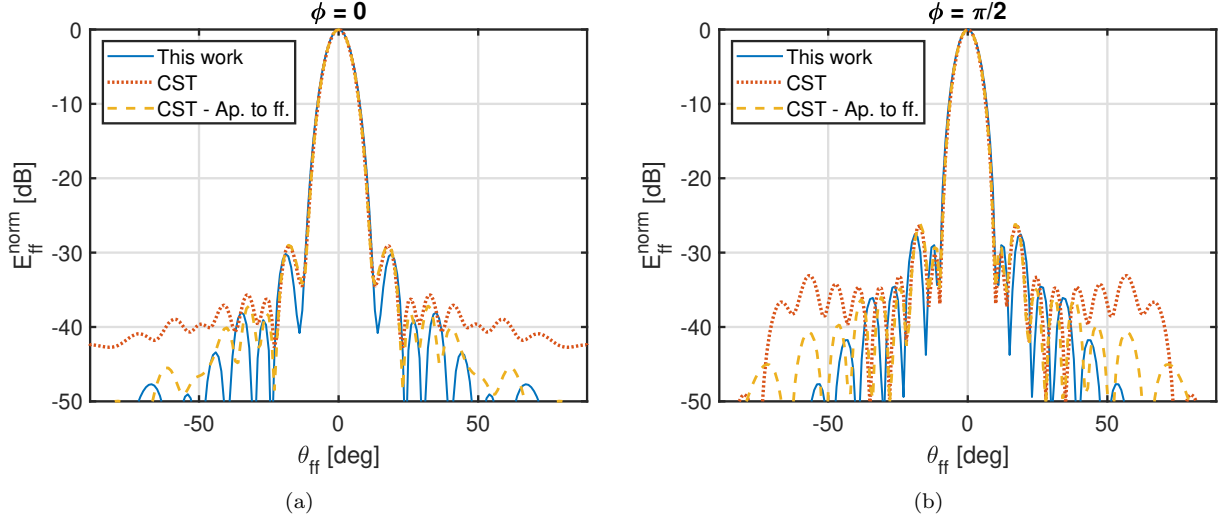


Figure 3.3: Far-field pattern of a lens with 2 matching layers above and below, 36 mm diameter, 5.7 mm height, and $n_{\text{side}} = 1$. Computed with CST and the analysis method proposed in this work. (a) $\phi = 0$ plane, (b) $\phi = \pi/2$ plane.

Aperture Fields

Figure 3.2 shows the total amplitude and the phase of the x -component of the E -field at the lens top aperture. The x -component is the main field contributing to the far field, since the feeding dipole is polarized along x . Both the results from the MATLAB code and CST are shown for comparison. Furthermore, η_{ATL} and η_{PEL} are reported above the amplitude and phase plots, respectively. It is important to note that all fields at the aperture are evaluated 1 mm above the interface of the last matching layer to avoid including reflections from that interface.

Figures 3.2a and 3.2b show the amplitude at the aperture for CST and the proposed simulation method, respectively, allowing a comparison between the two. Visually, the results are in good agreement. However, the CST simulation exhibits some oscillations that are not present in the results from the simulation tool. Despite this, the ATL efficiencies are very similar in both cases.

Figure 3.2d, showing the aperture (normalized) phase from the proposed simulation method, indicates a nearly flat phase front. A similar behavior is observed in the CST results, shown in Figure 3.2c, although some slightly varying ring-like features are present. These rings appear to correspond to the lobes observed in the CST amplitude distribution.

From the aperture phase and amplitude, it can be concluded that the fields computed by the model presented in this work agree well with the full-wave simulations. The far field can also be compared.

Far-Field Patterns

In Figure 3.3, the far-field patterns from CST and from the proposed tool are shown for the $\phi = 0$ and $\phi = \pi/2$ planes. The plots show the normalized amplitude of the electric far field as a function of the far-field angle θ_{ff} . In addition, the far field is computed from the CST aperture field and included for comparison. The difference between the red and yellow curves reflects the influence of spillover and diffraction effects at the PEC plane.

The tool's main lobe is shown to agree well with both CST results. In the $\phi = 0$ plane, the first sidelobe also shows good agreement, although it is slightly shifted in both amplitude and angle. The first null is deeper in the proposed simulation than in CST. At higher angles, larger discrepancies are observed between the tool and CST; however, these occur at levels below -35 dB relative to the maximum. In the $\phi = \pi/2$ plane, the first sidelobe is in very good agreement, while the second shows reasonable agreement. Furthermore, CST exhibits larger deviations from the other two results at higher angles. These discrepancies are attributed to two effects: rays exiting through the side of the lens and diffraction at the edges of the opening in the separation plane.

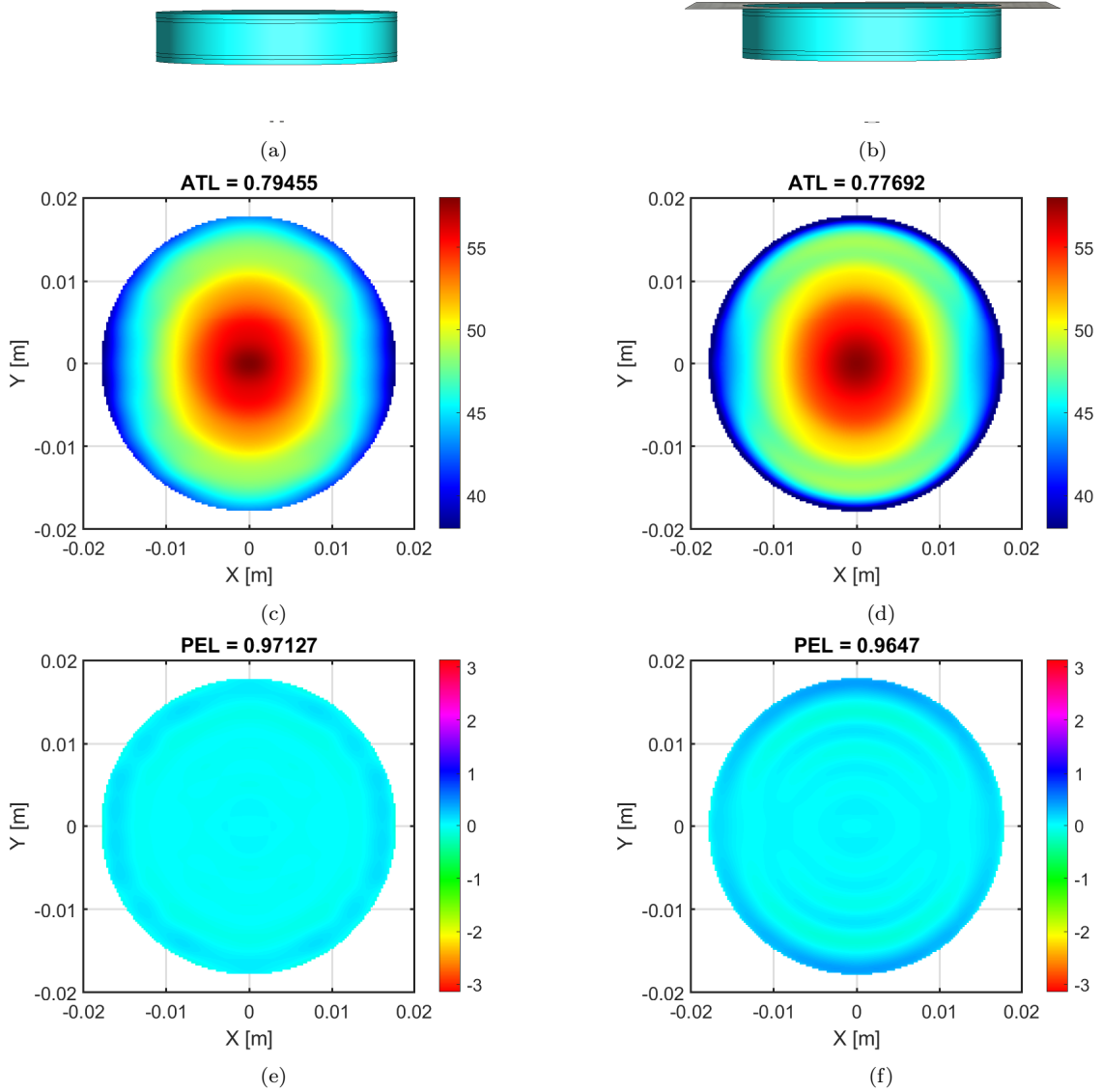


Figure 3.4: Comparison of the amplitude and phase of the x-oriented E-field at the aperture for a lens with two matching layers atop and below, 36 mm diameter, 5.7 mm height, and $n_{\text{side}} = 1$. CST simulation with (a), (c), and (d) no separation plane, and (b), (d), and (f) a separation plane present at the top of the lens.

In terms of maximum directivity, the tool agrees well with CST, yielding 27.4 dBi for the tool and 27.7 dBi for CST with the bottom plate.

3.2 Matching Layers, No or Top Metal Plate, $n_{\text{side}} = 1$

Aperture Fields

As seen in the previous section, the bottom separation plane causes unwanted diffraction effects, and power can still flow from the lens sides. Therefore, a simulation is performed where this plane is removed, and where the plane is moved to the top of the lens, as shown in Figure 3.4a and 3.4b respectively. The amplitude and phase on the lens top aperture for both these configurations are shown in Figure 3.4. Looking at the amplitudes, which are depicted in Figures 3.4c and 3.4d, it can be seen that the additional lobes disappear when the separation plate is removed. This shows that the absence of these lobes was not an error of the tool, but rather a failure to take certain effects into account. Furthermore, comparing the phases, shown in Figures 3.4d and 3.4f, the peaks and valleys have become much smaller without the metal plate. A small ring around the edge is still not fully in phase with the rest of the

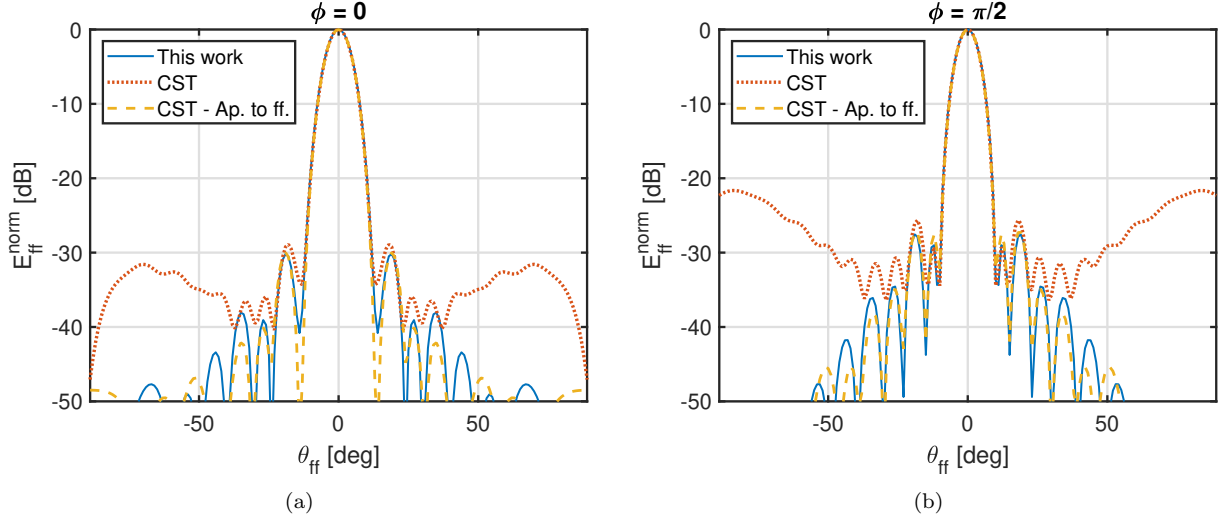


Figure 3.5: Comparison between CST and the method in this work for the far field for a lens with 2 matching layers atop and below, 36 mm diameter, 5.7 mm height, and $n_{\text{side}} = 1$. There is no separation plane present.

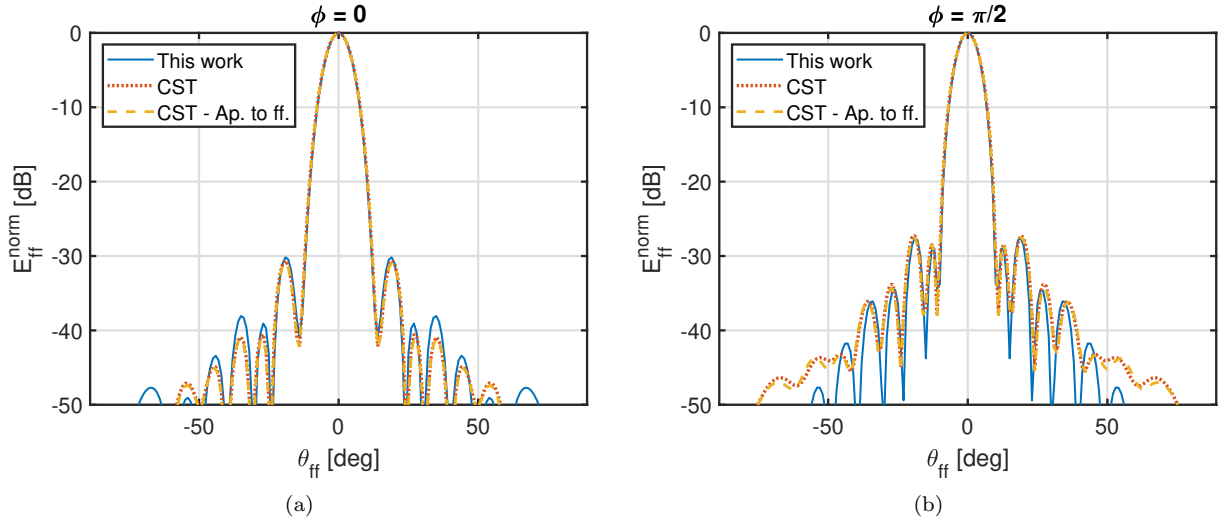


Figure 3.6: Comparison between CST and the method in this work for the far field for a lens with two matching layers atop and below, 36 mm diameter, 5.7 mm height, and $n_{\text{side}} = 1$. A separation plane is present at the top of the lens.

aperture. It is assumed that this is caused by the mismatch in dielectric permittivity resulting from the polynomial fit, which is more pronounced near the edges. The difference in η_{PEL} has also decreased. For the simulation with the top separation plate present, additional amplitude lobes and phase peaks similar to those observed for the bottom separation plate are visible. Consequently, the mismatch in both η_{PEL} and η_{ATL} is slightly larger than in the case without the separation plane.

Far-Field Patterns

The resulting far-field patterns can be seen in Figure 3.5, without a metal plate present, showing good agreement with CST for the main lobe and first side lobe. At larger angles, there is a large discrepancy between CST and the truncated apertures. This is because the fields propagating alongside the lens are also taken into account, rather than only the rays exiting the sides of the lens. As a result, the maximum directivity for the configuration without a separation plate is 25.7 dBi, which is significantly lower than the 27.4 dBi predicted by the model presented in this work. An additional CST simulation is performed in which a separation plate is placed above the lens. This plate blocks both the rays exiting the sides of the lens and those that completely miss the lens. Under these conditions, the CST far field and the far

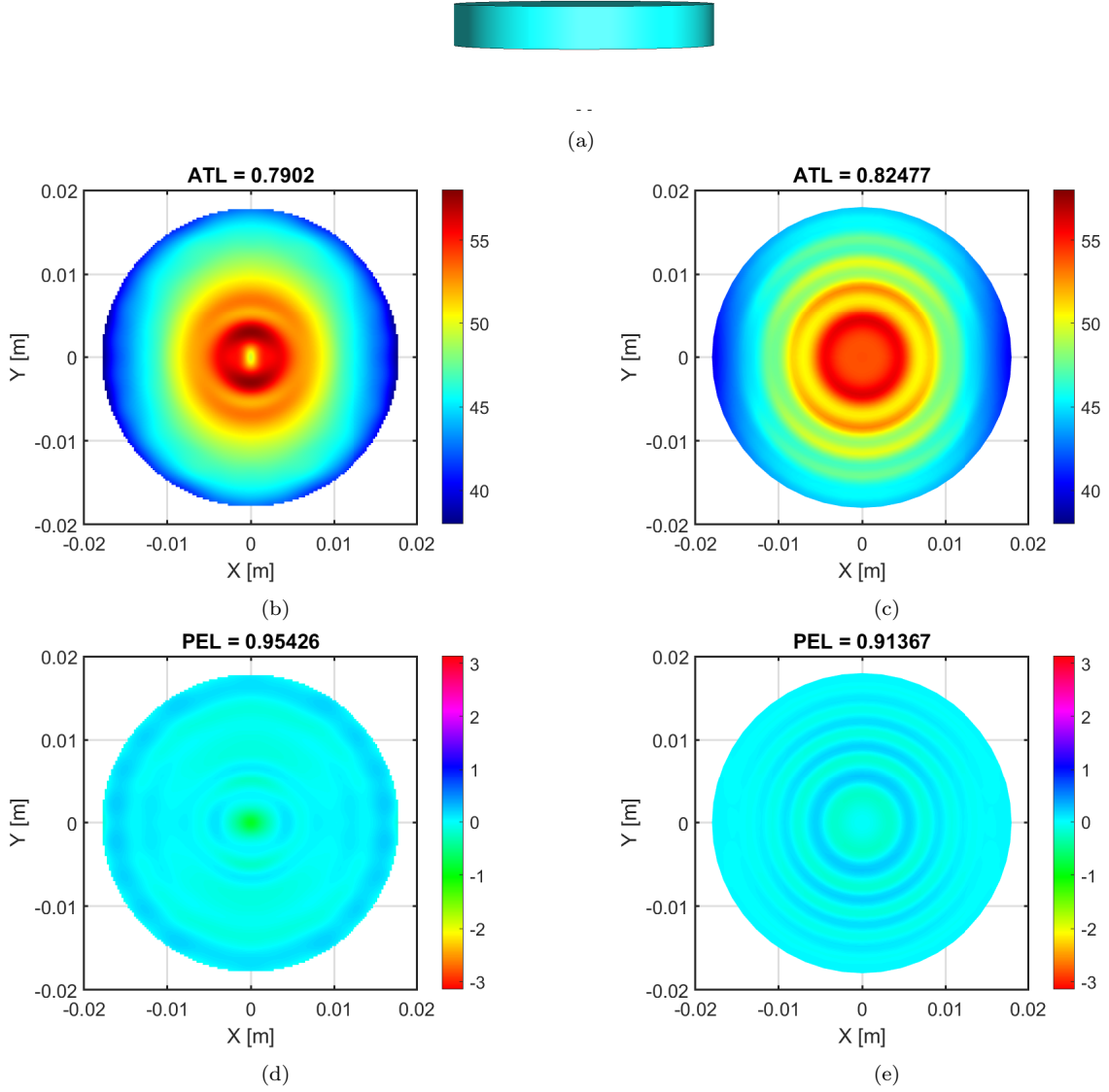


Figure 3.7: Comparison between CST and the method in this work for the amplitude and phase of the x-oriented field at the aperture for a lens without matching layers, 36mm diameter, 5.7mm height, and $n_{\text{side}} = 1$. (a) Geometry of CST simulation: a dipole and the lens, including a metal plate. (b) Amplitude of E-field from CST and (c) from the proposed analysis method. (d) Phase of E-field from CST and (e) from the proposed analysis method.

field computed from the CST aperture are in close agreement. Furthermore, the far field predicted by the proposed tool shows improved agreement, closely matching the CST results. This indicates that the tool accurately models the behavior of the rays exiting the top of the lens under these conditions. Only the far-field response at large observation angles deviates from the CST results, which can be attributed to diffraction caused by the separation plate. Nevertheless, the maximum directivity agrees closely, with the proposed tool predicting 27.3 dBi, resulting in a difference of only 0.1 dB compared with the CST simulation.

3.3 No matching layers, $n_{\text{side}} = 1$

Aperture Fields

Another case to consider is a lens with the same diameter and focal distance, but without any matching layers. No separation plane is included in this case. The amplitude and phase are plotted in Figure 3.7.

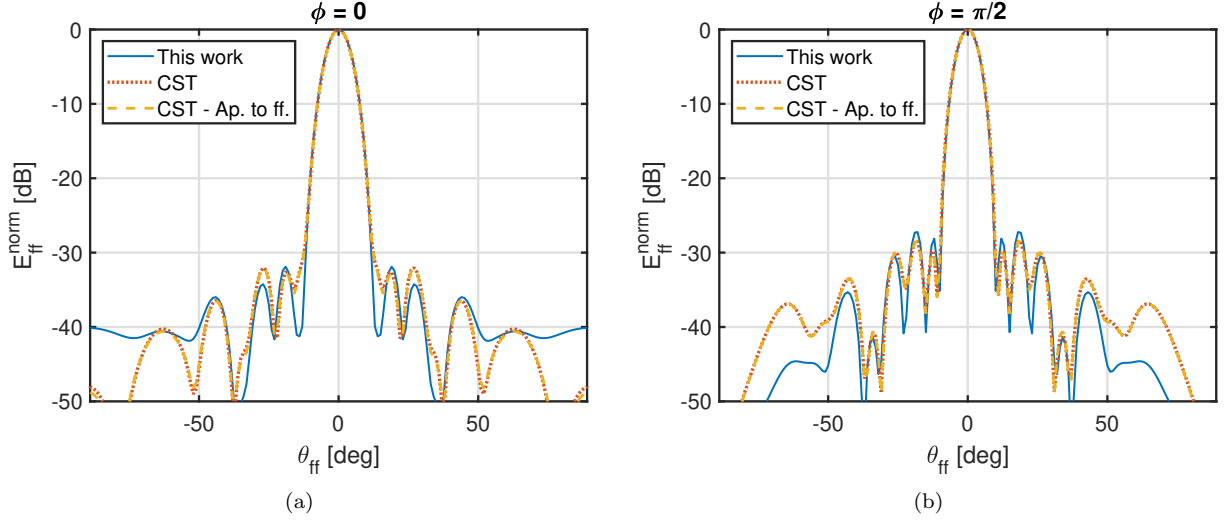


Figure 3.8: Comparison between CST and the method in this work for the far field for a lens without matching layers, 36mm diameter, 5.7mm height, and $n_{\text{side}} = 1$. A separation plane is present at the top of the lens.

An increase in amplitude and phase variations is observed for both CST and the tool, due to increased reflections at the lens-air interfaces. The discrepancy between the tool and CST is more visible. Figure 3.7d shows a relatively flat phase front, except for a large drop in the center and the same edge effects. The tool computes a phase distribution that varies more strongly, but it does not exhibit the central drop. Furthermore, a higher number of phase rings is present compared to the CST simulation. Similar differences are also observed in the amplitude plots. As a result, the differences between η_{ATL} and η_{PEL} increase, although they still remain in reasonable agreement.

Far-Field Patterns

The far-field patterns of this configuration are also compared, using a separation plane at the top, as this case yielded the best agreement in the far-field comparison. Figure 3.8 illustrates the results. In the $\phi = \pi/2$ plane, the results remain closely comparable across multiple sidelobes. In the $\phi = 0$ plane, a larger discrepancy is observed in the first null and the first sidelobe.

Nevertheless, the comparison shows that, for a simple flat lens without matching layers and with top metal plate, the tool still provides a good estimate of the main lobe and sidelobe levels.

