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Seismic inversion of soil damping and stiffness using multichannel analysis of surface wave measurements in the marine environment

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SUMMARY

Determination of soil material damping is known to be difficult and uncertain, especially in the offshore environment. Using an advanced inversion methodology based on multichannel spectral analysis, Scholte and Love wave measurements are used to characterize subsea soil from a North Sea site. After normalization, a determinant-based objective function is used in a genetic algorithm optimization to estimate the soil shear modulus. The inverted shear-modulus profile is comparable to previously published results for the same data, although a higher degree of certainty is achieved in the near-surface layers. The half-power bandwidth method is used for extracting the attenuation curve from the measurements and efficient reference data points are chosen based on wavelet compression. The material-damping ratio inversion is performed using a modified stochastic optimization algorithm. Accounting for measurement errors, the material-damping ratio profile is retrieved from the fundamental-mode Scholte wave with a high degree of certainty. Furthermore, a method is proposed for identifying the frequency dependence of the material-damping ratio from *in situ* measurements. No evidence for frequency dependence is found and the small-strain soil material-damping ratio at this site can be said to be frequency independent for the measured conditions.

Key words: Elasticity and anelasticity; Inverse theory; Controlled source seismology; Guided waves; Seismic attenuation; Surface waves and free oscillations.

1 INTRODUCTION

Material-damping ratio estimations are difficult and are rarely performed in marine environments where measurement campaigns are expensive and disturbances can be introduced in sampling and laboratory testing. Multichannel analysis of surface waves (MASW) is used by this research for estimating the small-strain dynamic properties of soil, with emphasis on the material-damping ratio. MASW is chosen to obtain measurements without disturbing the soil and due to the inherent benefit of averaging a large sample of soil as compared to point measurements like cone penetration tests or boreholes. MASW has been reported as particularly suitable for determining the material-damping ratio due to the low level of induced strain and linear response of the soil (Lai & Özcebe 2016). Multichannel measurements were shown to result in accurate dispersion curves, with a number of mechanisms allowing a higher quality of data to be collected compared to conventional spectral analysis of surface waves (Park 1999). Additionally, studies have shown a high sensitivity to the shallow shear wave velocity profile

and layering of the soil being measured, allowing these characteristics to be accurately estimated using MASW (Xia *et al.* 1999). In the marine environment, surface wave analysis has been combined with analysis of refracted waves in a combined inversion to reach estimation depths of around 200 m (Ritzwoller & Levshin 2002).

Retrieval of soil properties from measured data can be performed by solving an ‘inverse problem’. During inversions, model parameters are updated until the response predicted by a forward model matches well with the measured response. Different optimization algorithms and objective functions have been used to correlate the dispersion characteristics, that is frequency-dependent wave behaviour, with the shear-modulus profile. Classical objective functions have minimized the distance between measured spectral energy peaks, assumed to be modal wavenumber locations, and theoretical modal wavenumber locations (Gabriels *et al.* 1987). A determinant based approach was shown to offer significant reduction in computation requirements compared to the classical misfit function (Maraschini *et al.* 2010). A root finding algorithm is not required for the determinant misfit function because the determinant values of the

theoretical soil model candidates are minimized at the measured root locations. With MASW, measured data are typically analysed in the frequency–wavenumber domain as this allows a more clear separation and identification of wave modes than considering the raw signals in the time–space domain. This separation of waves allows for a more robust match with theoretical forward models, which benefits the inversion process (Foti 2015).

MASW has been used to retrieve the soil material-damping ratio profiles of onshore sites. Material damping estimates extracted from Pisa clay using MASW compared well to laboratory, borehole and cross-hole measurements (Foti 2003). Flaws and inconsistencies have surrounded both *in situ* and laboratory measurements of the material-damping ratio (Lai & Özcebe 2016). The Rayleigh-wave attenuation coefficients are usually far more sensitive to the shear wave material-damping ratio than to the pressure wave material-damping ratio. However, it was shown that shear and pressure wave material-damping ratios can be inverted separately (Xia *et al.* 2002). It is possible to make use of the weak coupling of shear-modulus and material-damping ratio to perform inversions sequentially. However, it is shown that a simultaneous coupled inversion of the shear-modulus and damping ratio can be more accurate (Rix *et al.* 1998; Lai & Rix 2002), though this may depend on the methodology used. The concept of the quality factor, as defined by Carcione *et al.* (1988), was further extended to the modal phase-damping ratio and used with an estimate of the complex wavenumber to determine the material-damping ratio (Misbah & Strobbia 2014). Numerical solvers are required to determine the complex root solutions of a damped theoretical soil model. Using linearized equations has been shown to result in inaccuracies which grow with larger material-damping ratios. An elegant method of using Cauchy's residue theorem to accurately find the roots of the fully non-linear equations was proposed (Lai & Rix 2002). However, the authors found that in practice the numerical precision required meant that this technique was no more efficient or accurate than a simpler brute force search in the neighbourhood of the linearized complex root locations. As for the accuracy of attenuation analysis of surface wave data, it has been used with single-mode approaches (Foti 2003), but multimodal attenuation analysis methods were shown to have improved accuracy (Xia *et al.* 2003). Spectral decomposition was used to separate the multiple modes and extract the modal damping with a half-bandwidth method, which is modified to reduce the influence of adjacent modes (Badsar *et al.* 2010).

For structural or geotechnical engineering application, soil properties are preferably collected within the same frequency range as the vibrations of the structure to be designed. Onshore measurements often include hammers and drop weights (Maraschini & Foti 2010; Misbah & Strobbia 2014), while in marine environments an airgun source is often used (Maraschini *et al.* 2010). Inversions often focus on the lower frequency data from the obtained measurements as these frequencies are closest to the vibration frequencies of engineered structures and have the largest depth penetration. At the same time, it is difficult to obtain active surface wave propagation measurements at very low frequencies (below 5 Hz) due to the prohibitively large forces being required from the source. Therefore, identifying the frequency dependence of the soil properties would be useful to allow broader application via extrapolation of the properties for low-frequency structures below 5 Hz. A frequency-dependent material-damping model has been proposed (Liu *et al.* 1976) in which the material-damping ratio increases linearly with frequency for low frequencies, remains constant over a middle range and then decreases linearly to zero for higher frequencies. Disagreement exists in literature about the

frequency dependence of material-damping ratios (Lai & Özcebe 2016). One study, which tested a clay specimen and sand specimens with varying percentages of bentonite–water mixture, showed that the material-damping ratio remains unchanged within a frequency range down to around 0.25–0.5 Hz (Khan *et al.* 2010), while other research on dry sand showed that there can be significant frequency-dependent variations in the material-damping ratio (Lin *et al.* 1996).

