

Multi-beam raindrop size distribution retrievals on the Doppler spectra

Christine Unal

Geoscience and Remote Sensing, TU-Delft Climate Institute, Stevinweg 1, 2628 CN Delft, Netherlands, c.m.h.unal@tudelft.nl



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1. Introduction

Acquiring the raindrop size distribution from radar data is still a challenge. Generally this distribution is retrieved using the reflectivity, Z , and the differential reflectivity, Z_{dr} , at S-band. The specific differential phase, K_{dp} , provides a third radar observable in the case of heavy precipitation to strengthen the retrieval technique.

For S-band profiling radars, K_{dp} cannot be used, mainly because of the limited number of range bins containing rain. The Z_{dr} measurement cannot be employed when the elevation is near the vertical, or in the case of drizzle and light rain. Therefore retrievals have to be performed on the measured Doppler spectra. For this purpose, the Doppler spectra of rain are modeled using Rayleigh scattering, the gamma distribution for the raindrop size, a size-shape relationship and a Gaussian kernel for the turbulence. Comparing the measured Doppler spectrum with the modeled one, a non-linear least-square fit technique is employed to obtain the parameters of the raindrop size distribution (D_0 , μ) and the turbulence broadening factor of the raindrop size spectrum (σ_0). The intercept parameter (N_w) and the radial wind component (v_0) are estimated by scaling the broadened drop size distribution (DSD) along the spectral reflectivity axis and the Doppler velocity axis respectively. This approach has been proposed in (Moisseev et al, 2006). So far, this technique has not been further developed, exploited and validated. It is our intention to investigate this methodology using the Doppler-polarimetric TARA radar to retrieve raindrop size distribution from drizzle to heavy precipitation with high spatial and time resolution. Since TARA can profile in three directions, three raindrop size distribution profiles are estimated, which can give insight in the microphysical homogeneity of the precipitation.

2. Microphysical model of raindrops

The Doppler spectrum, or spectral reflectivity $sZ_{HH}(v)$, can be expressed as the Doppler spectrum resulting from the raindrop size distribution, $sZ_{HH}(\tilde{v})$, convoluted by a Gaussian-shaped kernel of width σ_0 modeling spectral broadening. Spectral broadening has several causes, among them turbulence and wind shear in the radar resolution volume.

$$sZ_{HH,mod}(v) = \frac{1}{\sqrt{2\pi}\sigma_0} \int \exp\left[-\frac{(v-\tilde{v})^2}{2\sigma_0^2}\right] sZ_{HH}(\tilde{v}) d\tilde{v} \quad (\text{mm}^6 \text{m}^{-3}) \quad (1)$$

where v and $\tilde{v}$ are Doppler velocities and

$$sZ_{HH}(\tilde{v}) = N(D\{\tilde{v}\}) \sigma_{HH}(D\{\tilde{v}\}) \left| \frac{dD}{d\tilde{v}} \right| d\tilde{v} \quad (2)$$

The radar cross section, σ_{HH} , is calculated using Rayleigh-Gans scattering of spheroidal hydrometeors. It strongly depends on the raindrop equivolume diameter, D . The raindrop size distribution, $N(D)$, follows the gamma distribution:

$$N(D) = N_w f(\mu) \left(\frac{D}{D_0} \right)^\mu \exp\left[-(3.67 + \mu) \frac{D}{D_0}\right] \quad (\text{mm}^{-1} \text{m}^{-3}) \quad (3)$$

$$f(\mu) = \frac{6}{(3.67)^4} \frac{(3.67 + \mu)^{\mu+4}}{\Gamma(\mu+4)} \quad (4)$$

where the triplet (D_0 , N_w , μ) characterize the distribution. D_0 is the median volume diameter and μ the shape parameter. Finally, due to the contribution of the radial ambient wind, the Doppler spectrum experiences a shift of length v_0 along the Doppler velocity axis. A complete model schematic is given in Fig. 1.

