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# Adaptive tracking control for a class of disturbed nonlinear systems with unbounded time derivative for disturbance

Xiaojun Duan<sup>1</sup>, Xinmin Dong<sup>1</sup>, Zongcheng Liu<sup>2</sup>, Maolong Lv<sup>3</sup> and Wenqian Zhang<sup>2</sup>

**Abstract**—Adaptive tracking control problem for disturbed nonlinear systems with the time derivative of disturbance being unbounded is investigated in this paper. Different from the existing literatures, a new disturbance observer is constructively proposed with its parameters being functions rather than constants, which results in a new manner for our disturbance observer. The convergence of the new disturbance observer is then proved based on Lyapunov stability theorem. Moreover, it is proved that the tracking error of system can be regulated to arbitrary small by appropriately choosing the design functions and parameters. Finally, simulation results are given to demonstrate the effectiveness of designed method.

## I. INTRODUCTION

In the past years, many efforts have been made to attenuate the influence of uncertainties and disturbances [1-7], which aims at enhancing the robustness of controlled systems. It is well known that external disturbances, unmodeled dynamics and system uncertainties exist in a wide range of real control processes, which may cause the performance degradation and even the instability of the closed-loop control system, thus it is challenging to investigate disturbance estimation and rejection techniques in control systems societies. In [5-7], a typical method using robust adaptive compensator was used to attenuate the disturbances or uncertainties in systems. However, in these methods, the uncertainty or disturbance terms were assumed to be bounded by some unknown constants. Under this condition, fairly good tracking performances were achieved in these references. In order to investigate the unbounded disturbances or uncertainties, the disturbance observers were presented to approximate the disturbances or uncertainties. In [8], a disturbance observer-based dynamic surface control (DSC) approach was studied for the mobile wheeled inverted pendulum system with bounded lumped disturbance vector. In [9], an output feedback adaptive control method was proposed based on disturbance observer for uncertain non-affine nonlinear systems with unknown non-symmetric input saturation. In [10], a robust adaptive compensator was present by observing the unbounded disturbances, which were produced by system

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uncertainties, input nonlinearities and dead-zone. In [11], by introducing error observers, a composite neural dynamic surface control method has been presented to attenuate the influence by neural approximation error. Though these methods have achieved considerable performance for attenuating the disturbances, uncertainties or approximation errors, all the existing works are under the conditions that the disturbances, uncertainties or approximation errors are bounded or their derivatives are bounded. It is commonly seen in all the existing literatures that the derivative of disturbance is assumed to be bounded as long as a disturbance observer is constructed. This restrictive condition possibly severely limits the application of disturbance observer in real control systems.

Motivated by the above discussion, this paper presents a novel disturbance observer design method for the system with unbounded derivative for disturbance. The main contributions are summarized as follows: 1) The derivatives of disturbance, which may include system uncertainties and model errors, are no longer assumed to be bounded. Moreover, the disturbances are then not required to be bounded. A new disturbance observer is presented with the parameters being functions. 2) Based on Lyapunov stability theorem, the tracking error is proved to be able to converge to arbitrary small by appropriately choosing the designed functions and parameters.

The organization of this paper is as follows. The problem description of the disturbed nonlinear systems is addressed in Section 2. The disturbance observers and the corresponding adaptive controllers are designed in Section 3, and the convergence of tracking error is rigorously proved based on Lyapunov stability theorem. In Section 4, simulation examples are performed to demonstrate the effectiveness of the designed scheme. The concluding work is stated in Section 5.

## II. PROBLEM DESCRIPTION

$$\begin{cases} \dot{x}_i = x_{i+1}, \\ \dot{x}_n = f(x) + u + D(x, t) \\ y = x_1 \end{cases} \quad (1)$$

where  $x = [x_1, x_2, \dots, x_n]^T \in R^n$  denotes the state vector of the system;  $u \in R$  and  $y \in R$  are the control input and system output, respectively. The nonlinear function  $f(x)$  is system function.  $D(x, t)$  is the disturbance of system, which may include external or internal disturbances and system uncertainties. The control objective is to design adaptive tracking controller  $u$  such that the system output  $y$  follows

the desired trajectory  $y_d$  and the resulting tracking error can converge to a small neighborhood of the origin by appropriately choosing design parameters.