This research aims to demonstrate that MASW techniques can be used to generate a reliable estimate of the *in situ* soil material-damping ratio in the 0–50 m range in marine environments as this has only been done successfully in onshore applications (e.g. Xia *et al.* 2003, 2012). As shear-modulus estimation is required with or before material damping inversion, a method is proposed for normalizing the determinant of the surface wave eigenvalue problem to increase the robustness in shear-modulus inversions compared with using the non-normalized determinant. Damping estimation is performed on a marine data set which contains low-frequency content and multiple clearly visible higher modes. The efficiency and accuracy of damping inversions is improved through application of a wavelet compression technique to select the measured data points used in the inversion and a method is proposed to retrieve the frequency dependence of the measured soil. This research is applied to an offshore North Sea site where Scholte and Love waves were generated at the seabed via a hydraulically actuated linear shaker operating in the 2–60 Hz range (Vanneste *et al.* 2011), with the benefit that the shear-modulus can be benchmarked against results of other studies (Socco *et al.* 2011; Dong *et al.* 2013).

2 METHODS

In this section, the methodology of the research is presented. As the damping was found to have a minimal effect of the location of the modal wavenumbers used in the stiffness inversion, a decoupled inversion strategy is proposed. First the shear-modulus profile is estimated followed by the material-damping ratio profile. The shear-modulus inversion is based on the misfit function of Maraschini *et al.* (2010), which is minimized using a genetic algorithm. The material-damping ratio estimation is based on the approach of Badsar *et al.* (2010). The methods are tested and demonstrated on synthetic data and then applied to the data set from the offshore North Sea site (Vanneste *et al.* 2011). A method for retrieving the frequency dependency of the material-damping ratio is proposed as well.

2.1 Models

The equation for 3-D wave propagation in a homogeneous isotropic linear-elastic continuum forms the starting point of the forward models:

$$\rho \ddot{\mathbf{u}} = (\lambda + \mu) \nabla(\nabla \cdot \mathbf{u}) + \mu \nabla^2 \mathbf{u}, \quad (1)$$

where μ denotes the shear-modulus (also known as G), λ denotes Lamé's first constant, ρ denotes the density and $\mathbf{u}$ denotes the displacement vector.

Two types of forward models have been used to describe both Scholte and Love wave propagation characteristics. The Scholte wave propagation is modelled by assuming an axisymmetrical wave-field and after application of the Hankel transform, the solution can be obtained in the frequency (ω)—radial wavenumber (k_r) domain.

It is, for each specific layer, written in terms of the Helmholtz potentials (Achenbach 1973):

$$\tilde{\phi}^{H_0}(k_r, z, \omega) = A_S e^{-q_p z} + B_S e^{q_p z}, \quad q_p = \sqrt{k_r^2 - k_p^2}, \quad (2)$$

$$\tilde{\psi}^{H_1}(k_r, z, \omega) = C_S e^{-q_s z} + D_S e^{q_s z}, \quad q_s = \sqrt{k_r^2 - k_s^2}, \quad (3)$$

where A_S, B_S, C_S and D_S are the unknown constants that are different for each layer, and are found by applying boundary and interface conditions, see detailed derivations in Armstrong (2016). Furthermore, k_p and k_s are, respectively, the wavenumbers associated with the pressure and shear waves (different for each layer). The solutions or roots of the dispersion equation, which can be formed based on the boundary and interface conditions, are the modal surface wavenumbers, $k_{r,i}$, where i is the mode number index corresponding to first-mode and higher-mode generalized Scholte waves (Ewing *et al.* 1957). In a damped system, the roots are complex $k_{r,i}^* = k_{r,i} + i\alpha_{r,i}$, comprising the physical wavenumber $k_{r,i}$ and the modal attenuation $\alpha_{r,i}$. The imaginary part of $k_{r,i}^*$ can alternatively be expressed in terms of the real (Re) and imaginary (Im) parts of $k_{r,i}^*$

through the modal-damping ratio $D = \frac{\text{Im}[k_{r,i}^*]}{2 \text{Re}[k_{r,i}^*]}$ following Carcione *et al.* (1988) and Misbah & Strobbia (2014). Once the unknown constants from eqs (2) and (3) are solved, the displacements can be computed from the potentials:

$$\begin{pmatrix} \tilde{u}_r \\ \tilde{u}_\theta \\ \tilde{u}_z \end{pmatrix} = \begin{pmatrix} -k_r \tilde{\phi}^{H_0} - \partial_z \tilde{\psi}^{H_1} \\ 0 \\ \partial_z \tilde{\phi}^{H_0} + k_r \tilde{\psi}^{H_1} \end{pmatrix}, \quad (4)$$

where $\tilde{u}_r$ is the radial displacement, $\tilde{u}_\theta$ is the angular or tangential displacement and $\tilde{u}_z$ is the vertical displacement.

Considering the intrinsic directivity of the shear waves, the Love-wave propagation can be described using a 2-D model. The frequency (ω)—wavenumber (k_l) solution reads:

$$\tilde{u}_y(k_l, z, \omega) = A_L e^{-q_s z} + B_L e^{q_s z}, \quad q_s = \sqrt{k_l^2 - k_s^2}, \quad (5)$$

where $\tilde{u}_y$ is the horizontal transversal displacement, A_L and B_L are the unknowns which are found by applying boundary and interface conditions and k_l is the horizontal wavenumber. Specific Love modes that can be found by solving the associated dispersion equation are denoted $k_{l,i}^*$, as shown in Ravasio (2018). These two different forward models can be used to solve two separate inversion problems of which the results can be compared to give a higher degree of confidence in the results.

Throughout this paper, reference is made to a shear-modulus inversion, while material properties are quoted in terms of wave speeds of the undamped system. The direct relation between the shear wave speed, C_s , the compressional wave speed, C_p and the shear or compressional stiffness is expressed by the following relations:

$$C_s = \sqrt{\frac{\mu}{\rho}}, \quad C_p = \sqrt{\frac{\lambda + 2\mu}{\rho}}. \quad (6)$$

These wave velocities are related to the wavenumbers in eqs (2), (3) and (5) via the angular frequency ω :

$$k_s = \frac{\omega}{C_s}, \quad k_p = \frac{\omega}{C_p}. \quad (7)$$

Further, the intrinsic absorptive property of the soil known as hysteretic behaviour is modelled using the material-damping ratios which render the Lamé coefficients (and the wavenumbers)

complex-valued:

$$\mu^* = \mu(1 + i(2\zeta_s)), \quad \lambda^* = \lambda(1 + i(2\zeta_p)), \quad (8)$$

Although the material-damping ratio associated with the shear waves, ζ_s , and the one associated to the compressional wave, ζ_p , are in principle different, the sensitivity of the modal attenuation curves to ζ_s was analysed to be much higher than the sensitivity to ζ_p (Armstrong 2016). Hence, it is assumed throughout this paper that $\zeta_p = \zeta_s$ and forthwith it will only be referred to as the material-damping ratio, ζ . The assumption that $\zeta_p = \zeta_s$ has also been used by Badsar *et al.* (2010).

For clarity, it is important to distinguish that the material-damping ratio is therefore a model input or soil property (specific for each layer), while the modal damping ratio or modal attenuation are wave attributes that depend on the parameter values for all the layers including the material-damping ratio, shear-modulus, layer thickness and other parameters.

