

## Diagnostics, Prognostics and Selection Methodologies for Uninterrupted Functionality of K-Out-Of-N Systems

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$$h(t) = \frac{dF}{R dt} = \frac{-dR}{R dt} = \frac{f}{R} = \frac{\beta}{\alpha^\beta} \cdot t^{\beta-1} \quad (2)$$

The cumulative hazard rate  $H$  on log-log scale yields the Weibull plot (cf. Fig. 3):

$$H(t) = \int_0^t h(x) dx = -\ln(R(t)) = \left(\frac{t}{\alpha}\right)^\beta \quad (3)$$

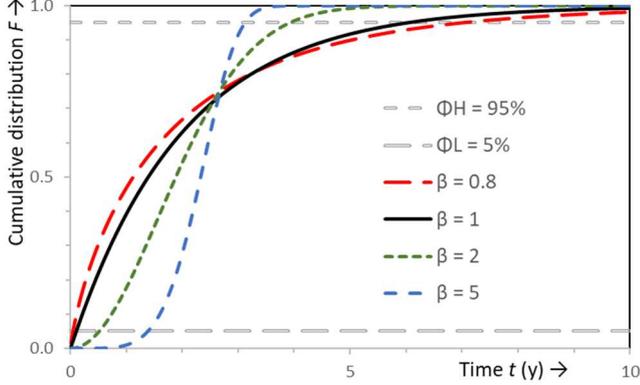


Fig. 2. Cumulative distribution functions  $F$  for various  $\beta$  values. For  $\beta > 1$ , the function is S-shaped. All four cases have the same mean lifetime  $\langle t \rangle = 2y$ .

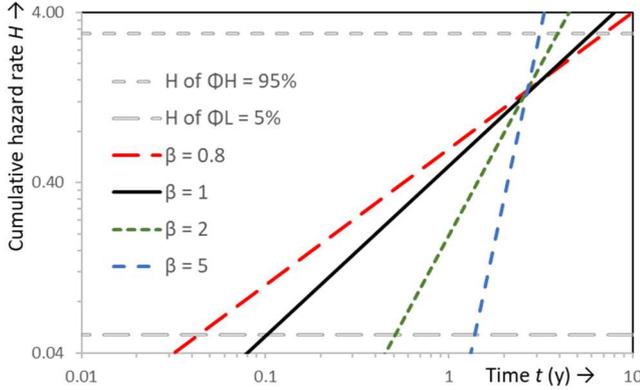


Fig. 3. The cumulative hazard functions  $H(t)$  on log-log scale for the same  $\beta$  values as in Fig. 2. This is the well-known Weibull plot.

A confidence limit  $C_\phi$  is defined as the time at which a given failure probability  $\Phi$  is reached. The difference between two confidence limits  $C_{\phi_H}$  and  $C_{\phi_L}$  is denoted the  $\Phi_H - \Phi_L$  confidence interval. A confidence limit  $C_\phi$  follows from (with  $F^{inv}$  the inverse of the cumulative function as in (1)):

$$C_\phi = F^{inv}(\phi) \quad (4)$$

As for the shape parameter:  $\beta < 1$  means a declining  $h$  as in child mortality,  $\beta = 1$  means a steady  $h$  as in random failure, and  $\beta > 1$  means an inclining  $h$  as in wear. The latter case, means that  $F$  is S-shaped (Fig. 2). Note, all cases in Fig. 2 have a  $\langle RUL \rangle = 2y$  at  $t=0$ . The higher the  $\beta$  value, the narrower the  $\Phi_H - \Phi_L$  confidence intervals, i.e., the shorter the period in which the  $\Phi_H - \Phi_L$  fraction fails. In Fig. 2,  $\Phi_H - \Phi_L = 90\%$  and  $5\%$  and  $95\%$  confidence limits intersect the curves. The impact of the  $\beta$  value on the widths of the intervals is evident. Similarly, Fig. 3 shows the intervals on log-scale.

The 3-parameter Weibull function adds a threshold parameter  $\tau$  as starting point. One of the applications is when aging is triggered by an external event. The 3 parameter form is achieved if lifetime  $t$  is replaced by  $(t - \tau)$  in (1), (2) & (3).

