

A STUDY OF RADIATION STRESS AND SET-UP IN THE NEARSHORE REGION

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ABSTRACT

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In this paper the variations of radiation stress and mean water level are studied for the two-dimensional shoaling and breaking of progressive, periodic waves on a plane, gently sloping laboratory beach. Observations of radiation stress and mean water level were made for two initial conditions. The observed variations are compared to the variations as calculated by a linear theory, James' nonlinear theory, and a new nonlinear theory based on Cokelet's work. It is shown that the nonlinear theories are qualitatively and quantitatively superior to the linear theory, thus indicating that the effects of finite amplitude are important for the present case.

INTRODUCTION

Phenomena observed in the surf zone, such as wave set-up and longshore currents, are successfully explained by the theory of radiation stress. This theory was developed by Longuet-Higgins and Stewart (1960, 1961, 1962, 1964), who introduced the name radiation stress for the excess flux of horizontal momentum due to the presence of waves. The variation of the radiation stress due to shoaling and breaking is balanced in the on-offshore direction by the slope of the mean water level (set-down and set-up). In the case of waves arriving at an angle to the shore, the variation across the surf zone of the wave-induced longshore momentum flux in a direction perpendicular to the coast provides a driving force for the longshore current.

Many studies of set-up and longshore currents that apply the concept of radiation stress use small-amplitude, linear wave theory. In the nearshore region, where the wave amplitude becomes comparable with the depth and the waves finally break, the small-amplitude assumption is violated. To account for the effects of finite amplitude James (1974) used a nonlinear approach, combining a third-order Stokes and a cnoidal theory, to calculate the radiation stress and set-up variations. The recent significant developments in the field of nonlinear theories enable us to present a new nonlinear

approach based on the nearly exact, irrotational wave theory of Cokelet (1977). However, all these approaches have in common that they apply theories for nonbreaking, plane-periodic waves under the assumption of a slowly varying wave motion and of a depth-determined wave motion in the surfzone.

The wave deformations before and after breaking and the presence of a turbulent, breaking motion in the surf zone indicate that the assumptions common to the approaches mentioned above are violated in the near-shore region. It is the aim of this study to compare observed variations of radiation stress and mean water level with those calculated by the linear and nonlinear approaches. The observations were made in a laboratory flume as a part of a study of the internal characteristics of waves breaking on a 1:40 slope. A preliminary analysis of the instantaneous velocity and pressure fields is given in Stive (1980). The internal pressures were measured only on a limited scale, so for this paper we have derived the radiation stress from the measured surface elevations and velocity fields by expressing the mean dynamic pressure field in terms of mean velocity properties.

The paper is made up as follows. First, the experiments are described. It is explained that laser doppler velocity measurements are not possible in aerated flow such as is met with in the crests of breaking waves. Secondly, we describe the extrapolation of the measured velocity fields in the crest regions to allow the integration over time and depth of the flow properties. Thirdly, these results are used to investigate some common approximations and the effects of nonlinearity and turbulence with respect to the mean momentum balance. Finally, we investigate and discuss the ability of the linear and nonlinear approaches to predict the variations of radiation stress and mean water level.

EXPERIMENTS

Arrangements

The experiments were conducted in a wave flume of the Delft Hydraulics Laboratory. The flume is 55 m long, 1 m wide and 1 m high. A plane, concrete beach with a 1:40 slope was installed (see Fig. 1a). To enable the installation of instruments in the bottom of the horizontal section, a false concrete floor was made which reduced the water depth in the horizontal section to 0.70 m.

Periodic waves with minimal free second-harmonic components were generated in a water depth of 0.85 m. To obtain a control signal for the wave generator which resulted in a maximum suppression of the free second harmonics, a simple trial and error procedure was applied, using Fontanet's theory (see e.g. Hulsbergen, 1974) as a first approximation.

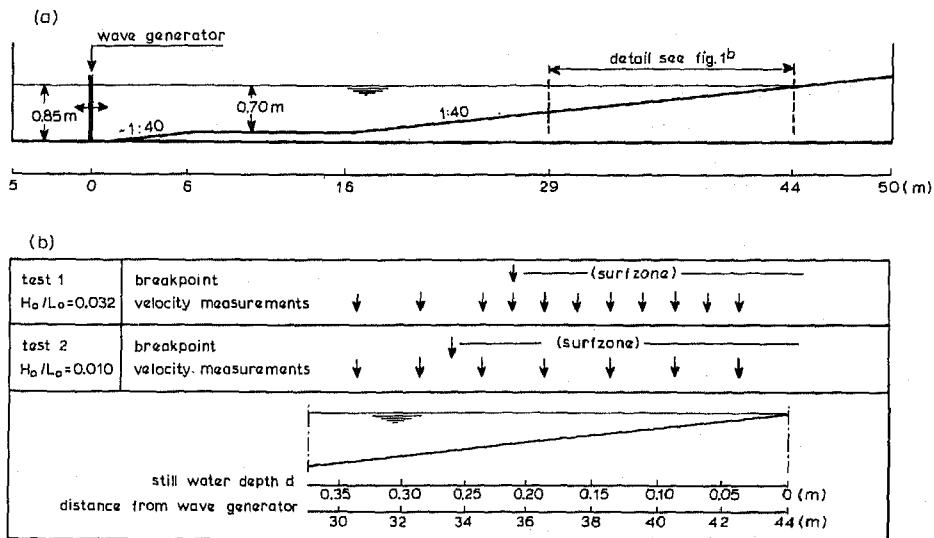


Fig. 1. a. Experimental set-up. b. Detail experimental set-up.

Instrumentation

Surface elevations were measured by means of resistance-type wave gauges. Three gauges were installed on a carriage to cover the horizontal section and most of the shoaling zone. Twelve fixed gauges were positioned near and in the surfzone. Calibrations confirmed that deviations from linearity were negligible, viz. 0.5% of the full-scale deflection, which amounted to 10, 20 or 50 cm depending on the wave heights.

Resistance-type wave gauges must be used with care in the case of sharply peaked waves. The conductivity measured by the gauges near a steep wave surface decreases noticeably, e.g. for a distance to an infinitely steep water front of 5 cm the decrease in conductivity is 1% and for a distance of 2 cm it is 5%. These effects are similar to the effects of an insulating partition. In general the accuracy of the measured surface elevations of sharply peaked waves is negligibly affected. However, when measuring significant gradients of the time mean surface elevations these effects were found to be non-negligible. It is for this reason that efforts to measure the mean water level by simply averaging the instantaneous surface elevations recorded by the wave gauges were not successful in the region around the breakpoint. Instead the mean water level was derived from the measured mean bottom pressures.

Also in the case of breaking waves, resistance-type wave gauges must be used with care; aeration influences their response. An independent study showed that the gauges measure a volume which is about 1.5 times the air volume less than actual volume of water and air. Based on photographic comparison it was concluded that — although locally the air content in the heavily

aerated crest region may reach 10% — the average air content over the region between the level of the wave troughs and the wave surface is less than 2%. This implies an underestimation of a maximum of 3% of the actual or 1% of the solid trough-to-surface elevation only, which affects the accuracy of the measured surface elevations negligibly.

The mean bottom pressures were measured using a system of taps (ϕ 3 mm), mounted flush with the bottom, connected by tubes to stilling wells. The combination of the relatively narrow tubes (ϕ 5 mm) and wide wells (ϕ 8 cm) causes sufficient damping of the water level fluctuation in the wells. The water level in the wells was read by pointgauges with an accuracy of 0.1 mm. The relation between the mean bottom pressure and the mean water level is considered in detail on p. 14.

The horizontal and vertical components of velocity were measured by means of a laser doppler velocimeter (LDV), which is described elsewhere (Godefroy, 1978). The device was mounted on a carriage such that any desired horizontal or vertical position along the flume could be reached. Accuracy of the LDV is merely determined by the stability of a main electronic component of the system, i.e. the frequency tracker. The inaccuracies thus introduced were found to amount to ± 1 cm/s.

