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CONSTRAINED RECURSIVE PARAMETER ESTIMATION FOR INSAR ARCS

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ABSTRACT

The growing availability of SAR data offers a real-time deformation monitoring opportunity, but data utilization can be inefficient. Our study introduces a mathematical framework using recursive least-squares and the wrapped phase, allowing efficient updates when new data arrives. This method also incorporates prior knowledge about signal smoothness for non-linear displacement estimation. Compared to the batch solution, our recursive approach achieves parameter estimation without storing past measurements while respecting signal smoothness constraints.

Index Terms— InSAR point scatterers, parameter estimation, recursive least-squares, smoothness constraints

1. INTRODUCTION

InSAR is a highly accurate method for monitoring surface and infrastructure deformation. Typically, parameters are estimated through batch processing of stacked interferograms. However, with frequent SAR acquisitions, achieving near real-time monitoring is challenging due to inefficient data utilization. The growing SAR data volume poses both numerical and methodological challenges. To address this, an efficient parameter estimation update strategy is essential for timely deformation monitoring without the need to reprocess the entire dataset when new SAR data becomes available. [1, 2]. Moreover, due to the non-uniqueness property of an InSAR solution, an implicit unwrapping solution is recommended instead of an explicit solution [3]. Fortunately, some assumptions can be made based on the data itself or a priori knowledge, which should be utilized to constrain the smoothness of the signal [4]. In this study, a method for the time series analysis of InSAR arcs between point scatterers (PS) is introduced, which enables the recursive estimation of parameters of interest. The method is based on the recursive least-squares concept and makes use of the variance-covariance matrix (VCM) of the estimation parameters, especially for non-linear displacements. The presented methodology adds a new acquisition with wrapped phase to the existing stack and updates the previous estimation while incorporating a smoothness constraint based on the acceleration of the displacement signal.

2. RECURSIVE LEAST-SQUARES FOR INSAR ARCS

In this section, we present the functional and stochastic model for InSAR parameter estimation using recursive least-squares and discuss the initialization and update strategy, outlining how we incorporate the wrapped phase with assumptions on ambiguity resolution for the absolute phase of new observations. We introduce a smoothness constraint in the recursive estimation based on expected acceleration.

2.1. Functional and stochastic model

When a steady-state model is assumed to be estimated for a temporal time series of an InSAR arc, the functional and stochastic model can be written as [5]

$$E\{\begin{bmatrix} \underline{\phi}_0 \\ \underline{\phi}_1 \\ \vdots \\ \underline{\phi}_T \end{bmatrix}\} = \begin{bmatrix} A_0 \\ A_1 \\ \vdots \\ A_T \end{bmatrix} x; D\{\begin{bmatrix} \underline{\phi}_0 \\ \underline{\phi}_1 \\ \vdots \\ \underline{\phi}_T \end{bmatrix}\} = \begin{bmatrix} Q_{\phi_0} & & & \\ & Q_{\phi_1} & & \\ & & \ddots & \\ & & & Q_{\phi_T} \end{bmatrix}, \tag{1}$$

where $E\{.\}$ is the expectation of the absolute double difference (DD) phase $\underline{\phi}_t$, shorthand for $\underline{\phi}_{i,j}^{0,t}$ ($t = 0, ..., T$), between epoch t_0 and t for the arc between PS i and j. x_t is the vector of unknown parameters, A_t is the $1 \times n$ matrix that transfers the parameters x into the expectation $E\{\underline{\phi}_t\}$, and n is the number of parameters per epoch. $D\{.\}$ is the dispersion of the model described by the VCM $Q_{\underline{\phi}_t}$. However, the parameters can be time varying, in this case, we introduce a transition matrix, $\Phi_{m,t}$, which is an $n \times n$ matrix that relates the unknowns per epoch, x_m ($m = 0, ..., T$), to the limited set of unknowns x_t for (arbitrary) epoch t, i.e.,