3.4 Matching Layers, $n_{\text{side}}=1.5$

Aperture Fields

As the lens material will most likely have a dielectric permittivity higher than that of free space, a set of simulations is performed in which n_{side} is set to 1.5. Figure 3.9 shows the comparison of the amplitude and phase, for a simulation without a metal plate. In the figure, an interesting effect is observed. Due to the permittivity contrast between the lens edges and the surrounding medium, internal reflections occur. As the field is mainly x -polarized, this reflection is stronger at the boundaries where $x = 0$ (TE incidence) and weaker at the boundaries where $y = 0$ (TM incidence). This is clearly visible in Figure 3.9a, which shows the aperture amplitude from the CST simulation. The reflection at the vertical lens boundaries is not accounted for in the tool, as can be seen in Figure 3.9c, which shows the tool-computed aperture amplitude. In terms of phase, this effect also has a strong influence, leading to significant variations at the outer rim of the aperture, as depicted in the CST and tool phase distributions in Figures 3.9d and 3.9e.

Far-Field Patterns

Figure 3.10 shows the far field of the tool and the CST simulation with a separation plane placed at the top of the lens, as this configuration yielded the best agreement for the $n_{\text{side}} = 1$ lens. In this case, the

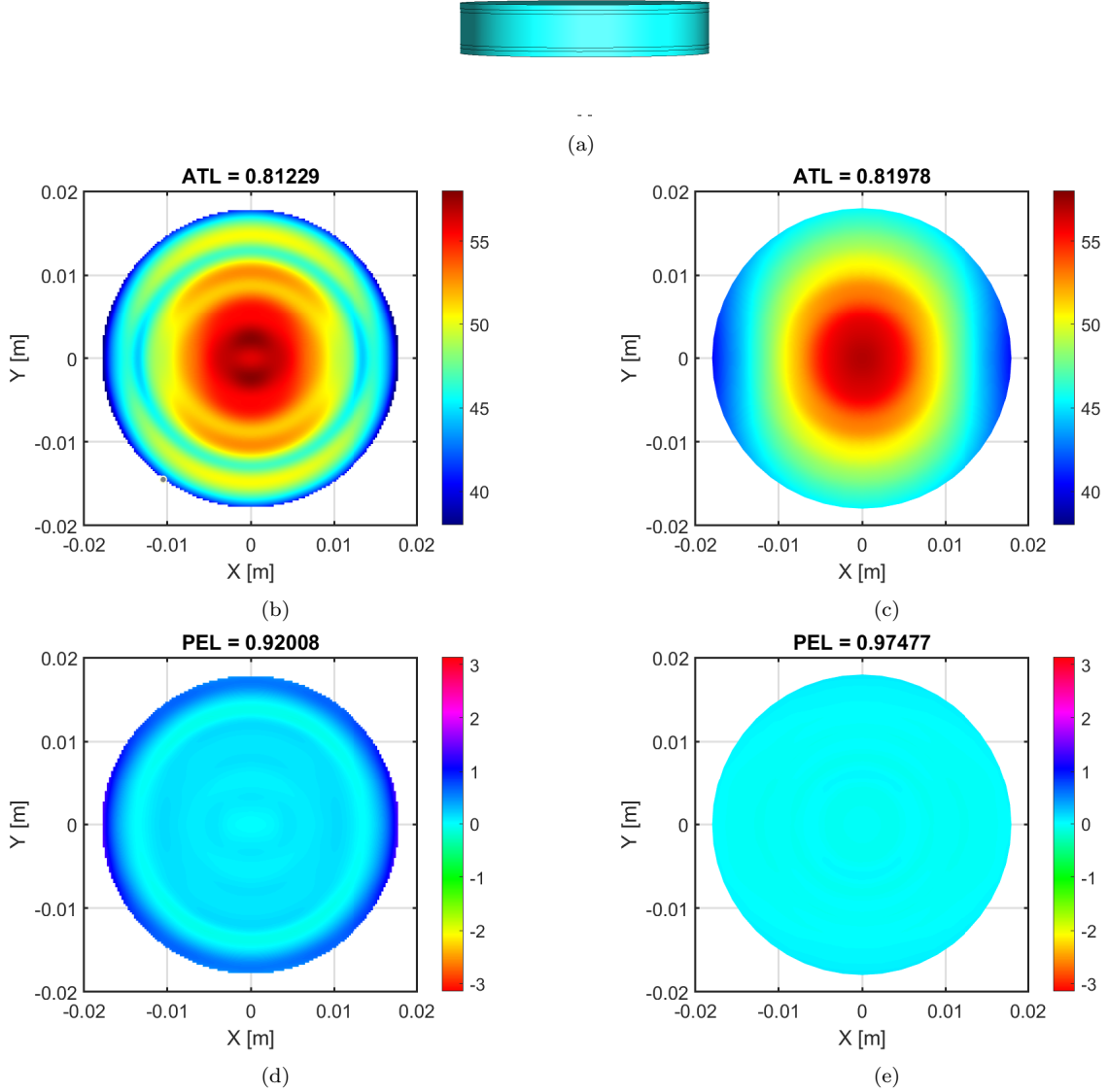


Figure 3.9: Comparison between CST and the method in this work for the amplitude and phase of the x-oriented field at the aperture for a lens with two matching layers atop and below, 36mm diameter, 5.7mm height, and $n_{\text{side}} = 1.5$. No separation plane is present.

agreement is slightly altered. The nulls in the CST sidelobes are less pronounced, which is not captured by the tool. However, the field levels remain around -40 , dB. This also leads to a reduction in taper efficiency of approximately 3% on average.

3.5 Discussion on Edge Effects

All simulations shown in this chapter have been performed using a short dipole with a wide radiation pattern. Because the feed is placed relatively close to the lens, a noticeable amplitude taper is present in the field at the bottom of the lens, which reduces perturbations due to internal reflections at the lens boundaries. As the dipole is moved further away, this taper decreases, and edge reflections may become more significant. In practice, however, more directive feeds are typically used, which introduce a comparable amplitude taper even at larger focal lengths. For further analysis, simulations with more directive feeds are performed.

Another option to dampen edge effects and thus obtain better approximations is to use a bottom plate with an inner radius smaller than that of the lens. This could be a shield for the rays exiting or reflecting on the side. However, this will introduce additional diffraction effects, which are also not accounted for.

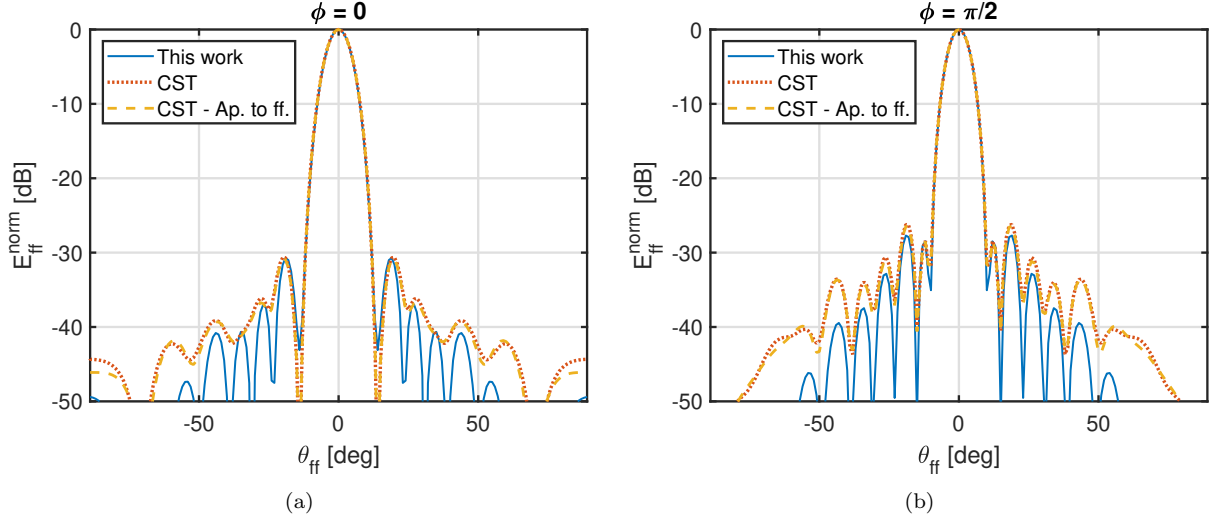


Figure 3.10: Comparison between CST and the method in this work for the far field for a lens without matching layers, 36mm diameter, 5.7mm height, and $n_{\text{side}} = 1.5$. A separation plane is present at the top of the lens.

3.6 Simulation Comparison: Directivity, Gain, Efficiencies

Table 3.1 presents all figures of merit discussed in Section 2.10 for each simulation performed using the tool and CST. It shows the differences between varying numbers of matching layers, varying minimum material refractive indices, and different separation planes. It can be observed, for example, that the reflection efficiency decreases significantly with fewer matching layers, and that the offset in taper efficiency increases, as stated previously. Furthermore, the difference in taper efficiency for $n_{\text{side}} = 1$ matches the expected behavior better than for $n_{\text{side}} = 1.5$.

It is important to note that the tool's directivity is computed only over the upper hemisphere. This is compensated for by the spillover efficiency when calculating the gain. The spillover efficiency is identical for all simulations, since neither the feed geometry nor the lens geometry is varied.

Finally, the computed gain is very similar to the CST results. For all separation planes, the gain is within 1 dB. Since both the top and bottom plates introduce diffraction effects that are not accounted for in the tool, the tool's gain is expected to match the CST simulation without a plate most closely. This is indeed the case for all simulations. The computed directivity from CST agrees well with the tool, except in the case without a metal plate, where the power not intersecting the lens is also included in the computation of the radiated power.

	ML	n_{side}	S.P.	η_{ATL}	η_{PEL}	η_{tap}	η_{refl}	η_{SO}	Dir. [dBi]	Gain [dBi]
SIM	2	1	-	0.815	0.976	0.795	0.971	0.328	27.4	22.4
CST	2	1	Bot. plate	0.809	0.965	0.781	-	-	27.7	22.6
CST	2	1	No plate	0.795	0.971	0.772	-	-	25.7	22.2
CST	2	1	Top plate	0.777	0.965	0.749	-	-	27.3	21.8
SIM	2	1.5	-	0.819	0.975	0.799	0.947	0.328	27.4	22.3
CST	2	1.5	Bot. plate	0.824	0.901	0.742	-	-	27.4	22.8
CST	2	1.5	No plate	0.812	0.920	0.747	-	-	25.6	22.3
CST	2	1.5	Top plate	0.803	0.918	0.737	-	-	27.3	22.0
SIM	0	1	-	0.825	0.970	0.800	0.699	0.328	27.5	21.0
CST	0	1	Bot. plate	0.803	0.940	0.755	-	-	27.7	21.7
CST	0	1	No plate	0.790	0.954	0.754	-	-	25.3	21.2
CST	0	1	Top plate	0.786	0.935	0.735	-	-	27.4	20.8

Table 3.1: Comparison between the proposed tool and CST results, including multiple figures of merit.



Chapter 4

Analysis of Off-Axis Feeds

Up to this point, all simulated lenses have been fed from the focal point. To achieve beam steering, the feed can be displaced from this position. This may involve a horizontal displacement, but for optimization purposes it can also require vertical displacement or rotation of the feed. This Chapter discusses these feed-offset implementations and the corresponding model modifications required to enable them.

The first important point to note is that, with a displaced feed, the tool can no longer rely on rotational symmetry for certain computations, which increases the computational time. For example, the binary search for the outermost ray must now be performed for each ϕ -cut rather than only once. This requirement is illustrated in Figure 4.1, which shows the ray tracing for a lens with an offset feed. In this case, the leftmost ray intersects the bottom of the lens closer to the edge than the rightmost ray. The same applies, for example, to the computation of spillover efficiency. Furthermore, matching-layer height compensation is not performed for a displaced feed; instead, the rays are still assumed to originate from the original focal point.

By displacing the feed, the spillover efficiency is inevitably affected. This may be detrimental, in which case it may be desirable to tilt the feed further toward the center of the lens. Therefore, the ability to simulate such a tilted feed in the tool is useful. A feed-rotation implementation has been integrated as discussed in Appendix A, which describes both the implementation and the validation of the feed-rotation functionality.

4.1 Polarization Tracking

For non-displaced feeds, the polarization at the top of the aperture can be computed by taking the polarization at the bottom and applying a Rodrigues rotation. This rotation is only applicable when

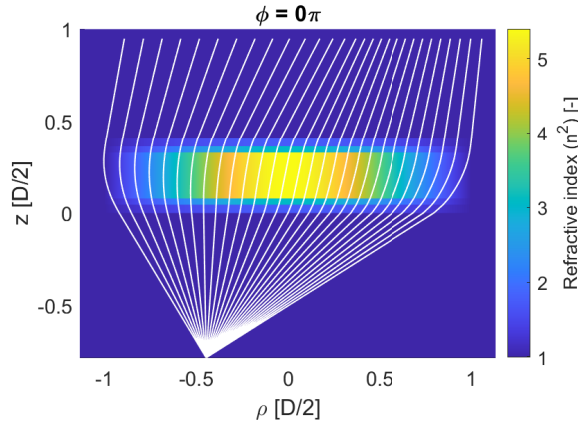


Figure 4.1: The raytracing of a 2-matching-layer lens, with the feed offset in the $-\hat{x}$ -direction



the rays bend within a single plane. This is not the case when the feed is displaced or when the lens is rotationally asymmetric.

Another approach is to track the polarization along the ray. This method is derived from the following equation for the electromagnetic field of a ray:

$$\frac{\partial \mathbf{E}}{\partial s} + \frac{1}{2} \left(\frac{1}{n} \nabla^2 S - \frac{\partial \ln \mu}{\partial s} \right) \mathbf{E} + (\mathbf{E} \cdot \nabla \ln n) \hat{\mathbf{p}}, \quad (4.1)$$

where S is the Eikonal, s the ray path, μ the magnetic permeability of the medium traversed by the ray, n the refractive index of the medium traversed by the ray and $\hat{\mathbf{p}}$ the unit tangent vector of the ray [42]. S , s , n and $\hat{\mathbf{p}}$ for this equation are the same as in (2.3) and (2.4), used for the ray-tracing formulation.

The $1/(n)\nabla^2 S$ term denotes the spreading, already taken into account by the ray tube theorem, and can therefore be neglected. Furthermore, if lens under test is assumed to have homogeneous permeability, the $\partial \ln \mu / \partial s$ term is zero. Adjusting it for tracking the polarization $\hat{\mathbf{e}}$, (4.1) is modified to

$$\frac{\partial \hat{\mathbf{e}}}{\partial s} = -(\hat{\mathbf{e}} \cdot \nabla \ln n) \hat{\mathbf{p}}. \quad (4.2)$$

This simple ODE can be added to the original set of ODEs used for ray tracing in order to track the polarization in parallel. As with the ray-tracing formulation, this ODE is only valid for slowly varying refractive indices. Therefore, at each interface, the change in polarization must be accounted for separately.

This is first done by finding the TE and TM unit vectors of the propagation vector, $\hat{\mathbf{e}}_{\perp}$ and $\hat{\mathbf{e}}_{\parallel}$, respectively. Taking the dot product of these unit vectors with the electric-field unit vector in the first medium yields the corresponding field magnitude components in the TE and TM polarizations, $|e_{\text{TE}}|$ and $|e_{\text{TM}}|$:

$$\begin{aligned} |e_{\text{TE}}| &= \hat{\mathbf{e}}_1 \cdot \hat{\mathbf{e}}_{\perp,1} = \hat{\mathbf{e}}_1 \cdot \frac{\hat{\mathbf{p}}_1 \times \hat{\mathbf{z}}}{\|\hat{\mathbf{p}}_1 \times \hat{\mathbf{z}}\|} \\ |e_{\text{TM}}| &= \hat{\mathbf{e}}_1 \cdot \hat{\mathbf{e}}_{\parallel,1} = \hat{\mathbf{e}}_1 \cdot \frac{\hat{\mathbf{e}}_{\perp,1} \times \hat{\mathbf{p}}_1}{\|\hat{\mathbf{e}}_{\perp,1} \times \hat{\mathbf{p}}_1\|}. \end{aligned} \quad (4.3)$$

Furthermore, the propagation vector in the second medium is found using

$$\begin{aligned} p_{2,x} &= p_{1,x} \\ p_{2,y} &= p_{1,y} \\ p_{2,z} &= \sqrt{\left(\frac{n_2}{n_1}\right)^2 p_{1,z}^2 + \left(\left(\frac{n_2}{n_1}\right)^2 - 1\right) (p_{1,x}^2 + p_{1,y}^2)}. \end{aligned} \quad (4.4)$$

For both modes, the transmission coefficients from medium 1 to medium 2 can be determined using

$$\begin{aligned} t_{\perp} &= \frac{2n_1 \cos(\theta_1)}{n_2 \cos(\theta_1) + n_1 \cos(\theta_2)} \\ t_{\parallel} &= \frac{2n_1 \cos(\theta_1)}{n_1 \cos(\theta_1) + n_1 \cos(\theta_2)}. \end{aligned} \quad (4.5)$$

To determine the electric-field unit vector in the second medium, the propagation vector is decomposed into the TE and TM unit vectors. These components are multiplied by their respective amplitudes and transmission coefficients, and subsequently normalized, as follows:

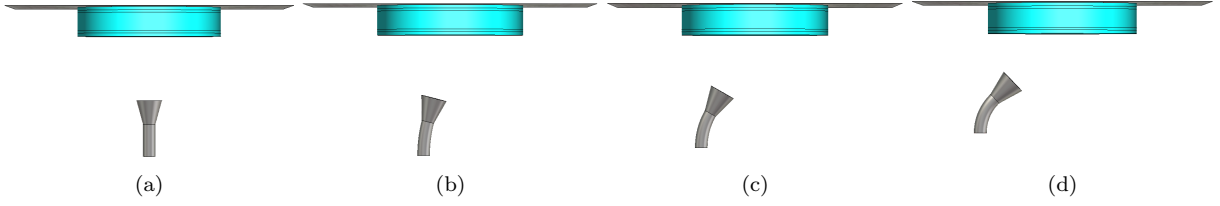


Figure 4.2: The lens under test with its feed displaced in x , z , and tilted, for Δx equal to (a) 0 mm, (b) 4 mm, (c) 8mm, and (d) 12 mm.

$$\hat{\mathbf{e}}_2 = \frac{|e_{\text{TE}}|t_{\perp}\hat{\mathbf{e}}_{\perp,2} + |e_{\text{TM}}|t_{\parallel}\hat{\mathbf{e}}_{\parallel,2}}{||(|e_{\text{TE}}|t_{\perp}\hat{\mathbf{e}}_{\perp,2} + |e_{\text{TM}}|t_{\parallel}\hat{\mathbf{e}}_{\parallel,2})||}. \quad (4.6)$$

By alternating between the polarization-tracking ODE and these computations, the polarization at the output of the lens can be determined.

Another important point is that, due to the varying rotational axes, TE and TM polarizations may interchange during rotation. This implies that an incident ray that is purely TE-polarized can acquire a TM component, and vice versa. Consequently, the model in which TE and TM components are fully separated, primarily used for the transmission-line formulation, does not strictly remain valid.

One way to obtain a more accurate representation is to decompose the rays into TE and TM modes at each interface. However, for computational simplicity, this approach is not considered here.

4.2 Validation of Off-Axis Feed Response

A set of CST simulations has been conducted to validate the tool's ability to simulate displaced feeds. For this purpose, a lens with two matching layers on both the top and bottom surfaces is used, with a diameter of 36 mm and $n_{\text{side}} = 1$. A conical horn feed is positioned such that its phase center lies at a focal distance of 16.054 mm. The conical horn operates at 70 GHz and has inner radii of 3 mm and 1.35 mm at the top and bottom, respectively, with a length of 6 mm. The horn is fed by a cylindrical waveguide, with TE_{11} mode along the x -direction, with a 1.35 mm inner radius and an 8 mm length.

This feed is displaced in the x -direction, with 0, 4, 8, and 12mm. The feed is not displaced further, as the tool indicates that internal reflections would occur beyond this point. Subsequently, the feed is displaced in the z -direction to fit the Petzval curve, keeping the feed closer to the lens focus. The Petzval curve states that the focal length is decreased by Δz according to [45, 46]

$$\Delta z = \frac{\Delta x^2}{2F}. \quad (4.7)$$

The feed is then rotated about its phase center along the y -axis, such that the broadside direction of the feed points toward the center of the bottom surface of the lens. The angle can be computed by

$$\theta_{\text{feedtilt}} = \arctan\left(\frac{\Delta x}{F - \Delta z}\right), \quad (4.8)$$

with F the original focal length. In CST, the cylindrical waveguide is bent because the waveguide port must be located on the $x = 0$, $y = 0$, or $z = 0$ planes. The feed with the bent waveguide is simulated individually and has been shown to provide a feed pattern similar to that of a tilted feed. The displaced feeds are depicted in Figure 4.2. The figure illustrates both the feed rotation and the vertical displacement applied as a result of the horizontal feed displacement.

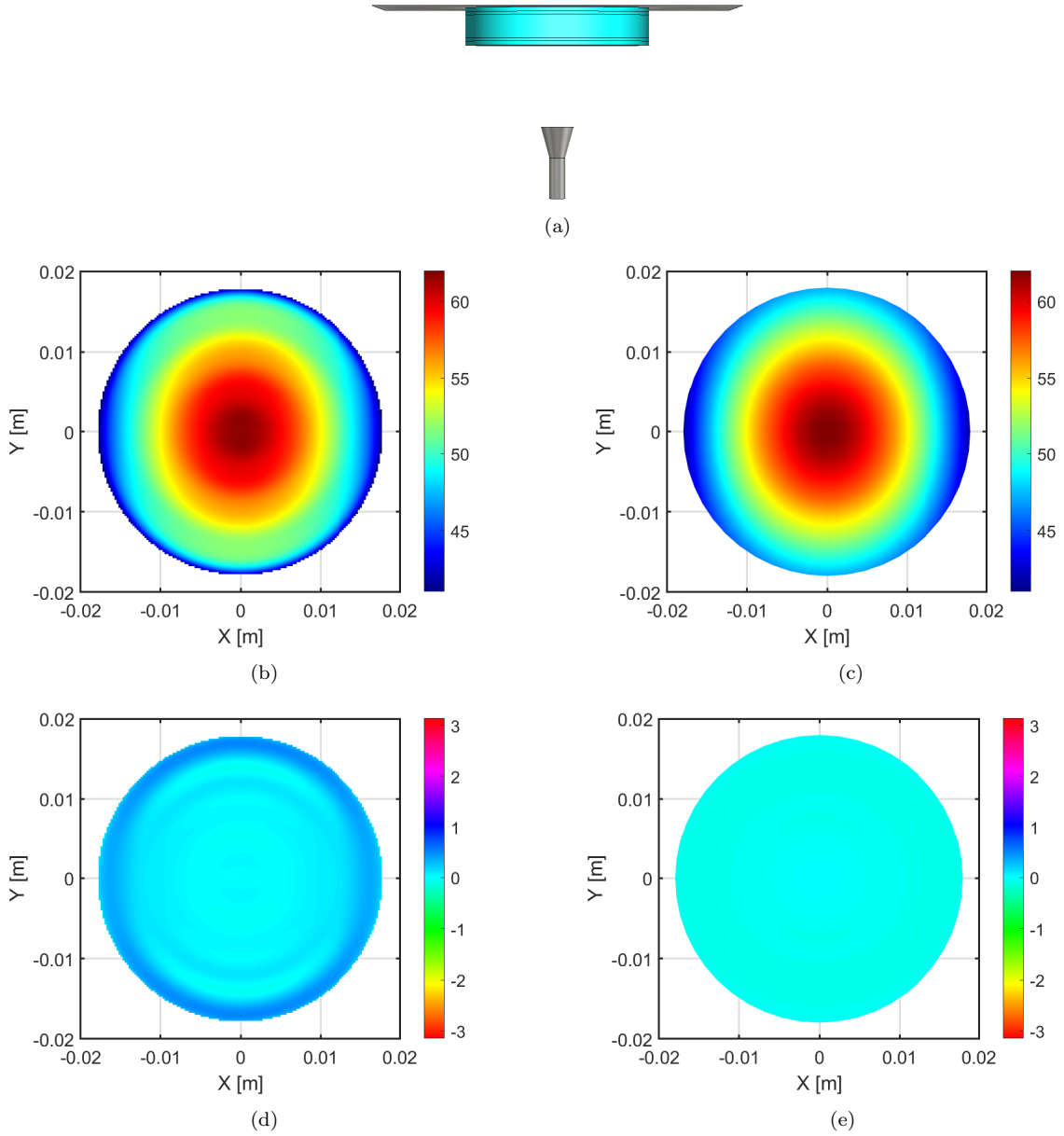


Figure 4.3: Comparison of the aperture fields between CST and the simulation tool for a lens with two matching layers atop and below, 36mm diameter, 5.7mm height, and 1 n_{side} , fed by a non-displaced conical feed. A separation plane is present at the top of the lens. The magnitude of the field for (b) CST and (c) this work, and the phase of E_x for (d) CST and (e) the tool.

