ALTERNATIVE RECIPROCITY RELATIONS FOR THE READ FLUX IN MAGNETIC RECORDING THEORY

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Abstract

With the aid of reciprocity relations relating a magnetic (scalar or vector) potential and one of the magnetic field quantities (magnetic flux density or magnetic field strength), alternative expressions for the read flux of a magnetic recording head are derived. They express the read flux of the head as a weighted product, over the front plane of the head, of its write field and certain field quantities that are related to the recorded pattern of magnetization.

Introduction

In the present paper some expressions are derived that yield alternatives to the usual expression [1]-[3] for the magnetic flux linked to the read coil of a magnetic recording head when a magnetization pattern recorded in a certain medium is present in front of the head. They express this read flux in terms of a surface integral over the boundary surface of a certain domain in the interior of which the recorded pattern of magnetization is present. One of the quantities entering into the surface integral is either a magnetic (vector or scalar) potential or a magnetic field quantity associated with the head's write field. Correspondingly the other one is either a magnetic field quantity or a magnetic (vector or scalar) potential that has the recorded magnetization pattern as its source. The quantities associated with the head's write field are the ones, that are caused by a unit current in the head's coil while the magnetically permeable matter in front of the head is present.

We restrict ourselves to media that are linear, time-invariant, isotropic, and instantaneously and locally reacting in their magnetic behavior. Conducting media are excluded, so all quantities in the configuration have the same time dependence. No specific assumptions are made to the type of inductive head or to the spatial distribution of the magnetization pattern.

In this way four novel expressions are obtained. They can be of practical importance in those cases when the magnetic potential or the magnetic field quantity associated with the head's write field can be measured on the boundary surface of the relevant domain (for example in the front plane of the head).

General Reciprocity Relations

Reciprocity theorems interrelate in a certain manner two admissible "states" that can be present in one and the same domain in space. Let D be an arbitrary bounded domain interior to the bounded closed surface ∂D and let $\underline{n}$ be the unit vector along the normal to ∂D, pointing away from D. The magnetic field equations in this domain D are, upon employing, as customary in magnetic recording theory, the quasi-static approximation of the electromagnetic field equations,

$$\underline{\nabla} \times \underline{H}(\underline{r},t) = \underline{J}(\underline{r},t), \tag{1}$$

$$\underline{\nabla} \times \underline{E}(\underline{r},t) = - \partial_t \underline{B}(\underline{r},t), \tag{2}$$

while

$$\underline{B}(\underline{r},t) = \mu_0[\underline{H}(\underline{r},t) + \underline{M}(\underline{r},t)]. \tag{3}$$

The magnetization is separated into a field-dependent induced part $\underline{M}_i$ and a field-independent permanent part $\underline{M}_p$:

$$\underline{M} = \underline{M}_i + \underline{M}_p. \tag{4}$$

The media that we consider have a scalar, field-independent susceptibility $\kappa(\underline{r})$, so

$$\underline{M}_i(\underline{r},t) = \kappa(\underline{r})\underline{H}(\underline{r},t). \tag{5}$$

At surfaces where this susceptibility changes abruptly, the tangential components of the electric and magnetic field strength are continuous, and the normal component of the magnetic flux density as well.

The electromagnetic state in the configuration has been reached by starting from a situation where no field is present, and sources have been switched on at the instant $t=t_0$ in the finite past. We define the vector potential $\underline{A}$ as

$$\underline{A}(\underline{r},t) = - \int_{t_0}^{t} \underline{E}(\underline{r},\tau)\, d\tau. \tag{6}$$

Substitution of (6) in (2) leads to

$$\underline{B}(\underline{r},t) = \underline{\nabla} \times \underline{A}(\underline{r},t). \tag{7}$$

In any subdomain free of electric current, (1) reduces to $\underline{\nabla} \times \underline{H}(\underline{r},t) = \underline{0}$, and in these domains a scalar magnetic potential Ψ can be introduced such that

$$\underline{H}(\underline{r},t) = - \underline{\nabla}\Psi(\underline{r},t). \tag{8}$$