$$x_m = \Phi_{m,t} x_t, \tag{2}$$

it implies that x_m can be parameterized in terms of one single vector x_t for all epochs m. As it is a rather stringent assumption, which will not be realistic for most practical applications, we add a difference vector d_t to model the dynamics of x_t, updating the state transition Eq. (2) to [5]

$$x_m = \Phi_{m,t} x_t + d_{m,t}. \tag{3}$$

The difference vector $d_{m,t}$ incorporates the changes (e.g., due to an acceleration) to the steady-state parameters and is of the same size and unit as x_t. It is not known deterministically and is therefore treated as pseudo-observation. Eqs. (1) and (3) can now be written as [5]

$$E\{\begin{bmatrix} \underline{\phi}_0 \\ \underline{d}_1 \\ \underline{\phi}_1 \\ \vdots \\ \underline{d}_k \\ \underline{\phi}_k \end{bmatrix}\} = \begin{bmatrix} A_0 & & & & \\ -\Phi_{1,0} & I & & & \\ & A_1 & & & \\ & & \ddots & & \\ & & & -\Phi_{k,k-1} & I \\ & & & & A_k \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_{k-1} \\ x_k \end{bmatrix};$$
$$D\{\begin{bmatrix} \underline{\phi}_0 \\ \underline{d}_1 \\ \underline{\phi}_1 \\ \vdots \\ \underline{d}_k \\ \underline{\phi}_k \end{bmatrix}\} = \begin{bmatrix} Q_{\phi_0} & & & & & \\ & Q_{d_1} & & & & \\ & & Q_{\phi_1} & & & \\ & & & \ddots & & \\ & & & & Q_{d_k} & \\ & & & & & Q_{\phi_k} \end{bmatrix}, \tag{4}$$

where Q_{d_m} is the VCM for the difference vector $d_{m,t}$.

2.2. Initialization

We need an initial value for the estimated parameters to start the recursive update. In order to successfully perform the initialization, especially regarding unwrapping, a sufficient number of acquisitions is required. Here we use the first 50 epochs to execute the initialization. When $t = T$, the batch solution using least-squares method for Eq. (1) is given by [5]

$$\hat{\underline{x}}_t = \left(\sum_{t=0}^{T} A_t^\intercal Q_{\phi_t}^{-1} A_t\right)^{-1} \left(\sum_{t=0}^{T} A_t^\intercal Q_{\phi_t}^{-1} \underline{\phi}_t\right);$$
$$Q_{\hat{x}_t} = \left(\sum_{t=0}^{T} A_t^\intercal Q_{\phi_t}^{-1} A_t\right)^{-1}. \tag{5}$$

As the absolute DD phase $\underline{\phi}$ is unknown, we apply the integer least-squares (ILS) [6] to resolve the ambiguity resolution, and the mathematical model can be written as

$$E\{\begin{bmatrix} \underline{\varphi} \\ \underline{b}_0 \end{bmatrix}\} = \begin{bmatrix} F_1 & B_1 \\ F_2 & B_2 \end{bmatrix} \begin{bmatrix} f \\ b \end{bmatrix};$$
$$D\{\begin{bmatrix} \underline{\varphi} \\ \underline{b}_0 \end{bmatrix}\} = \begin{bmatrix} Q_\varphi & 0 \\ 0 & Q_{b_0} \end{bmatrix}, \tag{6}$$

where $\underline{\varphi}$ and $\underline{b}_0$ are the DD phase and pseudo-observation with the corresponding VCM Q_φ and Q_{b_0}, f is the vector of the unknown integer ambiguities ($f \in \mathbb{Z}$), and b is the vector of unknown parameters of interest. F_1 is a $t \times t$ diagonal matrix with -2π on the diagonal, B_1 is a $t \times n$ matrix that transforms b into the expectation of $\underline{\phi}$, F_2 is an $n \times t$ zero matrix, and B_2 is an $n \times n$ identity matrix.

