Erasmus Mundus Programme
M.Sc. programme in
Coastal and Marine Management

CoMEM

Validation of extreme wave and crest height distribution

using laboratory simulation and field measurement data

Rahma Poeti Aniesah Djunaidi

July 2009
Thesis work done at the University of Southampton

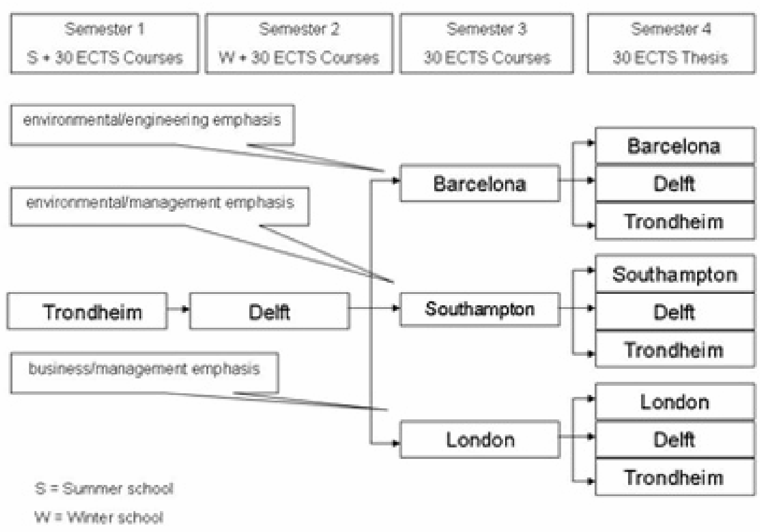
The Erasmus Mundus Master in Coastal and Marine Engineering and Management (CoMEM) is a two-year, English taught international Master's programme, in which five high-rated European universities participate. Focus is on the key issues involved in providing sustainable, environmentally friendly, legally and economically acceptable solutions to various problems in the CoMEM field.

The CoMEM MSc course is taught at five European universities:

* NTNU, Trondheim, Norway
* Delft Technical University, Delft, The Netherlands
* UPC, Barcelona, Spain
* City University, London, United Kingdom
* University of Southampton, Southampton, United Kingdom

The first year comprises two semesters of 30 ECTS each (course and project work), which will be spent at NTNU,Trondheim and Delft University respectively. This year provides academic and social coherence through a choice of compulsory and optional courses. It will establish a broad common foundation and prepare for the final year.

For the first half of the second year (one semester of 30 ECTS course and project work), three main routes with a particular emphasis are offered: either in environmental engineering at UPC, Barcelona, Spain, in environment and management at City University, London, or in business and management at the University of Southampton, Southampton, both in the United Kingdom. The third semester allows for specialisation in one of these three subjects. In the fourth and final semester a master's thesis at one of the three universities has to be made. The CoMEM programme therefore offers the following posibilities as illustrated in the diagram below:



Information regarding the CoMEM programme can be obtained from the programme director'
prof.dr. Marcel J.F. Stive
Delft University of Technolgoy
Faculty of Civil Engineering and Geosciencese
PO Box 5048
2600 GA Delft
The Netherlands
comem@tudelft.nl

UNIVERSITY OF SOUTHAMPTON

FACULTY OF ENGINEERING, SCIENCE AND MATHEMATICS

SCHOOL OF CIVIL ENGINEERING AND ENVIRONMENT

VALIDATION OF EXTREME WAVE AND CREST HEIGHT DISTRIBUTION USING LABORATORY SIMULATION AND FIELD MEASUREMENT DATA

RAHMA POETI ANIESAH DJUNAIDI

A dissertation submitted in partial fulfilment of the degree of

MSc in Coastal and Marine Engineering and Management

July 2009

TABLE OF CONTENTS

SUMMARY

This dissertation has focussed on the validation of extreme wave and crest height distribution. The empirical distribution was simulated based on laboratory and field measurement water elevation data. Both sets of data indicated the presence of abnormal wave ensuring the extreme condition aimed at in this study.

The literature review was performed to identify improved predictions of wave and crest height distribution. Wave height predictions which are involved in this study were based on a modified Weibull distribution. Along with these predictions, a later formula that involved nonlinear factor of wave steepness is also applied. Meanwhile, nonlinear factors have been increasingly involved in crest height prediction. The modified formulae which are compared include the nonlinear factor of water depth, wave steepness, and directional factor. In all cases, standard linear Rayleigh distribution is referred to in relation to how good the improved formulae are in predicting the distributions.

Based on wave height validation, Rayleigh shows better accuracy than the modified Weibull distributions. The standard Rayleigh distribution seems to fit the laboratory data, but deviates more widely from the field measurement data. The inadequacy of Rayleigh based on field validation showed the need of better prediction of nonlinear wave height in nature. Validation showed that the newly developed Rayleigh-Stokes prediction comes out with a slightly better prediction. Nonetheless, it still largely deviates from the observed distribution.

On the other hand, the inadequacy of the Rayleigh distribution is seen very clearly in relation to crest height validation. Newer nonlinear formulae are found to give a better prediction showing a stronger nonlinearity affect on crest height in nature. However these models show discrepancies from one another. It is possibly caused by the different methods underpinning the development of these formulae and the way nonlinear factors are included in their prediction.

ACKNOWLEDGEMENTS

I am sending my biggest gratitude to Prof John R. Chaplin as my supervisor for the patience and conscientiousness that he gave along the work. I would also like to thank Prof Robert Nichols as CoMEM coordinator program who tirelessly supervise the dissertation's progress of CoMEM group. I am giving my appreciation to Mr Marc Prevosto who has responded to my question concerning his previous work.

A sincere gratitude is also given to Mr. Gunawan Ariyanto for the assistance of understanding programming in MATLAB. My sincere gratitude is also sent to Mr. Rhahmadatul Hidayat for the numerical programs used in this study. And finally, I thank Ms. Sophie N Jain for her revision on my English.

DEDICATION

Alhamdulillah, thank you Allah, finally, my MSc dissertation is completed. I dedicate my work to my mother and father for being such a loving parents; my brother whom I care very much; all my housemates who cheer me up during the hectic period; all the CoMEM 2007/2009 whom I proud of; all CoMEM professors in NTNU, TUDelft, and University of Southampton for the precious knowledge; CoMEM ladies, especially Mariette, for the encouragement; European Union for the opportunity to see Europe; friends and relatives for their sincere wishes of my success; and every body who makes the work possible.

And Allah has extracted you from the wombs of your mothers not knowing a thing, and He made for you hearing and vision and hearts [i.e., intellectual] that perhaps you would be grateful

Quran: An-Nahl 16: 78

“So which of the favors of your Lord would you deny?”

Quran: Surah Ar Rahman 55:16

LIST OF FIGURES

LIST OF TABLES

LIST OF SYMBOLS

a	wave amplitude
a_c	highest crest height
a_{CN}	mode of N variables in Rayleigh-Stokes crest height distribution
a_N	mode of N variables
a_t	deepest trough heights
b_{CN}	scale parameter of N variables in Rayleigh-Stokes crest height distribution
b_N	scale parameter of N variables
b_2, b_3	coefficient of third order Stokes
C_{lin}	linear crest heights
C_r	non-linear crest heights
Cr_1	coefficient of significant wave height by Tucker
C_0,C_1,C_2	coefficient of Prevosto exceedance distribution
c	crest height
c_{diff}, c_{sum}	coefficient transfer functions determined by κ
cov	coefficient of variant
d	water depth
E	spectral bandwith of Ochi
E[x]	expected value of random variables x
e_0	transfer function of paddle
F(x)	cumulative distribution function
f_m	mean spectral frequency
f_p	peak period
f(x)	probability distribution function
$\widetilde{f}_m$	modified mean spectrum frequency of Prevosto
G(y)	probability of y smaller than certain value
g	gravity acceleration
g(x)	a particular function of x
H	wave height
H_{max}	maximum wave heights

H_{rms}	root mean square wave heights
$H_{1/3}$, H_s	significant wave heights
$\widetilde{H}_s$	modified significant wave height of Prevosto
iqr	difference between the third and first quartile
k	wave number
k_m	mean wave number
kr	kurtosis
k_1, k_2, k_3	number of interval in expressing statistical distribution
M	number of wave
m^k	k^{th} spectral moment
m_0	zeroth moment or variance of water surface elevation
N	total components of all events
N_z	total number of zero crossing
n	number of components in an event
Pr,p	probability
Q(x)	probability of exceedance of random variable x
Q_2	second quartile
R	wave steepness factor
R_D	distribution parameter of Rayleigh
$R_X(\tau)$	auto correlation function
r	range of data from minimum to maximum
S_1	wave steepness parameter of Forristall
$S\eta$	spectral density
$S_\eta^{PM}(f)$	spectral density of Pierson-Moskowitz
s	directional factor of Prevosto
sk	skewness
T	wave period
T_c	crest period
T^D,T^S	second order transfer functions
T_p	peak period
T_s	time duration
T_z	zero crossing periods
T_0	peak period

T_{01}	mean wave period
T_{02}	mean wave period
t	time
Ur	Ursell number of Forristall
Var	variant
X(t)	stochastic process
x	random variables, wave height in this case
x_{rms}	root mean square
y	random variables, wave height in this case
Z	normalised variable of Gaussian distribution
z	normalised variable of Rayleigh distribution
α	scale parameter
α_{fm}	modification factor of mean spectra frequency
α_{Hs}	modification factor of significant wave height of Prevosto
α_2	scale parameter of 2D Forristall
α_3	scale parameter of 3D Forristall
β	shape parameter of exceedance distribution
β_2	shape parameter of 2D Forristall
β_3	shape parameter of 3D Forristall
γ_1, γ_2	parameters of Haring crest height distribution
Δ	interval
ε	spectral width parameter
ζ	maxima symbol used by Ochi
η	water elevation
θ	scale parameter of exceedance distribution
κ	multiplication of wave number and water depth ($k_m d$)
μ	central value
ξ	normalized maxima of Ochi
Π	function that determines the value of c_{diff} in Prevosto formula
σ	standard deviation
τ	time difference
Φ	standard normal function
φ	wave phase

1 INTRODUCTION

Engineering problems are closely related to natural phenomena which behave randomly. Based on this fact, it is strongly suggested that field measurements are undertaken to complement every engineering activity. This effort will prevent the loss of time and money which might be caused by delays, such as caused by accidents, and other related factors.

Field observation will enable engineers to understand the characteristics geographical nature of a particular site. Random data which was obtained from the observation will be analysed by applying specific engineering procedures. The product of this analysis will then be used as load factor in designing various civil engineering structures.

In order to obtain useful parameters from the random observation, engineers need to utilise the theory of probability and statistics. The theory of probability helps engineers in getting the visual image of frequency of the random observed event. Likewise, the theory of statistics assists in developing characteristics of interest from the random processes in nature.

Figure 1-1 Offshore platform under storm conditions in Western Norway

Source: http://skywatch-media.com/2008/01/storm-hammers-norway-oil-platform-shut.html

Understanding the nature of ocean water is becoming increasingly important along with the advancement of offshore technology. The latest concern relating to ocean water is given to wave and crest height prediction. Recent studies focus on the extreme waves which have caused serious problem to the offshore operations.

Inappropriate design wave and crest height has caused serious damage to offshore platforms. Severe storms for example impaired or at least delayed offshore operation as depicted in Figure 1-1 where storms shut down an oil platform in western Norway. For this reason, it is important to have a precise prediction of the behavior of extreme waves.

The conditions which were described in the previous paragraph form the motive of this study. The natural event which will be discussed in this study is wind-generated waves under extreme conditions. Two primary wave parameters which are going to be compared are wave and crest height which will be presented in probability distribution. Focus will be given mainly to the uppermost region of wave and crest height value.

1.1 Background

Although it is visibly easy to recognise the presence of wave and crest qualitatively in nature, there are still few quantitative predictions of wave and crest height distribution (Forristall 2000). Many scientists have argued over the adequacy of first order linear prediction on wave and crest distribution. They believe that waves in nature are actually nonlinear so that standard linear distribution is no longer suitable to be used in practice.

Following their arguments, new modified empirical formulae were developed based on the second and the third order theories of Stokes. Initial improvements of standard linear wave height model were done by fitting the measurement data using Weibull’s distribution. Different scale and shape parameters of Weibull were suggested by Haring (1976 cited Nerzic and Prevosto 1998), Forristall (1978 cited ibid.), and Krogstad (1985 cited ibid.). A further study by Nerzic and Prevosto (ibid.) modified the linear prediction of Rayleigh using third order Stokes. The Rayleigh-Stokes model involves the nonlinear factor of wave steepness in its wave height prediction.

Along with the newly developed method for prediction of wave height, nonlinearity factors of wave crest were explored deeper in order to result in better prediction of crest height. The initial effort of developing crest height distribution was done by Jahn and Wheeler (1972 cited Prevosto 2000.) based on the linear transformation of Rayleigh. Nonlinear effect of water depth is considered in their formula. Later, scientists found that crest height is strongly influenced by the wave steepness: as a result, Tayfun (1980 cited Forristall 2000) and Huang (1986 cited ibid.) came out with a new formula derived from the narrow band model. Wave steepness is also considered by Kriebel-Dawson (1993 cited ibid) who developed crest height models from 2D irregular second order Stokes. Taking this a step further Forristall tried to create formulae that involved both water depth and wave steepness based on the perturbated narrow band model. One newest crest prediction of Prevosto which is based on the perturbated Weibull model will also be considered as the one and only formula that engages directional spreading. The Rayleigh-Stokes prediction for crest height is also discussed alongside the Ochi distribution which considered spectral width factor in the formula.

Validations have been done along with the development of wave and crest height prediction; nevertheless, there are still disagreements on how well these models fit the measurement data. Many factors cause the discrepancies such as the location of study, measurement techniques, and the characteristics of wave data used for validation.

Current validation uses laboratory simulation and field observation data. Six-day records which were taken from North Alwyn, North Sea present the fully developed multidirectional sea where wave interaction happens. On the other hand, hours of laboratory simulated water elevation records are considered to represent an idealised unidirectional irregular wave.

1.2 Objectives

This study is mainly aimed at validating new developed nonlinear prediction of wave and crest height distribution. There is still no one source of literature that presents all complete theories on the prediction of crest and wave height distribution. For this reason, an initial literature survey is also included as one of the aims. The objectives of this study are focused in the following points:

- Finding, studying and simulating new developed formulas of wave and crest height distributions

- Knowing how well the modified distributions of wave and crest height compared with the linear and nonlinear prediction using laboratory and field measurement data
- Analysing the causes of discrepancies between models and determining the factors that affect accuracy of wave and crest height probability.

1.3 Structure of the Report

The report is divided into five chapters. The first chapter is an introduction to the study including background, scoping, objectives, and the structure of the report.

The literature review is presented following the introduction. The literature review presents theories that support the work and analysis. They consist of the basic understanding of probability and statistical theory, the theory of irregular waves, and various predictions of wave and crest height distribution.

The laboratory simulation and field measurement were not conducted by the author because it is not the main aim of this study. Nevertheless, general information concerning the records will be explained in the third chapter. Chapter three will discuss the methodology which is used in the study.

The results, discussion and analysis are presented in the fourth chapter. This chapter presents the analysis of water elevation distribution (validation of Gaussian assumption), zero-crossing wave, validation of wave height distribution, and validation of crest height distribution. Relevant discussion and analysis follows the results of numerical analysis.

Finally, the overall conclusions and recommendations are presented in the last chapter. This chapter closes the study of "Validation of Extreme Wave and Height Distribution Using Laboratory Simulation and Field Measurement Data".

2 LITERATURE REVIEW

2.1 Probability and Statistic

Observation is an important part of planning to describe the natural behaviour of the observed object. This activity is done to optimise the time and the cost of the operation stage. Engineering deals with fluctuations of natural phenomena which are uncertain. The uncertainties happen even in the same location under the same condition. This is referred to as a random event. For this reason, site observation is necessary to compensate for the uncertainties in design and planning. Section 2.1 discusses the probability and stochastic theory of stochastic processes which is referred to many text books (eg. Wurdjanto 2004, Papoulis 1990, Berry and Lindgren 1990).

2.1.1 Stochastic Process

There are two ways to deal with uncertain phenomena. The first method is called the deterministic analysis (Naess 2007). This method requires a full history of the related event. Another approach is stochastic analysis. In stochastic analysis, the statistical concept is used to represent the event. This subchapter presents the latter where it will later be shown that it is wise to use the probability and statistic to deal with the uncertainty of nature.

Many physical events of interest to the engineering discipline are stochastic processes. On the examination of a number of records, it can be seen that each record is different; even when they are developed under the same conditions. It is hard to draw a pattern of the behaviour of these physical phenomena that are very irregular. The irregularity is clearly shown when a closer look of the record is taken. It is not impossible that striking features are found in the observation especially when data are zoomed. It shows that there are substantial differences between the records. Nevertheless, considering all records they appear somewhat similar to each other. For this reason, some basic assumptions are taken in analysing the stochastic processes.

Using mathematical definition, a particular phenomenon defined as $X(t)$ is called a stochastic process if $X(t)$ represents a random variable for each time value of t (ibid.). In this case t extends between certain intervals from possible extreme condition of negative infinite $-\infty$ and positive infinite ∞. In order to undertake a stochastic analysis of the physical events in nature, two fundamental assumptions are assumed. Those assumptions are that the stochastic process follows the stationary and ergodic laws. Each of these two definitions is explained below.

2.1.2 Definition of Probability

Probability can be defined as the possibility of occurrence of one specific event relative to the total events. Based on the fundamental theory of probability, a group of data consists of three main parts. The first part is sample point which is defined as a particular element of data denoted as x in Figure 2-1. One or more sample points clustered form a subset denoted as X. All subsets and sample points are located in the sample space symbolised as S.

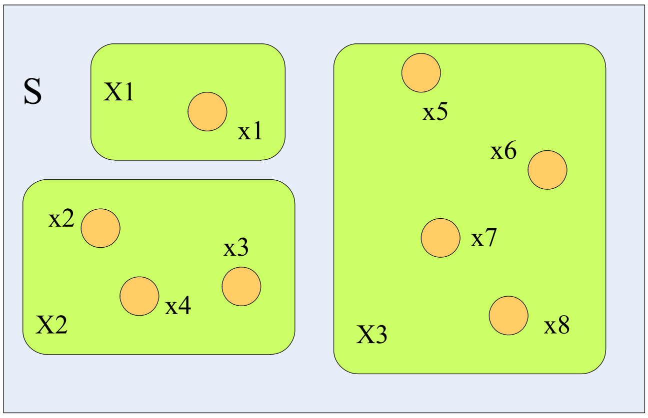


Figure 2-1 Illustration of data components in probability

The previous figure can also be expressed by the following mathematical term:

- x is the element of S denoted as $x \in S$
- X is the subset of S denoted as $X \subset S$
- x is the component the event X denoted as $x \in X$

In another words, each event may be represented by each of its elements (assuming $x_1 < x_2 < x_3 < x_4 < x_5 < x_6 < x_7 < x_8$) as follows:

- $X1$ is an event when $X = x_1$
- $X2$ is an event when $X = \{x_2, x_3, x_4\}$ or $\{x_2 < X < x_5\}$
- $X3$ is an event when $X = \{x_5, x_6, x_7, x_8\}$ or $\{x_5 < X < x_8\}$

Based on the previous definition, *X* refers to random variables. In this study, *X* can be replaced as wave height or crest height in the sample space where *x* is the value of each variable. The probability of a variable *X* is defined as the sum of *x* in the event, denoted as n, compared with the sum of *x* in all events, denoted as *N*. The mathematical term is written in the following:

$Pr\ (X) = n / N$ **(Equation 2-1)**

Where

$Pr\ (X)$	= the probability of an *X* event
n	= the number of components in the event
N	= the total components of all events

2.1.2.1 Cumulative Distribution Function

Before discussing cumulative distribution function, it is necessary to touch on a term of probability mass function, denoted by *p(x)*. Distribution mass function is a function that expresses the probability of random variable *X*. A mathematical expression of this distribution is noted in the following:

$p\ (x) = Pr\ (X{=}x)$ for all value of *x* **(Equation 2-2)**

Cumulative distribution function, usually denoted as *F(x)* is a function that expresses the probability distribution in the interval of -∞ to *x*. The mathematical expression of this type of distribution is written in the following:

$F\ (x) = Pr\ (X < x)$ **(Equation 2-3)**

$Pr\ (X \leq x)$ means probability of random variables less than or equal to *x*. In relation to distribution mass function, the probability is written as follows:

$$\Pr(X \leq x) = \sum_{x_i \leq x} p(x_i)$$ **(Equation 2-4)**

$$F(x) = \sum_{x_i \leq x} p(x_i)$$ **(Equation 2-5)**

For a specific interval, for example between *x* and *x + dx,* the cumulative distribution function is expressed in the following:

$$\Pr(x \le X \le x + dx) = \sum_{x_i \le x+dx} p(x_i) - \sum_{x_i \le x} p(x_i) \qquad \textbf{(Equation 2-6)}$$

2.1.2.2 Probability Density Function

Another way to show the probability of particular random variables beside probability mass function is probability density function, denoted as *f(x)*. Probability density function for specific interval *x* and *x* + *dx* is denoted as follows:

$$\begin{aligned} f(x) &= \frac{(F(x+dx) - F(x))}{dx} \\ &= \frac{dF(x)}{dx} \end{aligned} \qquad \textbf{(Equation 2-7)}$$

The previous mathematical term shows that probability density function is the first derivation of cumulative distribution function. Or from another approach, the cumulative distribution function is the integration of probability density function between -∞ and *x*.

$$F(x) = \int_{-\infty}^{x} f(x)dx \qquad \textbf{(Equation 2-8)}$$

In this integration, the probability is not represented by *f(x)* itself but by the notation of *f(x)dx* which is equal to the probability of *X* in the interval of -∞ and *x*, denoted by $Pr\ (x \le X \le x+dx)$. The relationship mathematically is written in the following:

$$\begin{aligned} \Pr(x \le X \le x + dx) &= \int_{-\infty}^{x+dx} f(x)dx - \int_{-\infty}^{x} f(x)dx \\ &= F(x+dx) - F(x) \end{aligned} \qquad \textbf{(Equation 2-9)}$$

The probability density function is usually presented in the form of a two dimensional Cartesian coordinate where the *x* and *y* axes show respectively the random variable *X* and its probability density function. The probability value can be obtained from the integration of the curve in a specific interval which is equal to the area beneath the curve of certain interval.

Data from the field measurement are discrete. To get a representative distribution of the data, it is necessary to determine an appropriate value of *dx*. The chosen value should be able to cope with the whole range of measured data to construct continuous distribution. Some theories which are used in this study to decide an appropriate number of intervals are depicted in the following:

- $k_1 = \sqrt{N}$ **(Equation 2-10)**
- $k_2 = 1 + 3.3 \log N$ (Sturges 1926) **(Equation 2-11)**
- $k_3 = \frac{r.N^{1/3}}{2.iqr}$ (Freedman and Diaconis 1981) **(Equation 2-12)**

  Where

  r = the range of minimum and maximum value of data

  N = the amount of data

  iqr = the difference between the third and first quartile

2.1.3 Statistics

In connection with probability theory, statistical terms are introduced. These statistical terms help to present the characteristic value of random variables. The terms which will be discussed in this chapter can generally be grouped into three, which are expected value, central value and distribution value. Each of these groups will be concisely explained below.

2.1.3.1 Expected Value

Concerning the continuity of data, there are two ways of presenting the expected value of particular random variables. For discrete data which has probability mass function $p(x_i)$, the expected value $E(x)$ is denoted as:

$$E[X] = \sum_{all\ x_i} x_i p(x_i)$$ **(Equation 2-13)**

While the expected value of continuous random variables is presented in the form of integration of its density function:

$$E[X] = \int_{-\infty}^{\infty} x f(x) dx$$ **(Equation 2-14)**

The expected values also apply for functional variables; for example $g(x)$, denoted as follows:

$$E[g(X)] = \sum_{all\ x_i} g(x_i)p(x_i) \quad \text{for discrete variables} \quad \textbf{(Equation 2-15)}$$

$$E[g(X)] = \int_{-\infty}^{\infty} g(x)f(x)dx \quad \text{for continuous variables} \quad \textbf{(Equation 2-16)}$$

In addition, and related to moment theory, the expected value of certain function $g(X) = X^k$ is equal to k^{th} moment:

$$m_k = E[g(X)] = E[X^k] \quad \textbf{(Equation 2-17)}$$

Along with the previous theory, the expected value of $g(X) = (X - \mu)^k$ is called k^{th} spectral moment which is denoted as follows:

$$\mu_k = E[g(X)] = E[(X - \mu)^k] \quad \textbf{(Equation 2-18)}$$

The relationship between the first three stages of moment and spectral is elaborated in the following:

Moment

$$m_1 = E[X] = \mu \quad \text{(mean value)} \quad \textbf{(Equation 2-19)}$$

$$m_2 = E[X^2] = \mu_2 + \mu^2 \quad \textbf{(Equation 2-20)}$$

$$m_3 = E[X^3] = \mu_3 + 3\mu\mu_3 + \mu^3 \quad \textbf{(Equation 2-21)}$$

Central Moment

$$\mu_1 = E[(X - \mu)] = 0 \quad \textbf{(Equation 2-22)}$$

$$\mu_2 = E[(X - \mu)^2] = m_2 - m_1 m_2 \quad \text{(variant)} \quad \textbf{(Equation 2-23)}$$

$$\mu_3 = E[(X - \mu)^3] = m_3 - 3m_1 m_2 + 2m_2 m_3 \quad \textbf{(Equation 2-24)}$$

2.1.3.2 Central Value

There are several terms which are used to define the central value of random variables. The most common term is known as mean value. Mean value represents the average of random variable *X* which is stated in both discrete and continuous data.

$$\mu_X = E[X] = \sum_{x_i} x_i p(x_i) = \frac{1}{N}\sum_{i=1}^{N} x_i \quad \textbf{(Equation 2-25)}$$

$$\mu_X = \int_{-\infty}^{\infty} x\, f(x) dx \qquad \text{(Equation 2-26)}$$

Another term correspondent with the dominant value in the distribution is called modus. Modus represents the most frequent value that occurs from random variable *X*. This statistical parameter shows the *X* value with the highest probability of occurrence.

Another central value which represents the middle value of sorted random variables, *X*, is known as median, denoted as $F(x_m) = 0.5$. In statistics, the term of quartile is also used. Quartiles divide data which have been sorted from the maximum value to the minimum value into four regions. As result, there are three quartile values of data. The most common is known as second quartile Q_2. It represents the value of the mid data, the same definition as median. Along with second quartile, Q_2, there are also first and third quartiles. They represent the mid variable of the first and the last one fourth of the sequence, respectively.

2.1.3.3 Distribution Values: Variance and Standard Deviation

There are three parameters which are generally used to show the distribution value of data. Those parameters are variant, standard deviation, and coefficient of variant. Variant is the second central moment of the expected value, denoted as *Var[X]*. (Note that it is different from variance definition of water surface elevation which is defined as the zero moment of the wave energy spectrum.)

$$\begin{aligned} Var[X] &= \sum_{x_i} (x_i - \mu_x)^2 \Pr(x_i) \\ &= \frac{1}{N} \sum_{i=1}^{N} (x_i - \mu_x)^2 \end{aligned} \qquad \text{(Equation 2-27)}$$

For continuous *X* with probability density function of *f(x),* the variant is formulated in the following:

$$Var[X] = \int_{-\infty}^{\infty} (x - \mu_x)^2 f(x) dx \qquad \text{(Equation 2-28)}$$

It can also be written in term of the mean square value and the square of the mean value as follows:

$$Var[X] = E(x^2) - \mu_x^2 \qquad \text{(Equation 2-29)}$$

The second distribution parameter known as standard deviation, denoted as σ_X, is obtained from the root of variant. In mathematics, it is stated as follows:

$$\sigma_X = \sqrt{Var(X)} \quad \textbf{(Equation 2-30)}$$

Both variant and standard deviation give information on how wide the distribution of the data is. In addition a dimensionless parameter is introduced. It is known as the coefficient of variation (*cov*) which is the ratio between the standard deviation and the mean value showing the relative deviation of data.

$$\text{cov} = \frac{\sigma_X}{\mu_X} \quad \textbf{(Equation 2-31)}$$

2.1.3.4 Distribution Values: Skewness and Kurtosis

Skewness is a statistical parameter that measures distortion of particular distribution and indicates direction of the distortion. For this purpose, skewness is related closely to the shape of the probability distribution; it indicates the asymmetry of the distribution. The value of skewness can be negative to positive. Positive skewness indicates that the tail part of the distribution is extending out to the right side; this type of skewness is called positively skewed. On the other hand, negative skewness indicates that the distribution is extending to the left, also known as negatively skewed.

Initial mathematical expression of skewness as written in Karl L. Wuensch's paper (2007) was proposed by Karl Pearson in 1895. Based on Pearson, skewness is measured by standardising the difference between the mean and the mode. The mathematical expression is depicted in the following:

$$sk = \frac{\mu - \text{mod}e}{\sigma} \quad \text{where sk denotes the skewness} \quad \textbf{(Equation 2-32)}$$

Nevertheless, sample modes cannot be used as an appropriate representation of population modes (Wuensch 2007). To overcome this barrier, Stuart and Ord in 1994 (ibid) suggested a new approach in estimating the difference between the mean and the mode by multiplying the difference between the mean and the median by three. Their suggestion was later re-written by many statisticians by excluding the three. Their formula causes the skewness value to vary between -1 and 1.

$$sk = \frac{(\mu - median)}{\sigma} \quad \textbf{(Equation 2-33)}$$

There are some other definitions of skewness proposed; however the one which is used in this study is known as Fisher's skewness. This type of skewness is calculated using the third moment about the mean.

$$sk = \frac{\sum (X - \mu)^3}{n\sigma^3} \quad \textbf{(Equation 2-34)}$$

The skewness shows the tendency of data deviation about the mean. The great value of skewness lies in the fact that it indicates the presence of outliers in the data. For a data sample, the Fisher's skewness is calculated as follows in a slightly different format:

$$sk = \frac{n}{(n-1)(n-2)} \sum \left(\frac{X - \mu}{\sigma} \right)^3 \quad \textbf{(Equation 2-35)}$$

Again, as discussed by Wuensch (ibid.), another distribution values is known as kurtosis. This parameter was introduced by Karl Pearson in 1905 in a simple mathematical form as follows:

$$kr = \frac{\sum (X - \mu)^4}{n\sigma^4} - 3 \quad \text{where } kr \text{ denotes the kurtosis} \quad \textbf{(Equation 2-36)}$$

Although it was introduced by Pearson, the above equation is commonly referred to as Fisher's kurtosis or kurtosis excess. Kurtosis is a measure of how flat the top of a particular symmetric distribution is compared with the normal distribution which has the same value of variance. For a data sample, an unbiased estimator of kurtosis is presented in the following:

$$kr = \frac{n(n+1)\sum (X - \mu)^4}{(n-1)(n-2)(n-3)} - \frac{3(n-1)^2}{(n-2)(n-3)} \quad \textbf{(Equation 2-37)}$$

When the value of kurtosis is less than zero, it shows a more flat-topped distribution; another term for this condition is 'platykurtic'. However, if the value of kurtosis is greater than zero, it means the distribution is less flat-topped 'leptokurtic'. The condition in between where kurtosis is equal to zero is referred to as an equally flat-topped distribution or 'mesokurtic'. Related to the size of the tail, leptokurtic is also referred to as the fat in the tails while platykurtic as the thin in the tails (Wuensch 2007).

2.1.4 Stationary and Ergodicity

Based on Naess (2007) there are actually several ways of defining stationary process. However, he chose to use the simplest definition. Stochastic process of $X(t)$ can be defined as (weakly) stationary if the following two conditions are fulfilled. The first condition is that the expected value of $X(t)$ denoted as $E[X(t)]$ is constant. It is actually the same as saying the mean value is constant.

$m_X = E[X(t)] = \text{constant}$ **(Equation 2-38)**

The second condition is that the expectation of random variable $X(t)$ times the expected value of t added with a particular time interval τ, denoted as $E[X(t)X(t+\tau)]$ is independent of t. It can also be written that $E[X(t)X(t+\tau)]$ only depends on the time interval τ. The value of $E[X(t)X(t+\tau)]$ is known as auto correlation function.

$R_X(\tau) = E[X(t)X(t+\tau)] = \text{function only of } \tau$ **(Equation 2-39)**

The second common assumption is that the stochastic process is ergodic. If an event is said to be ergodic, then it is also automatically defined as stationary. Naess (ibid.) defined that a stochastic process is called ergodic if every ensemble mean can be replaced by a time averaged over a single realisation. As a result, each and every realisation has the statistical properties of the whole ergodic process. Under this assumption, a person may calculate the statistical properties of one single time history to represent the whole process. Fortunately, these concepts can be applied to the observation analysis of physical phenomena of engineering interest.

2.1.5 Probability Distribution Function

Statistical properties are used to represent the characteristic of a data set. Each property will affect the shape of the distribution. The most common type of distribution is known as Rayleigh distribution, which has one parameter determining the shape. One parameter which influences the distribution is the mean square value (R_D). Some of the distributions are discussed briefly in the following:

1. <u>Normal Distribution (Gaussian Distribution)</u>

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma_x} \exp\left[-\frac{1}{2}\left(\frac{x-\mu_x}{\sigma_x} \right)^2 \right] \quad \textbf{(Equation 2-40)}$$

Where:

$$\mu_x = \frac{1}{N}\sum_{i=1}^{N} x_i \quad \textbf{(Equation 2-41)}$$

$$\sigma_x = \sqrt{\frac{1}{N}\sum_{i=1}^{n} (x_i - \mu_x)^2} \quad \textbf{(Equation 2-42)}$$

Normal distribution is characterised by following:

- symmetric at $x = \mu_x$ **(Equation 2-43)**
- $\Pr(x \le x^*) = F(x) = \int_{-\infty}^{x^*} \frac{1}{\sqrt{2\pi}\sigma_x} \exp\left[-\frac{1}{2}\left(\frac{x-\mu_x}{\sigma_x} \right)^2 \right] dx$ **(Equation 2-44)**
- $\Pr(a \le x \le b) = \int_{a}^{b} \frac{1}{\sqrt{2\pi}\sigma_x} \exp\left[-\frac{1}{2}\left(\frac{x-\mu_x}{\sigma_x} \right)^2 \right] dx$ **(Equation 2-45)**
- $\Pr(x \le \mu_x) = \Pr(x \ge \mu_x) = 0.5$ **(Equation 2-46)**

2. <u>Standard Normal Distribution</u>

Gaussian distribution with zero mean value and unit standard distribution is known as Standard Normal Distribution. This type distribution is written as follows:

$$f(z) = \frac{1}{2\pi} \exp\left[-\left(\frac{1}{2} \right) z^2 \right] \quad \textbf{(Equation 2-47)}$$

Where

$$Z = \frac{x-\mu_x}{\sigma_x} \quad \textbf{(Equation 2-48)}$$

The standard normal distribution is characterized as follow:

- symmetric at $z = 0$ **(Equation 2-49)**
- $\Pr = (z \le 0) = \Pr(z \ge 0) = 0.5$ **(Equation 2-50)**

- $F\left(z^*\right)=\Pr\left(Z \le z^*\right)=\Phi\left(z^*\right)=\int_{-\infty}^{z^*}\frac{1}{\sqrt{2\pi}}\exp\left[-\frac{z^2}{2}\right]dz \quad , z \ge 0$ **(Equation 2-51)**
- $\Pr\left(Z \ge z^*\right)=1-\Pr\left(Z \le +z^*\right)=1-\Phi\left(z^*\right) \qquad z^* \ge 0$ **(Equation 2-52)**
- $\begin{aligned}\Pr\left(Z \le -z^*\right)&=\Pr\left(Z \ge +z^*\right)\\ &=1-\Pr\left(Z \le +z^*\right)=1-\Phi\left(z^*\right); z^* \ge 0\end{aligned}$ **(Equation 2-53)**
- $\Pr\left(Z \ge -z^*\right)=\Pr\left(Z \le +z^*\right)=\Phi\left(z^*\right) \qquad z^* \ge 0$ **(Equation 2-54)**

3. Rayleigh Distribution

Related to wave height distribution, Rayleigh describes the peak to trough distribution of wave elevation. Cumulative and Density Rayleigh distribution are described as follows:

$$F(x)=\int_0^x \frac{2x}{R_D}\exp\left(-\frac{x^2}{R_D}\right)dx=1-\exp\left(-\frac{x^2}{R_D}\right) \quad \textbf{(Equation 2-55)}$$

$$f(x)=\frac{2x}{R_D}\exp\left(-\frac{x^2}{R_D}\right) \quad \textbf{(Equation 2-56)}$$

Where R_D notes the distribution parameters which is mathematically written as follows:

$$R_D=\frac{1}{N}\sum_{i=1}^{N}x_i^2 \quad \textbf{(Equation 2-57)}$$

The root R_D is known as root mean square which is denoted by x_{rms}.

4. Dimensional Rayleigh Distribution

Dimensional Rayleigh Distribution has a fixed shape which is written as follows:

$$f(z)=z\exp\left(-z^2\right) \quad \textbf{(Equation 2-58)}$$

where $z=\frac{x}{x_{rms}}$ **(Equation 2-59)**

This type of distribution was defined for ease in determining the value of $x_{1/n}$.

$$\Pr\left(x \ge x_p\right)=\int_{x_p}^{\infty}\frac{2x}{R_D}\exp\left(-\frac{x^2}{R_D}\right)dx=\frac{1}{n} \quad \textbf{(Equation 2-60)}$$

Hence the value of x_p can be determined if n is known.

$$\exp\left(-\frac{{x_p}^2}{R_D}\right)=\frac{1}{n} \quad \textbf{(Equation 2-61)}$$

$$\ln\left(\exp\left(-\frac{x_p^{\ 2}}{R_D}\right)\right) = \ln\left(\frac{1}{n}\right) \qquad \textbf{(Equation 2-62)}$$

$$-\frac{x_p^{\ 2}}{R_D} = -\ln n \qquad \textbf{(Equation 2-63)}$$

$$x_p = \sqrt{R_D(\ln n)} \qquad \textbf{(Equation 2-64)}$$

Mean value in Rayleigh distribution is represented by the first moment of its probability density function.

$$\overline{x_p} = \frac{\int_{x_p}^{\infty} xf(x)dx}{\int_{x_p}^{\infty} f(x)dx} \qquad \textbf{(Equation 2-65)}$$

2.2 Irregular Waves

A fundamental study of waves is known as linear wave theory. In this study, waves are considered to have a regular harmonic pattern. This simplification allows waves properties such as wave height, period, and direction to be quantified as deterministic values. Nevertheless realistic waves are random in nature. The real sea surface shows irregularities so that it is necessary to involve statistical calculation in determining its properties. This might complicate the process of determination; however, it represents the real nature of waves better. In contrast with the monochromatic wave that is considered to have constant properties, irregular wave is indicated by statistical variability; hence, the severity of the sea is denoted by these statistical properties.

Monochromatic wave is rarely found in nature. This simplified wave appears only in laboratory simulation. A wave in nature which is quite similar to a monochromatic wave is swell wave. A swell in deep water travels for a long distance from the place it was generated. Although swell physically looks like regular monochromatic waves, it is basically irregular in nature. When a storm happens, there will be locally generated wind sea which consists of short-crested waves and is highly irregular in nature. Ocean wave surface in a particular location might consist of only swell or only sea or mix of swell and sea.

Recent engineering practice uses linear wave theory to estimate the properties of irregular waves using the linear spectral methods (Krogstad et al 2000). This application considers the ocean surface as a combination of numbers of individual waves which are generated at different locations and travel into the observed location. It does, however, ignore the nonlinear interaction between waves. For this reason, the wave recorded at a specific location will not consist of wave repetition; however, it will show random and irregular water elevation. Even though it is possible to observe each wave component, each of them will have large differences in their properties. Therefore it is expedient to present the properties of the waves in a relatively short time scale by statistical terms known as short-term statistics. These properties are the ones that indicate the severity of a particular sea.

There are generally two approaches in dealing with irregular waves in a certain sea state of short-term statistics. The first method is the spectral method that involves Fourier Transform in its analysis. The second one is wave to wave (wave train) analysis. Wave train analysis uses the time history of sea surface at a particular point to define the wave properties.

2.2.1 Short-term Statistics

Short-term ocean wave is considered as a random process that depends on the time variable which is regulated based on probability theory (Chakrabarti 1987). In the case of wave analysis, the surface elevation is expressed as a stochastic process where it is constructed by random variables as a function of time. Two fundamental assumptions in implementing short term statistics are stationary and ergodic concepts. These two basic assumptions have been explained in the previous chapter.

Based on the stationary assumption, wave properties of particular wave record are considered invariant over a specific time range (few hours). In other words, the expected value of the random variables $X(t)$ is independent of time (time invariant). The next assumption of ergodicity allows the replacement of ensemble by time average of single realisation. Under this assumption, sample mean approaches the mean of the whole ensemble; and the variance of the sample approaches the variance of the whole record. The statistical parameters which are usually used are explained in the following.

Mean value of the process is defined as the expected value of *x(t)*. The process has constant mean based on stationary assumption.

$$\mu_x = E[x(t)] \quad \textbf{(Equation 2-66)}$$

Standard deviation is denoted by the square root of the variance which also has constant value due to stationary assumption.

$$\sigma_x = \sqrt{V[x(t)]} \quad \textbf{(Equation 2-67)}$$

Auto correlation function which is depicted in the following depends only on the time difference $\tau = t_2 - t_1$ based on ergodicity assumptions.

$$R(t_1, t_2) = E[x(t_1), x(t_2)] \quad \textbf{(Equation 2-68)}$$

Although the nature might not be as simple as this hypothesis, the concept of stationary and ergodicity is very important in the application of short term statistic. Another simplification is related to space. If the properties of particular event x(t) are also invariant in space, the process is called homogeneous. The stationary and homogeneous processes can be found in nature at specific times and space for only a few hours duration.

2.2.2 Wave Properties

There are two parameters which are commonly used as the indication of a particular wave record. Those parameters are wave period and wave height. Wave parameters of water surface elevation in time domain are defined based on zero-crossing analysis. In the time domain, one period of wave is defined as the distance between particular directions of zero-crossing (down-crossing or up-crossing) to the subsequent zero-crossing in the same direction. Additionally, one wave height is the difference between the maximum and the minimum water surface elevation in the pertinent wave.

Zero down-crossing is the transition of surface elevation from the level above the mean to the level below the mean (IAHR 1986). In contrast, the transition of water surface elevation from the level below the mean to the level above the mean is called zero-up crossing the wave. The difference between both zero-crossing definitions is illustrated in Figure 2-2. Revising the definition of wave height, different conventions have different definitions of wave height. It depends on whether the trough is referred to after or before the crest. According to the Permanent International Association of Navigation Congresses (PIANC) list, wave is the event between two successive down-crossings. This definition is analogous with the International Association of Hydraulic Research (IAHR) definition. Zero down-crossing is also the definition used by the majority of groups that deal with time domain wave analysis. For this reason, zero down-crossing wave definition is used in the determination of wave properties in this study.

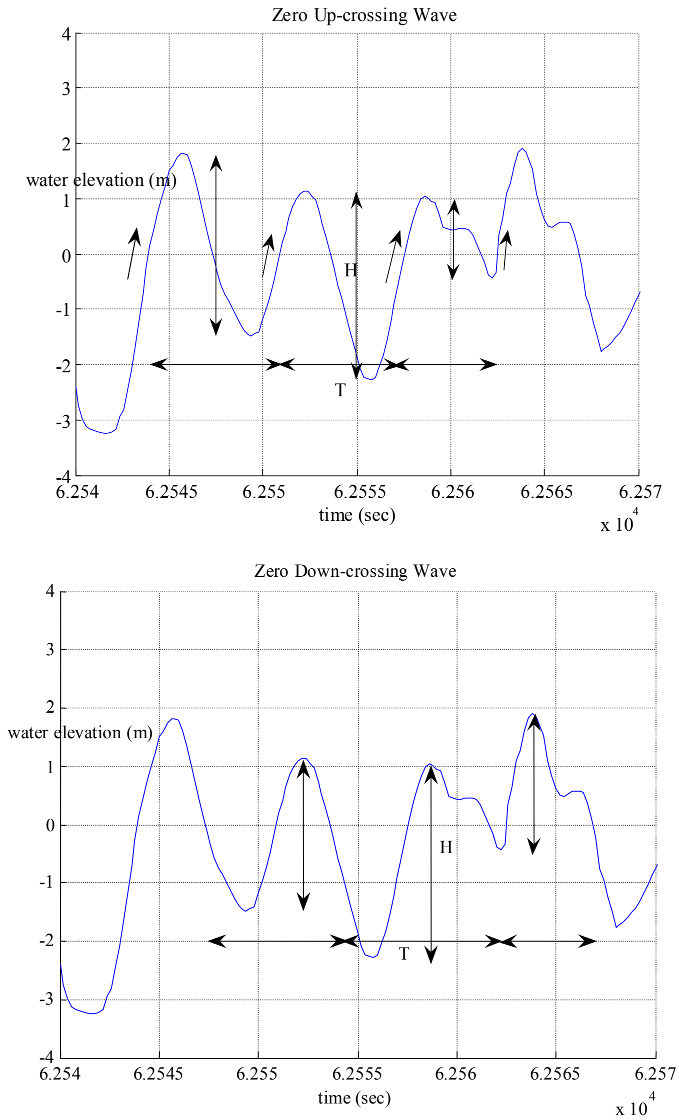


Figure 2-2 Zero up-crossing and down-crossing wave

Another term in time domain analysis is related to the local peaks and valleys of a wave. Any point in the record where its absolute surface elevation is higher than the adjacent elevations is named maxima. Maxima that occur on peak wave region must have a positive elevation so that they are called positive maxima. On the other hand, negative maxima are the ones located in the valley of a wave. As expected, a particular point in the time history is named minima when it has lower value compared with the adjacent elevations. Minima, so as with the maxima, are also divided into positive and negative based on their position with respect to mean water level. In addition, the highest positive of maxima is known as crest height while the deepest negative of minima is referred to as trough height.

The total time of the record is called duration. A particular wave duration consists of periods. There are generally two definitions of a wave period. The first type is known as the zero-crossing period which is the distance between two successive zero down-crossings in time domain analysis. The other one is called the crest period. This period is defined as the distance between two successive crests in the record.

Based on these definitions of wave parameters, sea state indicators are developed. These indicators are developed in statistical way which might be obtained directly from the time history of wave record or from its frequency domain representation. The following subsections describe those indicators. The most common used of significant wave height is discussed first.

2.2.2.1 Significant Wave Heights

Significant wave height was first introduced by Sverdrup and Munk in 1947 as the average wave height of the highest one third of all waves in a particular sea (Chakrabarti 1987). This definition was found to be close to the visual observation of wave height predicted by sailors.

The significant wave height may be determined from a wave record in three different ways. In general the preliminary step is to calculate the number of waves (crest to trough) in the record, sort the height of the waves and take one third highest value. The average of this group of wave height represents the value of significant wave height.

$$H_{1/3} = \frac{3}{N}\sum_{i=1}^{N/3} H_i$$ **(Equation 2-69)**

N = number of individual wave heights

H_i = record ranked highest to lowest

Another method in determining significant wave height was introduced by Tucker in 1963. Based on his study, significant wave height can be approximated directly from the wave record if the highest crest height (a_c) and the deepest trough height (a_t), and the total number of zero-crossings (N_z) are known.

$$H_s = \sqrt{2}Cr_1(a_c + a_t)$$ **(Equation 2-70)**

Where Cr_1 is a function of N_z

$$Cr_1 = (\ln N_Z)^{-1/2}\left[1 + 0.289(\ln N_Z)^{-1} - 0.247(\ln N_z)^{-2}\right]^{-1}$$ **(Equation 2-71)**

Significant wave height related to the total energy content in the wave spectrum is stated as a function of zeroth moment (m_0). Where mo is the same as the variance value of water surface elevation or in term of spectrum analysis, it is the same as the area under the wave energy density spectrum:

$$H_s = 4\sqrt{m_0}$$ **(Equation 2-72)**

2.2.2.2 Root Mean Square Wave Heights

Another parameter of wave height is the root mean square wave heights. Applying the same procedure for taking the wave height, each of the wave heights is then squared. After adding up all the square wave heights, the total is then divided by the total number of waves. Finally the root of the mean square wave height is calculated. In mathematical notation, root mean square is noted in the following formulation:

$$H_{rms} = \left[\frac{1}{N}\sum_{i=1}^{N} H_i^2\right]^{1/2}$$ **(Equation 2-73)**

On the other hand, root mean square wave height from the frequency domain is based on the zeroth moment (m_0) as follows:

$$H_{rms} = 2\sqrt{2m_0}$$ **(Equation 2-74)**

The root mean square of the wave elevation is the same as the standard deviation value:

$$\sigma = \sqrt{m_0} \qquad \textbf{(Equation 2-75)}$$

The discrete water elevation is obtained from the wave record at every time increment Δtm while the wave amplitudes are gained from the maxima and minima of the record. The standard deviation value for water elevation can be also written as the following:

$$\sigma = \left[\frac{1}{T_s} \sum_{T_s} \eta^2(t) \right]^{1/2} \qquad \textbf{(Equation 2-76)}$$

For simplification, Tucker generated the following formula based on the corrected wave elevation.

$$\sigma = 1.253 \overline{|\eta(t)|} \qquad \textbf{(Equation 2-77)}$$

2.2.2.3 Maximum Wave and Crest Heights

Wave height measurement which brings the most concern especially in the case of extreme conditions is maximum wave height. Maximum wave height is defined as the largest of all crests to adjacent trough value in the record (Chakrabarti 1987). It is also known as the measured maximum wave height.

Nevertheless, in the case of determining elevation of a platform above mean sea level, the real concern is the height of crest. It is possible that the maximum wave height is not the one causing the maximum crest height. For this reason, along with maximum wave height, designers are turning their attention to the height of maximum crest. Maximum crest is the highest water elevation in the record with respect to mean water level.

The most probable maximum wave height in a record can be estimated from the value of root mean square wave height or equivalently significant wave height. Assuming a narrow band spectrum of the record, Longuet-Higgins (1952 cited ibid.) derived a relationship between the most probable maximum wave height and the root mean square wave height for a specific number of waves which is depicted in the following formula:

$$H_{\max} = \left[\sqrt{\ln N} + \frac{0.2886}{\sqrt{\ln N}} \right] H_{rms}$$ **(Equation 2-78)**

This relationship can be used over a longer period of time by adjusting the value of N based on the mean of zero-crossing periods (T_z).

2.2.2.4 Average Wave Period

There are different definitions concerning the characteristic periods within a wave record. The most familiar terms are mean period, average zero-crossing period and peak period. Average wave period is the most common manifestation of a wave's characteristics.

There are two methods for determining the average wave period from time domain analysis. Both methods for average wave period are derived from the total duration of measurement (T_s). If $\overline{T}_z$ denotes the mean zero-crossing period and N_z symbolised the number of zero-crossing then the following relationship can be derived:

$$\overline{T}_z = \frac{T_s}{N_z}$$ **(Equation 2-79)**

On the other hand, if the total number of crest in the record is denoted by N_c, then the mean crest period $\overline{T}_c$ is expressed as follow.

$$\overline{T}_c = \frac{T_s}{N_c}$$ **(Equation 2-80)**

Based on previous expressions, the difference of both $\overline{T}_z$ and $\overline{T}_c$ determines the width of the spectrum. If their values are close together, this means most of the individual waves cross the zero level. This indicates that the wave has a narrow band spectrum which means the energy of the wave is concentrated in a small frequency band.

Another time domain wave period characteristic is known as significant wave period. As the significant wave height definition, significant wave period is also defined as the average of wave period of the highest one third in the record.

Taking the analysis from a frequency domain, mean wave period is calculated from the spectral moment denoted as m_n. The n^{th} order moment of an energy density spectrum is calculated based on the following relationship:

$$m_n = \int_0^{\infty} f^n S(f) df \qquad \textbf{(Equation 2-81)}$$

First definition of mean period T_{01} is stated as the ratio between zeroth spectral moment (m_0) and first spectral moment (m_1).

$$T_{0,1} = \frac{m_0}{m_1} \qquad \textbf{(Equation 2-82)}$$

Another definition of mean wave period of T_{02} is defined as the square root of zeroth spectral moment (m_0) divided by second spectral moment (m_2). Mathematical notation of T_{02} is presented in the following:

$$T_{0,2} = \sqrt{\frac{m_0}{m_2}} \qquad \textbf{(Equation 2-83)}$$

Another wave period characteristic based on frequency domain is the peak period denoted as T_0 or T_p. Peak period is the period at which the energy density spectrum peaks.

2.2.2.5 Spectral Width Parameters

As mentioned in the previous chapter, spectral width parameters is denoted as ε is a function of the mean zero-crossing and mean crest periods. The value of width parameter ranges from 0 to1. Small value of ε indicates a narrow banded spectrum; while large value of ε, near to 1, points a broad banded spectrum. In time domain analysis, spectral width parameter ε_t is estimated as follows:

$$\varepsilon_t^2 = 1 - \left(\frac{\overline{T}_c}{\overline{T}_z} \right)^2 \qquad \textbf{(Equation 2-84)}$$

This equation means that if local peaks follow a corresponding zero-crossing then the value of $\overline{T}_c$ is close to the value of $\overline{T}_z$ so that $\overline{T}_c / \overline{T}_c$ near to 1 which result in $\varepsilon_t \approx 0$. This condition represents a narrow band spectrum. On the other hand if numbers of local peaks are found (N_c is large) then the crest period $\overline{T}_c$ is small. As a result $\varepsilon_t \approx 1$ which indicates a broad-banded spectrum.

In frequency domain, spectral width parameter is a function of the spectral moment. The relationship is depicted in the following:

$$\varepsilon_s^2 = \frac{m_0 m_4 - m_2^2}{m_0 m_4} \qquad \textbf{(Equation 2-85)}$$

Chakrabarti (2001) noted that a person should be careful in calculating high degree of moment from the spectrum. Attention should be paid to the noises because they can amplify the value of higher moment calculation so that calculation should be limited to a reasonable finite frequency.

2.3 Wave and Crest Height Distribution

Many ocean engineers must be familiar with the Rayleigh distribution. The Rayleigh distribution is the standard form of wave height which was initially suggested by Longuet-Higgins in 1952 (cited Forristall 1978) based on two fundamental assumptions. Wave height distribution of Rayleigh arrives from the assumption that ocean surface elevation follows the Gaussian distribution and has a narrow-band spectrum.