It was assessed that an uncoupled inversion process maintains a sufficiently high degree of accuracy while improving the inversion process by allowing different strategies to be used for the shear-modulus and material-damping ratio estimation (Armstrong 2016). Hence, it was chosen to split the inversion for dynamic soil properties into 2 steps: a damping-independent method to find the shear-modulus profile, and an estimation of the material-damping ratio profile in a second step. An accurate shear-modulus profile is still important for estimating the material-damping ratio profile because of its dependence on the shear-modulus.

2.2 Stiffness inversion method

A key component in an inversion problem is the objective function. A poor or non-robust formulation may lead to futile results, while on the other hand, a robust, well-defined objective function can allow various methods to reach the correct result. In the shear-modulus inversion problem, the objective function is formulated based on Maraschini *et al.* (2010). It is referred to as a determinant misfit function since it is defined as the sum of the values of the normalized determinant at the modal locations picked in the measured data response:

$$\epsilon_d = \frac{1}{N} \sum_{j=1}^N s_j |M(f_j, k_j)|, \quad f_j = \frac{\omega_j}{2\pi}, \quad \sum_{j=1}^N s_j = 1, \quad (9)$$

where j is an index counting over the N data points picked from the measured response, ϵ_d is the total determinant error in the shear-modulus inversion, s_j is a weighting factor and M is the determinant of the layered soil matrix. The N data points may consist of multiple modes, but no distinction needs to be made except for weighting purposes. Eq. (9) holds for both Scholte and Love waves. Based on the formulation of the equations, the determinant is heavily dependent on the phase velocity $v = \frac{\omega}{k}$, with low phase velocities resulting in a determinant value tens of orders of magnitudes below the determinant at high phase velocities. This will cause a bias in the inversion process towards systems with high phase velocities. This is corrected via a normalization scheme, which first uses three or more angular test lines at fixed radii in the wavenumber–frequency domain, shown in red on Fig. 1(a), to compute the average dependence of the magnitude of the determinant on the phase velocity. This average dependence shown in Fig. 1(b) is plotted not versus phase velocity, but versus the angle relative to the wavenumber axis, $\theta = \tan^{-1}(\frac{\omega}{\omega_{max}} \frac{k_{max}}{k})$, for an easier comparison with

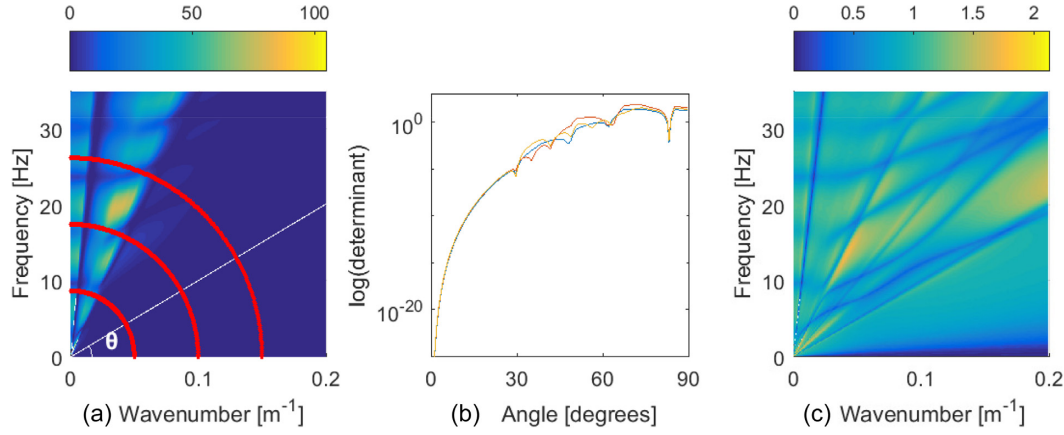


Figure 1. (a) Non-normalized determinant with test lines. (b) Determinant magnitude versus angle, $\theta = \tan^{-1}(\frac{\omega}{\omega_{max}} \frac{k_{max}}{k})$, from the wavenumber axis in panel (a). (c) Normalized determinant.

Fig. 1(a). The obtained dependence is used to normalize the determinant to a nominal level of approximately unity everywhere in the wavenumber–frequency domain which is not close to the zeros of the determinant. The zeros of the dispersive modal wavenumbers are not removed in the final normalization since the dependence is obtained from averaging the three test lines and the dispersive events cross each of the test lines at a different angle to the wavenumber axis. Additionally, non-dispersive events remain as the obtained dependence is further smoothed with a common moving-average filter (in this case the ‘smooth’ function from Matlab®) to increase normalization robustness. The purpose is to remove sudden dips in the normalization function and the exact smoothing function used is unlikely to be critical. The resulting normalized determinant in Fig. 1(c) shows a significantly clearer representation of the root locations, allowing the inversion to search successfully over a much broader range of shear wave velocities.

A genetic algorithm (Holland 1975) is used in the shear-modulus inversion. The cross-breeding and mutation of the population after each generation creates the necessary diversity to reach the global minimum and avoid local minima in the inversion process (de Winter 2014). Due to the determinant normalization, a very wide search velocity range can be specified in the initial search criteria. In order to improve convergence but allow this wide search range, a dynamic re-ranging has been applied, where after a specified number (e.g. 10) generations, the search range of each layer of the soil model is set to the maximum and minimum values of the top subset (e.g. 15 per cent) of the population. Some relaxation of this reduction is also provided such that the ranges do not shrink too quickly. Additionally, a layer stripping approach is implemented. Since the objective function tends to be most sensitive to the shallowest soil layers, the top layers are quickly estimated with a high degree of accuracy while lower layers may remain poorly estimated due to their weaker sensitivity in the objective function. Hence, after a fixed number of generations the estimation of the top active layer is frozen so that the inversion algorithm progressively moves its focus to estimating deeper layers. This helps all layers to be estimated with the highest possible accuracy even though the overall objective function has different sensitivities to different layers. Finally, when applicable, phased settings are used, where after a specified number of generations the reference data of the objective function is changed and correspondingly reweighted during the inversion run.

For example, the inversion begins with only the fundamental mode and after a specified number of generations measurement data for higher modes are added to the reference set and given a specific weighting as part of the objective function. This approach follows Armstrong (2016). The results of the shear-modulus inversion are presented in Section 3.

2.3 Damping inversion method

Once a reliable shear wave velocity profile has been determined, the material-damping ratio inversion can be performed. The attenuation analysis was inspired by Foti (2015). The attenuation coefficient has been chosen as the reference parameter for the material-damping ratio inversion on which to base the misfit function. The measured attenuation curve $A_i(\omega)$ of each Scholte wave mode is extracted by using the modified half-power bandwidth method (Badsar *et al.* 2010) in the f – k_r domain using the appropriate Hankel transform to correct for the geometric damping (Ravasio 2018):

$$A_i(\omega) = \frac{\Delta k_{r,i}(\omega)}{2\sqrt{\gamma^2 - 1}}, \quad (10)$$

where γ defines the height where the bandwidth $\Delta k_{r,i}(\omega)$ is calculated. The damping inversion is only conducted using Scholte waves as it proved difficult to properly correct for the geometrical spreading in the Love-wave data (see also Section 3.2). In eq. (10), γ is allowed to vary between 0 and 1 and should be chosen in order to avoid mixing of resonance peaks.

