One useful propeller mathematical model for MAV

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ABSTRACT

A model for Micro Air Vehicles (MAV) propeller performance evaluation is proposed and investigated. This model is derived on the basis of simple analytical relationships and is examined on a set of experimental data. It allows to predict the main propeller characteristics (for example, thrust coefficient, power coefficient, efficiency as functions of advanced ratio) and gives the method for investigation of the powerplant as whole at the stage of preliminary design.

1 INTRODUCTION

Micro Air Vehicles (MAVs) have some peculiarities compared to other aircrafts. First of all, the characteristic Reynolds numbers Re are small enough and correspond to the region of laminar-turbulent transition for the wing and below this region for the propellers. This leads to higher values of the wing drag and lower propeller efficiency compared to larger airplanes. The next one is rather high dimensions of propeller compared to the wing span (up to 1/3-1/2 of wing span). Thirdly, the amount of energy onboard corresponds to flight times of order of 0.5-1 hour. So, to make a small airplane with high performance a thorough investigation and optimal matching of all the aircraft components is required, and one of critical components is propeller.

To save time and money during the design process the preliminary investigation and calculation based on some mathematical models of aircraft components should be carried out. As for the wing, useful simple formulas for the preliminary design already exist. The situation with the propellers is not good enough. So, the investigation proposed is an attempt to create rather simple but useful mathematical model of MAV propeller for preliminary MAV design and investigations.

2 PROPELLER CHARACTERISTICS AND PROPELLERS INVESTIGATED

Any model is useful if it describes a wide range of objects in/or a wide range of conditions. So, our propeller model must give and predict the characteristics for various propeller blade shapes, angles of blade installation, rotational frequencies, flight velocities and Reynolds numbers Re. On the other hand, it is not required to describe all the propellers that man can make but only "good" ones because nobody will use "bad" propellers. So, authors have decided to make and verify the model on the basis of well known propellers with rather good characteristics (efficiency, thrust etc.).

Nowadays a huge amount of experimental data for the

propeller characteristics at low Re can be found in the articles, books and at the Internet sites, but authors have chosen only the sources with enough amount of required information ([1]-[6]).

First of all, data for the model of AV-31 propeller was chosen [1],[2]. This propeller is 20-sm copy of the larger propeller (of 1.1 meter diameter). One of the peculiarities of this model propeller is the ability to change the angle of blade installation. This small propeller was used in the experiments of Reynolds number influence on the propeller characteristics [1],[2]. As one of the authors took part in these investigations he possesses the required geometrical characteristics and enough experimental data for this propeller. Reynolds numbers in these experiments for the blade chord at ¾ propeller radius were $Re=5\cdot10^4 \div 9\cdot10^4$.

Other propeller data used in the investigation are those of Black Widow propeller [3] as this propeller has very high value of the efficiency for its range of Re (more than 80%). Also the shape of its blades is "unusual" comparing, for example, with model aircraft propellers (large chords, high pitch). Reynolds number range was $Re\approx10^4 \div 3\cdot10^4$

The last part of data chosen is taken from [4]-[5]. It is a database of model airplane propellers experimental investigations. Reynolds number range in these experiments was $Re=2.5\cdot10^4 \div 10^5$. Dimensionless chord length distribution along the radius is the same within one series of propeller (for example, for all the APS Thin Electric). Also for vast majority of propellers examined (APC, JWS, Kyosho, Graupner) the value of $r\cdot\beta(r)$ remains practically constant in the range of $0.2R<r<0.9R$ (r is the distance from rotational axis, R is propeller radius, $\beta(r)$ is angle of profile installation distribution along the blade). This fact is not valid only for Master Airscrew propellers. Such a parameters distribution is conventional for model airplane propellers as it gives simple way for design and fabrication of the propeller with acceptable efficiency but is not good enough. It gives zero induced drag (and torque) at zero thrust. But it is much more better to have minimal drag (and torque) and, as consequence, the highest efficiency at working regime.

To check data from [4,5] some data from [6] were used.

Very useful for the propeller performance description are dimensionless characteristics: thrust coefficient C_T, power coefficient C_P and efficiency η as function of advanced ratio λ:

$$\lambda = \frac{V}{nd},$$

$$C_T = \frac{T}{\rho n^2 d^4},$$

$$C_P = \frac{P}{\rho n^3 d^5},$$

$$\eta = \frac{C_T \lambda}{C_P},$$

where V – air velocity at infinity, n – frequency of propeller rotation, ρ – air density, d – propeller diameter, T – propeller thrust, P – propeller power.

