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CROSTRAN - A model for beach profile changes due to cross-shore sediment transport under random breaking waves

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CONTENTS

Abstract

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In the first phase of the detailed modelling of cross-shore sediment transport under random waves a model is constructed which adopts a vertically integrated transport description for sheetflow situations. The formulation of the transport as a function of the instantaneous velocity field is based on the approach of Bailard (1981). This approach assumes in essence simply that the instantaneous transport is proportional with some power of the instantaneous near-bottom velocity. Implementation of this transport description in a time-dependent model requires a formulation of the time-mean and some low order moments of the near-bottom velocity field. An ad-hoc formulation based on a monochromatic, second order Stokes wave representation is presented. The numerical research model CROSTRAN (acronym for cross-shore sediment transport) is based on the above formulations. Here the model is described and limitedly checked on its performance on the basis of available data. Some consequences for further study are indicated.

1. Introduction

The particular role of a nearly two-dimensional wave motion in the movement of sediment normal to the shore is poorly understood. It is generally assumed that a number of interaction mechanisms between this wave motion and the sediment motion contribute to the formation of the beach profile, also in the three-dimensional topographies that occur on a natural coast. Full account of all mechanisms can be taken when a description of both the horizontal velocity field, $u(x,z,t)$, and the sediment concentration field, $c(x,z,t)$, in space and time is available, so that the net cross-shore sediment transport, $\langle q(x)\rangle$, may be calculated from

$$\langle q(x)\rangle = \langle \int_d u(x,z,t).c(x,z,t)\ dz\rangle \qquad (1)$$

where the integration is performed over the instantaneous depth d and the brackets indicate time averaging. From the cross-shore variation of $\langle q(x)\rangle$ the bottom changes may be derived.

Visual and experimental observation of random waves on a two-dimensional beach indicates that one of the more important mechanisms under active surf conditions may be the transport of sediment by the time mean, seawards directed flow near the bottom induced by the breaking of waves. It was shown (Stive and Battjes, 1984) that this mechanism is so dominant that a vertically integrated model incorporating this mechanism alone describes the bottom variations in the surf zone to a satisfactory, first approximation. Extension of this model with other transport mechanisms is a logical step towards a more complete cross-shore sediment transport model. Here some first suggestions are made to extend the model with transport due to the asymmetry of the wave motion.

2. Transport formulation

In principle the net cross-shore sediment transport may be calculated from Equation (1). There are, however, two reasons persuading us to rely on a simplified, vertically integrated form of Equation (1). Firstly, our knowledge of the velocity and concentration field in time and space is very limited. Secondly, a simpler - but qualitative correct - formulation of the sediment transport provides a better insight in the mechanisms. Since we are interested in a transport formulation which takes also the effects of wave asymmetry into account, it is essential to adopt a formulation describing the transport instantaneously. A simple approach would be to assume that the instantaneous sediment transport rate, q, is proportional to some power of the local relative velocity between the bed and the fluid outside the boundary layer. For example,

$$q(t) = A\, u(t)\,|u(t)|^n \tag{2}$$

where $u(t) = u_b \cos\omega t$ with u_b the orbital velocity amplitude just outside the boundary layer and ω the angular frequency.

The latter approach has been elaborated consistently for surf zones on a plane sloping beach by Bailard (1981), who extended the work of Bailard and Inman (1981). Based on Bagnold's (1963) energetics concept these authors use as a starting point a description of the instantaneous sediment transport basically in the form of Equation (2), extended with the effect of a bottom slope. Bailard (1981) distinguishes between bedload transport in a granular-fluid shear layer of a thickness in the order of the wave boundary layer and suspended transport in a layer of greater thickness, typically in the order of several centimeters. For the bedload transport the power n as introduced by equation (2) is given by Bailard (1981) as 2, while for the suspended transport it is given as 3. Here his general two-dimensional horizontal formulation is reduced for application in the cross-shore direction which yields the instantaneous total load sediment transport equation (see also Bailard, 1982):

$$i(t) = i_B(t)+i_S(t) = \rho c_f \frac{\varepsilon_B}{\tan\phi}\left[|u(t)|^2\, u(t) - \frac{\tan\beta}{\tan\phi}\,|u(t)|^3\right] + \rho c_f \frac{\varepsilon_S}{w}\left[|u(t)|^3\, u(t) - \frac{\varepsilon_S}{w}\tan\beta\,|u(t)|^5\right] \qquad (3)$$

where i is the total cross-shore immersed weight sediment transport rate (composed of the bedload transport rate, i_B, and the suspended load transport rate, i_S), ρ is the water density, c_f is the drag coefficient for the bed, $\tan\beta$ is the slope of the bed, ϕ is the internal angle of friction of the sediment, w is the sediment's fall velocity and ε_B and ε_S are bedload and suspended load efficiencies, respectively. The efficiency factors ε_B and ε_S denote those (constant) fractions of the total power produced by the fluid motion which are expended in transporting. The immersed weight sediment transport rate is linked to the volumetric transport rate by

$$q = \frac{i}{(\rho_s-\rho)gN} \qquad (4)$$

where ρ_s is the sediment density, g the gravitational acceleration and N the local volume concentration of solids.

The above sediment transport formulation uses vertically integrated equations. As a consequence, the sediment transports are assumed to respond to the near bottom water velocity in an instantaneous, quasi-steady manner. This assumption is probably valid for bedload transport on a flat bed (except for a phase lag which is neglected for simplicity) because the bedload layer has a small thickness and it can respond quickly to the instantaneous shear stress. The suspended sediment transport, however, is distributed over a layer thickness of several centimeters. The characteristic time constant for this layer is the ratio of its thickness and the sediment fall velocity which is typically in the order of 1-2 seconds. For most natural beaches with prevailing plane bed conditions and incident wave periods of 5-10 seconds, it appears that the quasi-steady assumption is reasonable.

Another uncertainty in the transport formulation concerns the use of bedload and suspended load efficiency factors. Although constant values have been found adequate for certain types of flow (see Table 1), their variations with the type of flow considered leaves at least some quantitative uncertainty.

efficiency factor	steady stream flow (Bagnold, 1966)	longshore current flow (Bailard, 1981)	cross-shore current flow (Bailard, 1982)
ε_B	0.13	0.21	0.10
ε_S	0.01	0.025	0.020

Table 1 Estimates of the efficiency factors

For our present purpose, however, it is sufficient that the processes under consideration are described in a qualitative sense. A limited check on this is made in appendix A where the bedload and the suspended load due to random breaking waves as calculated with the present transport formulation is compared with the mean total load as measured in a laboratory flume. The results confirm that at least qualitatively there is a satisfactory agreement.

3. The cross-shore velocity field

Given the variation of the cross-shore velocity field the mean cross-shore sediment transport rate may in principle be calculated from the time averaged Equation (3):

$$\langle i \rangle = \rho\, c_f \frac{\varepsilon_B}{\tan\phi} [\langle |u|^2\, u\rangle - \frac{\tan\beta}{\tan\phi} |u|^3] +$$
$$\rho\, c_f \frac{\varepsilon_S}{w} [\langle |u|^3\, u\rangle - \frac{\varepsilon_S}{w} \tan\beta\, |u|^5] \qquad (5)$$

where the total velocity u is composed of a mean (overbar) and an oscillatory (tilde) flow component.:

$$u = \bar{u} + \tilde{u} \qquad (6)$$

Thus, the problem to be evaluated here is how to predict the cross-shore variation of the velocity moments appearing in Eq. (5).