A number of validations have been done to check the accuracy of the Rayleigh distribution on predicting the wave and crest height of ocean surface. However, there is still disagreement on how well the standard distribution matches the observed data (Forristall 1978). The emphasis is mainly on the high wave tail of the distribution because this extreme value was found to be the one which is responsible for the breakage of marine structures.

In his paper, Forristall (1978) found that the Rayleigh distribution consistently over-predicted the wave height, mainly in the highest waves. His finding confirms the result of Thomson (1974 cited ibid.) who discovered a significant deviation between the data taken from the Coastal Engineering Research Center (CERC) coast station and the Rayleigh distribution at the high wave end of the distribution. However there are differences in how far Rayleigh over-predicts the highest observed wave even from the same location. Using the same data taken from the

Gulf of Mexico, Earle came out with 2% while Haring resulted in 10% over-prediction of highest wave distribution (Forristall 1978).

The concern over inaccuracy of the standard wave and crest height prediction has motivated scientists and engineers to develop improved formulae in wave and crest height distribution. Ocean waves in nature have higher crest and lower trough. This nonlinear behaviour of waves is suspected to be the major factor that causes the mismatch of the standard prediction. Nevertheless, these improved formulae show a significant difference from one another which is intolerable for design purposes (Nerzic and Prevosto 1998).

Nerzic and Prevosto (ibid.) presume that the discrepancies between models are due to the differing t accuracy in presenting the wave properties. They found that the inaccuracy mostly happens in the prediction of rough sea states with steep waves. Their finding implicitly indicates that the linear representation of wave kinematics is inadequate, especially in predicting the extreme wave condition. The following sub-chapters present the development of prediction on wave and crest height distribution. These distributions will be elaborated on shortly.

2.3.1 Wave Height Distribution

In their paper, Nerzic and Prevosto (ibid.) undertook a brief survey on the development of wave and crest distribution. This sub-chapter represents the available wave height distribution taken from their survey as well as other distributions from various journals. The distributions are presented in exceeded probability of normalised variables.

2.3.1.1 Rayleigh Distribution

Rayleigh distribution is the standard form in predicting wave height and wave amplitude which is commonly used in practice. This distribution is derived from the Longuet-Higgins model which was initiated in 1952 (cited Forristall 1978). The model was developed based on its main assumption that sea surface is linear and follows the narrow-banded Gaussian process. The exceedance probability of Rayleigh is presented in the following.

$$Q(x) = \Pr ob(X > x) = \exp\left(-\left(\frac{x}{\theta} \right)^2 \right) \quad \textbf{(Equation 2-86)}$$

From the previous formula, Q is denoting the probability of exceedance of X which is a normalised wave height against significant wave height (H/H_s). The wave height defined in this formula is the difference between the trough and the adjacent peak of zero-crossing waves. The significant wave height is defined as the average of the highest one third wave height in the distribution. For the sake of simplicity, significant wave height is often expressed as $4\sqrt{m_0}$ .The term 'm_o' represents the zero-order moment of the wave spectrum which is equal to the variance of the sea surface elevation. However the analysis which will be done in this study will use the real definition of significant wave height. Lastly, the scale parameter for wave height distribution of Rayleigh which is denoted by θ is equal to 0.707.

The Rayleigh distribution has been used by many new empirical wave height distributions as a reference. These new distributions were developed from the studies in the North Sea and the Gulf of Mexico. They were fitted against Rayleigh and resulted in a scale parameter q. The value of q varies between 0.63 and 0.70. The earlier value of q is similar to the value proposed by Det Norske Veritas (DNV) in 1991. DNV proposed a value of 0.638 for the value of θ .

2.3.1.2 Weibull Distribution

While the Rayleigh distribution has only one scale parameter denoted by θ , Weibull distribution contains two determining parameters. Those parameters represent the scale and shape parameters which are denoted by θ and β respectively. In the Rayleigh distribution, β is equal to 2. Taking the more general form of Weibull, some proposed values of θ and β were introduced by scientists.

$$Q(x) = \Pr ob(X > x) = \exp\left(-\left(\frac{x}{\theta} \right)^\beta \right) \quad \textbf{(Equation 2-87)}$$

The above formula expresses the probability of exceedance based on the Weibull distribution. The probability depends on the normalised wave height (H/H_s). There are two determining factors in this distribution which are θ for scale parameter and β for shape parameter.

There have been scientists who tried to find the appropriate values of θ and β. They investigated it based on site measurements. In the late seventies, two values of the Weibull parameters were proposed. The values were derived based on the data taken from the Gulf of Mexico. Haring (1976 cited Nerzic and Prevosto 1998) proposed a semi-empirical model based on the Rayleigh distribution counting certain nonlinearities factor. The modified distribution proposed by Haring is presented in the following:

$$Q(x) = \Pr ob(X > x) = \exp(-2x^2(\theta + \beta x))$$ **(Equation 2-88)**

The wave measurement in the Gulf of Mexico is based on Haring's wave height distribution model. From the study which was done by Haring and his team, the proposed θ and β are respectively 0.968 and 0.176 (Nerzic and Prevosto 1998). Their study was then followed by Forristall who analysed 116 hours of hurricane generated waves in the Gulf of Mexico (Forristall 1978). He fitted the data empirically using generalised 2 parameters of Weibull and came out with the scale parameter of θ equal to 0.681 and shape parameter of β equal to 2.126 (Nerzic and Prevosto 1998).

Less than a decade after Haring and Forristall proposed their Weibull parameters, Krogstad in 1985 (cited ibid.) suggested another value of θ and β. His study was performed based on ocean data taken from three North Sea sites which have significant wave height of more than 5 metres. Krogstad found that the value of θ will be at the range of 0.73 and 0.75 while β varies between 2.37 and 2.50.

2.3.1.3 Rayleigh-Stokes

In their paper, Nerzic and Prevosto (ibid.) introduced a modified Rayleigh model of wave and crest height distribution. The model was developed considering the nonlinear factor of third-order Stokes expansion. They relate Rayleigh distribution with the shape and scale parameters of Gumbel. In the extreme statistic, asymptotic law for maxima based on Gumbel distribution is written:

$$G(y) = \Pr ob(X_N \leq y) = \exp\left(-\exp\left(-\frac{y - a_N}{b_N}\right)\right)$$ **(Equation 2-89)**

The previous distribution of Gumbel assumes that the number of variables, *N*, is high enough and follows the Weibull distribution. The mode of *N* variables is denoted as a_N while b_N is denoting the scale parameter. The parent distribution for both mode and scale parameters are:

$$Q(a_N) = \frac{1}{N}$$ **(Equation 2-90)**

$$b_N = \frac{1}{Nf(a_N)}$$ **(Equation 2-91)**

Q(x) in the previous equation is denoting the probability of exceedance.

From the parent distributions, considering two parameters of Weibull distribution, the value of mode and scale parameters for *N* variables are written as follows:

$$a_N = \theta(\log(N))^{1/\beta}$$ **(Equation 2-92)**

$$b_N = \frac{\theta}{\beta \log(N)^{1-(1/\beta)}} = \frac{a_N}{\beta \log(N)}$$ **(Equation 2-93)**

For Rayleigh distribution with β = 2, the mode and scale parameters are becoming:

$$a_N = 4\sqrt{mo}\sqrt{\left(\frac{\log(N)}{8}\right)}$$ **(Equation 2-94)**

$$b_N = \frac{a_N}{2\log(N)}$$ **(Equation 2-95)**

where m_0 and *N* denote the variance of ocean surface and the number of wave respectively. Taking the nonlinear effect of wave steepness from the third order Stokes, the mode and scale parameters of the non-normalised wave height of Gumbel distribution are formulated in the following:

$$a_{HN} = 2a_N\left(1 + b_3(k_m a_N)^2\right)$$ **(Equation 2-96)**

$$b_{HN} = 2a_N\left(1 + 3b_3(k_m a_N)^2\right)$$ **(Equation 2-97)**

For deep water condition, the value of b_3 is equal to 3/8. These values of mode and scale parameters are put back into the Gumbel distribution model.

$$G(y) = \Pr ob(X_N \le y) = \exp\left(-\exp\left(-\frac{y - a_{HN}}{b_{HN}}\right)\right) \quad \textbf{(Equation 2-98)}$$

2.3.2 Crest Height Distribution

So as with the prediction of wave height, new empirical and heuristic crest height distributions have been proposed and they are different from one another (Forristall 2000). Crest height distributions which are presented in the following are mainly taken from the survey which was done by Forristall in 2000. In addition, Prevosto (2000), Rayleigh-Stokes of Nerzic and Prevosto (2003), and Ochi (1998) crest height distribution are also included.

2.3.2.1 Rayleigh

Standard distribution of Rayleigh is the form of the first order theory where ocean surface is presented as Gaussian narrow band spectrum. Different papers present this distribution in a different notation which often confuses the reader. In order to make it consistent the crest will be denoted as 'c' here after.

$$Q(c) = \Pr ob(C > c) = \exp\left(-\left(\frac{c}{\theta}\right)^2\right) \quad \textbf{(Equation 2-99)}$$

Having the same form as the wave height distribution, *Q* is denoting the probability of exceedance of *C* which is a normalised crest height against significant wave height (c/H_s). Q is the same as one minus the probability distribution function (*Q = 1-F*). The significant wave height is calculated as the average of the highest one third wave height in the distribution. Finally the determining factor of the crest distribution, θ, is equal to the root of one per eight or equal to 0.354. Another way to present the Rayleigh distribution is depicted in the following:

$$P(\eta_C > \eta) = \exp\left[-8\frac{\eta^2}{H_s^2}\right] \quad \textbf{(Equation 2-100)}$$

Here, *P* represents the probability, η_c presents the wave height and the same symbol of H_s as significant wave height is used.

2.3.2.2 Jahns and Wheeler - Haring

In 1972, Jahns and Wheeler proposed an empirical modification of Rayleigh for wave-crest distribution taking into account the nonlinearities factor determined by the water depth. Their model includes elevated crest height at the moderate depth presented in the following formula (Prevosto 2000).

$$P(C > c) = \exp\left(-\frac{1}{2}\frac{c^2}{\sigma^2}\left(1-\gamma_1\frac{c}{d}\right)\left(\gamma_2-\frac{c}{d}\right)\right) \quad \textbf{(Equation 2-101)}$$

From the previous formula, d is denoting the water depth and σ signing the standard deviation of the ocean surface which can also be denoted as mo. However, they did not propose any specific value of γ_1 and γ_2.

Finally approximately four years later, Haring together with Heideman proposed the parameter value of γ_1 = 4.37 and γ_2 = 0.57 based on empirical fitting of 376 hours storm (Forristall 2000). The data was taken from the Gulf of Mexico, the North Sea and the Gulf of Alaska. Water depth factor is included in his formula; nonetheless, it does not depend on wave steepness. The coefficients are calculated based on wave measurement in the Gulf of Mexico.

2.3.2.3 Tayfun – Huang

In the eighties, Tayfun (1980 cited Forristall 2000) and Huang et al (1986 cited ibid.) tried to develop a crest height distribution formula based on second order Stokes model. They employed wave steepness factor into the distribution. Nevertheless, disagreement occurred between the authors concerning the exact formulation. A review by Tucker in 1991 on their formula is presented in the following (Forristall 2000).

$$P(C > c) = \exp\left\{-\frac{8}{R^2}\left[(1+2Rc/H_s)^{1/2}-1\right]^2\right\} \quad \textbf{(Equation 2-102)}$$

C in the above formula denotes crest height and H_s symbolises the significant wave height. An additional element in the Tayfun-Haring distribution is the inclusion of nonlinear factor of wave steepness. The wave steepness factor is denoted as *R* where *R* is equal to $k*H_s$.

2.3.2.4 Kriebel – Dawson

In the same year when Tucker published his review, Kriebel and Dawson (1993 cited ibid.) elaborated on a similar distribution as the one proposed by Tayfun and Haung. It was developed based on second-order Stokes in unidirectional narrow banded sea. The basic formulation of the nonlinear crest is depicted in the following:

$$c = a + \frac{1}{2}ka^2$$ **(Equation 2-103)**

Their formula contains the same parameter of *R* which represents the steepness factor. A perceptible difference is seen in that Kriebel and Dawson took out the square root form in their distribution (Forristall 2000). The formula is depicted as follows:

$$P(C > c) = \exp\left[-8\frac{c^2}{H_s^2}\right]\exp\left[8R\frac{c^3}{H_s^3}\right]$$ **(Equation 2-104)**

Nevertheless the above formula becomes negative when dealing with very large steepness. Improved distribution is later developed to encounter its defect. The new version was published in 1993 as follows:

$$P(C > c) = \exp\left[-8\frac{c^2}{H_s^2}\left(1 - \frac{1}{2}R\frac{c}{H_s}\right)^2\right]$$ **(Equation 2-105)**

2.3.2.5 Forristall

In the late nineties, Forristall (2000) developed another model of crest height distribution based on second order simulation of ocean waves (Wolfram 2003). Forristall developed his crest distribution from the two parameters of Weibull:

$$P(C > c) = \exp\left[-\left(\frac{c}{\alpha H_s}\right)^\beta\right]$$ **(Equation 2-106)**

Following earlier development of crest distribution, the modified Weibull should also contain water depth and steepness factors. For this reason, Forristall formulated each parameter as follows:

$$\alpha = \alpha_1 + \alpha_2 S_1 + \alpha_3 U_r$$ **(Equation 2-107)**

$$\beta = \beta_1 - \beta_2 S_1 - \beta_3 U_r + \beta_4 U_r^2$$ **(Equation 2-108)**

where S_1 denotes the steepness factor which is regulated by the significant wave height H_s and the mean wave period T_1. Along with steepness factor U_r which is known as Ursell number which is included to bring water depth factor into the formulation. Both elements are presented in the following.

$$S_1 = \frac{2\pi H_s}{g T_{01}^2}$$ **(Equation 2-109)**

$$U_r = \frac{H_s}{k_{01}^2 d^3}$$ **(Equation 2-110)**

The next attempt was to determine the value of α and β by fitting the formula to the simulated distribution based on Joint North Sea Wave Atmosphere Program (JONSWAP) spectrum. The fitting of the two dimensional simulation resulted in the following values:

$$\alpha_2 = 0.3536 + 0.2892 S_1 + 0.1060 U_r$$ **(Equation 2-111)**

$$\beta_2 = 2 - 2.1597 S_1 + 0.0968 U_r^2$$ **(Equation 2-112)**

While the three dimensional simulation came out with another values as follow:

$$\alpha_3 = 0.3536 + 0.2568 S_1 + 0.0800 U_r$$ **(Equation 2-113)**

$$\beta_3 = 2 - 1.7912 S_1 - 0.5302 U_r + 0.284 U_r^2$$ **(Equation 2-114)**

2.3.2.6 Prevosto

The next advancement in crest height distribution was suggested by Prevosto (2000). Prevosto's formulation is basically a nonlinear transformation of standard Rayleigh distribution (Wolfram and Venugopal 2003). The nonlinear element in the formulation is taken from the second order irregular wave. Basic formulation of the Prevosto nonlinear crest height is defined in the following:

$$C_r = C_{lin} + \left[T^D(f_m) + T^S(f_m)\right] C_{lin}^2 - T^D(f_m)\frac{H_s^2}{8}$$ **(Equation 2-115)**

C_{lin} denotes the linear crest height where in the second part it is multiplied by T^D and T^S that represents the second order transfer function. Both transfer functions contain nonlinear factor of water depth and mean wave number so that in the last component of C_r, wave steepness is accounted for. Both transfer functions are presented in the following:

$$T^D(f_m) = c_{diff}(\kappa)k_m \quad \textbf{(Equation 2-116)}$$

$$T^S(f_m) = c_{sum}(\kappa)k_m \quad \textbf{(Equation 2-117)}$$

Where k_m denotes the mean wave number calculated from the dispersion relation. The general form of dispersion relation is presented in the following. Wolfram and Venugopal (2003) misprinted the dispersion relation in their paper; thus, in this study, this is rectified by putting the square sign on the right hand side element in the following equation.

$$(2\pi f_m)^2 = gk_m \tanh(k_m d) \quad \textbf{(Equation 2-118)}$$

Both coefficients c_{diff} and c_{sum} are determined by κ which is equal to the multiplication between wave number and water depth ($\kappa = k_m d$). Both coefficients are presented in the following.

$$c_{diff}(\kappa) = \frac{\Pi(\kappa) + \kappa\left[1 - (\tanh\kappa)^2\right]}{\Pi(\kappa)^2 - 4\kappa(\tanh\kappa)} \quad \textbf{(Equation 2-119)}$$

$$c_{sum}(\kappa) = \frac{1}{4}\frac{2 + \left[1 - (\tanh\kappa)^2\right]}{(\tanh\kappa)^3} \quad \textbf{(Equation 2-120)}$$

$$\Pi(\kappa) = \tanh\kappa + \kappa\left[1 - (\tanh\kappa)^2\right] \quad \textbf{(Equation 2-121)}$$

Correction is made on equation 2-119 where it was printed as $4\kappa(\tanh\kappa)^2$ in Wolfram and Venugopal's (2003) paper (For corrections to the c_{diff} formula, refer to the appendix in the earlier Prevosto paper (Prevosto et al .2000))

In addition, Prevosto included the additional factor of spectral band width and directional spreading into the formula. These factors are enclosed together with the significant wave height and the mean frequency shown in the following.

$$\widetilde{H}_s = \alpha_{Hs} H_s \quad \textbf{(Equation 2-122)}$$

$$\widetilde{f}_m = \alpha_{fm} f_m \text{ with } f_m = \frac{1}{T_{02}} \quad \textbf{(Equation 2-123)}$$

where the modification factors denoted as α is expressed as follow.

$$\alpha_{Hs} = 1 - \frac{1}{2}\left[\tanh(kd) - 0.9\right]\sqrt{\frac{2}{1+s}}$$ **(Equation 2-124)**

$$\alpha_{fm} = \frac{1}{1.23}$$ **(Equation 2-125)**

The directional factor in the correction factor of significant wave height denoted as *s*, is the power of the equivalent cos^{2s} spreading function for the wave energy spectrum at the peak frequency. As a result, the nonlinear transformation of standard Rayleigh distribution was proposed by Prevosto as follows:

$$P(C < c) = \exp\left(-8\frac{\left\{\left(-C_1 + \sqrt{C_1^2 - 4\left[C_2\left(C_0 - C\right)\right]}\right)/\,2C_2\right\}^2}{H_s^2}\right)$$ **(Equation 2-126)**

Where

$$C_0 = -T^D\left(f_m\right)\frac{\widetilde{H}_s^2}{8}$$ **(Equation 2-127)**

$$C_1 = \alpha_{Hs}$$ **(Equation 2-128)**

$$C_2 = C_1^2\left[T^D\left(f_m\right) + T^S\left(f_m\right)\right]$$ **(Equation 2-129)**

Referring to Wolfram and Venugopal's (2003) paper, the author found two errors within the probability distribution formula of Prevosto printed in their paper. Correction is made for the coefficient C_2 depicted above. In the reference paper, it was written $C_2 = C_1^2\left[T^D\left(f_m\right) + T^S\left(f_m\right)\right]k_m$, after confirming directly with Marc Prevosto himself (personal written communication, June 10, 2009) that C_2 should be written as the one shown in equation-129. Wolfram and Venugopal has misprinted the formula of C_2 by re-include average wave number, k_m, which has been considered in the transfer functions. Furthermore, confusion was also shown in the probability equation written in Wolfram and Venugopal's paper. For this reason, a pair of brackets is added in equation 2-126 to mark that the division against $2C_2$ involved both $-C_1$ and the squared numbers.

The superiority of the Prevosto model is that it also includes the directional spreading. Additionally, it can also be applied to all water depths both in two dimensional or three dimensional events.

2.3.2.7 Rayleigh-Stokes

As mentioned in the previous subchapter concerning wave crest distribution, Nerzic and Prevosto (2003) introduced a modified Rayleigh model that considering the nonlinear factor of third-order Stokes expansion. Having the same definition of a_N and b_N, the modified mode and scale parameters of the non-normalised crest height of the Gumbel distribution are formulated in the following:

$$a_{CN} = a_N\left(1 + b_2(k_m a_N) + b_3(k_m a_N)^2\right) \quad \textbf{(Equation 2-130)}$$

$$b_{CN} = a_N\left(1 + 2b_2(k_m a_N) + 3b_3(k_m a_N)^2\right) \quad \textbf{(Equation 2-131)}$$

Having the same value of b_3 equal to 3/8, a new coefficient of b_2 is introduced which is equal to ½ for infinite water depth. The shape and scale parameters are used in the Gumbel distribution model for crest height as depicted in the following:

$$P(C \le c) = \exp\left(-\exp\left(-\frac{c - a_{CN}}{b_{CN}}\right)\right) \quad \textbf{(Equation 2-132)}$$

2.3.2.8 Ochi

Crest distribution of Ochi used in this study is based on prediction of maxima (Ochi 1998). The development of Ochi's formula is discussed very briefly in this subsection. The probability density equation of the positive maxima developed by Ochi is a function of zeroth spectral moment and spectral width density. If the maxima are denoted as ζ then the probability density function of the positive maxima as a function of normalised maxima $\xi = \frac{\zeta}{\sqrt{m_0}}$ is formulated in the following.

$$f(\xi) = \frac{2}{\left(1 + \sqrt{1-\varepsilon^2}\right)}\left[\frac{\varepsilon}{\sqrt{2\pi}}\exp\left(-\frac{\xi^2}{2\varepsilon^2}\right) + \sqrt{1-\varepsilon^2}\,\xi\exp\left(-\frac{\xi^2}{2}\Phi\left(\frac{\sqrt{1-\varepsilon^2}}{\varepsilon}\xi\right)\right)\right]$$

$$0 \le \xi < \infty$$

(Equation 2-133)

When assigning $\varepsilon = 0$, presenting narrow-band condition, previous equation forms the Rayleigh distribution. While for wide-band distribution with $\varepsilon = 1$, the probability distribution for infinite number of frequencies becomes

$$f(\xi) = \sqrt{\frac{2}{\pi}} \exp\left(-\frac{\xi^2}{2}\right)$$

(Equation 2-134)

$$0 \le \zeta < \infty$$

Note: These equations can also be applied for the negative minima.

In order to get the formula of distribution function, the previous formula of probability density function should be integrated. The integration of the probability density function of wave maxima as a function of normalised maxima $\xi = \frac{\zeta}{\sqrt{m_0}}$ is presented in the following equation:

$$F(\xi) = \frac{2}{1+\sqrt{1-\varepsilon^2}}\left[-\frac{1}{2}\left(1-\sqrt{1-\varepsilon^2}\right) + \Phi\left(\frac{\xi}{\varepsilon}\right) - \sqrt{1-\varepsilon^2}\exp\left(-\frac{\xi^2}{2}\right)\Phi\left(\frac{\sqrt{1-\varepsilon^2}}{\varepsilon}\xi\right)\right]$$

$$0 \le \xi < \infty$$

(Equation 2-135)

3 METHODOLOGY

This chapter presents the methodology used in validating the theoretical distribution of wave and crest distribution using field and laboratory measurements. Data collection is not part of the study; however, the general principal of both measurements will be discussed in this chapter. The work of the current study focuses on time history and statistical analysis using observation data. The analysis was executed in order to get the wave properties needed to simulate both empirical and theoretical distributions of wave and crest height.

The field measurement data used in this study is North Sea water elevation which was measured in November 1997. Water elevation was measured for a week during storm conditions using a wave height altimeter. Along with field measurements, laboratory simulations were carried out in the hydrodynamics laboratory of the University of Southampton as depicted in Figure 3-1. The laboratory wave generations were based on the empirical spectrum of Pierson Moskowitz.

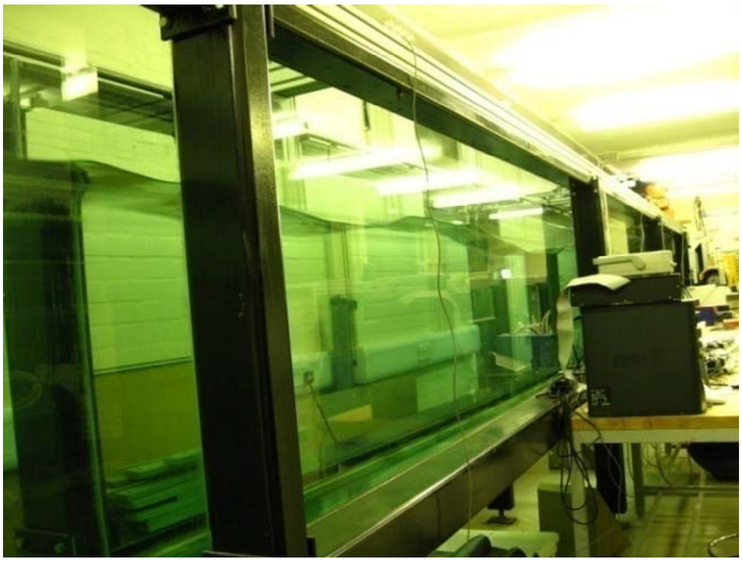


Figure 3-1 Hydrodynamics laboratory wave flume

Using both types of water elevation data, time history analysis was undertaken numerically in MATLAB. Water elevation data which are kept as DAT or BIN files are loaded into the interpreter to be analysed further. There are three types of laboratory data which differ based on predicted significant wave height. Two of them (50 mm and 75 mm) need to be calibrated; the calibration factor converts raw data in volt to meters. Laboratory data of 100 mm has been presented in meters as well as the field measurement data.

The first step after conversion (the two sets of laboratory data) is to ensure that the average water elevation is zero. After ensuring zero mean conditions, statistical properties of water elevation; in particular distribution factor of variance, were calculated to be used in later simulation of wave and crest distribution. Zero mean water elevation data are then analysed statistically in order to get the probability distribution. The empirical distribution of water elevation is then compared with the theoretical Gaussian distribution.

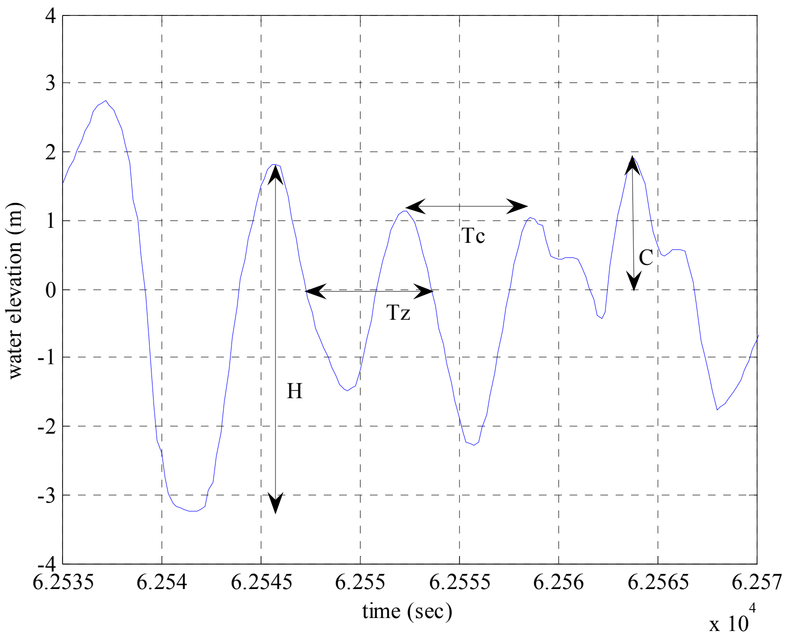


Figure 3-2 Wave properties used in this study

There are two types of waves based on zero-crossing definition which are zero up-crossing and zero down-crossing waves. Analysis of zero-crossing waves is presented to get some idea of how both definitions differ in the case of high waves. However, the down-crossing definition is used in this study following the definition used by most working groups. Based on this agreement, wave properties which are used in this study are shown in Figure 3-2. Based on agreed zero crossing definition, wave height, wave period, and crest height are acquired from each water elevation data.

Wave periods are processed statistically in order to get the mean period which later determines the wave number. This number is composed of the steepness factor of the theoretical prediction of nonlinear wave and crest height distribution. Wave and crest height which have been acquired are processed statistically to get the empirical probability distribution of exceedance. The distributions are first compared with the theoretical linear distribution of Rayleigh before being validated by the modified theoretical distribution.

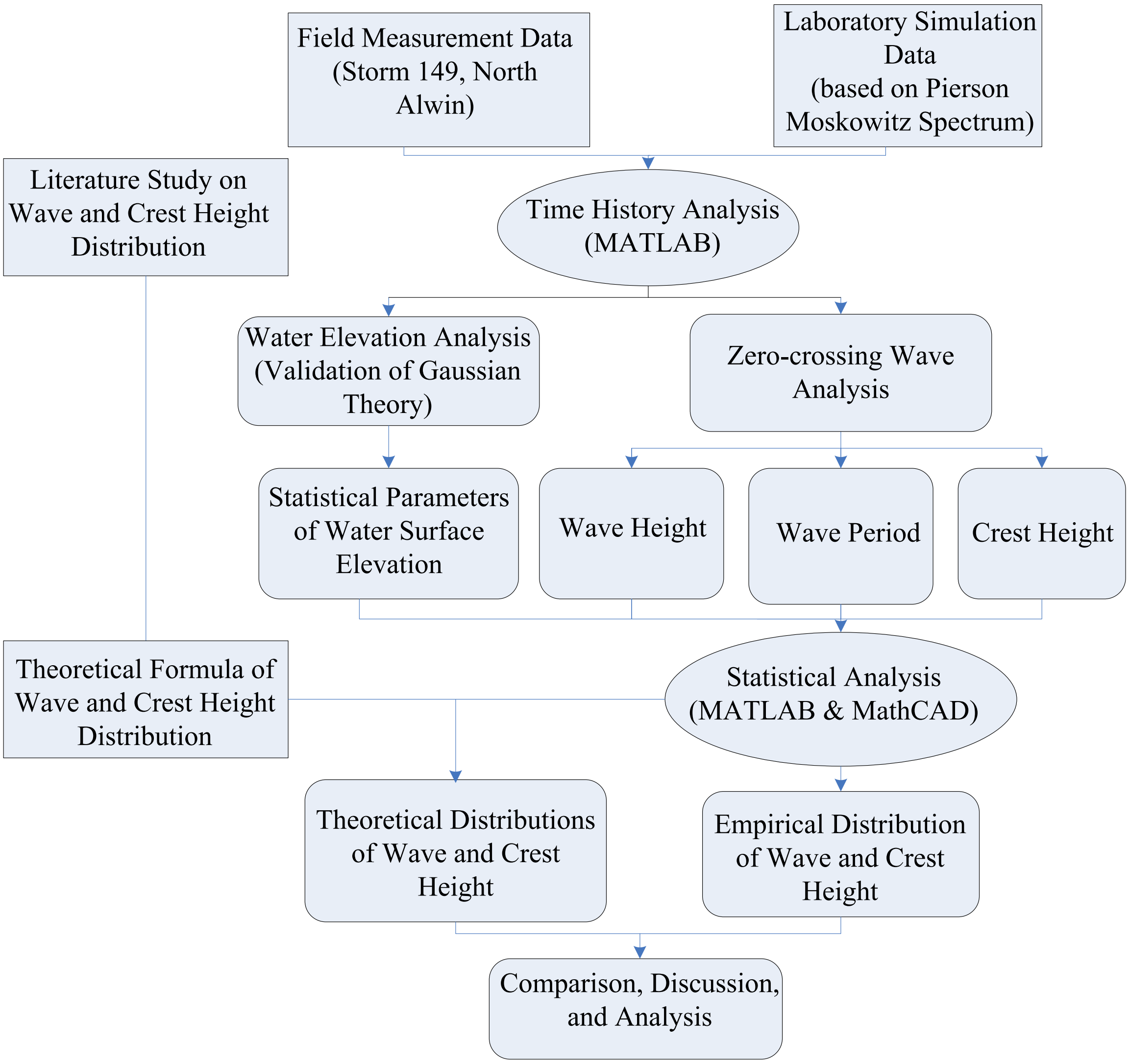


Figure 3-3 Diagram of the methodology used in this study

The improved formulae of wave and crest height distributions were gathered from various journals. Discussion concerning these formulae has been presented in the literature review. These theoretical probability distributions of exceedance are compared with the empirical distribution. The main focus is on the highest value of wave and crest height. Finally discussion and analysis are drawn from each comparison. Holistic methodology is shown in Figure 3-3.

3.1 Time Series Analysis: Water Surface Elevation and Wave Properties Determination

Analysis of water surface elevation using time series analysis is fundamental for scientists and engineers who work with physical oceanography. At this stage, a person no longer deals with idealised monochromatic waves. Time series analysis uses the real complex and random data of ocean surface elevation. As discussed in the literature study, random irregular ocean waves are presented as a result of the interaction of multiple single frequency waves with differing wavelengths, frequencies, wave height and direction of propagation (Whitford 2001b). Nevertheless, probability and statistics analysis is the method which is used in this study. The statistical properties of ocean waves are determined from the time history recorded by a stationary observer.

Water surface elevations of random waves are commonly presented as theoretical Gaussian probability density function. If η denotes the water elevation then the probability density function of Gaussian is expressed in the following:

$$p(\eta) = \frac{1}{\sigma_\eta \sqrt{2\pi}} \exp\left(\frac{-\eta^2}{2\sigma_\eta^2} \right) \qquad \textbf{(Equation 3-1)}$$

Nonetheless, the statistical distribution of water surface elevation is not the main interest of the engineers. Wave along with crest height, in this case, is considered to be the main concern for engineering purposes, especially their extreme values. For this reason, the following part describes the procedures for obtaining the wave and crest height distribution from time history of ocean surface elevation.

Before discussing the process of taking the wave and crest height, it is necessary to check whether the mean wave elevation is equal to zero. In the case where average water elevation is not zero, the record needs to be corrected to get zero mean water levels. In the case where the average is not zero, water elevation data should be firstly subtracted by the initial average. Wave height analysis was conducted by taking individual waves in the record; hence, the time series was, firstly, divided into individual waves. New waves begin when the water elevation crosses down the mean water level. The wave height of an individual wave is computed by taking the difference between the maximum and minimum water surface elevation of each individual waves. Along with this process, the crest height is acquired from the highest water elevation in each wave. The time duration of one individual wave defines the period of a wave.

As we are dealing with random waves, one record consists of many individual waves, where each record is associated with different wave height, crest height and period. Therefore, they have to be presented in the form of statistical parameters. The results of the statistical analysis of these parameters characterise the inherent signal.

Wave height distribution is different from the distribution of water elevation described previously. Longuet Higgins (1952 cited Whitford 2001b) found that the distribution of wave height can be approximated using the Rayleigh probability distribution.

$$p(H) = 2\frac{H}{H_{rms}^2}\exp\left[-\left(\frac{H^2}{H_{rms}^2}\right)\right]$$ **(Equation 3-2)**

where the root mean square wave height (H_{rms}) is defined as: $H_{rms} = \sqrt{\frac{1}{M}\sum_{j=1}^{M} H_j^2}$

with M being the number of waves under considerations and H_j is the j^{th} wave of the group M

3.1.1 Water Surface Elevation Analysis

Raw data which is used in this study is a random water surface elevation from field measurement and voltage measurement of laboratory simulation which is later converted into water elevation data. The field data consist of 412 records from approximately one week's measurements. Each measurement is 20 minutes long with the 5 water elevation data taken every second.

As mentioned previously, the mean water level should be zero, so that re-arrangement of water elevation data has to be conducted initially. In order to have sufficient wave components, sampling duration should be carefully considered. Short term statistics, which was discussed in the previous chapter, describes the probability of occurrence of wave height and wave crest that might occur during one particular observation at a stationary point.

Water surface analysis can be processed directly from the raw data. The most important parameters of the raw elevation data is the variance and standard deviation. Using these values, the distribution of water elevation, wave, and crest height will be normalised.

In the determination of bin (interval of data) size, maximum and minimum water elevation is investigated. It is important to include all the water elevation data. The difference between these data determines the width of each bin. After knowing the distribution of the surface elevation of each bin, the probability density function of the surface elevation data is presented. For a discrete data, the percentage of water elevation η in bin i is defined as: $\frac{N_i}{N_T}$

where N_i is the number of η values in bin i and N_T is the total number of η values in all bins.

The next step is to determine the probability density of bin i denoted as $p(\eta_i)$. It is less obvious compared with the previous definition. The idea is to create a representation value of each bin that will be equal to the percentage of $\frac{N_i}{N_T}$.

Something to be aware of is that the integral of probability density function must be equal to 1.

$$\int_{-\infty}^{\infty} p(\eta) d\eta \equiv 1 \qquad \textbf{(Equation 3-3)}$$

The probability density of bin i is expressed as follows:

$$p(\eta_i) = \frac{N_i}{N_T \Delta\eta} \qquad \textbf{(Equation 3-4)}$$

(where $\Delta\eta$ us the bin size and the units of $p(\eta)$ are m^{-1})

Where in the finite number of data, the probability density is expressed in the following.

$$\sum_{1=bn1}^{binNT} \frac{N_i}{N_T \Delta\eta} \Delta\eta = \sum \frac{N_i}{N_T} = 1$$ **(Equation 3-5)**

3.1.2 Wave and Crest Height Analysis

The first step in analysing the wave and crest height is to conduct the zero-crossing wave. Each zero-crossing wave has the wave height, crest height and period. Each of these was defined at the beginning of this subchapter. The properties which were gained are then sorted from lowest to highest. Similar to the method applied for water surface elevation, the bin width is determined based on the range of the maximum and minimum height. Afterwards, the probability distribution of wave and crest height are developed. Taking wave height as a representation the mathematical notation is presented as follow

$$p(H_i) = \frac{H_i}{H_T \Delta H}$$ **(Equation 3-6)**

where

H_i: the number of waves in bin i

H_T: the total number of waves in the record

ΔH is the bin width

The statistical distribution of wave and crest height are then compared with the theoretical Rayleigh distribution. The same step is conducted for constructing the cumulative distribution function taking the cumulative frequency of all data less than or equal to a particular bin. The percentage of waves having a height equal to or less than H is presented in the following:

$$P(H) = \int_0^H p(H) dH = 1 - \exp\left[-\left(\frac{H}{H_{RMS}}\right)^2\right]$$ **(Equation 3-7)**

Where the probability of exceedance is equal to one minus the cumulative distribution

$$1 - P(H) = \exp\left[-\left(\frac{H}{H_{rms}}\right)^2\right] 1 - P(H) = \exp\left[-\left(\frac{H}{H_{rms}}\right)^2\right]$$ **(Equation 3-8)**

3.2 Field Observation

In order to validate the available prediction of wave and crest height distribution, field measurement data is required. In this study, the wave data was taken from the North Alwyn Metocean Station in the northern most part of the North Sea. The measurement was taken at an approximate water depth of 130 metres. The gauges installation was intended as part of the research which is funded by the TotalFinaElf Corporation. This is one form of oil and gas companies' contribution to marine research.

The wave elevation data were measured using three Thorn EMI infrared laser wave height altimeters. The sensors are installed in three different positions. The first position denoted as M is called the Marex monitor. The second sensor was located on the North East corner monitor denoted by NEc. The last one was set on the walkway denoted by Ww. The layout is depicted in the following Figure 3-4 and the angles between monitor is illustrated in Figure 3-5.

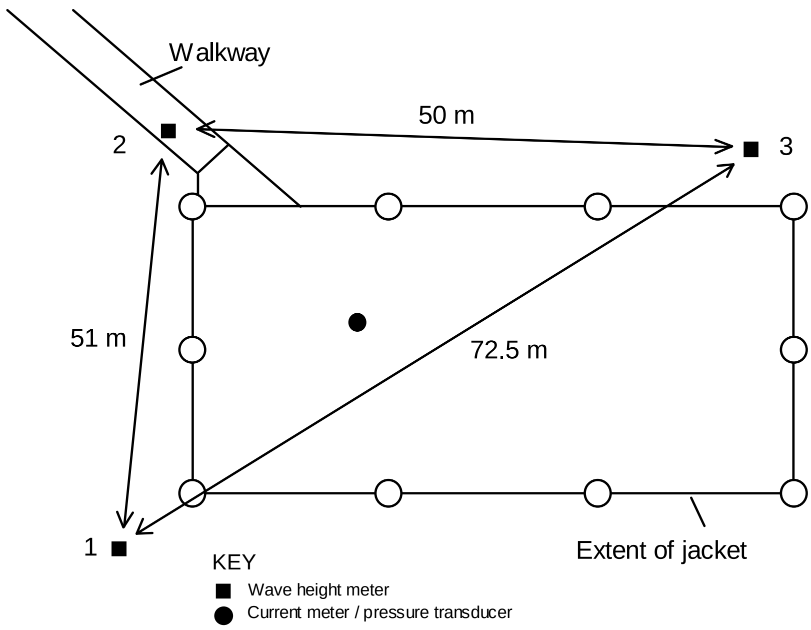
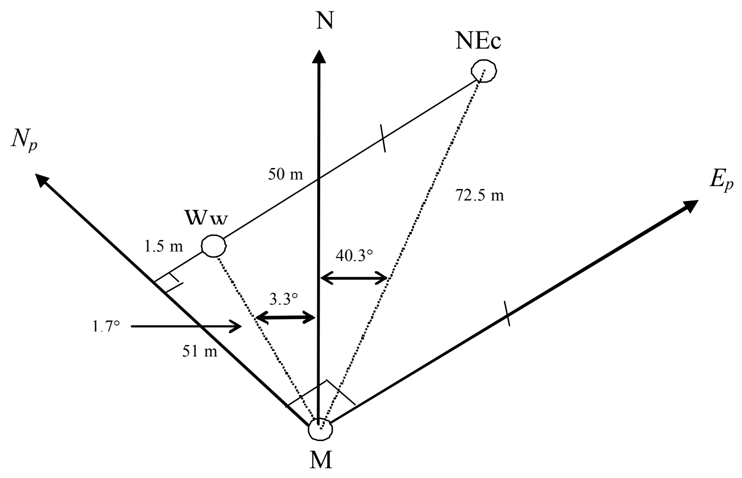


Figure 3-4 Sensor position on North Alwyn "A"



Note : Exaggerated for clarity. Np and Ep are platform north and platform east.

Figure 3-5 The angles between the three monitors

The wave elevation data which is used in this study is taken from the North Sea storm in November 1997. The storm is identified as storm 149. A whole week's data were taken between 16th and 22nd November 1997. There are approximately 412 pieces of data, each of which contains 6000 wave elevations. Each measurement was conducted for 20 minutes or 1200 second so that the interval between water elevations is 0.2 second. The significant wave height of the North East corner storm and its duration are shown in Figure 3-6 below.

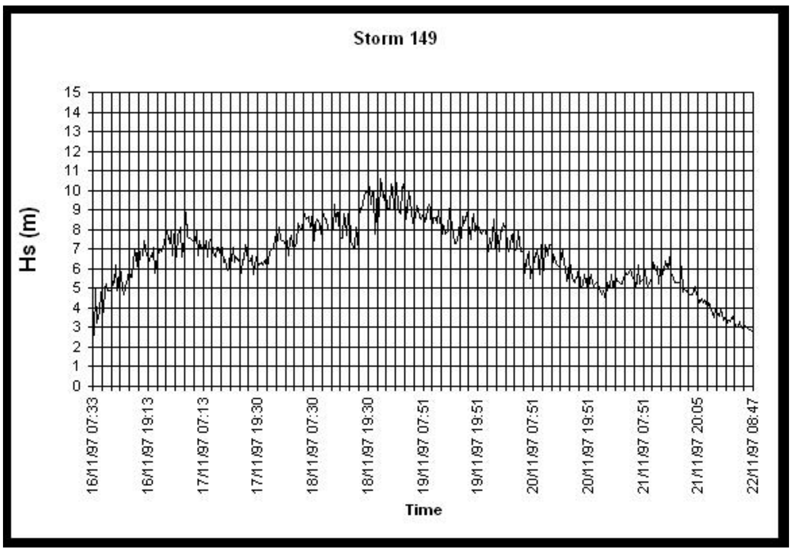


Figure 3-6 Storm 149: Significant wave height with storm duration

3.3 Laboratory Simulation

3.3.1 The Flume

The simulation of non-linear wave was done by a flume in The Hydrodynamics Laboratory, Department of Civil Engineering and Environment, University of Southampton. The flume dimension is approximately 12 metres x 0.5 metres x 1 metre for length, width, and height respectively. For reasons of visibility, the wave tank wall is made of glass material. The transparent wall is intended to make it easier for the users to observe the wave train.

Wave generation is done by a bottom hinged paddle at the left end of the wave flume. The paddle is connected to a mechanical set of spring and pulley. These give a hydrostatic load causing the forward and backward movement of the paddle. The work of the mechanical set is controlled by a numerical system. From this controller, a person can set the kind of wave that is going to be generated.

To ensure that there is no reflection of wave disturbing the simulation, the flume requires an absorption component. For this reason, a triangle shape of poly-ether foam shown in Figure 3-7 is used at the other end of the flume as a passive absorber.

Figure 3-7 Absorption system used in the wave flume

3.3.2 Wave Generation

Ocean waves can be regenerated in the laboratory using the identical energy spectra observation. The procedure of generating wave consists of two steps. The first step is to calculate the desired water elevation and position of the flume paddle (Miskovic 2008). The second step is to govern the paddles using the controlled signal which has been calculated for this purpose. In other words, the control signal of surface elevation should be calculated in the beginning. Later, this signal will be transferred to the control loop of wave maker when the simulation is started.

The simulation will be based on wind generated waves. In this case the wind transfers its energy to the ocean water. When the wave energy accumulation balances the dissipation then a sea state is developed. Dealing with the wave energy, the analysis is better interpreted using the spectral density function. The spectral density function which was used in this simulation is the Pierson-Moskowitz. The Pierson-Moskowitz is an empirical spectrum based on measurements in the North Atlantic Ocean. This spectrum represents the fully developed wind sea analysis properly. Empirical spectral density formula of Pierson-Moskowitz is presented in the following:

$$S_\eta^{PM}(f) = \frac{\alpha g^2}{(2\pi)^4 f^5} \exp\left(-\frac{5}{4} \left(\frac{f_p}{f} \right)^4 \right) \quad \textbf{(Equation 3-9)}$$

where f denotes the frequency, f_p symbolises the peak frequency, g is the gravitational acceleration, and α refers to scaling parameter which is equal to $8.1*10^{-3}$.

After spectrum generation, the next step is to construct the surface elevation. In order to build it, the spectrum should be divided into M frequencies bin with the same distance. M must be large enough to get smooth distributions. Each division presents a particular amplitude of the wave component. The final result is developed by summing each of these individual waves produced from the spectrum. Taking a random phase, simulated ocean surface elevation is expressed as follows:

$$\eta(k) = \sum_{f=f_1}^{f_M} \sqrt{2S_\eta(f)\Delta f} \cos(2\pi f \Delta t_k + \varphi(f)) \quad \textbf{(Equation 3-10)}$$

where Δf and Δt indicate the frequency interval and the time step of the control signal respectively, k denotes the time index and $\varphi(f)$ is the signal phase offset which is distributed between 0 and 2π.

The relationship between the surface elevation and the paddle motion is expressed by the Biesel equation (Miskovic 2008) in the following:

$$\eta(f) = 0 + ie_0 x(f) \quad \textbf{(Equation 3-11)}$$

where $\eta(f)$ notes the frequency domain water surface elevation, $x(f)$ represents the frequency domain paddle motion, and e_0 denotes the transfer function which depends on the type of the paddle used in the simulation. The previous expression shows the surface elevation immediately in front of the paddle. The spectrum of paddle motion can also be computed from the water surface spectrum using the following relationship:

$$S_\eta(f) = ie_0 S_x(f) \quad \textbf{(Equation 3-12)}$$

From the paddle motion spectrum, based on the same procedure in generating the surface elevation, paddle motion is defined as follows:

$$x(k) = \sum_{f=f_1}^{f_M} \sqrt{2S_x(f)\Delta f} \sin(2\pi f \Delta t_k + \varphi(f)) \quad \textbf{(Equation 3-13)}$$

3.3.3 Measurement Technique

The wave tank was initially filled w with water to a height of 70 cm. In this study, the laboratory simulation involves three types of wave with approximated significant wave of 50 mm, 75 mm and 100 mm. The frequency is set to 100 Hz so that there will be 100 of data in 1 second; or in other words, the time interval between the data is 0.01 sec. Each simulation was run for approximately 24 hours.

The observation techniques which are applied in the laboratory simulation include the Eulerian method. This method observes fluid parameters in a specific point in space as a function of time variable. It can be differentiated from the Lagrangian method that follows an individual fluid particle which flows through space and time.

The only thing that will be measured in the simulation is the water elevation. For this purpose, two wave gauges were installed. The wave gauges were put on vertical bars. Both bars are mounted on a transversal movable carriage on the top of the flume. The gauges measure the voltage, which is later calibrated to water elevation in metres. Each laboratory simulation uses two gauges which are named as gauge A and gauge B in this study. The calibration factors of gauge A and gauge B respectively are 0.03225 and 0.02443. The final output of the simulation will be a time history of wave elevation in metres.

4 RESULTS, DISCUSSIONS, AND ANALYSIS

This chapter presents the results of the numerical analysis which was undertaken to show how well the theoretical formula of wave and crest height distribution based on field and laboratory measurement. However, prior to the main issue under discussion in this study, there will be some discussion on how well the standard distribution of Gaussian and Rayleigh fit the measurements. Following validation of the standard distributions, discussion on zero-crossing definition will also be presented.

In general, the discussion will be divided into two sections, based on the type of data. The first type comprises field measurement which presents the three dimensional sea in nature under extreme conditions. The second one refers to two dimensional laboratory simulations which were generated based on the Pierson Moskowitz spectrum. Both conditions are considered to support research related to field investigation or laboratory simulation. Laboratory simulation that includes three wave types will be represented by one for the sake of optimum presentation.

4.1 Distribution of Water Surface Elevation

Chapter 4.1 discusses the behavior of water elevation taken from field and laboratory measurement. The field measurement was taken from North Alwyn in the North Sea during the 1997 storm. The data used in this study consists of more than two million water elevation units of data from -8.96m to 16.08m relative to the still water level. The measurements which were taken every 20 minutes with 5 Hz frequency were combined to be analysed numerically. The time history of six days combined water elevation data is shown in Figure 4-1.

The time history shown in Figure 4-1shows that, relative to mean water level, the crest heights are higher compared with the trough heights. This shows the nonlinear behaviour of wave in nature. Statistical analysis found that the skewness value of the distribution is around 0.3 (see Table 4-1). Non-zero positive skewness complements the nonlinear factor seen from the time history. Positive direction of water elevation's skewness indicates its tendency to have peaked crest and rounded trough.

It can also be seen that, at some points in time, there are extremely high water elevations recorded. It is not the concern of the current study to learn how they are generated; however, these crests given an indication of the extreme conditions involved in this study. These extreme crests will be the focus of later validation. With the presence of abnormal wave and the nonlinear nature of water surface, the next question will be how well the distribution is represented by Gaussian distribution.

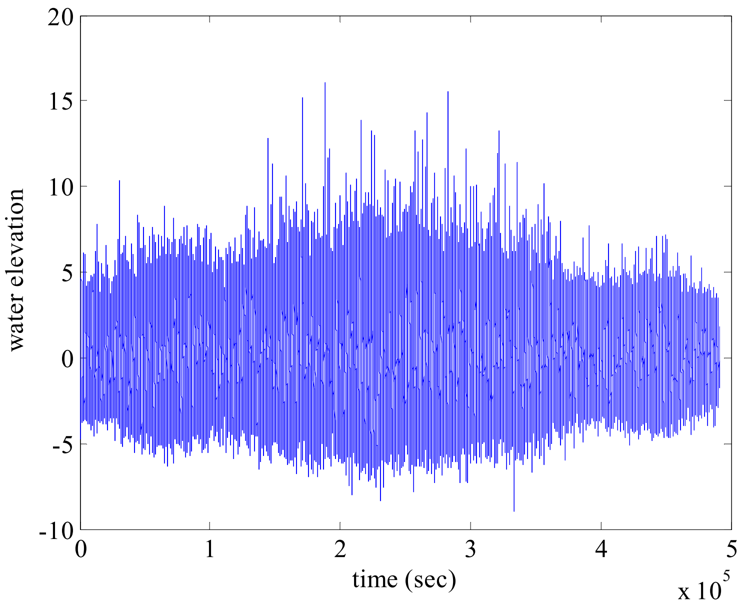


Figure 4-1 Time history of water elevation of field measurement

In order to answer the previous question, water elevation data were analysed statistically. Nevertheless, prior to the analysis, water elevation should be set at zero. The result of statistical analysis on water elevation data was shown as blue crosses in Figure 4-2. The theoretical linear probability density of water elevation data was calculated according to the theory of Gaussian distribution discussed in sub-chapter 2.1.5. The theoretical distribution was presented as a continuous red line in Figure 4-2. It confirms that that theoretical distribution of Gaussian generally fits the empirical distribution. However, impreciseness happens around the mean water level.

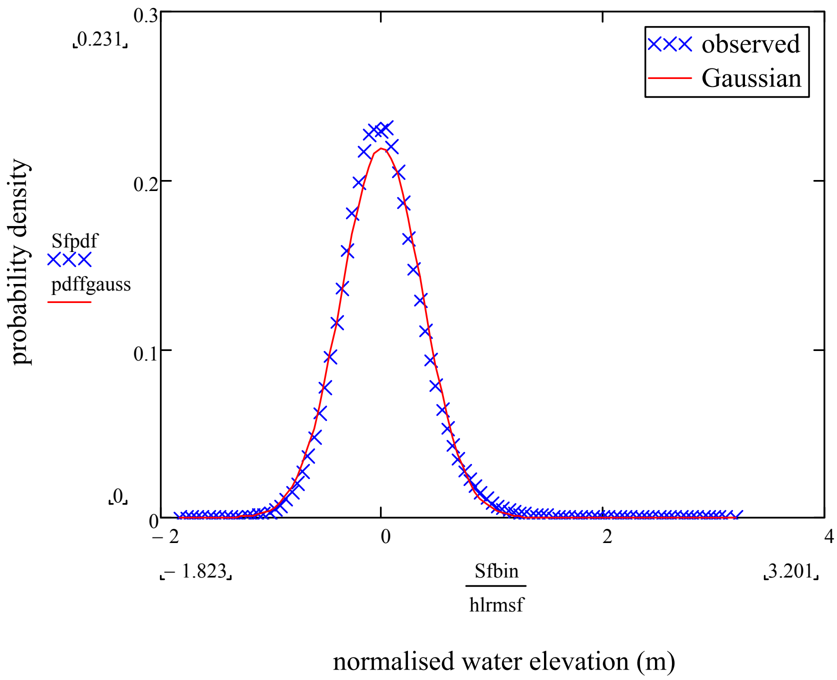


Figure 4-2 Probability density of water elevation of field measurement

Nevertheless, the interest is given to the highest water elevation. Figure 4-2 is unable to show clearly the accuracy of theoretical Gaussian distribution in extreme region. For this reason, the following Figure 4-3 is presented. In the following figure, the probability distribution is expressed as a logarithmic scale. Using logarithmic scale, it is shown more clearly that Gaussian distribution has mis-predicted the probability of highest wave region.