Figure 4.3 shows the aperture field results of the non-displaced feed, both for CST and this work. The edge effects introduced by the separation plane are still visible in the CST simulations but are not accounted for in the tool. This is evident in Figures 4.3b and 4.3c, where the CST results exhibit a drop in amplitude near the lens edge. Similarly, Figure 4.3d shows that the CST simulation exhibits ripples and an increase in phase near the edge, effects that are absent in the tool's results. However, the amplitude taper of the conical feed mitigates these edge effects. Overall, the phase response is very flat, and the peak amplitude is located at the center of the aperture.

The aperture amplitude and phase results for $\Delta x = 4$ mm are shown in Figure 4.4. Once again, the tool shows good agreement with the CST simulation. In both cases, the amplitude peak is slightly shifted from the center, and a nearly linear phase progression is observed across the aperture. As before, the phase agreement deteriorates near the aperture edge, which can be attributed to edge effects that are not accounted for by the tool.

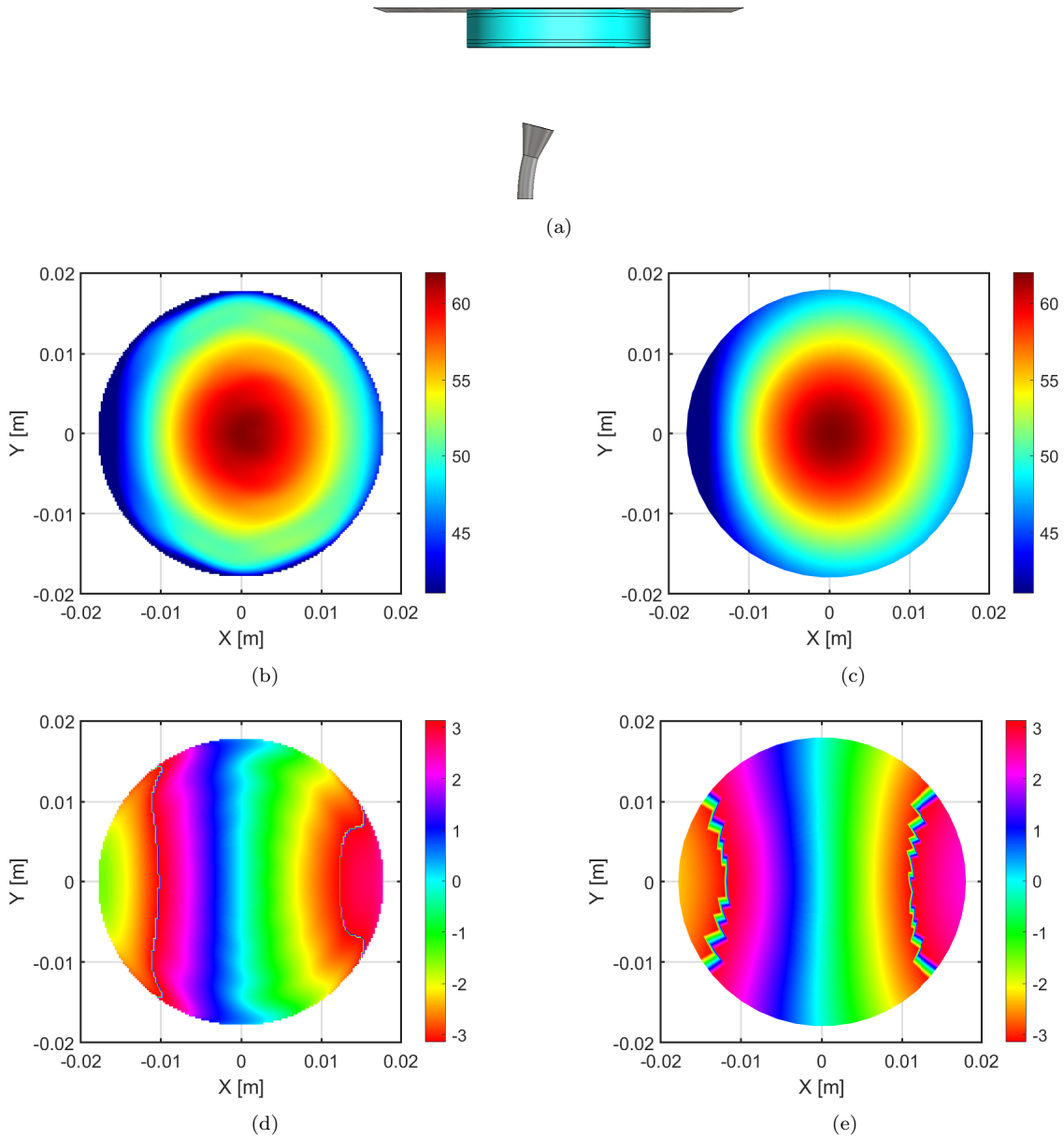


Figure 4.4: The comparison of the aperture fields between CST and the simulation of the tool for a lens with two matching layers atop and below, 36 mm diameter, 5.7 mm height, and 1 n_{side} , fed by a conical feed, displaced 4 mm. A separation plane is present at the top of the lens. The magnitude at the field for (b) CST and (c) this work, and the phase of E_x for (d) CST and (e) the tool.

The tool's simulation results for all feed displacements are summarized in Figure 4.5. The corresponding far-field directivity patterns, together with the CST results, are shown in Figure 4.5a. The actual values of the maximum directivity are tabulated in Table 4.1 and shown in Figure 4.5b, with their corresponding main beam angles. It can be observed that up to a displacement of $\Delta x = 8$ mm, the lens response is predicted accurately, with the maximum directivity agreeing with the CST results within 0.1 dB. For the case where $\Delta x = 12$ mm, the far-field lines do not perfectly coincide anymore. There are two possible explanations for this discrepancy. First, by vertically displacing the feed to keep it near the focal surface, the feed is also moved closer to the lens. As a result, the far-field assumption used for the feed may no longer be valid, leading to differences in the computed fields from the feed. Furthermore, by tilting the feed toward the bottom lens center, the main beam is directed more toward the top edge, possibly increasing the diffraction effects of the top plate.

Figures 4.5c and 4.5d show the aperture amplitude and phase distributions, respectively, for the $\phi = 0$

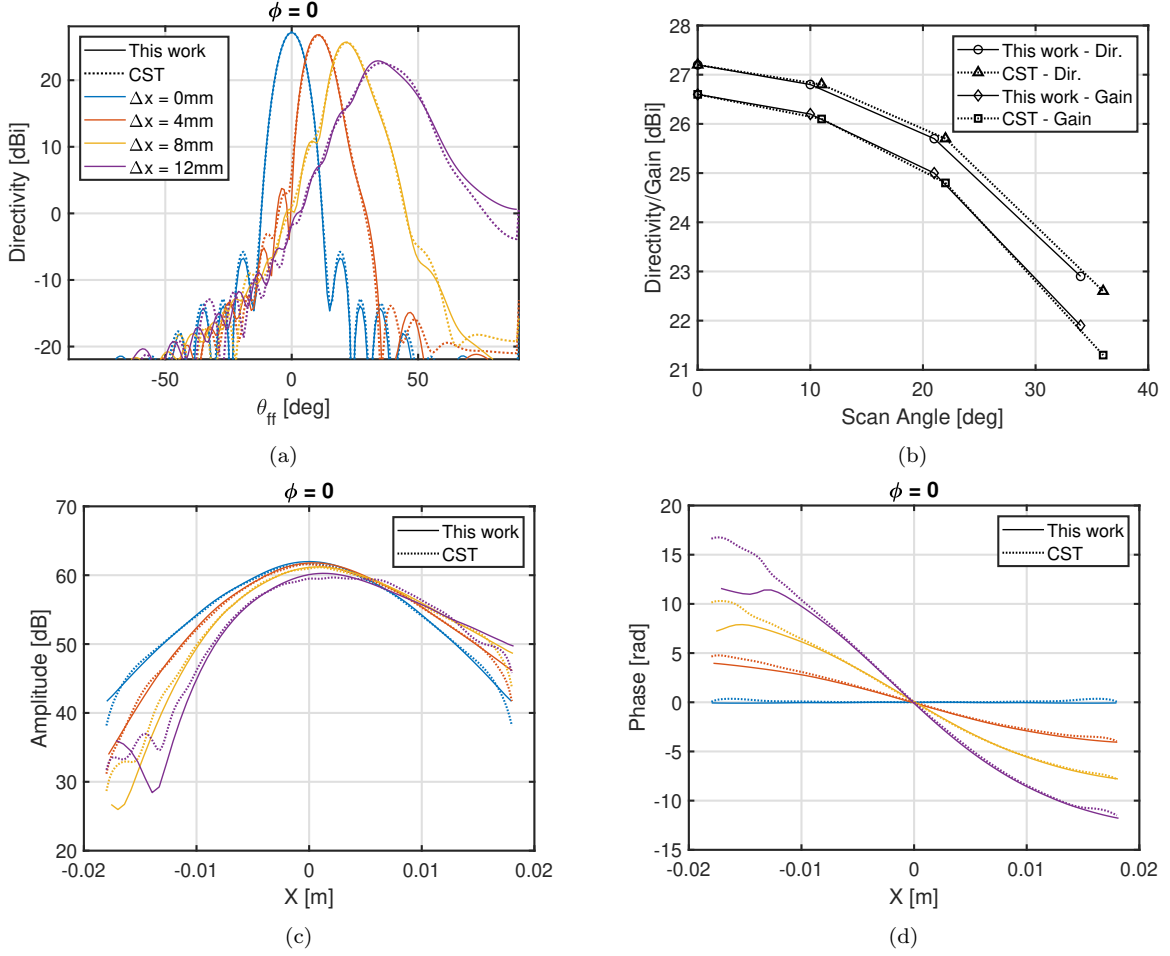


Figure 4.5: (a) The directivity of the far fields, (b) the directivity and gains for each displacement, (c) amplitude of the electric field at the aperture on $\phi = 0$, and (d) the phase of the x-component at the aperture on $\phi = 0$ for all displaced feeds in a consistent coloring scheme. A comparison is drawn between CST and this work.

cut. The largest discrepancies in both quantities occur near the edge of the aperture closest to the feed. Furthermore, these discrepancies become more pronounced as the feed displacement increases. Despite this effect, overall, the amplitude and phase appear to be well simulated by the model, compared to CST.

Figure 4.5b presents the maximum directivities and gains obtained with the proposed tool and CST for different feed offsets. Although the directivities agree almost perfectly, the scan angle (θ_{scan}) is consistently underestimated by the tool. This error increases slightly with increasing feed displacement. The gains, on the other hand, are consistently overestimated for the offset-feed configurations, with the discrepancy likewise increasing as the feed displacement increases. Despite these differences, the tool still provides very good estimates. Table 3.1 also showed that the tool overestimates the gain of configurations with a separation plane at the top, as opposed to without a plate. This could suggest that the error is indeed due to diffraction effects, which are even more pronounced with scanning.

Cross-polarization

Figure 4.6 shows the co-polarized and cross-polarized far-field components (denoted as *horizontal* and *vertical* in CST, respectively) for the same lens, evaluated in the $\phi = \pi/4$ cut. The third Ludwig's definition of co-polar and cross-polar is assumed [47]. The lens is fed by the same conical horn, which is excited by a cylindrical waveguide operating in the x -polarized TE_{11} mode. To demonstrate the ability of the proposed simulation tool to accurately track polarization for off-axis feeds, the feed is also displaced in the $\phi = \pi/4$ plane. This corresponds to $\Delta x = \Delta y = -5.66$ mm. In this case, the feed is neither displaced in the z -direction nor tilted toward the center of the lens. Figure 4.6a shows that, for the case without feed displacement, the first sidelobes of both the co-polarized and cross-polarized fields are

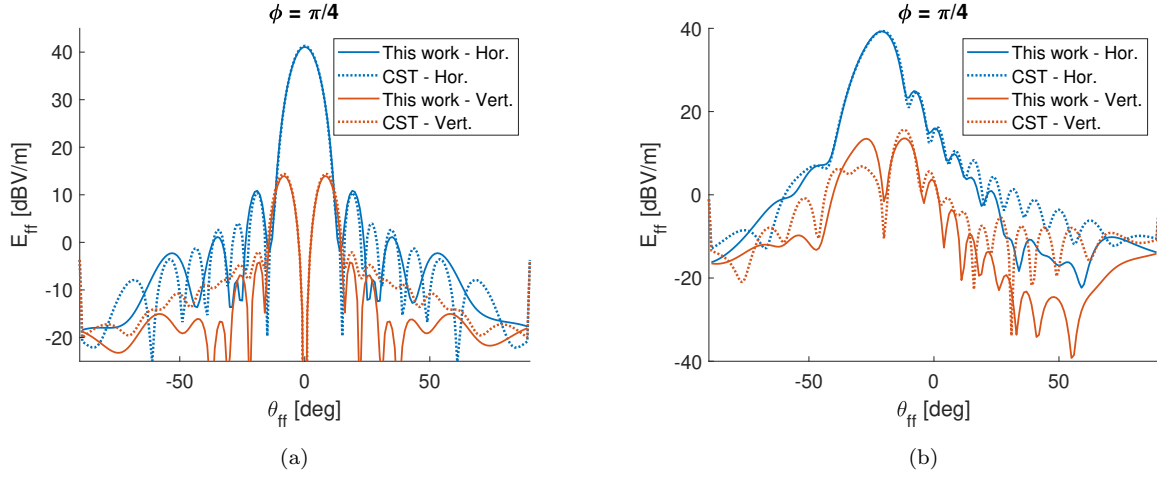


Figure 4.6: The horizontal and vertical polarization of the far field at the $\phi = \pi/4$ cut, of a lens, fed by a x-polarized TE₁₁ mod conical antenna, for (a) $\Delta x = \Delta y = 0 \text{ mm}$ and (b) $\Delta x = \Delta y = -5.66 \text{ mm}$. The lens is 36 mm in diameter, has a focal length of 16.054mm in both cases and a thickness of 5.7 mm, with two matching layers on top and at the bottom. The feed is not tilted in either case.

predicted accurately. Although the higher-order sidelobes do not match perfectly, their overall power levels are in good agreement with CST full-wave simulations.

Figure 4.6b shows the case with an offset feed. Again, the main lobes of both the co-polarized and cross-polarized fields are predicted accurately. However, at larger observation angles, the agreement with the CST results deteriorates. This discrepancy is more pronounced than in the case without feed displacement. One possible explanation is that the model does not fully account for the coupling between the TE and TM components as the rays undergo successive rotations. This may also affect the predicted far-field polarization. Furthermore, because the displaced feed is not tilted toward the lens center, a significant portion of the radiated power is directed toward the top plate. The resulting diffraction effects, which are not included in the model, may increase the radiated power at observation angles between 30 and 60°.

Change in Phase Error Efficiency Definition

The efficiency η_{PEL} , as defined in Section 2.10, is not a generic function but is specified for a pattern with maximum directivity at boresight; a new expression needs to be taken into account, where the main beam angle is considered. This is given by [44]

$$\eta_{\text{PEL}}(\theta, \phi) = \frac{(1 + \cos \theta)^2}{4} \frac{\left| \iint_s \mathbf{E}_{\text{ap}} e^{j\mathbf{k} \cdot \mathbf{r}'} ds \right|^2}{\left(\iint_s |\mathbf{E}_{\text{ap}}| ds \right)^2} \quad (4.9)$$

where $\mathbf{E}_{\text{ap}}$ is the electric field at the aperture, $\mathbf{r}'$ is the location of the field on the aperture, and

$$\mathbf{k} = k [\sin(\theta) \cos(\phi) \hat{\mathbf{x}} + \sin(\theta) \sin(\phi) \hat{\mathbf{y}} + \cos(\theta) \hat{\mathbf{z}}] .$$

If the aperture lies in the x-y plane,

$$\mathbf{k} \cdot \mathbf{r}' = k [x' \sin(\theta) \cos(\phi) + y' \sin(\theta) \sin(\phi)] .$$

Gain, Directivity, and Efficiencies

Table 4.1 shows the results of the displaced feed simulations in more detail. Once again, the taper efficiency is predicted accurately and remains within the expected range. The efficiency decreases with increasing feed displacement. Both the tool and CST show that mainly η_{PEL} decreases, resulting in a

	Separation plane	Δx [mm]	η_{ATL}	η_{PEL}	η_{tap}	η_{refl}	η_{SO}	Directivity [dBi]	Gain [dBi]	θ_{scan} [deg]
Tool	-	0	0.750	0.990	0.743	0.971	0.904	27.2	26.6	0
CST	Top plate	0	0.761	0.968	0.737	-	-	27.2	26.6	0
Tool	-	4	0.744	0.934	0.695	0.972	0.898	26.8	26.2	10
CST	Top plate	4	0.734	0.920	0.675	-	-	26.8	26.1	11
Tool	-	8	0.724	0.751	0.544	0.977	0.874	25.7	25.0	21
CST	Top plate	8	0.708	0.762	0.539	-	-	25.7	24.8	22
Tool	-	12	0.708	0.439	0.311	0.989	0.809	22.9	21.9	35
CST	Top plate	12	0.664	0.470	0.312	-	-	22.6	21.3	36

Table 4.1: A comparison of the tool and CST simulations, for different feed offsets, portraying a variety of figures of merit.

drop in directivity. The term η_{ATL} drops slightly, but not as much. These results demonstrate that the tool can help identify the dominant sources of scanning losses.

Furthermore, the feed tilt can be treated as a design parameter rather than being fixed toward the center of the bottom surface of the lens. It can then be optimized to minimize scanning losses. However, this requires balancing the trade-off between spillover efficiency and taper efficiency.

As stated previously, the directivity agrees well between both simulation systems up to the largest feed displacement considered. For the gain, only minor discrepancies are observed, while the results remain in good agreement overall. The simulation tool, however, appears to slightly underestimate the main beam direction, with this error increasing as the feed displacement increases.

Overall, these results indicate that, under the stated assumptions, the tool provides a good estimate of GRIN lens behavior under beam steering. However, the accuracy decreases as the steering angle increases.

4.3 Limitations of the Current Method

To show the potential and limitations of the simulation tool in this work, a slightly more complex lens is introduced [48]. This lens has a diameter of 10.714 mm and a core thickness of 1.5 mm. The same quarter-wavelength matching layers as discussed in Section 2.4 are applied, one at the bottom and one on top. The feed and bottom half-space are surrounded by plastic with a dielectric permittivity of $\epsilon_{r,in} = 2.34$, resulting in different matching layers on the top and bottom, as shown in Figure 4.7a. The feed is placed 5 mm away and is a cavity-backed double-slot array with 8 double-slot pairs, radiating at 140 GHz [48, 49]. The general schematics of this lens are also visible in Figure 4.7 [48].

The lens, however, is not rotationally symmetric in dielectric permittivity. The permittivity in the H-plane is designed such that the focal point lies at a distance of 5 mm, whilst that in the E-plane is designed for a focal point distance of 2.86 mm. These distributions are also created using the formulas from [28]. The permittivity of the lens is then formed according to

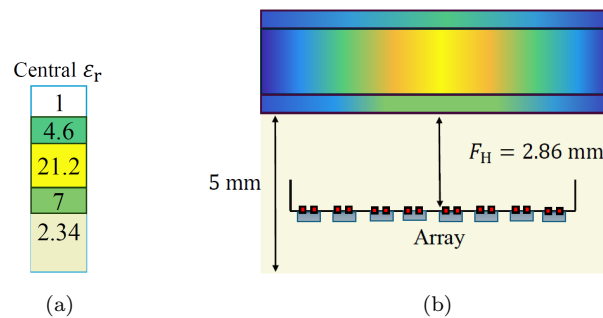


Figure 4.7: Schematic of the lens used for validation: (a) dielectric permittivities of the center and (b) side view of the array.

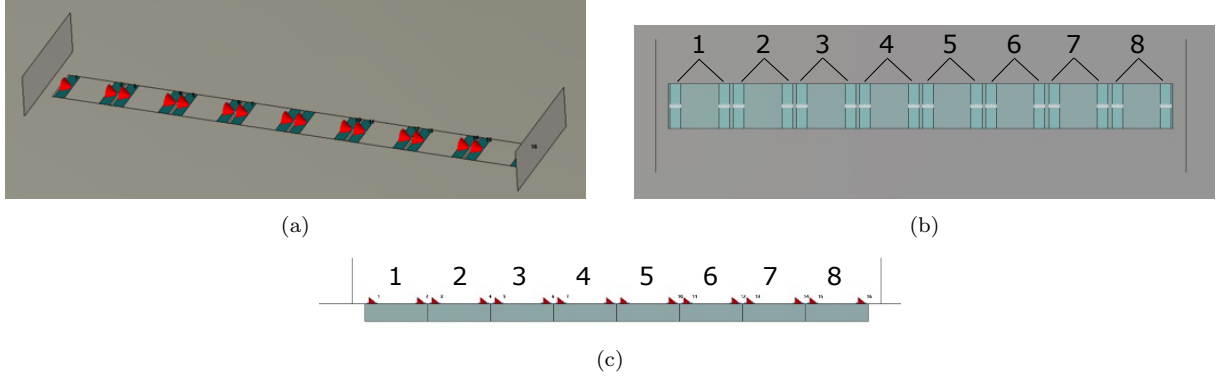


Figure 4.8: A CST capture of the cavity-backed double-slot array feed from a (a) 3D perspective view, (b) top view, and (c) side view.