We consider the DD phase $\underline{\varphi}_{\text{dd}}$ (i.e., $\underline{\varphi}_{i,j}^{0,t}$) as

$$\underline{\varphi}_{\text{dd}} = \underline{\phi}_{\text{dis}} + \underline{\phi}_{\Delta H} + \underline{\phi}_{\mu} + \underline{\phi}_{\text{noise}} + 2f\pi(f \in \mathbb{Z}), \tag{7}$$

where $\underline{\phi}_{\text{dis}}$, $\underline{\phi}_{\Delta H}$, $\underline{\phi}_{\mu}$ and $\underline{\phi}_{\text{noise}}$ represent the phase caused by non-thermal displacement, height difference, thermal expansion and noise, respectively. Thus the vector of unknown parameters b is

$$b = \begin{bmatrix} S & v & \Delta H & \mu \end{bmatrix}^\intercal, \tag{8}$$

where S is the atmosphere and noise of the mother image, v is the linear deformation rate, ΔH is the height difference between PS i and j, and μ is the thermal coefficient [7]. For each parameter of interest in b, a pseudo-observation is added in $\underline{b}_0$, and set to zero without a prior knowledge. The VCM Q_{b_0} contains a-priori chosen variances which provide soft bounds to the range of possible values for b. B_1 can be expressed as

$$B_1 = \begin{bmatrix} -\frac{4\pi}{\lambda} & -\frac{4\pi}{\lambda}t & -\frac{4\pi}{\lambda}\frac{B_t^\perp}{R\sin\theta_{\text{inc}}} & -\frac{4\pi}{\lambda}\Delta K_t \end{bmatrix}, \tag{9}$$

where λ is the wavelength of the satellite sensor. $B_t^\perp$ is the perpendicular baseline between the mother and daughter image at epoch t, R is the slant range between the orbit of the mother image and the scatterer, θ_{inc} is the iteratively updated incidence angle of the radar pulse, and ΔK_t is the relative temperature change between the epoch t_0 and t.

Q_φ is the VCM of $\underline{\varphi}$, and it is considered the same as Q_ϕ, which at epoch t can be simplified to

$$Q_{\phi_t} = \sigma^2_{\phi^{0,t}_{i,j}}, \tag{10}$$

where $\sigma_{\phi^{0,t}_{i,j}}$ is the a-priori standard deviation of the phase of arc i, j between epoch t_0 and t. We approximate it by the normalized median amplitude dispersion (NMAD) which is less vulnerable to outliers and calculated with

$$\text{NMAD} = \frac{\text{median}(|a_t - \tilde{a}|)}{\tilde{a}}, \tag{11}$$

where $\tilde{a}$ is the median of the amplitude time series and a_t is the amplitude of epoch t. The derived empirical relation between the NMAD and σ_ϕ is defined as

$$\sigma_\phi = 2.0\,\text{NMAD} - 5.233\,\text{NMAD}^2 + 21.11\,\text{NMAD}^3. \tag{12}$$

When the full time series is available, we detect the epochs where the behavior changes using amplitude data [8, 9], then, we calculate the NMAD for each partition between the detected epochs, and the partitioned σ_ϕ is used for the batch solution.

2.3. Recursive update

For the purpose of computing the present least-squares estimator $\hat{\underline{x}}_{t|t}$ (we denote the estimator $\hat{\underline{x}}_t$ as $\hat{\underline{x}}_{t|t}$, the estimator at epoch t, given the time series t_0: t), there is no need to store the previous observables $\underline{\phi}_t$. That is, the estimator $\hat{\underline{x}}_{t|t}$ can be computed directly from the previous estimator $\hat{\underline{x}}_{t-1|t-1}$, its corresponding VCM $Q_{\hat{x}_{t-1|t-1}}$, and the present observable $\underline{\phi}_t$. In this case, a solution is denoted as $\hat{\underline{x}}_{t|t}$ and is computed with Eq. (5). Once the initial $\hat{\underline{x}}_t$ is known, the updated $\hat{\underline{x}}_{t|t}$ can be recursively computed from the previous $\hat{\underline{x}}_{t|t}$ (i.e., $\hat{\underline{x}}_{t-1|t-1}$) and $\underline{\phi}_t$ using the recursive format of Eq. (4) [5], i.e., the time-update equations, which can be used for prediction,