During measurements the doppler signal may drop out temporarily for several reasons, such as the presence of air bubbles or large opacities. When this occurs, the last detected doppler frequency is held until the next doppler burst of about the same frequency is detected. These drop-outs occur when measuring above the level of the wave troughs and in water with entrained air such as is found in the crest of breaking waves. As a consequence, measurements are only possible up to lightly aerated regions. Estimates of the velocities in the aerated crest region were obtained by extrapolation, as will be described later.

Measurements and procedures

Prior to the laser doppler velocity measurements, several data runs of about three hours duration per wave condition were conducted which comprised the measurement of surface elevation and mean bottom pressure at positions with an interval of 1 m over the whole flume. The wave gauges were calibrated and referenced at each measurement position before and after each run. The data runs started after at least 10 min of wave generation to allow for the decay of initial low-frequency effects. Recordings of the surface elevations were made over 20 wave periods minimally. The water level in the wells connected to the pressure bottom taps was read until the level fluctuations were less than 0.2 mm over 30 min. Reflection, measured in the horizontal section of the flume, appeared to be small in the present experiments, viz. 4% in wave height. Satisfactory reproduction (average wave height deviations in the horizontal section smaller than 1%) could be realised.

Two velocity data runs of about 6 hours duration per wave condition were conducted, which comprised the measurement of the horizontal and vertical components of velocity at different levels in a limited number of cross-sections in and near the surf zone (see Fig. 1b). In each of these cross-sections the velocities were measured consecutively at different levels, i.e. non-simultaneously. To establish a common reference phase for the non-simultaneous measurements in one cross-section, the command signal of the wave generator and the surface elevation in the same cross-section were recorded simultaneously with the velocities.

All electric signals were recorded on analog tapes. At the same time they were digitised at 25 Hz by a mini-computer for limited processing, allowing a check on the data-acquisition immediately after the data runs. The analog tapes were digitised at 100 Hz for extensive processing.

Wave conditions

The experiments were restricted to two conditions (see Table I), which are referred to as test 1 and test 2. These conditions represent two types of initial breaking, that one usually finds on gently sloping beaches. The initial breaking behaviour of test 1 falls in the category spilling breaking, while that of test 2 falls in the category plunging breaking. As characterized by Svendsen et al. (1978), the rapid transitions of wave shape in the region right after breaking — the so-called outer region — develop soon, i.e. after a horizontal distance of several times the breaker depth, into the relatively well-organised, quasi-steady breaking motion of the inner region, which is virtually independent of the initial breaking behaviour. At that stage of their breaking motion, breakers on a beach may be described as spilling breakers or bores.

TABLE I

Wave conditions (wave height H , period T , wave length L , where the subscripts o, h and b denote deep water, horizontal section and breakpoint)

Test	H_o (m)	H_h (m)	H_b (m)	T (s)	H_o/L_o (-)
1	0.159	0.145	0.178	1.79	0.032
2	0.142	0.145	0.226	3.00	0.010

THE VELOCITY FIELD AND ITS EXTRAPOLATION

Quantification of the radiation stresses requires an integration over time and depth of the squared horizontal and vertical velocity. As explained before, laser doppler velocity measurements are not possible in the aerated crest regions of breaking waves, which prohibits a full time and depth integra-

tion inside the surf zone. Therefore, extrapolations were made of the flow field in the crest region of the breaking waves, with which we will mainly concern ourselves in this section.

From typical recordings of the horizontal and vertical velocities (denoted by u and w) in periodic shoaling and breaking waves (Stive, 1980), it was concluded that the non-breaking wave motion can be fully described by a periodic component (indicated by a tilde, i.e. $u = \tilde{u}$ and $w = \tilde{w}$). The breaking wave motion can be described as the sum of a purely periodic component and a non-periodic residual component (indicated by a prime, i.e. $u = \tilde{u} + u'$ and $w = \tilde{w} + w'$). The periodic component is obtained by averaging the signals at a fixed phase of a periodic reference signal over at least 20 waves. This procedure may be described as ensemble averaging. The major part of the residual component is formed by the turbulent fluctuation of the breaking process and is therefore termed the turbulent component. These fluctuations are non-stationary in time. The rms-value of the turbulent component at a fixed phase is obtained from the ensemble of at least 20 realisations of the turbulent process.

In the following we consider the extrapolation of the periodic components \tilde{u} and \tilde{w} and the turbulent components u'_{rms} and w'_{rms} separately. For the extrapolation of the periodic components use has been made of the observed quasi-stationarity of the mean wave motion in the inner breaking region. The extrapolation of the turbulent components is based on the characteristics of wake flows.

The periodic velocity field

The nature of the periodic flow fields in the inner region is described by a model for a quasi-steady breaking wave as proposed by Peregrine and Svendsen (1978). The model is schematically outlined in Fig. 2a. The horizontal periodic flow field resulting from this model, as interpreted and verified in Stive (1980), is refined here based on the results of the extrapolations in the crest.

The preservation of shape of the measured surface profiles suggests that for the extrapolation of the *horizontal* periodic velocity field the assumption of constant form is approximately correct. For constant form $\partial/\partial t = -c \partial/\partial x$, so that the balance equation of mass may be written as:

$$-c \frac{\partial}{\partial x} \int_{-d}^{\xi} \rho \, dz + \frac{\partial}{\partial x} \int_{-d}^{\xi} \rho \tilde{u} \, dz = 0 \quad (1)$$

Using $\int_{-d}^{\xi} \rho \tilde{u} \, dz = 0$ on account of an impermeable beach it follows that:

$$\int_{-d}^{\xi} \rho \tilde{u} \, dz = \rho c (\xi - \bar{\xi}) \quad (2)$$

From the measurements the velocity of propagation c and the surface elevation $\zeta - \bar{\zeta}$ are known. The value of the velocity integral so obtained yields a constraint that can be used in the extrapolation. The actual procedure applied is simply a linear extrapolation from the highest measurement level to the free surface such that eq. (2) is fulfilled. A check on the validity of eq. (2) in the trough region where $\int_{-d}^{\zeta} \rho \tilde{u} dz$ can be determined showed it to be a valid relation.

