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Geometrical effect of composite fermions

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According to recent theoretical and experimental work, the two-dimensional interacting electron gas in a strong magnetic field can be well described in terms of composite fermions for which the net magnetic field vanishes at a filling factor 1/2. We present a semiclassical theory of a geometrical effect of composite fermions that manifests itself by a strong suppression of the current through a gated sample under application of a very small gate voltage. This is explained by the strong bending of the classical ballistic trajectories of the composite fermions by the net weak magnetic field produced by the gate voltage.

As has been well known for a long time (see, for example, Ref. 1) the behavior of the two-dimensional (2D) electron system in quantizing magnetic fields in the vicinity of the filling factor $\nu=1/2$ ($\nu=2\pi\lambda^2 n_e$, $\lambda=\sqrt{\hbar c/eB}$ is magnetic length, n_e is electron density) is very different from the behavior near filling factors with odd denominators. In particular, the linear temperature dependence of ρ_{xx} around $\nu=1/2$ is a strong evidence for the absence of a gap in the energy spectrum of the system. The recent theory[2] (see also Ref. 3) attributes the origin of "the $\nu=1/2$ anomaly" to the existence of composite fermions (CF's), which are electrons with two magnetic flux quanta attached. The initial 2D system of strongly interacting electrons can be transformed into an equivalent system of fermions interacting with a Chern-Simons gauge field, which is equivalent to attaching to each electron a magnetic flux tube. In the mean field approximation CF's see an average value of a fictitious magnetic field arising from the flux tubes which is related to the mean value of the electron density by the equation $B_{\text{fic}}=4\pi\hbar c n_e/e$. Thus, within the mean field approach the average net (fictitious plus external B) magnetic field acting on the CF's is

$$\Delta B=B-\frac{4\pi\hbar c n_e}{e}. \tag{1}$$

For filling factor $\nu=1/2$ the average net magnetic field acting on the CF is zero, and the ground state of the system is a filled Fermi sea of CF's with $k_F=(4\pi n_e)^{1/2}=1/\lambda$. It was argued in Ref. 2 that gauge field fluctuations do not destroy the Fermi surface (at least for the Coulomb interaction between the electrons). The existence of a CF's Fermi surface already has been confirmed by several convincing experiments. A semiclassical behavior of CF's in an effective magnetic field ΔB was observed for small deviations of the external magnetic field from the value corresponding to $\nu=1/2$.[4]

We present here a semiclassical theory of a geometrical effect which CF's should exhibit. In contrast to the effects observed in Ref. 4, it is caused by a net classical *inhomogeneous* magnetic field. The main idea is based on the relation between the net magnetic field and the local electron density [Eq. (1)]. A change in the local electron density (by applying, for example, a gate voltage) causes an inhomogeneous net magnetic field. In the following we will consider a Hall bar with a gate across the sample (Fig. 1). Assume that $W\gg d\gg\lambda$, where W is the sample width and d is the length of the region covered by the gate. Furthermore, $l_{\text{tr}}\gg d$ (where l_{tr} is the mean free path for CF's) so that the CF's can cross the gate region *ballistically*. At the same time l_{tr} may be, of course, much smaller than W. Application of a gate voltage changes the electron density under the gate and thus leads to a net inhomogeneous magnetic field ΔB existing only in the gate region. The sign of this field can be arbitrary. This weak magnetic field has a strong influence on the resistance of the device: some of the CF's are backreflected without crossing the gate region (Fig. 1). It is clear that when the diameter of the cyclotron orbit in the net magnetic field is equal to $d(2\hbar c k_F/e\Delta B=d)$, the current will be completely blocked, because none of the CF's trajectories reach (without scattering) the opposite lead. We stress that the corresponding gate voltage and the net magnetic field can be very small: $V_g/V_{g,d}\simeq\Delta B/B\simeq\lambda/d\ll1$, where $V_{g,d}$ is the value of the voltage corresponding to complete depletion of electrons under the gate. For example, for $d=600$ nm, $\Delta B\approx0.2$ T suffices, while the external magnetic field is of the order of 9.5 T. We would like to point out that the above condition involves only the cyclotron radius of CF's, which is determined by the mean electron density and does not depend on the CF's effective mass.