The field quantities in the domain D in the two admissible states "a" and "b" in the reciprocity relations are denoted by the superscripts a and b, respectively. The expression $\underline{\nabla}\cdot(\underline{A}^a \times \underline{H}^b - \underline{A}^b \times \underline{H}^a)$ can then with the aid of (1), (3) and (7), be rewritten as

$$\underline{\nabla}\cdot(\underline{A}^a \times \underline{H}^b - \underline{A}^b \times \underline{H}^a)$$
$$= \mu_0 \underline{H}^b\cdot\underline{M}_p^a - \underline{A}^a\cdot\underline{J}^b - \mu_0 \underline{H}^a\cdot\underline{M}_p^b + \underline{A}^b\cdot\underline{J}^a, \tag{9}$$

provided that the condition $\underline{H}^b\cdot\underline{M}_i^a = \underline{H}^a\cdot\underline{M}_i^b$ is imposed (i.e., $\kappa^a(\underline{r}) = \kappa^b(\underline{r})$).

Integration of (9) over the domain D and application of Gauss' divergence theorem lead to

$$\int_{\underline{r}\in\partial D} \underline{n}\cdot(\underline{A}^a \times \underline{H}^b - \underline{A}^b \times \underline{H}^a)dA$$
$$= \int_{\underline{r}\in D} (\mu_0 \underline{H}^b\cdot\underline{M}_p^a - \underline{A}^a\cdot\underline{J}^b - \mu_0 \underline{H}^a\cdot\underline{M}_p^b + \underline{A}^b\cdot\underline{J}^a)dV. \tag{10}$$

Equation (10) is the global form of a reciprocity relation for domain D.

Similarly, manipulation of the expression $\underline{\nabla}\cdot(\Psi^a\underline{B}^b - \Psi^b\underline{B}^a)$ leads with the aid of (3)-(5),(8) and application to a current-free subdomain to

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$$-\int_{\underline{r}\in\partial D} \underline{n}\cdot(\Psi^a\underline{B}^b - \Psi^b\underline{B}^a)\,dA$$

$$= \mu_0 \int_{\underline{r}\in D} (\underline{H}^a\cdot\underline{M}^b_p - \underline{H}^b\cdot\underline{M}^a_p)\,dV. \qquad (11)$$

Equation (11) constitutes the global form of another reciprocity relation which holds for a current-free domain D.

In the reciprocity relation as expressed in (10) the volume integrals containing the electric current densities can be further reduced in case the currents flow in a thin-wire loop C. By using the relation

$$\underline{J}\,dV = I\,\underline{\tau}\,ds, \qquad (12)$$

where I denotes the electric current in the loop, $\underline{\tau}$ is the unit vector along the tangent to the center line of C, and dV is an elementary part of the loop with arc length ds along its center line. Since in the quasi-static approximation I does not vary along the wire, we have

$$\int_{\underline{r}\in D} (-\underline{A}^a\cdot\underline{J}^b + \underline{A}^b\cdot\underline{J}^a)\,dV$$

$$= -I^b\oint_C \underline{\tau}\cdot\underline{A}^a ds + I^a\oint_C \underline{\tau}\cdot\underline{A}^b ds. \qquad (13)$$

Let S denote any bounded two-sided surface that has C as boundary curve and let $\underline{\nu}$ be the unit vector along the normal to S such that $\underline{\nu}$ and $\underline{\tau}$ form a right-handed system. The magnetic flux Φ passing through the loop C is then defined by

$$\Phi = \int_{\underline{r}\in S} \underline{n}\cdot\underline{B}\,dA, \qquad (14)$$

which can be expressed in terms of $\underline{A}$ (cf. (7)) as

$$\Phi = \int_{\underline{r}\in S} \underline{n}\cdot\underline{\nabla}\times\underline{A}\,dA = \oint_C \underline{\tau}\cdot\underline{A}\,ds. \qquad (15)$$

The magnetic flux Φ is further related to the electromotive force e induced along the oriented closed contour C (cf. (6)) through

$$e = \oint_C \underline{\tau}\cdot\underline{E}\,ds = -\partial_t\Phi. \qquad (16)$$

Taking (13) – (15) into account, the reciprocity theorem (10), applied to the domain interior to ∂D, leads to

$$\Phi^a I^b - \Phi^b I^a = \int_{\underline{r}\in D} (\mu_0\underline{H}^b\cdot\underline{M}^a_p - \mu_0\underline{H}^a\cdot\underline{M}^b_p)\,dV$$

$$- \int_{\underline{r}\in\partial D} \underline{n}\cdot(\underline{A}^a\times\underline{H}^b - \underline{A}^b\times\underline{H}^a)\,dA. \qquad (17)$$

Until so far, the domain D, the boundary conditions on ∂D and the states "a" and "b" are arbitrary.