Conceptual simplifications follow by assuming that the oscillatory velocity is due to a single plane wave of frequency ω and some small nonlinear harmonics:

$$\tilde{u} = u_m \cos \omega t + u_{2m} \cos 2\omega t + u_{3m} \cos 3\omega t + \ldots \qquad (7)$$

in which $u_m > u_{2m} > u_{3m} > \ldots$

Using Equations (6) and (7) in Equation (5) yields:

$$\langle i \rangle = \rho\, c_f\, u_m^3 \frac{\varepsilon_B}{\tan\phi} [\psi_1 + \frac{3}{2}\delta_u - \frac{\tan\beta}{\tan\phi} (u3)^*] +$$
$$\rho\, c_f\, u_m^4 \frac{\varepsilon_S}{w} [\psi_2 + \delta_u (u3)^* - \frac{u_m}{w} \varepsilon_S \tan\beta\, (u5)^*] \qquad (8)$$

in which the relative current strength, δ_u, is

$$\delta_u = \bar{u}/u_m \qquad (9)$$

and the odd velocity moments, ψ_1 and ψ_2, are:

$$\psi_1 = \langle \tilde{u}^3 \rangle / u_m^3 \tag{10a}$$

$$\psi_2 = \langle |u|^3 \; \tilde{u} \rangle / u_m^4 \tag{10b}$$

The even velocity moments $(u3)^*$ and $(u5)^*$ are defined as:

$$(u3)^* = \langle |u|^3 \rangle / u_m^3 \tag{11a}$$

$$(u5)^* = \langle |u|^5 \rangle / u_m^5 \tag{11b}$$

Retaining first order in the relative current strength and odd moments only three velocity moments may be simplified further, i.e.

$$u_m^4 \; \psi_2 \simeq 2\bar{u} \langle |\tilde{u}|^3 \rangle + \langle |\tilde{u}|^3 \; \tilde{u} \rangle \tag{12}$$

and $u_m^3 (u3)^* \simeq |\bar{u}| \langle \tilde{u}^2 \rangle + \langle |\tilde{u}|^3 \rangle$ (13a)

$$u_m^5 (u5)^* \simeq |\bar{u}| \langle \tilde{u}^4 \rangle + \langle |\tilde{u}|^5 \rangle \tag{13b}$$

Inspection of the above expressions indicates that the following low order velocity moments are of importance:

- the four lowest even moments $\langle \tilde{u}^2 \rangle$, $\langle |\tilde{u}|^3$, $\langle \tilde{u}^4 \rangle$, $\langle |\tilde{u}|^5 \rangle$, which are non zero for symmetric velocities,
- the two lowest odd moments $\langle \tilde{u}^3 \rangle$, $\langle |\tilde{u}|^3 \; \tilde{u} \rangle$, which are zero for symmetric velocities.

The latter moments are the most difficult to estimate: they are nonzero only for nonlinear waves that actually occur nearshore. The shoreward velocities are typically stronger and of shorter duration than the offshore flows, leading to nonzero values for the odd moments. Calculation of the odd moments requires a nonlinear wave shoaling and decay model.

A theoretical evaluation of the even moments for both a monochromatic, linear sea (sinusoidal model) and a random, linear sea (Gaussian model) is given by Guza and Thornton (1985). The theoretical moments are compared to field observations from the NSTS study. A summary of observations and theory for the several cross-shore moments is given in Table 2 below. The moments are normalized by the local variance.

moment	observations Nov. 17	Nov. 20	theory Gaussian	sinusoid
$\langle\lvert\tilde{u}\rvert^3\rangle/\langle\tilde{u}^2\rangle^{3/2}$	1.60	1.69	1.60	1.20
$\langle\tilde{u}^4\rangle/\langle\tilde{u}^2\rangle^2$	2.86	3.50	3.00	1.50
$\langle\lvert\tilde{u}\rvert^5\rangle/\langle\tilde{u}^2\rangle^{5/2}$	7.77	8.58	6.38	1.92
$\langle\tilde{u}^3\rangle/\langle\tilde{u}^2\rangle^{3/2}$	0.55	0.50	0	0
$\langle\lvert\tilde{u}\rvert^3\tilde{u}\rangle/\langle\tilde{u}^2\rangle^2$	~1.20	~1.20	0	0
$\langle\tilde{u}^5\rangle/\langle\tilde{u}^2\rangle^{5/2}$	4.95	5.39	0	0

Table 2 Observed and theoretical velocity moments (after Guza and Thornton, 1985)

The above results indicate that even moments do not critically depend on cross-shore velocity asymmetry. This is due to the fact that also for symmetric velocities these terms are nonzero. At the present stage we will therefore rely on the Gaussian estimates for the even moments. The odd moments are zero for a symmetric velocity field, but can be nonzero for asymmetric (nonlinear) motions. Here we suggest the following ad-hoc formulation.

As indicated above calculation of the odd velocity moments requires a shoaling and decay model which predicts certain nonlinear properties of the presently considered random, breaking waves. A relevant nonlinear property is the asymmetry of the wave surface about the horizontal axis. For non-breaking waves this asymmetry may to a first approximation well be predicted on the basis of a horizontal bottom, nonlinear wave theory, assuming that due to gradual bottom variations the waves locally behave as on a horizontal bottom (see Flick, et al., 1981). However, in the horizontal bottom, nonlinear wave theories the phases of the harmonics are locked to zero and there is no vertical wave profile asymmetry possible. This asymmetry about the vertical plane is an essential property of the sawtooth shaped breaking waves in the surf zone. These theories are deficient in this respect and thus particularly unsuitable for calculations of odd velocity moments which depend critically on phase. To illustrate this we calculate the two lowest order odd moments assuming that the velocity fluctuation is described by a second order approximation with a locked but nonzero phase between the two components:

$$\tilde{u} = u_m \cos \omega t + u_{2m} \cos (2\omega t + \phi_2) \tag{14}$$

in which $u_m > u_{2m}$. After some algebraic manipulation it may be show that to lowest order the two odd velocity moments are given by:

$$\langle \tilde{u}^3 \rangle = \frac{3}{4} u_m^2 u_{2m} \cos \phi_2 \tag{15a}$$

$$\langle |\tilde{u}|^3 \tilde{u} \rangle = \frac{12}{5\pi} u_m^3 u_{2m} \cos \phi_2 \tag{15b}$$

An interesting perspective now arises when we combine these results with the following observations. In the inner surf zone where the breaking waves are quasi-steady the relative phase of the second harmonic increases smoothly toward the asymptotic value (see Flick et al., 1981):

$$\phi_2 \rightarrow \pi/2 \tag{16}$$

Thus, according to Eq. 15a, 15b, the odd velocity moents for breaking waves vanish ultimately.

At this point we may formulate an ad-hoc wave decay model which predicts linear and nonlinear properties necessary to derive the velocity moments. As a starting point Battjes and Janssen's (1978) wave decay model is adopted to predict the variance of the wave elevation in cross-shore direction. The propagation properties of this model are linear; the dissipation process due to breaking is based on a Gaussian wave description. Given the wave variance variation linear theory may be applied to provide the variation of the near-bottom velocity variance and thus the even velocity moments. In the random wave model there is a gradual transition in the breaking fraction of the wave field on a beach of monotoneously decreasing depth. Without the risk of discontinuities we may therefore safely estimate the odd velocity moments from the nonbreaking fraction of waves only and assume that the contribution of the breaking waves is negligible in view of the above conclusions. To provide results from this model we use the second order Stokes expansion with

$$\tilde{u} = u_m \cos \omega_p t + \frac{3}{4} \frac{u_m^2}{c} \sinh^{-2}(k_p h) \cos 2\omega_p t \qquad (17)$$

and choose $u_m = u_{rms}$ from the consideration that the monochromatic representation of the random wave field should have to same variance.

A more detailed description of this wave height decay and partly nonlinear kinematics model is given in Appendix C. Here we conclude with a comparison between observations of the undertow, the velocity variance and the skewness (i.e. the first odd velocity moment normalized by the variance, $\langle\tilde{u}^3\rangle/\langle\tilde{u}^2\rangle^{3/2}$) and calculations with the present model (see Figure 1 and 2). The observations