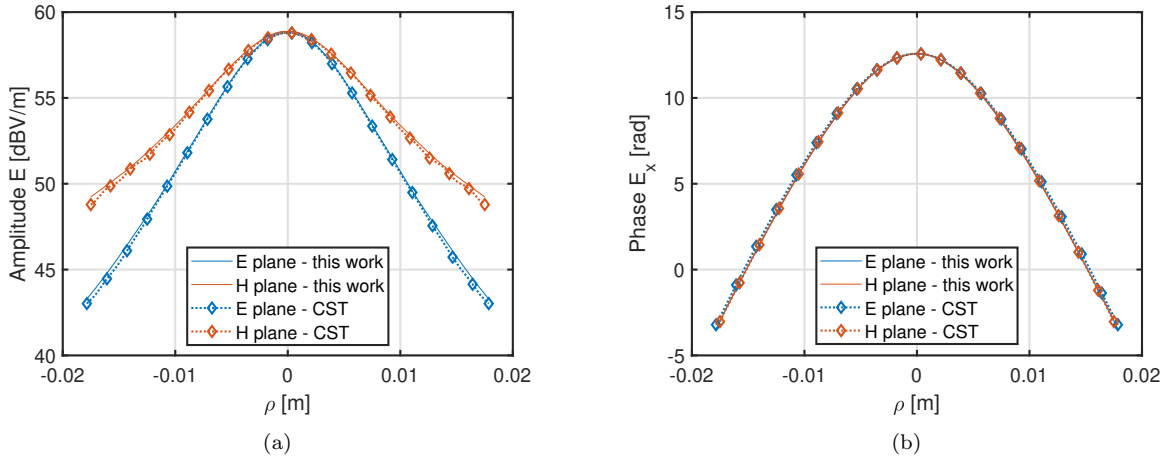


Figure 4.9: Comparison of (a) the total electric-field amplitude and (b) the phase of the x -component at the lens bottom, obtained using the far-field feed model and CST, for a short dipole feed at a 9 mm focal distance.

$$\varepsilon_r(\rho_{\text{lens}}, \phi_{\text{lens}}) = \cos(\phi_{\text{lens}})^2 \varepsilon_{r,E}(\rho_{\text{lens}}) + \sin(\phi_{\text{lens}})^2 \varepsilon_{r,H}(\rho_{\text{lens}}). \quad (4.10)$$

Important to note is that the permittivity of the H-plane is designed such that its center value matches that of the E-plane, by increasing $\varepsilon_{r,\text{side}}$ to 5.4. The permittivity distributions are implemented in a CST simulation using a third-order polynomial fit for the E-plane and a fifth-order fit for the H-plane.

Feed Modeling

Figure 4.8 shows the aforementioned feed in more detail. It illustrates the cavities for the double-slot pairs, separated by vertical metallic reflector plates. At the ends of the array, an additional set of reflector plates is placed for improve illumination efficiency of the lens. The CST results of each double-slot pair are combined to construct eight equivalent feeds that can be used within the proposed simulation tool. Alternatively, the slot antenna feed can be modeled using an efficient method of moments approach for planar slot antennas [50, 51].

For this feed, the far-field distance can be significantly larger compared to that of the short dipole feed or even the conical horn studied in the previous chapters. The feed modeling method, as described in Chapter 3, assumes that the lens is located in the far-field region of the feed and that the fields propagate spherically. To demonstrate this, Figure 4.9 compares the fields reconstructed from the far-field approximation of the dipole with the actual fields at the bottom of the lens. It shows that the lens is indeed in the far-field region of the dipole feed and that the electric fields at the bottom of the lens are reconstructed with sufficient accuracy.

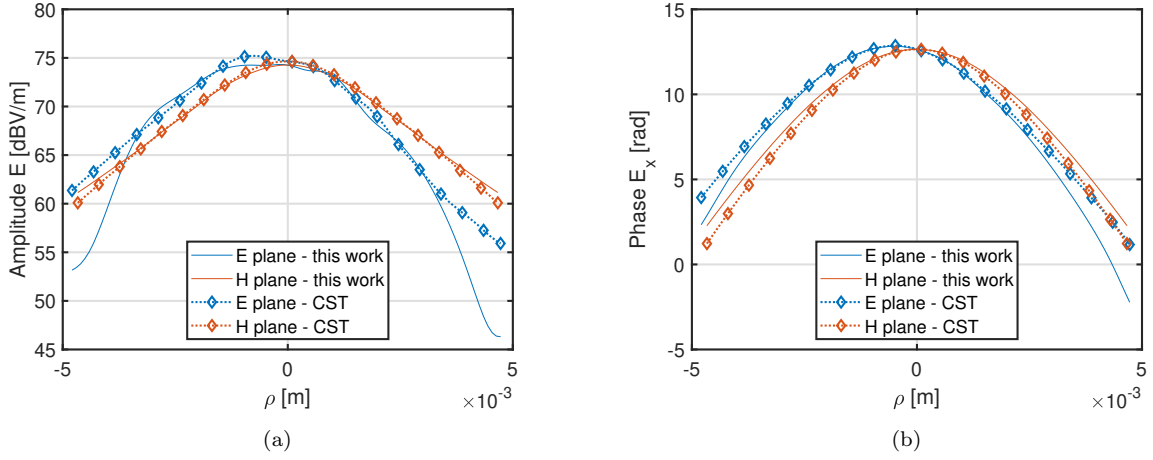


Figure 4.10: Comparison of (a) the total electric-field amplitude and (b) the phase of the x -component at lens bottom, obtained using the far-field feed model and CST, for the central element of the double-slot feed array at a focal distance of 2.86 mm.

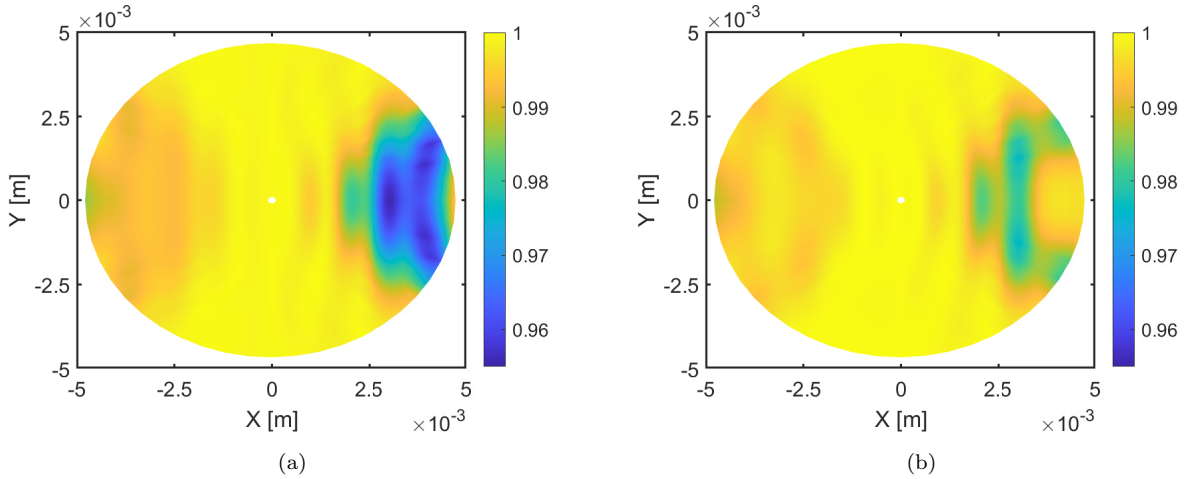


Figure 4.11: Normalized electric field amplitude for the component (a) orthogonal to the ray direction emanating from the feed point and (b) orthogonal to the Poynting vector.

However, with the increase in feed size and the reduction in focal length, the lens may no longer satisfy the far-field condition. Comparing the reconstructed fields with the CST results reveals a substantial discrepancy, as shown in Figure 4.10. In particular, near the edges of the lens bottom in the E-plane, the amplitude decreases significantly compared to the CST fields. Furthermore, the phase error increases gradually with distance from the center of the lens bottom. These results indicate that the far-field approximation to model the feed is not always applicable.

To address this issue, the near-field radiation of the feed is evaluated in CST in absence of the lens, on the plane corresponding to the bottom surface of the lens. It is important to note that in the near-field the electric field is not necessarily orthogonal to the rays originating from the feed. In this case, it is more appropriate to define the direction of propagation of the ray at each point as the time-averaged Poynting vector $\mathbf{S}$:

$$\mathbf{S} = \frac{1}{2} \Re(\mathbf{E}_{\text{bot}} \times \mathbf{H}_{\text{bot}}^*), \quad (4.11)$$

where the $*$ denotes the complex conjugate. Indeed, Figure 4.11a shows the normalized amplitude of the field component orthogonal to the ray to be different from the component orthogonal to the Poynting vector in Figure 4.11b.

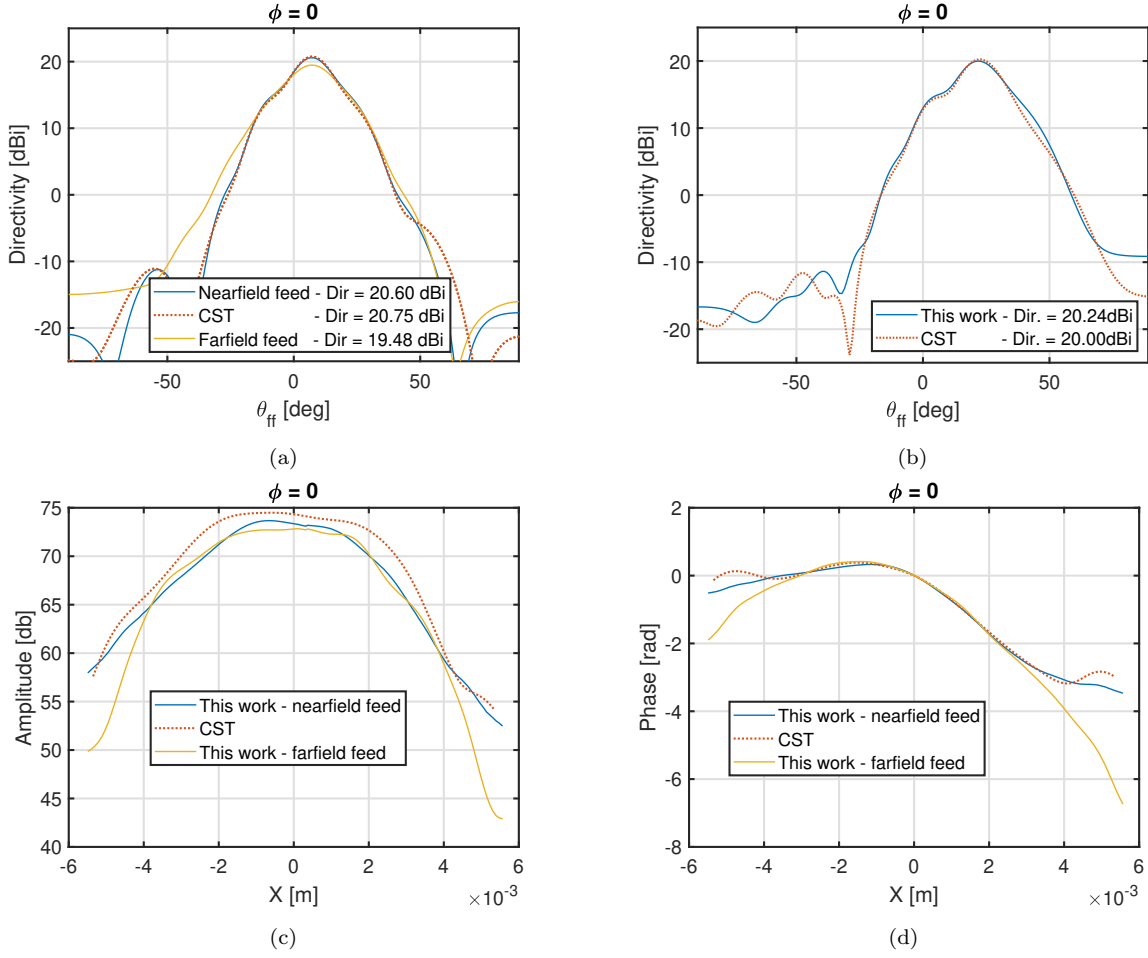


Figure 4.12: Comparison between CST and our method with near-field feed modeling. Directivity for (a) element 4 of the double-slot array and (b) element 3; (c) amplitude and (d) phase of the electric field at the top aperture for element 4. For element 4, also the results for the far-field approximation of the primary field are shown for comparison.

Moreover, when using the near-field method, the formula for the reflection efficiency, as described in (2.30) is generalized as

$$\eta_{\text{refl}} = 1 - \frac{\iint_{S_{\text{bot}}} |\mathbf{S}(x, y)| |S_{11}(x, y)|^2 ds}{\iint_{S_{\text{bot}}} |\mathbf{S}(x, y)| ds}. \quad (4.12)$$

The difference in the two feed modeling methods is shown in Figure 4.12. With the far-field approximation of the primary field, the directivity is 1 dB lower than that from CST, and the first null to the left is not reconstructed. On the contrary, the near-field method for the primary field provides a better agreement with CST in terms of directivity, pattern shape, and amplitude and phase of the aperture field. The near-field method still exhibits some discrepancy in the predicted amplitude compared to the CST results. This is likely because the ray-tube theorem assumes the electromagnetic field to be locally plane-wave in nature, an assumption that is not fully satisfied in the feed's near field.

This shows the necessity of the lens being in the feed's far field for the original feed-insertion method, but that this can be adequately solved with the near-field insertion method. The downside to this method is the tool's inflexibility. No feed rotation is possible, and for each vertical feed shift, another CST export needs to be executed.

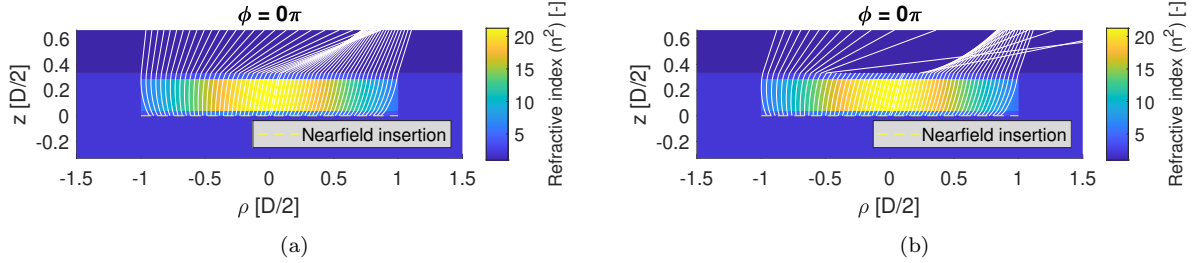


Figure 4.13: Ray-tracing on the $\phi = 0$ plane, for (a) the second outermost feed and (b) the outermost feed. The lens is 10.741 mm in diameter, has an original core thickness of 1.5mm, and a focal length of 2.86mm, inlaid in plastic with $\epsilon_r = 2.34$. The ray tracing starts from the Poynting vectors from the nearfield source insertion.

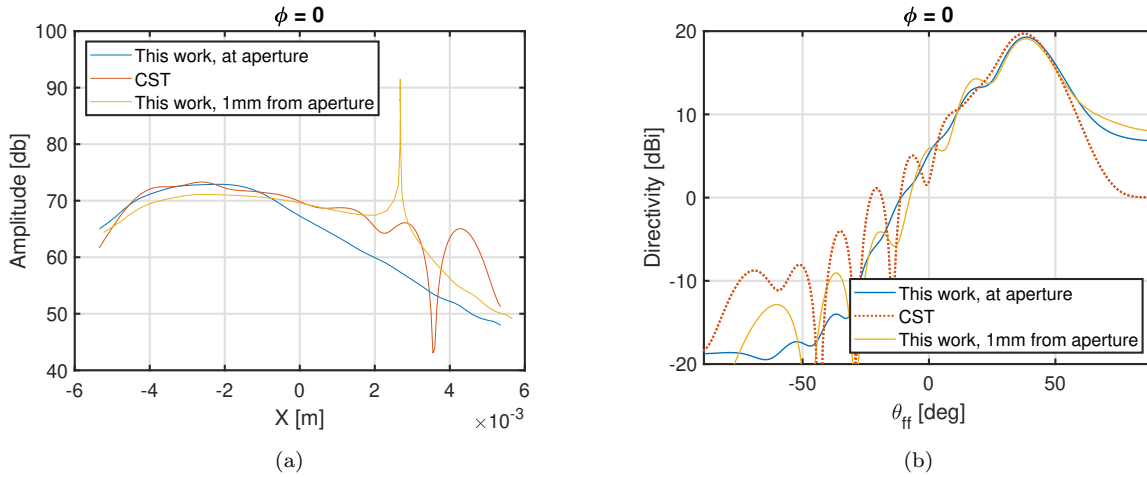


Figure 4.14: (a) Amplitude of electric field at the equivalence theorem plane and (b) far-field directivity for CST and this work, when the equivalence theorem plane is either at the lens top aperture or 1 mm above it.

Caustics and Internal Reflections

As discussed in Section 2.6, the Delaunay-triangulation-based ray-tube theorem breaks down in the presence of caustics or internal reflections. The first and second elements at the edge of the feed array provide a good illustration of this limitation. Figure 4.13a shows the ray tracing for the lens when illuminated by the second feed element. Unlike the previous cases, the rays originate from the bottom aperture, where the near-field source insertion is applied, rather than from the focal point. It can be seen that the rays to the right of the lens center are refracted more strongly toward the optical axis than those slightly to the left. As a result, the rays intersect just above the top aperture, forming a caustic. In the present case, this does not pose a problem because the equivalent currents are evaluated directly at the aperture. Figure 4.14 compares this with a scenario in which the equivalent currents are evaluated 1 mm above the aperture, corresponding to the export plane used in CST. The resulting amplitude distribution, shown in Figure 4.14a, exhibits a pronounced peak caused by the overlapping rays. This artifact also affects the far-field pattern, as shown in Figure 4.14b, although the effect is small because only a limited number of rays intersect. For larger feed displacements, however, the aberrations become more severe, increasing the associated error. Consequently, this limitation may become significant for more extreme scanning configurations.

Furthermore, Figure 4.13b depicts the ray tracing for the outermost feed, where it is evident that internal reflections occur. Hence, no useful electric field can be obtained at the lens top aperture. A solution to both of these issues could lie in the original ray tube theorem method, in which each ray would be accompanied by its own guide rays very close by. This way, the computation of the amplitude of a ray is

not dependent on the entire ray grid on the aperture, but only its own guide rays. This is computationally very costly, though, and should only be applied when internal reflections or caustics are detected.

In conclusion, the proposed simulation tool accurately predicts the far-field patterns for the four central feed pairs and provides adequate accuracy for the second-outermost pairs when the near-field feed-insertion method is used. However, for large off-axis displacement some limitations of the ray tracing approach are observed, and the structure cannot be reliably modeled.

4.4 Graphical User Interface

The model described in Chapters 2-4 was implemented in MATLAB, and a graphical user interface (GUI) was made. A screenshot of the GUI is shown in Figure 4.15.

The main inputs to the code are the lens geometry, the dielectric permittivity distribution, and the feed field. The lens permittivity can either be generated via the GUI or imported from a file if a previous design exists. Additionally, a `.txt` file with the far-field radiation pattern of the feed is required. When defining the focal length, it is important to take into account the phase center of the feed, which may not coincide with its physical position.

Furthermore, the addition of matching layers is supported, which automatically introduces quarter-wavelength transformers. The core height can be adjusted automatically to compensate for the presence of matching layers, although a manual compensation factor can also be specified. Finally, feed displacements and rotations can be defined.

When the user presses the start button, the GUI provides real-time updates on the current processing step. Once the computation is complete, a schematic of the lens with traced rays and a far-field radiation plot are displayed. The far-field plot can be shown for a selected ϕ -plane cut and may include both co- and cross-polarisation components. The directivity, spillover efficiency, taper efficiency, and reflection efficiency are also computed.

Several features that are currently not implemented but could be useful are:

- More general definition of matching layers.
- The implementation of the near-field feed insertion method.
- The possibility to perform sweeps over certain parameters, such as frequency, feed displacement, tilt, or lens geometry.

Nevertheless, this GUI provides a more user-friendly interface for operating the otherwise intricate code for basic flat-lens computations.

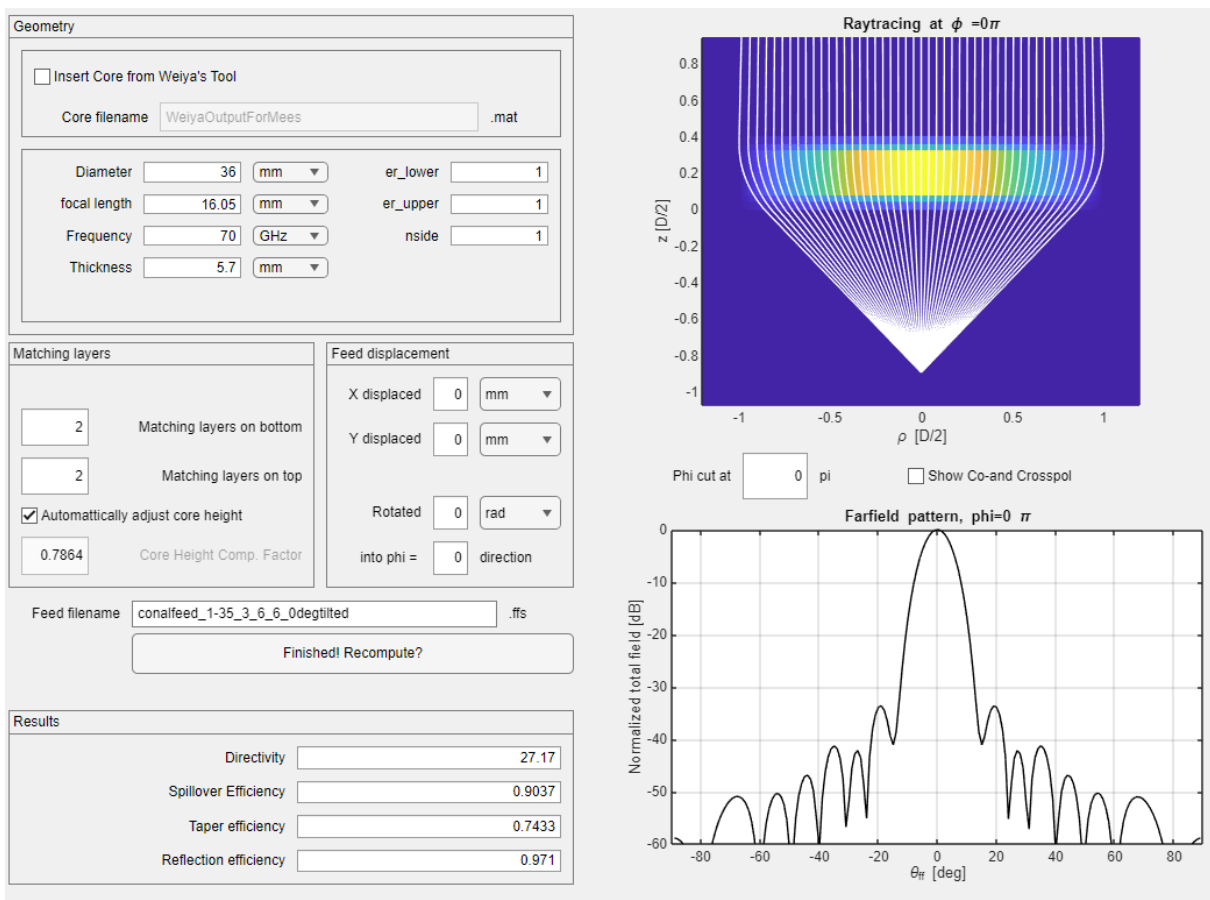


Figure 4.15: Screenshot of the GUI developed in this work.

