5 The Future of Single-Frequency Integer Ambiguity Resolution

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Abstract

The coming decade will bring a proliferation of Global Navigation Satellite Systems (GNSSs) that are likely to enable a much wider range of demanding applications compared to the current GPS-only situation. One such important area of application is single-frequency real-time kinematic (RTK) positioning. Presently, however, such systems lack real-time performance. In this contribution we analyze the ambiguity resolution performance of the single-frequency RTK model for different next generation GNSS configurations and positioning scenarios. For this purpose, a closed form expression of the single-frequency Ambiguity Dilution of Precision (ADOP) is derived. This form gives a clear insight into how and to what extent the various factors of the underlying model contribute to the overall performance. Analytical and simulation results will be presented for different measurement scenarios. The results indicate that low-cost, single-frequency Galileo+GPS RTK will become a serious competitor to its more expensive dual-frequency cousin.

Keywords

ADOP • Ambiguity resolution • Single-frequency RTK

1 Introduction

Global Navigation Satellite System (GNSS) ambiguity resolution (AR) is the process of resolving the unknown cycle ambiguities of the carrier phase data as integers. It is the key to high-precision GNSS parameter estimation. In order for AR to be successful, the probability of correct integer estimation needs to be sufficiently close to one. Whether or not this is the case depends on the strength of the underlying GNSS model and therefore on the number and type of signals observed, the number of satellites tracked, the relative receiver-satellite geometry, the length

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of the observational time window, the measurement precision, the dynamics of the positioning application and the need of having to include additional parameters like troposphere and/or ionosphere delays.

The coming decade will bring a proliferation of GNSSs (modernized GPS, Glonass, Galileo, Compass) that are likely to enable a much wider range of demanding applications compared to the current GPS-only situation due to the availability of many more satellites and signals. This contribution considers the application area of single-frequency real-time kinematic (RTK) positioning. Presently, low-cost single-frequency RTK systems lack real-time performance due to the weaknesses of the single-frequency GPS-only model, see e.g. Milbert (2005); Odijk et al. (2007); Takasu and Yasuda (2008). If low-cost single-frequency RTK would become feasible, a whole range of exciting applications awaits in e.g. the fast-evolving field of mobile Location Based Services, precision agriculture, surveying and mapping, e.g. Wirola et al. (2006); Denham et al. (2006); Saeki and Hori (2006); Millner et al. (2005).

In this contribution we analyze the ambiguity resolution performance of the single-frequency RTK model for different next generation GNSS configurations and for different positioning scenarios. For this purpose, first a closed form expression of the single-frequency Ambiguity Dilution of Precision (ADOP) is derived in Sect. 2. A performance analysis based on the ADOPs as well as empirical success rates is presented in Sect. 3. These results allow us to identify the circumstances that make successful single-frequency AR possible, as will be shown in the final Sect. 4.

2 Ambiguity Resolution

The key to rapid and high-precision GNSS positioning is the use of carrier-phase observations, which have mm-level precision while code observations only have a precision at the dm-level. In order to exploit the very precise carrier-phase measurements, first the unknown integer number of cycles of the observed carrier phase has to be resolved. The linearized double-difference GNSS model can be written as:

$$y = Bb + Aa + e, \quad b \in \mathbb{R}^{\nu}, \;\; a \in \mathbb{Z}^{n} \tag{5.1}$$

where y is the vector with double-differenced code and phase observables; b is the ν-vector with unknown real-valued parameters, such as the baseline increments, ionosphere and troposphere parameters; a is the n-vector with the unknown integer ambiguities; e is the noise vector. The matrices B and A link the unknown parameters to the observables. It is generally assumed that y follows the normal distribution, with zero-mean noise and the associated variance matrix Q_{yy} capturing the measurement precision.

Solving model (5.1) in a least-squares sense provides the so-called float solution, where the integer constraint on the carrier-phase ambiguities, i.e. $a \in \mathbb{Z}^n$, is not considered. This is done in a second step, the ambiguity resolution (AR) step, based on the float ambiguities $\hat{a}$ and associated variance matrix $Q_{\hat{a}\hat{a}}$. The integer least-squares (ILS) estimator is proven to be optimal in the sense that it maximizes the probability of correct integer estimation, Teunissen (1999). A well-known and efficient implementation of the ILS-principle is the LAMBDA method, Teunissen (1995). After resolving the integer ambiguities $\check{a}$, the final step is to adjust the float solution of b conditioned on the fixed integer solution. This provides the fixed baseline solution $\check{b}$.