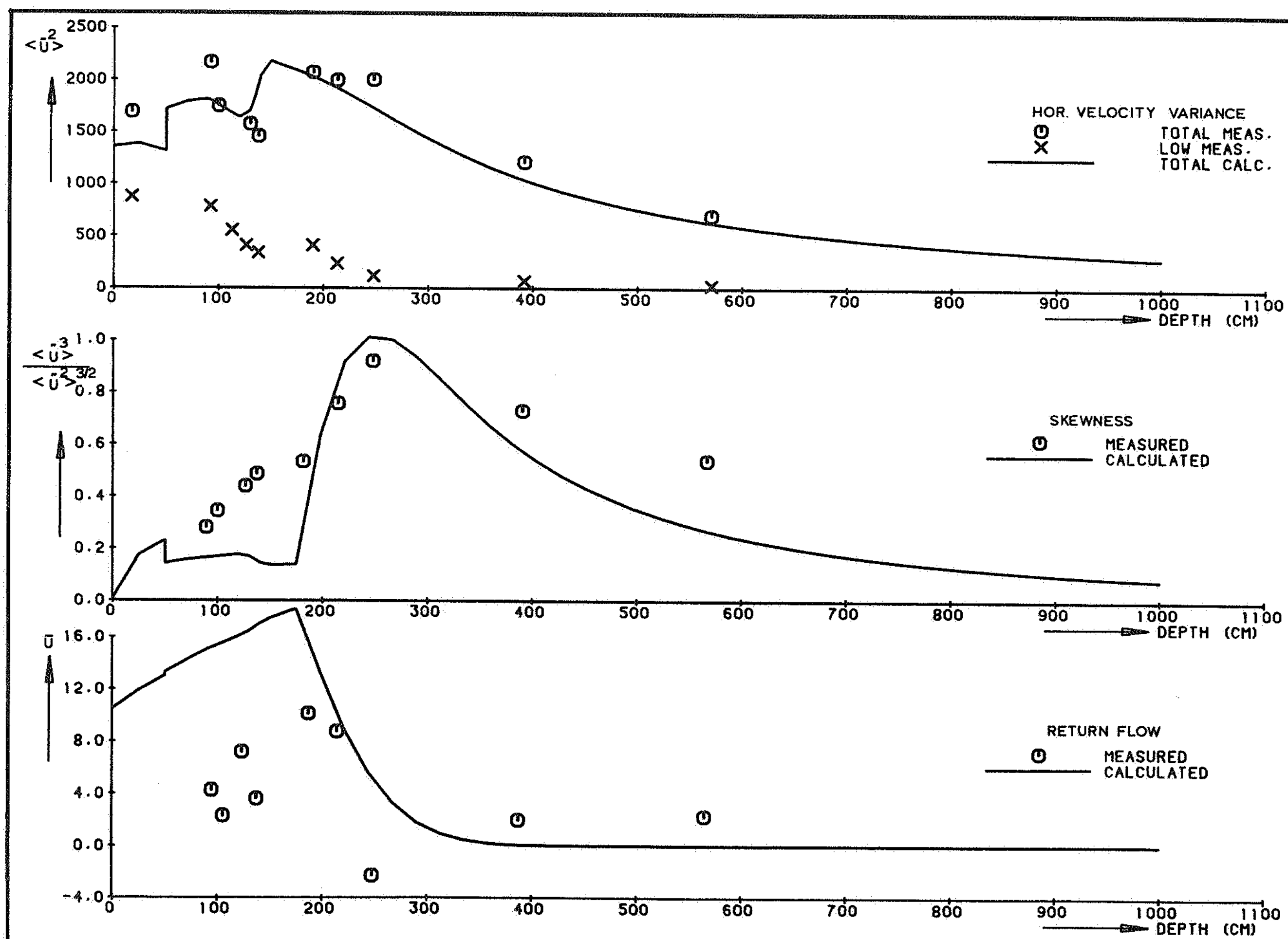


Fig. 1 Cross-shore velocity characteristics NSTS Torrey Pines measurements November 20 (after Guza and Thornton, 1985) compared to present theory

are by Guza and Thornton (1985) and Elgar and Guza (1985) and concern rather long wave conditions. The comparison shows that qualitatively the predictions are reasonable; quantitavely there are discrepancies indicating that improvements should be made. It is advised to extend the comparison with more narrow banded, wind wave data, eg. those of Egmond and new laboratory measurements. Appendix B describes which moments should be derived when field measurements are considered.

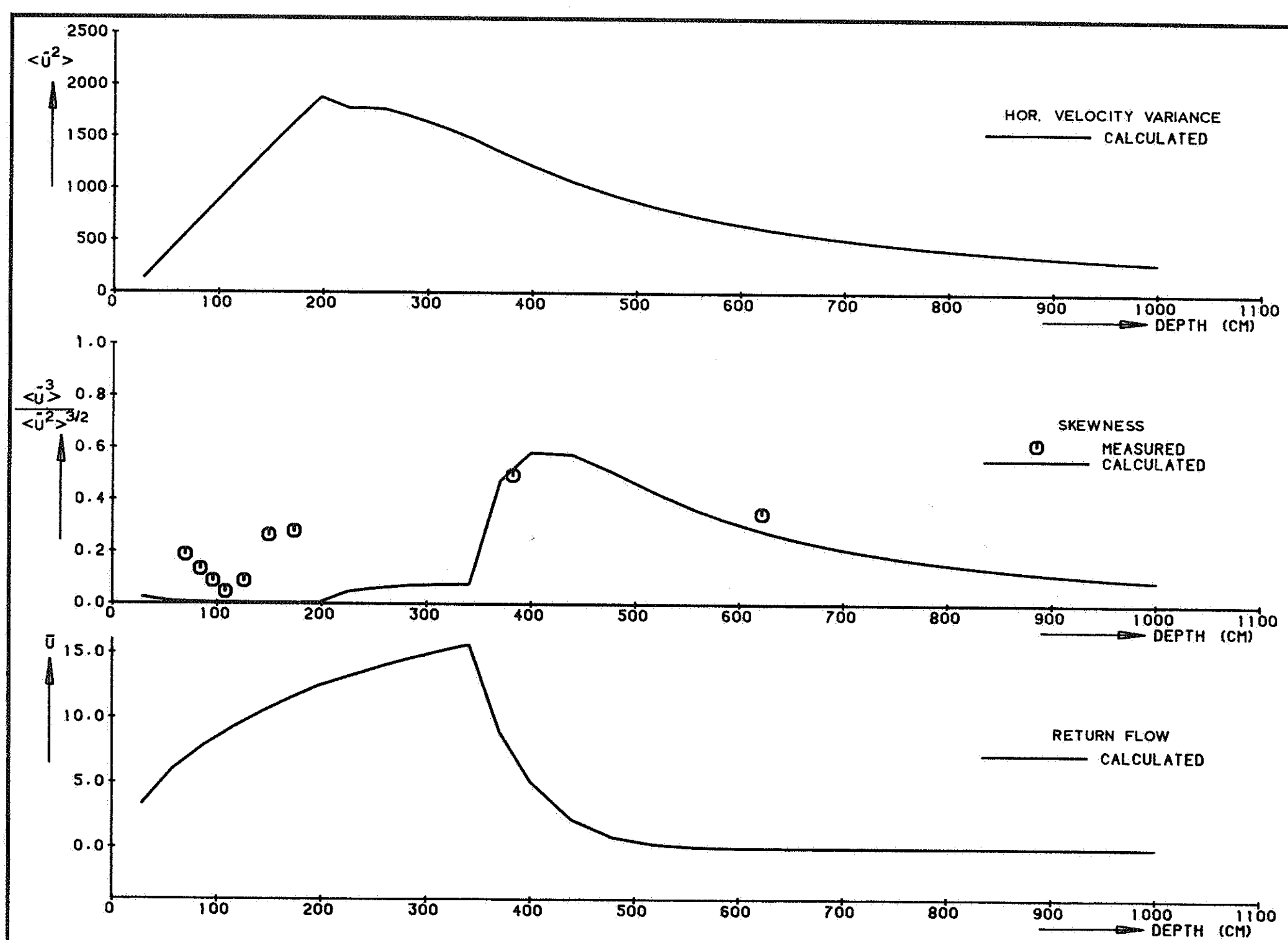


Fig. 2 Cross-shore velocity characteristics NSTS Santa Barbara measurements June 3 (after Elgar and Guza, 1985) compared to present theory

4. The computation of transport and bottom changes

In CROSTRAN the local mean, volumetric cross-shore sediment transport rate, $\langle q \rangle$, is calculated according to the following expressions, where use has been made of expressions (4) and (8)...(12):

$$\langle q \rangle = B_{as} \langle q_{as} \rangle + B_{un} \langle q_{un} \rangle - B_{sl} \langle q_{sl} \rangle \tag{18a}$$

$$\langle q_{as} \rangle = F_B \psi_1 + F_S \psi_2 \tag{18b}$$

$$\langle q_{un} \rangle = F_B \frac{3}{2} \delta_u + F_S 3\delta_u (u3)^* \tag{18c}$$

$$\langle q_{sl} \rangle = F_B \frac{\tan \beta}{\tan \phi} (u3)^* + F_S \frac{u_m}{w} \varepsilon_S \tan \beta (u5)^* \tag{18d}$$

$$F_B = \frac{c_f u_{rms} \varepsilon_B}{\Delta g N \tan\phi} \tag{18e}$$

$$F_S = \frac{c_f u_{rms} \varepsilon_S}{\Delta g N w} \tag{18f}$$

Here c_f is the drag coefficient equal to $\frac{1}{2} f_w$ with f_w the friction factor as defined in Stive and Battjes (1984) and B_{as}, B_{un} and B_{sl} are proportionality constants which should be $O(1)$ if the description is right. The free parameters in the above expressions are ε_B and ε_S which for cross-shore transport are given by Bailard (1982) on the basis of field observations as 0.10 and 0.02 respectively. These values are in principle adopted here.

