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# Wigner's Friend: Memory, Awareness & Observer-Dependent Realities

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#### **Abstract**

In this paper we analyze the Extended Wigner's Friend Scenario as presented by Baumann & Brukner. The conclusion—that awareness of any change in the Friend's 'internal record' is impossible—is argued to follow from the no-signaling principle. However, we show that this conclusion relies on combining two contradictory assumptions: 1) the lab is perfectly isolated, and 2) Wigner is a super-observer with complete control over the lab. Accepting both implies that the Friend's 'record' is quantum erased, making awareness impossible by definition. To model awareness properly, we introduce a notebook—a stable, unerasable record of the Friend's measurement result. This notebook forces a rejection of at least one of the original assumptions, resulting in a fundamentally different physical context. We demonstrate how this change affects the wavefunctions and joint measurement probabilities, revealing that Baumann & Brukner's reasoning effectively compares outcomes across incompatible contexts. Next, we investigate the physical nature of observers and measurements, proposing a more realistic model in which an observer's state consists of many quantum subsystems. Perfect isolation or complete control becomes implausible, and naturally leads stabilization of the state of the observer. These stable systems constitute an objective reality accessible to other observers, while unstable, erasable systems remain subjective and observer-relative. Our analysis supports an observer-dependent stance on facts in quantum mechanics, where both subjective and objective realities can coexist. This aligns closely with Relational Quantum Mechanics and provides a consistent framework for interpreting Wigner's Friend-type scenarios.

**Keywords:** Extended Wigner's Friend Scenario, Quantum Measurement Problem, Observer-Dependent Facts, Observers, Super-Observers, Modeling Memory Register, Quantum Erasure, Decoherence, Relational Quantum Mechanics, No-signaling Principle, Gedankenexperiment

# **Contents**

I	Introduction	3
2	Description of the Gedankenexperiment	5
	2.1 The Extended Wigner's Friend Scenario and Protocol	5
	2.2 Assumptions, Calculations and Conclusion	6
3	Analysis of the Gedankenexperiment	10
4	Introduction of the Notebook: Modeling Stable Memory	13
5	Implications & Discussion	15
6	Conclusion	17
A	Appendix	20
	A.1 Notation Change and Measurement Bases	20
	A.2 Model of F's State Change	20
	A.3 Model of F's State Change Without B's Measurement Result	20
В	Relational Quantum Analysis of the Gedankenexperiment	21
	B.1 F's Perspective	21
	B.2 B's Perspective	22
	B.3 W's Perspective	23
	B.4 C's Perspective	23

#### 1 Introduction

The recent paper by Frauchiger & Renner [1] instigated the discussion within the foundations of quantum theory and revived interest in Wigner's Friend Gedankenexperiments. These experiments confront the quantum measurement problem [2, 3]—the problem of uniting the deterministic evolution of isolated systems, described by the Schrödinger equation, with the probabilistic state-update rule applied after a measurement. In this Gedankenexperiment we consider 'Wigner's Friend', who is simultaneously treated as an observer and as a measurable quantum system [4]. The Friend performs a measurement on a quantum system inside an isolated laboratory, while Wigner, a so-called 'super-observer', describes this process using quantum formalism [5]. While the Friend perceives a definite outcome, Wigner still assigns a unitary evolution of the entangled state to the entirety of the lab, consisting of the Friend and the quantum system. This leads to the apparent contradiction between both their perspectives: while Wigner describes the joint system in a superposition, the Friend experiences a single definite result.

In recent years, several works have been published applying non-locality theorems (rejecting either locality or realism) to Wigner's Friend-type setups [6–12], shedding new light on the Gedankenexperiment. These so-called no-go theorems often rely on combining 'reasonable' assumptions, such as the *Universal Validity of Quantum Theory* (Q), *Locality* (L), *Freedom of Choice* (F), with the existence of *Observer-Independent Facts*, leading to a contradiction. It could then be concluded that maintaining this observer-independent stance on measurement results may be the limiting factor.

One interpretation that drops *Observer-Independent Facts* is Rovelli's relational quantum mechanics (RQM) [13–16]. RQM is inspired by relativity, which states that 'time' and 'space' are observer-dependent. Extending this idea, the theory suggests that *different observers can give different accounts of the actuality of the same physical property* [13, p.6], implying that a (quantum) 'state' is not an absolute fact, but only real in relation to the observer.

In this paper we analyze 'Observers in Superposition and the No-signaling Principle' by Baumann & Brukner [17]. Utilizing an Extended Wigner's Friend Scenario, including an additional observer Bob, it is argued that "Wigner's measurement will change the Friend's perception and internal record of her result." [18–21]. By assuming the no-signaling condition, thus prohibiting superluminal information transfer between two distant observers, it is concluded that awareness of any change in this 'internal record' by the Friend is impossible and that "no knowledge about the Friend's memory [internal record] before Wigner's measurement remains." [17, p.6].

We will argue that this conclusion appears to rely on combining two physical contexts that may not be simultaneously applicable. In Wigner's Friend-type scenarios, such as the one described in [17], the following two key assumptions are typically made: (1) Wigner is a super-observer with complete control over the lab, and (2) the lab is perfectly isolated from the environment. Accepting both implies that the Friend's internal record and measurement outcome are quantum erased due to Wigner's measurement on the entire lab. However, the model of 'awareness' proposed by [17] seems to assume that the Friend's internal record and measurement are definite results, denying any erasure and thus making the definition of awareness proposed by [17] impossible in Wigner's Friend-type scenarios. Because the contents of the internal memory are subject to erasure, we will refer to it as 'the state of the observer'.

To properly model the definiteness of awareness, we introduce a *notebook*—a stable, unerasable record of the Friend's measurement result. This addition changes the wavefunction and the joint probabilities, as it represents a different physical context in which at least one of the key assumptions (1) or (2) is rejected. We argue that Baumann & Brukner's reasoning implicitly utilizes these incompatible contexts, questioning the logical consistency behind the conclusion.

Next, we analyze what it means to be an 'observer' making a 'measurement' in a more physically realistic setting, using the notebook as a tool for interpretation. We assume that an observer's state is a macroscopic system, composed of many quantum subsystems. Given this large composition, the existence of stable subsystems or correlations with the stable environment is more likely, making the information effectively unerasable within these stable systems. These records constitute the 'objective reality' shared across observers. In contrast, the state of a completely unstable and isolated observer exists only relative

to that specific observer, thus forming a 'subjective reality' until its contents are erased, representing the observer-dependent facts necessary for the no-go scenarios we previously described [6–12]. Subjective and objective realities comply with the relational interpretation and we will show that both can coexist consistently within the framework of RQM.

This paper is structured as follows. In Section 2 we introduce the EWFS and reconstruct the reasoning by B&B. In Section 3 we analyze the made assumptions and show where the argumentation falls short. In Section 4 we introduce the notebook model and examine its implications. Section 5 discusses the physical meaning of an observer, measurement and decoherence. In Section 6 we conclude by supporting for observer-dependent facts within the RQM framework.

# 2 Description of the Gedankenexperiment

#### 2.1 The Extended Wigner's Friend Scenario and Protocol

We now describe the Extended Wigner's Friend Scenario (EWFS) and the protocol used by Baumann & Brukner (B&B) [17]. For clarity, some changes have been made to the original notation. These changes along with measurement bases for each observer are summarized in A.1.

The setup consists of a bipartite quantum state  $|\Phi\rangle_{S_1S_2}$ , containing two qubits  $S_1$  and  $S_2$ , and involves three observers: Wigner's Friend (F), Bob (B) and Wigner (W). From here on, we use shorthand notation to describe these systems and observers. The following is stated about measurements: "Measurements are described by entangling unitaries which correlate the respective observer (F and W) with the measured  $U_O: |i\rangle_S |r\rangle_O \to |i\rangle_S |I_i\rangle_O$  $\forall i$ , where  $|i\rangle$  are the eigenstates of the observable quantum system being measured and  $|I_i\rangle$  is the state of the observer having registered outcome i." [17, p. 7] and "the result encoded in the friend's state  $|\cdot\rangle_F$ , i.e. the entry in her memory register."[17, p. 7]. From these statements, we assume that B&B's definition of a measurement involves two actions: first, correlating the state of the observer  $|\cdot\rangle_O$  with the state of the observed system  $|\cdot\rangle_S$  and second, storing the outcome inside the observer's 'memory register'  $|\cdot\rangle_O$ . The terms 'memory register', 'memory' and '(internal) record' are used interchangeably throughout [17]. For clarity and consistency, we will refer to  $|\cdot\rangle_{O}$  exclusively as the state of observer O. We consider W to be a 'super-observer', who has full control over the isolated lab, consisting of the joint system  $\{S_1, F\}$ , and performs a measurement on this system in his own basis. All participating observers agree beforehand on the following measurement protocol (illustrated in Fig. 1):

(0.) At  $t = t_0$ , the following has been completed: the initial bipartite quantum state  $|\Phi\rangle_{S_1S_2}$  has been prepared in the following superposition of the computational basis  $\{|0\rangle, |1\rangle\}$ 

$$|\Phi\rangle_{S_1S_2} = \alpha |0\rangle_{S_1} |1\rangle_{S_2} + \beta |1\rangle_{S_1} |0\rangle_{S_2}$$

and qubits  $S_1$  and  $S_2$  are sent, and subsequently received by F inside the isolated laboratory, and by B respectively.