Correct integer estimation is essential to guarantee that $\check{b}$ will have cm-level precision. Hence, the probability of correct integer estimation, called success rate, is a valuable measure to assess the positioning performance. Unfortunately, no analytical expression is available to compute the ILS success rate exactly. Several approximations were proposed in the past, see Verhagen (2005). In this contribution empirical success rates based on Monte Carlo simulations will be used.

In Teunissen (1997) the Ambiguity Dilution of Precision (ADOP) was introduced as an AR performance measure. It is defined as:

$$\text{ADOP} = \sqrt{|Q_{\hat{a}\hat{a}}|^{\frac{1}{n}}} \tag{5.2}$$

The ADOP measure has the unit of cycles, and it is invariant to the decorrelating Z-transformation of the LAMBDA method. It is equal to the geometric mean of the standard deviations of the ambiguities if these would be completely decorrelated. Hence, the ADOP approximates the average precision of the transformed

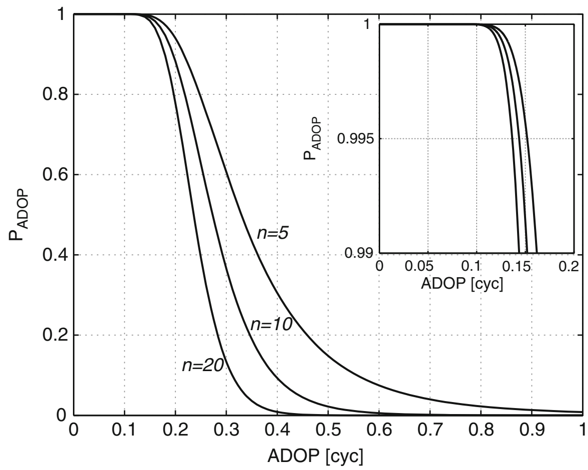

Fig. 5.1 P_{ADOP} as function of ADOP

ambiguities. The ADOP can also be used to get an approximation of the ILS success rate:

$$P(\check{a}=a) \approx P_{\mathrm{ADOP}} = \left(2\Phi\left(\frac{1}{2\mathrm{ADOP}}\right)-1\right)^{n} \tag{5.3}$$

Figure 5.1 shows the relation between ADOP and P_{ADOP} for different values of n. From this figure it can be concluded that for successful ambiguity resolution the ADOP should be smaller than 0.15 cycles.

It is possible to derive closed-form expressions for ADOP. In Odijk and Teunissen (2008) this was done for a hierarchy of multi-frequency single-baseline GNSS models. The closed-form expressions give a clear insight into how and to what extent the various factors of the underlying GNSS model contribute to the overall AR performance, see Odijk and Teunissen (2007). The closed-form expression for the ADOP of the single-frequency model corresponding to a moving receiver covering a short time span (no change in satellite geometry) can be derived as (see table 8 in Odijk and Teunissen (2008), use $j=1$):

$$\mathrm{ADOP} = \left[\frac{\sigma_\phi}{\lambda}\right][2sf]^{\frac{1}{2}}\left[\frac{\sum_{s=1}^{m} w_s}{\prod_{s=1}^{m} w_s}\right]^{\frac{1}{2(m-1)}} \times \left[1+\eta\cdot\frac{\kappa}{1+\kappa}\right]^{\frac{1}{2}}\left[1+\eta\cdot\frac{(2\kappa+1)^2}{1+\kappa(1+\eta)}\right]^{\frac{v}{2(m-1)}} \tag{5.4}$$

with:

- σ_ϕ undifferenced phase standard deviation [m]
- σ_p undifferenced code standard deviation [m]
- σ_ι undifferenced standard deviation of ionosphere observables [m]
- λ carrier wavelength [m]
- sf variance scale factor
- m number of satellites
- w_s elevation dependent weights, $s=1,\ldots,m$
- η $=\frac{\sigma_p^2}{\sigma_\phi^2}$ and $\kappa=\frac{\sigma_\iota^2}{\sigma_p^2}$

The ionosphere-weighted model, see e.g. Odijk (2002), is used where a priori information on the ionosphere delays is used in the form of ionosphere observables with standard deviation σ_ι depending on the baseline length. If the baseline is sufficiently short, the double difference ionosphere observables will become zero, and σ_ι is set to zero.

