A comprehensive review of fatigue of cementitious materials
Characterisation, modelling and mechanisms

Gan, Yidong; Yang, Shen; Zuo, Yibing; Schlangen, Erik; Shi, Boyuan; Šavija, Branko

DOI
10.1016/j.conbuildmat.2024.138962

Publication date
2024

Document Version
Final published version

Published in
Construction and Building Materials

Construction and Building Materials 452 (2024) 138962

Contents lists available at ScienceDirect

Construction and Building Materials

journal homepage: www.elsevier.com/locate/conbuildmat

A comprehensive review of fatigue of cementitious materials: Characterisation, modelling and mechanisms

Yidong Gan [a,b,*], Shen Yang [a], Yibing Zuo [a,**], Erik Schlangen [c], Boyuan Shi [d], Branko Šavija [c]

[a] School of Civil and Hydraulic Engineering, Huazhong University of Science and Technology, Wuhan 430074, China
[b] National Center of Technology Innovation for Digital Construction, Huazhong University of Science and Technology, Wuhan 430074, China
[c] Microlab, Faculty of Civil Engineering and Geosciences, Delft University of Technology, Delft CN 2628, The Netherlands
[d] China West Construction Academy of Building Materials Co., LTD, Chengdu 610299, China

ARTICLE INFO

Keywords:
Fatigue
Cementitious material
Fatigue damage
Characterisation
Modelling

ABSTRACT

This review provides a comprehensive analysis of the fatigue behaviour of cementitious materials, focusing on the characterisation, modelling, and mechanisms underlying their fatigue properties. It begins with a detailed exploration of how material composition and loading conditions influence fatigue performance, along with the underlying mechanisms that govern fatigue fracture processes. The review also synthesises current advanced experimental methods for characterising fatigue damage evolution, highlighting significant achievements and methodological innovations. It also examines theoretical approaches, summarising prominent models and emerging theories that deepen the understanding of fatigue behaviour in cementitious materials. Meanwhile, the paper reviews computational techniques based on fracture mechanics, damage mechanics, statistical analysis, machine learning algorithms and multiscale modelling, assessing their potential to improve the accuracy of fatigue predictions. Lastly, the review identifies crucial research gaps and outlines future directions, particularly in understanding fatigue-related coupling mechanisms to enhance the fatigue resistance and durability of cementitious materials.

1. Introduction

The word *fatigue* originated from the Latin expression *fatīgāre*, which means 'to exhaust'. Although it is commonly used to describe the physical or mental tiredness of people, the word *fatigue* has also become a widely accepted term in engineering vocabulary for describing the damage and failure of materials under repeated loading [1]. Studies on fatigue date back to the nineteenth century, during which several disastrous railroad accidents due to fatigue were reported [2]. These tragic accidents forced railway engineers to pay serious attention to fatigue issues in metallic materials. In the beginning, fatigue was thought to be a mysterious phenomenon because fatigue failure occurred abruptly without any visible warning [3]. Therefore, numerous investigations have been conducted to understand the nature of fatigue in metals. Noteworthy research on fatigue was done by the German engineer August Wöhler, who performed first systematic fatigue investigations on railway axles during the period 1852–1869 [4]. He developed the well-known Wöhler curve and recognised that fatigue is a fracture phenomenon occurring after a large numbers of load cycles with the loading magnitude much lower than the strength.

Despite the early attention on fatigue of metallic materials, the interest on fatigue of cement-based material lagged for nearly 40 years. Considére and De Joly conducted the first fatigue tests on mortar specimens [5,6]. Investigations on concrete fatigue, like those on metal fatigue, were also motivated by practical problems. In general, two types of fatigue loading can be distinguished [7], i.e. low-cycle fatigue with high stress levels (such as earthquakes, storms, etc.) and high-cycle fatigue with relatively low stress levels (traffic loading, wind and wave loading, etc.). Both types of fatigue loading lead to significant safety and durability challenges in numerous concrete structures [7,8]. Especially, fatigue has recently become a crucial concern in railways [9,10]. This is due to the advancement of ballastless concrete tracks, where the primary stress on the concrete stems from the cyclic loads generated by high-speed train operations. As the major load-bearing structure for

* Corresponding author at: School of Civil and Hydraulic Engineering, Huazhong University of Science and Technology, Wuhan 430074, China
** Corresponding author.
E-mail addresses: ygan@hust.edu.cn (Y. Gan), yangshen@hust.edu.cn (S. Yang), zuoyibing@hust.edu.cn (Y. Zuo), Erik.Schlangen@tudelft.nl (E. Schlangen), laoshiboyuan@163.com (B. Shi), B.Savija@tudelft.nl (B. Šavija).

https://doi.org/10.1016/j.conbuildmat.2024.138962
Received 28 June 2024; Received in revised form 21 September 2024; Accepted 27 October 2024
Available online 4 November 2024

high-speed trains, ballastless concrete tracks play a crucial role in ensuring the safety, stability, and comfort of train travel. In addition, concrete pylons for wind turbines, both onshore and offshore, are continuously exposed to cyclic loading and demand concrete designed with specific attention to fatigue resistance [11]. Therefore, it is important to understand the fatigue behaviour of concrete for these practical purposes.

Given that concrete is a quasi-brittle material with a multiscale heterogeneous structure, its fatigue mechanisms differ significantly from those of metallic materials. The process of fatigue crack propagation in concrete is highly intricate, involving damage accumulation at multiple length scales. The macroscopic fatigue performance of concrete not only depends on the material structure and fatigue properties of components at different scales, such as w/c ratio [12,13], porosity [14,15], binder type [16–18], addition of fibres [19–22] and aggregate type [23,24], but is also influenced by several external factors, including temperature [25], moisture content [25,26], stress level [27,28], loading frequency [29,30] and rest period [26,31], etc. A bibliometric analysis of literature on several keywords, such as "fatigue of concrete", "fatigue mechanisms of cementitious material", "fatigue model", "machine learning fatigue model", "multiscale fatigue model" and "stochastic fatigue model", etc. was carried out on the Scopus database and the results were examined in VOSviewer, which is shown in Fig. 1. Fig. 2 gives the annual scientifical productions on fatigue of cementitious materials from 1997 to 2024, sourced from journals, books and documents and other publications.

Fig. 1 shows that researches on the fatigue of different mixtures and structural elements, fatigue related durability and fatigue models are most prevalent in the scientific community. Moreover, there is an increase of publications regarding the fatigue of cementitious materials in these years, as is shown in Fig. 2. Over the past century, tremendous efforts have been devoted to concrete fatigue [32–35]. In contrast to the

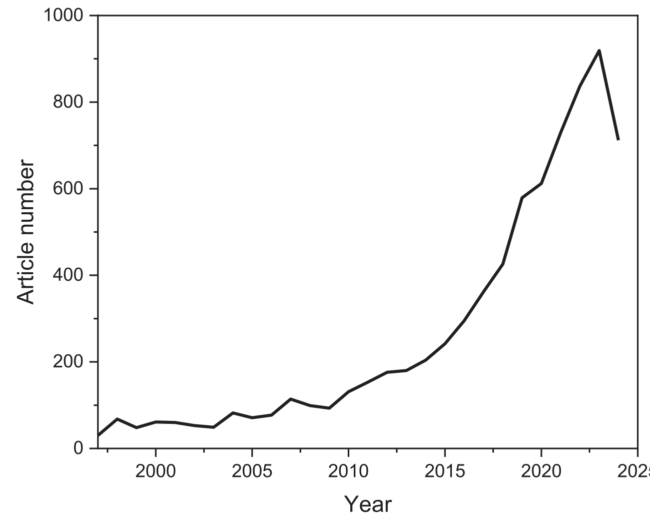

Fig. 2. Annual scientifical productions on fatigue of cementitious materials.

relatively well-understood mechanisms in metal fatigue, a clear understanding of concrete fatigue behaviour is still missing. Many important scientific and practical problems remain unsolved. These problems include at least: (1) how does the multiscale heterogeneous material structure of concrete affect the fatigue fracture and damage evolution; (2) how to accurately characterise fatigue damage; (3) how to properly model and predict the fatigue behaviour; (4) can machine learning methods help to predict and revel complex relationships in concrete fatigue performance?

This review will be directed toward research efforts on

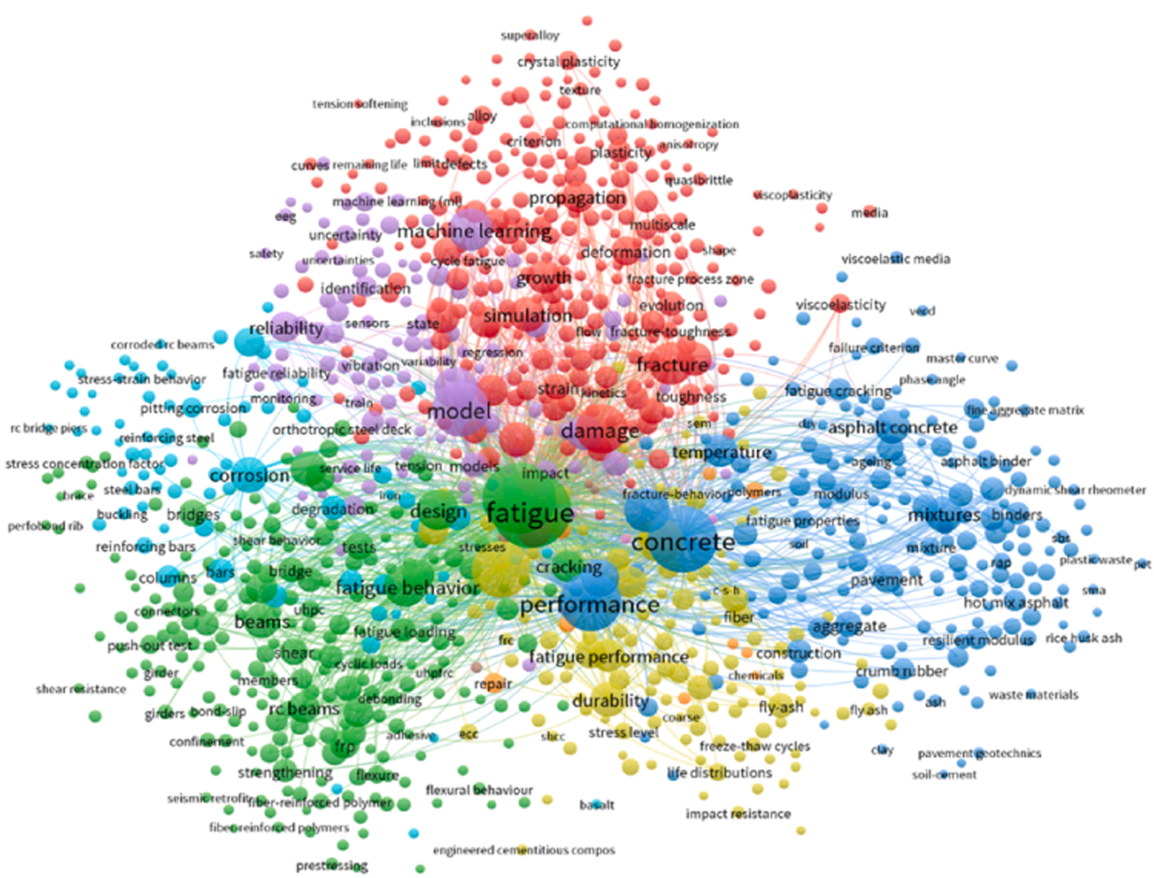

Fig. 1. Keyword network analysis of articles on fatigue of cementitious materials from the Scopus database.

understanding, evaluating, and simulating concrete fatigue. Both fundamental insights and advanced understanding of concrete fatigue will be provided in this review. The paper is structured as follows: first, an in-depth examination is conducted on various factors (including material compositions and environmental conditions) that impact the fatigue behaviour of concrete, along with explorations of their underlying mechanisms. Subsequently, a literature survey highlights recent experimental techniques for characterizing fatigue damage evolution. Additionally, advancements in simulating concrete fatigue are explored, and research utilizing machine learning approaches to predict fatigue performance is summarized. Finally, the paper concludes by outlining future needs for concrete fatigue studies.

2. Factors influencing fatigue behaviour of concrete

Fatigue behaviour of cementitious materials can be influenced by many factors, which can be broadly categorized into two groups. The first comprises factors related to material composition, including the w/c ratio, binder type, addition of aggregates and fibres, etc. Understanding how different materials and their proportions influence concrete fatigue helps to optimize the mix design for enhanced fatigue performance. The second category encompasses factors associated with sample size and loading scenarios, such as loading modes, loading frequency, rest period and variable amplitude, etc. Investigating these effects is crucial as it yields valuable insights into how concrete structures respond under realistic cyclic loading conditions, offering practical applications for construction and durability considerations. To offer a clear picture of how these factors influence the fatigue of concrete, a detailed review of each individual factor is undertaken in this section.

2.1. Factors for material composition

2.1.1. Effect of w/c (w/b) ratio

The water-to-cement (w/c) or water-to-binder (w/b) ratio stands out as the most influential parameter affecting the properties of concrete mixtures. In an early investigation of Kesler [36], negligible distinction in the fatigue performance of plain concrete specimens characterised by different w/c ratios was observed. Therefore, Kesler concluded that the w/c ratio has no effect on the flexural fatigue strength of concrete. However, Thomas [37] indicated that the flexural fatigue behaviour of concrete is influenced by w/c ratio. The fatigue strength is decreased for a low w/c ratio (0.2), but there is no discernible difference in fatigue strength of concrete within the w/c ratio range of 0.4–0.6.

Antrim [38] studied the impact of w/c ratio on the compressive fatigue performance of cement paste and plain concrete. The findings revealed that, for both paste and concrete, the number of cycles to failure is significantly higher when the w/c ratio is 0.45 compared to 0.70 at a given stress magnitude. This is primarily attributed to the increased porosity associated with a higher w/c ratio. It is important to note, however, that when plotting the percentage of static strength against fatigue life (*S-N* curve), there appears to be no distinguishable difference in the *S-N* curve with varying w/c ratios. This is because the w/c ratio can also markedly influence the static strength of cementitious materials [39].

Similar findings were reported by Klaiber and Lee [12], who investigated the flexural fatigue behaviour of concrete with w/c ratios ranging from 0.32 to 0.60. Their results indicated a decrease in fatigue life for concretes with lower w/c ratios (0.32), likely due to the increased shrinkage-induced damage at these ratios [40,41]. Therefore, both very low and high w/c ratios negatively impact the fatigue and mechanical properties of concrete. A low w/c ratio can lead to shrinkage-induced cracking, while a high w/c ratio results in increased porosity, reducing overall fatigue performance. Besides, Thomas et al. [23] investigated the fatigue performance of recycled aggregate concrete with different w/c ratios. It was found that using recycled aggregate reduces the ability to resist fatigue loadings, and this effect is more noticeable with lower w/c ratios, primarily due to the lower strength of the recycled aggregate. Oneschkow and Timmermann [42] suggested that the increase of w/c ratio leads to a higher porosity in mortar matrix and in the interfacial transition zone (ITZ). This increased porosity is identified as the primary factor resulting in inferior fatigue performance.

2.1.2. Effect of binder type

The binder type in concrete inherently defines the material's microstructure, performance, and durability. In a study by Tse et al. [16], the compressive fatigue properties of concretes containing fly ash (FA) were investigated. Different replacement ratios of FA were tested, and when compared on the basis of stress as a percentage of strength, no significant difference in fatigue strength was observed between concrete with 50 % cement replacement by FA and plain concrete. However, when compared based on the absolute value of the applied compressive stress, it was noted that the fatigue strength of concrete with 25 % or 50 % cement replacement by high-calcium FA was significantly higher than that of plain concrete. In contrast, the fatigue strength of concrete with low-calcium FA was much lower than that of plain concrete. The authors suggested that a 25 % replacement of cement with low-calcium FA or high-calcium FA with a 50 % replacement in concrete can lead to equivalent or even higher compressive and fatigue strengths at 28 days compared to plain concrete. Generally, high-calcium fly ash has a higher content of calcium oxide (CaO) and tends to enhance the early-age strength of concrete due to its higher calcium content during hydration process. The additional calcium can participate in hydration reactions, forming more C-S-H gel, which improves the density of microstructure, interfacial adhesion between matrix and aggregate, and overall mechanical properties of the concrete. The resulting lower porosity reduces the potential for fatigue crack initiation, while the improved interfacial adhesion between the matrix and aggregates strengthens resistance to fatigue crack propagation. Consequently, the material exhibits superior fatigue resistance. Harwalkar and Awanti [43] performed flexural fatigue tests on 28 days age high volume FA concrete and normal concrete. The findings indicate a slightly superior fatigue performance in normal (i.e., Portland cement) concrete compared to FA concrete. As documented in many researches [44–46], the static strength of concrete is often considered an indicator of the material's structural performance, with higher static strength typically associated with lower porosity, greater crack resistance, and consequently improved fatigue life.

Guo et al. [17] investigated the effect of different proportions of ground granulated blast-furnace slag (GGBS) on the flexural fatigue performance of 197 days old concretes. The study found that the incorporation of GGBS decreased the brittleness of concrete, resulting in higher fracture energy and a more tortuous cracking path. Additionally, the potential hydration and pozzolanic effect of GGBS in the matrix prolonged the fatigue life compared to that of plain concrete. Notably, the effect of GGBS on the fatigue behaviour of concrete was more pronounced at lower stress levels. Besides, Saini and Singh [47] studied the flexural fatigue performance of self-compacting concrete (SCC) incorporating silica fume (SF) and metakaolin (MK). The addition of SF and MK was found to significantly increase the endurance limit and fatigue life of concrete. It is suggested that, in addition to the pozzolanic activity of supplementary cementitious material (SCM), another positive impacts of blended cements on concrete stem from their filler ability, contributing to enhanced homogeneity and compactness. Moreover, the inclusion of 10 % MK proved to be particularly effective in improving the flexural fatigue performance of SCC.

In a study of Mun et al. [48], the effects of three binder types on the compressive fatigue performance of concrete were examined. The tested binder type includes ordinary Portland cement (OPC) binder composed of 20 % FA, and 50 % GGBS and alkali-activated (AA) binder composed of 50 % FA and 50 % GGBS. It was found that under, the same stress level, the fatigue life of high-volume SCM concrete and AA concrete is higher than that of the companion OPC concrete. The authors speculated that this is probably because of superior mechanical properties and lower creep coefficients of SCM and AA concretes, as was also suggested in [49,50]. Nevertheless, the test results also indicated a slower rate of fatigue damage evolution in OPC concrete, evidenced by lower incremental fatigue and residual strains at the minimum stress level compared to high-volume SCM or AA concrete mixtures. The authors suggested that the fatigue-creep coupling mechanism might play a role in this phenomenon.

Cândido et al. [51] investigated the flexural fatigue performance of geopolymer concrete made of blast furnace slag. The test results revealed a superior fatigue performance of geopolymer concrete compared to Portland cement concrete in terms of fatigue strength. Microstructural analysis further indicated better matrix/aggregate adhesion in the geopolymer as opposed to Portland cement concrete, potentially explaining its enhanced fatigue performance. The authors suggested that the improved performance of geopolymer concrete can be attributed to the more homogeneous geopolymer matrix and the absence of brittle particles of portlandite in the interface between the geopolymer matrix and aggregates.

Fig. 3 presents *S-N* data and corresponding fitted curves collected from available references. The solid lines represent the fitted curves derived from flexural fatigue data, while the dashed lines correspond to the curves obtained from compressive fatigue data. It can be seen from the Fig. 3 that there is a notable variation in *S-N* results mainly across different material compositions. The loading condition does not have a significant impact on the overall trend of the stress level versus cycle number curves. For instance, both the weakest results (lower left of the figure) from Harwalka and Awanti (2019, high-volume fly ash concrete) and the strongest results (upper right of the figure) from Saini and Singh (2020, recycled aggregate concrete with 50 % metakaolin) were obtained from flexural fatigue tests. This may indicate a relative uniformity in the fatigue response of cementitious materials, regardless of whether the loading is flexural or compressive. These findings could have important implications for engineering applications by offering a more unified understanding of material fatigue performance across varying loading conditions. Further analysis of fatigue data is recommended to confirm these trends and validate the observed similarities. In consideration of sustainability, it is generally agreed that the incorporation of supplementary cementitious materials is an advantageous strategy to enhance the fatigue performance of concrete. Notably, it can be seen from Fig. 3 that the inclusion of silica fume and metakaolin demonstrates particularly significant enhancements in this regard.

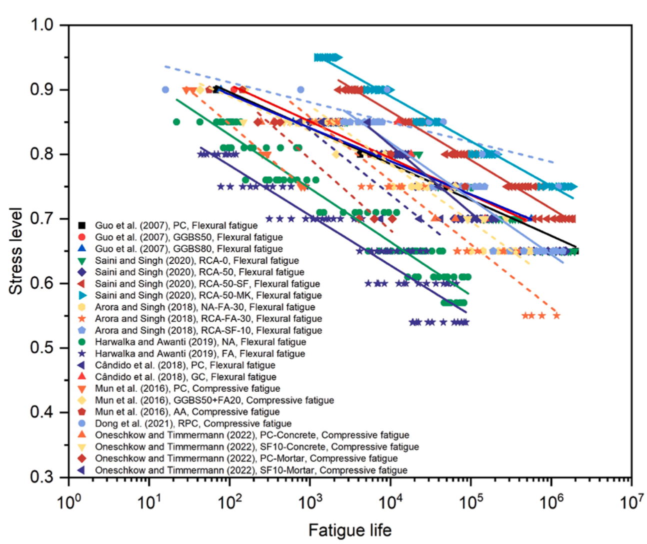

Fig. 3. *S-N* data and corresponding fitted curves collected from available references [17,42,43,47,48,51,52]. Note: the name of the authors, the year of publication, the type of binder and the loading conditions are listed; PC = Portland Cement, NA = normal aggregate concrete, GC = geopolymer concrete, RPC = reactive powder concrete, GGBS50 means granulated blast-furnace slag replacement ratio is 50 %, RCA-FA-30 means recycled aggregate concrete with the addition of 30 % fly ash; the solid line is the fitted curve for flexural fatigue data and the dash line is for compressive fatigue data.

2.1.3. Effect of aggregate

Aggregates, varying in size and type, constitute a major component of concrete, exerting a profound influence on its overall mechanical properties. Kasu et al. [53] studied the effect of aggregate size on the flexural fatigue behaviour of concrete. Concrete beams made of two different nominal maximum aggregate sizes (i.e., 10 mm and 20 mm) were tested. The results revealed that, at a stress level of 80 %, specimens with smaller aggregate sizes consistently demonstrate superior fatigue performance across varying loading frequencies. This might be attributed to the utilisation of larger size aggregates, resulting in a reduced surface area for the development of interfacial bonding, thereby introducing more heterogeneity in the concrete. Additionally, as the aggregate size decreases, better packing of the grains can be achieved. This improved packing also contributes to slowing down the crack propagation, resulting in longer fatigue life. Furthermore, enhanced packing also leads to fewer air voids or pores, which are typically the initiation points for cracks. Therefore, concrete containing smaller aggregates tends to exhibit superior fatigue performance.

