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A micromechanical model for predicting the dynamic properties of loose dry sands subjected to cyclic loading using DMA test

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ABSTRACT: Cyclic loading is characterized by load patterns applied at specific frequencies. It arises from natural events, such as seismic activity, which generates wave propagation through the soil and induces cyclic shearing. Under such conditions, contact forces between soil grains may shift and rearrange, allowing grain movement that can generate irreversible deformations. The dynamic response of soil deposits is primarily governed by the shear dynamic modulus G, the damping ratio ξ, and soil density ρ. This research focuses on the dynamic properties G and ξ of loose dry sand deposits, where low-density ρ and high void ratios e enhance particle sliding and rearrangement. This research aims to improve the understanding of the dynamic properties of loose dry sands by analyzing the degradation of G under cyclic loading. A novel micromechanical model based on Hertz–Mindlin theory is employed to simulate shear modulus degradation, explicitly incorporating particle roughness and contact interactions. Experimental data from shear rheometer tests on loose, dry sands are used to validate the model. The results show good agreement between experimental observations and model predictions, including very low confining pressures (≤ 10 kPa), highlighting the model's robustness and versatility.

KEYWORDS: Geotechnical engineering, granular materials, micromechanical modelling, modulus of elasticity, seismic engineering.

1 INTRODUCTION

Cyclic loading refers to uniform patterns applied at specific frequencies (Ishihara, 1996). Human activities such as traffic, wind acting on structures, construction processes, and mechanical compaction generate cyclic stresses in the ground. Natural sources, such as seismic events, also induce cyclic loading, causing shear-wave propagation through soils and leading to cyclic shearing (Wichtmann, 2016).

The dynamic response of soils to these loads depends on their physical properties and initial conditions. In dry sands, particle interactions occur primarily through direct contact between grains, without the presence of water (Caicedo et al., 2024; Zhehao, 2023; Huang et al., 2015). In such conditions, the variation of stiffness with strain amplitude is a key aspect of soil behavior (Seed and Idriss, 1971; Seed and Silver, 1972). Under relatively small strains, the dry sand stiffness remains nearly constant, suggesting no significant changes between the inter-grain contact forces and, hence, no changes in the soil fabric (Caicedo et al., 2024; Reddy, 2022). However, contact forces may shift and rearrange as strain increases, leaving some grains free to move and accumulating irreversible strains that reduce soil stiffness. This strain-dependent behavior changes with grain size, relative density, degree of saturation, confinement pressure, and Poisson ratio, among others (Kramer, 1996; Caicedo et al., 2024; Poblete et al., 2015). Specifically, loose dry sands at low confining pressures are often associated with geotechnical processes like slope instability, tunnels with shallow overburdens, and geotechnical structures subjected to static or cyclic loading under low confining stress (Huang, et al., 2015)

The soil's mechanical dynamic properties largely control its response to cyclic loading. Understanding dynamic soil behavior is key to engineering problems. The selection of testing techniques for measuring dynamic soil properties requires careful consideration and understanding of the specific situation. The tests should replicate the initial stress and cyclic loading conditions as closely as possible (Kramer, 1996).

The evaluation techniques can be, among others, experimental and theoretical modelling. In the experimental case, the combination of existing devices enables evaluation of the soil's elastic and elasto-plastic behavior over the strain range $\gamma=10^{-6}$ to 10^{-2}. However, obtaining a single consistent modulus–strain curve is not always possible due to differences in loading setups, sample preparation, and strain measurement methods (Villacreses et al., 2021; Chaparro-López et al., 2023). Therefore, using a single device capable of

This research uses the method developed by Chaparro-López et al. (2023), which applies a shear rheometer to perform Dynamic Mechanical Analysis (DMA) on loose dry sand. This technique enables the determination of dynamic parameters, like shear modulus G and damping ξ, across the strain range $\gamma=10^{-6}$ to 10^{-2} under varying confining pressures (i.e., $\sigma_3 \leq 10$ kPa), making it suitable for evaluating the dynamic behavior of surficial soil layers.

For theoretical modelling, a micromechanical model developed by Caicedo et al. (2024) is employed. The model assesses changes in shear modulus and damping ratio in loose sands subjected to low confining stresses. This situation prevails in low-gravity conditions, where rugosity effects at intergranular contacts are more significant. It is based on both the mechanical properties of the grains and the characteristics of the grain assembly, which is a step forward compared to models that rely solely on fitting experimental results.