In (5.4) sf is a scale factor, if $sf<1$ this can be either due to enhanced measurement precision, or due to an increased number of epochs k. In the first case it is assumed that the variance of code and phase observations is improved with the same factor sf. In the second case the scale factor would be equal to:

$$sf = \frac{1+\beta}{k-(k-2)\beta} \tag{5.5}$$

where $\beta (0 \le \beta < 1)$ describes the correlation parameter of a first-order autoregressive time process. Hence, $\beta=0$ means that time correlation is absent and

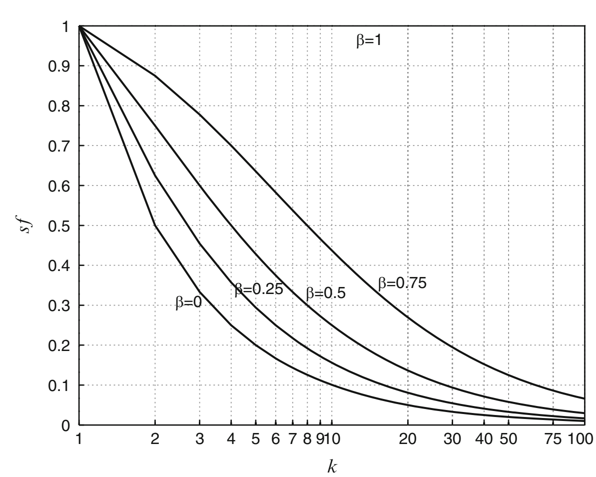

Fig. 5.2 Relation between the variance scale factor sf and number of epochs k for various time correlations β

$sf = \frac{1}{k}$, while $\beta = 1$ would mean that the observations are fully correlated between the epochs and $sf = 1$. Figure 5.2 shows the relation between the variance scale factor sf and the number of epochs k for various time correlations β.

3 Performance Analysis

An analysis of the ambiguity resolution performance is made based on the following assumptions:

$$\sigma_\phi = 2\,\text{mm},\ \sigma_p = 20\,\text{cm}$$
$$\sigma_\iota = 0, 4, 8\,\text{mm}$$
$$\lambda = 25.48\,\text{cm (L5 frequency)}$$
$$w_s = (1 + 10\exp(-e_s/10))^{-\frac{1}{2}}$$
$$v = 3 \text{ (no troposphere parameters estimated)}$$

with e_s the elevation of satellite s in degrees. A mask angle of 10° is used.

The three values of σ_ι are assumed to correspond to baseline lengths of <5, 10 and 20 km, respectively.

The future Galileo constellation is considered, as well as the combined GPS+Galileo constelation, where for GPS the nominal constellation of 24 satellites is used. A time span equal to the repeat orbit period of Galileo, approximately 10 days, is considered. Two different geographical locations are considered, both at longitude 3°E and latitudes 45°N and 75°N, respectively. The mid-latitude location is selected because on average the least number of satellites are visible while at the higher latitude of 75°N the opposite is true. Figure 5.3 shows the number of visible satellites and the skyplots for the two locations with the satellite tracks of both GPS and Galileo. Note that at higher latitudes the satellite geometry will generally be better as well, since satellites from all azimuths will be visible. The standard deviations of the code and phase observations are relatively conservative compared to the expected thermal noise characteristics of the future GNSS signals as presented in Simsky et al. (2006). Here we choose somewhat higher standard deviations to account for multipath and other residual effects, as well as to simulate the performance with low-grade receivers.

Figure 5.4 presents the mean ADOP as function of the number of satellites m with $sf = 1$ (i.e. the mean for each m is calculated over all instances that m satellites are visible during the 10-day period). The average values of the two locations are shown, since it turned out that the impact of the satellite geometry on the ADOP – third term in (5.4) – is averaged out and thus the results are nearly identical for the two different locations.