- (1.) At  $t = t_1$ , the following has been completed: F measures  $S_1$  in the same computational basis and the measurement result gets encoded inside F's state  $|\cdot\rangle_F$ .
- (2.) At  $t = t_2$ , the following has been completed: B measures  $S_2$  in the orthonormal basis

$$\begin{split} |\uparrow\rangle_{S_2} &:= \mu \, |0\rangle_{S_2} + \nu \, |1\rangle_{S_2} \,, \\ |\downarrow\rangle_{S_2} &:= \overline{\nu} \, |0\rangle_{S_2} - \overline{\mu} \, |1\rangle_{S_2} \end{split}$$

and the measurement result gets encoded inside B's state  $|\cdot\rangle_B$ .

(3.) At  $t = t_3$ , the following has been completed: W measures the lab consisting of the first qubit  $S_1$  and F in the following orthonormal basis

$$\begin{split} |\mathsf{ok}\rangle_{S_1F} &:= a\,|0\rangle_{S_1}\,|0\rangle_F + b\,|1\rangle_{S_1}\,|1\rangle_F\,,\\ |\mathsf{fail}\rangle_{S_1F} &:= \overline{b}\,|0\rangle_{S_1}\,|0\rangle_F - \overline{a}\,|1\rangle_{S_1}\,|1\rangle_F \end{split}$$

and the measurement result gets encoded inside W's state  $|\cdot\rangle_W$ .

Each measurement basis is orthonormal, meaning that  $|\alpha|^2 + |\beta|^2 = 1$ ,  $|\mu|^2 + |\nu|^2 = 1$  and  $|a|^2 + |b|^2 = 1$ , with  $\alpha, \beta, \mu, \nu, a, b \in \mathbb{C}$ . Now that the setup and protocol is clear, we will reproduce the calculations of the protocol and reasoning that support the eventual conclusion of [17].

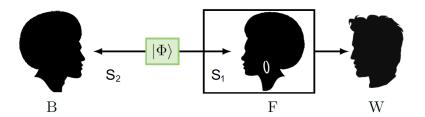


Figure 1 – A bipartite quantum state  $|\Phi\rangle_{S_1S_2}$  is emitted and received at  $t_0$ . At  $t_1$ , qubit  $S_1$  is measured by F and stored inside  $|\cdot\rangle_F$  using the computational basis. At  $t_2$ , qubit  $S_2$  is measured by B and stored inside  $|\cdot\rangle_B$  using  $\{\uparrow,\downarrow\}$ -basis. Subsequently, W makes his measurement using  $\{ok, fail\}$ -basis at  $t_3$  on the isolated lab consisting of the joint state  $\{S_1, F\}$ , thereby possibly altering the results stored in F's state  $|\cdot\rangle_F$  [17].

#### 2.2 Assumptions, Calculations and Conclusion

Before starting the calculations, it is important to note that Baumann & Brukner make many assumptions, which are not always explicitly stated. We will highlight these implicit assumptions to gain a broader understanding of the reasoning and the resulting conclusion presented in [17].

Using the protocol described in Section 2.1, the wavefunction at time  $t = t_i$  (with  $i \in \{0, 1, 2, 3\}$ ) can be calculated and will be represented in the following form:  $|\Psi\rangle^{t=t_i}$ . The wavefunction of an observer O, who is yet to make a measurement (ready-state) is denoted by  $|r\rangle_O$ . For example, when F measures the system  $S_1$ , its outcome will be registered inside her state  $|\cdot\rangle_F$  and correlated with the observable:  $|0\rangle_{S_1}|r\rangle_F \to |0\rangle_{S_1}|0\rangle_F$ . To enable W to be 'super-observer' and permit his measurement on the joint  $\{S_1, F\}$  system, the following assumption must be made: *Universal Validity of Quantum Theory* (Q): "The rules of quantum mechanics apply to all systems", thereby enabling us to measure an observer. The calculations corresponding to the protocol are as follows:

$$|\Psi\rangle^{t=t_0} = \left(\alpha |0\rangle_{S_1} |1\rangle_{S_2} + \beta |1\rangle_{S_1} |0\rangle_{S_2}\right) |r\rangle_F |r\rangle_B |r\rangle_W, \tag{1}$$

$$\begin{split} |\Psi\rangle^{t=t_{1}} &= \left(\alpha |0\rangle_{S_{1}} |0\rangle_{F} |1\rangle_{S_{2}} + \beta |1\rangle_{S_{1}} |1\rangle_{F} |0\rangle_{S_{2}}\right) |r\rangle_{B} |r\rangle_{W} \\ &= \left[\alpha |0\rangle_{S_{1}} |0\rangle_{F} \left(\overline{\nu} |\uparrow\rangle_{S_{2}} - \mu |\downarrow\rangle_{S_{2}}\right) \\ &+ \beta |1\rangle_{S_{1}} |1\rangle_{F} \left(\overline{\mu} |\uparrow\rangle_{S_{2}} + \nu |\downarrow\rangle_{S_{2}}\right) |r\rangle_{B} |r\rangle_{W}, \end{split}$$

$$(2)$$

$$|\Psi\rangle^{t=t_{2}} = \left[\alpha |0\rangle_{S_{1}} |0\rangle_{F} \left(\overline{\nu} |\uparrow\rangle_{S_{2}} |\uparrow\rangle_{B} - \mu |\downarrow\rangle_{S_{2}} |\downarrow\rangle_{B}\right) +\beta |1\rangle_{S_{1}} |1\rangle_{F} \left(\overline{\mu} |\uparrow\rangle_{S_{2}} |\uparrow\rangle_{B} + \nu |\downarrow\rangle_{S_{2}} |\downarrow\rangle_{B}\right) |r\rangle_{W} = \left[\alpha \left(\overline{a} |\text{ok}\rangle_{S_{1}F} + b |\text{fail}\rangle_{S_{1}F}\right) \left(\overline{\nu} |\uparrow\rangle_{S_{2}} |\uparrow\rangle_{B} - \mu |\downarrow\rangle_{S_{2}} |\downarrow\rangle_{B}\right) +\beta \left(\overline{b} |\text{ok}\rangle_{S_{1}F} - a |\text{fail}\rangle_{S_{1}F}\right) \left(\overline{\mu} |\uparrow\rangle_{S_{2}} |\uparrow\rangle_{B} + \nu |\downarrow\rangle_{S_{2}} |\downarrow\rangle_{B}\right) |r\rangle_{W},$$
(3)

$$\begin{split} |\Psi\rangle^{t=t_{3}} &= \left[\alpha\left(\overline{a}\left|\operatorname{ok}\right\rangle_{S_{1}F}\left|\operatorname{ok}\right\rangle_{W} + b\left|\operatorname{fail}\right\rangle_{S_{1}F}\left|\operatorname{fail}\right\rangle_{W}\right)\left(\overline{\nu}\left|\uparrow\right\rangle_{S_{2}}\left|\uparrow\right\rangle_{B} - \mu\left|\downarrow\right\rangle_{S_{2}}\left|\downarrow\right\rangle_{B}\right) \\ &+ \beta\left(\overline{b}\left|\operatorname{ok}\right\rangle_{S_{1}F}\left|\operatorname{ok}\right\rangle_{W} - a\left|\operatorname{fail}\right\rangle_{S_{1}F}\left|\operatorname{fail}\right\rangle_{W}\right)\left(\overline{\mu}\left|\uparrow\right\rangle_{S_{2}}\left|\uparrow\right\rangle_{B} + \nu\left|\downarrow\right\rangle_{S_{2}}\left|\downarrow\right\rangle_{B}\right) \\ &\left(\operatorname{Basis-change}\right) = \left(a\left|0\right\rangle_{S_{1}}\left|0\right\rangle_{F} + b\left|1\right\rangle_{S_{1}}\left|1\right\rangle_{F}\right)\left|\operatorname{ok}\right\rangle_{W}\left[\left|\uparrow\right\rangle_{S_{2}}\left|\uparrow\right\rangle_{B}\left(\beta\overline{\mu}\overline{b} + \alpha\overline{\nu}\overline{a}\right) + \left|\downarrow\right\rangle_{S_{2}}\left|\downarrow\right\rangle_{B}\left(\beta\nu\overline{b} - \alpha\mu\overline{a}\right)\right] \\ &+ \left(\overline{b}\left|0\right\rangle_{S_{1}}\left|0\right\rangle_{F} - \overline{a}\left|1\right\rangle_{S_{1}}\left|1\right\rangle_{F}\right)\left|\operatorname{fail}\right\rangle_{W}\left[\left|\uparrow\right\rangle_{S_{2}}\left|\uparrow\right\rangle_{B}\left(\alpha\overline{\nu}\overline{b} - \beta\overline{\mu}\overline{a}\right) - \left|\downarrow\right\rangle_{S_{2}}\left|\downarrow\right\rangle_{B}\left(\beta\nu\overline{a} + \alpha\mu\overline{b}\right)\right] \\ &= \left|0\right\rangle_{S_{1}}\left|0\right\rangle_{F}\left|\uparrow\right\rangle_{S_{2}}\left|\uparrow\right\rangle_{B}\left[\left(a\overline{b}\beta\overline{\mu} + \alpha\overline{\nu}|a|^{2}\right)\left|\operatorname{ok}\right\rangle_{W} + \left(\alpha\overline{\nu}|b|^{2} - a\overline{b}\beta\overline{\mu}\right)\left|\operatorname{fail}\right\rangle_{W}\right] \\ &+ \left|0\right\rangle_{S_{1}}\left|0\right\rangle_{F}\left|\downarrow\right\rangle_{S_{2}}\left|\downarrow\right\rangle_{B}\left[\left(a\overline{b}\beta\nu - \alpha\mu|a|^{2}\right)\left|\operatorname{ok}\right\rangle_{W} - \left(a\overline{b}\beta\nu + \alpha\mu|b|^{2}\right)\left|\operatorname{fail}\right\rangle_{W}\right] \\ &+ \left|1\right\rangle_{S_{1}}\left|1\right\rangle_{F}\left|\uparrow\right\rangle_{S_{2}}\left|\uparrow\right\rangle_{B}\left[\left(\beta\overline{\mu}|b|^{2} + \overline{a}b\alpha\overline{\nu}\right)\left|\operatorname{ok}\right\rangle_{W} - \left(\overline{a}b\alpha\overline{\nu} - \beta\overline{\mu}|a|^{2}\right)\left|\operatorname{fail}\right\rangle_{W}\right] \\ &+ \left|1\right\rangle_{S_{1}}\left|1\right\rangle_{F}\left|\downarrow\right\rangle_{S_{2}}\left|\downarrow\right\rangle_{B}\left[\left(\beta\nu|b|^{2} - \overline{a}b\alpha\mu\right)\left|\operatorname{ok}\right\rangle_{W} + \left(\beta\nu|a|^{2} + \overline{a}b\alpha\mu\right)\left|\operatorname{fail}\right\rangle_{W}\right]. \end{aligned} \tag{4}$$

In Eq. 4 the basis-change is made from the {ok, fail}-basis to the {0, 1}-basis to describe the outcome registered inside F's state  $|\cdot\rangle_F$  after W's measurement at  $t = t_3$ .