## B. Dynamics in Degradation and Failure Processes

The component hazard rates can vary with load (generated heat) and ambient stress (local temperature). E.g., overheating may damage power electronics followed by failure or timely cool-down (while not all damage may be undone). Degradation may be triggered after a time  $t = \tau$  and then raise the hazard rate. We discuss the dynamics of enhanced wear and random failure that may start from a trigger.

Fig. 4 shows a case of original random failure and two triggered processes: wear and random failure. Both have a zero  $h$  up to  $t = \tau$ , after which (2) applies (replace  $t$  by  $t - \tau$ ). Triggered wear may be due to another event (e.g., leak in cable jacket). Triggered random failure can be due to malfunction of a protection after which external random impact may cause failure that is unrelated to the component life time (so, random failure). The hazard rate jumps at  $t = \tau$  to a constant  $h$ .

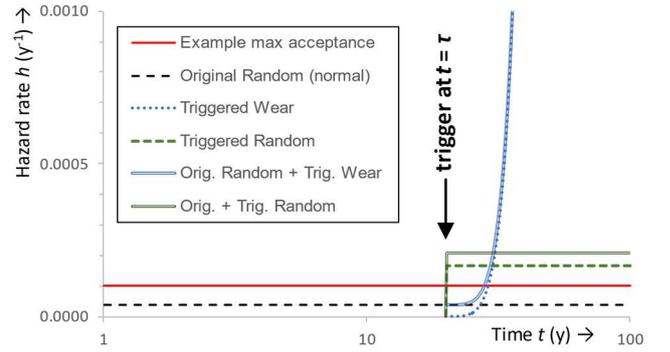


Fig. 4. Two example cases where triggered wear and triggered random failure add to the regular random failure. Both resulting hazard rates exceed the maximum acceptable hazard rate level here and intervention is needed.

If the aging process as such remains the same except for acceleration, then only its time scale changes. With a Weibull distribution  $\alpha$  changes, but  $\beta$  stays the same. Conversely, this might be used to test the validity of an accelerated aging test. An Acceleration Factor  $AF$  relates the reference  $\alpha_{ref}$  and the enhanced stress  $\alpha_{enh}$  as  $AF = \alpha_{ref} / \alpha_{enh}$ . Lifetime passes faster at enhanced stresses. Clock periods  $\Delta t_i$  with associated  $AF_i$  can be converted into an equivalent  $\Delta t_{eq,i}$  at reference stress. The resulting cumulative hazard rate  $H$  (3) can be expressed in terms of the reference stress and time  $t_{eq} = \sum \Delta t_{eq,i}$ . Then  $H(t_{eq})$  follows as:

$$H(t_{eq}) = \left(\frac{t_{eq}}{\alpha}\right)^\beta = \left(\frac{\sum_i \Delta t_i \cdot AF_i}{\alpha}\right)^\beta \quad (5)$$

This equation is valid for individual Weibull distributions provided  $\beta$  remains the same in all periods  $\Delta t_i$ .

## C. Diagnostics

Components in II.A and II.B are indistinguishable batch items with statistics of (1)-(5). Whether an at random selected item belongs to the weaker or the stronger part of a batch, comes with the probability of (1). If diagnostics can successfully distinguish soon-failing items from the rest, the statistics change significantly. Components with a similar diagnostic outcome form a (sub)group with their own statistics. Conversely, a component with a given diagnostic can be regarded to represent the (sub)group and, although it is a single item, we still can assign a statistical function like a hazard rate to it (as randomly selected from that group).

Condition assessment requires at least: a measurable side effect of the studied degradation, an adequate sensor system to measure the effect and, last but not least, intelligence to interpret the measurement output.

The intelligence may be built up by destructive testing or operational expertise until failure while keeping track of diagnostic output. However, it is noteworthy that hazard rates do not need to have a simple relationship with the amplitude of a sensor output. The interpretation of the diagnostic signals may concern characteristics such as phase shifts, the shape and multitude of peaks. Significant impact may also come from system impedance, a mix of sources or disturbing background noise, etc.. There may also be a critical level below which signals are not significant for diagnostics [5]. E.g., a clear wear phenomenon like electrical treeing along an insulator surface will have an increasing hazard rate, but may still produce a steady stream of partial discharge pulses [5].