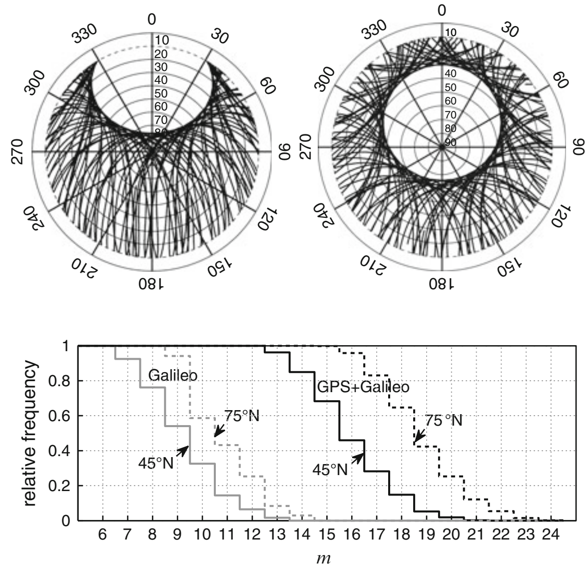

Fig. 5.3 *Top*: Skyplots (left for 45° latitude, right for 75° latitude) for one day with GPS and Galileo satellite tracks. *Bottom*: Relative frequencies that more than m satellites are visible for 10-day period

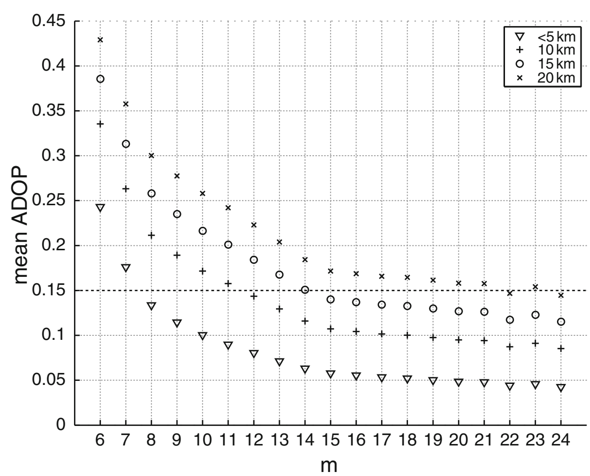

Fig. 5.4 Single-epoch, mean ADOPs [cycles] as function of number of satellites

From Fig. 5.1 it was concluded that an ADOP of 0.15 cycles was required for successful ambiguity resolution. Using this rule-of-thumb, it follows from Fig. 5.4 that 8 or more satellites are required with very

Table 5.1 Scale factor sf needed to obtain a success rate above 0.99 more than 99% of the time. The number between brackets is the corresponding number of epochs if $\beta = 0$

	Galileo		GPS+Galileo	
Baseline	45°N	75°N	45°N	75°N
<5 km	0.07 (*15*)	1 (*1*)	1 (*1*)	1 (*1*)
10 km	0.02 (*60*)	0.11 (*9*)	1 (*1*)	1 (*1*)
15 km	0.01 (*70*)	0.06 (*16*)	0.2 (*5*)	1 (*1*)

short baselines, more than 11 satellites with baselines of 10 km, and more than 14 satellites with baselines of 15 km. With longer baselines, single-epoch ambiguity resolution is generally not feasible. From Figs. 5.3 and 5.4 combined, it follows then that with very short baselines (<5 km) single-epoch, single-frequency RTK is possible with Galileo-only most of the time. However, for baselines up to 15 km this is only possible with GPS+Galileo.

Next, the AR performance is analyzed based on empirical success rates using Monte Carlo simulations, see e.g. Verhagen (2005). Table 5.1 presents the scale factor needed to obtain a success rate above 0.99 more than 99% of the time. The corresponding number of epochs if $\beta = 0$ is derived from Fig. 5.2, from which also follows that in the presence of time correlation more epochs are needed.

For baselines of 20 km and longer, single-frequency RTK is not feasible for large periods of time, and therefore the corresponding results are not shown in Table 5.1. Without time correlation and with 100 epochs of data, a success rate above 0.99 can be obtained during less than 75% of the time. However, for baselines shorter than 10 km instantaneous ambiguity resolution is possible with GPS+Galileo. At mid-latitudes the time to fix the ambiguities will often be longer with a baseline of 15 km, but is still rather short. With Galileo-only the time to fix depends very much on the satellite geometry and thus the location on Earth, but generally the time to fix will be more than 10 epochs with short baselines, and more than 50 epochs with baselines longer than 10 km.

4 Concluding Remarks

Single frequency RTK with the current GPS or future Galileo alone is only feasible with very short baselines (<5 km), and even then at some locations instantaneous ambiguity resolution will only be feasible for 65% of the time. At mid-latitudes more than 15 epochs of data are needed to guarantee a success rate above 0.99.

A dual-constellation GNSS will enhance the ambiguity resolution performance of single frequency RTK dramatically. Instantaneous success rates above 0.99 are obtained with baselines up to 15 km.

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