The usual expression follows from applying (17) to the domain interior to a large sphere S_Δ of radius Δ and center at the origin of the chosen coordinate system. The contribution from S_Δ vanishes in the limit $\Delta\to\infty$, since

$$\int_{\underline{r}\in S_\Delta} \underline{n}\cdot(\underline{A}^a\times\underline{H}^b - \underline{A}^b\times\underline{H}^a)\,dA = \text{Order}\ (\Delta^{-3})$$

$$\text{as } \Delta\to\infty. \qquad (18)$$

In this limit we obtain

$$\Phi^a I^b - \Phi^b I^a = \int_{\underline{r}\in D} (\mu_0\underline{H}^b\cdot\underline{M}^a_p - \mu_0\underline{H}^a\cdot\underline{M}^b_p)\,dV, \qquad (19)$$

where the right-hand side contains the permanent part of the magnetization only. The application of (19) to the entire R^3 with state "a", an "auxiliary" state characterized by $I^a \neq 0$, $\underline{M}^a_p = 0$, and state "b", the "reading" state denoted by the superscript R, characterized by $I^b = 0$, $\underline{M}^b_p = \underline{M}^R_p$, where $\underline{M}^R_p$ is the permanently recorded magnetization pattern, leads to

$$\Phi^R = \int_{\underline{r}\in D_p} \mu_0\underline{h}^a\cdot\underline{M}^R_p\,dV. \qquad (20)$$

Here, D_p denotes the domain occupied by the permanent magnetization $\underline{M}_p$, I the current in the coil, Φ the flux linked to the coil and $\underline{h}^a = \underline{H}^a/I^a$ a time-independent configurational quantity characteristic for the head's performance [2]. The use of lower case letters in the remaining text for quantities describing state "a" means that they are taken for $I^a = 1$. The absence of retardation and dispersion in the configuration makes that $\underline{H}^a$ and I^a have the same time dependence, so the lower case quantities are time-independent and describe a magnetostatic field.

Alternative Expressions for the Read Flux with the Magnetic Vector Potential

We now apply (17) to bounded domains and consider the case where on the boundary ∂D^+ of a bounded domain D^+ that completely contains the permanent magnetization distribution, additional boundary conditions are invoked. The domain D^+ may also contain induced magnetization occupying a subdomain D_i.

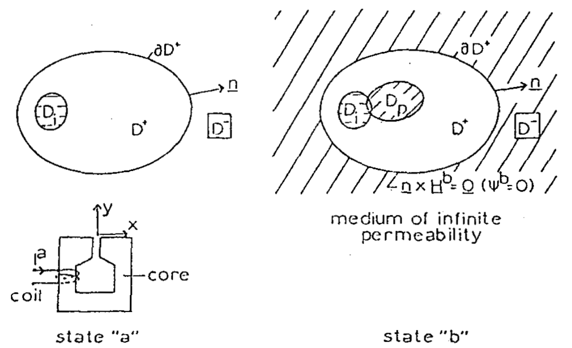

Fig. 1. The location of the domain and the two states for an alternative reciprocity relation.

The domain outside D^+ is denoted by D^- (Fig. 1). State "a" is taken to be the situation in which $\underline{M}_p^a = \underline{0}$, while $\underline{H}^a$ is the magnetic field strength due to a current I^a in the coil of the reproduce head that is located in domain D^-. For the latter state no specific boundary conditions on ∂D^+ are invoked, so the conditions of continuity of the tangential components of $\underline{A}$ and $\underline{H}$ hold across ∂D^+. State "b" is characterized by $\underline{M}_p^b = \underline{M}_p^R$, in which $\underline{M}_p^R$ is the recorded magnetization pattern; $\underline{M}_p^R$ is present in the domain D^+. Further, we subject the field in state "b" to the boundary condition $\underline{n} \times \underline{H}^b = \underline{0}$ on ∂D^+, where $\underline{n}$ is the unit vector along the outward normal to ∂D^+. With $\underline{h}^a = \underline{H}^a/I^a$ as configurational quantity, substitution of the assumptions for the states "a" and "b" in (10) leads to

$$\int_{\underline{r}\in\partial D^+} \underline{n}\cdot(\underline{A}^R \times \underline{h}^a)dA = \int_{\underline{r}\in D^+} \mu_0 \underline{h}^a \cdot \underline{M}_p^R dV. \tag{21}$$