$$\hat{\underline{x}}_{t|t-1} = \Phi_{t,t-1}\hat{\underline{x}}_{t-1|t-1} + \underline{d}_t;$$
$$Q_{\hat{x}_{t|t-1}} = \Phi_{t,t-1}Q_{\hat{x}_{t-1|t-1}}\Phi_{t,t-1}^\intercal + Q_{d_t}, \tag{13}$$

and the measurement-update equations, which show how to update the prediction $\hat{x}_{t|t-1}$ in order to include the new observable $\underline{\phi}_t$ and the corresponding VCM,

$$\hat{\underline{x}}_{t|t} = \hat{\underline{x}}_{t|t-1} + \left(Q^{-1}_{\hat{x}_{t|t-1}} + A_t^{\intercal} Q^{-1}_{\phi_t} A_t\right)^{-1} A_t^{\intercal} Q^{-1}_{\phi_t} \left(\underline{\phi}_t - A_t \hat{x}_{t|t-1}\right);$$
$$Q_{\hat{x}_{t|t}} = \left(Q^{-1}_{\hat{x}_{t|t-1}} + A_t^{\intercal} Q^{-1}_{\phi_t} A_t\right)^{-1}. \tag{14}$$

Estimations in batch can provide results after enough images are acquired and processed, while the recursive estimation can be executed parallel to data collection.

2.4. An assumption on the ambiguity resolution

The recursive equations are straightforward when the absolute DD phases, $\underline{\phi}_t$, are known, see Eq. (14), but unfortunately, they are unknown. Therefore, a temporal smoothness constraint is needed to aid the ambiguity resolution. To accomplish this, the assumption is made that the estimator of the absolute DD phase observation of the next epoch,

$$\hat{\underline{\phi}}_{t|t-1} = A_t \hat{\underline{x}}_{t|t-1}, \tag{15}$$

is within half a wave cycle of the actual observation, i.e.,

$$|\underline{\phi}_t - \hat{\underline{\phi}}_{t|t-1}| < \pi. \tag{16}$$

The wrapping operator $\mathcal{W}\{.\}$ is introduced as [10]

$$\mathcal{W}\left\{\underline{\phi}\right\} = \mathrm{mod}_{2\pi}(\underline{\phi} + \pi) - \pi, \tag{17}$$

where $\mathrm{mod}_{2\pi}$ is the modulo 2π operator. When Eq. (16) holds true, the wrapped observations can be used,

$$\mathcal{W}\left\{\underline{\varphi}_t - A_t \hat{\underline{x}}_{t|t-1}\right\} = \underline{\phi}_t - A_t \hat{\underline{x}}_{t|t-1}. \tag{18}$$

2.5. A smoothness constraint on correlated acceleration

Due to the non-uniqueness property of InSAR ambiguity resolution, smoothness constraints are required to find optimal solutions. A Gaussian-distributed zero-mean exponentially correlated acceleration is considered as a smoothness constraint in this study, and it can be given with the auto-covariance function,

$$V_{aa}(\Delta t) = \sigma^2_{\text{acc}} e^{-\frac{\Delta t}{L}}, \tag{19}$$

where Δt is the time interval between two epochs. The function is defined by correlation length L, and σ_{acc}, the standard deviation of the acceleration. L and σ_{acc} need to be approximated by a priori knowledge.

In the recursive update, considering the acceleration, the vector of unknown parameters x_t (adapted from b (Eq. (8))) is

$$x_t = \begin{bmatrix} D_t & v_t & a_t & \Delta H_t & \mu_t \end{bmatrix}^{\intercal}, \tag{20}$$

where D_t is the displacement compared to the reference epoch, v_t and a_t are the velocity and acceleration at epoch t. While including v_t and a_t in this vector might seem redundant since D_t already describes the displacement signal, it is of importance in the time-update step (Eq. (13)) to provide an a priori estimate for the next epoch. Thus A_t (adapted from B_1 (Eq. (9))) is

$$A_t = \begin{bmatrix} -\frac{4\pi}{\lambda} & 0 & 0 & -\frac{4\pi}{\lambda}\frac{B_t^{\perp}}{R \sin\theta_{\text{inc}}} & -\frac{4\pi}{\lambda}\Delta K_t \end{bmatrix}. \tag{21}$$