The modelled attenuation curve $\alpha_{r,i}$ is defined as the imaginary part of the complex modal wavenumber as discussed in Section 2.1. The misfit function for the material-damping ratio inversion is then defined as the weighted difference per frequency of the attenuation curve:

$$\epsilon_a = \frac{1}{N} \sum_{j=1}^N p_j \left| \frac{A(\omega_j) - \alpha_r(\omega_j)}{A(\omega_j)} \right|, \quad \sum_{j=1}^N p_j = 1, \quad (11)$$

where ϵ_a is the total error on the attenuation curve mismatch, N is the number of frequency points in the attenuation curve, p_j is a weighting factor, $A(\omega_j)$ is the measured attenuation coefficient of the j th data point and $\alpha_r(\omega_j)$ is the theoretical attenuation coefficient of

the j th point. The misfit function is here defined for the fundamental mode ($i = 1$) but can be extended to multiple modes.

If the attenuation curve is formed by points distributed at a uniform frequency spacing, then the misfit function becomes highly sensitive to the first layer material-damping ratio and poorly sensitive to the material-damping ratio of other layers. In order to equalize the sensitivity of the misfit function to all layers, a wavelet compression scheme was used on the attenuation curves (Armstrong 2016). The wavelet compression prioritizes keeping data points in areas of the attenuation curve where there is high shape curvature. It was found during synthetic inversion trials that to determine the correct solution, it is important to match the attenuation curve at these points of high shape curvature. By focusing on these locations, the objective function maintains a more balanced sensitivity to different layers. If a uniform material-damping ratio profile is assumed, it is observed that the frequency at which the high curvature occurs is dependent on the thickness, number of layers and relative shear-modulus distribution of the soil profile. Hence, this wavelet compression technique is performed on every shear-modulus profile for which the material-damping ratio is estimated, since the points to choose for the inversion will vary per soil profile. Furthermore, a modified stochastic algorithm (Rennard 2006) is applied in the material-damping ratio inversion. This method uses a random guess to distribute the trial population. With every iteration the method progressively reduces the size of the search domain, keeping only the domain which encompasses the best-fitting subset of the population. This utilizes the smoothness of the objective function and the discrete nature of the search domain in order to reduce the number of iterations and the randomness. The results of the wavelet compression and modified stochastic search algorithm the steps of the inversion are presented in Section 3.

2.4 Frequency dependency via Scale Factor Method

The material-damping ratio inversion is based on a frequency-independent or hysteretic soil material-damping ratio model. However, it might be more appropriate to determine the frequency dependency of the material-damping ratio based on the *in situ* measurements. Henceforth, a Scale Factor Method is proposed, which is used to estimate the frequency dependency of the material-damping ratio. The complex wavenumber, $k_{r,i}^* = k_{r,i} + i\alpha_{r,i}$, at one specific frequency of a given modal attenuation curve only depends on the input parameters at that specific frequency. Since the forward model is completely independent of adjacent frequencies, the soil model can facilitate any arbitrary dependence of material-damping ratio on frequency, if this relationship is incorporated into the forward model input parameters. The material-damping ratio inversion algorithm minimizes the distance between the modelled and measured attenuation curves in aggregate, so an enhancing step is to observe whether there is a trend of the mismatch versus frequency. This is done by computing the ratio between the measured attenuation curve and the attenuation curve corresponding to the best estimate of the inversion process. It has been shown that scaling the material-damping ratio results in the same scaling of the modal damping ratio, within linear approximations (Armstrong 2016). Thus it holds that the frequency-dependent ratio (i.e. scale factor) between the measured and hysteretic attenuation curves is directly equal to the frequency dependence of the material damping, assuming that all layers of the soil profile exhibit the same frequency dependence. The effectiveness of this technique and the

Table 1. Properties test soil profile.

Layer	Thickness [m]	C_p [m s ⁻¹]	C_s [m s ⁻¹]	ζ [-]	ρ [Mg m ⁻³]
Water	5	1500	0	0	1.0
1	3	1500	100	0.050	1.8
2	3	1500	200	0.035	2.2
3	3	1500	100	0.020	1.8
Half-space	∞	1500	400	0.010	2.3

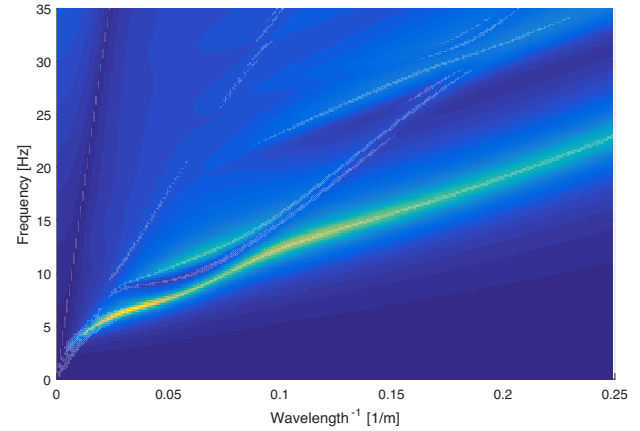


Figure 2. Test case soil profile $f-k_r$ synthetic spectrum for the Scholte wave model.

validity of the scale factor is demonstrated on the measured data in Section 3.3.

3 RESULTS

3.1 Synthetic case

Firstly, a simple synthetic case is considered in order to verify and test the shear-modulus and material-damping inversion algorithms. The layer thickness, C_p and ρ were taken as fixed constants at their correct values and only the shear-modulus and material-damping ratio are separately estimated. The soil profile chosen for this analysis presents a 5 m thickness water layer, three intermediate soil layers of 3 m and a half-space. The properties of the test soil profile are shown in Table 1. It represents a shear wave velocity profile that overall increases with depth (sometimes called ‘normally dispersive’) but with a weak middle layer (layer 3) and a soil material-damping profile that decreases with depth. It is important to highlight that for the Love wave model the water layer does not play any role since idealized fluids cannot sustain shear stresses. For the Scholte waves model it still plays some role since the pressure wave component exists. A Dirac impulse is used to generate the $f-k_r$ domain response spectrum for the Scholte waves shown in Fig. 2. The fundamental mode dominates the whole frequency range and is used as the only mode in the damping inversion, while, in the shear-modulus inversion, it is used along with the two higher modes. No noise is added to the synthetic inversions. For the measured data, however, the effect of uncertainties in modal wavenumber locations are implicitly addressed by the resulting bands of certainty in the inversion results (see Fig. 4).

