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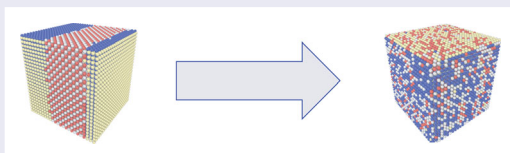
## Long-ranged interactions in bcc NbMoTaW high-entropy alloys

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### ABSTRACT

We reveal that in a prototypical bcc high-entropy alloy NbMoTaW chemical interactions are long ranged and highly frustrated. We show that this is the reason that bcc solid solutions in NbMoTaW can persist to low temperatures. The ab initio-computed long-ranged interactions strongly impact characteristic thermodynamic properties and ordering temperatures. This highlights the genuine importance of taking long-ranged chemical interactions into account for accurate theoretical predictions of high-entropy alloy properties.



### IMPACT STATEMENT

Long-ranged chemical interactions critically impact the thermodynamics and ordering temperature of NbMoTaW and are responsible that this HEA retains the bcc solid solution up to low temperatures.

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Refractory high-entropy alloys; ordering temperatures; ab initio

High entropy alloys (HEAs) are of great interest due to their excellent mechanical,[1–4] magnetic[5–7] and electronic properties.[8,9] Refractory HEAs, such as bcc NbMoTaW, possess extraordinary mechanical properties, comparable to current state-of-the-art nickel-based superalloys, [4,10–12] making them promising candidates for the next generation of high-temperature applications.

Despite their excellent materials properties, little is known about fundamental physical properties, for example, their ground states or the degree of chemical ordering at elevated temperatures. A fundamental conceptual aspect of HEAs is the presumption of a high degree of configurational disorder. Indeed, for NbMoTaW experimental studies reveal no indication of chemical ordering at room temperature as well as in the annealed state.[11] However, due to slow diffusivity, characteristic for refractory elements, chemical ordering is hardly ever approached under typical experimental conditions but can significantly influence creep properties in practical (long-term) applications. At the same time complementary theoretical studies addressing chemical ordering in

these alloys are so far limited due to the challenging nature inherent to the simulation of multi-component alloys.[13–18] In this paper we show that long-ranged chemical interactions, which have so far not been taken into account,[15–17] have an unexpected and dramatic effect on the degree of chemical ordering, causing chemical frustration and implying significant consequences for the ordering temperature as well as thermodynamic properties of HEAs.

NbMoTaW HEA feature very small size mismatch, none of the constituents deviates more than 2.5% in lattice parameter from the average value.[19] Relaxation effects can be presumed to be small, so that treatments taking the disordered bcc solid solution as reference state are a natural choice for studying the tendencies toward chemical order. Such a method is provided by the generalized perturbation method (GPM).[20] Alternately, a cluster expansion (CE) can be attempted. However, as a detailed study of the involved binary alloys indicates, such an expansion is already quite complex when there are just two constituents,[21] let alone when there are four, as is the case here. The efficiency of a GPM-based

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expansion can be intuited by considering that it pertains to one specific composition while a multi-component CE must describe the whole composition range in a quaternary alloy. Furthermore, a GPM expansion typically features strongly dominant effective pair interactions (EPIs) over effective multi-site interactions. The EPIs can be used to simulate configurational order–disorder processes through real-space Monte Carlo simulations. We will show below that long-ranged EPIs are of crucial importance in multi-component alloys. The long-ranged EPIs cause chemical frustration and affect the characteristics of the order–disorder transition.