The joint probability distributions for the contents of  $|\cdot\rangle_F$  and  $|\cdot\rangle_B$  are then obtained for  $t=t_2$  and  $t=t_3$  and are denoted by  $P_{t_i}(|\cdot\rangle_F,|\cdot\rangle_B)$ , where  $t_i \in \{t_2,t_3\}$ . The probabilities at  $t_2$  require no further explanation (see Eq. 3). At  $t_3$ , they are bit more involved (see Eq. 4), so an example will be provided below:

$$\begin{split} \mathbf{P}_{t_{3}}(|0\rangle_{F},|\uparrow\rangle_{B}) = & \left| a\overline{b}\beta\overline{\mu} + \alpha\overline{\nu}|a|^{2} \right|^{2} + \left| \alpha\overline{\nu}|b|^{2} - a\overline{b}\beta\overline{\mu} \right|^{2} \\ = & \left| ab\beta\mu \right|^{2} + \left| \alpha\nu|a|^{2} \right|^{2} + \left( a\overline{b}\beta\overline{\mu} \right) \left( \overline{\alpha}\nu|a|^{2} \right) + \left( \overline{a}b\overline{\beta}\mu \right) \left( \alpha\overline{\nu}|a|^{2} \right) \\ & + \left| \alpha\nu|b|^{2} \right|^{2} + \left| ab\beta\mu \right|^{2} - \left( \alpha\overline{\nu}|b|^{2} \right) \left( \overline{a}b\overline{\beta}\mu \right) - \left( \overline{\alpha}\nu|b|^{2} \right) \left( a\overline{b}\beta\overline{\mu} \right) \\ = & \left| \alpha\nu \right|^{2} \left( |a|^{4} + |b|^{4} \right) + 2|ab\beta\mu|^{2} + \left( a\overline{b}\alpha\beta\overline{\mu}\nu \right) \left( |a|^{2} - |b|^{2} \right) + \left( \overline{a}b\alpha\overline{\beta}\mu\overline{\nu} \right) \left( |a|^{2} - |b|^{2} \right) \\ = & \left| \alpha\nu \right|^{2} \left( |a|^{4} + |b|^{4} \right) + 2|ab\beta\mu|^{2} + \left( a\overline{b}\alpha\beta\overline{\mu}\nu + \overline{a}b\alpha\overline{\beta}\mu\overline{\nu} \right) \left( |a|^{2} - |b|^{2} \right) \\ = & \left| \alpha\nu \right|^{2} \left( |a|^{4} + |b|^{4} \right) + 2|ab\beta\mu|^{2} + 2\operatorname{Re}\left\{ a\overline{b}\alpha\beta\overline{\mu}\nu \right\} \left( |a|^{2} - |b|^{2} \right). \end{split}$$

The other calculations are similar and all results are summarized in Table 1. The joint probabilities will generally differ before and after W's measurement. To model this change in  $|\cdot\rangle_F$ , flipping probabilities  $q^{nm}$  are introduced by [17], where  $n \in \{0,1\}$  and  $m \in \{\uparrow,\downarrow\}$ : these represent the probability that F's result  $|n\rangle_F$  from  $t_1$  is flipped at  $t_3$  ( $n=0 \to 1$  or  $n=1 \to 0$ ), given B's registered result is  $|m\rangle_B$ . By applying these flipping probabilities to the probability distributions, a set of equations is obtained. The solutions for  $q^{nm}$  will depend 'non-trivially' on  $\mu$  and  $\nu$ , the parameters set by B's measurement settings (see A.2).

In addition to the flipping probabilities, the concept of 'perception' or 'awareness' of the change in F's state is introduced. As stated in [17, p.2]: "she [F] has an additional memory register [state] that can record whether or not a change has occurred in memory registers [state] storing the measurement result". Because of this awareness, F can deduce the contents of  $|\cdot\rangle_F$  at time  $t_2$  by combining her current registered result at time  $t_3$  with the information whether or not a change has occurred, i.e. F possesses full knowledge of the contents of her state  $|\cdot\rangle_F$  at both  $t_2$  and  $t_3$ .

Table 1 – The joint probabilities for measurement outcomes by F and B at times  $t = t_2$  and  $t = t_3$ , with  $\xi \equiv (|a|^2 - |b|^2) \operatorname{Re} \left\{ a \overline{b \alpha} \beta \overline{\mu} \nu \right\}$ . Table 1 is equivalent to TABLE I from [17, p. 4]

$P_{t_2}(F,B)$	$ \!\!\uparrow\rangle_B$	$ \downarrow\rangle_B$
$ 0\rangle_F$	$ \alpha v ^2$	$ \alpha\mu ^2$
$ 1\rangle_F$	$ \beta\mu ^2$	$ \beta \nu ^2$

$P_{t_3}(F,B)$	$ \!\!\uparrow\rangle_B$	$ \!\!\downarrow\rangle_B$
	$ \alpha v ^2 ( a ^4 +  b ^4)$	$ \alpha\mu ^2 ( a ^4 +  b ^4)$
$ 0\rangle_F$	$+2 \beta\mu ^2 ab ^2+2\xi$	$ +2 \beta\nu ^2 ab ^2-2\xi$
	$ \beta\mu ^2 ( a ^4 +  b ^4)$	$ \beta v ^2 ( a ^4 +  b ^4)$
$ 1\rangle_F$	$+2 \alpha v ^2 ab ^2-2\xi$	$+2 \alpha\mu ^2 ab ^2+2\xi$

Using the non-trivial  $\mu$ ,  $\nu$ -dependency of the flipping probabilities and awareness of change in F's state, it is argued that "no knowledge about the Friend's memory [state] before Wigner's measurement remains" [17, p. 6]. This conclusion is derived through the following steps:

- (I) Beforehand the prepared bipartite state  $|\Phi\rangle_{S_1S_2}$  and W's measurement settings (i.e., the values of  $\alpha$ ,  $\beta$ , a and b) are made known to all observers. B is allowed to choose between two different measurement settings,  $\{\mu_1, \nu_1\}$  and  $\{\mu_2, \nu_2\}$ . The specific values of both options are also known to everybody. B randomly selects one of the two measurement settings and keeps his choice to himself.
- (II) The protocol described in Section 2.1 is then repeated  $K \gg 1$  times. For all K iterations, the settings for  $|\Phi\rangle_{S_1S_2}$ , W, and the chosen setting for B remain fixed.
- (III) F calculates the flipping probabilities for both of B's measurement settings  $\{\mu_1, \nu_1\}$  and  $\{\mu_2, \nu_2\}$ . Assuming F has awareness of any change of her state, she can deduce which flipping probability matches her data and consequently knows B's chosen measurement setting. F can then communicate this information to W. If F and W's measurements are outside the future light cone of B's measurement, this information is transmitted at superluminal speeds, concluding that any awareness of change in F's state is impossible.

To make (I)-(III) more clear, the steps are illustrated in Fig. 2. It is important to emphasize that the number of iterations K is finite, meaning that this conclusion cannot be drawn with absolute certainty.

This reasoning implicitly relies on two notable assumption: *Freedom of choice* (F): "allowing observers to choose between different measurement settings" (applied by *B* in step (I)); and *Locality* (L): "observation made by one observer has no impact on the observation made by another observer who is space-like separated from them" (also called the no-signaling condition, applied to all observers in step (III)). Considering the implicit assumptions made by [17], the no-go theorem from [12, p.37] forbids *Observer-independent facts* in the Gedankenexperiment. We will return to this point in B.

Following the protocol, assumptions and line of reasoning, the conclusion is: "under the assumption of the no-signaling condition, no perception of the change in the Friend's memory [state] in Wigner's Friend experiments is possible.", i.e. "no knowledge about the memory [F's state] before Wigner's measurement remains." [17, p. 6]. In the next chapter, we will analyze the Gedankenexperiment more closely to examine whether the conclusion reached in [17] will hold.