The transition matrix $\Phi_{t,t-1}$ for exponentially correlated acceleration can be expressed as [5]

$$\Phi_{t,t-1} = \begin{bmatrix} 1 & \Delta t & L^2\left(-1 + \frac{\Delta t}{L} + e^{-\frac{\Delta t}{L}}\right) & 0 & 0 \\ 0 & 1 & L\left(1 - e^{-\frac{\Delta t}{L}}\right) & 0 & 0 \\ 0 & 0 & e^{-\frac{\Delta t}{L}} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \tag{22}$$

The difference vector d_t is deterministic with value 0 because of the zero-mean assumption. With auto-covariance function Eq. (19), the stochasticity of d_t, Q_{d_t}, is given by [5]

$$Q_{d_t} = \sigma^2_{\text{acc}} \begin{bmatrix} q_{11} & & & & symm. \\ q_{21} & q_{22} & & & \\ q_{31} & q_{32} & q_{33} & & \\ 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{23}$$

with

$$q_{11} = 2L^3\left[\Delta t - \frac{\Delta t^2}{L} + \frac{\Delta t^3}{3L^2} - 2e^{-\frac{\Delta t}{L}}\Delta t + \frac{L}{2}\left(1 - e^{-2\frac{\Delta t}{L}}\right)\right],$$
$$q_{21} = 2L^2\left[-\Delta t + \frac{1}{2L}\Delta t^2 + e^{-\frac{\Delta t}{L}}\Delta t - Le^{-\frac{\Delta t}{L}} + \frac{L}{2}\left(1 + e^{-2\frac{\Delta t}{L}}\right)\right],$$
$$q_{31} = 2L\left[-e^{-\frac{\Delta t}{L}}\Delta t + \frac{L}{2}\left(1 - e^{-2\frac{\Delta t}{L}}\right)\right],$$
$$q_{22} = 2L\left[\Delta t - \frac{3L}{2} + 2Le^{-\frac{\Delta t}{L}} - \frac{L}{2}e^{-2\frac{\Delta t}{L}}\right],$$
$$q_{32} = 2L\left[-e^{-\frac{\Delta t}{L}} + \frac{1}{2}\left(1 + e^{-2\frac{\Delta t}{L}}\right)\right],$$
$$q_{33} = \left[1 - e^{-2\frac{\Delta t}{L}}\right],$$

and it implicitly contains the expected smoothness of the displacement signal.

3. RESULTS

We apply the recursive least-squares method to analyze Sentinel-1 data over nine years in the Netherlands, comparing it to the batch solution and examining the impact of parameter variations on signal smoothness using exponentially correlated acceleration.

3.1. The batch solution and the recursive solution

Fig. 1 demonstrates the batch and recursive solutions for a specific arc. Fig. 1a shows the amplitude and the NMAD for PS i and j, respectively. The amplitude of PS j indicates anomalous behavior between 2017 and 2018, during which the batch solution detects a partition with a larger NMAD, while the recursive solution shows a peak in NMAD at the end of this period, gradually converging back to the original value. Fig. 1b presents the DD phase and the adjusted phase for both solutions. Clearly, the recursive solution fits the observations much better than the batch solution. Intriguingly, the DD observations with the proposed ambiguity resolution assumption in section 2.4 match the unwrapped solution from ILS, demonstrating the potential of updating using the wrapped phase. Fig. 1c shows the residuals, with most of them falling within the 95% confidence interval for the recursive solution. Fig. 1d illustrates the non-thermal displacement phase, revealing that a linear model in the batch solution fails to capture the anomalous signal, whereas the recursive solution effectively captures non-linear displacements. Figs. 1e and 1f present the instantaneous velocity and acceleration for both the batch and recursive solutions, using predefined parameters $\sigma_{\text{acc}} = 10$ mm/yr^2 and $L = 90$ days. Figs. 1g and 1i display the estimated height difference and thermal coefficient for both solutions. The recursive solution provides estimates comparable to the batch solution and converges toward the batch solution by the final epoch. Figs. 1h and 1j show the derived height and thermal components, which are consistent across both solutions.