When the parameters of the DSD are known, the reflectivity (Eq. 5), the liquid water content (Eq. 6) with $\rho_w = 10^{-3} \text{ g mm}^{-3}$ and the number of raindrops (Eq. 7) can be calculated:

$$Z = N_w f(\mu) \frac{\Gamma(7 + \mu)}{(3.67 + \mu)^{7 + \mu}} D_0^7 \quad (\text{mm}^6 \text{m}^{-3}) \quad (5)$$

$$LWC = \frac{\pi}{3.67^4} \rho_w N_w D_0^4 \quad (\text{g m}^{-3}) \quad (6)$$

$$N_t = N_w f(\mu) \frac{\Gamma(1 + \mu)}{(3.67 + \mu)^{1 + \mu}} D_0 \quad (\text{m}^{-3}) \quad (7)$$

Simulation of the rain Doppler spectra (S-band, Rayleigh scattering)

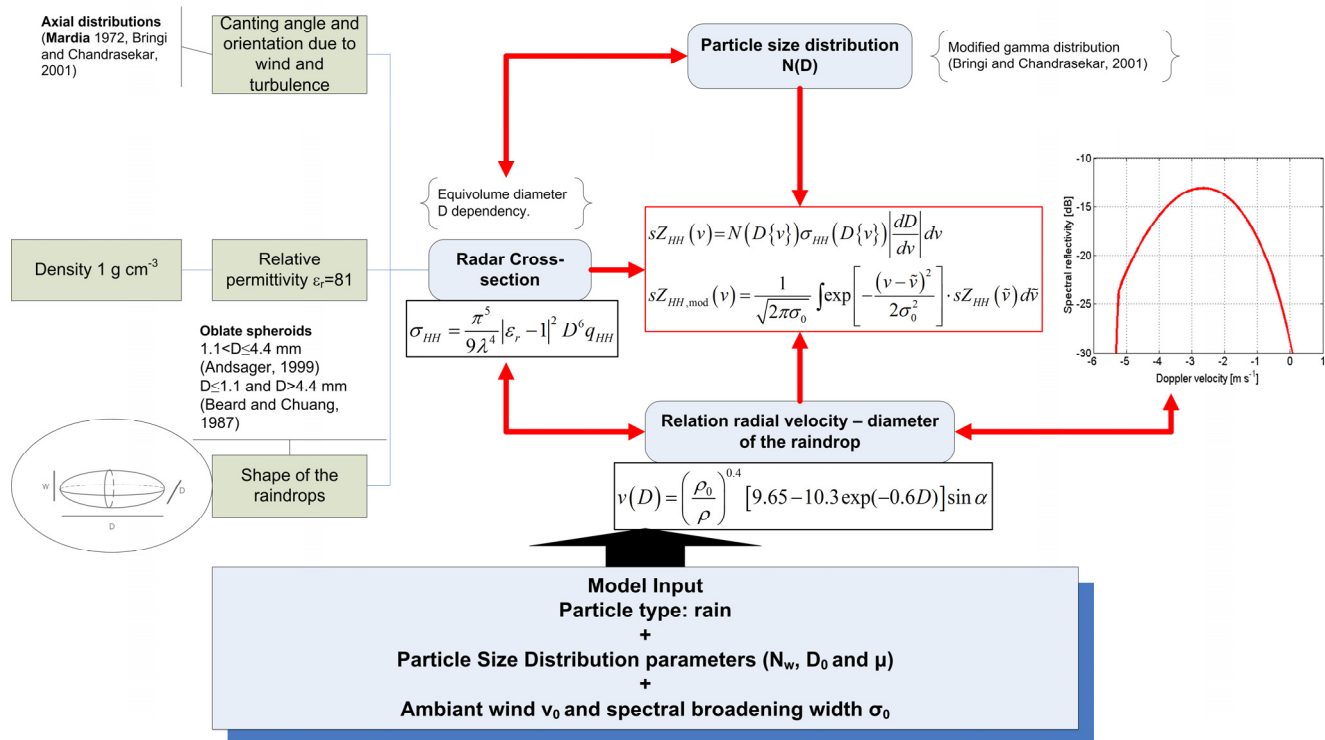


Fig. 1 Model schematic: this model simulates the Doppler spectra of rain for the polarization settings HH and VV at the elevation α .

3. Retrieval of the raindrop size distribution, the spectral broadening, and the radial wind velocity

The retrieval algorithm obtains the three parameters of the DSD (D_0 , N_w , μ) and the dynamic parameters (v_0 , σ_0) by fitting modeled spectra to measured spectra. An optimization procedure has to minimize the difference (or error) between the fitted spectrum and the measured spectrum by varying the five input parameters.

$$\min_{\Psi} \sum_{v=v_{\min}}^{v=v_{\max}} \left[10 \log(sZ_{HH}^{meas}(v)) - 10 \log(sZ_{HH}^{\text{mod}}(v, \Psi)) \right]^2 \quad (8)$$

where Ψ contains all three raindrop size distribution parameters, the spectral broadening and the ambient wind velocity. The five parameter minimization problem, Eq. (8), can be simplified by separating the retrieval of the intercept parameter:

$$sZ_{HH}^{meas}(v) = sZ_{HH}^{mod}(v, D_0, 1, \mu, v_0, \sigma_0) N_w + \varepsilon \quad (9)$$

This allows the derivation of estimates of the intercept parameters without nonlinear fitting. For this linear fit, the values of D_0 , μ and σ_0 have to be known and v_0 is set to 0. A second simplification of Eq. (8) is done on the estimation of v_0 . Assuming the other four parameters of the model are known, the shift between the modeled and the measured spectrum can be obtained by determining the lag of the cross correlation of the measured and the modeled spectrum $sZ_{HH}^{\text{mod}}(v, D_0, 1, \mu, 0, \sigma_0)N_w$.