It is well known that for the fixed Reynolds number these coefficients are independent of dimensions and velocities. Also for $Re>10^6$ these characteristics are independent also of Reynolds number, i.e. all the geometrically similar propellers in this range of Re have the same dimensionless characteristics. At lower Re the characteristics can change with Re (see, for example [1],[2]). Figures 1 and 2 show this change. Here $\beta_{0,75}$ is blade installation angle (commonly measured at 0.75 of the blade radius R). To see the change in characteristics one can compare, for example, C_T and efficiency at $\beta_{0,75}=20^0$ for 2500 and 5000 revolutions per minute (RPM)) (corresponding $Re=5\cdot10^4 \div 10^5$).
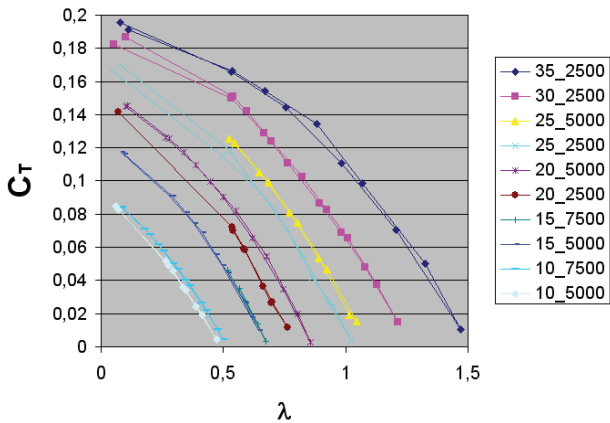

Figure 1: Thrust coefficient as function of advanced ratio and angle of blade inclination for AV-31 propeller (legend: $\beta_{0.75}$ _RPM)
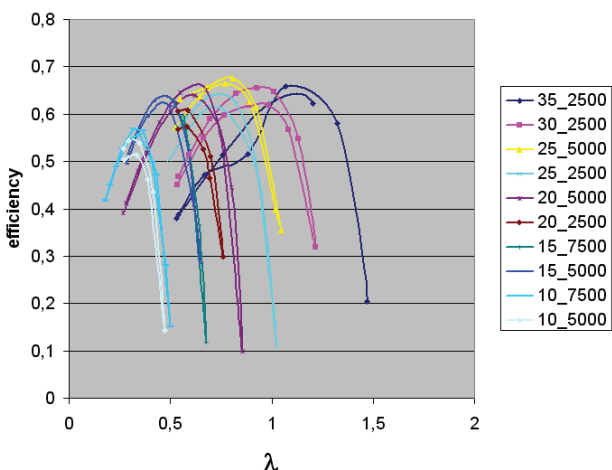

Figure 2: Efficiency as function of advanced ratio and angle of blade inclination for AV-31 propeller (legend: $\beta_{0.75}$ _RPM)

But this change is noticeable only for the propellers with thick blades. Figures 3 and 4 show the dimensionless characteristics of Black Widow propeller (derived from data of [3]), Reynolds number changes in the range of 10000-30000. It can be seen that for the propeller with thin blades

this change of characteristics with Re is negligible.

So, we can postulate that it is preferable to use propellers with thin blades on MAVs and one can use dimensionless coefficients without the correction on Re changes in this case.
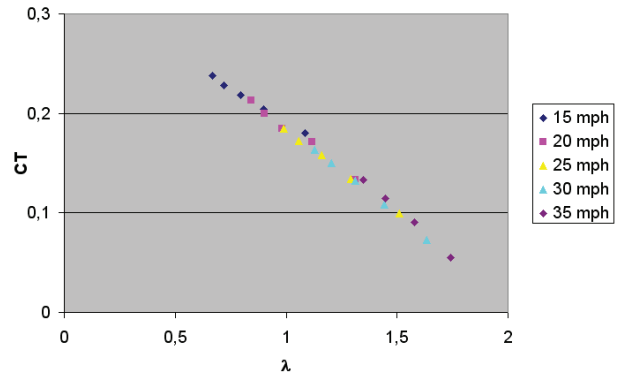

Figure 3: Thrust coefficient as function of advanced ratio for Black Widow propeller at various free stream velocities
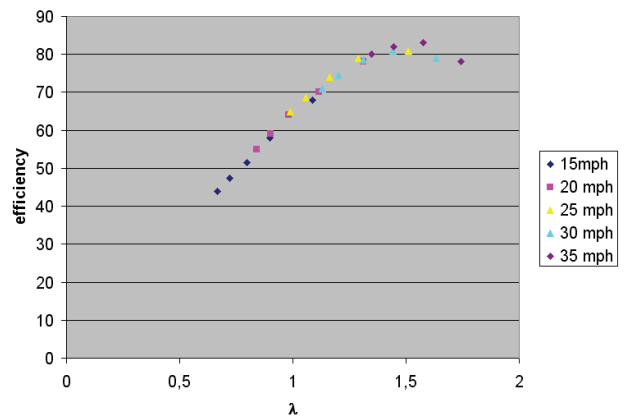