Without applied gate voltage the CF's do not feel a net magnetic field ($\nu=1/2$) and we assume that the current density across the sample is homogeneous. This assumption is natural for the metallic state in a zero effective magnetic

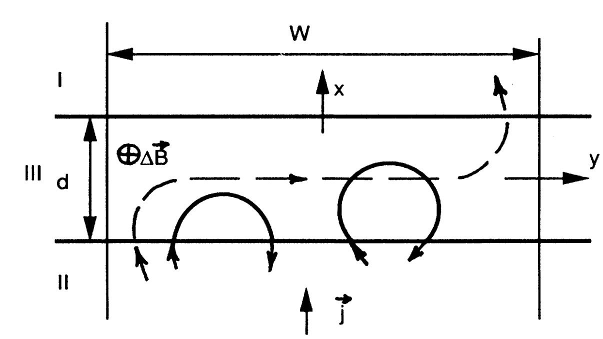

FIG. 1. Geometry of the structure considered. The dashed line shows the current direction in the absence of scattering in the gate region.

field, especially when the width of the sample W is much larger than CF mean free path.[5]

In order to calculate the current through the structure as a function of the net magnetic field in the gate region (or, which is the same, as a function of the gate voltage[6]), we need to solve the Boltzmann kinetic equation in three different regions (I–III, Fig. 1) and match the solutions at the boundaries of the gate region. The linearized kinetic equation for our problem is written as

$$\left(\mathbf{v}_\mathbf{p}\cdot\nabla_\mathbf{r}-\frac{e}{c}(\mathbf{v}_\mathbf{p}\times\Delta\mathbf{B})\cdot\nabla_\mathbf{p}\right)\delta f-e\mathbf{E}\cdot\mathbf{v}_\mathbf{p}\frac{\partial f_0}{\partial\epsilon}=\mathrm{St}\{\delta f\},\qquad(2)$$

where $\mathbf{p}$ is the kinetic momentum, $\epsilon_\mathbf{p}$ is the kinetic energy of a quasiparticle, $\delta f(\mathbf{r},\mathbf{p})$ is the local deviation from the equilibrium distribution $f_0(\epsilon_\mathbf{p})$, the effects of scattering are included in the collision integral $\mathrm{St}\{\delta f\}$, and $\Delta\mathbf{B}=\mathbf{0}$ in the regions of the leads ($|x|\geqslant d/2$). The electric field here is an effective field which includes also the field produced by the moving flux tubes.[5]

Since the magnitude of the effect in question depends on the value of the CF's mean free path, we need some estimation of this quantity. Fluctuations of the donor potential lead to variations of the electron density within the 2D plane, which in turn cause static fluctuations of the vector potential. Scattering by these gauge-field fluctuations is the main scattering mechanism for composite fermions.[2,3] It should be noted here that the theoretical value for the CF's mean free path[2] is much smaller than the value which follows from the experimental data.[4] The eikonal approximation (which is exact because of the small-angle nature of the CF's scattering by the donor potential) yields exactly the same value[7] for a transport scattering cross section as given by the calculation in the Born approximation,[2] which is actually a manifestation of a general theorem valid for small-angle scattering.[8] As a result, for the system of uncorrelated donors we obtain the same answer for the CF's transport mean free path as in Ref. 2. Without solving this dilemma, we will describe the scattering of CF's in the effective time approximation, taking the value of the transport mean free path $l_{\mathrm{tr}}=v_F\tau$ as a phenomenological parameter (which can be quite large):

$$\mathrm{St}\{\delta f\}=1/\tau(\langle\delta f\rangle-\delta f),\qquad(3)$$

where $\langle\delta f(\mathbf{r})\rangle=\int_0^{2\pi}(d\varphi/2\pi)\delta f(\mathbf{r},\mathbf{p})$ is the correction to the distribution function averaged over the momentum angle.

We did not find an exact analytical solution of the kinetic equation in the case when mean free paths in the leads and in the gate region have the same values. Instead, we consider here a model problem where the mean free path in the gate region, l_i, is much larger than the mean free path in the leads, l_{tr}: $l_i\gg l_{\mathrm{tr}}\gg d$. For ΔB corresponding to the condition $2r_c\approx d$ ($r_c=v_F/\omega_c,\omega_c=e\Delta B/mc$) (when the current through the structure is almost blocked) it is possible to obtain a simple analytical solution of the "ballistic" problem and subsequently take into account the weak scattering of CF's in the gate region by the perturbation theory. On the other hand, all the qualitative features of the phenomenon can be understood just by taking the limit of $l_i\sim l_{\mathrm{tr}}\gg d$ in the final solution.