Chapter 5

Study of Anisotropy and Output-Angle Offsets

5.1 Modeling ADL Anisotropy

Up to now, the model described has focused on GRIN lenses with an ideal inhomogeneous refractive index. One approach to realizing a slowly varying refractive index is to use Artificial Dielectrics (ADs) or Artificial Dielectric Layers (ADLs) [11]. These consist of a host material embedded with a periodic stratification of electrically small metal patches, thereby increasing the metamaterial's effective refractive index. The exact geometric layout of these patches (and their relative spacing) influences the susceptance of each layer and, in turn, the resulting increase in the effective refractive index. Hence, by tuning the size and spacing of these patches, the refractive index can be controlled. By gradually varying these geometries throughout the lens, a smoothly varying refractive index can be achieved [30, 31, 32, 33].

A characteristic of ADLs is that they do not behave isotropically; a plane wave incident at different angles experiences different effective refractive indices [11, 52]. This behavior can be modeled using a squared cosine dependence, which can be expressed as follows:

$$n_{\text{Ti}}^2(\theta) \approx (n^2(0^\circ) - n_{\text{Ti}}^2(90^\circ)) \cos^2(\theta) + n_{\text{Ti}}^2(90^\circ), \quad (5.1)$$

where n_{Ti} describes either the TE or TM mode [11].

Hence, to describe the behavior over the full range of angles of an ADL, one needs only the refractive index at the broadside and grazing incident angles. $n_{\text{Ti}}(0^\circ)$ is assumed to be known for square patches as $n(0^\circ)$ for both TE and TM modes and

$$\begin{aligned} n_{\text{TM}}^2(90^\circ) &= n_{\text{host}}^2 \\ n_{\text{TE}}^2(90^\circ) &= n_{\text{host}}^2 + \left(\frac{1}{k_0 d} \cos^{-1} \left(1 - \frac{\zeta k d B}{4} \right) \right)^2, \end{aligned} \quad (5.2)$$

where n_{host} is the refractive index of the host material, ζ is the impedance of the host material, k and k_0 are the wavenumbers in the host material and free space, respectively, and B is the susceptance of the ADL layer, expressed as

$$B = \frac{2 \cos(kd) - \cos(k_0 d n(0^\circ))}{\zeta \sin(kd)}, \quad (5.3)$$

with d the inter-layer spacing [11].

Note that the susceptance depends only geometrically on the inter-layer spacing. Taking a realistic value for d , such as 500 μm , one can compute the anisotropic refractive index of the slowly varying GRIN lens.

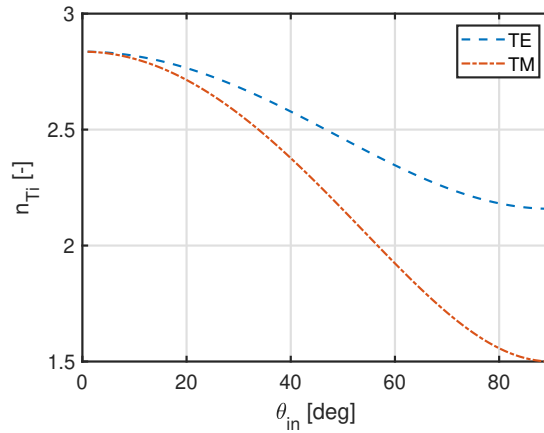


Figure 5.1: The anisotropic behavior of an ADL for the TE and TM modes, with a $n(0^\circ)$ of 2.86, n_{host} of 1.5, a frequency of 70GHz, and an inter-layer spacing of 500 μm .

Here, the spatially-dependent $n(x, y)$ of an original, ideal GRIN lens can be inserted as the $n(0^\circ)$ for each point, to create a spatially-, but also incident-angle-dependent refractive index. Figure 5.1 shows an example of how the refractive index changes with different incident angles. Here, it can be seen that at broadside incidence, TE and TM both experience the same effective refractive index of 2.86, but this drops for both modes as the angle increases. For TM, it indeed drops to the host material permittivity at the grazing angle, whereas for TE, this is higher because the field still interacts with the patches for 90° incidence.

Modeling the Anisotropy in CST

To examine the effects of ADL anisotropy on lens performance, CST simulations have been performed. Anisotropy, however, cannot be combined in CST with the analytical formula previously used to express the spatially variant refractive index. Therefore, the lens is subdivided into small cuboids, each with its own anisotropic refractive index, with an interlayer spacing of 500 μm . The width of these cuboids is set to $\lambda_0/10$, and the height is unaffected. CST requires a tensor input for the dielectric permittivity and permeability to describe anisotropic materials; hence, using the methods in Appendix B, these tensors are extracted from the raised cosines. The simulation is shown in Figure 5.2, where half of the lens is visible, indicating the different permittivities throughout by color. A 36-mm-diameter collimating lens with two matching layers on top and on the bottom is used, fed by a small dipole at a focal length of 7 mm. The lens is still cylindrical; however, for ease of integration, cuboids surround it in a square grid. The cuboids outside of the lens are equal to the background material, equivalently making a cylindrical lens. To isolate the effects of the anisotropy as much as possible, no metal plate is added.

A similar simulation is also run in which the cuboids are made of isotropic materials, with the intended refractive index at broadside. Figure 5.3 shows a comparison of these two simulations. Here, the fields

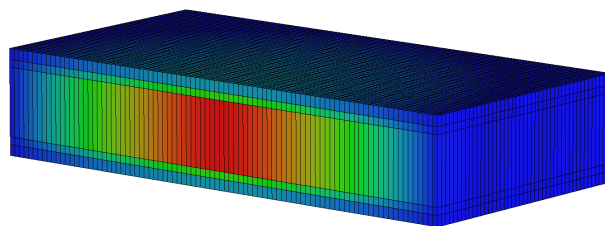


Figure 5.2: Half of the lens simulated in CST, cut in the x-plane. The lens consists of a multitude of $\lambda_0/10$ -wide bricks with different permittivities. The lens is 36 mm wide, fed by a dipole at a 7 mm focal length, and has 2 matching layers on top and 2 matching layers on the bottom, with an original core height of 5.7 mm. n_{side} is set to the host material, which is set to be 1.

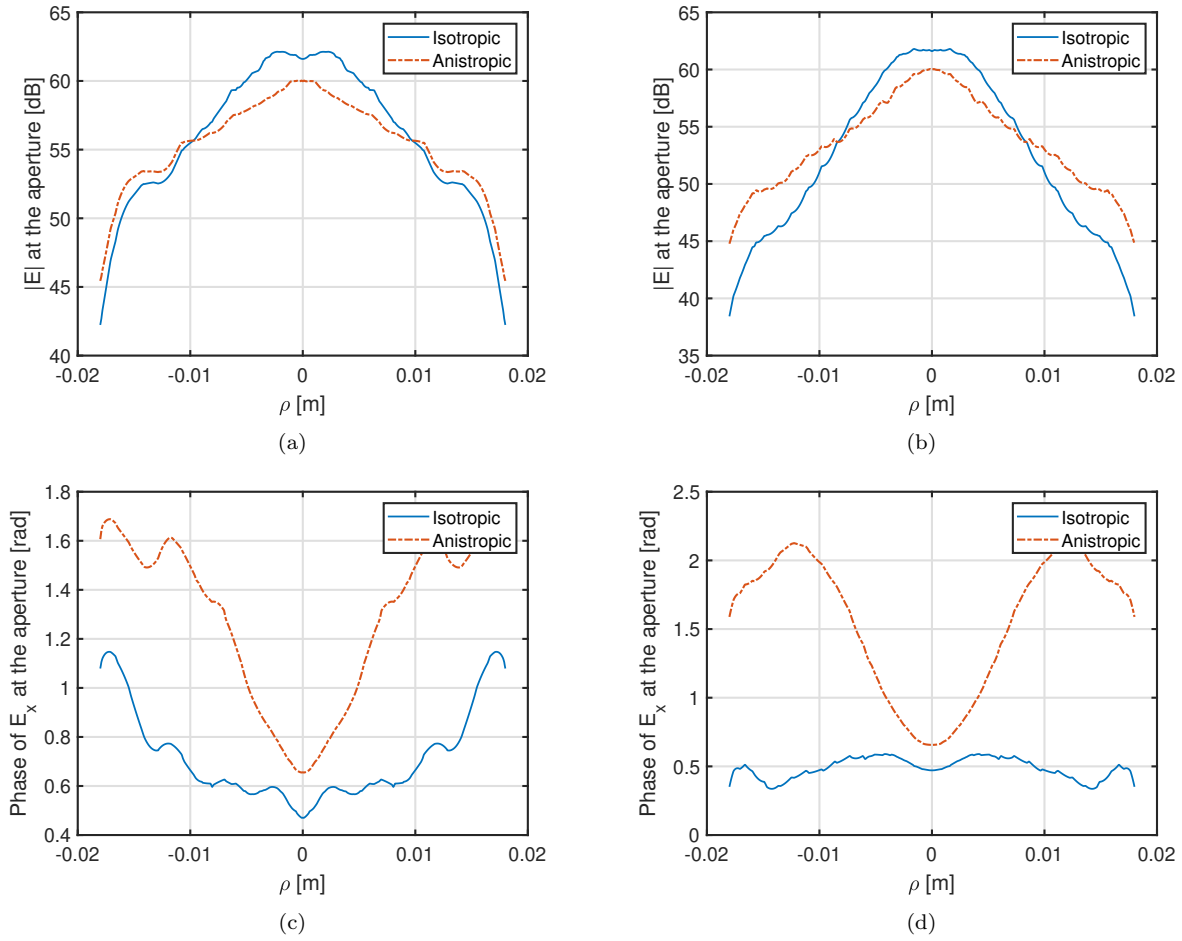


Figure 5.3: The comparison between the isotropic and anisotropic lens on the top aperture of the lens. (a) total field amplitude at $\phi = 0$ (b) total field amplitude at $\phi = \pi/2$ (c) the phase of E_x at $\phi = 0$, (d) the phase of E_x at $\phi = \pi/2$.

at the lens's top aperture are compared in both phase and amplitude. The phase of the main component is selected. It can be clearly seen in Figures 5.3c and 5.3d that, with the anisotropy, the desired flat phase is no longer present. Especially in the middle, a big drop in phase is visible. In the E-plane, at $\phi = 0$, the isotropic lens also does not show a very flat phase front, but this is due to the very small F-number. Furthermore, the anisotropic lens appears to have a smaller amplitude taper than its isotropic counterpart, shown in Figures 5.3a and 5.3b. Figure 5.4 shows the effects of these discrepancies on the far field.

The maximum directivity appears to be nearly unaffected, with a change of only 0.05 dB; other parameters have been influenced by the introduced anisotropy, however. We can see that the main beam is slightly narrower, decreasing the angular width in the E-plane from 9.1° to 8.0° , and from 7.7° to 7.2° in the H-plane. Additionally, the sidelobes have risen. In the E-plane, the shoulder has increased by 10.1 dB, and in the H-plane, the sidelobe level has increased by 8.9 dB. This indicates that the anisotropic behavior indeed affects the lens's radiation characteristics and should be taken into account.

Modeling the Anisotropy in the Tool

An attempt was made to incorporate anisotropic behavior into the simulation tool created in this thesis. This was done by changing the analytical formula for n to that of (5.1), with $n(0^\circ)$ the original desired refractive index polynomial. This yields a refractive index formula that varies with both space and the angle of incidence, which can be used for the ray tracing and subsequent steps. This implementation, however, is insufficient to incorporate the behavior. As shown in Figure 5.5, both the phase and amplitude

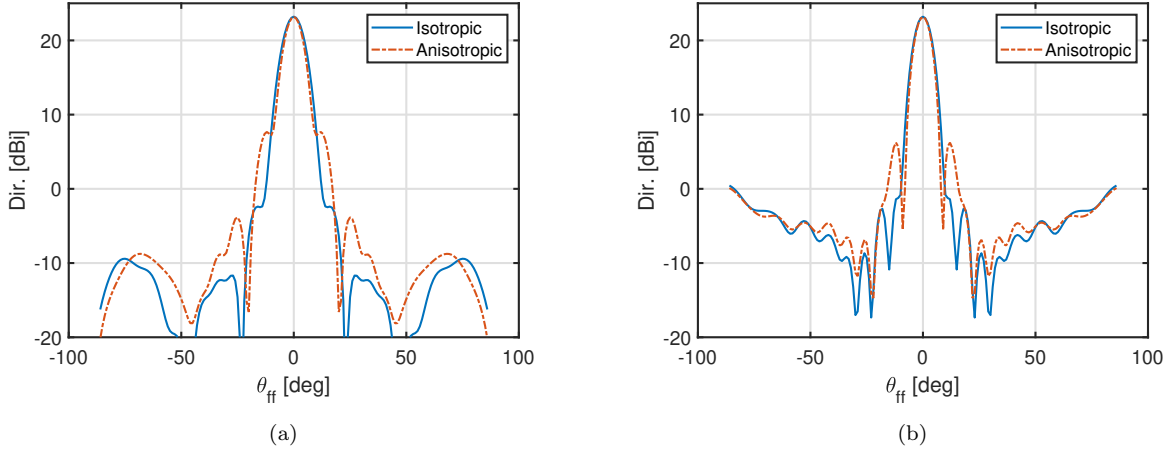


Figure 5.4: Comparison of directivities for a lens made of isotropic and anisotropic material. The lens is 36 mm wide, fed by a dipole at a 7 mm focal length, and has 2 matching layers on top and 2 matching layers on the bottom, with an original core height of 5.7 mm. The host material is set to be 1. (a) $\phi = 0$ plane, (b) $\phi = \pi/2$ plane.

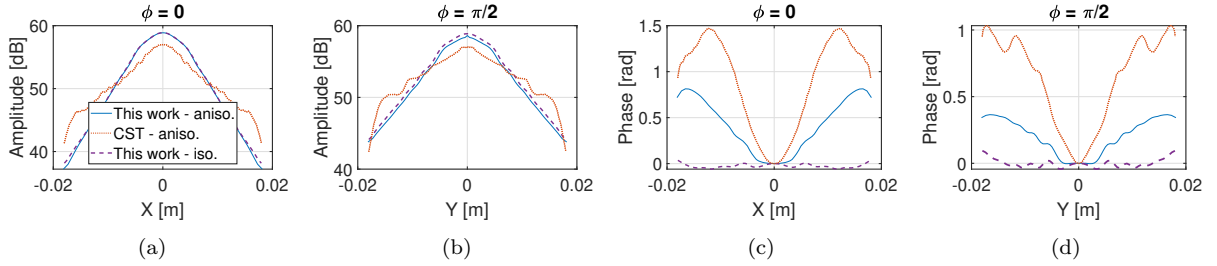


Figure 5.5: The comparison of the modeling of the fields on the top lens aperture for CST and the tool, for anisotropic and isotropic lenses. (a) total field amplitude at $\phi = 0$ (b) total field amplitude at $\phi = \pi/2$ (c) the phase of E_x at $\phi = 0$, and (d) the phase of E_x at $\phi = \pi/2$. The lens is 36 mm wide, fed by a dipole at a 7 mm focal length, and has 2 matching layers on top and 2 matching layers on the bottom, with an original core height of 5.7 mm. The host material is set to be 1.

at the lens aperture do not match. There is little change in the amplitude, but the phase is less flat. While the tool is more accurate when anisotropic effects are accounted for than when they are neglected, it still underestimates the degree of anisotropy. One reason this implementation might fail is that the raytracing ODEs for anisotropic materials are not obtained by simply substituting $n(x, y, z)$ by $n(x, y, z, \theta)$. For example, some steps used to derive the ODEs from the Eikonal equation assume a scalar dielectric permittivity, rather than a tensor, as is the case for anisotropic materials [35]. Therefore, the ray tracing might be flawed in this scenario [29].

Figure 5.6 shows the far-field patterns for CST anisotropy and this work, with and without anisotropy. The CST simulations have been performed with a top plate present, so that the tool more closely resembles the CST simulation. Comparing these patterns, one can see that the tool does not adequately model anisotropic behavior. Taking the behavior into account results in a rise in the shoulders and sidelobes. This is, however, a slight increase compared to the rise shown by CST. Hence, another approach is needed to account for ADL anisotropy in this tool, which is outside the scope of this thesis. Another suggestion is to look into isotropic metamaterials, such that these effects do not have to be modeled. This could, for example, be done by using 3D inclusions instead of the 2D patches. This again requires complex manufacturing, however [53].

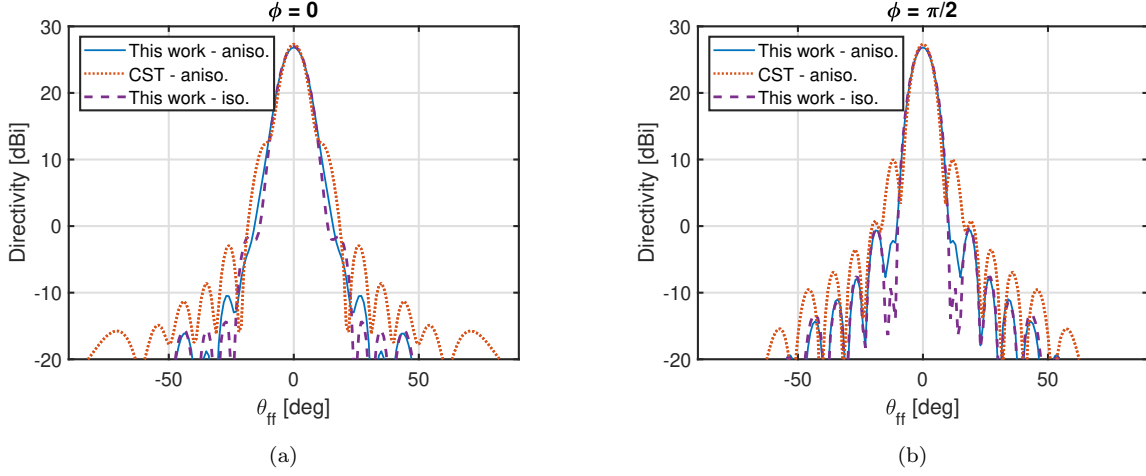


Figure 5.6: Comparison of far-field directivities of CST and the tool for an anisotropic and isotropic lens at (a) $\phi = 0$ and (b) $\phi = \pi/2$. The lens is 36 mm wide, fed by a dipole at a 7 mm focal length, has 2 matching layers on top and 2 matching layers on the bottom, and has a top plate present, with an original core height of 5.7 mm. The host material is set to be 1.

5.2 Output Angle Offset Compensation

For canonical lens designs, e.g., a collimating lens without any matching layers, one can use analytical expressions for the refractive index and obtain an excellent design. If more complex designs, e.g., with matching layers, are made, it might be more tedious to obtain a refractive index profile with similar performance. One way to evaluate a lens performance is to check whether the rays exit the lens in the desired direction. To improve such tedious designs, an automatic refractive-index compensation algorithm has been developed to adjust skewed rays into collimated rays.

An increase in the gradient along the path a ray traverses leads to a difference in the output angle:

$$\Delta \frac{\partial n(x, y)}{\partial \rho} \propto \Delta \theta_{\text{out}}.$$

Integrating both parts over ρ , this becomes

$$\Delta n \propto \int \Delta \theta_{\text{out}} d\rho.$$

This relation is used to apply a refractive-index correction to readjust the rays for collimation. Setting the desired output angle to zero, this relation leads to

$$\Delta n(\rho) = -\frac{1}{T} \int_0^\rho \theta_{\text{out}}(\rho) d\rho, \quad (5.4)$$

with $\theta_{\text{out}}(\rho)$ the output angle at a certain point ρ and T the thickness of the lens. Important to note is that this has only been researched for lenses without any matching layers. The $1/T$ scalar has been empirically found and is further outlined in Appendix C. The scalar seems to be independent of parameters such as diameter or focal length. The new permittivity $n'(\rho)$ will then be

$$n'(\rho) = n(\rho) + \Delta n(\rho), \quad (5.5)$$

with n the permittivity, from which the output angles are found, used for computation of Δn .

Example 1: Quadratic Error

To confirm this theory, a manual error is introduced into an originally collimating lens, designed with the equations given in [28]. In this case, the error is chosen to be a quadratic relation to the distance from

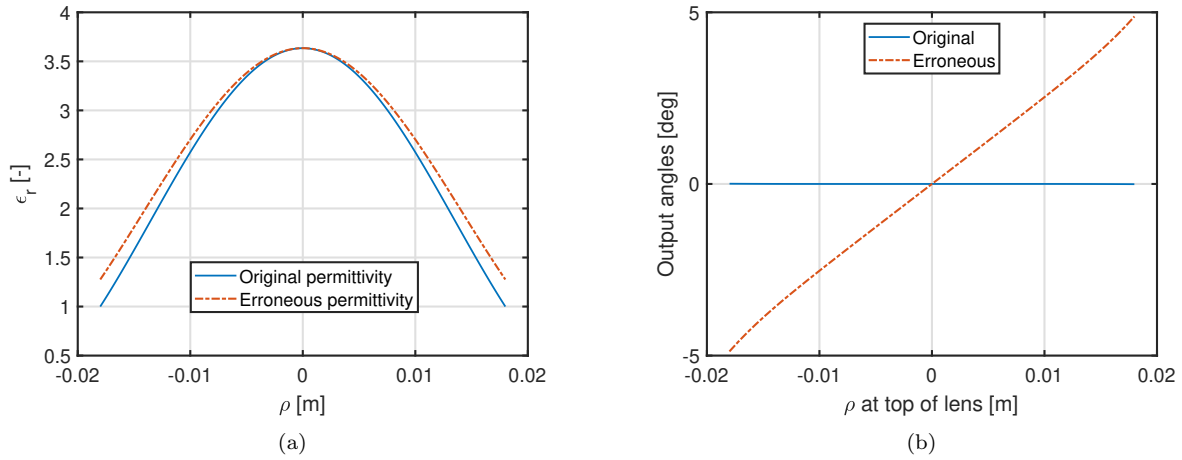


Figure 5.7: (a) The original dielectric permittivity distribution of a lens and the distribution with the error of $n_{\text{err}}(\rho) = 400\rho^2$ added to it. (b) The output angles of both distributions. The lens is designed to be collimating, with a diameter of 36 mm, a height of 5.7 mm, a focal distance of 27 mm, no matching layers, and n_{side} set to 1.

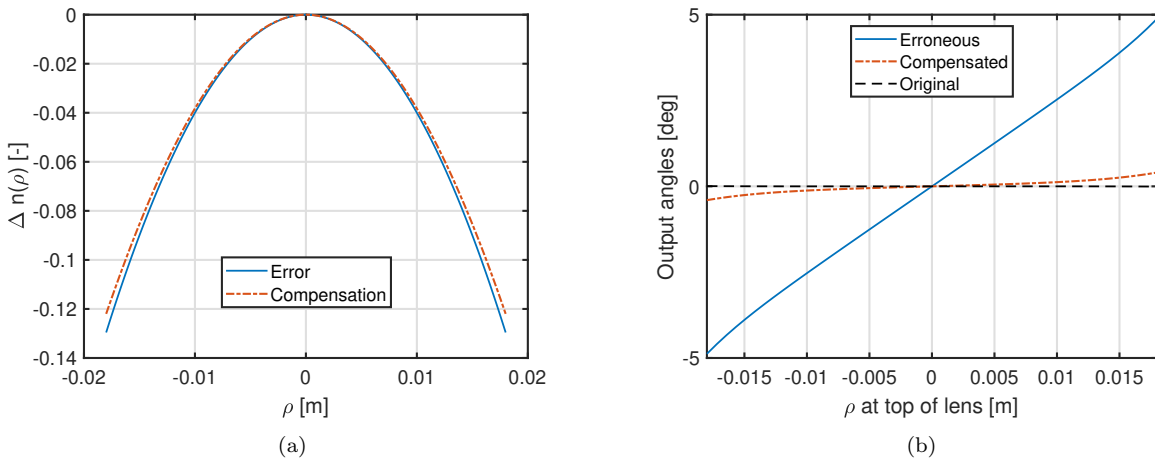


Figure 5.8: (a) The permittivity compensation compared to the opposite of the added error. (b) The results of the compensated lens compared with those of the erroneous and non-erroneous lenses.

the center. This error $n_{\text{err}}(\rho)$ of $400\rho^2$ is added to the refractive index profile of a 36-mm-diameter lens, as shown in Figure 5.7a. Ray tracing with this erroneous distribution yields the ray output angles shown in Figure 5.7b. It can be observed that the relation with ρ can be described as nearly linear. Integrating over this yields a nearly quadratic relation, which relates back to our original error.