Figure 4: Efficiency as function of advanced ratio for Black Widow propeller at various free stream velocities

3 THRUST COEFFICIENT

One can see from the graphs of $C_T(\lambda)$ (in Figures 1 and 3, for example) that there exist two characteristic regions of $C_T(\lambda)$. Near the zero values of thrust coefficient the dependence $C_T(\lambda)$ is linear:

(1) $C_T=C_{T0}-C_1\lambda$,

where C_{T0} and C_1 are some coefficients.

For the propeller with rather high blade inclination angles near the zero advanced ratio values the thrust coefficient is nearly constant (it can be explained by flow separation on the blade at high angles of attack).

The first region is more interesting in the case of high efficiency requirements. The second region is of interest for the hovering mode.

The behavior of the graph in the first region can be explained theoretically. Let's consider a small part of propeller blade (profile) at the distance r from the axis of rotation. If the angular velocity is ω, then the rotational velocity of the profile considered is ωr.
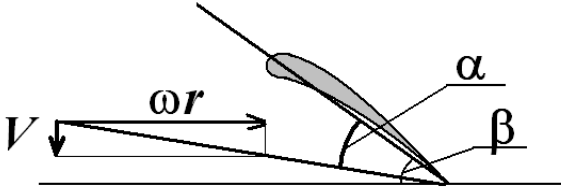
Figure 5: Angles of α and β definition for the propeller blade profile at distance r from the axis of rotation.

If the profile is at the angle β with respect to the plane of rotation (angle of inclination) (see Figure 5), then the angle of attack α for this profile is

$$\alpha = \beta(r) - \arcsin\left(\frac{V}{\sqrt{V^2 + (\omega r)^2}}\right).$$

If $V < \omega r$, the last formula can be simplified to

$$\alpha = \beta(r) - \frac{V}{\sqrt{V^2 + (\omega r)^2}}.$$

So, the thrust of this profile dT is

$$dT = C_L{}^{\alpha}\left(\beta(r) - \frac{V}{\sqrt{V^2 + (\omega r)^2}}\right) \times$$

$$\times \rho \frac{V^2 + (\omega r)^2}{2} \frac{\omega r}{\sqrt{V^2 + (\omega r)^2}} b(r)dr.$$

As $V < \omega r$, one can write this expression as

$$dT = C_L{}^{\alpha}\frac{\rho}{2}\left(\omega^2 \beta(r)b(r)r^2 dr - \omega V b(r)rdr\right).$$

From this formula one can obtain

$$(2)\quad C_{T0} = \frac{\pi^2}{8}C_L{}^{\alpha}\int_{-1}^{1}\beta(\overline{r})\overline{b}(\overline{r})\overline{r}^2 d\overline{r},$$

$$(3)\quad C_1 = \frac{\pi}{8}C_L{}^{\alpha}\int_{-1}^{1}\overline{b}(\overline{r})\overline{r}d\overline{r},$$

where letters with overline are dimensionless with respect to the propeller radius R.

One can also take into account the drag force on the profile. If the drag force coefficient at zero lift for profile is C_{D0}, the projection D_T of viscous drag force on the axis of thrust can be expressed as

$$D_T = \rho n^2 d^4 \lambda \frac{\pi}{8}C_{D0}\int_{-1}^{1}b(\overline{r})\overline{r}d\overline{r}.$$

Corresponding coefficient C_{1D} is

$$C_{1D} = \frac{\pi}{8}C_{D0}\int_{-1}^{1}b(\overline{r})\overline{r}d\overline{r}.$$

Assuming that C_{D0} is about 0.03 and $C_L{}^{\alpha}$ is about 2π (theoretical result for 2D flat plane), the value of C_{1D} is about 0.5% of C_1, and C_{1D} cannot be taken into account at the first approximation.

The last expression and formula (3) show that coefficient

C_1 must be practically the same for the series of propeller with the same $b(r)$ but various β(r). Moreover, from (3) C_1 must be equal for all the propellers with the same value of

$$C_L{}^{\alpha}\int_{-1}^{1}\overline{b}(\overline{r})\overline{r}d\overline{r}$$

Data from [1], [2], [4], [5] show that for small values of C_T this fact is valid within the experimental errors.