*Assumption 1* [12]: For the desired trajectory  $y_d$ , we assume that the reference signals  $y_d, \dot{y}_d, \ddot{y}_d, \dots, y_d^{(n)}$  are smooth and bounded, that is, there exists a positive constant  $B_0$  such that  $\Omega_0 := \left\{ (y_d, \dot{y}_d, \dots, y_d^{(n)}) : \sum_{i=0}^n (y_d^{(i)})^2 \leq B_0 \right\}$ .

*Assumption 2*: As for the unknown disturbance  $D(x, t)$ , we assume that there exists a known function  $\phi(x, t)$  such that the following equality holds.

$$\left| \dot{D}(x, t) \right| \leq \phi(x, t) \quad (2)$$

*Remark 1*: It is worth noticing that, the conditions  $|D(x, t)| \leq M_0$  or  $|\dot{D}(x, t)| \leq M_1$  with  $M_0$  and  $M_1$  being unknown positive constants are always used in the existing literatures. Especially,  $|\dot{D}(x, t)| \leq M_1$  is the basic condition for the designing of disturbance observer in the existing literatures, because it guarantees the varying rate of disturbance is large enough to track the variation of disturbance in the existing works, where a disturbance observer is proposed with all parameters being constant. Different from these previous results, it can be seen from Assumption 2 that we made the disturbance term and its derivative to be unbounded, which results from the fact that the function  $\phi(x, t)$  may be unbounded with the growth of  $x$  or  $t$ . Assumption 2 is more reasonable for real control systems, because the disturbance and its derivative can always be unbounded due to unmodeled dynamic and system uncertainties

### III. DISTURBANCE-OBSERVER-BASED ADAPTIVE TRACKING CONTROL

In this section, we will design a disturbance-observer-based adaptive tracking controller for system (1).

To begin with this work, we define

$$e = [e_1, e_2, \dots, e_n]^T, \quad e_i = x_i - y_d^{(i-1)} \quad (3)$$

In accordance with (3), the filtered tracking error of the disturbed nonlinear system (1) is defined as follows

$$s = \left( \frac{d}{dt} + q \right)^{n-1} e_1 = [\lambda_1, \lambda_2, \dots, \lambda_{n-1}, 1]e \quad (4)$$

where  $\lambda_i = C_{n-1}^{i-1} q^{n-i}$ , ( $i = 1, \dots, n-1$ ) and  $q > 0$  are positive constants, specified by designer.

*Remark 2*: It has been shown in [13] and [14] that definition (4) has the following properties:

a)  $s = 0$  defines a time-varying hyperplane in  $R^n$  on which the tracking error  $e_1$  converges to zero asymptotically;

b) If  $|s(t)| \leq C$ ,  $\forall t \geq 0$  with constant  $C > 0$ , then  $e(t)$  is bounded and will converge to  $\Omega_e$ , which is specified as

$$\Omega_e = \{e \mid |e_i| \leq 2^{i-1} q^{i-n} C, \quad i = 1, 2, \dots, n\}, \quad \forall t \geq T_0$$

with  $T_0 \geq 0$  a computable constant.

Subsequently, to confine  $e_1$  to a small neighborhood of origin, the regulation of  $s$  will be investigated in the following.

Differentiating (4) and using (1) and (3), yields

$$\dot{s} = f(x) + u + D(x, t) - y_d^{(n)} + \sum_{i=1}^{n-1} \lambda_i e_{i+1} \quad (5)$$

Consider the stabilization of system (5) and the following quadratic function candidate

$$V_s = \frac{1}{2} s^2 \quad (6)$$

The time derivative of  $V_s$  along (5) is

$$\dot{V}_s = s(f(x) + u + D(x, t)) + sY_d \quad (7)$$

where  $Y_d = -y_d^{(n)} + \sum_{i=1}^{n-1} \lambda_i e_{i+1}$ .