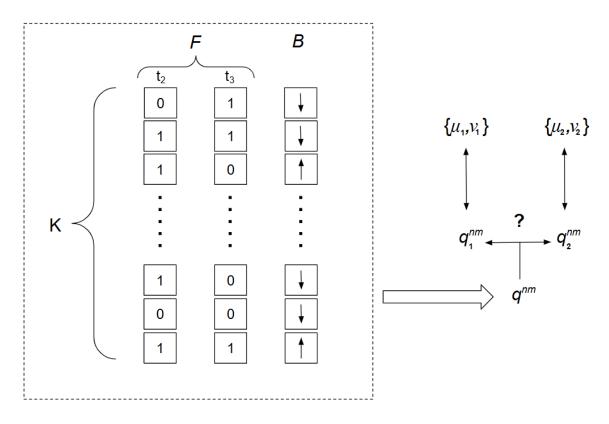


Figure 2 – Beforehand, B chooses one of the two measuring settings  $\{\mu_1, \nu_1\}$  or  $\{\mu_2, \nu_2\}$ . Assuming F has awareness of the change in her state  $|\cdot\rangle_F$ , she has full knowledge of its contents at times  $t_2$  and  $t_3$ . From the K iterations of the protocol, F uses this awareness and B's results to deduce the values of  $q^{nm}$ . F compares these to the flipping probabilities  $q_1^{nm}$  and  $q_2^{nm}$  belonging to B's measurement settings  $\{\mu_1, \nu_1\}$  or  $\{\mu_2, \nu_2\}$  respectively. Consequently, F knows (with certain probability) B's utilized measurement settings superluminally, leading to the contradiction.

# 3 Analysis of the Gedankenexperiment

In this chapter we will critically analyze the Gedankenexperiment and study the implications of the assumptions and definitions utilized in [17]. We will criticize three aspects of the reasoning, outlined as follows.

#### 1. Improper Application of Locality

The reasoning for achieving said conclusion relies heavily on the flipping probabilities  $q^{nm}$ . These probabilities quantify the extent to which F's state changes. However, [17] overlooks a crucial detail: this reasoning only works if F knows B's registered measurement outcome. Without access to this information, F cannot determine whether her state belongs to the  $|\uparrow\rangle_B$  or  $|\downarrow\rangle_B$  side of Table 1. In F's lack of this knowledge, Table 1 transforms into Table 2, shown below.

Table 2 – Probabilities for F's state at  $t = t_i$ , uninformed of B's result, denoted  $P_{t_i}(|\cdot\rangle_F)$ .

$P_{t_2}(F)$	$ \uparrow\rangle_B$ or $ \downarrow\rangle_B$
$ 0\rangle_F$	$ \alpha ^2$
$ 1\rangle_F$	$ oldsymbol{eta} ^2$

$P_{t_3}(F)$	$ \uparrow\rangle_B$ or $ \downarrow\rangle_B$
$ 0\rangle_F$	$ \alpha ^2 ( a ^4 +  b ^4) + 2 \beta ^2  ab ^2$
$ 1\rangle_F$	$ \beta ^2 ( a ^4 +  b ^4) + 2 \alpha ^2  ab ^2$

These results are obtained by summing  $P(|\cdot\rangle_F, |\uparrow\rangle_B) + P(|\cdot\rangle_F, |\downarrow\rangle_B)$  from Table 1. The  $\mu$ ,  $\nu$ -dependency in the flipping probabilities arises from joint probabilities containing B's state. Excluding B's result, this  $\mu$ ,  $\nu$ -dependency, and thus relevance for the argument of the flipping probabilities disappears (see A.3).

It is now clear that for the dependency argument to work, F must obtain B's registered measurement outcome. However, the acquisition of B's outcome doesn't happen superluminally, due to Locality (L). Therefore, any deduction of B's measurement setting by F or W must lie within B's future light cone. This suggests that the dependency-argument used in (III) can't be applied.

#### 2. F's Awareness of State Change

Another critical point is the definition of F's awareness of her state changing. According to [17, p.2], awareness means F possessing full knowledge of the contents of her state  $|\cdot\rangle_F$  at times  $t_2$  and  $t_3$ . But this notion is never explicitly modeled in the wavefunctions described in Section 2.2. Without this specifically modeled, F's state  $|\cdot\rangle_F$  at  $t_2$  is quantum erased by W's measurement at  $t_3$ . This erasure becomes apparent in Eq. 4, in which F's state  $|\cdot\rangle_F$  from  $t_2$  has vanished from the first two lines, thereby preventing F from having full knowledge of the contents of her state, contradicting the assumption of F's awareness. Since F's state is fully erased, the concept of 'change' becomes meaningless. Due to this erasure, one would expect an additional  $|r\rangle_F$  in the wavefunction, to remeasure  $S_1$  and store the outcome inside  $|\cdot\rangle_F$  after W's measurement, yet no such term appears in the calculations of Section 2.2.

The basis-change described in Eq. 4, is considered in [17] to represent F measurement and recording of its result inside her state after W's measurement. But this operation isn't in accordance with the definition of a measurement in [17] (see Section 2.1). This raises the question: how should F's state  $|\cdot\rangle_F$  be interpreted if it arises from a basis-change at  $t_3$ ?

To answer this question, we now demonstrate that this operation is mathematically equivalent to an alternative protocol, in which the following adjustments are made to the one described in Section 2.1:

(i) The measurement order by observer becomes B, W, F (instead of F, B, W followed by the basis-change).

(ii) Since the measurement order has changed, W only measures  $S_1$ , not the joint system  $\{S_1, F\}$ . W now is no longer a super-observer and his measurement bases change accordingly:

$$\begin{split} |0\rangle_{S_1} &:= \overline{a} \, |\mathrm{ok}\rangle_{S_1} + b \, |\mathrm{fail}\rangle_{S_1} \,, & |\mathrm{ok}\rangle_{S_1} &:= a \, |0\rangle_{S_1} + b \, |1\rangle_{S_1} \\ |1\rangle_{S_1} &:= \overline{b} \, |\mathrm{ok}\rangle_{S_1} - a \, |\mathrm{fail}\rangle_{S_1} \,, & |\mathrm{fail}\rangle_{S_1} &:= \overline{b} \, |0\rangle_{S_1} - \overline{a} \, |1\rangle_{S_1} \end{split}$$

With these changes in mind, we will derive the equivalence by computing the wavefunction at each time-step. According to (i), we start at  $t = t_0$  with the following wavefunction:

$$|\Psi\rangle^{t=t_0} = \left(\alpha\,|0\rangle_{S_1}\,|1\rangle_{S_2} + \beta\,|1\rangle_{S_1}\,|0\rangle_{S_2}\,\right)|r\rangle_B\,|r\rangle_W\,|r\rangle_F\;.$$

At  $t_1$ , B makes the first measurement on  $S_2$  in his basis

$$\begin{split} |\Psi\rangle^{t=t_1} &= \left[\alpha \; |0\rangle_{S_1} \left(\overline{\nu} \; |\uparrow\rangle_{S_2} \; |\uparrow\rangle_B - \mu \; |\downarrow\rangle_{S_2} \; |\downarrow\rangle_B \right) \\ &+ \beta \; |1\rangle_{S_1} \left(\overline{\mu} \; |\uparrow\rangle_{S_2} \; |\uparrow\rangle_B + \nu \; |\downarrow\rangle_{S_2} \; |\downarrow\rangle_B \right) \right] |r\rangle_W \; |r\rangle_F \\ &= \left[\alpha \left(\overline{a} \; |\text{ok}\rangle_{S_1} + b \; |\text{fail}\rangle_{S_1} \right) \left(\overline{\nu} \; |\uparrow\rangle_{S_2} \; |\uparrow\rangle_B - \mu \; |\downarrow\rangle_{S_2} \; |\downarrow\rangle_B \right) \\ &+ \beta \left(\overline{b} \; |\text{ok}\rangle_{S_1} - a \; |\text{fail}\rangle_{S_1} \right) \left(\overline{\mu} \; |\uparrow\rangle_{S_2} \; |\uparrow\rangle_B + \nu \; |\downarrow\rangle_{S_2} \; |\downarrow\rangle_B \right) \right] |r\rangle_W \; |r\rangle_F \; . \end{split}$$

Next, W will make his measurement at  $t = t_2$  using the bases described in (ii)

$$\begin{split} |\Psi\rangle^{t=t_2} &= \left[\alpha \left(\overline{a} \left| \mathrm{ok} \right\rangle_{S_1} \left| \mathrm{ok} \right\rangle_W + b \left| \mathrm{fail} \right\rangle_{S_1} \left| \mathrm{fail} \right\rangle_W \right) \left(\overline{\nu} \left| \uparrow \right\rangle_{S_2} \left| \uparrow \right\rangle_B - \mu \left| \downarrow \right\rangle_{S_2} \left| \downarrow \right\rangle_B \right) \\ &+ \beta \left(\overline{b} \left| \mathrm{ok} \right\rangle_{S_1} \left| \mathrm{ok} \right\rangle_W - a \left| \mathrm{fail} \right\rangle_{S_1} \left| \mathrm{fail} \right\rangle_W \right) \left(\overline{\mu} \left| \uparrow \right\rangle_{S_2} \left| \uparrow \right\rangle_B + \nu \left| \downarrow \right\rangle_{S_2} \left| \downarrow \right\rangle_B \right) \right] |r\rangle_F \\ &= \left(a \left| 0 \right\rangle_{S_1} + b \left| 1 \right\rangle_{S_1} \right) \left| \mathrm{ok} \right\rangle_W \left[ \left| \uparrow \right\rangle_{S_2} \left| \uparrow \right\rangle_B \left(\beta \overline{\mu} \overline{b} + \alpha \overline{\nu} \overline{a} \right) + \left| \downarrow \right\rangle_{S_2} \left| \downarrow \right\rangle_B \left(\beta \nu \overline{b} - \alpha \mu \overline{a} \right) \right] \\ &+ \left(\overline{b} \left| 0 \right\rangle_{S_1} - \overline{a} \left| 1 \right\rangle_{S_1} \right) \left| \mathrm{fail} \right\rangle_W \left[ \left| \uparrow \right\rangle_{S_2} \left| \uparrow \right\rangle_B \left(\alpha \overline{\nu} b - \beta \overline{\mu} a \right) - \left| \downarrow \right\rangle_{S_2} \left| \downarrow \right\rangle_B \left(\beta \nu a + \alpha \mu b \right) \right] |r\rangle_F \end{split}$$