The cross-shore variation of the local, mean sediment transport may now be calculated with the above expressions (18a...f) given the results of the wave height decay and kinematics model. Through application of the mass balance for the sediment (of which the properties are assumed constant) the bottom changes may be calculated. This procedure may be repeated for the new beach profile.

In the numerical evaluation of the above procedure a second order Runge-Kutta algorithm is used in the wave decay model and a modified Lax scheme in the bottom change calculations.

As a boundary condition on the waterline the present formulation yields $\langle q \rangle$ =

0. To simulate the smoothing effect of swash motion on the sediment transport near the waterline $\langle q(x) \rangle$ was damped starting from a depth of approximately half the initial wave height in proportion to the mean water depth.

5. Model verification

A laboratory measurement programme aimed at the verification of the present model CROSTRAN has not yet been conducted. Therefore we rely for the moment on otherwise available measurement results of which two cases are presented here. Moreover we present a preliminary comparison of model calculations with observed bar formation and deformation in an estuary region in the South of the Netherlands, the socalled Voordelta.

The first laboratory case concerns a nearly 1:40 sloped profile of medium sized sand in a large scale flume. The second case concerns a barred profile of relatively fine sand in a small scale flume. In both cases the bed deformations are relatively small, but do show an appreciable variation in cross-shore direction (see Figures 3 and 4). The bed deformations are hindcasted for two transport formulations, i.e. one in which only the undertow effect is taken into account and one in which in addition the effects of wave asymmetry and bed slope are taken into account.

The results achieved for the case of transport due to undertow alone are as described in Stive and Battjes (1984). It appears that with the exception of the region near the waterline the results in general improve when all three effects are included. The spatial phase shift of bottom changes in the large scale case is attributed to the observed convexity of the beach profile which was not accounted for since only the central cross-section was monitored. With respect to the choice of the value of the proportionality constants B_{as}, B_{un} and B_{sl} we note the following. If the model formulations are right and if Bailard's estimates of the efficiency factors ε_B, ε_S are sufficiently valid in the considered ranges of conditions the proportionality constants B_{as}, B_{un} and B_{sl} should be very close to 1. In hindcasting observations we have sofar accepted that deviations of a factor 2 are acceptable in the B values. Making this allowance yields satisfactory results. The above results were obtained with the following values for the proportionality constants.

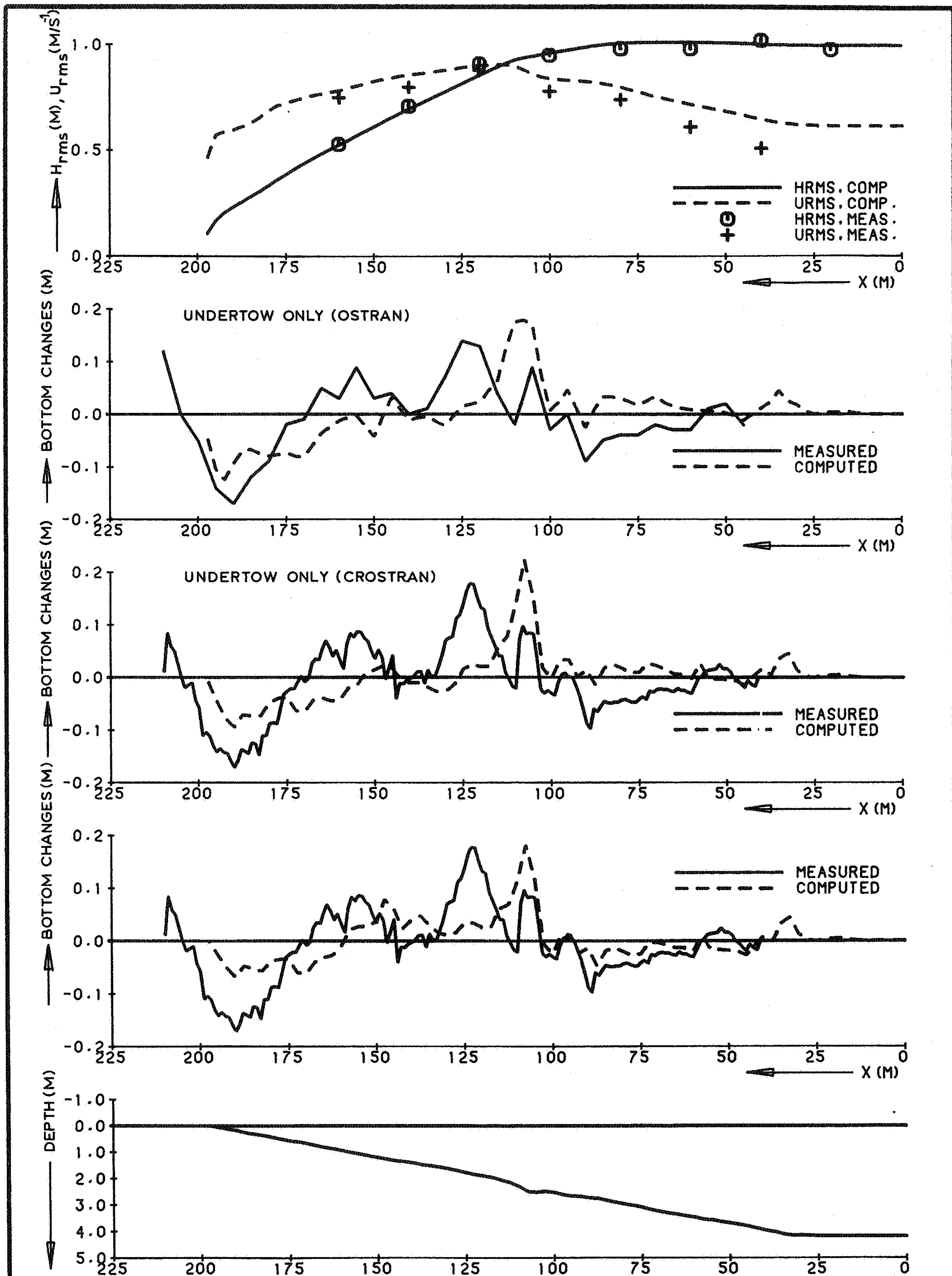


Fig. 3 Comparison between large scale laboratory measurements and present theoretical prediction of wave heights and bottom changes after 9 hrs.