To capture the chemical degree of freedom, the configurational ordering energy of the quaternary is mapped onto an Ising Hamiltonian employing EPIs as

$$\Delta E_{\text{conf}} = \frac{1}{2} \sum_{\mu\nu} \sum_p^{p_{\text{max}}} V_{\mu\nu}^{(p)} \sum_{n,m \in p} \delta c_{\mu}^{(n)} \delta c_{\nu}^{(m)}, \quad (1)$$

where the second sum runs over all pairs,  $p$ , and  $V_{\mu\nu}^{(p)}$  are the EPIs between distinct atomic species  $\mu$  and  $\nu$ . The first sum runs (in the ‘host’ picture) over the  $N-1$  species  $\mu, \nu$  and  $\delta c_{\mu}^{(n)}, \delta c_{\nu}^{(m)}$  denote the concentration variables at lattice sites  $n$  and  $m$ . The  $N-1$  independent concentration fluctuations  $\delta c_{\mu}^{(n)}$  are given as  $\delta c_{\mu}^{(n)} = c_{\mu}^{(n)} - c_{\mu}$  with concentration  $c_{\mu}$  (i.e. in the equi-atomic alloy  $c_{\mu} \equiv 0.25$ ). The  $c_{\mu}^{(n)}$  are site-occupation variables which equal 1 (0) if the atom at site  $i$  is (is not) occupied by species  $\mu$ . Higher order terms in Equation (1) are found to be small and are neglected.

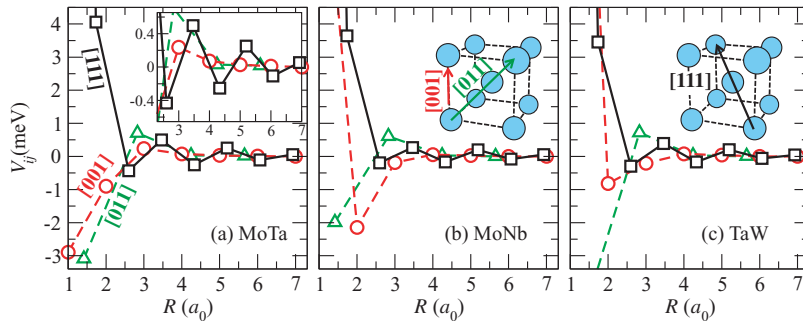
The EPIs have been computed using a density functional theory method employing the exact muffin-tin orbitals (EMTOs) [22] in the Lyngby version of the code.[23] The Brillouin zone integration has been performed employing a  $34 \times 34 \times 34$   $k$ -point mesh according to the Monkhorst-Pack scheme.[24] The lattice constant  $a_0$  has been chosen as  $3.235 \text{ \AA}$  being between the reported room temperature experimental

and theoretically derived  $T = 0 \text{ K}$  value. Chemical disorder is simulated in the single-site coherent potential approximation.[25–27] Screened Coulomb interactions have been taken into account by means of the screened generalized perturbation method.[28,29] Screening parameters are computed employing a large supercell containing 1024 atoms for the 4-component random alloy (with the first two Warren–Cowley short-range order parameters equal zero and the next 4 being less than 0.007 in absolute value for the 6 unique pairs in the considered alloy) using the locally self-consistent Greens function (LSGF) method [30,31] within the EMTO technique, the EMTO-LSGF (ELSGF).[32] Other technical details are as in [33].

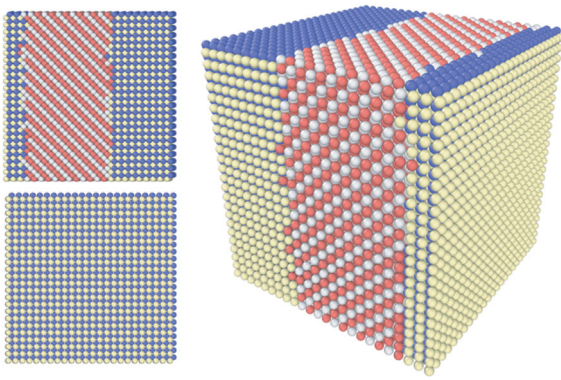
In Figure 1 we show the chemical interactions along three distinct crystallographic directions that is, [001], [011] and [111] for three selected atomic pairs of the HEA. We observe long-ranged and oscillating chemical interactions, particular in [111]-direction.