It is known that the major mechanisms responsible for the softening behaviour of quasi-brittle materials, like concrete, are microcracking and aggregate bridging. As the crack advances, the aggregate bridging action can be represented by an equivalent set of bridging stress that resists the crack growth, as depicted in Fig. 4. Under cyclic uniaxial tension, there is a deterioration in aggregate bridging, resulting in a progressive loss of mechanical strength [54]. Simon and Kishen [55] analytically evaluated the effect of fatigue stress ratio on the bridging stress, which is determined by equating the crack opening displacements obtained at the macroscale to the one determined at the mesoscale. It was found that for a given crack length, the bridging stress increases with decrease in the stress ratio. Moreover, the rate of decrease in aggregate bridging stress rises with an increase in stress ratio.

Scheiden and Oneschkow [56] conducted a systematic analysis of the influence of two types of coarse aggregate, i.e., basalt aggregate and granite aggregate, on the compressive fatigue behaviour of concrete. Two distinct mechanisms were observed in association with each type of aggregate. The concrete with granite aggregate achieved a higher number of cycles to failure than its basalt counterpart under higher stress levels, but this relationship was reversed under lower stress levels. Furthermore, the relationship of various damage indicators including stiffness, residual strain, and dissipated energy, in the two concrete mixtures also exhibits a reversal under higher stress levels. Acoustic emission measurements revealed a different damage mechanism in the granite concrete under lower stress levels compared to that observed in the basalt concrete. The authors recommend further research to delve into this aspect.

Several researchers also investigated the effect of recycled concrete aggregate (RCA) on the fatigue performance of concrete [35,57,58]. Arora and Singh [57] conducted a comparative analysis of the flexural fatigue behaviours between concrete incorporating 100 % RCA and natural aggregate (NA). Given the generally lower mechanical properties of RCA, inferior flexural fatigue performance in terms of fatigue strength and endurance limit was also observed in RCA concrete. Similar findings were reported in [23,35,58]. To improve the fatigue performance of RCA concrete, Feng et al. [59] introduced a modification using rubber particles. These particles were incorporated to replace the sand in the concrete, aiming to modify its microstructure. This enhancement was most pronounced when the rubber particle content reached 20 %. Sohel et al. [24] studied the flexural fatigue behaviours of ultra-lightweight cement composite (ULCC) and lightweight aggregate concrete (LWAC). The experimental results show that the flexural

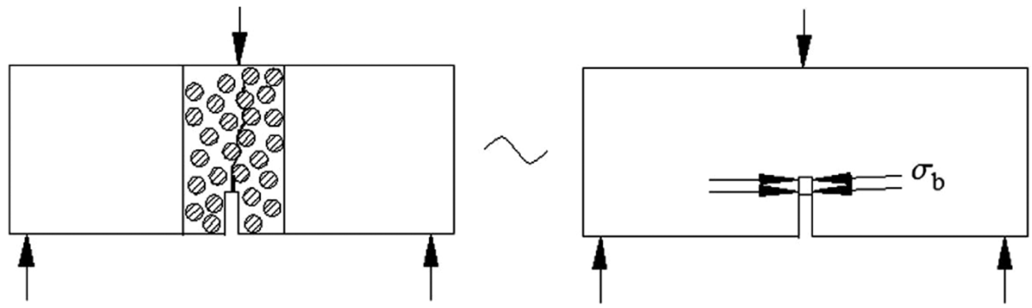

Fig. 4. Representation of the bridging mechanism in a concrete beam adapted from [55]. Note: σ_b represents the bridging stress.

fatigue performance of ULCC is better than that of LWAC, even though they both have similar static flexural strength. Fig. 5 shows the comparison of the *S-N* curves for three types of concrete, i.e., ULCC, LWAC and normal weight concrete (NWC).

2.1.4. Effect of fibre addition

The addition of fibres to concrete can have a remarkable impact on its fatigue performance [60–62]. As depicted in Fig. 6, under both static and cyclic loading conditions, fibres act as crack arrestors, effectively slowing down the initiation of cracks and their coalescence into macrocracks through mechanisms such as bridging, pull-out, and breakage. Stress redistribution also occurs as fibres help to distribute and dissipate the applied loads more effectively throughout the concrete matrix. The specific benefits may vary based on the type and dosage of fibre used, as well as the application and environmental conditions. Studies on the fatigue behaviour of plain and fibre-reinforced concrete have been reviewed by Lee and Barr [33]. It was shown in most studies that the inclusion of fibres can improve the fatigue performance of concrete. Grzybowski and Meyer [63] suggested that the addition of fibres has a dual effect on the cyclic behaviour of concrete. On one hand, fibres can bridge microcracks and retard their growth, thereby enhancing the composite's performance under cyclic loading. On the other hand, the presence of fibres increases the porosity and initial microcrack density, resulting in a decrease of strength. Therefore, the overall impact of these two competing effects depends on the fibre content.

Benard et al. [65] studied the fatigue behaviour of steel fibre-reinforced concrete (SFRC) under direct tension, showing that fatigue life increases with higher fiber volume fractions. Singh and Kaushik [66] conducted flexural fatigue tests on SFRC and demonstrated that steel fibres significantly enhance its flexural fatigue strength. Moreover, Yin and Hsu [67] found that SFRC exhibited significantly greater ductility compared to plain concrete under uniaxial and biaxial

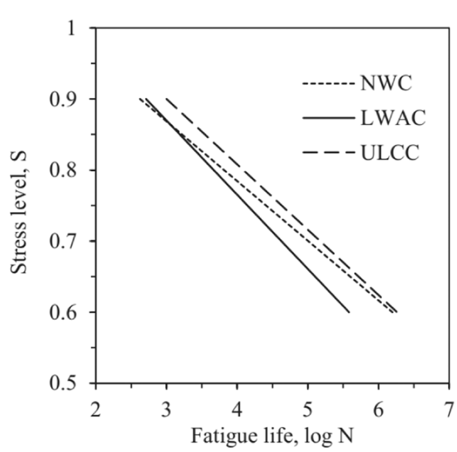

Fig. 5. The comparison of *S-N* curves for ULCC, LWAC, and NWC, adapted from [24].

compression fatigue loads, mainly by improving stress redistribution under fatigue loading. However, Cachim et al. [62] highlighted that fibre distribution, rather than just volume fraction, plays a crucial role in improving fatigue performance. It has been argued in [68] that excessive fibre content can distort the concrete matrix, creating pores and imperfections that facilitate crack initiation, leading to a reduction in fatigue life. Gao et al. [60] investigated the effect of fibre clustering, finding that increasing fibre volume from 0 % to 1.0 % improved fatigue life, but performance declined with 1.5 % volume due to fibre clusters weakening the beam. By using computed tomography, Vicente et al. [69] confirmed that fibres aligned perpendicularly to the loading axis significantly enhance fatigue behaviour. However, the presence of small pores near the fibres negatively impacts the overall mechanical properties. In addition, several studies on the influence of steel fibre shapes, such as straight, hooked, crimped, and twisted, concluded that spiral and hooked-end fibres provide superior bonding with the concrete matrix, offering better overall fatigue performance due to their interlocking features [70,71].

Choi et al. [61] investigated the compressive fatigue performance of fibre-reinforced lightweight concrete with high-volume SCM. The test results indicated a preference for polyvinyl acetate (PVA) fibre over amorphous steel (AS) fibre in enhancing the fatigue life of lightweight concrete. In cases of fatigue failure, AS fibres were observed to be pulled out along the macrocracks, while PVA fibres underwent rupture due to its lower mechanical properties. Additionally, the fatigue damage of PVA fibre concrete was found to be lower than that of AS fibre concrete, with this trend being more pronounced under higher maximum stress levels. Microfibers, exemplified by PVA fibres, exhibited greater effectiveness in limiting bond crack propagation along the interface between the cementitious matrix and lightweight aggregates compared to macrofibres like AS fibres. This phenomenon can be attributed to the superior dispersion characteristics of microfibers at the interface zone. Additionally, the inclusion of macrofibres leads to an increased number of pores and initial flaws in the cementitious matrix. The authors further recommended the hybridization of both types of fibres rather than using a single fibre type to enhance the fatigue resistance of concrete. Similar statements have also been reported by Cui et al. [72], who suggested the use of hybrid steel-polypropylene fibres in improving the fatigue behaviour of lightweight concrete. Saoudi and Bezzazi [73] studied the effect of fibre shape on the flexural fatigue of concrete by mixing short smooth steel fibre and hooked-end fibre. The study revealed that the fatigue life could be improved by adding 0.25 % of hooked-end and 1.5 % of smooth steel for all the studied stress levels.

Interestingly, a new fatigue failure mode of PVA fibre was discovered by Li et al [74]. By using scanning electron microscopy (SEM), the failure modes of PVA fibres on various regions of the static and fatigue failure surface were first examined. It seems that fibres suffered from crushing during the fatigue loading and there exists clear trace of friction on the failure surface, as shown in Fig. 7. One plausible explanation for the emergence of a new damage mode in the fibres could be attributed to the early-stage rupture of these damaged fibres during the fatigue failure process. Subsequently, the broken ends of the fibres may

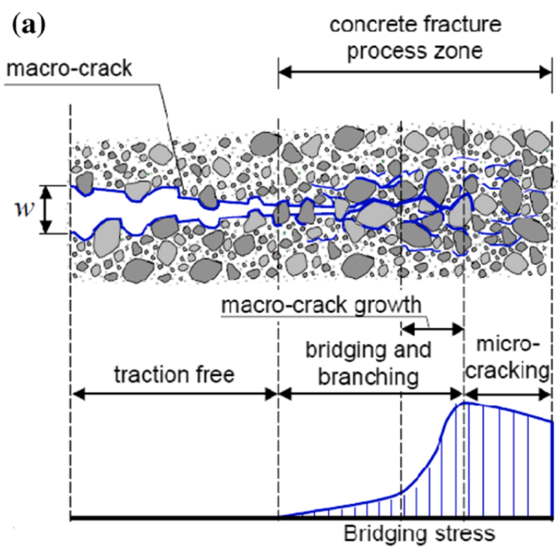

Fig. 6. Schematic description of the stress-crack opening relationship for (a) conventional and (b) fibre-reinforced concrete, adapted from [64]. Note: *w* is the crack width.

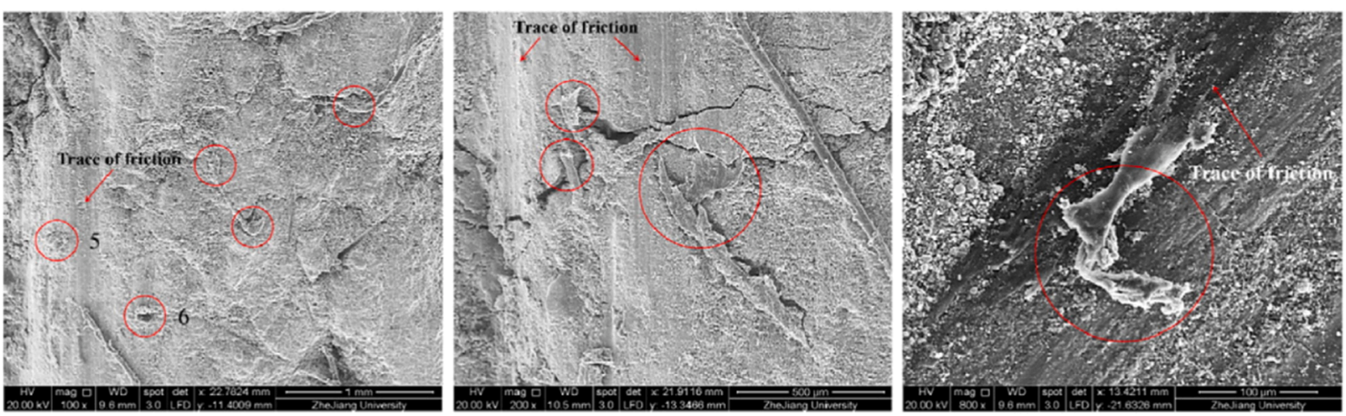

Fig. 7. SEM images of fatigue failure surface indicating the trace of friction of fibre during the fatigue loading, adapted from [74].

undergo repeated friction or crushing throughout the remaining fatigue life.

2.2. Influence of sample size and loading scenario

2.2.1. Effect of sample size

The size of a specimen can significantly influence its fatigue strength, with larger specimens typically exhibiting reduced fatigue life compared to smaller ones. This effect is of particular importance for large-scale structures like wind turbine pylons, bridge girders, and offshore platforms, where understanding the size effect can lead to more accurate fatigue life predictions and safer designs. The fatigue size effect of quasi-brittle materials was systematically studied in the works of Bažant and co-workers [75–77]. The fatigue crack growth rate, specifically referred to the Paris law coefficient, is found to exhibit a substantial size effect, analogous to the size effect of fracture under monotonic loading [75]. In other words, the fatigue damage evolution becomes slower for smaller samples and, consequently, the fatigue life tends to increase with the decreasing sample size [78]. Carpinteri and his colleagues [79–81] also explored the size effect using fractal analysis. By applying concepts from fractal geometry, they introduced new definitions for fracture energy and stress intensity factors, based on physical dimensions that deviate from classical models. Subsequently, they proposed a size-dependent crack growth law, which relates the crack growth rate to the range of the stress intensity factor. Ortega et al. [82] conducted a study on the size effect in compressive fatigue of fibre-reinforced concrete. Their findings revealed that as the specimen size increased, both fatigue life and failure strain decreased. Mena-Alonso et al. [83] found that smaller specimens exhibited significantly higher flexural fatigue strength, up to three orders of magnitude, in both plain and steel-fibre reinforced high-strength concrete. Additionally, the incorporation of fibres was shown to significantly mitigate the size effect on fatigue life. This is attributed to the increased ductility provided by the fibres, which shifts the brittle-ductile transition in steel-fibre reinforced concrete. This shift reduces the material's sensitivity to the size effect, enhancing its overall fatigue performance. A recent study by Gan et al. [13] on the fatigue testing of micro-cantilever beams revealed that, at the same stress level, the fatigue life of cement paste at the microscale is nearly two orders of magnitude longer than that at the macroscale, highlighting a pronounced size dependency in paste specimens. Unlike conventional laboratory-scale cement paste samples, microscale micro-cantilever beams are free from larger defects, such as meso-cracks and air voids, which significantly contribute to reduced fatigue resistance in larger specimens.

2.2.2. Effect of loading mode

Different loading modes, such as bending, tension, compression and compression-tension alternating modes, affect the fatigue performance of cementitious materials in distinct ways due to the varying nature of stress distribution. For direct comparison, it has been reported that the compressive fatigue resistances of cementitious materials are slightly higher than their tensile fatigue resistances at the same stress levels or magnitude [45], while it is also suggested in [7] that similar *S*-*N* equations can be used for both compressive and flexural fatigue. Many studies have been conducted on the compressive fatigue of cementitious materials, as it is the most commonly encountered loading mode in engineering practice [48,74,84,85]. The majority of these studies are concentrated on uniaxial fatigue compression, where the material is subjected to stress along a single axis. However, several research efforts have also explored the behaviour of cementitious materials under more complex loading conditions, such as biaxial or triaxial compression. These studies are crucial as they provide a deeper understanding of the material's performance when subjected to multidirectional stresses. Lü et al. [86] investigate the behaviour of concrete under biaxial compression and uniaxial fatigue. The experiments were conducted on cube specimens with three levels of lateral stress applied, defined as the ratio of horizontal stress to uniaxial compressive strength. The results indicate that increasing horizontal stress alters the failure mode and enhances the maximum vertical load-carrying capacity. In uniaxial compression tests, fractures typically occurred along the loading direction, causing the specimen to split into several short columns. In biaxial compression tests, failure was characterized by splitting along planes parallel to the specimen's free surfaces. The failure surfaces were consistent across both quasi-static and fatigue loading, with most fractures occurring at the interfaces between coarse aggregate and mortar. Concrete's fatigue strength under biaxial compression is consistently higher than under uniaxial compression across all load cycles. Similar results have also been reported in [67,87]. For the triaxial fatigue of concrete, Taliercio and Gobbi [88] were the pioneering researchers who conducted a thorough investigation of this behaviour. Their study demonstrated that, analogous to triaxial static compression tests, the fatigue life of concrete tends to increase with greater mean lateral confinement. This finding aligns with the understanding that the lateral confinement plays a positive role in improving the mechanical properties of concrete by resisting the lateral expansion of concrete under axial compression. This resistance reduces the tendency for concrete to spall or buckle, thereby increasing its overall strength. Moreover, confining pressure helps to limit the formation and propagation of microcracks within the concrete matrix and redistribute the applied loads more uniformly across the concrete section.

In addition, it has been reported that the stress state induced by wheel loads on the bridge deck slab is nearly biaxial, which significantly differs from uniaxial loading. Therefore, Shen and Brühwiler [89] studied the biaxial flexural fatigue behaviour of thin slab elements made of Ultra High Performance Fiber Reinforced Cementitious Composite (UHPFRC) by means of the ring-on-ring test method. Fatigue tests that result in failure reveal four distinct phases of damage evolution. Fig. 8 illustrates the four phases of damage evolution: an initial rapid progression leading to stabilization, followed by a slow and steady phase, then a stable phase with a faster rate of damage, and finally, a rapid deterioration culminating in failure. Gao et al. [90] conducted a comparison between the biaxial flexural fatigue (BFT) and four-point bending fatigue performance of ECC. Their findings revealed that ECC specimens subjected to biaxial bending exhibited a longer fatigue life compared to those under uniaxial bending. As the stress level increased, the fatigue life of the BFT specimens decreased more significantly than that of the bending specimens. Additionally, BFT specimens developed a greater number of microcracks than the bending specimens. However, Kim et al. [91] reported differing observations, finding no significant difference between uniaxial and biaxial flexural fatigue behaviour. This indicates the ongoing controversy in this area of research. Additionally, the complexity and labour-intensive nature of multi-axial fatigue loading experiments have led to a scarcity of experimental data in this area.

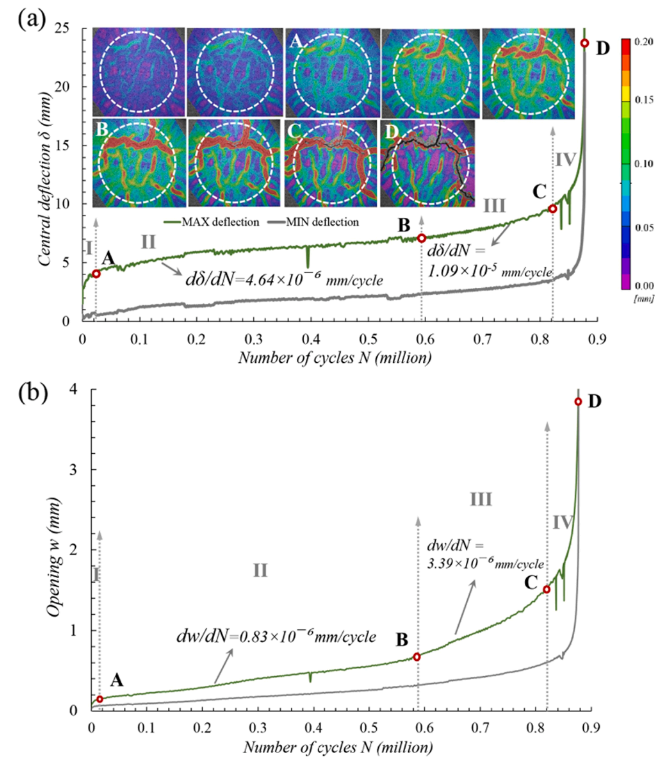

Fig. 8. Fatigue damage evolution of specimen at the stress level of 0.6 with a fatigue life of 0.88×10^6 (a) central deflection evolution and fracture process by DIC analysis; (b) evolution of fictitious crack opening. Adapted from [89].

In practice, the alternating stress between tension and compression is a common loading condition encountered in concrete structures. Cornelissen [92] investigated the effect of alternating tensile and compressive stresses on plain concrete. The results revealed that alternating between tension and compression significantly reduces the fatigue life of concrete. Subramaniam et al. [93] examined the fatigue behaviour of concrete under biaxial stresses in the compression-tension region using hollow cylindrical specimens subjected to torsional loading. It was found that the fatigue damage is localized to a single crack and the overall response of the material is primarily controlled by the growth of this crack. There is a strong correlation between the rate of decrease in stiffness in second phase of the fatigue damage evolution and the fatigue life of specimen. This relationship remains consistent regardless of the load range applied during the fatigue process.

Concerning the influence of loading modes on microcracking, it is generally accepted that the microcracking process in cementitious materials is primarily governed by Mode I, Mode II, or a combination of these fracture mechanics [94–96], similar to static fracture behaviour. Due to their brittle nature, cementitious materials are particularly susceptible to cracking under tensile stress. Even under compressive cyclic loading, crack propagation can be driven by localized tensile stresses at the microscale, further complicating the material's response to stress. Under cyclic compressive loading, the material can withstand higher stress levels before microcracks initiate, as these cracks tend to close under compression, slowing their growth and allowing the material to endure more cycles before failure. In flexural loading, the material experiences a stress gradient across its cross-section, with the upper part in compression and the lower part in tension. Under repeated flexural loading, microcracks typically initiate in the tensile zone and gradually propagate towards the compressive zone. As cyclic loading continues, these cracks grow and coalesce, ultimately leading to fatigue failure. Tensile loading subjects the entire cross-section to direct tensile stress, causing microcracks to initiate and propagate quickly. The absence of compressive zones to counteract crack growth leads to rapid fatigue failure. Substantial experimental evidence suggests that the evolution of fatigue damage, whether measured by strain or stiffness, exhibits consistent qualitative similarities across different loading modes. This consistency may suggest common underlying mechanisms in concrete fatigue fracture.

In addition, each loading mode interacts uniquely with the microstructure of cementitious materials, influencing the formation and growth of cracks. Flexural loading concentrates tensile stresses in specific zones, tensile loading exploits the material's inherent weakness in tension, and compressive loading benefits from the material's compressive strength. Compression-tension alternating loading, meanwhile, accelerates fatigue damage through repeated stress reversals. Understanding these effects is essential for designing cementitious structures that can withstand the demands of cyclic loading throughout their service life.

2.2.3. Effect of loading frequency

In practice, concrete structures undergo different loading frequencies during cyclic loading. The rate at which loads are applied and removed may influence the development of cracks, propagation of damage, and overall fatigue performance of the concrete structure. Graf and Brenen [97] investigated the effect of loading frequency on the fatigue life of concrete. They found that the loading frequency in the range between 4.5 and 7.5 Hz had little effect on the fatigue life, but when it decreased below 0.16 Hz the fatigue life decreased. Mudock [98] suggested that when the stress level was less than 75 % of the tensile strength, frequencies between 1 and 15 Hz had little influence. Similar results were also reported in [53]. The study of Sparks and Menzies [99] showed that for a stress level larger than 75 %, the loading frequency greatly affected the fatigue life, but when the stress level is lower than 75 %, frequencies between 0.1 and 100 Hz had no effect on the fatigue life. It was also argued that since the loading frequency determines the loading rate of fatigue tests, a higher loading rate may enhance the static and fatigue strength of cementitious materials [30,100].