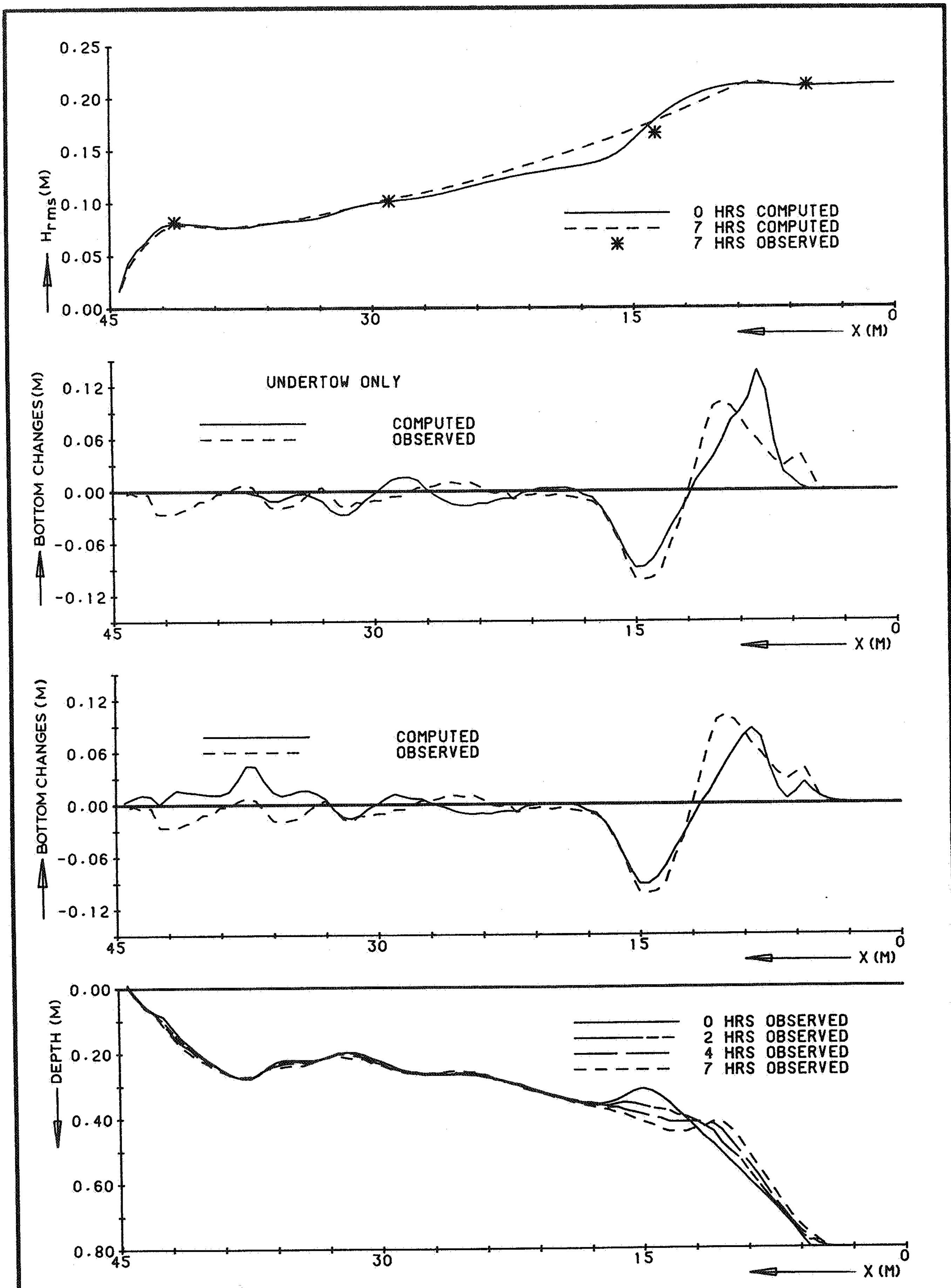


Fig. 4 Comparison between small scale laboratory measurements and present theoretical prediction of wave heights and bottom changes after 7 hrs.

B_{as}	B_{un}	B_{sl}	
large scale flume (fig. 3)	2.0	0.8	1.0
small scale flume (fig. 4)	2.0	0.9	1.0

Table 3 Choice of proportionality constants

The field comparison concerns the profile development that occurred after closure of one of the Southern Dutch estuaries. The profile deformation in cross-shore direction is appreciable (see Figure 5). The comparison between the hindcast results and the measurements is satisfactory, despite the fact that the wave climate and hydraulic conditions were schematized to one value for the incident wave characteristics and a fixed waterlevel. The proportionality constants B_{as}, $B_{un}$and B_{sl} were set at 1.0.

Finally, some characteristic parameters of the three cases are collected in Table 4 below.

case	profile diameter (μm)	grain (m)	$H_{rms,incident}$ (Hz)	f_p
large scale flume	plane	225	1.00	0.19
small scale flume	barred	100	0.21	0.39
field barred	225	1.50	0.17	

Table 4 Characteristic parameters hindcast cases

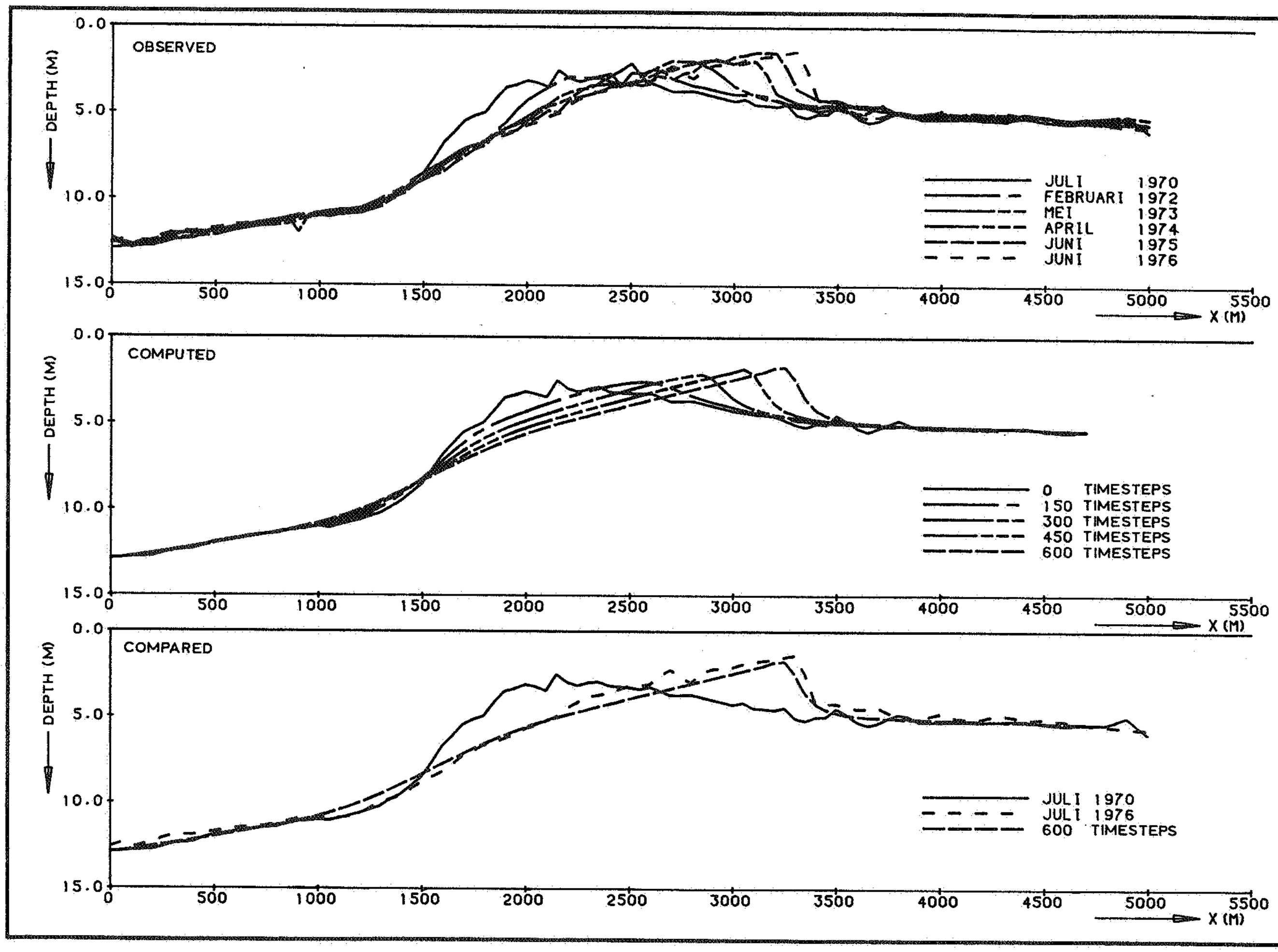


Fig. 5 Comparison between profile development at the Voordelta and present theoretical prediction

6. Discussion and conclusion

In this report a first suggestion is made to extend the earlier formulated model for offshore sediment transport due to undertow (Stive and Battjes, 1984) with the effects due to horizontal asymmetry in the wave motion. It appears that improvements in hindcasting the bottom changes may be obtained.

To arrive at these results it was necessary to model some low order odd moments of the near-bottom velocity field. An ad-hoc formulation based on a monochromatic, second order Stokes wave representation is shown to give a reasonable, first approximation to the odd velocity moments, but obviously the formulation needs improvement.

The odd velocity moments were readily used in the transport formulation after Bailard (1981). This concerns a vertically integrated description of the sediment transport in sheetflow conditions, which assumes that the instantaneous transport is proportional with some power of the instantaneous near-bottom velocity. The validity of this approach for natural surf zones needs further investigation. This requires study of the temporal and spatial variations of sediment load and/or sediment concentrations due to spatially varying waves in general and random waves breaking on a beach in particular.