Expressions (2)-(3) allow not only to explain the behavior of $C_T(\lambda)$, but also to obtain the values of C_{T0} and C_1 for the given β(r), $b(r)$ and profile shape. So, on the other hand, these formulas can be verified using the experimental data for propellers.

The verification was made by authors for a set of propellers from UIUC database [5]. Here, geometrical parameters (β(r), $b(r)$) and experimental data of APC, JWS, Kyosho, Graupner and Master Airscrew propellers can be found. The two problems were the absence of information on the profile shape of propeller "chords" and the value of $C_L{}^{\alpha}$ for profile. So, it was assumed that the curvature of the profile corresponds to the additional angle of attack of 0.05 radians (~3^0). As $C_L{}^{\alpha}$ is in both formulas then the ratio of C_{T0}/C_1 is independent of $C_L{}^{\alpha}$ and corresponds to λ for zero thrust coefficient (denote it as λ^0). Analysis shows that for the propeller examined the theoretical value coincides with the experimental data good enough, the errors are less than 5%. Also, it was found that for $C_L{}^{\alpha}=5$ the theoretical and experimental data of C_T coincide with good accuracy.

It should be mentioned that the comparison of theoretical and experimental data with the results from program PropCalc [7] shows that the theoretical predictions are not worse than numerical calculation from PropCalc, but only for the region of low C_T values.

Also one can see from the graph in Figure 1 that the values of λ^0 for zero thrust are nearly proportional to the angle of blade installation. This fact is very useful for the stage of the preliminary aircraft design as it can help to find the best propeller rather quickly. The abovementioned proportionality becomes evident if we look at the expressions for C_{T0} and C_1. As C_1 is independent of the angles of profile installation $\beta_{0.75}$, the value of λ^0 is determined by C_{T0}.

$$(4)\quad C_{T0} = C_1\lambda^0.$$

In turn, the value of C_{T0} is proportional to this angle (see (2)).

More accurate investigation shows that for zero angle of installation the corresponding value of λ^0 is not zero (it is due to the profile centerline curvature), but the shape of $\lambda^0(\beta_{0.75})$ remains linear function for the fixed Re. Figure 6 shows this dependence for AV-31 and APC Thin Electric 11" propellers.

Unfortunately, because of thick blade chords profile at AV-31 propeller one cannot obtain the right result of the Reynolds number influence on the function of $\lambda^0(\beta_{0.75})$, but the results for Black Widow propeller give the hope that this influence is small enough. (See Fig. 3, all the points are nearly on the one line.)
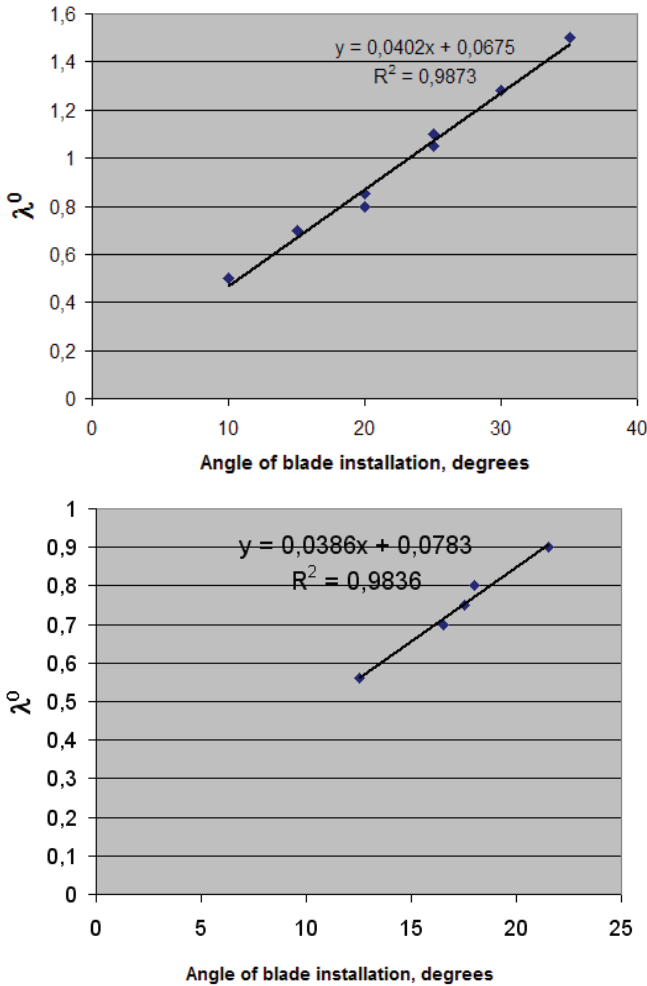
Figure 6: Zero-thrust advanced ratio as function of angle of blade installation for AV-31 (upper) and APC Thin Electric 11" (lower)

Results from [4], [5] show that λ_0 becomes ~5% lower when Re becomes 2 times lower for the same propeller. Also coefficient C_1 becomes slightly lower when Re becomes 2 times lower.