Medeiros et al. [85] investigated the loading frequency effect on the compressive fatigue behaviour of plain and fibre reinforced concrete. The results reveal a notable influence of loading frequency on fatigue of plain concrete, with the number of cycles to failure at 1/16 Hz being at least one order of magnitude lower than that at 4 Hz. Conversely, for polypropylene and steel fibre-reinforced concretes, the gap between the number of cycles to failure under low and high loading frequencies narrows, especially for steel fibre-reinforced concrete. This trend is attributed to the effective crack-bridging capability of fibres. Additionally, the findings highlight a direct correlation between the secondary strain rate and fatigue life, primarily dependent on the loading frequency. This relationship is valuable for estimating the fatigue life and conducting tests based on strain history without waiting for the final failure of specimen.

2.2.4. Effect of rest period

In view of the intermittent nature of traffic loading, the effect of the duration of the rest period was also studied by several researchers [26, 31,101,102]. Hilsdorf and Kesler [102] investigated the influence of rest period on the flexural fatigue behaviour of plain concrete at the age of 7 days. They found that the fatigue strength of plain concrete increases with the introduction of periodic rest periods, indicating the occurrence of partial recovery during the rest period. This beneficial effect was found to be a function of the length of the rest period when the length is less than 5 min. Further increasing the length of the rest period from 5 to 27 min did not affect the result. Similar beneficial effects of rest periods have been reported by other researchers [103,104]. Bennett and Muir [103] observed that rest periods of 24 h, following 190,000 load cycles, resulted in a 50 % recovery of residual strain. However, the strain recovery decreased to 10 % when the rest was applied after 3–4 million load cycles. This suggests that the extent of recovery is influenced by the degree of initial damage.

In addition, Raithby and Galloway [101] found that a very short rest period (only 2 s) had no effect on the fatigue strength of concrete. Nevertheless, it is clear that increasing rest period (up to several days or weeks) can raise the fatigue strength due to continuing hydration [98] or self-healing in the presence of water [105]. Several studies have also reported strength gains after fatigue testing, attributed to the ongoing hydration [34,76,106]. Garijo et al. [106] proposed that suggested that cyclic compressive loading activates the movement of water trapped in the mortar, causing chemical reactions that support autogenous self-healing. To test this, the authors examined intact and post-fatigue specimens using differential thermal analysis (DTA), thermogravimetric analysis (TGA), mercury intrusion porosimetry (MIP), and scanning electron microscopy with energy dispersive X-ray analysis (SEM-EDX). The TGA-DTA results showed new hydration pathways, with C-S-H gel and portlandite forming. MIP data showed a drop in pore volume, especially in fine pores, which supported the growth of new hydration products. SEM-EDX confirmed the makeup of these new microstructures. Moreover, the relaxation of residual stresses, originating from shrinkage and drying near inherent cracks, could explain the observed recovery [102,104]. It is hypothesized that fatigue-induced stresses contribute to the relaxation process, effectively "stress relieving" the microstructure and enhancing its resistance to fatigue.

2.2.5. Effect of variable stress amplitude

While most fatigue studies have focused on assessing the impact of constant amplitude loading, stress cycles in real concrete structures vary significantly in terms of stress magnitude and loading sequence [107]. Oh [108] conducted variable-amplitude fatigue tests on concrete beams, revealing that the fatigue failure of concrete is profoundly influenced by the magnitude and sequence of variable-amplitude cyclic loadings. When the magnitude of fatigue loading was gradually decreased, the resulting fatigue damage appeared more extensive compared to concrete subjected to increasing loading magnitudes. However, an experimental work conducted by Du et al. [109] suggested that with the increase of fatigue load amplitude, the stiffness degradation became faster. Similarly, Keerthana and Kishen [110] performed an experimental investigation into the effect of overload on concrete fatigue. They implemented a variable amplitude loading program with increasing steps. The maximum load amplitude was increased by 0.25 kN for every 250 cycles, starting from 0.25 kN. As the number of load cycles and load amplitude increases, microcracks coalesce and develop into larger cracks, ultimately leading to failure. There is a sudden acceleration in crack growth whenever the maximum amplitude in the load history is increased.

Based on the Digital Image Correlation (DIC) measurements, a theoretical interpretation of the observed loading sequence effect on flexural fatigue behaviour of concrete was performed by Baktheer and Becks [111], as shown in Fig. 9. Taking the Low stress level to High stress level (L-H) scenario as an example: after experiencing a certain amount of the loading cycles, the crack length increases, and the corresponding amplitude of stress intensity factor reaches the value of $\Delta K_{\text{I-L}}$, denoted as the stress intensity factor amplitude at low stress level. Note that the fatigue failure occurs when the critical stress intensity factor amplitudes ΔK_{IC} is reached. Upon the switch from (L) to (H) level, the stress intensity factor amplitude suddenly increases due to the increased applied load, potentially leading to crack growth following the load jump, reaching the value of $\Delta K_{\text{I-LH}}$. In the second stage with the higher loading range (H), the crack tends to grow faster than in the first stage, starting from a higher stress level $\Delta K_{\text{I-LH}}$ and reaching ΔK_{IC} earlier, thereby consuming the remaining fatigue life and potentially resulting in a reduced fatigue life. For more information regarding the theoretical explanation of loading sequence effect, the readers are referred to [111].

Unfortunately, the complex process of damage accumulation in concrete under random cyclic loading is still unclear, with only a limited number of cumulative fatigue damage hypotheses proposed for concrete

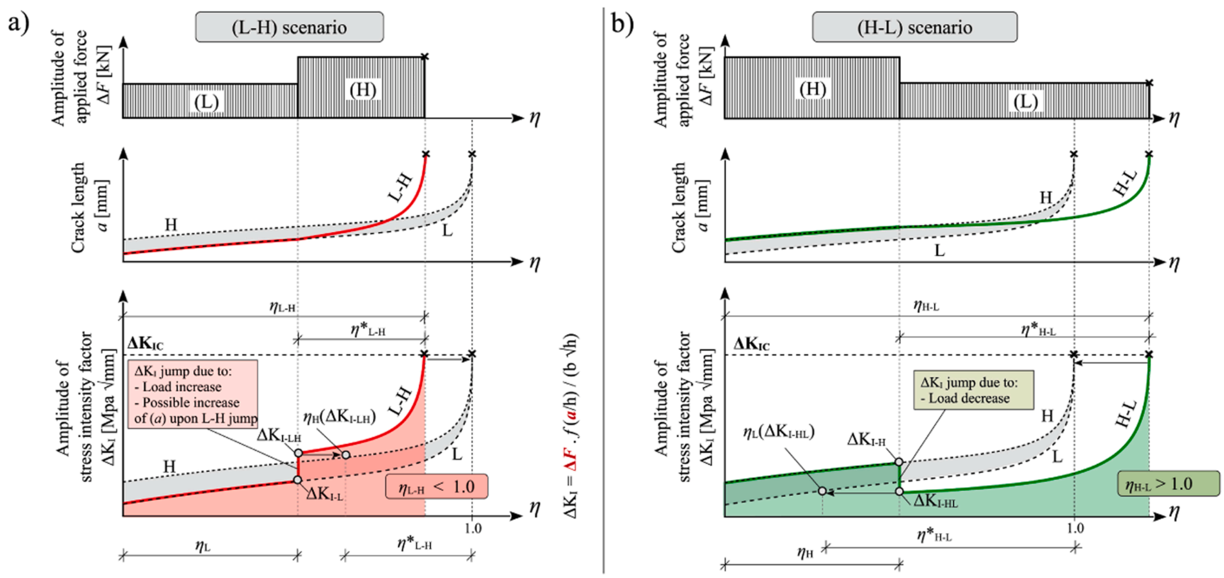

Fig. 9. Theoretical interpretation of the observed loading sequence effect on the flexural fatigue behaviour of concrete based on the actual stress state at the crack tip: (a) Low stress level to High stress level (L-H) scenario; (b) High stress level to low stress level (H–L) scenario, adapted from [111]. Note: ΔF is the amplitude of applied force, a is the crack length, ΔK_I is the amplitude of stress intensity factor, ΔK_{IC} is the critical stress intensity factor, η is the fatigue life, η^* is the remaining fatigue life and the subscripts L and H represent the low stress level stage and high stress level stage, respectively.

[108,110,112]. Among them, the well-known Palmgren–Miner (P-M) law is often applied to consider the effect of variable loading amplitude [113] and it states that if there are k different stress levels and the average number of cycles to failure at the i-th stress, S_i, is N_i, then the damage fraction, D, is:

$$D = \sum_{i=1}^{k} \frac{n_i}{N_i} \tag{2-1}$$

where n_i is the number of cycles accumulated at stress S_i. In general, fatigue failure occurs when the damage fraction D reaches 1. The applicability of this law in concrete fatigue has been examined by several studies [107,108,112]. Slemes [112] found that P-M law is useful for describing the cumulative damage due to variable-amplitude loadings. Similarly, Cornelissen and Reinhardt [107] also suggested that P-M law is a safe failure criterion in general. However, it was found by Holmen [114] that the variable amplitude loading appears to induce more damage than predicted by P-M law. This discrepancy is likely attributed to sequence effects inherent in variable loading, which are not considered in the law. Mohamadi et al. [115] also proposed that utilizing P-M law to predict the fatigue life of concrete is nonconservative, as indicated by the calculated 56 % error in their study.

Keerthana and Kishen [110] proposed an analytical model that integrates the concepts of damage and fracture mechanics within a thermodynamic framework. The principles of dimensional analysis and self-similarity are employed to derive the dual dissipation potential. The model was found to successfully reproduce the experimental results by accurately capturing the load sequence and overload effects for different sizes of concrete beams. In addition, several prediction models based on the continuum damage mechanism were also reported in [63, 116]. However, Baktheer and Becks [111] conducted a comparison of existing nonlinear damage accumulation rules and concluded that these rules exhibit limited validity as they ignore the phenomena of fatigue crack propagation and the associated stress redistribution. Additional experiments and theoretical analyses of the loading sequence effect on the damage accumulation process in concrete under variable loading amplitudes are still required.

3. Experimental characterisation of fatigue damage

Over the past few decades, numerous experimental techniques have been employed to assess fatigue damage in concrete. Depending on the way fatigue damage is disclosed, these techniques can be categorized into two types: direct and indirect approaches. Direct approaches encompass methodologies such as optical microscopy, scanning electron microscopy (SEM), and X-ray computed tomography (XCT), allowing for the visualisation of microcrack propagation under fatigue loading. On the other hand, indirect approaches involve measurements of stiffness degradation, acoustic emission, electrical resistivity, variations in pore structure, and transport properties. In these cases, fatigue damage is evaluated indirectly by monitoring changes in other properties.

This section provides a review of experimental characterizations of fatigue damage evolution using both direct and indirect methods, presenting and discussing insights gained from these approaches.

3.1. Direct approaches for fatigue damage characterisation

3.1.1. 2D microscopic observation

Direct microscopic observation allows for the examination of various characteristics of fatigue cracks, including size, number, and distribution. However, limitations in nano- and micro-structural characterisation techniques confine the study of fatigue fracture behaviour to a specific length scale. In the late 1980s, Saito [117] utilized a stereoscopic microscope to analyse microcracking in slices from concrete specimens subjected to static and fatigue tensile loadings. The results revealed only a slight increase in microcracks under cyclic loading, possibly due to the microscope's limited magnification (15–30×) used in his study. In contrast, Shah and Chandra [118], using a higher magnification microscope (200×), observed a substantial rise in microcracking, which is nearly three times more than that induced by static loading at the same stress level. Moreover, they also found that microcracking under cyclic compressive loading consisted of extensive branching in the paste matrix and a substantial amount of cracking at the aggregate-paste interface.

Similarly, Toumi et al. [119] observed a higher number of interface cracks using a SEM technique. More diffuse microcracks in the matrix were also detected compared to the static crack pattern. When plotting the measured crack length versus the number of cycles, a steady stage with a constant crack growth rate depending on the upper loading level can be identified. In addition, a new fatigue fracture mechanism was observed by Li et al. [74] using the SEM analysis, as mentioned in Section 2.1.4. A clear trace of 'friction' was observed in the fatigue failure surface. The authors speculated that this was attributed to the crushing and friction process between crack surfaces and fibres.

In general, crack development during fatigue loading can be divided into three stages: crack initiation (phase I), crack propagation (phase II), and unstable crack growth (phase III) stages [13], see Fig. 10. The first stage is the crack initiation stage. The initiation of a crack is always associated with stress concentrations, which may be due to flaws, surface discontinuities or pre-existing cracks. The crack initiation process in concrete can be ignored as there is already a large number of pre-existing cracks in the material even prior to any loading. The second stage is the stable crack propagation stage, which is characterised by slow and progressive growth of multiple cracks to a critical size. At this stage, the residual strain of concrete increases with an almost constant rate. Meanwhile, multiple cracks gradually propagate in the concrete with the increasing number of cycles, resulting in a decrease of stiffness. This phenomenon, known as the cyclic creep stage, was first identified by Sparks and Menzies [99]. They observed a strong correlation between the rate of strain increase per cycle during this second stage and the specimen's fatigue life, regardless of the load application rate. This relationship has since been extensively studied [106,120–122] and serves as a key model for fatigue damage evolution, providing a basis for predicting secondary strain and fatigue life. Garijo et al. [122] further confirmed that this correlation is independent of the loading frequency. In the final stage of concrete fatigue, when a sufficient number of cracks have rapidly propagated and coalesced, a continuous macrocrack is formed, eventually leading to the failure.

By using fluorescent microscopy and SEM, Thiele [123] examined the concrete damage at three fatigue stages. Examples of obtained microscopy and SEM images of the crack pattern in damaged concrete are shown in Fig. 11. It was found that both the crack length and crack width increase dramatically before fatigue failure. The author further assumed that damage also occurs on a smaller scale than considered in their research (presumably smaller than 1 μm). Similar findings have also been reported in [13,124]. In addition, the author noted pronounced crack patterns in the microstructure even before unloading, likely attributed to shrinkage during hydration. However, due to the destructive nature of microscopic investigations, it is not possible to follow up further development of crack patterns that had been identified in previous loading cycles.

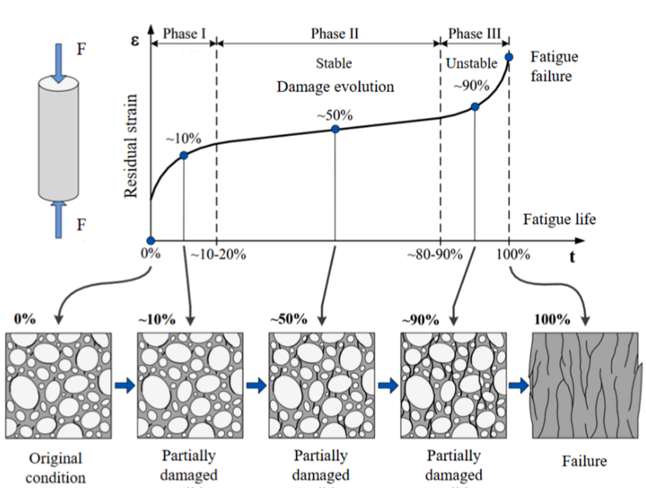

Fig. 10. Illustration of the fatigue damage evolution under compressive fatigue loading adapted from [123].

In a recent work performed by Schaan et al. [125], the transmission electron microscopy (TEM), combined with a focused ion beam (FIB) sample preparation, were employed to investigate the compressive fatigue damage in Ultra-High-Performance Concrete (UHPC). At the early stage of fatigue loading, several needle-shaped regions have formed with a length of 150 – 250 nm and a width of 20 – 35 nm in the bulk matrix, as is shown in Fig. 12. Meanwhile, the C-S-H compounds become denser during the fatigue process. Other components of the material, such as aggregate or unhydrated cement clinker, and silica fume particles, remain unaffected. These needle-shaped structural regions, which are initially believed to be the precursors of nanocracks, continuously grow in number and increasingly deplete with surrounding C-S-H as fatigue progresses. However, upon further investigation with the energy-dispersive X-ray spectroscopy (EDS) technique [126], the author concluded that these regions are related to locations where formation of ettringite occurs. It was argued that, given ettringite's ability to readily recrystallize in response to variations in environmental conditions, recurrent dissolution and crystallization may impose pressure on the adjacent material. This process can contribute to solidification and, ultimately, the development of nanocracks [126]. However, there is currently limited information on the fatigue fracture behaviour at the nanoscale in the existing literature.

In addition, Gan et al. [13] directly examined the microscopic fatigue behaviour of cement paste using micro-bending tests. Micro-cantilever beams were first fabricated and subjected to cyclic loading using a nanoindenter. The fracture surfaces were compared with statically failed samples using secondary electron (SE) microscopy. The fatigue fracture surfaces exhibited a higher density of cracks, predominantly observed in the fractured C-S-H phases. These cracks were concentrated around stiff particles (e.g., unhydrated cement particle and calcium hydroxide) or near pores. Moreover, the majority of these cracks were at the nano-scale, with widths less than 500 nm, as illustrated in Fig. 13. It was also proposed that the development of residual deformation in cement paste can be elucidated by the combined effects of viscoelastic deformation and the growth of fatigue cracking.

3.1.2. 3D X-ray computed tomography

Traditional microscopic observations of fatigue damage necessitate stopping the tests and cutting specimens to prepare samples for investigation. However, sample preparation may introduce unwanted damage that is challenging to distinguish from fatigue cracks. Therefore, the non-destructive X-ray computed tomography (XCT) technique emerged as a promising alternative. The XCT technique has been widely used to characterise the internal microstructure of materials, due to its non-destructive nature and ability to track and visualize 3D details of the microstructure, including different phases, interfaces, pores, and cracks [127,128].

Recently, several studies have employed the XCT technique to investigate the fatigue damage evolution in concrete. Sharma et al. [129] performed in-situ XCT tests on concrete specimens subjected to compressive cyclic loadings. The local fracture mechanisms, such as the crack opening and closing, crack tip extension and diversion and crack tip blunting were observed in the XCT images at different cycles. It was also revealed that there are many microcracks generated before the occurrence of macrocracks. Vicente et al. [69] examined the damage evolution in fibre-reinforced concrete subjected to the low-cycle fatigue loading. It was found that the specimens tested under monotonic loading display a crack pattern statistically equivalent to that observed under cyclic loading. This indicates that the internal micro-mechanisms of failure, including crack nucleation, growth, and distribution, observed in monotonic static compressive tests closely resemble those generated by low-cycle fatigue tests. Thus, this confirms the hypothesis of convergence to the initial distribution, suggesting that compressive tests represent a limit case of a cyclic test where failure occurs in the first cycle. This concept represents a paradigm in concrete technology and is generally accepted for all types of concrete. It has also been incorporated

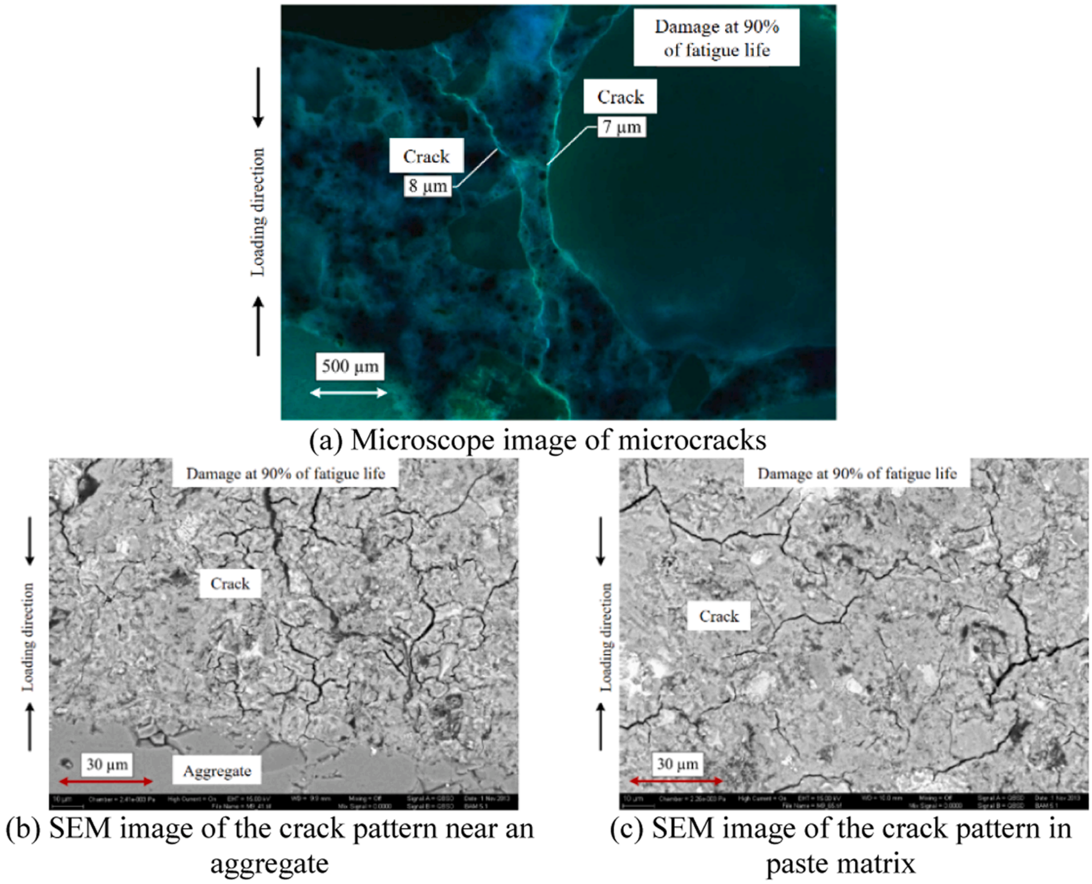

Fig. 11. (a) A microscope image of microcracks oriented parallel to the load direction with crack widths in a longitudinal section of a specimen that has been damaged for up to 90 % of the fatigue life; (b) SEM image of the crack pattern near an aggregate; (c) SEM image of the crack pattern in paste matrix. Adapted from [123].