Appendix A: Total mean sediment load

The purpose of the present appendix is to check Bailard's (1981) transport model with respect to the total mean sediment load in a qualitative sense through comparison with laboratory measurments of Bosman (1982). Bosman shows that sediment concentration distributions in random, breaking waves may be satisfactorily described by a "double layer" or "double first order" model for a rather wide variety of laboratory situations, such as horizontal and sloping bottom, no and net currents, non-breaking and breaking random waves. Based on a diffusion type description for each layer the model leads to an exponential concentration distribution as follows:

$$C(z) = \begin{cases} C_o e^{-z/\ell_1} & \text{for } z \leqslant A \\ C(A)\ e^{-(z-A)/\ell_1} = C_o\ e^{-A/\ell_1}\ e^{-(z-A)/\ell_2} & \text{for } z > A \end{cases} \qquad \text{(a.1)}$$

where A is the bottom layer thickness. Each layer has a constant turbulent viscosity, ε, and a fall velocity, w, for the sediment in that layer such that $\ell_1 \equiv \varepsilon_1/w_1$ and $\ell_2 \equiv \varepsilon_2/w_2$ (based on diffusion). Bosman's measurements show that the upward exponential decay of the concentration is so strong that the total sediment load, $\langle s \rangle$, is mainly confined to and determined by the bottom layer as follows:

$$\langle s \rangle = C_o \cdot \ell_1 \qquad \text{(a.2)}$$

where C_o is the bottom concentration and ℓ_1^{-1} is the relative concentration gradient in the bottom layer. It is this bottom layer of thickness A (approximately 5-8 centimeter in Bosman's experiments) that is described by Bailard's model, distinguishing within this bottom layer a bedload layer of thickness smaller than 1 centimeter and a suspended load layer of a few centimeter thickness.

The mean sediment load in the bottom layer according to Bailard's model may be determined as follows. Confining ourselves to situations with a horizontal bottom the instantaneous volumetric sediment flux may be written as:

$$q = \frac{i}{(\rho_s-\rho)gN} = \frac{\rho\ c_f}{(\rho_s-\rho)gN}\ [\frac{\varepsilon_B}{\tan\phi}\ |u|^2u + \frac{\varepsilon_S}{w}\ |u|^3u] \qquad \text{(a.3)}$$

Equation (a.3) expresses the sediment flux as the sediment load, s, times the instantaneous velocity, u. So the volumetric sediment load is given by

$$s = \frac{\rho\, c_f}{(\rho_s-\rho)gN}\left[\frac{\varepsilon_B}{\tan\phi}\,|u|^2 + \frac{\varepsilon_S}{w}\,|u|^3\right] \tag{a.4}$$

For modelling convenience we now use a monochromatic representation for the random nearbottom velocity field and we moreover assume that the motion is primary harmonic only, i.e.

$$u = u(t) = u_{rms}\cos\omega_p t \tag{a.5}$$

where u_{rms} is the rms nearbottom horizontal velocity amplitude and $\omega_p = 2\pi f_p$ is the relative peak frequency. For u_{rms} we apply the linear theoretical relation

$$u_{rms} = \frac{H_{rms}\omega_p}{2\sinh(k_p h)} \tag{a.6}$$

where H_{rms} is the rms wave height and k_p is the wave-number. Furthermore we assume the Rayleigh properties

$$u_s = \sqrt{2}u_{rms} \quad \text{and} \quad H_s = \sqrt{2}H_{rms} \tag{a.7}$$

It is noted that u_{rms} is the rms horizontal velocity amplitude which is related to the total variance of the fluctuating velocity with zero-mean, m_{o_u}, by

$$u_{rms} = (2m_{o_u})^{\frac{1}{2}} = \sqrt{2}u'_{rms} \tag{a.8}$$

where $m_{o_u} \equiv \int_0^\infty S_u(f)\,df$ with $S_u(f)$ the variance density spectrum of u and u'_{rms} the standard deviation of the fluctuating velocity with zero-mean.

Having defined the wave motion by (a.5) we may now calculate the time mean sediment load by taking the time mean of (a.4), i.e.

$$\langle s\rangle = \frac{\rho\, c_f}{(\rho_s-\rho)gN}\left[\frac{\varepsilon_B}{\tan\phi}\,\frac{1}{2}\,u^2_{rms} + \frac{\varepsilon_S}{w}\frac{4}{3\pi}\,u^3_{rms}\right] \tag{a.9}$$

For convenience we make the mean sediment load dimensionless with the median grain diameter, D, so

$$\frac{\langle s \rangle}{D} = \Psi \frac{c_f \varepsilon_B}{2\tan\phi\ N} + \Psi \frac{4 c_f \varepsilon_S}{3\pi N} \frac{u_{rms}}{w} \qquad \text{(a.10)}$$

where $\Psi \equiv \rho u_{rms}^2/(\rho_s - \rho) g D$ is the so-called sediment mobility parameter. A comparison between the mean sediment load as measured by Bosman (1982) on a horizontal bottom in random breaking waves and as calculated by the above expresion (a.10) is given in Figure (a.2). For the various parameters in (a.10) we have applied the values as indicated by Bailard (1982), i.e.

ε_B = 0.10
ε_S = 0.02
c_f = $\frac{1}{2} f_w = 0.005$
$\tan\phi$ = 0.63
N = 0.6

It appears that with these parameter values there is quantitatively an order of magnitude underestimation, but qualitatively there is a satisfactory agreement.

The reason for the underestimation is here not further investigated but it is a point which remains to be resolved.

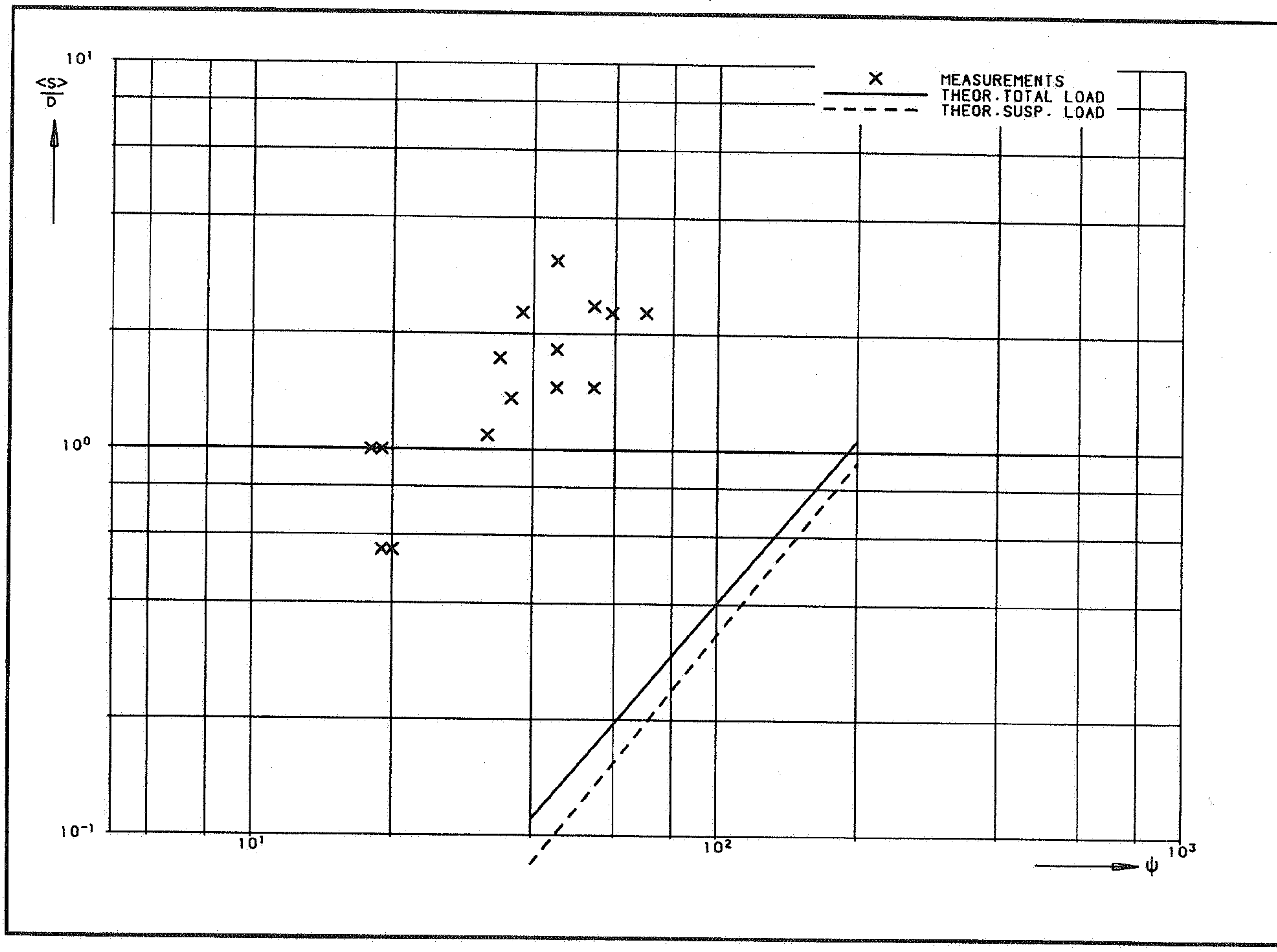
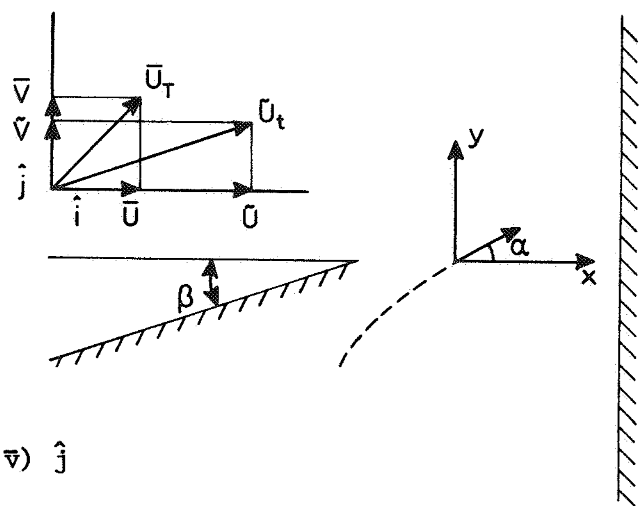