The separate estimation of the intercept parameter and the ambient wind velocity results in a three parameter nonlinear least squares problem. The three parameters are D_0 , μ and σ_0 . This approach is followed in (Moisseev et al, 2006) and (Spek et al, 2008). Because of multiple minima in the cost functions, an iterative cascaded retrieval algorithm, like in (Spek et al, 2008), is preferred to obtain them. This algorithm is illustrated in Fig. 2.

4. Quality of the retrieval technique

To get insight in the quality of the optimization procedure, the optimization is applied on simulated Doppler spectra. By comparing the input parameters used to create a simulated spectrum with the parameters obtained with the retrieval algorithm, conclusions can be drawn on their errors. To generate signals with real statistical properties, noise is added according to Chandrasekar et al. (1986). The values of the parameters are selected randomly from the intervals, given in Table 1. Like in (Moisseev et al.), 30 realizations of the Doppler power spectra are averaged to obtain an estimate of the true spectrum and the retrieval algorithm is only applied on the resulting spectrum when its corresponding reflectivity is between 10 and 55 dBZ. The root mean square deviation (RMSD) of each parameter is given in Table 1.

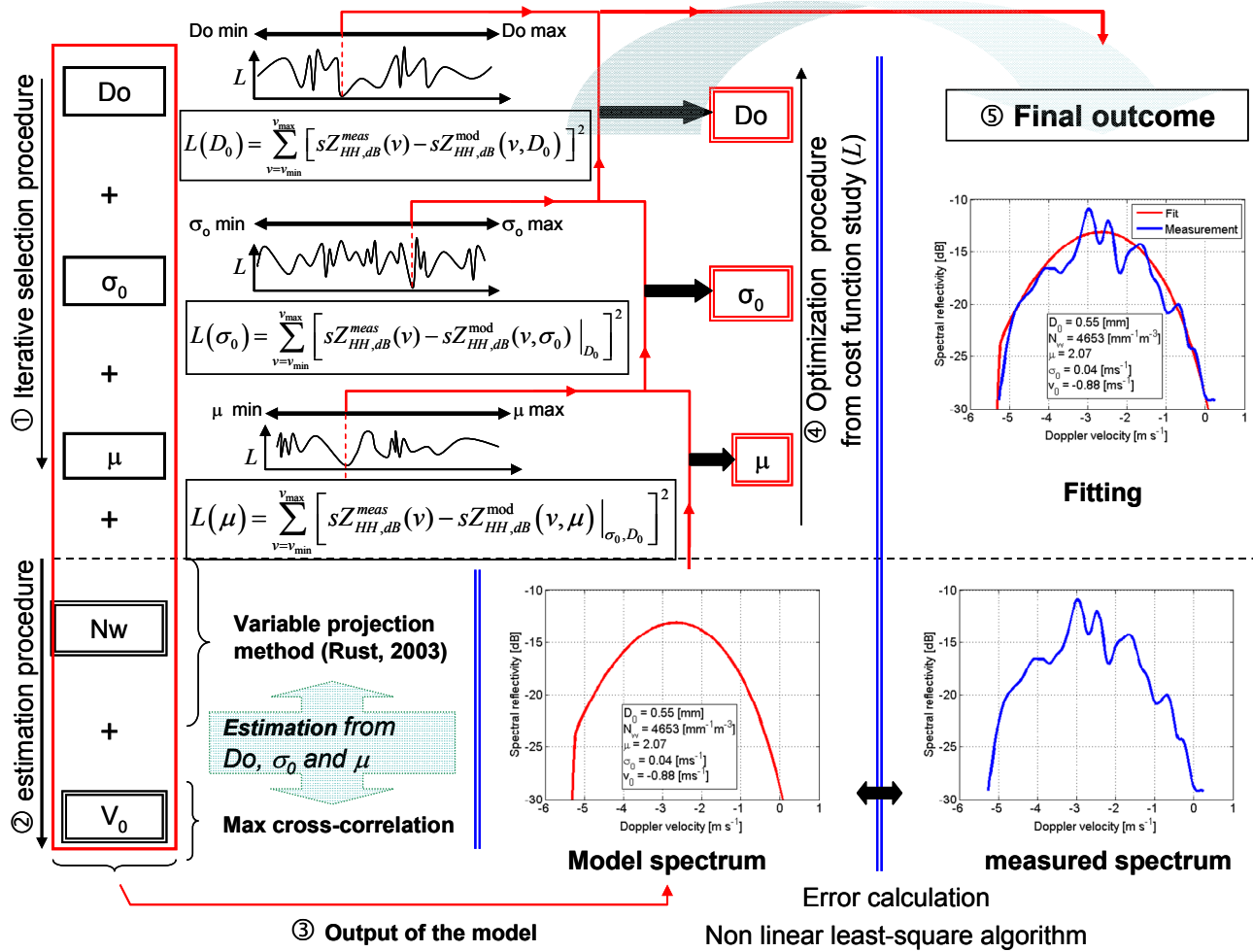


Fig. 2 Cascaded rain retrieval algorithm schematic where the cost function L is evaluated from spectral reflectivities expressed in dBZ.

The same exercise is carried out on integral parameters, the reflectivity, the liquid water content and the number of particles. The results are given in Table 2. The root mean square deviations in Tables 1-2 are estimated errors and do not include the uncertainties of the microphysical relationships of the model. The RMSD normalized to the mean (coefficient of variation) is given for the parameters of which the values are positive.