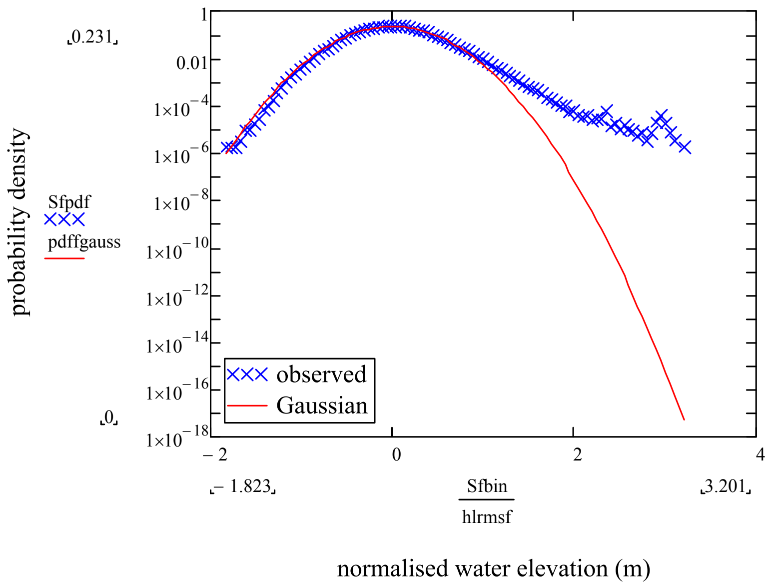


Figure 4-3 Probability density (log scale) of water elevation of field measurement

Along with water elevation analysis of field measurement data, identical computation was executed using the laboratory simulations data. The simulations were arranged into three different significant wave heights where each type was run for approximately seven hours. Figure 4-2 shows the time history of water elevation data taking 050A (significant wave height of 5 cm measured by gauge A) as a representation.

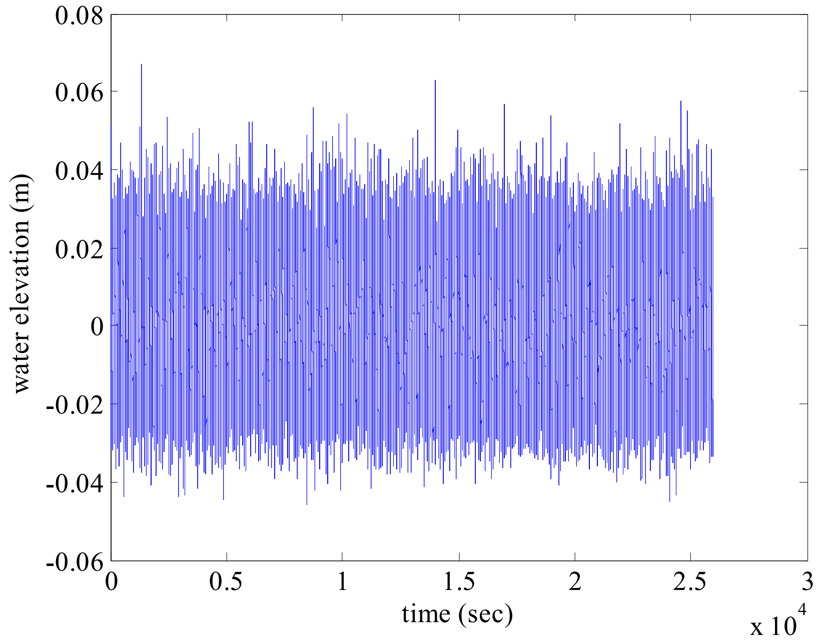


Figure 4-4 Time history of water elevation of laboratory simulation (050A)

Figure 4-4 above displays water elevation recorded over a seven hour period from one of the laboratory simulations. Through visual observation, the imbalance between the crest and trough is much less than the field measurement. The statistical calculation shown in Table 1 indicates the skewness value of 0.1. It shows a non-zero positive value, which is the same as the field analysis result; however, it is one third of the magnitude. Smaller skewness value confirms the fluctuation of laboratory water elevation which is less compared to that from the field measurement. Despite the fact that it is less skewed, laboratory data also shows some extreme crest heights at certain points in time.

Again, the same question is posed on how well the distribution of laboratory water elevation is presented by the linear Gaussian model. For this purpose, the next figure shows the laboratory simulation compared with the Gaussian prediction. The water elevation was analysed statistically using 100 bins of normalised water elevation value. Using the same symbol, the blue crosses represent the observed probability density of laboratory data and the continuous red line shows the theoretical Gaussian distribution. The Gaussian model fits the measured probability relatively well; however the most obvious imprecision is that Gaussian under-predicts the most likely water elevation around still water as shown in the following Figure 4-5.

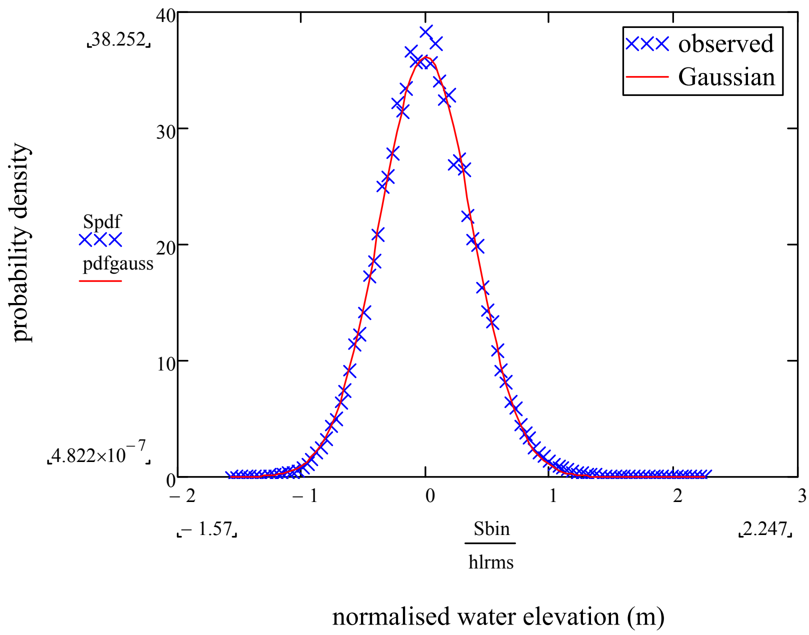


Figure 4-5 Probability density of water elevation of laboratory simulation

It cannot be seen clearly from the previous figure how the extreme values deviate from the Gaussian distribution. For this reason the probability density was converted into a logarithmic scale as depicted in Figure 4-6 below. Using a logarithmic scale, it is now more obvious how the Gaussian model fails to fit the observed probability of highest range water elevation. Nonetheless, taking a comparison between laboratory and field measurement, it is clear that the laboratory distribution fits the Gaussian prediction better than field measurement distribution. The highest wave is under-predicted by 10^3 for the laboratory distribution while it is under-predicted by up to 10^{11} in the case of field distribution.

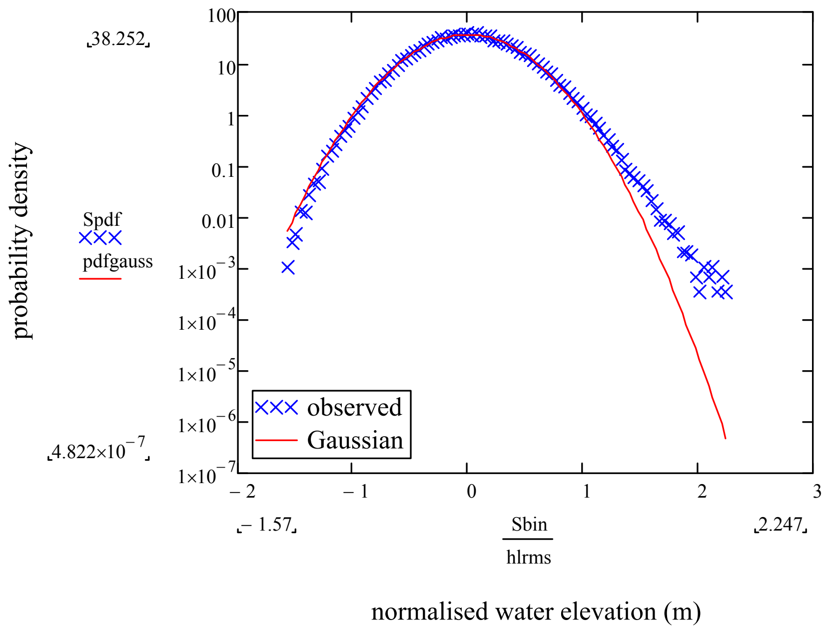


Figure 4-6 Probability density (log scale) of water elevation of laboratory simulation

Table 4-1 presents the statistical properties of both sets of water elevation data in order to get numerical parameters that can easily be compared. The table lists water elevation properties of skewness, kurtosis, and standard deviation of laboratory and field measurements.

Table 4-1 Statistical properties of water elevation for both laboratory and field measurement

Elevation Properties	Laboratory Simulation						Field Measurement
	050A	050B	075A	075B	100A	100B	
Skewness	0.105	0.092	0.134	0.129	0.159	0.193	0.3
Kurtosis	3.136	3.095	3.070	3.068	3.071	3.119	3.7
Std Dev (m)	0.011	0.011	0.018	0.017	0.027	0.024	1.8

Zero skewness shows that the distribution profile is symmetrical to still water level. This condition is basing the linear assumption of water level distribution. Positive skewness indicates that greater fluctuation of water elevation takes place above the mean sea level. The physical behaviour of wave that can be observed in nature due to positive skewness is the peaked crest and rounded trough. Comparing the skewness parameter of both types of data, the skewness of laboratory simulation is less than that of the field measurement. Therefore, it is concluded that field measurement data show higher nonlinearity compared with laboratory simulation data. How the nonlinearity affects the accuracy of extreme wave and crest height distribution will be discussed in sub-chapters 4.4 and 4.5.

The next statistical property which also affects the behaviour of water elevation distribution is the kurtosis value. This value represents the shape of the distribution to the Gaussian distribution which has kurtosis equal to 3. The kurtosis values which were calculated from the laboratory simulation are varied; nevertheless, they are still very near to the value of 3. This indicates that laboratory simulations are relatively well represented by Gaussian distribution. On the other hand, field measurement has a kurtosis value larger than that of the laboratory simulation. A comparison of the two shows that actually the distribution of water elevation in nature deviates from the ideal Gaussian distribution. Positive kurtosis which has a value bigger than 3 is known as leptokurtic. Leptokurtic distribution has more distribution concentrated on the tail part. Recent studies have shown that freak wave distribution of weakly non-Gaussian distribution can be predicted as a function of kurtosis (Onorato et el 2007; Goda 2000).

4.2 Zero-Crossing Wave

The distribution of water elevation which was discussed in the previous chapter does not require prior knowledge concerning zero-crossing concept. Nevertheless, in the case of wave height and wave period, we need to be aware of the concept of zero-crossing. This sub-chapter presents the comparison between two zero-crossing concepts.

There are two types of zero-crossing waves; they are zero up-crossing and zero down-crossing waves. Zero up-crossing waves are defined as water elevation between two successive transitions of surface elevation from the level below to the level above mean level. In contrast, zero down-crossing waves are water elevation between two successive transitions of surface elevation from the level above the mean to the level below the mean water elevation (IAHR 1986) as depicted in Figure 2-2.

The investigation was carried out using the comparison of probability distribution and probability of exceedance from both up-crossing and down-crossing wave heights. Firstly, the probability density of wave height from the field measurement data was simulated for both types of zero-crossing wave definitions. In order to get clearer view of the effect of the zero-crossing definition, especially in the highest region, the probability density is presented in logarithmic scale as depicted in Figure 4-7.

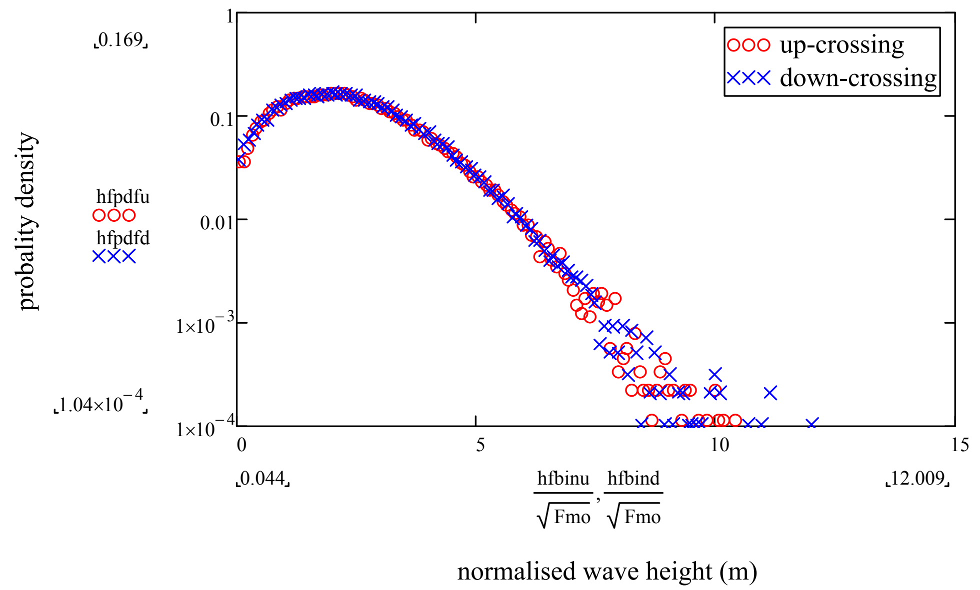


Figure 4-7 Probability density of zero-crossing field measurement data

Figure 4-7 shows the probability density of normalised wave height taken from the field's water elevation measurement. The red circles represent the probability density based on up-crossing waves; while the blue crosses indicate the probability of zero down-crossing waves. The above figure shows that the probability density of zero up-crossing and zero down-crossing are identical for small wave height, but diverse at the highest region of wave height.

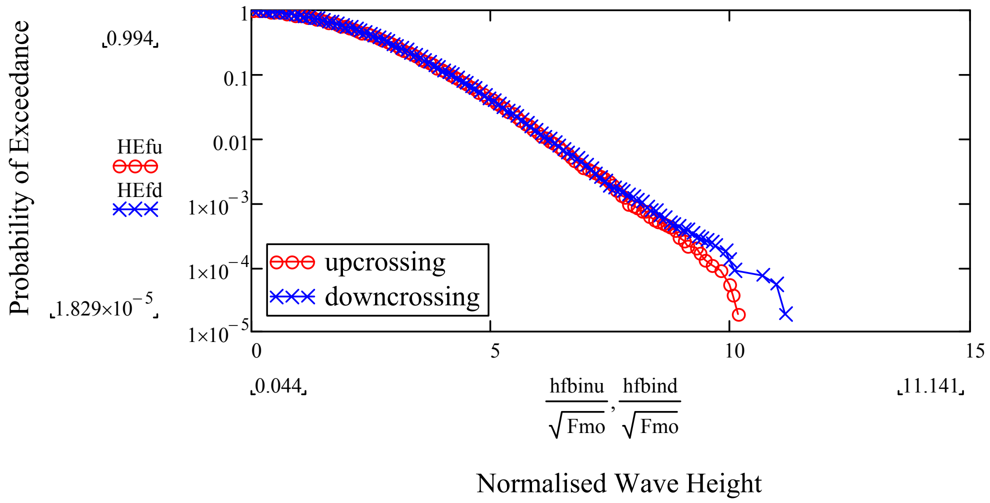


Figure 4-8 Probability of exceedance of zero-crossing field measurement data

In addition to the previous comparison, the zero-crossing wave height is expressed in the form of probability of exceedance. According to statistical analysis of exceedance probability, for the same wave height, zero down-crossing has a higher probability of being exceeded compared to zero up-crossing waves as shown in Figure 4-8.

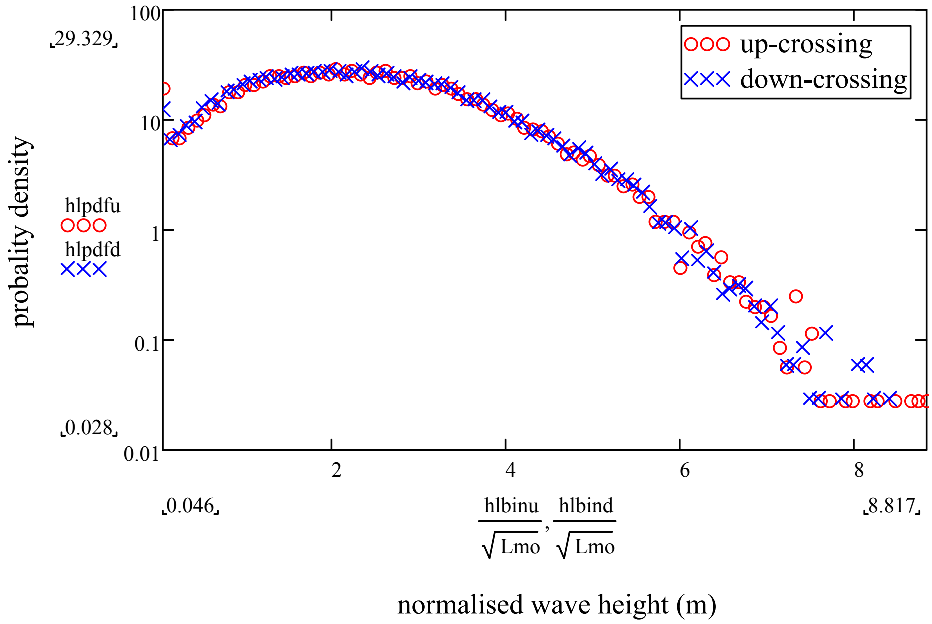


Figure 4-9 Probability density of zero-crossing laboratory simulation data

A similar result is found in the case of probability density taken from the laboratory simulation data. The most obvious deviation takes place at the highest region of wave height as depicted in Figure 4-9. Nevertheless, probability of exceedance of laboratory data shows that at extreme wave height, the up-crossing wave will have a higher probability of being exceeded, shown in Figure 4-10.

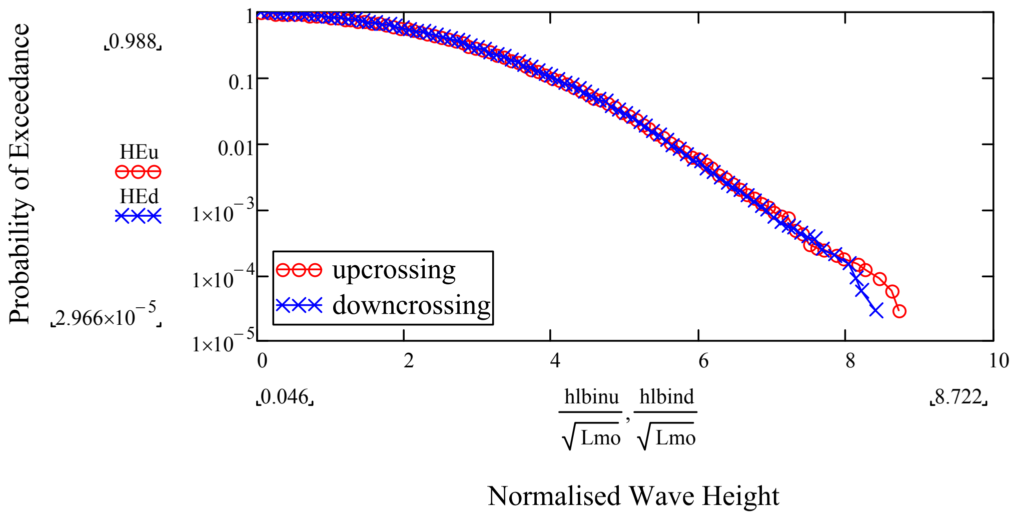


Figure 4-10 Probability of exceedance of zero-crossing laboratory simulation data

Further study needs to be done in order to investigate whether there is a tendency for one zero-crossing type to have a higher probability of exceedance than the other. In this study, it cannot be ensured that a similar pattern of probability of exceedance can be applied for every wave data. What this study is trying to put across is that zero-crossing definition is essential in the case of extreme wave height analysis. Nevertheless, this study uses zero down-crossing wave definition as recommended by IAHR (1986).

4.3 Abnormal Waves

Using the zero down-crossing definition, wave heights were derived from the water elevation records. In order to identify the presence of abnormal waves, attention is paid to the two main wave height parameters which are significant wave heights and maximum wave heights. Significant wave height was calculated based on the average wave height of the highest one third. Maximum wave height is the highest wave height in the record. There are still no particular rules concerning abnormal wave height definition which are generally accepted (Petrova and Soares 2008). Nonetheless, the current study operates within the limits defined by Dean (1990 cited Soares et al 2007) who states that an abnormal wave happens when the maximum wave height is bigger than twice the significant wave height, known as the Abnormality Index (AI).

Together with the abnormality definition based on wave height, there is also another definition based on crest height. Crest height is defined as the highest positive water elevation in each zero down-crossing wave. Each wave height corresponds to one specific value of crest. The most important feature of crest is the maximum crest. The crest height which is normalised with the significant wave height is named the crest amplification index (CI). There are different rules applied in defining the threshold value of CI to be categorised as abnormal wave. Nevertheless, this study uses the value of 1.3 as the maximum threshold suggested by in Soares et al (2007).

Taking the maximum wave height, significant wave height, and crest height from the record, the values of the AI and CI of each record was calculated. Based on the previous agreed thresholds of AI and CI, Table 4.2 shows the presence of abnormal waves in this study.

Table 4-2 Abnormal Index and Crest-Amplification Index

Wave Properties	Laboratory Simulation						Field Measurement
	050A	050B	075A	075B	100A	100B	
$H_{1/3}$ (m)	0.042	0.041	0.067	0.067	0.105	0.093	7.1
Hmax (m)	0.093	0.091	0.146	0.150	0.217	0.195	22.0
Hmax/$H_{1/3}$ (AI)	2.22	2.21	2.16	2.26	2.07	2.11	3.08
Cmax (m)	0.067	0.060	0.097	0.091	0.140	0.124	16.1
Cmax/$H_{1/3}$ (CI)	1.60	1.45	1.44	1.37	1.33	1.34	2.25

From the table above, the amplification index of the laboratory simulation varies from 2.07 to 2.26. Based on the threshold value put forward by Dean (1990 cited ibid.), these numbers shows the presence of abnormal wave in laboratory simulation. On the other hand, field measurements recorded maximum height that triples the value of its significant wave height. It shows an obvious presence of abnormal waves in field measurement data used in this study.

The previous table also presents values of the highest wave crests of laboratory simulation data that vary between 1.33 to 1.6 times the significant wave heights. In addition, the crest height of field measurement reaches up to 2.25 times the significant wave height. Based on the threshold value agreed previously, the existence of abnormal waves in the record is also confirmed by the crest amplification index.

4.4 Wave Height Distribution

Following the discussion of abnormal waves, this current chapter will now focus on the main objective of this study which is extreme wave height distribution. As the starting point, statistical wave properties will be presented. Prior to the validation of the modified distributions, the empirical distribution will be first compared with the linear Rayleigh distribution. The validation will show the degree of accuracy of Rayleigh's theory in wave height prediction. Finally, the improved wave height distribution formulae of Forristall (1978), Haring (1976 cited Prevosto 2000), Krogstad (1985 cited ibid.), and Rayleigh-Stokes (Nerzic and Prevosto 1998) (see subchapter 2.3.1) will be validated against the empirical distribution. Statistical parameters of wave heights are presented in Table 4-3.

Table 4-3 Statistical properties of wave height

Wave Height Stat. Prop	Laboratory Simulation						Field Measurement
	050A	050B	075A	075B	100A	100B	
mean μ (m)	0.026	0.025	0.042	0.041	0.066	0.058	4.320
std dev σ (m)	0.014	0.014	0.022	0.022	0.034	0.031	2.488
cov variance	0.536	0.560	0.524	0.545	0.514	0.530	0.576
skewness	0.499	0.416	0.482	0.409	0.441	0.444	0.787
kurtosis	3.034	3.000	-3.007	-3.086	-3.019	3.030	3.873

The previous table shows the statistical properties of wave height of each data type. Averaging the statistical property outcomes from gauge A and gauge B, the mean wave height of 050 laboratory data is 2.55 cm. Averaging also 075A and 075B, the mean wave height for 075 laboratory data is 4.15 cm. Finally, the average wave of 100A and 100B is is 6.2 cm. On the other hand, wave mean height from the field data is 4.32 m.

Standard deviation values from laboratory data are 0.85 cm, 1.4 cm, and 2 cm for 050, 075, and 100 respectively. Meanwhile, the standard deviation of field data is approximately 1.5 m. Based on the ratio between the standard deviation and the mean wave height, the covariance of each type of measurements is vary around 0.6.

Wave height skewness is much higher than the one calculated from the water elevation data. This condition comes out with the shifting of the tail of wave distribution to the right direction. Higher skewness might have been caused by the absolute value of wave height. On the other hand, wave height's kurtosis of the laboratory data indicates a small deviation from the normal distribution. However, field measurement shows large discrepancies from the standard distribution (standard distribution's skewness is equal to 3). Skewness from both types of data shows that field measurement data contains stronger non-linearity factors than laboratory simulation data.

4.4.1 Laboratory Simulation

Prior to the validation of extreme wave height distribution against the new, modified formulae, the laboratory empirical distribution is compared with the linear Rayleigh distribution. Laboratory data of 050 from gauge A is taken as a representative in the current discussion. Figure 4-11 shows the probability density of both empirical laboratory and theoretical Rayleigh distribution. The blue diamonds are the observed distribution while the red line is the standard Rayleigh distribution. From the figure, Rayleigh seems to fit the observed probability very well for wave height less than 6 cm. Higher wave heights are not predicted as well as the smaller ones; they tend to be scattered around the theoretical prediction of Rayleigh.

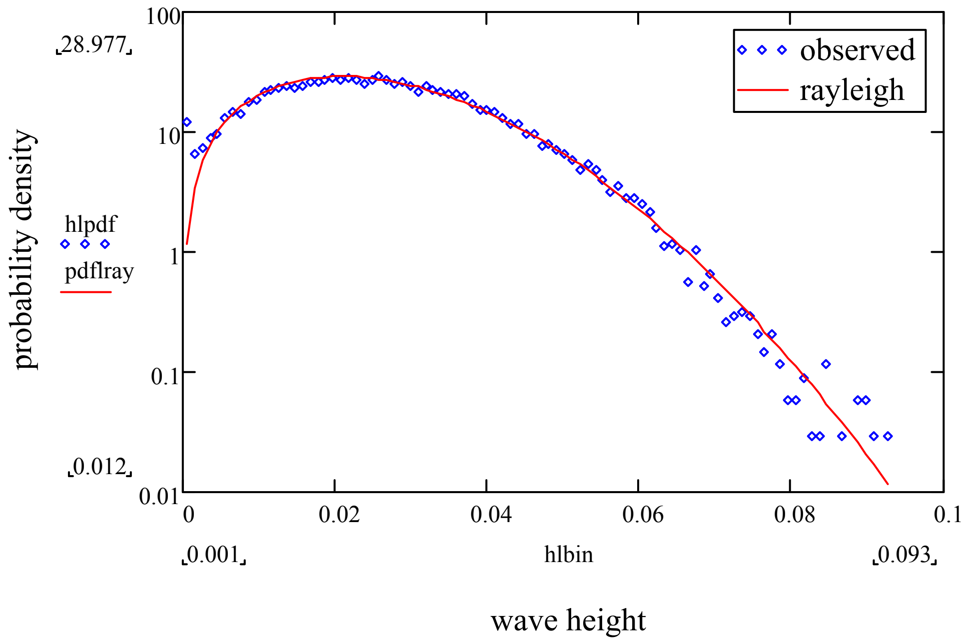
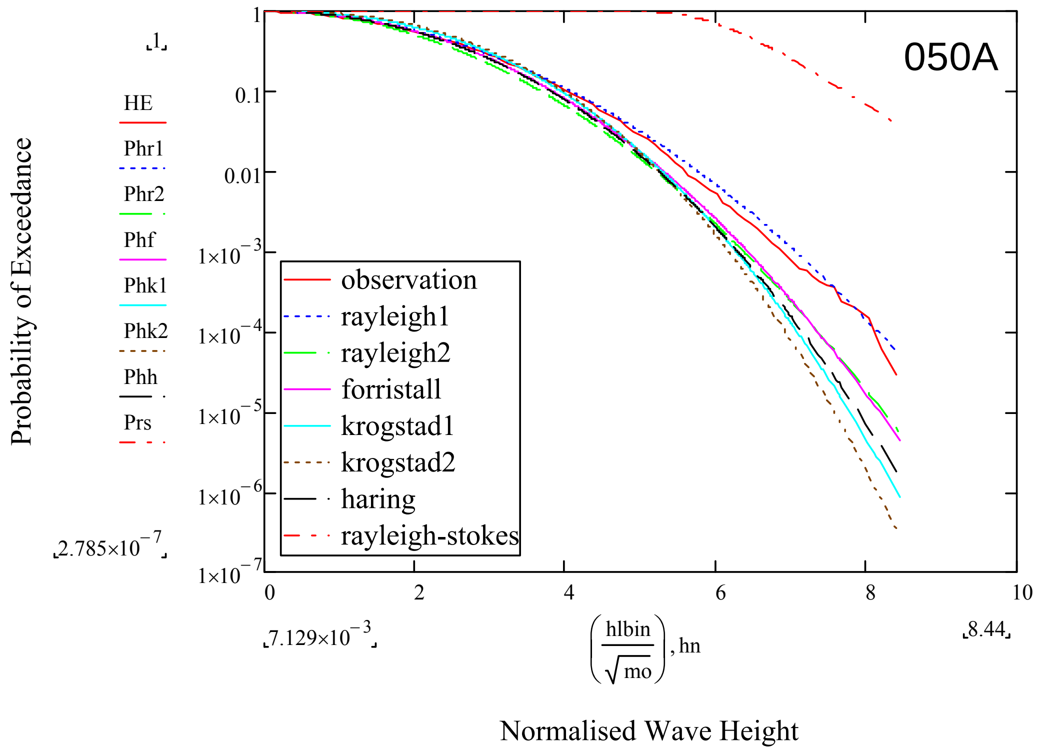


Figure 4-11 Probability density of wave height of laboratory data

The graph depicted above shows that Rayleigh does not give an accurate prediction of the largest wave. The validation includes the linear distribution of Rayleigh, Rayleigh-Stokes proposed by Nercic and Prevosto, and the modified Weibull distribution of Krogstad, Haring, and Forristall. In order to get a clearer validation of how these distributions fit the data, the comparison is presented as exceedance probability of normalised wave height as depicted in Figures 4-12, 4-13 and 4-14.

Numerical analysis results show that Rayleigh over-predicts the extreme wave height distribution of laboratory simulation which is denoted by the continuous red line. On the other side, the Krogstad, Haring and Forristall formulae were found to under-predict the observed distribution. The deviation can be seen to increase from Forristall, then Haring and finally Krogstad. Meanwhile, the nonlinear Rayleigh-Stokes formula that includes wave steepness factor is found to greatly over-predict the observation. A similar pattern was seen for all types of laboratory data. It is concluded that in the case of laboratory activities, linear Rayleigh distribution is still preferable in predicting the wave height distribution.



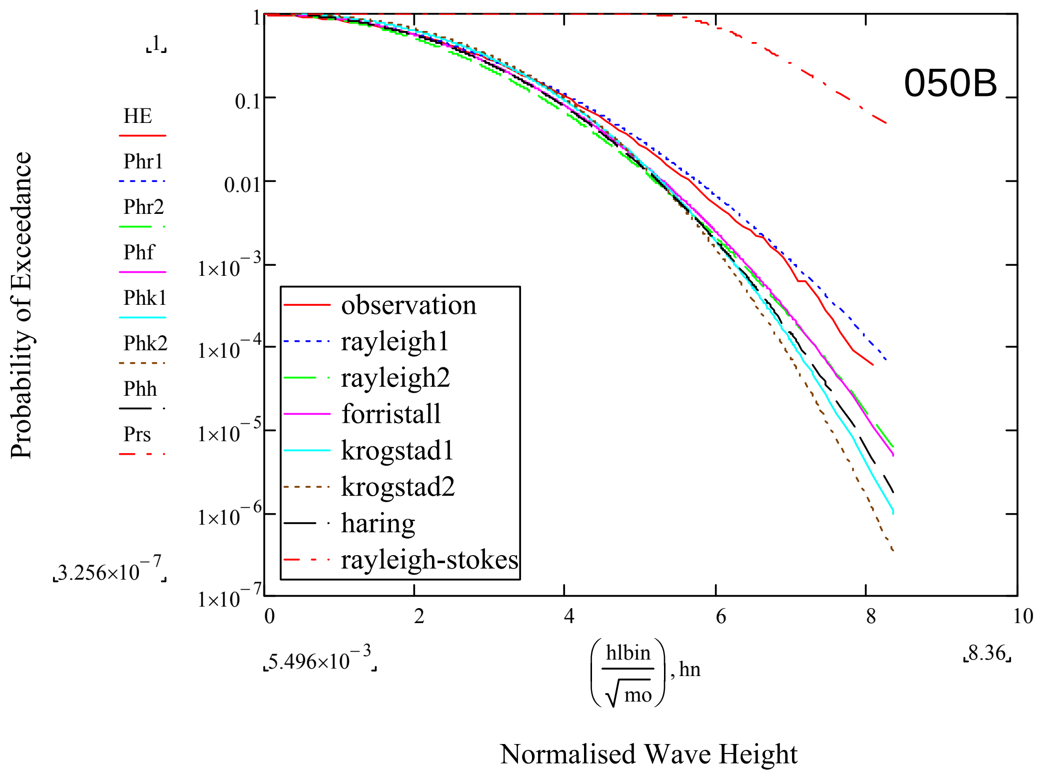
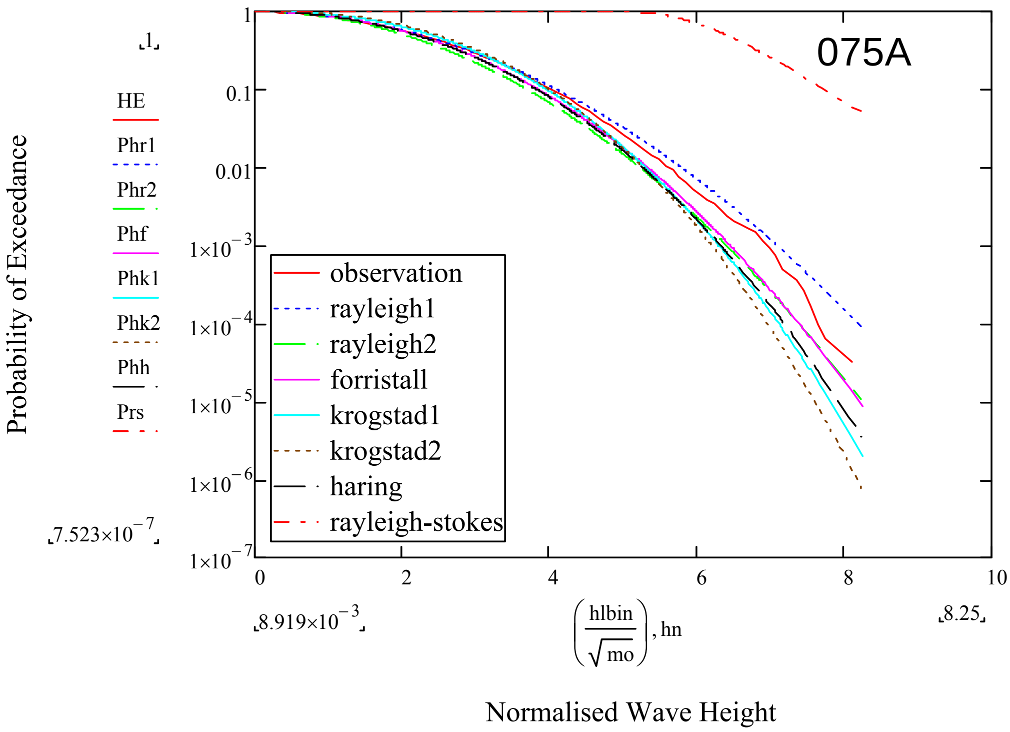


Figure 4-12 Probability of exceedance of normalised wave height 050A and 050B



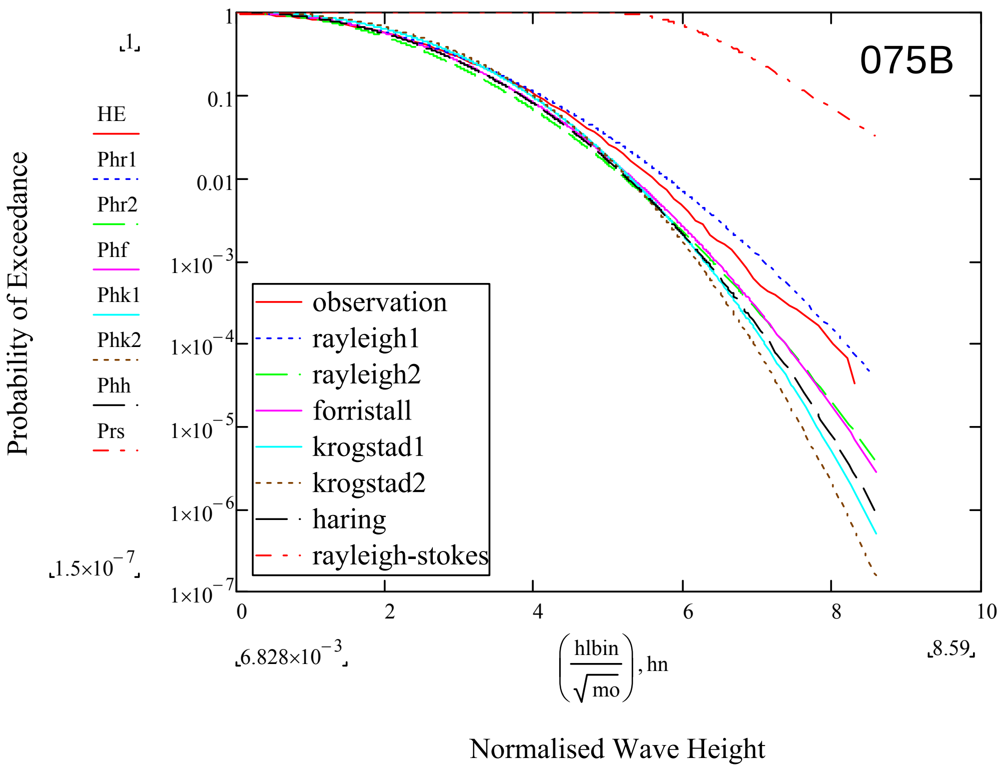
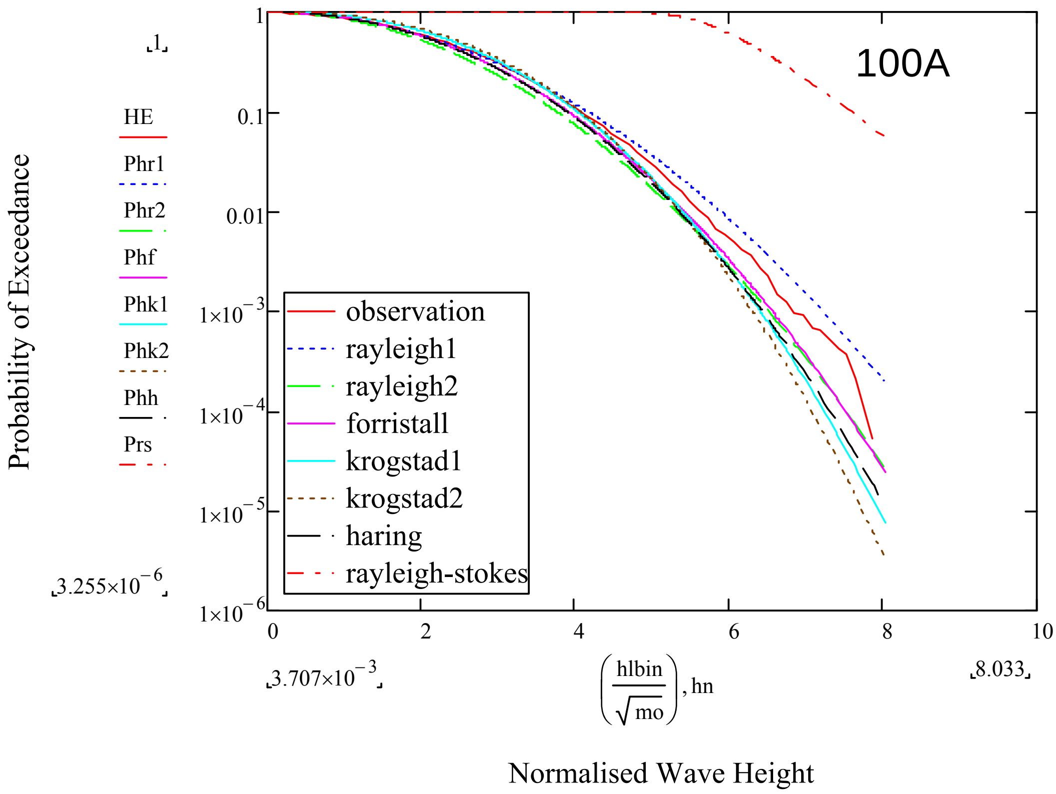


Figure 4-13 Probability of exceedance of normalised wave height 075A and 075B



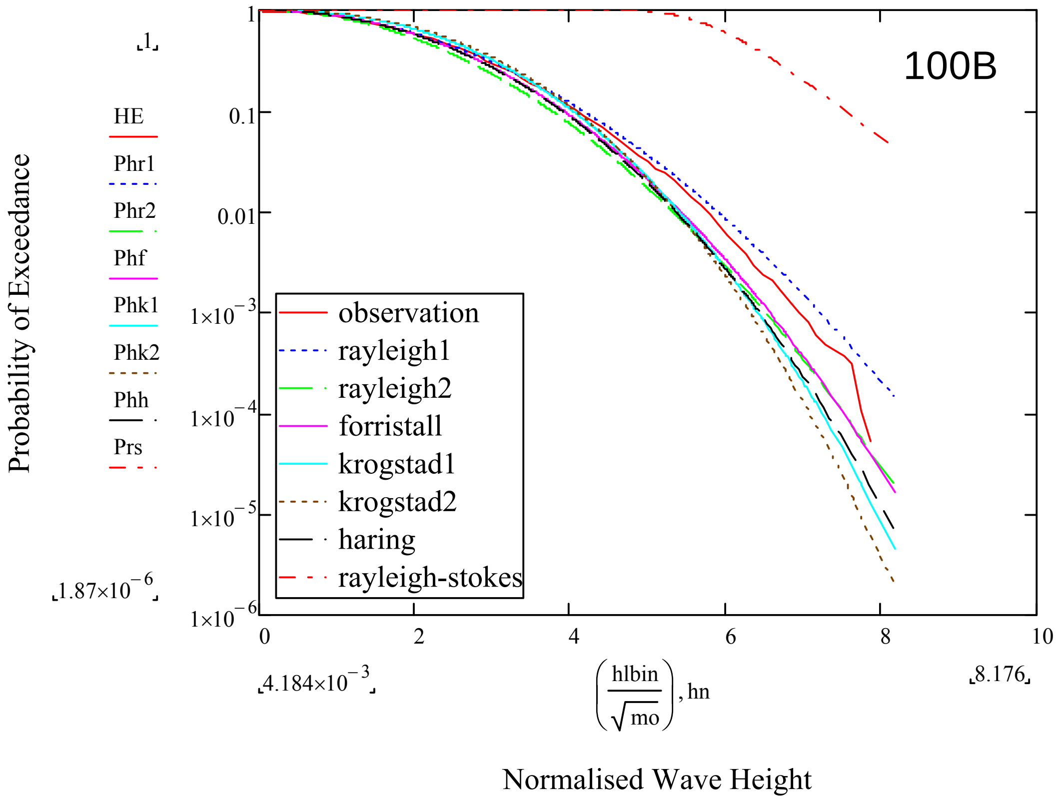


Figure 4-14 Probability of exceedance of normalised wave height 100A and 100B

4.4.2 Field Measurement

Following the laboratory analysis result, this current sub-chapter presents the validation using field measurement data. Figure 4-15 shows how well wave height distribution of Rayleigh fits the field measurement. The observed distribution is depicted as the blue diamonds while the Rayleigh prediction is described as the continuous red line. From the results of numerical simulation, it is found that for wave height less than 11m, the observed field data fits the Rayleigh distribution. Nonetheless, Rayleigh seems to fail in presenting the probability value of highest wave height region. Compared with laboratory data, it is obvious that field measurement data show a greater deviation from the Rayleigh distribution.

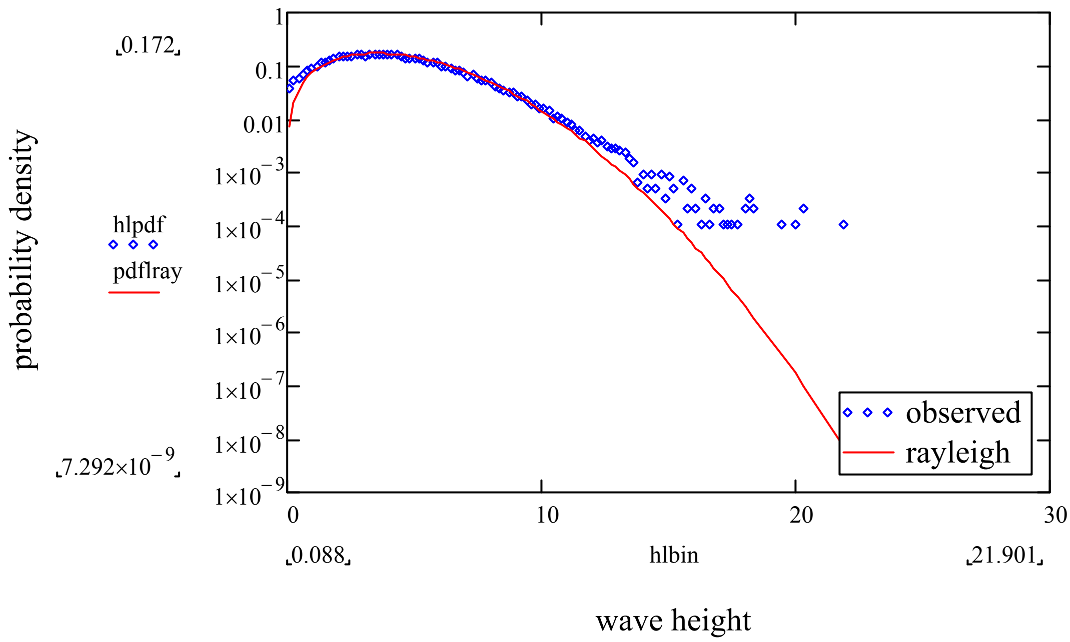


Figure 4-15 Probability density of wave height of field data

The inadequacy of the Rayleigh distribution in predicting the extreme field wave heights has motivated the development of improved formulae on wave height distribution. Figure 4-16 shows that the Rayleigh distribution under-predicts the field measurement distribution used in this study. However, the improved formula of Forristall, Haring and Krogstad does not give any better predictions. Meanwhile, the Rayleigh-Stokes formula seems to over-predict the wave height distribution with much less deviation compared with previous laboratory validation. It can be concluded that nonlinearity plays a significant role in nature so that nonlinear prediction of Rayleigh-Stokes shows a better prediction in the case of highest wave height region based on field measurement data.

The finding of the current validation - that Rayleigh distribution over-predicts the observed field measurement distribution - contradicts the previous statement by Forristall, who claimed that Rayleigh over-predicts the higher wave height distribution (Forristall 1978). This contradiction might be caused by several factors.

The first possible cause is location factor. Forristal's investigation was done in the Gulf of Mexico while the current study deals with the North Sea environment. The second factor might come from the variance of the ocean surface elevation. Having approximately the same number of waves, the data used in Forristall's formula has a variance that ranges from 0.36 to 11.02 m^2. It is much higher than the variance of storm 149 used in this study. Water elevation variance which was calculated in this study is approximately 3.3 m^2; much less than the one used by Forristall. The third factor that possibly causes the difference is the presence of abnormal waves. The normalised wave height in this study is higher than the one presented in Forristall (Forristal 1978). Nevertheless, Forristall's study did not bring up the abnormal waves issues. Clearer evidence can be found from the paper of Nervic and Prevosto (2003). In their study, they also indicate the over-prediction of Rayleigh. Nevertheless, the observed data which was used in their study does not involve abnormal waves. Similar findings relating to under-prediction of wave height were found by Soares et al (2007) who involved freak waves in their study.

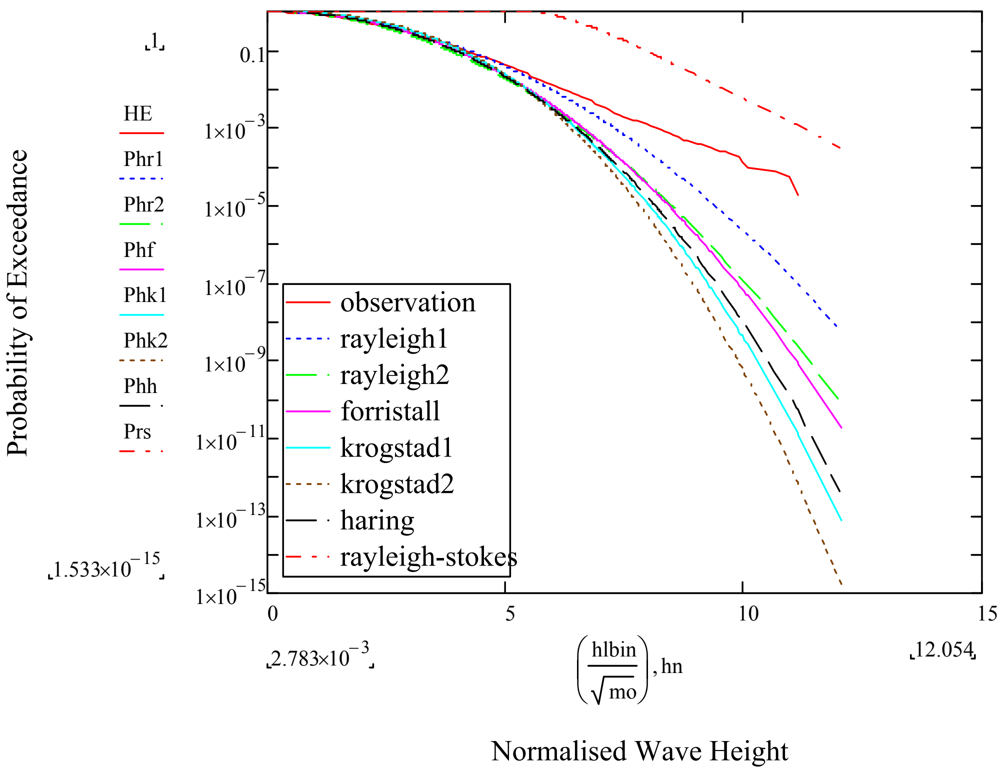


Figure 4-16 Probability of exceedance of normalised wave height form field measurement

4.5 Crest Height Distribution

Crest height is also a major factor to be considered in ocean engineering. Many scientists believe that crest height shows more nonlinearity compared with wave height. Before going on to the main discussion of crest distribution, it is best to present some general information concerning the statistical properties of crest height from all types of data used in this study. The statistical properties are presented in the following table 4-4.

Table 4-4 Statistical properties of crest height

Crest Height Stat. Prop	Laboratory Simulation						Field Measurement
	050A	050B	075A	075B	100A	100B	
mean μ (m)	0.014	0.013	0.023	0.022	0.036	0.032	2.265
std dev σ (m)	0.009	0.008	0.014	0.014	0.021	0.019	1.507
cov variance	0.616	0.636	0.606	0.623	0.582	0.603	0.665
skewness	0.592	0.519	0.544	0.533	0.560	0.602	1.036
kurtosis	3.382	3.167	3.201	3.172	3.236	3.341	5.180

After analysing the statistical properties of crest height compared with wave height, it is found that skewness (nonlinearity) values of crest are uniformly bigger than the wave height. This shows that crest height distortion is larger compared to wave height. Moreover, the kurtosis value that shows a bigger difference from the normal distribution value of 3 shows how crest nonlinearity affects the crest height more than wave height. It is obvious from the case of field measurement data how the crest distribution shows significant difference from the linear normal distribution. How this affects the behaviour of crest height distribution will be elaborated on further in the following part.

Nevertheless, prior to the discussion of crest height distribution, additional wave factors which will be involved in the empirical formulae are presented in Table 4-5. Those additional wave parameters are spectrum width, mean period, wave number and wave steepness.