Fig. a.1 Comparison between theoretical predicted (Bailard, 1981) mean sediment load and Bosman's (1982) measurements

Appendix B: Moments for nearshore sediment transport

The purpose of this appendix is to indicate which low order velocity moments derivable from field data are expected to be of potential importance for a sediment transport formulation. It is suggested that these moments are derived from the field data set of Egmond '82-'83 for further analysis.

As a starting point we consider the unsimplified form of Bailard's equations for the average cross and longshore sediment transport rates ($\langle i_x \rangle$, $\langle i_y \rangle$ respectively):



$$\vec{u}_t = (\tilde{u} + \bar{u})\hat{i} + (\tilde{v} + \bar{v})\ \hat{j}$$

$$|\vec{u}_t| = [\tilde{u}^2 + \tilde{v}^2 + \bar{u}^2 + \bar{v}^2 + 2(\tilde{u}\ \bar{u} + \tilde{v}\ \bar{v})]^{\frac{1}{2}}$$

$$\langle i_x \rangle = \rho\ c_f \frac{\varepsilon_B}{\tan\phi} [\langle |\vec{u}_t|^2\ (\tilde{u} + \bar{u})\rangle - \frac{\tan\beta}{\tan\phi} \langle |\vec{u}_t|^3] +$$

$$+ \rho\ c_f \frac{\varepsilon_S}{w} [\langle |\vec{u}_t|^3\ (\tilde{u} + \bar{u})\rangle - \frac{\varepsilon_S}{w} \tan\beta \langle |\vec{u}_t|^5\rangle] \qquad \text{(b.1a)}$$

$$\langle i_y \rangle = \rho\ c_f \frac{\varepsilon_B}{\tan\phi} [\langle |\vec{u}_t|^2\ (\tilde{v} + \bar{v})\rangle] + \rho\ c_f \frac{\varepsilon_S}{w} [\langle |\vec{u}_t|^3\ (\tilde{v} + \bar{v})] \qquad \text{(b.1b)}$$

The moments appearing in these equations depend on both mean and oscillatory currents. Splitting the velocity field into steady and unsteady components results in some terms that depend only on the oscillatory flow. Also, in the absence of mean cross-shore and longshore flows, several low order moments appear in the cross-shore transport rate, in which case the equations simplify to

$$\langle i_x \rangle = \rho\ c_f \frac{\varepsilon_B}{\tan\phi} [\langle \tilde{u}^3 \rangle - \frac{\tan\beta}{\tan\phi} \langle |\tilde{u}|^3 \rangle] + \rho\ c_f \frac{\varepsilon_S}{w} [\langle |\tilde{u}|^3\ \tilde{u}\rangle - \frac{\varepsilon_S}{w} \tan\beta \langle |\tilde{u}|^5\rangle] \qquad \text{(b.2)}$$

Obviously the following low order moments of the cross-shore velocity are of potential importance for a sediment transport formulation, i.e.

- the four lowest even moments $\langle\tilde{u}^2\rangle$, $\langle|\tilde{u}|^3\rangle$, $\langle\tilde{u}^4\rangle$, $\langle|\tilde{u}|^5\rangle$, which are nonzero for symmetric velocities,
- the three odd moments $\langle\tilde{u}^3\rangle$, $\langle|\tilde{u}|^3\ \tilde{u}\rangle$, $\langle\tilde{u}^5\rangle$, which are zero for symmetric velocities.

For the longshore velocity $\tilde{v}$ identical moments are of potential importance.

More specifically we identify the terms which appear in the generalized (random, directional) form of Bailard's idealized (monochromatic, unidirectional) equations (see Guza and Thornton, 1985):

$$\langle i_x\rangle = \rho\ c_f\ u_m^3\ \{\frac{\varepsilon_B}{\tan\phi}\ [\psi_1\cos\alpha_1 + \delta_u^3 + \delta_u\ (\tfrac{1}{2} + \cos^2\alpha_2 + \delta_u^2) + \delta_u\sin\alpha_3\cos\alpha_3$$

$$- \frac{\tan\beta}{\tan\phi}\ (u3)^*] + \frac{u_m}{w}\ \varepsilon_S\ [\psi_2\cos\alpha_5 + \delta_u(u3)^*] - \frac{u_m^2}{w^2}\ \varepsilon_S^2\ \tan\beta\ (u5)^*\} \qquad \text{(b.3a)}$$

$$\langle i_y\rangle = \rho\ c_f\ u_m^3\ \{\frac{\varepsilon_B}{\tan\phi}[\psi_1\sin\alpha_1 + \delta_v^3 + \delta_v\ (\tfrac{1}{2} + \sin^2\alpha_2 + \delta_u^2)$$

$$+ \delta_u\ \sin\alpha_3\cos\alpha_3] + \frac{u_m}{w}\ \varepsilon_S\ [\psi_2\sin\alpha_5 + \delta_v(u3)^*]\} \qquad \text{(b.3b)}$$

It is noted that for monochromatic, unidirectional plane waves all the α_n are equal. The relative steady current strengths are defined as

$$\delta = \bar{u}_T/u_m$$

$$\delta_u = \bar{u}_T\cos\Theta/u_m \qquad \text{(b.4)}$$

$$\delta_v = \bar{u}_T\ \sin\Theta/u_m$$

where $\bar{u}_T$ is the total mean current and Θ its local angle. The monochromatic amplitude, u_m, is related to the oscillatory variance

$$u_m^2 = 2[(\tilde{u}^2) + (\tilde{v}^2)] \qquad \text{(b.5)}$$

and the velocity moments ψ_1, ψ_2 (Eqs. 16a-b) by

$$u_m^3\ \psi_1 = \{\langle\tilde{u}(\tilde{u}^2 + \tilde{v}^2)\rangle^2 + \langle\tilde{v}(\tilde{u}^2 + \tilde{v}^2)\rangle^2\}^{\frac{1}{2}} \qquad \text{(b.6a)}$$

with $|\vec{u}_t| = [\tilde{u}^2 + \tilde{v}^2 + \tilde{u}^2 + \tilde{v}^2 + 2(\tilde{u}\bar{u} + \tilde{v}\bar{v})]^{\frac{1}{2}}$.

Averages must be over the entire recond length so

$$u_m^3 u_3^* = \langle|\vec{u}_t|^3\rangle \tag{b.7a}$$

$$u_m^5 u_5^* = \langle|\vec{u}_t|^5\rangle \tag{b.7b}$$

Some remaining angles are defined as

$$\tan\alpha_1 = \frac{\langle\tilde{v}^3 + \tilde{u}^2\tilde{v}\rangle}{\langle\tilde{u}^3 + \tilde{u}\tilde{v}^2\rangle} \tag{b.8a}$$

$$\tan\alpha_2 = \left(\frac{\langle\tilde{v}^2\rangle}{\langle\tilde{u}^2\rangle}\right)^{\frac{1}{2}} \tag{b.8b}$$

$$\sin\alpha_3 = \frac{4(\tilde{u}\tilde{v})}{u_m^2} \tag{b.8c}$$

$$\tan\alpha_5 = \frac{\langle|\vec{u}_t|^3\ \tilde{v}\rangle}{\langle|\vec{u}_t|^3\ \tilde{u}\rangle} \tag{b.8d}$$

Appendix C: The wave decay and kinematics model

The principal process in the coastal zone is the dissipation of the energy of incident windwaves and swell, due to wave breaking. Because of the randomness of wind-generated waves the occurrence of breaking at a fixed location is itself a random process. Realistic models for the prediction of the onshore variation of wave energy and radiation stress should take this randomness into account.