Table 1: Regions and root mean square deviations of parameters

Parameter	D_0	N_w	μ	σ_0	v_0
Region	0.2 – 3 mm	0 – 8000 $mm^{-1} m^{-3}$	-2 – 10	0.1 – 0.9 $m s^{-1}$	0 – 1.2 $m s^{-1}$
RMSD	0.12 mm	1350 $mm^{-1} m^{-3}$	0.67	0.04 $m s^{-1}$	0.18 $m s^{-1}$
CV(RMSD)	17%	54%		8.4%	28%

Table 2: Root mean square deviations of integral parameters

Parameter	Z	LWC	N_t
RMSD	0.30 dB	0.13 $g m^{-3}$	142 m^{-3}
CV(RMSD)	0.91%	22%	7.4%

5. Application of the retrieval technique on non-averaged Doppler spectra of rain

Because TARA has 3 beams for wind profiling, the retrieval technique can be applied on HH- and VV-spectra of the main beam (MB), which is dual-polarized, and on VV-spectra of the offset beams, OB1 and OB2, which are single-polarized. OB1 and OB2 are 15 deg. away from the main beam in two orthogonal directions. For the presented measurements, the elevations of MB, OB1 and OB2, are 75°, 90° and 69° respectively. Spectral polarimetric processing is performed to obtain noise- and clutter-free dealiased Doppler spectra (Unal and Moiseev 2004; Unal 2009) for the dual-polarized beam and classical spectral processing is carried out for the single-polarized beams. This full processing is implemented real-time for the TARA radar (Unal et al., 2012). Output examples are given in Fig. 3.

Prior to the retrieval algorithm, three supplementary processing steps are carried out. The first one consists of replacing single and dual-missing data using neighboring Doppler bins. Then an automatic low clipping level is estimated to ensure that the Doppler spectra have about the same spectral reflectivity value at the lower parts of the spectrum. Finally a light smoothing is performed. These steps are carried out on the spectra of Fig. 3, which results in the input spectra of Fig. 4.