Table 4-5 Additional wave properties used in crest distribution

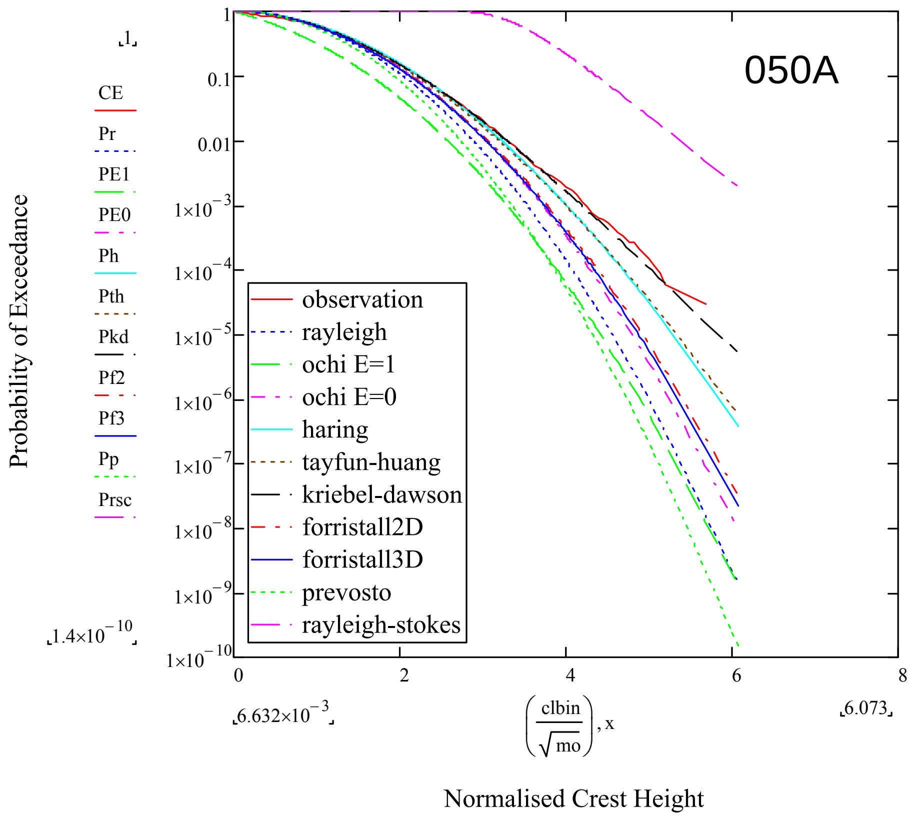
Wave Properties	Laboratory Simulation						Field Measurement
	050A	050B	075A	075B	100A	100B	
Ndzc	33719	33664	30911	30759	18857	18851	54673
Nmaxl	63119	152188	41682	109554	33931	72260	81115
bandwith ε	0.85	0.98	0.67	0.96	0.83	0.97	0.74
Tz (s)	0.770	0.772	0.929	0.934	1.192	1.192	9.0
kz	6.8	6.8	4.7	4.6	2.8	2.8	0.050
sz	0.045	0.044	0.050	0.049	0.047	0.042	0.057

Spectral bandwidth parameter, ε, is calculated based on the number of zero down-crossing waves (Ndzc) and the number of local positive maxima (Nmaxl). These parameters will be used in the calculation of crest height distribution. A large value of spectral bandwidth parameters shows that most of them have broad band-spectrum. Below the spectral width parameter, Tz denotes the value of zero-crossing wave period. This value is then used to calculate the wave number, kz, using dispersion relation. Wave steepness is then calculated based on wave number and significant wave height described earlier.

4.5.1 Laboratory Simulation

This sub-chapter presents the comparison of empirical crest height distribution and the observed laboratory distribution. Probability exceedance of crest from the laboratory simulation is denoted as the red line. Standard linear prediction of Rayleigh is represented as the broken blue line. From the numerical analysis, it is found that the Rayleigh prediction of crest height largely under-predicts the observation. The Rayleigh prediction is one of the lowest predictions on crest height along with the Ochi formula. Comparing both predictions of Ochi involving narrow-banded (E = 0) and broad-banded factors (E = 1), although the observation shows large spectral width parameters, it does not make Ochi prediction for wide-spectrum closer to the observation distribution. This indicates that spectral width parameter does not affect the crest distribution. This finding is equivalent to that from Cartwright's (1958 cited Forristall 1978) study. Cartwright succeeded in creating crest height distribution that fits the observation data without considering the spectral width parameter.

Developed based on the second order wave simulation, Forristall's prediction of crest height shows inadequacy in fitting the laboratory simulation. It is found to under-predict the distribution of crest height taken from laboratory simulation. Distributions which are found to be the closest to observation distribution are derived from the predictions of Haring-Jahns and Wheeler (1972 cited Prevosto 2000), Tayfun(1980 cited ibid.)-Huang (1986 cited ibid.), and Kriebel-Dawson (1993 cited ibid). The prediction of Haring (1972 cited ibid.) was developed based on empirical fitting. The factors which are involved in his prediction are variance of wave elevation and water depth. On the other hand, Tayfun (1980 cited ibid.) and Kriebel (1986 cited ibid.) consider the wave steepness factor in their prediction. The recently developed formula of Prevosto (Prevosto et al 2000) tends to show inconsistencies by sometimes over-predicting or under-predicting the observation. As seen also in wave height distribution, Rayleigh-Stokes (Nerzic and Prevosto 2003) consistently over-predicts the observation distribution with a large discrepancy. The analysis result of laboratory simulations is presented in the following Figure 4-17, Figure 4.18, and Figure 4-19.



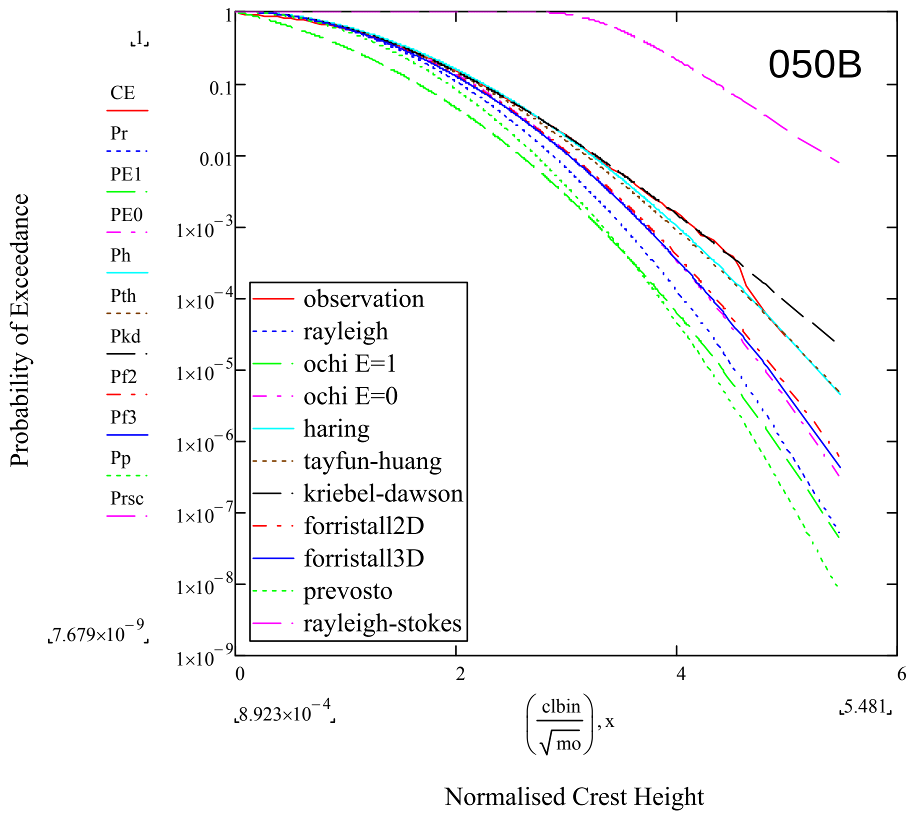
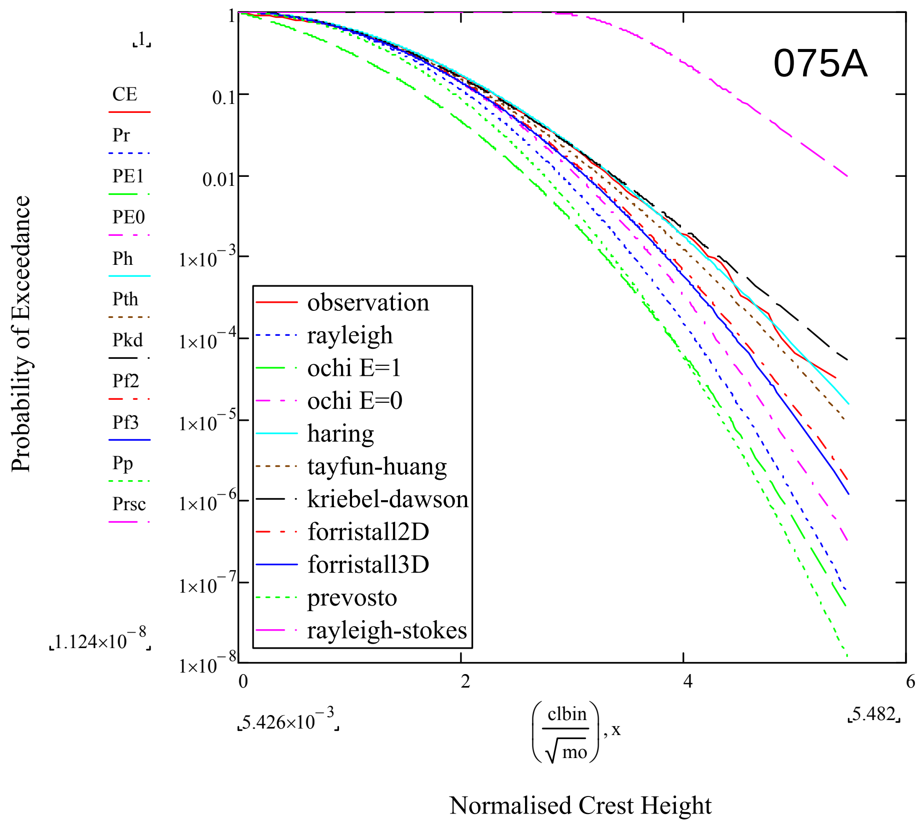


Figure 4-17 Probability of exceedance of normalised crest height 050A and 050B



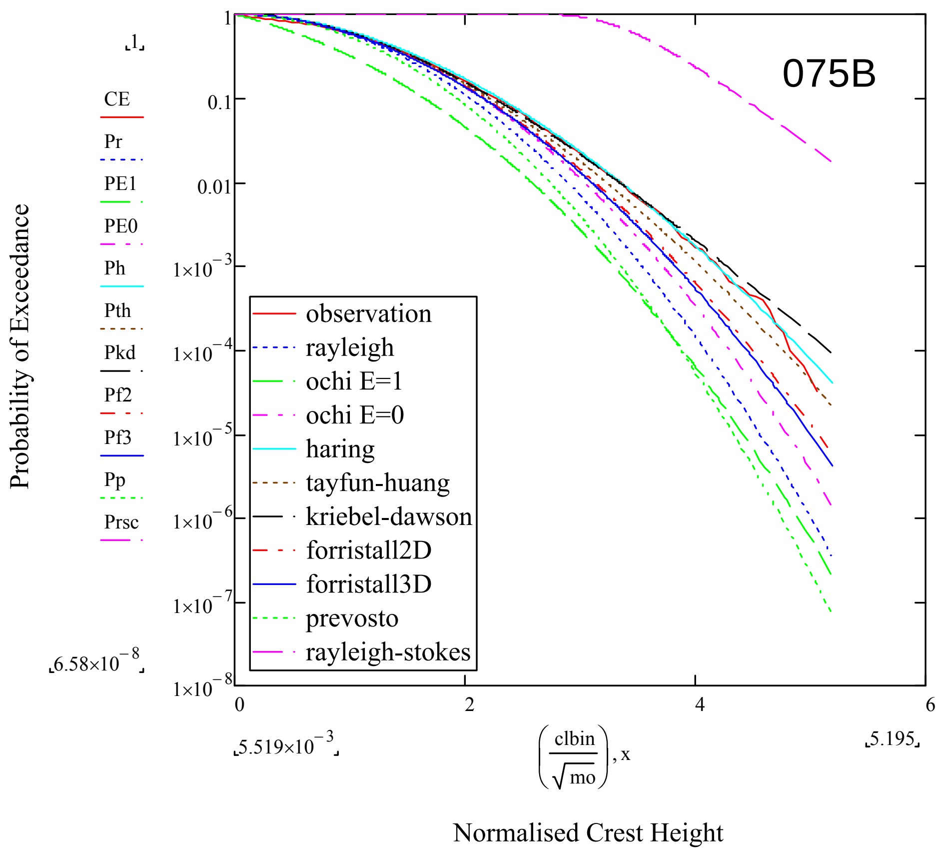


Figure 4-18 Probability of exceedance of normalised crest height 075A and 075B

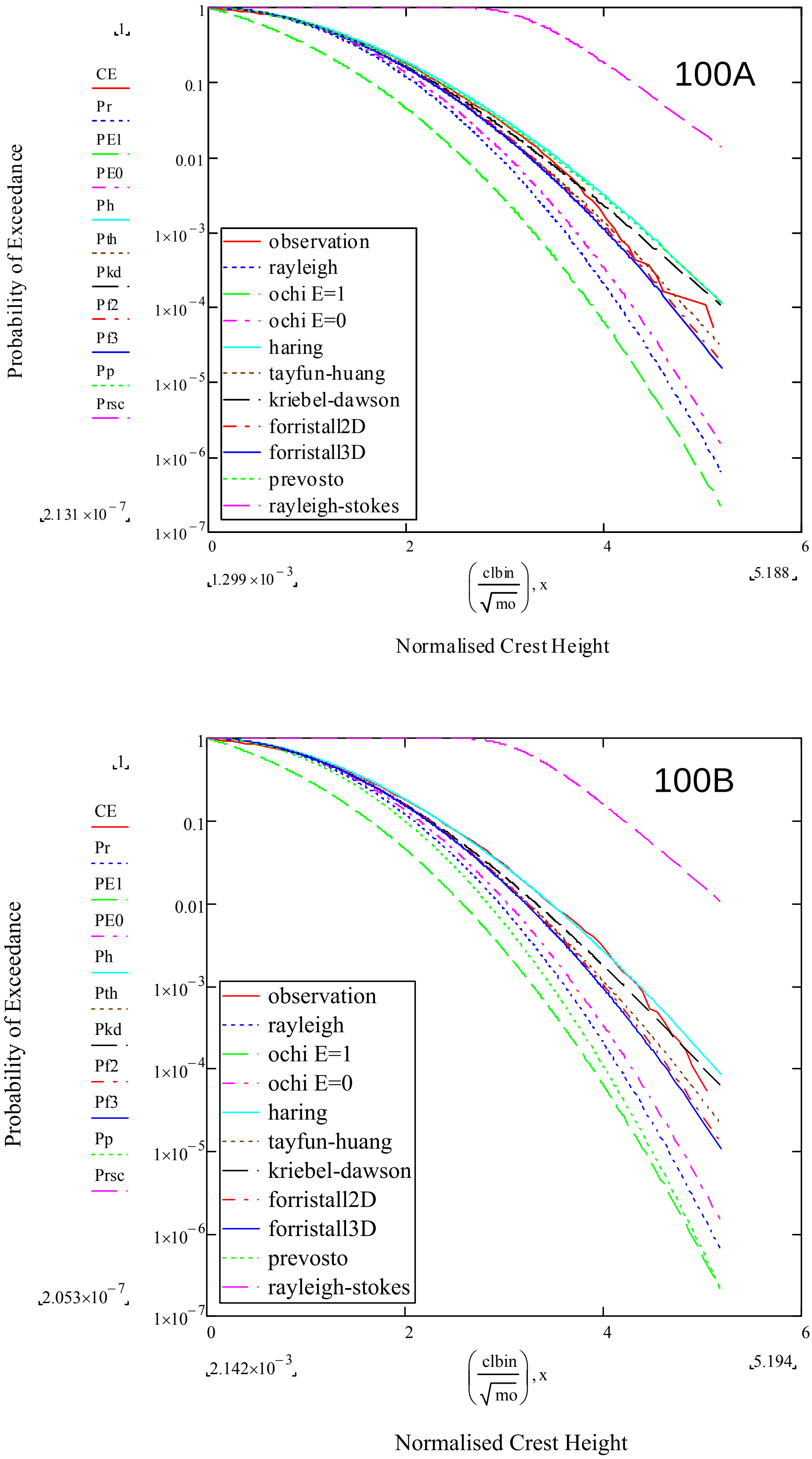


Figure 4-19 Probability of exceedance of normalised crest height 100A and 100B

4.5.2 Field Measurement

Following the investigation of crest distribution using the laboratory data, this sub-chapter presents the results of field measurement distribution. The observation of crest height distribution of field data is denoted as a red line. The Rayleigh crest distribution which is presented as a broken blue line is found to under-predict the field observation. It seems Rayleigh shows similar behaviour to laboratory validation of under-predicting the crest distribution. This finding relates to that of the previous study by Nerzic and Prevosto (2003). The same discovery was also made by Soares et al (2007).

The linear Ochi formulae are also found to under-predict the distribution of crest. However, Ochi and Rayleigh are not the ones that largely under-predict the crest distribution. It is found from the calculation that Prevosto has also greatly under-predicted the distribution. The empirical distribution of Haring is shown to give a better estimation compared with the linear prediction of Rayleigh; nevertheless, it is still far below the observed distribution. Tayfun-Huang and Kriebel Dawson are closer to the observed distribution but they still under-predict the distribution.

This time, Rayleigh-Stokes is found to fit well the highest region distribution of crest height. Rayleigh-Stokes shows significant differences when it is used with the laboratory data or field measurement. It somehow allows for a better prediction when it is used in predicting extreme height from field. Rayleigh-Stokes was developed based on the third-order Stokes expansion. It considers the nonlinear factor of wave steepness in the third order Stokes. This is possibly the reason why Rayleigh-Stokes fits the field data which have shown more nonlinearity better. The uppermost predictions come from Forristall's distributions. Showing opposite behaviour, this time Forristall over-predicts the field measurement.

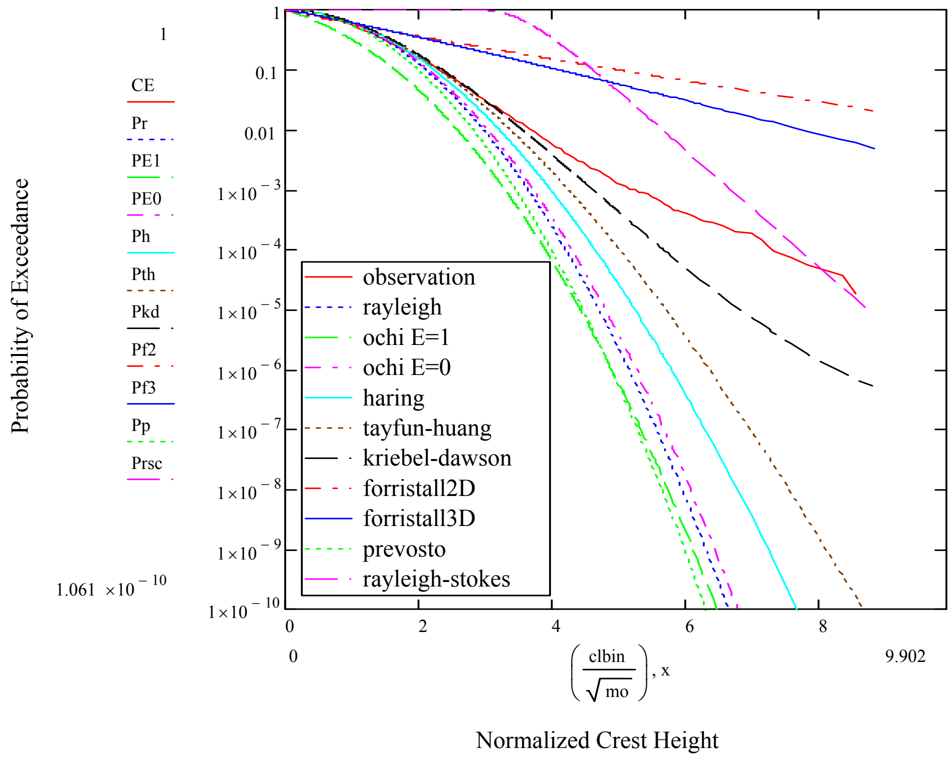


Figure 4-20 Probability of exceedance of normalised crest height of field measurement

5 CONCLUSIONS AND RECOMMENDATIONS

There are many uncertainties in nature that need to be quantified for engineering purposes. Therefore, it is the task of scientists and engineers to create a tool to quantify the randomness of nature. Luckily, today's rapid advancements in technology enable people to acquire a better understanding of these random phenomena. Numerical and mechanical equipment has been created to support scientific projects related to engineering needs.

Concerning the random phenomenon of offshore waves, there is increasing interest in developing a better understanding of their behavior. Distribution formulae have been developed theoretically and empirically. Validations have been done for many years; nevertheless, there is still disagreement on how well the theory fits the observed distribution. Most recent concern has been with the prediction of wave and crest height distribution in extreme sea conditions.

After studying several journals articles on wave and crest height distributions, the current study proposed the validation of six formulae of wave height (Rayleigh type1, Rayleigh type2 Forrsitall, Krogstad, Haring and Rayleigh-Stokes) and nine predictions of crest heights (Rayleigh, Ochi, Haring, Tayfun-Huang, Kriebel-Dawson, Forristall 2D, Forristall 3D, Prevosto, Rayleigh-Stokes). The principal ideas of these formulae are discussed in the literature review. Most modified formulae presented in this study involved nonlinearity factors of water depth, wave steepness, and directional spectra. In general, new formulae are developed to improve the performance of the linear Rayleigh distribution. Although validation had been undertaken for some of these formulae when they were formulated, further validation is required to check the accuracy of these new formulae in the case of the extreme wave.

Two types of data are used in this study representing two different interests. The first one comprises laboratory simulation data based on the Pierson Moskowitz spectrum. The results of laboratory validation are expected to support any laboratory research that requires wave or crest height prediction. Another data is field measurement of water elevation from the North Alwyn region in the North Sea. Validation of field measurement data can be used as the groundwork for choosing an appropriate formula for ocean engineering interests such as design purposes.

Chapter 4 shows the validation results on wave and crest height distributions. In general, validation showed that the nonlinearity factor was engaged more in field measurement than laboratory simulation. Visual observation of time history and statistical parameters supported this result. On the other hand, crest height seemed to be more influenced by the nonlinearity factors than wave height distribution.

Validation was taken with the standard linear distribution of Rayleigh as reference of other formulae. Empirical distribution of wave and crest height was sorted and the statistical distribution was calculated numerically before being validated against the theoretical formulae.

In the case of wave height distribution, focusing on the largest wave, the Rayleigh distribution was found to over-predict the observed laboratory simulation; contrarily, it under-predicted the field measurement distribution. From the laboratory simulation, Rayleigh was found to be the closest theoretical formula to the observed data showing the linearity of laboratory wave height distribution. Therefore the Rayleigh distribution is recommended in the case of laboratory simulation to predict the wave height distribution. On the other hand, Rayleigh was found to under-predict wave height distribution of field measurement data. Although Rayleigh has shown under-prediction of the field wave height distribution, it was found to fit better than the modified Weibull distribution of Forrsitall, Krogstad and Haring. Nevertheless, Rayleigh-Stokes was found to show better prediction in this case. This shows the nonlinearity involvement in the case of wave in nature.

When the Rayleigh crest distribution was validated against lab and field measurement data, both resulted in severe under-prediction against observed distribution. This shows stronger nonlinearity effects in crest distribution than wave height distribution. In this case, the improved non-linear formula seems to fit the data better. Kriebel-Dawson showed the closest prediction of crest height of laboratory simulation. It indicates the involvement of nonlinear factor of wave steepness in predicting laboratory crest height. The Kriebel Dawson model deviated further from field distribution. In general all predictions tended to give a larger inaccuracy against field data. However, Kriebel-Dawson showed better prediction compared with Rayleigh, Ochi, Haring and Tayfun-Huang. In the validation of the field measurement, Forristall seemed to over-predict the data. The Forristall prediction of largest wave height deviated 10^2 from the observed field distribution. The closest prediction was found to be derived from the Rayleigh-Stokes distribution. Although it greatly over-predicted the laboratory crest height, Rayleigh-Stokes managed to give the most accurate prediction of the highest crest height from the field data. Interestingly, the Prevosto formula that includes the directional factor did not give a better prediction even when compared with the linear theory.

New modified crest height prediction in this study included the nonlinear factors in a different way. The Haring formula which includes water depth did not give the required accuracy in extreme crest height prediction. Better accuracy was performed by formulae that considered the wave steepness factor (Tayfun-Huang and Kriebel-Dawson). However, the discrepancy between the two theories is caused by the mathematical construction of the formulae. A higher degree of steepness was involved in Kriebel-Dawson (see equations 2-102 and 2-105). The prediction of Forristall that involved both water depth and wave steepness was less accurate for the laboratory simulation data, but showed better fitting for the field measurement compared with the Tayfun-Huang and Kriebel-Dawson models. This might be caused by the development of parameters used by Forristall, which were based on field measurement fitting. Rayleigh-Stokes ,which was found to give the best prediction of largest crest height, showed a sharp degradation of exceedance probability which might result in large under-prediction in a very large crest. Surprisingly, the Prevosto prediction of crest height that considered the wave direction factor and steepness factor was found to show a significant deviation from the observed crest height even when compared with the linear theory.

Data which is used in current study did not set to provide variety of wave steepness. Referring to table 4.5, the steepness values are relatively invariant. Further study can be focused on the effect of wave steepness by delivering simulations that address steepness variety. Because it has only used deep water data, the current study is not informed on the water depth effect. For this reason, further study can involve shallow water data to examine the effect of water depth. Different results of laboratory and field validation showed that behaviour differences in both conditions influenced the distribution. The behaviour differences might come from directional effect and wave interaction which needs to be studied further.

REFERENCES

Berry, Donald A, and Lindgren, Bernard W., 1990. Statistics : theory and methods. Brooks/Cole Publishing Company. Pacific Grove, California, USA

Chakrabarti, S.K, 2001. Hydrodynamics of Offshore Structures, Computational Mechanics Publications, WIT Press, Southampton.

Forristall, G.Z., 2000. Wave Crest Distribution: Observation and Second-Order Theory. Journal of Geophysical Research 30, 1931–1943.

Forristall, G.Z., 1978. On the statistical distribution of wave heights in a storm. Journal of Geophysical Research 830 (C5), 2353–2358.

Freedman, David, and Diaconis, Persi, 1981. On the histogram as a density estimator: L2 theory. Probability Theory and Related Fields. 57(4), 453-476

Goda Y. 2000. Random seas and design of maritime structures. Advanced series of ocean engineering volume 15. World Scientific.

International Association of Hydraulics Research (IAHR), 1986. List of sea parameters.

Krogstad, Harald E., and Arntsen, Øivind A, 2000. Linear wave theory Part b. Random waves and wave statistics. Norwegian University of Science and Technology. Trondheim, Norway

Miskovis, Ivan, Eskinja, Zdravko, and Horvat, Krunoslav. Wavemaker Control System for Irregular Developed Sea Waves Generation. Marine Research and Advanced Technologies. Croatio

Mori, Nobuhito, Janssen, Peter A.E.M, and Onorato, Miguel. 2007. Freak Wave Prediction from Spectra. Maximum Wave Height Prediction in 1D waves (10th international workshop on wave hindcasting and forecasting and coastal hazard symposium)

Naess, A., 2007. An Introduction to Random Vibrations. Centre for Ships and Ocean Structures. Norwegian University of Science and Technology. Trondheim, Norway.

Nerzic,R., Prevosto, M., 1998. A Weibull-Stokes model for the distribution of maximum wave and crest heights. The international Journal of Offshore and Polar Engineering 8 (2), 90-101

Ochi, M.K. 1998 Ocean Waves – The Stochastic Approach, Cambridge Ocean Technology Series 6, Cambridge.

Papoulis, Athanasios., 1990. Probability & statistics. Prentice-Hall. Englewood Cliffs, New Jersey, USA

Petrova, P., and Soares, C.Guedes.,2008. Maximum wave crest and height statistics of irregular and abnormal waves in an offshore basin. CENTEC, Technical University of Lisbon, Lisboa, Portugal. Applied Ocean Research 30, 144 -152

Petrova, P., Chevneva, Z., and Soares, C.Guedes, 2007. On the adequacy of second order models to predict abnormal waves. Ocean Engineering 34, 956-961

Prevosto, Marc., Krogstad, Harald. E., and Robin, A., 2000, ‘‘Probability distributions for maximum wave and crest heights, Coastal Engineering 40, 329–360

Sturges, H.A., 1926. The choice of a class interval. Journal of American Statisticians Association (21), 65-66

Whitford, Dennis J., Vieira, Mario E., and Waters, Jennifer K., 2001b. Teaching time – series analysis. II. Wave height and water surfaces elevation probability distributions. American Journal of Physics 69 (4), 497- 504

Wolfram, J., and Venugopal, V., 2003. Crest height statistics of storm waves in deep water. Proc.Instn Mech.Engrs Vol.217 Part M:J.Engineering for the Maritime Environment, 213-229

Wuensch, K. L., 2007. Skewness, Statistics and The Normal Curve. http://core.ecu.edu/psyc/wuenschk/docs30/Skew-Kurt.doc.

Wurjanto, Andojo., 2004. Statistic Analysis and Probabilistic (Analisis Statistik dan Probabilitas). Civil Engineering Department. ITB Publisher

BIBLIOGRAPHY

Al-Humoud, J., Tayfun, M.A., and Askar, H., 2002. Distribution of nonlinear wave crests. Ocean Engineering 29, 1929–1943.

Baldock, T.E., Swan, C. and Taylor, P.H., 1996. A Laboratory Study of Nonlinear Surface Waves on Water. The Royal Society London A 354, 649 – 676

Boccotti, 2000. Paolo Wave Mechanics for Ocean Engineering. Elsevier Science B.V, Netherland

Coles, Stuart. 2001. An Introduction to Statistical Modeling of Extreme Values. Springer. London

Doucet, Y, Labetrie, J, and Thebault, J,. 1987. Validation of Stochastic Environmental Design Criteria in the Frigg Site. Graham and Trotman. United Kongdom.

Hughes, Steven A. 1993. Physical Modelling and Laboratory Techniques in Coastal Engineering. World Scientific. USA

Jha A, Winterstein S. Nonlinear random ocean waves: Prediction and comparison with data. In: Proceedings of the ETCE/OMAE. 2000 joint conference energy for the new millennium. USA: ASME; 2000. Paper ETCE/OMAE2000-6125.

Leblond, Paul H. 1978. Waves in the Ocean. Elsevier Scientific. Netherland

Magnusson, Anne Karin, Jenkins, Alastair, Niedermayer, Andreas, and Nieto-Borge ,José Carlos. 2003. Extreme wave statistics from time-series data. Proceedings of MAXWAVE Final Meeting October 8-10, 2003 Geneva, Switzerland

McClelland, Bramlette, and Reifel, Michael D., 1986. Planning and Design of Fixed Offshore Platforms. Van Nostrand Reinhold Company. New York

Prevosto, Marc, and Forristall, George Z., 2004. Statistics of wave crests from models vs. measurements. Journal of offshore mechanics and artic engineering 126, 43-50

Prpić-Oršić, Jasna, and Turk, Anton, 2004. Fitting of extreme wave probability distribution function using genetic algorithm. XVI Symposium SORTA

Sarpkaya, T., and Isaacson, M, 1981 Mechanics of Wave Forces on Offshore Structures, Van Nostrand Reinhold Company Inc., New York.

Stansell, Paul, 2005. Distribution of extreme wave, crest and trough height measured in the North Sea. Ocean Engineering 32, 1015-1036

Tawn, Jonathan Angus. 1988. Extreme Value Theory with Oceanographic Applications. Department of Mathematic. University of Surrey.

Whitford, Dennis J., Vieira, Mario E., and Waters, Jennifer K., 2001a. Teaching time – series analysis. I. Finite Fourier analysis of ocean waves. American Journal of Physics 69 (4), 490- 496

APPENDICES I

MATLAB CALCULATION

ON ZERO DOWN-CROSSING WAVES EXTRACTION AND

STATISTICAL DISTRIBUTIONS OF WAVE AND CREST HEIGHTS

```
clear all
data=load('SurfaceEl50A.out');       %Any output files (Wave and Crest
mydata=sort(data);                   %height) that needs to be
n=length(mydata);                    %expressed in form of distribution

%calculating the bin interval
Numbin=100;                          %Number of bin is predetermined
r=max(mydata)-min(mydata);
int=r/Numbin;
center1=0.5*int;
cbin=(min(mydata)-center1):int:(max(mydata)+center1);
bin=cbin';
lbin=length(cbin);

% calculate frequency of each bin
P=zeros(2000000,1);
k=0;
freq=zeros(lbin,1);
for i=1:lbin
    for j=1:n
        if mydata(j)<=(bin(i)+center1)&& mydata(j)>(bin(i)-center1)
            k=k+1;
            P(k)=mydata(j);
        else
            continue
        end
    end
    nzP=nonzeros(P);
    freq(i)=length(nzP);
    k=0;
    P=zeros(2000000,1);
end

% calculate frequency cumulative of each wave height bin
frcum=cumsum(freq)/n;

% plot graphic
% plot(cbin,freq,'b.')
% plot(cbin,frcum,'b.')

% Saving the Output in ASCII form
save SEBin50A.out bin -ASCII
save SEFreq50A.out freq -ASCII
save SEFrcum50A.out frcum -ASCII
```

```
clear all
data=load('CrestHeight050B.out');        %Any output files (Wave and Crest
mydata=sort(data);                       %height) that needs to be
n=length(mydata);                        %expressed in form of distribution

% calculate the number of bin
% first method
nbin1=sqrt(n);
% second method
nbin2=1+(3.3*log10(n));
% third method
r=max(mydata)-min(mydata);
q1=round(0.25*n);
Q1=mydata(q1,1);
q3=round(0.75*n);
Q3=mydata(q3,1);
nbin3=r*n^(1/3)/(2*(Q3-Q1));
% calculating the average of three methods
nbinave=roundn(1/3*(nbin1+nbin2+nbin3));

%calculating the bin interval
int=max(mydata)/nbinave;
center1=0.5*int;
cbin=center1:int:(max(mydata)+center1);
bin=cbin';
lbin=length(cbin);

% calculate frequency of each wave height bin
P=zeros(10000,1);
k=0;
freq=zeros(lbin,1);
for i=1:lbin
    for j=1:n
        if mydata(j)<=(bin(i)+center1)&& mydata(j)>(bin(i)-center1)
            k=k+1;
            P(k)=mydata(j);
        else
            continue
        end
    end
    nzP=nonzeros(P);
    freq(i)=length(nzP);
    k=0;
    P=zeros(10000,1);
end

% calculate frequency cumulative of each wave height bin
frcum=cumsum(freq)/n;

% plot graphic
% plot(cbin,freq,'b.')
% plot(cbin,frcum,'b.')

% Saving the Output in ASCII form
save CrestBin050B.out bin -ASCII
save CrestFreq050B.out freq -ASCII
save CrestFrcum050B.out frcum -ASCII
```

```
clear all
fid = fopen('spec24050out.bin','r');   %Open binary data
mydata = fread(fid,'float');           %Read binary data and
                                       %define data type as float
%Dimension of data file
n=length(mydata);
gaugeA=zeros(2700000,1);                %Provide empty vector with
k=0;                                    %spesific dimension

%FIRST: Cluster the data from each gauges to one variable
%Calculate the statistical properties
for i=2:3:n-1                           %Taking the data from binary file
    k=k+1;
    gaugeA(k)=0.03225*mydata(i);        %Multiply with calibtration factor
end
DataA=nonzeros(gaugeA);
MeanA=mean(DataA);                      %Initial mean value

%Making the data average equal to zero
%and calculate the statistical properties
dataA=DataA-MeanA;
meanA=mean(dataA);
stdA=std(dataA);
varA=var(dataA);
skewA=skewness(dataA);
kurtA=kurtosis(dataA);

%Saving the Output in ASCII form
save SurfaceEl50A.out dataA -ASCII

%SECOND: Getting The Crest and Wave Height
%Define temporary vectors
P=zeros(1000,1);                %Positive elevation
N=zeros(1000,1);                %Negative elevation
wHeight=zeros(1000000,1);       %Wave height
cHeight=zeros(1000000,1);       %Crest height
T=zeros(1000000,1);             %Period
p=0;q=0;r=0;s=0;t=0;
state=0;
% initial state = 0, positive state = 1, and negative state = -1

m=length(dataA);
for j=1:m
    % The Algorithm must be started from negative state
    %(zero down-crossing wave)
    if state == 0 && dataA(j) <= 0
        state = -1;
        q=q+1;
        N(q)=dataA(j);                  % push data to N vector
    elseif state == -1 && dataA(j) <= 0
            q=q+1;
            N(q)=dataA(j);              % push data to N vector
    elseif state == -1 && dataA(j) >0
            state= 1;
            p=p+1;
            P(p)=dataA(j);              % push data to P vector
    elseif  state == 1 && dataA(j) >0
            p=p+1;
            P(p)=dataA(j);              % push data to P vector
    elseif state == 1 && dataA(j) <= 0
```

```
            tempWave= max(P) - min(N);      % taking the wave height
            r=r+1;
            wHeight(r)=tempWave;
            tempCrest= max(P);              % taking the crest height
            s=s+1;
            cHeight(s)=tempCrest;
            t=t+1;                          % taking the wave period
            T(t)=0.01*(length(nonzeros(P))+length(nonzeros(N)));
            % flush all data, & ready to start from the beginning state
            % again (the negative state)
            q=0;
            p=0;
            N=zeros(1000,1);
            q=q+1;
            N(q)=dataA(j);
            P=zeros(1000,1);
            state=-1;
    end
end

WHeight=nonzeros(wHeight);
CHeight=nonzeros(cHeight);
WPeriod=nonzeros(T);

% Saving the Output in ASCII form
save WaveHeight050A.out WHeight -ASCII
save CrestHeight050A.out CHeight -ASCII
save Period050A.out WPeriod -ASCII

%THIRD: Taking the positive maxima and plot the time history
Maxi=zeros(1000000,1);
Point=zeros(1000000,1);
y=0;x=0;

for j=2:m-1
      if dataA(j) > 0 && dataA(j) > dataA(j-1) && dataA(j) > dataA(j+1)
        y=y+1;
        Point(y)=(j-1)*0.01;
        x=x+1;
        Maxi(x)=dataA(j);
      else
          continue
      end
end

Maxim=nonzeros(Maxi);               %Positive maxima
Po=nonzeros(Point);                 %Pertinent time of maxima
Nmaxim=length(Maxim);               %Number of Maxima

% plot the time series
timeplot=0:0.01:((m-1)*0.01);
plot(timeplot,dataA);
hold on
plot(Po,Maxim,'r*')
```

```
clear all
fid = fopen('spec24050out.bin','r');    %Open binary data
mydata = fread(fid,'float');            %Read binary data and
                                        %define data type as float
%Dimension of data file
n=length(mydata);
gaugeB=zeros(2700000,1);                 %Provide empty vector with
k=0;                                     %spesific dimension

%FIRST: Cluster the data from each gauges to one variable
%Calculate the statistical properties
for i=3:3:n                              %Taking the data from binary file
    k=k+1;
    gaugeB(k)=0.02443*mydata(i);         %Multiply with calibtration factor
end
DataB=nonzeros(gaugeB);
MeanB=mean(DataB);                       %Initial mean value

%Making the data average equal to zero
%and calculate the statistical properties
dataB=DataB-MeanB;
meanB=mean(dataB);
stdB=std(dataB);
varB=var(dataB);
skewB=skewness(dataB);
kurtB=kurtosis(dataB);

%Saving the Output in ASCII form
save SurfaceEl50B.out dataB -ASCII

%SECOND: Getting The Crest and Wave Height
%Define temporary vectors
P=zeros(1000,1);                %Positive elevation
N=zeros(1000,1);                %Negative elevation
wHeight=zeros(1000000,1);       %Wave height
cHeight=zeros(1000000,1);       %Crest height
T=zeros(1000000,1);             %Period
p=0;q=0;r=0;s=0;t=0;
state=0;
% initial state = 0, positif state = 1, and negatif state = -1

m=length(dataB);
for j=1:m
    % The Algorithm must be started from negative state
    if state == 0 && dataB(j) <= 0
        state = -1;
        q=q+1;
        N(q)=dataB(j);                     % push data to N vector
    elseif state == -1 && dataB(j) <= 0
            q=q+1;
            N(q)=dataB(j);                 % push data to N vector
    elseif state == -1 && dataB(j) >0
            state= 1;
            p=p+1;
            P(p)=dataB(j);                 % push data to P vector
    elseif  state == 1 && dataB(j) >0
            p=p+1;
            P(p)=dataB(j);                 % push data to P vector
    elseif state == 1 && dataB(j) <= 0
            tempWave= max(P) - min(N);     % taking the wave height
```

```
            r=r+1;
            wHeight(r)=tempWave;
            tempCrest= max(P);                % taking the crest height
            s=s+1;
            cHeight(s)=tempCrest;
            t=t+1;                            % taking the wave period
            T(t)=0.01*(length(nonzeros(P))+length(nonzeros(N)));
            % flush all data, & ready to start from the beginning state
            % again (the negative state)
            q=0;
            p=0;
            N=zeros(1000,1);
            q=q+1;
            N(q)=dataB(j);
            P=zeros(1000,1);
            state=-1;
    end
end

WHeight=nonzeros(wHeight);
CHeight=nonzeros(cHeight);
WPeriod=nonzeros(T);

% Saving the Output in ASCII form
save WaveHeight050B.out WHeight -ASCII
save CrestHeight050B.out CHeight -ASCII
save Period050B.out WPeriod -ASCII

%THIRD: Taking the positive maxima and plot the time history
Maxi=zeros(1000000,1);
Point=zeros(1000000,1);
y=0;x=0;

for j=2:m-1
      if dataB(j) > 0 && dataB(j) > dataB(j-1) && dataB(j) > dataB(j+1)
        y=y+1;
        Point(y)=(j-1)*0.01;
        x=x+1;
        Maxi(x)=dataB(j);
      else
          continue
      end
end

Maxim=nonzeros(Maxi);                 %Positive maxima
Po=nonzeros(Point);                   %Pertinent time of maxima
Nmaxim=length(Maxim);                 %Number of Maxima

% plot the time series
timeplot=0:0.01:((m-1)*0.01);
plot(timeplot,dataB);
hold on
plot(Po,Maxim,'r*')
```

```
clear all
fid = fopen('spec24075out.bin','r');    %Open binary data
mydata = fread(fid,8640000,'float');    %Read binary data and
                                        %define data type as float
%Dimension of data file
n=length(mydata);
gaugeA=zeros(3000000,1);                %Provide empty vector with
k=0;                                    %spesific dimension

%FIRST: Cluster the data from each gauges to one variable
%Calculate the statistical properties
for i=2:3:n-1                           %Taking the data from binary file
    k=k+1;
    gaugeA(k)=0.03225*mydata(i);        %Multiply with calibtration factor
end
DataA=nonzeros(gaugeA);
MeanA=mean(DataA);                      %Initial mean value

%Making the data average equal to zero
%and calculate the statistical properties
dataA=DataA-MeanA;
meanA=mean(dataA);
stdA=std(dataA);
varA=var(dataA);
skewA=skewness(dataA);
kurtA=kurtosis(dataA);

%Saving the Output in ASCII form
save SurfaceEl75A.out dataA -ASCII

%SECOND: Getting The Crest and Wave Height
%Define temporary vectors
P=zeros(1000,1);                %Positive elevation
N=zeros(1000,1);                %Negative elevation
wHeight=zeros(1000000,1);       %Wave height
cHeight=zeros(1000000,1);       %Crest height
T=zeros(1000000,1);             %Period
p=0;q=0;r=0;s=0;t=0;
state=0;
% initial state = 0, positive state = 1, and negative state = -1

m=length(dataA);
for j=1:m
    % The Algorithm must be started from negative state
    if state == 0 && dataA(j) <= 0
        state = -1;
        q=q+1;
        N(q)=dataA(j);                      % push data to N vector
    elseif state == -1 && dataA(j) <= 0
            q=q+1;
            N(q)=dataA(j);                  % push data to N vector
    elseif state == -1 && dataA(j) >0
            state= 1;
            p=p+1;
            P(p)=dataA(j);                  % push data to P vector
    elseif  state == 1 && dataA(j) >0
            p=p+1;
            P(p)=dataA(j);                  % push data to P vector
    elseif state == 1 && dataA(j) <= 0
            tempWave= max(P) - min(N);      % taking the wave height
```

```
            r=r+1;
            wHeight(r)=tempWave;
            tempCrest= max(P);                % taking the crest height
            s=s+1;
            cHeight(s)=tempCrest;
            t=t+1;                            % taking the wave period
            T(t)=0.01*(length(nonzeros(P))+length(nonzeros(N)));
            % flush all data, & ready to start from the beginning state
            % again (the negative state)
            q=0;
            p=0;
            N=zeros(1000,1);
            q=q+1;
            N(q)=dataA(j);
            P=zeros(1000,1);
            state=-1;
    end
end

WHeight=nonzeros(wHeight);
CHeight=nonzeros(cHeight);
WPeriod=nonzeros(T);

% Saving the Output in ASCII form
save WaveHeight0751A.out WHeight -ASCII
save CrestHeight0751A.out CHeight -ASCII
save Period0751A.out WPeriod -ASCII

%THIRD: Taking the positive maxima and plot the time history
Maxi=zeros(1000000,1);
Point=zeros(1000000,1);
y=0;x=0;

for j=2:m-1
      if dataA(j) > 0 && dataA(j) > dataA(j-1) && dataA(j) > dataA(j+1)
        y=y+1;
        Point(y)=(j-1)*0.01;
        x=x+1;
        Maxi(x)=dataA(j);
      else
          continue
      end
end

Maxim=nonzeros(Maxi);                 %Positive maxima
Po=nonzeros(Point);                   %Pertinent time of maxima
Nmaxim=length(Maxim);                 %Number of Maxima

% plot the time series
timeplot=0:0.01:((m-1)*0.01);
plot(timeplot,dataA);
hold on
plot(Po,Maxim,'r*')
```

```
clear all
fid = fopen('spec24075out.bin','r');     %Open binary data
mydata = fread(fid,8640000,'float');     %Read binary data and
                                         %define data type as float
%Dimension of data file
n=length(mydata);
gaugeB=zeros(3000000,1);                 %Provide empty vector with
k=0;                                     %spesific dimension

%FIRST: Cluster the data from each gauges to one variable
%Calculate the statistical properties
for i=3:3:n                              %Taking the data from binary file
    k=k+1;
    gaugeB(k)=0.02443*mydata(i);         %Multiply with calibtration factor
end
DataB=nonzeros(gaugeB);
MeanB=mean(DataB);                       %Initial mean value

%Making the data average equal to zero
%and calculate the statistical properties
dataB=DataB-MeanB;
meanB=mean(dataB);
stdB=std(dataB);
varB=var(dataB);
skewB=skewness(dataB);
kurtB=kurtosis(dataB);

%Saving the Output in ASCII form
save SurfaceEl75B.out dataB -ASCII

%SECOND: Getting The Crest and Wave Height
%Define temporary vectors
P=zeros(1000,1);                 %Positive elevation
N=zeros(1000,1);                 %Negative elevation
wHeight=zeros(1000000,1);        %Wave height
cHeight=zeros(1000000,1);        %Crest height
T=zeros(1000000,1);              %Period
p=0;q=0;r=0;s=0;t=0;
state=0;
% initial state = 0, positive state = 1, and negative state = -1

m=length(dataB);
for j=1:m
    % The Algorithm must be started from negative state
    if state == 0 && dataB(j) <= 0
        state = -1;
        q=q+1;
        N(q)=dataB(j);                         % push data to N vector
    elseif state == -1 && dataB(j) <= 0
            q=q+1;
            N(q)=dataB(j);                     % push data to N vector
    elseif state == -1 && dataB(j) >0
            state= 1;
            p=p+1;
            P(p)=dataB(j);                     % push data to P vector
    elseif  state == 1 && dataB(j) >0
            p=p+1;
            P(p)=dataB(j);                     % push data to P vector
    elseif state == 1 && dataB(j) <= 0
            tempWave= max(P) - min(N);         % taking the wave height
```

```
            r=r+1;
            wHeight(r)=tempWave;
            tempCrest= max(P);                % taking the crest height
            s=s+1;
            cHeight(s)=tempCrest;
            t=t+1;                            % taking the wave period
            T(t)=0.01*(length(nonzeros(P))+length(nonzeros(N)));
            % flush all data, & ready to start from the beginning state
            % again (the negative state)
            q=0;
            p=0;
            N=zeros(1000,1);
            q=q+1;
            N(q)=dataB(j);
            P=zeros(1000,1);
            state=-1;
    end
end

WHeight=nonzeros(wHeight);
CHeight=nonzeros(cHeight);
WPeriod=nonzeros(T);

% Saving the Output in ASCII form
save WaveHeight0751B.out WHeight -ASCII
save CrestHeight0751B.out CHeight -ASCII
save Period0751B.out WPeriod -ASCII

%THIRD: Taking the positive maxima and plot the time history
Maxi=zeros(1000000,1);
Point=zeros(1000000,1);
y=0;x=0;

for j=2:m-1
      if dataB(j) > 0 && dataB(j) > dataB(j-1) && dataB(j) > dataB(j+1)
        y=y+1;
        Point(y)=(j-1)*0.01;
        x=x+1;
        Maxi(x)=dataB(j);
      else
          continue
      end
end

Maxim=nonzeros(Maxi);                 %Positive maxima
Po=nonzeros(Point);                   %Pertinent time of maxima
Nmaxim=length(Maxim);                 %Number of Maxima

% plot the time series
timeplot=0:0.01:((m-1)*0.01);
plot(timeplot,dataB);
hold on
plot(Po,Maxim,'r*')
```

```
clear all
mydata = load('spec24100out1.dat');     %Open DAT data

%Dimension of data file
n=length(mydata);
gaugeA=zeros(2500000,1);                %Provide empty vector with
k=0;                                    %spesific dimension

%FIRST: Cluster the data from each gauges to one variable
%Calculate the statistical properties
for i=1:n                               %Taking the data from binary file
    k=k+1;
    gaugeA(k)=mydata(i,1);              %Multiply with calibtration factor
end
DataA=nonzeros(gaugeA);
MeanA=mean(DataA);                      %Initial mean value

%Making the data average equal to zero
%and calculate the statistical properties
dataA=DataA-MeanA;
meanA=mean(dataA);
stdA=std(dataA);
varA=var(dataA);
skewA=skewness(dataA);
kurtA=kurtosis(dataA);

%Saving the Output in ASCII form
save SurfaceEl100A.out dataA -ASCII

%SECOND: Getting The Crest and Wave Height
%Define temporary vectors
P=zeros(1000,1);                %Positive elevation
N=zeros(1000,1);                %Negative elevation
wHeight=zeros(1000000,1);       %Wave height
cHeight=zeros(1000000,1);       %Crest height
T=zeros(1000000,1);             %Period
p=0;q=0;r=0;s=0;t=0;
state=0;
% initial state = 0, positive state = 1, and negative state = -1

m=length(dataA);
for j=1:m
    % The Algorithm must be started from negative state
    if state == 0 && dataA(j) <= 0
        state = -1;
        q=q+1;
        N(q)=dataA(j);                          % push data to N vector
    elseif state == -1 && dataA(j) <= 0
            q=q+1;
            N(q)=dataA(j);                      % push data to N vector
    elseif state == -1 && dataA(j) >0
            state= 1;
            p=p+1;
            P(p)=dataA(j);                      % push data to P vector
    elseif  state == 1 && dataA(j) >0
            p=p+1;
            P(p)=dataA(j);                      % push data to P vector
    elseif state == 1 && dataA(j) <= 0
            tempWave= max(P) - min(N);          % taking the wave height
            r=r+1;
```

```
            wHeight(r)=tempWave;
            tempCrest= max(P);              % taking the crest height
            s=s+1;
            cHeight(s)=tempCrest;
            t=t+1;                          % taking the wave period
            T(t)=0.01*(length(nonzeros(P))+length(nonzeros(N)));
            % flush all data, & ready to start from the beginning state
            % again (the negative state)
            q=0;
            p=0;
            N=zeros(1000,1);
            q=q+1;
            N(q)=dataA(j);
            P=zeros(1000,1);
            state=-1;
    end
end

WHeight=nonzeros(wHeight);
CHeight=nonzeros(cHeight);
WPeriod=nonzeros(T);

% Saving the Output in ASCII form
save WaveHeight100A.out WHeight -ASCII
save CrestHeight100A.out CHeight -ASCII
save Period100A.out WPeriod -ASCII

%THIRD: Taking the positive maxima and plot the time history
Maxi=zeros(1000000,1);
Point=zeros(1000000,1);
y=0;x=0;

for j=2:m-1
      if dataA(j) > 0 && dataA(j) > dataA(j-1) && dataA(j) > dataA(j+1)
        y=y+1;
        Point(y)=(j-1)*0.01;
        x=x+1;
        Maxi(x)=dataA(j);
      else
          continue
      end
end

Maxim=nonzeros(Maxi);                %Positive maxima
Po=nonzeros(Point);                  %Pertinent time of maxima
Nmaxim=length(Maxim);                %Number of Maxima

% plot the time series
timeplot=0:0.01:((m-1)*0.01);
plot(timeplot,dataA);
hold on
plot(Po,Maxim,'r*')
```

```
clear all
mydata = load('spec24100out1.dat');      %Open DAT data

%Dimension of data file
n=length(mydata);
gaugeB=zeros(2500000,1);                %Provide empty vector with
k=0;                                    %spesific dimension

%FIRST: Cluster the data from each gauges to one variable
%Calculate the statistical properties
for i=1:n                               %Taking the data from binary file
    k=k+1;
    gaugeB(k)=mydata(i,2);              %Multiply with calibtration factor
end
DataB=nonzeros(gaugeB);
MeanB=mean(DataB);                      %Initial mean value

%Making the data average equal to zero
%and calculate the statistical properties
dataB=DataB-MeanB;
meanB=mean(dataB);
stdB=std(dataB);
varB=var(dataB);
skewB=skewness(dataB);
kurtB=kurtosis(dataB);

%Saving the Output in ASCII form
save SurfaceEl100B.out dataB -ASCII

%SECOND: Getting The Crest and Wave Height
%Define temporary vectors
P=zeros(1000,1);                %Positive elevation
N=zeros(1000,1);                %Negative elevation
wHeight=zeros(1000000,1);       %Wave height
cHeight=zeros(1000000,1);       %Crest height
T=zeros(1000000,1);             %Period
p=0;q=0;r=0;s=0;t=0;
state=0;
% initial state = 0, positive state = 1, and negative state = -1

m=length(dataB);
for j=1:m
    % The Algorithm must be started from negative state
    if state == 0 && dataB(j) <= 0
        state = -1;
        q=q+1;
        N(q)=dataB(j);                      % push data to N vector
    elseif state == -1 && dataB(j) <= 0
            q=q+1;
            N(q)=dataB(j);                  % push data to N vector
    elseif state == -1 && dataB(j) >0
            state= 1;
            p=p+1;
            P(p)=dataB(j);                  % push data to P vector
    elseif  state == 1 && dataB(j) >0
            p=p+1;
            P(p)=dataB(j);                  % push data to P vector
    elseif state == 1 && dataB(j) <= 0
            tempWave= max(P) - min(N);      % taking the wave height
            r=r+1;
```

```
            wHeight(r)=tempWave;
            tempCrest= max(P);                % taking the crest height
            s=s+1;
            cHeight(s)=tempCrest;
            t=t+1;                            % taking the wave period
            T(t)=0.01*(length(nonzeros(P))+length(nonzeros(N)));
            % flush all data, & ready to start from the beginning state
            % again (the negative state)
            q=0;
            p=0;
            N=zeros(1000,1);
            q=q+1;
            N(q)=dataB(j);
            P=zeros(1000,1);
            state=-1;
    end
end

WHeight=nonzeros(wHeight);
CHeight=nonzeros(cHeight);
WPeriod=nonzeros(T);

% Saving the Output in ASCII form
save WaveHeight100B.out WHeight -ASCII
save CrestHeight100B.out CHeight -ASCII
save Period100B.out WPeriod -ASCII

%THIRD: Taking the positive maxima and plot the time history
Maxi=zeros(1000000,1);
Point=zeros(1000000,1);
y=0;x=0;

for j=2:m-1
      if dataB(j) > 0 && dataB(j) > dataB(j-1) && dataB(j) > dataB(j+1)
        y=y+1;
        Point(y)=(j-1)*0.01;
        x=x+1;
        Maxi(x)=dataB(j);
      else
          continue
      end
end

Maxim=nonzeros(Maxi);                %Positive maxima
Po=nonzeros(Point);                  %Pertinent time of maxima
Nmaxim=length(Maxim);                %Number of Maxima

% plot the time series
timeplot=0:0.01:((m-1)*0.01);
plot(timeplot,dataB);
hold on
plot(Po,Maxim,'r*')
```

```
clear all
mydata = load('ALL.dat');        %Open DAT data

%Dimension of data file
n=length(mydata);

%Making the data average equal to zero
%and calculate the statistical properties
MeanFM=mean(mydata);
dataFM=mydata-MeanFM;
meanfm=mean(dataFM);
stdfm=std(dataFM);
varfm=var(dataFM);
skewfm=skewness(dataFM);
kurtfm=kurtosis(dataFM);

%Saving the Output in ASCII form
save SurfaceElFM.out dataFM -ASCII

%SECOND: Getting The Crest and Wave Height
%Define temporary vectors
P=zeros(1000,1);                %Positive elevation
N=zeros(1000,1);                %Negative elevation
wHeight=zeros(1000000,1);       %Wave height
cHeight=zeros(1000000,1);       %Crest height
T=zeros(1000000,1);             %Period
p=0;q=0;r=0;s=0;t=0;
state=0;
% initial state = 0, positive state = 1, and negative state = -1

m=length(dataFM);
for j=1:m
    % The Algorithm must be started from negative state
    if state == 0 && dataFM(j) <= 0
        state = -1;
        q=q+1;
        N(q)=dataFM(j);                          % push data to N vector
    elseif state == -1 && dataFM(j) <= 0
            q=q+1;
            N(q)=dataFM(j);                      % push data to N vector
    elseif state == -1 && dataFM(j) >0
            state = 1;
            p=p+1;
            P(p)=dataFM(j);                      % push data to P vector
    elseif  state == 1 && dataFM(j) >0
            p=p+1;
            P(p)=dataFM(j);                      % push data to P vector
    elseif state == 1 && dataFM(j) <= 0
            tempWave= max(P) - min(N);          % taking the wave height
            r=r+1;
            wHeight(r)=tempWave;
            tempCrest= max(P);                  % taking the crest height
            s=s+1;
            cHeight(s)=tempCrest;
            t=t+1;                              % taking the wave period
            T(t)=0.2*(length(nonzeros(P))+length(nonzeros(N)));
            % flush all data, & ready to start from the beginning state
            % again (the negative state)
            q=0;
            p=0;
```

```
            N=zeros(1000,1);
            q=q+1;
            N(q)=dataFM(j);
            P=zeros(1000,1);
            state=-1;
    end
end

WHeight=nonzeros(wHeight);
CHeight=nonzeros(cHeight);
WPeriod=nonzeros(T);

% Saving the Output in ASCII form
save WaveHeightFM.out WHeight -ASCII
save CrestHeightFM.out CHeight -ASCII
save PeriodFM.out WPeriod -ASCII

%THIRD: Taking the positive maxima and plot the time history
Maxi=zeros(1000000,1);
Point=zeros(1000000,1);
y=0;x=0;

for j=2:m-1
      if dataFM(j) > 0 && dataFM(j) > dataFM(j-1) && dataFM(j) > dataFM(j+1)
        y=y+1;
        Point(y)=(j-1)*0.01;
        x=x+1;
        Maxi(x)=dataFM(j);
      else
          continue
      end
end

Maxim=nonzeros(Maxi);                  %Positive maxima
Po=nonzeros(Point);                    %Pertinent time of maxima
Nmaxim=length(Maxim);                  %Number of Maxima

% plot the time series
timeplot=0:0.01:((m-1)*0.01);
plot(timeplot,dataFM);
hold on
plot(Po,Maxim,'r*')
```

APPENDICES II

MATHCAD CALCULATION

ON WAVE AND CREST HEIGHT DISTIRBUTIONS

DISTRIBUTION OF WATER SURFACE ELEVATION

Surface elavation of laboratory simulation 050A

surface elevation bin

Sbin :=

	0
0	-0.046
1	-0.045
2	-0.044
3	-0.043
4	...

frequency

Sfreq :=

	0
0	3
1	9
2	13
3	37
4	...

cumulative distribution

Scdf :=

	0
0	$1.155 \cdot 10^{-6}$
1	$4.62 \cdot 10^{-6}$
2	$9.624 \cdot 10^{-6}$
3	$2.387 \cdot 10^{-5}$
4	...