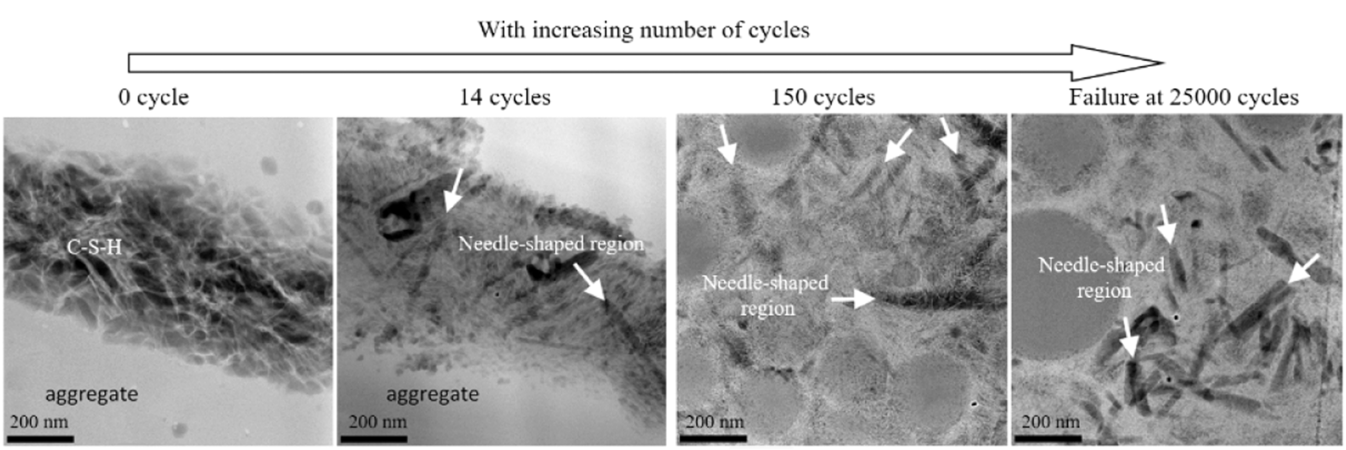

Fig. 12. High-angle annular dark field STEM images of UHPC cement paste under the compressive fatigue loading adapted from [125].

into the Model Code 2010 [130].

Obara et al. [131] introduced a novel approach for damage estimation utilizing the XCT technique. They employed Three-Dimensional Medial Axis Analysis (3DMA) to assess damage evolution in concrete specimens subjected to cyclic loading. The loading regime involved variable amplitude loading with a step-wise increase in the maximum load. Burn number and medial axis parameters in 3DMA were utilized to evaluate crack width and length, respectively. Larger values of these parameters generally signify a higher degree of damage. The resulting damage distributions in concrete specimens with varying stress levels are depicted in Fig. 14, indicating a damage localization phenomenon. It was concluded that the XCT with 3DMA method is useful to evaluate damage in a specimen while under loading. However, only cracks with a width of more than 50 µm could be detected reliably by their XCT equipment.

Fan and Sun [132] also applied the XCT technique with a spatial resolution of 1 μm^3/voxel to investigate compressive fatigue damage evolution in cylindrical concrete specimens of 50 mm in diameter and 100 mm in height. The fatigue crack at different stages can be clearly visualized, as shown in Fig. 15(a). The observed fatigue crack expanded

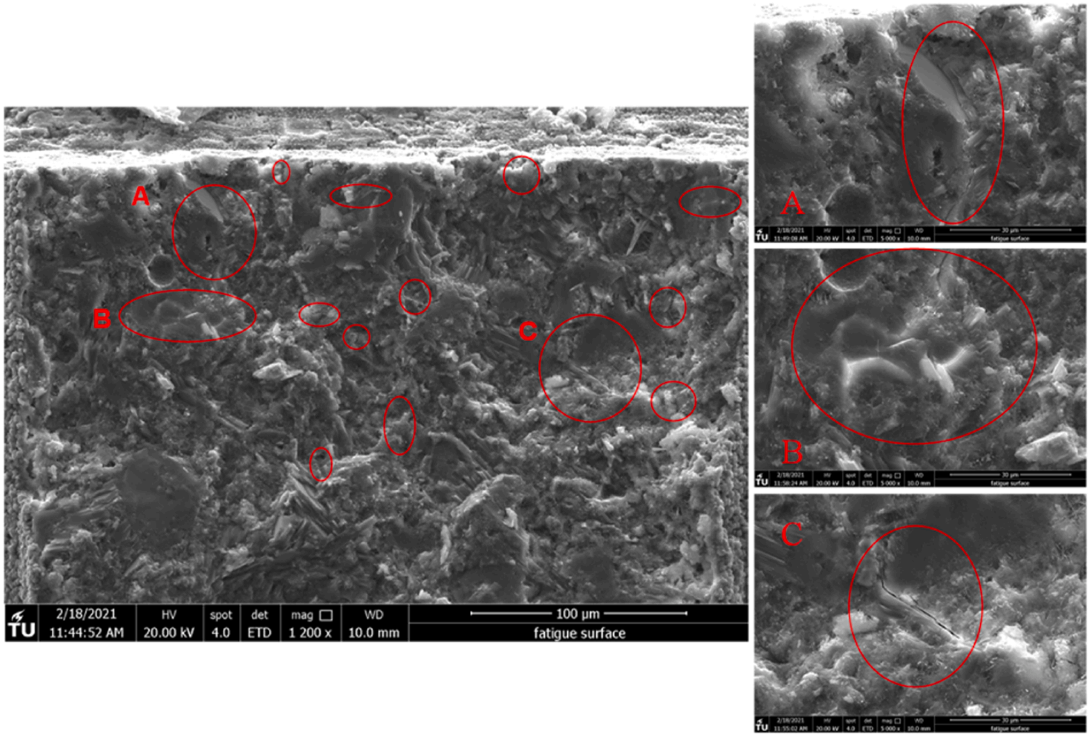

Fig. 13. SE images of fatigue fractured surfaces of micro-cantilever beams and marked nanoscale cracks, adapted from [13].

with the increasing number of cycles, and the crack width was in the range of 0.6–1.7 mm at 90 % of the fatigue life. The change of porosity of the specimen was also determined based on XCT images and used as an indicator to describe the degree of damage, see Fig. 15 (b). It was found that the porosity gradually increased and reached a value of 20 % at 90 % of the fatigue life. However, it should be noted that the resolutions of most commercially available XCT instruments are often not sufficient to detect extremely small fatigue-induced damage. Therefore, the XCT can only detect relatively wide cracks shortly before fatigue failure [13, 124].

3.2. *Indirect approaches for damage characterisation*

3.2.1. *Development of stiffness and strain*

In addition to direct observations, the assessment of fatigue damage evolution in cementitious materials can be achieved through indirect approaches. The developments of strain and stiffness were focused on as a first step towards a more mechanism-orientated analysis of concrete fatigue. In general, the initiation and propagation of cracks lead to a continuous reduction in stiffness as well as an increase in maximum strain. As stiffness and strain can be readily derived from stress-strain curves, many researchers have employed these two parameters as indicators of fatigue damage [114,133,134]. Both the global stiffness and maximum strain evolutions under fatigue loading follow an S-shaped curve, characterised by three stages: the first stage involves a rapid drop in stiffness and a rapid increase in strain, followed by a second stage characterised by a steady and gradual change in both parameters, and the third stage denotes a dramatic decline in stiffness and an increase in strain. Typical evolution curves of stiffness and strain are shown in Fig. 16. Kolluru et al. [133] observed that the measured crack growth during fatigue loading shows a trend similar to the change in stiffness. The degradation rate of stiffness largely depends on the loading frequency and maximum stress level [30,135]. By considering the visco-elastic deformation during the fatigue loading, many researchers [13, 120,122] suggested that the creep mechanism also plays an important role in the development of strain in concrete. Note that these two macroscopic damage indicators can only offer an overall trend for the damage evolution, lacking detailed information on the distribution of damage.

3.2.2. *Acoustic emission technique*

To obtain additional information regarding the damage distribution and propagation inside the specimen, several researchers have applied the acoustic emission (AE) technique to evaluate the fatigue crack growth in concrete [124,136–138]. This non-destructive technique enables real-time monitoring of microscopic processes as well as crack propagation inside the concrete. Different AE parameters, such as the amplitude, energy, count and duration, are often considered in terms of their quantitative distribution to detect the degree of generated damage and crack distribution [136,137]. Recently, Oneschkow et al. [138] investigated the fatigue behaviour of mortar and concrete samples using the AE technique. A higher AE activity was observed for concrete compared to mortar due to the presence of coarse aggregate. Moreover, a gradually increasing AE activity in the concrete was observed at the lower stress level, which cannot be reflected by the change of specimen stiffness. This indicated a damage process at very small scales (perhaps on a sub-microscale) during the fatigue process.

Deresse et al. [139] suggested that the AE energy-based method is the most appropriate for quantifying the size of the fracture process zone (FPZ) during fatigue damage progression. The authors observed that under low-cycle loading, the gradual progression of damage may not always occur, and abrupt failure, similar to monotonic loading, can occur even after several load cycles. The evolution of the cumulative AE events during load-cycle fatigue tests showed different failure modes, i.e., progressive failures and abrupt failures. Material heterogeneity and high stress magnitude in stepwise low-cycle fatigue loading were identified as the causes for the variation in failure modes. This was further elucidated using AE-based FPZ analyses, which demonstrated a slowly growing FPZ for samples that failed progressively and a sudden formation of FPZ for samples that failed abruptly.

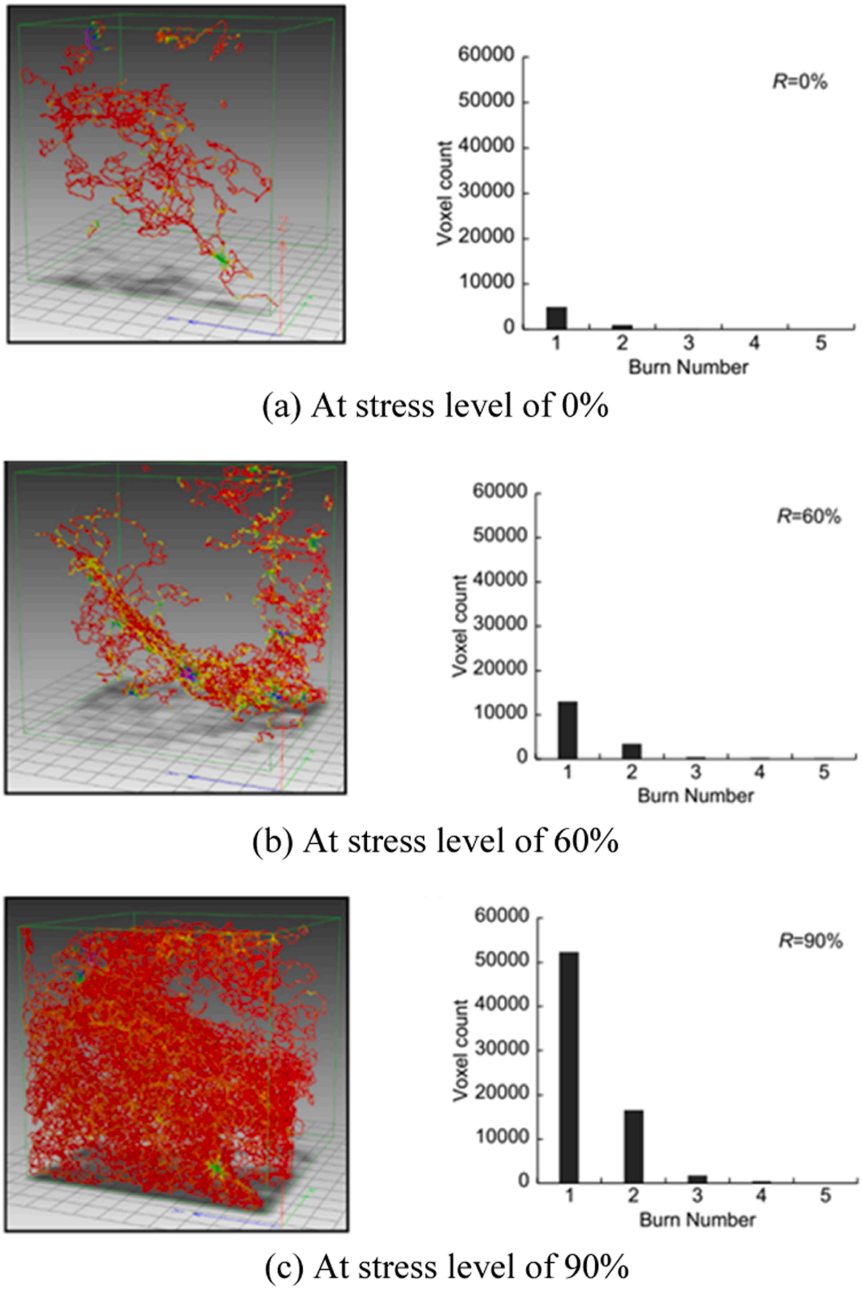

Fig. 14. The distribution of medial axis (the warm color is a small burn number and the cold color represents a large burn number; the red color represents burn number 1) and the amount of burn number in cubic concrete specimens (10 × 10 × 10 mm^3) under the stress level of (a) 0 %, (b) 60 % and (c) 90 % adapted from [131].

3.2.3. Electrical resistivity measurements

Cao and Chung [140] attempted to monitor fatigue damage by measuring the electrical resistivity of mortar specimens. It was found that the baseline resistivity irreversibly increased as cycling progressed due to damage generation (i.e., microcracking), which was most significant in the first few cycles, and diminished as cycling progressed. Later, a follow-up study on concrete fatigue conducted by the authors [141] showed that the measured resistivity decreased during the loading stage of the compressive fatigue cycle due to defect diminution (i.e., closing of microcracks). It then increased during the unloading stage due to the defect extension (i.e., propagation or enlargement of microcracks). Additionally, an examination of the results from cement paste and mortar samples revealed that the interface between the paste and aggregate played a role in microcracks closing behaviour. As an alternative approach for non-destructive real-time monitoring of damage accumulation, electrical resistivity measurement offers new perspectives on the evolution of fatigue damage in concrete. However, this technique is limited to evaluating only the overall trend of fatigue damage evolution.

3.2.4. Pore structure changes

A study of Zhang [15] demonstrates that several characteristics of the pore structure, such as the porosity and specific surface area, can be used as micro- or mesoscopic indicators to evaluate the macroscopic damage of concrete under fatigue loading. The testing results based on mercury intrusion porosimetry (MIP), helium pycnometry and nitrogen adsorption methods showed that, under flexural fatigue loading, the magnitudes of all these indicators increased almost linearly with the

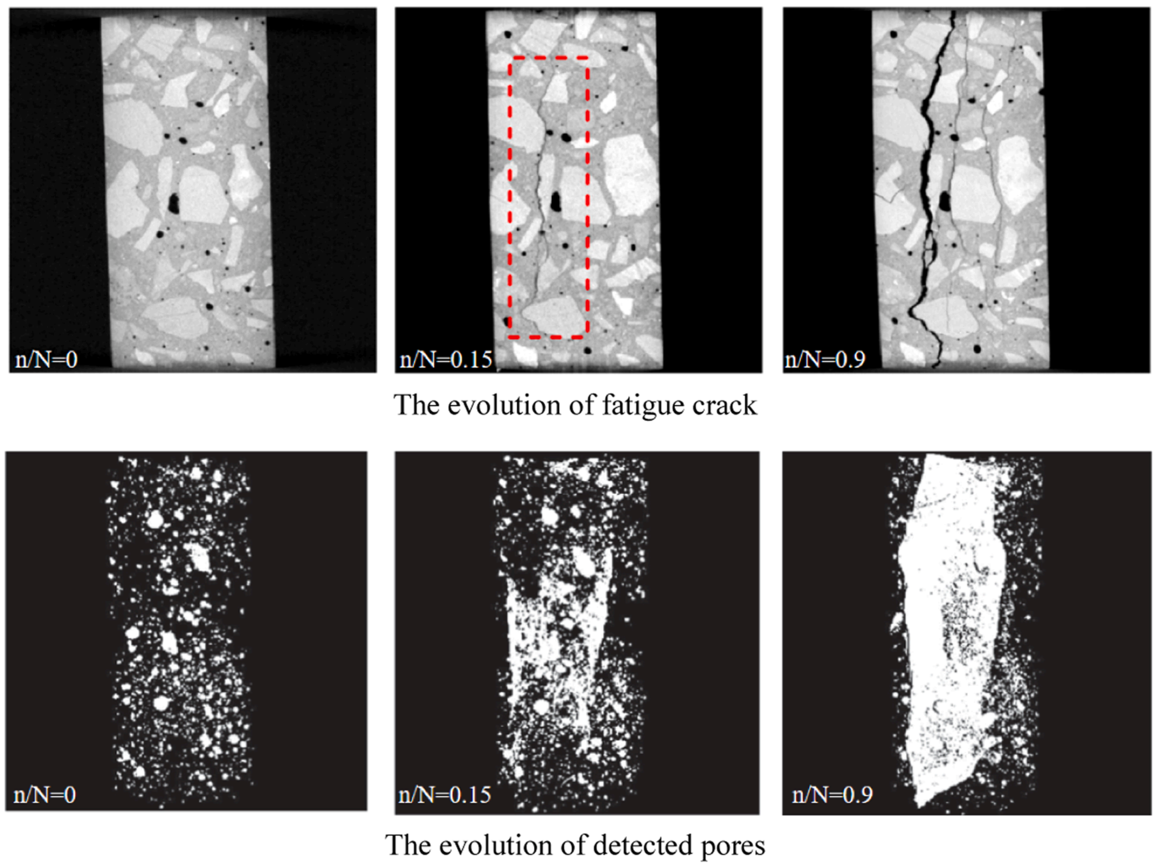

Fig. 15. The ICT images for (a) the fatigue damage evolution in concrete specimens with the size of 50 mm in diameter and 100 mm in height under compressive fatigue loading and (b) the distribution of detected pores under the stress level of 80 % adapted from [132].

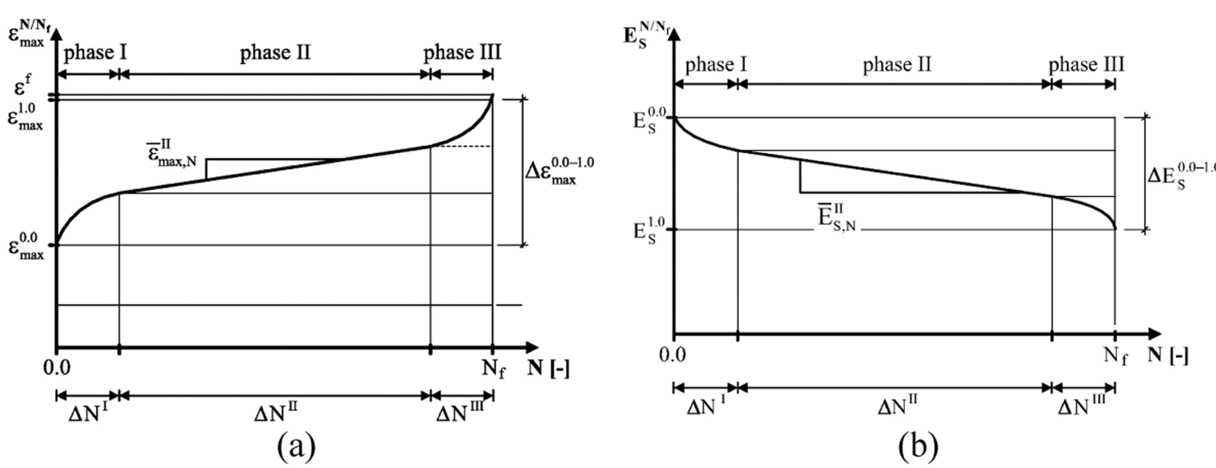

Fig. 16. The schematics of the development of (a) maximum strain and (b) stiffness during the fatigue loading, adapted from [134]. Note ε_{max} is the maximum strain at the upper loading, E_s is the stiffness and N is the number of cycles.

increasing number of cycles. Both porosity in mortar (mainly macro pores) and interface between mortar and coarse aggregates (interfacial cracks) developed at a similar rate. The experimental results also showed good correlations between macroscopic mechanical properties (e.g., residual bending strength and Young's modulus) and micro- or mesoscopic parameters (porosity and mean pore size distribution).

By using the MIP, Schaan et al. [125] also observed an increase of pore volume in the 30 – 500 nm pore radius range in cement paste specimen due to fatigue loading. Similar findings have also been reported by Shen et al. [142]. However, a continuous and accurate characterisation of the pore structure subjected to fatigue loadings is still a challenging task. Instead of direct examination of the pore structure, the fatigue damage evolution may also be reflected by the variation of transport properties. Fu et al. [143] investigated the chloride penetration in fatigue-damaged concrete with various loading levels. A quantitative correlation between the residual strain and the apparent chloride diffusivity of fatigue-damaged concrete has been established. In general, the chloride diffusivity increases with fatigue damage. It was found that when the maximum fatigue load is greater than 30–35 % of the ultimate tensile load, the chloride penetration dramatically increased, suggesting that a substantial damage had been generated. Similarly, Desmettre and Charron [144] examined the real time variations of water permeability of normal-strength concrete and fibre-reinforced concrete under cyclic loading. It was found that the addition of fibres could arrest the fatigue crack growth as indicated by the smaller growth of water permeability compared to that in normal-strength concrete. However, due to the presence of water in the specimens in these tests, the transport properties may be affected by the self-healing process and, therefore, may not be a precise indicator for the fatigue damage.

3.3. General remarks

To enhance readability, Table 1 provides a summary of the experimental techniques used to characterize concrete fatigue damage, highlighting the key advantages and limitations of each method. Numerous macroscopic fatigue damage indicators have been proposed to monitor the overall fatigue damage evolution of concrete under cyclic loading, indicating that fatigue damage manifests at the macroscopic level through stable crack growth. The recent use of XCT offers an opportunity to continuously assess fatigue damage evolution at lower scales, shedding light on the influence of mesoscopic features. Nevertheless, the understanding at the nano- and microscopic levels, where the accumulations of fatigue damage in different components and their contributions to global damage occur, remains largely speculative. Advanced techniques may be necessary to characterize fatigue behaviour at these scales.

Table 1
Summary of experimental technique for characterisation of fatigue damage.

Ref	Experimental Method	Main Conclusions on Fatigue Damage Characterisation	Advantages and Disadvantages
[7,13,74,99,117–119,123,124–126]	Optical Microscopy and Scanning Electron Microscopy	More diffuse microcracking, with increased branching in the paste matrix and at the aggregate-paste interface. Revealing new fatigue fracture mechanisms.	Direct observation of fatigue induced microcracking. Destructive nature of microscopy and limited resolution to detect smaller cracks.
[69,129,131,132]	3D X-ray Computed Tomography	Revealing local fracture mechanisms, damage extent, and crack paths under cyclic loading.	Non-destructive nature and ability to track and visualize 3D details of the cracking. Limited resolution for large sample and time-consuming testing procedure.
[13,30,114,124,133,134]	Stiffness and Residual Strain	S-shaped damage evolution defined by three stages and prediction of fatigue life and damage evolution using the secondary strain rate.	Easy to measure but insensitive to small-scale crack growth. Unable to capture fatigue fracture mechanisms occurring at the micro or nanoscale.
[124,136–139]	Acoustic Emission	Additional insights on fatigue crack growth include quantitative methods for detecting damage degree and crack distribution.	Indicator of local cracking behaviours. Difficulty in interpretation of results. Unable to reflect the fatigue fracture mechanism happenings at the lower scale.
[140,141]	Electrical Resistivity	Most significant damage generation in the initial cycles and decreased with continued cycling. Observations of defect reduction and extension under compressive fatigue loading.	Non-destructive real-time monitoring of damage accumulation. Limited to evaluate the overall trend of fatigue damage evolution.
[15,125,142–144]	Mercury Intrusion Porosimetry and Transport Properties	Linear relationship between fatigue damage and variation of pore structure. Correlation with chloride diffusion and water permeability.	Sensitive to small variation of microstructural characteristics. Affected by the presence of water and self-healing process.