and finally F will make his measurement on qubit  $S_1$  at  $t = t_3$ . Rearranging the equation we obtain

$$\begin{split} |\Psi\rangle^{t=t_{3}} &= |0\rangle_{S_{1}} \, |0\rangle_{F} \, |\uparrow\rangle_{S_{2}} \, |\uparrow\rangle_{B} \left[ \left( a\overline{b}\beta\overline{\mu} + \alpha\overline{\nu}|a|^{2} \right) |\operatorname{ok}\rangle_{W} + \left( \alpha\overline{\nu}|b|^{2} - a\overline{b}\beta\overline{\mu} \right) |\operatorname{fail}\rangle_{W} \right] \\ &+ |0\rangle_{S_{1}} \, |0\rangle_{F} \, |\downarrow\rangle_{S_{2}} \, |\downarrow\rangle_{B} \left[ \left( a\overline{b}\beta\nu - \alpha\mu|a|^{2} \right) |\operatorname{ok}\rangle_{W} - \left( a\overline{b}\beta\nu + \alpha\mu|b|^{2} \right) |\operatorname{fail}\rangle_{W} \right] \\ &+ |1\rangle_{S_{1}} \, |1\rangle_{F} \, |\uparrow\rangle_{S_{2}} \, |\uparrow\rangle_{B} \left[ \left( \beta\overline{\mu}|b|^{2} + \overline{a}b\alpha\overline{\nu} \right) |\operatorname{ok}\rangle_{W} - \left( \overline{a}b\alpha\overline{\nu} - \beta\overline{\mu}|a|^{2} \right) |\operatorname{fail}\rangle_{W} \right] \\ &+ |1\rangle_{S_{1}} \, |1\rangle_{F} \, |\downarrow\rangle_{S_{2}} \, |\downarrow\rangle_{B} \left[ \left( \beta\nu|b|^{2} - \overline{a}b\alpha\mu \right) |\operatorname{ok}\rangle_{W} + \left( \beta\nu|a|^{2} + \overline{a}b\alpha\mu \right) |\operatorname{fail}\rangle_{W} \right]. \end{split}$$

The result, Eq. 5, is identical to Eq. 4. We thus conclude that the basis-change is equivalent to the measurement protocol described in (i), in which F's state prior to W's measurement has no existence

and is only defined after W's measurement, due to its quantum erasure. Since F's state  $|\cdot\rangle_F$  is only defined after W's measurement and it being another protocol, any comparison of probability distributions between  $t_2$  and  $t_3$  by a basis-change as described in Section 2.2 consequently seems impossible.

## 3. Quantum Erasure of 'Recorded' Measurements

As stated before, B&B's definition of a measurement involves correlating the state of the observer  $|\cdot\rangle_O$  with the state of the observed system  $|\cdot\rangle_S$  and storing the outcome inside  $|\cdot\rangle_O$ , e.g.  $|0\rangle_{S_1}|r\rangle_F \to |0\rangle_{S_1}|0\rangle_F$ . But as discussed in **2.** F's Awareness of State Change, F's state is quantum erased by W's measurement, making awareness impossible.

If we want to model F's awareness properly, then the measurement outcome must genuinely be recorded, meaning unaffected by quantum erasure. This record should be modeled as definite, unerasable, and unchangeable. However, [17] uses the term 'recorded' to refer to F's state  $|\cdot\rangle_F$ , which is susceptible to quantum erasure. To distinguish between erasable and unerasable models, we introduce the notion of a notebook: an definite, stable, and unchangeable record of F's measurement result.

In summary: 1. Since F's necessary acquisition of B's measurement outcome cannot happen superluminally, the argument appears to fail; 2. F's state  $|\cdot\rangle_F$  at  $t_2$  is quantum erased, so no meaningful comparison of states is possible; and 3. To maintain awareness, the outcomes must be stored in a definite, stable and unchangeable model of a record, which we call a *notebook*. In the next chapter, we will apply this notebook to the Gedankenexperiment to examine its consequences.

# 4 Introduction of the Notebook: Modeling Stable Memory

As we saw in Section 3, to utilize F's awareness it is necessary to model the state of an observer in such a way that quantum erasure of the contents of F's state from  $t_1$  is prevented, meaning that the recordings are definite and unchangeable. The model for this stable record will from now be referred to as a *notebook*, denoted by N.

Only observer F will be assigned a notebook N for her measurement result, as it is this result that would otherwise be erased during the protocol. N records the measurement outcome simultaneously with F's measurement on  $S_1$  at  $t = t_1$ , and its contents will be identical to F's state. For clarification: Every reference by [17] to the 'internal memory register', 'memory register' or 'memory' of observer O is denoted by  $|\cdot\rangle_O$  and called 'state of observer O'. This state is considered unstable, meaning its contents may be subject to quantum erasure. These contents are a 'subjective fact' to that specific observer O from the moment of getting encoded until it is quantum erased. In contrast, a notebook, denoted by  $|\cdot\rangle_N$ , contains definite information that becomes an 'objective fact'; accessible to all observers once they read out N, and thereby functioning as the 'real' memory in practice. The acquisition of the information inside the notebook will of-course still comply with Locality (L).

The awareness of change in F's state described in [17] involves a comparison. To retain F's measurement outcome from  $t_1$ , we need to utilize a notebook. However, this makes the comparison itself pointless, as N's contents are definite and remain unchanged, even after W's measurement. As seen in 3, the described basis-change cannot be used to reconstruct a measurement on  $S_1$  after W's measurement. It can only be interpreted using the definite information that is encoded in N.

The contents of a notebook can always be accessed, allowing other observers to infer new information from F's recorded measurement at later stages of the experiment. The ability to share measured result, aligns with the scientific approach, as it allows us to test our beliefs by directly measuring it.

We now carry out the calculations of the protocol, incorporating the notebook N. The associated wavefunction at  $t = t_i$  will be denoted as  $|\Psi_N\rangle^{t=t_i}$ , to indicate N's inclusion. The initial state is prepared at  $t = t_0$ , and the first measurement is made by F at  $t = t_1$  on  $S_1$ , simultaneously writing the result down in N.

$$|\Psi_N\rangle^{t=t_1} = \left(\alpha \,|0\rangle_{S_1}\,|0\rangle_F\,|0\rangle_N\,|1\rangle_{S_2} + \beta \,|1\rangle_{S_1}\,|1\rangle_F\,|1\rangle_N\,|0\rangle_{S_2}\,\right)|r\rangle_B\,|r\rangle_W\,.$$

After a basis-change, B performs a measurement on  $S_2$  at  $t = t_2$ :

$$\begin{split} |\Psi_N\rangle^{t=t_2} &= \left[\alpha \; |0\rangle_{S_1} \; |0\rangle_F \; |0\rangle_N \left(\overline{\nu} \; |\uparrow\rangle_{S_2} \; |\uparrow\rangle_B - \mu \; |\downarrow\rangle_{S_2} \; |\downarrow\rangle_B \right) \\ &+ \beta \; |1\rangle_{S_1} \; |1\rangle_F \; |1\rangle_N \left(\overline{\mu} \; |\uparrow\rangle_{S_2} \; |\uparrow\rangle_B + \nu \; |\downarrow\rangle_{S_2} \; |\downarrow\rangle_B \right) \right] |r\rangle_W \, . \end{split}$$

At  $t = t_3$ , W makes his measurement on the joint system  $\{S_1, F\}$ 

$$|\Psi_{N}\rangle^{t=t_{3}} = |\operatorname{ok}\rangle_{S_{1}F} |\operatorname{ok}\rangle_{W} \left[ \left( \beta \overline{\mu} \overline{b} |1\rangle_{N} + \alpha \overline{\nu} \overline{a} |0\rangle_{N} \right) |\uparrow\rangle_{S_{2}} |\uparrow\rangle_{B} \right.$$

$$\left. + \left( \beta \nu \overline{b} |1\rangle_{N} - \alpha \mu \overline{a} |0\rangle_{N} \right) |\downarrow\rangle_{S_{2}} |\downarrow\rangle_{B} \right]$$

$$\left. + |\operatorname{fail}\rangle_{S_{1}F} |\operatorname{fail}\rangle_{W} \left[ \left( \alpha \overline{\nu} b |0\rangle_{N} - \beta \overline{\mu} a |1\rangle_{N} \right) |\uparrow\rangle_{S_{2}} |\uparrow\rangle_{B} \right.$$

$$\left. - \left( \beta \nu a |1\rangle_{N} + \alpha \mu b |0\rangle_{N} \right) |\downarrow\rangle_{S_{2}} |\downarrow\rangle_{B} \right]$$

$$(6)$$

and we are done with the calculation. To simplify the comparison, rewriting Eq. 4 without the notebook yields

$$|\Psi\rangle^{t=t_{3}} = |\operatorname{ok}\rangle_{S_{1}F} |\operatorname{ok}\rangle_{W} \left[ \left( \beta \overline{\mu} \overline{b} + \alpha \overline{\nu} \overline{a} \right) |\uparrow\rangle_{S_{2}} |\uparrow\rangle_{B} \right. \\ \left. + \left( \beta \nu \overline{b} - \alpha \mu \overline{a} \right) |\downarrow\rangle_{S_{2}} |\downarrow\rangle_{B} \right]$$

$$+ |\operatorname{fail}\rangle_{S_{1}F} |\operatorname{fail}\rangle_{W} \left[ \left( \alpha \overline{\nu} b - \beta \overline{\mu} a \right) |\uparrow\rangle_{S_{2}} |\uparrow\rangle_{B} \right.$$

$$\left. - \left( \beta \nu a + \alpha \mu b \right) |\downarrow\rangle_{S_{2}} |\downarrow\rangle_{B} \right]$$

$$(7)$$

highlighting the difference between Eq. 6 (with N) and Eq. 7 (without N). Due to the addition of the notebook, the wavefunction at  $t = t_3$  has altered, affecting the joint probabilities concerning B and W's measurement results from  $t = t_3$ . These probabilities are summarized in Table 3.