Caicedo et al. (2024) developed the contact theory model to assess the maximum shear modulus, building on the works of Bachrach et al. (2000), Bahrami et al. (2005), and Butt et al. (2015). The Hashin–Shtrikman upper bound was then used to combine two types of behaviors: one involving intergrain slippage at the microscopic level and another accounting for shear strength localization.

This research presents results for TP-Lisbon sand, a uniform subangular sand with a mean diameter D_{50}=0.2 mm, obtained using the shear rheometer and modelled with the micromechanical model developed by Caicedo et al. (2024). First, the methodology is presented, including the material's physical and particle shape properties, followed by a comparison of the experimental and theoretical results.

2 METHODS

The methodology includes selecting the material and characterizing its roughness properties. The experimental procedure is then described, followed by the formulation of the theoretical model. The experimental methods follow Chaparro López et al. (2023), and the theoretical model follows Caicedo et al. (2024).

2.1 *Material*

The experimental tests are carried out on dry sand, TP-Lisbon. It has a uniform grain size distribution and a mean diameter D_{50}=0.2mm, maximum and minimum void ratio of e_{max}=1.01, e_{min}=0.63 (Molina-Gomez, 2023) (see Figure 1-a). TP-Lisbon is subangular sand with sphericity of 0.61 and roundness of 0.50 as shown in Figure 1-b, based on the shape chart from Santamarina & Cho (2004). The shape parameters are obtained using SEM images in the shape analysis procedure. These are analyzed using a MATLAB algorithm by Zheng & Hryciw (2015).

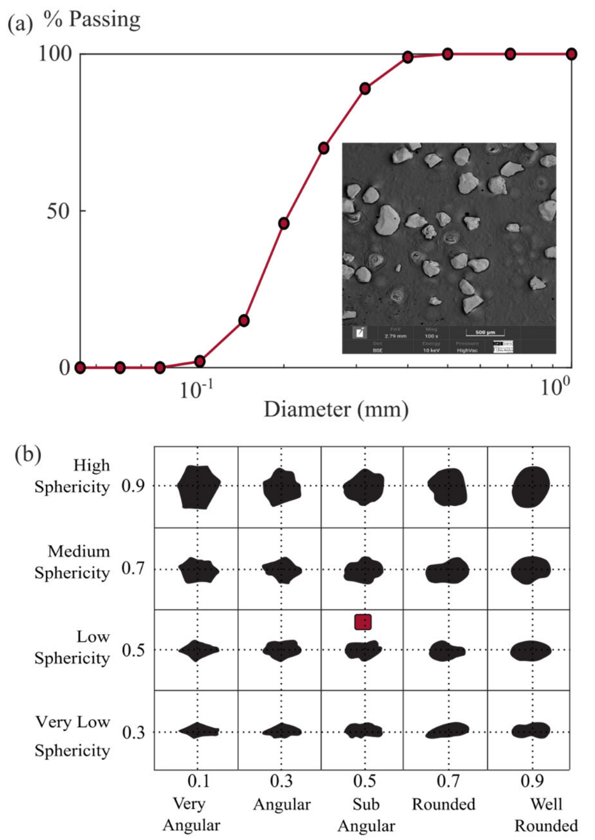

Figure 1. (a) Grain Size distribution of TP-Lisbon. (b) Shape parameter evaluation.

2.2 *Experimental Setup*

Rheometers that apply cyclic strains are known as Dynamic Mechanical Analysis (DMA) testers (Villacreses et al., 2020). The test involves applying a harmonic oscillatory angular strain with frequency and amplitude control while measuring the required stress to maintain the harmonic motion and the sample rotation with precisions of 10^{-9} N m and 40 nrad, respectively (TA Instruments, 2006). The measuring principle of the DMA is like the cyclic torsional test currently used to characterize cyclic soil behavior.

For dry sands, the sand sample in the shear rheometer is held on two metallic caps, tightened by two O-rings, and enclosed by a handmade latex membrane. Initially, the piston directly contacts the sample, and the membrane fits onto the cap. Although the membrane may introduce additional confining stress, this effect can be disregarded when the vacuum is applied, as the membrane becomes stress-free. However, in larger deformations where the specimen is fully degraded, the membrane remains embedded in the cap, resulting in recorded shear corresponding to the membrane. The confining pressure system is activated through a miniature hole on the bottom cap, as shown in Figure 2.