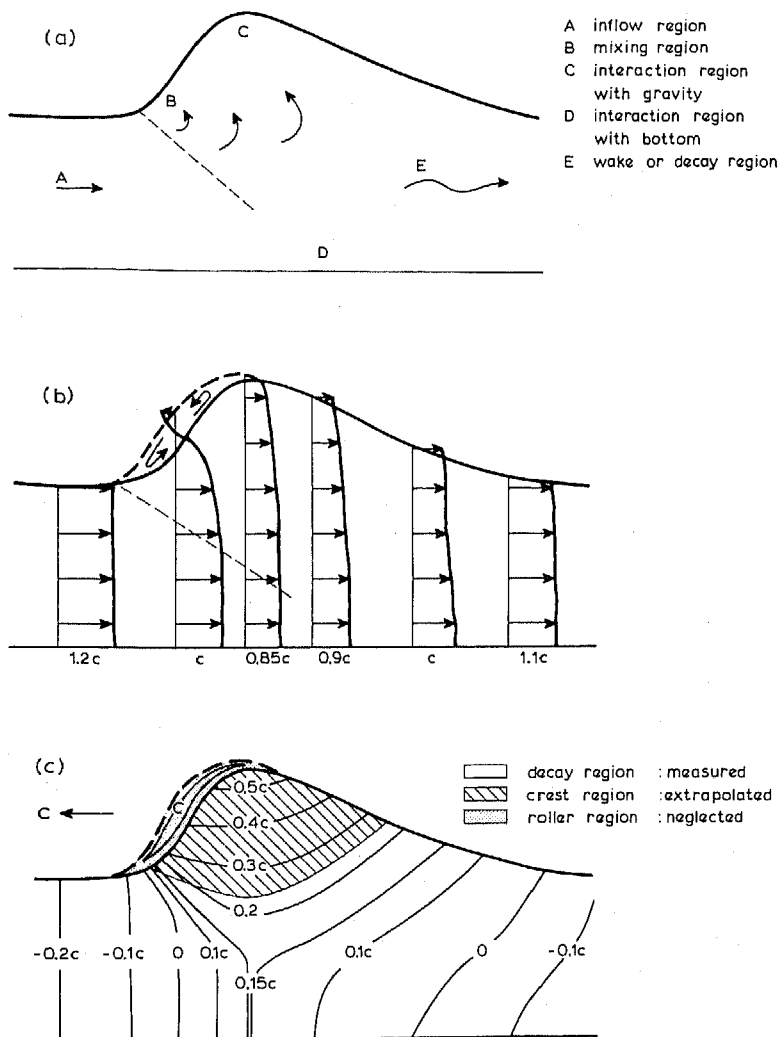


Fig. 2. a. Model of a quasi-steady breaking wave (after Peregrine and Svendsen, 1978) for a reference frame moving with the wave. b. Horizontal velocity field of a quasi-steady breaking wave for a reference frame moving with the wave. c. Horizontal velocity field of a quasi-steady breaking wave for a fixed frame of reference.

The extrapolation of the *vertical* periodic velocities is based on the kinematic boundary condition at the free surface and again the constant form assumption.

At the free surface:

$$\frac{dz}{dt} = \frac{\partial \zeta}{\partial t} + \frac{\partial \zeta}{\partial x} \frac{dx}{dt}, \quad (3)$$

introducing $\frac{\partial}{\partial x} = -\frac{1}{c} \frac{\partial}{\partial t}$, we obtain

$$\tilde{w} = \frac{\partial \zeta}{\partial t} \left(1 - \frac{\tilde{u}}{c}\right) \quad (4)$$

From the measurements c and $\frac{\partial \zeta}{\partial t}$ are known. Using the extrapolated magnitude of \tilde{u} at the surface, we directly obtain the vertical velocity \tilde{w} at the surface. A linear interpolation was used to estimate the values of \tilde{w} between the highest measurement level and the surface. This was satisfactorily checked in the trough region again.

The extrapolated horizontal flow field (see Fig. 2b,c) shows the flow characteristics of Peregrine and Svendsen's model. However, the velocities obtained by the extrapolation do not reach a magnitude c at the wave front and crest as expected from their model. The explanation is that our linear extrapolation method does not reveal the presence of the surface roller, which — based on visual observation — in our experiments stretches out over the whole wave front. As indicated by Peregrine and Svendsen (1978), the surface roller is only a small part of the flow and does not play a dominant role in the dynamics of the wave. It merely acts as a trigger to initiate the turbulence. Its contribution to the momentum flux is assumed to be negligible.

We have chosen for the linear extrapolation since it is a simple procedure applicable to every velocity profile of the wave and at the same time justifiable in view of the accuracies achieved on the basis of the constant form assumption.

It is remarked that the above extrapolation procedures were based on the wave characteristics of the inner breaking region. Based on a sensitivity analysis we have estimated that the extrapolation procedure introduces an uncertainty in the periodic contribution to the mean momentum flux of 10% maximally. For the measurements made at the transition of outer to inner region, i.e. at cross-section 36.5 m for test 1 and 34.5 m for test 2, the above procedures are less applicable. The uncertainties in the momentum flux contributions at these locations are estimated to be greater by at least a factor of 2 than those mentioned above, as will be discussed later.

The turbulent velocity field

The measured turbulent velocity fields have been shown to agree — at least qualitatively — with Peregrine and Svendsen's model (Stive, 1980). This specifically applies to the wake region, which takes up the major part of the turbulent flow. We have based the extrapolation of the turbulent velocities in the crest region on the flow characteristics of wakes.

Wakes belong to a well-known class of self-preserving free shear flows. One of the characteristics to which the self-preservation hypothesis is applicable is the velocity defect, i.e. the difference of the local velocity \tilde{u} and the undisturbed stream velocity \tilde{u}_0 . The velocity defect profile is invariant with respect to x , if expressed in terms of the local length scale $l = l(x)$ and the local maximum velocity defect $\tilde{u}_s = \tilde{u}_s(x)$ at the center of the wake, in which x is the distance in which the wake develops. Under the assumption of a constant eddy-viscosity — the value of which is determined experimentally — the velocity defect for plane wakes is found to be (Tennekes and Lumley, 1974):

$$(\tilde{u}_0 - \tilde{u})/\tilde{u}_s = \exp(-\frac{1}{2}\alpha\xi^2) \quad (5)$$

with $\xi = l/z$, in which z is the cross-stream distance from the center line of the wake. Setting $\alpha = 1$ defines the local length scale: $(\tilde{u}_0 - \tilde{u})/\tilde{u}_s \cong 0.6$ at $z = l$ (see Fig. 3).

Application of the self-preservation hypothesis to the Reynolds stress and assuming a close relation between turbulent energy $\overline{q^2}$ and Reynolds stress restricted to the region $\xi > 1$ leads to (Tennekes and Lumley, 1974):

$$\overline{q^2} = 0.6 \tilde{u}_s^2 \xi \exp(-\frac{1}{2}\xi^2) \quad \text{for } \xi > 1 \quad (6)$$

This agrees well with experiments. For $0 \leq \xi < 1$ the experiments show a nearly constant behaviour of $\overline{q^2} \cong (0.6 \tilde{u}_s)^2$. However, in the present model the free surface is regarded as a symmetry line of the wake flow. In contrast to a full wake, gravity suppresses our turbulent motions at the water surface. Therefore, it is assumed here that eq. (6) is also valid for $0 \leq \xi < 1$. Now introducing the ratio $\overline{u'^2} : \overline{w'^2} : \overline{v'^2}$ as given by Townsend (1976) for plane wakes, i.e. 0.43 : 0.32 : 0.25, we can set the ratio of turbulent intensity to the local maximum velocity defect as (see also Fig. 3):

$$u'_{\text{rms}}/\tilde{u}_s = 0.50 \xi^{\frac{1}{2}} \exp(-\frac{1}{4}\xi^2) \quad (7)$$

$$w'_{\text{rms}}/\tilde{u}_s = 0.44 \xi^{\frac{1}{2}} \exp(-\frac{1}{4}\xi^2) \quad (8)$$

The extrapolation of the turbulent velocities is based on these results as follows. The local maximum velocity defect \tilde{u}_s can be determined from the horizontal velocity at the free surface (the "center" of the wake) and the horizontal velocity in the undisturbed flow region, \tilde{u}_0 , for which we have taken the velocities measured at the lowest level. Thus, based on the above ratios, the distribution of the turbulent intensities is known as a function of ξ .