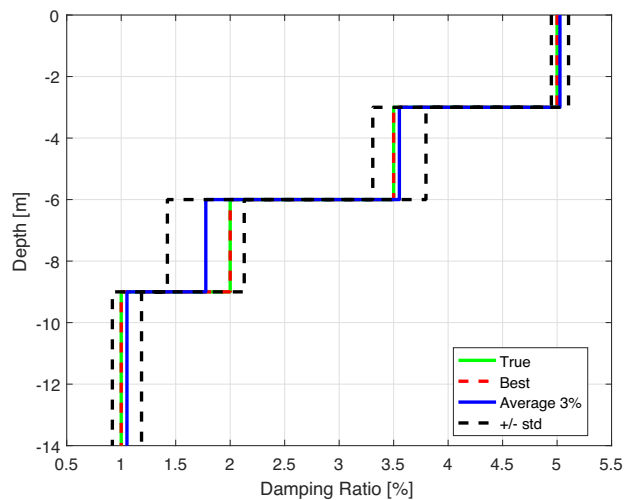
Table 2 presents the inversion settings used in the shear-modulus inversion. N_{GEN} is the number of generations in the genetic algorithm. N_{POP} is the number of trial soil profiles in the population. N_{DAD} is the number of parents, N_{CON} is the number of contestants

Table 2. Stiffness inversion settings for the test case.

N_{GEN}	N_{POP}	N_{DAD}	N_{CON}	p_{FM}	Phase setting	Rerange	Layer strip
70	60	20	35	0.25	No	Yes, top 15 per cent	Yes

Table 3. Damping inversion settings for the test case. SD refers to 'standard deviation' indicating that after every iteration the re-ranging is done where the search boundaries are reset to 1 standard deviation from the mean value based on the top 3 per cent of the population. N_{Iter} is the number of iterations. N_{Mem} is the number of trial soil profiles in the population.

N_{Iter}	N_{Mem}	Rerange
3	625	Yes, with SD

**Figure 3.** Material damping inversion results for the synthetic test soil profile.

taking part in the tournament selection. p_{FM} is the probability that one of the properties will be mutated. Parents pass on their genes to the next generation and N_{DAD} is taken as one third of N_{POP} by default. Mutation properties are randomly selected using a uniform distribution from within the search range.

The shear-modulus profile is successfully identified and the overall error is about 1.1 per cent. The profile is not shown graphically as the true profile, mean and best estimate and the confidence range do not significantly differ to the extent they are visually distinguishable in a plot of the values over the depth. Further results can be seen in Ravasio (2018).

Table 3 shows the inversion settings used in the damping inversion. Fig. 3 shows the results in terms of material-damping profile. It is observed that the best profile matches the true synthetic profile perfectly, and that the average error of the top 3 per cent of the population is lower than 5 per cent. At the end of the three iterations, the confidence intervals are reduced by more than 80 per cent from the initial blind guess. This highlights quick convergence of the inversion algorithm.

Table 4. Shear-modulus inversion settings for the genetic algorithm.

N_{Gen}	N_{Pop}	N_{Dad}	N_{Con}	p_{FM}	Phased setting	Re-range	Layer strip
100	240	80	35	0.25	Yes	Yes, top 15 per cent	Yes

3.2 Measured data

In the experiment (Vanneste *et al.* 2011), a custom built shear wave vibrator has been used. This vibrator has a 3.25 m diameter suction caisson which penetrates 2.5 m into the soil and contains a linear hydraulic actuator with a 3700 kg mass. The working range of frequencies is 2–60 Hz and the shaker was designed to give a uniform energy content in the frequency range 10–55 Hz. The receivers were placed with an interval of 25 m on a 600 m long array and the cable was dragged to achieve an effective receiver spacing of 2.5 m. The source is placed within the measurement array at 450 m and only 450 m of the signal has been used for this research. Sweeps were generated in both in-line and cross-line directions to the cable orientation in order to observe respectively the Scholte and Love waves on the seabed. Two displacement components are used in the present analysis called respectively U_z and U_y ; the capitals are used here to emphasize that these are measured responses, as shown in the top of Fig. 5, not the modelled counterparts (see eqs 4 and 5). U_z is the vertical displacement component generated by in-line sweeps and is used for the Scholte wave inversion, while U_y refers to the horizontal cross-line displacement obtained by cross-line sweeps and is used for the Love wave inversion. The Scholte wave data were taken into the wavenumber domain using the Hankel transform in order to correct for the geometric spreading and only focus on the material-damping in the $f-k_r$ domain for the inversion. The order of the Bessel function is equal to 1 following Vostroukhov *et al.* (2004). In accordance with the common practice in dispersion analysis to get the frequency–wavenumber spectrum containing Love waves only, the Love-wave data (U_y) in the space domain were transformed to the wavenumber domain by applying the Fourier transform over space (Vanneste *et al.* 2011; Socco *et al.* 2011; Dong *et al.* 2013). The reason is that suppressing completely the geometrical spreading associated with the Love wave in the transformation to the wavenumber domain is known to be difficult (Lai & Özcebe 2016), and would probably require the combination of the in-line and cross-line displacement components (Vostroukhov *et al.* 2004). In the current paper, the Love-wave data can thus only be used for the shear-modulus inversion, not for the material-damping ratio inversion. The measured data after transformation to the frequency–wavenumber domain are shown in the middle plots of Fig. 5. The amplitude spectra clearly show fundamental-mode Scholte and Love waves, as well as one higher-mode Scholte and two higher-mode Love waves.

In the shear-modulus inversion, for both Scholte and Love waves, the best modal match with the measured data set was obtained with a soil system consisting of 12 layers overlying a half-space soil system. The inversion settings are reported in Table 4 and the layer thickness, C_p and ρ were taken as fixed constants during the inversion. Determination of the number of layers and layer thicknesses, which result in the best inversion, was performed as a prior step to this final shear-modulus inversion which assumes these parameters as fixed (Armstrong 2016). The inversion resulted in excellent convergence using phased settings, layer stripping and dynamic re-ranging. In the first phase, only the fundamental mode is considered. After 30 generations, the phased settings are used to introduce one higher mode for the Scholte wave inversion and two higher modes

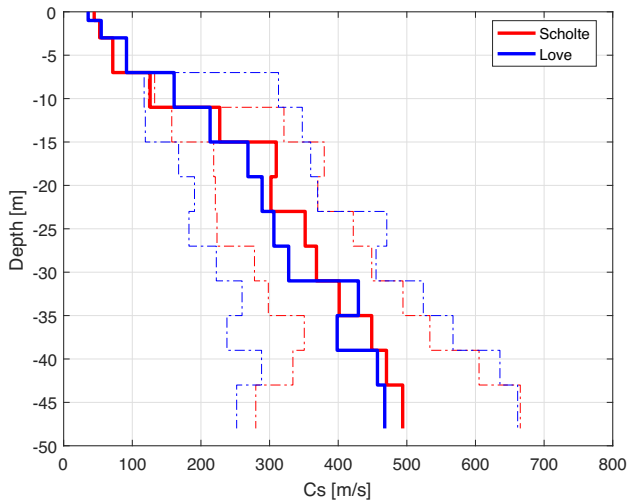


Figure 4. Shear-modulus profile estimated from the North Sea data set based on Scholte waves (red line) and Love waves (blue line). The solid line shows the mean estimate of the multimodal inversion while the dotted lines show the range of values from the top 15 per cent of the population.