4. Models and theories for fatigue analysis

Besides experimental investigations, many models and theories have been developed over the past decades to predict and simulate the fatigue behaviour of cementitious materials. A review of the existing fatigue models is given in the following sections.

4.1. Wöhler curve

Fatigue of materials is commonly characterised by an *S-N* curve (stress level versus fatigue life), also known as the Wöhler curve [2]. The stress level is generally calculated by dividing the maximum fatigue stress by the static strength of an identical specimen. If N is plotted on a logarithmic scale and S on a linear scale, the *S-N* curve will become approximately a straight line. To consider the effect of minimum fatigue stress on the fatigue life, Aas-Jakobsen [145] proposed a modified *S-N* relationship incorporating the stress ratio R (the ratio of minimum fatigue stress to the maximum fatigue stress), which is based on the observed linear relationship between S and R. Moreover, due to the significant scatter in fatigue life data, a family of *S–N* curves with a certain probability of failure, known as *S-N-P* plots, is also used [146,147]. The *S-N* curve is found to be largely affected by the mix composition of concrete and the loading history. Generally, considerable experimental data is required to draw the *S-N* curves. These experimentally obtained *S-N* curves are widely adopted and remain the primary design tool for predicting the fatigue life of concrete structures. However, the major drawback of the *S-N* method is that it is purely phenomenological, without any information regarding the physical processes occurring in the microstructure. The lack of generality of this method limits its application for different concrete mixtures. Moreover, it has been recognised that designers should not only know when fatigue failure occurs, but also how much damage has been accumulated during the structure's service life [148]. Therefore, researchers aim to accomplish a more ambitious goal, i.e., to predict, or to at least understand, the fatigue damage evolution or propagation of cracks at lower scales rather than just observing the macroscopic results.

4.2. Paris' law

Thanks to the advent of fracture mechanics, designers and engineers are able to investigate the fatigue phenomenon on a theoretical basis [77,149]. Since the fatigue of material involves initiation and propagation of microcracks, fracture mechanics is thought to be a useful tool for studying the fatigue fracture process. In general, there are three stages for the fatigue fracture process [33,123], as shown in Fig. 10. Most studies into the fatigue phenomenon focus on the crack propagation stage as it generally accounts for the largest portion of the fatigue life [120,149]. According to experimental observations on metallic materials, Paris and Erdogan [150] proposed a law to describe the fatigue crack growth rate by relating it to the amplitude of the stress intensity factor range ΔK:

$$\frac{da}{dN} = C(\Delta K)^m \tag{2-2}$$

where C and m are experimentally obtained constants dependent on the material, da/dN is the crack growth per cycle. Meanwhile, the Paris' law is well-known and its modifications have gained wide acceptance for the application in metals. The first study attempting to apply the Paris' law for concrete was conducted by Baluch et al. [151]. Following their study, many numerical and analytical models based on the Paris' law have been developed to predict the fatigue life of concrete [75,152,153]. It should be noted that the Paris' Law can be adapted to investigate the fracture behaviour of fatigue, but it may not be applicable to compressive fatigue. Additionally, the previously mentioned cyclic-creep curve resembles the Paris' Law. The ε-N curve and its derivative are more suitable for studying concrete fatigue under various types of fatigue loading.

In addition, several attempts have been made to modify the original Paris' law to improve the prediction accuracy. For instance, Bažant and Xu [77] proposed a size-adjusted Paris' law to consider the size effect of fracture in concrete specimens. The effect of variable amplitude cyclic loading was also included in the Paris' law by Slowik et al. [154]. In a recent theoretical study by Le and Bažant [155], the Paris' law was physically justified for brittle and quasi-brittle materials based on atomic fracture mechanics and an energetic analysis, which relates the fatigue kinetics of a macrocrack to the growth rate of nanocracks. This law was then used to relate the probability distribution of critical stress amplitude to the probability distribution of fatigue lifetime. It was found that the proposed theoretical model yields a power-law relation for the S-N curve. In their work, the propagation of a nanocrack is assumed to be the result of the fracture between nanoscale elements [156]. Such nanoscale elements can be either an atomic lattice block representing a crystal grain of brittle ceramics, see Fig. 17(a), or a completely disordered system of nanoparticles of the calcium silicate hydrate (C-S-H) in concrete, see Fig. 17(b). For more details regarding the physical explanations and formulations of the nanoscale crack propagation under the cyclic loading, the reader is referred to [155–157].

However, it is known that in front of the crack tip in concrete there exists a so-called fracture process zone (FPZ), which is assumed to consist of many closely spaced microcracks [158,159]. This zone offers resistance to the crack growth owing to various toughening mechanisms, such as particle interlock, aggregate bridging, surface friction and crack branching, among others [160]. Due to the presence of a large fracture process zone, the Paris' law based on the linear elastic fracture mechanics may give erroneous results even with carefully modified versions [160,161]. Hence, for a better understanding of the fatigue crack propagation phenomenon in concrete, the nonlinear behaviour in front of the crack tip needs to be properly described [162].

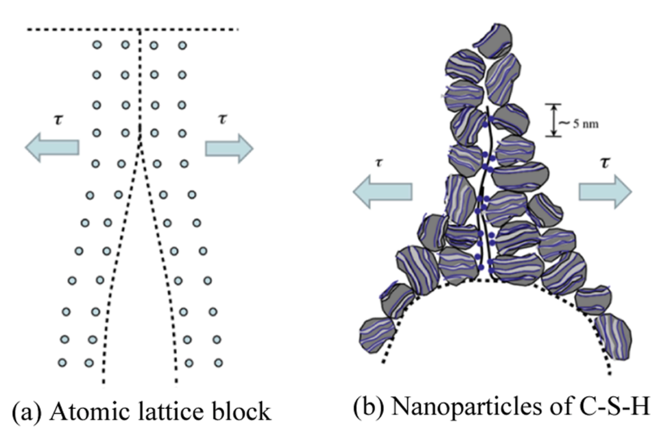

Fig. 17. The assumed fracture of the nanoscale element for (a) an atomic lattice block; (b) a disordered system of nanoparticles of the calcium silicate hydrate adapted from [155].

4.3. Cohesive zone model

In the work of Hillerborg et al. [163], a non-linear fracture mechanics model has been proposed to simulate the complex tensile fracture behaviour in concrete under monotonic loading conditions. In this model, the fracture process zone is modelled as a fictitious crack that can transfer cohesive stresses, as is shown in Fig. 18(a). In reality, this cohesive crack corresponds to the diffused microcracking zone in concrete with some remaining ligaments that allow the stress transfer. The maximum cohesive stress that can be transmitted is assumed to be a decreasing function of the crack-opening displacement [163]. Note that, for simplicity, a linear or a bilinear softening curve is often assumed [164,165]. Therefore, the crack growth can be readily simulated as a consequence of continuous de-bonding of the fracture process zone [165–167]. Moreover, the energy balance can also be considered in the model. As is depicted in Fig. 18(b), the amount of energy absorbed per unit crack area in widening the crack from zero to ω_1 can be calculated as the area between the curve and the coordinate axes. The proposed cohesive crack model is found to yield realistic results regarding the crack formation and propagation under the monotonic loading [163].

Due to its success in monotonic fracture cases, the principle of the cohesive zone model was then applied for simulating the fatigue fracture of concrete [54,168–170]. Based on Hillerborg's model, an extended version considering the cyclic loading conditions was proposed by Gylltoft [171]. By introducing a cyclic softening stress-strain relationship, the model is able to describe the progressive tensile fatigue failure of concrete. Afterwards, many attempts have been made to modify the shape of the cyclic softening curve [54,168,169]. Among them, Hordijk [169] proposed the continuous function model (CFM), which provides an accurate approximation of the complete unloading–reloading cycle, see Fig. 19. Based on a close inspection of experimental results, four analytical expressions were used for the CFM and each of them represents a certain stage of the cyclic constitutive law, i.e., (I) the post-peak envelop curve, (II) the unloading curve, (III) the gap on the envelop curve and (IV) the reloading curve. Hordijk [169] found that after one unloading-reloading cycle, the intersection point between the reloading curve and the envelop curve will not be the original unloading point. This deviation (ω_{inc}) is thought to be caused by the material degradation during the loading cycle and can be determined using the following expression:

$$\frac{\omega_{\mathrm{inc}}}{\omega_{\mathrm{c}}} = 0.1\left(\frac{\omega_{\mathrm{eu}}}{\omega_{\mathrm{c}}}\right)\left\{\ln\left(1+3\frac{\sigma_{\mathrm{eu}}-\sigma_{\mathrm{L}}}{f_{\mathrm{t}}}\right)\right\} \tag{2-3}$$

where f_{t} is the tensile strength of concrete. Other parameters can be found in Fig. 19. An appealing feature of this approach is that the analytical expression can be easily implemented as a mathematical subroutine in numerical simulations. This provides new solutions to simulate the complex fatigue fracture behaviour of concrete.

However, most cohesive crack models focus on describing the behaviour of a single macroscopic crack in concrete. Only recently, Gong et al. [172] developed a mesoscopic fatigue model based on the Rigid Body Spring Method (RBSM). The constitutive laws used for the cyclic loading, which are similar to the principle of cohesive zone model, were developed at normal and shear directions for the mortar and interfacial transition zone (ITZ). The fracture pattern and the strain history under tensile fatigue loading can be obtained by this model, as shown in Fig. 20. A good agreement was found between the simulation results and experimental data, in terms of S-N curve and strain history. But still, the knowledge regarding the fatigue fracture mechanisms at lower scales and the effects of microstructural features on the growth of multiple cracks is rather limited. Therefore, more efforts are still needed with respect to the multiscale investigation of concrete fatigue.

A recent work conducted by Gan et al. [173] combined the innovative experimental technique and lattice fracture modelling to investigate the effect of microstructure on the fatigue behaviour of cement paste.

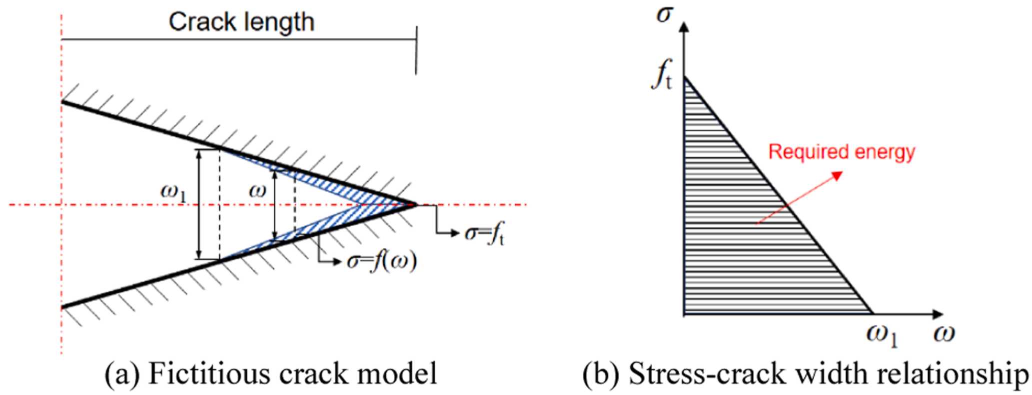

Fig. 18. The fictitious crack model adapted from [163] (a) the assumed stress distribution near the crack; (b) the assumed linear relationship between the stress and the crack opening displacement.

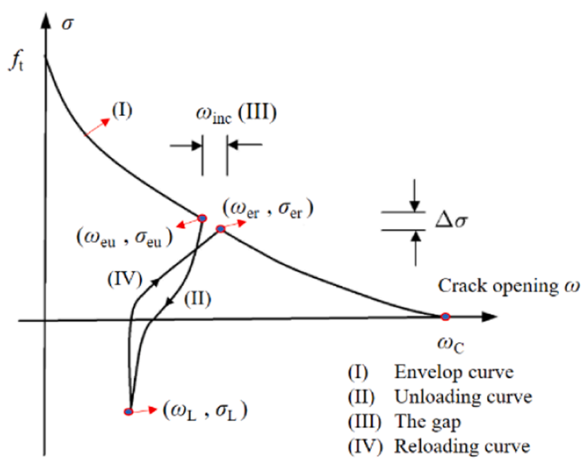

Fig. 19. The description of continuous function model adapted from [169].

The realistic microstructure of cement paste with different w/c ratios were obtained from XCT images and mapped to the lattice model. Similar to the cohesive model, a cyclic constitutive law and fatigue damage evolution law were proposed for different phases in cement paste. Furthermore, the model accounted for the simulation of residual deformation development by incorporating the creep and fatigue compliance of C-S-H phases. The proposed model replicates experimental outcomes, demonstrating proficiency in reproducing key parameters such as *S-N* curves, stiffness degradation and residual deformation. The effects of stress level and heterogeneity of the microstructure on the fatigue fracture pattern can be properly studied by the model, as is shown in Fig. 21. The findings suggest that this microscopic model lays a solid foundation for the multiscale analysis of concrete fatigue.

4.4. Damage mechanics

An alternative method to consider the fatigue damage evolution in concrete is the damage mechanics. Damage mechanics has been extensively used to model the progressive degradation of mechanical properties of materials caused by the propagation and coalescence of microcracks, microvoids and similar defects [174–176]. Damage mechanics offers a theoretical framework to understand and predict the progressive deterioration of materials under cyclic loading. In this approach, damage variables need to be introduced to describe the degree of deterioration of the material. Afterwards, constitutive equations of damage evolution for these variables are formulated in the framework of thermodynamics [175]. These equations are calibrated and validated with experimental data to model different damage processes [177–180]. It is known that the concrete subjected to fatigue loadings will experience gradual accumulation of damage in terms of the microcracking, which manifests itself in the degradation of mechanical properties such as stiffness. Therefore, the damage mechanics approach may be suitable for describing the fatigue behaviour of concrete.

Over past decades, many phenomenological macroscopic damage models have been developed [180–184]. Suaris et al. [180] proposed a stress-based damage model for monotonic and cyclic behaviour of concrete. The damage evolution law was obtained using an approach similar to the bounding surface plasticity theory [185]. The macroscopic behaviour of concrete under compression, tension and cyclic loading can be well predicted by this model. By combining the concepts of damage mechanics and fracture mechanics, Fathima and Kishen [183] proposed

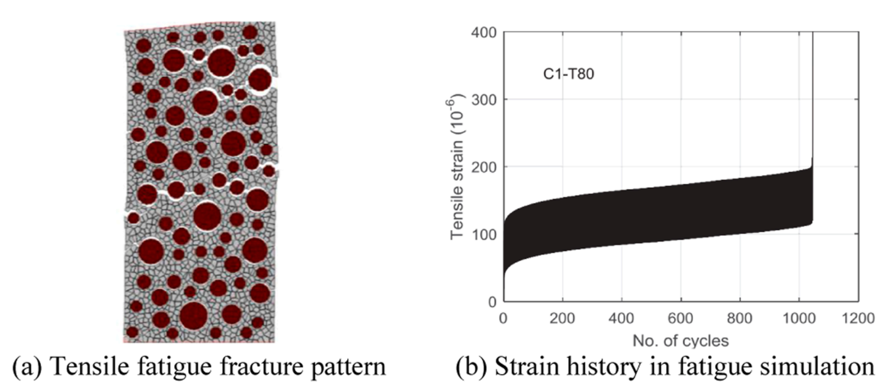

Fig. 20. The numerical results of mesoscopic model based on RBSM for tensile fatigue adapted from [172]: (a) an example of the tensile fatigue fracture pattern (red particles represent aggregate and grey for mortar) (b) the strain history in fatigue simulation under 80 % of tensile strength.

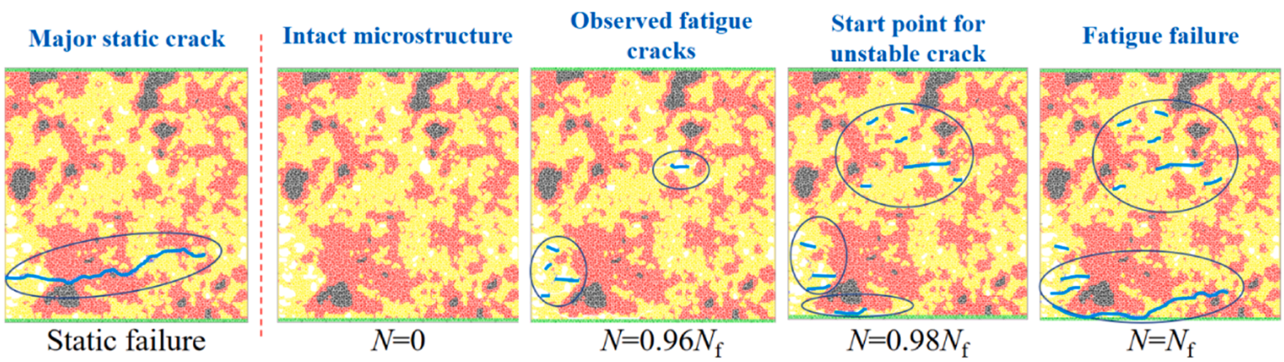

Fig. 21. Simulated fatigue fracture behaviours of cement pastes (w/c 0.4) under the the stress level of 50 %, adapted from [173].

a damage model for the case of constant amplitude cyclic loading. In their work, the damage evolution law was proposed through a closed form expression for dual dissipation potential, which was derived using the principles of thermodynamics and dimensional analysis. Progressive stiffness reduction due to the fatigue loading can be captured by the model. The same authors further extended the analytical model to consider the variable amplitude cyclic loading in [110]. In the work of Liang et al. [186], the fatigue behaviour of large reinforced concrete structures was simulated by implementing the constitutive damage model into the finite element analysis. The damage distribution and the potential fatigue damage failure pattern of concrete structures can be reasonably predicted by the model.

In addition to the macroscopic models, a few mesoscopic fatigue damage models have been proposed recently [187–189]. Grassl and Rempling [187] developed a damage-plasticity interface constitutive model to describe the irreversible strain under the cyclic loading. This calibrated interface model was then inserted into the mesoscale simulation of concrete by considering an idealized heterogeneous mesostructure. It has been demonstrated that the typical hysteresis loops for cyclic loading and the macroscopic damage evolution can be modeled as a structural result of localized irreversible strains on the constitutive level. A different strategy to consider the fatigue damage evolution was proposed by Sun and Xu [188]. In their work, a fatigue damage variable was established by homogenizing the collective effect of microscale cracks, while crack growth was determined based on the Paris' law. The mesoscopic heterogeneity of concrete was also introduced by introducing probability distributions to material properties. The effectiveness of the model was verified by comparing calculated results with experimental results.

Even though the macroscopic and mesoscopic damage phenomena of concrete can be predicted quite reliably with damage mechanics, the underlying physical origin of fatigue at the microscale is still not clear. Moreover, only very limited information regarding the damage evolution of basic constituents and their effects on the overall fatigue behaviour of concrete can be found in available literature.

4.5. Statistical models

One of the main challenges in fatigue prediction of cementitious material is the significant scatter in the fatigue life, which can differ by several orders of magnitude. Understanding the source of scatter is essential for ensuring the reliable prediction of fatigue performance of concrete structures. The scatter in fatigue results can stem from a variety of sources, such as the complex material inhomogeneity, specimen geometry, environmental factors, experimental procedures and operator variability [190]. The challenge of addressing and interpreting the wide scatter in fatigue test results has led to the adoption of empirical approaches based on *S-N* curves, as presented in design codes such as Eurocode 2 [191] and Model Code 2010 [130]. However, these deterministic methods overlook the probabilistic nature of concrete fatigue. To address this, statistical models are considered more appropriate for predicting fatigue behaviour. There is a general consensus among researchers that fatigue results predominantly follow a Weibull distribution, typically with two or three parameters [122,192–195]. Castillo and Fernández-Canteli [196] further demonstrated that, based on the weakest-link hypothesis, compatibility conditions, and asymptotic properties, only the Weibull and Gumbel distributions are fundamentally suited for modelling fatigue behaviour. Saucedo et al. [195] applied a three-parameter Weibull cumulative distribution function to account for the statistical distribution of static compressive strength, which served as the basis for the subsequent fatigue probabilistic model. They further incorporated the effects of loading frequency and stress ratio into the model, which was validated through fatigue tests on cubic specimens under varying stress ratios and frequencies. The results demonstrated that the proposed model could accurately predict the fatigue life for both plain and fibre-reinforced concretes across different loading conditions. The Weibull distribution is considered the most suitable for fatigue and has also been adopted in [192–194] to model the flexural fatigue life of concrete at various stress levels.

Based on the stochastic damage mechanics, Li and Guo [197] proposed a unified damage model to predict the behaviour of concrete under both monotonic and cyclic loading. The micro-meso stochastic fracture model (MMSF), which accounts for the nonlinear mechanical behaviour and randomness of concrete [198], is considered as an energy dissipation element. In this model, the Representative Volume Element (RVE) is conceptualized as an assembly of parallel micro-springs, constrained by rigid plates at both ends. The MMSF model assumes that fracture strain is a random field, consistent with the theory of random media [199]. The stochastic evolution of damage at the macroscale emerges from the random fracture of these micro-springs. To accurately describe the multiscale energy dissipation, the model employs rate process theory alongside a crack hierarchy framework. Fig. 22 illustrates the energy dissipation process of micro-springs within a typical tensile sample. Under monotonic loading, the energy dissipation of micro-springs with varying fracture strains starts gradually and then rises sharply, as shown in Fig. 22 (a). In contrast, fatigue loading results in a ladder-like increase in energy dissipation across cycles, as depicted in Fig. 22 (b). Micro-springs with higher fracture strains endure more cycles, likely due to their better resistance to damage accumulation. These observations highlight the critical role of energy dissipation in understanding the mechanical behaviour of concrete under different loading conditions. A similar model has been applied in [161], indicating that the variability in fatigue life can be attributed to the inherent randomness in the mesoscopic structure of the material. This further underscores the importance of accounting for this randomness in predictive models of concrete fatigue.