Applying (5.4) to this output angle yields the permittivity pattern as shown in Figure 5.8a. This is compared to the opposite of the added error. It can be seen that these are fairly similar. Adding this compensation to the erroneous permittivity and performing the ray tracing yields the output angles given in Figure 5.8b. It can be seen that even though the compensated output angles do not perfectly match the original ones, there is still a significant improvement. The compensation is slightly undercompensating and could be adjusted manually by adding a small factor. Furthermore, this small difference could be explained by the imperfect linear behavior of the output angles.

Example 2: Sinusoidal Error

Another, slightly more complicated error is introduced: $n_{\text{err}}(\rho) = -0.05 \cos(80\pi\rho)$. The permittivities with and without this error are shown in Figure 5.9a and the resulting output angles in Figure 5.9b.

Looking at Figure 5.10a, it can be seen that the integral of the output angles is indeed near-sinusoidal

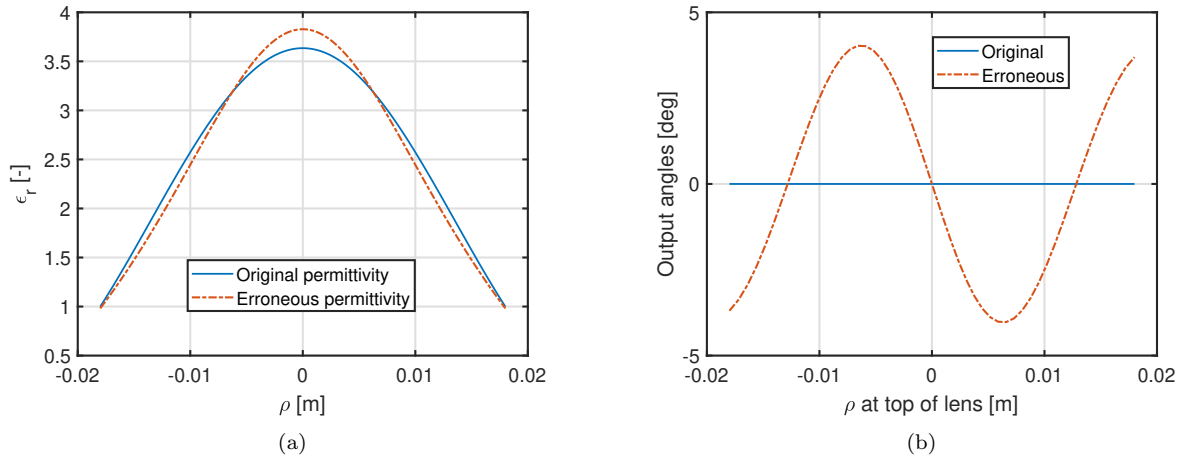


Figure 5.9: (a) The original dielectric permittivity distribution of a lens and the distribution with the error of $n_{\text{err}}(\rho) = -0.05 \cos(80\pi\rho)$ added to it. (b) The output angles of both distributions. The lens is designed to be collimating, with a diameter of 36 mm, a height of 5.7 mm, a focal distance of 27 mm, no matching layers, and n_{side} set to 1.

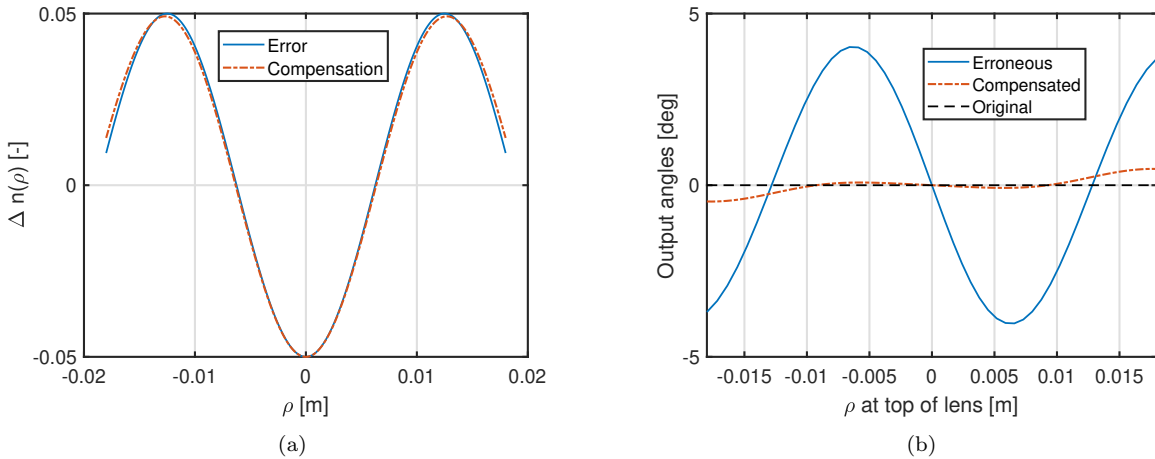


Figure 5.10: (a) The permittivity compensation compared to the opposite of the added error. (b) The results of the compensated lens compared to the erroneous and original lens.

and overlaps the error relatively well. The more to the edges, the fit is slightly less perfect, however. This is also evident in the output angles shown in Figure 5.10b, where, at the edges, the output angles are not exactly zero. In this case, it cannot be improved by adding a scalar. Yet, the compensation is a significant improvement.

Handling More Complex Cases

There is a downside to the compensation formulation. The output angle is influenced by the entire ray path, but it is assigned to only one point in ρ , at the end of the path; for the compensation, only this point is taken into account. For most lenses, the rays do not traverse very much horizontally, so the ray path is relatively close to the point of influence. However, in some cases, the ray path might be very horizontal. Then it could be that the error is mainly present at the beginning of the path, whilst the compensation still would be applied at the end of the path, quite a long way further. To illustrate this a bit more, another combination of lens and error is selected: $n_{\text{err}}(\rho) = 0.05 \cos(160\pi\rho)$ and the feed is moved to a focal length of 7mm. The latter ensures that the rays are more horizontal, whilst the former creates a more rapidly varying refractive index error. The permittivities and output angles for this error are shown in Figure 5.11.

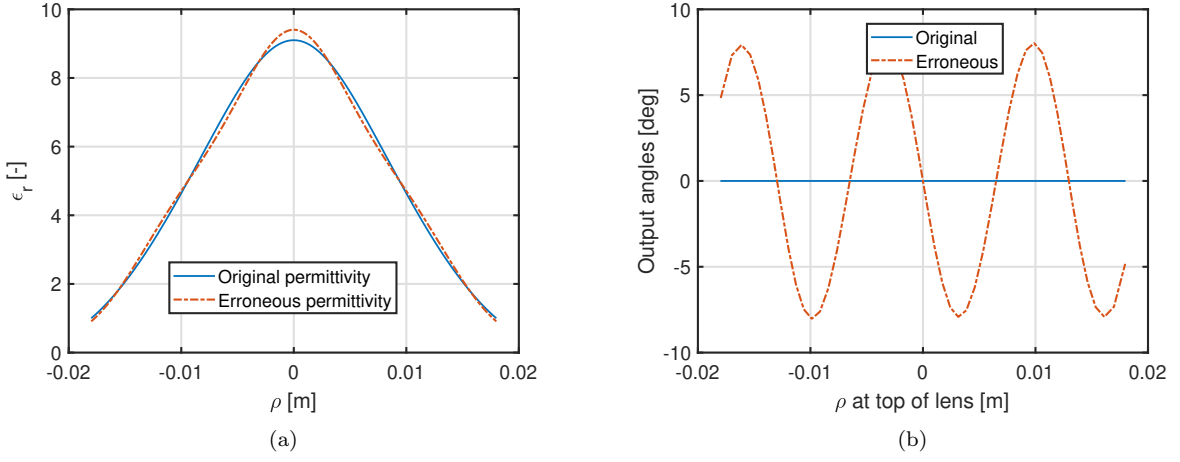


Figure 5.11: (a) The original dielectric permittivity distribution of a lens and the distribution with the error of $n_{\text{err}}(\rho) = -0.05 \cos(160\pi\rho)$ added to it. (b) The output angles of both distributions. The lens is designed to be collimating, with a diameter of 36 mm, a height of 5.7 mm, a focal distance of 7 mm, no matching layers, and n_{side} set to 1.

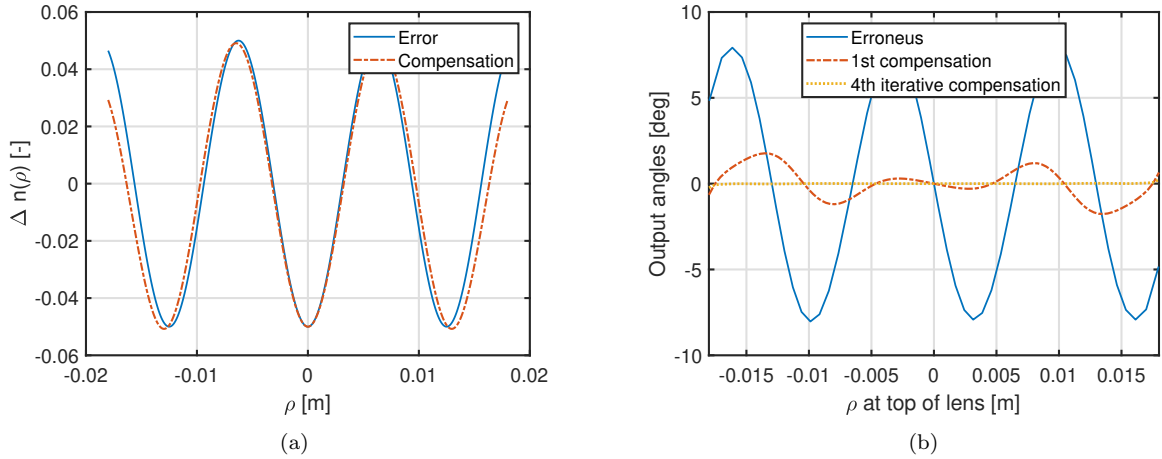


Figure 5.12: (a) The permittivity compensation compared to the opposite of the added error. (b) The results of the erroneous lens compared to the compensated lens, with one or four iterations.

Looking at Figure 5.12a, the described effect is visible, where the error precedes the compensation for larger angles. The point to which the path effects are assigned lies further than is required. This effect is also slightly visible in Figures 5.10 and 5.8a, but is much less present. The result on the compensated output angle is therefore also not as great for this small F/D as depicted in Figure 5.12b.

The algorithm is nevertheless found to be iteratively applicable. Figure 5.12b showcases the influence of this iteration, where the algorithm has been applied three more times, yielding very flat output angles. One can thus set a threshold for the output angles and iterate the compensation algorithm until that threshold is met. This can be based on a least-squares difference with the target output angles, the maximum difference alone, or any other method the user suggests. This shows that the algorithm can be applied to a multitude of lenses that do not fully meet the requirements, based on ray output angles. Even when the compensation appears flawed, it can be resolved by applying additional compensation rounds, still yielding excellent results. However, it is important for the user to validate the newly provided permittivity distribution. It could be that this compensation results in a distribution that is not physically possible or falls outside the design parameters, e.g., some values fall below $n = 1$ or n_{host} . In this case, the compensation could be offset to adjust for this.

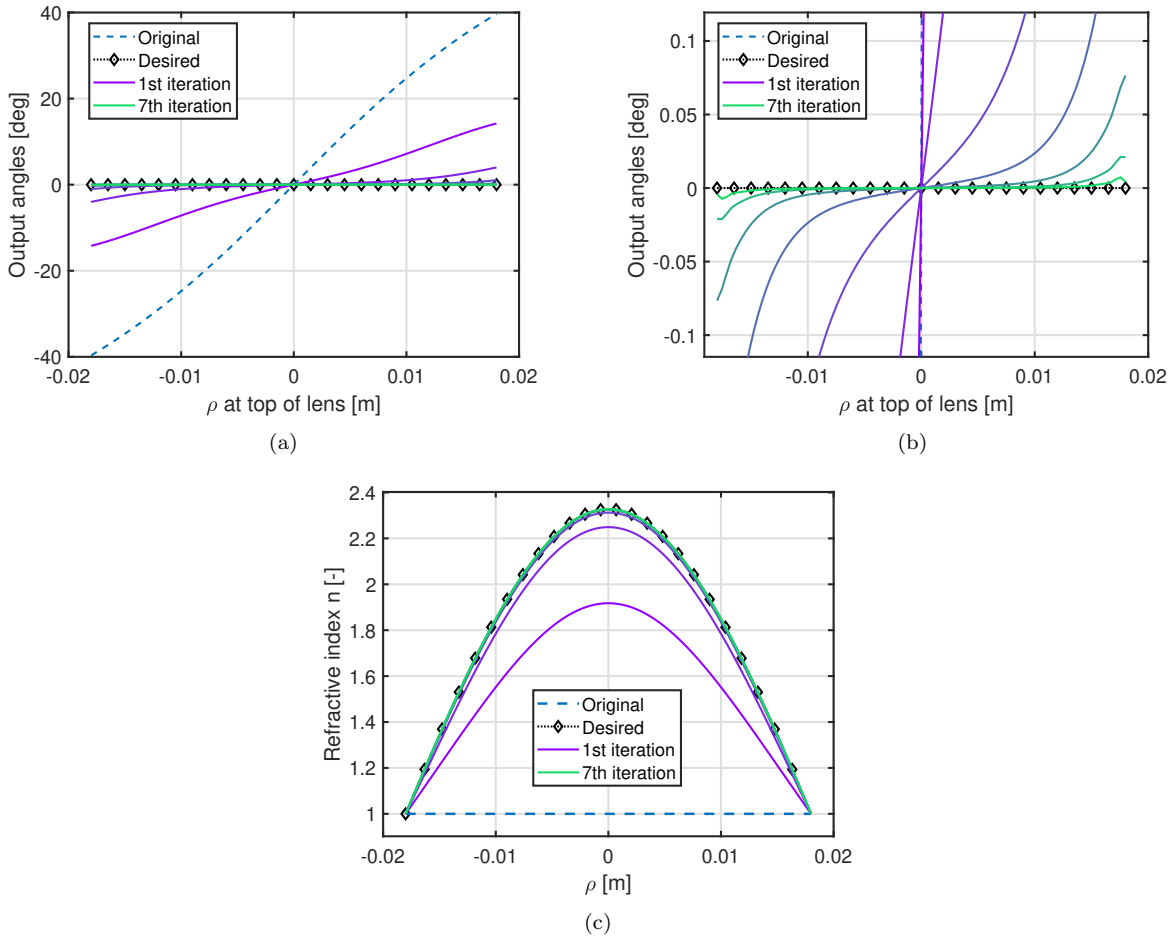


Figure 5.13: The results of the iterative program to design a collimating lens, with $D = 36$ mm, $T = 5.7$ mm, $n_{\text{side}} = 1$, and $F = 16$ mm. The output angles for each iteration are shown on (a) full scale and (b) small scale, with (c) the corresponding refractive index distributions.

Creating Lens from Desired Output Angles

The algorithm can be extended not only to compensate for errors in the permittivity profile, but also to synthesize a permittivity distribution corresponding to a desired output angular distribution. If one knows the desired output angles for a given lens design, this algorithm can be applied iteratively to obtain the appropriate permittivity distribution. Figure 5.13 shows an example of this, where a lens is designed for collimation. The lens is originally set to have an overall refractive index of 1, a thickness of 5.7 mm, a diameter of 36 mm, and a focal length of 16 mm. Figure 5.13a indicates the output angles of this original, homogeneous slab, going up to 40° . A first iteration already creates a lens design that is a significant improvement to the homogeneous slab, reducing the output angles to below 20° . The refractive index distribution for this is shown in Figure 5.13c. This figure also shows the desired distribution, as given by equations in [28]. Because the output angles are very large, the first iteration is quite off. The seventh iteration, as shown in Figure 5.13b, yields a distribution that collimates the rays to below 0.01° . Figure 5.13c shows that the permittivity indeed overlaps well with the desired shape. This shows that this algorithm could be used to design lenses within minutes if the desired output angles are known. This way, one is less bound to specific lens designs, such as collimation. It is again important, however, that the user imposes constraints on the algorithm, such as the refractive index at the boundaries.

This algorithm could be further tested and applied to matched-layer lenses. Another place this could be applied to is the height compensation for the matched-layer lenses, as discussed in Section 2.5. Instead of changing the thickness, one could alter the core lens permittivity distribution using the described



technique above. With flat lenses, however, it is often desirable to reduce thickness to improve system integration [54, 55]. Hence, the reduction is still applied. The output angle compensation can still be applied to the remaining small errors.

Chapter 6

Conclusion and Future Work

6.1 Conclusion

This thesis presented an efficient semi-analytical framework for the analysis of gradient-index (GRIN) flat lens antennas. The proposed approach combines geometrical optics ray tracing, a Delaunay-triangulation-based implementation of the ray tube theorem, transmission-line modeling, and physical optics to predict the aperture fields and far-field radiation characteristics of electrically large GRIN lenses. Furthermore, the applicability is extended, implementing generalized quarter-wavelength matching-layer synthesis, automatic lens-thickness compensation to account for matching layers, polarization tracking, and a transmission-line formulation that captures propagation through multilayer lens structures. The framework was subsequently extended to analyze lenses illuminated by off-axis feeds. Feed displacement and rotation were incorporated into the ray-tracing formulation, enabling the efficient evaluation of beam steering and scan performance.

The framework was extensively validated against CST for both center-fed and off-axis-fed lenses, showing good agreement between the predicted aperture fields and radiation patterns. For broadside configurations, the proposed method predicts the peak directivity within approximately 0.1 dB of CST, whereas this accuracy is reduced to 0.2-0.5 dB for large beam-scanning cases. The validation also identified the primary limitations of the model, such as the presence of caustics, total internal reflection, and diffraction effects, thereby defining its range of applicability. Furthermore, the complete analysis procedure was integrated into a MATLAB-based graphical user interface, providing an accessible design and evaluation tool for GRIN lens antennas.

In addition, the influence of anisotropy in artificial dielectric layers was investigated. While CST simulations demonstrated that anisotropy can significantly affect lens performance, the results showed that a scalar refractive-index formulation is insufficient to accurately capture these effects. Finally, a novel output-angle-based refractive-index compensation method was introduced, which was shown to effectively correct profile errors and can also be used as an iterative design method to synthesize GRIN lens profiles.

Overall, this work provides an accurate and computationally efficient alternative to full-wave analysis for electrically large GRIN flat lenses. Once the feed has been characterized separately in CST, the complete lens analysis can typically be performed in less than two minutes, compared to the multiple hours required for a full-wave simulation of the complete feed-and-lens system. This enables rapid design iterations and optimization without sacrificing practical accuracy, making it a valuable tool for the development of GRIN flat lens antennas.

6.2 Future Work

Even though the simulation tool shows promising results, more can be improved or added:

- Both feed displacements and different types of lenses create a non-flat phase front. This is yet to be taken into account for automated height compensation, as opposed to only collimation.
- Furthermore, the matching layers are now fixed to be based on quarter-wavelength transformers. The option to incorporate arbitrary permittivities and thicknesses could increase flexibility. This could enable matching layer structures with Chebyshev and binomial frequency responses.
- Multiple articles claim that parallelization could significantly increase computational speed [56, 57]. In MATLAB's parallel computing environment, workers are independent MATLAB computational engines that run in the background. An early version of the algorithm has been parallelized, but without much improvement. A possible explanation is that workers spend nearly as much time organizing and communicating tasks as they do executing them. This can happen when the computations are not very heavy, and mainly just a lot. For the current ray tracing algorithm, this is indeed the case. It might be worth vectorizing the computations for all rays into a single large matrix, rather than using two nested for loops over ρ and ϕ . In this way, one computation is larger, performing many small ones at once, reducing the percentage of time spent on the workers' organization. Besides, MATLAB is particularly well-suited for matrix computations.
- Another way to speed up the ray tracing is to apply rotational symmetry. When the feed is not offset and the lens is rotationally symmetric, so are the rays. Therefore, it would suffice to compute only one slice and map it to the other cuts.
- With the introduction of off-axis feeds, TE and TM mode-switching throughout a ray path was also introduced. For now, the model does not include a solution to take this into account. Nevertheless, the model does show promising results without this. However, it might be worth investigating further.
- The Delaunay-triangulation-based ray tube theorem is a fast and effective method for computing the divergence factor of individual rays. However, there are multiple cases where it cannot produce legitimate results. This is for internal reflections and caustics. A possible solution could be to automate the detection of these two errors, rerun the ray tracing with very tight guide rays for each original ray, and apply the ray tube theorem accordingly. This way, the lack of rays due to internal reflections does not affect the Delaunay triangulation for all other rays. Additionally, at very small scales, these rays might not cross, reducing the effect of caustics. This would significantly increase simulation time; however, it could possibly yield results, as opposed to none. Another possible approach to finding the divergence factor is to dismiss the ray tube theorem altogether. According to [56], the divergence can be tracked using an additional set of ODEs, based on the wavefront curvature radii. This is more mathematically involved, but further research could possibly show it to be a good fit for the simulation tool in this work. This latter suggested solution still requires a Delaunay-triangulated mesh to compute the far field.
- Automatically detecting when rays are badly sampled, even without caustics, and then locally sampling more could already be very useful. This would reduce the redundancy built in to prevent bad sampling.
- The output angle offset compensation shows promising results for single-slab lenses. As GRIN lenses usually contain matching layers, it is worth extending the algorithm to cover them as well.
- The ultimate goal of this tool is to simulate GRIN lenses made from ADLs. The unit cells, in which ADLs are usually subdivided, are already in place in the tool. The UC-layer stack could easily be converted to an ADL stack. This could then be solved by finding the ABCD matrices from the patches' susceptances, following [30, 31, 32, 33]. Additionally, the synthesis of ADL UC stacks to achieve a desired dielectric permittivity remains missing. This is required to make a CST model that can be used to confirm the simulation model. Using the algorithm described in [58], the UCs of ADLs can be created. Furthermore, the anisotropic behavior has been shown to have a significant effect, which needs to be taken into account in the computations.
- Next, it would also be helpful to account for rays exiting from the sides of the lens, as in some



physical configurations this is simply the case. This requires a better sampling algorithm and some tweaks to multiple parts of the code.

- Lastly, the GUI could be extended further to enable a range of design flexibilities and perspectives. Examples could be a frequency response, sweeping over displacement or other parameters, manually creating matching layers, and changing the computational speed and, inversely, the accuracy by adjusting the polynomial fit order.

In short, there is much more this work could be improved on to create a more versatile and useful tool for daily design processes.



Appendix A

Feed Rotation Implementation

An essential step in beam steering is the ability to steer the lens feed into the desired direction. This appendix illustrates both the mathematical operations performed to reach this ability and its validation.