for the Love wave inversion. For the Scholte wave inversion the fundamental mode was weighted 70 per cent and the higher mode 30 per cent while for the Love wave inversion the fundamental mode was weighted at 40 per cent and the two higher modes at 30 per cent each. The shear-modulus inversion results are shown in Figs 4 and 5. In the spectral plots of Fig. 5 we observe that the fundamental mode is matched very well for both Scholte and Love models and the two higher modes have a good visual shape agreement but the exact locations are not fully matched. In this inversion a higher weighting has been given to the fundamental mode, as it was used for the damping inversion. If matching higher modes more accurately is of importance, it may be beneficial to incorporate into the misfit function a measurement of the distance between the measured and modelled modal wavenumber locations, rather than using only the determinant based misfit. The estimated shear-modulus profile also agrees well with other research on the same data set where results were produced independently via a Monte Carlo inversion technique (Vanneste *et al.* 2011) and a Bayesian inversion (Dong *et al.* 2013). The shear wave profile presents an overall linearly increasing trend and a good correspondence between Scholte and Love results (Fig. 4). The average difference between the two shear wave velocity profiles is lower than 12 per cent. Table 5 shows the mean value of the top 15 per cent of the population for the shear wave velocity and the minimum and maximum estimations for each layer.

The material-damping ratio inversion requires increased computational resources with respect to the shear-modulus inversion and so the number of layers of the forward model had to be reduced from 11 to 6. The shear-modulus inversion was reperformed and it was verified that the obtained shear-modulus profile leads to an acceptable result. In the material-damping ratio inversion only the fundamental mode attenuation curve has been used. Table 6 shows the damping inversion settings. The layer thickness, C_p , ρ and the shear-modulus were taken as fixed constants, shown in Table 7, during the damping inversion. The results of the damping inversion are reported in Fig. 6. Only Scholte wave results are shown, because, as indicated above, the transformation applied to the Love wave data does not suppress the geometrical spreading. The results indicate a material-damping profile which increases in the first 20 m and

is constant for the rest of the visible depth. The magnitude of the material-damping profile is plausible according to the range seen from other published results (Foti 2003; Xia *et al.* 2003; Badsar *et al.* 2010; Lai & Özcebe 2016). Table 7 shows the mean value of the top 3 per cent of the population for the material-damping ratio ζ , and the minimum and maximum estimations for each layer, which clearly indicates that the accuracy of the estimation is high.

3.2.1 Modal damping ratio curve analysis

A qualitative analysis of the modal damping ratio curves enables a better intuitive understanding of the attenuation data. Fig. 7 shows the extracted modal damping curves from the North Sea site. These are used as a reference for the present analysis. Three different soil configurations for the forward model are generated. The first configuration (Fig. 8) contains a material-damping ratio which is linearly increasing with depth from 1 per cent to 7 per cent while the second (Fig. 9) shows a material-damping ratio which is linearly decreasing with depth from 7 to 1 per cent. The third configuration (Fig. 10) contains a constant value of material-damping ratio, 3 per cent, for each layer and is called homogeneous profile. For each configuration, the modal damping ratio curves are computed. By visual comparison, it is possible to observe that the extracted curves for the North Sea site (Fig. 7) have characteristics which are most like the increasing profile with some similarities to the homogeneous profile. This enhances the confidence that the resulting material-damping ratio profile of the North Sea site as presented in Fig. 6 is a combination of an increasing and a homogeneous profile.

3.3 Frequency dependence analysis

A frequency dependency analysis of the material-damping ratio is performed by using the results obtained with the Scholte wave and applying the Scale Factor Method introduced in Section 2.4. The left-hand panel of Fig. 11 shows the comparison between the attenuation curve obtained by the average of the top 3 per cent of the final population of the inversion process and the measured curve extracted from the U_z response. The frequency dependent scale factor is computed as the ratio between the measured and theoretical attenuation curves and is shown in the right-hand panel of Fig. 11. It can be observed that the scaling factor is very weakly dependent on frequency in the considered frequency range.

Since the scaling factor hardly deviates from unity, it is possible to conclude that, in the frequency range of the present measurements, the material-damping ratio can be assumed to be frequency independent. Recalling the model proposed by Liu *et al.* (1976), it can be said that the actual data highlight only the region associated to the constant part of the relation. This finding is in agreement with the findings of Khan *et al.* (2010) who found that the material-damping ratio is relatively frequency-independent in the range 1–100 Hz, even though sand and clay exhibited different trends. However, as other research have obtained contradictory findings (Lin *et al.* 1996), further research with measurements at different locations and soil types are needed.

3.4 Overestimation of the material-damping ratio

In MASW, the limited amount of receivers causes a spatial truncation of the infinite signal, resulting in modified integration limits of the Hankel transform and introducing a ‘windowing’ effect which