4 POWER COEFFICIENT AND EFFICIENCY

For the propellers at high Reynolds numbers the following semi-empirical formula exists [8]:

$$\frac{C_P}{\lambda^2} = a\frac{C_T}{\lambda^2} + b \, .$$

This expression can be rewritten in the form of

(5) $C_P = aC_T + b\lambda^2 \, .$

For the power and thrust this formula looks like

$P = nd(aT + \rho bd^2 V^2)$

Authors examined these formulas on the experimental data [3], [5], [6] for low Re propellers. It was found that real dependency of $C_P(C_T(\lambda), \lambda)$ is not a straight line for some kinds of propellers, but its curvature is small enough so one can assume it as straight line at least for the region of high efficiency. As an example, for the Black Widow propeller it is "very good" straight line, for set of APC Electric, Graupner and Master Airscrew propellers the graph angle of incidence becomes lower at high C_T/λ^2, but at least for APC 8×8.5 propeller this angle becomes little higher at high

C_T/λ^2. Also one must keep in mind that the measurements near zero λ give higher value of the relative error, so the higher values of the error will be in the verification of formula (5) for low values of λ.

For the region near the zero values of λ formula (5) becomes

$C_P \approx aC_T \, ,$

and (as C_P and C_T at this region do not change significantly) the expression for efficiency can be written as

(6) $\eta \approx \dfrac{\lambda}{a} \, .$

One can see that the graph of $\eta(\lambda)$ is linear at low λ (see, for example, Figure 2), so expression (6) can be really used for this region of λ. Also one can use it for value of the coefficient a determination (if required).

But all the above mentioned in this chapter facts are valid for the fixed propeller and fixed Re. So, first of all, it is interesting to test this dependency for various Reynolds numbers.

Figure 7 shows the corresponding data for Black Widow propeller [3]. Shown in Figure 8 is the result of APC Thin Electric 11×10 propeller data comparison [5] for various frequencies of rotation. The lowest frequency for APC 11×10 (3000 RPM) corresponds to $Re \approx 36000$. One can see that the change of Re from 36000 to ~60000 gives practically no effect, the graphs are practically the same.
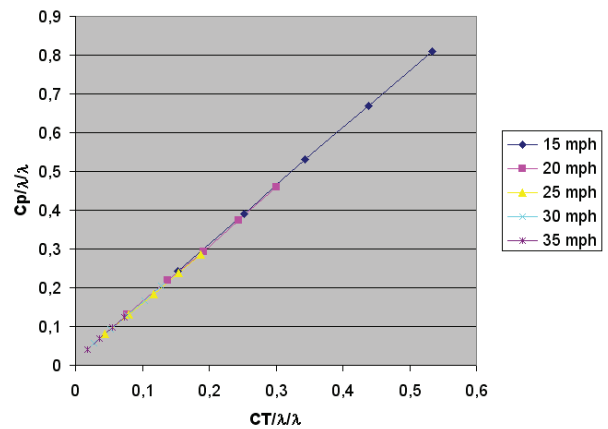

Figure 7: C_P/λ^2 as function of C_T/λ^2 for Black Widow propeller
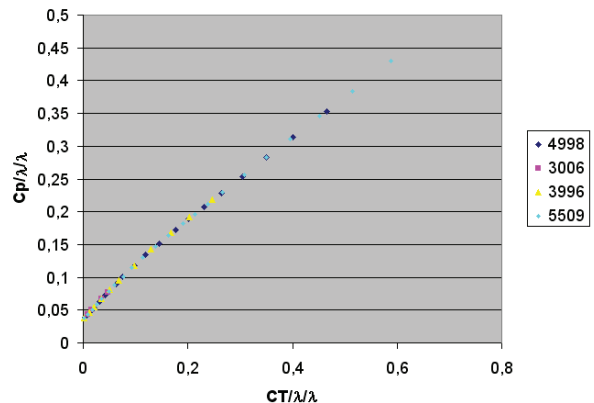

Figure 8: C_P/λ^2 as function of C_T/λ^2 for APC 11×10 propeller for various RPM

Also it is rather important to know the behavior of a and b coefficients with blade inclination angle change. In other words, the question is what is the difference of these coefficients, for example, for APC 11×5.5 and APC 11×10 propellers.

Figure 9 shows this dependency for APCE 11" propellers family.