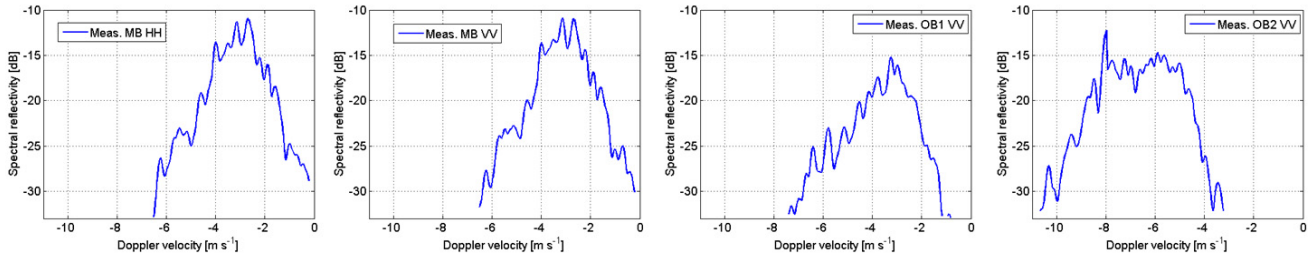


Fig. 3 Multi-beam measured Doppler spectra. The vertical beam is OB1. The height related to the radar is the same (826 m) but the radar resolution volumes are separated by 214 m. This separation has to be compared with the radar resolution resulting from the antenna beamwidth, which is around 30 m for the considered ranges. The full width of the Doppler spectra is about 6 m s⁻¹ but the range of Doppler velocities clearly differs of 1 m s⁻¹ for OB1-MB and -3 m s⁻¹ for OB1-OB2 due to the radial component of the wind ($-v_0$).

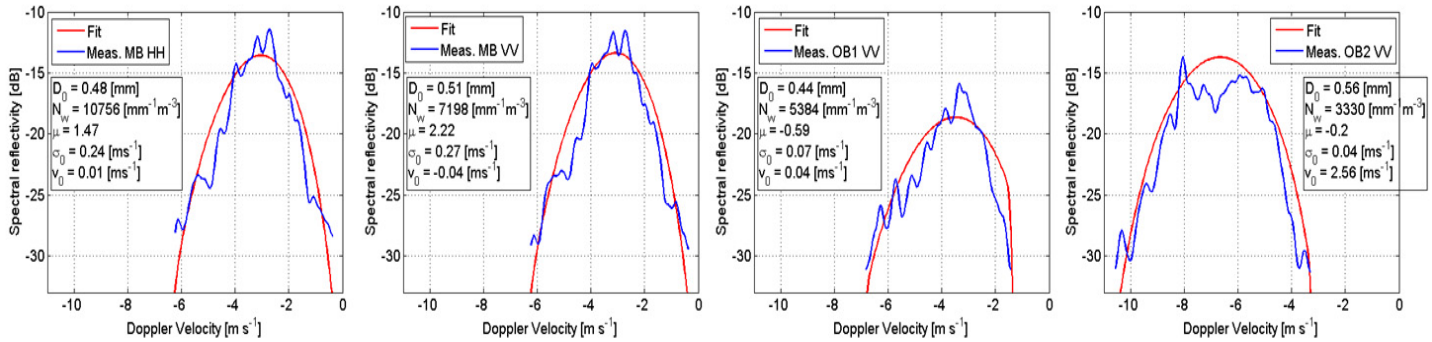


Fig. 4 Input Doppler spectra clipped and lightly smoothed. The clipping level is -28.3, -29.4, -31.1 and -31.3 dB for main beam HH and VV, OB1 and OB2, respectively. The fit and the results of the retrieval algorithm are given. Compared to the other beams, a significant Doppler shift of $-v_0$ (-2.56 m s⁻¹) is found for OB2, which was expected.

To strengthen the retrieval algorithm, two modifications have been implemented. The first one is the reduction of the search interval of μ , which is now $[-1, 5]$. The second one is the reduction of the search interval of D_0 using the reflectivity value. Varying the drop size distribution parameters leads to simulated reflectivity values. After a large number of runs, an interval of median volume diameters can be associated to an interval of reflectivity values, assuming that N_w varies from $10^{2.5}$ to 10^5 . For example, if the measured reflectivity is between 0 and 10 dBZ, like in the example of Figs. 3-4, the search interval of D_0 is $[0.3, 0.9]$ mm. Results of the retrieval algorithms are given in Fig. 4. Note that results for the main beam Doppler spectra HH and VV differ while the radar resolution volumes are the same. Those differences are comparable with the RMSD estimated in Table 1, except for N_w .

6. Retrieval results comparison using HH and VV Doppler spectra

Because the main beam probe the same medium with two different polarization settings, the retrieval algorithm is applied on the Doppler spectra HH and VV. We expect the same retrieval results with an error comparable to the RMSD values of Tables 1-2. The retrieval algorithm is applied on 15552 non-averaged Doppler spectra, HH and VV, acquired with a time resolution of 2.9 s and a height resolution of 14.5 m. They represent stratiform light rain during 3.5 min between 200 m and 1750 m. The results are given in Fig. 5 (odd panels). The correlation coefficient (corr) and the coefficient of variation (CV) of the RMSD are indicated with the RMSD in each scatter-plot. Concerning the microphysical retrievals, the median volume diameter is well retrieved, the error on the shape parameter is ± 1 and the RMSD of both the intercept parameter and the number of raindrops is much too large. Despite its strong dependency in D_0 (D_0^4), the liquid water content estimate is affected by the large error on N_w . Concerning the dynamical retrievals, the radial wind velocity estimate is reasonable and the spectrum width broadening is not well retrieved.