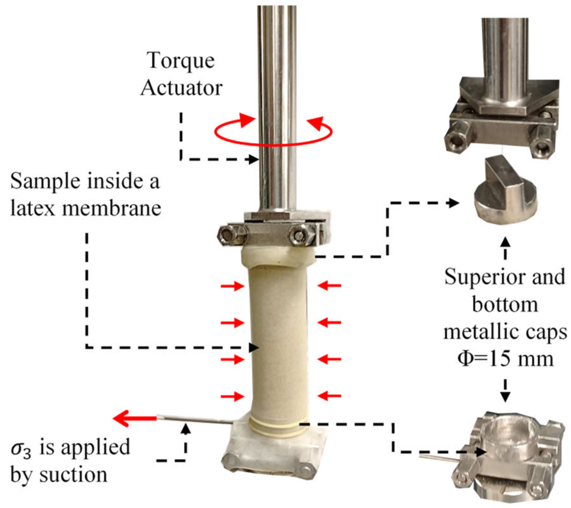

Figure 2. Experimental Set-up.

The system device components have been designed especially for this test. First, the hand-made latex membrane is made using a rigorous process with a smooth wood mold and submerged in liquid latex. The bottom and top caps are aluminum and match the diameter of the rheometer pedestal. A rigid polyvinyl chloride slice pipe supports the latex membrane while the sand sample is being prepared. Table 1 shows the dimensions of the shear rheometer components.

Table 1. System device components.

Component	Dimensions	Material
Bottom and Top Cap	Diameter: 15 mm	Aluminum
Membrane	Diameter: 15mm, thickness: 0.8 mm	Latex
Rigid slice pipe	Diameter: 15mm, height: 45 mm	PVC

The sample preparation is the most critical and challenging step due to the sample's small dimensions and low density. The preparation process is carefully controlled to ensure replicability across all samples. The DMA sample is prepared within a latex membrane and encapsulated within a rigid, sliced pipe. The sliced pipe determines the sample dimensions, resulting in a 15 mm diameter sample. The space between the pedestal and the piston restricts the sample height to 45 mm.

In order to evaluate the dynamic properties of the loose dry sand, the DMA tests are conducted on the TP-Lisbon sand with a relative density of D_r= 0.26 +/- 0.05, subjected to a strain range of γ= [10^{-6}:10^{-2}] and maintain a constant shear strain rate frequency of f=1 Hz over ten harmonic signals. A confining pressure of σ_3=10 kPa was tested to evaluate the low confining pressure.

2.3 *Roughness Analysis*

Surface roughness refers to the small-scale irregularities on a particle's surface, as illustrated in Figure 3-a. It can be measured along a single profile or across a surface map composed of multiple parallel profiles. Roughness is commonly characterized by the root mean square height (R_q), as shown in Equation 1, where z_i is the height at point i, $\bar{z}$ is the mean height, and N is the total number of measured points. (Bhushan, 2013; ASME, 2019).

$$R_q=\sqrt{\frac{1}{N}\sum_{i=1}^{N}(z_i-\bar{z})^2} \tag{1}$$

The value R_q represents the statistical deviation of surface elevations from a mean plane and captures the average height of asperities (Bhushan, 2013; ASME, 2019). These asperities influence the mechanical behavior at interparticle contacts by modifying the actual contact area and reducing it.

In this study, asperities height is determined from Atomic Force Microscopy (AFM) images obtained at the Microscopy Laboratory of Universidad de Los Andes. The analysis uses Gwyddion software to process the 3D surface scans. Roughness is measured over the full area of each particle. The parameter Rq is computed over the entire scanned area. The parameter Rq is computed over the entire scanned area, giving a value of 0.3 µm. In the micromechanical model, this value is taken as the asperity height, σ_{rms}=0.3µm.

2.4 *Theoretical Model*

The primary objective is to model the results from the Dynamic Mechanical Analysis (DMA) using the micromechanical model developed by Caicedo et al. (2024). This model is based on Hertz-Mindlin contact theory and incorporates the effects of particle roughness at intergranular contacts.

The micromechanical model evaluates changes in shear modulus and damping ratio in loose sands under low confining pressures, conditions where rugosity effects become more relevant. It relies not only on the intrinsic mechanical properties of the grains but also on the configuration of the granular assembly, which represents an improvement over approaches that only fit experimental data.