Bin interval

$sbinl := Sbin_1 - Sbin_0$

$sbinl = 1.13 \times 10^{-3}$

$Nbl := length(Sbin)$

$Nbl = 101$

Probability density

$NSfreq := length(Sfreq)$

$$Sfrel := \frac{Sfreq}{\sum (Sfreq)}$$

$$Spdf := \frac{Sfreq}{\sum (Sfreq \cdot sbinl)}$$

Probability of exceedance

$SE := 1 - Scdf$

Standar deviation of water elevation data

$stds := 0.011045$ [Matlab analysis]

Gaussian Distribution

$$p(\eta) = \frac{1}{\sigma_\eta \sqrt{2\pi}} \exp\left(\frac{-\eta^2}{2\sigma_\eta^2}\right)$$

$i := 0 .. (length(Sbin) - 1)$

$$pdfgauss_i := \left(\frac{1}{stds \cdot \sqrt{2\pi}}\right) \cdot \exp\left[\frac{-(Sbin_i)^2}{2 \cdot (stds)^2}\right]$$

ZERO CROSSING WAVE HEIGHT

Zero crossing wave height in meter (hlr)

hlr :=

	0
0	$1.339 \cdot 10^{-3}$
1	$9.448 \cdot 10^{-4}$
2	$1.26 \cdot 10^{-3}$
3	$9.448 \cdot 10^{-4}$
4	$3.858 \cdot 10^{-3}$
5	$8.661 \cdot 10^{-3}$
6	...

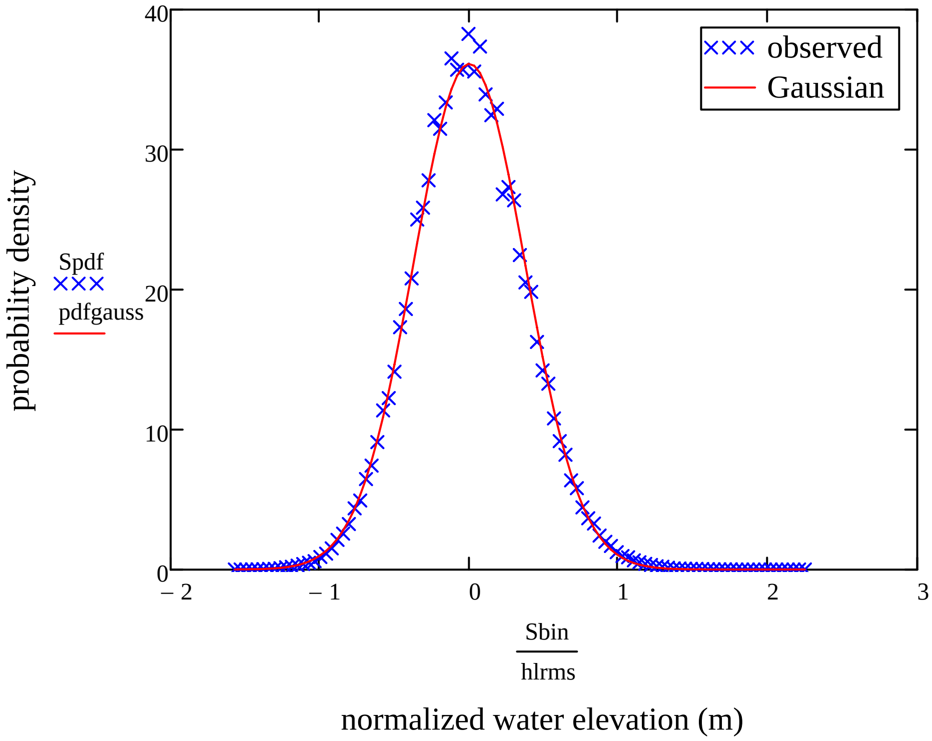
$hl := sort(hlr)$ $hl2 := hl^2$

Number of wave

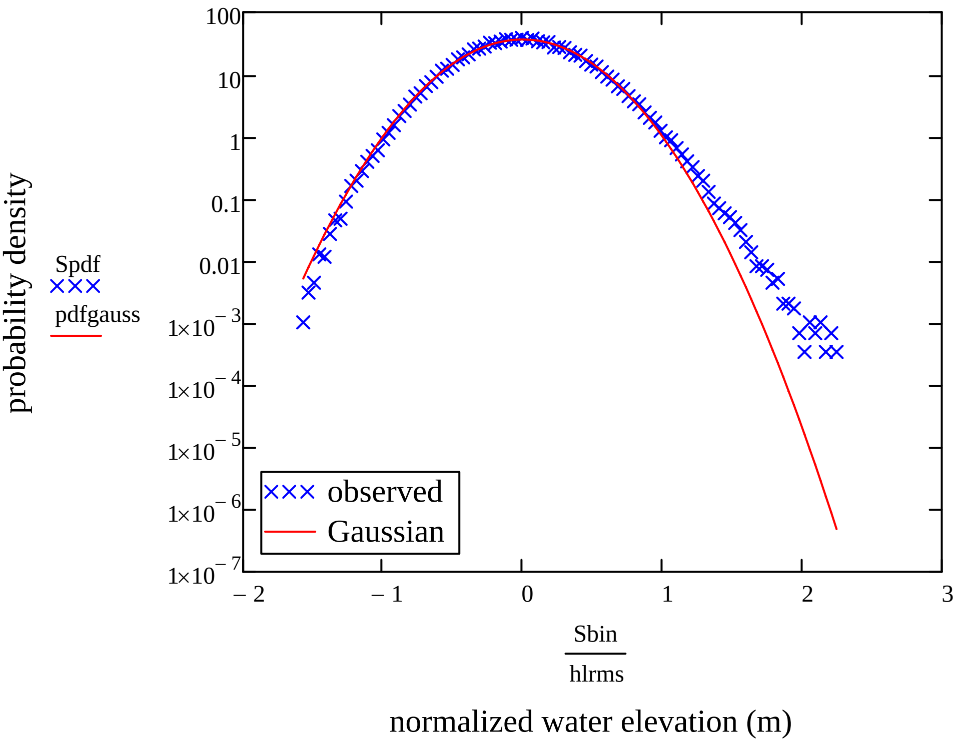
$Nhl := length(hl)$ $Nhl = 3.372 \times 10^4$

Root mean square wave height

$$hlrms := \sqrt{\left[\frac{1}{Nhl} \cdot \left(\sum hl2\right)\right]}$$ $hlrms = 0.03$



observed
Gaussian
probability density
Spdf
pdfgauss
40
30
20
10
0
− 2
− 1
0
1
2
3
Sbin
hlrms
normalized water elevation (m)



observed
Gaussian
probability density
Spdf
pdfgauss
100
10
1
0.1
0.01
1×10⁻³
1×10⁻⁴
1×10⁻⁵
1×10⁻⁶
1×10⁻⁷
− 2
− 1
0
1
2
3
Sbin
hlrms
normalized water elevation (m)

Surface elavation of field measurement

surface elevation bin

Sfbin :=

	0
0	-9.088
1	-8.838
2	-8.587
3	-8.337
4	...

$sfbinl := Sfbin_1 - Sfbin_0$

$sfbinl = 0.25$

$Nfbl := length(Sfbin)$

$Nfbl = 100$

frequency

Sffreq :=

	0
0	1
1	1
2	1
3	2
4	...

$NSffreq := length(Sffreq)$

$$Sffrel := \frac{Sffreq}{\sum(Sffreq)}$$

$$Sfpdf := \frac{Sffreq}{\sum(Sffreq \cdot sfbinl)}$$

cumulative distribution

Sfcdf :=

	0
0	$4.075 \cdot 10^{-7}$
1	$8.15 \cdot 10^{-7}$
2	$1.222 \cdot 10^{-6}$
3	$2.037 \cdot 10^{-6}$
4	...

$SfE := 1 - Sfcdf$

Standar deviation of water elevation data

$sftds := 1.8237$ [Matlab analysis]

Gaussian Distribution

$$p(\eta) = \frac{1}{\sigma_\eta \sqrt{2\pi}} \exp\left(\frac{-\eta^2}{2\sigma_\eta^2}\right)$$

$i := 0 .. (length(Sfbin) - 1)$

$$pdffgauss_i := \left(\frac{1}{sftds \cdot \sqrt{2\pi}}\right) \cdot \exp\left[\frac{-\left(Sfbin_i\right)^2}{2 \cdot (sftds)^2}\right]$$

ZERO CROSSING WAVE HEIGHT

Zero crossing wave height in meter (hlr)

hlrf :=

	0
0	4.15
1	4.475
2	3.754
3	0.655
4	3.947
5	2.734
6	...

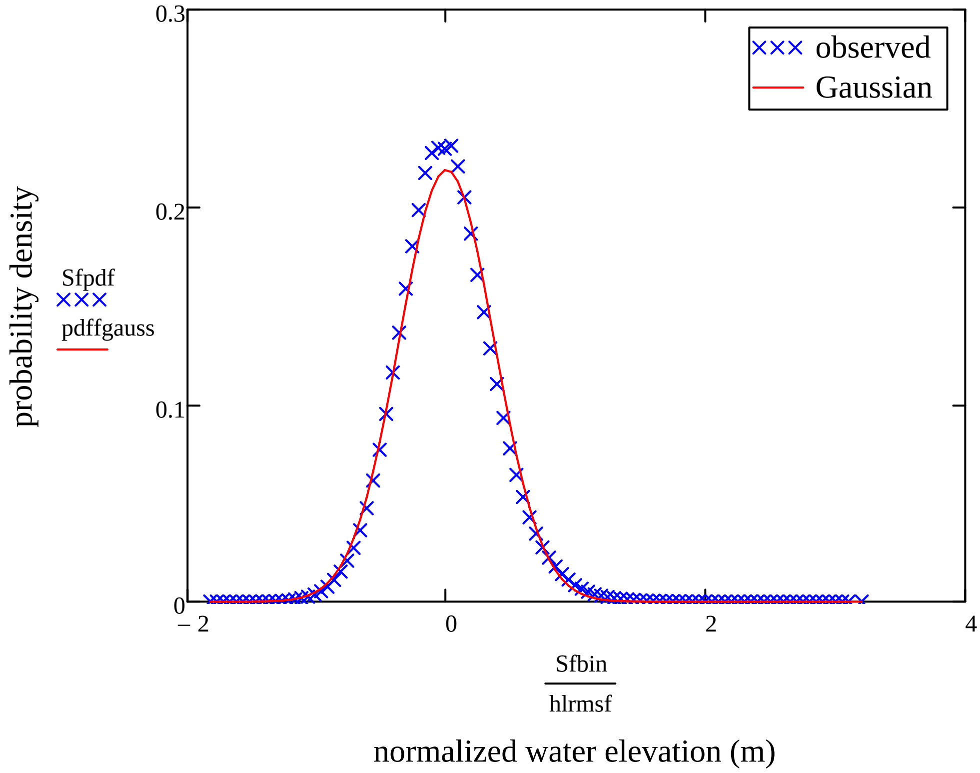
$hlf := sort(hlrf)$ $hlf2 := hlf^2$

Number of wave

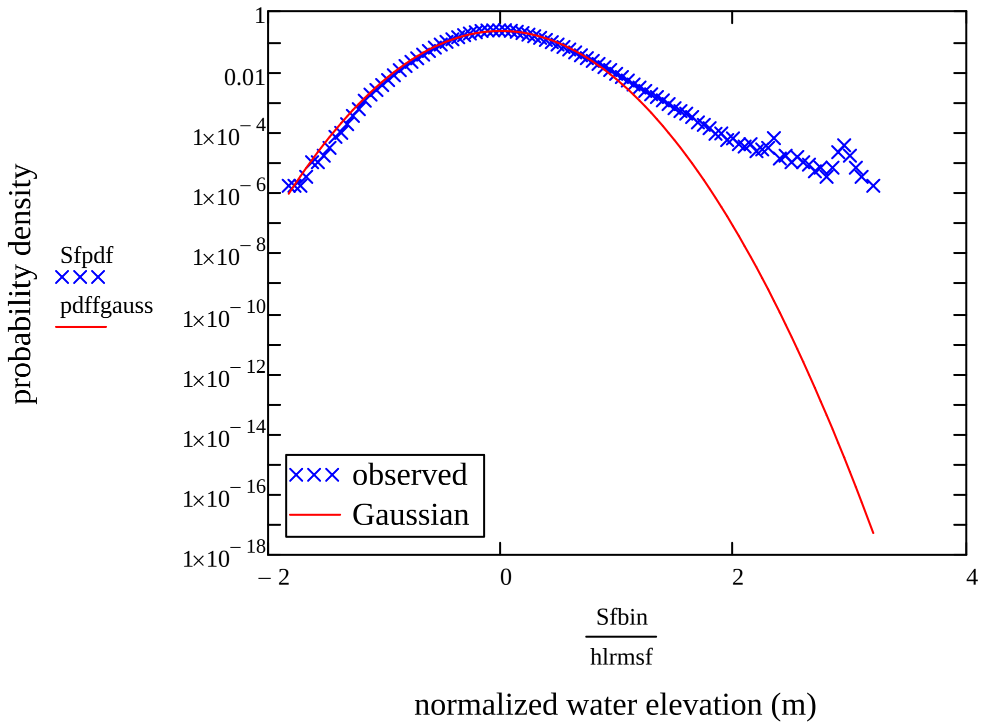
$Nhlf := length(hlf)$ $Nhlf = 5.467 \times 10^4$

Root mean square wave height

$$hlrmsf := \sqrt{\left[\frac{1}{Nhlf} \cdot \left(\sum hlf2\right)\right]}$$ $hlrmsf = 4.985$



0.3
0.2
0.1
0
− 2
0
2
4
observed
Gaussian
probability density
Sfpdf
pdffgauss
Sfbin
hlrmsf
normalized water elevation (m)



1
0.01
1×10^− 4
1×10^− 6
1×10^− 8
1×10^− 10
1×10^− 12
1×10^− 14
1×10^− 16
1×10^− 18
− 2
0
2
4
observed
Gaussian
probability density
Sfpdf
pdffgauss
Sfbin
hlrmsf
normalized water elevation (m)

ZERO-CROSSING STUDY

Comparison of Up- and Down-Crossing Wave

Laboratory Simulation 050A

$Lmo := 0.00012199$ variance of water surface elevation

$Ld := 0.7$ water depth in meter

ZERO UP-CROSSING WAVE

hlru :=

	0
0	$1.339\cdot10^{-3}$
1	$1.339\cdot10^{-3}$
2	$9.448\cdot10^{-4}$
3	$1.26\cdot10^{-3}$
4	$1.024\cdot10^{-3}$
5	$6.141\cdot10^{-3}$
6	0.014
7	0.015
8	0.025
9	...

$hlu := sort(hlru)$ $hlu2 := hlu^2$

Number of wave

$Nhlu := length(hlu)$ $Nhlu = 33720$

Root mean square

$$hlrmsu := \sqrt{\left[\frac{1}{Nhlu}\cdot\left(\sum hlu2\right)\right]}$$

$hlrmsu = 0.0296$

Maximum wave height

$hlmu := max(hlu)$

$hlmu = 0.0972$

Significant wave height

$$Nhsu := round\left[\left(\frac{2}{3}\right)\cdot Nhlu\right]$$

$Nhsu = 22480$

$i := (Nhsu - 1) .. (Nhlu - 1)$

$hlu_{(Nhsu-1)} = 0.0314$

$hlu_{(Nhlu-1)} = 0.0972$

$hlu_i =$

0.0314
...

$$sumhsu := \sum_{i = Nhsu-1}^{(Nhlu-1)} hlu_i$$

$sumhsu = 472.0225$

$$hsu := \frac{sumhsu}{(Nhlu - Nhsu + 1)}$$

$hsu = 0.042$

Wave Height Distribution

hlbinu :=

	0
0	$5.207\cdot10^{-4}$
1	$1.562\cdot10^{-3}$
2	$2.604\cdot10^{-3}$
3	$3.645\cdot10^{-3}$
4	$4.687\cdot10^{-3}$
5	...

hlfrequ :=

	0
0	678
1	241
2	246
3	302
4	352
5	...

hlcdfu :=

	0
0	0.02
1	0.027
2	0.035
3	0.044
4	0.054
5	...

$binlu := 2 \cdot hlbinu_0$

$binlu = 0.001$

$Nblu := length(hlbinu)$

$Nblu = 90$

$Nhlfrequ := length(hlfrequ)$

$$hlfrelu := \frac{hlfrequ}{\sum (hlfrequ)}$$

$$hlpdfu := \frac{hlfrequ}{\sum (hlfrequ \cdot binlu)}$$

$Nlcdfu := length(hlcdfu)$

$HEu := 1 - hlcdfu$

ZERO DOWN-CROSSING WAVE

hlrd :=

	0
0	$1.339 \cdot 10^{-3}$
1	$9.448 \cdot 10^{-4}$
2	$1.26 \cdot 10^{-3}$
3	$9.448 \cdot 10^{-4}$
4	$3.858 \cdot 10^{-3}$
5	$8.661 \cdot 10^{-3}$
6	0.017
7	0.013
8	0.048
9	...

$hld := sort(hlrd)$ $\quad hld2 := hld^2$

Number of waves

$Nhld := length(hld)$ $\quad Nhld = 33719$

Root mean square wave height

$$hlrmsd := \sqrt{\left[\frac{1}{Nhld} \cdot \left(\sum hld2\right)\right]}$$

$hlrmsd = 0.0296$

COMPARISON

$hlrmsu = 0.0296$

Maximum wave height

$hlmd := max(hld)$

$hlmd = 0.0932$

$hlmu = 0.0972$

Significant wave height

$$Nhsd := round\left[\left(\frac{2}{3}\right) \cdot Nhld\right]$$

$Nhsd = 22479$

$i := (Nhsd - 1) .. (Nhld - 1)$

$hld_{(Nhsd-1)} = 0.0314$

$hld_{(Nhld-1)} = 0.0932$

$hld_i =$

0.0314
...

$$sumhsd := \sum_{i = Nhsd-1}^{(Nhld-1)} hld_i$$

$sumhsd = 471.7858$

$$hsd := \frac{sumhsd}{(Nhld - Nhsd + 1)}$$

$hsd = 0.042$

COMPARISON

$hsu = 0.042$

Wave Height Distribution

hlbind :=

	0
0	$5.066 \cdot 10^{-4}$
1	$1.52 \cdot 10^{-3}$
2	$2.533 \cdot 10^{-3}$
3	...

hlfreqd :=

	0
0	420
1	224
2	256
3	...

hlcdfd :=

	0
0	0.012
1	0.019
2	0.027
3	...

$binld := 2 \cdot hlbind_0$

$binld = 0.001$

$Nbld := length(hlbind)$

$Nbld = 89$

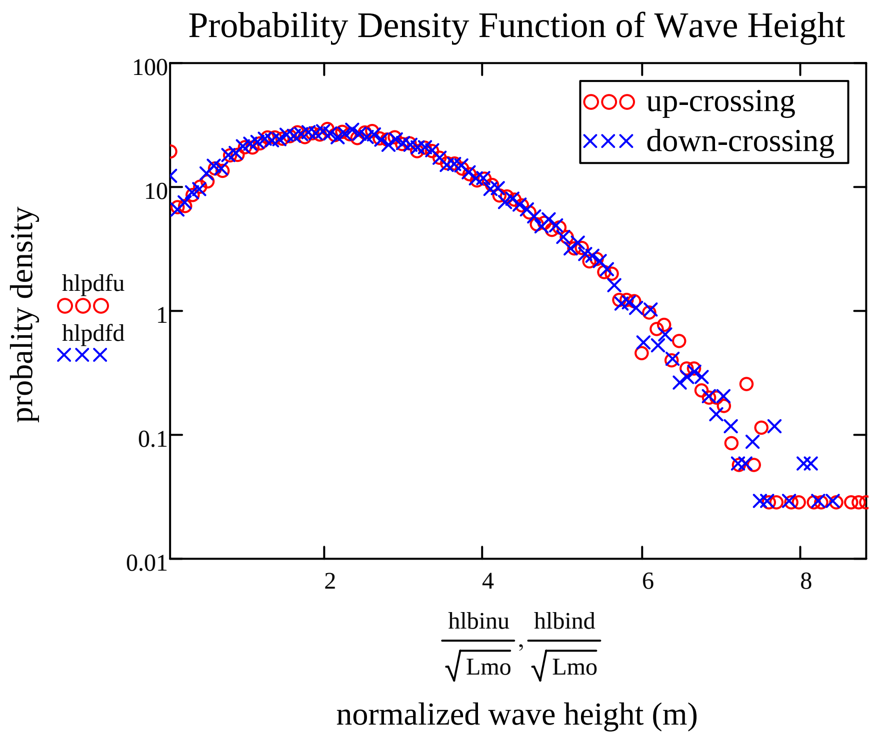
$Nhlfreqd := length(hlfreqd)$

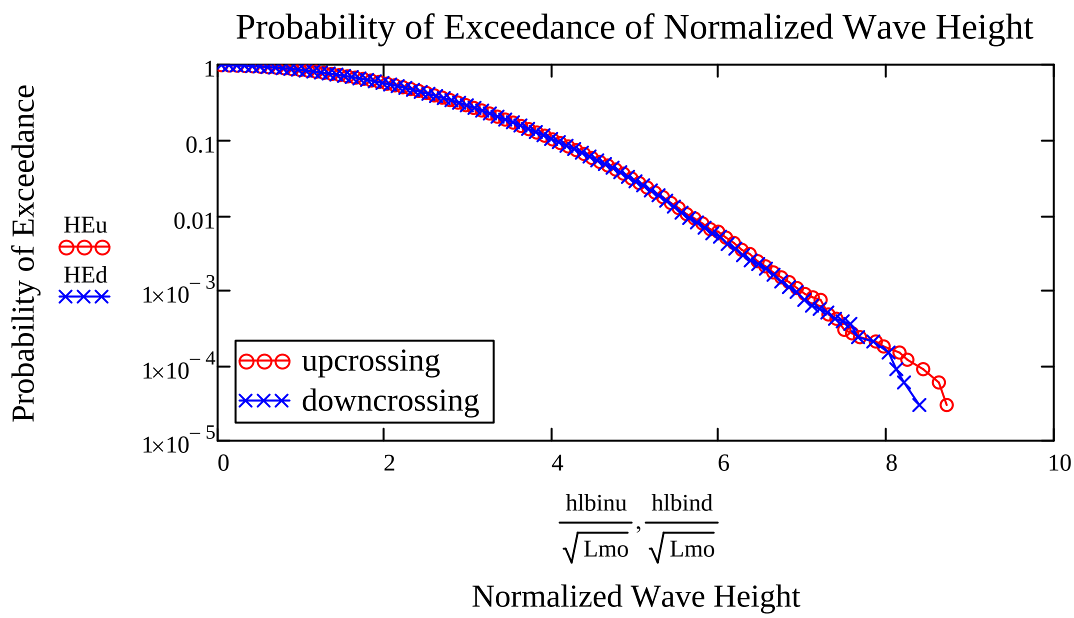
$$hlfreld := \frac{hlfreqd}{\sum(hlfreqd)}$$

$$hlpdfd := \frac{hlfreqd}{\sum(hlfreqd \cdot binld)}$$

$Nlcdfd := length(hlcdfd)$

$HEd := 1 - hlcdfd$





Field Measurement

$Fmo := 3.3257$ variance of water surface elevation

$Fd := 130$ water depth in meter

ZERO UP-CROSSING WAVE

hfru :=

	0
0	5.563
1	4.326
2	1.872
3	2.357
4	3.465
5	3.006
6	1.781
7	3.669
8	2.409
9	...

$hfu := sort(hfru)$ $hfu2 := hfu^2$

Number of wave

$Nhfu := length(hfu)$ $Nhfu = 54673$

Root mean square

$$hfrmsu := \sqrt{\left[\frac{1}{Nhfu}\cdot\left(\sum hfu2\right)\right]}$$

$hfrmsu = 4.9747$

Maximum wave height

$hfmu := max(hfu)$

$hfmu = 18.9794$

Significant wave height

$$Nhfsu := round\left[\left(\frac{2}{3}\right)\cdot Nhfu\right]$$

$Nhfsu = 36449$

$i := (Nhfsu - 1) .. (Nhfu - 1)$

$hfu_{(Nhfsu-1)} = 5.1219$

$hfu_{(Nhfu-1)} = 18.9794$

$hfu_i =$

5.1219
...

$$sumhfsu := \sum_{i = Nhfsu-1}^{(Nhfu-1)} hfu_i$$

$sumhfsu = 129799.8068$

$$hfsu := \frac{sumhfsu}{(Nhfu - Nhfsu + 1)}$$

$hfsu = 7.1221$

Wave Height Distribution

hfbinu :=

	0
0	0.079
1	0.238
2	0.397
3	0.556
4	...

hffrequ :=

	0
0	313
1	313
2	428
3	568
4	...

hfcdfu :=

	0
0	$5.725\cdot10^{-3}$
1	0.011
2	0.019
3	0.03
4	...

$binfu := 2 \cdot hfbinu_0$

$binfu = 0.1588$

$Nbfu := length(hfbinu)$

$Nbfu = 114$

$Nhffrequ := length(hffrequ)$

$$hffrelu := \frac{hffrequ}{\sum(hffrequ)}$$

$$hfpdfu := \frac{hffrequ}{\sum(hffrequ \cdot binfu)}$$

$Nfcdfu := length(hfcdfu)$

$HEfu := 1 - hfcdfu$

ZERO DOWN-CROSSING WAVE

hfrd :=

	0
0	4.15
1	4.475
2	3.754
3	0.655
4	3.947
5	2.734
6	2.372
7	2.483
8	3.576
9	...

$hfd := sort(hfrd)$ $\quad hfd2 := hfd^2$

Number of waves

$Nhfd := length(hfd)$ $\quad Nhfd = 54673$

Root mean square wave height

$$hfrmsd := \sqrt{\left[\frac{1}{Nhlu} \cdot \left(\sum hlu2\right)\right]}$$

COMPARISON

$hfrmsd = 0.0296$ $\quad hfrmsu = 4.9747$

Maximum wave height

$hfmd := max(hfd)$

$hfmd = 21.9815$ $\quad hfmu = 18.9794$

Significant wave height

$$Nhfsd := round\left[\left(\frac{2}{3}\right) \cdot Nhfd\right]$$

$Nhfsd = 36449$

$i := (Nhfsd - 1) .. (Nhfd - 1)$

$hfd_{(Nhfsd-1)} = 5.1379$

$hfd_{(Nhfd-1)} = 21.9815$

$hfd_i =$

5.1379
...

$$sumhfsd := \sum_{i = Nhfsd-1}^{(Nhfd-1)} hfd_i$$

$sumhfsd = 130168.2955$

$$hfsd := \frac{sumhfsd}{(Nhfd - Nhfsd + 1)}$$

COMPARISON

$hfsd = 7.1423$ $\quad hfsu = 7.1221$

Wave Height Distribution

hfbind :=

	0
0	0.088
1	0.264
2	0.44
3	...

hffreqd :=

	0
0	360
1	514
2	573
3	...

hfcdfd :=

	0
0	$6.585 \cdot 10^{-3}$
1	0.016
2	0.026
3	...

$binfd := 2 \cdot hfbind_0$

$binfd = 0.1759$

$Nbfd := length(hfbind)$

$Nbfd = 108$

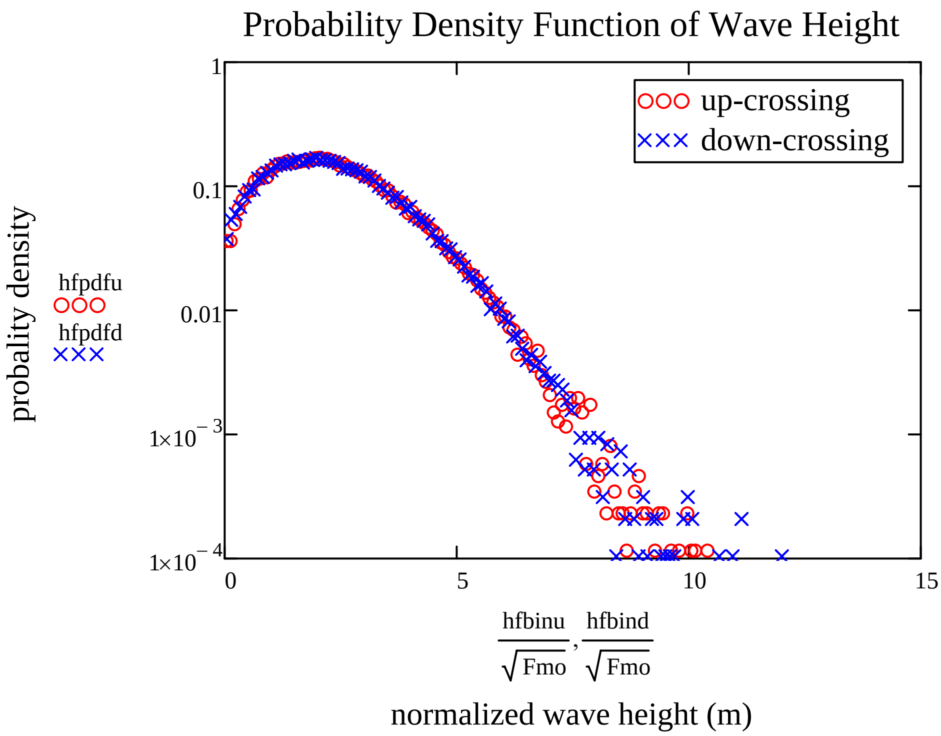
$Nhffreqd := length(hffreqd)$

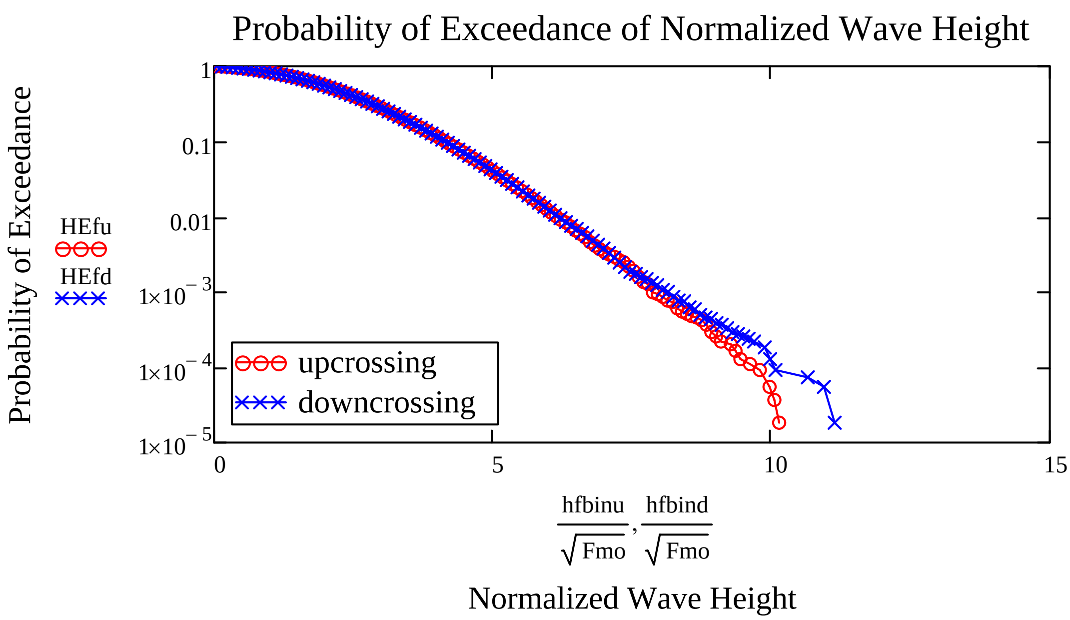
$$hffreld := \frac{hffreqd}{\sum(hffreqd)}$$

$$hfpdfd := \frac{hffreqd}{\sum(hffreqd \cdot binfd)}$$

$Nfcdfd := length(hfcdfd)$

$HEfd := 1 - hfcdfd$





LABORATORY SIMULATION 050A

Variance of water surface elevation

$mo := 0.00012199$

Water depth in meter

$d := 0.7$

ZERO CROSSING WAVE PERIOD

Tw :=

	0
0	0.92
1	0.22
2	0.21
3	0.22
4	1.29
5	...

$Twm := \text{mean}(Tw)$

$Twm = 0.7703$ sec

dispersion relation of deep water condition

$k := 4\cdot\frac{\pi^2}{Twm^2 9.81}$

$k = 6.7819$

cek $\tanh(k\cdot d) = 0.9998$

ZERO CROSSING WAVE HEIGHT

Zero crossing wave height in meter (hlr)

hlr :=

	0
0	$1.339\cdot10^{-3}$
1	$9.448\cdot10^{-4}$
2	$1.26\cdot10^{-3}$
3	$9.448\cdot10^{-4}$
4	$3.858\cdot10^{-3}$
5	$8.661\cdot10^{-3}$
6	0.017
7	0.013
8	...

$hl := \text{sort}(hlr)$ $\qquad hl2 := hl^2$

Number of wave

$Nhl := \text{length}(hl)$ $\qquad Nhl = 33719$

Statistical properties

$\text{mean}(hl) = 0.0261$

$\text{stdev}(hl) = 0.014$

$\text{skew}(hl) = 0.4991$

$\text{kurt}(hl) = 0.0342$

Root mean square wave height

$$hlrms := \sqrt{\left[\frac{1}{Nhl}\cdot\left(\sum hl2\right)\right]}$$

$hlrms = 0.0296$

Normalized wave height

$$hn := \frac{hl}{\sqrt{mo}}$$

Significant wave height

$Nhs := \text{round}\left[\left(\frac{2}{3}\right)\cdot Nhl\right]$ $\qquad Nhs = 22479$

$i := (Nhs - 1) .. (Nhl - 1)$

$hl_{(Nhs-1)} = 0.0314$ $\qquad hl_{(Nhl-1)} = 0.0932$

$hl_i =$

0.0314
0.0314
0.0314
0.0314
0.0314
0.0314
...

$$sumhs := \sum_{i = Nhs-1}^{(Nhl-1)} hl_i \qquad sumhs = 471.7858$$

$$hs := \frac{sumhs}{(Nhl - Nhs + 1)} \qquad hs = 0.042$$

comparison with theoritical significant wave height

$$hss := 4 \cdot \sqrt{mo} \qquad hss = 0.0442$$

Maximum wave height

$hlm := max(hl)$

$hlm = 0.0932$

comparison with theoritical Rayleigh formula of maximum wave height

$$hmax := \sqrt{\frac{\ln(Nhl)}{2}} \cdot hs$$

$hmax = 0.0958$

OBSERVED WAVE HEIGHT DISTRIBUTION

Wave height bin

$hlbin :=$

	0
0	$5.066 \cdot 10^{-4}$
1	$1.52 \cdot 10^{-3}$
2	$2.533 \cdot 10^{-3}$
3	$3.547 \cdot 10^{-3}$
4	$4.56 \cdot 10^{-3}$
5	$5.573 \cdot 10^{-3}$
6	$6.586 \cdot 10^{-3}$
7	$7.6 \cdot 10^{-3}$
8	$8.613 \cdot 10^{-3}$
9	...

Wave height frequency

$hlfreq :=$

	0
0	420
1	224
2	256
3	309
4	328
5	439
6	506
7	481
8	617
9	...

Cumulative frequency

$hlcdf :=$

	0
0	0.012
1	0.019
2	0.027
3	0.036
4	0.046
5	0.059
6	0.074
7	0.088
8	0.106
9	...

Bin interval

$binl := 2 \cdot hlbin_0$

$binl = 0.001$

Number of bin

$Nbl := length(hlbin)$

$Nbl = 89$

Probability density

$$hlfrel := \frac{hlfreq}{\sum(hlfreq)}$$

$$hlpdf := \frac{hlfreq}{\sum(hlfreq \cdot binl)}$$

Probability of exceedance

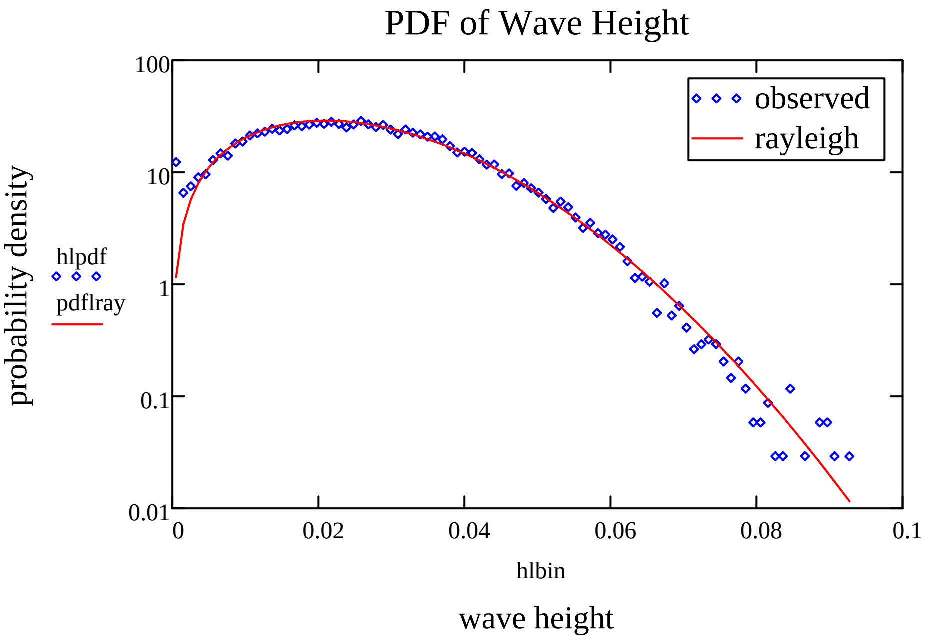
$HE := 1 - hlcdf$

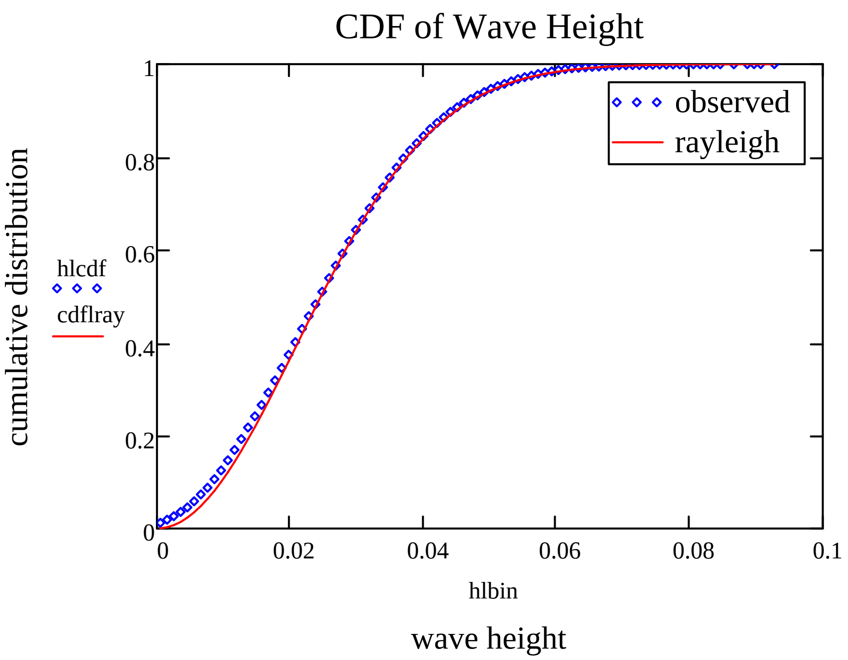
RAYLEIGHT DISTRIBUTION

$i := 0 .. (\text{length}(\text{hlbin}) - 1)$

Rayleigh Distribution

$$\text{pdflray}_i := \left[\frac{(2 \cdot \text{hlbin}_i)}{\text{hlrms}^2}\right] \cdot \exp\left[\frac{-(\text{hlbin}_i)^2}{\text{hlrms}^2}\right] \qquad \text{cdflray}_i := 1 - \exp\left[\frac{-(\text{hlbin}_i)^2}{\text{hlrms}^2}\right]$$





EMPIRICAL WAVE HEIGHT DISTRIBUTION [COMPARISON]

Rayleigh

$$Phr1 := \exp\left[-\left(\frac{hl}{0.707\cdot hs}\right)^2\right]$$

$$Phr2 := \exp\left[-\left(\frac{hl}{0.638\cdot hs}\right)^2\right]$$

Forristall

$$Phf := \exp\left[-\left(\frac{hl}{0.681\cdot hs}\right)^{2.126}\right]$$

Krogstad

$$Phk1 := \exp\left[-\left(\frac{hl}{0.73\cdot hs}\right)^{2.37}\right]$$

$$Phk2 := \exp\left[-\left(\frac{hl}{0.75\cdot hs}\right)^{2.5}\right]$$

Haring

$$Phh1 := 0.968 + 0.176\cdot\frac{hl}{hs} \qquad Phh2 := \overrightarrow{\left[-2\left(\frac{hl}{hs}\right)^2\cdot Phh1\right]}$$

$$Phh := \exp(Phh2)$$

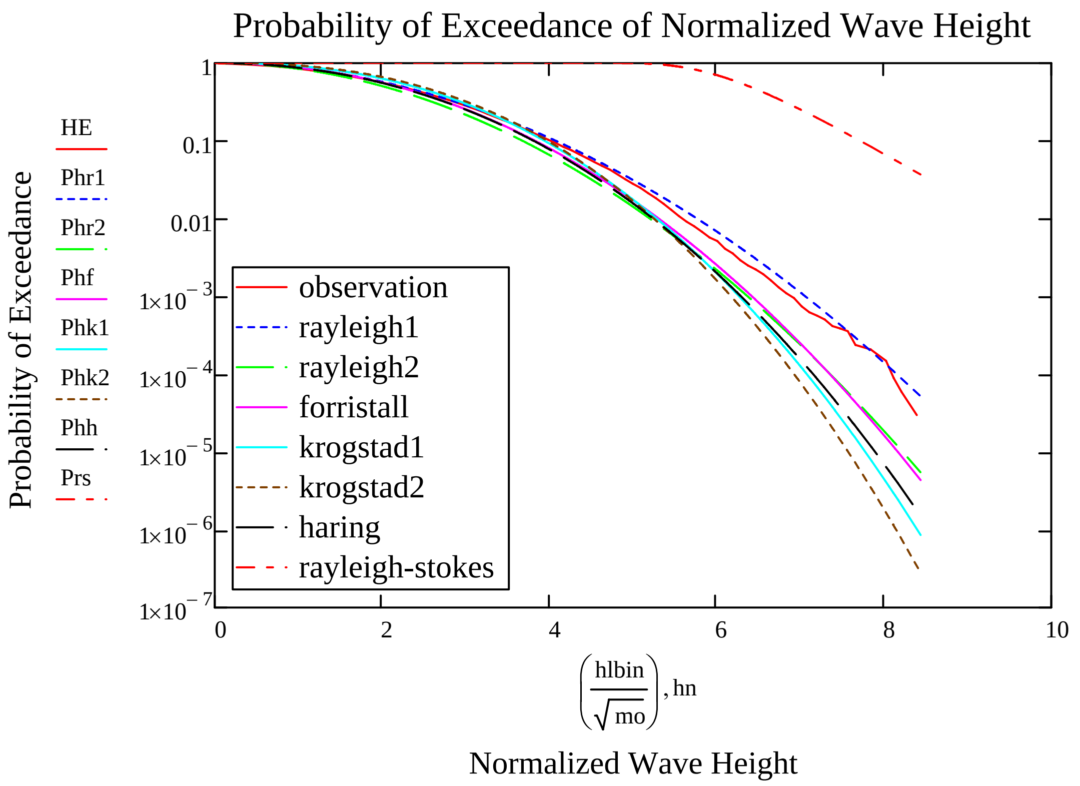
Rayleigh Stokes

$$an := 4\cdot\sqrt{mo}\cdot\sqrt{\left(\frac{\log(Nhl)}{8}\right)} \qquad bn := \frac{an}{2\cdot\log(Nhl)} \qquad B2 := \frac{1}{2} \qquad B3 := \frac{3}{8}$$

$$ahn := 2\cdot an\cdot\left[1 + B3\cdot(k\cdot an)^2\right] \qquad ahn = 0.0677$$

$$bhn := 2\cdot bn\cdot\left[1 + 3\cdot B3\cdot(k\cdot an)^2\right] \qquad bhn = 0.0078$$

$$Prs := 1 - \exp\left[-\exp\left[-\left(\frac{hl - ahn}{bhn}\right)\right]\right]$$



ZERO CROSSING CREST HEIGHT

Zero crossing crest height in meter (clr)

clr :=

	0
0	0.00133
1	0.00094
2	0.00125
3	0.00094
4	0.00377
5	0.00629
6	0.00984
7	...

Sorted crest — cl := sort(clr)

Number of crest — Ncl := length(cl) — Ncl = 33719

Maximum crest height — max(cl) = 0.0671

Statistical Properties

mean(cl) = 0.0138

stdev(cl) = 0.0085

skew(cl) = 0.5917

kurt(cl) = 0.3817

Normalized crest height

$$x := \frac{cl}{\sqrt{mo}}$$

OBSERVED CREST HEIGHT DISTRIBUTION

Crest height bin

clbin :=

	0
0	$3.433\cdot10^{-4}$
1	$1.03\cdot10^{-3}$
2	$1.717\cdot10^{-3}$
3	$2.403\cdot10^{-3}$
4	$3.09\cdot10^{-3}$
5	$3.776\cdot10^{-3}$
6	$4.463\cdot10^{-3}$
7	$5.15\cdot10^{-3}$
8	$5.836\cdot10^{-3}$
9	...

Crest height frequency

clfreq :=

	0
0	$1.291\cdot10^{3}$
1	743
2	658
3	571
4	682
5	691
6	750
7	656
8	811
9	...

Cumulative frequency

clfcum :=

	0
0	0.038
1	0.06
2	0.08
3	0.097
4	0.117
5	0.137
6	0.16
7	0.179
8	0.203
9	...