It can be seen from (7) that there is an unbounded unknown term  $D(x, t)$ , which made the control design impossible for system (1). Therefore, we need to design a disturbance observer to estimate it. Since the time derivative of  $D(x, t)$  is bounded by function  $\phi(x, t)$ , not a constant. Accordingly, the nonlinear disturbance observer for it should be designed with varying parameter, which is very different from the existing approaches.

To design the new disturbance observer, an auxiliary variable is introduced as follows:

$$z = D(x, t) - k(x, t)s \quad (8)$$

where  $k(x, t)$  is a design function or so-called varying parameter for the new disturbance observer. The new disturbance observer will be given later.

From (5) and (8), it follows that the time derivative of  $z$  is

$$\begin{aligned} \dot{z} &= \dot{D}(x, t) - k(x, t)\dot{s} - \dot{k}(x, t)s \\ &= \dot{D}(x, t) - k(x, t)(f(x) + u + D(x, t) + Y_d) - \dot{k}(x, t)s \\ &= \dot{D}(x, t) - k(x, t)(f(x) + u + k(x, t)s + z + Y_d) \\ &\quad - \dot{k}(x, t)s \end{aligned} \quad (9)$$

Accordingly, the estimate of immediate variable  $z$  is given as follows

$$\dot{\hat{z}} = -k(x, t)(f(x) + u + k(x, t)s + \hat{z} + Y_d) - \dot{k}(x, t)s \quad (10)$$

with the design function  $k(x, t)$  chosen as

$$k(x, t) = k_0 + \phi(x, t) \quad (11)$$

where  $k_0$  is any positive constant, and  $\hat{z}$  is the estimate of  $z$ .

Then, the new disturbance observer is designed as

$$\hat{D} = \hat{z} + k(x, t)s \quad (12)$$

where  $\hat{D}$  is the estimate of  $D(x, t)$ .

*Remark 3:* It should be noticed that, in the proposed disturbance observer (12), function  $k(x, t)$  and  $\dot{k}(x, t)$  are used to estimate the fast varying disturbance, while these are always constant in the existing methods. Therefore, compared with the traditional disturbance observer, our method can be used for a wider range of disturbed control systems.

Define

$$\tilde{D} = D(x, t) - \hat{D} = z - \hat{z} = \tilde{z} \quad (13)$$

Differentiating (13) and using (9) and (10), yields

$$\dot{\tilde{D}} = \dot{z} - \dot{\hat{z}} = \dot{D}(x, t) - k(x, t)\tilde{D} = \dot{D}(x, t) - (k_0 + \phi(x, t))\tilde{D} \quad (14)$$

Thus, we design the disturbanceobserver-based adaptive tracking controller as follows

$$u = -k_1 s - f(x) - \hat{D} - Y_d \quad (15)$$

where  $k_1$  is a positive design parameter.

So far, we have completed the design of disturbance-observer-based adaptive tracking control for the nonlinear system (1) with disturbance and its derivative being unbounded. Then, we will give the analysis of stability and tracking performance in the following theorem.

*Theorem 1:* Consider the closed-loop system consisting of the disturbed nonlinear system (1) satisfying Assumptions 1-2, the disturbance-observer-based adaptive tracking controller is chosen as (15), and the disturbance observer is designed as (12). Then, for  $k(x, t) = k_0 + \phi(x, t)$ ,  $k_0 > 0$  and  $k_1 = 0.5 + c_1$  with  $c_1$  being any positive constant, the closed-loop system is stability and the following properties are guaranteed: the filtered tracking error  $s$  and tracking error  $e$  will eventually converge to compact sets  $\Omega_c$  and  $\Omega_e$ , respectively, defined by

$$\Omega_c = \{s \mid |s| \leq C\}$$

$$\Omega_e = \{e \mid |e_i| \leq 2^{i-1} q^{i-n} C, \quad i = 1, 2, \dots, n\}$$

where  $C > 0$  is a constant related to the design parameters.