A.1 Feed Rotation Operation

It is assumed that the feed rotates about an axis in the $x - y$ -plane, thereby pointing the feed more towards the center of the lens. This axis can be found by

$$\hat{\mathbf{u}} = \begin{bmatrix} -\sin(\phi_{\text{rot}dir}) \\ \cos(\phi_{\text{rot}dir}) \\ 0 \end{bmatrix}, \quad (\text{A.1})$$

through the feeds phase center, with

$$\phi_{\text{rot}dir} = \tan^{-1} \left(\frac{\Delta y}{\Delta x} \right) + \pi,$$

taking the four quadrants into account. Using the Rodrigues rotation, as discussed in Section 2.7, the feed can be tilted over this axis. The rotation can also be formatted into one rotational matrix

$$\mathbf{R} = \begin{bmatrix} c + u_x^2(1-c) & u_x u_y(1-c) - u_z s & u_x u_y(1-c) + u_y s \\ u_y u_z(1-c) + u_z s & c + u_y^2(1-c) & u_y u_z(1-c) - u_x s \\ u_x u_z(1-c) - u_y s & u_y u_z(1-c) + u_x s & c + u_z^2(1-c) \end{bmatrix}, \quad (\text{A.2})$$

where c and s are $\cos(\theta_{\text{rot}dir})$ and $\sin(\theta_{\text{rot}dir})$, respectively.

For this feed rotation, first, the far field of the feed is imported and brought back to the focal plane, assuming spherical spreading, as discussed in (3.1). Secondly, the original axes system is rotated in Cartesian coordinates.

$$\begin{bmatrix} x_{\text{rot}} \\ y_{\text{rot}} \\ z_{\text{rot}} \end{bmatrix} = \mathbf{R} \begin{bmatrix} x \\ y \\ z \end{bmatrix}. \quad (\text{A.3})$$

This Cartesian system is then brought back to the spherical coordinate system. This rotation shifts the fields pointed towards the original coordinate system towards the new coordinates, effectively steering the magnitudes towards the right direction. Next, the feed fields are interpolated to fit on the ray coordinates, using MATLAB's `scatteredInterpolant` function. Lastly, the actual field vectors are also rotated, using

$$\begin{bmatrix} E_{x,\text{rot}} \\ E_{y,\text{rot}} \\ E_{z,\text{rot}} \end{bmatrix} = \mathbf{R} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}. \quad (\text{A.4})$$

This way, the feed is rotated in both the field direction and orientation and interpolated onto the correct grid.

A.2 Validation of Feed Rotation

To validate the mathematical operations described above, in CST, a conical horn with an inner diameter of 3 mm and 1.35 mm at the top and bottom, respectively, and a length of 6 mm is simulated. This horn is fed by a TE_{11} x -polarised cylindrical waveguide of 6 mm long. The far field is exported with the origin in the feed's phase center. Next, using the MATLAB algorithm, the feed is rotated 45° about the x -axis for the figures in Section A.2 and about the $x = y$ -axis for the figures in Section A.2. In CST, the far field axes are rotated about the same axes as the MATLAB rotations. The corresponding far fields are also exported and compared with the results from MATLAB's rotation. Figures A.1 and A.3 portray the comparisons between the total fields, whilst Figures A.2 and A.4 depict the x -, y -, and z -components of the fields. Comparing CST's rotation and this work's rotation, there are nearly no differences, validating the algorithm.

Rotation of 45° about the x -Axis

In this example, the feed is rotated 45° about the x -axis in the z -plane of the phase center.

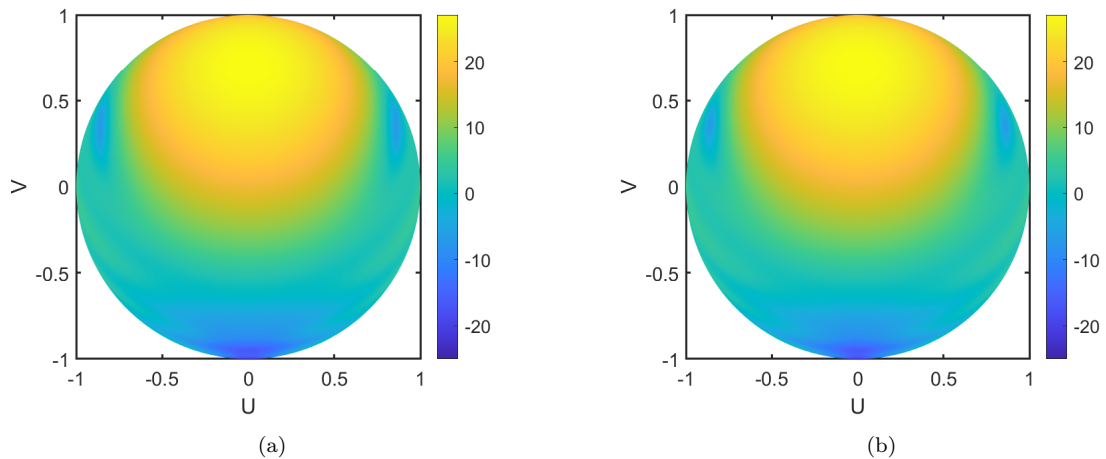


Figure A.1: The total electric far-field pattern of a conical feed, with an inner diameter of 3 mm and 1.35 mm at the top and bottom, respectively, and a length of 6 mm, fed by an x -polarised circular waveguide of 6 mm long. The feed is rotated 45° about the x -axis (a) in CST and (b) with the tool's MATLAB algorithm.

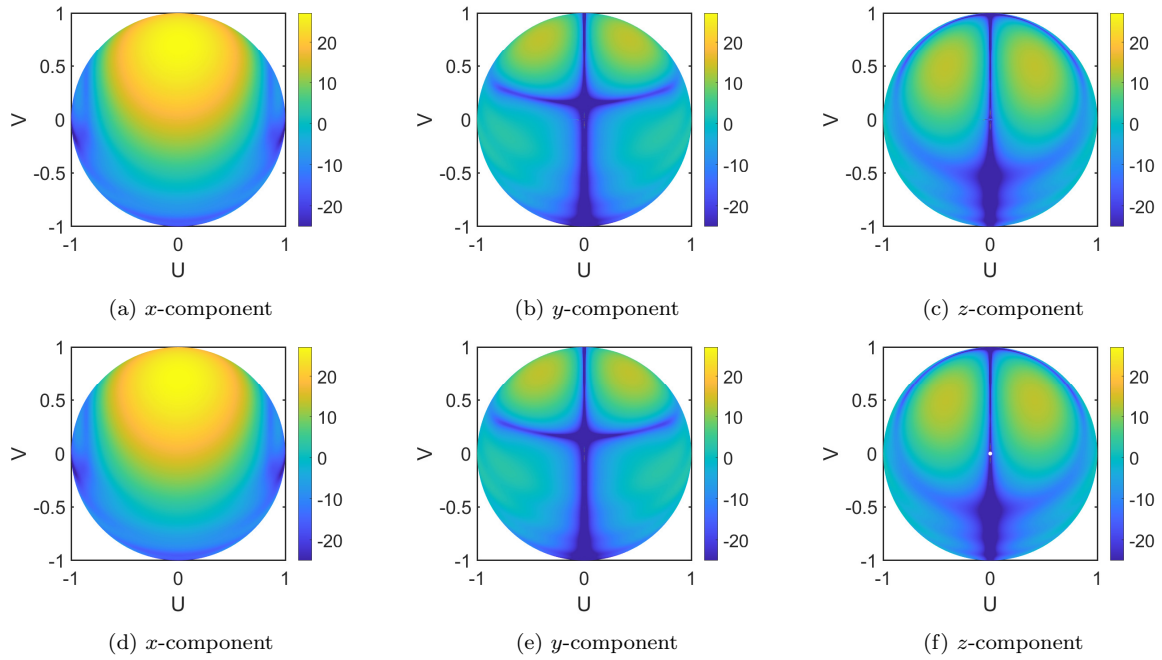


Figure A.2: The x -, y - and z -components of the electric far-field pattern of a conical feed, with an inner diameter of 3 mm and 1.35 mm at the top and bottom, respectively, and a length of 6 mm, fed by an x -polarised circular waveguide of 6 mm long. The feed is rotated 45° about the x -axis (a)-(c) in CST and (d)-(f) with the tool's MATLAB algorithm.

Rotation of 45° about the $x = y$ Axis

In this example, the feed is rotated 45° about the $x = y$ -axis in the z -plane of the phase center.

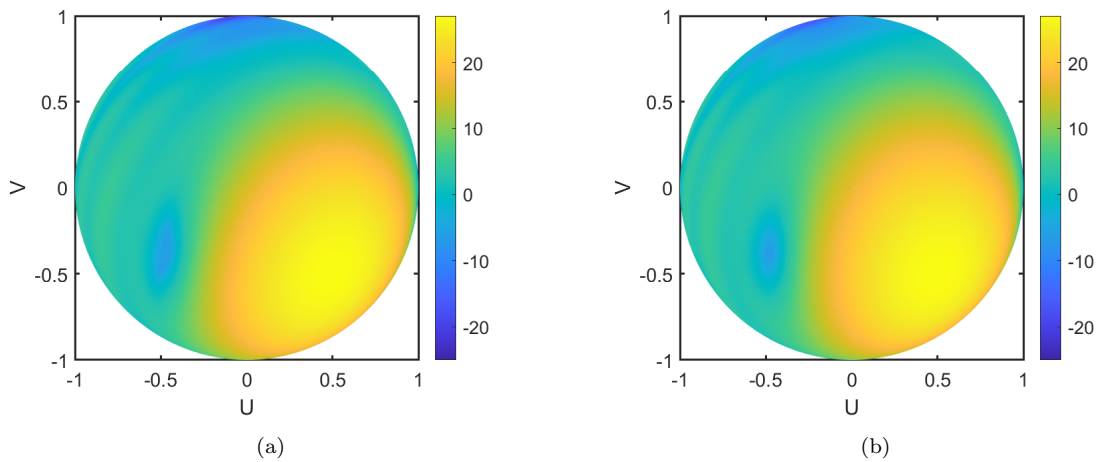


Figure A.3: The total electric far-field pattern of a conical feed, with an inner diameter of 3 mm and 1.35 mm at the top and bottom, respectively, and a length of 6 mm, fed by an x -polarised circular waveguide of 6 mm long. The feed is rotated 45° about the $x = y$ -axis (a) in CST and (b) with the tool's MATLAB algorithm.

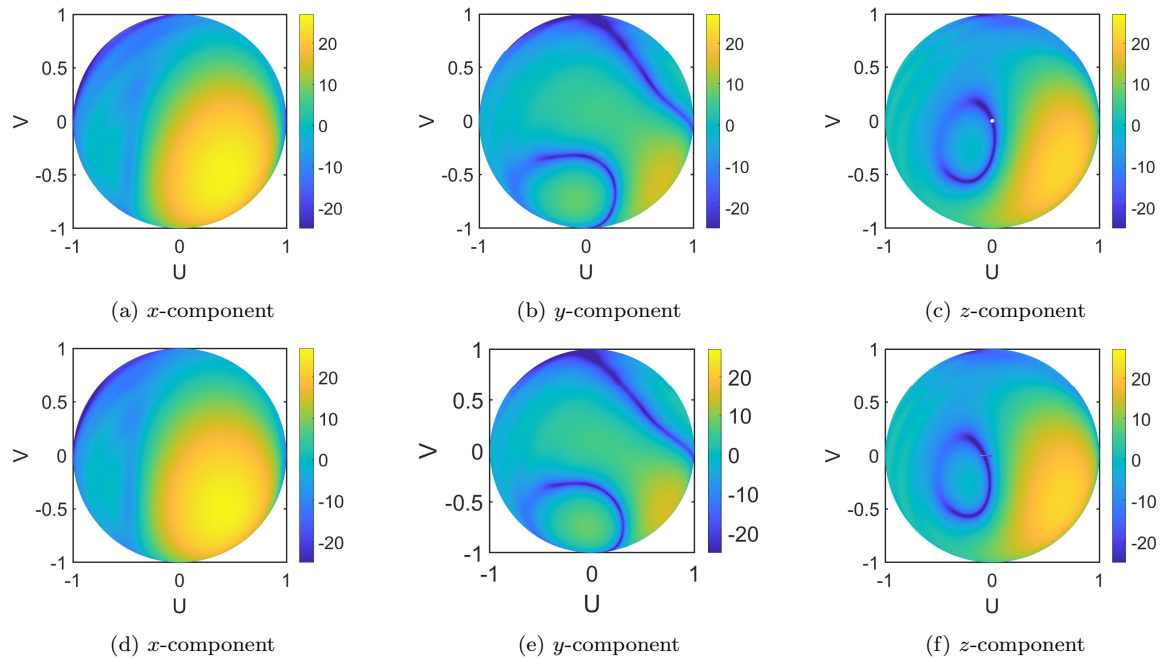


Figure A.4: The x -, y - and z -components of the electric far-field pattern of a conical feed, with an inner diameter of 3 mm and 1.35 mm at the top and bottom, respectively, and a length of 6 mm, fed by an x -polarised circular waveguide of 6 mm long. The feed is rotated 45° about the $x = y$ -axis (a)-(c) in CST and (d)-(f) with the tool's MATLAB algorithm.

Appendix B

Anisotropy Tensor Extraction

To import anisotropic materials into CST, the material must be expressed in tensor values $\bar{\bar{\epsilon}}_r$ and $\bar{\bar{\mu}}_r$ for the permittivity and permeability, respectively. This chapter provides two methodologies for extracting such tensors from different possible material slabs.

B.1 Tensor Extraction using S-parameters Under Different Incidence Angles

A method to extract the tensor permittivity and permeability values $\epsilon_{r,x}$, $\epsilon_{r,y}$, $\epsilon_{r,z}$, $\mu_{r,x}$, $\mu_{r,y}$ and $\mu_{r,z}$ is suggested by [59] and [58]. This uses the S-parameters for an incident plane wave on a slab of the material at two different incident angles: broadside and oblique. In this set of examples, $\theta_1^{\text{oblique}}$, as shown in Figure B.1 [59], is set to $\pi/4$.

The S-parameters can be computed for different slabs of different materials using the ABCD-matrix method described in Section 2.8. This can be extended to ADLs using the equations in [30], [31], and [33]. From these S-parameters, the parameters in (B.1a)-(B.1d) can be computed.

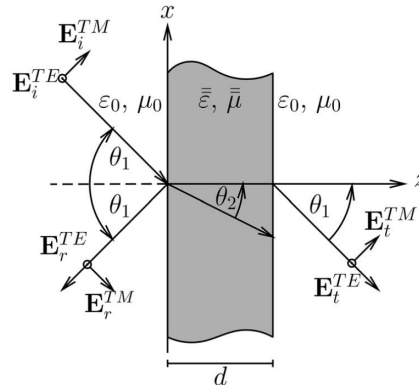


Figure B.1: The geometry of the slab under plane wave incidence.



$$\eta^{\text{TE}} = \pm \sqrt{\frac{(1 + S_{11}^{\text{TE}})^2 - (S_{21}^{\text{TE}})^2}{(1 - S_{11}^{\text{TE}})^2 - (S_{21}^{\text{TE}})^2}} \sec \theta_1 \quad (\text{B.1a})$$

$$\eta^{\text{TE}} = \pm \sqrt{\frac{(1 + S_{11}^{\text{TM}})^2 - (S_{21}^{\text{TM}})^2}{(1 - S_{11}^{\text{TM}})^2 - (S_{21}^{\text{TM}})^2}} \cos \theta_1 \quad (\text{B.1b})$$

$$n^{\text{TE}} = \sqrt{\left(\frac{\log |\zeta^{\text{TE}}| + j [\angle(\zeta^{\text{TE}}) + 2\pi m]}{-jk_0 d} \right)^2 + \sin^2 \theta_1} \quad (\text{B.1c})$$

$$n^{\text{TM}} = \sqrt{\left(\frac{\log |\zeta^{\text{TM}}| + j [\angle(\zeta^{\text{TM}}) + 2\pi \bar{m}]}{-jk_0 d} \right)^2 + \sin^2 \theta_1}, \quad (\text{B.1d})$$

where m and $\bar{m}$ are the branch indices, d is the structure's height, and

$$\zeta^{\text{TE}} = \frac{S_{21}^{\text{TE}}}{1 - S_{11}^{\text{TE}} (\eta^{\text{TE}} \cos \theta_1 - 1) / (\eta^{\text{TE}} \cos \theta_1 + 1)} \quad (\text{B.2a})$$

$$\zeta^{\text{TM}} = \frac{S_{21}^{\text{TM}}}{1 - S_{11}^{\text{TM}} (\eta^{\text{TM}} / \cos \theta_1 - 1) / (\eta^{\text{TM}} / \cos \theta_1 + 1)} \quad (\text{B.2b})$$

[59].

From these, the permittivities can be computed using:

$$\varepsilon_{r,x} = \frac{n^{\text{TM}}}{\eta^{\text{TM}}} \Big|_{\theta_1=0} \quad (\text{B.3a})$$

$$\varepsilon_{r,y} = \frac{n^{\text{TE}}}{\eta^{\text{TE}}} \Big|_{\theta_1=0} \quad (\text{B.3b})$$

$$\mu_{r,x} = (n^{\text{TE}} \eta^{\text{TE}}) \Big|_{\theta_1=0} \quad (\text{B.3c})$$

$$\mu_{r,y} = (n^{\text{TM}} \eta^{\text{TM}}) \Big|_{\theta_1=0} \quad (\text{B.3d})$$

There is, however, a certain ambiguity in the values of the branch indices m and $\bar{m}$ in (B.1). [58] and [59] state that m and $\bar{m}$ are obtained by equating $\varepsilon_{r,z}^{(1)}$ and $\varepsilon_z^{(2)}$, and $\mu_z^{(1)}$ and $\mu_z^{(2)}$, shown in (B.4). Once obtained, they can be inserted to find the other tensor values using the following equations:

$$\varepsilon_{r,z}^{(1)} = \varepsilon_{r,x} \frac{\sin^2 \theta_1}{\sin^2 \theta_1 - (n^{\text{TM}})^2 + (n^{\text{TM}}|_{\theta_1=0})^2} \quad (\text{B.4a})$$

$$\varepsilon_{r,z}^{(2)} = \frac{\sin^2 \theta_1}{(n^{\text{TM}} \eta^{\text{TM}})|_{\theta_1=0} - \varepsilon_{r,x} (\eta^{\text{TM}})^2} \quad (\text{B.4b})$$

$$\mu_{r,z}^{(2)} = \frac{\sin^2 \theta_1}{(n^{\text{TE}} / \eta^{\text{TE}})|_{\theta_1=0} - \mu_{r,x} / (\eta^{\text{TE}})^2} \quad (\text{B.4c})$$

$$\mu_{r,z}^{(1)} = \mu_{r,x} \frac{\sin^2 \theta_1}{\sin^2 \theta_1 - (n^{\text{TE}})^2 + (n^{\text{TE}}|_{\theta_1=0})^2}. \quad (\text{B.4d})$$

Figure B.2 shows the extracted z -components of ε_r and μ_r values for different slab thicknesses. In this case, $m = \bar{m} = 0$ was used. It shows the necessity of using the correct branch cuts. Here, a homogeneous isotropic slab with a permittivity of $\varepsilon_r = 7.5$ and a permeability of $\mu_r = 1$ is introduced. It can be seen

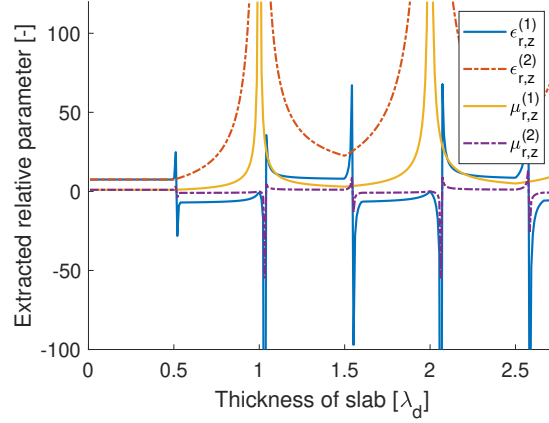


Figure B.2: Extraction of tensor z-components, with m and $\bar{m}$ set to zero.

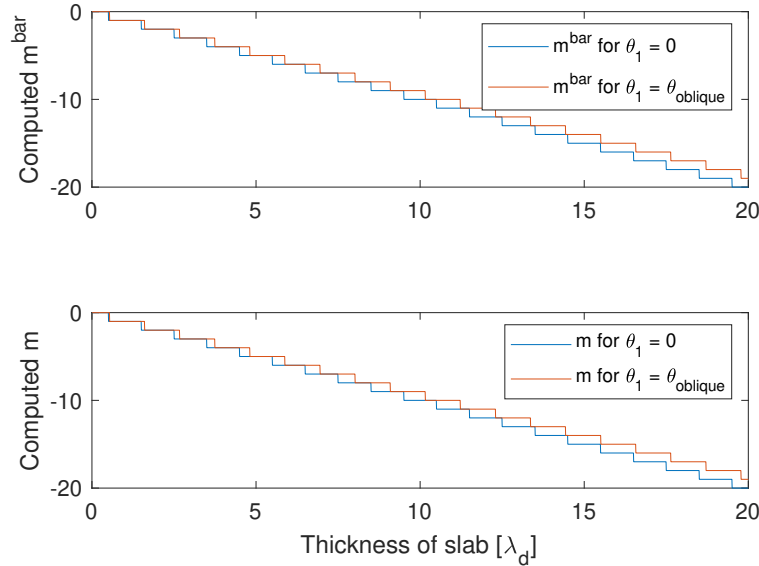


Figure B.3: The required branch indices m and $\bar{m}$ for both incident angles, over different slab thicknesses. The isotropic homogeneous material under test has $\varepsilon_r = 4$ and $\mu_r = 1$, with $\theta_{oblique}$ set to $\pi/4$.

that at small thicknesses, $\varepsilon_{r,z}^{(1)}$, $\varepsilon_{r,z}^{(2)}$, $\mu_{r,z}^{(1)}$ and $\mu_{r,z}^{(2)}$ indeed equate and give the original values of the slab. As the slab thickness increases, certain oscillations appear. This is because a wave has traveled multiple wavelengths inside the medium, increasing the branch index that must be used.

By computing and equating $\varepsilon_{r,z}^{(1)}$, $\varepsilon_{r,z}^{(2)}$, $\mu_{r,z}^{(1)}$ and $\mu_{r,z}^{(2)}$, cycling through different m and $\bar{m}$, the correct indices can be found for different slab thicknesses, counting up from 0. The required indices for another homogeneous isotropic slab under test are shown in Figure B.3, where ε_r is set to 4. An important notion, not explicitly mentioned in [59] and [58], is that m and $\bar{m}$ can differ for both angles of incidence.

Because, with an angle present, $k_{d,z} < k_d$. This results in a longer time of the wavefront in the medium, virtually a longer vertical wavelength than at broadside incidence. Therefore, the required m and $\bar{m}$ may differ for oblique and broadside incidence.