Bin interval

$lbin := 2\cdot clbin_0$

$lbin = 0.0007$

Number of bin

$Ncbl := length(clbin)$

$Ncbl = 86$

Probability density

$$clfrel := \frac{clfreq}{\sum clfreq}$$

$$clpdf := \frac{clfreq\cdot\sqrt{mo}}{\sum(clfreq\cdot lbin)}$$

Probability of exceedance

$CE := 1 - clfcum$

EMPIRICAL CREST HEIGHT DISTRIBUTION [COMPARISON]

Rayleigh

$$Pr := \exp\left(-8\frac{cl^2}{hs^2}\right)$$

Ochi

$E0 := 0.1$ $\quad \alpha0 := \sqrt{\left(1 - E0^2\right)}$ $\quad$ narrow band

$E1 := 1$ $\quad \alpha1 := \sqrt{\left(1 - E1^2\right)}$ $\quad$ broad band

probability density function of E=1

$$pa := \exp\left(\frac{-x^2}{2}\right)$$

$$pb1 := \overrightarrow{(\alpha1\cdot x\cdot pa)}$$

$$pc1 := 0.5 + 0.5 \cdot erf\left(\frac{\alpha 1}{E1} \cdot \frac{x}{\sqrt{2}}\right)$$

$$pd1 := \overrightarrow{(pb1 \cdot pc1)}$$

$$p1 := \frac{2}{(1 + \alpha 1)} \cdot \left(\frac{E1}{\sqrt{2\pi}} \cdot exp\left(\frac{-x^2}{2E1^2}\right) + pd1\right)$$

cumulative distribution function E=1

$$pe1 := \alpha 1 \cdot exp\left(\frac{-x^2}{2}\right)$$

$$pf1 := 0.5 + 0.5\,erf\left(\frac{\alpha 1}{E1} \cdot \frac{x}{\sqrt{2}}\right)$$

$$pg1 := \overrightarrow{(pe1 \cdot pf1)}$$

$$P1 := \frac{2}{(1 + \alpha 1)} \cdot \left[\frac{-1}{2}(1 - \alpha 1) + \left(0.5 + 0.5\,erf\left(\frac{x}{E1 \cdot \sqrt{2}}\right)\right) - pg1\right]$$

$$PE1 := 1 - P1$$

probability density function E=0

$$pb0 := \overrightarrow{(\alpha 0 \cdot x \cdot pa)}$$

$$pc0 := 0.5 + 0.5 \cdot erf\left(\frac{\alpha 0}{E0} \cdot \frac{x}{\sqrt{2}}\right)$$

$$pd0 := \overrightarrow{(pb0 \cdot pc0)}$$

$$p0 := \frac{2}{(1 + \alpha 0)} \cdot \left(\frac{E0}{\sqrt{2\pi}} \cdot exp\left(\frac{-x^2}{2E0^2}\right) + pd0\right)$$

cumulative distribution function E=0

$$pe0 := \alpha 0 \cdot exp\left(\frac{-x^2}{2}\right)$$

$$pf0 := 0.5 + 0.5\,erf\left(\frac{\alpha 0}{E0} \cdot \frac{x}{\sqrt{2}}\right)$$

$$pg0 := \overrightarrow{(pe0 \cdot pf0)}$$

$$P0 := \frac{2}{(1 + \alpha 0)} \cdot \left[\frac{-1}{2}(1 - \alpha 0) + \left(0.5 + 0.5\,erf\left(\frac{x}{E0 \cdot \sqrt{2}}\right)\right) - pg0\right]$$

$$PE0 := 1 - P0$$

Haring

$$Pha := \overrightarrow{\left[4.37\cdot\left(\frac{cl}{d}\right)\cdot\left(0.57 - \frac{cl}{d}\right)\right]} \qquad Phb := \overrightarrow{\left[-\left(\frac{1}{2}\right)\cdot\left(\frac{cl^2}{mo}\right)\cdot(1 - Pha)\right]}$$

$$Ph := \exp(Phb)$$

Tayfun and Huang

$$RR := k\cdot hs \qquad RR = 0.2846$$

$$Pth := \exp\left[\frac{-8}{RR^2}\left(\sqrt{1 + 2\cdot RR\cdot\frac{cl}{hs}} - 1\right)^2\right]$$

Kriebel and Dawson

$$Pkda := \left(1 - \frac{1}{2}RR\cdot\frac{cl}{hs}\right)^2$$

$$Pkdb := -8\cdot\frac{cl^2}{hs^2}$$

$$Pkdc := \overrightarrow{(Pkda\cdot Pkdb)}$$

$$Pkd := \exp(Pkdc)$$

Forristall

$$Sl := \left(\frac{2\pi\, hs}{9.81Twm}\right) \qquad Sl = 0.0349$$

$$Ur := \frac{hs}{k^2\, d^3} \qquad Ur = 0.0027$$

Two - dimensional

$$a2 := 0.3536 + 0.2892Sl + 0.1060Ur \qquad a2 = 0.364$$

$$b2 := 2 - 2.1597Sl + 0.0968Ur^2 \qquad b2 = 1.9246$$

$$Pf2 := \exp\left[-\left(\frac{cl}{a2\cdot hs}\right)^{b2}\right]$$

Three - dimensional

$$a3 := 0.3536 + 0.2568Sl + 0.0800Ur \qquad a3 = 0.3628$$

$$b3 := 2 - 1.7912Sl - 0.5302Ur + 0.284Ur^2 \qquad b3 = 1.9361$$

$$Pf3 := \exp\left[-\left(\frac{cl}{a3\cdot hs}\right)^{b3}\right]$$

Prevosto

$ss := 1$ unidirectional wave

$ahs := 1 - \left(\frac{1}{2}\right)\cdot(\tanh(k\cdot d) - 0.9)\cdot\sqrt{\frac{2}{1 + ss}}$ $ahs = 0.9501$ Directional Factor

$afm := \frac{1}{1.23}$ $afm = 0.813$ Spectral Bandwith

$T02 := \frac{Twm}{1.2}$ $fm := \frac{1}{T02}$ $fm = 1.5578$

modified significant wave height hs

$hsp := ahs\cdot hs$ $hsp = 0.0399$

modified mean frequency fm

$fmp := afm\cdot fm$ $fmp = 1.2665$

modified wave number (dispersion relation)

$kmp := \frac{(2\cdot\pi\cdot fm)^2}{9.81}$ $kmp = 9.7659$ cek $\tanh(kmp\cdot d) = 1$

dimensionless depth

$kap := kmp\cdot d$ $kap = 6.8361$

$PI := \tanh(kap) + kap\cdot\left[1 - (\tanh(kap))^2\right]$ $PI = 1$

Second order coefficients

$cdiff := \frac{\left[PI + kap\cdot\left[1 - (\tanh(kap))^2\right]\right]}{\left(PI^2\right) - 4\cdot kap\cdot\tanh(kap)}$ $cdiff = -0.038$

$csum := \left(\frac{1}{4}\right)\cdot\frac{\left[2 + \left[1 - (\tanh(kap))^2\right]\right]}{(\tanh(kap))^3}$ $csum = 0.5$

Second order transfer functions

$TD := cdiff\cdot k$ $TD = -0.2574$

$TS := csum\cdot k$ $TS = 3.391$

Non-linear crest components

$C0 := TD\cdot\frac{hsp^2}{8}$ $C0 = -0.0001$

$C1 := ahs$ $C1 = 0.9501$

$C2 := C1^2\cdot(TD + TS)$ $C2 = 2.8285$

$Cr := cl + (TD + TS)\cdot cl^2 - TD\cdot\frac{hs^2}{8}$

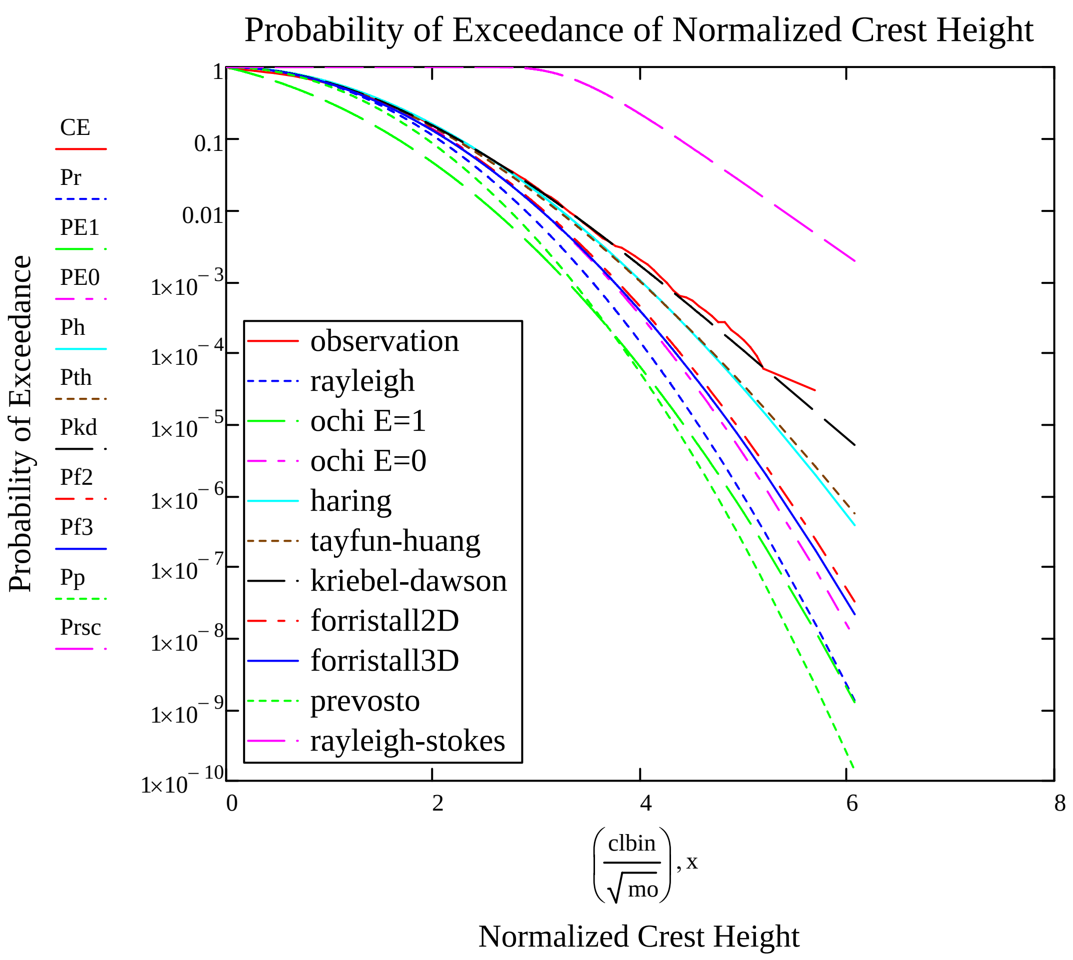
$$Pp := \exp\left[\frac{-8}{hs^2}\cdot\left[\frac{\left[-C1 + \sqrt{C1^2 - 4\cdot[C2\cdot(C0 - Cr)]}\right]}{2C2}\right]^2\right]$$

Rayleigh Stokes

$$acn := an\cdot\left[1 + B2\cdot(k\cdot an) + B3\cdot(k\cdot an)^2\right]$$

$$bcn := bn\cdot\left[1 + 2\cdot B2\cdot(k\cdot an) + 3\cdot B3\cdot(k\cdot an)^2\right]$$

$$Prsc := 1 - \exp\left[-\exp\left[\frac{-(cl - acn)}{bcn}\right]\right]$$



LABORATORY SIMULATION 050B

Variance of water surface elevation

$mo := 0.00011775$

Water depth in meter

$d := 0.7$

ZERO CROSSING WAVE PERIOD

Tw :=

	0
0	0.02
1	0.11
2	0.02
3	0.02
4	1.08
5	0.4
6	0.74
7	...

$Twm := mean(Tw)$

$Twm = 0.7717$ sec

dispersion relation of deep water condition

$$k := 4 \cdot \frac{\pi^2}{Twm^2 9.81} \qquad k = 6.7576$$

ZERO CROSSING WAVE HEIGHT

Zero crossing wave height in meter (hlr)

hlr :=

	0
0	$1.193 \cdot 10^{-4}$
1	$2.982 \cdot 10^{-4}$
2	$4.175 \cdot 10^{-4}$
3	$7.754 \cdot 10^{-4}$
4	$3.936 \cdot 10^{-3}$
5	$3.28 \cdot 10^{-3}$
6	$7.634 \cdot 10^{-3}$
7	0.014
8	0.016
9	...

$hl := sort(hlr) \qquad hl2 := hl^2$

Number of wave

$Nhl := length(hl) \qquad Nhl = 33664$

Statistical properties

$mean(hl) = 0.0252$

$stdev(hl) = 0.0141$

$skew(hl) = 0.4163$

$kurt(hl) = 0$

Root mean square wave height

$$hlrms := \sqrt{\left[\frac{1}{Nhl} \cdot \left(\sum hl2\right)\right]}$$

$hlrms = 0.0288$

Normalized wave height

$$hn := \frac{hl}{\sqrt{mo}}$$

Significant wave height

$$Nhs := round\left[\left(\frac{2}{3}\right)\cdot Nhl\right] \qquad Nhs = 22443$$

$$i := (Nhs - 1)\,..\,(Nhl - 1)$$

$$hl_{(Nhs-1)} = 0.0307$$

$$hl_{(Nhl-1)} = 0.0907$$

$hl_i =$

0.0307
0.0307
0.0307
0.0307
0.0307
0.0307
...

$$sumhs := \sum_{i = Nhs-1}^{(Nhl-1)} hl_i \qquad sumhs = 460.2459$$

$$hs := \frac{sumhs}{(Nhl - Nhs + 1)} \qquad hs = 0.041$$

comparison with theoritical significant wave height

$$hss := 4\cdot\sqrt{mo} \qquad hss = 0.0434$$

Maximum wave height

$$hlm := max(hl)$$

$$hlm = 0.0907$$

comparison with theoritical Rayleigh formula of maximum wave height

$$hmax := \sqrt{\frac{ln(Nhl)}{2}}\cdot hs$$

$$hmax = 0.0936$$

OBSERVED WAVE HEIGHT DISTRIBUTION

Wave height bin

hlbin :=

	0
0	$4.962\cdot10^{-4}$
1	$1.489\cdot10^{-3}$
2	$2.481\cdot10^{-3}$
3	$3.473\cdot10^{-3}$
4	$4.466\cdot10^{-3}$
5	$5.458\cdot10^{-3}$
6	$6.451\cdot10^{-3}$
7	$7.443\cdot10^{-3}$
8	$8.436\cdot10^{-3}$
9	...

Wave height frequency

hlfreq :=

	0
0	$1.318\cdot10^{3}$
1	223
2	214
3	301
4	305
5	338
6	458
7	493
8	514
9	...

Cumulative frequency

hlcdf :=

	0
0	0.039
1	0.046
2	0.052
3	0.061
4	0.07
5	0.08
6	0.094
7	0.108
8	0.124
9	...

Bin interval

$binl := 2 \cdot hlbin_0$

$binl = 0.001$

Number of bin

$Nbl := length(hlbin)$

$Nbl = 87$

Probability density

$Nhlfreq := length(hlfreq)$

$$hlfrel := \frac{hlfreq}{\sum(hlfreq)}$$

$$hlpdf := \frac{hlfreq}{\sum(hlfreq \cdot binl)}$$

Probability of exceedance

$HE := 1 - hlcdf$

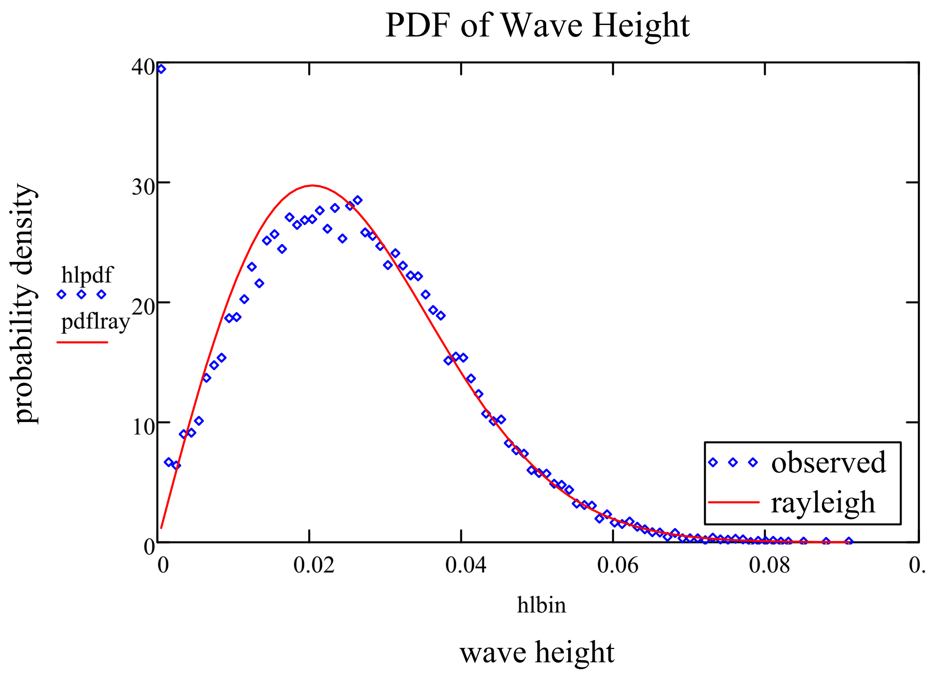
RAYLEIGHT DISTRIBUTION

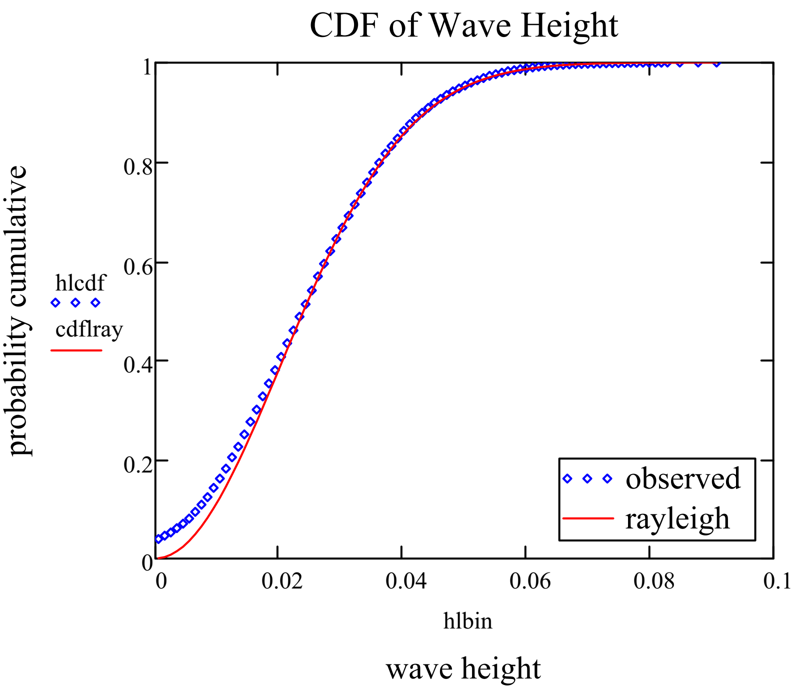
$i := 0 .. (length(hlbin) - 1)$

Rayleigh Distribution

$$pdflray_i := \left[\frac{(2 \cdot hlbin_i)}{hlrms^2}\right] \cdot exp\left[\frac{-(hlbin_i)^2}{hlrms^2}\right]$$

$$cdflray_i := 1 - exp\left[\frac{-(hlbin_i)^2}{hlrms^2}\right]$$





EMPIRICAL WAVE HEIGHT DISTRIBUTION [COMPARISON]

Rayleigh

$$Phr1 := \exp\left[-\left(\frac{hl}{0.707 \cdot hs}\right)^2\right]$$

$$Phr2 := \exp\left[-\left(\frac{hl}{0.638 \cdot hs}\right)^2\right]$$

Forristall

$$Phf := \exp\left[-\left(\frac{hl}{0.681 \cdot hs}\right)^{2.126}\right]$$

Krogstad

$$Phk1 := \exp\left[-\left(\frac{hl}{0.73 \cdot hs}\right)^{2.37}\right]$$

$$Phk2 := \exp\left[-\left(\frac{hl}{0.75 \cdot hs}\right)^{2.5}\right]$$

Haring

$$Phh1 := 0.968 + 0.176 \cdot \frac{hl}{hs}$$

$$Phh2 := \overrightarrow{\left[-2\left(\frac{hl}{hs}\right)^2 \cdot Phh1\right]}$$

$$Phh := \exp(Phh2)$$

Rayleigh Stokes

$$an := 4\cdot\sqrt{mo}\cdot\sqrt{\left(\frac{\log(Nhl)}{8}\right)} \qquad an = 0.0327$$

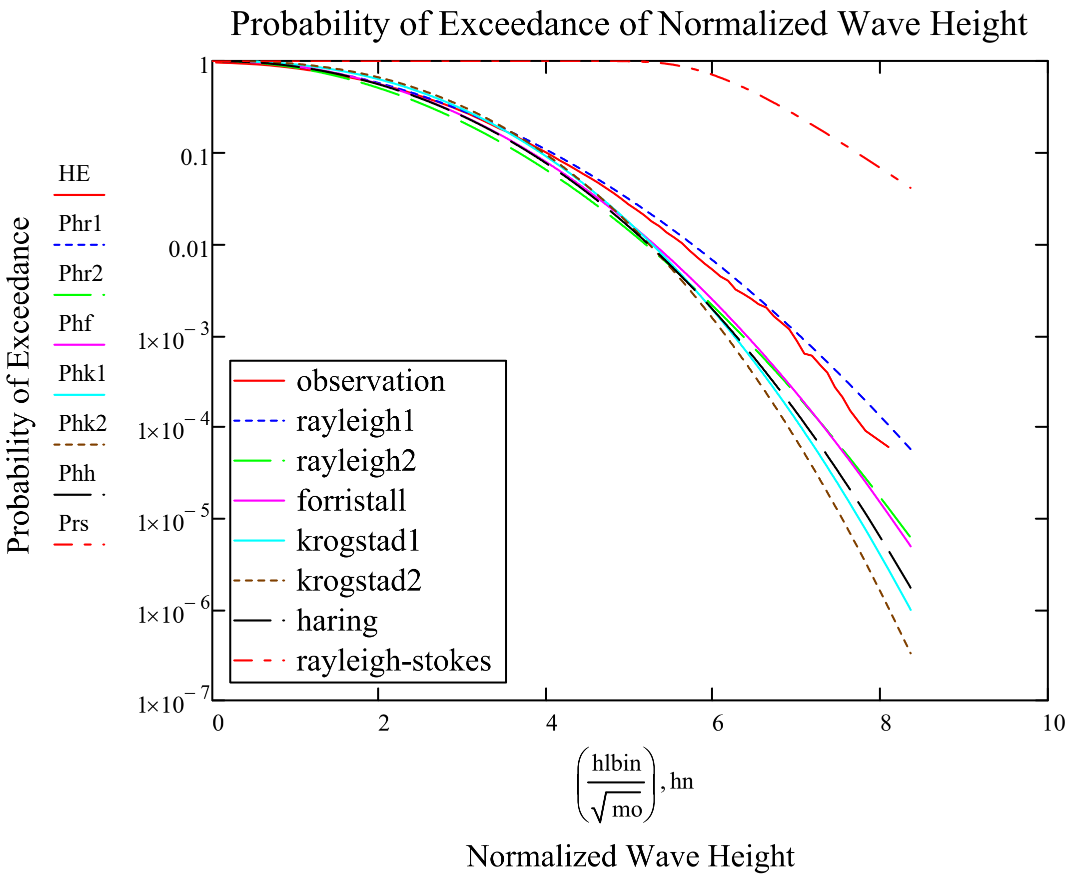
$$bn := \frac{an}{2\cdot\log(Nhl)} \qquad bn = 0.0036$$

$$B2 := \frac{1}{2} \qquad B3 := \frac{3}{8}$$

$$ahn := 2\cdot an\cdot\left[1 + B3\cdot(k\cdot an)^2\right]$$

$$bhn := 2\cdot bn\cdot\left[1 + 3\cdot B3\cdot(k\cdot an)^2\right]$$

$$Prs := 1 - \exp\left[-\exp\left[-\left(\frac{hl - ahn}{bhn}\right)\right]\right]$$



ZERO CROSSING CREST HEIGHT

Zero crossing crest height in meter (clr)

clr :=

	0
0	$9.68253\cdot10^{-6}$
1	$6.93261\cdot10^{-5}$
2	0.00019
3	0.00037
4	0.00335
5	$9.68253\cdot10^{-6}$
6	0.00758
7	0.008
8	0.01027
9	...

Sorted crest $cl := sort(clr)$

Number of crest $Ncl := length(cl)$ $Ncl = 33664$

Maximum crest height $max(cl) = 0.0595$

Statistical Properties

$mean(cl) = 0.0132$

$stdev(cl) = 0.0084$

$skew(cl) = 0.5191$

$kurt(cl) = 0.1672$

Normalized crest height

$$x := \frac{cl}{\sqrt{mo}}$$

OBSERVED CREST HEIGHT DISTRIBUTION

Crest height bin

clbin :=

	0
0	$3.159\cdot10^{-4}$
1	$9.477\cdot10^{-4}$
2	$1.58\cdot10^{-3}$
3	$2.211\cdot10^{-3}$
4	$2.843\cdot10^{-3}$
5	$3.475\cdot10^{-3}$
6	$4.107\cdot10^{-3}$
7	$4.739\cdot10^{-3}$
8	$5.371\cdot10^{-3}$
9	...

Crest height frequency

clfreq :=

	0
0	$2.074\cdot10^{3}$
1	600
2	539
3	605
4	540
5	607
6	581
7	651
8	699
9	...

Cumulative frequency

clfcum :=

	0
0	0.062
1	0.079
2	0.095
3	0.113
4	0.129
5	0.147
6	0.165
7	0.184
8	0.205
9	...

Bin interval

$lbin := 2\cdot clbin_0$

$lbin = 0.0006$

Number of bin

$Ncbl := length(clbin)$

$Ncbl = 83$

Probability density

$$clfrel := \frac{clfreq}{\sum clfreq}$$

$$clpdf := \frac{clfreq\cdot\sqrt{mo}}{\sum(clfreq\cdot lbin)}$$

Probability of exceedance

$CE := 1 - clfcum$

EMPIRICAL CREST HEIGHT DISTRIBUTION [COMPARISON]

Rayleigh

$$Pr := \exp\left(-8\,\frac{cl^2}{hs^2}\right)$$

Ochi

$$E0 := 0.1 \qquad \alpha 0 := \sqrt{\left(1 - E0^2\right)}$$

$$E1 := 1 \qquad \alpha 1 := \sqrt{\left(1 - E1^2\right)}$$

probability density function of E=1

$$pa := \exp\left(\frac{-x^2}{2}\right)$$

$$pb1 := \overrightarrow{(\alpha 1 \cdot x \cdot pa)}$$

$$pc1 := 0.5 + 0.5 \cdot \mathrm{erf}\left(\frac{\alpha 1}{E1} \cdot \frac{x}{\sqrt{2}}\right)$$

$$pd1 := \overrightarrow{(pb1 \cdot pc1)}$$

$$p1 := \frac{2}{(1 + \alpha 1)} \cdot \left(\frac{E1}{\sqrt{2\pi}} \cdot \exp\left(\frac{-x^2}{2E1^2}\right) + pd1\right)$$

cumulative distribution function E=1

$$pe1 := \alpha 1 \cdot \exp\left(\frac{-x^2}{2}\right)$$

$$pf1 := 0.5 + 0.5\,\mathrm{erf}\left(\frac{\alpha 1}{E1} \cdot \frac{x}{\sqrt{2}}\right)$$

$$pg1 := \overrightarrow{(pe1 \cdot pf1)}$$

$$P1 := \frac{2}{(1 + \alpha 1)} \cdot \left[\frac{-1}{2}(1 - \alpha 1) + \left(0.5 + 0.5\,\mathrm{erf}\left(\frac{x}{E1 \cdot \sqrt{2}}\right)\right) - pg1\right]$$

$$PE1 := 1 - P1$$

probability density function E=0

$$pb0 := \overrightarrow{(\alpha 0 \cdot x \cdot pa)}$$

$$pc0 := 0.5 + 0.5 \cdot \mathrm{erf}\left(\frac{\alpha 0}{E0} \cdot \frac{x}{\sqrt{2}}\right)$$

$$pd0 := \overrightarrow{(pb0 \cdot pc0)}$$

$$p0 := \frac{2}{(1 + \alpha 0)} \cdot \left(\frac{E0}{\sqrt{2\pi}} \cdot \exp\left(\frac{-x^2}{2E0^2}\right) + pd0\right)$$

cumulative distribution function E=0

$$pe0 := \alpha0 \cdot \exp\left(\frac{-x^2}{2}\right)$$

$$pf0 := 0.5 + 0.5\,\text{erf}\left(\frac{\alpha0}{E0} \cdot \frac{x}{\sqrt{2}}\right)$$

$$pg0 := \overrightarrow{(pe0 \cdot pf0)}$$

$$P0 := \frac{2}{(1+\alpha0)} \cdot \left[\frac{-1}{2}(1-\alpha0) + \left(0.5 + 0.5\,\text{erf}\left(\frac{x}{E0 \cdot \sqrt{2}}\right)\right) - pg0\right]$$

$$PE0 := 1 - P0$$

Haring

$$Pha := \overrightarrow{\left[4.37 \cdot \left(\frac{cl}{d}\right) \cdot \left(0.57 - \frac{cl}{d}\right)\right]}$$

$$Phb := \overrightarrow{\left[-\left(\frac{1}{2}\right) \cdot \left(\frac{cl^2}{mo}\right) \cdot (1 - Pha)\right]}$$

$$Ph := \exp(Phb)$$

Tayfun and Huang

$RR := k \cdot hs$ $\qquad$ $RR = 0.2771$

$$Pth := \exp\left[\frac{-8}{RR^2}\left(\sqrt{1 + 2 \cdot RR \cdot \frac{cl}{hs}} - 1\right)^2\right]$$

Kriebel and Dawson

$$Pkda := \left(1 - \frac{1}{2} RR \cdot \frac{cl}{hs}\right)^2$$

$$Pkdb := -8 \cdot \frac{cl^2}{hs^2}$$

$$Pkdc := \overrightarrow{(Pkda \cdot Pkdb)}$$

$$Pkd := \exp(Pkdc)$$

Forristall

$Sl := \left(\frac{2\pi\, hs}{9.81 Twm}\right)$ $\qquad$ $Sl = 0.034$

$Ur := \frac{hs}{k^2 d^3}$ $\qquad$ $Ur = 0.0026$

Two - dimensional

$a2 := 0.3536 + 0.2892Sl + 0.1060Ur$ $\quad a2 = 0.3637$

$b2 := 2 - 2.1597Sl + 0.0968Ur^2$ $\quad b2 = 1.9265$

$$Pf2 := \exp\left[-\left(\frac{cl}{a2 \cdot hs}\right)^{b2}\right]$$

Three - dimensional

$a3 := 0.3536 + 0.2568Sl + 0.0800Ur$ $\quad a3 = 0.3626$

$b3 := 2 - 1.7912Sl - 0.5302Ur + 0.284Ur^2$ $\quad b3 = 1.9376$

$$Pf3 := \exp\left[-\left(\frac{cl}{a3 \cdot hs}\right)^{b3}\right]$$

Prevosto

$ss := 1$ unidirectional wave

$ahs := 1 - \left(\frac{1}{2}\right) \cdot (\tanh(k \cdot d) - 0.9) \cdot \sqrt{\frac{2}{1 + ss}}$ $\quad ahs = 0.9501$ Directional Factor

$afm := \frac{1}{1.23}$ $\quad afm = 0.813$ Spectral Bandwith

$T02 := \frac{Twm}{1.2}$ $\quad fm := \frac{1}{T02}$ $\quad fm = 1.555$

modified significant wave height hs

$hsp := ahs \cdot hs$ $\quad hsp = 0.039$

modified mean frequency fm

$fmp := afm \cdot fm$ $\quad fmp = 1.2642$

modified wave number (dispersion relation)

$kmp := \frac{(2 \cdot \pi \cdot fm)^2}{9.81}$ $\quad kmp = 9.7309$ cek $\tanh(kmp \cdot d) = 1$

dimensionless depth

$kap := kmp \cdot d$ $\quad kap = 6.8116$

$PI := \tanh(kap) + kap \cdot \left[1 - (\tanh(kap))^2\right]$ $\quad PI = 1$

Second order coefficients

$$cdiff := \frac{\left[PI + kap \cdot \left[1 - (\tanh(kap))^2\right]\right]}{\left(PI^2\right) - 4 \cdot kap \cdot \tanh(kap)} \quad cdiff = -0.0381$$

$$csum := \left(\frac{1}{4}\right) \cdot \frac{\left[2 + \left[1 - (\tanh(kap))^2\right]\right]}{(\tanh(kap))^3} \quad csum = 0.5$$

Second order transfer functions

$TD := cdiff \cdot k$ $\quad TD = -0.2575$

$TS := csum \cdot k$ $\quad TS = 3.3788$

Non-linear crest components

$C0 := TD \cdot \frac{hsp^2}{8}$ $\quad C0 = -0$

$C1 := ahs$ $\quad C1 = 0.9501$

$C2 := C1^2 \cdot (TD + TS)$ $\quad C2 = 2.8175$

$$Cr := cl + (TD + TS) \cdot cl^2 - TD \cdot \frac{hs^2}{8}$$

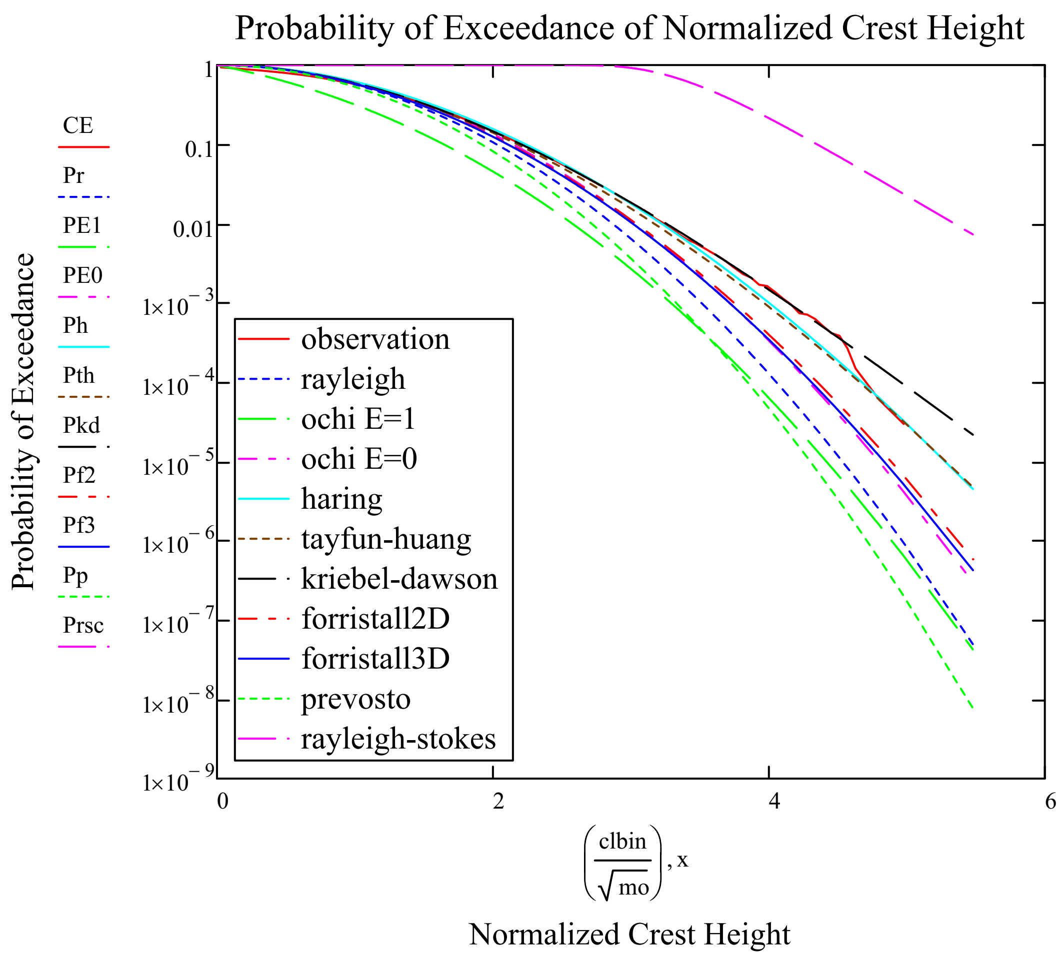
$$Pp := \exp\left[\frac{-8}{hs^2} \cdot \left[\frac{\left[-C1 + \sqrt{C1^2 - 4 \cdot [C2 \cdot (C0 - Cr)]}\right]}{2C2}\right]^2\right]$$

Rayleigh Stokes

$$acn := an \cdot \left[1 + B2 \cdot (k \cdot an) + B3 \cdot (k \cdot an)^2\right]$$

$$bcn := bn \cdot \left[1 + 2 \cdot B2 \cdot (k \cdot an) + 3 \cdot B3 \cdot (k \cdot an)^2\right]$$

$$Prsc := 1 - \exp\left[-\exp\left[\frac{-(cl - acn)}{bcn}\right]\right]$$



Probability of Exceedance of Normalized Crest Height
Probability of Exceedance
CE
Pr
PE1
PE0
Ph
Pth
Pkd
Pf2
Pf3
Pp
Prsc
1
0.1
0.01
1×10⁻³
1×10⁻⁴
1×10⁻⁵
1×10⁻⁶
1×10⁻⁷
1×10⁻⁸
1×10⁻⁹
0
2
4
6
observation
rayleigh
ochi E=1
ochi E=0
haring
tayfun-huang
kriebel-dawson
forristall2D
forristall3D
prevosto
rayleigh-stokes
(clbin / √mo), x
Normalized Crest Height

LABORATORY SIMULATION 075A

Variance of water surface elevation

$mo := 0.00031172$

Water depth in meter

$d := 0.7$

ZERO CROSSING WAVE PERIOD

Tw :=

	0
0	0.17
1	0.15
2	0.13
3	0.24
4	0.28
5	0.36
6	...

$Twm := mean(Tw)$

$Twm = 0.9286$ sec

dispersion relation of deep water condition

$$k := 4 \cdot \frac{\pi^2}{Twm^2 9.81}$$

$k = 4.6673$

ZERO CROSSING WAVE HEIGHT

Zero crossing wave height in meter (hlr)

hlr :=

	0
0	$6.299 \cdot 10^{-4}$
1	$4.724 \cdot 10^{-4}$
2	$6.299 \cdot 10^{-4}$
3	$6.299 \cdot 10^{-4}$
4	$9.448 \cdot 10^{-4}$
5	$1.26 \cdot 10^{-3}$
6	$7.874 \cdot 10^{-4}$
7	...

$hl := sort(hlr)$ $\quad hl2 := hl^2$

Number of wave

$Nhl := length(hl)$ $\quad Nhl = 30911$

Statistical properties

$mean(hl) = 0.0422$

$stdev(hl) = 0.0221$

$skew(hl) = 0.4823$

$kurt(hl) = -0.0067$

Root mean square wave height

$$hlrms := \sqrt{\left[\frac{1}{Nhl} \cdot \left(\sum hl2\right)\right]}$$

$hlrms = 0.0477$

Normalized wave height

$$hn := \frac{hl}{\sqrt{mo}}$$

Significant wave height

$$Nhs := round\left[\left(\frac{2}{3}\right) \cdot Nhl\right]$$

$Nhs = 20607$

$i := (Nhs - 1) .. (Nhl - 1)$

$hl_{(Nhs-1)} = 0.0509$ $\quad hl_{(Nhl-1)} = 0.1457$

$hl_i =$

0.0509
0.0509
0.0509
0.0509
0.0509
...

$$sumhs := \sum_{i = Nhs-1}^{(Nhl-1)} hl_i \qquad sumhs = 694.4459$$

$$hs := \frac{sumhs}{(Nhl - Nhs + 1)} \qquad hs = 0.0674$$

comparison with theoritical significant wave height

$$hss := 4\cdot\sqrt{mo} \qquad hss = 0.0706$$

Maximum wave height

$hlm := max(hl)$

$hlm = 0.1457$

comparison with theoritical Rayleigh formula of maximum wave height

$$hmax := \sqrt{\frac{ln(Nhl)}{2}}\cdot hs$$

$hmax = 0.1532$

OBSERVED WAVE HEIGHT DISTRIBUTION

Wave height bin

hlbin :=

	0
0	$8.272\cdot10^{-4}$
1	$2.482\cdot10^{-3}$
2	$4.136\cdot10^{-3}$
3	$5.791\cdot10^{-3}$
4	$7.445\cdot10^{-3}$
5	$9.1\cdot10^{-3}$
6	0.011
7	0.012
8	0.014
9	...

Wave height frequency

hlfreq :=

	0
0	280
1	176
2	175
3	287
4	302
5	406
6	457
7	564
8	538
9	...

Cumulative frequency

hlcdf :=

	0
0	$9.058\cdot10^{-3}$
1	0.015
2	0.02
3	0.03
4	0.039
5	0.053
6	0.067
7	0.086
8	0.103
9	...

Bin interval

$binl := 2\cdot hlbin_0$

$binl = 0.0017$

Number of bin

$Nbl := length(hlbin)$

$Nbl = 85$

Probability density

$Nhlfreq := length(hlfreq)$

$$hlfrel := \frac{hlfreq}{\sum(hlfreq)}$$

$$hlpdf := \frac{hlfreq}{\sum(hlfreq\cdot binl)}$$

Probability of exceedance

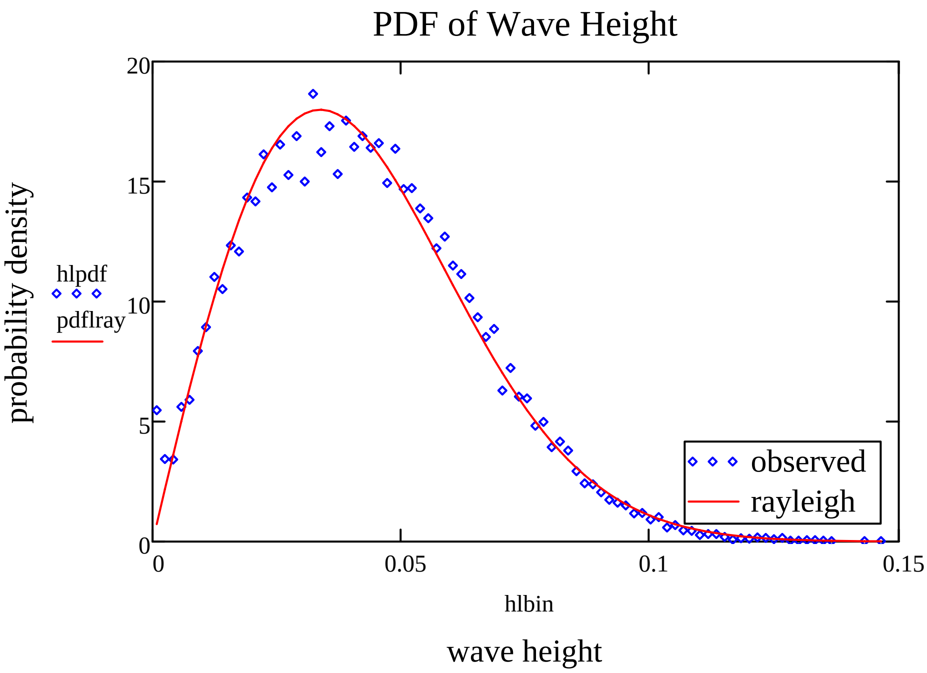
$Nlcdf := length(hlcdf)$

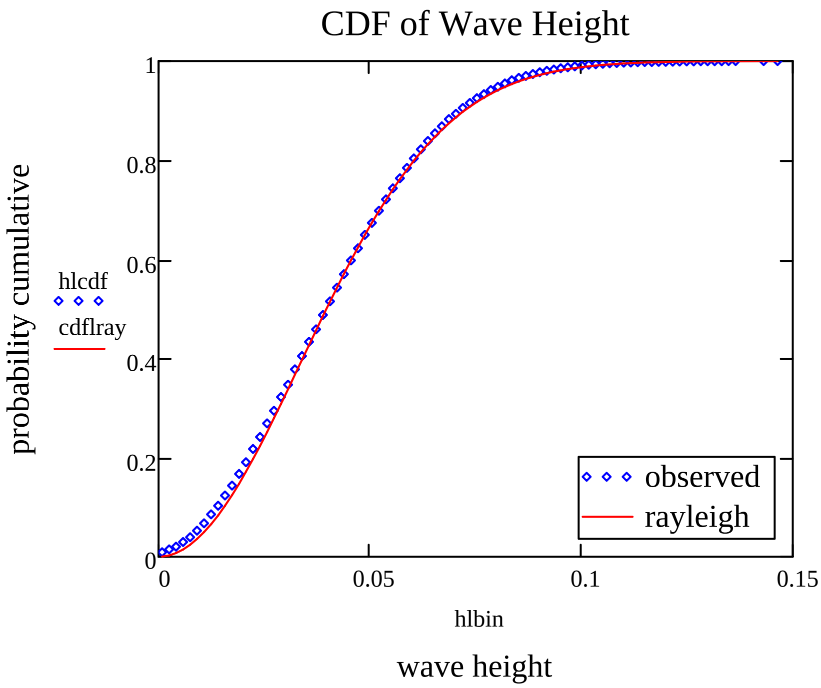
$HE := 1 - hlcdf$

$i := 0 .. (\text{length}(\text{hlbin}) - 1)$

Rayleigh Distribution

$$\text{pdflray}_i := \left[\frac{(2 \cdot \text{hlbin}_i)}{\text{hlrms}^2}\right] \cdot \exp\left[\frac{-(\text{hlbin}_i)^2}{\text{hlrms}^2}\right] \qquad \text{cdflray}_i := 1 - \exp\left[\frac{-(\text{hlbin}_i)^2}{\text{hlrms}^2}\right]$$





EMPIRICAL WAVE HEIGHT DISTRIBUTION [COMPARISON]

Rayleigh

$$\mathrm{Phr1} := \exp\left[-\left(\frac{hl}{0.707 \cdot hs}\right)^2\right]$$

$$\mathrm{Phr2} := \exp\left[-\left(\frac{hl}{0.638 \cdot hs}\right)^2\right]$$

Forristall

$$\mathrm{Phf} := \exp\left[-\left(\frac{hl}{0.681 \cdot hs}\right)^{2.126}\right]$$

Krogstad

$$\mathrm{Phk1} := \exp\left[-\left(\frac{hl}{0.73 \cdot hs}\right)^{2.37}\right]$$

$$\mathrm{Phk2} := \exp\left[-\left(\frac{hl}{0.75 \cdot hs}\right)^{2.5}\right]$$

Haring

$$\mathrm{Phh1} := 0.968 + 0.176 \cdot \frac{hl}{hs}$$

$$\mathrm{Phh2} := \overrightarrow{\left[-2\left(\frac{hl}{hs}\right)^2 \cdot \mathrm{Phh1}\right]}$$

$$\mathrm{Phh} := \exp(\mathrm{Phh2})$$

Rayleigh Stokes

$$an := 4 \cdot \sqrt{mo} \cdot \sqrt{\left(\frac{\log(Nhl)}{8}\right)}$$

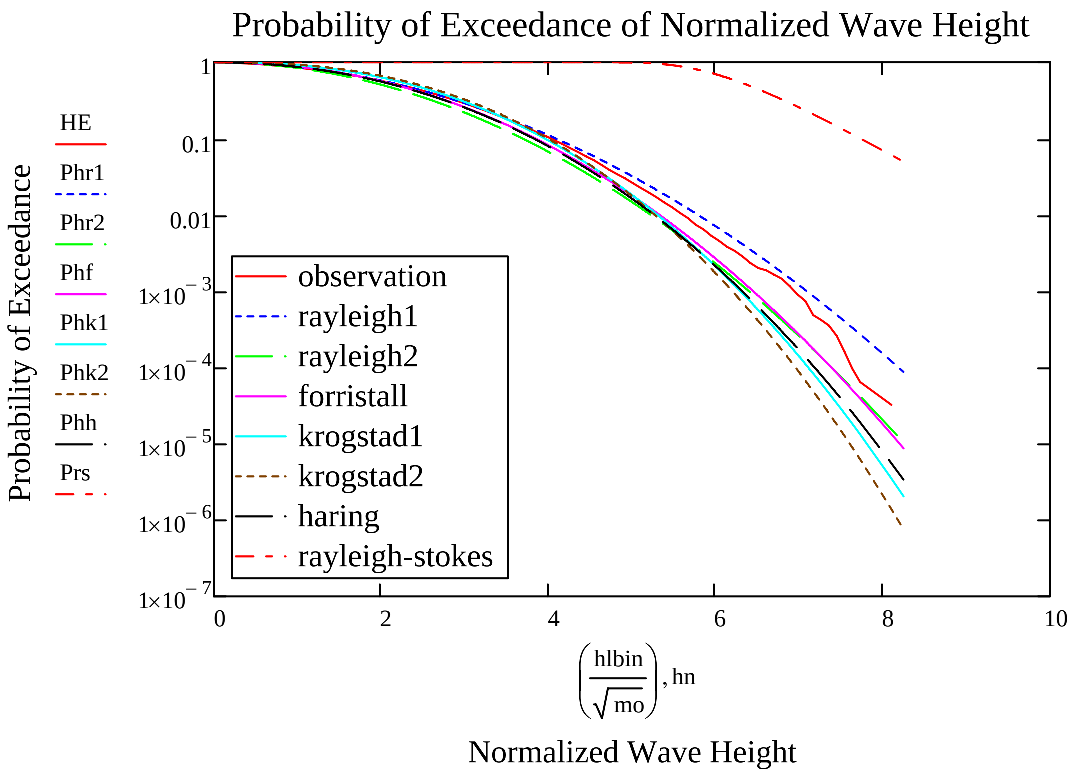
$$bn := \frac{an}{2 \cdot \log(Nhl)}$$

$$B2 := \frac{1}{2} \qquad B3 := \frac{3}{8}$$

$$ahn := 2 \cdot an \cdot \left[1 + B3 \cdot (k \cdot an)^2\right]$$

$$bhn := 2 \cdot bn \cdot \left[1 + 3 \cdot B3 \cdot (k \cdot an)^2\right]$$

$$\mathrm{Prs} := 1 - \exp\left[-\exp\left[-\left(\frac{hl - ahn}{bhn}\right)\right]\right]$$



ZERO CROSSING CREST HEIGHT

Zero crossing crest height in meter (clr)

clr :=

	0
0	9.58003·10⁻⁵
1	9.58003·10⁻⁵
2	0.00057
3	9.58003·10⁻⁵
4	0.00057
5	0.00073
6	9.58003·10⁻⁵
7	...

Sorted crest $cl := sort(clr)$

Number of crest $Ncl := length(cl)$ $Ncl = 30911$

Maximum crest height $max(cl) = 0.0968$

Statistical Properties

$mean(cl) = 0.0226$

$stdev(cl) = 0.0137$

$skew(cl) = 0.5435$

$kurt(cl) = 0.201$

Normalized crest height

$$x := \frac{cl}{\sqrt{mo}}$$

OBSERVED CREST HEIGHT DISTRIBUTION

Crest height bin

clbin :=

	0
0	$5.352\cdot10^{-4}$
1	$1.606\cdot10^{-3}$
2	$2.676\cdot10^{-3}$
3	$3.746\cdot10^{-3}$
4	$4.817\cdot10^{-3}$
5	$5.887\cdot10^{-3}$
6	$6.957\cdot10^{-3}$
7	$8.028\cdot10^{-3}$
8	$9.098\cdot10^{-3}$
9	...

Crest height frequency

clfreq :=

	0
0	$1.014\cdot10^{3}$
1	521
2	570
3	611
4	658
5	602
6	556
7	608
8	709
9	...

Cumulative frequency

clfcum :=

	0
0	0.033
1	0.05
2	0.068
3	0.088
4	0.109
5	0.129
6	0.147
7	0.166
8	0.189
9	...