*Proof:* To analyze the stability of tracking error of the closed-loop system firstly, we consider the convergence of  $\tilde{D}$  by using the following Lyapunov function candidate:

$$V_{\tilde{D}} = \frac{1}{2} \tilde{D}^2 \quad (16)$$

By noting (14), the time derivative of  $V_{\tilde{D}}$  is

$$\dot{V}_{\tilde{D}} = \tilde{D}\dot{\tilde{D}}(x, t) - (k_0 + \phi(x, t))\tilde{D}^2 \quad (17)$$

From (2), we have

$$\left| \dot{D}(x, t) \right| \leq \phi(x, t) \quad (18)$$

It follows from (17) and (18) that

$$\begin{aligned} \dot{V}_{\tilde{D}} &= -\frac{k_0 + \phi(x, t)}{2} \tilde{D}^2 - \left( \frac{k_0 + \phi(x, t)}{2} \tilde{D}^2 - \tilde{D}\dot{D}(x, t) \right) \\ &\leq -\frac{k_0 + \phi(x, t)}{2} \tilde{D}^2 - \left( \frac{k_0 + \phi(x, t)}{2} \tilde{D}^2 - \left| \tilde{D}\dot{D}(x, t) \right| \right) \end{aligned} \quad (19)$$

It is easily known that, if  $\left| \tilde{D} \right| \geq \frac{2|\dot{D}(x, t)|}{k_0 + \phi(x, t)}$ , then

$$\frac{k_0 + \phi(x, t)}{2} \tilde{D}^2 - \left| \tilde{D}\dot{D}(x, t) \right| \geq 0 \quad (20)$$

and therefore we have

$$\dot{V}_{\tilde{D}} \leq -\frac{k_0 + \phi(x, t)}{2} \tilde{D}^2 \leq 0 \quad (21)$$

This fact implies the following equality always holds

$$\left| \tilde{D} \right| \leq \frac{2 \left| \dot{D}(x, t) \right|}{k_0 + \phi(x, t)} \quad (22)$$

which can be further rewritten as

$$\left| \tilde{D} \right| \leq \frac{2\phi(x, t)}{k_0 + \phi(x, t)} = 2 \left( 1 - \frac{k_0}{k_0 + \phi(x, t)} \right) \leq 2 \quad (23)$$

In the above equality, (18) is used.

Considering (7) and (15), the time derivative of  $V_s$  is

$$\dot{V}_s = -k_1 s^2 + s\tilde{D} \quad (24)$$

Noting (24), we have

$$\dot{V}_s \leq -k_1 s^2 + \left| s\tilde{D} \right| \quad (25)$$

By using the completion of squares, we have

$$s\tilde{D} \leq \frac{s^2}{2} + \frac{\tilde{D}^2}{2} \quad (26)$$

Using (23) and (26), we can rewrite (25) as

$$\begin{aligned} \dot{V}_s &\leq -k_1 s^2 + \frac{s^2}{2} + \frac{\tilde{D}^2}{2} \\ &\leq -c_1 s^2 + \frac{\tilde{D}^2}{2} \\ &\leq -\alpha V_s + 2 \end{aligned} \quad (27)$$

where  $\alpha = 2c_1$ .

Integrating (27) over  $[0, t]$ , we have

$$V_s(t) \leq (V_s(0) - \phi) e^{-\alpha t} + \phi \quad (28)$$

where  $\phi = 2/\alpha$ . From (28), it is known that  $V_s$ ,  $s$  and  $\tilde{D}$  are bounded. Considering the definition of  $V_s$  in (6) and applying (28), the following inequality hold:

$$\frac{1}{2} s^2 = V_s(t) \leq V_s(0) + \phi \quad (29)$$

which implies that

$$\left| s \right| \leq \sqrt{2(V_s(0) + \phi)} \quad (30)$$

Therefore, it follows from (30) and Remark 2 that  $e_1, e_2, \dots, e_n$  are bounded.