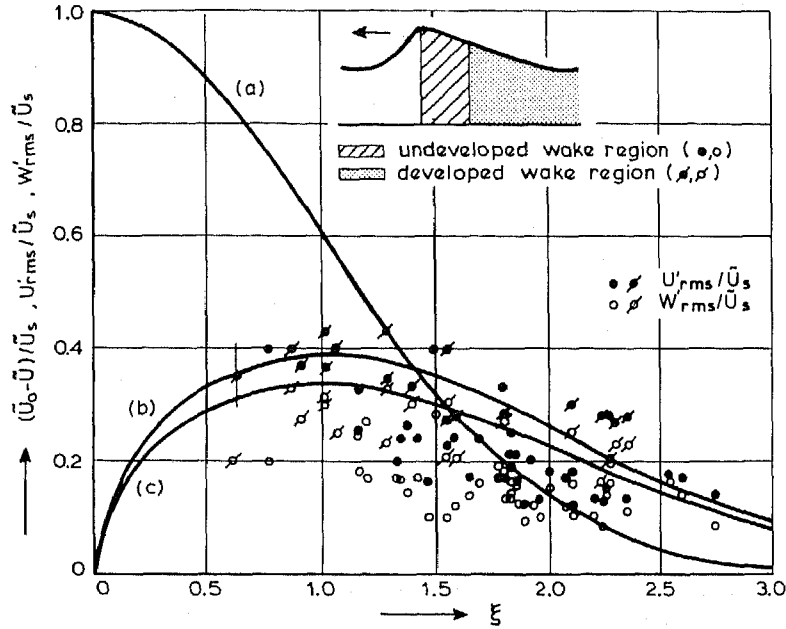


Fig. 3. Velocity defect and turbulent intensity; solid line: theory for plane wakes (after Townsend, 1976, and Tennekes and Lumley, 1974): (a) eq. (5), (b) eq. (7), (c) eq. (8); $\bullet, \phi, \circ, \phi$ observations for breaking waves inner region.

We may eliminate ξ with eq. (5) and find the distribution of the turbulent intensities as a function of $\tilde{u}_0 - \tilde{u}$. Since the periodic horizontal velocity \tilde{u} is known over the full depth from the periodic extrapolation procedure, the turbulent velocity field is now determined.

The extrapolation of the turbulent velocity field in the crest region is mainly based on the assumed similarity to a wake flow. A check on the extrapolation procedure was made in the wake flow region not affected by aeration (Fig. 3). The results indicate that the turbulent contributions will be somewhat overestimated due to undeveloped turbulent intensities in the crests (note that the error bar indicates the estimated uncertainty of the measurements). Since the contribution of the crest region is dominant, the estimation of the total turbulent contribution may be expected to have a low accuracy, say $\pm 50\%$ of the extrapolated value. However, it will be shown that the contribution to the total momentum flux is 5% maximally, so that the uncertainty introduced in the total momentum flux is negligible.

RADIATION STRESS AND SET-UP

In this section we will first consider the mean momentum balance with the aim of verifying some common approximations and to investigate the effects of nonlinearity and breaking-induced turbulence. Secondly, we describe

linear and nonlinear approaches to calculate the on-offshore variation of radiation stress and mean water level. Thirdly, a comparison is made between the calculations and laboratory observations for two wave conditions.

Before doing so, we recall that we consider the simple case of plane, periodic waves normally incident to a gently sloping beach. A Cartesian coordinate frame is used with horizontal coordinates x, y and vertical coordinate z , with x positive in the direction of wave propagation and z vertically upward, while $z = 0$ is chosen at the still-water level. Since the motion is assumed homogeneous in the y -direction, we will in the following consider the two-dimensional motion in the x, z -plane. The position of the fixed, impermeable bottom is given by $z = -d(x)$ and that of the free surface by $z = \zeta(x, t)$, where t is time. Furthermore we define the density $\rho = 0$ and the pressure $p = 0$ for $z > \zeta(x, t)$, while the time mean of the properties in the region above the level of the wave troughs is taken over a full wave period. In view of these definitions, we can exchange the frequently appearing order of integration over depth and time, such that for any flow property q : $\int_{-d}^{\zeta} q \, dz = \int_{-d}^{\zeta_c} \bar{q} \, dz$, where ζ_c is the level of the wave crest and an overbar denotes time averaging. For ζ_c we have used the ensemble mean crest level, neglecting the contributions due to the turbulent fluctuations of the surface.

The mean momentum balance

The time mean balance equation for horizontal momentum of a vertical column of water extending from the bottom to the surface reads:

$$\frac{d}{dx} \int_{-d}^{\zeta_c} (\bar{p} + \rho \bar{u}^2) \, dz = \bar{p}_b \frac{dd}{dx} - \bar{\tau}_b \quad (9)$$

with \bar{p} the mean pressure, \bar{p}_b the mean bottom pressure and $\bar{\tau}_b$ the mean shear stress at the bottom. Furthermore, the radiation stress is defined as:

$$S_{xx} = \int_{-d}^{\zeta_c} (\bar{p} + \rho \bar{u}^2) \, dz - \int_{-d}^{\bar{\zeta}} \rho g (\bar{\zeta} - z) \, dz \quad (10)$$

We now follow Mei (1973) in defining a mean dynamic pressure on the bottom, being the difference between the mean pressure and the mean hydrostatic pressure on the bottom:

$$\bar{P}_b \equiv \bar{p}_b - \rho g (\bar{\zeta} + d) \quad (11)$$

Similarly, we define a mean dynamic pressure at a level z within the fluid being the difference between the mean pressure and the mean hydrostatic pressure at a level z within the fluid:

$$\bar{P} \equiv \bar{p} - \rho g (\bar{\zeta} - z) \quad (12)$$

Inserting the definitions (11) and (12) in eqs. (9) and (10) we obtain:

$$\bar{P}_b \frac{dd}{dx} - \rho g(d + \bar{\xi}) \frac{d\bar{\xi}}{dx} - \frac{dS_{xx}}{dx} - \bar{\tau}_b = 0 \quad (13)$$

where

$$S_{xx} = \int_d^{\xi_c} (\bar{P} + \rho \overline{u^2}) dz + \frac{1}{2} \rho g (\bar{\xi} - \bar{\xi})^2 \quad (14)$$

The mean dynamic pressure terms \bar{P}_b and \bar{P} may be expressed in terms of mean velocity properties by using the mean balance equation of vertical momentum of a vertical column of water extending from the bottom to the surface and from a level z within the fluid to the surface, respectively, as follows:

$$\bar{P}_b = \frac{d}{dx} \int_d^{\xi_c} \rho \overline{uw} dz - \rho \overline{w_b^2} \quad (15)$$

and

$$\bar{P} = \frac{d}{dx} \int_z^{\xi_c} \rho \overline{uw} dz - \rho \overline{w^2} \quad (16)$$

Except for the mean bottom shear stress $\bar{\tau}_b$ all terms in the above eqs. (13) to (16) were experimentally quantified, i.e. from the measured and partly extrapolated mean velocity properties and the measured mean and variance of the surface elevations. The distributions of the mean velocity properties $\rho \overline{u^2}$, $\rho \overline{u'^2}$, $\rho \overline{w^2}$, $\rho \overline{w'^2}$ and $\rho \overline{uw}$, where $\rho \overline{u^2} = \rho \overline{u'^2} + \rho \overline{u'^2}$ and $\rho \overline{w^2} = \rho \overline{w'^2} + \rho \overline{w'^2}$ (assuming zero correlation between periodic and turbulent motion), are presented in Fig. 4a–e. These experimental results enable us to investigate the validity of the common approximations for the mean dynamic pressure and to investigate the effects of turbulence and nonlinearity on the mean momentum balance.