One can see that values of a and b change with the propeller pitch change. To make more accurate investigation it is better to use data for AV-31 propeller. Figure 10 shows the values of a as functions of λ^0. (Here "eff" corresponds to the points near maximal efficiency, "all" corresponds to all the points from experiment.) One can that a is nearly proportional to λ^0:

(7) $a = k\lambda^0$,

where k is proportionality coefficient.

The same result can be obtained also from other data. For example, Figure 11 shows the efficiency as function of advanced ratio for various angles of blade installation [9]. It can be easily derived from these graphs the same dependency (7).
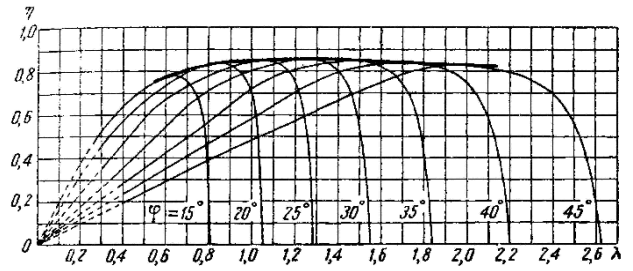

Figure 11: Efficiency as function of λ and $\beta_{0.75}$ for one of propellers from [9].

Characteristic values of coefficient k obtained from data analysis are 0.5-0.9, and characteristic values of coefficient b are about 0.02-0.08. It is evident that coefficient b can be found from experimental data with less accuracy as coefficient a. So, it is rather difficult to understand the b dependency on λ^0 (or $\beta_{0.75}$). The coefficient b obtained on the basis of experimental data for AV-31 is shown in Figure 12. One can see the uncertainty in its definition. So, the alternative ways of b definition must be found. One of them will be proposed below. But on the basis of data analyzed one can assume that for rather high angles of installation (and, as consequence, high λ^0) the value of b does not change significantly.
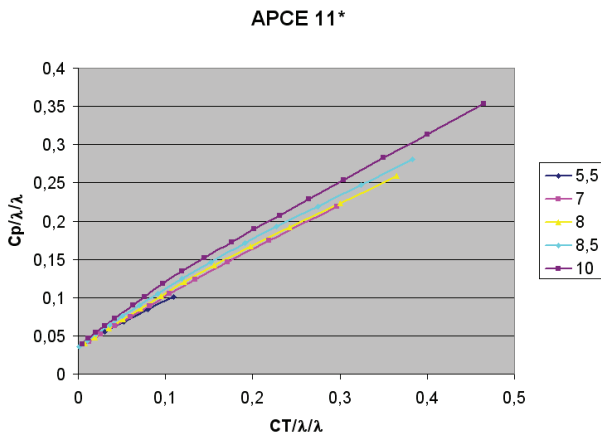

APCE 11*

Figure 9: C_P/λ^2 as function of C_T/λ^2 for APCE 11" of various pitches

Figure 10: Coefficient a as function of λ^0 for AV-31 propeller for various RPM.
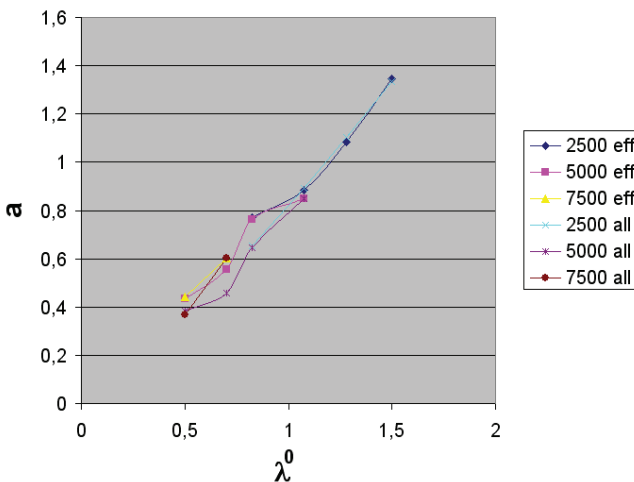
Figure 12: Coefficient b as function of $\beta_{0.75}$ for AV-31 propeller for various RPM.

5 ANALYTICAL INVESTIGATION OF MAXIMAL EFFICIENCY

One can obtain a set of useful results on the basis of equations (1)-(7). First of all, it is easy to find the value of advanced ratio λ_{eff} corresponding to the maximum of propeller efficiency η_{max}. Substituting (1) into (5) and setting its derivation with respect to λ to zero one can find

(8) $\lambda_{eff} = \dfrac{C_{T0}\left(C_1 a - \sqrt{C_{T0}ab}\right)}{C_1^2 a - C_{T0}b} = \dfrac{C_{T0}\sqrt{a}}{C_1\sqrt{a} + \sqrt{C_{T0}b}}.$

Corresponding value of η_{max} is

(9) $\eta_{max} = \dfrac{C_{T0}}{C_1 a + 2\sqrt{C_{T0}ab}}.$

Calculations based on data from [3]-[6] show the good matching of these formulas with experimental data.