Bin interval

$lbin := 2\cdot clbin_0$

$lbin = 0.0011$

Number of bin

$Ncbl := length(clbin)$

$Ncbl = 83$

Probability density

$$clfrel := \frac{clfreq}{\sum clfreq}$$

$$clpdf := \frac{clfreq\cdot\sqrt{mo}}{\sum (clfreq\cdot lbin)}$$

Probability of exceedance

$CE := 1 - clfcum$

EMPIRICAL CREST HEIGHT DISTRIBUTION [COMPARISON]

Rayleigh

$$Pr := \exp\left(-8\frac{cl^2}{hs^2}\right)$$

Ochi

$E0 := 0.1$ $\qquad \alpha0 := \sqrt{\left(1 - E0^2\right)}$

$E1 := 1$ $\qquad \alpha1 := \sqrt{\left(1 - E1^2\right)}$

probability density function of E=1

$$pa := \exp\left(\frac{-x^2}{2}\right)$$

$$pb1 := \overrightarrow{(\alpha1\cdot x\cdot pa)}$$

$$pc1 := 0.5 + 0.5\cdot erf\left(\frac{\alpha1}{E1}\cdot\frac{x}{\sqrt{2}}\right)$$

$$pd1 := \overrightarrow{(pb1 \cdot pc1)}$$

$$p1 := \frac{2}{(1+\alpha1)} \cdot \left(\frac{E1}{\sqrt{2\pi}} \cdot \exp\left(\frac{-x^2}{2E1^2} \right) + pd1 \right)$$

cumulative distribution function E=1

$$pe1 := \alpha1 \cdot \exp\left(\frac{-x^2}{2} \right)$$

$$pf1 := 0.5 + 0.5\,\mathrm{erf}\left(\frac{\alpha1}{E1} \cdot \frac{x}{\sqrt{2}} \right)$$

$$pg1 := \overrightarrow{(pe1 \cdot pf1)}$$

$$P1 := \frac{2}{(1+\alpha1)} \cdot \left[\frac{-1}{2}(1-\alpha1) + \left(0.5 + 0.5\,\mathrm{erf}\left(\frac{x}{E1 \cdot \sqrt{2}} \right) \right) - pg1 \right]$$

$$PE1 := 1 - P1$$

probability density function E=0

$$pb0 := \overrightarrow{(\alpha0 \cdot x \cdot pa)}$$

$$pc0 := 0.5 + 0.5 \cdot \mathrm{erf}\left(\frac{\alpha0}{E0} \cdot \frac{x}{\sqrt{2}} \right)$$

$$pd0 := \overrightarrow{(pb0 \cdot pc0)}$$

$$p0 := \frac{2}{(1+\alpha0)} \cdot \left(\frac{E0}{\sqrt{2\pi}} \cdot \exp\left(\frac{-x^2}{2E0^2} \right) + pd0 \right)$$

cumulative distribution function E=0

$$pe0 := \alpha0 \cdot \exp\left(\frac{-x^2}{2} \right)$$

$$pf0 := 0.5 + 0.5\,\mathrm{erf}\left(\frac{\alpha0}{E0} \cdot \frac{x}{\sqrt{2}} \right)$$

$$pg0 := \overrightarrow{(pe0 \cdot pf0)}$$

$$P0 := \frac{2}{(1+\alpha0)} \cdot \left[\frac{-1}{2}(1-\alpha0) + \left(0.5 + 0.5\,\mathrm{erf}\left(\frac{x}{E0 \cdot \sqrt{2}} \right) \right) - pg0 \right]$$

$$PE0 := 1 - P0$$

Haring

$$Pha := \overrightarrow{\left[4.37 \cdot \left(\frac{cl}{d} \right) \cdot \left(0.57 - \frac{cl}{d} \right) \right]} \qquad Phb := \overrightarrow{\left[-\left(\frac{1}{2} \right) \cdot \left(\frac{cl^2}{mo} \right) \cdot (1 - Pha) \right]}$$

$$Ph := \exp(Phb)$$

Tayfun and Huang

$RR := k \cdot hs$ $RR = 0.3145$

$$Pth := \exp\left[\frac{-8}{RR^2}\left(\sqrt{1 + 2 \cdot RR \cdot \frac{cl}{hs}} - 1\right)^2\right]$$

Kriebel and Dawson

$$Pkda := \left(1 - \frac{1}{2} RR \cdot \frac{cl}{hs}\right)^2$$

$$Pkdb := -8 \cdot \frac{cl^2}{hs^2}$$

$$Pkdc := \overrightarrow{(Pkda \cdot Pkdb)}$$

$$Pkd := \exp(Pkdc)$$

Forristall

$$Sl := \left(\frac{2\pi\, hs}{9.81 Twm}\right) \qquad Sl = 0.0465$$

$$Ur := \frac{hs}{k^2\, d^3} \qquad Ur = 0.009$$

Two - dimensional

$a2 := 0.3536 + 0.2892Sl + 0.1060Ur$ $a2 = 0.368$

$b2 := 2 - 2.1597Sl + 0.0968Ur^2$ $b2 = 1.8996$

$$Pf2 := \exp\left[-\left(\frac{cl}{a2 \cdot hs}\right)^{b2}\right]$$

Three - dimensional

$a3 := 0.3536 + 0.2568Sl + 0.0800Ur$ $a3 = 0.3663$

$b3 := 2 - 1.7912Sl - 0.5302Ur + 0.284Ur^2$ $b3 = 1.912$

$$Pf3 := \exp\left[-\left(\frac{cl}{a3 \cdot hs}\right)^{b3}\right]$$

Prevosto

$ss := 1$ unidirectional wave

$$ahs := 1 - \left(\frac{1}{2}\right) \cdot (\tanh(k \cdot d) - 0.9) \cdot \sqrt{\frac{2}{1 + ss}}$$ $ahs = 0.9515$ Directional Factor

$$afm := \frac{1}{1.23}$$ $afm = 0.813$ Spectral Bandwith

$T02 := \frac{Twm}{1.2}$ $fm := \frac{1}{T02}$ $fm = 1.2923$

modified significant wave height hs

$hsp := ahs \cdot hs$ $hsp = 0.0641$

modified mean frequency fm

$fmp := afm \cdot fm$ $fmp = 1.0507$

modified wave number (dispersion relation)

$kmp := \frac{(2\cdot\pi\cdot fm)^2}{9.81}$ $kmp = 6.7209$ cek $\tanh(kmp\cdot d) = 0.9998$

dimensionless depth

$kap := kmp \cdot d$ $kap = 4.7046$

$PI := \tanh(kap) + kap\cdot\left[1 - (\tanh(kap))^2\right]$ $PI = 1.0014$

Second order coefficients

$$cdiff := \frac{\left[PI + kap\cdot\left[1 - (\tanh(kap))^2\right]\right]}{\left(PI^2\right) - 4\cdot kap\cdot\tanh(kap)}$$ $cdiff = -0.0563$

$$csum := \left(\frac{1}{4}\right)\cdot\frac{\left[2 + \left[1 - (\tanh(kap))^2\right]\right]}{(\tanh(kap))^3}$$ $csum = 0.5003$

Second order transfer functions

$TD := cdiff \cdot k$ $TD = -0.2628$

$TS := csum \cdot k$ $TS = 2.3352$

Non-linear crest components

$C0 := TD\cdot\frac{hsp^2}{8}$ $C0 = -0.0001$

$C1 := ahs$ $C1 = 0.9515$

$C2 := C1^2\cdot(TD + TS)$ $C2 = 1.8761$

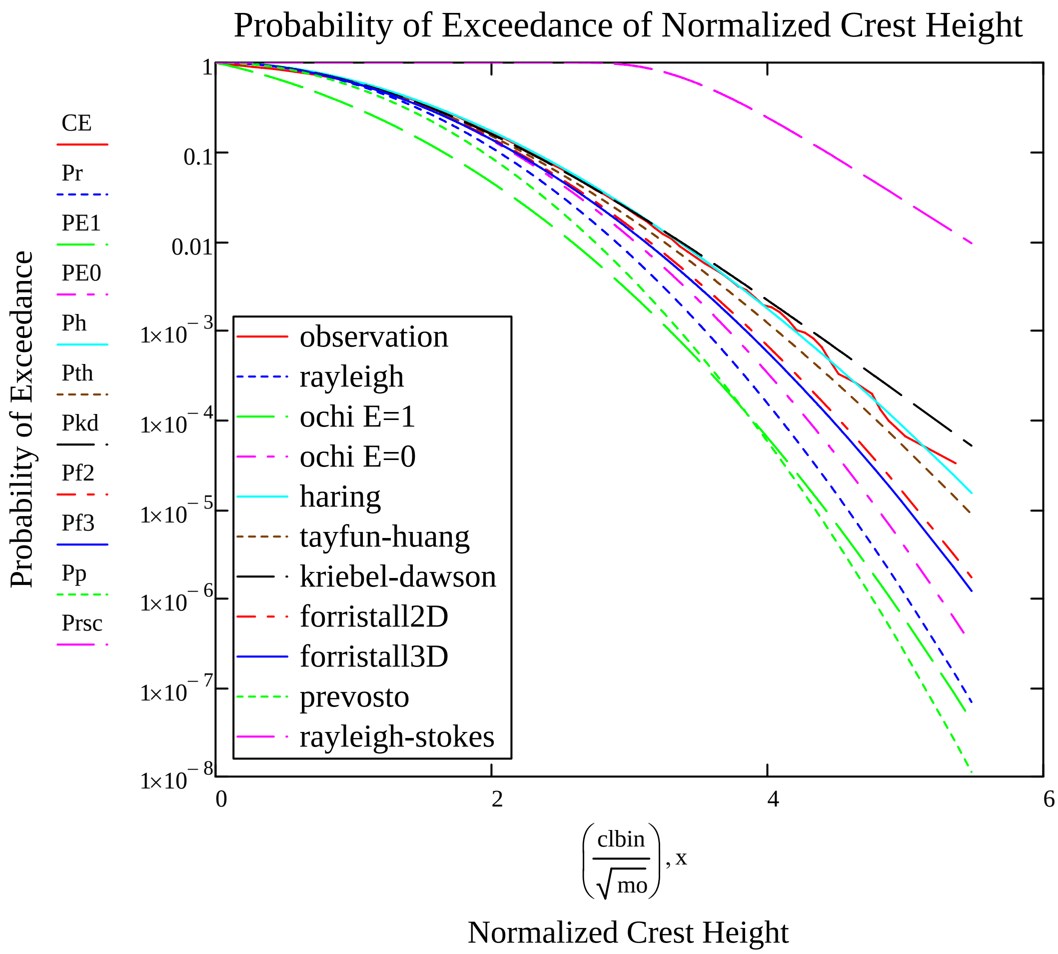
$Cr := cl + (TD + TS)\cdot cl^2 - TD\cdot\frac{hs^2}{8}$

$$Pp := \exp\left[\frac{-8}{hs^2}\cdot\left[\frac{\left[-C1 + \sqrt{C1^2 - 4\cdot[C2\cdot(C0 - Cr)]}\right]}{2C2}\right]^2\right]$$

Rayleigh Stokes

$acn := an\cdot\left[1 + B2\cdot(k\cdot an) + B3\cdot(k\cdot an)^2\right]$ $bcn := bn\cdot\left[1 + 2\cdot B2\cdot(k\cdot an) + 3\cdot B3\cdot(k\cdot an)^2\right]$

$$Prsc := 1 - \exp\left[-\exp\left[\frac{-(cl - acn)}{bcn}\right]\right]$$



Probability of Exceedance of Normalized Crest Height
Probability of Exceedance
CE
Pr
PE1
PE0
Ph
Pth
Pkd
Pf2
Pf3
Pp
Prsc
1
0.1
0.01
1×10^−3
1×10^−4
1×10^−5
1×10^−6
1×10^−7
1×10^−8
0
2
4
6
observation
rayleigh
ochi E=1
ochi E=0
haring
tayfun-huang
kriebel-dawson
forristall2D
forristall3D
prevosto
rayleigh-stokes
(clbin/√mo), x
Normalized Crest Height

LABORATORY SIMULATION 075B

Variance of water surface elevation

$mo := 0.00030518$

Water depth in meter

$d := 0.7$

ZERO CROSSING WAVE PERIOD

Tw :=

	0
0	0.06
1	0.02
2	0.11
3	0.02
4	0.02
5	...

$Twm := mean(Tw)$

$Twm = 0.9341$ sec

dispersion relation of deep water condition

$$k := 4 \cdot \frac{\pi^2}{Twm^2 9.81} \qquad k = 4.6123$$

ZERO CROSSING WAVE HEIGHT

Zero crossing wave height in meter (hlr)

hlr :=

	0
0	$4.771 \cdot 10^{-4}$
1	$1.193 \cdot 10^{-4}$
2	$8.35 \cdot 10^{-4}$
3	$2.386 \cdot 10^{-4}$
4	$2.386 \cdot 10^{-4}$
5	$7.157 \cdot 10^{-4}$
6	$2.386 \cdot 10^{-4}$
7	$1.193 \cdot 10^{-4}$
8	...

$hl := sort(hlr) \qquad hl2 := hl^2$

Number of wave

$Nhl := length(hl) \qquad Nhl = 30759$

Statistical properties

$mean(hl) = 0.0411$

$stdev(hl) = 0.0224$

$skew(hl) = 0.409$

$kurt(hl) = -0.0855$

Root mean square wave height

$$hlrms := \sqrt{\left[\frac{1}{Nhl} \cdot \left(\sum hl2\right)\right]}$$

$hlrms = 0.0469$

Normalized wave height

$$hn := \frac{hl}{\sqrt{mo}}$$

Significant wave height

$$Nhs := round\left[\left(\frac{2}{3}\right) \cdot Nhl\right] \qquad Nhs = 20506$$

$i := (Nhs - 1) .. (Nhl - 1)$

$hl_{(Nhs-1)} = 0.05 \qquad hl_{(Nhl-1)} = 0.1501$

$hl_i =$

0.05
0.05
0.05
0.05
0.05
...

$$sumhs := \sum_{i = Nhs-1}^{(Nhl-1)} hl_i \qquad sumhs = 681.726$$

$$hs := \frac{sumhs}{(Nhl - Nhs + 1)} \qquad hs = 0.0665$$

comparison with theoritical significant wave height

$$hss := 4 \cdot \sqrt{mo} \qquad hss = 0.0699$$

Maximum wave height

$hlm := max(hl)$

$hlm = 0.1501$

comparison with theoritical Rayleigh formula of maximum wave height

$$hmax := \sqrt{\frac{\ln(Nhl)}{2}} \cdot hs$$

$hmax = 0.1511$

OBSERVED WAVE HEIGHT DISTRIBUTION

Wave height bin

hlbin :=

	0
0	$8.478 \cdot 10^{-4}$
1	$2.543 \cdot 10^{-3}$
2	$4.239 \cdot 10^{-3}$
3	$5.935 \cdot 10^{-3}$
4	$7.63 \cdot 10^{-3}$
5	$9.326 \cdot 10^{-3}$
6	0.011
7	0.013
8	0.014
9	...

Wave height frequency

hlfreq :=

	0
0	835
1	173
2	217
3	287
4	373
5	413
6	479
7	516
8	564
9	...

Cumulative frequency

hlcdf :=

	0
0	0.027
1	0.033
2	0.04
3	0.049
4	0.061
5	0.075
6	0.09
7	0.107
8	0.125
9	...

Bin interval

$binl := 2 \cdot hlbin_0$

$binl = 0.0017$

Number of bin

$Nbl := length(hlbin)$

$Nbl = 85$

Probability density

$Nhlfreq := length(hlfreq)$

$$hlfrel := \frac{hlfreq}{\sum(hlfreq)}$$

$$hlpdf := \frac{hlfreq}{\sum(hlfreq \cdot binl)}$$

Probability of exceedance

$HE := 1 - hlcdf$

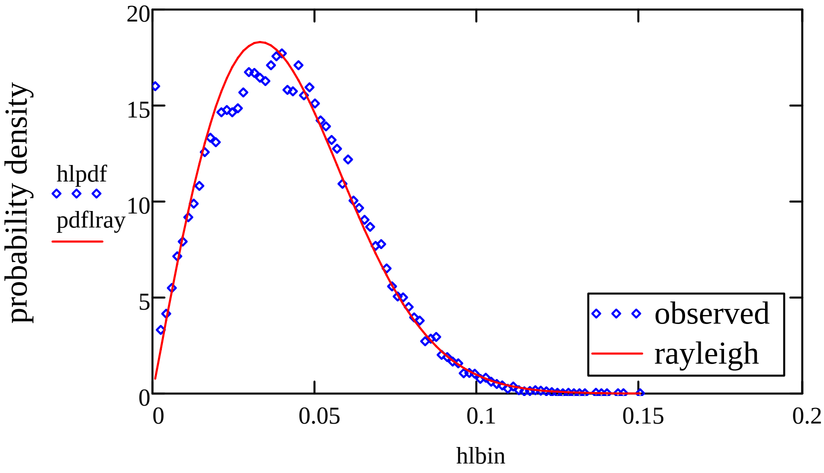
RAYLEIGHT DISTRIBUTION

$i := 0 .. (\text{length}(hlbin) - 1)$

Rayleigh Distribution

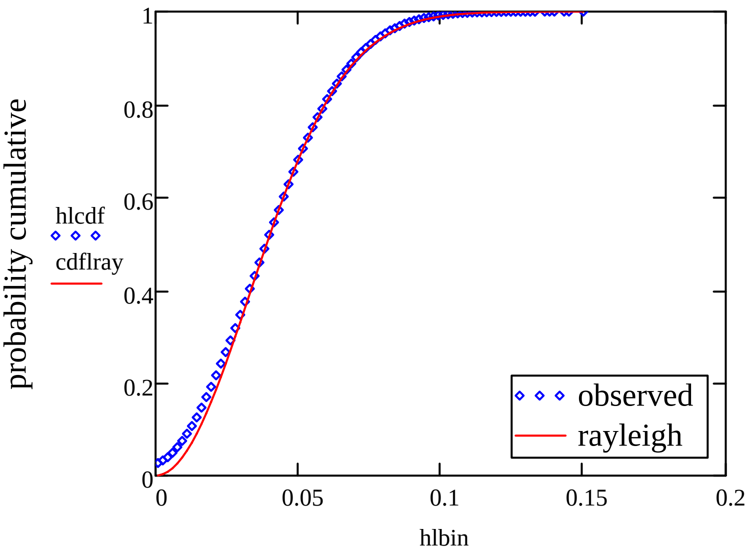
$$pdflray_i := \left[\frac{(2 \cdot hlbin_i)}{hlrms^2}\right] \cdot \exp\left[\frac{-(hlbin_i)^2}{hlrms^2}\right] \qquad cdflray_i := 1 - \exp\left[\frac{-(hlbin_i)^2}{hlrms^2}\right]$$

PDF of Wave Height



wave height

CDF of Wave Height



wave height

EMPIRICAL WAVE HEIGHT DISTRIBUTION [COMPARISON]

Rayleigh

$$Phr1 := \exp\left[-\left(\frac{hl}{0.707 \cdot hs}\right)^2\right]$$

$$Phr2 := \exp\left[-\left(\frac{hl}{0.638 \cdot hs}\right)^2\right]$$

Forristall

$$Phf := \exp\left[-\left(\frac{hl}{0.681 \cdot hs}\right)^{2.126}\right]$$

Krogstad

$$Phk1 := \exp\left[-\left(\frac{hl}{0.73 \cdot hs}\right)^{2.37}\right]$$

$$Phk2 := \exp\left[-\left(\frac{hl}{0.75 \cdot hs}\right)^{2.5}\right]$$

Haring

$$Phh1 := 0.968 + 0.176 \cdot \frac{hl}{hs} \qquad Phh2 := \overrightarrow{\left[-2\left(\frac{hl}{hs}\right)^2 \cdot Phh1\right]}$$

$$Phh := \exp(Phh2)$$

Rayleigh Stokes

$$an := 4 \cdot \sqrt{mo} \cdot \sqrt{\left(\frac{\log(Nhl)}{8}\right)}$$

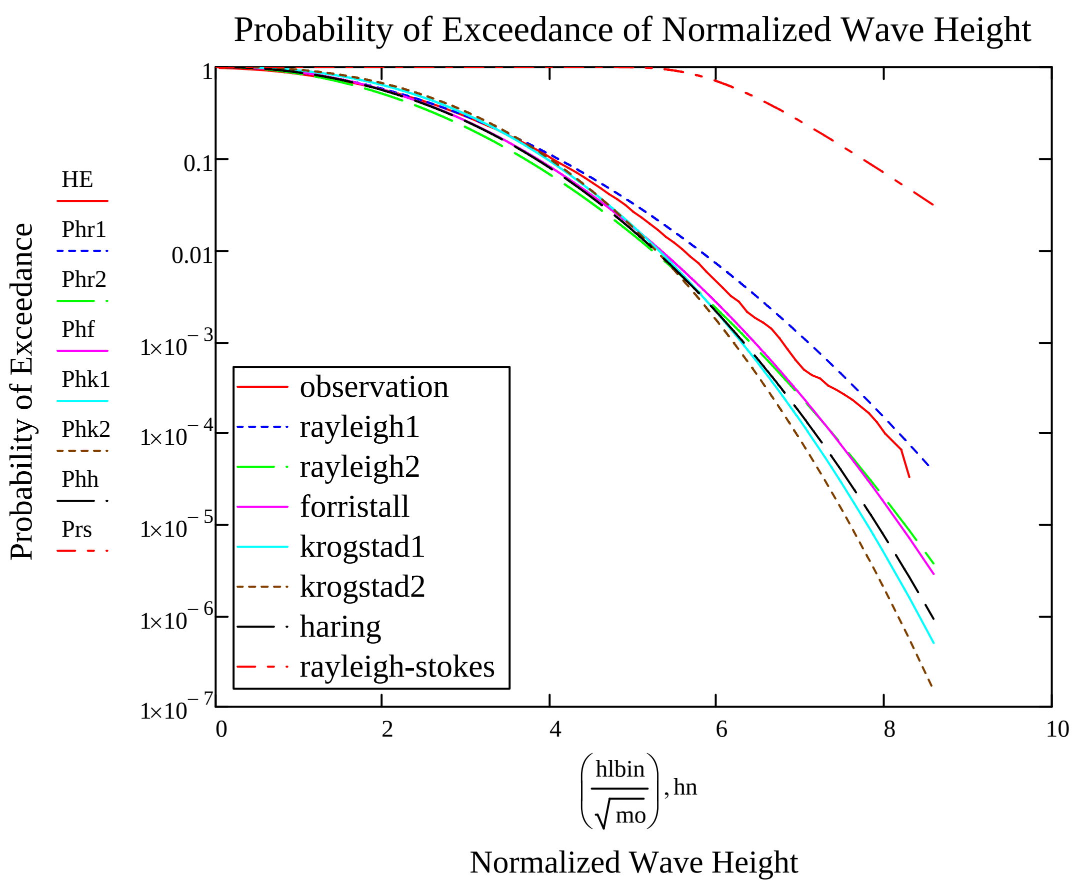
$$bn := \frac{an}{2 \cdot \log(Nhl)}$$

$$B2 := \frac{1}{2} \qquad B3 := \frac{3}{8}$$

$$ahn := 2 \cdot an \cdot \left[1 + B3 \cdot (k \cdot an)^2\right]$$

$$bhn := 2 \cdot bn \cdot \left[1 + 3 \cdot B3 \cdot (k \cdot an)^2\right]$$

$$Prs := 1 - \exp\left[-\exp\left[-\left(\frac{hl - ahn}{bhn}\right)\right]\right]$$



ZERO CROSSING CREST HEIGHT

Zero crossing crest height in meter (clr)

clr :=

	0
0	0.00033
1	$9.64121 \cdot 10^{-5}$
2	0.00045
3	0.00022
4	$9.64121 \cdot 10^{-5}$
5	0.00033
6	0.00022
7	$9.64121 \cdot 10^{-5}$
8	0.00069
9	...

Sorted crest — cl := sort(clr)

Number of crest — Ncl := length(cl) — Ncl = 30759

Maximum crest height — max(cl) = 0.0908

Statistical Properties

mean(cl) = 0.022

stdev(cl) = 0.0137

skew(cl) = 0.5326

kurt(cl) = 0.1723

Normalized crest height

$$x := \frac{cl}{\sqrt{mo}}$$

OBSERVED CREST HEIGHT DISTRIBUTION

Crest height bin

clbin :=

	0
0	$5.107\cdot10^{-4}$
1	$1.532\cdot10^{-3}$
2	$2.554\cdot10^{-3}$
3	$3.575\cdot10^{-3}$
4	$4.596\cdot10^{-3}$
5	$5.618\cdot10^{-3}$
6	$6.639\cdot10^{-3}$
7	$7.661\cdot10^{-3}$
8	$8.682\cdot10^{-3}$
9	...

Crest height frequency

clfreq :=

	0
0	$1.435\cdot10^{3}$
1	587
2	505
3	534
4	614
5	509
6	614
7	585
8	670
9	...

Cumulative frequency

clfcum :=

	0
0	0.047
1	0.066
2	0.082
3	0.1
4	0.119
5	0.136
6	0.156
7	0.175
8	0.197
9	...

Bin interval

$lbin := 2\cdot clbin_0$

$lbin = 0.001$

Number of bin

$Ncbl := length(clbin)$

$Ncbl = 86$

Probability density

$$clfrel := \frac{clfreq}{\sum clfreq}$$

$$clpdf := \frac{clfreq\cdot\sqrt{mo}}{\sum(clfreq\cdot lbin)}$$

Probability of exceedance

$CE := 1 - clfcum$

EMPIRICAL CREST HEIGHT DISTRIBUTION [COMPARISON]

Rayleigh

$$Pr := \exp\left(-8\,\frac{cl^2}{hs^2}\right)$$

Ochi

$E0 := 0.1 \qquad \alpha0 := \sqrt{\left(1 - E0^2\right)}$

$E1 := 1 \qquad \alpha1 := \sqrt{\left(1 - E1^2\right)}$

probability density function of E=1

$$pa := \exp\left(\frac{-x^2}{2}\right)$$

$$pb1 := \overrightarrow{(\alpha1\cdot x\cdot pa)}$$

$$pc1 := 0.5 + 0.5\cdot erf\left(\frac{\alpha1}{E1}\cdot\frac{x}{\sqrt{2}}\right)$$

$$pd1 := \overrightarrow{(pb1\cdot pc1)}$$

$$p1 := \frac{2}{(1+\alpha 1)} \cdot \left(\frac{E1}{\sqrt{2\pi}} \cdot \exp\left(\frac{-x^2}{2E1^2} \right) + pd1 \right)$$

cumulative distribution function E=1

$$pe1 := \alpha 1 \cdot \exp\left(\frac{-x^2}{2} \right)$$

$$pf1 := 0.5 + 0.5\,\mathrm{erf}\left(\frac{\alpha 1}{E1} \cdot \frac{x}{\sqrt{2}} \right)$$

$$pg1 := \overrightarrow{(pe1 \cdot pf1)}$$

$$P1 := \frac{2}{(1+\alpha 1)} \cdot \left[\frac{-1}{2}(1-\alpha 1) + \left(0.5 + 0.5\,\mathrm{erf}\left(\frac{x}{E1 \cdot \sqrt{2}} \right) \right) - pg1 \right]$$

$$PE1 := 1 - P1$$

probability density function E=0

$$pb0 := \overrightarrow{(\alpha 0 \cdot x \cdot pa)}$$

$$pc0 := 0.5 + 0.5 \cdot \mathrm{erf}\left(\frac{\alpha 0}{E0} \cdot \frac{x}{\sqrt{2}} \right)$$

$$pd0 := \overrightarrow{(pb0 \cdot pc0)}$$

$$p0 := \frac{2}{(1+\alpha 0)} \cdot \left(\frac{E0}{\sqrt{2\pi}} \cdot \exp\left(\frac{-x^2}{2E0^2} \right) + pd0 \right)$$

cumulative distribution function E=0

$$pe0 := \alpha 0 \cdot \exp\left(\frac{-x^2}{2} \right)$$

$$pf0 := 0.5 + 0.5\,\mathrm{erf}\left(\frac{\alpha 0}{E0} \cdot \frac{x}{\sqrt{2}} \right)$$

$$pg0 := \overrightarrow{(pe0 \cdot pf0)}$$

$$P0 := \frac{2}{(1+\alpha 0)} \cdot \left[\frac{-1}{2}(1-\alpha 0) + \left(0.5 + 0.5\,\mathrm{erf}\left(\frac{x}{E0 \cdot \sqrt{2}} \right) \right) - pg0 \right]$$

$$PE0 := 1 - P0$$

Haring

$$Pha := \overrightarrow{\left[4.37 \cdot \left(\frac{cl}{d} \right) \cdot \left(0.57 - \frac{cl}{d} \right) \right]} \qquad Phb := \overrightarrow{\left[-\left(\frac{1}{2} \right) \cdot \left(\frac{cl^2}{mo} \right) \cdot (1 - Pha) \right]}$$

$$Ph := \exp(Phb)$$

Tayfun and Huang

$$RR := k \cdot hs \qquad RR = 0.3066$$

$$Pth := \exp\left[\frac{-8}{RR^2}\left(\sqrt{1 + 2\cdot RR\cdot\frac{cl}{hs}} - 1\right)^2\right]$$

Kriebel and Dawson

$$Pkda := \left(1 - \frac{1}{2}RR\cdot\frac{cl}{hs}\right)^2 \qquad Pkdb := -8\cdot\frac{cl^2}{hs^2}$$

$$Pkdc := \overrightarrow{(Pkda\cdot Pkdb)}$$

$$Pkd := \exp(Pkdc)$$

Forristall

$$Sl := \left(\frac{2\pi\, hs}{9.81Twm}\right) \qquad Sl = 0.0456$$

$$Ur := \frac{hs}{k^2\, d^3} \qquad Ur = 0.0091$$

Two - dimensional

$$a2 := 0.3536 + 0.2892Sl + 0.1060Ur \qquad a2 = 0.3677$$

$$b2 := 2 - 2.1597Sl + 0.0968Ur^2 \qquad b2 = 1.9016$$

$$Pf2 := \exp\left[-\left(\frac{cl}{a2\cdot hs}\right)^{b2}\right]$$

Three - dimensional

$$a3 := 0.3536 + 0.2568Sl + 0.0800Ur \qquad a3 = 0.366$$

$$b3 := 2 - 1.7912Sl - 0.5302Ur + 0.284Ur^2 \qquad b3 = 1.9135$$

$$Pf3 := \exp\left[-\left(\frac{cl}{a3\cdot hs}\right)^{b3}\right]$$

Prevosto

$ss := 1$ unidirectional wave

$$ahs := 1 - \left(\frac{1}{2}\right)\cdot(\tanh(k\cdot d) - 0.9)\cdot\sqrt{\frac{2}{1 + ss}} \qquad ahs = 0.9516$$ Directional Factor

$$afm := \frac{1}{1.23} \qquad afm = 0.813$$ Spectral Bandwith

$$T02 := \frac{Twm}{1.2} \qquad fm := \frac{1}{T02} \qquad fm = 1.2847$$

modified significant wave height hs

$$hsp := ahs\cdot hs \qquad hsp = 0.0633$$

modified mean frequency fm

$$fmp := afm\cdot fm \qquad fmp = 1.0444$$

modified wave number (dispersion relation)

$$kmp := \frac{(2\cdot\pi\cdot fm)^2}{9.81}$$ $$kmp = 6.6417$$ cek

$$\tanh(kmp\cdot d) = 0.9998$$

dimensionless depth

$$kap := kmp\cdot d$$ $$kap = 4.6492$$

$$PI := \tanh(kap) + kap\cdot\left[1 - (\tanh(kap))^2\right]$$ $$PI = 1.0015$$

Second order coefficients

$$cdiff := \frac{\left[PI + kap\cdot\left[1 - (\tanh(kap))^2\right]\right]}{\left(PI^2\right) - 4\cdot kap\cdot\tanh(kap)}$$ $$cdiff = -0.057$$

$$csum := \left(\frac{1}{4}\right)\cdot\frac{\left[2 + \left[1 - (\tanh(kap))^2\right]\right]}{(\tanh(kap))^3}$$ $$csum = 0.5004$$

Second order transfer functions

$$TD := cdiff\cdot k$$ $$TD = -0.2631$$

$$TS := csum\cdot k$$ $$TS = 2.3078$$

Non-linear crest components

$$C0 := TD\cdot\frac{hsp^2}{8}$$ $$C0 = -0.0001$$

$$C1 := ahs$$ $$C1 = 0.9516$$

$$C2 := C1^2\cdot(TD + TS)$$ $$C2 = 1.8515$$

$$Cr := cl + (TD + TS)\cdot cl^2 - TD\cdot\frac{hs^2}{8}$$

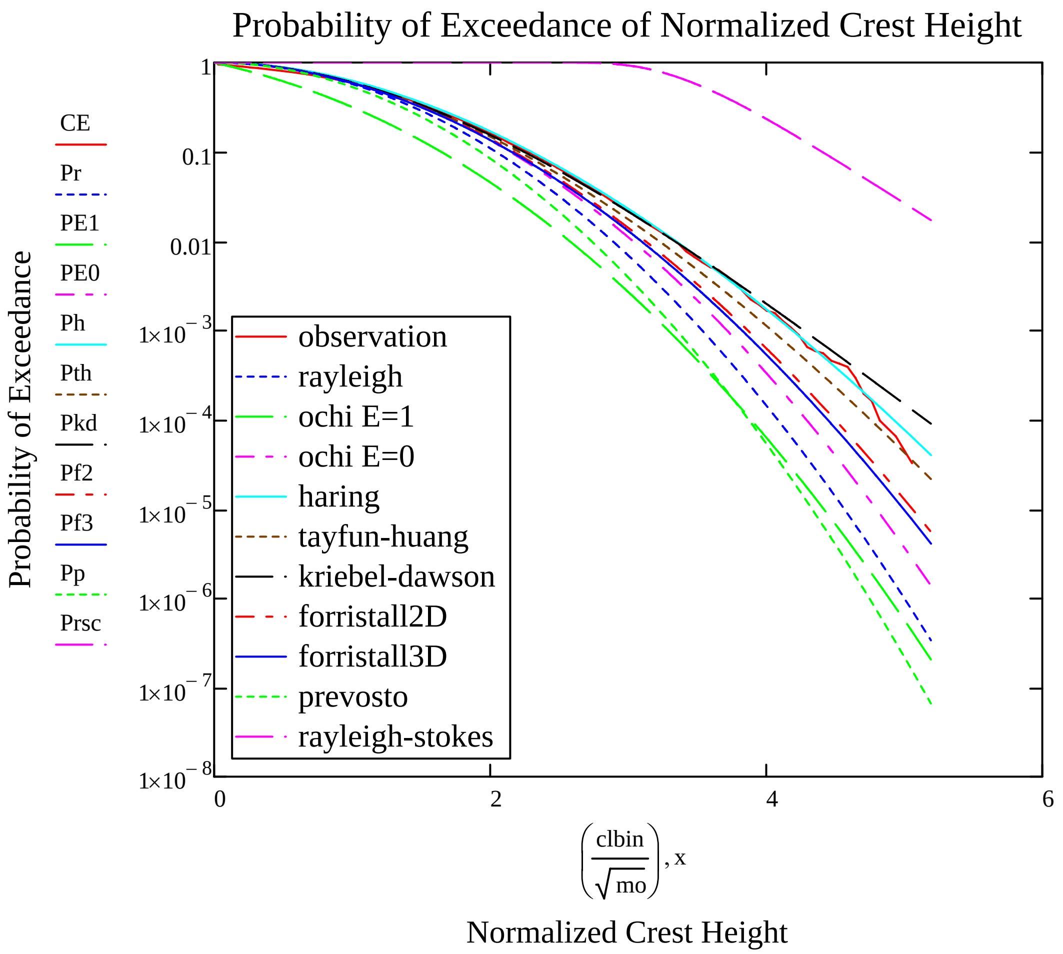
$$Pp := \exp\left[\frac{-8}{hs^2}\cdot\left[\frac{\left[-C1 + \sqrt{C1^2 - 4\cdot[C2\cdot(C0 - Cr)]}\right]}{2C2}\right]^2\right]$$

Rayleigh Stokes

$$acn := an\cdot\left[1 + B2\cdot(k\cdot an) + B3\cdot(k\cdot an)^2\right]$$

$$bcn := bn\cdot\left[1 + 2\cdot B2\cdot(k\cdot an) + 3\cdot B3\cdot(k\cdot an)^2\right]$$

$$Prsc := 1 - \exp\left[-\exp\left[\frac{-(cl - acn)}{bcn}\right]\right]$$



Probability of Exceedance of Normalized Crest Height
Probability of Exceedance
CE
Pr
PE1
PE0
Ph
Pth
Pkd
Pf2
Pf3
Pp
Prsc
1
0.1
0.01
1×10⁻³
1×10⁻⁴
1×10⁻⁵
1×10⁻⁶
1×10⁻⁷
1×10⁻⁸
0
2
4
6
observation
rayleigh
ochi E=1
ochi E=0
haring
tayfun-huang
kriebel-dawson
forristall2D
forristall3D
prevosto
rayleigh-stokes
(clbin/√mo), x
Normalized Crest Height

LABORATORY SIMULATION 100A

Variance of water surface elevation

$mo := 0.00072767$

Water depth in meter

$d := 0.7$

ZERO CROSSING WAVE PERIOD

Tw :=

	0
0	4.64
1	1.56
2	1.57
3	1.57
4	1
5	...

$Twm := mean(Tw)$

$Twm = 1.1915$

dispersion relation of deep water condition

$k := 4 \cdot \frac{\pi^2}{Twm^2 9.81}$ $\quad k = 2.8347$

ZERO CROSSING WAVE HEIGHT

Zero crossing wave height in meter (hlr)

hlr :=

	0
0	$7.5 \cdot 10^{-3}$
1	0.056
2	0.129
3	0.079
4	0.015
5	0.118
6	0.062
7	0.054
8	0.01
9	...

$hl := sort(hlr)$ $\quad hl2 := hl^2$

Number of wave

$Nhl := length(hl)$ $\quad Nhl = 18857$

Statistical properties

$mean(hl) = 0.0662$

$stdev(hl) = 0.034$

$skew(hl) = 0.441$

$kurt(hl) = -0.0188$

Root mean square wave height

$$hlrms := \sqrt{\left[\frac{1}{Nhl} \cdot \left(\sum hl2\right)\right]}$$

$hlrms = 0.0745$

Normalized wave height

$$hn := \frac{hl}{\sqrt{mo}}$$

Significant wave height

$Nhs := round\left[\left(\frac{2}{3}\right) \cdot Nhl\right]$ $\quad Nhs = 12571$

$i := (Nhs - 1) .. (Nhl - 1)$

$hl_{(Nhs-1)} = 0.0794$ $\quad hl_{(Nhl-1)} = 0.2167$

$hl_i =$

0.0794
0.0794
0.0794
0.0794
0.0794
...

$$sumhs := \sum_{i = Nhs-1}^{(Nhl-1)} hl_i$$ $sumhs = 658.5759$

$$hs := \frac{sumhs}{(Nhl - Nhs + 1)}$$ $hs = 0.1048$

comparison with theoritical significant wave height

$$hss := 4 \cdot \sqrt{mo}$$ $hss = 0.1079$

Maximum wave height

$hlm := max(hl)$

$hlm = 0.2167$

comparison with theoritical Rayleigh formula of maximum wave height

$$hmax := \sqrt{\frac{\ln(Nhl)}{2}} \cdot hs$$

$hmax = 0.2324$

OBSERVED WAVE HEIGHT DISTRIBUTION

Wave height bin

hlbin :=

	0
0	$1.527 \cdot 10^{-3}$
1	$4.581 \cdot 10^{-3}$
2	$7.636 \cdot 10^{-3}$
3	0.011
4	0.014
5	0.017
6	0.02
7	0.023
8	0.026
9	...

Wave height frequency

hlfreq :=

	0
0	203
1	120
2	154
3	235
4	250
5	323
6	343
7	429
8	409
9	...

Cumulative frequency

hlcdf :=

	0
0	0.011
1	0.017
2	0.025
3	0.038
4	0.051
5	0.068
6	0.086
7	0.109
8	0.131
9	...

Bin interval

$binl := 2 \cdot hlbin_0$

$binl = 0.0031$

$Nbl := length(hlbin)$

$Nbl = 70$

Probability density

$Nhlfreq := length(hlfreq)$

$$hlfrel := \frac{hlfreq}{\sum(hlfreq)}$$

$$hlpdf := \frac{hlfreq}{\sum(hlfreq \cdot binl)}$$

Probability of exceedance

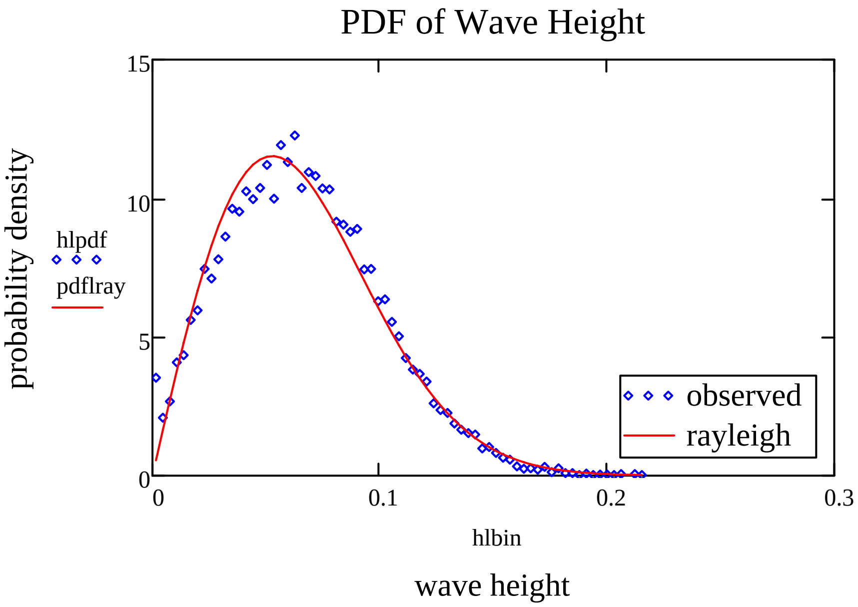
$Nlcdf := length(hlcdf)$

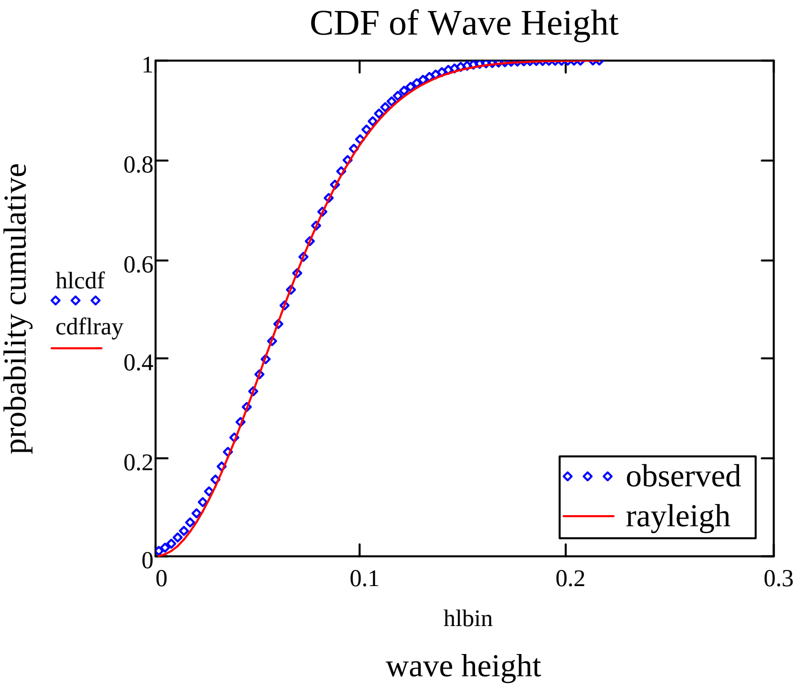
$HE := 1 - hlcdf$

$i := 0 .. (\text{length}(hlbin) - 1)$

Rayleigh Distribution

$$pdflray_i := \left[\frac{(2 \cdot hlbin_i)}{hlrms^2}\right] \cdot \exp\left[\frac{-(hlbin_i)^2}{hlrms^2}\right] \qquad cdflray_i := 1 - \exp\left[\frac{-(hlbin_i)^2}{hlrms^2}\right]$$





EMPIRICAL WAVE HEIGHT DISTRIBUTION [COMPARISON]

Rayleigh

$$Phr1 := \exp\left[-\left(\frac{hl}{0.707\cdot hs}\right)^2\right]$$

$$Phr2 := \exp\left[-\left(\frac{hl}{0.638\cdot hs}\right)^2\right]$$

Forristall

$$Phf := \exp\left[-\left(\frac{hl}{0.681\cdot hs}\right)^{2.126}\right]$$

Krogstad

$$Phk1 := \exp\left[-\left(\frac{hl}{0.73\cdot hs}\right)^{2.37}\right]$$

$$Phk2 := \exp\left[-\left(\frac{hl}{0.75\cdot hs}\right)^{2.5}\right]$$

Haring

$$Phh1 := 0.968 + 0.176\cdot\frac{hl}{hs}$$

$$Phh2 := \overrightarrow{\left[-2\left(\frac{hl}{hs}\right)^2\cdot Phh1\right]}$$

$$Phh := \exp(Phh2)$$

Rayleigh Stokes

$$an := 4\cdot\sqrt{mo}\cdot\sqrt{\left(\frac{\log(Nhl)}{8}\right)}$$

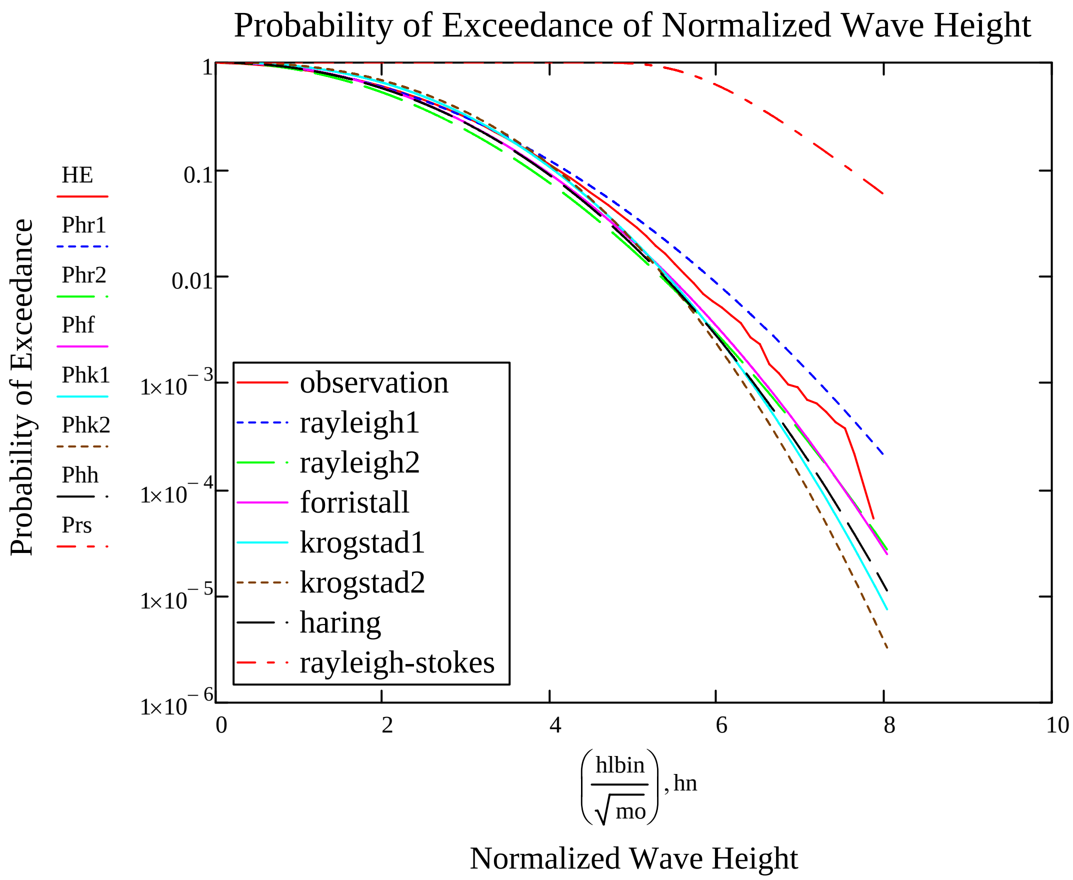
$$bn := \frac{an}{2\cdot\log(Nhl)}$$

$$B2 := \frac{1}{2} \qquad B3 := \frac{3}{8}$$

$$ahn := 2\cdot an\cdot\left[1 + B3\cdot(k\cdot an)^2\right]$$

$$bhn := 2\cdot bn\cdot\left[1 + 3\cdot B3\cdot(k\cdot an)^2\right]$$

$$Prs := 1 - \exp\left[-\exp\left[-\left(\frac{hl - ahn}{bhn}\right)\right]\right]$$



ZERO CROSSING CREST HEIGHT

Zero crossing crest height in meter (clr)

clr :=

	0
0	0.00334
1	0.03694
2	0.07664
3	0.01794
4	0.00964
5	0.06984
6	0.01284
7	0.02604
8	...

Sorted crest $cl := sort(clr)$

Number of crest $Ncl := length(cl)$ $Ncl = 18857$

Maximum crest height $max(cl) = 0.1399$

Statistical Properties

$mean(cl) = 0.0361$

$stdev(cl) = 0.021$

$skew(cl) = 0.5597$

$kurt(cl) = 0.2356$

Normalized crest height

$$x := \frac{cl}{\sqrt{mo}}$$

OBSERVED CREST HEIGHT DISTRIBUTION

Crest height bin

clbin :=

	0
0	$9.619\cdot10^{-4}$
1	$2.886\cdot10^{-3}$
2	$4.809\cdot10^{-3}$
3	$6.733\cdot10^{-3}$
4	$8.657\cdot10^{-3}$
5	0.011
6	0.013
7	0.014
8	0.016
9	...

Crest height frequency

clfreq :=

	0
0	577
1	374
2	324
3	311
4	348
5	358
6	434
7	447
8	491
9	...

Cumulative frequency

clfcum :=

	0
0	0.031
1	0.05
2	0.068
3	0.084
4	0.103
5	0.122
6	0.145
7	0.168
8	0.194
9	...

Bin interval

$lbin := 2\cdot clbin_0$

$lbin = 0.0019$

Number of bin

$Ncbl := \mathrm{length}(clbin)$

$Ncbl = 67$

Probability density

$$clfrel := \frac{clfreq}{\sum clfreq}$$

$$clpdf := \frac{clfreq\cdot\sqrt{mo}}{\sum (clfreq\cdot lbin)}$$

Probability of exceedance

$CE := 1 - clfcum$

EMPIRICAL CREST HEIGHT DISTRIBUTION [COMPARISON]

Rayleigh

$$Pr := \exp\left(-8\frac{cl^2}{hs^2}\right)$$

Ochi

$E0 := 0.1 \qquad \alpha0 := \sqrt{\left(1 - E0^2\right)}$

$E1 := 1 \qquad \alpha1 := \sqrt{\left(1 - E1^2\right)}$

probability density function of E=1

$$pa := \exp\left(\frac{-x^2}{2}\right)$$

$$pb1 := \overrightarrow{(\alpha1\cdot x\cdot pa)}$$

$$pc1 := 0.5 + 0.5\cdot \mathrm{erf}\left(\frac{\alpha1}{E1}\cdot\frac{x}{\sqrt{2}}\right)$$

$$pd1 := \overrightarrow{(pb1\cdot pc1)}$$

$$p1 := \frac{2}{(1+\alpha1)}\cdot\left(\frac{E1}{\sqrt{2\pi}}\cdot\exp\left(\frac{-x^2}{2E1^2}\right)+pd1\right)$$

cumulative distribution function E=1

$$pe1 := \alpha1\cdot\exp\left(\frac{-x^2}{2}\right)$$

$$pf1 := 0.5 + 0.5\,\mathrm{erf}\left(\frac{\alpha1}{E1}\cdot\frac{x}{\sqrt{2}}\right)$$

$$pg1 := \overrightarrow{(pe1\cdot pf1)}$$

$$P1 := \frac{2}{(1+\alpha1)}\cdot\left[\frac{-1}{2}(1-\alpha1)+\left(0.5+0.5\,\mathrm{erf}\left(\frac{x}{E1\cdot\sqrt{2}}\right)\right)-pg1\right]$$

$$PE1 := 1 - P1$$

probability density function E=0

$$pb0 := \overrightarrow{(\alpha0\cdot x\cdot pa)}$$

$$pc0 := 0.5 + 0.5\cdot\mathrm{erf}\left(\frac{\alpha0}{E0}\cdot\frac{x}{\sqrt{2}}\right)$$

$$pd0 := \overrightarrow{(pb0\cdot pc0)}$$

$$p0 := \frac{2}{(1+\alpha0)}\cdot\left(\frac{E0}{\sqrt{2\pi}}\cdot\exp\left(\frac{-x^2}{2E0^2}\right)+pd0\right)$$

cumulative distribution function E=0

$$pe0 := \alpha0\cdot\exp\left(\frac{-x^2}{2}\right)$$

$$pf0 := 0.5 + 0.5\,\mathrm{erf}\left(\frac{\alpha0}{E0}\cdot\frac{x}{\sqrt{2}}\right)$$

$$pg0 := \overrightarrow{(pe0\cdot pf0)}$$

$$P0 := \frac{2}{(1+\alpha0)}\cdot\left[\frac{-1}{2}(1-\alpha0)+\left(0.5+0.5\,\mathrm{erf}\left(\frac{x}{E0\cdot\sqrt{2}}\right)\right)-pg0\right]$$

$$PE0 := 1 - P0$$

Haring

$$Pha := \overrightarrow{\left[4.37\cdot\left(\frac{cl}{d}\right)\cdot\left(0.57-\frac{cl}{d}\right)\right]} \qquad Phb := \overrightarrow{\left[-\left(\frac{1}{2}\right)\cdot\left(\frac{cl^2}{mo}\right)\cdot(1-Pha)\right]}$$

$$Ph := \exp(Phb)$$

Tayfun and Huang

$RR := k \cdot hs \qquad RR = 0.2969$

$$Pth := \exp\left[\frac{-8}{RR^2}\left(\sqrt{1 + 2 \cdot RR \cdot \frac{cl}{hs}} - 1\right)^2\right]$$

Kriebel and Dawson

$$Pkda := \left(1 - \frac{1}{2} RR \cdot \frac{cl}{hs}\right)^2$$

$$Pkdb := -8 \cdot \frac{cl^2}{hs^2}$$

$$Pkdc := \overrightarrow{(Pkda \cdot Pkdb)}$$

$$Pkd := \exp(Pkdc)$$

Forristall

$Sl := \left(\frac{2\pi\, hs}{9.81 Twm}\right) \qquad Sl = 0.0563$

$Ur := \frac{hs}{k^2 d^3} \qquad Ur = 0.038$

Two - dimensional

$a2 := 0.3536 + 0.2892 Sl + 0.1060 Ur \qquad a2 = 0.3739$

$b2 := 2 - 2.1597 Sl + 0.0968 Ur^2 \qquad b2 = 1.8785$

$$Pf2 := \exp\left[-\left(\frac{cl}{a2 \cdot hs}\right)^{b2}\right]$$

Three - dimensional

$a3 := 0.3536 + 0.2568 Sl + 0.0800 Ur \qquad a3 = 0.3711$

$b3 := 2 - 1.7912 Sl - 0.5302 Ur + 0.284 Ur^2 \qquad b3 = 1.8794$

$$Pf3 := \exp\left[-\left(\frac{cl}{a3 \cdot hs}\right)^{b3}\right]$$

Prevosto

$ss := 1$ unidirectional wave

$ahs := 1 - \left(\frac{1}{2}\right) \cdot (\tanh(k \cdot d) - 0.9) \cdot \sqrt{\frac{2}{1 + ss}} \qquad ahs = 0.9686$ Directional Factor

$afm := \frac{1}{1.23} \qquad afm = 0.813$ Spectral Bandwith

$T02 := \frac{Twm}{1.2} \qquad fm := \frac{1}{T02} \qquad fm = 1.0071$

modified significant wave height hs

$hsp := ahs \cdot hs$ $\quad hsp = 0.1015$

modified mean frequency fm

$fmp := afm \cdot fm$ $\quad fmp = 0.8188$

modified wave number (dispersion relation)

$$kmp := \frac{(2 \cdot \pi \cdot fm)^2}{9.81} \qquad kmp = 4.0819$$

cek

$\tanh(kmp \cdot d) = 0.9934$

dimensionless depth

$kap := kmp \cdot d$ $\quad kap = 2.8573$

$$PI := \tanh(kap) + kap \cdot \left[1 - (\tanh(kap))^2\right] \qquad PI = 1.0309$$

Second order coefficients

$$cdiff := \frac{\left[PI + kap \cdot \left[1 - (\tanh(kap))^2\right]\right]}{\left(PI^2\right) - 4 \cdot kap \cdot \tanh(kap)} \qquad cdiff = -0.1038$$

$$csum := \left(\frac{1}{4}\right) \cdot \frac{\left[2 + \left[1 - (\tanh(kap))^2\right]\right]}{(\tanh(kap))^3} \qquad csum = 0.5133$$

Second order transfer functions

$TD := cdiff \cdot k$ $\quad TD = -0.2942$

$TS := csum \cdot k$ $\quad TS = 1.4551$

Non-linear crest components

$$C0 := TD \cdot \frac{hsp^2}{8} \qquad C0 = -0.0004$$

$C1 := ahs$ $\quad C1 = 0.9686$

$C2 := C1^2 \cdot (TD + TS) \cdot kmp$ $\quad C2 = 4.4452$

$$Cr := cl + (TD + TS) \cdot cl^2 - TD \cdot \frac{hs^2}{8}$$

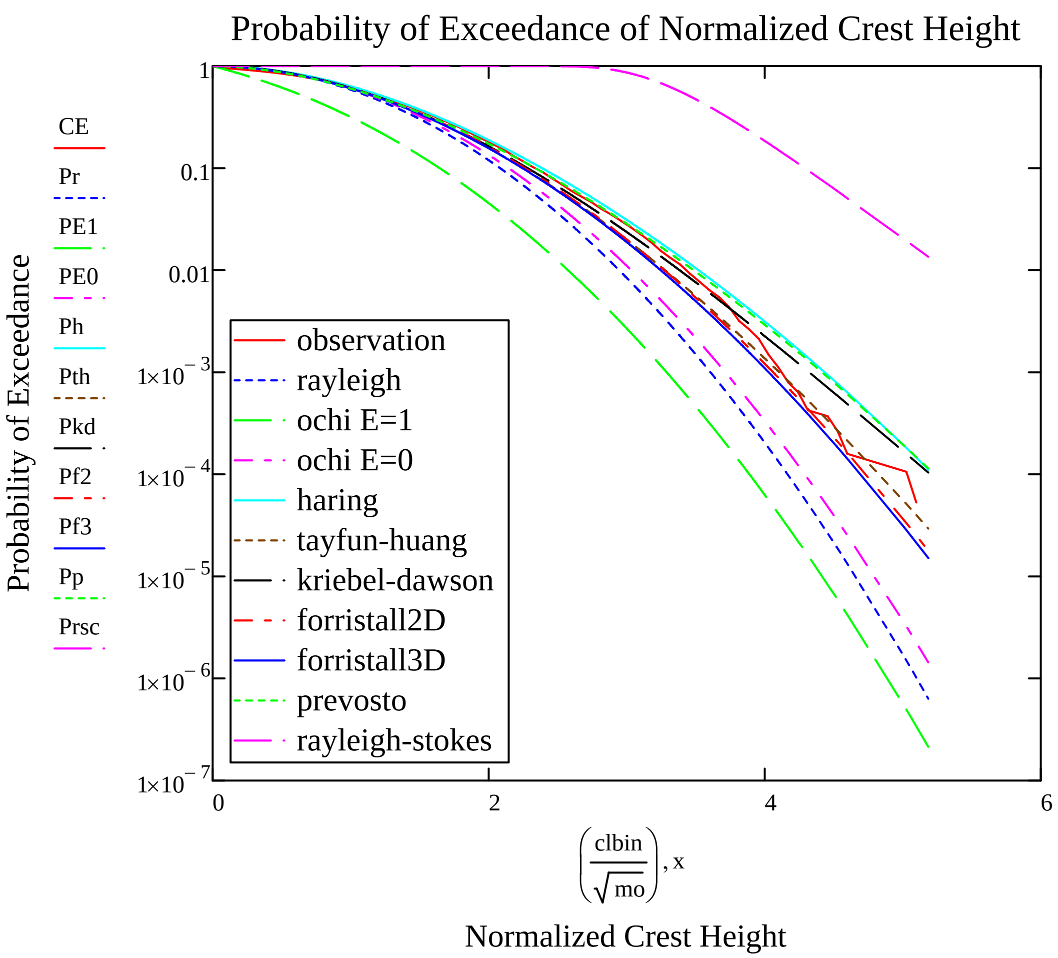
$$Pp := \exp\left[\frac{-8}{hs^2} \cdot \left[\frac{\left[-C1 + \sqrt{C1^2 - 4 \cdot [C2 \cdot (C0 - Cr)]}\right]}{2C2}\right]^2\right]$$

Rayleigh Stokes

$$acn := an \cdot \left[1 + B2 \cdot (k \cdot an) + B3 \cdot (k \cdot an)^2\right]$$

$$bcn := bn \cdot \left[1 + 2 \cdot B2 \cdot (k \cdot an) + 3 \cdot B3 \cdot (k \cdot an)^2\right]$$

$$Prsc := 1 - \exp\left[-\exp\left[\frac{-(cl - acn)}{bcn}\right]\right]$$



Probability of Exceedance of Normalized Crest Height
Probability of Exceedance
CE
Pr
PE1
PE0
Ph
Pth
Pkd
Pf2
Pf3
Pp
Prsc
1
0.1
0.01
1×10^−3
1×10^−4
1×10^−5
1×10^−6
1×10^−7
0
2
4
6
observation
rayleigh
ochi E=1
ochi E=0
haring
tayfun-huang
kriebel-dawson
forristall2D
forristall3D
prevosto
rayleigh-stokes
(clbin / √mo), x
Normalized Crest Height

LABORATORY SIMULATION 100B

Variance of water surface elevation

$mo := 0.00057124$

Water depth in meter

$d := 0.7$

ZERO CROSSING WAVE PERIOD

Tw :=

	0
0	8.28
1	1.65
2	1.62
3	1.14
4	1.31
5	...