Longuet-Higgins and Stewart (1962) have shown that, after neglecting some terms of higher than second order in wave amplitude, for simple, progressive waves on a gently sloping bottom, the mean dynamic pressure at the bottom and within the fluid may be approximated by $\bar{P}_b = 0$ and $\bar{P} = -\rho \overline{w^2}$, respectively. These approximations are based on a comparison to lowest order in wave amplitude of the momentum balance terms $\bar{P}_b dd/dx$ and dS_{xx}/dx and of the radiation stress terms \bar{P} and $\rho \overline{u^2}$. They may be checked for the nearshore motion on basis of the measurements. The magnitude of these terms as measured around the breakpoint (see Tables II and III) is taken to be representative for an experimental comparison. From the results it can be concluded that $\bar{P}_b dd/dx$ is negligibly small compared to dS_{xx}/dx and that only the term $\rho \overline{w^2}$ of the mean dynamic pressure \bar{P} cannot be neglected in general compared to $\rho \overline{u^2}$,

TABLE II

Comparison of mean momentum balance terms

mean momentum balance terms	$\bar{p}_b \frac{dd}{dx}$	$\frac{dS_{xx}}{dx}$
composing terms	$\left(\frac{d}{dx} \int_{-d}^{\zeta_c} \rho \overline{uw} dz \right) \frac{dd}{dx}$	$\frac{d}{dx} \int_{-d}^{\zeta_c} \rho u^2 dz$
boundary value at most	$\overline{\rho w}_{\max} (d + \zeta_c) \frac{dd}{dx}$	$\overline{\rho u^2}_{\min} \frac{dd}{dx}$ at least
magnitude at breakpoint (Nm^{-2}) at most	$4 \frac{dd}{dx}$ at most	$(2 - 20) \frac{dd}{dx}$ at least
see Figure	4e	4c + d

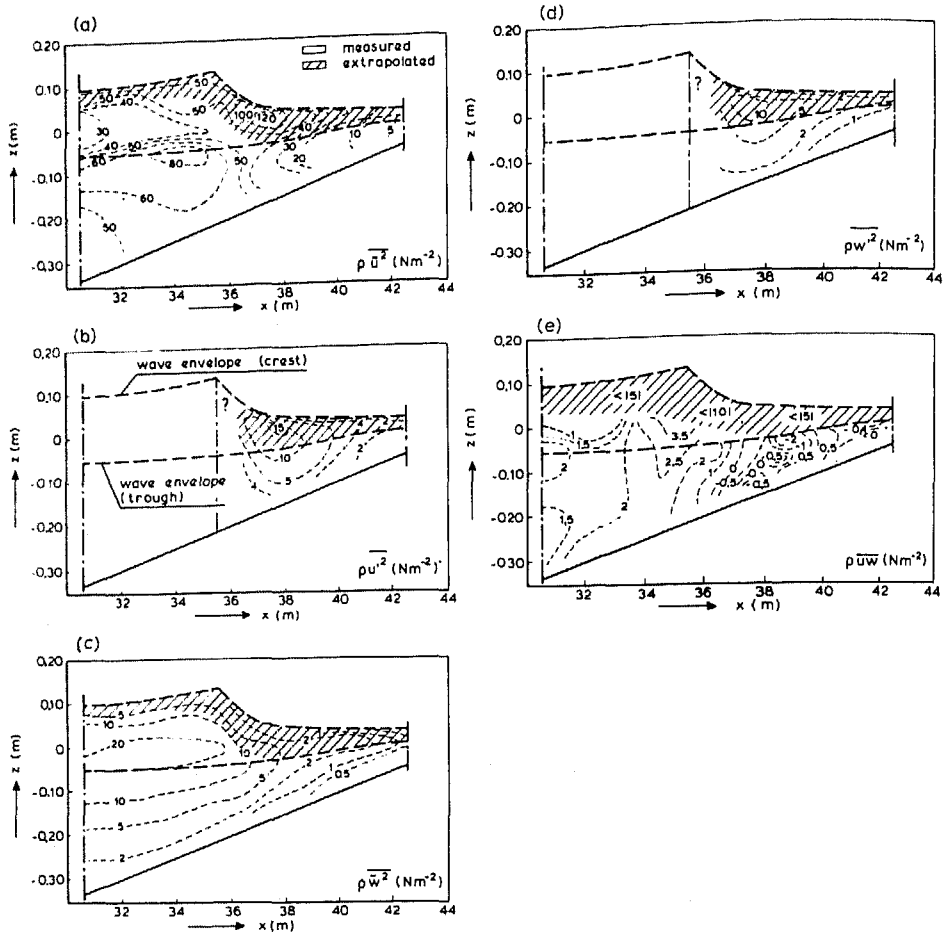


Fig. 4a, b, c, d, e. Isolines of mean velocity properties for test 1.

which experimentally confirms the validity of the second-order approximations of the mean dynamic pressures, even for the nonlinear nearshore motion on a gently sloping beach. Hence the mean bottom pressure can be approximated by the mean hydrostatic pressure at the bottom:

$$\bar{p}_b = \rho g (\bar{\zeta} + d) \quad (17)$$

and the mean momentum balance can be written:

$$\rho g (d + \bar{\zeta}) \frac{d\bar{\zeta}}{dx} + \frac{dS_{xx}}{dx} + \bar{\tau}_b = 0 \quad (18)$$

where

$$S_{xx} = \int_{-d}^{\zeta_c} (\rho \overline{u^2} - \rho \overline{w^2}) dz + \frac{1}{2} \overline{\rho g(\zeta - \bar{\zeta})^2} \quad (19)$$

The reduced expression for the radiation stress of eq. (19) is seen to consist of two terms, both of which have been measured. The second term, which is due to the fluctuations of the hydrostatic part of the pressure, is simply obtained from the variance of the measured surface elevations. The first term, which is due to the fluctuating velocities, is obtained through depth integration of the mean velocity property $\rho \overline{u^2} - \rho \overline{w^2}$, which may be separated in a periodic and a turbulent contribution, viz. $(\rho \overline{\tilde{u}^2} - \rho \overline{\tilde{w}^2}) + (\rho \overline{u'^2} - \rho \overline{w'^2})$. From the distributions of these terms as presented in Fig. 4a-d two important conclusions can be drawn. First, it can be observed that substantial contributions to the squared horizontal periodic velocity $\rho \overline{\tilde{u}^2}$ are found in the region above the level of wave troughs. Since this term dominates the magnitude of the radiation stress due to the fluctuating velocities, it can be concluded that finite amplitude effects on the radiation stress are important in the nearshore region. Secondly, it can be observed that the squared turbulent horizontal and vertical velocities $\rho \overline{u'^2}$ and $\rho \overline{w'^2}$ amount to 10% to 20% and 5% to 15% of the squared horizontal periodic velocity term. Since it is the difference, i.e. 5%, between these turbulent terms that determines the turbulent contribution to the radiation stress in the surf zone, it can be concluded that the effects of breaking-induced turbulence are weak.

We next discuss the mean bottom shear stress. The magnitude of this stress can be estimated using eq. (18), since the first two terms of this equation are

TABLE III

Comparison of radiation stress terms

radiation stress terms	\bar{P}	$\overline{\rho u^2}$
composing terms	$\frac{d}{dx} \int_z^{\zeta_c} \rho \overline{uw} dz$	$\overline{\rho w^2}$
boundary value	$\rho \overline{uw}_{\max} (d + \zeta_c)$ at most	$\overline{\rho u^2}_{\min}$ at least
magnitude at breakpoint (Nm^{-2})	4 at most	2 - 20 at least
see Figure	4e	4c + d