Among statistical methods, Bayesian inference is regarded an appropriate tool for fatigue analysis [200]. It applies Bayes' theorem to update the probability of a hypothesis as new evidence or information becomes available. This approach relies on prior knowledge,

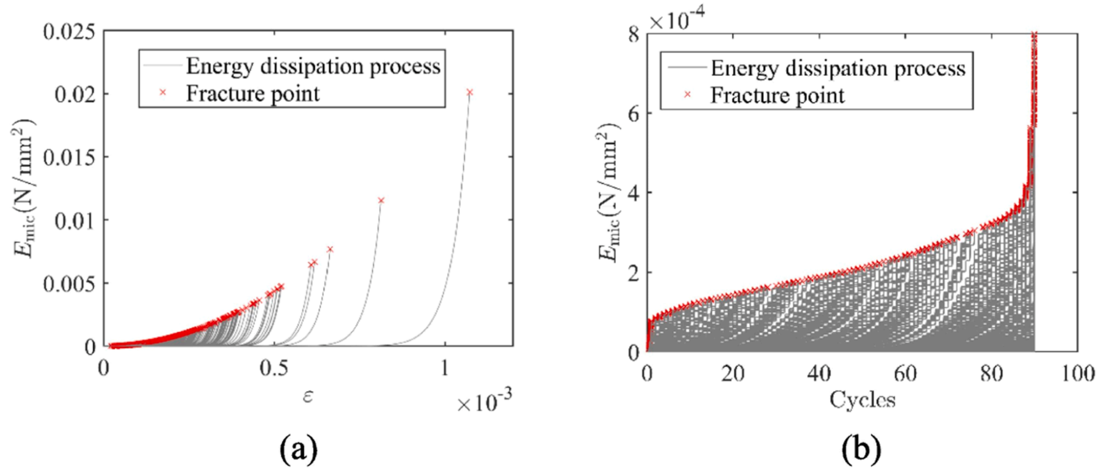

Fig. 22. The simulated energy dissipation processes of tensile micro-springs (a) Monotonic loading (b) Fatigue loading. Adapted from [197].

represented by a prior distribution, to estimate posterior probabilities as additional data is incorporated. Ge and Kim [201] developed a predictive model for the service life of reinforced concrete (RC) structures exposed to chloride attack and fatigue. They improved the model's precision by updating its parameters through Bayesian inference and employing the Markov-chain Monte-Carlo (MCMC) sampling method. This approach significantly reduced errors and uncertainties in the predictions. Similarly, Wu et al. [202] introduced a probabilistic model for predicting the fatigue life of RC beams in chloride environments. By incorporating Bayesian inference to account for statistical uncertainty and model parameter variability, their model accurately predicts fatigue life using a probabilistic *S-N* curve. The results show that increasing the number of update points enhances the alignment between predictions and experimental data, while also decreasing statistical uncertainty.

4.6. Machine learning models

Numerical investigations into fatigue damage accumulation in concrete often require significant computational costs, owing to the necessity of considering various loading scenarios and complex multiscale heterogeneous material structures. To address this issue, several researchers have turned to machine learning (ML) based methods. Zhang et al. [203] employed ML techniques to enhance the understanding and prediction of this complex issue. Their approach encompasses the utilization of various learning algorithms, i.e., the random forest (RF), support vector machine (SVM), and artificial neural networks (ANNs) models. The study extensively compiles experimental data from existing literature, utilizing this information to train the machine learning models specifically for estimating the fatigue life. After training process, it appeared that the random forest machine learning model exhibited superior performance in delineating the complex relationships among the experimental variables. When it comes to the prediction of fatigue life of concrete, the random forest model also demonstrated significantly higher accuracy when compared with conventional theoretical models. Son and Yang [204] also used ML models (e.g., random forest, neural network, gradient boosting, and AdaBoost models) to predict the fatigue life of plain concrete under uniaxial compression. It was found that excluding the sustained strength of the concrete variable, initially considered as the seventh input variable in Zhang et al.'s study [203], led to enhancements in both the Mean squared error (MSE) and determination coefficient R^2 values. Furthermore, the gradient boosting model yielded a minimum error result with a high R^2 value of 0.753, indicating a high level of accuracy in outcome prediction. Lu et al. [205] used SVM, Gaussian process (GP), neural network (NN), and RF to evaluate the fatigue damage of slender coastal bridges. Parametric probabilistic models for vehicles are developed using extensive long-term field measurements, incorporating stochastic loadings from wind and waves. These models are parameterized to accommodate a range of loading scenarios, providing comprehensive input parameters. ML models output equivalent fatigue damage accumulation considering the coupled vehicle-bridge-wind-wave (VBWW) system and stress analysis via multiscale FEA. Various training strategies yield fatigue life for critical details under changing coastal conditions. Testing reveals GP algorithm's superior performance over others, though all show reasonable predictive capability.

Due to the nonlinear influence of interconnected factors on fatigue, there is still no consensus regarding the quantitative measurement of these factors. Therefore, Chen et al. [206] employed the Bayesian regularized backpropagation neural network (BR-BPNN) to predict concrete behaviour subjected to tensile fatigue. The Bayesian BR-BPNN possesses significant advantages due to its unique self-learning capability and strong generalization capacity. The model effectively predicted logarithmic fatigue life, demonstrating good agreement with test results and conventional data fitting curves. This reliability underscores its efficacy in forecasting concrete tensile fatigue behaviour.

Fathalla et al. [207] using a multi-scale simulation along with the pseudo-cracking method to estimate the remaining fatigue life of real reinforced concrete bridge decks. Their estimation is based on the surface crack patterns observed during site inspections, wherein crack location, orientation, and width are identified as primary factors influencing remaining life. The proposed methodology is outlined in Fig. 23. To facilitate quick diagnosis of remaining fatigue life on-site, an artificial neural network (ANN) is constructed to correlate fatigue life with observed cracks. This ANN is built using extensive data encompassing various crack patterns and widths, including artificially generated patterns associated with the firm coupling of shear and flexure. This comprehensive approach ensures robustness and reliability in the ANN model, capable of handling indeterminate crack events that may arise in the future.

Although machine learning techniques can provide highly accurate predictions, the complexity of the trained models often causes them to behave like black boxes, without adequate explanation. This can be a potential obstacle for practical applications in decision-making and troubleshooting. Most ML models may not inherently focus on the underlying mechanisms, they can uncover complex relationships and patterns in large datasets that might be difficult to detect otherwise. These insights can then guide further investigation into the causal factors, providing a valuable starting point for more in-depth analysis. Moreover, there are some advanced interpretable ML models available in the references [208–210], even though most of them are currently not specifically focus on the fatigue of cementitious material. Yang et al. [211] applied interpretable machine learning model to predict and

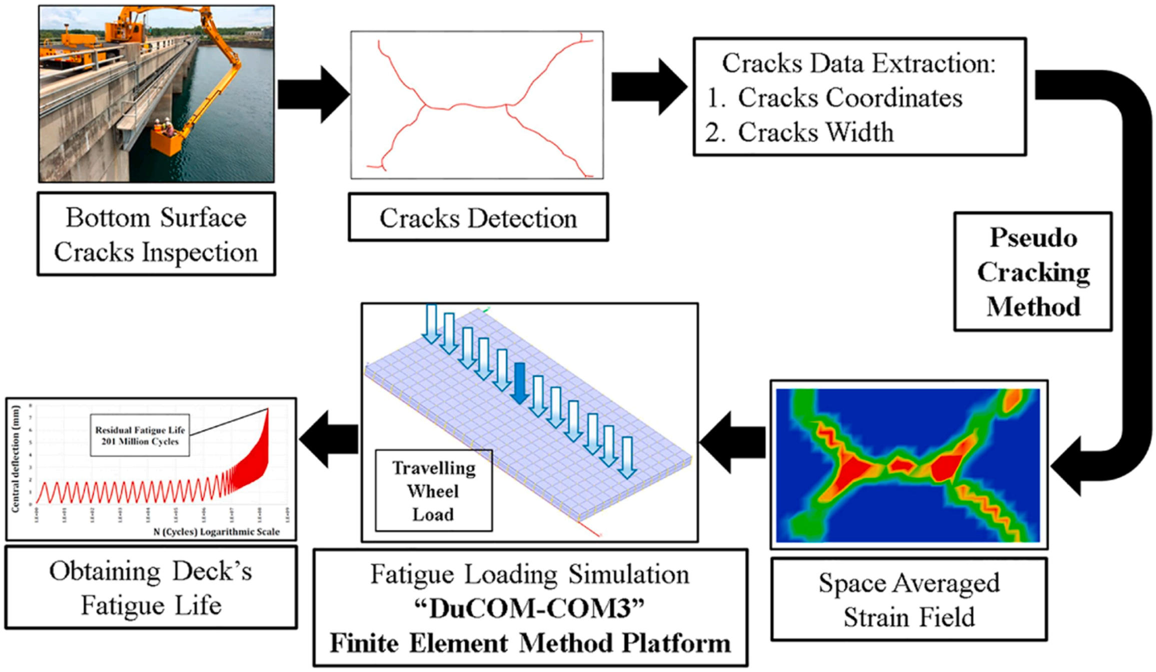

Fig. 23. Overview of remaining fatigue life prediction methodology, adapted from [207].

formulate the low-cycle fatigue design for reinforced high-strength concrete (RHSC). Sensitivity analysis is initially employed to methodically examine the effects of changes in input parameters on system outcomes. Utilizing K-fold cross-validation ensures the importance of data randomization in machine learning, leading to consistent and unbiased model evaluations. Their findings highlight that, when properly utilized, regression analysis can significantly improve the durability and lifespan of RHSC structures subjected to dynamic loadings. Frie et al. [209] applied SHapley Additive exPlanations (SHAP) for enhancing the interpretability of data-driven fatigue strength predictions. Their findings reveal that SHAP not only provides a robust feature sensitivity analysis but also aligns with established physical principles from materials processing and fatigue theory. This is particularly noteworthy given the complexity of high-dimensional, cross-correlated fatigue feature spaces and the inherent data heterogeneity, which includes variations in materials, component designs and loading conditions. These interpretable ML models are designed to provide insights into the reasoning behind their predictions, making the decision-making process more transparent and understandable. By offering explanations for how certain predictions are made, these models help researchers gain a better understanding of the factors driving the observed outcomes. It is believed that their methodologies can be adapted and extended to the research on fatigue of cementitious materials. In addition, ML techniques that combined with physics-based models can enhance predictive accuracy and efficiency [212–215]. A physics-based ML model combines data, partial differential equations (PDEs), and mathematical frameworks and it is trained to perform supervised learning tasks while adhering to physical laws expressed through general nonlinear equations. By integrating these elements, physics-based ML models ensure that predictions remain consistent with established physical principles, enhancing their robustness and reliability in various applications. By combining a fracture mechanics based model and adaptive ML algorithm, Wang et al. [216] proposed physics-guided machine learning frameworks to predict the fatigue life of additively manufactured (AM) metals. These hybrid ML models leverage the detailed morphological characteristics of critical defects, offering a statistical framework that outperforms purely data-driven approaches. The proposed integration frameworks deliver acceptable prediction accuracy for all 3 AM materials studied.

4.7. Multiscale modelling scheme

To efficiently cope with the multiscale nature of concrete, multiscale modelling schemes are often applied [217]. The aim of a multiscale model is to establish a relation between macroscopically observed phenomena and their underlying origins at lower scales. When addressing the multiscale heterogeneity of concrete, it is advantageous to incorporate material structures and local properties of fundamental components at various scales into the model to enhance prediction accuracy [218]. In recent decades, great progress has been made in multiscale modelling strategies of static fracture of concrete and examples of such models can be found in literature [217,219–221]. However, comparatively little attention has been given to simulating the fatigue fracture behaviour of concrete in a multiscale manner. This can be attributed to two primary reasons. First, fatigue cracking behaviour is inherently more complex than static fracture. Understanding the static mechanical properties of concrete is fundamental before analysing the more intricate aspects of fatigue fracture [169]. Consequently, efforts initially focus on developing mature and reliable multiscale modelling schemes for static fracture analysis, with the intention of extending them to simulate fatigue behaviours [173,219]. Another reason is that the knowledge regarding the fatigue behaviour of cementitious materials at nano- and microscales remains limited. As discussed earlier, there are technical challenges associated with experimentally characterizing fatigue damage evolution at these lower scales. Thus far, most fatigue models concentrate solely on macroscopic or mesoscopic fatigue damage evolution, as these are easier to measure and compare. Nevertheless, advancing techniques and adopting appropriate multiscale modelling schemes are necessary to predict the macroscopic fatigue properties of concrete accurately.

Several efforts in the multiscale analysis of concrete fatigue have been made to enhance our understanding of damage progression. Sun et al. [222] developed a FEM based multiscale framework, integrating interactive mesoscopic and macroscopic models to analyse fatigue damage evolution in concrete specimens. The stochastic characteristics of concrete mesoscopic structure was considered in the model by assigning random aggregate shape and location, which result in different crack initiation and the fracture mode. Simulation results reveal that fatigue cracks predominantly initiate near the ITZ before progressing into the cement mortar matrix, forming finite-sized cracks. Fig. 24 shows the simulated damage distribution in concrete near 90 % of the fatigue life under different stress levels. By considering both microscopic and mesoscopic features, Gan et al. [223] developed two scales models to investigate the fatigue fracture of cementitious materials. The effect of microstructural features and microscopic aggregate roughness on the fracture behaviour of ITZ was investigated. Moreover, the effect of ITZ properties and stress level on the fatigue damage evolution of mortar was also studied. It was found that the increase of the ITZ strength can significantly enhance the fatigue properties of mortar by modifying the evolution of damage and fracture processes, ultimately extending its fatigue life. In general, a weak ITZ induces localised damage during fatigue loading, whereas a strong ITZ promotes a more homogenized mortar, resulting in a more evenly distributed damage pattern.

Simon and Kishen [160] developed an analytical model for predicting the entire crack growth curve in plain concrete, drawing on principles of dimensional analysis and self-similarity, analogous to human population growth dynamics. This model is based on a multiscale approach, where the authors derived a linearized stress intensity factor (SIF) for concrete, incorporating fracture process zone mechanisms such as aggregate bridging and microcracking. The derived SIF incorporates both the bridging zone and the microcrack at the macrocrack tip, providing a more robust framework for accurately capturing the fracture behaviour of concrete.

To account for the stochastic nature of concrete fatigue, a two-scale stochastic fatigue damage model was developed by Wang and Li [224]. The model introduces a new micro-damage evolution law in the framework of the microscopic stochastic fracture model [225], incorporating a key material parameter termed "nano-to-micro comprehensive fracture energy" to capture the randomness and nonlinear behaviour of concrete. The evolution law of the micro damage is established by a multiscale energy analysis inspired by the work of [155, 157]. The proposed model successfully predicts the fatigue life, stiffness degradation, and the complete stress-strain response of concrete.

4.8. General remarks

Table 2 summarises assumptions, limitations and major conclusions of previously mentioned fatigue models. Various types of models have been developed to describe the macroscopic fatigue behaviour by formulating laws governing the propagation of local cracks. These include semi-empirical equations with theoretical underpinnings [155, 157] and purely phenomenological models [54,168,169]. These models offer both qualitative insights and quantitative predictions of concrete fatigue with certain reliability. In addition, several attempts have been made by employing ML models to predict the fatigue life of concrete. While some progress has been made, there are still unclear underlying mechanisms and interactions between variables that require further interpretation. The common issue of previous mentioned models is that they often overlook the complex microstructural mechanisms inherent in concrete, such as material properties and structures at different scales. Consequently, their applicability may be limited when different material compositions are employed. Moreover, extensive experimental evidence has demonstrated that numerous microscopic (or mesoscopic) features of cementitious materials can profoundly influence macroscopic fatigue behaviour. This necessitates a more sophisticated multiscale modelling approach to address the complex fracture and deformation behaviour of cementitious materials under cyclic loading conditions.

5. Synthesis of new ideas

From an in-depth review of the fatigue behaviour of cementitious materials, several new concepts and key aspects have come to the forefront:

Firstly, it is acknowledged that the origin of fatigue damage is suited at nano/microscale. The majority of research focuses on macroscopic indicators to describe global fatigue damage. There is limited knowledge about how fatigue damage accumulates at lower scales. Advanced characterization techniques and innovative testing methods, such as TEM, nanoindentation and XCT techniques, are needed to characterize and understand fatigue properties at these levels, which can provide better insights into the fatigue behaviour of concrete and can also be integrated into multiscale models for better accuracy. Especially recommended are techniques that enable real-time monitoring of damage evolution and provide detailed insights on how different loading conditions affect fatigue performance. Moreover, microscopic features of cementitious materials significantly influence the macroscopic fatigue behaviour of concrete. Establishing quantitative relationships between these microscopic features and the overall fatigue performance is essential. This includes understanding the impact of the ITZ properties, aggregate characteristics and other microstructural features on the fatigue behaviour of concrete. Moreover, the mechanisms underlying the

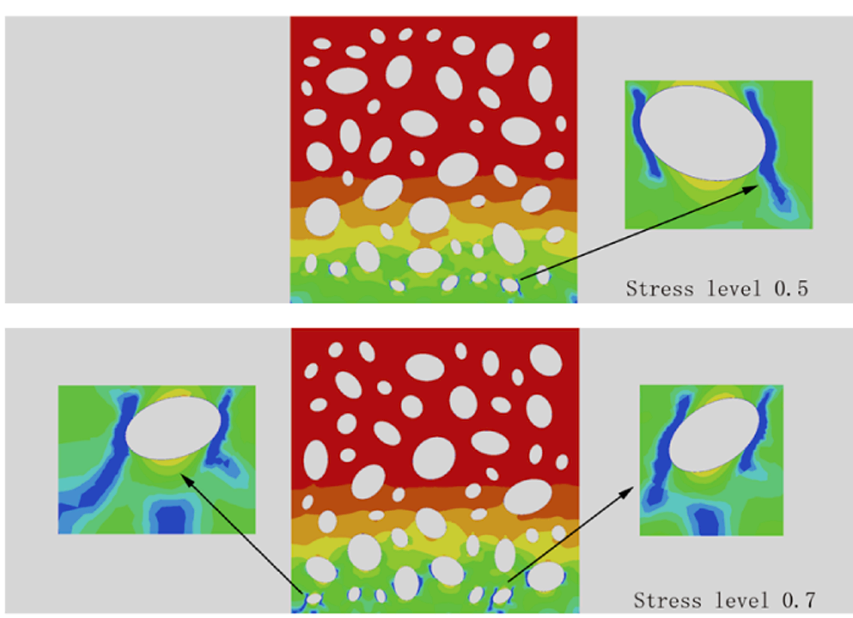

Fig. 24. The simulated damage distribution in concrete near 90 % of the fatigue life under different stress levels [222].

Table 2
Summary of different fatigue models.

Ref	Fatigue Model	Assumptions	Limitations	Major Implications and Achievements
[2,75, 145–147,150, 151,153]	Wöhler Curve	Purely phenomenological method without considering the underlying physical mechanisms.	Limited generality due to empirical nature and requiring extensive testing results.	Establishment of empirical law between fatigue life and stress level with a certain probability of failure. Widely adopted in design codes.
[75,77,120, 149–157]	Paris' Law	Linear elastic fracture mechanics-based relationship between the fatigue crack growth rate and stress intensity factor range.	Inappropriate to predict the non-linear FPZ behaviour.	Size-adjusted parameter for predicting size effect of concrete fatigue. Description of fracture between nanoscale C-S-H particles and linking macrocrack fatigue kinetics to nanocrack growth rates.
[163–173]	Cohesive Zone Model	Non-linear fracture mechanics-based semi-empirical approach by assuming the FPZ as a fictitious crack that can transfer cohesive stresses.	Limited to describe only one main crack.	Description of the progressive fatigue fracture using cyclic softening stress-strain relationships. Consideration of microstructural features on the fracture behaviour.
[174–189]	Damage Mechanics Models	By formulating constitutive equations of damage evolution in the framework of thermodynamics.	Most phenomenological damage evolution laws derived macroscopically.	Theoretically robust analysis of global degradation of material properties and capable of considering localised damage evolution.
[122,161, 190–202]	Statistical Models	Consideration of stochastic nature of concrete fatigue by assuming different probability distributions.	Data dependency and may overlook the effect of microstructural features.	Variability in fatigue results originates from the inherent randomness of concrete. Accounting for statistical uncertainty and model parameter variability.
[203–216]	Machine Learning Models	Utilisation of learning algorithms to make data-driven predictions.	Massive data requirement and computational demand, "Black box" and limited physical insight.	Revealing complex relationships and improving robustness and prediction accuracy using interpretable ML and physics-based ML models.
[217–225]	Multiscale Models	Separation of scales and homogenisation with appropriate upscaling method.	High computational demand and lack of robust fatigued damage evolution law across scales.	Accounting for concrete's multiscale heterogeneity. Establishing quantitative links between macroscale fatigue performance and underlying origins at lower scales.

coupling between fatigue and creep in concrete are poorly understood. Investigating how these phenomena interact under cyclic loading conditions is crucial for developing more accurate predictive models.

Besides, employing a multiscale modelling approach to study the fatigue of cementitious materials is essential for a more comprehensive understanding of concrete's fatigue behaviour. This method considers interactions across various scales of material structure, from microscopic to macroscopic levels. Current models often struggle to link these complex microstructural mechanisms with macroscopic phenomena, which are crucial for accurately predicting fatigue life and damage evolution in concrete. Therefore, efforts should focus on developing and validating multiscale models that incorporate the intricate interactions between different scales of concrete structure. The integration of ML techniques offers a promising direction for predicting the fatigue life of concrete. ML models can effectively capture complex relationships between various material compositions and their overall fatigue performance. However, enhancing the reliability and practical applicability of ML models requires improved interpretation of the underlying mechanisms and interactions among different variables within these models.

There is limited understanding of how fatigue interacts with various durability issues in concrete, such as freeze-thaw cycles, sulfate attack and alkali-silica reaction. Investigating these interactions under cyclic loading conditions is crucial for developing more accurate predictive models. Despite extensive research on the mechanisms of concrete fatigue, there remains a gap in knowledge about these coupled interactions and the principles governing fatigue damage accumulation and evolution. Advancing the field requires focused investigation into these coupling mechanisms and the development of more robust models for fatigue damage evolution.