Table 3 – The joint probabilities for measurement outcomes by B and W at  $t = t_3$ . F not using and using notebook N are the left and right tables respectively. Probabilities are denoted by  $P_{t_3}(|\cdot\rangle_B, |\cdot\rangle_W)$  and they apply to all observers except F, with  $\eta = a\overline{b\alpha}\beta\overline{\mu}\nu$ .

$P_{t_3}(B,W)$	$\ket{\mathrm{ok}}_W$	$ { m fail} angle_W$
	$ \beta\mu b ^2 +  \alpha\nu a ^2$	$ \alpha vb ^2 +  \beta \mu a ^2$
$ \!\uparrow\rangle_B$	$+2\operatorname{Re}\{\eta\}$	$-\operatorname{Re}\{\eta\}$
	$ \beta vb ^2 +  \alpha \mu a ^2$	$ \beta va ^2 +  \alpha \mu b ^2$
$ \!\downarrow\rangle_B$	$-2\operatorname{Re}\{\eta\}$	$+\operatorname{Re}\{\eta\}$

$P_{t_3}(B,W)$	$ { m ok}\rangle_W$	$ \mathrm{fail}\rangle_W$
$ \!\!\uparrow\rangle_B$	$ \beta\mu b ^2 +  \alpha\nu a ^2$	$ \alpha vb ^2 +  \beta \mu a ^2$
$ \!\!\downarrow\rangle_B$	$ \beta vb ^2 +  \alpha \mu a ^2$	$ \beta va ^2 +  \alpha \mu b ^2$

What is remarkable is that the mere existence of the notebook N changes the joint measurement probabilities. Since the probability distributions differ, B and W can, in principle, deduce from their measurement outcomes whether F used her notebook. One might ask: could this lead to signaling? We argue against this. To notice the change in distributions, one must obtain W's measurement outcome. Since W jointly measures F, W is always within the future light cone of F's measurement and her notebook N. Nevertheless, it is remarkable that B and W together can deduce whether F secretly used a notebook.

In the next chapter, we show that correctly modeling awareness using the notebook, constitutes a different physical context to the standard Wigner's Friend-type scenario: one in which either *W* is not a super-observer, or the measured lab is not a perfectly isolated system. Following this, we will explore the defining characteristics for a measurement, the (state of the) observer or super-observer, and notebooks.

# 5 Implications & Discussion

In the paper the following question was posed by [17]: "What properties the internal thoughts of observers in superposition [F's state] must have in order to be compatible with known physical principles?" [17, p. 1]. Based on the assumptions and reasoning, B&B concluded that "no knowledge about the memory [F's state] before Wigner's measurement remains." [17, p. 6]. However, as stated in Section 3, we criticize the argumentation leading to this conclusion. We will now argue that the reasoning by [17] ultimately relies on combining two distinct contradictory physical contexts. Wigner's Friend-type scenarios, such as the one described in 2.1, typically rely on two crucial assumptions [5]:

- 1) The lab is perfectly isolated from the environment.
- 2) W is a super-observer with complete control over the lab.

As described in Section 3, accepting both assumptions forbids F from having any knowledge about her state  $|\cdot\rangle_F$  prior to W's measurement. This conclusion is identical to the conclusion made by [17], but follows directly from quantum erasure, without relying on the disputable argumentation from Section 2.2.

To retain any knowledge of her state  $|\cdot\rangle_F$  before W's measurement, we introduced a notebook N in Section 4, in which F's measurement outcome becomes encoded and thereby unerasable. However, assuming that N's contents remain stable, while simultaneously complying with assumptions 1) and 2), leads to a contradiction. We will demonstrate this by considering two cases:

Case 1: Suppose *N* is stable and assumption 1) holds. Then the stable notebook must be located inside the lab. But if *W* performs a measurement in a different basis on the entire lab, the notebook—being part of the lab—will be erased along with it. This contradicts assumption 2), as *W* would no longer have complete control over the isolated lab.

Case 2: Suppose N is stable and assumption 2) holds. Since a measurement by W in a different basis would erase all information in the lab, the stability of the notebook implies that it must lie outside the lab. But this means that information about F's measurement outcome has leaked into the environment, violating assumption 1).

Therefore, to retain F's measurement outcome prior to W's measurement in the notebook N, at least one of the assumptions 1) or 2) must be rejected. This contextual inconsistency explains the difference between the two wavefunctions and their associated probability distributions: Eq. 7 corresponds to the typical Wigner's Friend-type scenario, where both assumptions 1) and 2) are held, while Eq. 6 applies to the modified scenario involving the notebook, where at least one assumption is rejected. These equations represent two different contradictory contexts, and assuming both simultaneously—specifically in the model of 'awareness'—leads to inconsistencies in the reasoning by [17].

We will now try to characterize the nature of 'observers' and 'measurements'. B&B stated that "The 'friend' was in that case a single photon, for which the assignment of the notion 'observer' or 'observation' has little physical meaning" [17, p.1], yet modeled each observer's state analogous to a photon. From a more physically realistic standpoint, an observer's state should consist of many quantum subsystems in which the measurement result is encoded multiple times [22]:

$$|\cdot\rangle_{F^*} = |\cdot\rangle_{F_1} \otimes ... \otimes |\cdot\rangle_{F_n}$$

where each  $|\cdot\rangle_{F_i}$  represents a part of the total observer system  $F^*$ . A measurement made by this observer on the observed system  $S_1$ , would give rise to the correlation of the form:

$$|\psi\rangle = \alpha \, |0\rangle_{S_1} \, |0\rangle_{F_1} \dots |0\rangle_{F_n} + \beta \, |1\rangle_{S_1} \, |1\rangle_{F_1} \dots |1\rangle_{F_n} \, .$$

Here, the state of the system  $S_1$  becomes entangled with the macroscopic observer  $F^*$ , modeled as a composition of n quantum subsystems. In this entangled state, the measurement outcome is effectively copied into each of the  $F_i$ , where  $i \in \{1, ..., n\}$ . Consequently, it is more likely that at least one degree of freedom interacts with the environment, or fall outside the control of W. This makes it difficult to

1) perfectly isolate and 2) completely control the entire state of  $F^*$ . This implies the existence of a subset of states that compose  $|\cdot\rangle_{F^*}$  that is stable and effectively acts as the notebook— $\{F_{k_1},\ldots,F_{k_m}\}\subseteq \{F_1,\ldots,F_n\}$ , with  $\{k_1,\ldots,k_m\}\subseteq \{1,\ldots,n\}$  and  $n,m\in\mathbb{N}$ :

$$|\cdot\rangle_N:=|\cdot\rangle_{F_{k_1}}\otimes\ldots\otimes|\cdot\rangle_{F_{k_n}}$$

This effectively models a stable subcomponent of the observer's state, which behaves like a notebook that preservers the measurement outcome. This stabilization of subsystems, often through interaction with the environment, is known as *decoherence* and explains how information becomes robust and persistent at the macroscopic level.

We again address the question proposed by [17, p.1]: "What properties the internal thoughts of observers in superposition [F's state] must have in order to be compatible with known physical principles?". An observer's measurement is described by correlating the state of the measured system (e.g.  $S_1$ ) with the state of the observing system. If the observing system is completely unstable and perfectly isolated (as F is assumed to be), the result won't be recorded in a stable system and will eventually be quantum erased. This defines a kind of 'subjective reality', existing only relative to that specific observer and persists only until its quantum erasure, thereby occurring more frequently on a microscopic level. This is sometimes called a 'pre-measurement', and will only correlate with the measured system, not generating any 'facts' [6, p.1]. If instead the observing system is (partially) stable or entangled with a stable environment (as in the case of W), the result becomes unerasable and thus accessible to others inside these stable systems. This constitutes an 'objective reality'—defined as facts available to all observers within the experimental framework and will occur more frequently on a macroscopic level. The transition from subjective to objective reality is caused by decoherence and aligns closely with the relational understanding of physical reality (RQM).

Throughout the paper, it is suggested that an observer-dependent stance on facts is essential in Wigner's Friend-type scenarios, as argued by [12, p.37]. This allows observers-or any quantum system that correlates-to have 'subjective' (observer-relative) or 'objective' (inter-observer) realities. One interpretation of quantum mechanics that allows observer's subjectivity is *Relational Quantum Mechanics* (RQM). We will explore this interpretation in detail in B.

#### 6 Conclusion

In Wigner's Friend-type scenarios such as [17], two key assumptions are made: 1) Wigner is a superobserver with complete control over the lab, and 2) that the lab is perfectly isolated from the environment. We have shown that these assumptions are mutually incompatible if the observer's state is to remain stable and accessible. The conclusion reached by [17] relies on adopting both of these contradictory contexts at once, suggesting logical inconsistencies in the reasoning.