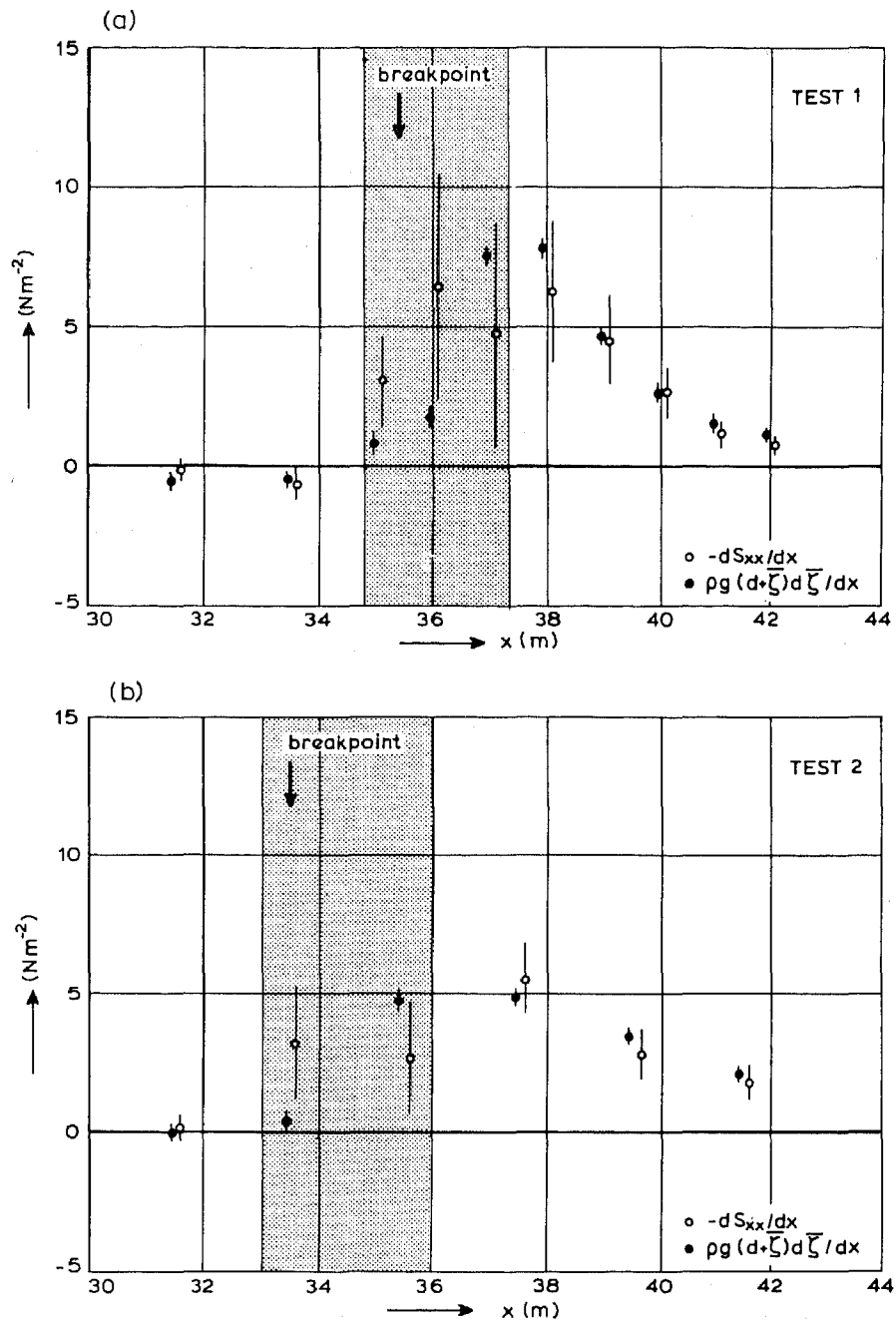


Fig. 5a, b. Observed gradients of radiation stress and mean water level.

known from the measurements. The first term, the net horizontal pressure force due to the slope of the mean water level, is obtained from the measured mean bottom pressures. The second term is found as the gradient of the radiation stresses obtained from the measured mean velocity properties and variance of the surface elevations at the cross-sections indicated in Fig. 1b as described above. The variations of these two terms are presented in Fig. 5a,b. The error bars of the radiation stress gradients represent the estimated uncertainties in the mean squared velocity terms due to the extrapolations made in the crest regions.

Apart from the region just before and right after breaking (the shaded area) the averaged bottom shear stress, that is the difference between the gradients of radiation stress and mean water level, is negligible within the accuracy of our measurements. For the region of rapidly changing wave shape around the initial breaking point, there appears to be a discrepancy, which would indicate a sudden increase of mean bottom shear stress. We are not convinced that this discrepancy is real. Contrary to the rapid changes at the surface the bottom flow field undergoes a gradual change only (see Fig. 4a), which would also cause a gradual increase of $\bar{\tau}_b$. Possibly we may have underestimated the experimental uncertainties around the breakpoint somewhat. This pertains both to the measurement of the radiation stress and the mean bottom pressure. Regarding the radiation stress, it is recalled that the extrapolation procedure of the mean flow properties is not well applicable in a region of rapid changes; also some underestimation of the variance of the surface elevations may be expected due to the sharply peaked waves. Regarding the mean bottom pressure reference is made to Battjes (1974), where the hydrodynamical aspects of the bottom pressure measurement system are discussed; Battjes concluded that due to nonlinearity of the system, the mean water level in the wells is not necessarily the same as that in the flume due to asymmetry in the back-and-forth motions, which are most pronounced around the breakpoint.

From the above we conclude that the present experiments justify the common neglect of the mean bottom shear stress in the mean horizontal momentum balance, with some reserve made for the region around the breakpoint. In this region the experimental inaccuracy is too high to draw a pertinent conclusion.

Linear and nonlinear theories

Many studies in which the nearshore momentum balance is evaluated use small-amplitude, linear wave theory (e.g. Longuet-Higgins and Stewart, 1963, 1964). To account for the effects of finite amplitude, James (1974) used a combination of nonlinear theories. These studies are all based on three main assumptions: (a) the bottom slope is so small that locally the waves behave as if the water were of constant depth; (b) outside the surf zone the shoreward flux of energy is constant; and (c) inside the surf zone the breaking wave motion is depth controlled and this motion can be sufficiently described by a

theory for non-breaking limit waves, i.e. breakers can be modelled by non-breaking waves of *maximum amplitude*.

Here we introduce an additional nonlinear approach based on the work of Cokelet (1977). This approach also assumes (a) and (b), while assumption (c) differs regarding the limiting property, i.e. breakers can be modelled by non-breaking waves of *maximum energy flux*.

In the following we will describe some details of the three approaches.

The *linear approach* applies the well-known linear expression for the relevant wave properties. For the radiation stress this yields:

$$S_{xx} = \left(\frac{1}{2} + 2kd/\sinh 2kd\right) \frac{1}{8} \rho g H^2 \quad (20)$$

Equation (20) expresses the radiation stress in terms of local wave height, wave number and water depth. Based on the assumptions (a) and (b), the so-called shoaling assumptions, the variations of wave height outside the surf zone can be calculated. These shoaling calculations were performed starting from the horizontal section of the flume. The theoretical breaker position is chosen at the point where the linear wave height over water depth ratio reaches an assumed maximum, which is commonly taken as 0.8 (see e.g. Longuet-Higgins, 1970). Besides this value also a value of 0.6 is used in the present study which yields better quantitative results as will be discussed later. Inside the surf zone these ratios are kept constant in accordance with assumption (c). In combination with the linear dispersion relation, which determines the local wave number, the radiation stress can now be calculated from the wave height variations as a function of the water depth using eq. (20). Outside the surf zone the still-water depth is taken as the water depth, whereas inside the surf zone the mean water depth is taken, since the wave induced set-up is no longer small compared to the still-water depth.

The *nonlinear approach* of James (1974) applies a combination of theories. We shall only describe some qualitative aspects of his approach; the reader is referred to the original paper for the details.

James uses a third-order Stokes theory for deep and intermediate water depths and a hyperbolic approximation of the cnoidal theory for shallow water. The relative water depth at which the change from the one theory to the other is made depends on the parameter $S = HL^2/d^3$. For deep-water wave steepnesses $H_0/L_0 < 0.09$ the transition point is always reached in the shoaling region. Outside the surf zone the variations of wave height and radiation stress can be determined as a function of mean water depth based on the shoaling assumptions. However, at the transition point between the theories, continuity in the energy flux leads to discontinuities in the wave heights up to 10% and even higher in the momentum fluxes. Here we have avoided the discontinuity problem by not performing shoaling calculations from the horizontal portion of the flume, but rather confining our calculations to the hyperbolic domain which covers our radiation stress measurements.