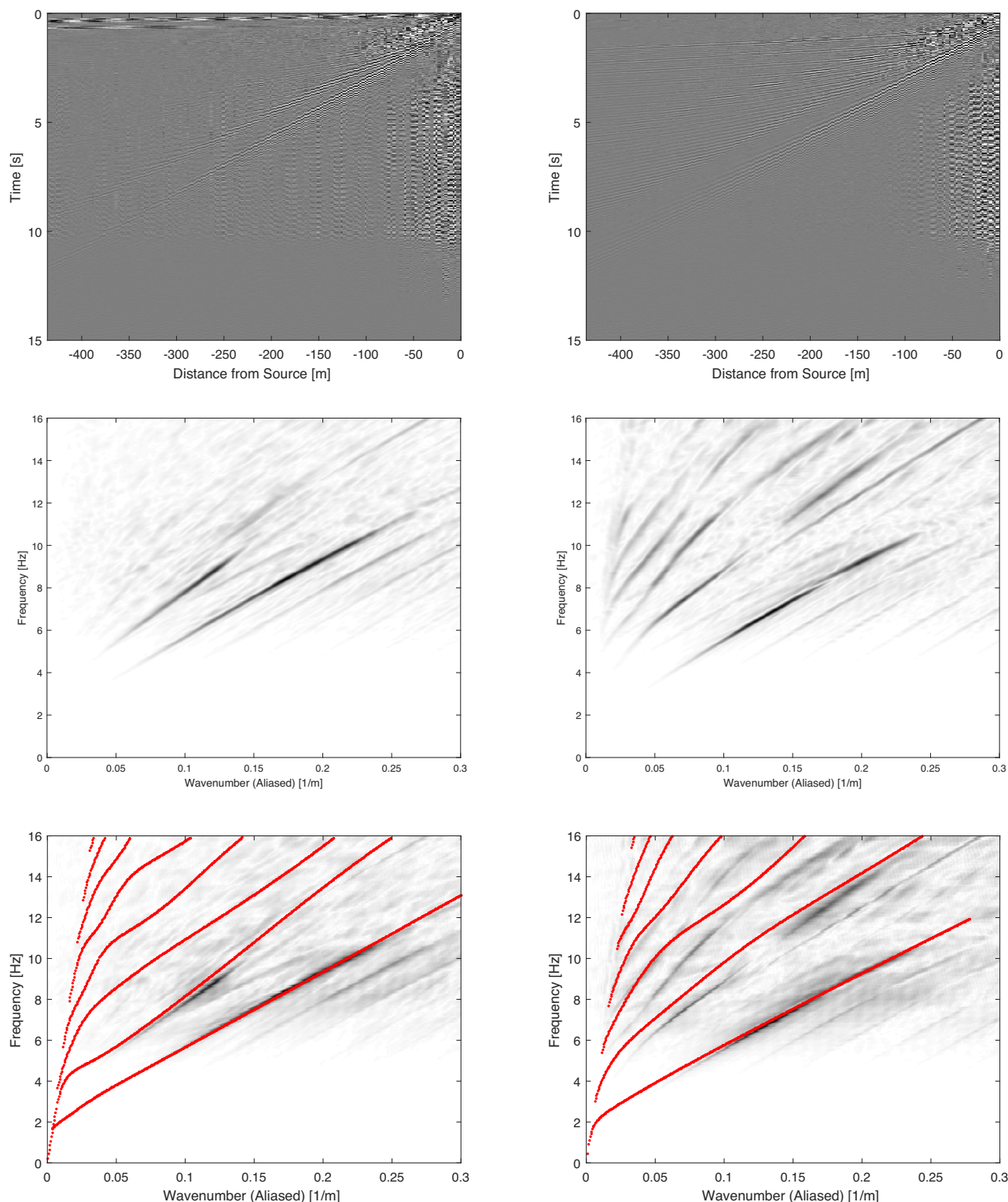


Figure 5. Top panel: measured data in time–space domain (Left-hand panel: $U_z(x, t)$, Right-hand panel: $U_y(x, t)$). Mid panel: measured data in frequency–wavenumber domain (Left-hand panel: $\tilde{U}_z(k_r, \omega)$, Right-hand panel: $\tilde{U}_y(k_r, \omega)$). Bottom panel: model-based modal wavenumber locations of the shear-modulus inversion plotted over measured data. (Left-hand panel: Scholte wave, Right-hand panel: Love wave).

causes a widening of the peaks in the $f-k_r$ domain. The truncation results in an overestimation of the material-damping profile in the inversion. To correct this error and improve the reliability of the damping estimation, the error is estimated using the theoretical

model. The full-waveform response is obtained by convolving the $f-k_r$ domain response signal and the Hankel transform of a boxcar function representing the limited amount of receivers. The full-waveform response is computed in the $f-k_r$ domain with $\Delta k_r = 5$

Table 5. North Sea soil profile C_s mean top 15 per cent and maximum and minimum estimations for the Scholte wave.

Layer	Thickness [m]	C_p [m s ⁻¹]	ρ [kg m ⁻³]	Scholte - C_s		
				Mean [m s ⁻¹]	Min [m s ⁻¹]	Max [m s ⁻¹]
Water	364.6	1500	1025	0	0	0
1	1	1500	1650	44.5	44.1	44.6
2	2	1500	1700	55.2	52.2	52.7
3	4	1500	1800	71.5	71.4	71.8
4	4	1500	1800	126.0	123.5	132.6
5	4	1500	1900	227.4	157.5	321.0
6	4	1500	1900	309.8	218.6	379.6
7	4	1500	2000	302.0	221.0	370.3
8	4	1500	2000	351.9	223.4	422.0
9	4	1500	2100	368.3	278.0	448.9
10	4	1500	2100	401.4	298.2	494.4
11	4	1500	2100	448.8	350.6	533.5
12	4	1500	2100	470.3	334.0	605.2
Half-space	∞	1500	2100	493.9	279.9	665.3

Table 6. Damping inversion settings for the North Sea dataset. SD refers to ‘standard deviation’ indicating that at every update the search boundaries are reset to 1 standard deviation from the mean value based on the top 3 per cent of the population.

N_{iter}	Scholte	
	N_{Mem}	Rerange
12	78125	Yes, with SD

Table 7. Reduced soil profile ζ mean top 3 per cent and maximum and minimum estimations for the Scholte wave inversion.

Layer	Thickness [m]	C_p [m s ⁻¹]	ρ [Mg m ⁻³]	Scholte			
				ζ [per cent]	Min [per cent]	Max [per cent]	
Water	364.6	1500	1.0	0	-	-	-
1	3	1500	1.7	50.5	0.91	0.90	0.92
2	4	1500	1.8	74.5	1.46	1.39	1.49
3	4	1500	1.8	96.8	3.34	3.04	3.49
4	8	1500	1.9	268.3	3.87	3.76	4.04
5	12	1500	2.0	338.5	3.69	3.47	3.99
6	12	1500	2.1	347.2	3.68	3.46	3.90
Half-space	∞	1500	2.1	473.8	3.73	3.55	3.96

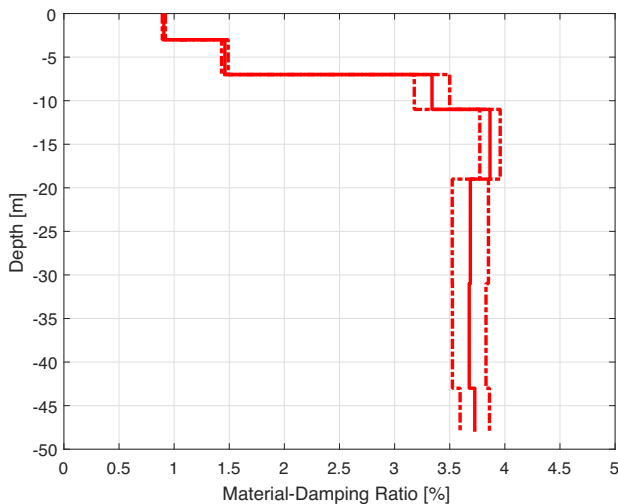


Figure 6. Scholte wave material-damping ratio profile estimate for the North Sea data set.

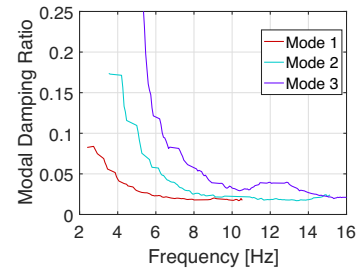


Figure 7. Extracted modal damping curves from the North Sea data set.

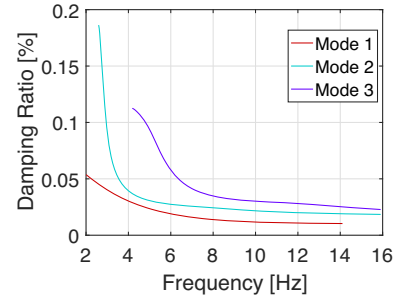


Figure 8. Modal damping curve of the increasing profile.

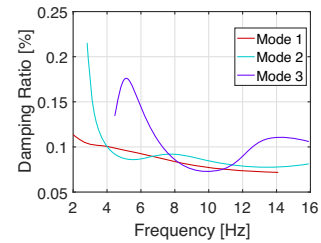


Figure 9. Modal damping curve of the decreasing profile.