Combination of (4), (7) and (8) gives

(10) $\lambda_{eff} = \dfrac{\sqrt{C_1 k}}{\sqrt{C_1 k} + \sqrt{b}}\,\lambda^0.$

So, within our model, the value of λ_{eff}/λ^0 remains constant

for all the propellers of the same blade geometry and various angles of blade inclination. This fact is valid not only for the propellers examined. For example, the graph from [9] can lead to the same conclusion (see Figure 11).

Combination of (4), (7) and (9) leads to

$$(11) \quad \eta_{max} = \frac{C_1}{C_1 k + 2\sqrt{C_1 kb}} = \frac{\sqrt{C_1}}{\sqrt{C_1} k + 2\sqrt{kb}} .$$

One can see that this expression is explicitly independent of λ^0, but it can depend on λ^0 through b and, may be, through C_1. From Figures 2 and 11 it is seen that the value of η_{max} remains constant within rather wide range of λ^0 (0.8–1.5), corresponding to the working range of λ^0 for the propellers. On the other hand, this coincidence proves our assumption made above that the value of b does not change significantly with λ^0 for rather high λ^0.

The expression for C_T at λ_{eff} can be found from (1), (4), (10) as

$$(12) \quad C_{Teff} = \frac{\sqrt{b}}{\sqrt{C_1} k + \sqrt{b}} C_1 \lambda^0 .$$

It is useful to compare the values of η_{max}, λ_{eff} and C_{Teff} through formulas (10), (11), (12) and experimental values.

For Black Widow propeller [3] a=1.49, b=0.0134, C_1=0.168, k=0.71, λ^0=2.1. For these values η_{max}=0.83, experimental value is 0.82-0.83; C_{Teff}=0.088, experimental value is ~0.9; λ_{eff}/λ^0=0.75, experimental value is ~0.74.

For AV-31 propeller with $\beta_{0.75}$=35^0 a=1.34, b=0.025, C_1=0.205, k=0.89, λ^0=1.5; η_{max}=0.65 (0.656 experiment); C_{Teff}=0.085 (0.0987 experiment); λ_{eff}/λ^0=0.73 (0.69 experiment).

For APC Thin Electric 11×8.5 ($\beta_{0.75}$=18^0) a=0.605, b=0.052, C_1=0.2124, k=0.756, λ^0=0.8; η_{max}=0.619 (0.622 experiment); C_{Teff}=0.061 (0.0675 experiment); λ_{eff}/λ^0=0.64 (0.625 experiment).

So, one can see that the analytical results are rather close to the experiment. So, the conclusion can be made that the model proposed is adequate enough.

We need to know the values of C_1, k and b. Using the formulas obtained the required coefficients can be defined. Coefficient C_1 can be found if we know the geometry of propeller, but assume that we have no such information. Assume that we have the experimental data of $C_T(\lambda)$ and $\eta(\lambda)$. From the first graph one can obtain C_1 using formula (1). From the second graph with the help of (6) and (7) one can find the values of a and k. One of formulas (10) or (11) gives then the value of b. The second of (10) or (11) can be used to check and correct the results if necessary.

It should be emphasized that only three coefficients C_1, k and b must be known to describe the characteristics of all the "good" propellers with the same blade geometry for all the "good" operating regimes. So, formulas obtained can be used for the whole powerplant characteristics determination in a wide range of flight conditions.

6 PROPELLER MATCHING FOR MAV

Now we have enough knowledge to proceed to the main task of this investigation. Assume that we have the aircraft that fly at the velocity V and it produces drag D, and we need to find the propeller with maximal efficiency and with predefined diameter d.

First of all, we must choose the propeller family (with the same blade geometries but different $\beta_{0.75}$) that provide the maximal efficiency (above we have seen that this value of efficiency is practically constant in rather wide range of λ^0). The next step is to match the angle of blade inclination $\beta_{0.75}$ (or, in other words, λ^0).

As the drag must be equal to the thrust (D=T), then with the help of (10) and (11) one can obtain

$$(13) \quad \lambda^0 = \frac{\rho V^2 d^2}{kT} \left(\sqrt{C_1 kb} + b \right) .$$

This expression gives us enough information for the best choice of propeller. The only thing we must do is to check if this value of λ^0 is within the region of maximal η_{eff}. If this value is outside this region one must change the propeller diameter.