6. Conclusions

Based on the presented state of the art on fatigue of concrete, the following conclusions can be drawn:

(1) The macroscopic fatigue performance of cementitious materials in terms of fatigue life, fracture behaviour and damage evolution, is fundamentally influenced by its internal microstructural features, especially ITZ properties, aggregate characteristics and fibre additions, as well as external factors like specimen size, loading modes and stress amplitude. To fully understand the complex fatigue phenomenon, more efforts are needed to explore the origins and propagation mechanisms of fatigue cracks at lower scales.
(2) The existing literature primarily focuses on developing macroscopic indicators to describe global fatigue damage evolution in concrete. Non-destructive methods, such as in-situ high-resolution XCT, offer the ability to directly visualise microcrack propagation and distribution, providing deeper insights into fatigue damage accumulation. While indirect indicators can give a global assessment of fatigue damage, caution is needed in interpreting these results, especially when self-healing and ongoing hydration processes occur simultaneously. Additionally, more advanced techniques, such as TEM and nanoindentation, are required to characterise fatigue properties at smaller scales.
(3) Traditional fatigue models have distinct advantages and limitations, incorporating fracture mechanics, damage mechanics, and stochastic analysis. However, some of them may struggle to capture the link between complex microstructural effects and macroscopic behaviour, reducing their predictive accuracy and generality. Machine learning tools, particularly interpretable and physics-based models, may uncover complex relationships and significantly enhance prediction accuracy. In addition, multiscale modelling offers promising potential to address the heterogeneous nature of cementitious materials, yet this approach remains underexplored due to limited understanding of fatigue mechanisms and lack of robust upscaling methods of damage evolution. Further efforts are recommended to integrate physics-based models with data-driven tools, combining their strengths to improve the understanding and prediction of concrete fatigue behaviour.
(4) Understanding the interactions between fatigue and various durability issues in cementitious materials remains insufficient. Significant knowledge gaps persist regarding these coupled phenomena and the rules governing fatigue damage accumulation. Future studies should concentrate on investigating these coupling mechanisms and developing comprehensive models that precisely depict the complex evolution of fatigue damage in concrete exposed to various durability challenges.

These conclusions underscore the importance of integrating advanced characterisation techniques, multiscale modelling and machine learning tool to enhance the understanding and fatigue prediction of cementitious materials. Further research in these areas will contribute to more durable and reliable concrete structures.

CRediT authorship contribution statement

Yidong Gan: Writing – review & editing, Writing – original draft, Visualization, Supervision, Methodology, Funding acquisition, Data curation, Conceptualization. **Branko Šavija:** Writing – review & editing, Supervision, Funding acquisition, Conceptualization. **Erik Schlangen:** Writing – review & editing, Supervision, Conceptualization. **Boyuan Shi:** Writing – review & editing, Methodology. **Shen Yang:** Writing – review & editing, Data curation. **Yibing Zuo:** Writing – review & editing, Supervision, Investigation, Funding acquisition.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

The author is an Editorial Board Member/Editor-in-Chief/Associate Editor/Guest Editor for *Construction and Building Materials* and was not involved in the editorial review or the decision to publish this article.

Acknowledgements

Yidong Gan would like to acknowledge the funding supported by the National Natural Science Foundation of China (Grant No. 52408261). Yibing Zuo would like to gratefully acknowledge the Fundamental Research Funds for the Natural Science Foundation of Hubei Province [Grant No. 2022CFB048]. Branko Šavija acknowledges the financial support of the European Research Council (ERC) within the framework of the ERC Starting Grant Project "Auxetic Cementitious Composites by 3D printing (ACC-3D)", Grant Agreement Number 101041342".

Data Availability

Data will be made available on request.

References

[1] S. Suresh, Fatigue of materials, Cambridge university press, 1998.
[2] W. Schütz, A history of fatigue, Eng. Fract. Mech. 54 (1996) 263–300.
[3] J. Schijve, Fatigue of structures and materials in the 20th century and the state of the art, Int J. Fatigue 25 (2003) 679–702.
[4] A. Wöhler, Versuche zur Ermittlung der auf die Eisenbahnwagenachsen einwirkenden Kräfte und die Widerstandsfähigkeit der Wagen-Achsen, Z. F. üR. Bauwes. 10 (1860) 583–614.
[5] A. Considère, Influence des armatures métalliques sur les propriétés des mortiers et bétons, Gauthier-Villars, 1899.
[6] D. Joly, La resistance et l,"'elasticite des"'ciments Portland, Ann. Pons Chauss-.-. 16 (1898) 198–244.
[7] T.T.C. Hsu, Fatigue of plain concrete, J. Proc. (1981) 292–305.
[8] L. Gao, C.T.T. Hsu, Fatigue of concrete under uniaxial compression cyclic loading, Acids Mater. J. 95 (1998) 575–581.
[9] Y. Yang, G. Wu, Z.-S. Wu, Fatigue Performance of ballastless track slabs reinforced with BFRP and SFCB, J. Compos. Constr. 21 (2017) 04016069.
[10] Y. Yang, G. Zhang, G. Wu, D. Cao, Study on fatigue damage laws and life prediction of CRTS-II ballastless track slab, Eng. Struct. 252 (2022) 113659.
[11] J.-H. Fan, W.-D. Wang, Y.-L. Shi, L. Zheng, Low-cycle fatigue behaviour of concrete-filled double skin steel tubular (CFDST) members for wind turbine towers, Thin-Walled Struct. 205 (2024) 112384.
[12] F.W. Klaiber, D.Y. Lee, The effects of air content, water-cement ratio, and aggregate type on the flexural fatigue strength of plain concrete, American Concrete Institute, ACI Special Publication. SP-075, 1982, pp. 111–131.
[13] Y. Gan, H. Zhang, Y. Zhang, Y. Xu, E. Schlangen, K. van Breugel, B. Šavija, Experimental study of flexural fatigue behaviour of cement paste at the microscale, Int J. Fatigue 151 (2021).
[14] M.A. Vicente, D.C. González, J. Mínguez, M.A. Tarifa, G. Ruiz, R. Hindi, Influence of the pore morphology of high strength concrete on its fatigue life, Int J. Fatigue 112 (2018) 106–116.
[15] B. Zhang, Relationship between pore structure and mechanical properties of ordinary concrete under bending fatigue, Cem. Concr. Res 28 (1998) 699–711.
[16] E.W. Tse, D.Y. Lee, F.W. Klaiber, Fatigue behavior of concrete containing fly ash, Special Publication. SP-091, 1986, pp. 273–289.
[17] L.P. Guo, W. Sun, K.R. Zheng, H.J. Chen, B. Liu, Study on the flexural fatigue performance and fractal mechanism of concrete with high proportions of ground granulated blast-furnace slag, Cem. Concr. Res 37 (2007) 242–250.
[18] X. Chen, L. Xu, J. Bu, Dynamic tensile test of fly ash concrete under alternating tensile–compressive loading, Mater. Struct. /Mater. Et. Constr. 51 (2018) 20.
[19] S. Müller, V. Mechtcherine, Fatigue behaviour of strain-hardening cement-based composites (SHCC, Cem. Concr. Res 92 (2017) 75–83.
[20] J. Qiu, E. Yang, Micromechanics-based investigation of fatigue deterioration of engineered cementitious composite (ECC, Cem. Concr. Res 95 (2017) 65–74.
[21] S.J. Stephen, R. Gettu, Fatigue fracture of fibre reinforced concrete in flexure, Mater. Struct. /Mater. Et. Constr. 53 (2020) 1–11.
[22] C.D. Johnston, R.W. Zemp, Flexural fatigue performance of steel fiber reinforced concrete. Influence of fiber content, aspect ratio, and type, Acids Mater. J. 88 (1991) 374–383.
[23] C. Thomas, I. Sosa, J. Setién, J.A. Polanco, A.I. Cimentada, Evaluation of the fatigue behavior of recycled aggregate concrete, J. Clean. Prod. 65 (2014) 397–405.
[24] K.M.A. Sohel, K. Al-Jabri, M.H. Zhang, J.Y.R. Liew, Flexural fatigue behavior of ultra-lightweight cement composite and high strength lightweight aggregate concrete, Constr. Build. Mater. 173 (2018) 90–100.
[25] J. Ren, D. Li, Y. Xu, J. Huang, W. Liu, Fatigue behaviour of rock asphalt concrete considering moisture, high-temperature, and stress level, Int. J. Pavement Eng. 0 (2021) 1–11.
[26] J.W. Galloway, H.M. Harding, K.D. Raithby, Effects of moisture changes on flexural and fatigue strength of concrete, 1979.
[27] B.T. Huang, Q.H. Li, S.L. Xu, W. Liu, H.T. Wang, Fatigue deformation behavior and fiber failure mechanism of ultra-high toughness cementitious composites in compression, Mater. Des. 157 (2018) 457–468.
[28] T.T.C. Hsu, Fatigue and microcracking of concrete, Mat. ériaux Et. Constr. 17 (1984) 51–54.
[29] B. Zhang, D.V. Phillips, K. Wu, Effects of loading frequency and stress reversal on fatigue life of plain concrete, Mag. Concr. Res. 48 (1996) 361–375.
[30] X. Chen, J. Bu, X. Fan, J. Lu, L. Xu, Effect of loading frequency and stress level on low cycle fatigue behavior of plain concrete in direct tension, Constr. Build. Mater. 133 (2017) 367–375.
[31] R.B. Tait, Fatigue and fracture of cement mortar, (1984).
[32] Z. Ding, D.C. Feng, X. Ren, J. Wang, Physically based constitutive modeling of concrete fatigue and practical numerical method for cyclic loading simulation, Eng. Fail Anal. 101 (2019) 230–242.
[33] M.K. Lee, B.I.G. Barr, An overview of the fatigue behaviour of plain and fibre reinforced concrete, Cem. Concr. Compos 26 (2004) 299–305.
[34] J.-K. Kim, Y.-Y. Kim, Experimental study of the fatigue behavior of high strength concrete, Cem. Concr. Res 26 (1996) 1513–1523.
[35] S. Arora, S.P. Singh, Fatigue strength and failure probability of concrete made with RCA, Mag. Concr. Res. 69 (2017) 55–67.
[36] C.E. Kesler, Effect of speed of testing on flexural fatigue strength of plain concrete, Proc., Highw. Res. Board (1953) 251–258.
[37] T.L. Thomas, The effects of air content, water-cement ratio, and aggregate type on the flexural fatigue strength of plain concrete, Iowa State University, 1979.
[38] J.D. Antrim, A study of the mechanism of fatigue in cement paste and plain concrete, Purdue University, 1964.
[39] Y. Gan, H. Zhang, B. Šavija, E. Schlangen, K. van Breugel, Static and fatigue tests on cementitious cantilever beams using nanoindenter, Micro (Basel) 9 (2018).
[40] P. Lura, Autogenous deformation and internal curing of concrete, Delft University of Technology, PhD Thesis, 2003.
[41] K. van Breugel, T.A. van Beek, Ageing of old and modern concrete structures-Observations and research, Rev. ALCONPAT 7 (2017).
[42] N. Oneschkow, T. Timmermann, Influence of the composition of high-strength concrete and mortar on the compressive fatigue behaviour, Mater. Struct. /Mater. Et. Constr. 55 (2022).
[43] A.B. Harwalkar, S.S. Awanti, Fatigue Behavior of High Volume Fly Ash Concrete under Constant Amplitude and Compound Loading, Int. J. Civ. Eng. Technol. (IJCIET) 3 (2012) 404–414.
[44] S.H. Mai, F. Le-corre, G. Foret, B. Nedjar, A continuum damage modeling of quasi-static fatigue strength of plain concrete, Int J. Fatigue 37 (2012) 79–85.
[45] A.D. Morris, G.G. Garrett, A comparative study of the static and fatigue behaviour of plain and steel fibre reinforced mortar in compression and direct tension, Int. J. Cem. Compos. Lightweight Concr. 3 (1981).
[46] M. Aboubakar, Y. Sato, T. Ayano, K. Niitani, Experimental and numerical investigation of static and fatigue behavior of mortar with blast furnace slag sand as fine aggregates in air and water, Constr. Build. Mater. 143 (2017) 429–443.
[47] B.S. Saini, S.P. Singh, Flexural fatigue strength prediction of self compacting concrete made with recycled concrete aggregates and blended cements, Constr. Build. Mater. 264 (2020).
[48] J.-S. Mun, K.-H. Yang, S.-J. Kim, Tests on the Compressive Fatigue Performance of Various Concretes, J. Mater. Civ. Eng. 28 (2016).
[49] R.S. Ghosh, J. Timusk, Creep of fly ash concrete, J. Proc. (1981) 351–357.
[50] C. Shi, D. Roy, P. Krivenko, Alkali-activated cements and concretes, CRC press, 2003.
[51] V.S. Cândido, A.C.R.Da Silva, N.T. Simonassi, E.S. Lima, F.S.Da Luz, S. N. Monteiro, Mechanical and microstructural characterization of geopolymeric concrete subjected to fatigue, J. Mater. Res. Technol. 7 (2018) 566–570.

[52] S. Dong, Y. Wang, A. Ashour, B. Han, J. Ou, Uniaxial compressive fatigue behavior of ultra-high performance concrete reinforced with super-fine stainless wires, Int J. Fatigue 142 (2021) 105959.
[53] S.R. Kasu, S. Deb, N. Mitra, A.R. Muppireddy, S.R. Kusam, Influence of aggregate size on flexural fatigue response of concrete, Constr. Build. Mater. 229 (2019).
[54] H.W. Reinhardt, H.A.W. Cornelissen, D.A. Hordijk, Tensile tests and failure analysis of concrete, J. Struct. Eng. 112 (1986) 2462–2477.
[55] K.M. Simon, J.M.C. Kishen, Influence of aggregate bridging on the fatigue behavior of concrete, Int J. Fatigue 90 (2016) 200–209.
[56] T. Scheiden, N. Oneschkow, Influence of coarse aggregate type on the damage mechanism in high-strength concrete under compressive fatigue loading, Struct. Concr. 20 (2019) 1212–1219.
[57] S. Arora, S.P. Singh, Analysis of flexural fatigue failure of concrete made with 100% coarse recycled concrete aggregates, Constr. Build. Mater. 102 (2016) 782–791.
[58] J. Xiao, H. Li, Z. Yang, Fatigue behavior of recycled aggregate concrete under compression and bending cyclic loadings, Constr. Build. Mater. 38 (2013) 681–688.
[59] F. Liu, L.Y. Meng, G.F. Ning, L.J. Li, Fatigue performance of rubber-modified recycled aggregate concrete (RRAC) for pavement, Constr. Build. Mater. 95 (2015) 207–217.
[60] D. Gao, Z. Gu, C. Wei, C. Wu, Y. Pang, Effects of fiber clustering on fatigue behavior of steel fiber reinforced concrete beams, Constr. Build. Mater. 301 (2021).
[61] S.J. Choi, J.S. Mun, K.H. Yang, S.J. Kim, Compressive fatigue performance of fiber-reinforced lightweight concrete with high-volume supplementary cementitious materials, Cem. Concr. Compos 73 (2016) 89–97.
[62] P.B. Cachim, J.A. Figueiras, P.A.A. Pereira, Fatigue behavior of fiber-reinforced concrete in compression, Cem. Concr. Compos 24 (2002) 211–217.
[63] M. Grzybowski, C. Meyer, Damage accumulation in concrete with and without fiber reinforcement, Acids Mater. J. 90 (1993).
[64] I. Löfgren, Fibre-reinforced Concrete for Industrial Construction-a fracture mechanics approach to material testing and structural analysis, PhD Thesis, Chalmers University of Technology, 2005.
[65] B. Isojeh, M. El-Zeghayar, F.J. Vecchio, Fatigue behavior of steel fiber concrete in direct tension, J. Mater. Civ. Eng. 29 (2017) 04017130.
[66] S.P. Singh, S.K. Kaushik, Flexural fatigue analysis of steel fiber-reinforced concrete, Mater. J. 98 (2001) 306–312.
[67] W. Yin, T.T.C. Hsu, Fatigue behavior of steel fiber reinforced concrete in uniaxial and biaxial compression, Mater. J. 92 (1995) 71–81.
[68] E. Poveda, G. Ruiz, H. Cifuentes, R.C. Yu, X. Zhang, Influence of the fiber content on the compressive low-cycle fatigue behavior of self-compacting SFRC, Int J. Fatigue 101 (2017) 9–17.
[69] M.A. Vicente, G. Ruiz, D.C. Gonzalez, J. Minguez, M. Tarifa, X. Zhang, CT-Scan study of crack patterns of fiber-reinforced concrete loaded monotonically and under low-cycle fatigue, Int J. Fatigue 114 (2018) 138–147.
[70] L. Soufeiani, S.N. Raman, M.Z.Bin Jumaat, U.J. Alengaram, G. Ghadyani, P. Mendis, Influences of the volume fraction and shape of steel fibers on fiber-reinforced concrete subjected to dynamic loading – A review, Eng. Struct. 124 (2016) 405–417.
[71] B. Li, L. Xu, Y. Shi, Y. Chi, Q. Liu, C. Li, Effects of fiber type, volume fraction and aspect ratio on the flexural and acoustic emission behaviors of steel fiber reinforced concrete, Constr. Build. Mater. 181 (2018) 474–486.
[72] T. Cui, B. Ning, X. Shi, J. Li, Flexural fatigue behavior of hybrid steel-polypropylene fiber reinforced high-strength lightweight aggregate concrete, Constr. Build. Mater. 377 (2023).
[73] N. Saoudi, B. Bezzazi, Flexural fatigue failure of concrete reinforced with smooth and mixing hooked-end steel fibers, Cogent Eng. 6 (2019).
[74] Q. Li, B. Huang, S. Xu, B. Zhou, R.C. Yu, Compressive fatigue damage and failure mechanism of fiber reinforced cementitious material with high ductility, Cem. Concr. Res 90 (2016) 174–183.
[75] K. Kirane, Z.P. Bažant, Size effect in Paris law and fatigue lifetimes for quasibrittle materials: modified theory, experiments and micro-modeling, Int J. Fatigue 83 (2016) 209–220.
[76] Z.P. Bazant, W.F. Schell, Fatigue fracture of high-strength concrete and size effect, Acids Mater. J. 90 (1993) 472.
[77] Z.P. Bazant, K. Xu, Size effect in fatigue fracture of concrete, Acids Mater. J. 88 (1991) 390–399.
[78] J. Zhang, V.C. Li, H. Stang, Size Effect on Fatigue in Bending of Concrete, J. Mater. Civ. Eng. 13 (2001) 446–453.
[79] A. Carpinteri, A. Spagnoli, S. Vantadori, A multifractal analysis of fatigue crack growth and its application to concrete, Eng. Fract. Mech. 77 (2010) 974–984.
[80] A. Carpinteri, A. Spagnoli, S. Vantadori, Size effect in S – N curves: a fractal approach to finite-life fatigue strength, Int J. Fatigue 31 (2009) 927–933.
[81] L.P. Guo, W. Sun, A. Carpinteri, Investigation on fatigue damage mechanism of modern concrete microstructure by lattice model, 1st Int. Conf. Microstruct. Relat. Durab. Cem. Compos. (2008) 1179–1188.
[82] J.J. Ortega, G. Ruiz, E. Poveda, D.C. González, M. Tarifa, X.X. Zhang, R.C. Yu, M.Á. Vicente, Á. de la Rosa, L. Garijo, Size effect on the compressive fatigue of fibre-reinforced concrete, Constr. Build. Mater. 322 (2022) 126238.
[83] Á. Mena-Alonso, D.C. González, J. Mínguez, M.A. Vicente, Size effect on the flexural fatigue behavior of high-strength plain and fiber-reinforced concrete, Constr. Build. Mater. 411 (2024) 134424.
[84] S. Viswanath, J.M. LaFave, D.A. Kuchma, Concrete compressive strain behavior and magnitudes under uniaxial fatigue loading, Constr. Build. Mater. 296 (2021) 123718.
[85] A. Medeiros, X. Zhang, G. Ruiz, R.C. Yu, M.D.S.L. Velasco, Effect of the loading frequency on the compressive fatigue behavior of plain and fiber reinforced concrete, Int J. Fatigue 70 (2015) 342–350.
[86] P. Lü, Q. Li, Y. Song, Behavior of concrete under nonproportional biaxial fatigue stresses with one constant, Acids Mater. J. 104 (2007) 3.
[87] E.C.M. Su, T.T.C. Hsu, Biaxial Compression Fatigue and the Discontinuity of Concrete, Mater. J. 85 (1988) 178–188.
[88] A.L.F. Taliercio, E. Gobbi, Experimental investigation on the triaxial fatigue behaviour of plain concrete, Mag. Concr. Res. 48 (1996) 157–172.
[89] X. Shen, E. Brühwiler, Biaxial flexural fatigue behavior of strain-hardening UHPFRC thin slab elements, Int J. Fatigue 138 (2020) 105727.
[90] S. Gao, J. Zhang, H. Zhang, Y. Zhu, Study on biaxial bending fatigue performance of Engineered Cementitious Composites (ECC), J. Build. Eng. 84 (2024) 108478.
[91] J. Kim, C. Yi, S.-J. Lee, G. Zi, Flexural fatigue behaviour of concrete under uniaxial and biaxial stress, Mag. Concr. Res. 65 (2013) 757–764.
[92] H.A.W. Cornelissen, Fatigue failure of concrete in tension, HERON 29 (4) (1984) 1984.
[93] K.V. Subramaniam, J.S. Popovics, S.P. Shah, Fatigue response of concrete subjected to biaxial stresses in the compression-tension region, ACI Structural Journal 96 (6) (1999) 663–669.
[94] M. Jia, Z. Wu, R.C. Yu, X. Zhang, Experimental investigation of mixed mode I–II fatigue crack propagation in concrete using a digital image correlation method, Eng. Fract. Mech. 272 (2022) 108712.
[95] M. Jia, Z. Wu, X. Jiang, R.C. Yu, X. Zhang, Y. Wang, Modified Paris law for mode I fatigue fracture of concrete based on crack propagation resistance, Theor. Appl. Fract. Mech. 131 (2024) 104383.
[96] M. Jia, Z. Wu, H. Wang, R.C. Yu, X. Zhang, Analytical method for predicting mode I crack propagation process of concrete under low-cycle fatigue loading, Eng. Fract. Mech. 286 (2023) 109320.
[97] O. Graf, E. Brenner, Versuche zur Ermittlung der Widerstandsfähigkeit von Beton gegen oftmals wiederholte Druckbelastung, Mitt. Der Dtsch. Mater. üfungsanstalten (1940) 416.
[98] J.W. Murdock, A critical review of research on fatigue of plain concrete, University of Illinois Engineering Experiment Station Bulletin, 1965, p. 25.
[99] P.R. Sparks, J.B. Menzies, The effect of rate of loading upon the static and fatigue strengths of plain concrete in compression, Mag. Concr. Res. 25 (1973) 73–80.
[100] Y. Gan, C.R. Rodriguez, E. Schlangen, K. van Breugel, B. Šavija, Assessing strain rate sensitivity of cement paste at the micro-scale through micro-cantilever testing, Cem. Concr. Compos 121 (2021).
[101] K.D. Raithby, J.W. Galloway, Effects of moisture condition, age, and rate of loading on fatigue of plain concrete, Special Publication. SP-041, 1974, pp. 15–34.
[102] H.K. Hilsdorf, C.E. Kesler, Behavior of concrete in flexure under varying repeated loads, TAM R 172, 1960.
[103] E.W. Bennett, S.E.S.J. Muir, Some fatigue tests of high-strength concrete in axial compression, Mag. Concr. Res. 19 (1967) 113–117.
[104] R.B. Tait, Fatigue and fracture of cement mortar, PhD Thesis, University of Cape Town, 1984.
[105] C. Desmettre, J.P. Charron, Water permeability of reinforced concrete subjected to cyclic tensile loading, Acids Mater. J. 110 (2013) 79–88.
[106] L. Garijo, Á. De La Rosa, G. Ruiz, J.J. Ortega, Autogenous self-healing induced by compressive fatigue in carbonated hydraulic lime mortars, J. Build. Eng. 80 (2023) 108035.
[107] H.A.W. Cornelissen, H.W. Reinhardt, Uniaxial tensile fatigue failure of concrete under constant-amplitude and programme loading, Mag. Concr. Res. 36 (1984) 216–226.
[108] B.H. Oh, Cumulative Damage Theory of Concrete under Variable-Amplitude Fatigue Loadings, Acids Mater. J. 88 (1991) 41–48.
[109] Y. Du, J. Wei, J. Yuan, Y. Lai, D. Sun, Experimental research on fatigue behavior of prestressed concrete beams under constant-amplitude and variable-amplitude fatigue loading, Constr. Build. Mater. 259 (2020) 119852.
[110] K. Keerthana, J.M. Chandra Kishen, An experimental and analytical study on fatigue damage in concrete under variable amplitude loading, Int J. Fatigue 111 (2018) 278–288.
[111] A. Baktheer, H. Becks, Fracture mechanics based interpretation of the load sequence effect in the flexural fatigue behavior of concrete using digital image correlation, Constr. Build. Mater. 307 (2021).
[112] A.J.M. Slemes, Miner's rule with respect to plain concrete variable amplitude tests, Special Publication. SP-075, 1982, pp. 343–372.
[113] M.A. Miner, Cumulative damage in fatigue, (1945).
[114] J.O. Holmen, Fatigue of concrete by constant and variable amplitude loading, 75, ACI Special Publication, 1982, pp. 71–110.
[115] M.R. Mohamadi, J.A. Mohandesi, M. Homayonifar, Fatigue behavior of polypropylene fiber reinforced concrete under constant and variable amplitude loading, J. Compos Mater. 47 (2013) 3331–3342.
[116] I.M. Vega, M.A. Bhatti, W.A. Nixon, A non-linear fatigue damage model for concrete in tension, Int. J. Damage Mech. 4 (1995) 362–379.
[117] M. Saito, Characteristics of microcracking in concrete under static and repeated tensile loading, Cem. Concr. Res 17 (1987) 211–218.
[118] Surendra P. Shah, Sushil Chandra, Fracture of Concrete Subjected To Cyclic and Sustained Loading, J. Am. Concr. Inst. 67 (1970) 816–827.
[119] A. Toumi, A. Bascoul, A. Turatsinze, Crack propagation in concrete subjected to flexural-cyclic loading, Mater. Struct. /Matia Et. Constr. 31 (1998) 451–458.
[120] Z.P. Bazant, M.H. Hubler, Theory of cyclic creep of concrete based on Paris law for fatigue growth of subcritical microcracks, J. Mech. Phys. Solids 63 (2014) 187–200.