In addition, according to (6) and (28), the following inequality holds:

$$\lim_{t \rightarrow \infty} |s| \leq C \quad (31)$$

where  $C = \sqrt{2\varphi}$ . It can be concluded from Remark 2 that  $s$  and  $e$  will eventually converge to compact sets  $\Omega_c$  and  $\Omega_e$ . Since the size of  $\varphi$  can be minimized by increasing the design parameters  $k_0$  and  $k_1$ , the compact sets  $\Omega_c$  and  $\Omega_e$  can be made as small as desired by appropriately choosing the design parameters. This completes the proof.

*Remark 4:* According to (10) and (11), it is easy to find that  $\dot{k}(x, t)$  exist in the disturbance observer. For the case that  $\dot{k}(x, t)$  is constant, the derivatives are easy to obtain. If  $\dot{k}(x, t)$  is not constant, we can use a first order sliding mode differentiator present in [15] to approach the value of  $\dot{k}(x, t)$ , which can almost preserve the tracking performance of the designed closed-loop system.

#### IV. SIMULATION RESULTS

In this section, two simulation examples are given to demonstrate the effectiveness of the designed method.

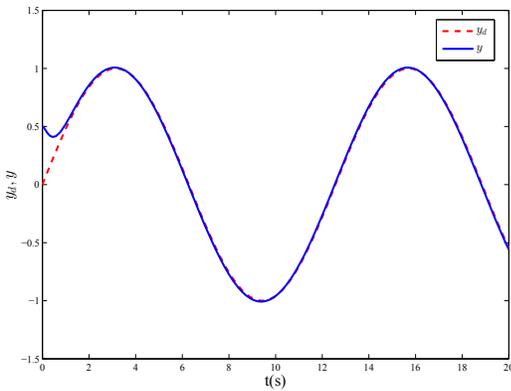


Fig. 1. System output  $y$  and desired trajectory  $y_d$  of Example 1

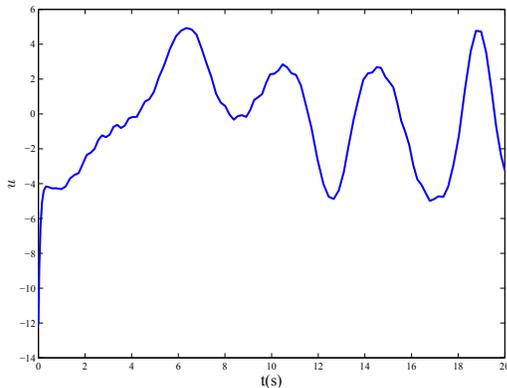


Fig. 2. Control input  $u$  of Example 1

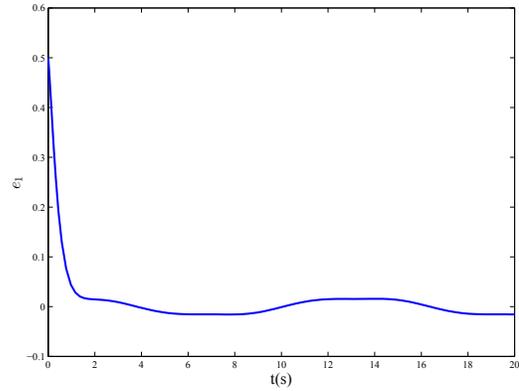


Fig. 3. Tracking error  $e_1$  of Example 1

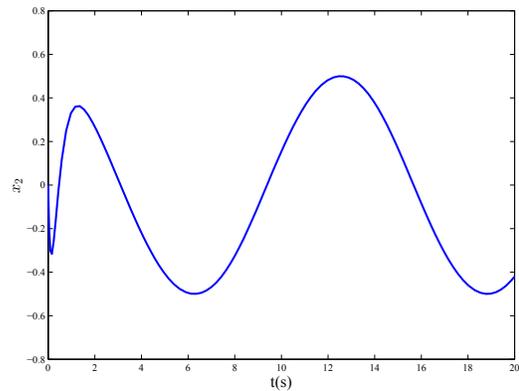


Fig. 4. System state  $x_2$  of Example 1

**Example 1:** To illustrate the validity of the proposed control scheme, consider a class of Duffing-Holmes chaotic systems with disturbance and its derivative being unbounded as follows:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f(x, t) + u + D(x, t) \\ y = x_1 \end{cases} \quad (32)$$