Let us present first the zeroth order solution with respect to the small parameter d/l_i. We consider the linear regime in the applied source-drain voltage eV and the net effective magnetic field in the interval $2r_c\geqslant d$ ($2r_c-d\ll d$). We may then neglect the influence of the electric field in the gate region on the quasiparticle trajectories and determine the distribution function in each point within the gate region just from consideration of the classical ballistic paths of CF's in the net magnetic field:

$$f^{(0)}(x,\mathbf{p})=f_0\left(\epsilon_\mathbf{p}+\frac{eV}{2}\,\eta(x,\mathbf{p})\right),$$

$$\eta(x,\mathbf{p})=\begin{cases}-1 & \text{if } \varphi_2(x)\leqslant\varphi\leqslant\varphi_1(x)\\ +1 & \text{otherwise,}\end{cases}\qquad(4)$$

where φ is the angle between the CF momentum $\mathbf{p}$ and the x axis and $f_0(\epsilon_\mathbf{p}\pm eV/2)$ is the distribution function of CF's entering the gate region from lead II (I). The distribution function (4) gives the solution far from the boundaries of the system: $y+W/2\gg r_c, W/2-y\gg r_c$. For $\varphi_1(x),\varphi_2(x)$ we have

$$\sin\varphi_1(x)=-1+\frac{d/2-x}{r_c},\quad \sin\varphi_2(x)=1-\frac{d/2+x}{r_c}.\qquad(5)$$

The current density in the x direction is given by the formula

$$j_x=e\int\frac{d^2p}{(2\pi\hbar)^2}\,v_F\cos\varphi f^{(0)}(x,\mathbf{p}),\qquad(6)$$

and using the results of Eqs. (4) and (5) we finally have

$$j_x=-2\frac{e^2k_FV}{(2\pi)^2\hbar}\left(1-\frac{d}{2r_c}\right).\qquad(7)$$

Thus, when the condition $2r_c=d$ is fulfilled ($r_c\gg\lambda$), the current through the interior region is blocked. However, current can still flow in the regions of width $\sim d$ nearby the boundaries $y=\pm W/2$ (being directed along the y axis in the interior of the sample; see Fig. 1). Indeed, assuming specular scattering of CF's at the boundaries, we can obtain that the current density in the x direction on line $x=+d/2$ is equal to zero (at $2r_c=d$) everywhere except the interval of y of the order of d near $y=+W/2$, where it is of the order of $e^2k_FV/(2\pi)^2\hbar$. Concerning the current density on line $x=-d/2$, we have the symmetry relation $j_x(y)|_{x=+d/2}=j_x(-y)|_{x=-d/2}$. On the other hand, using distribution function (4) we can calculate the current density in the gate region in the y direction. It is given by Eq. (6) with the cosine replaced by a sine function. As a result, we get at $2r_c=d$:

$$j_y(x)=-2\frac{e^2k_FV}{(2\pi)^2\hbar}\sqrt{1-\left(\frac{2x}{d}\right)^2}.$$

Thus, under these conditions the current flows along the path indicated by the dashed line in Fig. 1 (through the corners of the gate region). For the value of this residual current we have $I_{\mathrm{res}}\sim(e^2V/\hbar)k_Fd$, and for the ratio of residual current to the current value I_0 at $\Delta B=0$ we have $I_{\mathrm{res}}/I_0\sim(d/W)\times(L/l_{\mathrm{tr}})$, where L is the sample length in the x direction (when $2r_c\ll d$, we should substitute r_c for d in these formulas). For the current value I_0 we have used the relation

$I_0=\sigma VW/L$, $\sigma=(e^2/4\pi\hbar)(k_F l_{\rm tr})$, which is correct when the sample length is larger than the CF's mean free path $l_{\rm tr}$, and, as a result, there is a local relation between the current density and electric field. Besides, it explicitly uses the fact that the quasiparticle conductivity tensor is diagonal at filling factor 1/2.[2] Thus, we see that for the trivial reason that we have three resistances in series (regions I–III, Fig. 1) the magnitude of the effect in question depends on the total length of the sample. For not too long samples $L \ll W$, which can be achieved in practice using multiple-strip gated structures (see discussion below), the value of the residual current is much smaller than the initial current.

Let us now take into account the weak scattering of CF's in the gate region by the perturbation theory. In order to get the correction to the distribution function $f^{(1)}(x,\mathbf{p})$ due to the scattering in the interior of the gate region we set $f^{(0)}(x,\mathbf{p})$ [Eq. (4)] and $\langle f^{(0)}(x)\rangle=\int_0^{2\pi}(d\varphi/2\pi)f^{(0)}(x,\mathbf{p})$ into collision term (3) of kinetic equation (2). The corrected solution can then be expressed in the form of an integral along the trajectory of CF motion in the net magnetic field:

$$f^{(1)}(x,\mathbf{p})=-\frac{1}{\tau_i}\int_{-t(\varphi)}^{0}dt_1[f^{(0)}(x(t_1),\mathbf{p}(t_1))-\langle f^{(0)}(x(t_1))\rangle],$$

where t_1 is the time of motion. The quantities $x(t_1)$ and $\mathbf{p}(t_1)$ are, respectively, the coordinate and momentum on the trajectory of the CF:

$$\mathbf{p}(t_1)=m\,\frac{d\mathbf{r}(t_1)}{dt_1}, \quad \frac{d\mathbf{p}(t_1)}{dt_1}=-\frac{e}{mc}[\mathbf{p}(t_1)\times\Delta\mathbf{B}].$$