[121] D.C. Gonzalez, A. Mena, G. Ruiz, J.J. Ortega, E. Poveda, J. Minguez, R. Yu, Á. De La Rosa, M.A. Vicente, Size effect of steel fiber–reinforced concrete cylinders under compressive fatigue loading: Influence of the mesostructure, Int J. Fatigue 167 (2023) 107353.

[122] L. Garijo, J.J. Ortega, G. Ruiz, Á. De La Rosa, X.X. Zhang, Effect of loading frequency on the fatigue life in compression of natural hydraulic lime mortars, Theor. Appl. Fract. Mech. 118 (2022) 103201.

[123] M. Thiele, Experimentelle Untersuchung und Analyse der Schädigungsevolution in Beton unter hochzyklischen Ermüdungsbeanspruchungen, Bundesanstalt für Materialforschung und-prüfung (BAM), 2016.

[124] N. Oneschkow, T. Scheiden, M. Hüpgen, C. Rozanski, M. Haist, Fatigue-induced damage in high-strength concrete microstructure, Materials 14 (2021).

[125] G. Schaan, S. Rybczynski, F. Schmidt-Döhl, M. Ritter, Fatigue of concrete examined on the nanoscale, TEM Studies of Fatigue-Induced Changes in the Cement Paste of UHPC, Imag. Microsc. 2 (2020) 25–27.

[126] S. Rybczynski, G. Schaan, M. Dosta, M. Ritter, F. Schmidt-Döhl, Discrete element modeling and electron microscopy investigation of fatigue-induced microstructural changes in ultra-high-performance concrete, Materials 14 (2021).

[127] S. Brisard, M. Serdar, P.J.M. Monteiro, Multiscale X-ray tomography of cementitious materials: A review, Cem. Concr. Res 128 (2020).

[128] Z. Yang, W. Ren, R. Sharma, S. McDonald, M. Mostafavi, Y. Vertyagina, T. J. Marrow, In-situ X-ray computed tomography characterisation of 3D fracture evolution and image-based numerical homogenisation of concrete, Cem. Concr. Compos 75 (2017) 74–83.

[129] R. Sharma, W. Ren, S.A. Mcdonald, Z.J. Yang, Micro-mechanisms of concrete failure under cyclic compression: X-ray tomographic in-situ observations, : Proc. 9th Int. Conf. Fract. Mech. Concr. Concr. Struct. (Fram. -9), Berkeley, CA, USA (2016).

[130] FIB-Fédération Internationale du Béton, FIB Model Code 2010. Final Draft, vol. 1–2., Lausanne, Switzerland: FIB Bulletin. (2012) 65–66.

[131] Y. Obara, I. Tanikura, J. Jung, R. Shintani, S. Watanabe, Evaluation of micro-damage of concrete specimens under cyclic uniaxial loading by X-ray CT method, J. Adv. Concr. Technol. 14 (2016) 433–443.

[132] Z. Fan, Y. Sun, Detecting and evaluation of fatigue damage in concrete with industrial computed tomography technology, Constr. Build. Mater. 223 (2019) 794–805.

[133] S.V. Kolluru, E.F. O'Neil, J.S. Popovics, S.P. Shah, Crack propagation in flexural fatigue of concrete, J. Eng. Mech. 126 (2000) 891–898.

[134] N. Oneschkow, Fatigue behaviour of high-strength concrete with respect to strain and stiffness, Int J. Fatigue 87 (2016) 38–49.

[135] B. Isojeh, M. El-Zeghayar, F.J. Vecchio, Concrete Damage under Fatigue Loading in Uniaxial Compression, Acids Mater. J. 114 (2017).

[136] M.N. Noorsuhada, An overview on fatigue damage assessment of reinforced concrete structures with the aid of acoustic emission technique, Constr. Build. Mater. 112 (2016) 424–439.

[137] S.G. Shah, J.M. Chandra Kishen, Use of acoustic emissions in flexural fatigue crack growth studies on concrete, Eng. Fract. Mech. 87 (2012) 36–47.

[138] N. Oneschkow, T. Timmermann, S. Löhnert, Compressive Fatigue Behaviour of High-Strength Concrete and Mortar: Experimental Investigations and Computational Modelling, Materials 15 (2022).

[139] N. Eshetu Deresse, M. Sarem, H. Nasser, S. François, E. Verstrynge, Fatigue fracture quantification in brittle cementitious materials using acoustic emission testing and digital image correlation, Constr. Build. Mater. 400 (2023).

[140] J. Cao, D.D.L. Chung, Minor damage of cement mortar during cyclic compression monitored by electrical resistivity measurement, Cem. Concr. Res 31 (2001) 1519–1521.

[141] J. Cao, D.D.L. Chung, Defect dynamics and damage of concrete under repeated compression, studied by electrical resistance measurement, Cem. Concr. Res 31 (2001) 1639–1642.

[142] A. Shen, S. Lin, Y. Guo, T. He, Z. Lyu, Relationship between flexural strength and pore structure of pavement concrete under fatigue loads and Freeze-thaw interaction in seasonal frozen regions, Constr. Build. Mater. 174 (2018) 684–692.

[143] C. Fu, H. Ye, X. Jin, D. Yan, N. Jin, Z. Peng, Chloride penetration into concrete damaged by uniaxial tensile fatigue loading, Constr. Build. Mater. 125 (2016) 714–723.

[144] C. Desmettre, J.-P. Charron, Water Permeability of Reinforced Concrete Subjected to Cyclic Tensile Loading, Acids Mater. J. 110 (2013).

[145] K. Aas-Jakobsen, Fatigue of concrete beams and columns, Division of Concrete Structures, Norwegian Inst. of Technology, University, 1970.

[146] J.T. McCall, Probability of fatigue failure of plain concrete, J. Proc. (1958) 233–244.

[147] M. Do, O. Chaallal, P. Aïtcin, Fatigue Behavior of High-Performance Concrete, J. Mater. Civ. Eng. 5 (1993) 96–111.

[148] K. Matsumoto, Y. Sato, T. Ueda, L. Wang, Mesoscopic analysis of mortar under high-stress creep and low-cycle fatigue loading, J. Adv. Concr. Technol. 6 (2008) 337–352.

[149] J.L. Le, J. Manning, J.F. Labuz, Scaling of fatigue crack growth in rock, Int. J. Rock. Mech. Min. Sci. 72 (2014) 71–79.

[150] P. Paris, F. Erdogan, A critical analysis of crack propagation laws, J. Basic Eng. 85 (1963) 528–533.

[151] M.H. Baluch, A.B. Qureshy, A.K. Azad, Fatigue crack propagation in plain concrete. in: Fracture of Concrete and Rock, Springer, 1989, pp. 80–87.

[152] A. Mohammadipour, K. Willam, A numerical lattice method to characterize a contact fatigue crack growth and its Paris coefficients using configurational forces and stress-life curves, Comput. Methods Appl. Mech. Eng. 340 (2018) 236–252.

[153] T. Sain, J.M.C. Kishen, Damage indices for failure of concrete beams under fatigue, Eng. Fract. Mech. 75 (2008) 4036–4051.

[154] V. Slowik, G.A. Plizzari, V.E. Saouma, Fracture of concrete under variable amplitude fatigue loading, Acids Mater. J. 93 (1996) 272–283.

[155] J.-L. Le, Z.P. Bažant, Unified nano-mechanics based probabilistic theory of quasibrittle and brittle structures: II. Fatigue crack growth, lifetime and scaling, J. Mech. Phys. Solids 59 (2011) 1322–1337.

[156] J.-L. Le, Z.P. Bažant, M.Z. Bazant, Unified nano-mechanics based probabilistic theory of quasibrittle and brittle structures: I. Strength, static crack growth, lifetime and scaling, J. Mech. Phys. Solids 59 (2011) 1291–1321.

[157] D. Zhaodong, L. Jie, A physically motivated model for fatigue damage of concrete, Int. J. Damage Mechnics 27 (2018) 1192–1212.

[158] J.G.M. Van Mier, Concrete fracture: a multiscale approach, CRC press, 2012.

[159] Z.P. Bažant, Concrete fracture models: Testing and practice, Eng. Fract. Mech. 69 (2001) 165–205.

[160] K.M. Simon, J.M.C. Kishen, A multiscale approach for modeling fatigue crack growth in concrete, Int J. Fatigue 98 (2017) 1–13.

[161] Y. Wang, Physical stochastic damage model for concrete subjected to fatigue loading, Int J. Fatigue 121 (2019) 191–196.

[162] H.W. Reinhardt, Fracture Mechanics of an Elastic Softening Material Like Concrete, Heron 29 (1984) 1–42.

[163] A. Hillerborg, M. Modéer, P.E. Petersson, Analysis of crack formation and crack growth in concrete by means of fracture mechanics and finite elements, Cem. Concr. Res 6 (1976) 773–781.

[164] J. van Mier, Classical Fracture Mechanics Approaches, Concr. Fract. (2012) 11–34.

[165] B. Yang, S. Mall, K. Ravi-Chandar, A cohesive zone model for fatigue crack growth in quasibrittle materials, Int J. Solids Struct. 38 (2001) 3927–3944.

[166] M. Corrado, J.F. Molinari, Effects of residual stresses on the tensile fatigue behavior of concrete, Cem. Concr. Res 89 (2016) 206–219.

[167] S.A. Ponnusami, S. Turteltaub, S. van der Zwaag, Cohesive-zone modelling of crack nucleation and propagation in particulate composites, Eng. Fract. Mech. 149 (2015) 170–190.

[168] D.Z. Yankelevsky, H.W. Reinhardt, Uniaxial Behavior of Concrete in Cyclic Tension, J. Struct. Eng. 115 (1989) 166–182.

[169] D. Hordijk, Local approach to fatigue of concrete, PhD Thesis, Delft University of Technology, 1991, p. 216.

[170] W. Ren, Z. Yang, R. Sharma, C. Zhang, P.J. Withers, Two-dimensional X-ray CT image based meso-scale fracture modelling of concrete, Eng. Fract. Mech. 133 (2015) 24–39.

[171] K. Gylltoft, A fracture mechanics model for fatigue in concrete, Mat. ériaux Et. Constr. 17 (1984) 55–58.

[172] F. Gong, T. Ueda, Y. Wang, D. Zhang, Z. Wang, Mesoscale simulation of fatigue behavior of concrete materials damaged by freeze-thaw cycles, Constr. Build. Mater. 144 (2017) 702–716.

[173] Y. Gan, H. Zhang, M. Liang, E. Schlangen, K. van Breugel, B. Šavija, A numerical study of fatigue of hardened cement paste at the microscale, Int J. Fatigue 151 (2021) 106401.

[174] E. Papa, A. Taliercio, Anisotropic damage model for the multiaxial static and fatigue behaviour of plain concrete, Eng. Fract. Mech. 55 (1996) 163–179.

[175] J. Lemaitre, How to use damage mechanics, Nucl. Eng. Des. 80 (1984) 233–245.

[176] D. Krajcinovic, Damage mechanics, Mech. Mater. 8 (1989) 117–197.

[177] J. Lemaitre, A continuous damage mechanics model for ductile fracture, Journal of Engineering Materials and Technology, Trans. ASME 107 (1985) 83–89.

[178] J.L. Chaboche, Anisotropic Creep Damage in the Framework of Continuum Damage Mechanics, Trans. Int. Conf. Struct. Mech. React. Technol. (1983).

[179] F.A. Leckie, A course on damage mechanics, Springer Science & Business Media, 1998.

[180] W. Suaris, C. Ouyang, V.M. Fernando, Damage Model for Cyclic Loading of Concrete, J. Eng. Mech. 116 (1990) 1020–1035.

[181] G.D. Nguyen, A thermodynamic approach to non-local damage modelling of concrete, Int J. Solids Struct. 45 (2008) 1918–1934.

[182] J. Mazars, G. Pijaudier-Cabot, Continuum Damage Theory—Application to Concrete, J. Eng. Mech. 115 (1989) 345–365.

[183] K.M.P. Fathima, J.M.C. Kishen, A thermodynamic framework for the evolution of damage in concrete under fatigue, Arch. Appl. Mech. 85 (2015) 921–936.

[184] K.M. Pervaiz Fathima, J.M. Chandra Kishen, A thermodynamic framework for fatigue crack growth in concrete, Int J. Fatigue 54 (2013) 17–24.

[185] Y.F. Dafalias, Bounding Surface Plasticity. I: Mathematical Foundation and Hypoplasticity, J. Eng. Mech. 112 (1986) 966–987.

[186] J. Liang, X. Nie, M. Masud, J. Li, Y.L. Mo, A study on the simulation method for fatigue damage behavior of reinforced concrete structures, Eng. Struct. 150 (2017) 25–38.

[187] P. Grassl, R. Rempling, A damage-plasticity interface approach to the meso-scale modelling of concrete subjected to cyclic compressive loading, Eng. Fract. Mech. 75 (2008) 4804–4818.

[188] B. Sun, Z. Xu, An efficient numerical method for meso-scopic fatigue damage analysis of heterogeneous concrete, Constr. Build. Mater. 278 (2021) 122395.

[189] B. Wu, Z. Li, K. Tang, K. Wang, Microscopic multiple fatigue crack simulation and macroscopic damage evolution of concrete beam, Appl. Sci. (Switz.) 9 (2019).

[190] J.J. Ortega, G. Ruiz, R.C. Yu, N. Afanador-García, M. Tarifa, E. Poveda, X. Zhang, F. Evangelista, Number of tests and corresponding error in concrete fatigue, Int J. Fatigue 116 (2018) 210–219.

[191] CEN/TC250., EN 1992–1-1:2004, Eurocode 2: Design of concrete structures - Part 1- 1: General rules and rules for buildings, Brussels, Belgium. (2004).

[192] H. Li, M. Zhang, J. Ou, Int. J. Fatigue Flexural Fatigue Perform. Concr. Contain. nano-Part. Pavement 29 (2007) 1292–1301.
[193] B.H. Oh, Fatigue life distributions of concrete for various stress levels, Mater. J. 88 (1991) 122–128.
[194] Y. Zhang, Q. Sun, H. Xin, Y. Liu, J.A.F.O. Correia, F. Berto, Probabilistic flexural fatigue strength of ultra-lightweight cement concrete and high strength lightweight aggregate concrete, Int J. Fatigue 158 (2022) 106743.
[195] L. Saucedo, R.C. Yu, A. Medeiros, X. Zhang, G. Ruiz, A probabilistic fatigue model based on the initial distribution to consider frequency effect in plain and fiber reinforced concrete, Int J. Fatigue 48 (2013) 308–318.
[196] E. Castillo, A. Fernández-Canteli, A Unified Statistical Methodology for Modeling Fatigue Damage, Springer, Netherlands, 2009.
[197] J. Li, C. Guo, A unified stochastic damage model for concrete under monotonic and fatigue loading, Int J. Fatigue 175 (2023) 107766.
[198] J. Li, X. Ren, Stochastic damage model for concrete based on energy equivalent strain, Int J. Solids Struct. 46 (2009) 2407–2419.
[199] H. Liu, X. Ren, J. Li, Indentation tests based multi-scale random media modeling of concrete, Constr. Build. Mater. 168 (2018) 209–220.
[200] Y. Ma, L. Wang, J. Zhang, Y. Xiang, Y. Liu, Bridge Remaining Strength Prediction Integrated with Bayesian Network and In Situ Load Testing, J. Bridge Eng. 19 (2014), 04014037.1-04014037.11.
[201] B. Ge, S. Kim, Probabilistic service life prediction updating with inspection information for RC structures subjected to coupled corrosion and fatigue, Eng. Struct. 238 (2021) 112260.
[202] J. Wu, B. Zhang, J. Xu, L. Jin, B. Diao, Probabilistic fatigue life prediction for RC beams under chloride environment considering the statistical uncertainty by Bayesian updating, Int J. Fatigue 173 (2023) 107680.
[203] W. Zhang, D. Lee, J. Lee, C. Lee, Residual strength of concrete subjected to fatigue based on machine learning technique, Struct. Concr. 23 (2022) 2274–2287.
[204] J. Son, S. Yang, A new approach to machine learning model development for prediction of concrete fatigue life under uniaxial compression, Appl. Sci. (Switz.) 12 (2022).
[205] Q. Lu, J. Zhu, W. Zhang, Quantification of fatigue damage for structural details in slender coastal bridges using machine learning-based methods, J. Bridge Eng. 25 (2020) 04020033.
[206] H. Chen, Z. Sun, Z. Zhong, Y. Huang, Fatigue factor assessment and life prediction of concrete based on bayesian regularized BP neural network, Materials 15 (2022).
[207] E. Fathalla, Y. Tanaka, K. Maekawa, Remaining fatigue life assessment of in-service road bridge decks based upon artificial neural networks, Eng. Struct. 171 (2018) 602–616.
[208] M. Liang, Z. Chang, Z. Wan, Y. Gan, E. Schlangen, B. Šavija, Interpretable ensemble-machine-learning models for predicting creep behavior of concrete, Cem. Concr. Compos 125 (2022).
[209] C. Frie, A. Riza Durmaz, C. Eberl, Exploration of materials fatigue influence factors using interpretable machine learning, Fatigue Fract. Eng. Mater. Struct. (2024).
[210] B. Hilloulin, V.Q. Tran, Interpretable machine learning model for autogenous shrinkage prediction of low-carbon cementitious materials, Constr. Build. Mater. 396 (2023) 132343.
[211] D.-S. Yang, D. Xue, H. Xu, W. Duan, Low-cycle fatigue design for reinforced high-strength concrete under high compressive stress, Case Stud. Constr. Mater. 20 (2024) e03078.
[212] S.R. Vadyala, S.N. Betgeri, J.C. Matthews, E. Matthews, A review of physics-based machine learning in civil engineering, Results Eng. 13 (2022) 100316.
[213] C. Yu, M. Yu, X. Li, L. Xu, S. Liu, J. Ye, Uniaxial constitutive model for fiber reinforced concrete: A physics-based data-driven framework, Constr. Build. Mater. 406 (2023) 133377.
[214] S. Huzni, I.B.M. Ibrahim, S. Fonna, R. Pidaparti, Physics-based surrogate model for reinforced concrete corrosion simulation, Results Eng. 16 (2022) 100659.
[215] J. Chen, Y. Liu, Probabilistic physics-guided machine learning for fatigue data analysis, Expert Syst. Appl. 168 (2021) 114316.
[216] L. Wang, S.-P. Zhu, C. Luo, D. Liao, Q. Wang, Physics-guided machine learning frameworks for fatigue life prediction of AM materials, Int J. Fatigue 172 (2023) 107658.
[217] H. Zhang, B. Šavija, S.C. Figueiredo, E. Schlangen, Experimentally validated multi-scale modelling scheme of deformation and fracture of cement paste, Cem. Concr. Res 102 (2017) 175–186.
[218] H. Zhang, Experimentally validated multi-scale fracture modelling scheme of cementitious materials, Delft University of Technology, PhD Thesis, 2019.
[219] Z. Qian, Multiscale modeling of fracture processes in cementitious materials, Delft University of Technology, PhD Thesis, 2012.
[220] M. Hlobil, V. Šmilauer, G. Chanvillard, Micromechanical multiscale fracture model for compressive strength of blended cement pastes, Cem. Concr. Res 83 (2016) 188–202.
[221] V.P. Nguyen, M. Stroeven, L.J. Sluys, An enhanced continuous-discontinuous multiscale method for modeling mode-I cohesive failure in random heterogeneous quasi-brittle materials, Eng. Fract. Mech. 79 (2012) 78–102.
[222] X. Sun, X. Guo, L. Guo, B. Wu, Y. Wang, Multiscale analysis of concrete damage and crack propagation under high cycle loading, Int J. Comput. Methods 17 (2020) 1844007.
[223] Y. Gan, M. Liang, E. Schlangen, K. van Breugel, B. Šavija, Two scale models for fracture behaviours of cementitious materials subjected to static and cyclic loadings, Constr. Build. Mater. 426 (2024) 136107.
[224] Y. Wang, J. Li, A two-scale stochastic damage model for concrete under fatigue loading, Int J. Fatigue 153 (2021) 106508.
[225] L.I. Jie, Z. Qi-Yun, Study of Stochastic Damage Constitutive Relationship for Concrete Material, Journal of Tongji University. (2001).