with

$$f(x, t) = -p_1 x_1 - p_2 x_2 - x_1^3 + q \cos(\omega t)$$

$$D(x, t) = x_1^3 + x_1$$

where  $D(x, t)$  is the disturbance. Then, we choose the design function  $k(x, t) = (3x_1^2 + 1)(x_2^2 + 1) + 2$ . According to our method, the nonlinear disturbance observer is designed as

$$\hat{D} = \dot{\hat{z}} + k(x, t)s$$

$$\dot{\hat{z}} = -k(x, t)(f(x) + u + k(x, t)s + \hat{z} + Y_d) - \dot{k}(x, t)s$$

and the disturbance-observer-based adaptive tracking controller is designed as

$$u = -5s - f(x, t) - \hat{D} - Y_d$$

where  $Y_d = -y_d^{(2)} + 3e_2$ .

For the purpose of simulation, we suppose that  $p_1 = 0.3 + 0.2 \sin(10t)$ ,  $q_0 = 5 + 0.1 \cos(t)$ ,  $p_2 = 0.2 + 0.2 \cos(5t)$  and  $\omega = 0.5 + 0.1 \sin(t)$ . The initial conditions are set as:  $[x_1(0), x_2(0)] = [0.5, 0]$ ,  $\hat{D}(0) = \hat{z}(0) = 0$ . Let the desired trajectory be  $y_d = \sin(0.5t)$ . The simulation results are reported in Fig. 1-4. From Fig. 1, it can be seen that the system output  $y$  tracks  $y_d$  very well. Fig. 2-3 show the control input  $u$ , tracking error  $e_1$  and system state  $x_2$ , respectively.

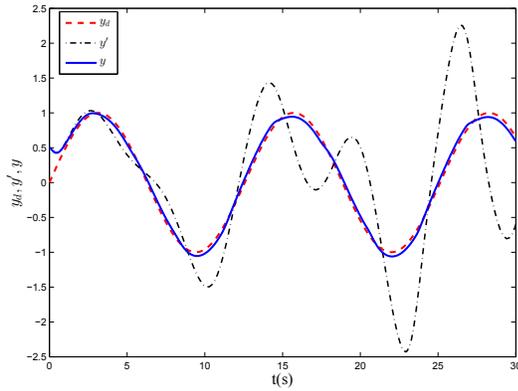


Fig. 5. System output  $y$ ,  $y'$  and desired trajectory  $y_d$  of Example 2

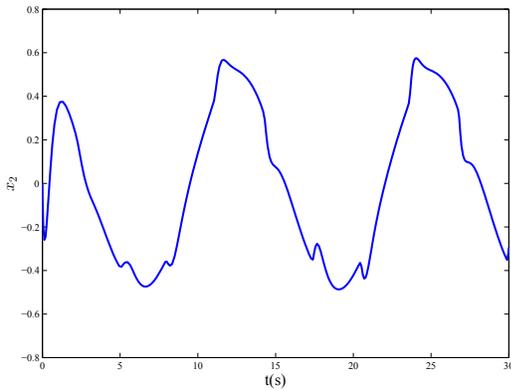


Fig. 6. System state  $x_2$  of Example 2

Example 2: To further show the superior tracking performance of the proposed method, we using the model (32) with an unbounded derivative for disturbance as follows

$$D(x, t) = x_1^3 + x_1 + 2t \sin(t)$$

The initial conditions and other parameters are the same as Example 1. It can be easily verified that  $|\dot{D}(x, t)|$  grows