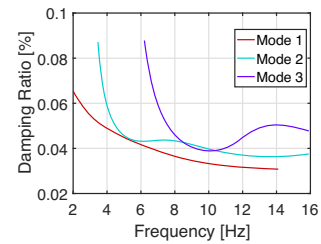


Figure 10. Modal damping curve of the homogeneous profile.

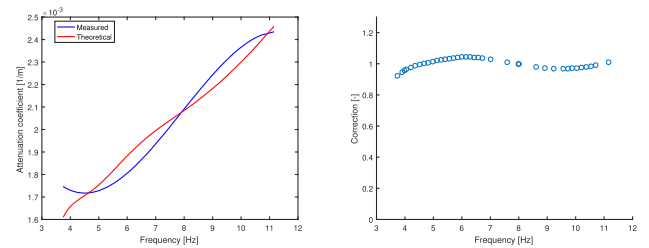


Figure 11. Frequency dependence of the material-damping ratio for Scholte waves. Left-hand panel: comparison between the measured and theoretical attenuation curves for the fundamental mode of the Scholte wave. Right-hand panel: ratio (between the measured and theoretical attenuation curves) versus frequency.

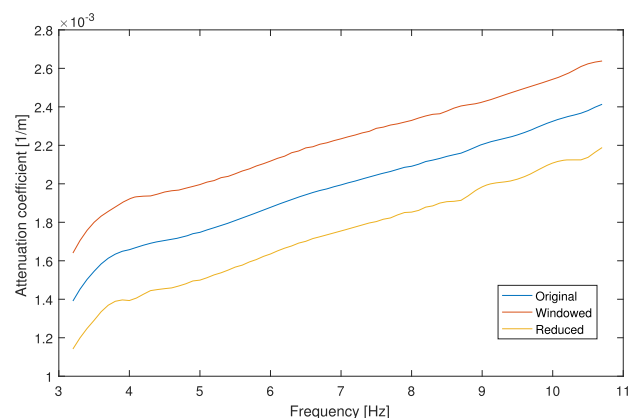


Figure 12. Comparison between the attenuation curves of the fundamental-mode Scholte wave.

$\times 10^{-4} \text{ m}^{-1}$. The Hankel-transformed boxcar function is calculated with the same resolution as it was observed that this is sufficient. Moreover, the wavenumber range was taken six times larger than the full-waveform response to properly incorporate the tails of the curve.

In this analysis, it is assumed that it is possible to estimate the ‘non-truncated’ attenuation curve (in Fig. 12 called ‘reduced’) by evaluating the effect of the windowing on the theoretical model with the profile which has already been estimated without considering the effect of windowing (in Fig. 12 called ‘original’). The increase in measured attenuation, due to windowing effects, is estimated on the theoretical model as the difference between the model ‘original’ curve (blue) and the theoretical ‘windowed’ curve (red). This is then subtracted from the measured curve (also blue) to compute a curve with reduced attenuation values (yellow) that is the estimated true attenuation coefficient of the measured data without spatial truncation. Fig. 12 shows the comparison between the three attenuation curves, where the ‘original’ (blue) curve has a double role representing both the measured attenuation curve, which inherently includes windowing, and the first inversion result of modelled curve, which excludes windowing. The reduced attenuation curve (yellow) is subsequently used as an input for a new damping inversion performed using the Scholte wave model. The results of the inversion are presented in Fig. 13, in which the profile obtained in this section is compared with the one obtained in Fig. 6, where the effect of truncation was not corrected. The new estimate is significantly lower for the entire considered depth.

The mean overestimation is about 32 per cent and it is possible to observe that the highest reduction, in the range 35–47 per cent, is found for the deeper layers. This could have been expected, as truncation affects the longer wavelengths (i.e. lower frequencies) more since long wavelengths require a greater spatial distance to decay to negligible values. The larger amplitudes at the end of the measurement window cause a greater truncation, leading to higher overestimation of the material-damping in the lower layers.

4 CONCLUSIONS

Two different types of surface waves were measured in the North Sea data set: Scholte and Love waves. Using two separate forward models, the dispersion curves and modal wavenumbers of the Scholte and Love waves were computed, which were subsequently used in a two-step inversion to retrieve the dynamic properties of the

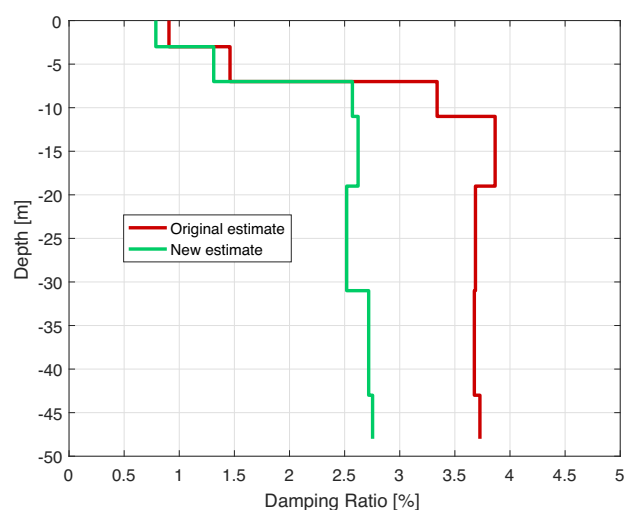


Figure 13. Comparison between the original damping profile and the one (corrected) obtained considering the influence of spatial windowing for Scholte wave inversion.

soil system: shear-modulus and material-damping ratio. The shear-modulus inversion results have shown a good correspondence to previously published results and a perfect alignment of the fundamental mode between modelled and measured dispersion curves as well as a good visual shape agreement for two higher modes. The two shear wave velocity profiles present a similar magnitude and trend, a linear increase of the shear-modulus with depth. The average difference between the two shear wave velocity profiles is lower than 12 per cent. The shear-modulus profile identified from the Scholte wave was the starting point of the material-damping ratio inversion where the estimation was performed for the fundamental mode. This has shown excellent results with an average misfit error for the best-estimate profile lower than 3 per cent in terms of matching the measured attenuation curve. Finally, it was shown that the material damping can, for the current site and conditions, be accurately modelled with a frequency independent assumption. It should be emphasized that the latter result is strictly valid for the considered frequency range (4–11 Hz). For even lower frequencies, which are not easily excited using active sources such as vibrators, the frequency dependence of the *in situ* material-damping ratio could be assessed using surface waves excited by passive noise sources (Weemstra 2013; van Dalen *et al.* 2014, 2015).

Future work should focus, among other things, on the use of the Love waves for material-damping ratio inversion. Incorporating the Love waves in the inversion process may further increase the accuracy of the inversion results. The suppression of the geometrical spreading of the waves will be challenging, but the unique data set is worth the effort.

Finally, this research has broad applicability, both in marine foundations such as offshore wind turbine support structures and in further improving geophysical exploration where characterization of near-surface layers may improve the ability to image deeper layers.

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