Battjes and Janssen (1978), hereafter referred to as BJ, presented an approach in which the mean local rate of energy dissipation is modelled, based on that occurring in a bore and on the local probability of wave breaking. The result is used as a sink in the energy balance, which is subsequently integrated to obtain the wave energy as a function of onshore distance. A few laboratory experiments performed by BJ, including cases with a bar-trough profile, indicated a very promising degree of agreement.

The essence of BJ's model is the estimation of the time-averaged rate of dissipation of wave energy per unit area due to breaking (D). Two aspects are distinguished: the rate of energy dissipation in periodic breaking waves, and the probability of occurrence of breaking waves of given height in a random wave field.

The energy dissipation in breaking waves is modelled after that in a bore of the same height. For periodic waves with frequency f and breaking waveheight H_b in water of mean depth h, BJ arrive at the following order-of-magnitude estimate for the mean dissipation rate per unit area:

$$D \sim \tfrac{1}{4}\, f \rho g\, H_b^2 \qquad (c.1)$$

For application to random waves, the expected value of D (written as $\bar{D}$) must be estimated, taking into account the randomness of the waves and the fact that not all the waves passing the point considered are breaking. In this estimate, BJ have used characteristic values for the frequency and the breaking waveheights, and they have derived a prognostic equation for the local fraction of breaking waves.

The characteristic frequency used is f_p, the frequency at the peak of the energy spectrum of the incident waves.

The mean square of H_b is equated by BJ to the square of the nominal, depth-limited height of periodic waves (H_m) in water of the local mean depth. BJ use a Miche-type expression for H_m, adapted through the inclusion of a parameter γ to account for influences of bottom slope and mean wave steepness:

$$H_m = 0.88\ k_p^{-1} \tanh (\gamma\ k_p h/0.88) \tag{c.2}$$

in which $k_p = 2\pi/L_p$ is the wave number calculated on the basis of the linear theory dispersion equation for gravity waves with frequency f_p.

To determine the local fraction of breaking waves (Q), BJ assume that the cumulative probability distribution of all waveheights (breaking or non-breaking) is of the Rayleigh-type, cut off discontinously at $H = H_m$. This was shown to imply the following relation between Q and H_{rms}/H_m, in which H_{rms} is the runs rms of all waveheights:

$$\frac{1 - Q}{-\ln Q} = \left(\frac{H_{rms}}{H_m}\right)^2 \tag{c.3}$$

Substituting the approximations mentioned above in the averaged equation (c.1), and writing the order-of-magnitude relation in the form of an equation, gives

$$\overline{D} = \tfrac{1}{4}\ \alpha\ Q\ f_p\ \rho g\ H_m^2 \tag{c.4}$$

in which α is a coefficient which is expected to be of order 1. It is pointed out that $\overline{D}$ varies with H_{rms} through Q.

To close the model, $\overline{D}$ is used as a sink in the wave energy balance, which in its most reduced form (statistically steady, uniform alongshore, no other sources or sinks than $\overline{D}$) can be written as

$$\frac{\partial P_x}{\partial x} + \overline{D} = 0 \tag{c.5}$$

P_x is the onshore energy flux per unit width, approximated as $_{Ecg}$, in which

$E = \frac{1}{8} \rho g H^2_{rms}$ and c_g is the group velocity according to the linear theory for $f = f_p$. The energy balance (c.5) is integrated simultaneously with the balance of onshore momentum (not reproduced here), resulting in the simultaneous determination of the onshore variation of H_{rms} and of the set-up of the mean water level ($\bar{\eta}$).

In a follow up to Battjes and Janssen, Battjes and Stive (1985) have described a calibration of the theoretical model for the wave energy dissipation and resulting wave energy variation. It is shown that the theoretical model has effectively one adjustable parameter. Optimal values of this coefficient have been determined. These vary slightly in a physically realistic range with the incident wave steepness. A parameterization of this dependence is presented so that the model can be used for prediction. Using this parameterization, the overall performance of the model has been evaluated. The coefficient of correlation between predicted and observed H_{rms}-values is 0.98; the model bias is not significantly different from zero, and the rms relative error is 0.06.

The near-bed wave kinematics are derived from the wave energy variation through local application of a horizontal bottom wave theory. So, it is assumed that locally the waves behave as if they were on a horizontal bottom. The prediction of the sediment transport requires the following estimates of near-bed kinematics in the random breaking wave field:

a) the undertow, i.e. the total velocity time-averaged over at least one wave group
b) the even oscillatory velocity moments $\langle\tilde{u}^2\rangle$, $\langle|\tilde{u}|^3\rangle$, $\langle\tilde{u}^4\rangle$
c) the odd oscillatory velocity moments $\langle\tilde{u}^3\rangle$, $\langle|\tilde{u}|^3\,\tilde{u}\rangle$.

<u>item a: the undertow</u>

It is assumed that in a random wave field breaking on a beach the majority of the breaking waves has a quasi-steady depth-similar flow field as described by Stive and Wind (1982) for breaking, periodic waves. Based on the dimensionless flow field presented there and adopting the observation that the flow profile is rather uniform over the lower depths the return flow in a periodic, breaking wave field is simply modelled as:

$$\bar{u}_{br,periodic} = 1/10\ (g/d)^{\frac{1}{2}}\ H_b \qquad \text{(c.6)}$$

where g is the acceleration of gravity, d the water depth and H_b the breaking wave height.

In random waves on a beach the fraction of waves breaking at a point (Q) varies with position. Battjes and Janssen (1978) have presented an implicit expression (Eq. c.3) for Q as a function of the ratio of the rms wave height (H_{rms}) to a local breaking height, which in turn is primarily depth-controlled. Qualitatively this expression is in accordance with laboratory observations; quantitatively the following adjustment is suggested:

$$\tilde{Q} = 6\,Q \qquad \text{for } \tilde{Q} \leq 0.8 \tag{c.7}$$

$$\tilde{Q} = 0.8 + 0.35\,[(H_{rms}/H_{max})^{\frac{1}{2}} - 0.43] \qquad \text{for } 0.8 < \tilde{Q} \leq 1$$

The return flow velocity in a random, breaking wave field is simply modelled here as

$$\bar{u}_{br,random} = \bar{u}_{br,periodic} \cdot \tilde{Q} \tag{c.8}$$

<u>item b: the even velocity moments</u>

It is known from surf zone observations (see eg. Van Heteren and Stive, 1984) that linear Gaussian theory provides a reasonable estimate (i.e. 10%-20% conservative) of the lowest even moment $\langle\tilde{u}^2\rangle$. Table 2 of Chapter 3 indicates that also for the higher even moments these linear, Gaussian estimates are reasonable. For a first approximation we therefore rely on the linear, Gaussian model.

<u>item c: the odd velocity moments</u>

As described in Chapter 3 the linear, Gaussian model predicts zero magnitude for the odd velocity moments. For a first approximation the described ad-hoc, nonlinear model is adopted which assumes (1) that only the nonbreaking wave fraction $(1 - \tilde{Q})$ contributes to the odd velocity moments and (2) that this contribution may be derived from Eq. 15a,b using a monochromatic second order Stokes representation (Eq. 17). The resulting expressions are:

$$\langle \tilde{u}^3 \rangle = \frac{9}{16} \frac{u_{rms}^4}{c} \sinh^{-2}(k_p h) \; (1 - \tilde{Q}) \; A_3$$

$$\langle |\tilde{u}|^3 \tilde{u} \rangle = \frac{12}{5\pi} \frac{u_{rms}^5}{c} \sinh^{-2}(k_p h) \; (1 - \tilde{Q}) \; A_4$$

where A_3 and A_4 are constant scaling factors which incorporate yet unquantified effects due to adopting a monochromatic, unidirectional representation for the random wave fields considered, directional in the fiels cases and unidirectional in the laboratory cases. Based on the velocity data described by Bailard (1982) A_3 and A_4 are set at 0.5.

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