The investigation of the profiles of the retrieved parameters shows that they are noisy. It is necessary to apply a height-smoothing in order to interpret the profiles (Fig. 6). The even panels of Fig. 5 give the scatter-plots between retrievals for the same radar resolution volume when height-smoothing has been applied. As expected, the retrievals HH and VV are better comparable. The error on the shape parameter is decreased to ± 0.4 , the RMSD of N_w and N_t is still large but strongly improved. The retrieval algorithm combined with height-smoothing of the results shows a good consistency of D_0 , μ , v_0 and LWC.

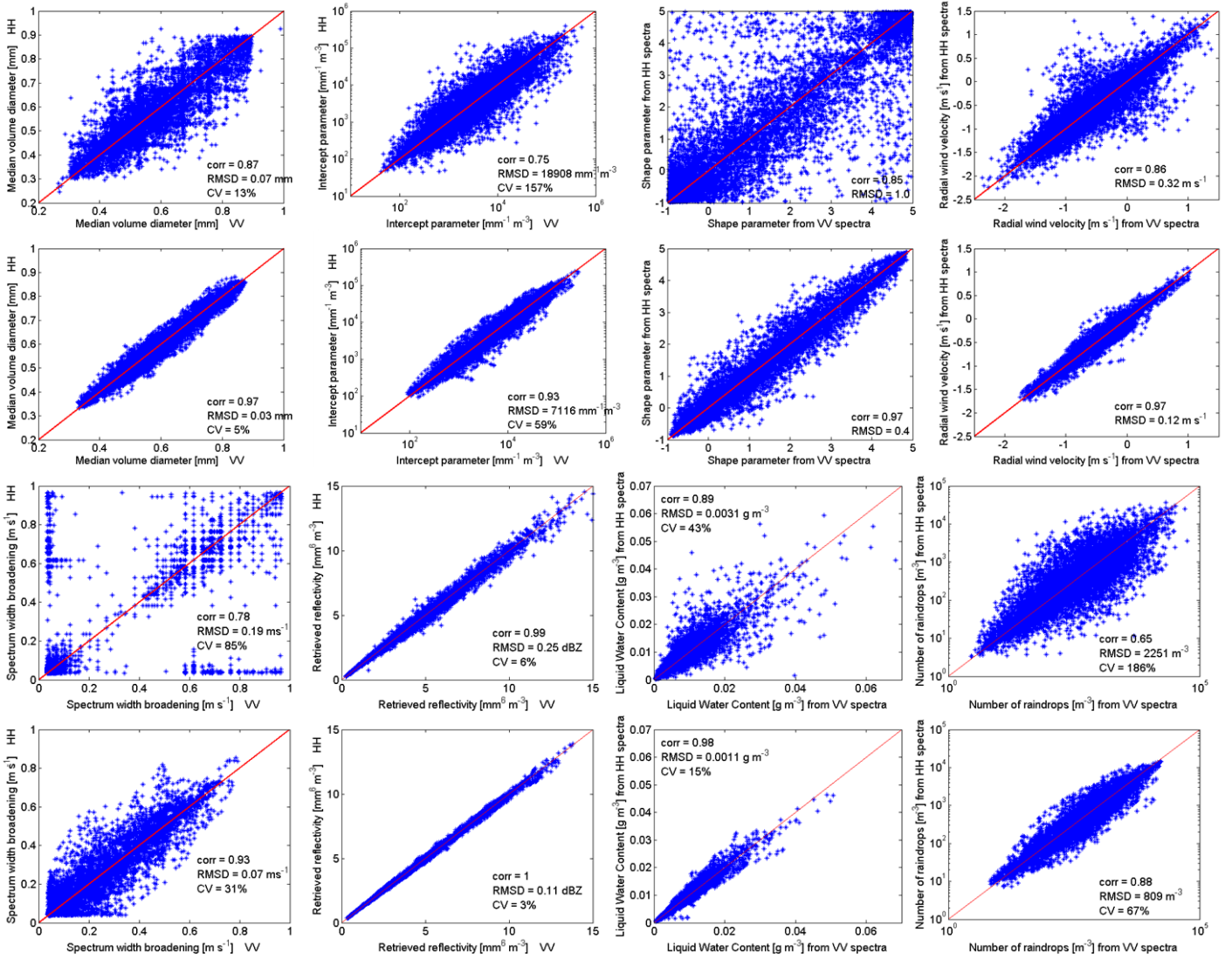


Fig. 5 Scatter plots of the retrieval results for the same radar resolution volume. The variations of the microphysical and dynamical parameters are rather large (odd panel). The even panels show the height-smoothed retrieved results.