We mainly focus on thermally accelerated aging in power electronics. Temperature is often a good indicator of the component health as both insulation reliability and junction functionality usually have a distinct temperature range for operation. Temperature monitoring of individual power electronics components may unambiguously identify near failure and the temperature rise can also be used to construct a hazard rate as a function of time and from that percentiles and the mean of the *RUL*. In the following we will assume that we are able to estimate  $h(t)$  with sufficient accuracy.

### III. SYSTEM PERFORMANCE

In comparison to single components above, redundant and repairable systems aim to increase resilience balancing:

1. Component hazard rate, constant  $\lambda$  or varying  $h(t)$
2. Redundancy by configuration
3. Repair rate, constant  $\mu$  or varying  $\mu(t)$
4. System smartness enabling remedial intervention

As for the hazard rate, it was duly discussed above. What may be added, is that with given redundancy and repair options, component qualities are to be balanced with the system requirements. We consider both  $\lambda$  and  $h(t)$  in this study. Both depend on stresses as in (2), (3) and (5).

The concept of redundancy by  $k$ -out-of- $n$  systems is that  $n$  components are parallel and a switch selects  $k$  working items (Fig. 5). Whether or not the  $k/n$  switch is a discrete object, it is an essential system part that might fail on its own. At this stage, we ignore its vulnerability and assume ideal performance: correct, timely and uninterrupted.

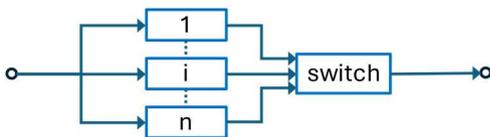


Fig. 5. The principle of  $k/n$  systems. The switch is to select a set of working  $k$ -out-of- $n$  parallel components.

A fundamental design aspect is whether or not the system is repairable after component failure or near-failure. If it is, an equilibrium may develop between failure and repair rates. Occasionally the system may be down. The equilibrium is often quantified through the measures Availability and Mean Time between Failures.

If the system has one or more absorbing fault states, it reaches an end (of life) after  $n-k+1$  failures ultimately. The system reliability  $R_S(t)$  decays from 1 to 0. Still, redundancy can be very useful to extend the total system life depending on the hazard rates, redundancy configuration and repair rates.

### IV. EXPERIMENTAL WORK

The framework aims at uninterrupted functioning of  $k/n$  redundant systems under dynamic stresses and timely deselect suspect items for recovery or replacement. Our primary goal is to build a framework with a chain of practices (Fig. 1). In the following we discuss our approach and some explorative results of a Markov model with time varying transition rates.

#### A. Approach

Redundant systems are often studied with Markov chains and constant transition rates. The differential equations can then be solved by Laplace transforms [6]. We also applied this to semi-constant rates, e.g., when the rates show little change within the prognostic horizon [7]. In our case, however, time varying load and ambient stress leads to transition rates that vary in time. Some other studies also discuss time variation in Markov models (cf. [8,9,10]).

In our approach, we assume that the relation between relevant processes and diagnostics are tested in labs and/or during operation. We also assume that diagnostics have an established relationship with  $h(t)$  employing Weibull models [5]. In [7] we discussed Markov models for 1-out-of- $n$  redundancy with constant transition rates, semi-constant transition rates with bathtub models and also involved defect sub-populations. At this stage, we focus on random and wear processes that may vary in time due to separate components. This results in acceleration factors (section II.B), that can vary and differ for the components that build the system.

The  $k/n$  redundant systems are studied with a Markov model, where the transition rates are time functions. The state probabilities are calculated by numerically solving the differential equations in Julia programs.

A specific case is studied that consists of  $n=3$  components of which  $k=2$  must work (Fig. 6). The initial, fully functional state is  $S_0$ . All three components C1, C2 and C3 endure a competition of two processes: random failure ( $\alpha_r=\lambda=10^{-4} \text{ y}^{-1}$ ,  $\beta_r=1$ ) and wear ( $\alpha_w=100 \text{ y}^{-1}$ ,  $\beta_w=3$ ).