Long-ranged chemical interactions are, for instance, also observed in fcc CuPd alloys [34] which can be traced back to Fermi surface nesting effects. Although this behavior in the present HEA might also originate from Fermi surface nesting effects, which are known to be present in the pure refractory elements,[35] in a disordered multi-component alloy lifetime effects would weaken such surface nesting characteristics. More intuitive is a tight-binding argument connected to the moments of the Hamiltonian, which favors linear graphs along the strongest hopping integrals [20] in bcc along [111].

Based on the computed pair interactions, the ground state of the NbMoTaW HEA, being unknown so far, has been predicted. We first employ the MC method to equilibrate the system at a low temperature of 10 K. A snapshot of the equilibrated simulation cell is presented in Figure 2. We find that the HEA separates at low temperatures into B2(Mo;Ta) and B32(Nb;W) as illustrated in Figure 2. This is consistent with previous



**Figure 1.** Pair interactions in the HEA NbMoTaW in [001] (circles), [011] (triangles) and [111] (squares) direction for (a) Mo-Ta, (b) Mo-Nb and (c) Ta-W. Long-ranged chemical interactions in [111] are highlighted in subfigure (a). First nearest-neighbor interactions are not shown to facilitate visualization.

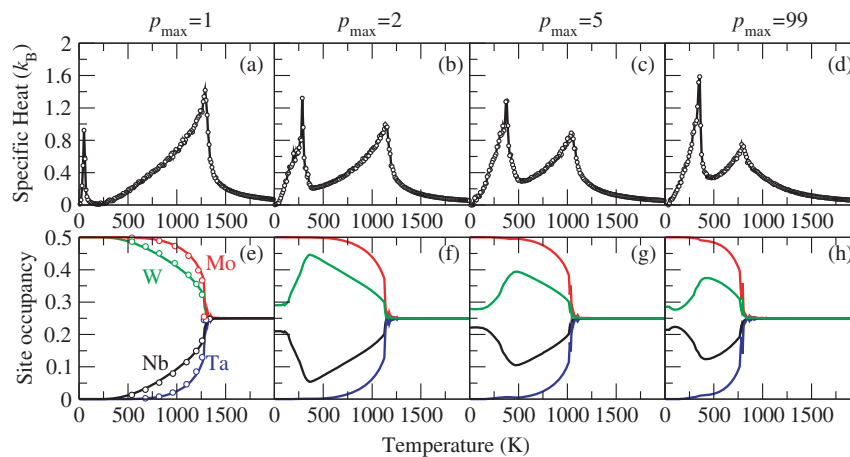


**Figure 2.** The ground state of the HEA NbMoTaW consists of B2(Mo;Ta) (blue and yellow) and B32(Nb;W) (red and silver). The simulation box contained 27,648 atoms.

binary considerations.[21] In order to verify our findings obtained by the MC simulations we performed in addition an extensive ground state search over more than a quarter of a million structures by considering all possible ordered structures containing  $\leq 12$  atoms per primitive unit cell and translation vectors  $< 2 a_0$ . No other ordered structure has been found being lower in energy compared to the phase separated B2(Mo;Ta)/B32(Nb;W) alloy.

We now turn to the prediction of finite-temperature properties. Our key results are shown in Figure 3. The MC simulations have been performed for a box containing 5488 atoms with periodic boundary conditions from 2000 K down to 10 K, including 1, 2, 5 and 99 EPIs where the latter can be regarded as the scenario for a well-converged data set with respect to included EPIs. The configurational contribution to the specific heat, evaluated via the energy fluctuations, is presented in the first row in Figure 3(a)–(d). Two phase transitions are identified for all considered cases. Since B32 ordering requires