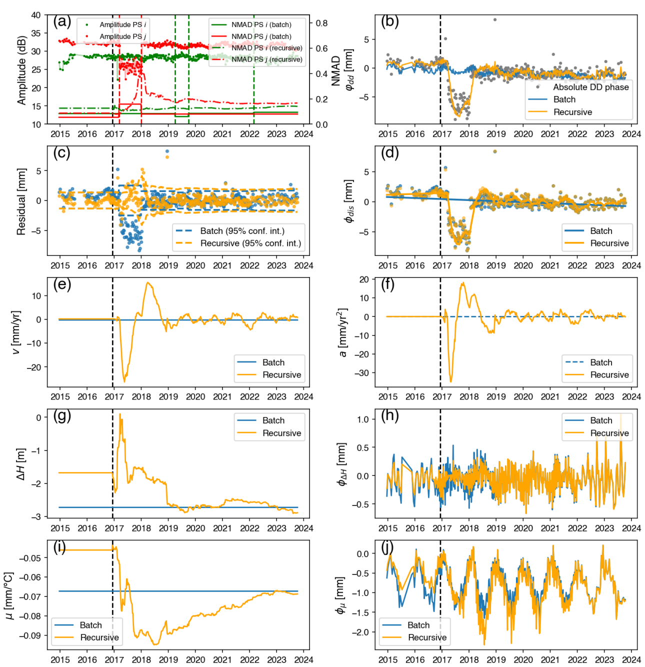

Fig. 1. The batch solution and recursive solution of an arc between two PS from Sentinel-1 data. (a) The amplitude and NMAD. (b) The DD phase and adjusted DD phase; (c) The residuals in (b); (d) The non-thermal displacement phase; (e)-(f) The instantaneous velocity and instantaneous acceleration; (g)-(h) The height difference and the derived component; (i)-(j) The thermal coefficient and the derived component. The black dash lines in (a)-(j) show the last (50th) epoch of the initialization.

3.2. Demonstration of the smoothness constraint

Here we investigate the influence of the constraint on the smoothness of the signal (Fig. 2). Figs. 2a-d show the DD phase and adjusted DD phase of an arc without constraints and with different constraints, and Figs. 2e-f show the corresponding instantaneous velocity and instantaneous acceleration. Note that the standard deviation of the derived acceleration may not be identical with the predefined σ_{acc} due to the restriction of certain ambiguity level of the DD phase. From Figs. 2a and 2b, it is shown that with a constraint, a more reasonable ambiguity resolution is derived. And from Figs. 2b and 2c, it is shown that a bigger σ_{acc} leads to a bigger displacement as it allows a bigger variation of the acceleration. In addition, from Figs. 2c and 2d, a longer L is more likely to result in a bigger fluctuation of displacement (acceleration). Moreover, Figs. 2b and 2d show similar estimates, even though σ_{acc} and L are both different, indicating that these two parameters can play an equivalent role to some extent. Furthermore, the unwrapping solutions in Figs. 2b-d all seem reasonable, and the solutions depend on the constraint we impose. Therefore, our method highlights the importance of smoothness constraints, and implicit unwrapping is highly recommended.