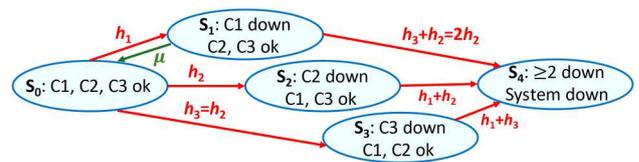


Fig. 6. Markov model used in this particular example case.

C1 suffers enhanced stresses (e.g., external heating) up to  $t=0.01 \text{ y}$ . This is taken here to cause accelerated aging with a single (hazard rate) enhancement factor  $EF$ . From Section II.B follows that  $EF=AF^\beta$ . For a random process,  $\beta=1$  and in our wear case,  $\beta=3$ . For convenience, both  $EF$  are 5000.

When C1 is deselected for high temperatures, it is considered failed (state  $S_1$ ), switched off and recovers with rate  $\mu$ . State  $S_2$  and  $S_3$  concern failure of C2 resp. C3. Any second failure means the system stops functioning, i.e., is down. This is malfunction and the system is out of active service. This state is taken to be the absorbing state  $S_4$ .

The parameters are chosen to be in a typical range for power electronics, but the chosen failure scenarios are merely an example for working with time varying transition rates. Except for the principles, the exercise is not meant to be characteristic in general. The configuration, scenarios and rates can be adjusted at will in future work.

The differential equations for the system in Fig. 6 are:

$$h_2(t) = h_3(t) = \left( \frac{\beta}{\alpha^\beta} \cdot t^{\beta-1} + \lambda \right) \quad (6)$$

$$h_1(t) = AF_w^\beta \cdot \frac{\beta}{\alpha^\beta} \cdot t^{\beta-1} + AF_r \cdot \lambda = EF h_2(t) \quad (7)$$

$$\frac{dP_0}{dt} = -(h_1 + h_2 + h_3) \cdot P_0 + \mu \cdot P_1 \quad (8)$$

$$\frac{dP_1}{dt} = h_1 \cdot P_0 - (\mu + 2 \cdot h_2) \cdot P_1 \quad (9)$$

$$\frac{dP_2}{dt} = h_2 \cdot P_0 - (h_1 + h_2) \cdot P_2 \quad (10)$$

$$\frac{dP_3}{dt} = h_3 \cdot P_0 - (h_1 + h_3) \cdot P_3 \quad (11)$$

$$\frac{dP_4}{dt} = 2 \cdot h_2 \cdot P_1 + (h_1 + h_2) \cdot (P_2 + P_3) \quad (12)$$

It may be noted that not only  $h_1, h_2$  and  $h_3$  vary in time, but also  $AF_w$  and  $AF_r$ . Also  $\mu$  is taken a constant, but will probably also be taken a function of time in follow-up work.

### B. Results

For  $EF=5000$  compared to no acceleration, various repair rates  $\mu$  were simulated. Fig. 7 shows the probabilities  $P_1$  (C1 failure while C2 and C3 keep the system up) and  $P_4$  (system down). The graph shows that the lifetime of the system is impacted by the enhanced wear that builds up damage, but that fast repair counteracts this. The random component returns to the original level, but wear only partly, as some damage always remains due to wear. The findings are in agreement with the expectations. The built numeric solver appeared to fulfill the requirements for the time varying transition rates.

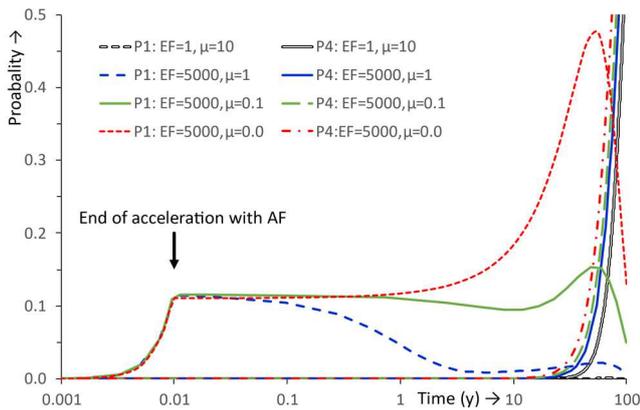


Fig. 7. State probabilities  $P_1$  and  $P_4$  for having Component 1 at deselection level respectively having the system down. Component 1 suffered enhanced stress causing an acceleration factor  $AF$  until  $t=0.01$  y. The repair rate for Component 1 is  $\mu$  ( $y^{-1}$ ). Normally  $\mu$  will be much faster, lowering  $P_1$  and  $P_4$ .