However, on account of (20) the right-hand side of (21) equals Φ^R and hence

$$\Phi^R = \int_{\underline{r}\in\partial D^+} \underline{n}\cdot(\underline{A}^R \times \underline{h}^a)dA. \tag{22}$$

In (22), $\underline{n} \times \underline{A}^R$ is the tangential component of the magnetic vector potential of the magnetic field in D^+, that is caused by the permanent magnetization $\underline{M}_p^R$ in the read situation, but is now subject to the condition $\underline{n} \times \underline{H}^b = 0$ on ∂D^+. The latter field can be envisaged as the one that would be present in the domain D^+, if in D^- a medium of infinite permeability were present.

Similarly, we obtain for the same domain D^+, with the same characterization of state "a", but now for the state "b" subject to the boundary condition $\underline{n} \times \underline{A}^b = 0$ on ∂D^+, a different expression for Φ^R (Fig. 2). With $\underline{a}^a = \underline{A}^a/I^a$ as configurational quantity, substitution of

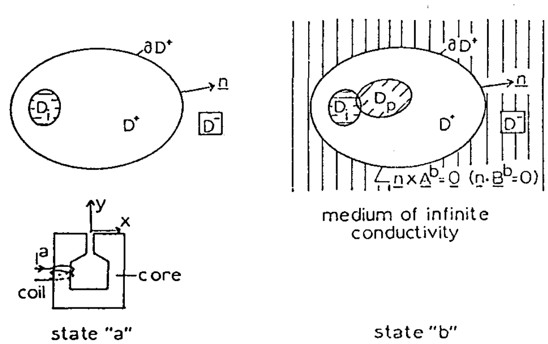

Fig. 2. The location of the domain and the two states for an alternative reciprocity relation.

the assumptions for states "a" and "b" in (10) leads to

$$\int_{\underline{r}\in\partial D^+} \underline{n}\cdot(\underline{H}^R \times \underline{a}^a)dA = \int_{\underline{r}\in D^+} \mu_0 \underline{h}^a \cdot \underline{M}_p^R dV. \tag{23}$$

However, on account of (20) we have

$$\Phi^R = \int_{\underline{r}\in\partial D^+} \underline{n}\cdot(\underline{H}^R \times \underline{a}^a)dA, \tag{24}$$

in which, $\underline{n} \times \underline{H}^R$ is the tangential part of the magnetic field strength of the magnetic field on ∂D^+, that is caused by the permanent magnetization $\underline{M}_p^R$ in the read situation, but is now subject to the condition $\underline{n} \times \underline{A}^b = \underline{0}$ on ∂D^+ (i.e. $\underline{n} \times \underline{E}^b = \underline{0}$ on ∂D^+). The latter field can be envisaged as the one that would be present in D^+, if in D^- a medium of infinite conductivity were present.

In the expressions (22) and (24) for the read flux the vector potential $\underline{A}$ is explicitly present. To obtain a value for the read flux from these alternative expressions a value for this vector potential is required. Now, the vector potential is, when it satisfies the above stated conditions, determined up to the gradient of a scalar function of position. In the alternative expressions for the read flux ((22) and (24)) the vector potential may vary with the gradient of a scalar potential without affecting the result. Taking in (22) e.g.

$$\underline{A} = \underline{A}' + \underline{\nabla}\phi, \tag{25}$$

in which ϕ is a scalar potential, we obtain for the read flux

$$\Phi^R = \int_{\underline{r}\in\partial D^+} \underline{n}\cdot\{(\underline{A}^R)' \times \underline{h}^a\}dA + \int_{\underline{r}\in\partial D^+} \underline{n}\cdot(\underline{\nabla}\phi^R \times \underline{h}^a)dA. \tag{26}$$