Following James' procedure for comparison with measurements, the position of the theoretical breakpoint was made to coincide with the point where the theoretical breaking wave height over mean water depth ratio equals the measured wave height over mean water depth ratio. At the theoretical breakpoint a maximum wave height over trough depth ratio is reached of 0.85, a figure agreeing well with the theoretical maximum for solitary waves. From experiments on spilling breakers on gentle slopes, James concluded that for all practical purposes breaking waves can be considered to behave as limit nonbreaking waves on a locally constant water depth. This supports the use of the hyperbolic theory for the limit wave throughout the surf zone (assumption c) in order to obtain the wave height and radiation stress as a function of mean water depth.

Our nonlinear approach is based on Cokelet's wave theory (1977) and uses the results of shoaling calculations performed with this theory by Sakai and Battjes (1980). We shall neither describe Cokelet's method of solution to the nonlinear, irrotational wave problem nor the details of Sakai and Battjes' method in applying the theory to the shoaling problem; again we refer to the original papers. Here we shall only describe how their results may be applied to calculate the radiation stress as a function of the mean water depth.

From Sakai and Battjes' shoaling curves the wave height outside the surf zone (starting from the horizontal section of the flume) can be found directly as a function of mean water depth. As shown by Cokelet for arbitrary water depths the energy flux, like many properties, has a maximum for waves lower than the highest. Consequently, as Sakai and Battjes show, waves maintaining a constant energy flux on a gradually decreasing water depth reach a depth where the energy flux is maximum and the gradient of the wave height variation becomes infinite. At this depth the breakpoint is assumed. For smaller depths, i.e. in the surf zone, we have assumed that due to a gradual dissipation ("spilling breaking") the waves at each depth adjust to the condition of maximum energy flux. The resulting wave height variation can also be found from the results of Sakai and Battjes, since their curve connecting the breakpoints of the shoaling curves describes the relative wave height to mean water depth variation for waves of maximum energy flux.

The wave heights outside and inside the surf zone so obtained as a function of mean water depth can be used as input parameter in Cokelet's tables to find the radiation stress variations. In accordance with Sakai and Battjes' procedure for the energy flux, the radiation stress in Cokelet's reference frame, S_{xx_c} , is transformed to a reference frame which is stationary with respect to the beach. In this reference frame the mean mass transport is zero on account of the impermeability of the beach, whereas it has a non-zero value, I_c , in Cokelet's formulation. For the radiation stress this gives:

$$S_{xx} = S_{xx_c} - I_c^2 / [\rho(d + \bar{\xi})] \quad (21)$$

After describing the three approaches to obtain the radiation stress as a

function of mean water depth, we are now left with describing how to obtain the radiation stress and mean water level as a function of horizontal distance normal to the beach. This merely involves integration of the mean momentum balance (eq. 18), in which we have neglected the mean bottom shear stress. For the linear and our nonlinear approach the integration was performed in shoreward direction from the horizontal section of the flume. For James' nonlinear approach the integration was performed in shoreward and — limited to the hyperbolic domain — in seaward direction from the theoretical breakpoint.

The method of integration of the mean momentum balance may also be applied to calculate the radiation stress variation *indirectly* from the measured variation of mean water level in addition to the radiation stress variation calculated *directly* from the measured variation of the mean flow properties. In this case the integration is performed from the mean water line in seaward direction. The radiation stresses calculated directly and indirectly from the measurements are referred to in the following as direct and indirect observations.

Discussion of results

The observed and calculated variations of radiation stress and mean water level for both tests are presented in Fig. 6a—d. The error bars of the directly observed radiation stress represent the uncertainty in the mean squared velocity terms due to the extrapolations made in the crest regions. Apart from the region right after the breakpoint, the outer breaking region, a good agreement is found between the direct and indirect observations, indicating that the mean bottom shear stress is negligibly small. As discussed before, the cause of the discrepancy in the outer region is not believed to be due to a sudden increase of the mean bottom shear stress but rather to unknown, additional experimental inaccuracies in that region. We therefore conclude that, with some reserve for the outer region, the on-offshore variations of radiation stress are simply balanced by the variations in mean water level. As a result the comparison between observed and calculated variations of radiation stress can be discussed parallel to the comparison between observed and calculated variations of mean water level.

For a qualitative comparison of observations and calculations two features of the observed variations of radiation stress and mean water level are important, i.e. (1) the maximum of the radiation stress just before the breakpoint and the corresponding minimum in the mean water level (maximum set-down) just before the breakpoint; and (2) the shoreward decreasing gradient of the mean water level in the surf zone, which is more pronounced for waves of higher incident steepness H_0/L_0 . Both these characteristics are predicted by the nonlinear theories, contrary to the linear theory. The linear theory yields a maximum in radiation stress and set-down at the breakpoint and a practically constant gradient of the mean water level in the surf zone.

The nonlinear theories are qualitatively very similar. They differ only slightly with respect to the position and magnitude of the maximum in radiation stress and set-down relative to the radiation stress and mean water level at the breakpoint. The present experiments do not allow for a comparison on these points.

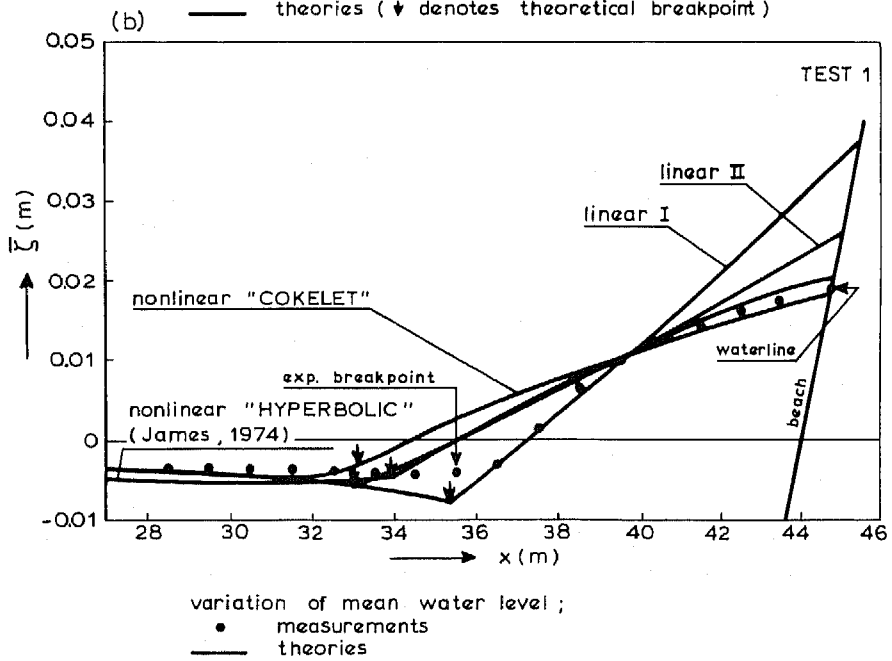
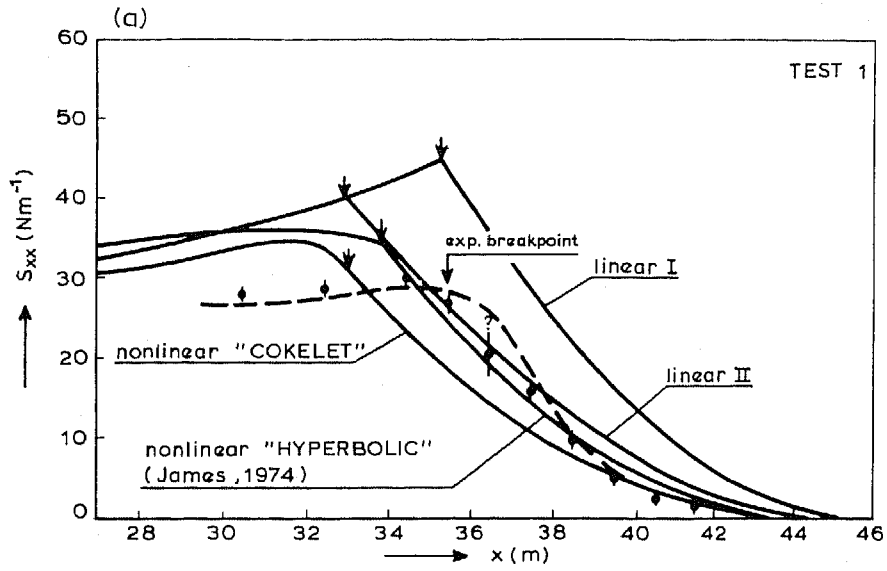
For a quantitative comparison of observations and calculations, two properties of the theories are important, i.e.: (a) the predicted magnitude of the radiation stress at the breakpoint; and (b) the predicted position of the breakpoint. Both nonlinear approaches overestimate the radiation stress at the break point by 25% at most, whereas the linear approach gives an overestimation of about 70% for $H/d = 0.8$ (linear-I) and 50% for $H/d = 0.6$ (linear-II). The overestimation is a primary cause of discrepancy between observations and calculations. In this respect the nonlinear approaches are superior to the linear approach. Further quantitative discrepancies between the observations and nonlinear calculations are mainly due to the poor prediction of the position of the breakpoint. Here James' approach is in somewhat closer agreement with the observations due to its closer prediction of the breakpoint. It is interesting to note that obviously the conclusion of Sakai and Battjes (1980) that: "although Cokelet's theory may be an exact theory for waves of constant form, its use for the calculation of wave shoaling does not necessarily give more reliable results near the break point than does the use of an approximate theory" also applies with regard to the radiation stress.