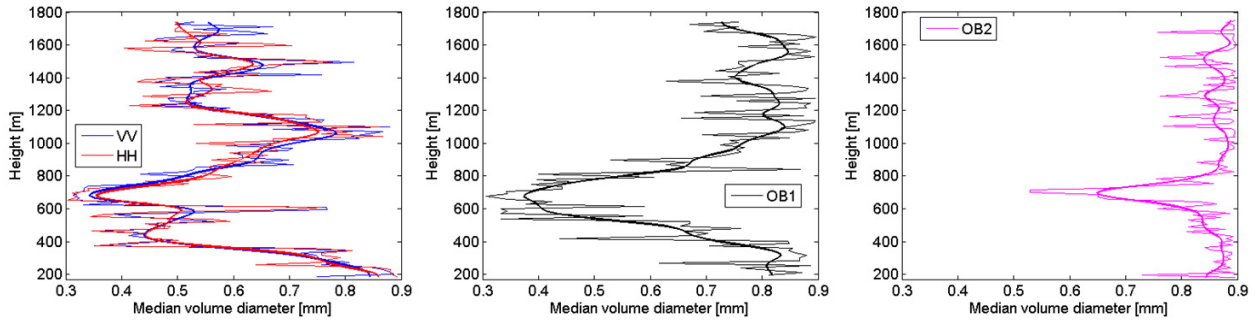


Fig. 6 Example of multi-beam profiles of median volume diameter

7. Retrieval results: first multi-beam comparisons

At the start of the radar far-field (200 m), we expect to obtain retrieval results similar for the three probing beams. At 300 m, the radar resolution volumes are separated by 79 m. The parameters of the 3-beams raindrop size distributions are given in Fig. 7. They reasonably agree. However, the retrieval of D_0 for the beam OB2 differs in absolute value from the D_0 values of the two other beams. Because wind measurements can be performed with TARA, we can compare the 3.5 min averaged radial wind component retrieved ($-v_\theta$) with the radial component of the mean horizontal wind. This is done in Fig. 9. The retrieval ($-v_\theta$) is clearly underestimated. Consequently, the retrieval of D_0 is expected to be overestimated for OB2 and underestimated for MB. This can be already seen in Fig. 7. If the contribution of the 10.5 min averaged horizontal wind radial component is subtracted to the Doppler velocities of the Doppler spectrum before the retrieval procedure, the retrieval of D_0 may be improved. This is demonstrated in Fig. 8, where the agreement of the multi-beam raindrop size distributions is much better.

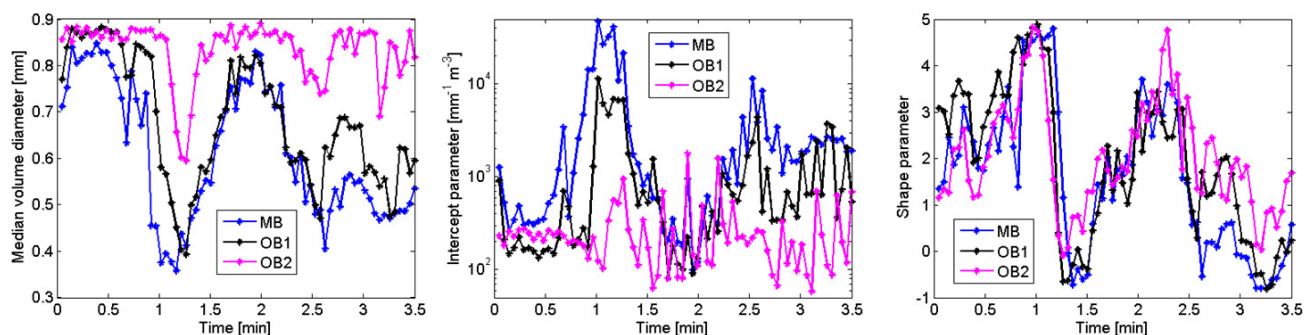


Fig. 7 Comparison of the multi-beam raindrop size distributions at the height 300 m. The time resolution is 2.92 s.