The last integral can be rewritten as

$$\int_{\underline{r}\in\partial D^+} \underline{n}\cdot(\underline{\nabla}\phi^R \times \underline{h}^a)dA = \int_{\underline{r}\in\partial D^+} \underline{n}\cdot(\underline{\nabla} \times \phi^R\underline{h}^a)dA - \int_{\underline{r}\in\partial D^+} \underline{n}\cdot\phi^R(\underline{\nabla} \times \underline{h}^a)dA. \tag{27}$$

Now both integrals on the right-hand side of (27) vanish. The first one due to the fact that ∂D^+ is a closed surface where application of Stokes' theorem to parts of it leads to cancellation of the results. The second one gives no contribution, because $\underline{\nabla} \times \underline{h}^a = \underline{0}$ in D^+. In the same way it can be shown that (24) is not affected by this change in the vector potential.

In cases of practical interest, D^+ is chosen to be the half-space $D^+ = \{\underline{r} \in R^3;\ y > 0\}$ in front of the head's front plane. Then the expressions for the fields to be calculated in the "reading" state R with boundary conditions on ∂D^+ (i.e. now, $y = 0$) can be obtained explicitly with the aid of the method of images.

Alternative Expressions for the Read Flux with the Magnetic Scalar Potential

Again the bounded domain D^+, that contains the permanent magnetization distribution, is considered. As this domain is a current-free one, we can also employ the magnetic scalar potential in it. Applying (11) to D^+ with the same characterization of states "a" and "b" (Fig. 1), but with the additional boundary condition $\underline{n} \times \underline{H}^b = \underline{0}$ on ∂D^+ and using (20) leads to

$$\Phi^R = -\int_{\underline{r}\in\partial D^+} \underline{n}\cdot\psi^a\underline{B}^R \, dA, \qquad (28)$$

in which $\underline{n}\cdot\underline{B}^R$ is the normal component of the magnetic flux density of the magnetic field caused by the permanent magnetization $\underline{M}_p^R$ in the read situation in D^+ under the boundary condition that Ψ^b is constant on ∂D^+.

Similarly, we obtain with the boundary condition $\underline{n}\cdot\underline{B}^b=0$ on ∂D^+ (Fig. 2) the result

$$\Phi^R = \int_{\underline{r}\in\partial D^+} \underline{n}\cdot\underline{b}^a\Psi^R \, dA, \qquad (29)$$

in which Ψ^R is the scalar potential of the magnetic field, that is caused by the permanent magnetization $\underline{M}_p^R$ in the read situation, but is subject to the condition $\underline{n}\cdot\underline{B}^b=0$ on ∂D^+. The field in the reading state in (28) can as in (22) be envisaged as the one that would be present in D^+ if in D^- a medium of infinite permeability were present, while this field in (29) can be envisaged as the one that would be present in D^+ if in D^- a medium of infinite conductivity were present.

In the expressions (28) and (29) for the read flux the scalar potential Ψ is explicitly present. This scalar potential is with the prescribed boundary conditions determined up to an additive constant. In the expressions (28) and (29) for the read flux the scalar potential may vary with a constant without affecting the result.

In the practical case that D^+ is chosen to be the half-space $D^+=\{\underline{r} \in R^3;\ y>0\}$ in front of the head's front plane the read flux in expression (28) can directly be interpreted as the integral over the head's front plane of the magnetic flux density emanating from the magnetization pattern weighted by the scalar potential characteristic for the magnetic head [4].

Conclusion

Starting from the general reciprocity relations for a bounded domain several equivalent forms of the reciprocity theorem of magnetic recording theory are presented. By an appropriate choice of the domain and of the boundary conditions to which the magnetic potentials and/or the magnetic field quantities are subjected these equivalent forms follow from the general reciprocity relations for the bounded domain. These relations hold for any distribution of magnetization in space and time in a bounded domain. The application of the reciprocity relation to an unbounded domain leads to the usual expression for the read flux which consists of a volume integral, whereas the application to an appropriate bounded domain leads to an expression for the read flux consisting of a surface integral. The results obtained in [4] have been extended and generalized. In [4] only the expression (28) was obtained for the special case of domain D^+ as the half-space in front of the head. Since all expressions are equivalent, conclusions can be reached with any of the expressions. However, the quantities occurring in the expressions and the domains over which one has to integrate differ. Depending on the purpose of the analysis, one expression will be more convenient to analyze and thus give more insight than another one.

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