The value $t_1=0$ corresponds to the point x where the CF has momentum $\mathbf{p}(\varphi)$ and $t(\varphi)$ is the time of motion along the trajectory from the point x inside the gate region to the boundary of the gate region with one of the leads. Below we shall consider only the case $2r_c=d$. For the current density in the x direction in the interior of the gate region (far from the boundaries $|y|=W/2$) we get

$$\begin{aligned} j_x&=\frac{e^2k_FV}{(2\pi)^2\hbar}\,\frac{d}{2l_i}\int_{\pi/2}^{3\pi/2}d\varphi\cos\varphi\\ &\quad\times\int_0^{3\pi-2\varphi}d\varphi_1\left\{\frac{1}{2}+\frac{1}{\pi}\arcsin[-1+\sin\varphi-\sin(\varphi_1+\varphi)]\right\}\\ &=-1.27\left(\frac{d}{l_i}\right)\frac{e^2k_FV}{(2\pi)^2\hbar}. \end{aligned} \tag{8}$$

It should be noted that Eq. (8) was actually derived under the assumption that the applied source-drain voltage drops in the gate region (more precisely, over the length $\sim l_{\rm tr}$ across the gate region) or, in other words, that the resistance of the gate region (at $2r_c=d$) is larger than the resistances of the leads. In general, only a fraction $\tilde{V}$ of the applied voltage drops over this region. In this case the current density can be estimated just by substituting $\tilde{V}$ for V in Eq. (8). Using the relation between the current density and the electric field in the leads, $j_x=\sigma E_x$, $\sigma=(e^2/4\pi\hbar)(k_F l_{\rm tr})$, we can relate $\tilde{V}$ and V and finally obtain

$$j_x=j_0\frac{1}{\left(1+\alpha\,\dfrac{l_il_{\rm tr}}{Ld}\right)}, \tag{9}$$

where j_0 is the current density at $\Delta B=0$ and α is a numerical coefficient of the order of unity. So, we see that in the limit of $l_i\sim l_{\rm tr}$ the effect in question is large for $Ld<l_{\rm tr}^2$. It is clear that the magnitude of the effect can be enhanced considerably in the case of multiple-gate structures where effective L is the period of the structure.[9]

It should be noted that, besides the magnetic field, CF's are also affected by some potential barrier in the gate region. But for the value of the gate voltage considered here the height of this barrier is $\Delta\varphi\sim E_F\lambda/d\ll E_F$, where E_F is the CF's Fermi energy, and the spatial scale of its variation nearby the boundaries of the gate region is $a\gg\lambda$ $(a\ll d)$; therefore, the influence of this barrier on the transport is exponentially small (a is the distance between the 2D plane and the top of the heterostructure).

The effect considered should be strong in 2D ballistic point contacts, where all applied voltage drops in the region of the constriction. On application of a negative voltage to the split gate (which defines the constriction), an additional spot of net magnetic field in the constriction region is created. The most striking feature of the phenomenon in this geometry is that the effect should be visible even when the width of the electronic sea in the constriction region W is larger than depletion length $l_{\rm dep}$. Indeed, estimating the density reduction in the constriction region as $\delta n/n\sim l_{\rm dep}/W$, we obtain for the cyclotron radius of CF's in the net magnetic field within the constriction $r_c\sim\lambda W/l_{\rm dep}$. Thus, we have $\lambda\ll r_c\ll W$ if $\lambda\ll l_{\rm dep}\ll W$. Therefore, the cyclotron radius can be much smaller than the width of the contact even in the case of weak depletion in the center of the constriction $(W\gg l_{\rm dep})$.

We believe that the phenomenon described is specific for the quantum state with filling factor 1/2. For comparison, if we consider the same geometry for the case of external magnetic field close to zero, then application of small gate voltage will have a negligible effect on the current. In this sense, the present phenomenon is analogous to the influence on the transport of an external magnetic field applied locally as a strip. Some aspects of this problem were investigated in several papers.[10] As regards the condition that the region of the net magnetic field should be passed by CF's ballistically, it seems that this condition is not crucial and the phenomenon should persist when the CF's mean free path is smaller than the length d of the gated strip (but larger than r_c), though probably less pronounced.

In conclusion, we have proposed a phenomenon which is a consequence of the composite fermion picture and another manifestation of extremely unusual properties of the state at filling factor 1/2. On the other hand, it seems that the experimental realization of the idea suggested is quite simple.

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[6] A relation between the effective magnetic field and the applied gate voltage is $\Delta B=2\Phi_0\kappa V_g/4\pi ea$, where Φ_0 is the flux quantum, κ is the dielectric constant, and a is the distance between the 2D plane and the top of the heterostructure.

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