For better understanding, the difference is calculated mathematically for this specific slab. For broadside incidence, $k_{0,\rho} = 0$ and $k_{0,z} = k_0$. In the medium

$$\begin{aligned} k_d &= \sqrt{\varepsilon_r^{iso}} k_0 = 2k_0 \\ k_{d,\rho} &= k_{0,\rho} = 0, \end{aligned}$$

hence

$$k_{d,z} = k_{0,z} = 2k_0.$$

One could state that the virtual vertical wavelength λ_v^{broad} is

$$\lambda_v^{\text{broad}} = \frac{2\pi}{k_{d,z}} = \frac{\pi}{k_0}.$$

With an oblique incidence angle of $\frac{\pi}{4}$,

$$k_{0,\rho} = k_{0,z} = \frac{1}{\sqrt{2}} k_0.$$

This leads to

$$\begin{aligned} k_{d,\rho} &= k_{0,\rho} = \frac{1}{\sqrt{2}} k_0, \\ k_{d,z} &= -j\sqrt{-(k_d^2 - k_{d,\rho}^2)} = -j\sqrt{-(4k_0^2 - \frac{1}{2}k_0^2)} = \sqrt{3.5}k_0. \end{aligned}$$

Eventually, the virtual vertical wavelength λ_v^{obliq} can be stated to be

$$\lambda_v^{\text{obliq}} = \frac{2\pi}{\sqrt{3.5}k_0},$$

which is smaller than λ_v^{broad} .

This can be seen in Figure B.3. Here the m and $\bar{m}$ drop to -1 after $0.5\lambda_d = 0.5\lambda_v^{\text{broad}}$ for the broadside incidence and after $0.533\lambda_d$, which is $0.5\lambda_v^{\text{obliq}}$. For the isotropic slab, this drop repeats after each λ_v .

Hence, rather than using the same $\bar{m}$ for both angles, a slightly more nuanced approach is required. At first, the $\bar{m}$ filled in for the oblique and broadside incidence is expected to be the same (which might hold for thin slabs and small angles), and $\varepsilon_{r,z}^{(1)}$ and $\varepsilon_{r,z}^{(2)}$ are computed for a set of $\bar{m}$'s. If at one of the $\bar{m}$ values the $\varepsilon_{r,z}$'s coincide, the correct $\bar{m}$ is found, and the remaining tensor components can be computed. However, if no $\bar{m}$ was found within a reasonable range of $\bar{m}$'s, a set of $\bar{m}$ -offsets for the oblique incidence can be tried to find the correct $\bar{m}$. If that succeeds, the corresponding $\bar{m}$ filled in for the broadside incidence, without the offset, is used to compute the $\varepsilon_{r,x}$ and $\varepsilon_{r,y}$. This is because there, only broadside incidence is required for computation. For $\varepsilon_{r,z}$, the same $\bar{m}$ and $\bar{m}$ -offset are to be used. This procedure is to be repeated for $\mu_{r,z}$, to determine the required m .

This method of separating m and $\bar{m}$ for both incidence angles that is introduced to compute the z-components of $\bar{\varepsilon}_r$ and $\bar{\mu}_r$ is applied to the example shown in Figure B.2. As shown in Figure B.4, a constant $\varepsilon_{r,z}$ of 7.5 and $\mu_{r,z}$ of 1 are extracted.

The same procedure is applied to ADLs. The example stack is shown in Figure B.5a. The period of the patches is 1.5 mm or $0.025 \lambda_0$ at a center frequency of 5 GHz. The gaps between the patches are 0.6 mm or $0.01 \lambda_0$. Between the patches are dielectric slabs with $\varepsilon_r = 1.2$ and a height of 0.6 mm, or $0.01 \lambda_0$. The resulting $\varepsilon_{r,z}$ and $\mu_{r,z}$ extraction is shown in Figure B.5b. Here it is observed that not for all frequencies, $\varepsilon_{r,z}^{(1)}$ and $\varepsilon_{r,z}^{(2)}$ match. After about 4 GHz, this is no longer the case. The same holds for $\mu_{r,z}$ above 10 GHz. This could be due to the patch being assumed electrically small when computing the S-parameters, which may not hold at higher frequencies. The period at 10 GHz is still $0.05 \lambda_0$ and $0.055 \lambda_d$, so it would still be expected to hold. Hence, this technique would mostly work when computing at low frequencies, but is not reliable across all frequencies.

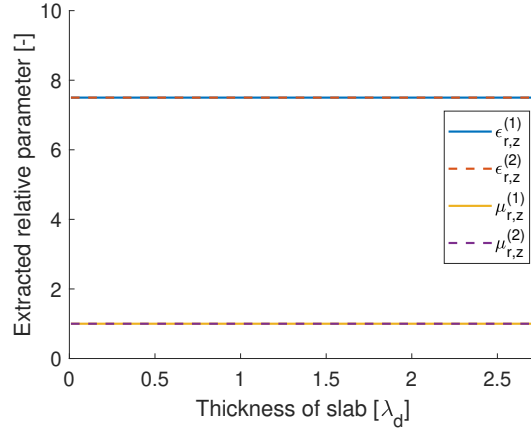


Figure B.4: Extraction of tensor z -components, with the correct m and $\tilde{m}$.

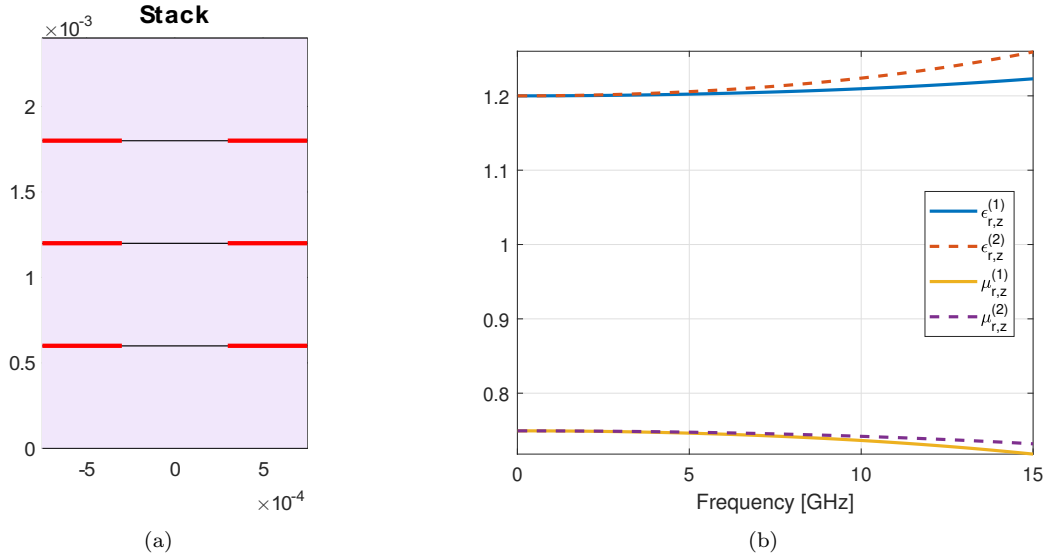


Figure B.5: (a) The ADL stackup, with (b) its corresponding z -component extraction for different frequencies

B.2 Relationship Between Permittivity and Tensors for ADLs

Another, less generic way to compute the tensors exists, which presents simple equations, particularly for ADLs.

$$\varepsilon_{r,x} = n_{Ti}(0^\circ)^2 \quad (\text{B.5a})$$

$$\varepsilon_{r,y} = n_{Ti}(0^\circ)^2 \quad (\text{B.5b})$$

$$\varepsilon_{r,z} = n_{TM}(90^\circ)^2 \quad (\text{B.5c})$$

$$\mu_{r,x} = 1 \quad (\text{B.5d})$$

$$\mu_{r,y} = 1 \quad (\text{B.5e})$$

$$\mu_{r,z} = \frac{1}{1 - n_{TE}(90^\circ)^2 + \varepsilon_{r,x/y}} \quad (\text{B.5f})$$

Using these equations, the tensors can be exported directly from the dielectric anisotropic patterns at each point.

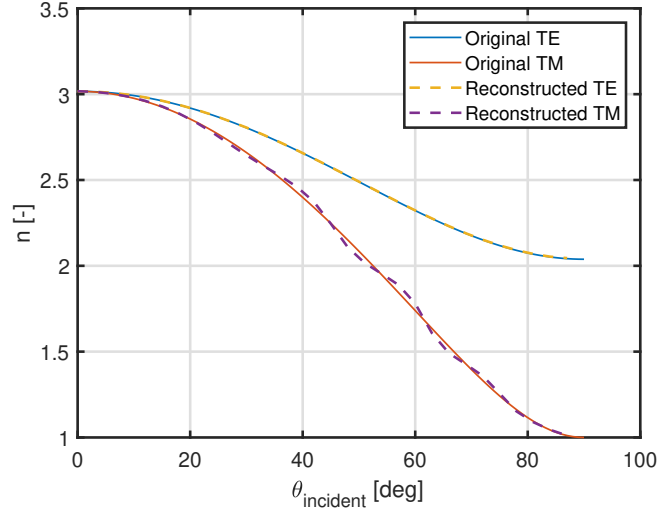


Figure B.6: The tensor-based reconstructed anisotropic profile for both TE and TM, alongside the original profiles, from which the tensors are extracted.

B.3 Validating the Tensor Extraction

To validate the tensor extraction, in CST, a 3 mm thick unit cell is illuminated with TE and TM plane waves at different incidence angles ($\theta_{incident}$), ranging from 0° to 80° in 10° increments. This unit cell is made of an example material with

$$\bar{\epsilon} = \begin{bmatrix} 9.09913 & \\ & 9.09913 \\ & & 1 \end{bmatrix}, \quad \bar{\mu} = \begin{bmatrix} 1 & \\ & 1 \\ & & 0.168216 \end{bmatrix}.$$

From these plane-wave illuminations, the S-parameters are extracted, in particular the phase of S_{21} . In MATLAB, for each illumination angle and mode, the equivalent refractive index is again extracted from these phases. This is done by computing the phase of S_{21} using the ABCD matrix method for an estimated n , then varying n until it matches the CST phase. To avoid the ambiguity of different amounts of wavelengths, the computation is first started at $\theta_{incident} = 0$, as for this n is known to be 3.02. From here, the angles are increased, and the guess for n for that angle is slowly decreased until the S_{21} phases are matched. This is repeated for both modes.

Figure B.6 shows this reconstructed refractive index anisotropy profile over the different incidence angles, for both TE and TM. Alongside it, the original profiles, from which the tensors are extracted, are plotted. It can be observed that both reconstructed profiles are in good agreement with the original ones, indicating that the tensor extraction is valid.

Appendix C

Empirical Proof for the $1/T$ Factor

This Appendix presents empirical evidence for the $1/T$ factor in Section (5.4), where T is the thickness of the lens. For this, multiple configurations were studied, in which the lens diameter, focal length, and thickness are varied, keeping the other parameters constant. First, a collimating permittivity distribution is computed for each lens geometry following the equations in [36]. Next, an error term, $\Delta n(\rho) = 400\rho^2$, is added to the distribution. The ray tracing is then performed along a single ϕ -cut to compute the corresponding output angles. These errors are then compensated for using (5.4) and (5.5), where the $1/T$ factor is present. Section C.1 portrays the results for a change in focal length, in Section C.2 the diameter is changed, and Section C.3 shows the case where the thickness is modified.

Looking at Figures C.1, C.2 and C.3, where the output angles are plotted as a function of ρ for the erroneous lenses and the compensated ones with different geometries, it can be observed that, no matter the diameter, focal length, and lens height, and thus different erroneous output angles, the compensated permittivity collimates very well. The $1/T$ factor compensates for the dependency on height, while D and F appear to have little direct effect on the required compensation.

C.1 The Dependency on the Focal Length

This section shows the results for varying the focal length from 6 to 34 mm, whilst keeping the lens diameter and height constant. These are set to 36 and 5.7 mm, respectively. This is depicted in Figure C.1, where the output angles as a function of ρ are plotted for the erroneous and compensated lenses.

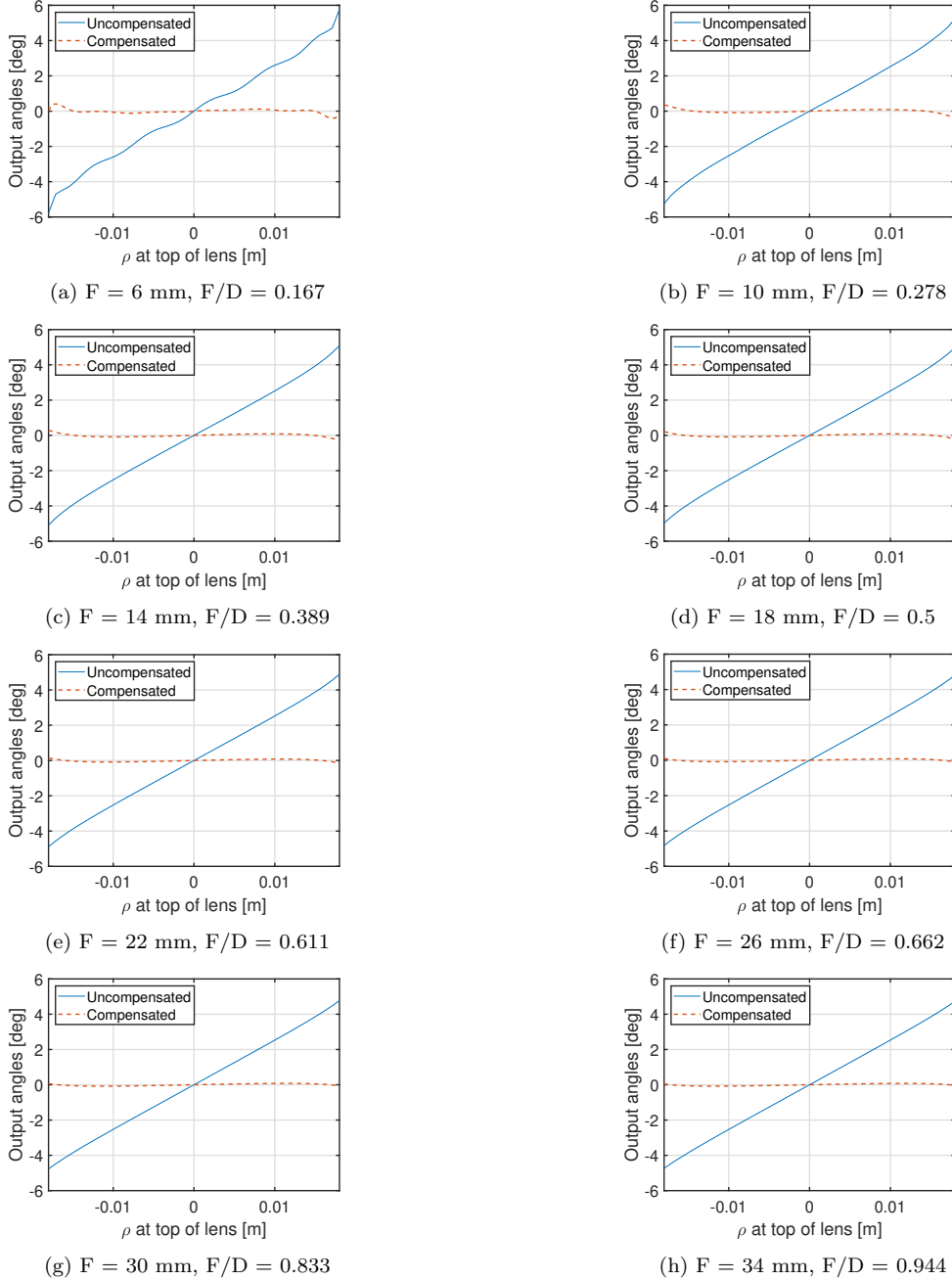


Figure C.1: The output angles of the original and compensated permittivity, for varying focal lengths F , for a lens with a 36 mm diameter, 5.7 mm thickness without matching layers.



C.2 The Dependency on the Diameter

Here, the diameter is the parameter that is changed, from 15 to 50 mm, with a focal length of 15 mm and a height of 5.7 mm, whose compensated output angles are plotted in Figure C.2.

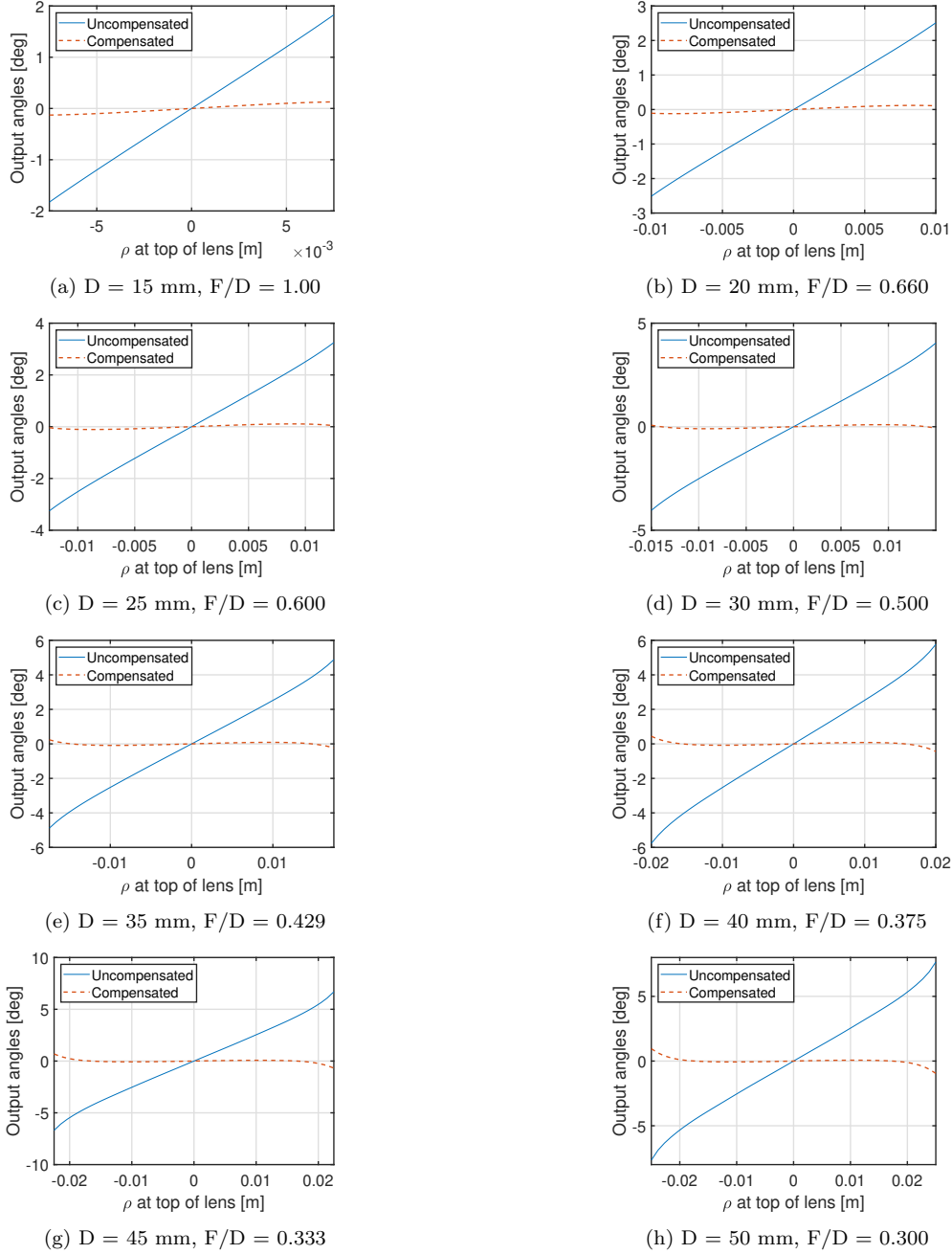


Figure C.2: The output angles of the original and compensated permittivity, for varying diameters D , for a lens with a 15 mm focal length and 5.7 mm thickness without matching layers.

C.3 The Dependency on the Thickness

Lastly, the height is varied between 2 and 9 mm, with the lens diameter of 36 mm and the focal length of 15 mm held constant. The resulting output angles are graphed in Figure C.3.

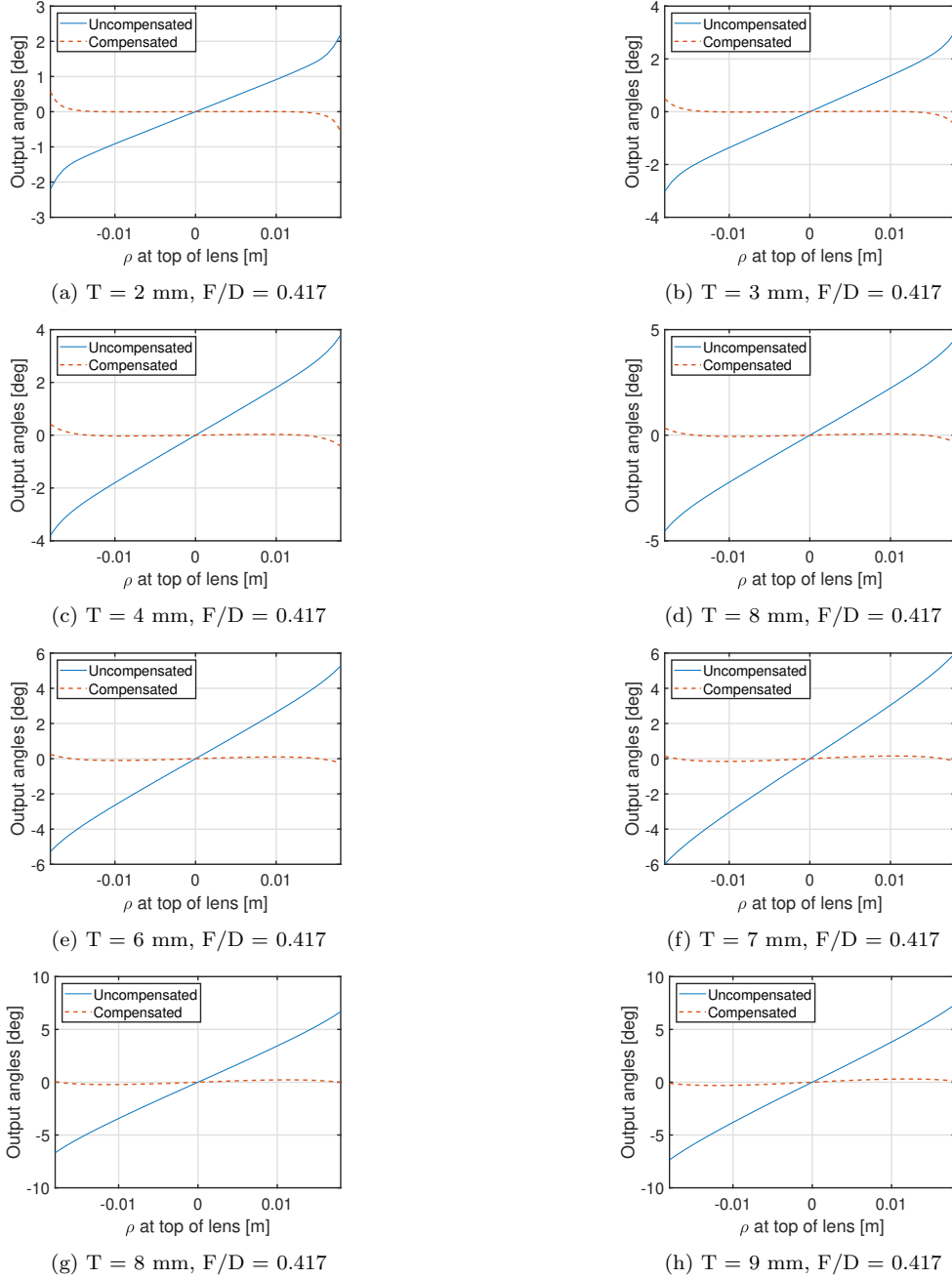


Figure C.3: The output angles of the original and compensated permittivity, for varying lens thicknesses T , for a lens with a 36 mm diameter and a focal length of 15 mm, without matching layers.

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