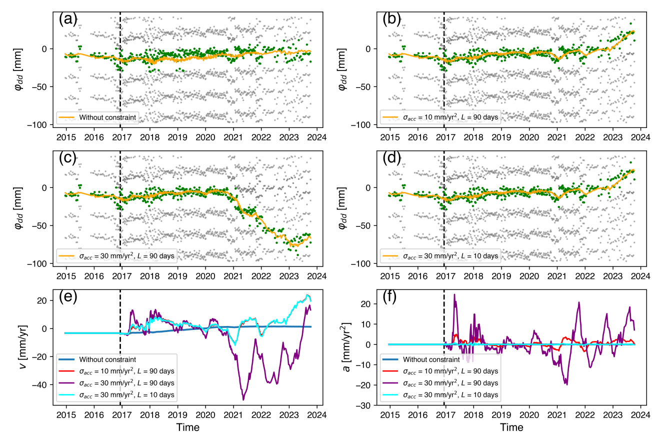

Fig. 2. The influence of the constraint on the smoothness of the signal for an arc. (a) The DD phase and adjusted DD phase without constraints; (b)-(d) show constraints with $[\sigma_{\text{acc}}, L] = [10, 90]$ (b), $[30, 90]$ (c), and $[30, 10]$ (d) in [mm/yr^2] and [days], and the gray dots in (a)-(d) indicate the ambiguity levels; (e)-(f) The corresponding instantaneous velocity and instantaneous acceleration of the signals shown in (a)-(d).

4. CONCLUSION

Recursive least-squares combined with smoothness constraints shows great potential for parameter estimation of InSAR arcs between PS, which contributes to near real-time deformation monitoring. The proposed approach updates the existing dataset when a new observation is available without the need to store the previous observations using the wrapped phase. It demonstrates consistent results compared to the batch solution and manifests an advantage in modeling non-linear displacements. Furthermore, a constraint based on exponentially correlated acceleration is incorporated into the recursive estimation to constrain the smoothness of the displacement signal.

5. REFERENCES

[1] P. S. Marinkovic, F. J. van Leijen, G. Ketelaar, and R. F. Hanssen, "Recursive Data Processing and Data Volume Minimization for PS-InSAR," in *IGARSS 2005 - 2005 IEEE International Geoscience and Remote Sensing Symposium*, 2005, vol. 4, pp. 2697–2700.

[2] F. Hu, F. J. van Leijen, L. Chang, J. Wu, and R. F. Hanssen, "Combined Detection of Surface Changes and Deformation Anomalies Using Amplitude-augmented Recursive InSAR Time Series," *IEEE Trans Geosci Remote Sens*, vol. 60, pp. 1–16, 2022.

[3] Y. Wang, W. S. Brouwer, F. J. Van Leijen, and R. F. Hanssen, "Non-Parametric InSAR Time Series Analysis of Arcs Using Complex B-Splines," in *IGARSS 2023 - 2023 IEEE International Geoscience and Remote Sensing Symposium*, 2023, pp. 8238–8241.

[4] Q. Verburg, "QUInSAR: Temporal Parameter and Ambiguity Estimation Using Recursive Least-Squares: A Methodology for Persistent Scatterer Interferometry," MSc. thesis, TU Delft, the Netherlands, 2017.

[5] P. J. G. Teunissen, *Dynamic Data Processing: Recursive Least-squares*, Delft University Press, Delft, 1st edition, 2001.

[6] P. J. G. Teunissen, "Least-Squares Estimation of the Integer GPS Ambiguities," in *Invited Lecture, Section IV Theory and Methodology, the General Meeting of the International Association of Geodesy, Beijing, China*, August 1993.

[7] L. Chang, R. P. B. J. Dollevoet, and R. F. Hanssen, "Nationwide Railway Monitoring Using Satellite SAR Interferometry," *IEEE Journal of Selected Topics in Applied Earth Observations and Remote Sensing*, vol. 10, no. 2, pp. 596–604, 2017.

[8] C. Truong, L. Oudre, and N. Vayatis, "Selective review of offline change point detection methods," *Signal Processing*, vol. 167, pp. 107299, 2020.

[9] W. S. Brouwer, Y. Wang, F. J. van Leijen, and R. F. Hanssen, "On the Stochastic Model for InSAR Single Arc Point Scatterer Time Series," in *IGARSS 2023 - 2023 IEEE International Geoscience and Remote Sensing Symposium*, 2023, pp. 7902–7905.

[10] R. F. Hanssen, *Radar Interferometry: Data Interpretation and Error Analysis*, Kluwer Academic Publishers, Dordrecht, 2001.