To account for contact roughness, this study uses Atomic Force Microscopy (AFM) images to estimate the height of asperities in Fontainebleau and TP Lisbon sands. These measurements are introduced as input parameters to adjust the contact mechanics and stiffness in the theoretical formulation. Following Bahrami et al. (2005) and Butt et al. (2015), the model incorporates the reduction in stiffness caused by surface asperities and considers intergrain sliding and shear strain localization. The shear modulus considering the intergrain sliding $G(\tau^M_{mob})$ is obtained by Equation 2., where $G^{\neq L}_{RS}$ is shear modulus for the volumetric proportion of grains that do not experience localization, G^L_{RS} is the shear modulus for the volumetric proportion of grains undergoing localization, ψ is the volumetric proportion of grains that do not experience localization, and K_R bulk modulus of the granular arrangement considering the coordination number *CN*.

$$G(\tau^M_{mob})=G^{\neq L}_{RS}+\frac{1-\psi}{\left(G^L_{RS}-G^{\neq L}_{RS}\right)^{-1}+\frac{2\psi\left(K_R+2G^{\neq L}_{RS}\right)}{5G^{\neq L}_{RS}\left(K_R+\frac{4}{3}G^{\neq L}_{RS}\right)}} \tag{2}$$

3 RESULTS

The experimental results for σ_3 = 10 kPa are modeled using the micromechanical formulation developed by Caicedo et al. (2024), using material parameters derived from the mineralogical, morphological, and mechanical characterization of TP-Lisbon sand. According to Molina-Gómez (2023), the sand consists of 78 percent quartz, 8 percent orthoclase, 8 percent muscovite, and 6 percent albite, a composition used to assign a grain shear modulus of 30 GPa and a Poisson's ratio of 0.12, consistent with published values for quartz-rich materials.

The microhardness of the asperities is set to 6.2 GPa, and the average asperity height obtained from AFM measurements is σ_{rms} = 0.3 µm. The model uses a micro-friction coefficient µ = 0.23, following Caicedo et al. (2024), and a macroscopic friction coefficient μ^M = 0.60, derived from a repose angle of 31°. The coordination number is estimated by combining two approaches: the porosity-based relation CN × n ≈ 2.86 proposed by Caicedo (2019), validated through datasets linking porosity and coordination number (Fei & Narsilio, 2020; Park & Santamarina, 2023; Wright et al., 2021), and the shape-based expression by Wright et al. (2021), CN = 1.88 S + 18.05 A + 2.64, which yields CN = 5.2; averaging both provides a representative coordination number for TP-Lisbon sand at D_r ≈ 0.26 and n ≈ 0.47.

Using these inputs, the micromechanical formulation reproduces the initial stiffness and captures the overall shape of the degradation curve. At small strains ($\gamma \leq 10^{-5}$), it matches the experimental stiffness plateau and predicts the maximum shear modulus with reasonable accuracy. Although the two experimental repetitions exhibit some scatter, the model closely follows the mean trend and reflects the progressive degradation in stiffness associated with particle sliding and rearrangement. The model slightly underestimates the shear modulus in the intermediate strain range ($10^{-5} < \gamma < 10^{-3}$), but it captures the global response under low confining pressure. These results highlight the importance of explicitly incorporating grain-scale interactions and surface roughness into the formulation.

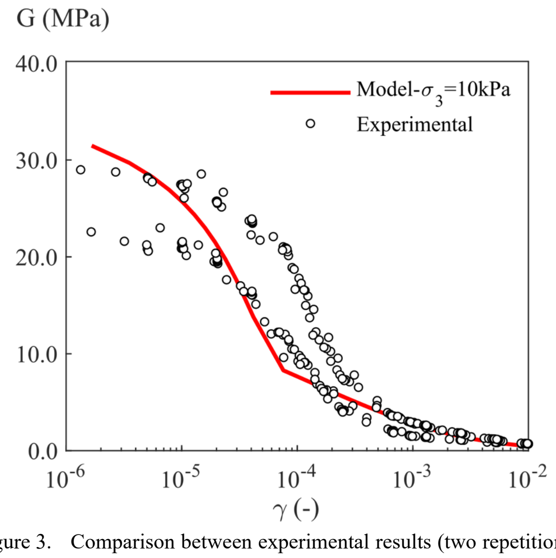

Figure 3. Comparison between experimental results (two repetitions) and micromechanical model prediction for TP-Lisbon.

4 CONCLUSIONS

This study presents a micromechanical modeling approach capable of predicting the degradation of shear modulus in loose dry sands subjected to cyclic loading. The model incorporates grain-scale parameters such as coordination number and asperity height, which are estimated through SEM and AFM imaging, respectively.

Results from DMA tests on TP-Lisbon sand show good agreement with the model predictions under low confining pressures. The model successfully reproduces the initial stiffness and captures the degradation trend across several orders of strain magnitude. Incorporating particle roughness was key to achieving accurate predictions at small strains, confirming the relevance of contact-scale parameters in dynamic soil behavior. These findings support the use of micromechanical models as a predictive tool for sands subjected to low confining pressures.

5 ACKNOWLEDGEMENTS

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