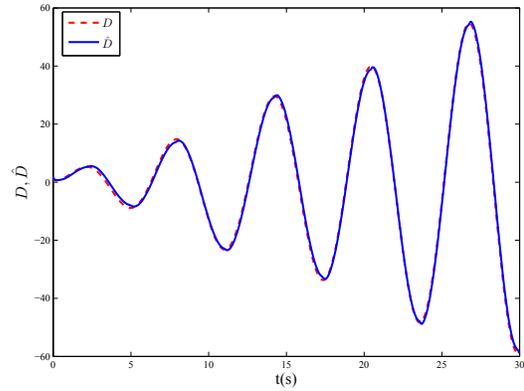


Fig. 7. Disturbance  $D$  and its estimate  $\hat{D}$  of Example 2

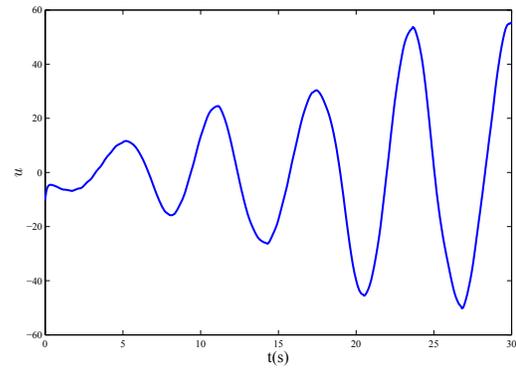


Fig. 8. Control input  $u$  of Example 2

with time  $t$ , and therefore it is unbounded. Then, we choose the design function  $k(x, t)$  as

$$k(x, t) = 2 + \phi(x, t) = 2 + \theta(x, t) \tanh \theta(x, t)$$

with

$$\theta(x, t) = 3x_1^2 x_2 + x_2 + 2 \sin(t) + 2t \cos(t)$$

According to our method, the nonlinear disturbance observer is designed as

$$\hat{D} = \hat{z} + k(x, t)s$$

$$\dot{\hat{z}} = -k(x, t)(f(x) + u + k(x, t)s + \hat{z} + Y_d) - \dot{k}(x, t)s$$

and the disturbance-observer-based adaptive tracking controller is designed as

$$u = -5s - f(x, t) - \hat{D} - Y_d$$

where  $Y_d = -y_d^{(2)} + 3e_2$ .

In the traditional disturbance observer design method,  $k(x, t)$  is always chosen as a constant. For comparison,

we use the traditional-disturbance-observer-based adaptive tracking controller for this example with the same conditions of our method. The simulation results are present in Fig. 5-8. The comparison results of traditional-disturbance-observer-based method and our method are shown in Fig. 5, where  $y'$  is system output of traditional-disturbance-observer-based method and  $y$  is system output of our method. It can be seen from Fig. 5 that the tracking performance of our method is much better than the traditional one, since the system output  $y$  tracks  $y_d$  very well even in the case that the derivative of disturbance grows with time  $t$ . Fig. 6-8 show the system state  $x_2$ , disturbance  $D$  and its estimate  $\hat{D}$ , and control input  $u$  of our method, respectively. From Fig. 7, it can be seen that the designed disturbance observer in our method can approach the growing disturbance.

## V. CONCLUSIONS

A novel disturbance observer is designed for a class of disturbed nonlinear systems with the disturbance and its time derivative being unbounded functions. In the sequel, a disturbance-observer-based adaptive tracking control is then proposed. The new disturbance observer is designed with parameters being functions, which is critical to solve the problem of fast variation for disturbance. Based on Lyapunov stability theorem, it is proved that, under the action of the new disturbance observer, the disturbance can be estimated and therefore the tracking error of system can be regulated to arbitrary small. Finally, two simulation examples are presented to verify the effectiveness of proposed approach. It can be seen from the simulation results that the tracking performances of the new disturbance observer and the disturbance-observer-based adaptive tracking controller are very well.

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