Finally, it is remarked that reduction of the H/d ratio to 0.6 (linear-II calculation) was found to yield a quantitative agreement between observations and linear calculations approximating that of the nonlinear calculations. This is due to the more seaward prediction of the breakpoint which compensates for the overestimation of the radiation stress at the breakpoint. The following must, however, be noted. Where the other approaches have applied a more or less theoretical initial breaking criterion, i.e. for the linear-I and James' nonlinear approach the theoretical maximum wave height over waterdepth ratio for solitary waves and for the present nonlinear approach the theoretical maximum wave height for arbitrary waves, the linear-II breaking criterion is chosen merely to obtain closer agreement with the observed radiation stress and mean water level variations in the surf zone.

CONCLUSIONS

Observations of radiation stress and mean water level were made for the case of periodic waves normally incident to a gently sloping beach. The radiation stresses were in fact calculated from measurements of the velocity field and the surface elevations.

Based on the observations some approximations with respect to the mean momentum balance were investigated. Experimental confirmation was found of the common mean dynamic pressure approximations and, with some reserve for the outer breaking region, of the neglect of the time mean bottom shear stress.



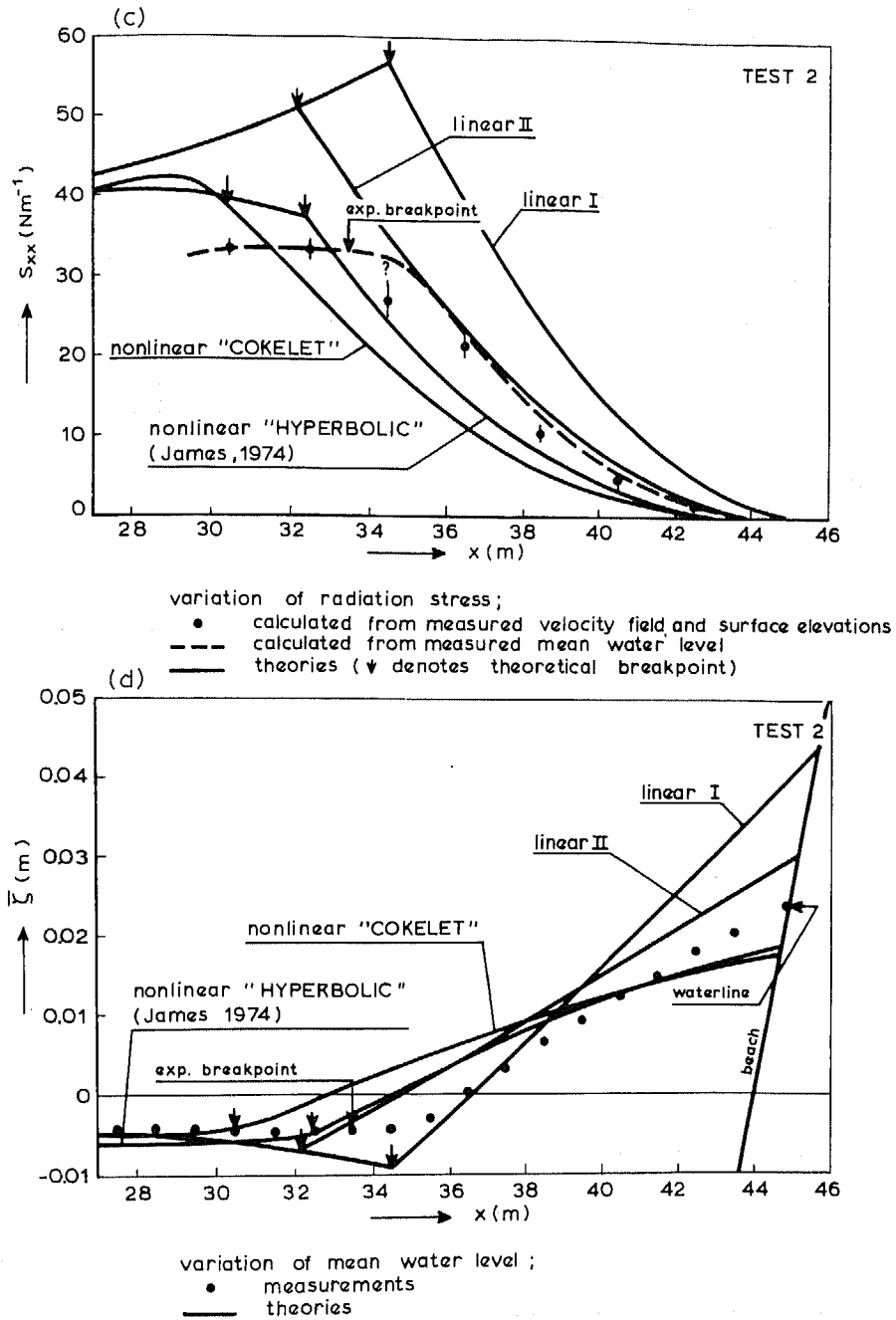


Fig. 6. a and c. Variation of radiation stress (tests 1 and 2). b and d. Variation of mean water level (tests 1 and 2).

The observed variations of radiation stress and mean water level were compared to the variations as calculated by a linear theory, James' nonlinear theory and a new nonlinear theory based on Cokelet's work.

The nonlinear theories are qualitatively superior to the linear theory, which is due to their prediction of two important features of the observed variations of radiation stress and mean water level, i.e. the maximum in radiation stress and set-down just before breaking and the decreasing slope of mean water level in the surf zone. These features are not predicted by the linear theory. The nonlinear predictions are nearly identical in these qualitative respects.

The nonlinear theories are also quantitatively superior to the linear theory. This is due to their close prediction of the radiation stress magnitude at the breakpoint compared to the overprediction by the linear theory. It is, however, the poor prediction of the position of the breakpoint by the nonlinear theories that withstands a close quantitative agreement with the observations. In this respect James' nonlinear theory is somewhat better than our nonlinear theory.

Finally, it was found that reducing the breaking wave height over water depth ratio from 0.8 to 0.6 in the linear calculations yielded quantitative results approximating those of the nonlinear theories.

The above results confirm the conclusions that the effects of finite amplitude are important for the on-offshore variation of radiation stress and mean water level, while the effects of the breaking-induced turbulence are less important. These conclusions were drawn beforehand from the measured distributions of the mean squared horizontal and vertical velocities.

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