at least second nearest-neighbor pair interactions, a B2(Mo;Ta) and B2(Nb;W) ground state is observed for the special case of  $p_{\max} = 1$ , whereas for  $p_{\max} \geq 2$ , a B2(Mo;Ta) and B32(Nb;W) separation is found (Figure 2). Above  $\approx 300$ K, a B2(Mo,W;Ta,Nb) ordering is observed consistent with previous works.[13–17] Note that the determination of first-order phase transition temperatures could be subject to hysteresis effects within MC simulations, which might thus affect the estimated transition temperature. If only nearest-neighbor interactions are taken into account, the B2 order–disorder transition is found at  $\approx 1300$ K (Figure 3(a)). The corresponding site occupancies clearly reveal the order–disorder transition as well as the dominant role of Mo-Ta pairs. The computed site occupancies are in excellent agreement with previous calculations by Huhn.[15] The transition temperature changes, however, dramatically if long-ranged interactions (Figure 1) are taken into account. For  $p_{\max} = 5$  (which corresponds to a cutoff radius of  $1.73 a_0$  for the interactions), the critical temperature is already decreased by  $\approx 20\%$  to  $\approx 1050$  K. By observing Figure 1 it becomes clear, however, that even five coordination shells cannot provide an adequate representation of the long-ranged nature of chemical interactions in this HEA. Eventually, if the long-ranged tail of interactions is taken into account, the ordering temperature is further reduced down to  $\approx 750$  K, which is only  $\approx 60\%$  of the predicted value based on nearest-neighbor interactions only. We can therefore conclude that the refractory NbMoTaW HEA does not retain a bcc solid solution down to low temperatures because the interactions are so small and short ranged, but because they are long ranged and frustrated. Therefore, their considerable strength does not lead to highly stable ordered states.



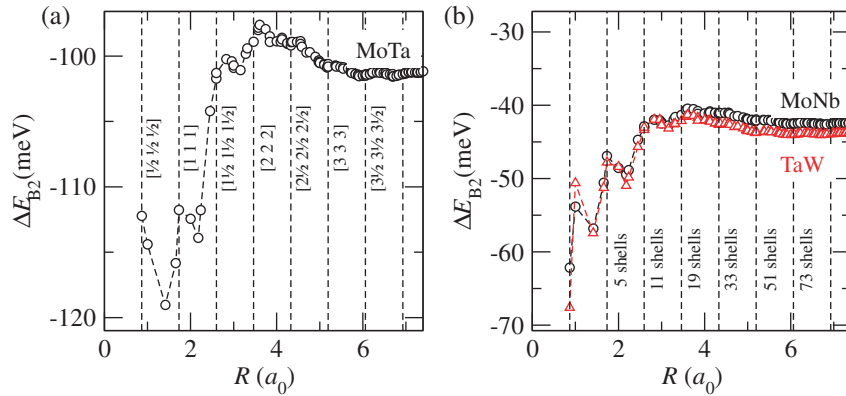
**Figure 3.** Specific heat capacity (a)–(d) and site occupation (e)–(h) for different scenarios of included pair interactions. The B2 ordering temperature is decreased by almost a factor of 2 if long-ranged interactions are taken into account. For nearest-neighbor interactions ( $p_{\max} = 1$ ), the computed site occupancies (e) are compared with previous results obtained in [15].

We can also directly relate the observed strong suppression of the B2 order–disorder transition temperature to the dependence of the respective B2 ordering energies,  $\Delta E_{B2}(R)$ , of the involved pairs on the number of included shells. This is highlighted in Figure 4, where  $\Delta E_{B2}$  is shown for the three B2-favoring pairs MoTa (a) as well as MoNb and TaW (b). The suppression of the B2-ordering temperature when increasing the number of included shells goes along with the increase in  $\Delta E_{B2}$  for all three individual pairs. In fact, all three ordering energies clearly reveal the fingerprints of the long-ranged nature of the underlying chemical interactions shown in Figure 1.