We demonstrated that retaining the contents of the Friend's state prior to Wigner's measurement, requires rejecting at least one of the assumption 1) or 2). This distinction directly clarifies the difference between the two resulting wavefunctions and associated probabilities: Eq. 7 corresponds to the standard Wigner's Friend context, while Eq. 6 represents a physical setup in which a stable system has been preserved.

Furthermore, we proposed a physically realistic model of an 'observer' as a composite quantum system. We argued that, in practice, maintaining perfect isolation and complete control is highly improbable on the macroscopic level. This naturally leads to the phenomenon of *decoherence*, in which certain subsystems become stable themselves or by correlating with the environment. These stable records form what we have called 'objective reality'—facts accessible across observers within the experimental context.

Our analysis supports an observer-dependent stance on facts in quantum theory. Measurement outcomes can result in either 'subjective' realities—when recorded in unstable systems—or 'objective' realities—when recorded in stable systems. This perspective aligns with interpretations like Relational Quantum Mechanics (RQM), which explicitly allows for facts to be relative to observers and is analyzed in B.

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# A Appendix

#### A.1 Notation Change and Measurement Bases

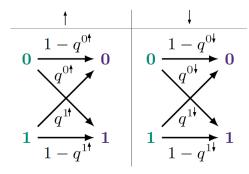
All the changes made to the notation and all the different measurement bases of all the observers are described in Table 4 below.

Notation Paper	Used Notation		
0>	0>		
1>	1>	$ \Phi\rangle_{S_1,S_2} = \alpha  0\rangle_{S_1}  1\rangle_{S_2} + \beta  1\rangle_{S_1}  0\rangle_{S_2}$	-
$ B=0\rangle$		$ \uparrow\rangle_{S_2} = \mu  0\rangle_{S_2} + \nu  1\rangle_{S_2}$	$ 0\rangle_{S_2} = \overline{\mu}  \uparrow\rangle_{S_2} + \nu  \downarrow\rangle_{S_2}$
$ B=1\rangle$	$ \downarrow\rangle$	$ \downarrow\rangle_{S_2} = \overline{\nu}  0\rangle_{S_2} - \overline{\mu}  1\rangle_{S_2}$	$ 1\rangle_{S_2} = \overline{\nu}  \uparrow\rangle_{S_2} - \mu  \downarrow\rangle_{S_2}$
$ W=1\rangle$	ok>	$ \operatorname{ok}\rangle_{S_1F} = a  0\rangle_{S_1}  0\rangle_F + b  1\rangle_{S_1}  1\rangle_F$	$ 0\rangle_{S_1} 0\rangle_F = \overline{a}  \text{ok}\rangle_{S_1F} + b  \text{fail}\rangle_{S_1F}$
$ W=2\rangle$	fail>	$ \operatorname{fail}\rangle_{S_1F} = \overline{b}  0\rangle_{S_1}  0\rangle_F - \overline{a}  1\rangle_{S_1}  1\rangle_F$	$ 1\rangle_{S_1} 1\rangle_F = \overline{b}  \text{ok}\rangle_{S_1F} - a  \text{fail}\rangle_{S_1F}$
1	$S_1$		
2	$S_2$		

Table 4 – Notation changes and bases of the observers.

#### A.2 Model of F's State Change

Figure 3 – The results encoded in F's state at  $t_2$  and  $t_4$  are depicted in green and purple respectively, with their associated flipping probabilities. The figure is split into B's two different measurement outcomes.



The flipping probability  $q^{nm}$  with  $n \in \{0,1\}$  and  $m \in \{\uparrow,\downarrow\}$  are pictured in Fig. 3 on the left. Combining  $q^{nm}$  and the change of the joint probabilities of the measured results encoded in the observer's state, produces the set of equations below. If we take a closer look, we can see that the first two and last two equations are equivalent to each other, with  $\xi \equiv (|a|^2 - |b|^2) \operatorname{Re} \left\{ a \overline{b \alpha} \beta \overline{\mu} \nu \right\}$ . The solutions for the flipping probabilities  $q^{nm}$  are dependent on the  $\xi$ -term and are thus also dependent on  $\mu$  and  $\nu$ .

$$\begin{split} & \left(1-q^{0\uparrow}\right)|\alpha\nu|^2+q^{1\uparrow}|\beta\mu|^2=|\alpha\nu|^2\Big(|a|^4+|b|^4\Big)+2|ab|^2|\beta\mu|^2+2\xi\\ & \left(1-q^{1\uparrow}\right)|\beta\mu|^2+q^{0\uparrow}|\alpha\nu|^2=|\beta\mu|^2\Big(|a|^4+|b|^4\Big)+2|ab|^2|\alpha\nu|^2-2\xi\\ & \left(1-q^{0\downarrow}\right)|\alpha\mu|^2+q^{1\downarrow}|\beta\nu|^2=|\alpha\mu|^2\Big(|a|^4+|b|^4\Big)+2|ab|^2|\beta\nu|^2-2\xi\\ & \left(1-q^{1\downarrow}\right)|\beta\nu|^2+q^{0\downarrow}|\alpha\mu|^2=|\beta\nu|^2\Big(|a|^4+|b|^4\Big)+2|ab|^2|\alpha\mu|^2+2\xi \end{split}$$

#### **A.3** Model of F's State Change Without B's Measurement Result

We will now define the flipping probability  $q^n$  with  $n \in \{0, 1\}$ : the probability that F's state  $|n\rangle_F$  switches. Combining  $q^n$  and the outcome probabilities of encoded result in F's state, leads to the set of equations

$$(1 - q^{0})|\alpha|^{2} + q^{1}|\beta|^{2} = |\alpha|^{2}(|a|^{4} + |b|^{4}) + 2|\beta|^{2}|ab|^{2}$$
(8)

$$(1 - q^{1})|\beta|^{2} + q^{0}|\alpha|^{2} = |\beta|^{2}(|a|^{4} + |b|^{4}) + 2|\alpha|^{2}|ab|^{2}.$$
(9)

These two equations are also equivalent. It is clear that any  $\mu$ ,  $\nu$ -dependency of  $q^n$  is now gone.

# **B** Relational Quantum Analysis of the Gedankenexperiment

In this appendix, we will analyze the Gedankenexperiment described by [17] within the framework of *Relational Quantum Mechanics* (RQM), first introduced by Rovelli [13]. RQM proposes that "different observers can give different accounts of the actuality of the same physical property" [13, p.6]. That is, quantum states and the properties they describe are not absolute facts but only meaningful in relation to the observer. Properties possessed by any system S are defined relative to another interacting (or observing) system A that is influenced by these properties and correlates, thereby forming A's subjective physical reality. If the interacting system A keeps a stable record of the information exchange with S, it ensures consistency for different observers, constituting an objective truth within the context of the experiment, allowing meaningful scientific exchange of information.

In accordance with RQM, any conclusions are only drawn from the information the specific observer possesses or has acquired. We will review the protocol and calculations from each observer's perspective: F, B, W and an external observer C. The external observer should report identical to the experimenters, in the case their conclusion would be experimentally tested. Every observer will model the protocol from their view, by choosing a tensor product of Hilbert spaces. They don't describe their own Hilbert space, but only consider their measurement outcome (as this information is directly known to that specific observer). For example, the notation of the wavefunction in the perspective of B at time  $t = t_i$  will be denoted by

$$|\Psi\rangle_{S_1FS_2W}^{B,t=t_i} \in \mathcal{H}_{S_1} \otimes \mathcal{H}_F \otimes \mathcal{H}_{S_2} \otimes \mathcal{H}_W.$$

Only the external observer C (the theoretical experimenter) will describe all the components of the Gedankenexperiment. This will reproduce the setup in Section 2.1 and the wavefunction described in Section 2.2, but now interpreted relationally. He won't measure any of the systems, but can still acquire information from others. In this analysis (Q), (F) and (L) are still assumed, the state of the observer is not explicitly protected from erasure, and the stable notebook N will not be included. In each observer's perspective, we will analyze both their possible measurement outcomes. Furthermore, it is important to note that from F's perspective, a complete analysis is not possible. When she tries to describe W's measurement of the joint system  $\{S_1, F\}$ , she will run into difficulties, since the 'self-measurement' would quantum erase her own measurement [23–26]. The calculations and analysis in every perspective will be provided below, beginning with the Friend's perspective.

## **B.1** *F*'s Perspective

First we will analyze the Gedankenexperiment in the perspective of F. This analysis will continue until W is ready to measure. From F's perspective she will interact with  $S_1$  at  $t = t_1$  and performs measurement and encodes the result inside  $|\cdot\rangle_F$ . Depending on her outcome, the state collapses accordingly and F would describe:

$$-\left.|0\right\rangle_{S_{1}} \text{ with probability } |\alpha|^{2} \colon \left.|\Psi\right\rangle_{S_{1}S_{2}BW}^{F,t=t_{1}} = \left.|0\right\rangle_{S_{1}} \left.|1\right\rangle_{S_{2}} \left.|r\right\rangle_{B} \left.|r\right\rangle_{W},$$

 $-\left|1\right\rangle_{S_{1}} \text{ with probability } \left|\beta\right|^{2} \colon \left|\Psi\right\rangle_{S_{1}S_{2}BW}^{F,t=t_{1}} = \left|1\right\rangle_{S_{1}}\left|0\right\rangle_{S_{2}}\left|r\right\rangle_{B}\left|r\right\rangle_{W}.$ 

Now B measures the spin of  $S_2$ , described by

$$-|0\rangle_{S_1}$$
 with probability  $|\alpha|^2$ :  $|\Psi\rangle_{S_1S_2BW}^{F,t=t_2} = |0\rangle_{S_1} \left(\overline{\nu}|\uparrow\rangle_{S_2}|\uparrow\rangle_B - \mu|\downarrow\rangle_{S_2}|\downarrow\rangle_B\right)|r\rangle_W$ ,

$$-\left|1\right\rangle_{S_{1}} \text{ with probability } |\beta|^{2} \colon \left|\Psi\right\rangle_{S_{1}S_{2}BW}^{F,t=t_{2}} = \left|1\right\rangle_{S_{1}} \left(\overline{\mu}\left|\uparrow\right\rangle_{S_{2}}\left|\uparrow\right\rangle_{B} + \nu\left|\downarrow\right\rangle_{S_{2}}\left|\downarrow\right\rangle_{B}\right) |r\rangle_{W}.$$

In F's perspective, the mathematical framework for RQM doesn't allow a measurement by W at  $t = t_3$  on the lab  $\{S_1, F\}$ , since its a form of self-measurement and the interaction causes quantum erasure of  $|\cdot\rangle_F$ . F is now unable to compare result and any reasoning relying on awareness of change between  $t_1$  and  $t_3$  fails under RQM. These calculations describe F's subjective reality, until the inevitable erasure.