Also from the last formula and (10) one can obtain

$$(14) \quad \lambda_{eff} = \frac{\rho V^2 d^2}{kT} \sqrt{C_1 kb}$$

for the propeller chosen.

From this, the required frequency for such propeller is

$$(15) \quad n_{eff} = \frac{V}{d \lambda_{eff}} = \frac{kT}{\rho V d^3 \sqrt{C_1 kb}}$$

Formulas (13) - (15) lead to the conclusion that lower values of thrust for the fixed diameter and velocity require higher angles of blade installation and, as consequence, lower rotational frequencies.

7 EXPERIMENTAL ERRORS AND INACCURACY INFLUENCE ON MATHEMATICAL MODEL

All the data used are obtained in experiments. So, there can be experimental errors in this data. For example, the results from [5] and [6] slightly differ from each other (one can compare the maximal values of efficiency in both experiments). Unfortunately, authors have not found the values of errors in most of experimental data used in this investigation.

So, on the one hand, the analytical model and corresponding expressions can be not very accurate as they use some experimental data and verified on experimental data. On the other hand, authors used the experimental data from different researchers for different propellers. As the main results are nearly the same one can hope that the mathematical model proposed is rather adequate.

Also at the stage of preliminary design it's better to have less accurate but simpler and quicker methods rather than more accurate but more complex and slower methods. So, anybody who uses the above formulas must keep in mind that this method do not provide the best accuracy and must be used only for the first approximation. In the case of higher accuracy requirements one must use more complex methods but can use the "simple results" as the starting point.

8 FUTURE WORK

Now authors are working on the theory for b and k determination through the propeller geometrical parameters and flow conditions. So, they hope to obtain these expressions in their future work.

9 CONCLUSION

Obtained is the mathematical model of propeller's thrust and power coefficients as functions of geometrical and kinematical parameters for the preliminary MAV design. It needs minimal set of parameters required and can be used for rapid determination of propeller characteristics. This model was verified on a set of experimental data and has shown the adequate accuracy in the range of $Re=30000\div100000$. On the basis of this model the method for the best propeller choosing is proposed. Formulas obtained can be used for the whole powerplant characteristics determination in a wide range of flight conditions.

Investigation conducted also allows to make the following recommendations concerning the propeller design.

First of all, for low-Re propellers thin blades are preferable as they give higher value of efficiency.

Next, for the best efficiency the angle of blade installation is proportional to flight velocity squared, to propeller diameter squared and inversely proportional to thrust.

It was also found that model aircraft propellers are not good enough as they does not provide maximal efficiency because of nonoptimal shape of the blade (blade twist).

Finally, it should be noted that additional special experiments are required for more thorough investigation of Re and λ^0 influence on C_1, k, b.

REFERENCES

[1] Ya. Flaxman, Al. Lipin, S. Ostroukhov, M. Ustinov, A. Shustov, A. Kornushenko, S. Serokhvostov. Experimental Investigations Of Propellers At Low Reynolds Numbers. In Proc. of *3rd US-European Competition and Workshop on Micro Air Vehicle Systems (MAV07), Toulouse, France,* 2007

[2] А.В. Липин, С.П. Остроухов, С.В. Серохвостов, М.В. Устинов, А.В. Шустов, Я.Ш. Флаксман. Экспериментальное исследование зависимости характеристик воздушного винта от числа Рейнольдса. *Ученые записки ЦАГИ,* XXXVIII (3-4), 102-110, 2007.

[3] J.M. Grasmeyer, M.T. Keennon. Development of the Black Widow Micro Air Vehicle. AIAA Paper No. 2001-0127, 2001

[4] J.B.Brandt, M.S.Selig. Propeller performance data at low Reynolds numbers. In 49th AIAA Aerospace Sciences Meeting, Orlando, FL, 2011, AIAA 2011-1255

[5] J.B.Brandt, M.S.Selig. Small-Scale Propeller Performance at Low Speeds – Online Database. http://www.ae.illinois.edu/m-selig/props/propDB.html, 2010

[6] M.P. Merchant, L.S. Miller. Propeller Performance Measurement for Low Reynolds Number UAV Applications. In 44th AIAA Aerospace Sciences Meeting and Exhibit, Reno, Nevada, 2006

[7] PropCalc program. http://www.drivecalc.de/PropCalc/index.html

[8] http://www.allstar.fiu.edu/aero/BA-Background.htm

[9] Александров В.Л. Воздушные винты. М., Оборонгиз, 1951