$Twm := mean(Tw)$

$Twm = 1.1923$

dispersion relation of deep water condition

$k := 4 \cdot \frac{\pi^2}{Twm^2 9.81}$ $\qquad k = 2.8309$

ZERO CROSSING WAVE HEIGHT

Zero crossing wave height in meter (hlr)

hlr :=

	0
0	0.018
1	0.08
2	0.103
3	0.052
4	0.058
5	0.06
6	0.068
7	...

$hl := sort(hlr)$ $\qquad hl2 := hl^2$

Number of wave

$Nhl := length(hl)$ $\qquad Nhl = 18851$

Statistical properties

$mean(hl) = 0.0581$

$stdev(hl) = 0.0308$

$skew(hl) = 0.4436$

$kurt(hl) = 0.0302$

Root mean square wave height

$$hlrms := \sqrt{\left[\frac{1}{Nhl} \cdot \left(\sum hl2\right)\right]}$$

$hlrms = 0.0658$

Normalized wave height

$$hn := \frac{hl}{\sqrt{mo}}$$

Significant wave height

$Nhs := round\left[\left(\frac{2}{3}\right) \cdot Nhl\right]$ $\qquad Nhs = 12567$

$i := (Nhs - 1) .. (Nhl - 1)$

$hl_{(Nhs-1)} = 0.0699$ $\qquad hl_{(Nhl-1)} = 0.1954$

$hl_i =$

0.0699
0.0699
0.0699
0.0699
0.0699
...

$$sumhs := \sum_{i = Nhs-1}^{(Nhl-1)} hl_i$$ $sumhs = 583.5509$

$$hs := \frac{sumhs}{(Nhl - Nhs + 1)}$$ $hs = 0.0928$

comparison with theoritical significant wave height

$$hss := 4 \cdot \sqrt{mo}$$ $hss = 0.0956$

Maximum wave height

$hlm := max(hl)$

$hlm = 0.1954$

comparison with theoritical Rayleigh formula of maximum wave height

$$hmax := \sqrt{\frac{\ln(Nhl)}{2}} \cdot hs$$

$hmax = 0.206$

OBSERVED WAVE HEIGHT DISTRIBUTION

Wave height bin

hlbin :=

	0
0	$1.371 \cdot 10^{-3}$
1	$4.112 \cdot 10^{-3}$
2	$6.853 \cdot 10^{-3}$
3	$9.595 \cdot 10^{-3}$
4	0.012
5	0.015
6	0.018
7	0.021
8	0.023
9	...

Wave height frequency

hlfreq :=

	0
0	421
1	123
2	175
3	195
4	245
5	291
6	336
7	400
8	457
9	...

Cumulative frequency

hlcdf :=

	0
0	0.022
1	0.029
2	0.038
3	0.048
4	0.061
5	0.077
6	0.095
7	0.116
8	0.14
9	...

Bin interval

$binl := 2 \cdot hlbin_0$

$binl = 0.0027$

$Nbl := length(hlbin)$

$Nbl = 70$

Probability density

$Nhlfreq := length(hlfreq)$

$$hlfrel := \frac{hlfreq}{\sum (hlfreq)}$$

$$hlpdf := \frac{hlfreq}{\sum (hlfreq \cdot binl)}$$

Probability of exceedance

$Nlcdf := length(hlcdf)$

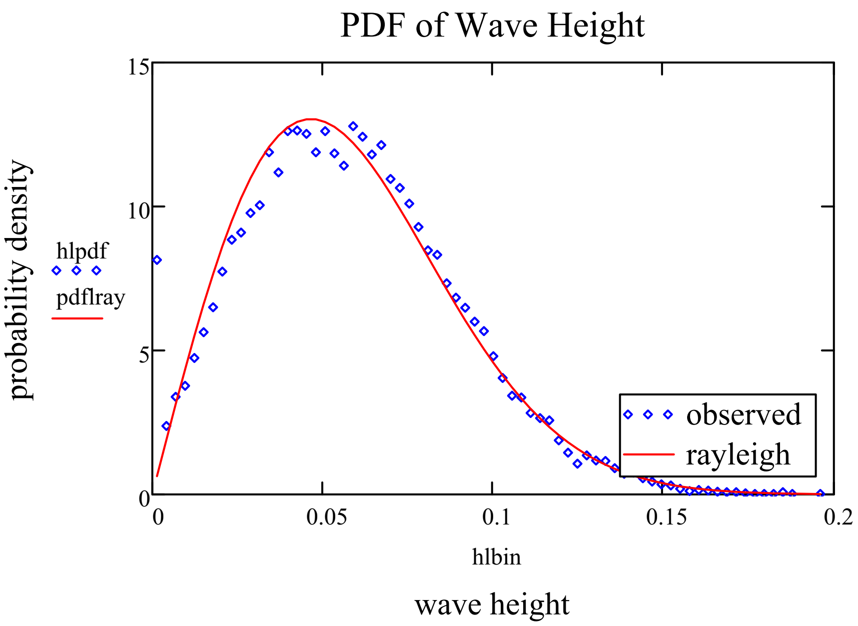
$HE := 1 - hlcdf$

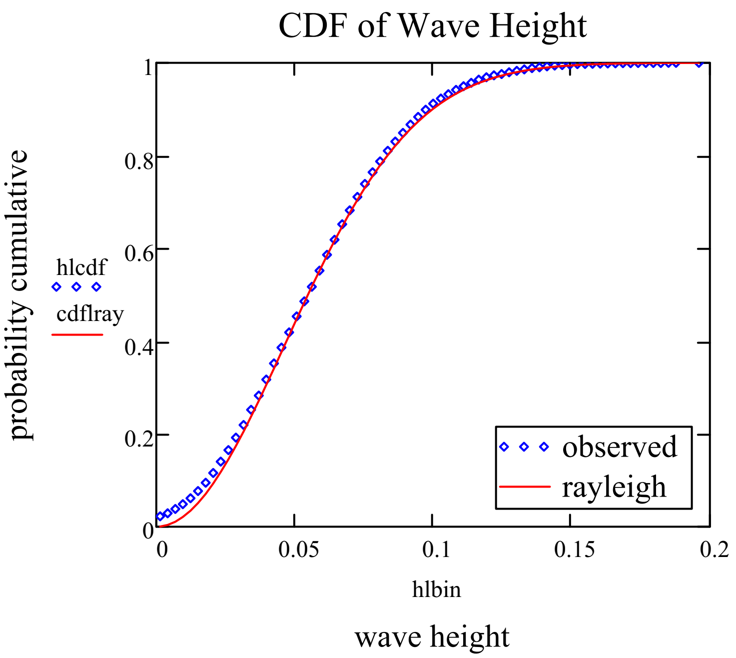
$i := 0 .. (\mathrm{length}(hlbin) - 1)$

Rayleigh Distribution

$$pdflray_i := \left[\frac{(2 \cdot hlbin_i)}{hlrms^2}\right] \cdot \exp\left[\frac{-(hlbin_i)^2}{hlrms^2}\right]$$

$$cdflray_i := 1 - \exp\left[\frac{-(hlbin_i)^2}{hlrms^2}\right]$$





EMPIRICAL WAVE HEIGHT DISTRIBUTION [COMPARISON]

Rayleigh

$$Phr1 := \exp\left[-\left(\frac{hl}{0.707 \cdot hs}\right)^2\right]$$

$$Phr2 := \exp\left[-\left(\frac{hl}{0.638 \cdot hs}\right)^2\right]$$

Forristall

$$Phf := \exp\left[-\left(\frac{hl}{0.681 \cdot hs}\right)^{2.126}\right]$$

Krogstad

$$Phk1 := \exp\left[-\left(\frac{hl}{0.73 \cdot hs}\right)^{2.37}\right]$$

$$Phk2 := \exp\left[-\left(\frac{hl}{0.75 \cdot hs}\right)^{2.5}\right]$$

Haring

$$Phh1 := 0.968 + 0.176 \cdot \frac{hl}{hs}$$

$$Phh2 := \overrightarrow{\left[-2\left(\frac{hl}{hs}\right)^2 \cdot Phh1\right]}$$

$$Phh := \exp(Phh2)$$

Rayleigh Stokes

$$an := 4 \cdot \sqrt{mo} \cdot \sqrt{\left(\frac{\log(Nhl)}{8}\right)}$$

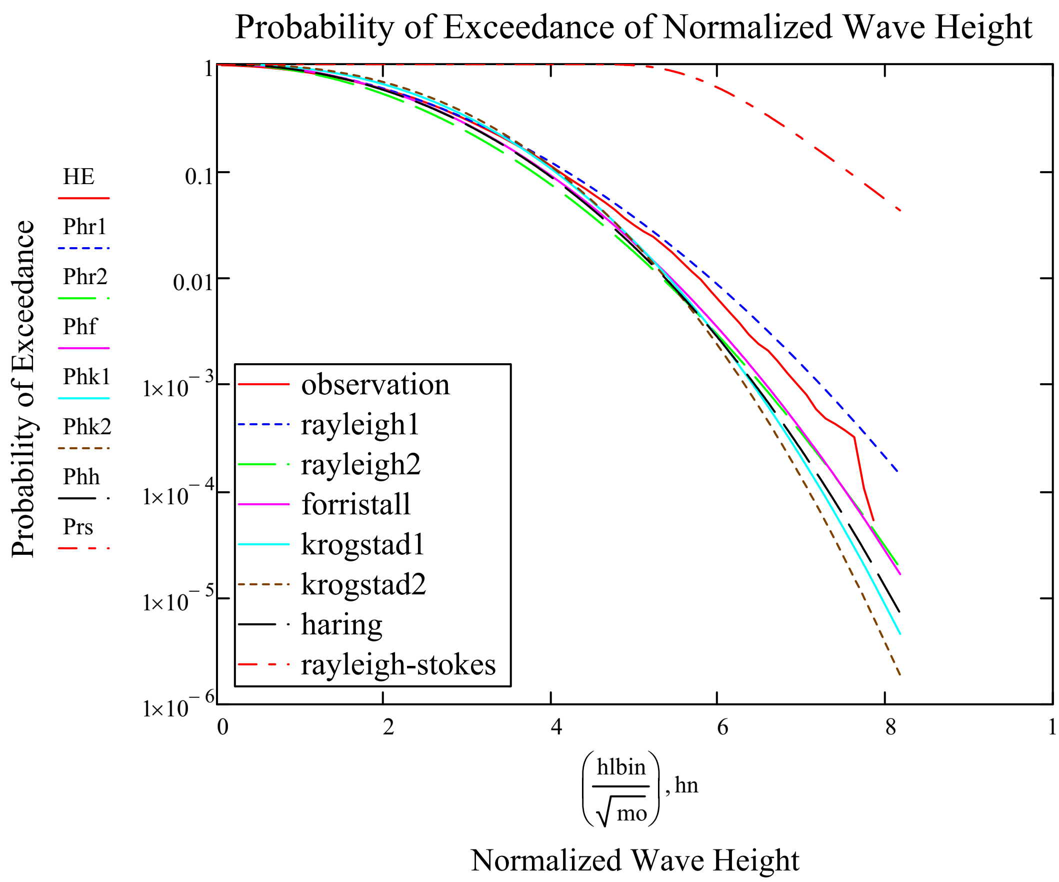
$$bn := \frac{an}{2 \cdot \log(Nhl)}$$

$$B2 := \frac{1}{2} \qquad B3 := \frac{3}{8}$$

$$ahn := 2 \cdot an \cdot \left[1 + B3 \cdot (k \cdot an)^2\right]$$

$$bhn := 2 \cdot bn \cdot \left[1 + 3 \cdot B3 \cdot (k \cdot an)^2\right]$$

$$Prs := 1 - \exp\left[-\exp\left[-\left(\frac{hl - ahn}{bhn}\right)\right]\right]$$



ZERO CROSSING CREST HEIGHT

Zero crossing crest height in meter (clr)

clr :=

	0
0	0.01395
1	0.04775
2	0.04865
3	0.02015
4	0.03075
5	0.03245
6	0.03265
7	0.01465
8	0.01575
9	...

Sorted crest $cl := sort(clr)$

Number of crest $Ncl := length(cl)$ $Ncl = 18851$

Maximum crest height $max(cl) = 0.1242$

Statistical Properties

$mean(cl) = 0.0317$

$stdev(cl) = 0.0191$

$skew(cl) = 0.6018$

$kurt(cl) = 0.3407$

Normalized crest height

$$x := \frac{cl}{\sqrt{mo}}$$

OBSERVED CREST HEIGHT DISTRIBUTION

Crest height bin

clbin :=

	0
0	$8.555\cdot10^{-4}$
1	$2.567\cdot10^{-3}$
2	$4.278\cdot10^{-3}$
3	$5.989\cdot10^{-3}$
4	$7.7\cdot10^{-3}$
5	$9.411\cdot10^{-3}$
6	0.011
7	0.013
8	0.015
9	...

Bin interval

$lbin := 2\cdot clbin_0$

$lbin = 0.0017$

Number of bin

$Ncbl := length(clbin)$

$Ncbl = 71$

Crest height frequency

clfreq :=

	0
0	780
1	323
2	334
3	320
4	359
5	382
6	413
7	437
8	492
9	...

Probability density

$$clfrel := \frac{clfreq}{\sum clfreq}$$

$$clpdf := \frac{clfreq\cdot\sqrt{mo}}{\sum(clfreq\cdot lbin)}$$

Cumulative frequency

clfcum :=

	0
0	0.041
1	0.059
2	0.076
3	0.093
4	0.112
5	0.133
6	0.154
7	0.178
8	0.204
9	...

Probability of exceedance

$CE := 1 - clfcum$

EMPIRICAL CREST HEIGHT DISTRIBUTION [COMPARISON]

Rayleigh

$$Pr := \exp\left(-8\frac{cl^2}{hs^2}\right)$$

Ochi

$E0 := 0.1 \qquad \alpha0 := \sqrt{\left(1 - E0^2\right)}$

$E1 := 1 \qquad \alpha1 := \sqrt{\left(1 - E1^2\right)}$

probability density function of E=1

$$pa := \exp\left(\frac{-x^2}{2}\right)$$

$$pb1 := \overrightarrow{(\alpha1\cdot x\cdot pa)}$$

$$pc1 := 0.5 + 0.5\cdot erf\left(\frac{\alpha1}{E1}\cdot\frac{x}{\sqrt{2}}\right)$$

$$pd1 := \overrightarrow{(pb1\cdot pc1)}$$

$$p1 := \frac{2}{(1+\alpha1)}\cdot\left(\frac{E1}{\sqrt{2\pi}}\cdot\exp\left(\frac{-x^2}{2E1^2}\right) + pd1\right)$$

cumulative distribution function E=1

$$pe1 := \alpha 1 \cdot \exp\left(\frac{-x^2}{2}\right)$$

$$pf1 := 0.5 + 0.5\,\mathrm{erf}\left(\frac{\alpha 1}{E1} \cdot \frac{x}{\sqrt{2}}\right)$$

$$pg1 := \overrightarrow{(pe1 \cdot pf1)}$$

$$P1 := \frac{2}{(1 + \alpha 1)} \cdot \left[\frac{-1}{2}(1 - \alpha 1) + \left(0.5 + 0.5\,\mathrm{erf}\left(\frac{x}{E1 \cdot \sqrt{2}}\right)\right) - pg1\right]$$

$$PE1 := 1 - P1$$

probability density function E=0

$$pb0 := \overrightarrow{(\alpha 0 \cdot x \cdot pa)}$$

$$pc0 := 0.5 + 0.5 \cdot \mathrm{erf}\left(\frac{\alpha 0}{E0} \cdot \frac{x}{\sqrt{2}}\right)$$

$$pd0 := \overrightarrow{(pb0 \cdot pc0)}$$

$$p0 := \frac{2}{(1 + \alpha 0)} \cdot \left(\frac{E0}{\sqrt{2\pi}} \cdot \exp\left(\frac{-x^2}{2E0^2}\right) + pd0\right)$$

cumulative distribution function E=0

$$pe0 := \alpha 0 \cdot \exp\left(\frac{-x^2}{2}\right)$$

$$pf0 := 0.5 + 0.5\,\mathrm{erf}\left(\frac{\alpha 0}{E0} \cdot \frac{x}{\sqrt{2}}\right)$$

$$pg0 := \overrightarrow{(pe0 \cdot pf0)}$$

$$P0 := \frac{2}{(1 + \alpha 0)} \cdot \left[\frac{-1}{2}(1 - \alpha 0) + \left(0.5 + 0.5\,\mathrm{erf}\left(\frac{x}{E0 \cdot \sqrt{2}}\right)\right) - pg0\right]$$

$$PE0 := 1 - P0$$

Haring

$$Pha := \overrightarrow{\left[4.37 \cdot \left(\frac{cl}{d}\right) \cdot \left(0.57 - \frac{cl}{d}\right)\right]}$$

$$Phb := \overrightarrow{\left[-\left(\frac{1}{2}\right) \cdot \left(\frac{cl^2}{mo}\right) \cdot (1 - Pha)\right]}$$

$$Ph := \exp(Phb)$$

Tayfun and Huang

$RR := k \cdot hs$ $\qquad\qquad RR = 0.2628$

$$Pth := \exp\left[\frac{-8}{RR^2}\left(\sqrt{1 + 2 \cdot RR \cdot \frac{cl}{hs}} - 1\right)^2\right]$$

Kriebel and Dawson

$$Pkda := \left(1 - \frac{1}{2} RR \cdot \frac{cl}{hs}\right)^2$$

$$Pkdb := -8 \cdot \frac{cl^2}{hs^2}$$

$$Pkdc := \overrightarrow{(Pkda \cdot Pkdb)}$$

$$Pkd := \exp(Pkdc)$$

Forristall

$$Sl := \left(\frac{2\pi\, hs}{9.81 Twm}\right) \qquad Sl = 0.0499$$

$$Ur := \frac{hs}{k^2 d^3} \qquad Ur = 0.0338$$

Two - dimensional

$a2 := 0.3536 + 0.2892Sl + 0.1060Ur$ $\qquad a2 = 0.3716$

$b2 := 2 - 2.1597Sl + 0.0968Ur^2$ $\qquad b2 = 1.8924$

$$Pf2 := \exp\left[-\left(\frac{cl}{a2 \cdot hs}\right)^{b2}\right]$$

Three - dimensional

$a3 := 0.3536 + 0.2568Sl + 0.0800Ur$ $\qquad a3 = 0.3691$

$b3 := 2 - 1.7912Sl - 0.5302Ur + 0.284Ur^2$ $\qquad b3 = 1.8931$

$$Pf3 := \exp\left[-\left(\frac{cl}{a3 \cdot hs}\right)^{b3}\right]$$

Prevosto

$ss := 1$ $\qquad$ unidirectional wave

$$ahs := 1 - \left(\frac{1}{2}\right) \cdot (\tanh(k \cdot d) - 0.9) \cdot \sqrt{\frac{2}{1 + ss}} \qquad ahs = 0.9686$$ Directional Factor

$$afm := \frac{1}{1.23} \qquad afm = 0.813$$ Spectral Bandwith

$T02 := \frac{Twm}{1.2}$ $\quad fm := \frac{1}{T02}$ $\quad fm = 1.0065$

modified significant wave height hs

$hsp := ahs \cdot hs$ $\quad hsp = 0.0899$

modified mean frequency fm

$fmp := afm \cdot fm$ $\quad fmp = 0.8183$

modified wave number (dispersion relation)

$kmp := \frac{(2 \cdot \pi \cdot fm)^2}{9.81}$ $\quad kmp = 4.0765$ $\quad$ cek $\tanh(kmp \cdot d) = 0.9934$

dimensionless depth

$kap := kmp \cdot d$ $\quad kap = 2.8535$

$PI := \tanh(kap) + kap \cdot \left[1 - (\tanh(kap))^2\right]$ $\quad PI = 1.031$

Second order coefficients

$$cdiff := \frac{\left[PI + kap \cdot \left[1 - (\tanh(kap))^2\right]\right]}{\left(PI^2\right) - 4 \cdot kap \cdot \tanh(kap)} \qquad cdiff = -0.104$$

$$csum := \left(\frac{1}{4}\right) \cdot \frac{\left[2 + \left[1 - (\tanh(kap))^2\right]\right]}{(\tanh(kap))^3} \qquad csum = 0.5134$$

Second order transfer functions

$TD := cdiff \cdot k$ $\quad TD = -0.2944$

$TS := csum \cdot k$ $\quad TS = 1.4535$

Non-linear crest components

$C0 := TD \cdot \frac{hsp^2}{8}$ $\quad C0 = -0.0003$

$C1 := ahs$ $\quad C1 = 0.9686$

$C2 := C1^2 \cdot (TD + TS)$ $\quad C2 = 1.0875$

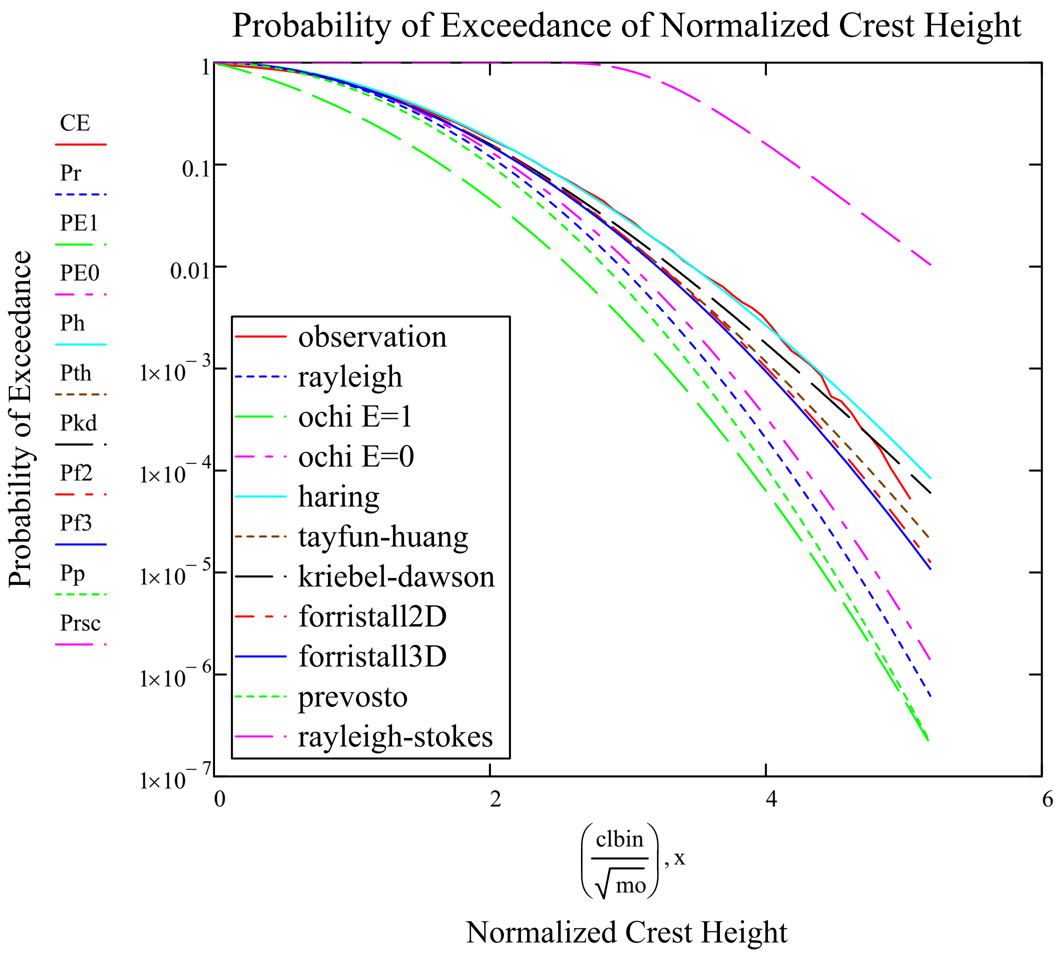
$$Cr := cl + (TD + TS) \cdot cl^2 - TD \cdot \frac{hs^2}{8}$$

$$Pp := \exp\left[\frac{-8}{hs^2} \cdot \left[\frac{\left[-C1 + \sqrt{C1^2 - 4 \cdot [C2 \cdot (C0 - Cr)]}\right]}{2C2}\right]^2\right]$$

Rayleigh Stokes

$acn := an \cdot \left[1 + B2 \cdot (k \cdot an) + B3 \cdot (k \cdot an)^2\right]$ $\quad bcn := bn \cdot \left[1 + 2 \cdot B2 \cdot (k \cdot an) + 3 \cdot B3 \cdot (k \cdot an)^2\right]$

$$Prsc := 1 - \exp\left[-\exp\left[\frac{-(cl - acn)}{bcn}\right]\right]$$



Probability of Exceedance of Normalized Crest Height
Probability of Exceedance
CE
Pr
PE1
PE0
Ph
Pth
Pkd
Pf2
Pf3
Pp
Prsc
1
0.1
0.01
1×10⁻³
1×10⁻⁴
1×10⁻⁵
1×10⁻⁶
1×10⁻⁷
0
2
4
6
observation
rayleigh
ochi E=1
ochi E=0
haring
tayfun-huang
kriebel-dawson
forristall2D
forristall3D
prevosto
rayleigh-stokes
(clbin/√mo), x
Normalized Crest Height

FIELD MEASUREMENT

Variance of water surface elevation

$mo := 3.3257$

Water depth in meter

$d := 130$

ZERO CROSSING WAVE PERIOD

Tw :=

	0
0	3.8
1	9
2	8.8
3	2.6
4	6.6
5	10
6	...

$Twm := \text{mean}(Tw)$

$Twm = 8.9769$

dispersion relation of deep water condition

$$k := 4 \cdot \frac{\pi^2}{Twm^2 9.81}$$

$k = 0.0499$

$\tanh(k \cdot d) = 1$

ZERO CROSSING WAVE HEIGHT

Zero crossing wave height in meter (hlr)

hlr :=

	0
0	4.15
1	4.475
2	3.754
3	0.655
4	3.947
5	2.734
6	2.372
7	2.483
8	...

$hl := \text{sort}(hlr)$ $\quad hl2 := hl^2$

Number of wave

$Nhl := \text{length}(hl)$ $\quad Nhl = 54673$

Statistical properties

$\text{mean}(hl) = 4.3195$

$\text{stdev}(hl) = 2.4877$

$\text{skew}(hl) = 0.7872$

$\text{kurt}(hl) = 0.8728$

Root mean square wave height

$$hlrms := \sqrt{\left[\frac{1}{Nhl} \cdot \left(\sum hl2\right)\right]}$$

$hlrms = 4.9847$

Normalized wave height

$$hn := \frac{hl}{\sqrt{mo}}$$

Significant wave height

$$Nhs := \text{round}\left[\left(\frac{2}{3}\right) \cdot Nhl\right]$$

$Nhs = 36449$

$i := (Nhs - 1) .. (Nhl - 1)$

$hl_{(Nhs-1)} = 5.1379$ $\quad hl_{(Nhl-1)} = 21.9815$

$hl_i =$

5.1379
5.1381
5.1381
5.1381
5.1381
...

$$sumhs := \sum_{i = Nhs-1}^{(Nhl-1)} hl_i \qquad sumhs = 130168.2955$$

$$hs := \frac{sumhs}{(Nhl - Nhs + 1)} \qquad hs = 7.1423$$

comparison with theoritical significant wave height

$$hss := 4 \cdot \sqrt{mo} \qquad hss = 7.2946$$

Maximum wave height

$hlm := \max(hl)$

$hlm = 21.9815$

comparison with theoritical Rayleigh formula of maximum wave height

$$hmax := \sqrt{\frac{\ln(Nhl)}{2}} \cdot hs$$

$hmax = 16.6808$

OBSERVED WAVE HEIGHT DISTRIBUTION

Wave height bin

hlbin :=

	0
0	0.088
1	0.264
2	0.44
3	0.616
4	0.792
5	0.967
6	1.143
7	1.319
8	1.495
9	...

Wave height frequency

hlfreq :=

	0
0	360
1	514
2	573
3	653
4	792
5	893
6	903
7	$1.11 \cdot 10^{3}$
8	$1.138 \cdot 10^{3}$
9	...

Cumulative frequency

hlcdf :=

	0
0	$6.585 \cdot 10^{-3}$
1	0.016
2	0.026
3	0.038
4	0.053
5	0.069
6	0.086
7	0.106
8	0.127
9	...

Bin interval

$binl := 2 \cdot hlbin_0$

$binl = 0.1759$

Number of bin

$Nbl := length(hlbin)$

$Nbl = 108$

Probability density

$Nhlfreq := length(hlfreq)$

$$hlfrel := \frac{hlfreq}{\sum(hlfreq)}$$

$$hlpdf := \frac{hlfreq}{\sum(hlfreq \cdot binl)}$$

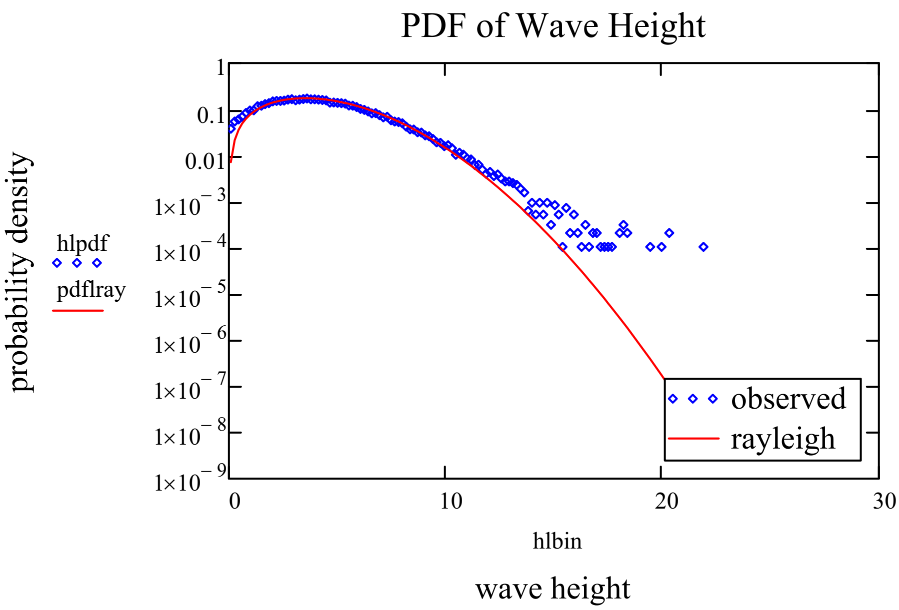
Probability of exceedance

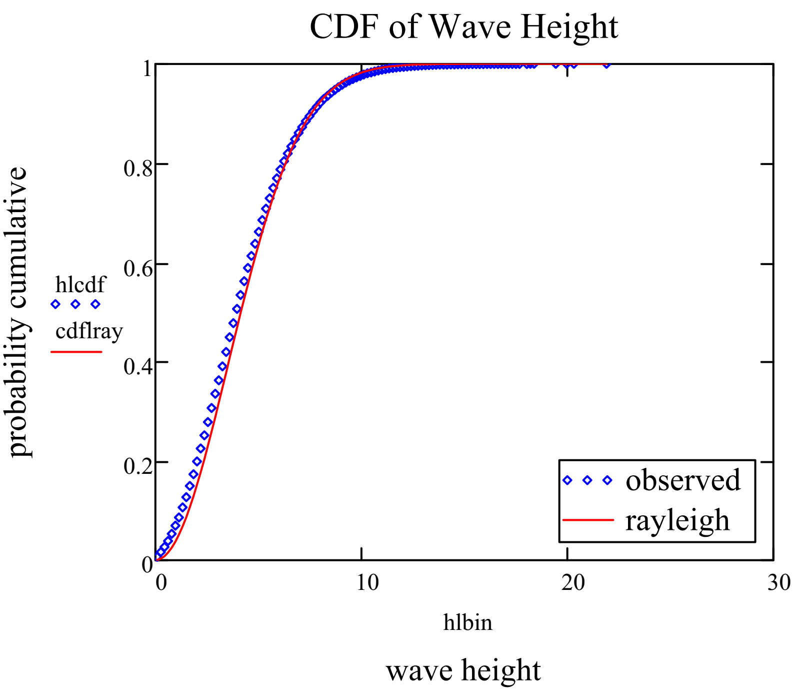
$HE := 1 - hlcdf$

$i := 0 .. (\text{length}(hlbin) - 1)$

Rayleigh Distribution

$$pdflray_i := \left[\frac{(2 \cdot hlbin_i)}{hlrms^2}\right] \cdot \exp\left[\frac{-(hlbin_i)^2}{hlrms^2}\right] \qquad cdflray_i := 1 - \exp\left[\frac{-(hlbin_i)^2}{hlrms^2}\right]$$





EMPIRICAL WAVE HEIGHT DISTRIBUTION [COMPARISON]

Rayleigh

$$\mathrm{Phr1} := \exp\left[-\left(\frac{hl}{0.707\cdot hs}\right)^2\right]$$

$$\mathrm{Phr2} := \exp\left[-\left(\frac{hl}{0.638\cdot hs}\right)^2\right]$$

Forristall

$$\mathrm{Phf} := \exp\left[-\left(\frac{hl}{0.681\cdot hs}\right)^{2.126}\right]$$

Krogstad

$$\mathrm{Phk1} := \exp\left[-\left(\frac{hl}{0.73\cdot hs}\right)^{2.37}\right]$$

$$\mathrm{Phk2} := \exp\left[-\left(\frac{hl}{0.75\cdot hs}\right)^{2.5}\right]$$

Haring

$$\mathrm{Phh1} := 0.968 + 0.176\cdot\frac{hl}{hs} \qquad \mathrm{Phh2} := \overrightarrow{\left[-2\left(\frac{hl}{hs}\right)^2\cdot \mathrm{Phh1}\right]}$$

$$\mathrm{Phh} := \exp(\mathrm{Phh2})$$

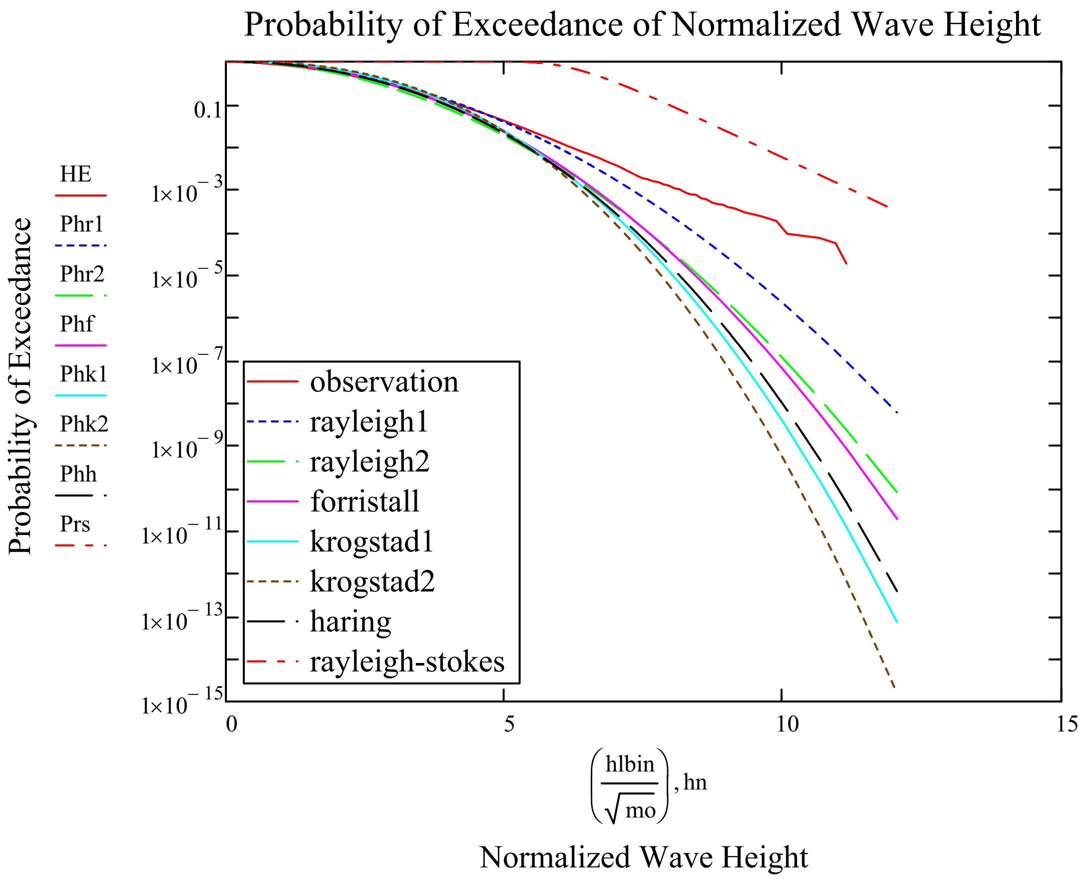
Rayleigh Stokes

$$an := 4\cdot\sqrt{mo}\cdot\sqrt{\left(\frac{\log(Nhl)}{8}\right)} \qquad bn := \frac{an}{2\cdot\log(Nhl)} \qquad B2 := \frac{1}{2} \qquad B3 := \frac{3}{8}$$

$$ahn := 2\cdot an\cdot\left[1 + B3\cdot(k\cdot an)^2\right] \qquad ahn = 11.5581$$

$$bhn := 2\cdot bn\cdot\left[1 + 3\cdot B3\cdot(k\cdot an)^2\right] \qquad bhn = 1.2896$$

$$\mathrm{Prs} := 1 - \exp\left[-\exp\left[-\left(\frac{hl - ahn}{bhn}\right)\right]\right]$$



ZERO CROSSING CREST HEIGHT

Zero crossing crest height in meter (clr)

clr :=

	0
0	3.44065
1	2.35235
2	1.78056
3	0.56368
4	2.15379
5	1.42218
6	0.78799
7	...

Sorted crest — cl := sort(clr)

Number of crest — Ncl := length(cl) — Ncl = 54673

Maximum crest height — max(cl) = 16.0834

Statistical Properties

mean(cl) = 2.2648

stdev(cl) = 1.5065

skew(cl) = 1.0359

kurt(cl) = 2.1798

Normalized crest height

$$x := \frac{cl}{\sqrt{mo}}$$

OBSERVED CREST HEIGHT DISTRIBUTION

Crest height bin

clbin :=

	0
0	0.059
1	0.178
2	0.296
3	0.415
4	0.533
5	0.652
6	0.77
7	0.889
8	1.008
9	...

Crest height frequency

clfreq :=

	0
0	$1.554\cdot10^3$
1	$1.459\cdot10^3$
2	$1.274\cdot10^3$
3	$1.269\cdot10^3$
4	$1.261\cdot10^3$
5	$1.34\cdot10^3$
6	$1.363\cdot10^3$
7	$1.314\cdot10^3$
8	$1.575\cdot10^3$
9	...

Cumulative frequency

clfcum :=

	0
0	0.028
1	0.055
2	0.078
3	0.102
4	0.125
5	0.149
6	0.174
7	0.198
8	0.227
9	...

Bin interval

$lbin := 2\cdot clbin_0$

$lbin = 0.1185$

Number of bin

$Ncbl := length(clbin)$

$Ncbl = 111$

Probability density

$$clfrel := \frac{clfreq}{\sum clfreq}$$

$$clpdf := \frac{clfreq\cdot\sqrt{mo}}{\sum(clfreq\cdot lbin)}$$

Probability of exceedance

$CE := 1 - clfcum$

EMPIRICAL CREST HEIGHT DISTRIBUTION [COMPARISON]

Rayleigh

$$Pr := \exp\left(-8\frac{cl^2}{hs^2}\right)$$

Ochi

$E0 := 0.1$ $\quad \alpha0 := \sqrt{\left(1 - E0^2\right)}$ $\quad$ narrow band

$E1 := 1$ $\quad \alpha1 := \sqrt{\left(1 - E1^2\right)}$ $\quad$ broad band

probability density function of E=1

$$pa := \exp\left(\frac{-x^2}{2}\right)$$

$$pb1 := \overrightarrow{(\alpha1\cdot x\cdot pa)}$$

$$pc1 := 0.5 + 0.5\cdot erf\left(\frac{\alpha1}{E1}\cdot\frac{x}{\sqrt{2}}\right)$$

$$pd1 := \overrightarrow{(pb1\cdot pc1)}$$

$$p1 := \frac{2}{(1+\alpha 1)}\cdot\left(\frac{E1}{\sqrt{2\pi}}\cdot\exp\left(\frac{-x^2}{2E1^2}\right)+pd1\right)$$

cumulative distribution function E=1

$$pe1 := \alpha 1\cdot\exp\left(\frac{-x^2}{2}\right)$$

$$pf1 := 0.5 + 0.5\,\mathrm{erf}\left(\frac{\alpha 1}{E1}\cdot\frac{x}{\sqrt{2}}\right)$$

$$pg1 := \overrightarrow{(pe1\cdot pf1)}$$

$$P1 := \frac{2}{(1+\alpha 1)}\cdot\left[\frac{-1}{2}(1-\alpha 1)+\left(0.5+0.5\,\mathrm{erf}\left(\frac{x}{E1\cdot\sqrt{2}}\right)\right)-pg1\right]$$

$$PE1 := 1 - P1$$

probability density function E=0

$$pb0 := \overrightarrow{(\alpha 0\cdot x\cdot pa)}$$

$$pc0 := 0.5 + 0.5\cdot\mathrm{erf}\left(\frac{\alpha 0}{E0}\cdot\frac{x}{\sqrt{2}}\right)$$

$$pd0 := \overrightarrow{(pb0\cdot pc0)}$$

$$p0 := \frac{2}{(1+\alpha 0)}\cdot\left(\frac{E0}{\sqrt{2\pi}}\cdot\exp\left(\frac{-x^2}{2E0^2}\right)+pd0\right)$$

cumulative distribution function E=0

$$pe0 := \alpha 0\cdot\exp\left(\frac{-x^2}{2}\right)$$

$$pf0 := 0.5 + 0.5\,\mathrm{erf}\left(\frac{\alpha 0}{E0}\cdot\frac{x}{\sqrt{2}}\right)$$

$$pg0 := \overrightarrow{(pe0\cdot pf0)}$$

$$P0 := \frac{2}{(1+\alpha 0)}\cdot\left[\frac{-1}{2}(1-\alpha 0)+\left(0.5+0.5\,\mathrm{erf}\left(\frac{x}{E0\cdot\sqrt{2}}\right)\right)-pg0\right]$$

$$PE0 := 1 - P0$$

Haring

$$Pha := \overrightarrow{\left[4.37\cdot\left(\frac{cl}{d}\right)\cdot\left(0.57-\frac{cl}{d}\right)\right]} \qquad Phb := \overrightarrow{\left[-\left(\frac{1}{2}\right)\cdot\left(\frac{cl^2}{mo}\right)\cdot(1-Pha)\right]}$$

$$Ph := \exp(Phb)$$

Tayfun and Huang

$RR := k \cdot hs \qquad RR = 0.3567$

$$Pth := \exp\left[\frac{-8}{RR^2}\left(\sqrt{1 + 2 \cdot RR \cdot \frac{cl}{hs}} - 1\right)^2\right]$$

Kriebel and Dawson

$$Pkda := \left(1 - \frac{1}{2} RR \cdot \frac{cl}{hs}\right)^2$$

$$Pkdb := -8 \cdot \frac{cl^2}{hs^2}$$

$$Pkdc := \overrightarrow{(Pkda \cdot Pkdb)}$$

$$Pkd := \exp(Pkdc)$$

Forristall

$$Sl := \left(\frac{2\pi\, hs}{9.81 Twm}\right) \qquad Sl = 0.5096$$

$$Ur := \frac{hs}{k^2\, d^3} \qquad Ur = 0.0013$$

Two - dimensional

$a2 := 0.3536 + 0.2892Sl + 0.1060Ur \qquad a2 = 0.5011$

$b2 := 2 - 2.1597Sl + 0.0968Ur^2 \qquad b2 = 0.8994$

$$Pf2 := \exp\left[-\left(\frac{cl}{a2 \cdot hs}\right)^{b2}\right]$$

Three - dimensional

$a3 := 0.3536 + 0.2568Sl + 0.0800Ur \qquad a3 = 0.4846$

$b3 := 2 - 1.7912Sl - 0.5302Ur + 0.284Ur^2 \qquad b3 = 1.0865$

$$Pf3 := \exp\left[-\left(\frac{cl}{a3 \cdot hs}\right)^{b3}\right]$$

Prevosto

$ss := 1$

$ahs := 1 - \left(\frac{1}{2}\right) \cdot (\tanh(k \cdot d) - 0.9) \cdot \sqrt{\frac{2}{1 + ss}} \qquad ahs = 0.95$ Directional Factor

$afm := \frac{1}{1.23} \qquad afm = 0.813$ Spectral Bandwith

$T02 := \frac{Twm}{1.2} \qquad fm := \frac{1}{T02} \qquad fm = 0.1337$

modified significant wave height hs

$hsp := ahs \cdot hs$ $\qquad hsp = 6.7852$

modified mean frequency fm

$fmp := afm \cdot fm$ $\qquad fmp = 0.1087$

modified wave number (dispersion relation)

$kmp := \frac{(2 \cdot \pi \cdot fm)^2}{9.81}$ $\qquad kmp = 0.0719$ $\quad$ cek $\quad tanh(kmp \cdot d) = 1$

dimensionless depth

$kap := kmp \cdot d$ $\qquad kap = 9.3485$

$PI := tanh(kap) + kap \cdot \left[1 - (tanh(kap))^2\right]$ $\qquad PI = 1$

Second order coefficients

$cdiff := \frac{\left[PI + kap \cdot \left[1 - (tanh(kap))^2\right]\right]}{\left(PI^2\right) - 4 \cdot kap \cdot tanh(kap)}$ $\qquad cdiff = -0.0275$

$csum := \left(\frac{1}{4}\right) \cdot \frac{\left[2 + \left[1 - (tanh(kap))^2\right]\right]}{(tanh(kap))^3}$ $\qquad csum = 0.5$

Second order transfer functions

$TD := cdiff \cdot k$ $\qquad TD = -0.0014$

$TS := csum \cdot k$ $\qquad TS = 0.025$

Non-linear crest components

$C0 := TD \cdot \frac{hsp^2}{8}$ $\qquad C0 = -0.0079$

$C1 := ahs$ $\qquad C1 = 0.95$

$C2 := C1^2 \cdot (TD + TS)$ $\qquad C2 = 0.0213$

$Cr := cl + (TD + TS) \cdot cl^2 - TD \cdot \frac{hs^2}{8}$

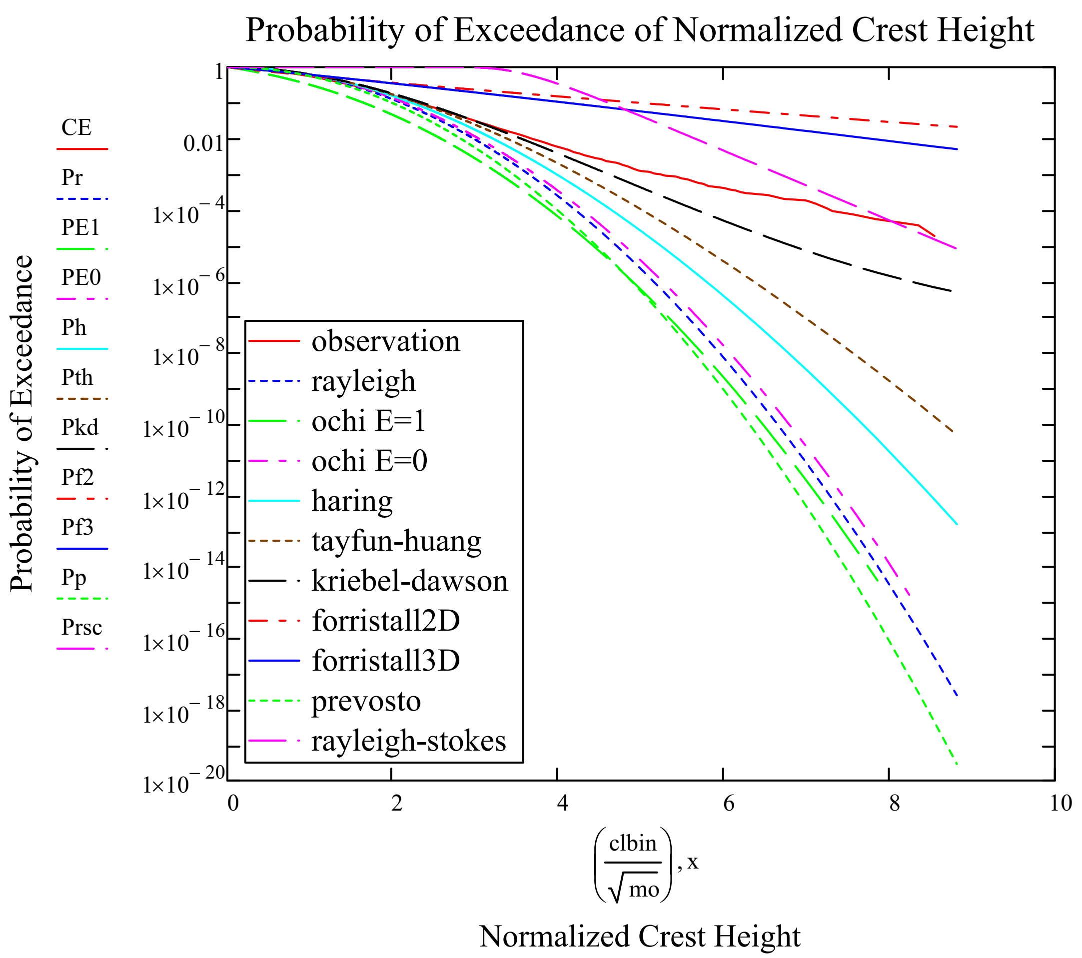
$$Pp := exp\left[\frac{-8}{hs^2} \cdot \left[\frac{\left[-C1 + \sqrt{C1^2 - 4 \cdot [C2 \cdot (C0 - Cr)]}\right]}{2C2}\right]^2\right]$$

Rayleigh Stokes

$acn := an \cdot \left[1 + B2 \cdot (k \cdot an) + B3 \cdot (k \cdot an)^2\right]$

$bcn := bn \cdot \left[1 + 2 \cdot B2 \cdot (k \cdot an) + 3 \cdot B3 \cdot (k \cdot an)^2\right]$

$Prsc := 1 - exp\left[-exp\left[\frac{-(cl - acn)}{bcn}\right]\right]$


Probability of Exceedance of Normalized Crest Height
Probability of Exceedance
CE
Pr
PE1
PE0
Ph
Pth
Pkd
Pf2
Pf3
Pp
Prsc
1
0.01
1×10^−4
1×10^−6
1×10^−8
1×10^−10
1×10^−12
1×10^−14
1×10^−16
1×10^−18
1×10^−20
0
2
4
6
8
10
observation
rayleigh
ochi E=1
ochi E=0
haring
tayfun-huang
kriebel-dawson
forristall2D
forristall3D
prevosto
rayleigh-stokes
(clbin/√mo), x
Normalized Crest Height