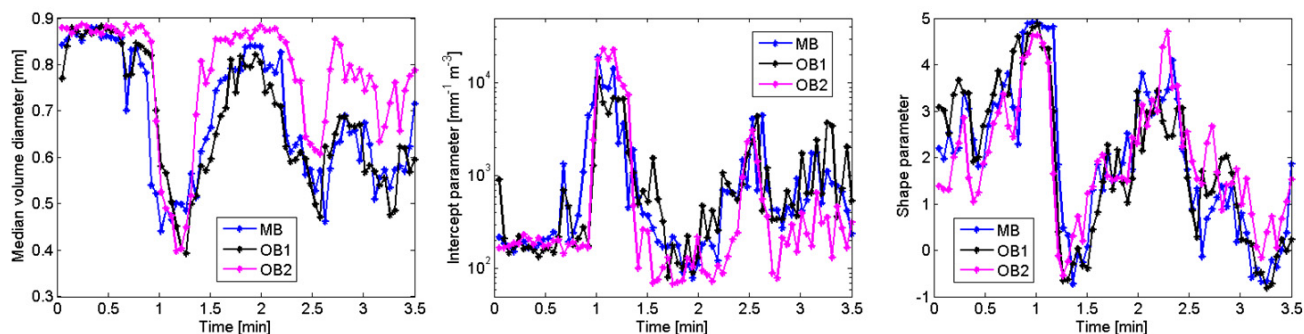


Fig. 8 Comparison of the multi-beam raindrop size distributions at the height 300 m when the mean horizontal wind radial component has been removed in the Doppler spectrum before the retrieval procedure.

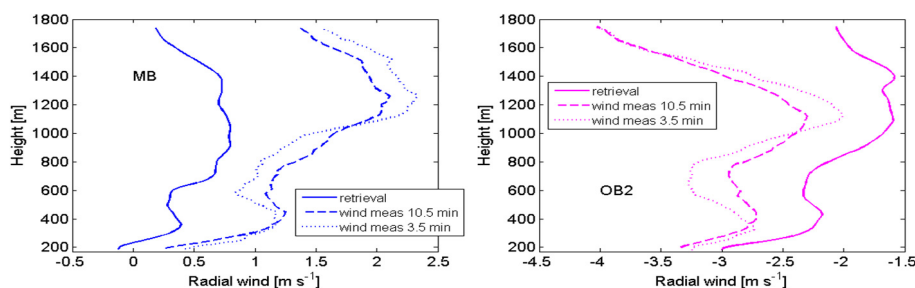


Fig. 9 Radial component of measured mean horizontal wind (averaged on 3.5 and 10.5 min) and retrieved radial wind

8. Preliminary conclusions

When the differential reflectivity cannot be used because of single-polarized beam, near-vertical profiling or very light precipitation, the raindrop size distribution results obtained from the measurement of Doppler spectra show that too many variables need to be retrieved (D_0 , N_{ws} , μ , v_0 , σ_0). With the proposed retrieval technique, the correct estimation of D_0 is crucial. A good candidate to replace Z_{dr} is the radial component of the mean horizontal wind. With this knowledge, the Doppler spectra can be compensated for the horizontal wind influence. This strongly improves the estimation of D_0 . Consequently the other parameters to retrieve are directly improved as well.

References

- Chandrasekar, V., V. Bringi, and P. Brockwell, 1986: Statistical properties of dual-polarized radar signals. Preprints, 23d Conf. on Radar Meteorology, Snowmass, CO, Amer. Meteor. Soc., 193-196.
- Moisseev, D. N., V. Chandrasekar, C. M. H. Unal, and H. W. J. Russchenberg, 2006: Dual-polarization spectral analysis for retrieval of effective raindrop shapes. *J. Atmos. Oceanic Technol.*, **23**, 1682-1695.
- Spek, A., C. Unal, D. Moisseev, H. Russchenberg, V. Chandrasekar, and Y. Dufournet, 2008: A new technique to categorize and retrieve the microphysical properties of ice particles above the melting layer using radar dual polarization spectral analysis. *J. Atmos. Oceanic Technol.*, **25**, 482-497.
- Unal, C., 2009: Spectral polarimetric radar clutter suppression to enhance atmospheric echoes. *J. Atmos. Oceanic Technol.*, **26**, 1781-1797.
- Unal, C. M. H., and D. N. Moisseev, 2004: Combined Doppler and polarimetric radar measurements: correction for spectrum aliasing and non simultaneous polarimetric measurements. *J. Atmos. Oceanic Technol.*, **21**, 443-456.
- Unal, C., Y. Dufournet, T. Otto, and H. Russchenberg, 2012: The new real-time measurement capabilities of the profiling TARA radar. Preprints, Seventh European Conf. on Radar in Meteorology and Hydrology (ERAD), Toulouse, France.