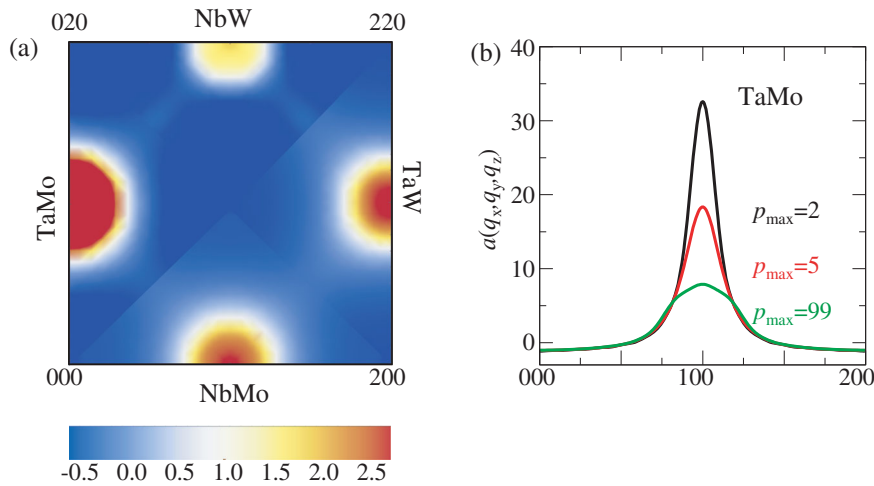
The long-ranged nature of chemical interactions has further consequences for the appearance of short-range order (SRO) at elevated temperatures. We show in Figure 5(a) the SRO parameters  $a(q_x, q_y, q_z)$  at 1300

K computed from the MC simulations including long-ranged interactions ( $p_{\max} = 99$ ). The four quarters of the SRO still clearly reveal the signature of B2(Mo,W;Ta,Nb) ordering, where Mo-Ta pairs are the most dominant SRO contributors followed by Ta-W, Nb-Mo, and Nb-W. Nb-W pairs also reveal fingerprints of the B32-ordering tendencies causing a second (weaker) maximum at  $a(\frac{1}{2} \frac{1}{2} \frac{1}{2})$  (not shown). In Figure 5(b) the SRO parameter in [100] direction is shown for different scenarios of included chemical interactions. The strong impact of the predicted magnitude of SRO on the included number of interactions is consistent with the computed specific heat capacity contribution in Figure 3(a)–(d) and reveals once more the importance of taking long-ranged interactions into account when simulating HEAs at elevated temperatures.

We finally note that the employed interactions derived from the homogeneous disordered alloy represent a good



**Figure 4.** B2 ordering energies depending on the range of included interactions for MoTa (a), MoNb and TaW both (b) The suppression of the B2 order–disorder transition temperature is directly related to the increase in the B2 ordering energies of the individual dominant ordering pairs.



**Figure 5.** Pairwise short-range order parameter in the HEA at 1300 K computed from Monte Carlo simulations. (a) SRO reveals signatures of the B2(Mo,W;Ta,Nb) ordering. Ta-Mo pairs show the strongest SRO tendencies. (b) SRO parameters for TaMo for various scenarios of included chemical interactions reveal the necessity of including long-ranged interactions.



choice for simulating the experimentally relevant high-temperature order–disorder transition, where the alloy remains completely homogeneous.

In contrast to common belief that a few chemical pair interactions are sufficient to describe chemical ordering in transition metal alloys,[20] the prototype bcc NbMoTaW HEA reveals long-ranged interactions causing chemical frustration. The ground state of bcc NbMoTaW is predicted to consist of B2(Mo;Ta) and B32(Nb;W) and the alloy reveals a B2(Mo,W;Ta,Nb) ordering at ambient temperatures. At elevated temperatures, chemical interactions cause chemical frustration and suppress the B2 ordering down to low temperatures. We demonstrate that truncated interactions can have serious consequences for the prediction of thermodynamic properties and ordering effects. Surprisingly, the origin of the appearance of an NbMoTaW solid solution at elevated temperatures is not the presence of short-ranged but rather of competing, long-ranged and frustrated interactions. This new and unexpected insight will greatly facilitate theoretical modeling of the new and promising class of HEAs.

## Disclosure statement

No potential conflict of interest was reported by the authors.

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