#### **B.2** B's Perspective

Secondly we take on B's point of view. After the bipartite joint particle state has been prepared at  $t = t_0$ , F's measurement of  $S_1$  at  $t = t_1$  is described using entanglement by:

$$|\Psi\rangle_{S_1FS_2W}^{B,t=t_1} = \left(\alpha\;|0\rangle_{S_1}\;|0\rangle_F\;|1\rangle_{S_2} + \beta\;|1\rangle_{S_1}\;|1\rangle_F\;|0\rangle_{S_2}\;\right)|r\rangle_W\;.$$

At  $t = t_2$ , B performs his measurement and updates the wavefunction accordingly. The two possible outcomes are modeled and after normalization described by:

 $-|\uparrow\rangle_{S_2}$  with probability  $|\alpha\nu|^2 + |\beta\mu|^2$ :

$$|\Psi\rangle_{S_1FS_2W}^{B,t=t_2} = \frac{1}{\sqrt{|\alpha\nu|^2 + |\beta\mu|^2}} |\uparrow\rangle_{S_2} \left[\alpha\overline{\nu} |0\rangle_{S_1} |0\rangle_F + \beta\overline{\mu} |1\rangle_{S_1} |1\rangle_F \right] |r\rangle_W,$$

 $-\mid\downarrow\rangle_{S_2}$  with probability  $|\alpha\mu|^2 + |\beta\nu|^2$ :

$$|\Psi\rangle_{S_1FS_2W}^{B,t=t_2} = \frac{1}{\sqrt{\left|\alpha\mu\right|^2 + \left|\beta\nu\right|^2}} \left|\downarrow\right\rangle_{S_2} \left[ -\alpha\mu\left|0\right\rangle_{S_1}\left|0\right\rangle_F + \beta\nu\left|1\right\rangle_{S_1}\left|1\right\rangle_F \right] |r\rangle_W \,.$$

B can describe W's subsequent measurement. At  $t = t_3$ , W makes his measurement on the joint state  $\{S_1, F\}$  using his own basis, resulting in the following wavefunctions:

 $-\mid\uparrow\rangle_{S_2}$  with probability  $|\alpha\nu|^2 + |\beta\mu|^2$ :

$$\begin{split} |\Psi\rangle_{S_1FS_2W}^{B,t=t_3} &= \frac{1}{\sqrt{|\alpha\nu|^2 + |\beta\mu|^2}} \,|\!\uparrow\rangle_{S_2} \left[ \left( \overline{a}\alpha\overline{\nu} + \overline{b}\beta\overline{\mu} \right) |\!\operatorname{ok}\rangle_{S_1F} \,|\!\operatorname{ok}\rangle_W \right. \\ &\left. + \left( b\alpha\overline{\nu} - a\beta\overline{\mu} \right) |\!\operatorname{fail}\rangle_{S_1F} \,|\!\operatorname{fail}\rangle_W \,\right], \end{split}$$

 $-\mid\downarrow\rangle_{S_2}$  with probability  $|\alpha\mu|^2 + |\beta\nu|^2$ :

$$\begin{split} |\Psi\rangle_{S_{1}FS_{2}W}^{B,t=t_{3}} &= -\frac{1}{\sqrt{|\alpha\mu|^{2}+|\beta\nu|^{2}}} \left|\downarrow\right\rangle_{S_{2}} \left[ \left(\alpha\mu\overline{a}-\beta\nu\overline{b}\right) \left|\text{ok}\right\rangle_{S_{1}F} \left|\text{ok}\right\rangle_{W} \right. \\ &\left. + \left(\alpha\mu b + \beta\nu a\right) \left|\text{fail}\right\rangle_{S_{1}F} \left|\text{fail}\right\rangle_{W} \right]. \end{split}$$

B's description at  $t = t_3$  lacks any information about F's state, meaning that F's encoded result from  $t_1$  does not constitute a physical fact for B. In contrast, since B's own outcome survives until the end of the experiment, it remains stable and thereby forms an objective reality within the context of the experiment.

#### **B.3** W's Perspective

After the preparation of bipartite state at  $t = t_0$ , F makes a measurement at  $t = t_1$  and W describes it as:

$$|\Psi\rangle_{S_1FS_2B}^{W,t=t_1} = \left(\alpha\;|0\rangle_{S_1}\;|0\rangle_F\;|1\rangle_{S_2} + \beta\;|1\rangle_{S_1}\;|1\rangle_F\;|0\rangle_{S_2}\,\right)|r\rangle_B\;.$$

At  $t = t_2$ , B makes his measurement on qubit  $S_2$ :

$$\begin{split} |\Psi\rangle_{S_{1}FS_{2}B}^{W,t=t_{2}} = &\alpha |0\rangle_{S_{1}} |0\rangle_{F} \left(\overline{\nu} |\uparrow\rangle_{S_{2}} |\uparrow\rangle_{B} - \mu |\downarrow\rangle_{S_{2}} |\downarrow\rangle_{B}\right) \\ + &\beta |1\rangle_{S_{1}} |1\rangle_{F} \left(\overline{\mu} |\uparrow\rangle_{S_{2}} |\uparrow\rangle_{B} + \nu |\downarrow\rangle_{S_{2}} |\downarrow\rangle_{B}\right). \end{split}$$

At  $t = t_3$ , W performs his joint measurement on the lab  $\{S_1, F\}$  in the  $\{ok, fail\}$ -basis and updates the wavefunctions to:

 $-|ok\rangle_{S_1F}$  with probability  $|\alpha a|^2 + |\beta b|^2$ :

$$\begin{split} |\Psi\rangle_{S_{1}FS_{2}B}^{W,t=t_{3}} &= \frac{1}{\sqrt{|\alpha a|^{2} + |\beta b|^{2}}} \left| \text{ok} \right\rangle_{S_{1}F} \left[ \left( \beta \overline{\mu} \overline{b} + \alpha \overline{\nu} \overline{a} \right) |\uparrow\rangle_{S_{2}} |\uparrow\rangle_{B} \right. \\ &\left. + \left( \beta \nu \overline{b} - \alpha \mu \overline{a} \right) |\downarrow\rangle_{S_{2}} |\downarrow\rangle_{B} \right], \end{split}$$

 $-|\text{fail}\rangle_{S_1F}$  with probability  $|\alpha b|^2 + |\beta a|^2$ :

$$\begin{split} |\Psi\rangle_{S_1FS_2B}^{W,t=t_3} &= \frac{1}{\sqrt{|\alpha b|^2 + |\beta a|^2}} \left| \mathrm{fail} \right\rangle_{S_1F} \left[ \left( \alpha \overline{\nu} b - \beta \overline{\mu} a \right) |\uparrow\rangle_{S_2} |\uparrow\rangle_B \right. \\ &\left. - \left( \beta \nu a + \alpha \mu b \right) |\downarrow\rangle_{S_2} |\downarrow\rangle_B \right]. \end{split}$$

W does not assign a definite value to F's state prior to his measurement. It remains in superposition until W makes his measurement at  $t_3$ . Any information regarding F's measurement result from  $t_1$  gets quantum erased, making this result not an objective fact. In contrast, W's measurement outcome survives the end of the experiment and defines an objective fact within the experimental context. The tension between F and W's description of  $S_1$  is what Rovelli refers to as 'partial descriptions'. Treating partial descriptions as absolute may lead to contradictions, as illustrated in the model of F's awareness.

#### **B.4** C's Perspective

External observer C plays the role of a theoretical experimenter, who describes the entire protocol without performing any measurement. The wavefunctions he assigns are identical to those calculated in in Section 2.2. Since C does not interact with any systems, he cannot access measurement results directly and must rely on reports from other observers. In particular, after W's measurement at  $t_3$ , C cannot determine F's result from  $t_1$ , unless it was recorded in a stable state (like the notebook introduced from Section 4). Without a stable record, no meaningful comparison between F's measurements is possible and and remains subjective to F.

#### **Conclusion of the Relational Analysis**

We have now fully described the Gedankenexperiment within the framework of Relational Quantum Mechanics (RQM), analyzing it from the perspective of each observer. We critize that conclusion by [17], since it arises from comparing the contents of F's state across different times. Such comparisons are meaningless unless the information is recorded in a stable system. The key mistake is treating observer-relative facts as absolute. The awareness argument proposed by [17] faces difficulties because it implicitly assumes an absolute, observer-independent reality, something RQM explicitly rejects. Within the relational view, facts are observer-dependent and only become objective through shared stable records, making the model of awareness by [17] problematic.