Additionally, the cumulative hazard rate  $H$  of the system is shown in Fig. 8 for the cases in Fig. 7. This representation is useful for studying the quality requirements of the total system and the impact of its history.

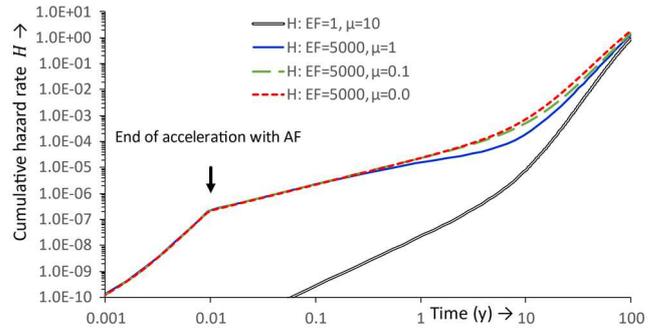


Fig. 8. The cumulative hazard rate for various repair rate and compared to non accelerated degradation

## V. CONCLUSIONS

Establishing the relation between diagnostics and deterioration, can be very challenging. Various wear phenomena (like PD) regularly do not allow a firm *RUL* prediction, because the hazard rate as a time function remains uncertain [5]. This impacts Health Index models [11,12]. Still, such diagnostics are very useful for alerts and identifying PD. This study focuses on preventing overheating. Temperature sensors seem to be better at forecasting  $h(t)$ , but, e.g., their precise positioning can have a significant impact on the prognostics effectivity.

The differential equation solver completes the framework and chain of data analytics practices that were (re)developed in the past years. Further study will focus on models like digital twins for power electronics in the electrical energy supply and drivers for automotive applications.

## VI. REFERENCES

- [1] IEC 62539(E):2007 IEEE Std 930-2004. "IEC/IEEE Guide for the statistical analysis of electrical insulation breakdown data" (IEC Central Office, Geneva, 2007. doi 10.1109/IEEESTD.2007.4288250)
- [2] R. Ross, A-J. de Graaf, P. Ypma and M. Ross, "Condition assessment after early failures in power equipment despite successfully passed factory acceptance and commissioning tests", Cigre 2024 Paris Session, 2024, paper 10556.
- [3] R. Ross, Reliability analysis for asset management of electric power grids, Wiley IEEE Press, ISBN 978-1-119-12519-8, 2019.
- [4] R. Ross, 2022, "Weibull analytics of observed and suspended failure data", Proc. CMD 2022, 13-18 Nov. 2022, Kitakyushu.
- [5] R. Ross, "Strategic role of diagnostics in asset management under ageing, climate change & earthquake", INMR World Congress 2023, Bangkok, Thailand.
- [6] IEC 61165. Application of Markov techniques, (IEC Central Office, Geneva, July 2006). ISBN 2-8318-8625-2
- [7] R. Ross, 2022, "Prognostics and Health Management for Power Electronics and Electrical Power Systems", Proceedings CMD 2022, 13-18 Nov. 2022, Kitakyushu.
- [8] N.C. Viswanath, "Transient study of Markov models with time-dependent transition rates," Operational Research (2022), Springer-Verlag GmbH Germany, 2022, pp. 2209–2243.
- [9] W. Whitt, "Time-Varying Queues," Queueing Models and Service Management Vol. 1, No. 2, 2018, pp.079-164.
- [10] L.V. Green, P.J. Kolesar, and W. Whitt, "Coping with time-varying demand when setting staffing requirements for a service system," Prod. and Oper. Mgt, Vol. 16, No. 1, POMS, 2007, pp. 13–39.
- [11] CIGRE WG A2.49. TB 761 - Condition assessment of power transformers. Paris: CIGRE, 2019. 158 p.
- [12] CIGRE WG B3.48. TB 858 - Asset health indices for equipment in existing substations. Paris: CIGRE, 2021. 167 p.